EE 476
Homework 3

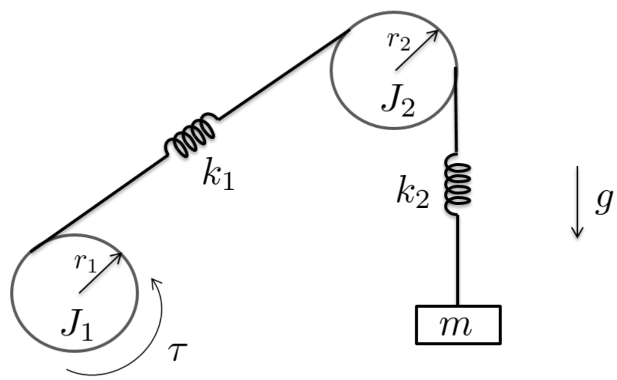


Figure 1: Two pulley system

Exercise 1 (40 points)**.** With respect to Figure 1 assume:

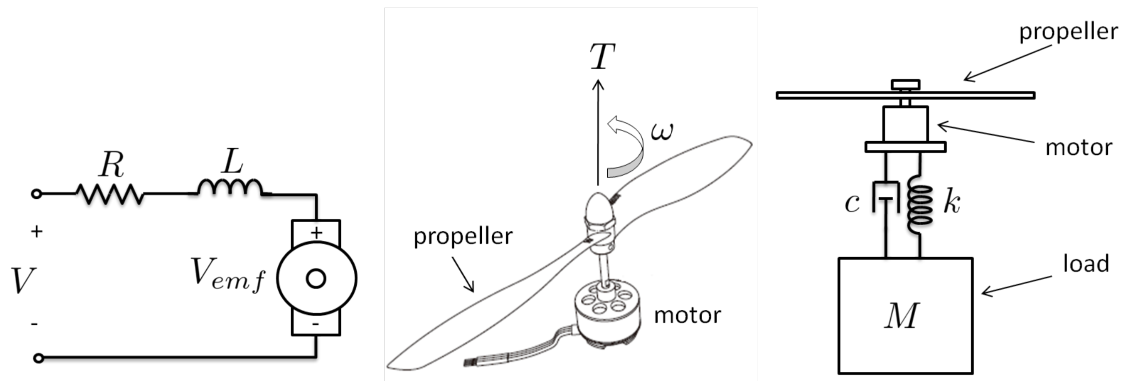
- the two pulleys have inertia J_1 and J_2, and radius r_1 and r_2 respectively
- the line between pulley 1 and pulley 2 has some flex with spring constant k_1
- the line between pulley 2 and the mass m has some flex with spring constant k_1
- there is a torque τ acting on pulley 1 in a counterclockwise direction as shown
- the acceleration of gravity is g in the direction shown

We wish to derive the differential equations for the system in Figure 1. Do so using

1. the Newtonian approach
2. the Lagrangian approach

and verify that the differential equations you obtain using each approach are equivalent.

Exercise 2 (20 points)**.** An inextensible string of length l is fixed at one end and a bob of mass m is attached at another. The bob swings in $\mathbf{R}^3$ (Cartesian coordinates) and the string remains taut. Find the Lagrangian for the system and the equations of motion using spherical coordinates (r, θ, ϕ). *Hint: what does the string staying taut mean about the motion in spherical coordinates?*



(a) Motor Circuit (b) Motor & propeller (c) Motor & Propeller & Load

Figure 2: Motor Circuit and Physical Setup

Exercise 3 (40 points)**.** Assume that we attach a propeller to a DC motor. See Figure 2(a) and 2(b). The motor is fixed to a small platform, and this platform is then attached to a larger load mass via a semi-flexible link. See Figure 2c. Assume that the load mass only moves in the up/down (z) direction for simplicity (i.e. only the motor+propeller rotate). Also assume the following:

- The motor inductance is negligable, i.e. $L = 0$.
- The motor torque is $Q_m = k_v(i - i_0 \operatorname{sgn}(\omega))$ where i_0 is what is often times called zero-load current.
- The motor back-emf voltage is given by $V_{emf} = k_v \omega$.
- The motor and propeller have combined moment of inertia J.
- The motor and propeller and small platform have mass m, while the load mass is M.
- The semi-flexible link can be represented as a typical spring and damping using k and c respectively.
- The thrust produced by the propeller is given by $T = k_t \omega^2$.
- The torque generated by the propeller drag is $Q_p = k_d \omega^2$.
- The acceleration of gravity is g and pointing 'down' in Figure 2c

(a) Derive the nonlinear equations governing the system dynamics using the Lagrangian methodology.

(b) Derive the nonlinear equations governing the system dynamics using the Newtonian methodology.

(c) Find the input voltage $\bar{V}$ to reach equilibrium with zero load velocity, essentially 'hovering'. Is it possible to reach an equilibrium with all states zero? Explain.