EE 476
LTI System Transformation Review

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Given the LTI system:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

we can find an equivalent system with a different state variable–keeping the same inputs and outputs–using a transformation matrix. We can also change the inputs and outputs of the system using essentially the same idea.

State Transformation $(x \leftrightarrow \tilde{x})$ For any invertible matrix T we can define $\dot{\tilde{x}} = Tx \Rightarrow x = T^{-1}\dot{\tilde{x}}$. Since T is a constant matrix, we have that $\dot{\tilde{x}} = T\dot{x} = TAx + TBu$. Using the fact that $x = T^{-1}\tilde{x}$ we can express the equivalent LTI system:

$$\dot{\tilde{x}} = TAT^{-1}\tilde{x} + TBu$$
$$y = CT^{-1}\tilde{x} + Du$$

Output Transformation $(y \leftrightarrow \tilde{y})$ For any invertible matrix M we can define $\tilde{y} = My \Rightarrow y = M^{-1}\tilde{y}$, and we can then express the LTI system with transformed output:

$$\dot{x} = Ax + Bu$$
$$\tilde{y} = MCx + MDu$$

Input Transformation $(u \leftrightarrow \tilde{u})$ For any invertible matrix W we can define $\tilde{u} = Wu \Rightarrow u = W^{-1}\tilde{u}$, and we can then express the LTI system with transformed input:

$$\dot{x} = Ax + BW^{-1}\tilde{u}$$
$$y = Cx + DW^{-1}\tilde{u}$$

Combined Transformation We can combine any of the previous transformations together. For example, if we want to perform all 3 types we would have the system:

$$\dot{\tilde{x}} = TAT^{-1}\tilde{x} + TBW^{-1}\tilde{u}$$
$$\tilde{y} = MCT^{-1}\tilde{x} + MDW^{-1}\tilde{u}$$

Remark 1. Note that although we can form transformations using arbitrary invertible matrices, for any given application the matrices used will depend on the problem at hand. That is, they will not be *arbitrary* but determined by the original system and the desired outcome.

For example, one of the most straightforward cases is where we want to change the units of some states and/or inputs and/or outputs. In this case the transformation matrices will simply be diagonal matrices made up of the appropriate unit conversion factors.