Problem 1 Consider a second-order bandpass transfer function expressed three different ways

\[ T(s) = \frac{b_s}{s^2 + a_s s + a_0} = \frac{K_s}{s^2 + s \frac{a_h}{Q} + a_0^2} = \frac{H_s}{(s + \alpha + j\beta)(s + \alpha - j\beta)} \]

Assume a circuit is designed so that the peak gain is 10, the BW=1KHz and the peak gain occurs at 10KHz.

Sensitivity analysis is often used to predict how the performance of a circuit is affected by components in the circuit. The argument is made that if the sensitivity to a component is low, the circuit is not adversely affected by small changes in the component value. However, different sensitivities are often considered such as coefficient sensitivities, \( \omega_0 \) sensitivities, pole sensitivities, etc. Assume four different structures have sensitivities \( S_\omega = 5 \), \( S_\alpha = 5 \), \( S_\omega = 5 \), and \( S_\alpha = 5 \). Which of the four structures would be least affected by variations in the component x?

Problem 2 Consider the paper referenced below.

a) Identify the new integrator structure introduced in the paper
b) In terms of process parameters and implementation, compare the performance of this integrator with TAC structures discussed in class
c) Derive the transfer function of the second-order biquad introduced in the paper

Current-Mode, WCDMA Channel Filter With In-Band Noise Shaping

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Problem 3 A new filter structure was introduced in Fig. 2b of the following paper. The authors claim that the circuit of Fig. 2a is the standard filter.

a) Are the “standard” and the new filters integrator based? If so, give a block diagram that shows this.
b) Derive the transfer function for both filters
c) Will the signal swing benefits be obtained if a Bipolar process is used instead of a MOS process to build the transconductance stages?

Low-Power and Widely Tunable Linearized Biquadratic Low-Pass Transconductor-C Filter

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Problem 4  Consider the paper referenced below
a) What performance benefits to the authors claim for current-mode filters?
b) The authors identify two new integrator structures. Are they integrator based?
c) Compare the performance of one of the two filter structures with the OTA-C filter you used for your project in the laboratory.

Problem 5  Obtain the 3,2 order Pade approximation of the 6th-order normalized Butterworth lowpass transfer function.

Problem 6  The Chebyshev polynomials with an even index number are even functions of x. Can the even-indexed Chebyshev polynomials be used to obtain a new type of Chebyshev approximation that still maintains the equal ripple characteristics in the pass band and has a magnitude squared function \( H_A(\omega) = \frac{1}{1 + \varepsilon^2 C_n(\omega)} \) ?

Problem 7  The Chebyshev and the Butterworth approximations both have some interesting properties. There are different ways that some of the characteristics of both the BW and the CC approximations can be combined together. One such way to consider is with the n-th order magnitude squared function

\[
H_A(\omega) = \frac{1}{1 + \varepsilon^2 \left[ C_{n-m}^2(\omega) \omega^{2m} \right]} \quad \text{where } 0 \leq m \leq n
\]

In this case, when \( m=0 \) it becomes a pure CC approximation, when \( m=n \) it becomes a pure BW approximation but it can be argued that when \( m \) is between these extremes it has properties that are more like BW or more like CC depending on whether \( m \) is closer to 0 or closer to \( n \).
a) Does the inverse mapping to $T_A(s)$ exist for this magnitude-squared approximating function
b) If the answer to part a) is yes, obtain $T_A(s)$ for all values of $m$ if $n=4$
c) If the answer to a) is yes, compare this transitional approximation for all values of $m$ for $n=4$ in the pole domain, in the magnitude domain, and in the phase domain.
d) Comment on whether this approximation is useful.

Problem 8  The Chebyshev polynomials with an even index number are even functions of $x$. Define the shifted Chebyshev polynomial by the expression

$$C_{ns}(x) = \frac{C_n(x) + 1}{2}.$$  

Can the even-indexed shifted Chebyshev polynomials be used to obtain a new type of Chebyschev approximation that still maintains an equal ripple characteristic in the pass band and has a magnitude squared function

$$H_A(\omega) = \frac{1}{1 + \varepsilon^2 C_{ns}(\omega)}.$$

Problem 9  Consider the following amplifier where the op amp is assumed to be ideal. Assume the amplifier is to be designed to have a gain of $16 \pm 0.5\%$ and the layout of the resistors uses a common-centroid geometry to cancel gradient effects. Assume the resistors are created by using a series connection of a unit resistors of 50 ohms and area $5\mu m^2$. Assume the matching characteristics for closely-placed interdigitized resistors is characterized by the parameter $A_R=.01\mu^{-1}$.

a) Determine the area for the resistors and the yield if the nominal value of $R_2$ is $3.2K$.
b) How does the area and yield change if the nominal value of $R_2$ is increased to $32K$
**Problem 10** In a previous homework assignment, the size of R and C for a first-order lowpass filter were determined to minimize the area when laid out in an integrated circuit. Consider now a requirement where two of these filters are to be designed and that the poles of these two filters are to be closely matched and are to be nominally equal to $\omega_0$. To achieve this, a common-centroid layout is used for the resistors and a common centroid layout is used for the capacitors. Assume the matching characteristics of closely-coupled resistors is characterized by the parameter $A_R$ and that of closely-coupled capacitors is characterized by the parameter $A_C$ and the resistance density and capacitance density are respectively $D_R$ and $D_C$ where the resistors are of fixed width $W_R$. Assume that each of the two resistors is realized by a cascade of series connection of resistors of width $W_R$ (that is, a larger number of resistors can not be connected in parallel to form either of the resistors).

a) Determine the area of the resistor, $A_{RES}$, and the area of the capacitor, $A_{CAP}$ to satisfy the nominal $\omega_0$ requirement while minimizing the variance of the difference between the two band edges, $\omega_01$ and $\omega_02$.

b) How does the area required for minimizing variance compare to that required for minimizing area if $A_R = 0.01\text{u}^{-1}$ and $A_C = 0.005\text{u}^{-1}$?

c) How much would the variance drop if the sizing strategy to achieve minimum area were used instead of the sizing strategy to minimize variance? Assume the same values of $A_R$ and $A_C$ as used in part b).

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**Problem 11** In the laboratory the standard deviation of the GB for a group of the op amps that are ideally identical was measured. In the datasheet, little information is typically given about the statistics of performance parameters though these statistics often affect the performance characteristics of a filter using the op amps to some degree. Assume that an op amp is a two-stage Miller compensated MOS structure shown below. The GB for this op amp is given by

$$GB = \frac{g_{m}}{C_C}$$

where $g_{m}$ is the transconductance gain of the differential input pair and where $C_C$ is the compensation capacitor. Assume the biasing current sources are ideal.
a) Analytically determine the standard deviation of the GB of this op amp. In this derivation, assume the W,L, μn, μp, and C_{OX} are constant. Assume the threshold voltages of the devices are random variables characterized by the parameter \( A_{VT0} = 30 \text{mV} \cdot \mu \) and the compensation capacitor is a random variable characterized by the parameter \( A_{C} = 0.005 \text{u}^{-1} \). In this derivation, assume that the compensation capacitor is nominally 2pF, the capacitance density is 5fF/um², the nominal excess bias voltage on all transistors is 200mV, the gate area of M₁ is 10µ² and the gate area of M₃ is 30µ².

b) Repeat part a) if the gate the gate areas of M₁ and M₂ are increased by a factor of 100 and all other device sizes remain the same.


**Problem 12**   Obtain the 4th order Butterworth transfer function that has a 3dB band edge of 5kHz. Using any graphics package you choose, plot the response to show that you meet the specifications.

**Problem 13**   Repeat Problem 12 for the Chebyschev approximation

**Problem 14,15** Consider the filter design requirements depicted below where the forbidden region in the transfer function magnitude is the blue hashed region. Assume that \( A_{RN} = -3 \text{dB} \) and \( A_{SN} = -25 \text{dB} \).

a) Obtain the minimum order BW approximation that meets these specifications.

b) Obtain the minimum order CC approximation that meets these specifications

Using any graphics package you choose, plot the response to show that you meet the specifications.
Problem 16  Consider the filter design requirements depicted below where the forbidden region in the transfer function magnitude is the blue hashed region. Assume that $A_{RN} = -3\text{dB}$ and $A_{SN} = -25\text{dB}$. Obtain the minimum order BW approximation that meets these specifications. Using any graphics package you choose, plot the response to show that you meet the specifications.

Problem 17  The LM 13700 has an expression in the datasheet that indicated the transconductance of the device is given by the expression $g_m = \frac{I_{ABC} q}{2kT}$ where $q$ is the charge of an electron, $k$ is Boltzman’s constant, $T$ is temperature in K and $I_{ABC}$ is the bias current. The circuit structure of this OTA, exclusive of the linearizing diodes, is drawn below. Derive this expression. Assume the emitter area of all transistors is the same and that $\beta_n = \beta_p = 100$. 
Problem 18  It was stated in class that the linearity of the OTA of the previous problem is independent of the magnitude of the differential input provided the differential input voltage is less than 25mV. Prove this.

Problem 19  Consider the OTA architecture of Problem 17
a) Derive an expression for the transconductance gain of the OTA above if the bipolar transistors are all replaced with MOS devices. Assume all n-channel devices have $W_n=50\mu m$ and all p-channel devices have $W_p=150\mu m$. The lengths of all devices are 5um.

b) If $V_{DD}=-V_{SS}=10V$, determine the range of $I_{ABC}$ over which the devices will all remain in saturation if the common mode input voltage is 0V.

Problem 20
Derive an expression for and compare the integrator Q factor for the following two noninverting integrators.
Problem 21
Compare the integrator Q factor of the zero sensitivity Miller inverting integrator with that of the standard Miller Integrator.

Problem 22
a) Give the schematic of the single ended Tow-Thomas biquad filter based upon OTA-C integrators
b) Give the schematic of the fully differential Tow-Thomas biquad filter based upon OTA-C integrators
Problem 23  Consider the two bandpass filters shown. One is a two-integrator loop structure and the other is the bridged-T feedback configuration.

a) Derive the transfer function of the two circuits if the op amps are ideal.

b) Determine the sensitivity of the Q and the sensitivity of the pole of the second circuit with respect to $C_1$ and with respect to $R_1$.

c) From the results in part b), analytically estimate the change in the pole Q and compare numerically with the actual change in the pole Q due to the finite GB of the op amp as $\tau_0\omega_0$ varies from 0 to 0.1.

d) Plot the root locus of the poles of the resonator bandpass filter for different values of Q and compare with that of the bridged T network. Both circuits are shown below. For the bridged-T feedback network, assume $C_1=C_2$.

Problem 24  Determine the passive sensitivities of the $\omega_0$ for Circuit 2 in the previous problem with respect to all of the impedances in the network. If it is homogeneous of some order in the impedances, give the order and verify that the appropriate summed sensitivities are consistent with the order of homogeneity.