Lecture 9:

Folded-Cascode Amplifiers
Current Mirror Op Amps
Basic Op Amp Design

• Fundamental Amplifier Design Issues
• Single-Stage Low Gain Op Amps
• Single-Stage High Gain Op Amps
• Other Basic Gain Enhancement Approaches
• Two-Stage Op Amp
Review from last lecture:

High output impedance quarter-circuits

Regulated Cascode Amplifier
or “Gain Boosted Cascode”

- A is usually a simple amplifier, often the reference op amp with + terminal connected to the desired quiescent voltage
- Assume biased with a dc current source (not shown) at drain of $M_3$
Review from last lecture:

**Gain-Boosted Telescopic Cascode Op Amp**
(with or w/o current mirror counterpart circuits)

**Advantages:**

- Significant increase in dc gain

**Limitations:**

- Signal swing ($4V_{DSAT}+V_T$ between $V_{DD}$ and $V_{SS}$)
- Reduction in GB power efficiency
  - some current required to bias “A” amplifiers
- Additional pole in “A” amplifier
  - may add requirements for some compensation
- Area Overhead for 4 transistors and 4 amplifiers
  - actually minor concern since performance will usually justify these resources
Are there other useful high output impedance circuits that can be used for the quarter circuit?

\[ A_{V0} = \frac{-G_{M1}}{2(G_1 + G_2)} \]

\[ BW = \frac{G_1 + G_2}{C_L} \]

\[ GB = \frac{G_{M1}}{2C_L} \]
What circuit is this?

Cascode Amplifier
What circuit is this?

Small-signal circuit structures are identical!

- Cascode Amplifier
  - Often termed a “Folded Cascode Amplifier”
  - Same small-signal performance as other
  - $V_{OUT}$ swing $V_{DSAT1} - V_{DSAT2}$ could be small or negative
  - But a biasing problem!!
Folded Cascode Amplifier

Biased Folded Cascode Amplifier

Must have $I_{D1} = I_{B1} - I_{B2} > 0$
Implementation of Biased Folded Cascode Amplifier?

![Biased Folded Cascode Amplifier Diagram]
Analysis of Biased Folded Cascode

\[
\begin{align*}
V_{\text{OUT}} (g_{o3} + sC_L) + g_{m3} V_3 &= V_X g_{o3} \\
V_X (g_{o1} + g_{o3} + g_{o5}) + g_{m1} V_1 - g_{m3} V_3 &= V_{\text{OUT}} g_{o3} \\
V_3 &= -V_X \\
V_1 &= V_{\text{IN}}
\end{align*}
\]

\[
\begin{align*}
V_{\text{OUT}} (g_{o3} + sC_L) + (g_{m3} + g_{o3}) V_3 &= 0 \\
+g_{m1} V_{\text{IN}} &= V_3 (g_{m3} + g_{o1} + g_{o3} + g_{o5}) + V_{\text{OUT}} g_{o3}
\end{align*}
\]

\[
\frac{V_{\text{OUT}}}{V_{\text{IN}}} \approx - \frac{g_{m1}}{sC_L + (g_{o1} + g_{o5}) g_{o3}/g_{m3}}
\]

How can this be seen by inspection?

• First observe if all $g_o$’s are 0, $G_M = g_{m1}$
• Then observe $M_3$ “cascodes” the impedance $g_{o1} + g_{o5}$
Biased Folded Cascode Quarter Circuit

\[
\frac{V_{\text{OUT}}}{V_{\text{IN}}} \approx - \frac{g_{m1}}{sC_L + (g_{o1} + g_{o5}) \frac{g_{o3}}{g_{m3}}}
\]

\[
A_{V0} \approx \frac{g_{m1}}{(g_{o1} + g_{o5}) g_{o3}} g_{m3}
\]

\[
GB \approx \frac{g_{m1}}{C_L}
\]
## Basic Amplifier Structure Comparisons

(ideal current source biasing)

<table>
<thead>
<tr>
<th>Small Signal Parameter Domain</th>
<th>Common Source</th>
<th>( A_{v0} \approx \frac{g_m}{g_o} )</th>
<th>GB ( \approx \frac{g_m}{C_L} )</th>
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<tr>
<td>Cascode</td>
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<td>Regulated Cascode</td>
<td>( A_{v0} \approx \frac{g_{m1} g_{m3}}{g_{o1} g_{o3}} ) A</td>
<td>GB ( \approx \frac{g_m}{C_L} )</td>
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<tr>
<td>Folded Cascode</td>
<td>( A_{v0} \approx \frac{g_{m1} g_{m3}}{(g_{o1} + g_{o5}) g_{o3}} )</td>
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Basic Amplifier Structure Comparisons

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<td>$GB = \left( \frac{2P}{V_{DD}C_L} \right) \left( \frac{1}{V_{EB}} \right)$</td>
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<td>$GB = \left( \frac{2P}{V_{DD}C_L} \right) \left( \frac{1-\theta}{V_{EB1}} \right)$</td>
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<td>$GB = \left( \frac{2P}{V_{DD}C_L} \right) \left[ \frac{\theta}{V_{EB1}} \right]$</td>
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<td>$\Theta = \text{fraction of current of } M_5 \text{ that is in } M_1$</td>
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Biased Folded-Cascode Amplifier

Quarter Circuit

Counterpart Circuit
Folded-Cascode Operational Amplifier

QUARTER CIRCUIT

Op Amp
Folded-Cascode Operational Amplifier (redrawn)

These transistors pair-wise form a current source and one in each pair can be removed
Folded Cascode Op Amp

- Needs CMFB Circuit for $V_{B4}$
- Either single-ended or differential outputs
- Can connect counterpart as current mirror to eliminate CMFB
- Folding caused modest deterioration of $A_{V0}$ and GB energy efficiency
- Modest improvement in output swing
Folded Cascode Op Amp  
(Single-ended Output)

\[
A_v(s) \approx -\frac{g_{mEQ}}{sC_L + g_{OEQ}}
\]

\[
A_v \approx \frac{g_{mEQ}}{g_{OEQ}}
\]

\[
GB \approx \frac{g_{mEQ}}{C_L}
\]

\[
g_{mEQ} = g_{m1}
\]

\[
g_{OEQ} \approx (g_{O1} + g_{O5}) \frac{g_{O3}}{g_{m3}} + (g_{O7}) \frac{g_{O9}}{g_{m9}}
\]

\[
A_v \approx \frac{g_{m1}}{(g_{O1} + g_{O5}) \frac{g_{O3}}{g_{m3}} + (g_{O7}) \frac{g_{O9}}{g_{m9}}}
\]

\[
GB = \frac{g_{m1}}{C_L}
\]
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<th>$A_{v0} = \frac{1}{2} \frac{g_{m1}}{g_{o1} + g_{o3}}$</th>
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<td>Telescopic Cascode</td>
<td>$A_o = \frac{g_{m1}}{2 g_{o1} \frac{g_{o3}}{g_{m3}} + g_{o7} \frac{g_{o5}}{g_{m5}}}$</td>
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<td>Regulated Cascode</td>
<td>$A_o \approx \frac{g_{m1}}{2 g_{o1} \frac{g_{o3}}{g_{m3} A_1} + g_{o7} \frac{g_{o9}}{g_{m9} A_3}}$</td>
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## Operational Amplifier Structure Comparison

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<tr>
<td><strong>Telescopic Cascode</strong></td>
<td>( A_{V0} = \frac{2}{V_{EB1}(\lambda_1 \lambda_3 V_{EB3} + \lambda_5 \lambda_7 V_{EB5})} )</td>
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Folded Cascode Op Amp (Single-ended Output)

$$A_{V0} \approx \frac{g_{m1}}{\left( g_{o1} + g_{o5} \right) \frac{g_{o3}}{g_{m3}} + \left( g_{o7} \right) \frac{g_{o9}}{g_{m9}}}$$

$$GB = \frac{g_{m1}}{C_L}$$

How many degrees of freedom are there?

What is a practical design parameter set?

DOF? 9 DOF

$$\{I_T, W_1/L_1, W_5/L_5, W_3/L_3, W_9/L_9, W_7/L_7, V_{B1}, V_{B2}, V_{B3}\}$$

Practical Design Parameters

$$\{P, \theta, V_{EB1}, V_{EB3}, V_{EB5}, V_{EB7}, V_{EB9}, V_{B2}, V_{B3}\}$$

where $$\theta = \frac{I_T}{(I_T + I_{T2})}$$
Textbook reference:

Some of the material we have been discussing appears in Chapter 3, some in Chapter 5, and some in Chapter 6 of the Martin and Johns text.

In particular, the telescopic and folded cascode structures are referred to as advanced op amps and appear in later chapters of the text.
Folded Gain-boosted Cascode Amplifier

\[ A_o \approx \frac{-g_{m1}}{g_{o1}} \times \frac{g_{o3}}{g_{m3}} \]

\[ GB = \frac{g_{m1}}{2C_L} \]

- with ideal current source bias
- modest improvement in output swing
Folded Gain-boosted Cascode Amplifier

\[
\frac{V_{OUT}}{V_{IN}} \approx \frac{-g_{m1}}{sC_L + \frac{(g_{o1} + g_{o5})g_{o3}}{g_{m3}A}}
\]

\[
A_0 \approx \frac{-g_{m1}g_{m3}A}{(g_{o1} + g_{o5})g_{o3}}
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\[
GB = \frac{g_{m1}}{C_L}
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modest improvement in output swing
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# Basic Amplifier Structure Comparisons

## Practical Parameter Domain

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**Notation:**
- $\lambda_{1}, \lambda_{5}$: Parameters related to the transistors
- $\lambda_{3}$: Parameter related to the load
- $A$: Amplification factor
- $P$: Power
- $C$: Capacitance
- $V$: Voltage
- $\theta$: Fraction of current
- $\Theta$: Percentage of power

**Notes:**
- $\Theta_{1}$: Percentage of total power in $A$
- $\Theta_{2}$: Fraction of current of $M_{5}$ that is in $M_{1}$
Folded Gain-boosted Telescopic Cascode Op Amp

\[ A_o \approx \frac{-g_{m1}}{2} \]

\[ (g_{o1} + g_{o5}) \frac{g_{o3}}{A_3 g_{m3}} + g_{o7} \frac{g_{o9}}{A_1 g_{m9}} \]

\[ GB = \frac{g_{m1}}{2C_L} \]

- Needs CMFB Circuit for \( V_{B4} \)
- Either single-ended or differential outputs
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Summary of Folded Amplifier Performance

• +  Modest improvement in output signal swing (from $5 \, V_{DS\, SAT}$ to $4 \, V_{DS\, SAT}$)
• +  Can directly feed output back to input to create buffer
• -  Deterioration in $A_{V0}$ (maybe 30% or more)
• -  Deterioration in GB power efficiency (can be significant)
• -  Minor increase in circuit size
End of Lecture 9