EE 435

Lecture 9:

High-Gain Single-Stage Op Amps
Review from last-last lecture:

Determination of 2-port parameters

Determination of \( \{g_{o1}, g_{o2}, g_{M1}, g_{M2}\} \)

Method 2: Load Termination Approach

Express the gain \( A(s) \) in form

\[
A(s) = \frac{a_0}{sC_L + b_0}
\]

Observe

\[
V_2(g_{o2} + sC_L) + g_{M2}V_{TST} = 0
\]

\[
A(s) = \frac{V_2(s)}{V_{TST}(s)} = -\frac{g_{M2}}{sC_L + g_{o2}}
\]

(must express in integer-monic form)
Review from last-last lecture:

Are there other useful high output impedance circuits that can be used for the quarter circuit?

\[
A_{VO} = \frac{-G_{M1}}{2(G_1 + G_2)}
\]

\[
BW = \frac{G_1 + G_2}{C_L}
\]

\[
GB = \frac{G_{M1}}{2C_L}
\]
High output impedance quarter-circuits

Regulated Cascode Amplifier
or “Gain Boosted Cascode”

(A is usually a simple amplifier, often the reference op amp with + terminal connected to the desired quiescent voltage)
Gain-Boosted Telescopic Cascode Op Amp

\[
A_o = \frac{-g_{m1}}{2} \frac{A_1 g_{o3}}{g_{m3}} + \frac{g_{o5} A_3 g_{o7}}{g_{m7}}
\]

\[
GB = \frac{g_{m1}}{2C_L}
\]

This is modestly less efficient at generating GB because now power is consumed in both the cascode devices and the boosting amplifier.
Are there other useful high output impedance circuits that can be used for the quarter circuit?

\[
A_{VO} = \frac{-G_{M1}}{2(G_1 + G_2)}
\]

\[
BW = \frac{G_1 + G_2}{C_L}
\]

\[
GB = \frac{G_{M1}}{2C_L}
\]
What circuit is this?

Cascode Amplifier
Often termed a “Folded Cascode Amplifier”
Same small-signal performance as other
But a biasing problem!!
Biased Folded Cascode Amplifier

Folded Cascode Amplifier

Biased Folded Cascode
Implementation of Biased Folded Cascode Amplifier?

Biased Folded Cascode

Implementation of Biased Folded Cascode
Analysis of Biased Folded Cascode

$$V_{OUT} (g_{o3} + sC_L) + g_{m3} V_3 = V_X g_{o3}$$
$$V_X (g_{o1} + g_{o3} + g_{o5}) + g_{m1} V_1 - g_{m3} V_3 = V_{OUT} g_{o3}$$

$$V_3 = -V_X$$
$$V_1 = V_{IN}$$

$$V_{OUT} (g_{o3} + sC_L) + (g_{m3} + g_{o3}) V_3 = 0$$
$$+ g_{m1} V_{IN} = V_3 (g_{m3} + g_{o1} + g_{o3} + g_{o5}) + V_{OUT} g_{o3}$$

$$\frac{V_{OUT}}{V_{IN}} \approx \frac{g_{m1}}{sC_L + (g_{o1} + g_{o5}) \frac{g_{o3}}{g_{m3}}}$$

How can this be seen by inspection?

- First observe if all $g_o$’s are 0, $G_M=g_{m1}$
- Then observe $M_3$ “cascodes” the impedance $g_{o1}+g_{o5}$
Biased Folded Cascode Quarter Circuit

\[
\frac{V_{\text{OUT}}}{V_{\text{IN}}} \approx \frac{-g_{m1}}{sC_L + (g_{o1} + g_{o5}) \left( \frac{g_{o3}}{g_{m3}} \right)}
\]

\[
A_{V0} = \frac{g_{m1}}{(g_{o1} + g_{o5}) g_{o3}} \frac{g_{m3}}{C_L}
\]

\[
\text{GB} = \frac{g_{m1}}{C_L}
\]
## Basic Amplifier Structure Comparisons
(ideal current source biasing)

<table>
<thead>
<tr>
<th></th>
<th>Small Signal Parameter Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Common Source</strong></td>
<td>$A_{vo} = \frac{g_m}{g_o}$</td>
</tr>
<tr>
<td><strong>Cascode</strong></td>
<td>$A_{vo} = \frac{g_{m1} g_{m3}}{g_{o1} g_{o3}}$</td>
</tr>
<tr>
<td><strong>Regulated Cascode</strong></td>
<td>$A_{vo} \approx \frac{g_{m1} g_{m3}}{g_{o1} g_{o3}} A$</td>
</tr>
<tr>
<td><strong>Folded Cascode</strong></td>
<td>$A_{vo} = \frac{g_{m1} g_{m3}}{(g_{o1} + g_{o5}) g_{o3}}$</td>
</tr>
</tbody>
</table>
## Basic Amplifier Structure Comparisons

<table>
<thead>
<tr>
<th>Practical Parameter Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Common Source</strong></td>
</tr>
<tr>
<td>( A_{vo} = \left( \frac{2}{\lambda} \right) \left( \frac{1}{V_{EB}} \right) )</td>
</tr>
<tr>
<td><strong>Cascode</strong></td>
</tr>
<tr>
<td>( A_{vo} = \left( \frac{4}{\lambda_1 \lambda_3} \right) \left( \frac{1}{V_{EB_1} V_{EB_3}} \right) )</td>
</tr>
<tr>
<td><strong>Regulated Cascode</strong></td>
</tr>
<tr>
<td>( \Theta = \text{pct power in } A )</td>
</tr>
<tr>
<td><strong>Folded Cascode</strong></td>
</tr>
<tr>
<td>( \Theta = \text{fraction of current of } M_5 \text{ that is in } M_1 )</td>
</tr>
</tbody>
</table>
Biased Folded-Cascode Amplifier

Quarter Circuit

Counterpart Circuit
Folded-Cascode Operational Amplifier

QUARTER CIRCUIT

Op Amp
Folded-Cascode Operational Amplifier (redrawn)

These transistors pair-wise form a current source and one in each pair can be removed
Folded Cascode Op Amp

- Needs CMFB Circuit for $V_{B4}$
- Either single-ended or differential outputs
- Can connect counterpart as current mirror to eliminate CMFB
- Folding caused modest deterioration of $A_{V0}$ and GB energy efficiency
- Modest improvement in output swing
Folded Cascode Op Amp
(Single-ended Output)

\[ A_v(s) \approx -\frac{g_{mEQ}}{sC_L + g_{OEQ}} \]

\[ A_v \approx \frac{g_{mEQ}}{g_{OEQ}} \]

\[ GB \approx \frac{g_{mEQ}}{C_L} \]

\[ g_{mEQ} = g_{m1} \]

\[ g_{OEQ} \approx \left( g_{o1} + g_{o5} \right) \frac{g_{o3}}{g_{m3}} + \left( g_{o7} \right) \frac{g_{o9}}{g_{m9}} \]

\[ A_v \approx \frac{g_{m1}}{\left( g_{o1} + g_{o5} \right) \frac{g_{o3}}{g_{m3}} + \left( g_{o7} \right) \frac{g_{o9}}{g_{m9}}} \]

\[ GB = \frac{g_{m1}}{C_L} \]
# Operational Amplifier Structure Comparison

<table>
<thead>
<tr>
<th>Reference Op Amp</th>
<th>( A_{VO} = \frac{1}{2} \frac{g_{m1}}{g_{o1} + g_{o3}} )</th>
<th>GB = ( \frac{g_{m1}}{2C_L} )</th>
<th>SR = ( \frac{I_T}{2C_L} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Telescopic Cascode</td>
<td>( A_o = \frac{2}{g_{o1} g_{o3} + g_{o7} g_{o9}} )</td>
<td>GB = ( \frac{g_{m1}}{2C_L} )</td>
<td>SR = ( \frac{I_T}{2C_L} )</td>
</tr>
<tr>
<td>Regulated Cascode</td>
<td>( A_o \approx \frac{2}{g_{o1} g_{o3} + g_{o7} g_{o9}} )</td>
<td>GB = ( \frac{g_{m1}}{2C_L} )</td>
<td>SR = ( \frac{I_T}{2C_L} )</td>
</tr>
<tr>
<td>Folded Cascode</td>
<td>( A_o = \frac{2}{(g_{o1} + g_{o5}) g_{o3} + g_{o7} g_{o9}} )</td>
<td>GB = ( \frac{g_{m1}}{2C_L} )</td>
<td>SR = ( \frac{I_T}{2C_L} )</td>
</tr>
</tbody>
</table>
Operational Amplifier Structure Comparison

<table>
<thead>
<tr>
<th>Practical Parameter</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reference Op Amp</strong></td>
<td>$A_{v0} = \frac{1}{\lambda_1 + \lambda_3} \left( \frac{1}{V_{EB1}} \right)$, $GB = \left( \frac{P}{2V_{DD}C_L} \right) \cdot \left[ \frac{1}{V_{EB1}} \right]$, $SR = \frac{P}{2V_{DD}C_L}$</td>
</tr>
<tr>
<td><strong>Telescopic Cascode</strong></td>
<td>$A_{v0} = \frac{2}{V_{EB1}(\lambda_1 \lambda_3 V_{EB3} + \lambda_5 \lambda_7 V_{EB5})}$, $GB = \left( \frac{P}{2V_{DD}C_L} \right) \cdot \left[ \frac{1}{V_{EB1}} \right]$, $SR = \frac{P}{2V_{DD}C_L}$</td>
</tr>
</tbody>
</table>
| **Regulated Cascode** | $\Theta = \text{pct power in } A$
| $A_{v0} \approx \frac{2}{V_{EB1} \left( \frac{\lambda_1 \lambda_3 V_{EB3}}{A_1} + \frac{\lambda_5 \lambda_7 V_{EB7}}{A_3} \right)}$, $GB = \left( \frac{P(1-\Theta)}{2V_{DD}C_L} \right) \cdot \left[ \frac{1}{V_{EB1}} \right]$, $SR = \frac{P(1-\Theta)}{2V_{DD}C_L}$ |
| **Folded Cascode** | $\Theta = \text{fraction of current of } M_5$ that is in $M_1$
| $A_{v0} = \frac{2\Theta}{V_{EB1} \left( (\Theta \lambda_1 + \lambda_5) \lambda_3 V_{EB3} + (1-\Theta) \lambda_6 \lambda_7 V_{EB9} \right)}$, $GB = \left( \frac{P}{2V_{DD}C_L} \right) \cdot \left[ \frac{\Theta}{V_{EB1}} \right]$, $SR = \frac{\Theta P}{2V_{DD}C_L}$ |
Folded Cascode Op Amp (Single-ended Output)

\[ A_{v0} \approx \frac{g_{m1}}{(g_{o1} + g_{o5}) g_{o3} + (g_{o7}) g_{o9}} \]

\[ GB = \frac{g_{m1}}{C_L} \]

How many degrees of freedom are there?

What is a practical design parameter set?

DOF? 9 DOF
\{I_T, W_1/L_1, W_5/L_5, W_3/L_3, W_9/L_9, W_7/L_7, V_{B1}, V_{B2}, V_{B3}\}

Practical Design Parameters
\{P, \theta, V_{EB1}, V_{EB3}, V_{EB5}, V_{EB7}, V_{EB9}, V_{B2}, V_{B3}\}
where \( \theta = I_T/(I_T+I_{T2}) \)
Textbook reference:

Some of the material we have been discussing appears in Chapter 3, some in Chapter 5, and some in Chapter 6 of the Martin and Johns text.

In particular, the telescopic and folded cascode structures are referred to as advanced op amps and appear in later chapters of the text.
Folded Gain-boosted Cascode Amplifier

\[ A_o \approx \frac{-g_{m1}}{(g_{o1}) \frac{g_{o3}}{A g_{m3}}} \]

\[ GB = \frac{g_{m1}}{2C_L} \]

- with ideal current source bias
- modest improvement in output swing
Folded Gain-boosted Cascode Amplifier

\[ \frac{V_{\text{OUT}}}{V_{\text{IN}}} \approx \frac{-g_{m1}}{sC_L + \left(\frac{g_{o1} + g_{o5}}{g_{m3}}\right)g_{o3}} \]

\[ A_0 \approx \frac{-g_{m1}g_{m3}A}{\left(\frac{g_{o1}}{g_{o5}}\right)g_{o3}} \]

\[ GB = \frac{g_{m1}}{C_L} \]

modest improvement in output swing
**Basic Amplifier Structure Comparisons**

<table>
<thead>
<tr>
<th>Structure</th>
<th>Small Signal Parameter Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Common Source</td>
<td>$A_{vo} = \frac{g_m}{g_o}$</td>
</tr>
<tr>
<td></td>
<td>$GB = \frac{g_m}{C_L}$</td>
</tr>
<tr>
<td>Cascode</td>
<td>$A_{vo} = \frac{g_{m1} g_{m3}}{g_{o1} g_{o3}}$</td>
</tr>
<tr>
<td></td>
<td>$GB = \frac{g_{m1}}{C_L}$</td>
</tr>
<tr>
<td>Regulated Cascode</td>
<td>$A_{vo} \approx \frac{g_{m1} g_{m3}}{g_{o1} g_{o3}} A$</td>
</tr>
<tr>
<td></td>
<td>$GB = \frac{g_{m1}}{C_L}$</td>
</tr>
<tr>
<td>Folded Cascode</td>
<td>$A_{vo} = \frac{g_{m1} g_{m3}}{(g_{o1} + g_{o5}) g_{o3}}$</td>
</tr>
<tr>
<td></td>
<td>$GB = \frac{g_{m1}}{C_L}$</td>
</tr>
<tr>
<td>Folded Regulated Cascode</td>
<td>$A_{vo} = \frac{g_{m1} g_{m3}}{(g_{o1} + g_{o5}) g_{o3}} A$</td>
</tr>
<tr>
<td></td>
<td>$GB = \frac{g_{m1}}{C_L}$</td>
</tr>
</tbody>
</table>
## Basic Amplifier Structure Comparisons

<table>
<thead>
<tr>
<th>Practical Parameter Domain</th>
<th>Common Source</th>
<th>Cascode</th>
<th>Regulated Cascode</th>
<th>Folded Cascode</th>
<th>Folded Regulated Cascode</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_{VO} \approx \frac{2}{\lambda} \frac{1}{V_{EB}} )</td>
<td>( A_{VO} = \frac{4}{\lambda_1 \lambda_3} \frac{1}{V_{EB1} V_{EB3}} )</td>
<td>( A_{VO} = \frac{4}{\lambda_1 \lambda_3} \frac{A}{V_{EB1} V_{EB3}} )</td>
<td>( A_{VO} \approx \frac{4 \theta}{(\theta \lambda_1 + \lambda_5) \lambda_3 V_{EB1} V_{EB3}} )</td>
<td>( A_{VO} \approx \frac{A_4 \theta_2}{(\theta_2 \lambda_1 + \lambda_5) \lambda_3 V_{EB1} V_{EB3}} )</td>
<td>( \Theta_1 = \text{pct of total power in } A ) ( \Theta_2 = \text{fraction of current of } M_5 \text{ that is in } M_1 )</td>
</tr>
<tr>
<td>GB = ( \frac{2P}{V_{DD} C_L} ) ( \frac{1}{V_{EB}} )</td>
<td>GB = ( \frac{2P}{V_{DD} C_L} ) ( \frac{1}{V_{EB1}} )</td>
<td>GB = ( \frac{2P}{V_{DD} C_L} ) ( \frac{1 - \theta}{V_{EB1}} )</td>
<td>GB = ( \frac{2P}{V_{DD} C_L} ) ( \frac{\theta}{V_{EB1}} )</td>
<td>GB = ( \frac{2P}{V_{DD} C_L} ) ( \frac{\theta_2 (1 - \theta_1)}{V_{EB1}} )</td>
<td>( \Theta_1 = \text{pct of total power in } A ) ( \Theta_2 = \text{fraction of current of } M_5 \text{ that is in } M_1 )</td>
</tr>
</tbody>
</table>
Folded Gain-boosted Telescopic Cascode Op Amp

\[ A_o \approx \frac{-g_{m1}}{2} \]

\[ GB = \frac{g_{m1}}{2C_L} \]

- Needs CMFB Circuit for \( V_{B4} \)
- Either single-ended or differential outputs
- Can connect counterpart as current mirror to eliminate CMFB
- Folding caused modest deterioration in GB efficiency and gain
- Modest improvement in output swing
## Operational Amplifier Structure Comparison

<table>
<thead>
<tr>
<th>Reference Op Amp</th>
<th>$A_{vo} = \frac{1}{2} g_{m1} \left( g_{o1} + g_{o3} \right)$</th>
<th>$GB = \frac{g_{m1}}{2C_L}$</th>
<th>$SR = \frac{I_T}{2C_L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Telescopic Cascode</td>
<td>$A_o = \frac{g_{m1}}{2} \left( \frac{g_{o3}}{g_{m3}} + \frac{g_{o7}}{g_{m5}} \right)$</td>
<td>$GB = \frac{g_{m1}}{2C_L}$</td>
<td>$SR = \frac{I_T}{2C_L}$</td>
</tr>
<tr>
<td>Regulated Cascode</td>
<td>$A_o \approx \frac{g_{m1}}{2} \left( \frac{g_{o3}}{g_{m3} A_1} + \frac{g_{o7}}{g_{m9} A_3} \right)$</td>
<td>$GB = \frac{g_{m1}}{2C_L}$</td>
<td>$SR = \frac{I_T}{2C_L}$</td>
</tr>
<tr>
<td>Folded Cascode</td>
<td>$A_o = \frac{g_{m1}}{2} \left( \frac{g_{o1} + g_{o5}}{g_{m3}} \right) \left( \frac{g_{o3}}{g_{m9} A_3} + \frac{g_{o7}}{g_{m9} A_9} \right)$</td>
<td>$GB = \frac{g_{m1}}{2C_L}$</td>
<td>$SR = \frac{I_T}{2C_L}$</td>
</tr>
<tr>
<td>Folded Regulated Cascode</td>
<td>$A_o = \frac{g_{m1}}{2} \left( \frac{g_{o1} + g_{o5}}{g_{m3} A_3} + \frac{g_{o7}}{g_{m9} A_9} \right)$</td>
<td>$GB = \frac{g_{m1}}{2C_L}$</td>
<td>$SR = \frac{I_T}{2C_L}$</td>
</tr>
</tbody>
</table>
Summary of Folded Amplifier Performance

• +  Modest improvement in output signal swing (from $5 \text{ V}_{\text{DS SAT}}$ to $4 \text{ V}_{\text{DS SAT}}$)

• -  Deterioration in $A_{\text{V0}}$ (maybe 30% or more)

• -  Deterioration in GB power efficiency (can be significant)

• -  Minor increase in circuit size
End of Lecture 9