EE 435

Lecture 9:

High-Gain Single-Stage Op Amps
**Background**

**Review from last-last lecture:**

**Determination of 2-port parameters**

Determination of \( \{ g_{o1}, g_{o2}, g_{M1}, g_{M2} \} \)

**Method 2  Load Termination Approach**

**General Two-Port Network**

\[
\begin{align*}
V_1 & \quad g_{M1} \quad V_2 \\
g_{M2} \quad V_TST & \quad g_{O2}
\end{align*}
\]

express the gain \( A(s) \) in form

\[
A(s) = \frac{a_0}{sC_L + b_0}
\]

**General Two-Port Network**

observe

\[
V_TST \quad g_{M2} \quad V_2 (g_{o2} + sC_L) + g_{M2} V_TST = 0
\]

\[
A(s) = \frac{V_TST(s)}{V_2(s)} = -\frac{g_{M2}}{sC_L + g_{o2}}
\]

(must express in integer-monic form)
Review from last-last lecture:

Are there other useful high output impedance circuits that can be used for the quarter circuit?

\[ A_{VO} = \frac{-G_{M1}}{2(G_1 + G_2)} \]

\[ BW = \frac{G_1 + G_2}{C_L} \]

\[ GB = \frac{G_{M1}}{2C_L} \]
Review from last-last lecture:

**Telescopic Cascode Op Amp**

- Differential Output
- CMFB to establish $V_{B1}$ or $V_{B5}$ needed
- Tail current generally generated with current mirror
Review from last-last lecture:

Telescopic Cascode Op Amp

n-channel inputs

p-channel inputs
Are there other high output impedance circuits that can be used as quarter circuits?
Are there other high output impedance circuits that can be used as quarter circuits?

I recall the regulated cascode circuits have this property.
High output impedance quarter-circuits

Regulated Cascode Amplifier
or “Gain Boosted Cascode”

(A is usually a simple amplifier, often the reference op amp with + terminal connected to the desired quiescent voltage)
Analysis of Regulated Cascode Amplifier

\[
\begin{align*}
V_{OUT} (g_{o3} + sC_L) + g_{m3} V_2 &= V_X g_{o3} \\
V_X (g_{o1} + g_{o3}) + g_{m1} V_1 - g_{m3} V_2 &= V_{OUT} g_{o3} \\
V_2 &= -AV_X - V_X \\
V_1 &= V_{IN}
\end{align*}
\]

\(V_X, V_1\) and \(V_2\) can be eliminated from these 4 equations.
Analysis of Regulated Cascode Amplifier

\[ V_{OUT} \left( g_{o3} + sC_L \right) + g_{m3} V_2 = V_x g_{o3} \]
\[ V_x (g_{o1} + g_{o3}) + g_{m1} V_1 - g_{m3} V_2 = V_{OUT} g_{o3} \]
\[ V_2 = -A V_x - V_x \]
\[ V_1 = V_{IN} \]
\[ V_{OUT} (g_{o3} + sC_L) - g_{m3} V_x (1 + A) = V_x g_{o3} \]
\[ V_x (g_{o1} + g_{o3}) + g_{m1} V_{IN} + g_{m3} V_x (1 + A) = V_{OUT} g_{o3} \]

\[
\frac{V_{OUT}}{V_{IN}} = \frac{-g_{m1} \left( g_{o3} + g_{m3} [1+ A] \right)}{sC_L \left( g_{o1} + g_{o3} + g_{m3} [1+ A] \right) + g_{o1} g_{o3}} \approx \frac{-g_{m1} g_{m3} [1+ A]}{sC_L g_{m3} [1+ A] + g_{o1} g_{o3}} = \frac{-g_{m1}}{sC_L + \frac{g_{o1} g_{o3}}{g_{m3} [1+ A]}}
\]

for \( A \) large:

\[
\frac{V_{OUT}}{V_{IN}} \approx \frac{-g_{m1}}{sC_L + g_{o1} \left( \frac{g_{o3}}{g_{m3}} \right) \left( \frac{1}{A} \right)}
\]

[Diagram of the Regulated Cascode Amplifier with labeled components and equations.]
High output impedance quarter-circuits

Output conductance has been decreased even more!

\[ g_{OEQ} \approx g_{o1} \left[ \frac{g_{o3}}{g_{m3}} \frac{1}{A} \right] \]

\[ g_{mEQ} \approx g_{m1} \]

\[ A_V \approx \frac{-g_{m1}}{sC_L + g_{o1} \left( \frac{g_{o3}}{g_{m3}A} \right)} \]

\[ A_O \approx \left( -\left( \frac{g_{m1}}{g_{o1}} \right) \right) \left[ \frac{g_{m3}A}{g_{o3}} \right] \]

\[ GB \approx \frac{g_{m1}}{C_L} \]

Regulated Cascode Amplifier or “Gain Boosted Cascode”

Same GB as for previous two circuits
Gain-Boosted Telescopic Cascode Op Amp

Needs CMFB Circuit for $V_{b1}$
Either single-ended or differential outputs
Can connect counterpart as current mirror to eliminate CMFB
Use differential op amp to facilitate biasing of cascode device
Gain-Boosted Telescopic Cascode Op Amp

Single-ended operation

\[ g_{OQC} = \]  
\[ g_{OCC} = \]  
\[ g_{mQC} = \]
Gain-Boosted Telescopic Cascode Op Amp

This is modestly less efficient at generating GB because now power is consumed in both the cascode devices and the boosting amplifier.
Gain-Boosted Telescopic Cascode Op Amp

\[ A_0 = \frac{-g_{m1}}{g_{o1} \frac{g_{o3}}{A_1 g_{m3}} + g_{o5} \frac{g_{o7}}{A_3 g_{m7}}} \]

\[ GB = \frac{g_{m1}}{C_L} \]

This is modestly less efficient at generating GB because now power is consumed in both the cascode devices and the boosting amplifier.

Elimination of need for CMFB Circuit
A minimum of $5 \ V_{DSAT}$ drops between $V_{DD}$ and $V_{SS}$

This establishes a lower bound on $V_{DD}-V_{SS}$ and it will be reduced by the p-p signal swing on the output.
Gain-Boosted Telescopic Cascode Op Amp
(with or w/o current mirror counterpart circuits)

Advantages:

Significant increase in dc gain

Limitations:

- Signal swing ($4V_{D_{SAT}}+V_T$ between $V_{DD}$ and $V_{SS}$)
- Reduction in GB power efficiency
  - some current required to bias “A” amplifiers
- Additional pole in “A” amplifier
  - may add requirements for some compensation
- Area Overhead for 4 transistors and 4 amplifiers
  - actually minor concern since performance will usually justify these resources
Are there other useful high output impedance circuits that can be used for the quarter circuit?

\[ A_{V0} = \frac{-G_{M1}}{2(G_1 + G_2)} \]

\[ BW = \frac{G_1 + G_2}{C_L} \]

\[ GB = \frac{G_{M1}}{2C_L} \]
What circuit is this?

Cascode Amplifier
Often termed a “Folded Cascode Amplifier”
Same small-signal performance as other
But a biasing problem !!
Biased Folded Cascode Amplifier

Folded Cascode Amplifier

Biased Folded Cascode
Implementation of Biased Folded Cascode Amplifier?
Analysis of Biased Folded Cascode

\[
\begin{align*}
V_{\text{OUT}} (g_{o3} + sC_L) + g_{m3} V_3 &= V_X g_{o3} \\
V_X (g_{o1} + g_{o3} + g_{o5}) + g_{m1} V_1 - g_{m3} V_3 &= V_{\text{OUT}} g_{o3} \\
V_3 &= -V_X \\
V_1 &= V_{\text{IN}} \\
V_{\text{OUT}} (g_{o3} + sC_L) + (g_{m3} + g_{o3}) V_3 &= 0 \\
+ g_{m1} V_{\text{IN}} &= V_3 (g_{m3} + g_{o1} + g_{o3} + g_{o5}) + V_{\text{OUT}} g_{o3} \\
\frac{V_{\text{OUT}}}{V_{\text{IN}}} &\approx -\frac{g_{m1}}{sC_L + (g_{o1} + g_{o5}) \frac{g_{o3}}{g_{m3}}}
\end{align*}
\]

How can this be seen by inspection?

- First observe if all \( g_o \)'s are 0, \( G_M = g_{m1} \)
- Then observe \( M_3 \) “cascodes” the impedance \( g_{o1} + g_{o5} \)
Biased Folded Cascode Quarter Circuit

\[ \frac{V_{\text{OUT}}}{V_{\text{IN}}} \approx - \frac{g_{m1}}{sC_L + \left(g_{o1} + g_{o5}\right) \frac{g_{o3}}{g_{m3}}} \]

\[ A_{V0} \approx \frac{g_{m1}}{\left(g_{o1} + g_{o5}\right) g_{o3}} \frac{g_{m3}}{g_{m3}} \]

\[ GB \approx \frac{g_{m1}}{C_L} \]
# Basic Amplifier Structure Comparisons

(ideal current source biasing)

<table>
<thead>
<tr>
<th>Small Signal Parameter Domain</th>
<th>Common Source</th>
<th>Cascode</th>
<th>Regulated Cascode</th>
<th>Folded Cascode</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_{v0} )</td>
<td>( \frac{g_m}{g_o} )</td>
<td>( \frac{g_{m1} g_{m3}}{g_{o1} g_{o3}} )</td>
<td>( \frac{g_{m1} g_{m3}}{g_{o1} g_{o3}} )</td>
<td>( \frac{g_{m1} g_{m3}}{(g_{o1} + g_{o5}) g_{o3}} )</td>
</tr>
<tr>
<td>( GB )</td>
<td>( \frac{g_m}{C_L} )</td>
<td>( \frac{g_m}{C_L} )</td>
<td>( \frac{g_m}{C_L} )</td>
<td>( \frac{g_m}{C_L} )</td>
</tr>
</tbody>
</table>
## Basic Amplifier Structure Comparisons

<table>
<thead>
<tr>
<th>Practical Parameter Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Common Source</strong></td>
</tr>
<tr>
<td>( A_{vo} = \left( \frac{2}{\lambda} \right) \left( \frac{1}{V_{EB}} \right) )</td>
</tr>
<tr>
<td>( GB = \left( \frac{2P}{V_{DD}C_L} \right) \left( \frac{1}{V_{EB}} \right) )</td>
</tr>
<tr>
<td><strong>Cascode</strong></td>
</tr>
<tr>
<td>( A_{vo} = \left( \frac{4}{\lambda_1\lambda_3} \right) \left( \frac{1}{V_{EB1}V_{EB3}} \right) )</td>
</tr>
<tr>
<td>( GB = \left( \frac{2P}{V_{DD}C_L} \right) \left( \frac{1}{V_{EB1}} \right) )</td>
</tr>
<tr>
<td><strong>Regulated Cascode</strong></td>
</tr>
<tr>
<td>( \Theta = \text{pct power in } A )</td>
</tr>
<tr>
<td>( A_{vo} \approx \left( \frac{4}{\lambda_1\lambda_3} \right) \left( \frac{A}{V_{EB1}V_{EB3}} \right) )</td>
</tr>
<tr>
<td>( GB = \left( \frac{2P}{V_{DD}C_L} \right) \left( \frac{1 - \Theta}{V_{EB1}} \right) )</td>
</tr>
<tr>
<td><strong>Folded Cascode</strong></td>
</tr>
<tr>
<td>( \Theta = \text{fraction of current of } M_5 \text{ that is in } M_1 )</td>
</tr>
<tr>
<td>( A_{vo} \approx \left( \frac{4\Theta}{(\Theta\lambda_1 + \lambda_5)\lambda_3 V_{EB1}V_{EB3}} \right) )</td>
</tr>
<tr>
<td>( GB = \left( \frac{2P}{V_{DD}C_L} \right) \left[ \frac{\Theta}{V_{EB1}} \right] )</td>
</tr>
</tbody>
</table>
Biased Folded-Cascode Amplifier

Quarter Circuit

Counterpart Circuit
Folded-Cascode Operational Amplifier

QUARTER CIRCUIT

Op Amp
Folded-Cascode Operational Amplifier (redrawn)

These transistors pair-wise form a current source and one in each pair can be removed.
Folded Cascode Op Amp

- Needs CMFB Circuit for $V_{B4}$
- Either single-ended or differential outputs
- Can connect counterpart as current mirror to eliminate CMFB
- Folding caused modest deterioration of $A_{V0}$ and GB energy efficiency
- Modest improvement in output swing
Folded Cascode Op Amp (Single-ended Output)

\[
A_V(s) \approx -\frac{g_{mEQ}}{sC_L + g_{OEQ}}
\]

\[
A_{V0} \approx \frac{g_{mEQ}}{g_{OEQ}}
\]

\[
GB \approx \frac{g_{mEQ}}{C_L}
\]

\[
g_{mEQ} = g_{m1}
\]

\[
g_{OEQ} \approx \left( g_{o1} + g_{o5} \right) \frac{g_{o3}}{g_{m3}} + \left( g_{o7} \right) \frac{g_{o9}}{g_{m9}}
\]

\[
A_{V0} \approx \frac{g_{m1}}{\left( g_{o1} + g_{o5} \right) \frac{g_{o3}}{g_{m3}} + \left( g_{o7} \right) \frac{g_{o9}}{g_{m9}}}
\]

\[
GB = \frac{g_{m1}}{C_L}
\]
## Operational Amplifier Structure Comparison

<table>
<thead>
<tr>
<th>Structure</th>
<th>Gain Amplification ($A_{vo}$)</th>
<th>Gain Bandwidth (GB)</th>
<th>Settling Time (SR)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reference Op Amp</strong></td>
<td>$A_{vo} = \frac{1}{2} \frac{g_{m1}}{g_{o1} + g_{o3}}$</td>
<td>$GB = \frac{g_{m1}}{2C_L}$</td>
<td>$SR = \frac{I_T}{2C_L}$</td>
</tr>
<tr>
<td><strong>Telescopic Cascode</strong></td>
<td>$A_o = \frac{2}{g_{o1} g_{o3} + g_{o7} g_{o5} g_{m3}}$</td>
<td>$GB = \frac{g_{m1}}{2C_L}$</td>
<td>$SR = \frac{I_T}{2C_L}$</td>
</tr>
<tr>
<td><strong>Regulated Cascode</strong></td>
<td>$A_o \approx \frac{2}{g_{o1} g_{o3} + g_{o7} g_{o9}}$</td>
<td>$GB = \frac{g_{m1}}{2C_L}$</td>
<td>$SR = \frac{I_T}{2C_L}$</td>
</tr>
<tr>
<td><strong>Folded Cascode</strong></td>
<td>$A_o = \frac{2}{(g_{o1} + g_{o5}) g_{o3} + g_{o7} g_{o9}}$</td>
<td>$GB = \frac{g_{m1}}{2C_L}$</td>
<td>$SR = \frac{I_T}{2C_L}$</td>
</tr>
</tbody>
</table>
## Operational Amplifier Structure Comparison

<table>
<thead>
<tr>
<th>Practical Parameter</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reference Op Amp</strong></td>
<td>$A_{V_0} = \left[ \frac{1}{\lambda_1 + \lambda_3} \right] \left( \frac{1}{V_{EB1}} \right)$</td>
</tr>
<tr>
<td><strong>Telescopic Cascode</strong></td>
<td>$A_{V_0} = \frac{2}{V_{EB1}(\lambda_1 \lambda_3 V_{EB3} + \lambda_5 \lambda_7 V_{EB3})}$</td>
</tr>
<tr>
<td><strong>Regulated Cascode</strong></td>
<td>$\Theta = \text{pct power in } A$</td>
</tr>
<tr>
<td><strong>Folded Cascode</strong></td>
<td>$\Theta = \text{fraction of current of } M_5$ that is in $M_1$</td>
</tr>
</tbody>
</table>

$\Theta = \text{fraction of current of } M_5$ that is in $M_1$
Folded Cascode Op Amp (Single-ended Output)

\[ A_{v0} \approx \frac{g_{m1}}{(g_{o1} + g_{o5}) g_{o3}} + \frac{g_{o7}}{g_{m9}} \]

\[ GB = \frac{g_{m1}}{C_L} \]

How many degrees of freedom are there?
What is a practical design parameter set?

DOF ? 9 DOF
\{I_T, W_1/L_1, W_5/L_5, W_3/L_3, W_9/L_9, W_7/L_7, V_{B1}, V_{B2}, V_{B3}\}

Practical Design Parameters
\{P, \theta, V_{EB1}, V_{EB3}, V_{EB5}, V_{EB7}, V_{EB9}, V_{B2}, V_{B3}\}
where \( \theta = I_T/(I_T+I_{T2}) \)
Textbook reference:

Some of the material we have been discussing appears in Chapter 3, some in Chapter 5, and some in Chapter 6 of the Martin and Johns text.

In particular, the telescopic and folded cascode structures are referred to as advanced op amps and appear in later chapters of the text.
Folded Gain-boosted Cascode Amplifier

\[ A_0 \approx \frac{-g_{m1}}{(g_{o1})g_{o3}A_{g}g_{m3}} \]

\[ GB = \frac{g_{m1}}{2C_L} \]

- with ideal current source bias
- modest improvement in output swing
Folded Gain-boosted Cascode Amplifier

\[
\frac{V_{\text{OUT}}}{V_{\text{IN}}} \approx \frac{-g_{m1}}{sC_L + \frac{(g_{o1} + g_{o5})g_{o3}}{g_{m3}A}}
\]

\[
A_0 \approx \frac{-g_{m1}g_{m3}A}{(g_{o1} + g_{o5})g_{o3}}
\]

\[
GB = \frac{g_{m1}}{C_L}
\]

modest improvement in output swing
## Basic Amplifier Structure Comparisons

<table>
<thead>
<tr>
<th>Small Signal Parameter Domain</th>
<th>Common Source</th>
<th>Cascode</th>
<th>Regulated Cascode</th>
<th>Folded Cascode</th>
<th>Folded Regulated Cascode</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{vo}$</td>
<td>$A_{vo} = \frac{g_m}{g_o}$</td>
<td>$A_{vo} = \frac{g_{m1} g_{m3}}{g_{o1} g_{o3}}$</td>
<td>$A_{vo} \approx \frac{g_{m1} g_{m3}}{g_{o1} g_{o3}} A$</td>
<td>$A_{vo} = \frac{g_{m1} g_{m3}}{(g_{o1} + g_{o5}) g_{o3}}$</td>
<td>$A_{vo} = \frac{g_{m1} g_{m3} A}{(g_{o1} + g_{o5}) g_{o3}}$</td>
</tr>
<tr>
<td>$GB$</td>
<td>$GB = \frac{g_m}{C_L}$</td>
<td>$GB = \frac{g_{m1}}{C_L}$</td>
<td>$GB = \frac{g_{m1}}{C_L}$</td>
<td>$GB = \frac{g_{m1}}{C_L}$</td>
<td>$GB = \frac{g_{m1}}{C_L}$</td>
</tr>
</tbody>
</table>
### Basic Amplifier Structure Comparisons

#### Practical Parameter Domain

<table>
<thead>
<tr>
<th>Structure</th>
<th>$A_{VO}$</th>
<th>$GB$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Common Source</strong></td>
<td>$A_{VO} = \left( \frac{2}{\lambda} \right) \frac{1}{V_{EB}}$</td>
<td>$GB = \left( \frac{2P}{V_{DD}C_L} \right) \left( \frac{1}{V_{EB}} \right)$</td>
</tr>
<tr>
<td><strong>Cascode</strong></td>
<td>$A_{VO} = \left( \frac{4}{\lambda_1\lambda_3} \right) \frac{1}{V_{EB1}V_{EB3}}$</td>
<td>$GB = \left( \frac{2P}{V_{DD}C_L} \right) \left( \frac{1}{V_{EB1}} \right)$</td>
</tr>
<tr>
<td><strong>Regulated Cascode</strong></td>
<td>$A_{VO} \approx \left( \frac{4}{\lambda_1\lambda_3} \right) \frac{A}{V_{EB1}V_{EB3}}$</td>
<td>$GB = \left( \frac{2P}{V_{DD}C_L} \right) \left( 1 - \theta \right) \left( \frac{1}{V_{EB1}} \right)$</td>
</tr>
<tr>
<td><strong>Folded Cascode</strong></td>
<td>$A_{VO} \approx \left( \frac{4\theta}{(\theta\lambda_1 + \lambda_5)\lambda_3 V_{EB1}V_{EB3}} \right)$</td>
<td>$GB = \left( \frac{2P}{V_{DD}C_L} \right) \left( \frac{\theta}{V_{EB1}} \right)$</td>
</tr>
<tr>
<td><strong>Folded Regulated Cascode</strong></td>
<td>$A_{VO} \approx \left( \frac{A4\theta_2}{(\theta_2\lambda_1 + \lambda_5)\lambda_3 V_{EB1}V_{EB3}} \right)$</td>
<td>$GB = \left( \frac{2P}{V_{DD}C_L} \right) \left( \frac{\theta_2 \left( 1 - \theta_1 \right)}{V_{EB1}} \right)$</td>
</tr>
</tbody>
</table>

- $\Theta$ = fraction of current of $M_5$ that is in $M_1$
- $\Theta_1$ = pct of total power in $A$
- $\Theta_2$ = fraction of current of $M_5$ that is in $M_1$
Folded Gain-boosted Telescopic Cascode Op Amp

\[
A_o \approx \frac{-g_{m1}}{2} \left( g_{o1} + g_{o5} \right) \frac{g_{o3}}{A_3 g_{m3}} + g_{o7} \frac{g_{o9}}{A_7 g_{m9}}
\]

\[
GB = \frac{g_{m1}}{2C_L}
\]

- Needs CMFB Circuit for \( V_{B4} \)
- Either single-ended or differential outputs
- Can connect counterpart as current mirror to eliminate CMFB
- Folding caused modest deterioration in GB efficiency and gain
- Modest improvement in output swing
## Operational Amplifier Structure Comparison

<table>
<thead>
<tr>
<th>Reference Op Amp</th>
<th>$A_{vo} = \frac{1}{2} \frac{g_{m1}}{g_{o1} + g_{o3}}$</th>
<th>$GB = \frac{g_{m1}}{2C_L}$</th>
<th>$SR = \frac{I_T}{2C_L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Telescopic Cascode</td>
<td>$A_o = \frac{g_{m1}}{2} \frac{g_{o3}}{g_{m3}A_1} + \frac{g_{o7}}{g_{m5}} \frac{g_{o9}}{g_{m9}A_3}$</td>
<td>$GB = \frac{g_{m1}}{2C_L}$</td>
<td>$SR = \frac{I_T}{2C_L}$</td>
</tr>
<tr>
<td>Regulated Cascode</td>
<td>$A_o \approx \frac{g_{m1}}{2} \frac{g_{o3}}{g_{m3}A_1} + \frac{g_{o7}}{g_{m9}A_3}$</td>
<td>$GB = \frac{g_{m1}}{2C_L}$</td>
<td>$SR = \frac{I_T}{2C_L}$</td>
</tr>
<tr>
<td>Folded Cascode</td>
<td>$A_o = \frac{g_{m1}}{2} \frac{(g_{o1} + g_{o5})}{(g_{m3}A_1 + g_{m9}A_3)}$</td>
<td>$GB = \frac{g_{m1}}{2C_L}$</td>
<td>$SR = \frac{I_T}{2C_L}$</td>
</tr>
<tr>
<td>Folded Regulated Cascode</td>
<td>$A_o = \frac{g_{m1}}{2} \frac{(g_{o1} + g_{o5})}{(g_{m3}A_3 + g_{m9}A_9)}$</td>
<td>$GB = \frac{g_{m1}}{2C_L}$</td>
<td>$SR = \frac{I_T}{2C_L}$</td>
</tr>
</tbody>
</table>
Summary of Folded Amplifier Performance

- Modest improvement in output signal swing (from $5 \, V_{DS\, SAT}$ to $4 \, V_{DS\, SAT}$)
- Can directly feed output back to input to create buffer
- Deterioration in $A_{V0}$ (maybe 30% or more)
- Deterioration in GB power efficiency (can be significant)
- Minor increase in circuit size
End of Lecture 9