LECTURE 17

REVIEW :

- Most dynamical-systems can be described by

$$\frac{dx}{dt} = f(x, u) \quad \text{- - - -} \quad (NL)$$

- Equilibrium points:

x^* is a equilibrium point of the dynamics (NL) if

$$f(x^*, 0) = 0.$$

- LINEARIZATION:

Suppose we want to Linearize (NL) about the operating point $(\bar{x},\bar{u})$. Then the linearized dynamics are described by

$$\mathring{x} = A(x-\bar{x}) + B(u-\bar{u}).$$

$$A = \left.\frac{\partial f(x,u)}{\partial x}\right|_{(\bar{x},\bar{u})} \; ; \quad B = \left.\frac{\partial f}{\partial u}\right|_{\substack{x=\bar{x}\\ u=\bar{u}}}.$$

By defining $\begin{aligned} z &= x-\bar{x} \\ v &= u-\bar{u} \end{aligned}$ we have

$$\mathring{z} = Az + Bv.$$

STABILITY IN THE STATE-SPACE SETTING

- Stability is a concept about equilibrium points.

- Let x^* be an equilibrium point of the system

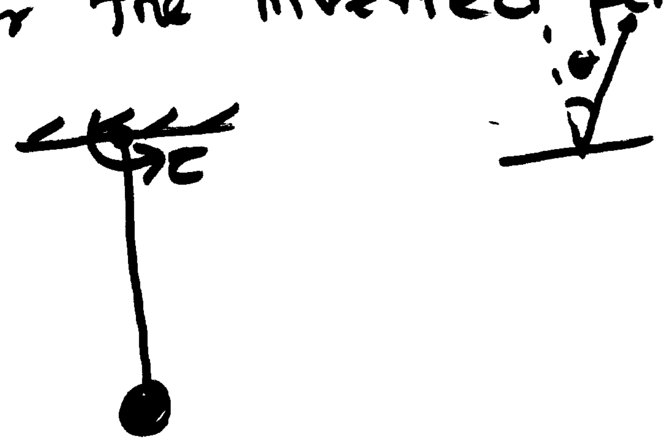
$$\dot{x} = f(x, u).$$

That is $f(x^*, 0) = 0$.

When can we say that the equilibrium point is stable?.

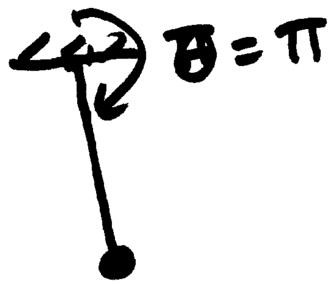
Answer: If we perturb the state from it equilibrium point by a small amount the dynamics comes back to the equilibrium point.

Example: Consider the inverted pendulum



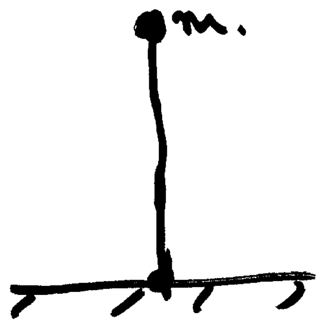
We have seen it has two distinct equilibrium points:
$(\theta=0, \dot{\theta}=0)^T$ and $(\theta=\pi, \dot{\theta}=0)^T$.

Suppose we perturb the pedulum from the equilibrium



position shown above. Then, if there is damping it will eventually come back to the above position. Thus, intuitively it seems that $(\theta = \pi, \dot{\theta} = 0)$ is a stable equilibrium pont.

Similarly, the position



$(\theta = 0, \dot{\theta} = 0)$ seems to be an unstable equilibrium point. Because, if we provide a slight perturbation to the pendulum from the position shown, it never recovers back to the same position; it falls down.

Mathematically ☹ the following are the

definitions:

Stability: consider $\dot{x} = f(x,u)$. The equilibrium point x^*, with $f(x^*,0)=0$ is said to be stable if, for all $\varepsilon>0$, there is a $\delta>0$, such that

$\text{distance}(x_0,x^*) < \delta$ implies $\text{dist}(x(t),x^*) < \varepsilon$

where $x(t)$ is the solution to

$$\frac{dx}{dt} = f(x,0) \; ; \quad x(0)=x_0 .$$

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Asymptotic stability:

The equilibrium point is said to be asymptotically stable if it is stable

and $\exists\ \delta > 0$ such that distance$(x_0, x^*) < \delta$

implies distance$(x(t), x^*) \to 0$ as $t \to \infty$

$x(t)$ being the solution of

$$\frac{dx}{dt} = f(x, 0) \; ; \; x(0) = x_0 .$$