"diff"

Wednesday, April 13th 2011

State estimators

- Sometimes, there are a lot of states

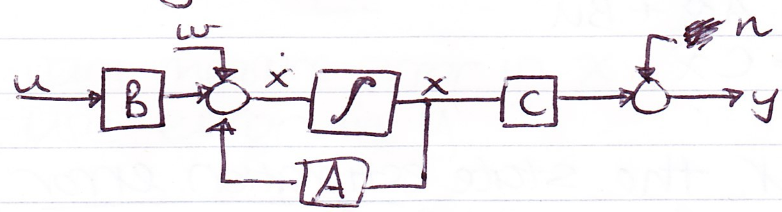
Problem: given $\dot{x} = Ax + Bu + B_w w$, $y = Cx + n$ (sensors)

sensors have noise

process noise

· example: flapping of helicopter wings

$$\rightarrow \quad \dot{x} = Ax + Bu + B_w \cdot w$$
$$y = Cx + n$$



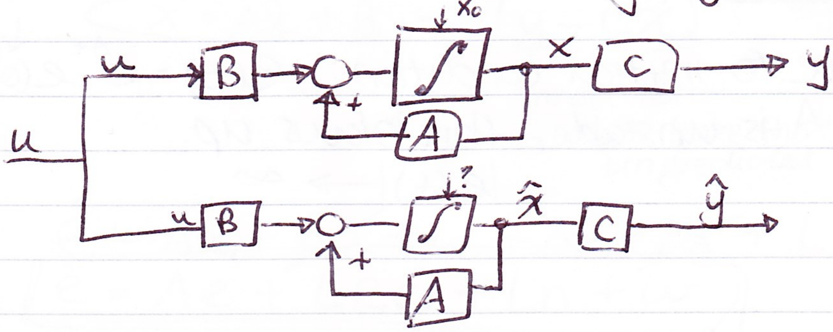
* want to build a system $E(y, u) \rightarrow \hat{x}$ (state estimator)

so that $\hat{x}(t) \approx x(t)$

__1st Attempt__:

Assume $w, n = 0 \quad \forall t$

How about the following system?



Claim: good estimator; this will do the job (i.e. $\hat{x} \triangleq x$)? **No!** (especially for unstable systems)

Why? Initial conditions are different

actual system: we don't really know exact initial conditions

- will make transient responses different

Assume A is unstable

$$\dot{x} = Ax + Bu$$
$$y = Cx$$
$$\dot{\hat{x}} = A\hat{x} + Bu$$
$$\hat{y} = C\hat{x}$$

Consider the state estimation error

$$e(t) = x(t) - \hat{x}(t)$$
$$\dot{e}(t) = \dot{x}(t) - \dot{\hat{x}}(t)$$

$$= Ax + \cancel{Bu} - A\hat{x} - \cancel{Bu}$$
$$= A(x - \hat{x})$$
$$= Ae$$

$$\therefore \boxed{\dot{e} = Ae}$$

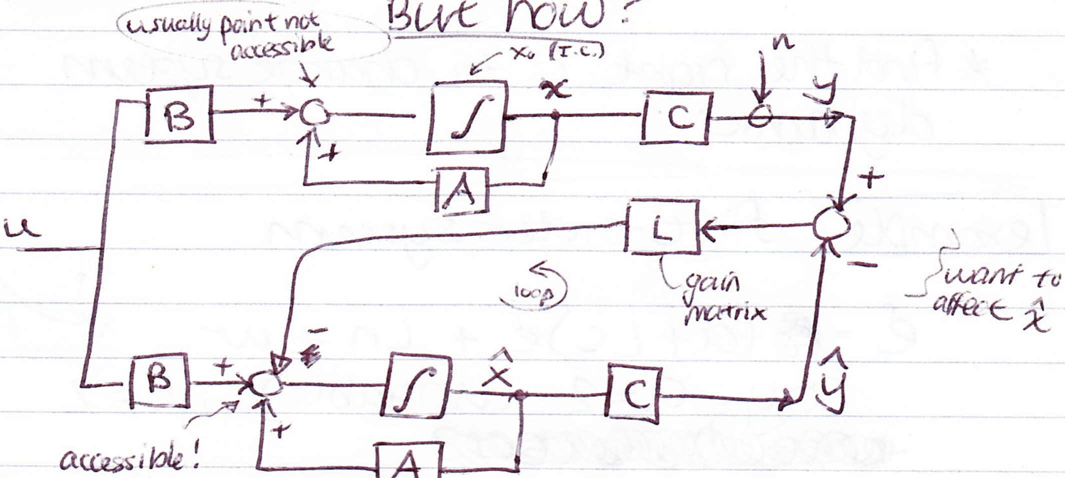
Response to initial condition: $e(t) = e^{At} \overset{?}{e(0)}$

- if A is unstable, this blows up.

$$|e(t)| \to \infty$$

How can we use the sensors?

2nd attempt: using sensor measurements. But how?



- can't measure error in x
- use error in y

Derived Equations for system ↑

$$\dot{x} = Ax + Bu + w$$
$$\dot{\hat{x}} = A\hat{x} + Bu - L(y - \tilde{y})$$
$$y = Cx + n$$
$$\tilde{y} = C\hat{x}$$

$$\Rightarrow \dot{x} = Ax + Bu + w$$
$$\dot{\hat{x}} = A\hat{x} + Bu - L\underbrace{(y - C\hat{x})}_{\text{innovation}}$$

estimator

innovation
- what's there that was unpredicted

$$\dot{e} = Ax + \cancel{Bu} + w - A\hat{x} - \cancel{Bu} + L(Cx - C\hat{x} + n)$$
$$\boxed{\dot{e} = Ae + LCe + Ln + w}$$

$$\boxed{\dot{e} = (A + LC)e + Ln + w}$$

(compared to $\dot{e}(t) = Ae(t)$)

* find the right L to change system dynamics

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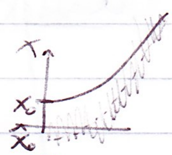
example first-order system

$$\dot{e} = (a + Lc)e + Ln + w$$



say: $a = 2$ (unstable), $c = 2$

- choose $L = -2 \Rightarrow (a + Lc) = -2$ (stable!)

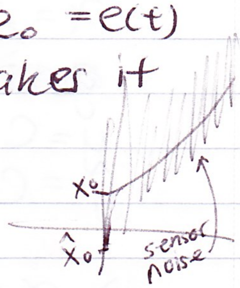
If $w = 0$, $n = 0$, then $\dot{e} = -2e$

$$\Rightarrow e(t) = e^{-2t} e_0$$

Why not $L = -200$ then?

$$2 - 200(2) = -398 \Rightarrow e^{-398t} e_0 = e(t)$$

Because: pushes noise up, makes it huge w/ huge L



* how large you can push L will depend on the quality of your sensor

* If we know our sensor noise, can pick an optimal L