EE 508
Lecture 1

Introduction to Course

Catalog Course Description:

E E 508. Filter Design and Applications. (3-3) Cr. 4. *Prereq: 501.* Filter design concepts. Approximation and synthesis. Transformations. Continuous-time and discrete time filters. Discrete, active and integrated synthesis techniques.

Instructor: Randy Geiger

contact information:

294-7745

rlgeiger@iastate.edu

course linked at: www.randygeiger.org

Course Coverage

- Filter design process
- Approximation Problem
- Synthesis
- Acitve and passive realizations
- Integrated Applications
  - Discrete-time filters
    - (SC and digital)
  - Continutous-time filters
- PLLs (if time permits)

Major emphasis will be placed on methods for implementing filters on silicon

COURSE INFORMATION

Room: Lecture - 1157 Sweeny
Labs - 2046 Coover
-
Time: Lecture - MWF 10:00 – 10:50
Laboratory - Arranged

Lecture Instructor:
Randy Geiger
2133 Coover
Voice: 294-7745
e-mail: rlgeiger@iastate.edu
Office Hours: I maintain an open-door policy, will reserve 11:00 to 12:00 MWF specifically for students in EE 508. Appointments are welcomed too.

Course Description:

Filter design concepts. Approximation and synthesis. Transformations. Continuous-time and discrete time filters. Discrete, active and integrated
synthesis techniques

Course Web Site **http:/class.ee.iastate.edu/ee508/**

Homework assignments, lecture notes, laboratory assignments, and other course support materials will be posted on this WEB site. Students will be expected to periodically check the WEB site for information about the course.

Required Test:

There is no required text for this course. There are a large number of books that cover portions of the material that will be discussed in this course and some follow. Part of these focus on the concepts of filter design and some of the best are not new. Those that focus more on integrated applications are mostly rather narrow in scope.

Grading: Points will be allocated for several different parts of the course. A letter grade will be assigned based upon the total points accumulated. The points allocated for different parts of the course are as listed below:

2 Exams	100 pts each
Homework	100 pts.total
Lab and Lab Reports	100 pts.total
Design Project	100 pts. total

Laboratory:

There will be weekly laboratory experiments. Students will be expected to bring parts kits such as those used in EE 230 and EE 330. To the maximum extent possible, students will be expected to work individually in the laboratory.

The design project will be the design of an integrated filter structure. Expectations will be to carry the design through post layout simulation. The option for fabricating this integrated circuit will be available to students in the class.

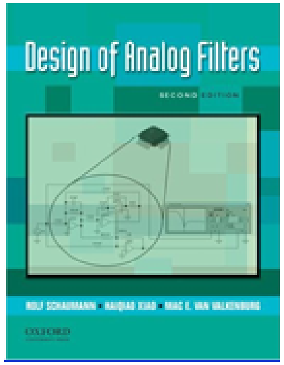
Homework:

Homework assignments are due at the beginning of the class period on the designated due dates. Late homework will be accepted, without penalty, up until 5:00 p.m. on the due date in Room 2133 Coover.

Additional Comments

I encourage you to take advantage of the e-mail system on campus to communicate about any issues that arise in the course. I typically check my e-mail several times a day. Please try to include “EE 508" in the subject field of any e-mail message that you send so that they stand out from what is often large volumes of routine e-mail messages.

Reference Texts:



Design of Analog Filters – Second Edition, by Schaumann and Van Valkenburg, Oxford, 2009.

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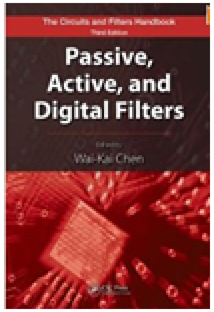
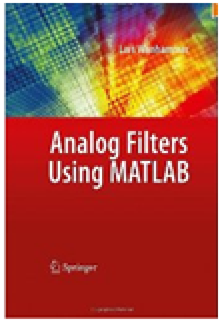
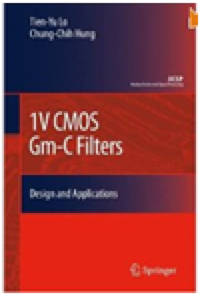


Passive, Active, and Digital Filters, by Wai-Kai Chen, CRC Press, 2009.

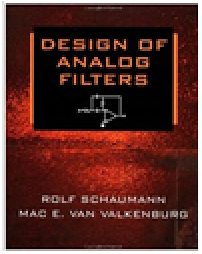


Analog Filters Using MATLAB, by Wanhammar, Springer, 2009.



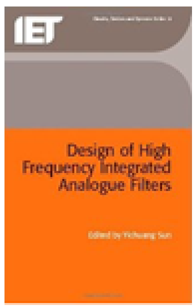
1V CMOS Gm-C Filters, by Lo and Hung, Springer, 2009.

Passivc and Active Filters, Theory and Implementaitons, by Wai-Kai Chen, Wiley, 1986.



Design of Analog Filters, by Schaumann and Van Valkenburg, Oxford, 2001.

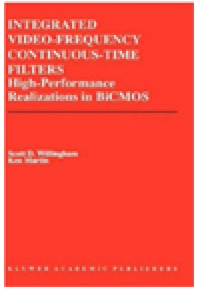
Switched-Capacitor Techniques for High-Accuracy Filter and ADC Design, by Quinn and van Roermund, Springer, 1997.



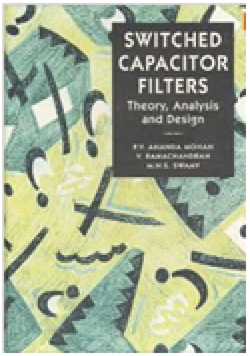
Design of high frequency integrated analogue filters, by Sun, IEE, 2002.

High-Performance CMOS Continuous-Time Filters, by Silva-Martinez, Steyaert, and Sansen, Kluwer, 1993

Introduction to the Design of Transconductor-Capacitor Filters, by Kardontchik, Kluwer, 1992.

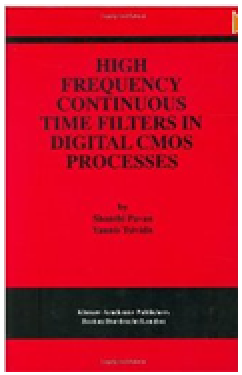


Integrated Video-Frequency Continuous-Time Filters: High-Performance Realizations in BiCMOS, by Willingham and Martin, Kluwer, 1995.

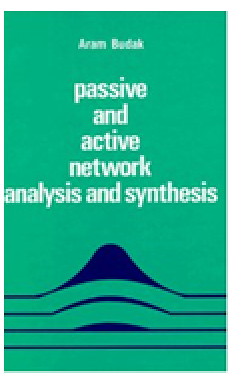


Switched Capacitor Filtrs, Theory, Analysis, and Design, by Mohan, Ramachandran and Swamy, Prentice Hall, 1995.

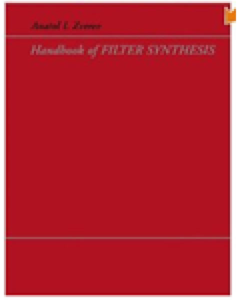
Continuous-Time Active Filter Design, by Deliyannis, Sun, and Fidler, CRC Press, 1998.



High Frequency Continuous-Time Filters in Digital CMOS Processes, by Tsividis and Springer, 2000.



Passive and Active Network Analysis and Synthesis, by Budak, Waveland Press, 1991.



Handbook of Filter Synthesis by Zverev, Wiley, 1967 and 2005.

Digital Filters, Analysis, Design, and Applications, Second Edition, by Antoniou, McGraw Hill, 1993.

Introduction to the Theory and Design of Acitve Filters, by Huelsman and Allen, McGraw Hill, 1980.

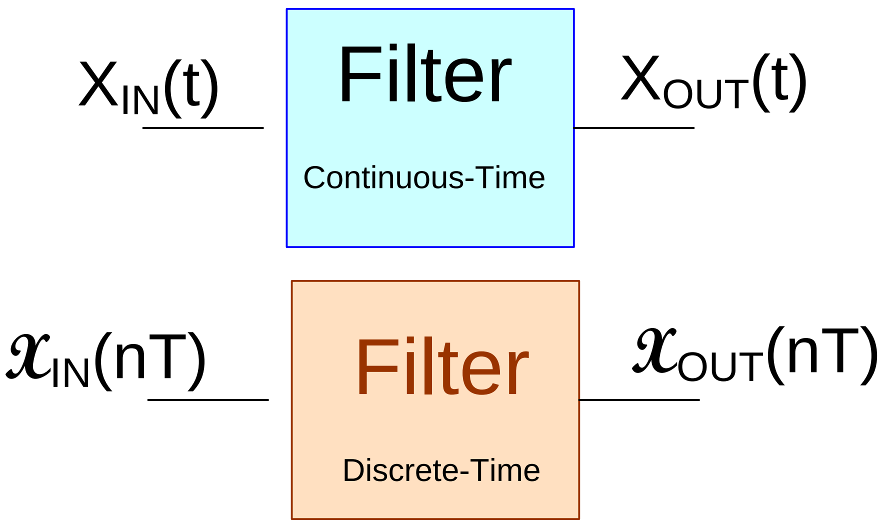
MOS Switched-Capacitor and Continuous-Time Integrated Circuits and Systems, by Unbehauen and Cichocki, Springer, 1989.

What is a filter?

Conceptual definition:

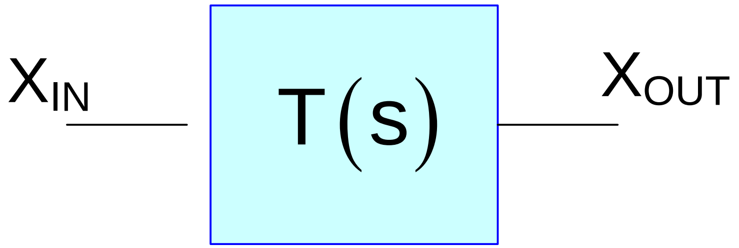
A filter is an amplifier or a system that has a frequency dependent gain

Note:
Implicit assumption is made in this definition that the system is linear. In this course, will restrict focus to filters that are ideally linear



Filters can be continuous-time or discrete-time

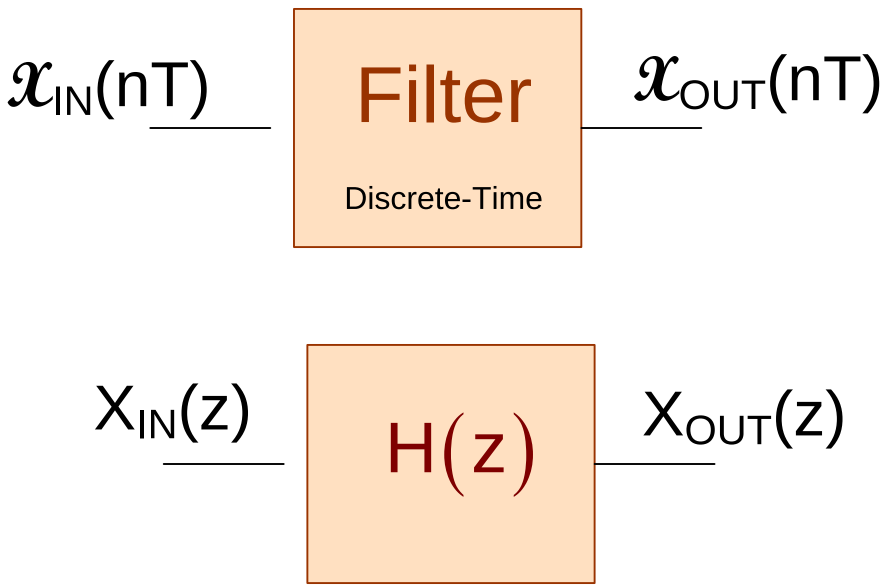
Continuous-time filters



Continuous-time filters are often characterized in the frequency domain

$$T(s) = \frac{X_{OUT}(s)}{X_{IN}(s)}$$

Discrete-time filters



Discrete-time filters are often characterized in the frequency domain

$$H(z) = \frac{X_{OUT}(z)}{X_{IN}(z)}$$

Observations:

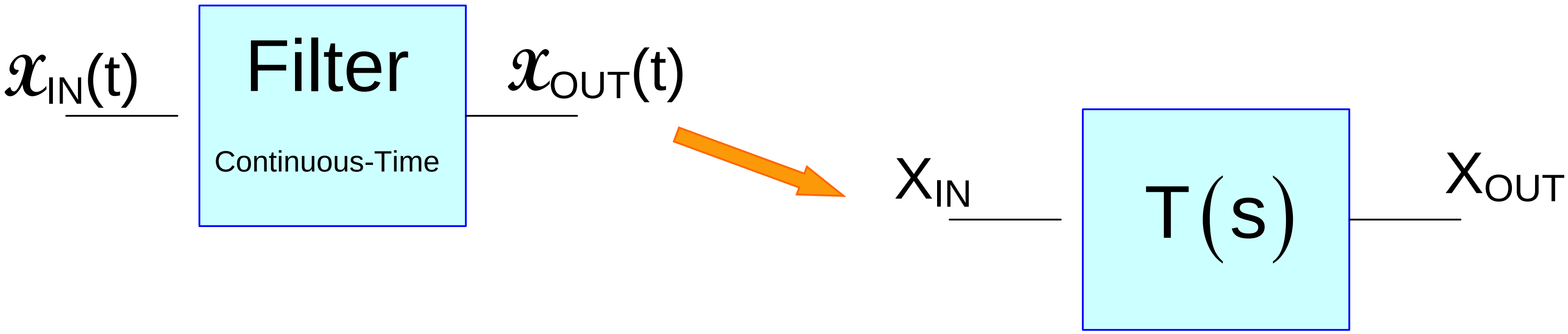
- Some (if not most) filters will exhibit some undesired nonlinearities
- Frequency response characteristics often of most interest in filters but in some filters, other characteristics may be of interest

Time delay
Spectral leakage
Inter-modulation distortion

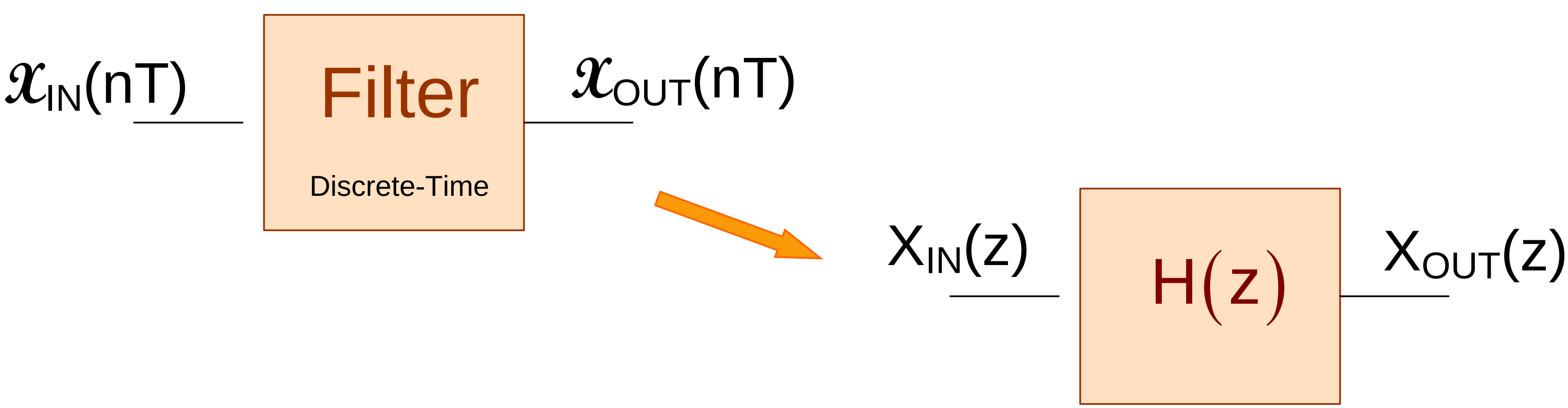
- Some classes of nonlinear circuits that are also termed “filters” and that have fundamentally different operational characteristics exist (but are not covered in this course)

Median Filters
Log Domain Filters
…

Most classical filter applications stipulate gain vs f or phase vs f as the desired operating characteristics

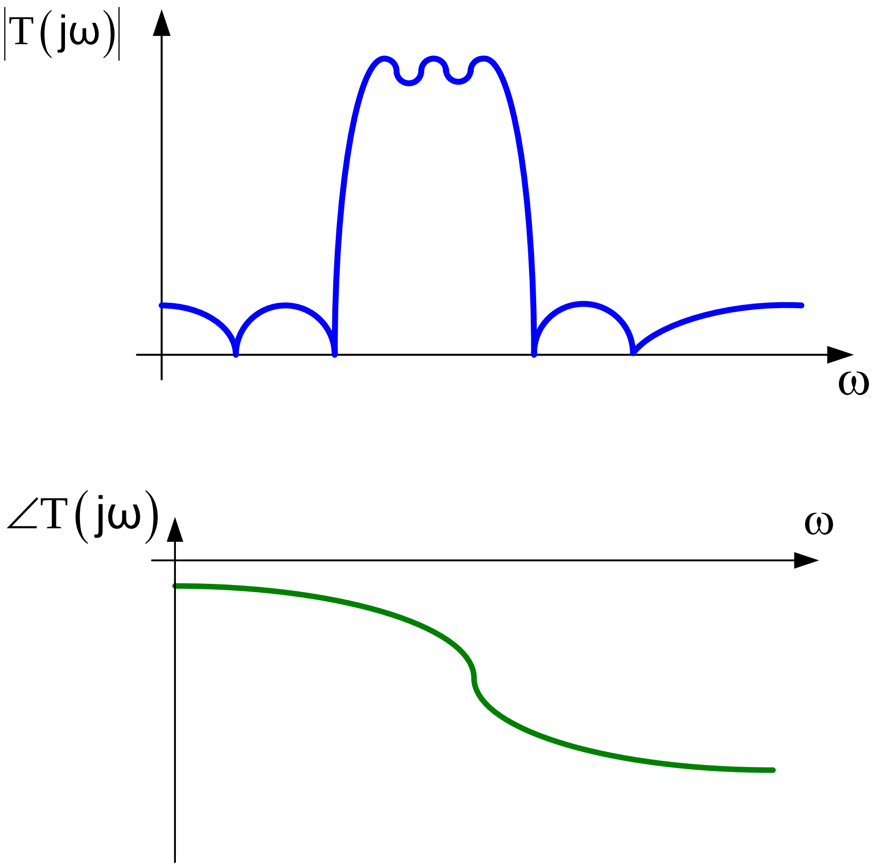


$$T(s) = \frac{X_{OUT}(s)}{X_{IN}(s)}$$



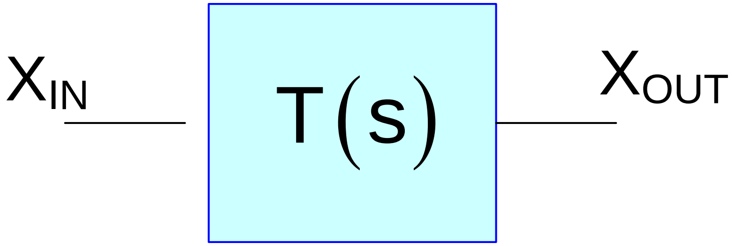
$$H(z) = \frac{X_{OUT}(z)}{X_{IN}(z)}$$

Representation of magnitude and phase characteristics of a filter:



Key properties of filters:

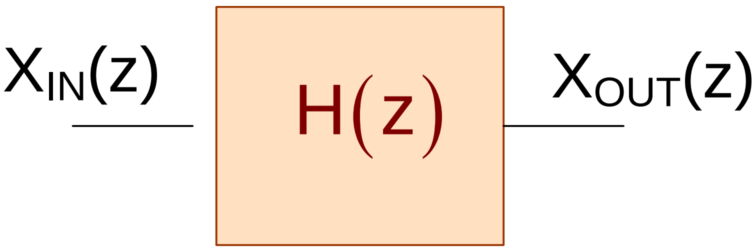
Transfer functions of continuous-time filters with finite number of lumped elements are rational fractions with real coefficients



$$T(s)=\frac{\sum_{i=1}^{m} a_i s^i}{\sum_{i=1}^{n} b_i s^i}=\frac{N(s)}{D(s)}$$

Key properties of filters:

Transfer functions of discrete-time filters with finite number of real additions are rational fractions with real coefficients



$$H(z) = \frac{\sum_{i=1}^{m} a_i z^i}{\sum_{i=1}^{n} b_i z^i} = \frac{N(z)}{D(z)}$$

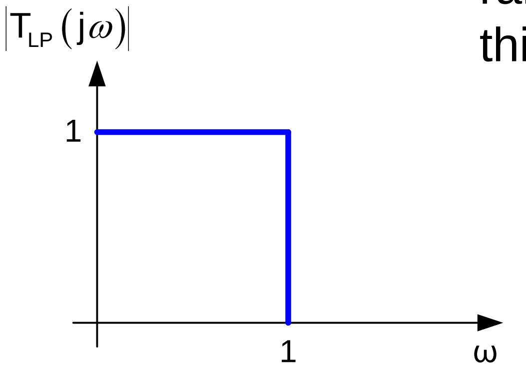
Key properties of filters:

Transfer functions of any realizable filter (finite elements) have no discontinuities in either the magnitude or phase response

Is this property good or bad?

BAD !

Often want filters that will perfectly pass a signal in some frequency range and perfectly block it outside this range



Ideal lowpass filter

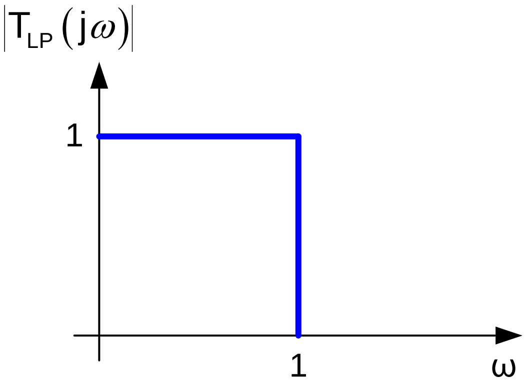
Key properties of filters:

Transfer functions of any realizable filter (finite elements) have no discontinuities in either the magnitude or phase response

Often system designer will “want” overly challenging specifications but really only “need” something somewhat less demanding

Critical that the circuit and system designer agree upon an appropriate relaxed filter requirement so overall system performance is met and designg time and circuit cost is acceptable

Observations:



Ideal lowpass filter

The closer the designer comes to realizing the ideal lowpass characteristics, the more complicated and expensive the design becomes

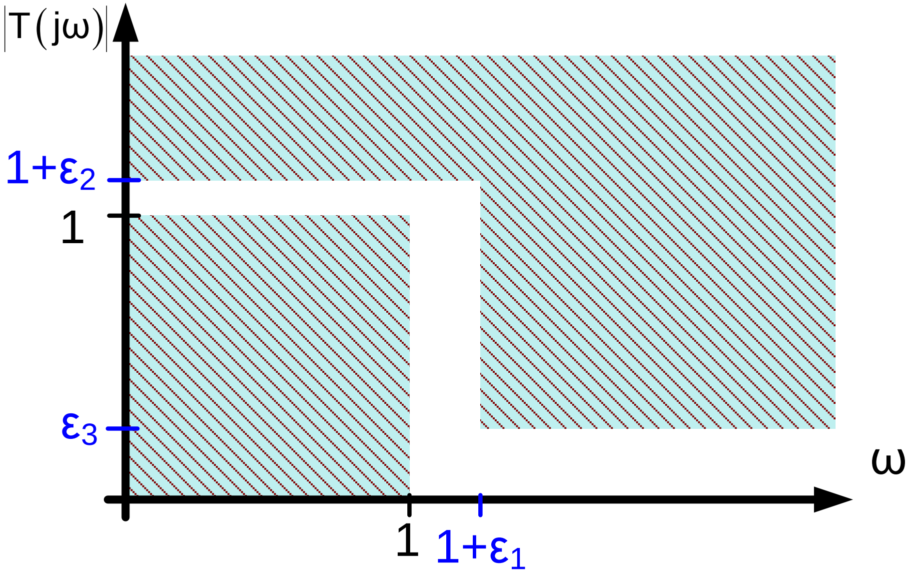
Filter applications often have strict requirements on where major changes in magnitude or phase occur

Window of transition from “pass-band” to “stop-band” often very narrow

Filter design field has received considerable attention by engineers for about 8 decades

- Passive RLC
- Vacuum Tube Op Amp RC
- Active Filters (Integrated op amps, R,C)
- Digital Implementation (ADC,DAC,DSP)
- Integrated Filters (SC)
- Integrated Filters (Continuous-time and SC)

Filter specifications often given by bounds for acceptable characteristics in frequency domain

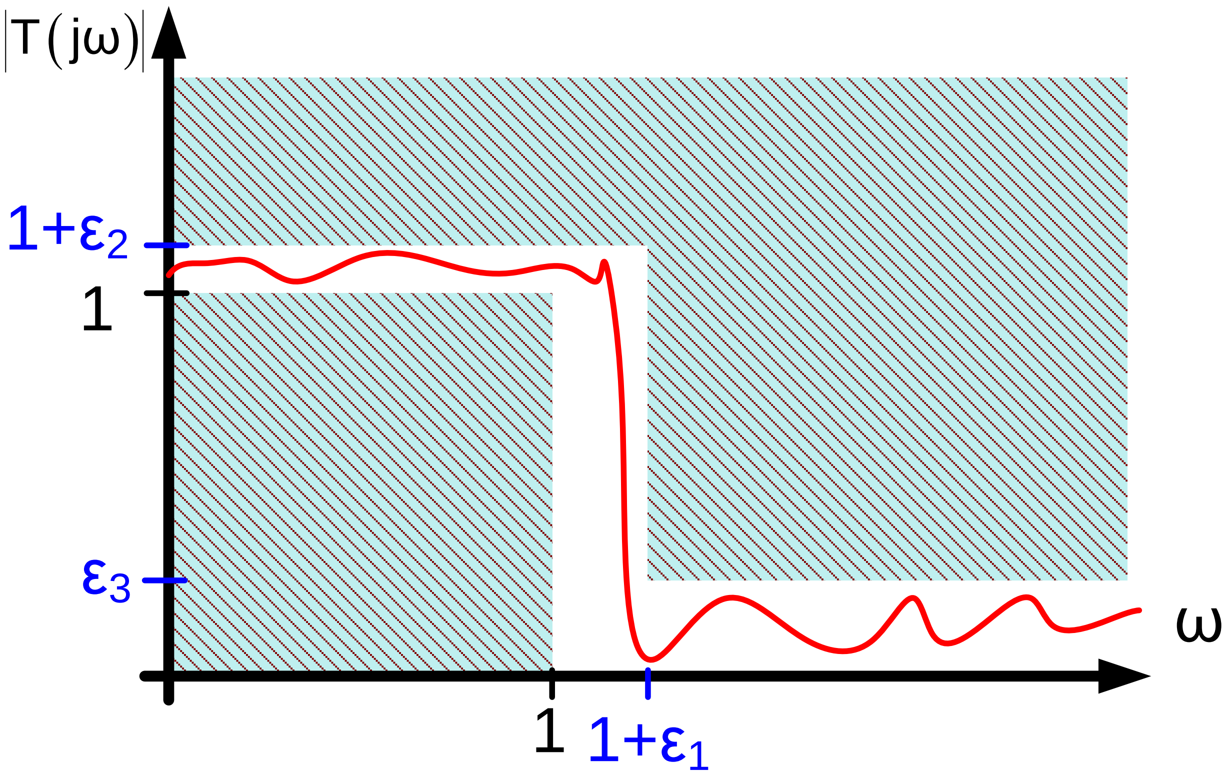


This example characterized by the three parameters $\{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$

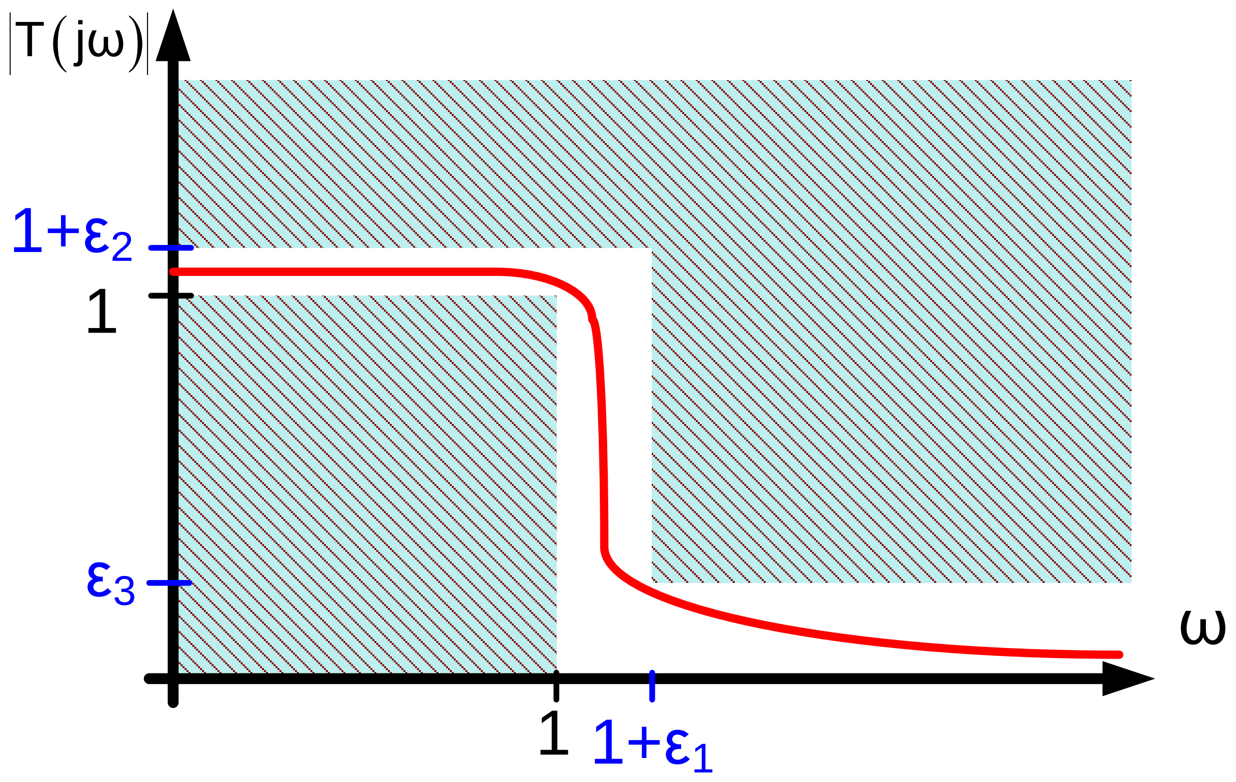
Any circuit that has a transfer function that does not enter the forbidden region is an acceptable solution from a performance viewpoint

Filter design must provide margins for component tolerance, temperature dependence, and aging

Any circuit that has a transfer function that does not enter the forbidden region is an acceptable solution from a performance viewpoint



Any circuit that has a transfer function that does not enter the forbidden region is an acceptable solution from a performance viewpoint



- Minor changes in specifications can have significant impact on cost and effort for implementing a filter

- Work closely with the filter user to determine what filter specifications are really needed

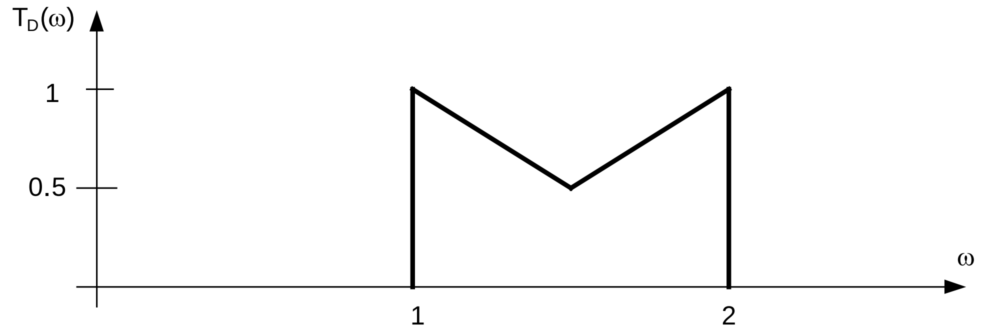
- This will become increasingly important as many (most) system designers in the future will have weak background in filter issues

EE 508
HW 1
Fall 2012

Short Assignment – due Friday of this week

The seemingly simple problem of obtaining a rational fraction that approximates a desired transfer function can become quite involved and, with the exception of a few standard approximations, there is still often no known technique for obtaining a transfer function. In this assignment, you will be asked to use whatever techniques you have available to obtain a transfer function that approximates a given magnitude response. A metric defined below will be used to assess how good your approximation is for this assignment.

Consider the desired "M" transfer function shown below where the frequency axis is linear.



Mathematically, the desired transfer function magnitude is characterized by the function

$$T_D(\omega) = \begin{cases} 0 & 0 \le \omega \le 1 \\ 2-\omega & 1 < \omega \le 1.5 \\ \omega - 1 & 1.5 < \omega \le 2 \\ 0 & \omega > 2 \end{cases}$$

Obtain a rational fraction approximation, T(s), to this transfer function magnitude. Your approximation is constrained to $m + n \le 6$ where m is the degree of the numerator polynomial and n is the degree of the denominator polynomial. That is

$$\mathbf{T(s)} = \frac{\sum_{i=1}^{m} a_i s^i}{\sum_{i=1}^{n} b_i s^i}$$

The "goodness" of your approximation should be assessed by the L_2 norm defined by

$$\varepsilon = \int_{\omega=0}^{4} \left| \left|T(j\omega)\right| - T_D(\omega) \right|^2 d\omega$$

You should include your approximation T(s), a plot of the magnitude of your transfer function along with that of $T_D(\omega)$ with a linear frequency axis for $0 < \omega < 5$, the value you obtain for ε, and a brief description of how you obtained your approximation.

Keep track of your time and spend at most 3 hours on this assignment. The major purpose of this assignment is to establish an appreciation for the approximation problem.

End of Lecture 1

EE 508
Lecture 2

Filter Design Process

Filter design field has received considerable attention by engineers for about 8 decades

- Passive RLC
- Vacuum Tube Op Amp RC
- Active Filters (Integrated op amps, R,C)
- Digital Implementation (ADC,DAC,DSP)
- Integrated Filters (SC)
- Integrated Filters (Continuous-time and SC)

Review from Last Time

Filter: Amplifier or system that has a frequency-dependent gain

- Filters are ideally linear devices
- Characteristics usually expressed as either desired frequency response or time domain response
- Transfer functions filters with finite number of lumped elements are rational fractions with real coefficients
- Transfer functions of any realizable filter (finite elements) have no discontinuities in either the magnitude or phase response

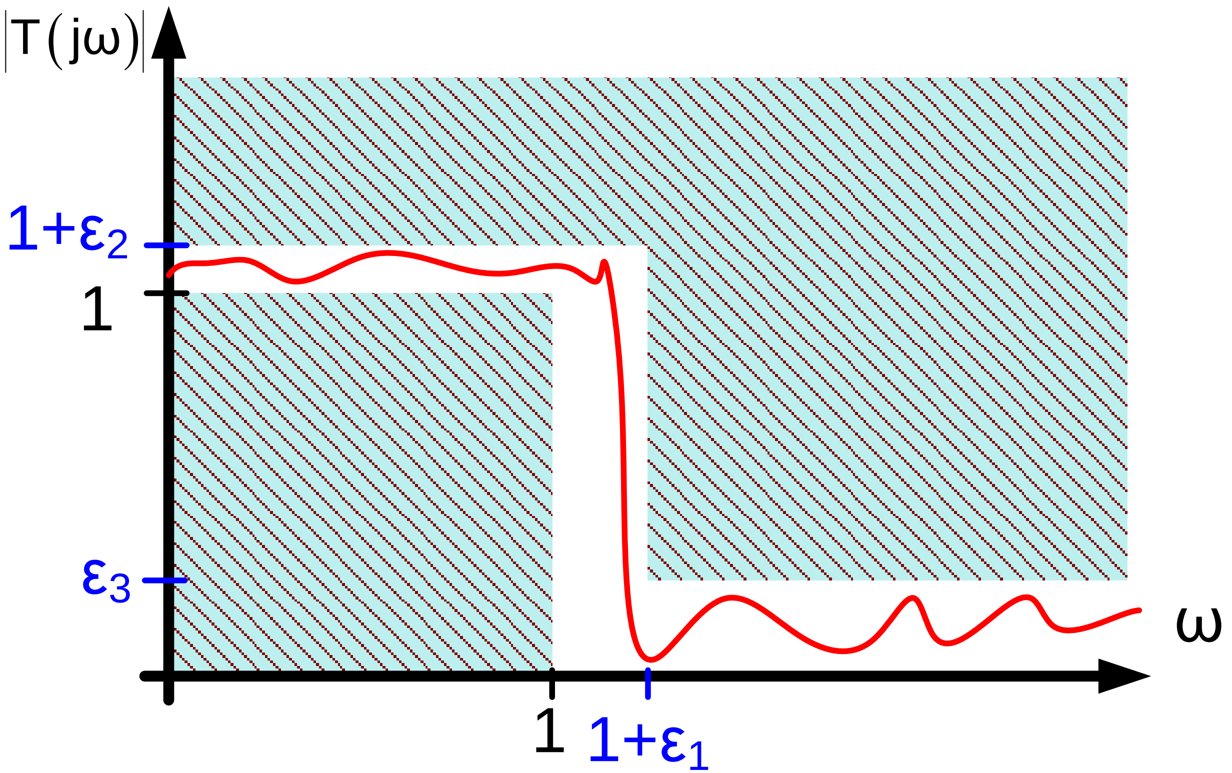
Review from Last Time

$$T(s) = \frac{\sum_{i=1}^{m} a_i s^i}{\sum_{i=1}^{n} b_i s^i} = \frac{N(s)}{D(s)}$$

$$H(z) = \frac{\sum_{i=1}^{m} a_i z^i}{\sum_{i=1}^{n} b_i z^i} = \frac{N(z)}{D(z)}$$

Review from Last Time

Any circuit that has a transfer function that does not enter the forbidden region is an acceptable solution from a performance viewpoint



Review from Last Time

- Minor changes in specifications can have significant impact on cost and effort for implementing a filter
- Work closely with the filter user to determine what filter specifications are really needed

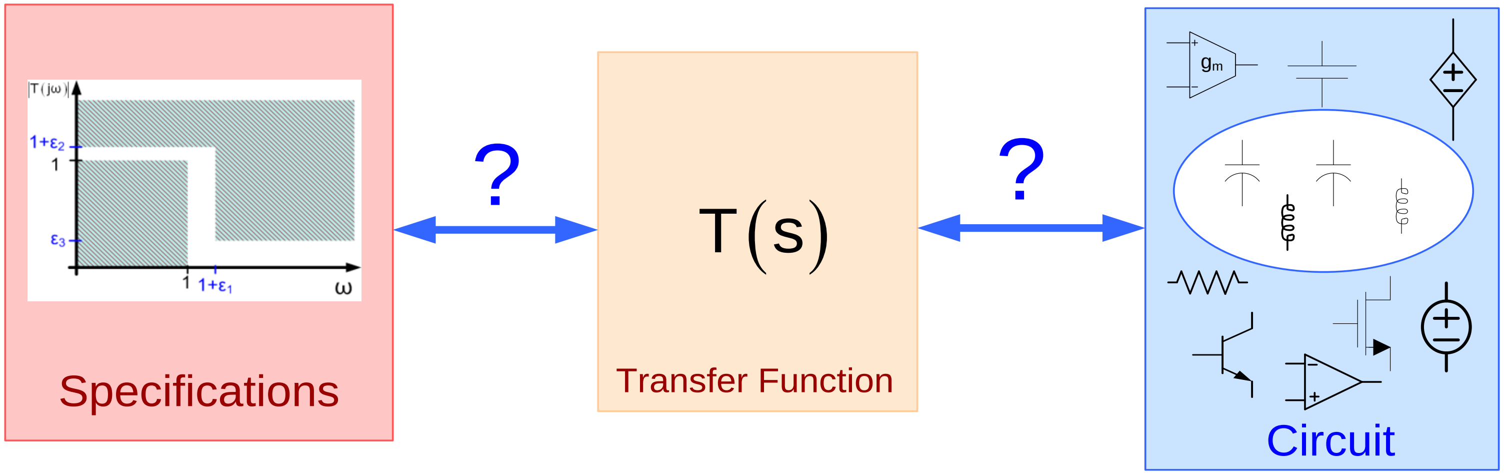
Is there a systematic way to design filters?

Observations:

- All filter circuits with a finite number of lumped elements have a transfer function that is a rational fraction in s
- All digital filters have a transfer function that is a rational fraction in z
- Most (ideally all) of the characteristics of a filter are determined by the transfer function

Is there a systematic way to design filters?

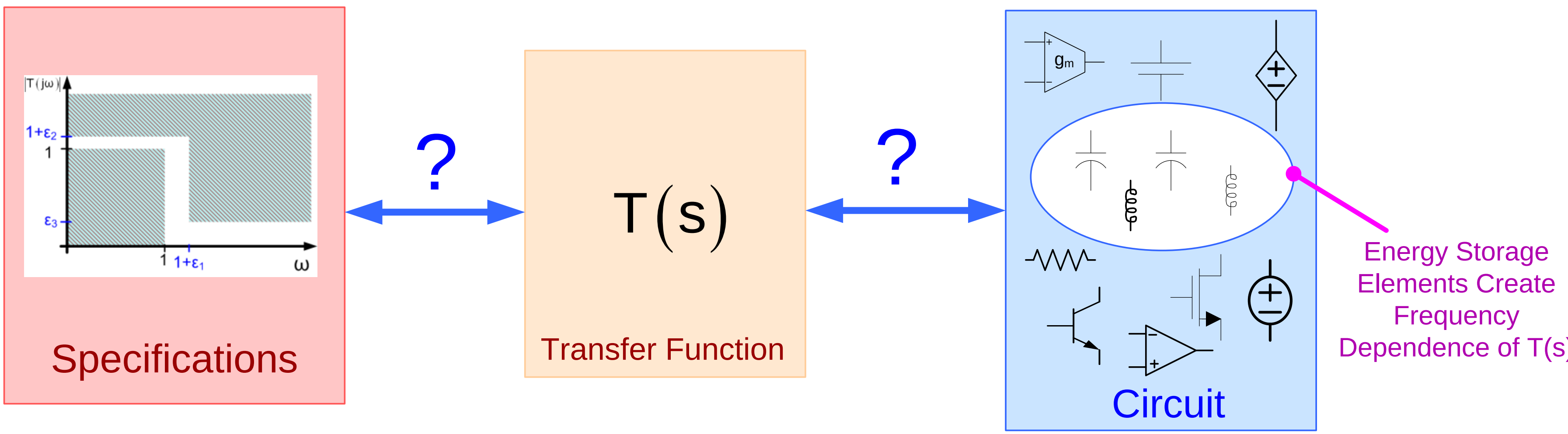
(Consider continuous-time first)



Filter Design Strategy: Use the transfer function as an intermediate step between the Specifications and Circuit Implementation

Is there a systematic way to design filters?

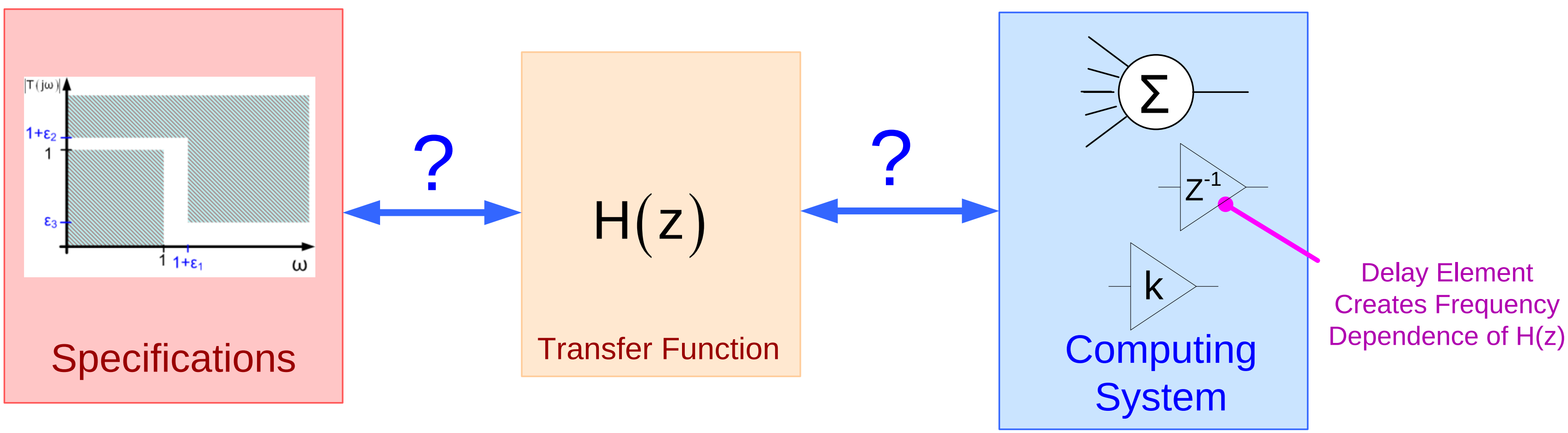
(Consider continuous-time first)



Filter Design Strategy: Use the transfer function as an intermediate step between the Specifications and Circuit Implementation

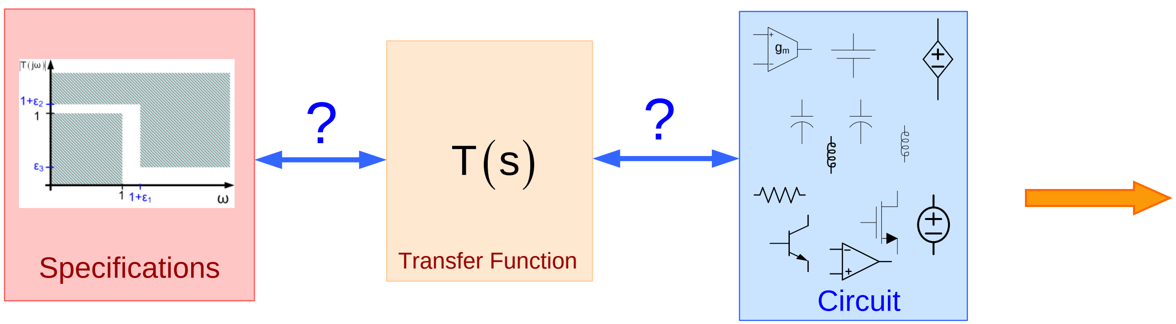
Is there a systematic way to design filters?

(Consider discrete-time domain)



Filter Design Strategy: Use the transfer function as an intermediate step between the Specifications and Circuit Implementation

Filter Design Process



Filter Design Strategy: Use the transfer function as an intermediate step between the Specifications and Circuit Implementation

Establish Specifications

- possibly $T_D(s)$ or $H_D(z)$
- magnitude and phase characteristics or restrictions
- time domain requirements

↓

Approximation

- obtain acceptable transfer functions $T_A(s)$ or $H_A(z)$
- possibly acceptable realizable time-domain responses

↓

Synthesis

- build circuit or implement algorithm that has response close to $T_A(s)$ or $H_A(z)$
- actually realize $T_R(s)$ or $H_R(z)$

↓

Filter

Filter Design Process

Establish Specifications

- possibly $T_D(s)$ or $H_D(z)$
- magnitude and phase characteristics or restrictions
- time domain requirements

Must understand the real performance requirements

↓

Approximation

- obtain acceptable transfer functions $T_A(s)$ or $H_A(z)$
- possibly acceptable realizable time-domain responses

Obtain an acceptable approximating function

↓

Synthesis

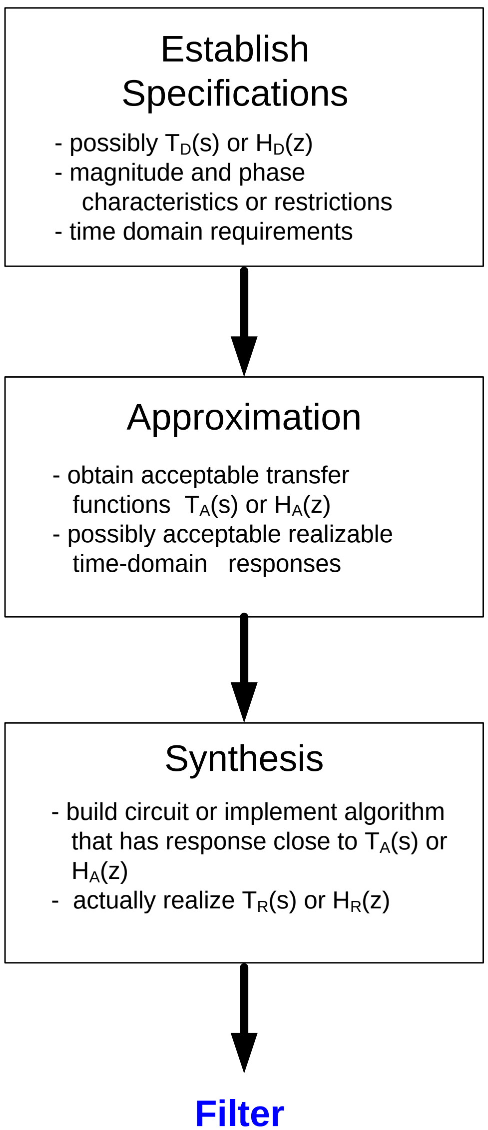
- build circuit or implement algorithm that has response close to $T_A(s)$ or $H_A(z)$
- actually realize $T_R(s)$ or $H_R(z)$

Design (synthesize) a practical circuit or system that has a transfer function close to the acceptable transfer function

↓

Filter

Filter Design Process



Must understand the real performance requirements

- **Many acceptable specifications for a given application**
- **Some much better than others**
- **But often difficult to obtain even one that is useful**

Obtain an acceptable approximating function

- **Many acceptable approximating functions for a given specification**
- **Some much better than others**
- **But often difficult to obtain even one!**

Design (synthesize) a practical circuit or system that has a transfer function close to the acceptable transfer function

- **Many acceptable circuits for a given approximating function**
  Some much better than others
- **But often difficult to obtain even one!**

Important to make good decisions at each step in the filter design process because poor decisions will not be absolved in subsequent steps

Filter Design Process

Establish Specifications

- possibly $T_D(s)$ or $H_D(z)$
- magnitude and phase characteristics or restrictions
- time domain requirements

↓

Approximation

- obtain acceptable transfer functions $T_A(s)$ or $H_A(z)$
- possibly acceptable realizable time-domain responses

↓

Synthesis

- build circuit or implement algorithm that has response close to $T_A(s)$ or $H_A(z)$
- actually realize $T_R(s)$ or $H_R(z)$

↓

Filter

- Order of approximating function directly affects cost of implementation
- Number of energy storage elements in circuit is equal to the order of T(s) (neglecting energy storage element loops)
- High Q poles and zeros adversely affect cost (because component tolerances become tight)
- Cost of implementation (synthesis) is essentially independent of the quality of the approximation if the order is fixed
- Major effort over several decades was focused on the approximation problem

Filter Design Process

Establish Specifications

- possibly $T_D(s)$ or $H_D(z)$
- magnitude and phase characteristics or restrictions
- time domain requirements

Approximation

- obtain acceptable transfer functions $T_A(s)$ or $H_A(z)$
- possibly acceptable realizable time-domain responses

Synthesis

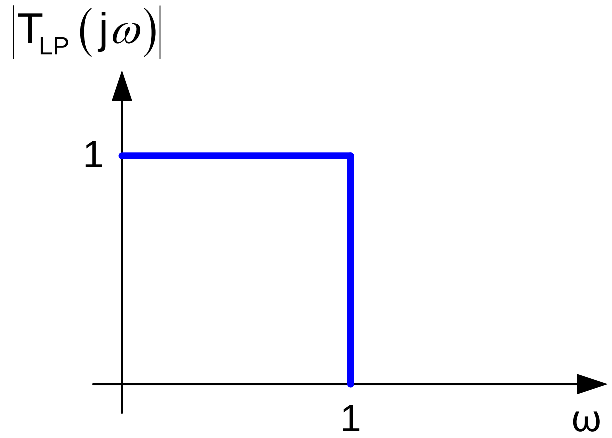
- build circuit or implement algorithm that has response close to $T_A(s)$ or $H_A(z)$
- actually realize $T_R(s)$ or $H_R(z)$

Filter

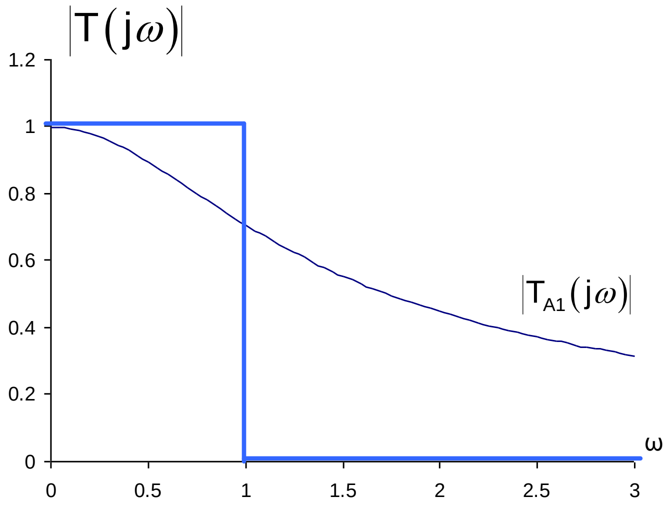
Some realizations are much better than others

- Cost
- Sensitivity
- Tunability
- Parasitic Effects
- Linearity
- Area
- **Major effort over several decades focused on synthesis problem**

Example:
Design a filter that approximates the ideal lowpass filter



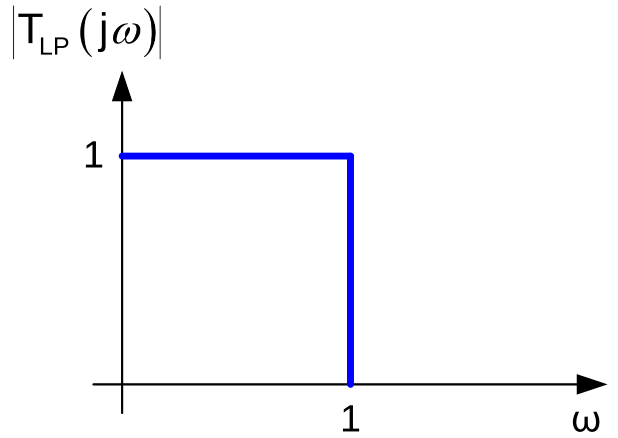
Desired filter response



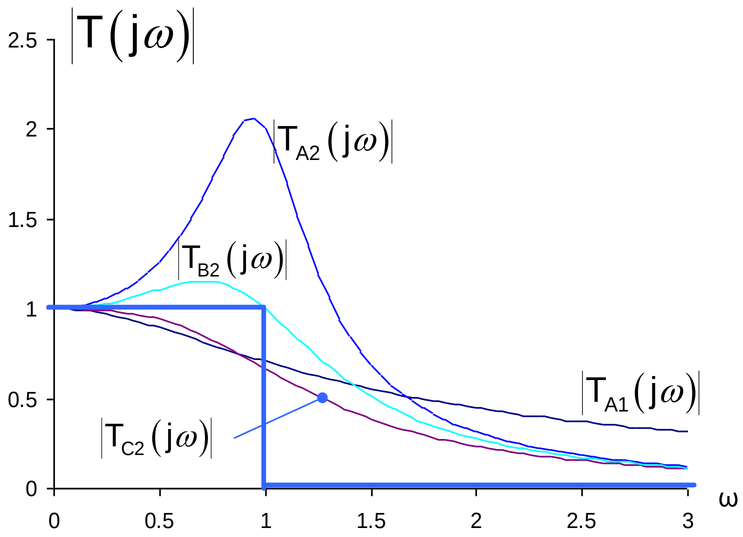
$$T_{A1} = \frac{1}{s+1}$$

One approximating function

Example:
Design a filter that approximates the ideal lowpass filter



Desired filter response



$$T_{A1}=\frac{1}{s+1}$$

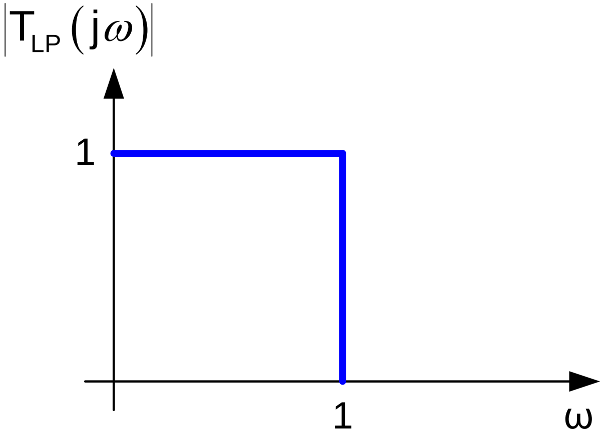
$$T_{A2}=\frac{1}{s^2+0.5s+1}$$

$$T_{B2}=\frac{1}{s^2+s+1}$$

$$T_{C2}=\frac{1}{s^2+1.5s+1}$$

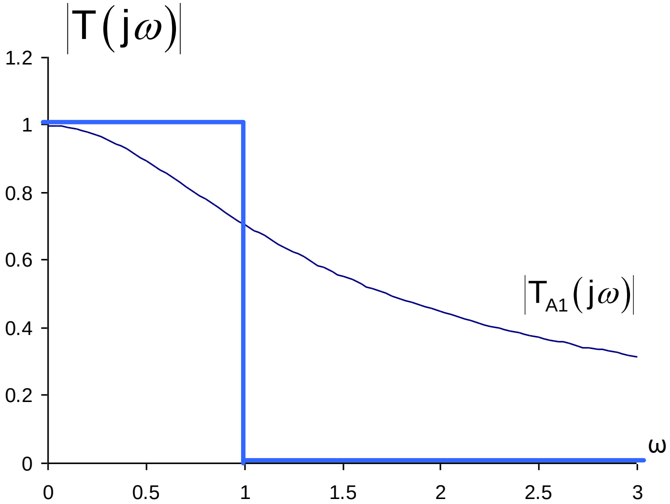
Some additional approximating functions

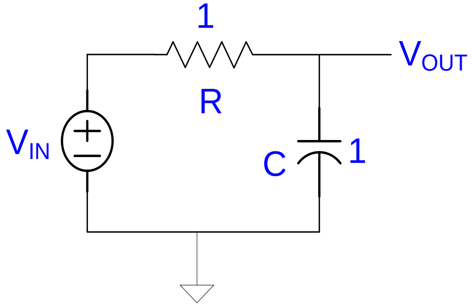
Example:
Design a filter that approximates the ideal lowpass filter



Desired filter response

$$T_{A1} = \frac{1}{s+1}$$



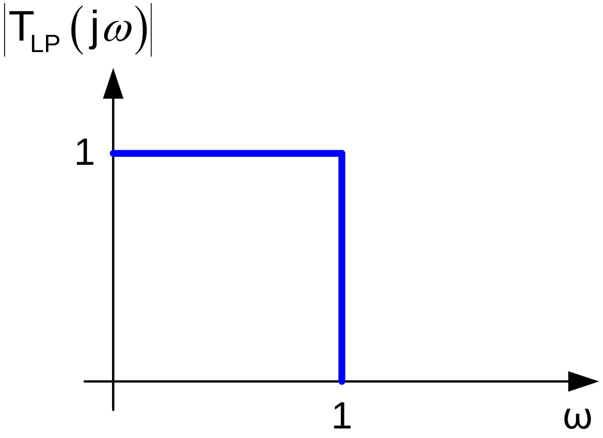


$$T(s) = \frac{1}{1+RCs}$$

A circuit that realize T_{A1}

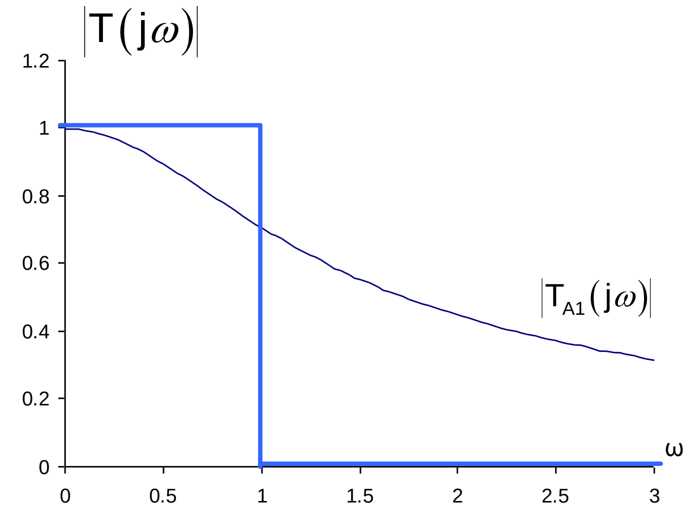
But not practical because C is too large!

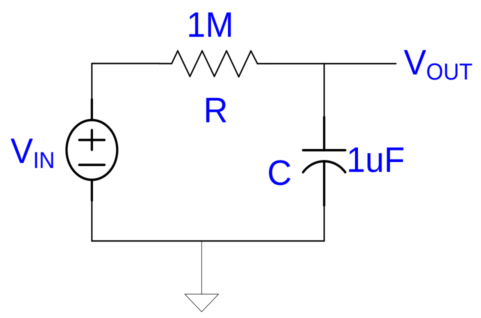
Example:
Design a filter that approximates the ideal lowpass filter



Desired filter response

$$T_{A1} = \frac{1}{s+1}$$





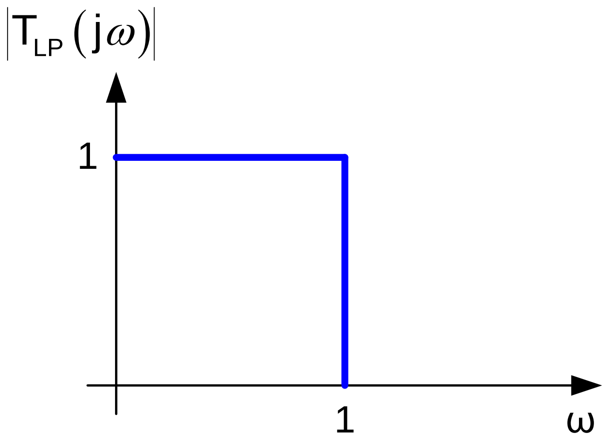
A circuit that realize T_{A1}

$$T(s) = \frac{1}{1+RCs}$$

More practical (C must not be electrolytic)!

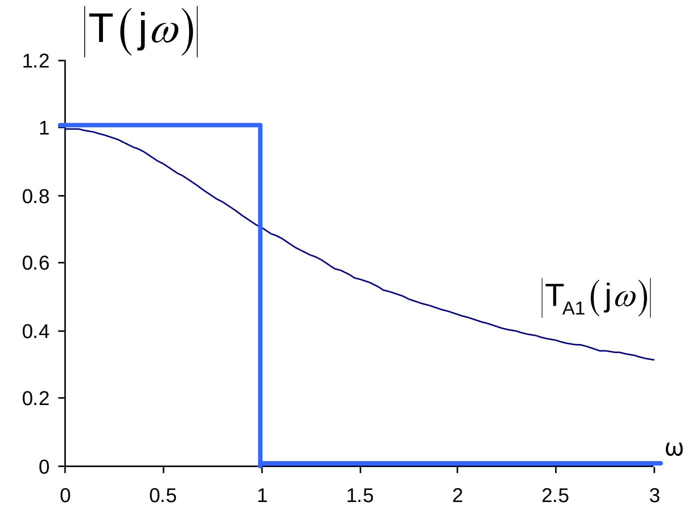
Example:

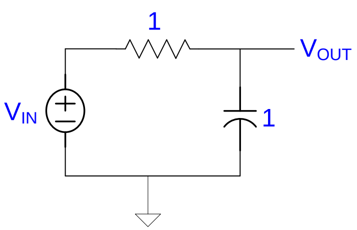
Design a filter that approximates the ideal lowpass filter



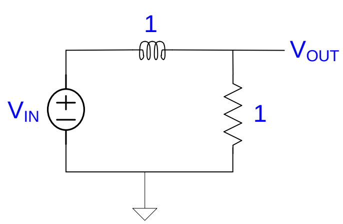
Desired filter response

$$T_{A1}=\frac{1}{s+1}$$

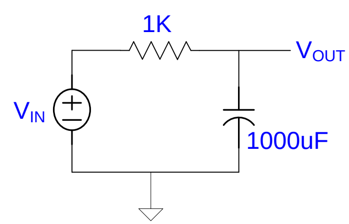




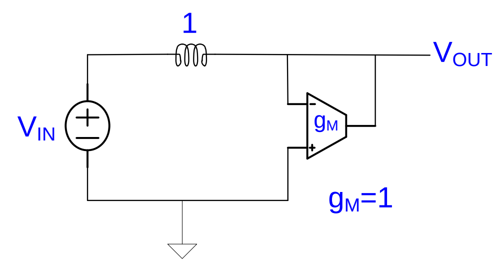
$$T(s)=\frac{1}{1+RCs}$$



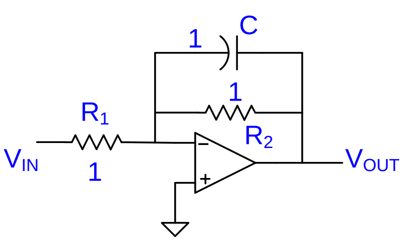
$$T(s)=\frac{1}{1+\frac{L}{R}s}$$



$$T(s)=\frac{1}{1+RCs}$$



$$T(s)=\frac{1}{1+g_mLs}$$

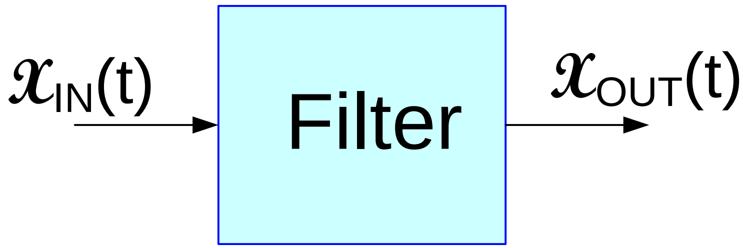


$$T(s)=\frac{-\frac{R_2}{R_1}}{1+R_2Cs}$$

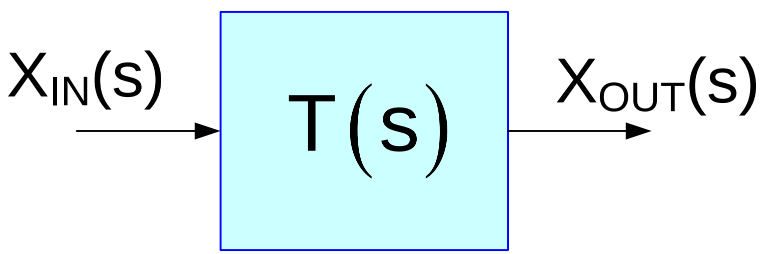
Some additional circuits that realize T_{A1}

Time Domain and Frequency Domain Characterization

Filters always operate in the time domain



Filters often characterized/designed in the frequency domain



$$T(s) = \frac{X_{OUT}(s)}{X_{IN}(s)} \longrightarrow T(s) = \frac{\sum_{i=0}^{m} a_i s^i}{\sum_{i=0}^{n} b_i s^i}$$

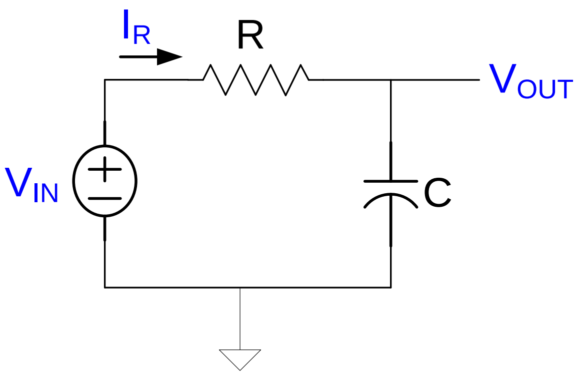
$$T(s) = \frac{\mathcal{L}(x_{OUT}(t))}{\mathcal{L}(x_{IN}(t))} \longrightarrow ?$$

$m \le n$

Time Domain and Frequency Domain Characterization

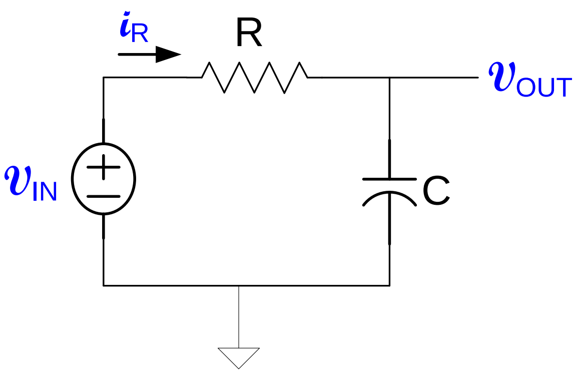
Example:

Frequency Domain



$$\left.\begin{array}{l} I_R = \dfrac{V_{IN} - V_{OUT}}{R} \\ I_R \bullet \dfrac{1}{sC} = V_{OUT} \end{array}\right\} \rightarrow \quad T(s) = \frac{V_{OUT}(s)}{V_{IN}(s)} = \frac{1}{1+RCs} = \frac{1}{1+b_1 s}$$

Time Domain



$$\left.\begin{array}{l} i_R = \dfrac{v_{IN} - v_{OUT}}{R} \\ i_R = C\dfrac{dv_{OUT}}{dt} \end{array}\right\} \rightarrow \quad \frac{dv_{OUT}}{dt} = \left(\frac{1}{RC}\right)v_{IN} - \left(\frac{1}{RC}\right)v_{OUT}$$

$$\frac{dv_{OUT}}{dt} = \left(\frac{1}{b_1}\right)v_{IN} - \left(\frac{1}{b_1}\right)v_{OUT}$$

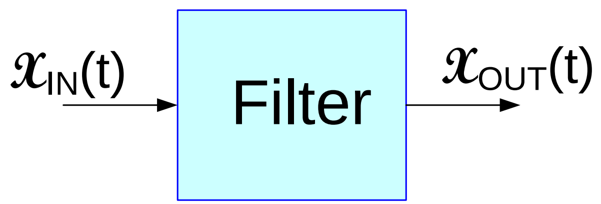
Taking the Laplace transform of the differential equation, we obtain

$$\left.\begin{array}{l} \mathcal{L}\left(\dfrac{dv_{OUT}}{dt}\right) = \left(\dfrac{1}{b_1}\right)\mathcal{L}(v_{IN}) - \left(\dfrac{1}{b_1}\right)\mathcal{L}(v_{OUT}) \\ sV_{OUT} = \left(\dfrac{1}{b_1}\right)V_{IN} - \left(\dfrac{1}{b_1}\right)V_{OUT} \end{array}\right\} \quad T(s) = \frac{V_{OUT}}{V_{IN}} = \frac{1}{1+b_1 s}$$

Time Domain and Frequency Domain Characterization

Generalizing from the previous example:

Time Domain



Elements in filter are {R's, C's,L's, indep sources, dep sources}

Assume n energy storage elements and no energy storage element loops in the circuit

The relationship between $\mathcal{X}_{OUT}(t)$ and $\mathcal{X}_{IN}(t)$ can always be expressed by a single time-domain differential equation as

$$\frac{d^n v_{OUT}}{dt^n} = \sum_{k=0}^{m} \alpha_k \frac{d^k v_{IN}}{dt^k} - \sum_{k=0}^{n-1} \beta_k \frac{d^k v_{OUT}}{dt^k}$$

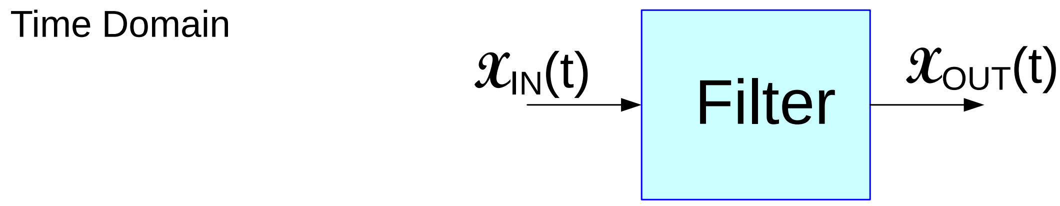
where the α_k and β_k are constants dependent on the values of the circuit elements

Taking the Laplace transform of this differential equation, we obtain

$$s^n V_{OUT} = \sum_{k=1}^{m} \alpha_k s^k V_{IN} - \sum_{k=1}^{n} \beta_k s^k V_{OUT}$$

Time Domain and Frequency Domain Characterization

Generalizing from the previous example:



$$s^n V_{OUT} = \sum_{k=0}^{m} \alpha_k s^k V_{IN} - \sum_{k=1}^{n-1} \beta_k s^k V_{OUT}$$

If we define $\beta_n = 1$, this can be rewritten as

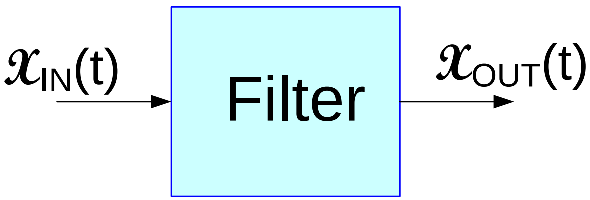
$$\left(\sum_{k=0}^{n} \beta_k s^k \right) V_{OUT} = \sum_{k=0}^{m} \alpha_k s^k V_{IN}$$

Thus, the transfer function can be written as

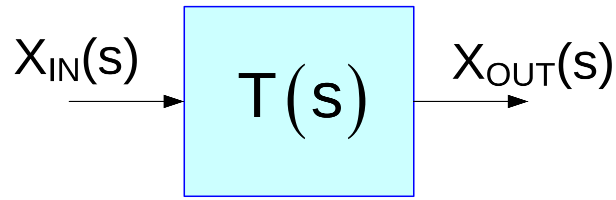
$$T(s) = \frac{V_{OUT}}{V_{IN}} = \frac{\sum_{k=0}^{m} \alpha_k s^k}{\sum_{k=0}^{n} \beta_k s^k}$$

Time Domain and Frequency Domain Characterization

Time Domain



Frequency Domain



$$\frac{d^n v_{OUT}}{dt^n} = \sum_{k=0}^{m} \alpha_k \frac{d^k v_{IN}}{dt^k} - \sum_{k=0}^{n-1} \beta_k \frac{d^k v_{OUT}}{dt^k}$$

$$T(s) = \frac{\sum_{k=0}^{m} \alpha_k s^k}{\sum_{k=0}^{n} \beta_k s^k}$$

$$T(s) = \frac{\sum_{i=0}^{m} a_i s^i}{\sum_{i=0}^{n} b_i s^i}$$

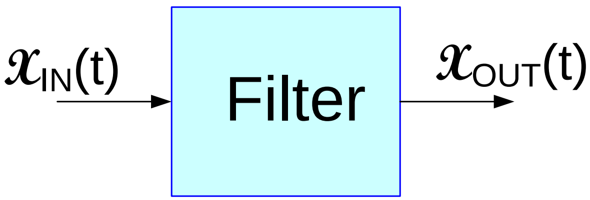
How do the α_k and β_k parameters relate to the a_k and b_k paramaters?

If we normalize the frequency-domain solution so that $b_n=1$, then

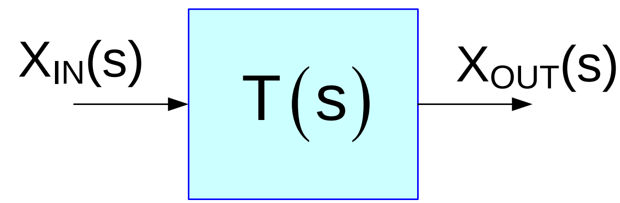
$\alpha_k=a_k$ and $\beta_k=b_k$ for all k

Time Domain and Frequency Domain Characterization

Time Domain



Frequency Domain



$$\frac{d^n v_{OUT}}{dt^n} = \sum_{k=0}^{m} \alpha_k \frac{d^k v_{IN}}{dt^k} - \sum_{k=0}^{n-1} \beta_k \frac{d^k v_{OUT}}{dt^k}$$

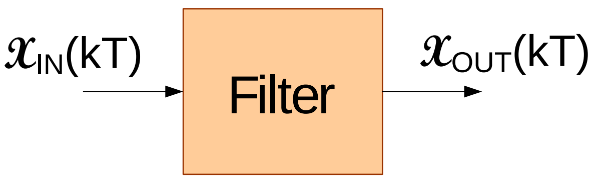
$$T(s) = \frac{\sum_{i=0}^{m} a_i s^i}{\sum_{i=0}^{n} b_i s^i}$$

Thus, the time-domain characterization of a filter which can be expressed as a single differential equation can be obtained directly from the transfer function T(s) obtained from a frequency-domain analysis of the circuit

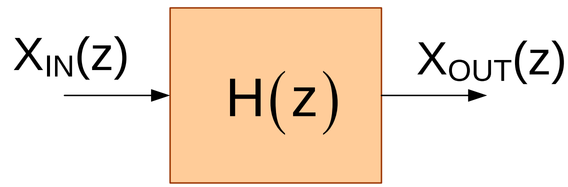
This differential equation does not contain any initial condition information

Time Domain and Frequency Domain Characterization

Time Domain



Frequency Domain



$$v_{OUT}(nT) = \sum_{k=0}^{m} \alpha_k v_{IN}((n-k)T) - \sum_{k=1}^{n-1} \beta_k v_{OUT}((n-k)T)$$

If we define $\beta_n=1$, this can be rewritten as

$$H(z) = \frac{\sum_{k=0}^{m} \alpha_k z^k}{\sum_{k=0}^{n} \beta_k z^k}$$

$$H(z) = \frac{\sum_{i=0}^{m} a_i z^i}{\sum_{i=0}^{n} b_i z^i}$$

How do the α_k and β_k parameters relate to the a_k and b_k paramaters?

If we normalize the frequency-domain solution so that $b_n=1$, then

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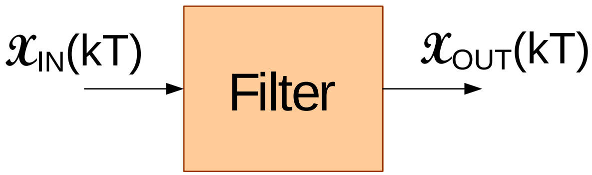
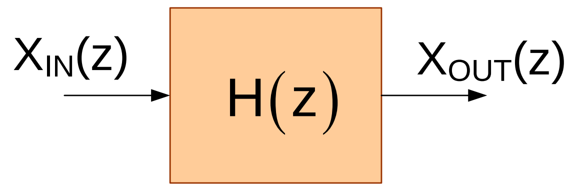
Time Domain and Frequency Domain Characterization

Time Domain



Frequency Domain



$$v_{OUT}(nT) = \sum_{k=0}^{m} \alpha_k v_{IN}((n-k)T) - \sum_{k=1}^{n-1} \beta_k v_{OUT}((n-k)T)$$

$$H(z) = \frac{\sum_{i=0}^{m} a_i z^i}{\sum_{i=0}^{n} b_i z^i}$$

Thus, the time-domain characterization of a filter which can be expressed as a single difference equation can be obtained directly from the transfer function H(s) obtained from a frequency-domain analysis of the circuit

This difference equation does not contain any initial condition information

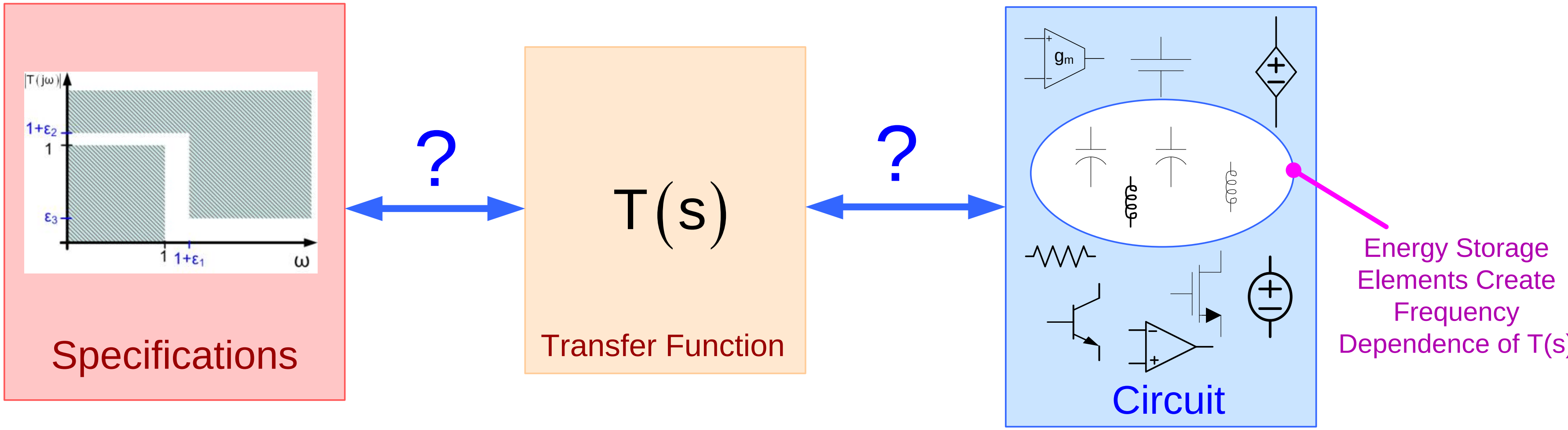
EE 508
Lecture 3

Filter Concepts/Terminology

Basic Properties of Electrical Circuits

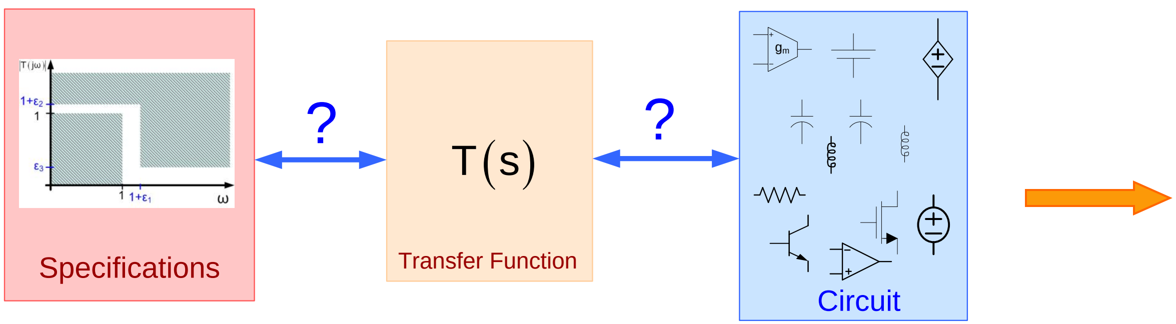
Review from Last Time

Is there a systematic way to design filters?



Filter Design Strategy: Use the transfer function as an intermediate step between the Specifications and Circuit Implementation

Review from Last Time



Filter Design Strategy: Use the transfer function as an intermediate step between the Specifications and Circuit Implementation

Filter Design Process

Establish Specifications

- possibly $T_D(s)$ or $H_D(z)$
- magnitude and phase characteristics or restrictions
- time domain requirements

Approximation

- obtain acceptable transfer functions $T_A(s)$ or $H_A(z)$
- possibly acceptable realizable time-domain responses

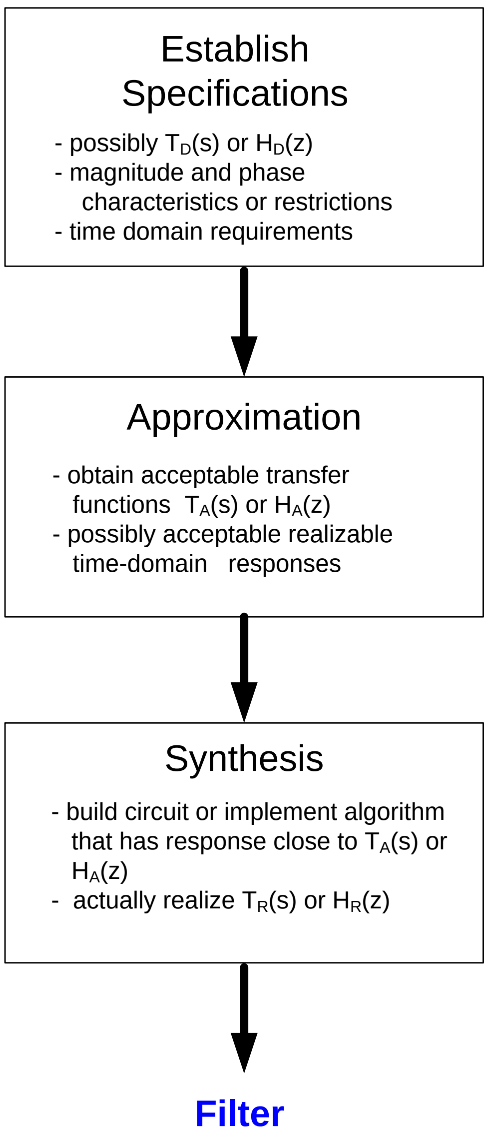
Synthesis

- build circuit or implement algorithm that has response close to $T_A(s)$ or $H_A(z)$
- actually realize $T_R(s)$ or $H_R(z)$

Filter

Filter Design Process

Review from Last Time



Must understand the real performance requirements

- **Many acceptable specifications for a given application**
- **Some much better than others**
- **But often difficult to obtain even one that is useful**

Obtain an acceptable approximating function

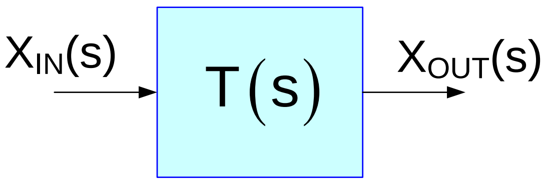
- **Many acceptable approximating functions for a given specification**
- **Some much better than others**
- **But often difficult to obtain even one!**

Design (synthesize) a practical circuit that has a transfer function close to the acceptable transfer function

- **Many acceptable circuits for a given approximating function**
  Some much better than others
- **But often difficult to obtain even one!**

Important to make good decisions at each step in the filter design process because poor decisions will not be absolved in subsequent steps

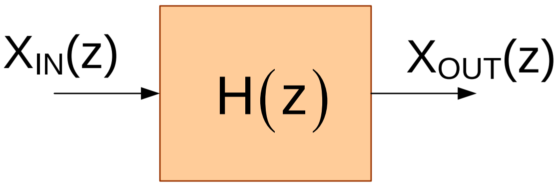
Filter Concepts and Terminology



$$T(s) = \frac{\sum_{i=0}^{m} a_i s^i}{\sum_{i=0}^{n} b_i s^i} = \frac{N(s)}{D(s)}$$

- A polynomial is said to be “integer monic” if the coefficient of the highest-order term is 1
- If D(s) is integer monic, then N(s) and D(s) are unique
- If D(s) is integer monic, then the a_k and b_k terms are unique
- The roots of N(s) are termed the zeros of the transfer function
- The roots of D(s) are termed the poles of the transfer function
- If N(s) and D(s) are of orders m and n respectively, then there are m zeros and n poles in T(s)

Filter Concepts and Terminology



$$H(z) = \frac{\sum_{i=0}^{m} a_i z^i}{\sum_{i=0}^{n} b_i z^i} = \frac{N(z)}{D(z)}$$

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- If D(z) is integer monic, then N(z) and D(z) are unique
- If D(z) is integer monic, then the a_k and b_k terms are unique
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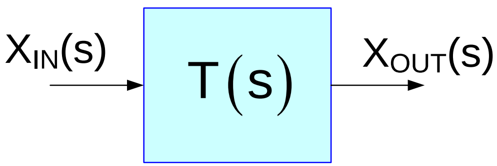
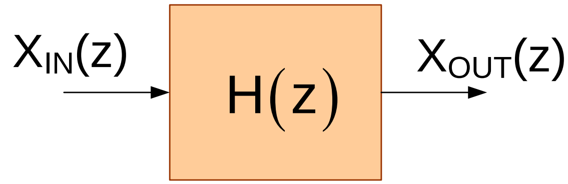
Filter Concepts and Terminology





$$T(s) = \frac{\sum_{i=0}^{m} a_i s^i}{\sum_{i=0}^{n} b_i s^i} = \frac{N(s)}{D(s)}$$

$$H(z) = \frac{\sum_{i=0}^{m} a_i z^i}{\sum_{i=0}^{n} b_i z^i} = \frac{N(z)}{D(z)}$$

- Key Theorem: The continuous-time filter is stable iff all poles lie in the open left half of the s-plane

- Key Theorem: The discrete-time filter is stable iff all poles lie in the open unit circle

- The zeros of T(s) need not lie in the left half plane to maintain stability

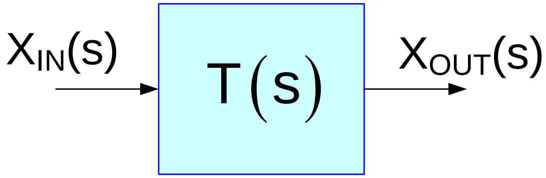
- The zeros of H(z) need not lie in the open unit circle to maintain stability

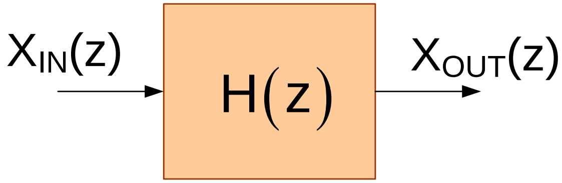
Filter Concepts and Terminology

Minimum Phase Property

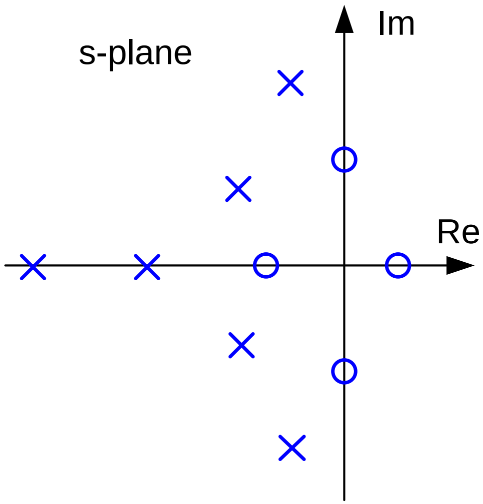
- An s-domain rational fraction is termed minimum-phase if all poles and all zeros have a non-positive real part
- An s-domain rational fraction is minimum-phase if it has no poles or zeros in the RHP or on the imaginary axis
- A z-domain rational fraction is minimum-phase if the magnitude of all poles ad zeros are less that 1
- A z-domain rational fraction is minimum-phase iff no poles or zeros lie on or outside of the unit circcle

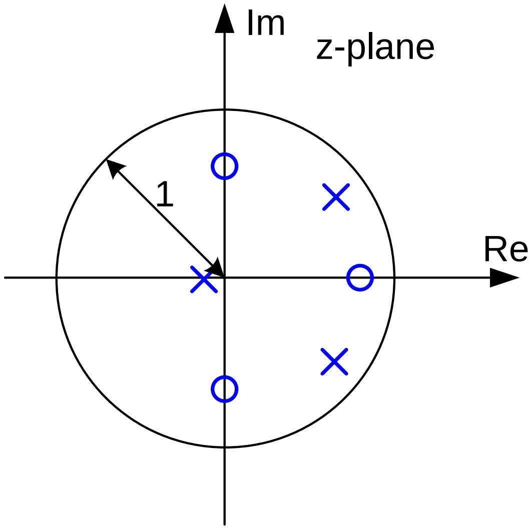
Filter Concepts and Terminology



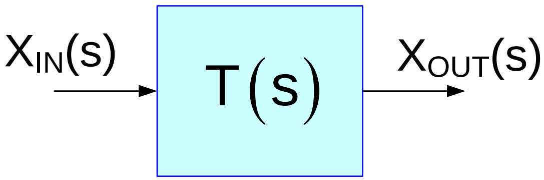


Pole-zero Plots





Filter Concepts and Terminology



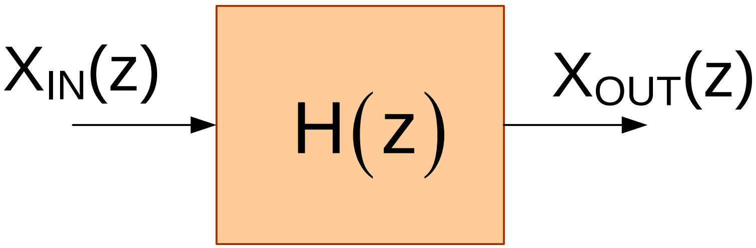
$$T(s) = \frac{\sum_{i=0}^{m} a_i s^i}{\sum_{i=0}^{n} b_i s^i} = \frac{N(s)}{D(s)}$$

- If T(s) is a rational fraction with poles and/or zeros in the RHP, then $\tilde{T}(s)$ obtained by reflecting all RHP roots around the imaginary axis back into the LHP has the following properties

a) minimum phase
b) stable
c) $\left|\tilde{T}(s)\right|_{s=j\omega} = \left|T(s)\right|_{s=j\omega}$ for all ω

Note the phase of T(s) and $\tilde{T}(s)$ will differ

Filter Concepts and Terminology



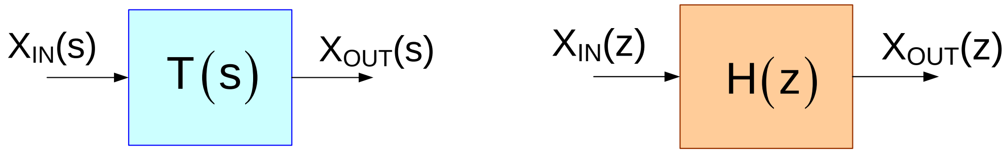
$$H(z) = \frac{\sum_{i=0}^{m} a_i z^i}{\sum_{i=0}^{n} b_i z^i} = \frac{N(z)}{D(z)}$$

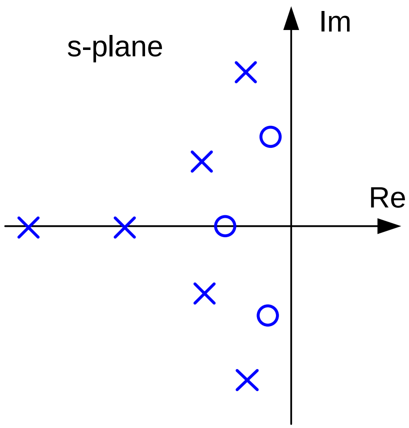
If H(z) is a rational fraction with poles and/or zeros outside the unit circle, then $\tilde{H}(z)$ obtained by reflecting all roots outside the unit circle back into the unit circle by the complex conguate reciprocal reflection and then scaling the transfer function by the magnitude of the reciprocal of the root has the following properties

a) minimum phase
b) stable
c) $\left|\tilde{H}(z)\right|_{z=e^{j\omega T}} = \left|H(z)\right|_{z=e^{j\omega T}}$ for all ω

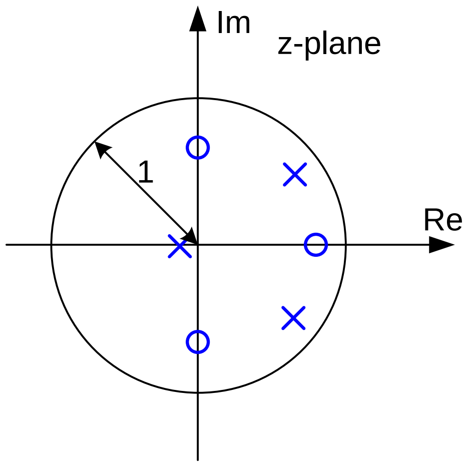
Note the phase of H(z) and $\tilde{H}(z)$ will differ

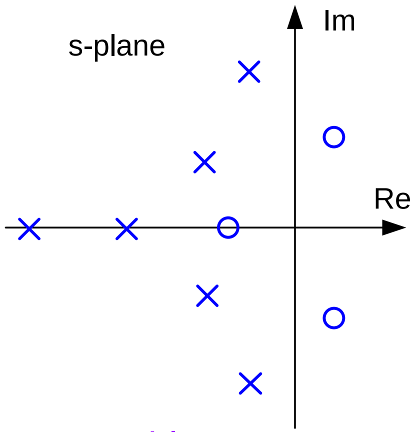
Filter Concepts and Terminology



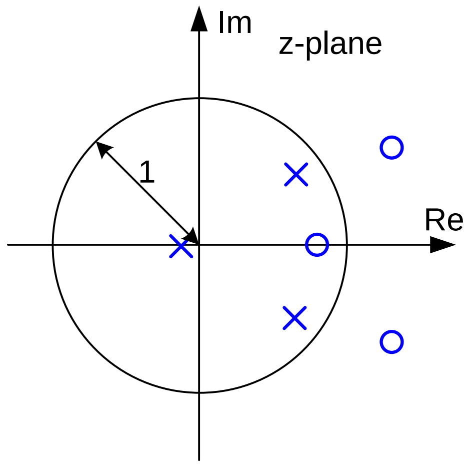


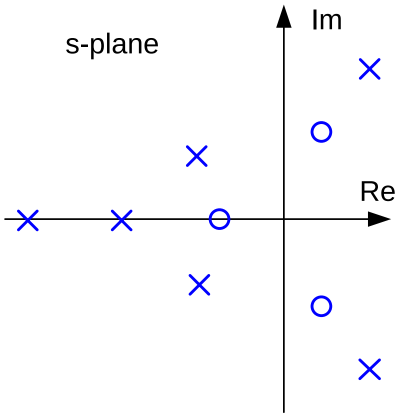
- Stable
- Minimum Phase



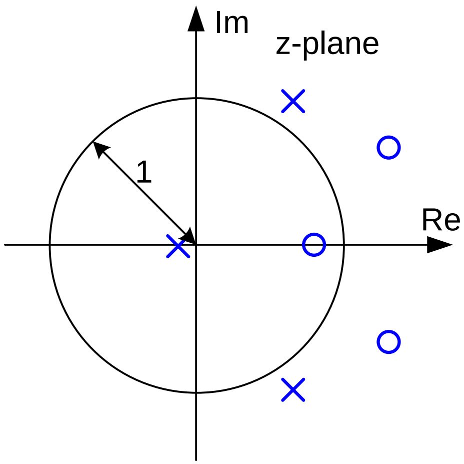


- Stable
- Not minimum Phase





- Not stable
- Not minimum Phase



Example: Non-minimum Phase Transfer Function

$$T(s) = \frac{s-1}{s+1}$$

$$\left|T(j\omega)\right| = \sqrt{\frac{\omega^2 + (-1)^2}{\omega^2 + 1^2}} = 1$$

$$\angle T(j\omega) = \frac{\tan^{-1}\left(\frac{\omega}{-1}\right)}{\tan^{-1}\left(\frac{\omega}{1}\right)}$$

Beware that arctan function in multi-valued and in CAD tools give “a” principle value that may or may not consider the quadrant of the two arguements

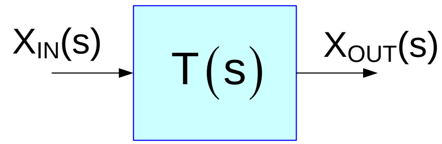
Example: Non-minimum Phase Transfer Function

$$T(s) = \frac{s^2 - as + 1}{s^2 + as + 1}$$

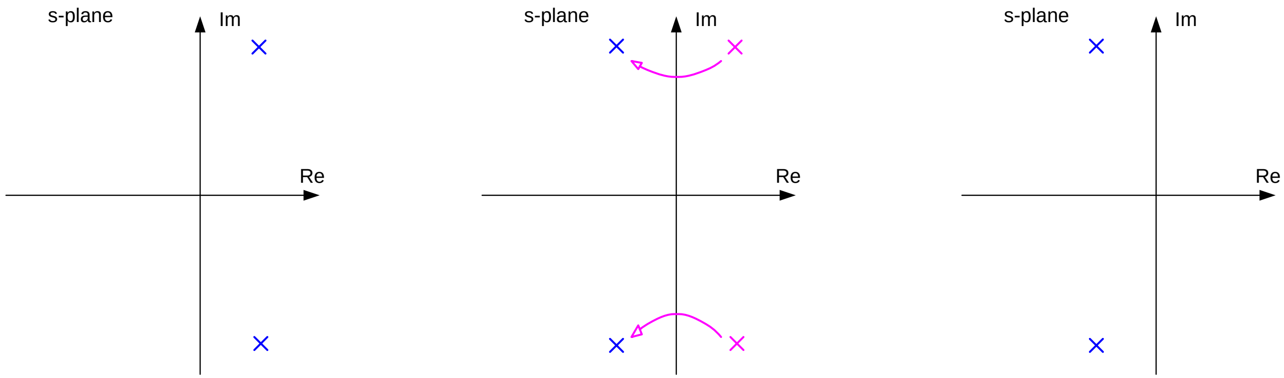
$$\left|T(j\omega)\right| = \sqrt{\frac{\left(1-\omega^2\right)^2 + a^2\omega^2}{\left(1-\omega^2\right)^2 + (-a)^2\omega^2}} = 1$$

$$\angle T(j\omega) = \frac{\tan^{-1}\left(\frac{-a\omega}{1-\omega^2}\right)}{\tan^{-1}\left(\frac{a\omega}{1-\omega^2}\right)}$$

Filter Concepts and Terminology



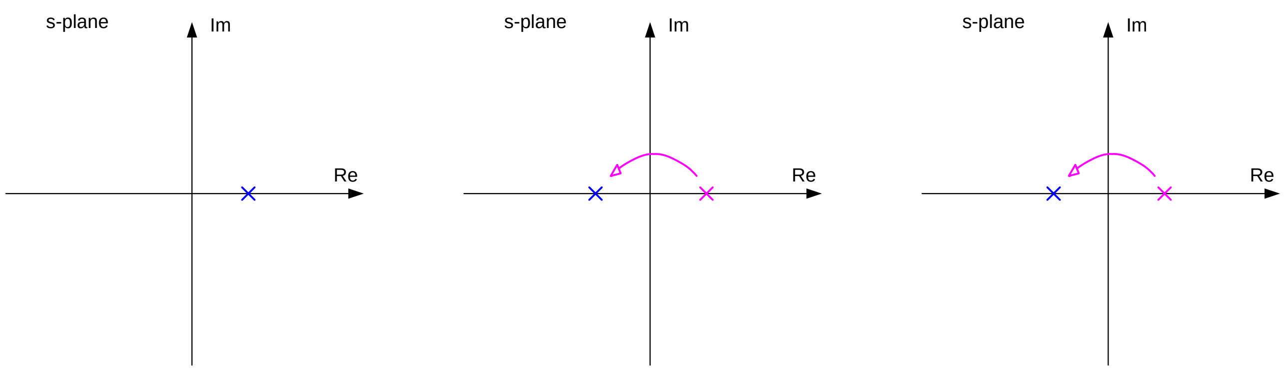
Reflecting poles and zeros to maintain stability or establish minimum phase



Not minimum Phase

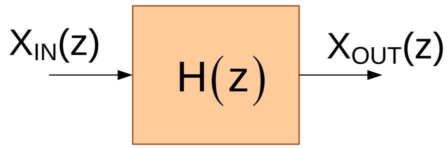
Reflection

Minimum Phase

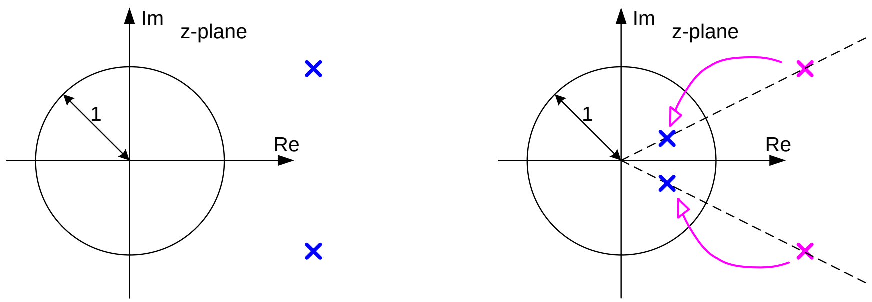


Note: magnitude of real part is preserved in reflection, imaginary part remains unchanged

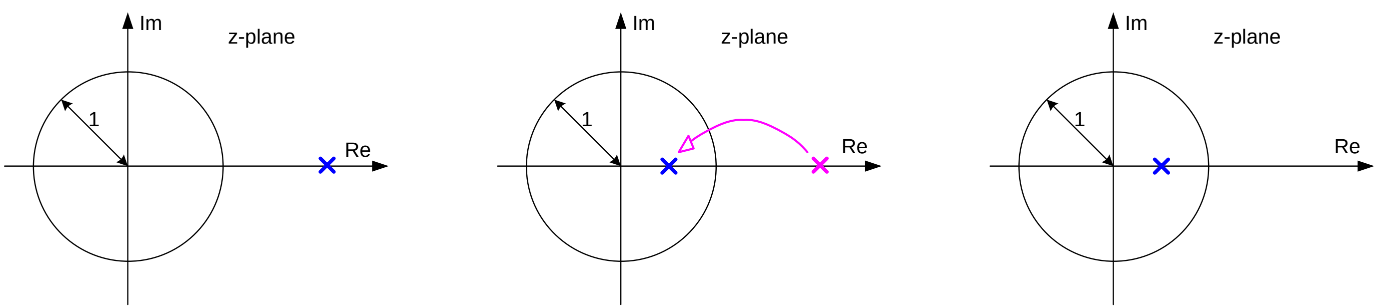
Filter Concepts and Terminology



Reflecting poles and zeros to maintain stability or establish minimum phase

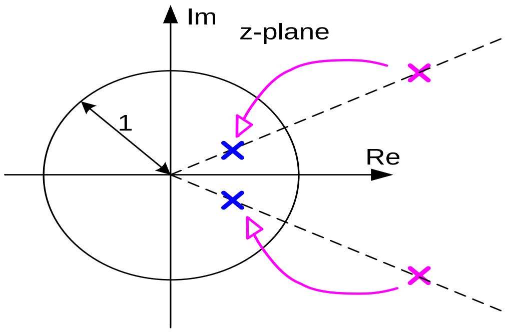


Not minimum Phase Reflection Minimum Phase



Note: complex conjugate reciprocal reflection maintains angle but magnitude of reflected root is the reciprocal of the magnitude of the original root

Complex Conjugate Reciprocal Reflection



Express X in polar form as

$$X = \mathrm{R}\, e^{j\theta}$$

The complex conjugate reciprocal reflection is

$$X_{CCRT} = \mathrm{R}^{-1}\, e^{j\theta}$$

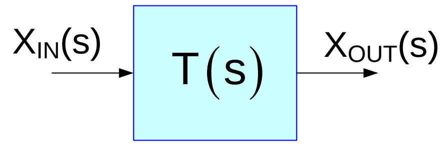
End of Lecture 3

EE 508
Lecture 4

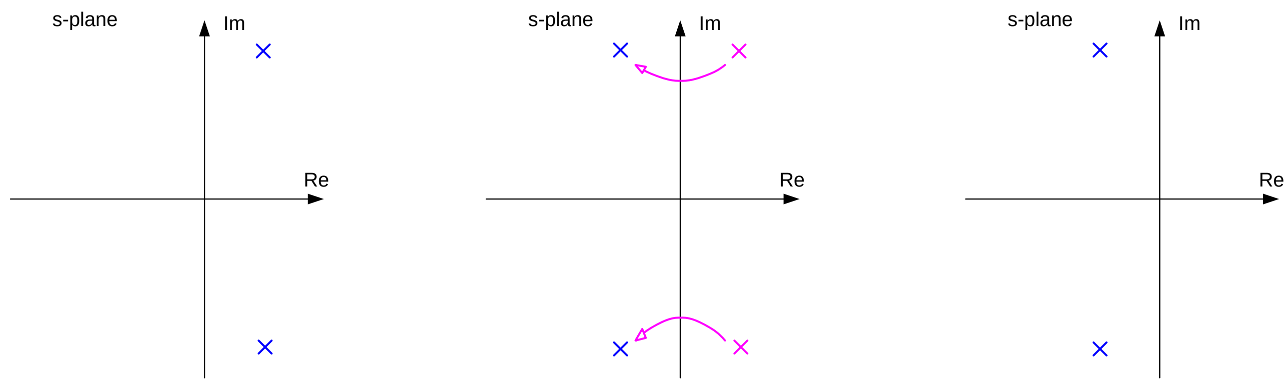
Filter Concepts/Terminology

Basic Properties of Electrical Circuits

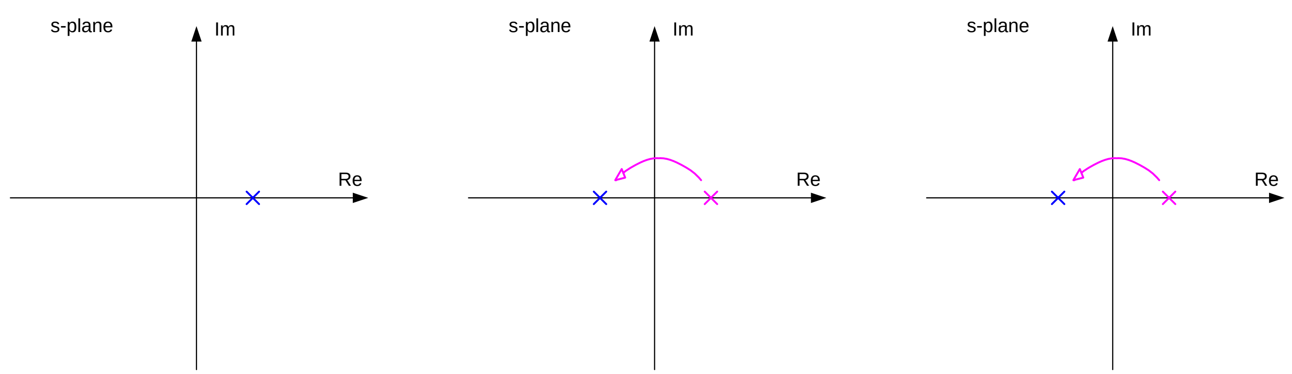
Filter Concepts and Terminology



Reflecting poles and zeros to maintain stability or establish minimum phase



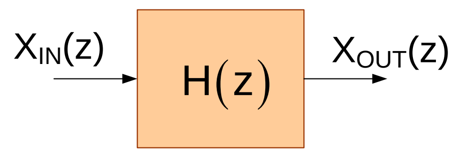
Not minimum Phase   Reflection   Minimum Phase



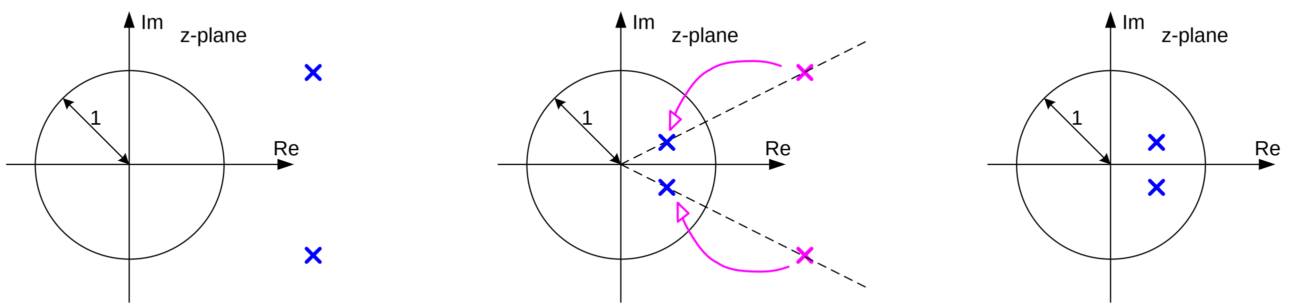
Note: magnitude of real part is preserved in reflection, imaginary part remains unchanged

Review from Last Time

Filter Concepts and Terminology



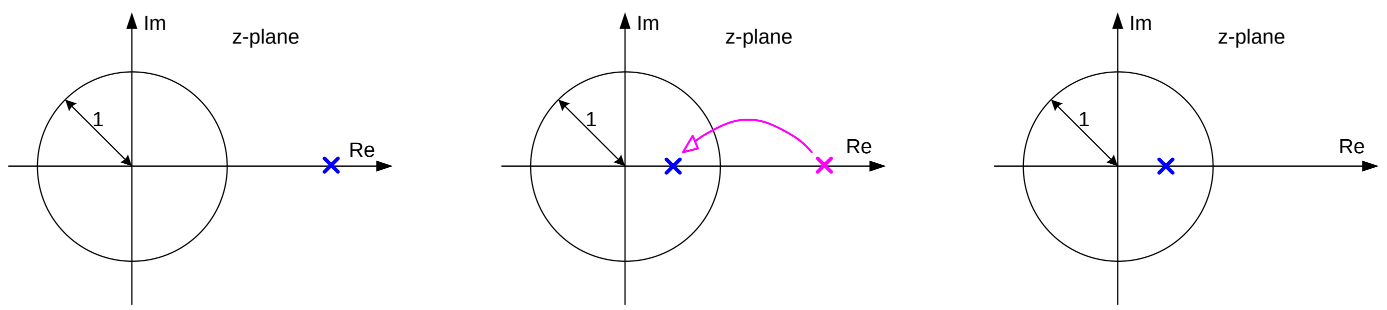
Reflecting poles and zeros to maintain stability or establish minimum phase



Not minimum Phase

Reflection

Minimum Phase



Note: complex conjugate reciprocal reflection maintains angle but magnitude of reflected root is the reciprocal of the magnitude of the original root

Filter Concepts and Terminology

- 2-nd order polynomial characterization
- Biquadratic Factorization
- Op Amp Modeling
- Stability and Instability
- Roll-off characteristics
- Dead Networks
- Root Characterization
- Scaling, normalization, and transformation

2-nd order polynomial characterization

$$s^2+as+b \qquad \{a,b\}$$

$$s^2+\frac{\omega_o}{Q}s+\omega_0^2 \qquad \{\omega_o,Q\}$$

$$s^2+2\zeta\omega_0 s+\omega_0^2 \qquad \{\zeta,\ \omega_o\}$$

$$s^2+(p_1+p_2)s+p_1p_2 = (s+p_1)(s+p_2) \qquad \{p_1,p_2\}$$

with complex conjugate roots

$$s^2+2\alpha s+\alpha^2+\beta^2 = (s+\alpha+j\beta)(s+\alpha-j\beta) \qquad \{\alpha,\beta\}$$

$$s^2+2r\cos(\theta)s+r^2 = (s+re^{j\theta})(s+re^{-j\theta}) \qquad \{r,\theta\}$$

2-nd order polynomial characterization

$\{a,b\}$ $\{\omega_o,Q\}$ $\{\zeta, \omega_o\}$ $\{p_1,p_2\}$

$\{\alpha,\beta\}$ $\{r,\theta\}$

Alternate equivalent parameter sets

Widely used interchangeably

Easy mapping from one to another

Defined irrespective of whether polynomial appears in numerator or denominator of transfer function

If order is greater than 2, often multiple root pairing options so these parameter sets will not be unique for a given polynomial or transfer function

If cc roots exist, these will almost always be paired together (unique)

Biquadratic Factorization

$$T(s)=\frac{\sum_{i=1}^{m} a_i s^i}{\sum_{i=1}^{n} b_i s^i} = \frac{N(s)}{D(s)}$$

If m or n is even, integer-monic polynomials N(s) or D(s) can be expressed as

$$P(s) = \sum_{i=0}^{k} c_i s^i = \prod_{i=1}^{k/2} (s^2+d_{1i}s+d_{2i})$$

If m or n is odd, integer-monic polynomials N(s) or D(s) can be expressed as

$$P(s) = \sum_{i=0}^{k} c_i s^i = (s+d_0)\prod_{i=1}^{(k-1)/2} (s^2+d_{1i}s+d_{2i})$$

- These are termed quadratic factorizations

- If both N(s) and D(s) are expressed as quadratic factorizations, quadratic pairs can be grouped to obtain a Biquadratic factorization of T(s)

Biquadratic Factorization

Pole and zero pairings can always be made so that all coefficients
In the biquadratic factorizations are real

In general, the biquadratic factorizations are not unique

-If roots are real, multiple choices for first-order factor and remaining roots
can be partitioned into groups of 2 in different ways

-Complex conjugate root pairs are generally grouped together so that all
Coefficients are real

Biquadratic Factorization

If n is even,

$$T(s)=\frac{\sum_{i=1}^{m} a_i s^i}{\sum_{i=1}^{n} b_i s^i} = K \bullet \prod_{i=1}^{n/2} T_{BQi}(s)$$

If n is odd,

$$T(s)=\frac{\sum_{i=1}^{m} a_i s^i}{\sum_{i=1}^{n} b_i s^i} = K \bullet \left(\frac{a_{10}s+a_{00}}{s+b_{00}}\right) \bullet \prod_{i=1}^{(n-1)/2} T_{BQi}(s)$$

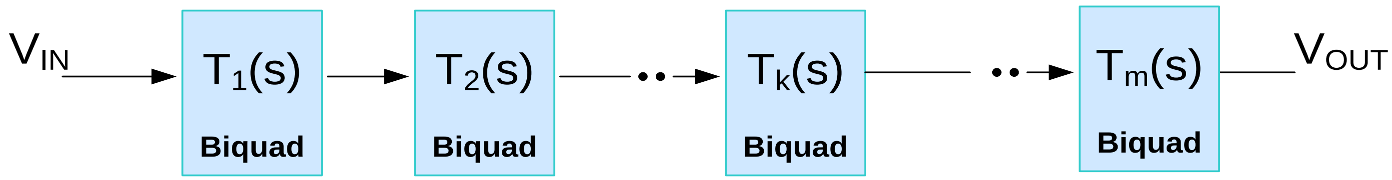
where $$T_{BQi}(s)=\frac{a_{2i}s^2 + a_{1i}s+a_{0i}}{s^2 + b_{1i}s+b_{0i}}$$

and where K is a real constant and all coefficients are real (some may be 0)

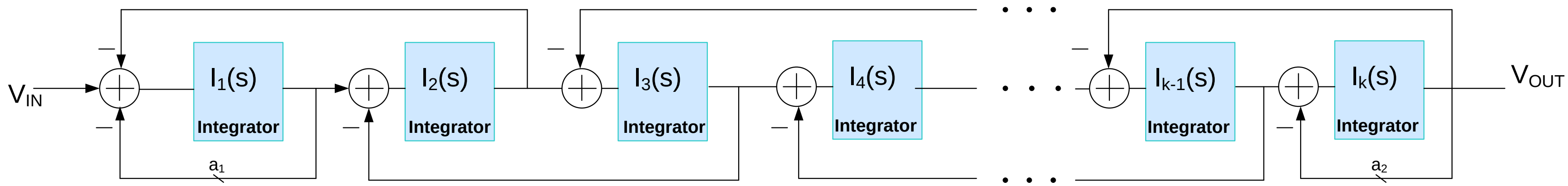
- Factorization is not unique
- H(z) factorizations not restricted to have m<n
- Each biquatratic factor can be represented by any of the 6 alternative parameter sets in the numerator or denominator

Common Filter Architectures

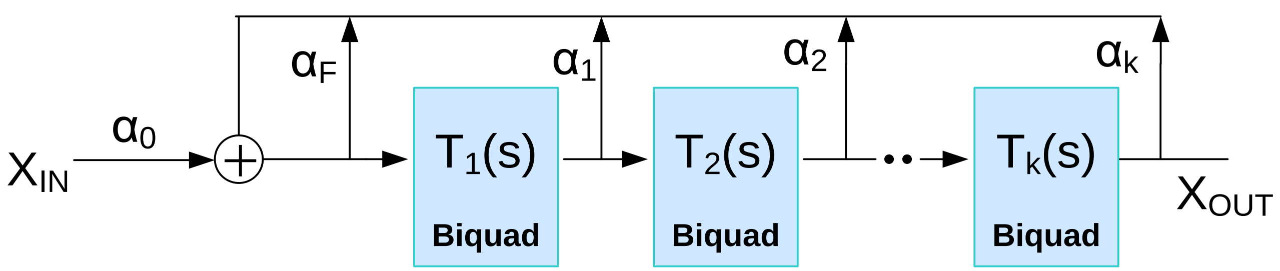
Cascaded Biquads



Leapfrog



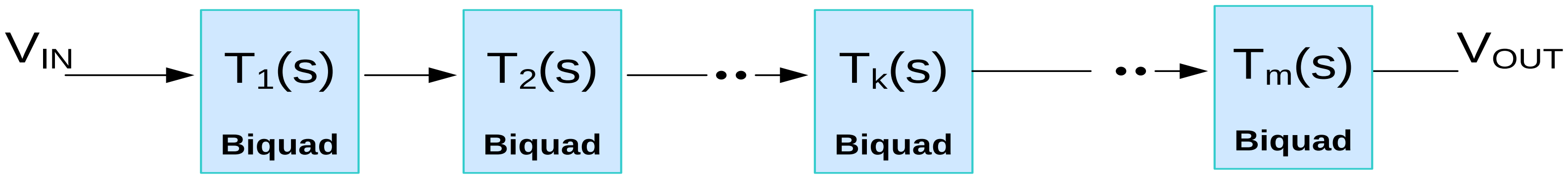
Multiple-loop Feedback



- Three classical filter architectures are shown
- The Cascaded Biquad and the Leapfrog approaches are most common

Common Filter Architectures

Cascaded Biquads



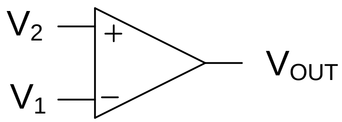
$$T(s) = T_1 T_2 \bullet\bullet T_m$$

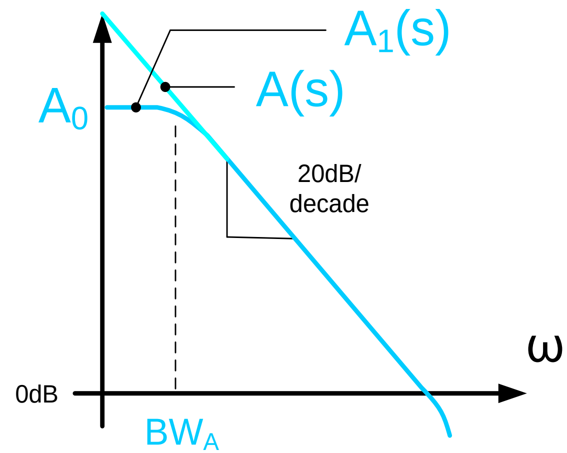
- Sequence in Cascade often affect performance
- Different biquadratic factorizations will provide different performance
- Although some attention was given to the different alternatives for biquadratic factorization, a solid general formulation of the cascade sequencing problem or the biquadratic factorization problem never evolved

Gain, Bandwidth and GB

Frequency Dependent Model of Op Amps

Most op amps are designed so that they behave as a first-order circuit at frequencies up to the unity gain frequency or beyond





$$A_1(s) = \frac{GB}{s + BW_A}$$

$$GB = A_0 \bullet BW_A$$

where $$BW_A = \frac{GB}{A_0} \ll 1$$

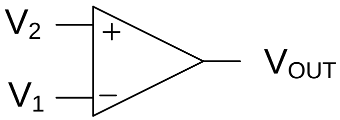
Can usually model with a more-simplified gain expression

$$A(s) = \frac{GB}{s}$$

Adequate model for most applications

Gain, Bandwidth and GB

Effects of GB on closed-loop Amplifiers



$$A_1(s)=\frac{GB}{s+BW_A}$$

$$GB = A_0 \bullet BW_A$$

$$A(s)=\frac{GB}{s}$$

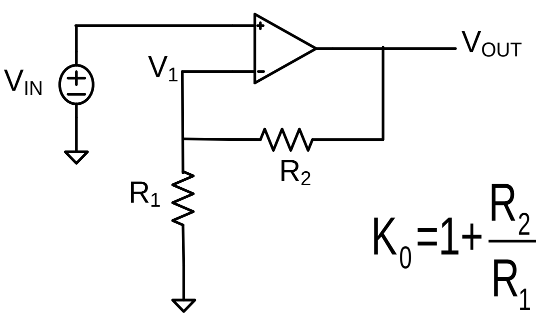
Adequate model for most applications

$$\left.\begin{aligned} V_1 &= \frac{V_{OUT}}{K_0} \\ V_{OUT} &= A(s)(V_1-V_{IN}) \\ A(s) &= \frac{GB}{s+BW_A} \end{aligned}\right\}$$

$$A_{FB}(s)=\frac{V_{OUT}}{V_{IN}}=\frac{K_0}{s\frac{K_0}{GB}+\left(1+K_0\frac{BW_A}{GB}\right)}$$

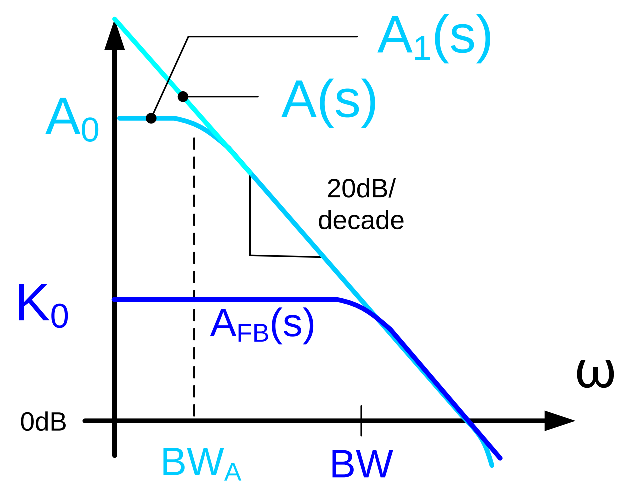
$$A_{FB}(s)\simeq\frac{K_0}{1+s\frac{K_0}{GB}}$$

$$BW=\frac{GB}{K_0}$$



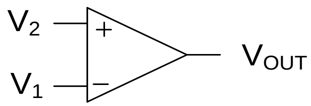
$$K_0=1+\frac{R_2}{R_1}$$

Basic Noninverting Amplifier



Gain, Bandwidth and GB

Effects of GB on closed-loop Amplifiers



$$A_1(s)=\frac{GB}{s+BW_A}$$

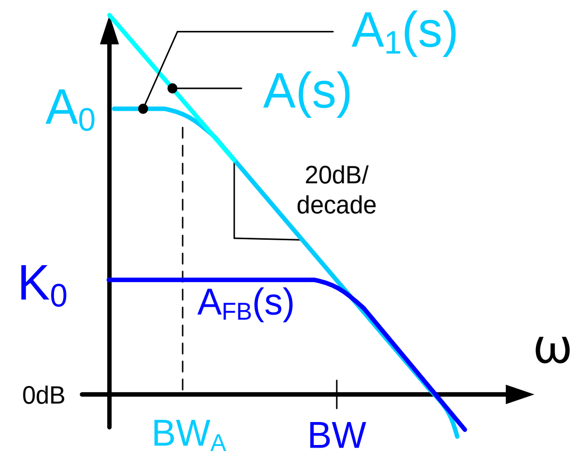
$$GB = A_0 \bullet BW_A$$

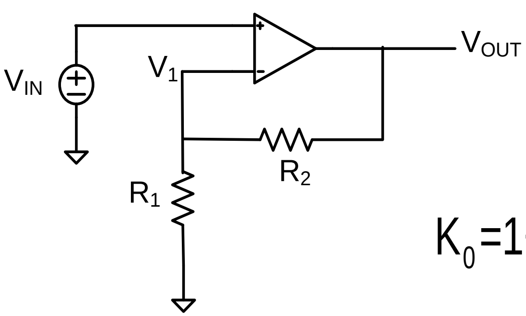
$$A(s)=\frac{GB}{s}$$

Adequate model for most applications

$$A_{FB}(s) \simeq \frac{K_0}{1+s\frac{K_0}{GB}}$$

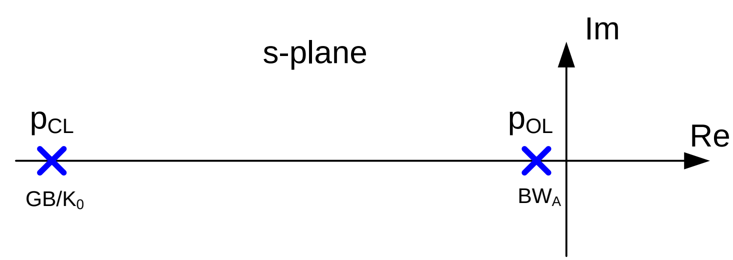
$$BW = \frac{GB}{K_0}$$





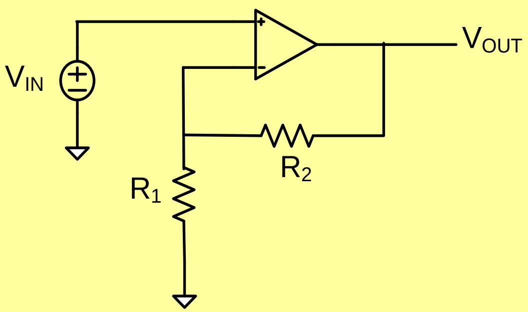
$$K_0=1+\frac{R_2}{R_1}$$

Basic Noninverting Amplifier



Gain, Bandwidth and GB

Summary of Effects of GB on Basic Inverting and Noninverting Amplifiers

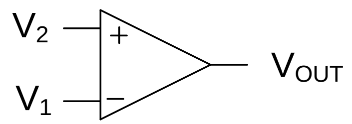


Basic Noninverting Amplifier

$$K_0 = 1 + \frac{R_2}{R_1}$$

$$BW = \frac{GB}{K_0}$$

$$A_{FB}(s) = \frac{K_0}{1 + s\frac{K_0}{GB}}$$

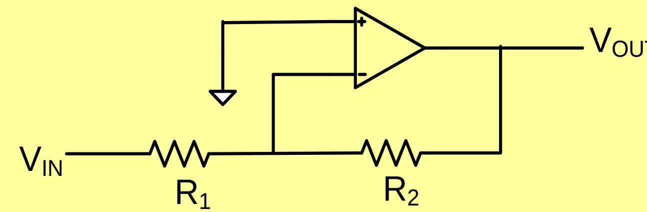


$$A_1(s) = \frac{GB}{s + BW_A}$$

$$GB = A_0 \bullet BW_A$$

$$A(s) = \frac{GB}{s}$$

Adequate model for most applications

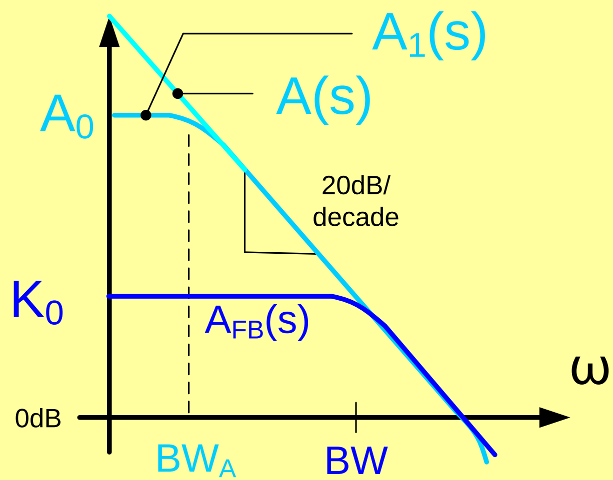


Basic Inverting Amplifier

$$K_0 = \frac{R_2}{R_1}$$

$$BW = \frac{GB}{1 + K_0}$$

$$A_{FB}(s) = -\frac{K_0}{1 + s\frac{(1 + K_0)}{GB}}$$



Stability and Instability

True or False?

An unstable circuit will oscillate

False – unstable circuits will either latch up or oscillate. Latch-up is often the consequence of saturating nonlinearities of circuits that have positive real axis poles

Achieving stability is a major goal of the filter designer

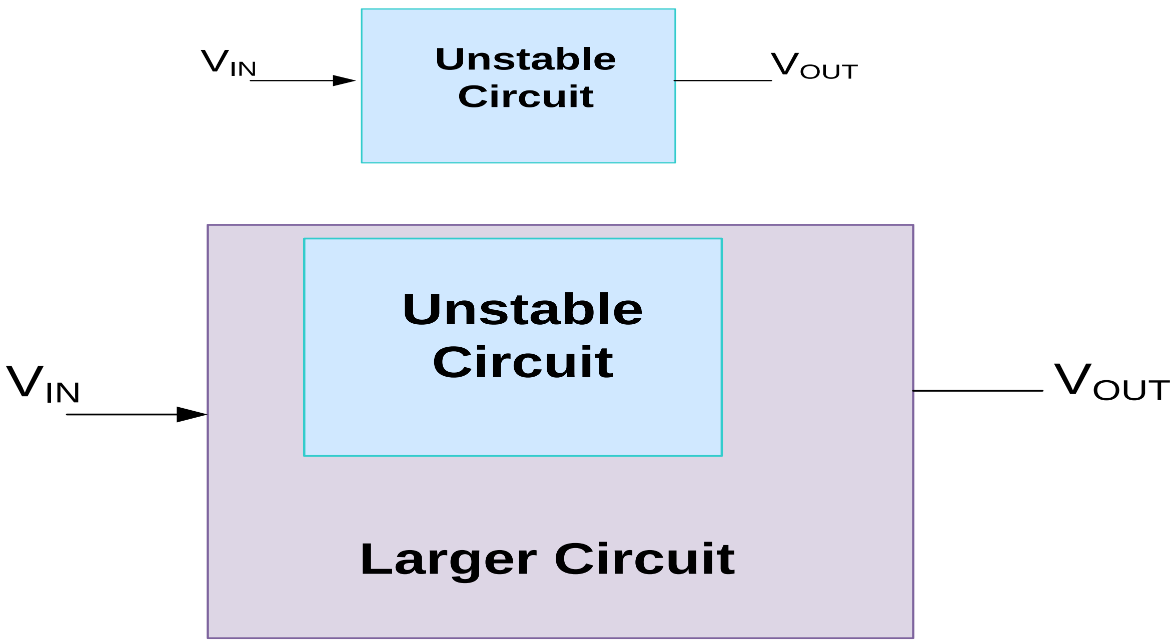
False – a filter is usually of little practical use if there are concerns about stability

Unstable circuits are of little use in designing filters

False – will discuss details later

Theorem:

If a circuit is unstable, then if this circuit is included as a subcircuit in a larger circuit structure, the larger circuit will also be unstable.

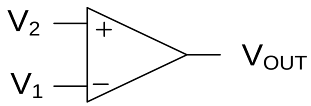


Proof ?:

Consider First Some Related Concepts

Gain, Bandwidth and GB

Consider “positive feedback” closed-loop amplifier

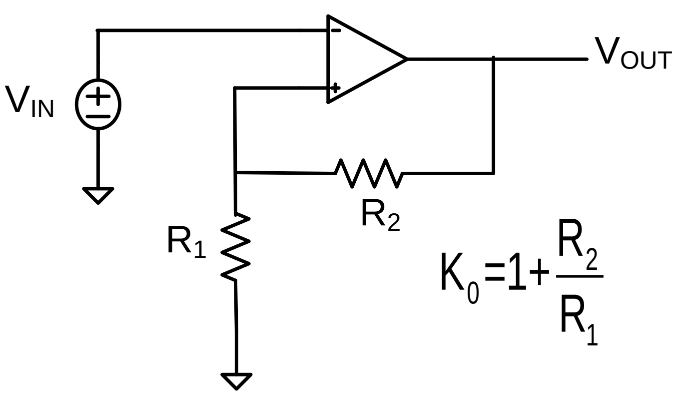


$$A_1(s)=\frac{GB}{s+BW_A}$$

$$GB = A_0 \bullet BW_A$$

$$A(s)=\frac{GB}{s}$$

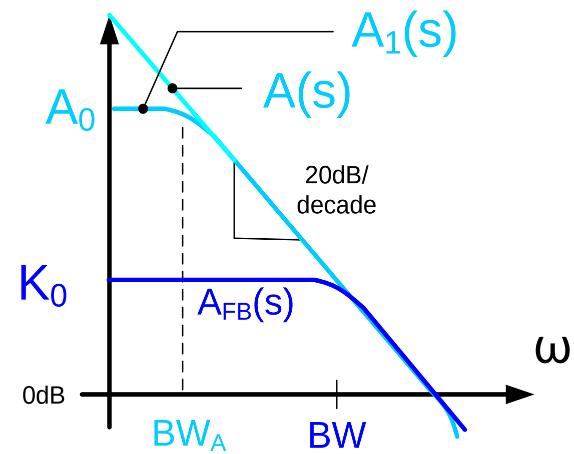
Adequate model for most applications

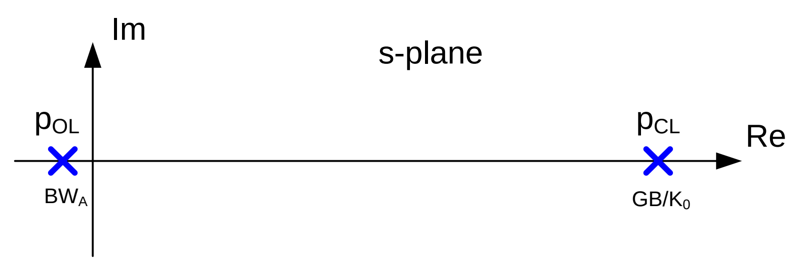


$$K_0=1+\frac{R_2}{R_1}$$

$$A_{FB}(s)\simeq\frac{K_0}{1-s\frac{K_0}{GB}}$$

$$BW=\frac{GB}{K_0}$$

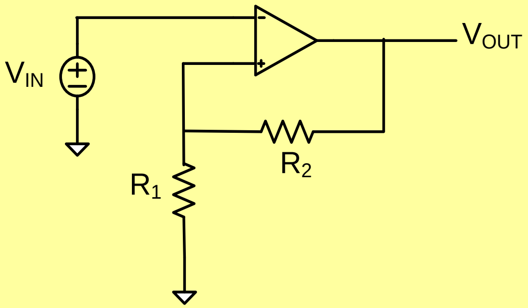




Feedback Amplifier is Unstable !

Gain, Bandwidth and GB

Summary of Effects of GB on Basic Inverting and Noninverting Amplifiers with “Positive Feedback”

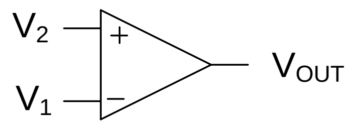


Basic Noninverting Amplifier

$$K_0 = 1 + \frac{R_2}{R_1}$$

$$BW = \frac{GB}{K_0}$$

$$A_{FB}(s) = \frac{K_0}{1 - s\frac{K_0}{GB}}$$

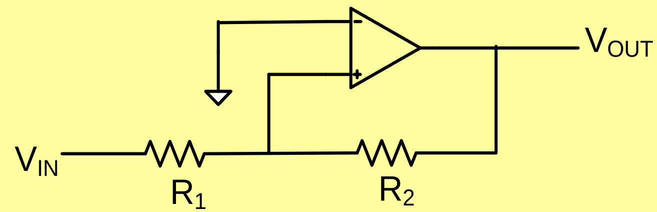


$$A_1(s) = \frac{GB}{s + BW_A}$$

$$GB = A_0 \bullet BW_A$$

$$A(s) = \frac{GB}{s}$$

Adequate model for most applications

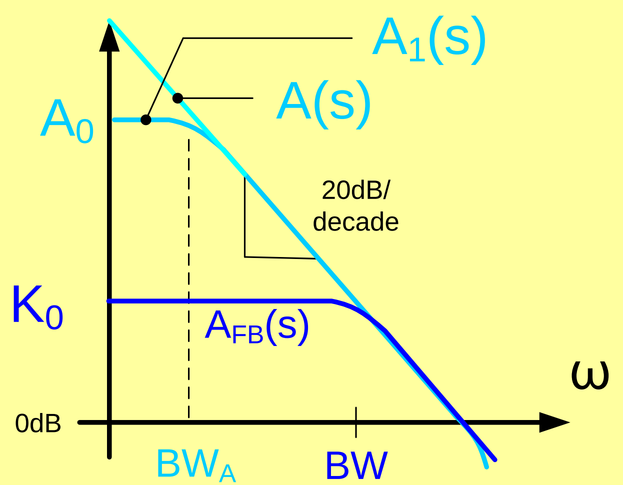


Basic Inverting Amplifier

$$K_0 = \frac{R_2}{R_1}$$

$$BW = \frac{GB}{1 + K_0}$$

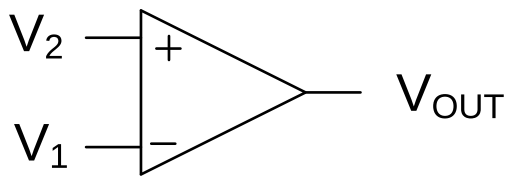
$$A_{FB}(s) = -\frac{K_0}{1 - s\frac{(1+K_0)}{GB}}$$



Both FB Amplifiers are Unstable

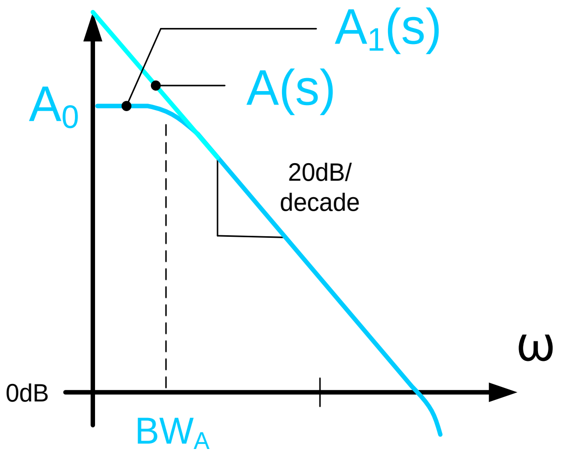
Gain, Bandwidth and GB

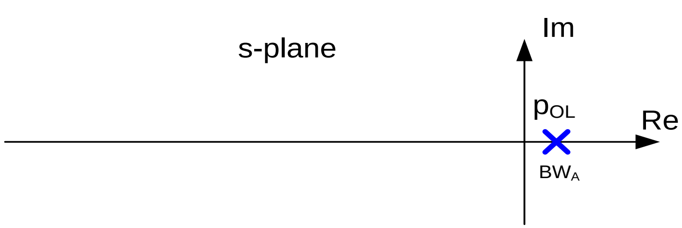
Consider Op Amp with RHP Pole (Unstable Op Amp)



$$A_1(s) = \frac{GB}{s - BW_A}$$

$$|GB| = A_0 \bullet BW_A$$

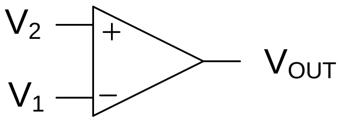




Op Amp is Unstable, dc gain is negative

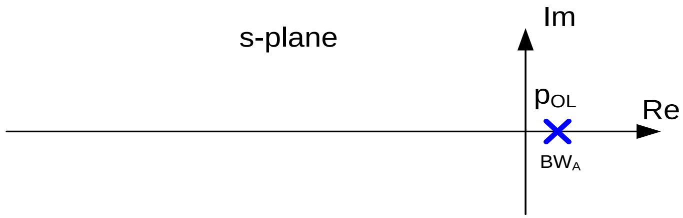
Gain, Bandwidth and GB

Consider Op Amp with RHP Pole (Unstable Op Amp)



$$A_1(s) = \frac{GB}{s - BW_A}$$

$$|GB| = A_0 \bullet BW_A$$



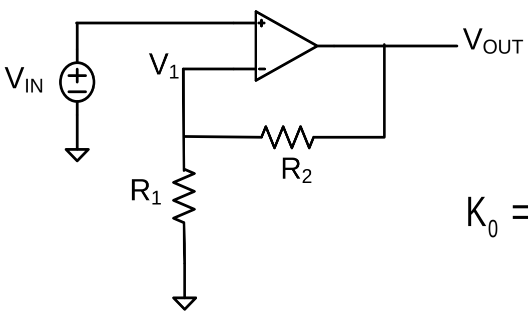
$$V_1 = \frac{V_{OUT}}{K_0}$$

$$V_{OUT} = A(s)(V_1 - V_{IN})$$

$$A(s) = \frac{GB}{s - BW_A}$$

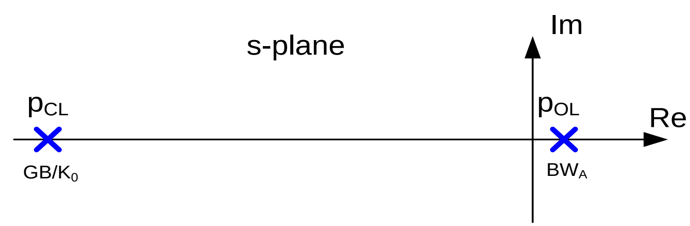
$$A_{FB}(s) = \frac{V_{OUT}}{V_{IN}} = \frac{K_0}{s\frac{K_0}{GB} + \left(1 - K_0\frac{BW_A}{GB}\right)}$$

$$A_{FB}(s) \simeq \frac{K_0}{1 + s\frac{K_0}{GB}} \qquad BW = \frac{GB}{K_0}$$



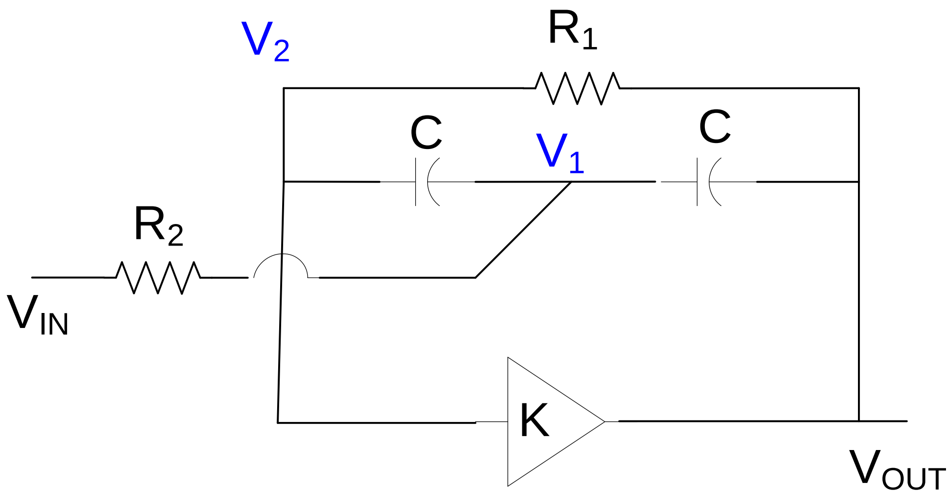
$$K_0 = 1 + \frac{R_2}{R_1}$$

Basic Noninverting Amplifier



- Feedback Amplifier is stable and performs very well!
- Serves as counter-example for “Theorem”!

Consider another Filter Example:



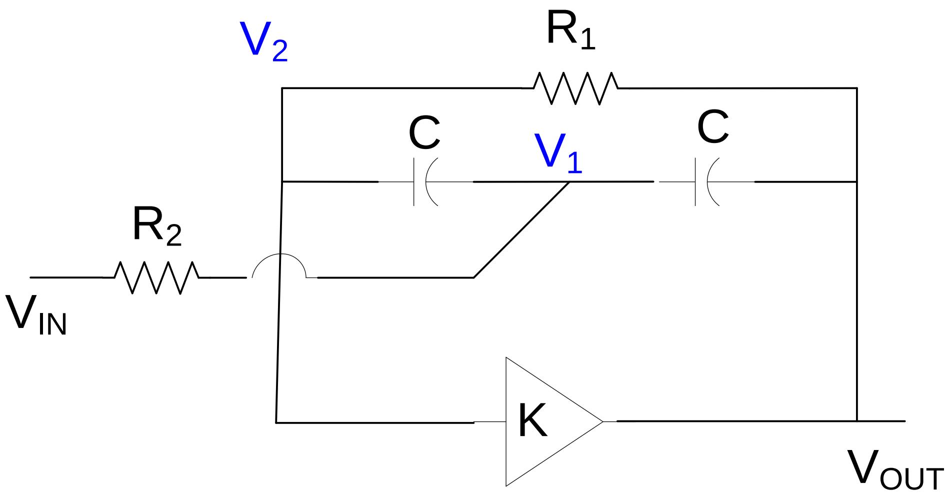
$$V_1\left(sC+sC+G_2\right) = V_{IN}G_2+V_2sC+V_{OUT}sC$$

$$V_2\left(sC+G_1\right) = V_1sC+V_{OUT}G_1$$

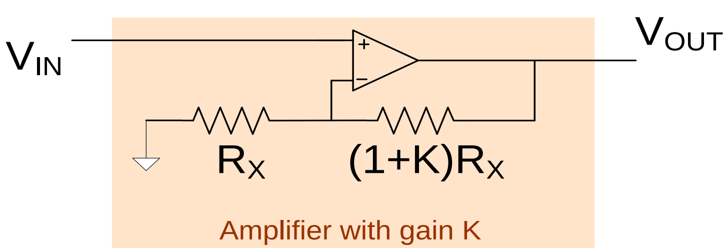
$$V_{OUT} = KV_2$$

$$T(s) = \frac{s\left(\frac{K}{CR_2[1-K]}\right)}{s^2+s\left(\frac{2}{CR_1}-\frac{1}{CR_2[1-K]}\right)+\frac{1}{C^2R_1R_2}}$$

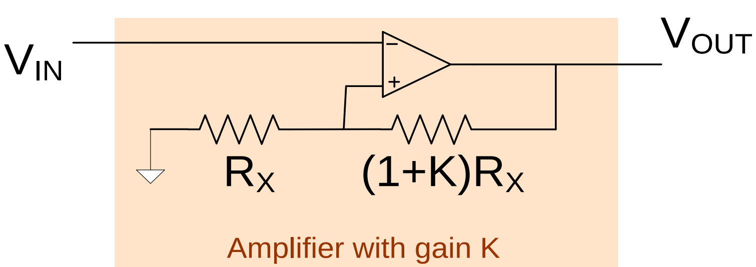
Consider Filter Example:



$$T(s) = \frac{s\left(\frac{K}{CR_2[1-K]}\right)}{s^2+s\left(\frac{2}{CR_1}-\frac{1}{CR_2[1-K]}\right)+\frac{1}{C^2R_1R_2}}$$



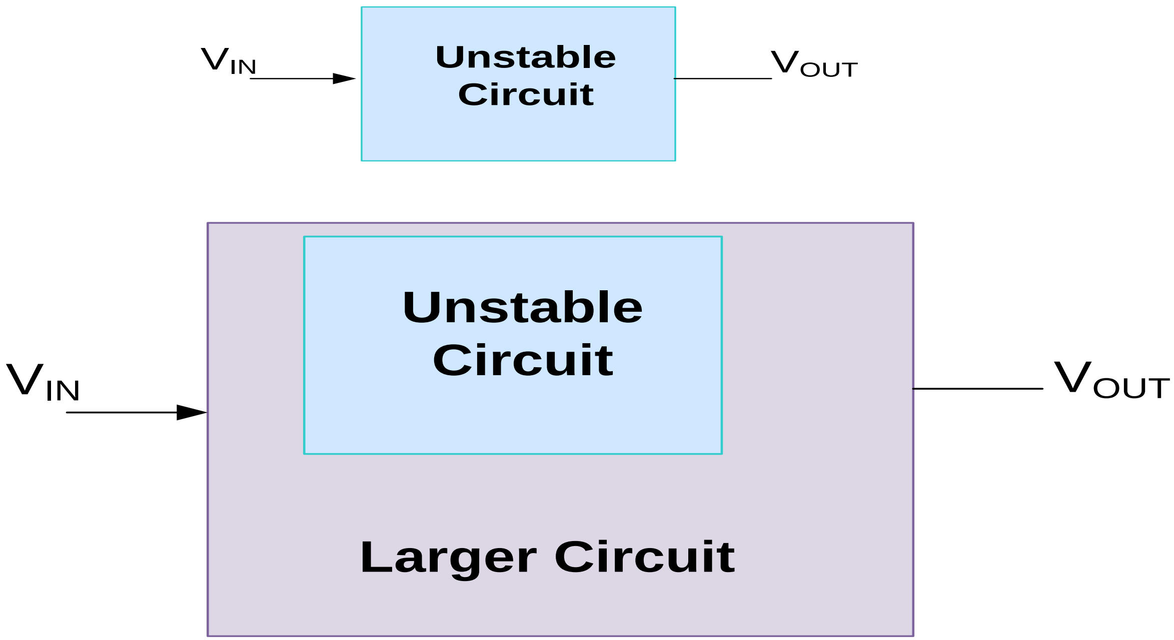
- Stable Amplifier
- But if used in above, filter will be unstable



- Unstable Amplifier
- But if used in above, filter will be stable
- Serves as another counter example for “theorem”

Theorem:

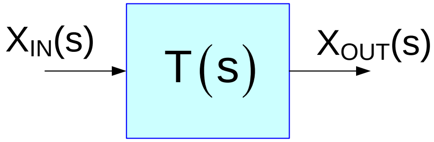
If a circuit is unstable, then if this circuit is included as a subcircuit in a larger circuit structure, the larger circuit will also be unstable.

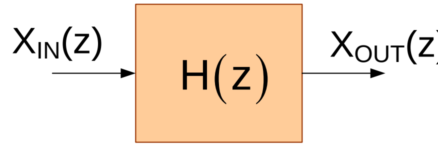


Proof:

This theorem is not valid though many circuit and filter designers believe it to be true !

Filter Concepts and Terminology





Stability Issues:

Is stability or instability good or bad?

Often there is an impression that instability is bad - but why?

Some observations:

- An unstable filter does not behave as a filter
- Unstable filter circuits are often used as waveform generators
- If an unstable circuit is embedded in a larger system, the larger system may be stable or it may be unstable
- If a stable circuit is embedded in a larger system, the larger system may be stable or it may be unstable
- Digital latches, RAMs, etc. are unstable amplifiers
- Some of the best filter circuits include an embedded unstable filter

Stability or Instability is neither good or bad, but it is important for the designer to be aware of the opportunities and limitations associated with this issue

End of Lecture 4

EE 508
Lecture 5

Filter Concepts/Terminology

Basic Properties of Electrical Circuits

Review from Last Time

2-nd order polynomial characterization

s^2+as+b $\{a,b\}$

$s^2+\frac{\omega_o}{Q}s+\omega_0^2$ $\{\omega_o,Q\}$

$s^2+2\zeta\omega_0 s+\omega_0^2$ $\{\zeta, \omega_o\}$

$s^2+(p_1+p_2)s+p_1p_2 = (s+p_1)(s+p_2)$ $\{p_1,p_2\}$

with complex conjugate roots

$s^2+2\alpha s+\alpha^2+\beta^2 = (s+\alpha+j\beta)(s+\alpha-j\beta)$ $\{\alpha,\beta\}$

$s^2+2r\cos(\theta)s+r^2 = (s+re^{j\theta})(s+re^{-j\theta})$ $\{r,\theta\}$

Review from Last Time

Biquadratic Factorization

If n is even,

$$T(s)=\frac{\sum_{i=1}^{m} a_i s^i}{\sum_{i=1}^{n} b_i s^i} = K \bullet \prod_{i=1}^{n/2} T_{BQi}(s)$$

If n is odd,

$$T(s)=\frac{\sum_{i=1}^{m} a_i s^i}{\sum_{i=1}^{n} b_i s^i} = K \bullet \left(\frac{a_{10}s+a_{00}}{s+b_{00}}\right) \bullet \prod_{i=1}^{(n-1)/2} T_{BQi}(s)$$

where $$T_{BQi}(s)=\frac{a_{2i}s^2+a_{1i}s+a_{0i}}{s^2+b_{1i}s+b_{0i}}$$

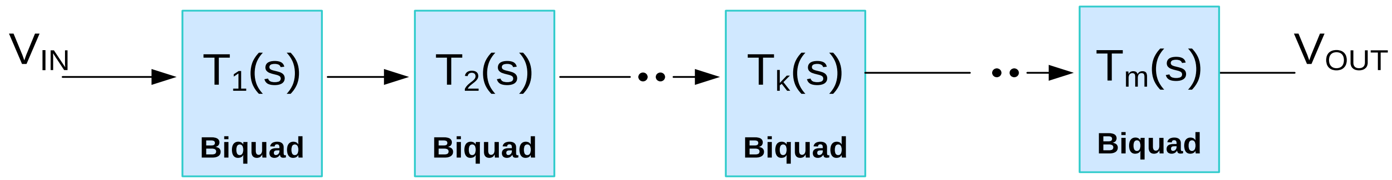
and where K is a real constant and all coefficients are real (some may be 0)

- Factorization is not unique
- H(z) factorizations not restricted to have m<n
- Each biquatratic factor can be represented by any of the 6 alternative parameter sets in the numerator or denominator

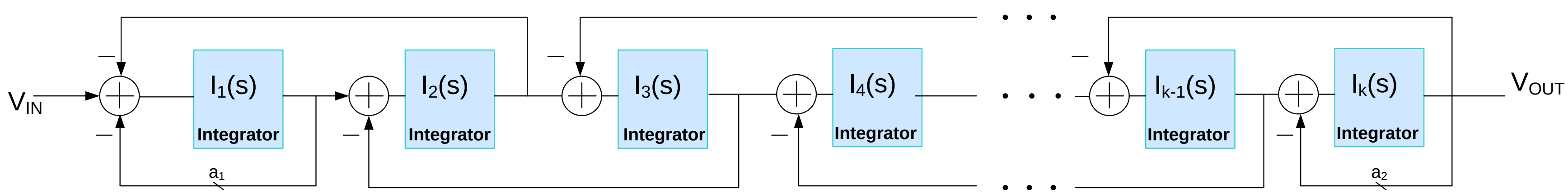
Review from Last Time

Common Filter Architectures

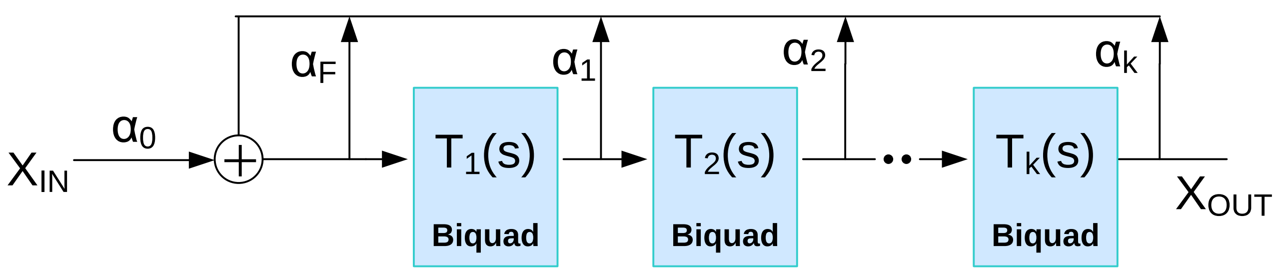
Cascaded Biquads



Leapfrog



Multiple-loop Feedback

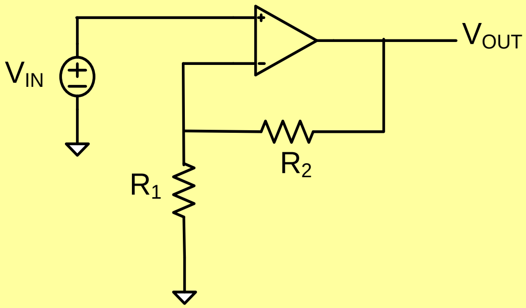


- Three classical filter architectures are shown
- The Cascaded Biquad and the Leapfrog approaches are most common

Review from Last Time

Gain, Bandwidth and GB

Summary of Effects of GB on Basic Inverting and Noninverting Amplifiers

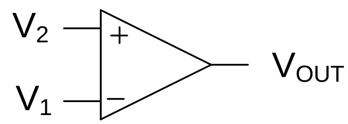


Basic Noninverting Amplifier

$$K_0 = 1 + \frac{R_2}{R_1}$$

$$BW = \frac{GB}{K_0}$$

$$A_{FB}(s) = \frac{K_0}{1 + s\frac{K_0}{GB}}$$

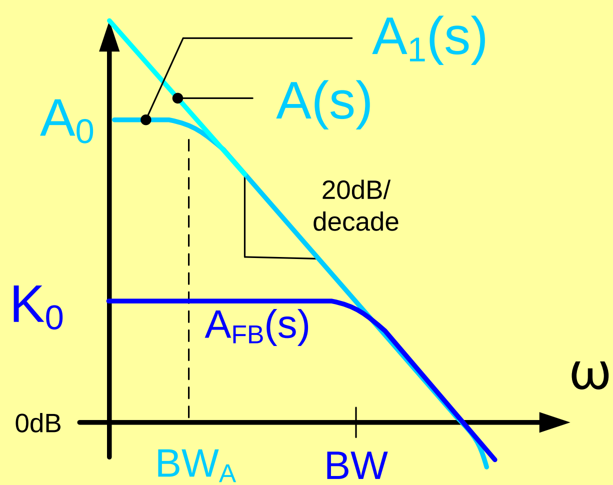


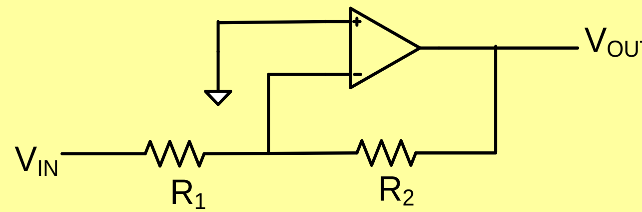
$$A_1(s) = \frac{GB}{s + BW_A}$$

$$GB = A_0 \bullet BW_A$$

$$A(s) = \frac{GB}{s}$$

Adequate model for most applications





Basic Inverting Amplifier

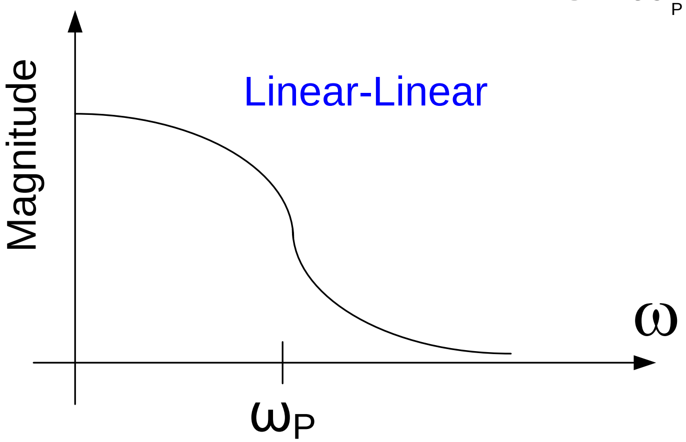
$$K_0 = \frac{R_2}{R_1}$$

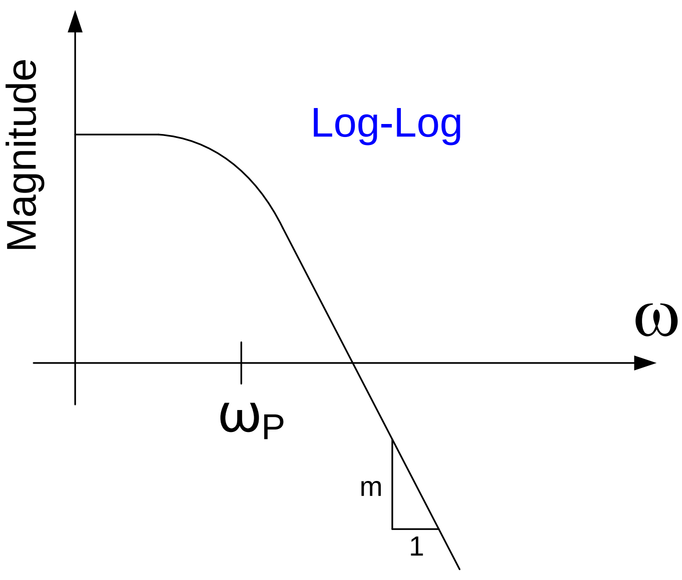
$$BW = \frac{GB}{1 + K_0}$$

$$A_{FB}(s) = -\frac{K_0}{1 + s\frac{(1 + K_0)}{GB}}$$

Single-pole roll-off characterization

Consider: $T(s) = \frac{\omega_P}{s+\omega_P}$





$$T(j\omega) = \frac{\omega_P}{j\omega+\omega_P}$$

$$\left|T(j\omega)\right| = \frac{\omega_P}{\sqrt{\omega^2+\omega_P^2}}$$

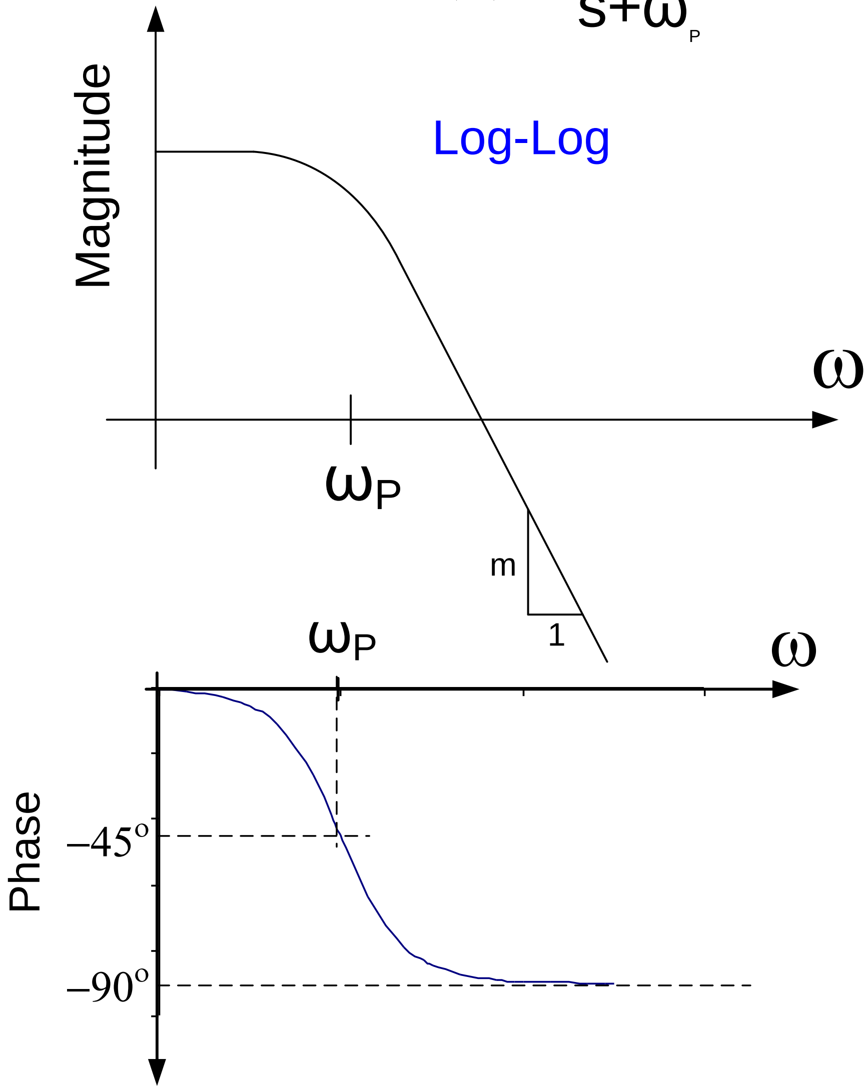
m = - 20dB/decade

m = -6dB/octave

$$\angle T(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_P}\right)$$

Single-pole roll-off characterization

Consider: $T(s) = \frac{\omega_P}{s+\omega_P}$



$$T(j\omega) = \frac{\omega_P}{j\omega+\omega_P}$$

$$\left|T(j\omega)\right| = \frac{\omega_P}{\sqrt{\omega^2+\omega_P^2}}$$

$$\angle T(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_P}\right)$$

Roll-off characterization

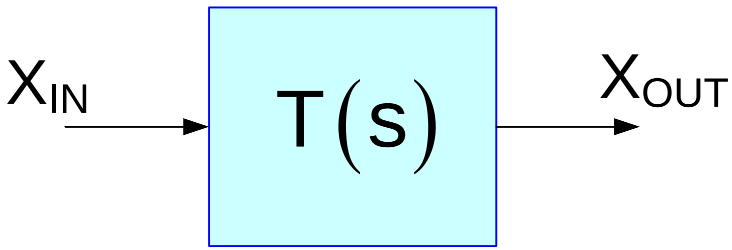
At frequencies well-past a pole or zero, each LHP pole (real or complex) causes a roll-off in magnitude on a log-log axis of -20dB/decade and each LHP zero causes a roll-off of +20dB/decade

At frequencies of magnitude comparable to that of a pole or zero, it is not easy to predict the roll-off in the magnitude characteristics by some simple expression

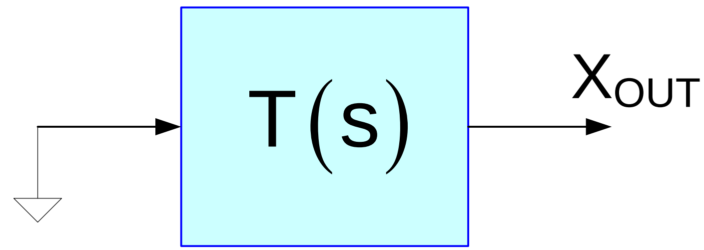
Distortion in Filters

- Magnitude Distortion
  - frequency dependent change in gain of a circuit (usuallybad if building amplifier but critical if building a filter)
- Phase Distortion
  - a circuit has phase distortion if the phase of the transfer function is not linear with frequency
- Nonlinear Distortion
  - Presence of frequency components in the outut that are not present in the input (generally considered bad in filters but necessary in many other circuits)

Dead Networks



$$T(s) = \frac{N(s)}{D(s)}$$



$$D(s)$$

The "dead network" of any linear circuit is obtained by setting ALL independent sources to zero.

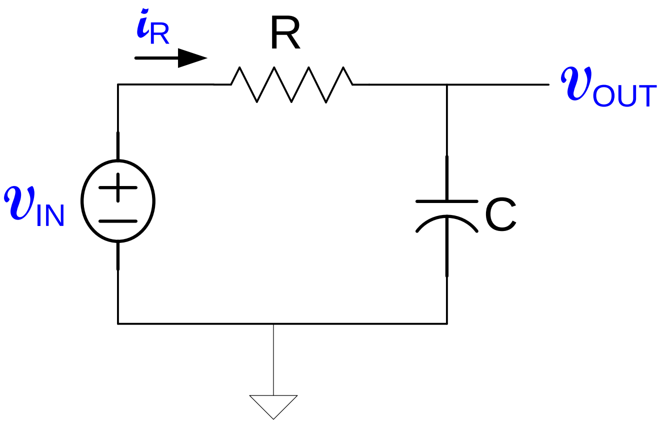
- Replace independent current sources with opens
- Replace independent voltage sources with shorts
- Dependent sources remain intact

D(s) is characteristic of the dead network and is independent of where the excitation is applied or where the response is measured

D(s) is the same for ALL transfer functions of a given "dead network"

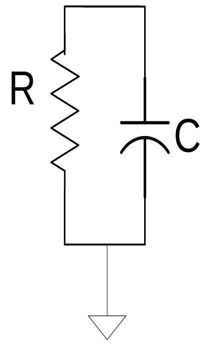
Dead Networks

Example:



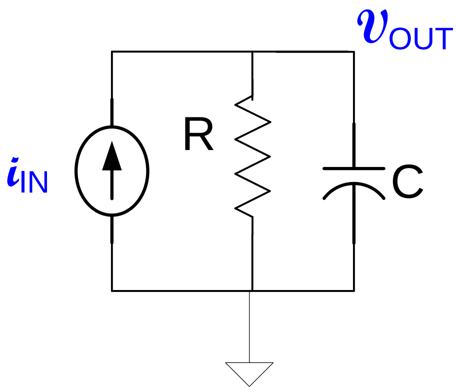
$$T(s) = \frac{1}{1+RCs}$$

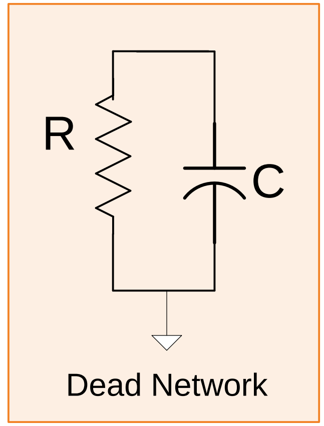
$$D(s) = 1+RCs$$



Dead Network

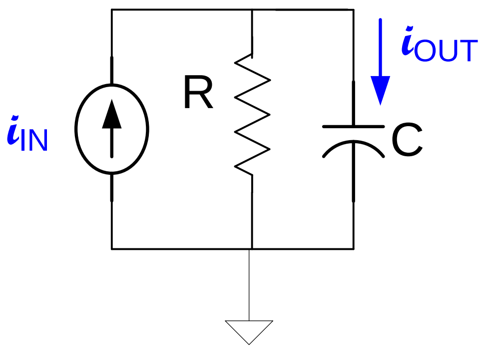
Dead Networks

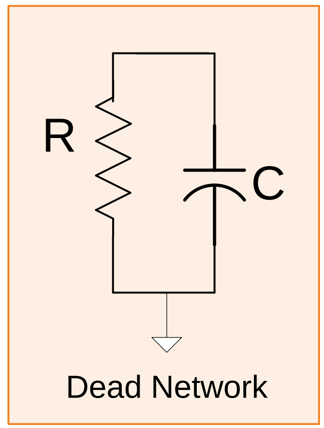




$$\frac{v_{OUT}}{i_{IN}} = T(s) = \frac{R}{1+RCs}$$

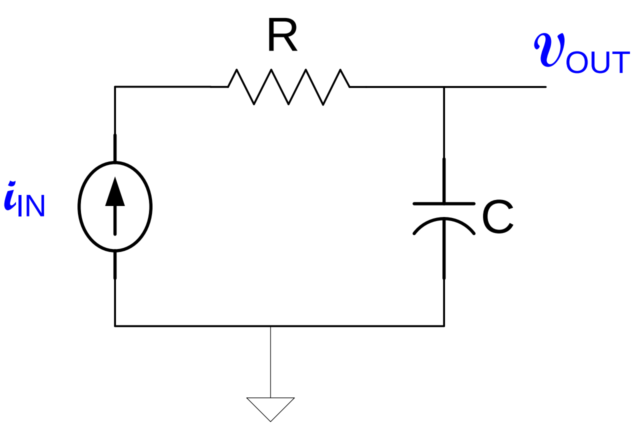
$$D(s) = 1+RCs$$

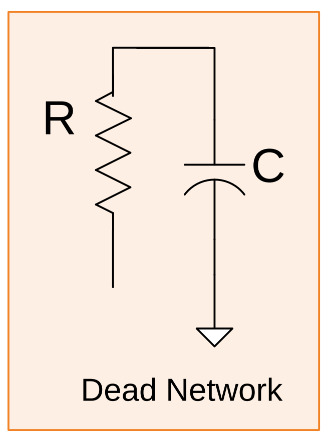




$$\frac{i_{OUT}}{i_{IN}} = T(s) = \frac{RCs}{1+RCs}$$

$$D(s) = 1+RCs$$



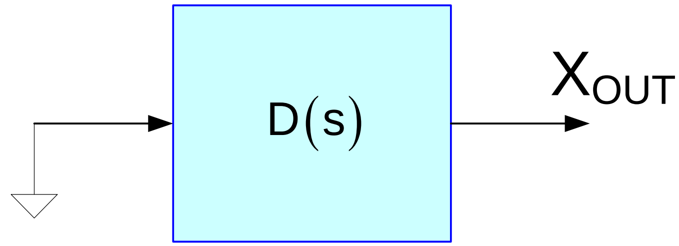


$$\frac{v_{OUT}}{i_{IN}} = T(s) = \frac{1}{Cs}$$

$$D(s) = Cs$$

Note: This has a different dead network!

D(s) is the same for ALL transfer functions of a given "dead network"



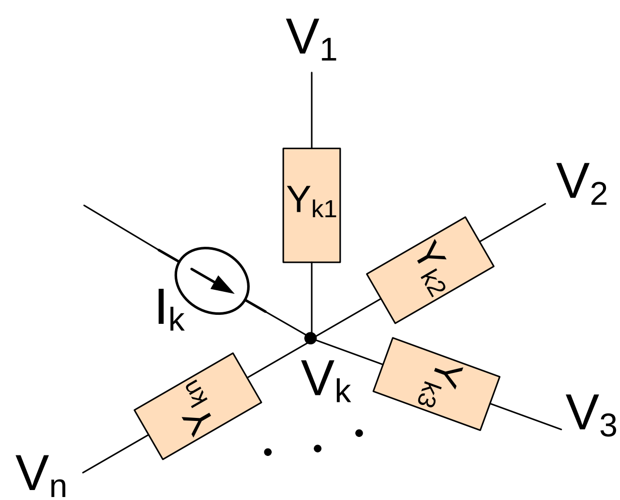
This is an important observation. Why is it true?

Plausibility argument:

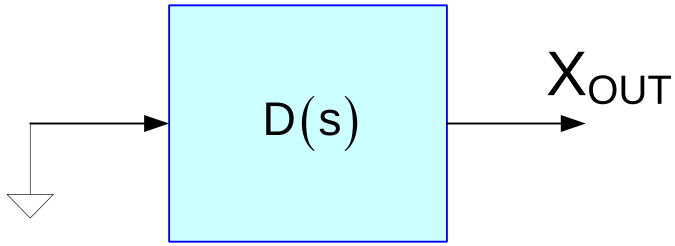
Consider a network with only admittance elements and independent current sources

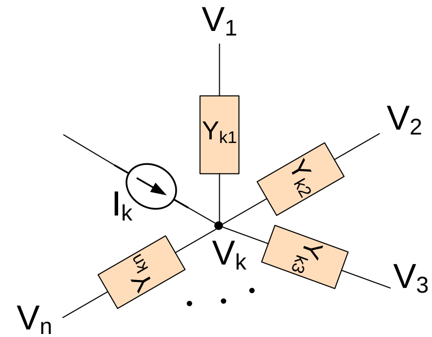
At node k, can write the equation

$$\sum_{\substack{i=1 \\ i \neq k}}^{n} Y_{ki}\left(V_k - V_i\right) = I_k$$



D(s) is the same for ALL transfer functions of a given "dead network"





Plausibility argument:

Doing this at each node results in the set of equations

$$\begin{bmatrix} Y_{11} & Y_{12} & \cdots\cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots\cdots & Y_{2n} \\ . & & & \\ . & & & \\ Y_{n1} & Y_{n2} & \cdots\cdots & Y_{nn} \end{bmatrix} \bullet \begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ . \\ V_n \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ . \\ . \\ . \\ I_n \end{bmatrix}$$

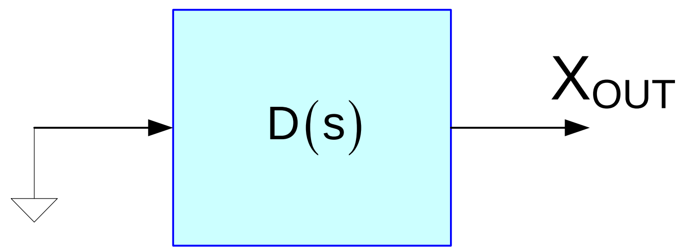
In matrix form

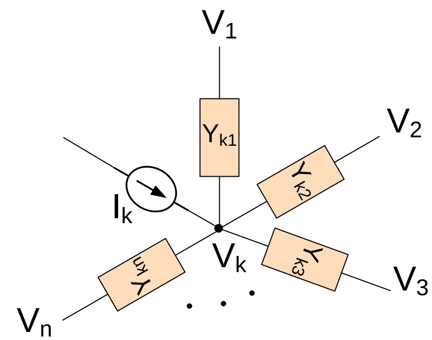
$$\mathbf{Y} \bullet \mathbf{V} = \mathbf{I}$$

The nodal voltages are given by

$$\mathbf{V} = \mathbf{Y}^{-1} \bullet \mathbf{I}$$

D(s) is the same for ALL transfer functions of a given "dead network"





Plausibility argument:

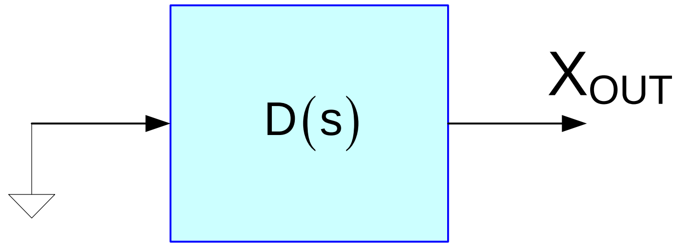
$$\mathbf{V} = \mathbf{Y}^{-1} \bullet \mathbf{I}$$

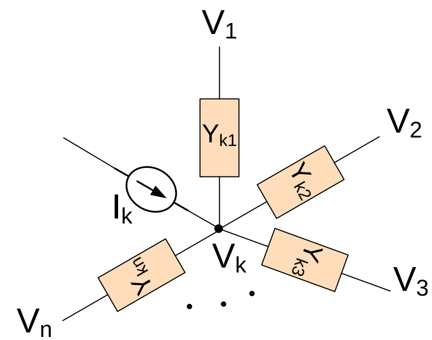
The nodal voltage V_k in this solution is given by the ratio of two determinates where the one in the numerator is obtained by replacing the kth column with the excitation vector and the one in the denominator is the determinate of the indefinite admittance matrix **Y**

$$V_k = \frac{\begin{vmatrix} Y_{11} & Y_{12} & ..I_1. & Y_{1n} \\ Y_{21} & Y_{22} & ..I_2. & Y_{2n} \\ . & & & \\ . & & & \\ Y_{n1} & Y_{n2} & ..I_n. & Y_{nn} \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} & ..... & Y_{1n} \\ Y_{21} & Y_{22} & ..... & Y_{2n} \\ . & & & \\ . & & & \\ Y_{n1} & Y_{n2} & ..... & Y_{nn} \end{vmatrix}}$$

Note the denominator is the same for all nodal voltages and is independent of the excitations – that is, it is dependent only upon the dead network

D(s) is the same for ALL transfer functions of a given “dead network”





Plausibility argument:

Note the denominator is the same for all nodal voltages and is independent of the excitations – that is, it is dependent only upon the dead network

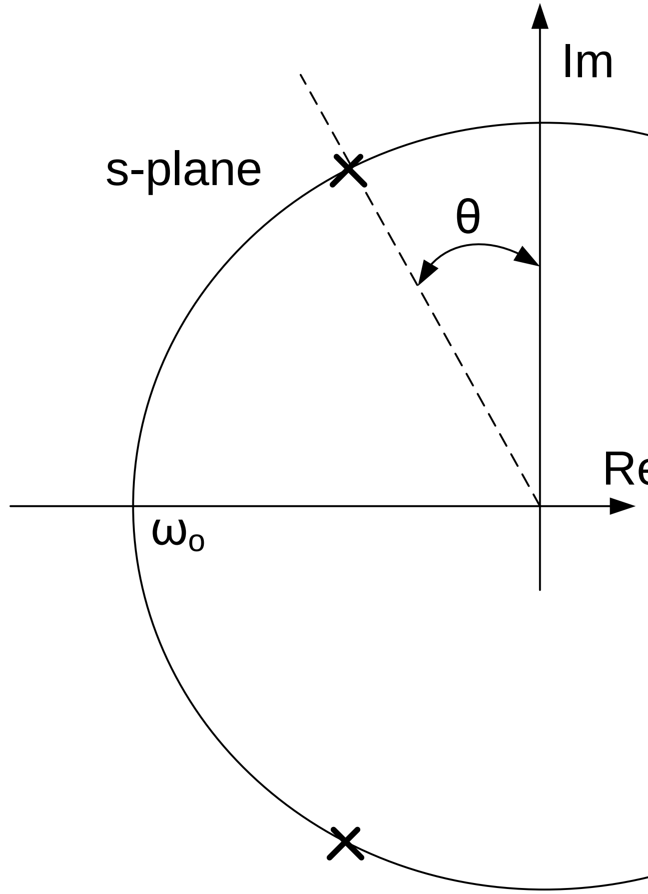
Thus all branch voltages and all branch currents have the same denominator and this (after multiplying through by the correct power of s to make V_k a rational fraction) is the characteristic polynomial D(s)

This concept can be extended to include independent voltage sources as well as dependent sources

$$V_k = \frac{\begin{vmatrix} Y_{11} & Y_{12} & ..I_1. & Y_{1n} \\ Y_{21} & Y_{22} & ..I_2. & Y_{2n} \\ . & & & \\ . & & & \\ Y_{n1} & Y_{n2} & ..I_n. & Y_{nn} \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} & ..... & Y_{1n} \\ Y_{21} & Y_{22} & ..... & Y_{2n} \\ . & & & \\ . & & & \\ Y_{n1} & Y_{n2} & ..... & Y_{nn} \end{vmatrix}}$$

Root characterization in s-plane

(for complex-conjugate roots)

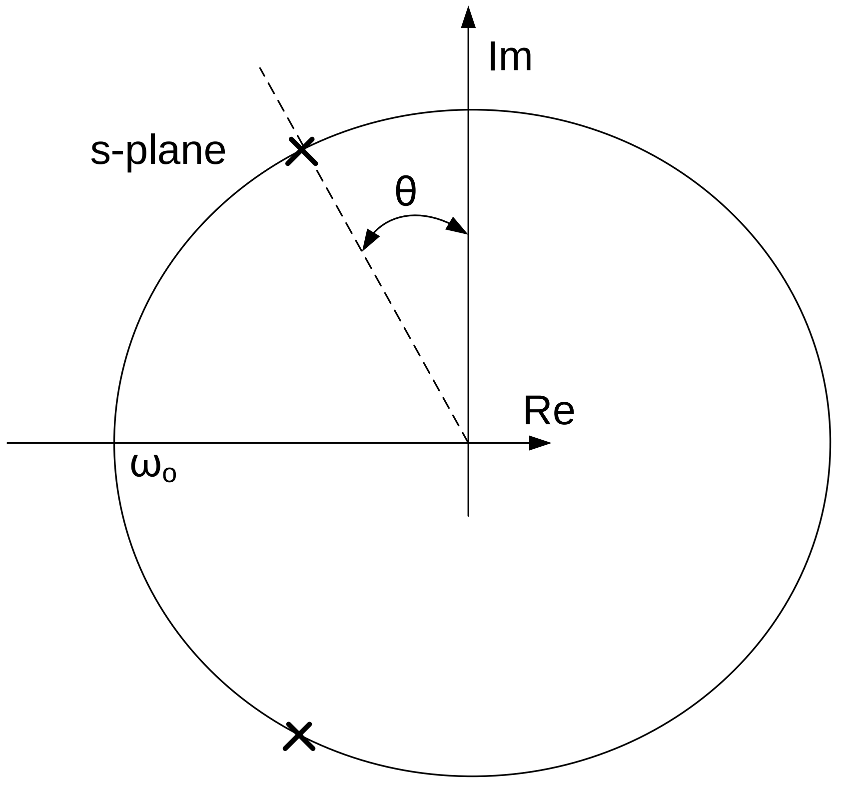


$$s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2$$

1-1 relationship between angle θ and Q of root

For low Q, θ is large
For high Q, θ is small

Root characterization in s-plane
(for complex-conjugate roots)



$$s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2$$

for θ=90°, Q=1/√2

roots located at $$s = -\frac{\omega_0}{2Q} \pm \frac{1}{2}\sqrt{\left(\frac{\omega_0}{Q}\right)^2 - 4\omega_0^2} = \omega_0\left(-\frac{1}{2Q} \pm \frac{1}{2}\sqrt{\left(\frac{1}{Q}\right)^2 - 4}\right)$$

$$\theta = \tan^{-1}\left(4Q^2 - 1\right)$$

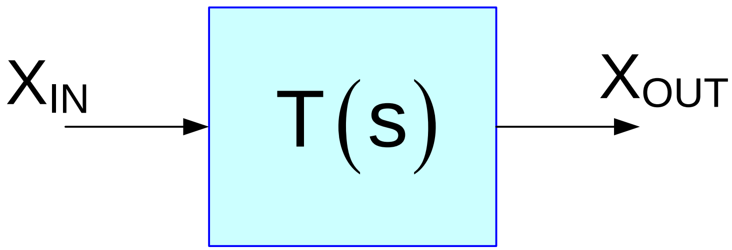
End of Lecture 5

EE 508
Lecture 6

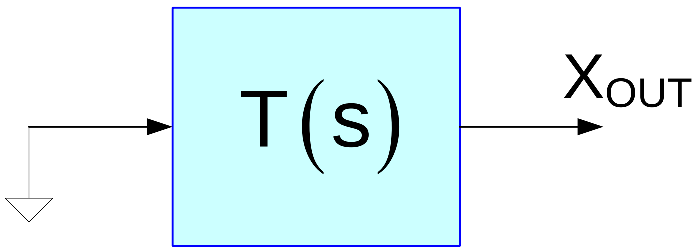
Scaling, Normalization and Transformation

Review from Last Time

Dead Networks



$$T(s) = \frac{N(s)}{D(s)}$$



D(s)

The “dead network” of any linear circuit is obtained by setting ALL independent sources to zero.

- Replace independent current sources with opens
- Replace independent voltage sources with shorts
- Dependent sources remain intact

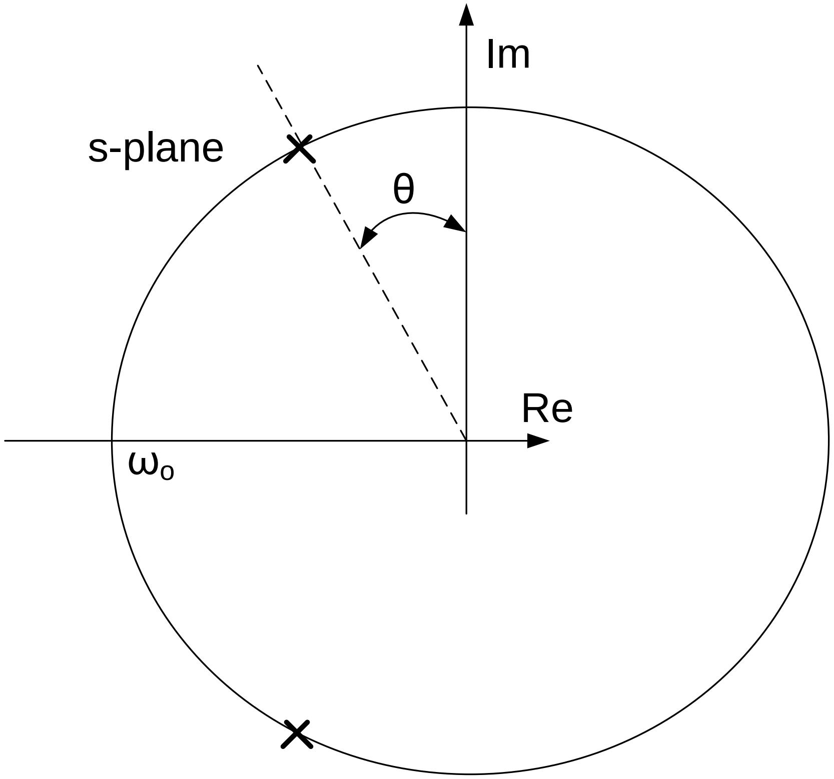
D(s) is characteristic of the dead network and is independent of where the excitation is applied or where the response is measured

D(s) is the same for ALL transfer functions of a given “dead network”

Review from Last Time

Root characterization in s-plane

(for complex-conjugate roots)



$$s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2$$

for θ=90°, Q=1/√2

roots located at

$$s = -\frac{\omega_0}{2Q} \pm \frac{1}{2}\sqrt{\left(\frac{\omega_0}{Q}\right)^2 - 4\omega_0^2} = \omega_0\left(-\frac{1}{2Q} \pm \frac{1}{2}\sqrt{\left(\frac{1}{Q}\right)^2 - 4}\right)$$

$$\theta = \tan^{-1}\left(4Q^2 - 1\right)$$

Scaling, Normalization and Transformations

- Frequency scaling
- Frequency Normalization
- Impedance scaling
- Transformations
  - LP to BP
  - LP to HP
  - LP to BR

Scaling, Normalization and Transformations

Frequency normalization: $$s_n = \frac{s}{\omega_0}$$

Frequency scaling: $$s = \omega_0 s_n$$

Purpose:

ω_0 independent approximations

ω_0 independent synthesis

Simplifies analytical expressions for T(s)

Simplifies component values in synthesis

Use single table of normalized filter circuits for given normalized approximating function

Note: The normalization subscript “n” is often dropped

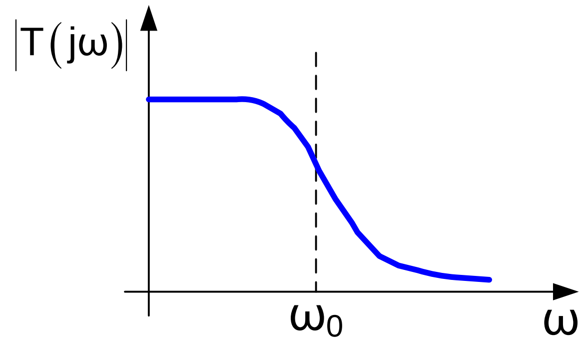
Frequency normalization/scaling example

$$T(s) = \frac{6000}{s + 6000}$$

Define ω_0=6000

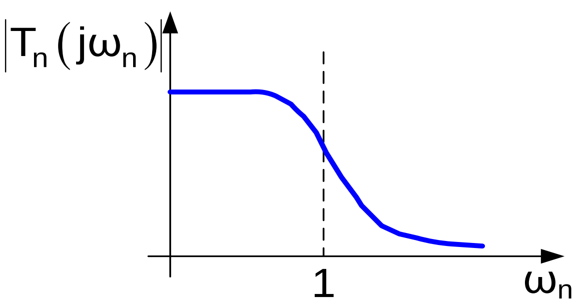
$$s_n = \frac{s}{\omega_0}$$

$$T(s) = \frac{\omega_0}{s + \omega_0}$$



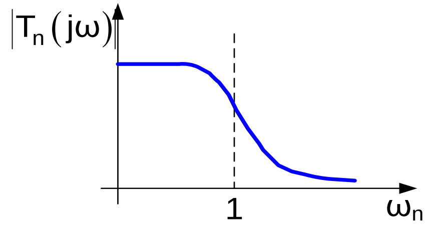
Normalized transfer function:

$$T_n(s_n) = \frac{1}{s_n + 1}$$

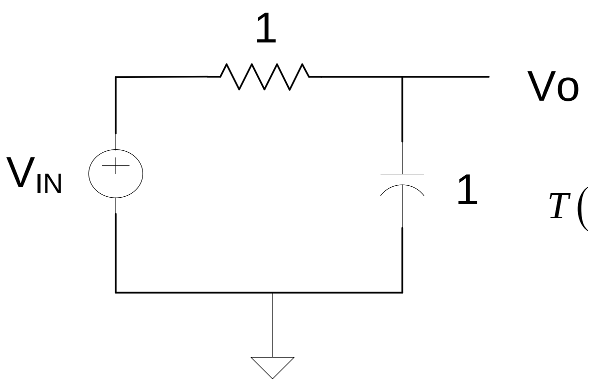


Frequency normalization/scaling example

$$T_n(s_n) = \frac{1}{s_n + 1}$$



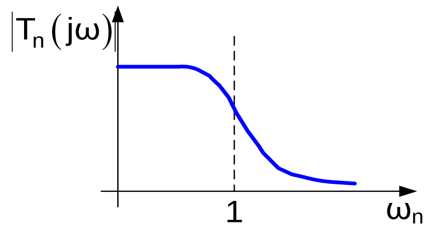
Synthesis of normalized function



$$T(s) = \frac{1}{s+1}$$

Frequency normalization/scaling example

$$T_n(s_n) = \frac{1}{s_n + 1}$$



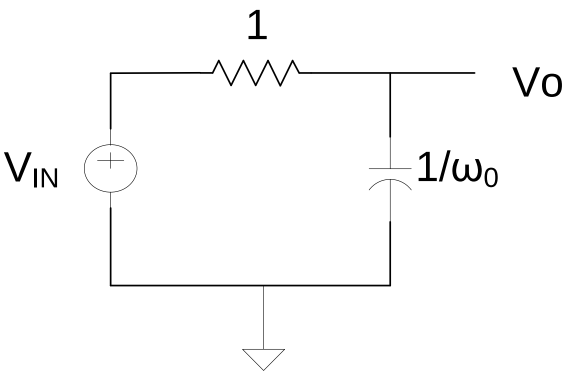
Frequency scaling by ω_0 (of transfer function)

$$s = \omega_0 s_n$$

$$T(s) = \frac{1}{\left(\frac{s}{\omega_0}\right) + 1} \rightarrow T(s) = \frac{\omega_0}{s + \omega_0}$$

Frequency scaling by ω_0 (actually magnitude of ω_0) (scale all energy storage elements in circuit)

$$C = C_n / \omega_0$$

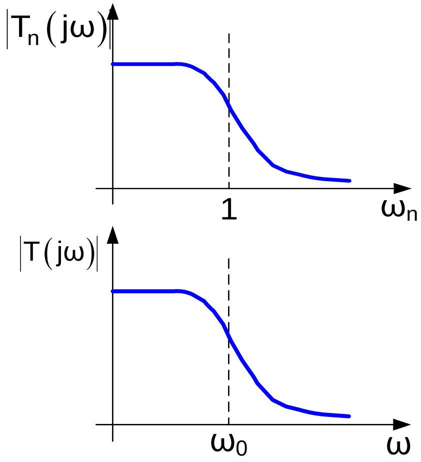


$$\rightarrow T(s) = \frac{\omega_0}{s + \omega_0}$$

Frequency normalization/scaling example

$$T_n(s_n) = \frac{1}{s_n + 1}$$

$$T(s) = \frac{\omega_0}{s + \omega_0}$$



Frequency scaling / normalization does not change the shape of the transfer function, it only scales the frequency axis linearly

The frequency scaled circuit can be obtained from the normalized circuit simply by scaling the frequency dependent impedances (up or down) by the scaling factor

This makes the use of filter design tables for the design of lowpass filters practical whereby the circuits in the table all have a normalized band edge of 1 rad/sec.

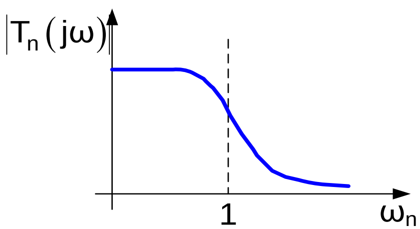
Anatol I. Zverev

Handbook of FILTER SYNTHESIS

Frequency normalization/scaling

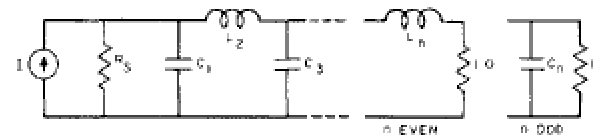
Example: Table for passive LC ladder Butterworth filter with 3dB band edge of 1 rad/sec and resistive source/load terminations

$$T_n(s_n) = \frac{1}{s_n + 1}$$



LOW PASS ELEMENT VALUES

BUTTERWORTH RESPONSE

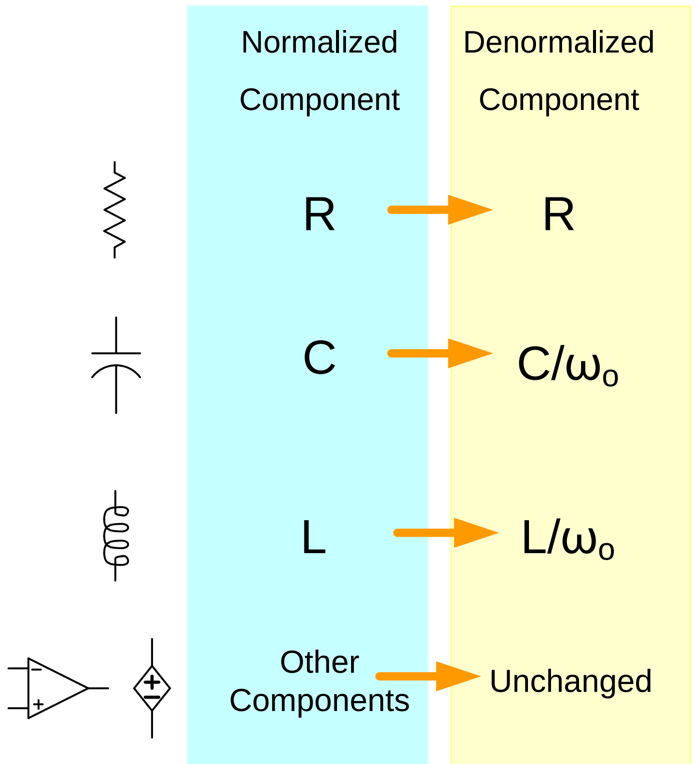


n	R_s	C_1	L_2	C_3	L_4
2	1.0000	1.4142	1.4142		
	1.1111	1.0353	1.9352		
	1.2500	0.8485	2.1213		
	1.4286	0.6971	2.4387		
	1.6667	0.5657	2.8284		
	2.0000	0.4483	3.3461		
	2.5000	0.3419	4.0951		
	3.3333	0.2447	5.3126		
	5.0000	0.1557	7.7067		
	10.0000	0.0743	14.8138		
	INF.	1.4142	0.7071		
3	1.0000	1.0000	2.0000	1.0000	
	0.9000	0.8082	1.6332	1.5994	
	0.8000	0.8442	1.3840	1.9259	
	0.7000	0.9152	1.1652	2.2774	
	0.6000	1.0225	0.9650	2.7024	
	0.5000	1.1811	0.7789	3.2612	
	0.4000	1.4254	0.6062	4.0642	
	0.3000	1.8380	0.4396	5.3634	
	0.2000	2.6687	0.2842	7.9102	
	0.1000	5.1672	0.1377	15.4554	
	INF.	1.5000	1.3333	0.5000	
4	1.0000	0.7654	1.8478	1.8478	0.7654
	1.1111	0.4657	1.5924	1.7439	1.4690
	1.2500	0.3983	1.6946	1.5110	1.8109
	1.4286	0.3251	1.8615	1.2913	2.1752
	1.6667	0.2693	2.1029	1.0824	2.6131
	2.0000	0.2175	2.4524	0.8826	3.1868
	2.5000	0.1692	2.9858	0.6911	4.0094
	3.3333	0.1237	3.8826	0.5072	5.3381
	5.0000	0.0804	5.6835	0.3307	7.9397
	10.0000	0.0392	11.0942	0.1616	15.6421
	INF.	1.5307	1.5772	1.0824	0.3827
n	$1/R_s$	L_1	C_2	L_3	C_4

Frequency normalization/scaling

The frequency scaled circuit can be obtained from the normalized circuit simply by scaling the frequency dependent impedances (up or down) by the scaling factor

Component denormalization by factor of ω_0



Component values of energy storage elements are scaled down by a factor of ω_0

Desgin Strategy

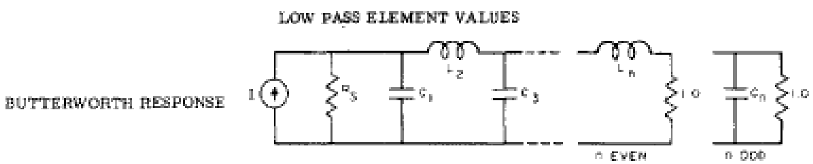
Theorem: A circuit with transfer function T(s) can be obtained from a circuit with normalized transfer function $T_n(s_n)$ by denormalizing all frequency dependent components.

$$C \rightarrow C/\omega_o$$

$$L \rightarrow L/\omega_o$$

Example: Design a V-V passive 3rd-order Lowpass Butterworth filter with a 3-db band-edge of 1K rad/sec and equal source and load terminations.

(from the BW approximation which will be discussed later:) $T(s)=\frac{1}{s^3+2s^2+2s+1}$



n	R_s	C_1	L_2	C_3	L_4
2	1.0000	1.4142	1.4142		
	1.1111	1.0353	1.4352		
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	2.0000	0.4483	3.3461		
	2.5000	0.3419	4.0951		
	3.3333	0.2447	5.3126		
	5.0000	0.1557	7.7067		
	10.0000	0.0743	14.8138		
	INF.	1.4142	0.7071		
3	1.0000	1.0000	2.0000	1.0000	
	[illegible]	[illegible]	[illegible]	[illegible]	
	0.8000	0.8442	1.3840	1.9259	
	0.7000	0.9152	1.1652	2.2774	
	0.6000	1.0225	0.9650	2.7024	
	0.5000	1.1811	0.7789	3.2612	
	0.4000	1.4254	0.6042	4.0642	
	0.3000	1.8380	0.4396	5.3634	
	0.2000	2.6687	0.2862	7.9102	
	0.1000	5.1672	0.1377	15.4554	
	INF.	1.5000	1.3333	0.5000	
4	1.0000	0.7654	1.8478	1.8478	0.7654
	1.1111	0.4657	1.5924	1.7439	1.4690
	1.2500	0.3982	1.6946	1.5110	1.8109
	1.4286	0.3251	1.8618	1.2913	2.1752
	1.6667	0.2693	2.1029	1.0824	2.6131
	2.0000	0.2175	2.4524	0.8826	3.1868
	2.5000	0.1692	2.9858	0.6911	4.0094
	3.3333	0.1237	3.8826	0.5072	5.1381
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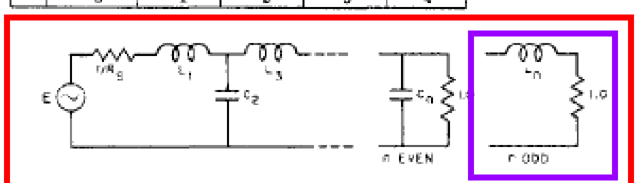
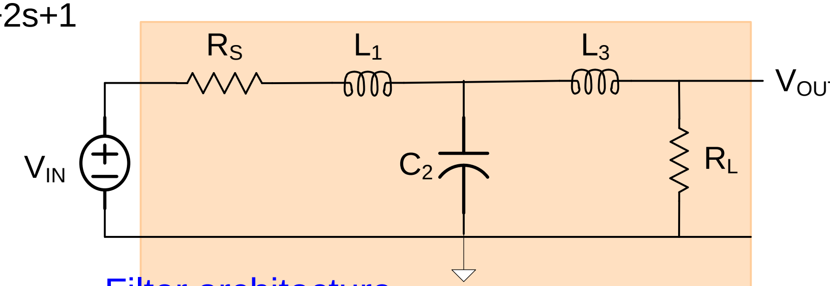
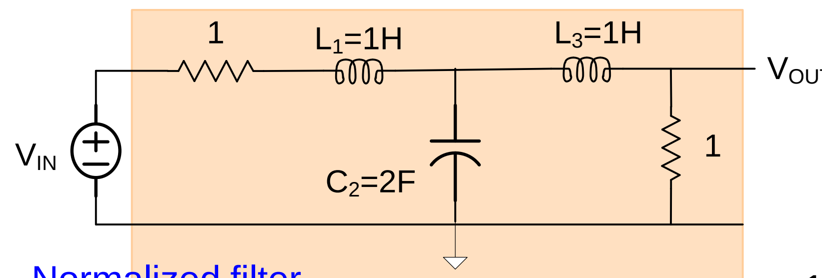
	10.0000	0.0392	11.0942	0.1616	15.6421
	INF.	1.5307	1.5772	1.0824	0.3827
n	$1/R_s$	L_1	C_2	L_3	C_4





Filter architecture

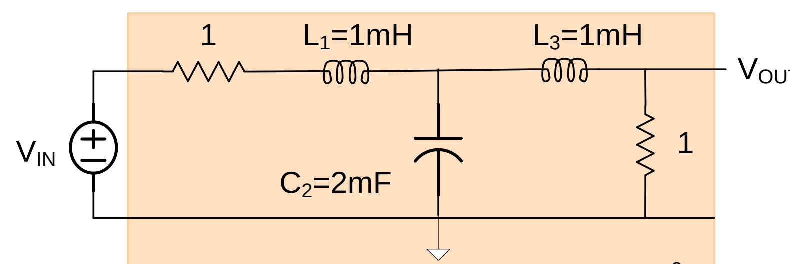


Normalized filter

$T(s)=K\frac{1}{s^3+2s^2+2s+1}$

$C \rightarrow C/\theta$

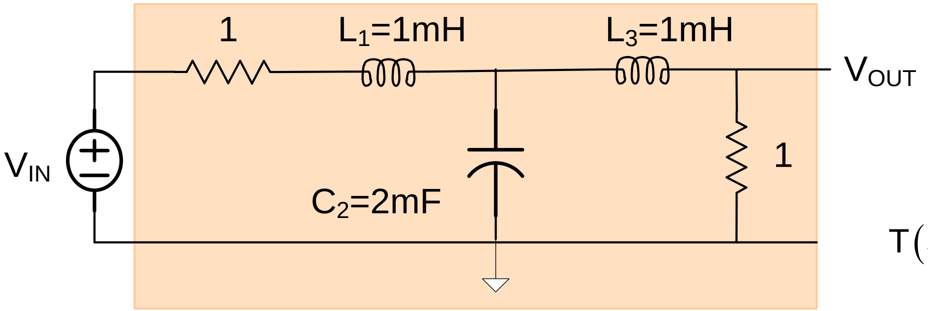
$L \rightarrow L/\theta$



Denormalized filter

$T(s)=K\frac{10^9}{s^3+2\bullet10^3s^2+2\bullet10^6s+10^9}$

Example: Design a V-V passive 3rd-order Lowpass Butterworth filter with a band-edge of 1K Rad/Sec and equal source and load terminations.



$$T(s)=K\frac{10^9}{s^3+2\bullet 10^3 s^2+2\bullet 10^6 s+10^9}$$

Is this solution practical?

Some component values are too big and some are too small !

Filter Concepts and Terminology

- Frequency scaling
- Frequency Normalization
- Impedance scaling
- Transformations
  - LP to BP
  - LP to HP
  - LP to BR

Impedance Scaling

Impedance scaling of a circuit is achieved by multiplying ALL impedances in the circuit by a constant

$R \rightarrow \theta R$

$C \rightarrow C/\theta$

$L \rightarrow L\theta$

$A \rightarrow$
- θA for transresistance gain
- A for dimensionless gain
- A/θ for transconductance gain

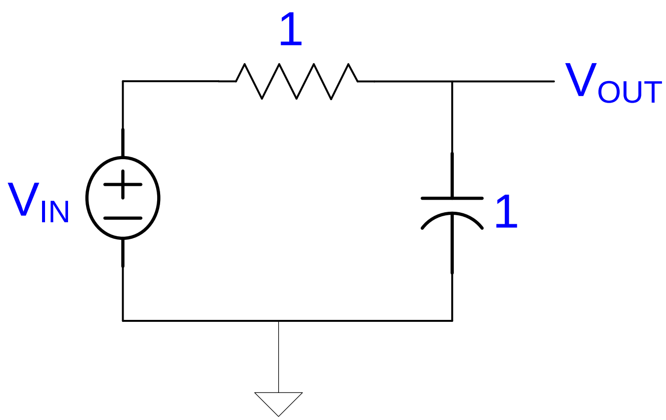
Impedance Scaling

Theorem: If all impedances in a circuit are scaled by a constant θ, then

a) All dimensionless transfer functions are unchanged
b) All transresistance transfer functions are scaled by θ
c) All transconductance transfer functions are scaled by θ^{-1}

Impedance Scaling

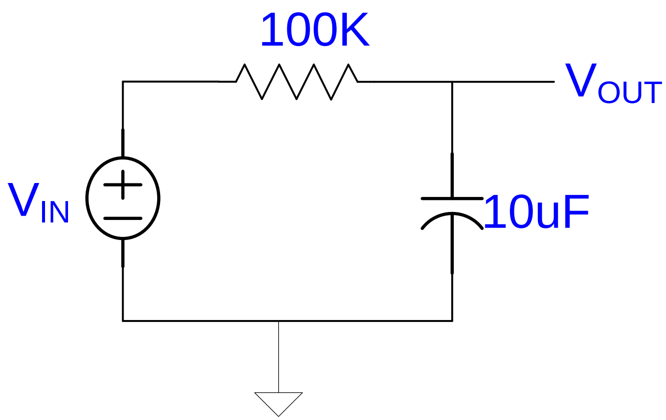
Example:



$$T(s) = \frac{1}{s+1}$$

T(s) is dimensionless

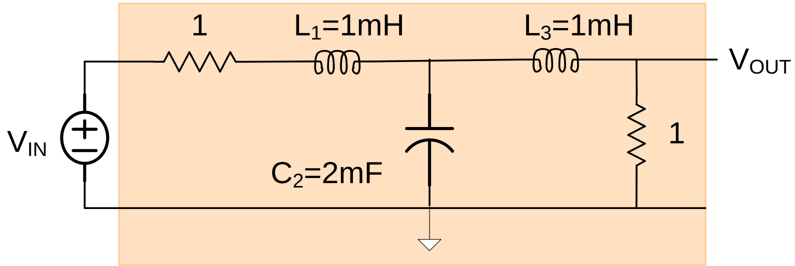
Impedances scaled by $\theta = 10^5$



$$T(s) = \frac{1}{s+1}$$

Note second circuit much more practical than the first

Example: Design a V-V passive 3rd-order Lowpass Butterworth filter with a band-edge of 1K Rad/Sec and equal source and load terminations.



$$T(s)=K\frac{10^9}{s^3+2\bullet10^3s^2+2\bullet10^6s+10^9}$$

Is this solution practical?

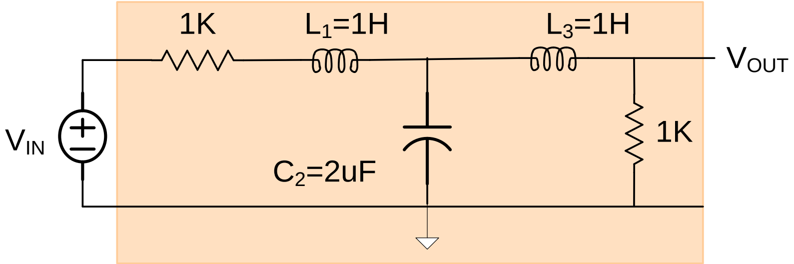
Some component values are too big and some are too small !

Impedance scale by θ=1000

$R \rightarrow \theta R$

$C \rightarrow C/\theta$

$L \rightarrow \theta L$



$$T(s)=K\frac{10^9}{s^3+2\bullet10^3s^2+2\bullet10^6s+10^9}$$

Component values more practical

Typical approach to lowpass filter design

1. Obtain normalized approximating function
2. Synthesize circuit to realize normalized approximating function
3. Denormalize circuit obtained in step 2
4. Impedance scale to obtain acceptable component values

End of Lecture 6

EE 508
Lecture 7

Degrees of Freedom

The Approximation Problem

Review from Last Time

Desgin Strategy

Theorem: A circuit with transfer function T(s) can be obtained from a circuit with normalized transfer function $T_n(s_n)$ by denormalizing all frequency dependent components.

$C \rightarrow C/\omega_o$

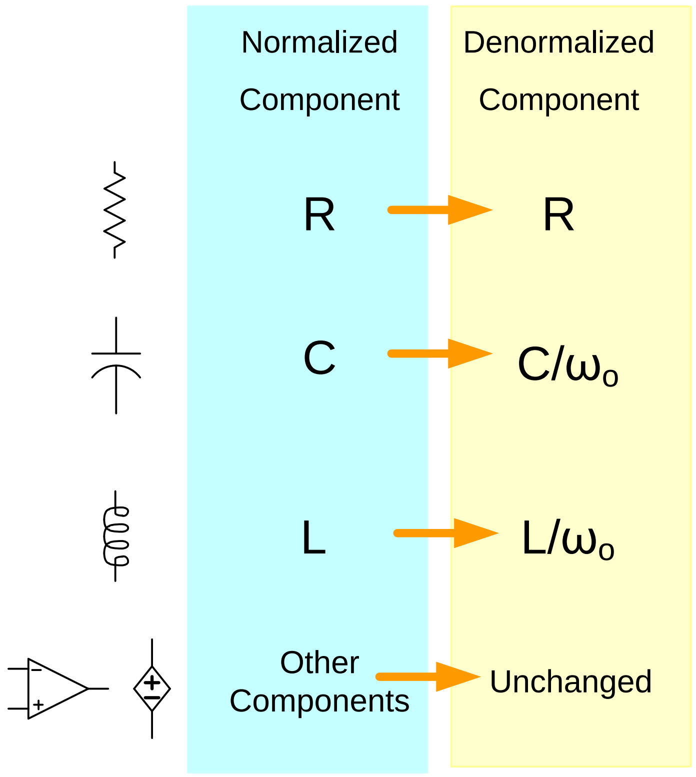
$L \rightarrow L/\omega_o$

Review from Last Time

Frequency normalization/scaling

The frequency scaled circuit can be obtained from the normalized circuit simply by scaling the frequency dependent impedances (up or down) by the scaling factor

Component denormalization by factor of ω_0



Component values of energy storage elements are scaled down by a factor of ω_0

Impedance Scaling

Theorem: If all impedances in a circuit are scaled by a constant θ, then

a) All dimensionless transfer functions are unchanged
b) All transresistance transfer functions are scaled by θ
c) All transconductance transfer functions are scaled by θ^{-1}

Review from Last Time

Impedance Scaling

Impedance scaling of a circuit is achieved by multiplying ALL impedances in the circuit by a constant

$R \rightarrow \theta R$

$C \rightarrow C/\theta$

$L \rightarrow L\theta$

$A \rightarrow$

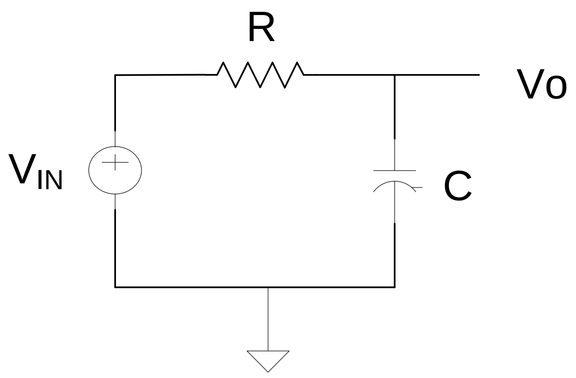
- θA for transresistance gain
- A for dimensionless gain
- A/θ for transconductance gain

Review from Last Time

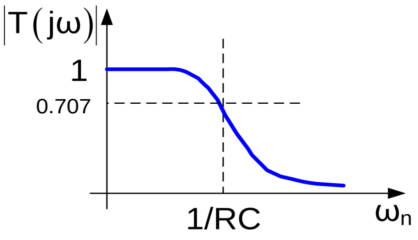
Typical approach to lowpass filter design

1. Obtain normalized approximating function
2. Synthesize circuit to realize normalized approximating function
3. Denormalize circuit obtained in step 2
4. Impedance scale to obtain acceptable component values

Degrees of Freedom



$$T(s)=\frac{V_O}{V_{IN}}=\frac{1}{RCs+1}$$



Circuit has two design variables: {R,C}

Circuit has one key controllable performance characteristic: $\omega_0=\frac{1}{RC}$

If ω_0 is specified for a design, circuit has

2 design variables
1 constraint

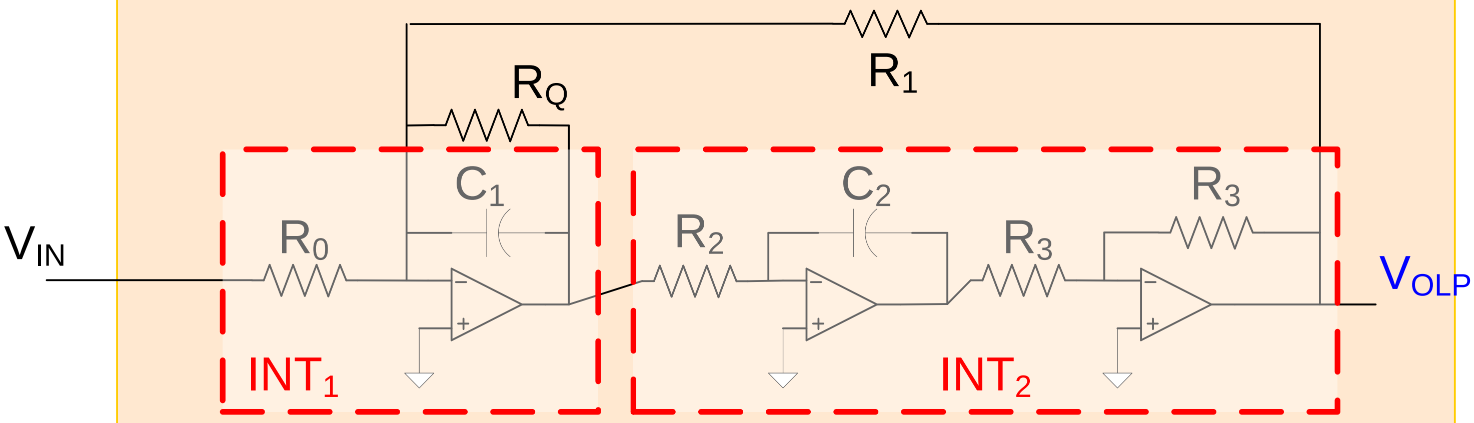
1 Degree of Freedom

Performance/Cost strongly affected by how degrees of freedom in a design are used !

Example: Design a 2nd order lowpass Butterworth filter with 3dB passband attenuation, a dc gain of 5, and a 3dB bandedge of 4KHz

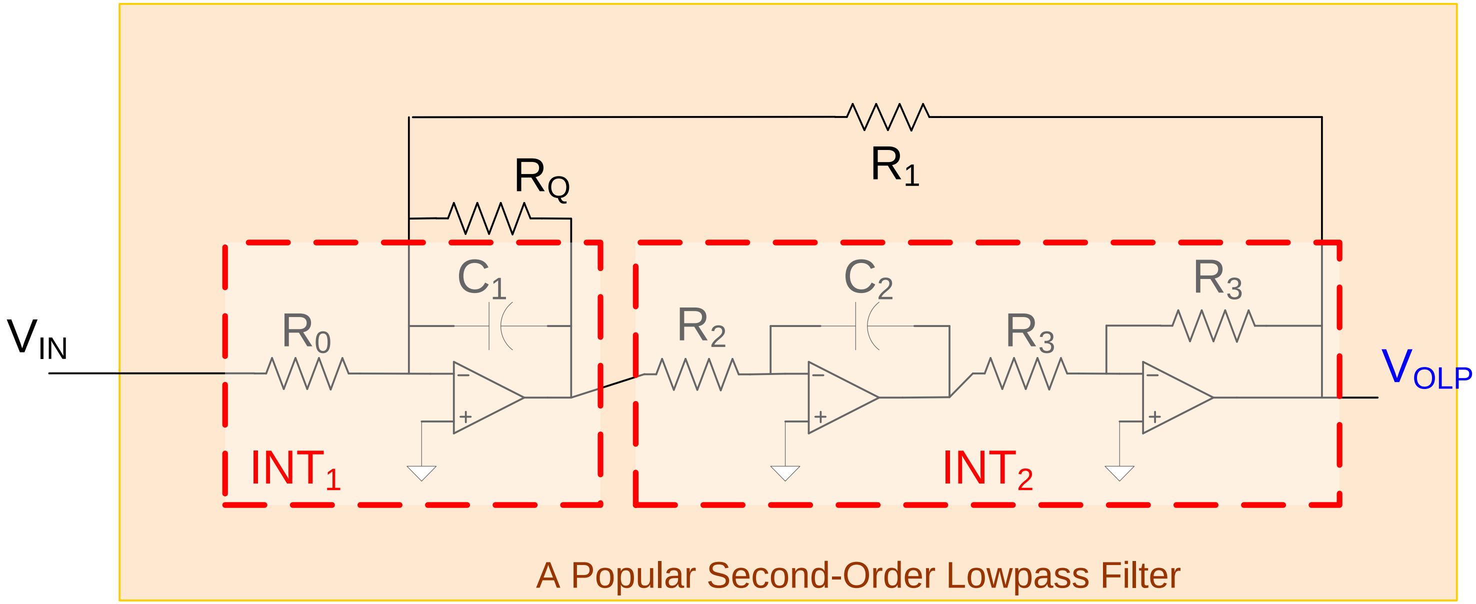
Note: We have not discussed the Butterworth approximation yet so some details here will be based upon concepts that will be developed later

$$T_{BWn} = \left(\frac{1}{s^2+\sqrt{2}s+1}\right)\bullet 5$$



A Popular Second-Order Lowpass Filter

Example: Design a 2nd order lowpass Butterworth filter with 3dB passband attenuation, a dc gain of 5, and a 3dB band edge of 4KHz



A Popular Second-Order Lowpass Filter

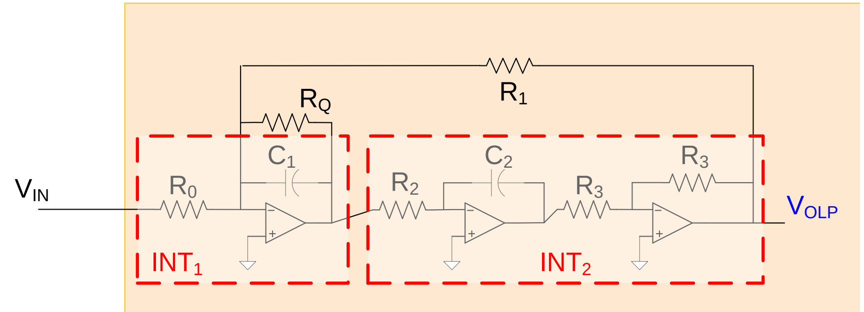
$$T(s) = \frac{\frac{1}{R_2R_0C_1C_2}}{s^2+s\left(\frac{1}{R_QC_1}\right)+\frac{1}{R_2R_1C_1C_2}}$$

$$\omega_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}} \qquad Q = \frac{R_Q}{\sqrt{R_1R_2}}\sqrt{\frac{C_1}{C_2}}$$

7 design variables and only two constraints (ignoring the gain right now)

Circuit has 5 Degrees of Freedom!

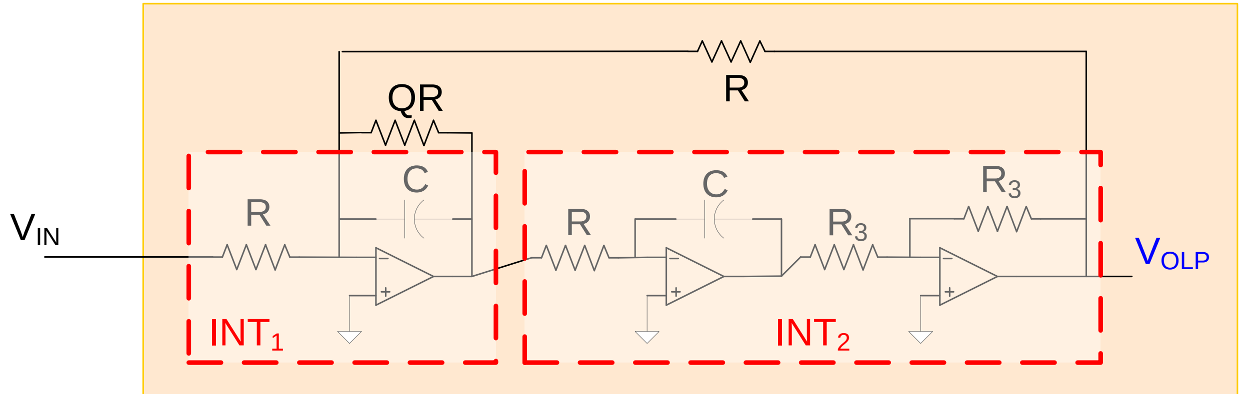
Example: Design a 2nd order lowpass Butterworth filter with 3dB passband attenuation, a dc gain of 5, and a 3dB band edge of 4KHz



A Popular Second-Order Lowpass Filter

If $C_1=C_2=C$ and $R_1=R_2=R_0=R$, this reduces to

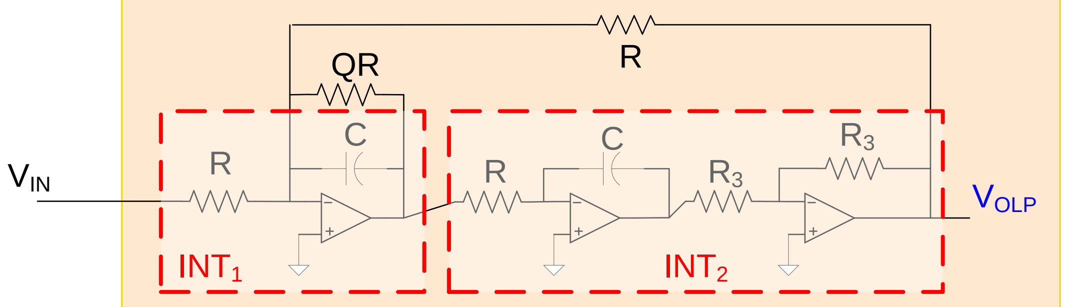
$$T(s)=\frac{\frac{1}{(RC)^2}}{s^2+s\left(\frac{R}{R_Q}\frac{1}{RC}\right)+\frac{1}{(RC)^2}}$$



A Popular Second-Order Lowpass Filter

How many degrees of freedom remain? 2

Example: Design a 2^{nd} order lowpass Butterworth filter with 3dB passband attenuation, a dc gain of 5, and a 3dB bandedge of 4KHz



A Popular Second-Order Lowpass Filter

$$T(s) = \frac{\frac{1}{(RC)^2}}{s^2 + s\left(\frac{R}{R_Q}\frac{1}{RC}\right) + \frac{1}{(RC)^2}}$$

$$\omega_0 = \frac{1}{RC}$$

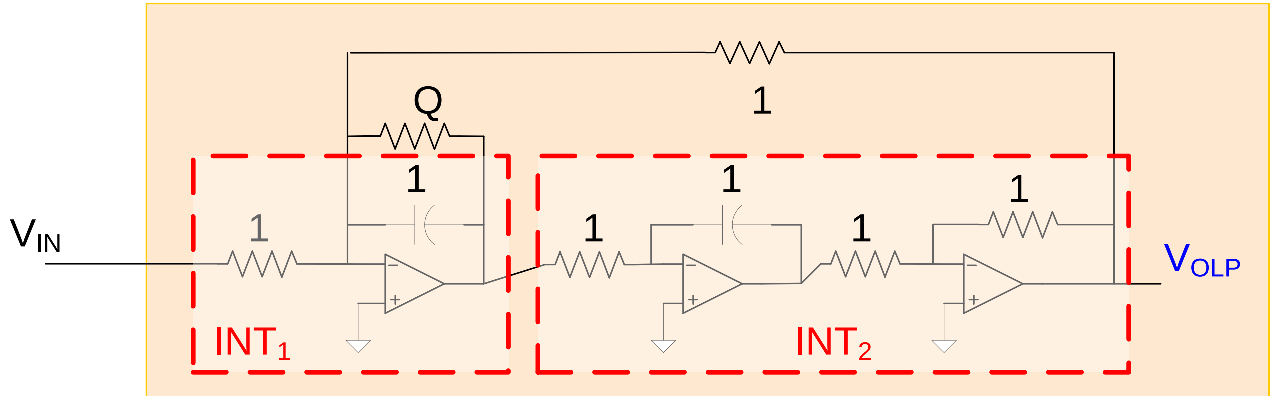
$$Q = \frac{R_Q}{R}$$

Normalizing by the factor ω_0, we obtain

$$T(s_n) = \frac{1}{s^2 + s\left(\frac{1}{Q}\right) + 1}$$

Setting $R=C=R_3=1$ obtain the following normalized circuit

Example: Design a 2nd order lowpass Butterworth filter with 3dB passband attenuation, a dc gain of 5, and a 3dB bandedge of 4KHz



A Popular Second-Order Lowpass Filter

$$T(s_n) = \frac{1}{s^2 + s\left(\frac{1}{Q}\right) + 1}$$

$$\omega_{0n} = 1$$

Must now set $Q = \frac{1}{\sqrt{2}}$

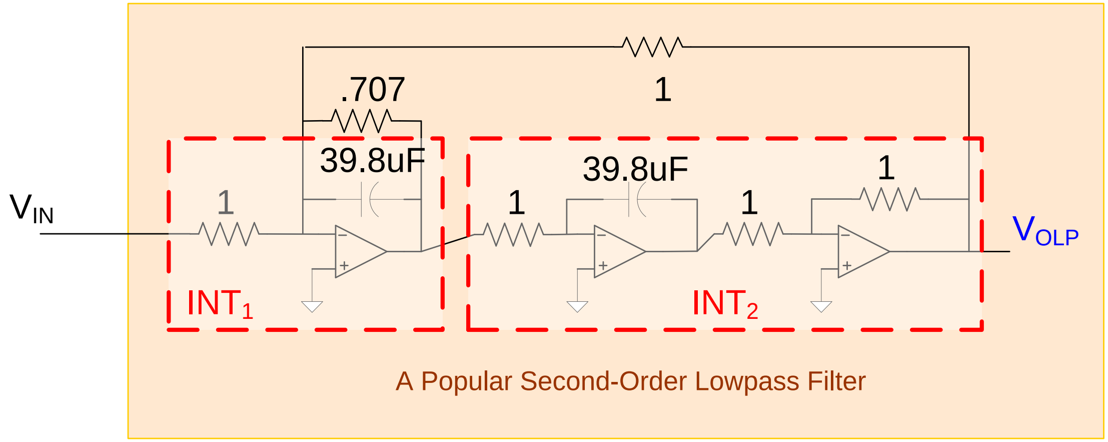
Now we can do frequency scaling   C → C/ω_o

L → L/ω_o

C=1 → 1/(2π•4K) = 39.8uF

Example: Design a 2nd order lowpass Butterworth filter with 3dB passband attenuation, a dc gain of 5, and a 3dB bandedge of 4KHz

Denormalized circuit with bandedge of 4 KHz



This has the right transfer function (but unity gain)

Can now do impedance scaling to get more practical component values

R → θR

C → C/θ

L → θL

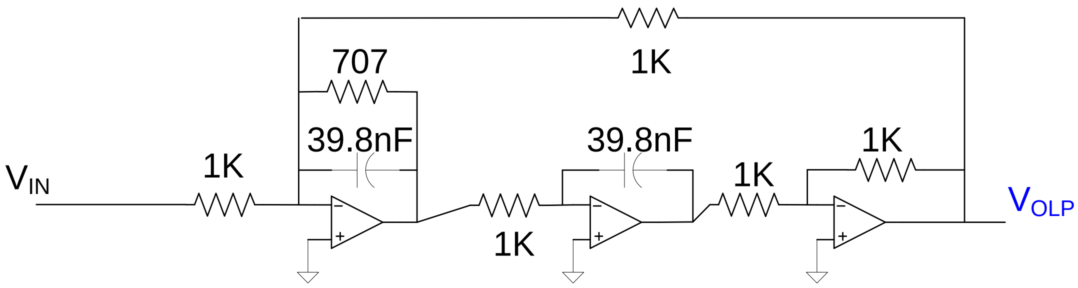
A good impedance scaling factor may be θ=1000

R → 1K

C → 39.8nF

Example: Design a 2nd order lowpass Butterworth filter with 3dB passband attenuation, a dc gain of 5, and a 3dB bandedge of 4KHz

Denormalized circuit with bandedge of 4 KHz



This has the right transfer function (but unity gain)

To finish the design, preceed or follow this circuit with an amplifier with a gain of 5 to meet the dc gain requirements

Review from Last Time

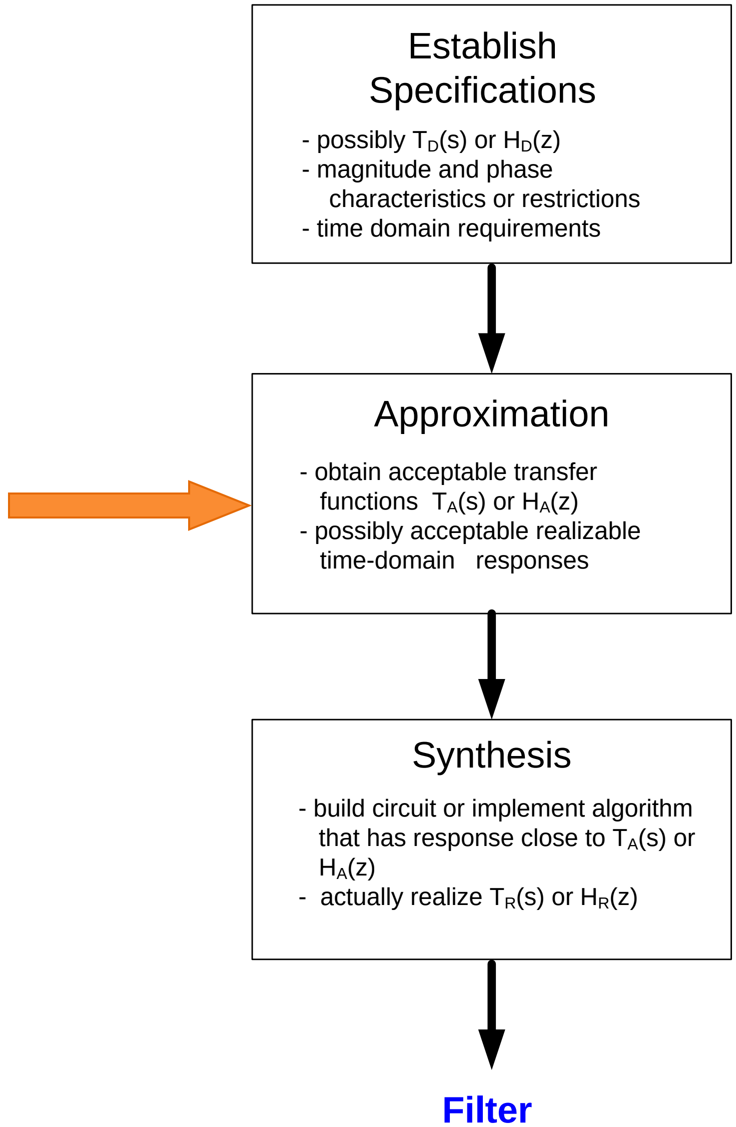
Filter Concepts and Terminology

- Frequency scaling
- Frequency Normalization
- Impedance scaling
- Transformations
  - LP to BP
  - LP to HP
  - LP to BR

It can be shown the standard HP, BP, and BR approximations can be obtained by a frequency transformation of a standard LP approximating function

Will address the LP approximation first, and then provide details about the frequency transformations

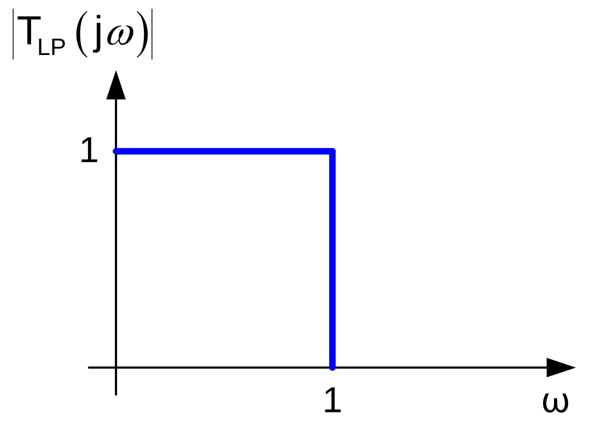
Filter Design Process



The Approximation Problem

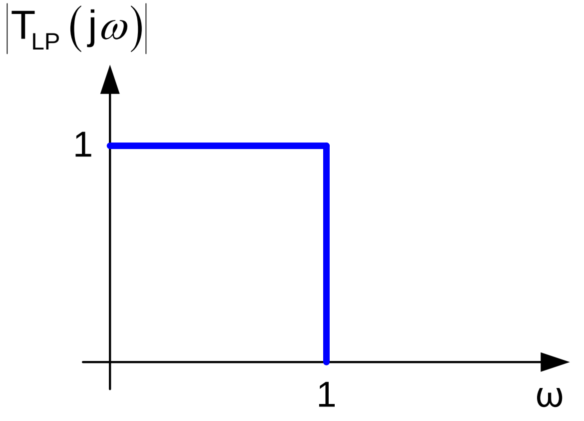
The goal in the approximation problem is simple, just want a function $T_A(s)$ or $H_A(z)$ that meets the filter requirements.

Will focus primarily on approximations of the standard normalized lowpass function



- Frequency scaling will be used to obtain other LP band edges
- Frequency transformations will be used to obtain HP, BP, and BR responses

The Approximation Problem



$$T_A(s) = ?$$

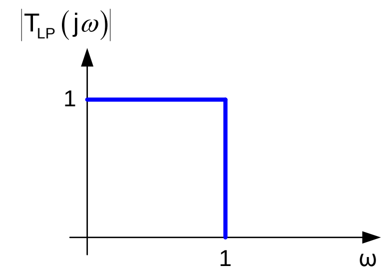
$T_A(s)$ is a rational fraction in s

$$T(s) = \frac{\sum_{i=0}^{m} a_i s^i}{\sum_{i=0}^{n} b_i s^i}$$

Rational fractions in s have no discontinuities in either magnitude or phase response

No natural metrics for $T_A(s)$ that relate to magnitude and phase characteristics (difficult to meaningfully compare $T_{A1}(s)$ and $T_{A2}(s)$)

The Approximation Problem



Approach we will follow:

→ Magnitude Squared Approximating Functions $H_A(\omega^2)$

- Inverse Transform $H_A(\omega^2) \to T_A(s)$
- Collocation
- Least Squares
- Pade Approximatins
- Other Analytical Optimization
- Numerical Optimization
- Canonical Approximations
  - → Butterworth (BW)
  - → Chebyschev (CC)
  - → Elliptic
  - → Thompson

Magnitude Squared Approximating Functions

$$T(s) = \frac{\sum_{i=0}^{m} a_i s^i}{\sum_{i=0}^{n} b_i s^i}$$

$$T(j\omega) = \frac{\sum_{i=0}^{m} a_i (j\omega)^i}{\sum_{i=0}^{n} b_i (j\omega)^i}$$

$$T(j\omega) = \frac{a_o + a_1(j\omega) + a_2(j\omega)^2 + \ldots + a_m(j\omega)^m}{b_o + b_1(j\omega) + b_2(j\omega)^2 + \ldots + b_n(j\omega)^n}$$

$$T(j\omega) = \frac{\left[a_o - a_2\omega^2 + a_4\omega^4 + ..\right] + j\left[a_1\omega - a_3\omega^3 + a_5\omega^5 + \ldots\right]}{\left[b_o - b_2\omega^2 + b_4\omega^4 + ..\right] + j\left[b_1\omega - b_3\omega^3 + b_5\omega^5 + \ldots\right]}$$

$$T(j\omega) = \frac{\left[\sum_{\substack{0 \le k \le m \\ k\,\text{even}}} a_k\omega^k\right] + j\left[\omega \sum_{\substack{0 \le k \le m \\ k\,\text{odd}}} a_k\omega^{k-1}\right]}{\left[\sum_{\substack{0 \le k \le n \\ k\,\text{even}}} b_k\omega^k\right] + j\left[\omega \sum_{\substack{0 \le k \le n \\ k\,\text{odd}}} b_k\omega^{k-1}\right]}$$

$$T(j\omega) = \frac{\left[F_1(\omega^2)\right] + j\left[\omega F_2(\omega^2)\right]}{\left[F_3(\omega^2)\right] + j\left[\omega F_4(\omega^2)\right]}$$

where F_1, F_2, F_3 and F_4 are even functions of ω

Magnitude Squared Approximating Functions

$$T(s) = \frac{\sum_{i=0}^{m} a_i s^i}{\sum_{i=0}^{n} b_i s^i}$$

$$T(j\omega) = \frac{\left[F_1(\omega^2)\right] + j\left[\omega F_2(\omega^2)\right]}{\left[F_3(\omega^2)\right] + j\left[\omega F_4(\omega^2)\right]}$$

$$\left|T(j\omega)\right| = \sqrt{\frac{\left[F_1(\omega^2)\right]^2 + \omega^2\left[F_2(\omega^2)\right]^2}{\left[F_3(\omega^2)\right]^2 + \omega^2\left[F_4(\omega^2)\right]^2}}$$

Thus $\left|T(j\omega)\right|$ is an even function of ω

It follows that $\left|T(j\omega)\right|^2$ is a rational fraction in ω^2 with real coefficients

Since $\left|T(j\omega)\right|^2$ is a real variable, natural metrics exist for comparing approximating functions to $\left|T(j\omega)\right|^2$

Magnitude Squared Approximating Functions

$$T(s) = \frac{\sum_{i=0}^{m} a_i s^i}{\sum_{i=0}^{n} b_i s^i}$$

If a desired magnitude response is given, it is common to find a rational fraction in ω^2 with real coefficients, denoted as $H_A(\omega^2)$, that approximates the desired magnitude squared response and then obtain a function $T_A(s)$ that satisfies the relationship $\left|T_A(j\omega)\right|^2 = H_A(\omega^2)$

$H_A(\omega^2)$ is real so natural metrics exist for obtaining $H_A(\omega^2)$

$$H_A(\omega^2) = \frac{\sum_{i=0}^{2l} c_i \omega^{2i}}{\sum_{i=0}^{2k} d_i \omega^{2i}}$$

Obtaining $T_A(s)$ from $H_A(\omega^2)$ is termed the inverse mapping problem

But how is $T_{\underline{A}}(s)$ obtained from $H_A(\omega^2)$?

Inverse mapping problem:

$$T_A(s) \xrightarrow[\text{well defined}]{} H_A(\omega^2) \qquad H_A(\omega^2) = |T_A(j\omega)|^2$$

$$T_A(s) \xleftarrow{\textbf{?}} H_A(\omega^2)$$

Consider an example:

$$T_1(s) = s + 1 \searrow$$

$$H_A(\omega^2) = 1 + \omega^2$$

$$T_1(s) = s - 1 \nearrow$$

Thus, the inverse mapping in this example is not unique !

Inverse mapping problem:

$$T_A(s) \longrightarrow H_A(\omega^2) \qquad\qquad H_A(\omega^2) = |T_A(j\omega)|^2$$

$$T_A(s) \overset{?}{\longleftarrow} H_A(\omega^2)$$

Some observations:

- If an inverse mapping exists, it is not necessarily unique

- If an inverse mapping exists, than a minimum phase inverse mapping exists and it is unique (within all-pass factors)

- The mapping from $T_A(s)$ to $H_A(\omega^2)$ increases order by a factor of 2

- Any inverse mapping from $H_A(\omega^2)$ to $T_A(s)$ will reduce order by a factor of 2 (within all-pass factors)

Example:

$$H_A\left(\omega^2\right)=\frac{2\omega^2+1}{\omega^4 + 2\omega^2 + 1} \longrightarrow T_A\left(s\right)=\frac{\sqrt{2}s+1}{\left(s+1\right)\left(s+1\right)}$$

Example:

$$H_A\left(\omega^2\right)=\frac{\omega^2-1}{\omega^4 + 2\omega^2 + 1} \longrightarrow ?$$

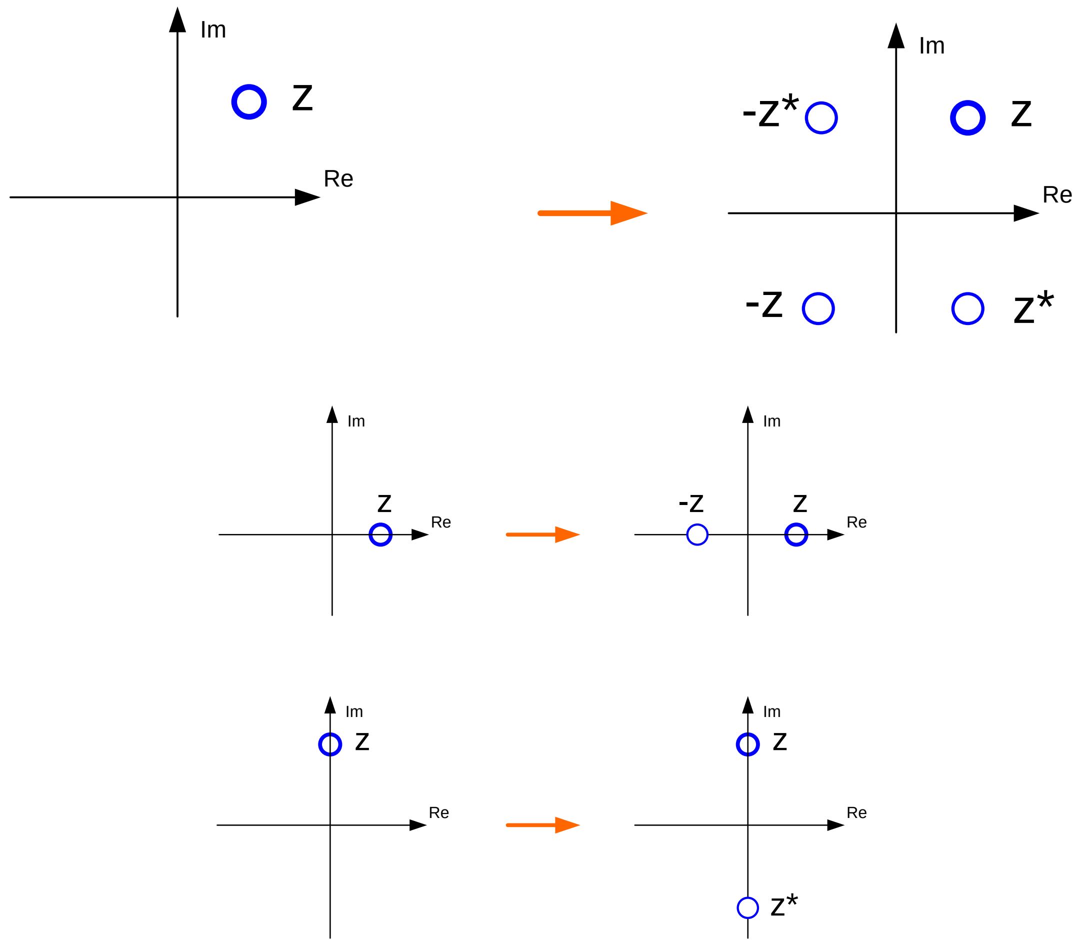
Inverse mapping does not exist !

It can be shown that many even rational fractions in ω^2 do not have an inverse mapping back to the s-domain !

Often these functions have a magnitude squared response that does a good job of approximating the desired filter magnitude response

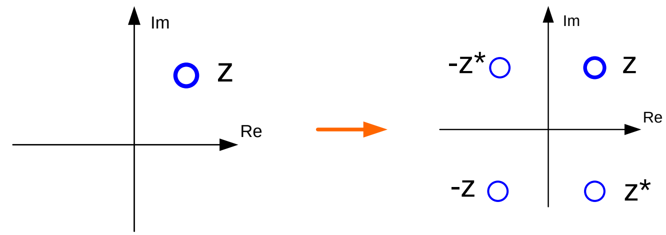
If an inverse mapping exists, there are often several inverse mappings that exist

Observation: If z is a zero (pole) of $H_A(\omega^2)$, then –z, z*, and –z* are also zeros (poles) of $H_A(\omega^2)$



Thus, roots come as quadruples if off of the axis and as pairs if they lay on the axis

Observation: If z is a zero (pole) of $H_A(\omega^2)$, then –z, z*, and –z* are also zeros (poles) of $H_A(\omega^2)$



Proof:

Consider an even polynomial in ω^2 with real coefficients $$P(\omega^2) = \sum_{i=0}^{m} a_i \omega^{2i}$$

At a root, this polynomial satisfies the expression $$P(\omega^2) = \sum_{i=0}^{m} a_i \omega^{2i} = 0$$

Replacing ω with –ω, we obtain

$$P\left([-\omega]^2\right) = \sum_{i=0}^{m} a_i [-\omega]^{2i} = \sum_{i=0}^{m} a_i \left[-1^2\right]^i [\omega]^{2i} = \sum_{i=0}^{m} a_i [\omega]^{2i} = 0 \quad \Longrightarrow \quad -\omega \text{ is a root of } P(\omega^2)$$

Recall (xy)*=x*y* and $\left(x^n\right)^* = \left(x^*\right)^n$

Taking the complex conjugate of $P(\omega^2)=0$ we obtain

$$P(\omega^2)^* = \sum_{i=0}^{m} \left(a_i \omega^{2i}\right)^* = \sum_{i=0}^{m} \left(a_j^*\right)\left(\omega^{2i}\right)^* = \sum_{i=0}^{m} \left(a_j^*\right)\left(\left(\omega^*\right)^{2i}\right) = 0$$

Since a_i is real for all I, it thus follows that

$$\sum_{i=0}^{m} \left(a_j\right)\left(\left(\omega^*\right)^{2i}\right) = 0 \quad \Longrightarrow \quad \omega^* \text{ is a root of } P(\omega^2)$$

Theorem: If $H_A(\omega^2)$ is a rational fraction with real coefficients with no poles or zeros of odd multiplicity on the real axis, then there exists a real number H_0 such that the function

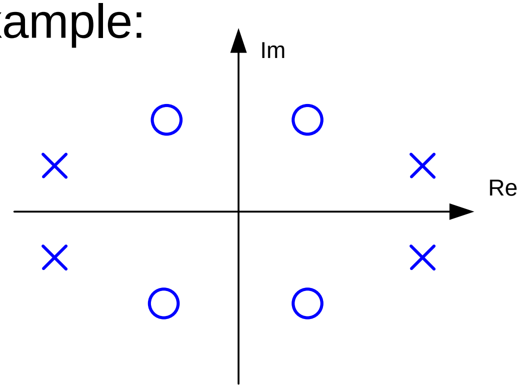
$$T_{AM}(s) = \frac{H_0\,(s-jz_1)(s-jz_2)\bullet\ldots\bullet(s-jz_m)}{(s-jp_1)(s-jp_2)\bullet\ldots\bullet(s-jp_n)}$$

is a minimum phase rational fraction with real coefficents that satisfies the relationship

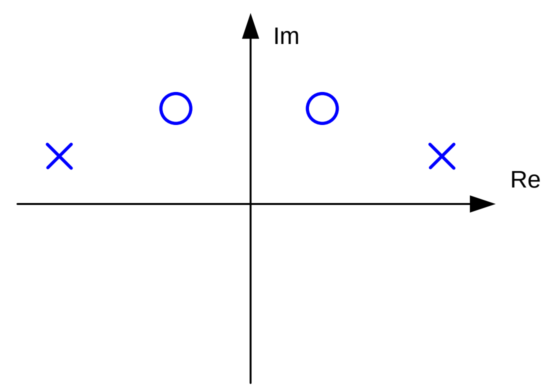
$$\left|T_{AM}(j\omega)\right| = \sqrt{H_A(\omega^2)}$$

where $\{z_1, z_2, \ldots z_m\}$ are the upper half-plane zeros of $H_A(\omega^2)$ and exactly half of the real axis zeros,
and where where $\{p_1, p_2, \ldots p_n\}$ are the upper half-plane poles of $H_A(\omega^2)$ and exactly half of the real axis poles.

Example:



Roots of $H_A(\omega^2)$



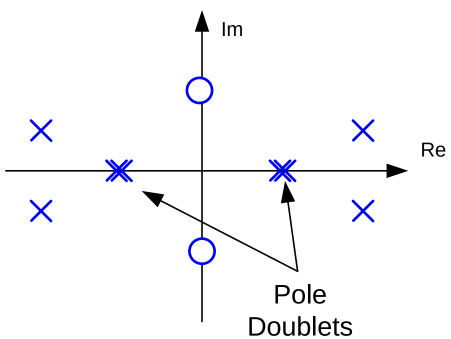
Roots that Appear in $T_{AM}(s)$

$$H_A\left(\omega^2\right)=\frac{H_0^2\left[(\omega-z_1)(\omega-z_2)\bullet\ldots\bullet(\omega-z_m)\right]\bullet\left[(\omega+z_1)(\omega+z_2)\bullet\ldots\bullet(\omega+z_m)\right]}{\left[(\omega-p_1)(\omega-p_2)\bullet\ldots\bullet(\omega-p_n)\right]\bullet\left[(\omega+p_1)(\omega+p_2)\bullet\ldots\bullet(\omega+p_n)\right]}$$

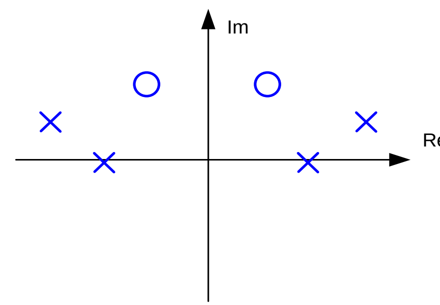
If inverse exists

$$T_{AM}(s)=\frac{H_0(s-jz_1)(s-jz_2)\bullet\ldots\bullet(s-jz_m)}{(s-jp_1)(s-jp_2)\bullet\ldots\bullet(s-jp_n)}$$

Example:

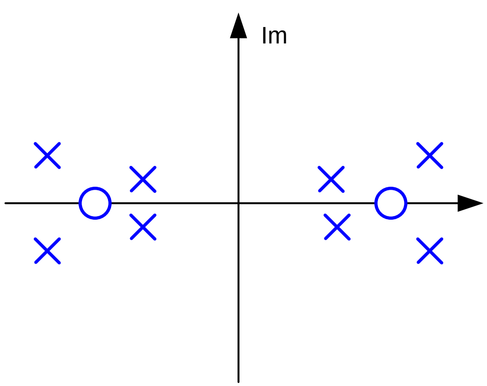


Roots of $H_A(\omega^2)$



Roots that appear in $T_{AM}(s)$

Example:

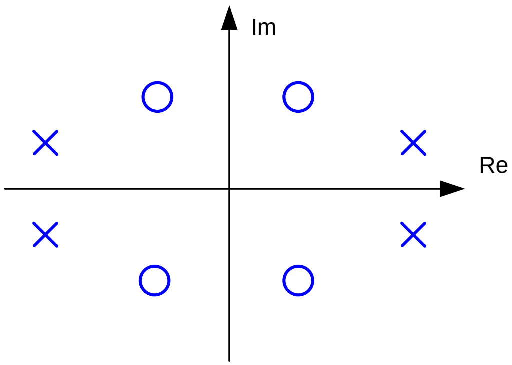


Inverse does not exist because zeros are of odd multiplicity on the real axis

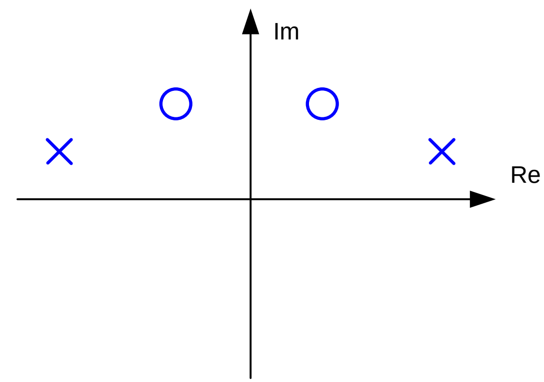
$$H_A\left(\omega^2\right) = \frac{H_0^2\left[(\omega-z_1)(\omega-z_2)\bullet\ldots\bullet(\omega-z_m)\right]\bullet\left[(\omega+z_1)(\omega+z_2)\bullet\ldots\bullet(\omega+z_m)\right]}{\left[(\omega-p_1)(\omega-p_2)\bullet\ldots\bullet(\omega-p_n)\right]\bullet\left[(\omega+p_1)(\omega+p_2)\bullet\ldots\bullet(\omega+p_n)\right]}$$

If inverse exists

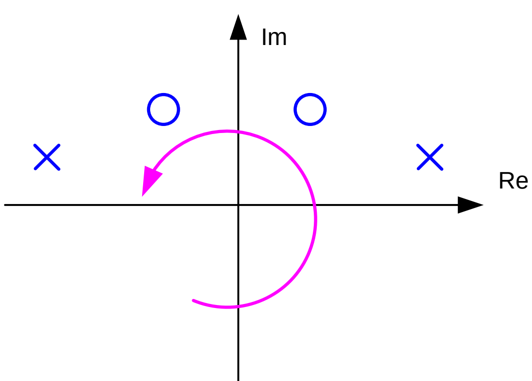
$$T_{AM}(s) = \frac{H_0(s-jz_1)(s-jz_2)\bullet\ldots\bullet(s-jz_m)}{(s-jp_1)(s-jp_2)\bullet\ldots\bullet(s-jp_n)}$$



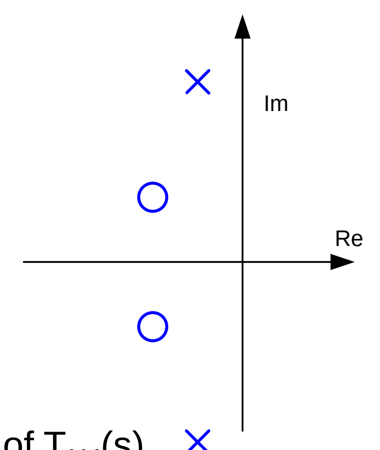
Roots of $H_A(\omega^2)$



Roots that appear in $T_{AM}(s)$



Rotate roots by 90°



Roots of $T_{AM}(s)$

$$H_A\left(\omega^2\right)=\frac{H_0^2\left[(\omega\text{-}z_1)(\omega\text{-}z_2)\bullet\ldots\bullet(\omega\text{-}z_m)\right]\bullet\left[(\omega+z_1)(\omega+z_2)\bullet\ldots\bullet(\omega+z_m)\right]}{\left[(\omega\text{-}p_1)(\omega\text{-}p_2)\bullet\ldots\bullet(\omega\text{-}p_n)\right]\bullet\left[(\omega+p_1)(\omega+p_2)\bullet\ldots\bullet(\omega+p_n)\right]}$$

If inverse exists

$$T_{AM}(s)=\frac{H_0\,(s\text{-}jz_1)(s\text{-}jz_2)\bullet\ldots\bullet(s\text{-}jz_m)}{(s\text{-}jp_1)(s\text{-}jp_2)\bullet\ldots\bullet(s\text{-}jp_n)}$$

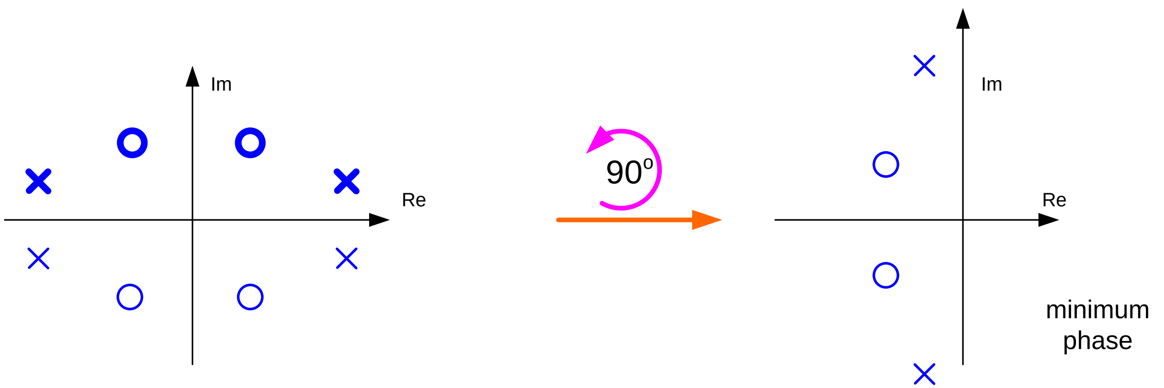
Observations:

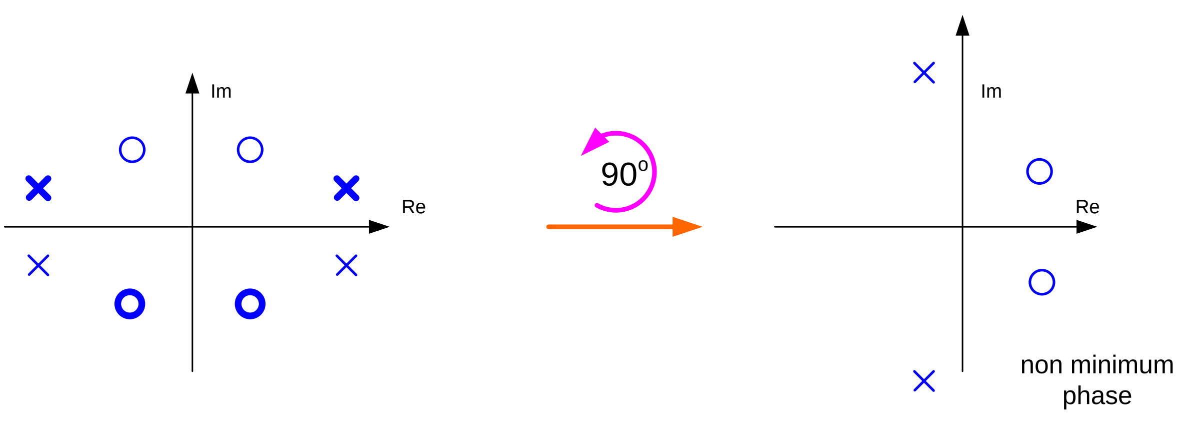
- Coefficients of $T_{AM}(s)$ are real
- If x is a root of $H_A(\omega^2)$, then jx is a root of $T_{AM}(s)$
- Multiplying a root by j is equivalent to rotating it by 90° cc in the complex plane
- Roots of $T_{AM}(s)$ are obtained from roots of $H_A(\omega^2)$ by multiplying by j
- Roots of $T_{AM}(s)$ are upper half-plane roots and exactly half of real axis roots all rotated cc by 90°
- If a root of $H_A(\omega^2)$ has odd multiplicity on the real axis, the inverse mapping does not exist
- Other (often many) inverse mappings exist but are not minimum phase
  (These can be obtained by reflecting any subset of the zeros or poles around the imaginary axis into the RHP)

$$H_A\left(\omega^2\right) = \frac{H_0^2\left[(\omega-z_1)(\omega-z_2)\bullet\ldots\bullet(\omega-z_m)\right]\bullet\left[(\omega+z_1)(\omega+z_2)\bullet\ldots\bullet(\omega+z_m)\right]}{\left[(\omega-p_1)(\omega-p_2)\bullet\ldots\bullet(\omega-p_n)\right]\bullet\left[(\omega+p_1)(\omega+p_2)\bullet\ldots\bullet(\omega+p_n)\right]}$$

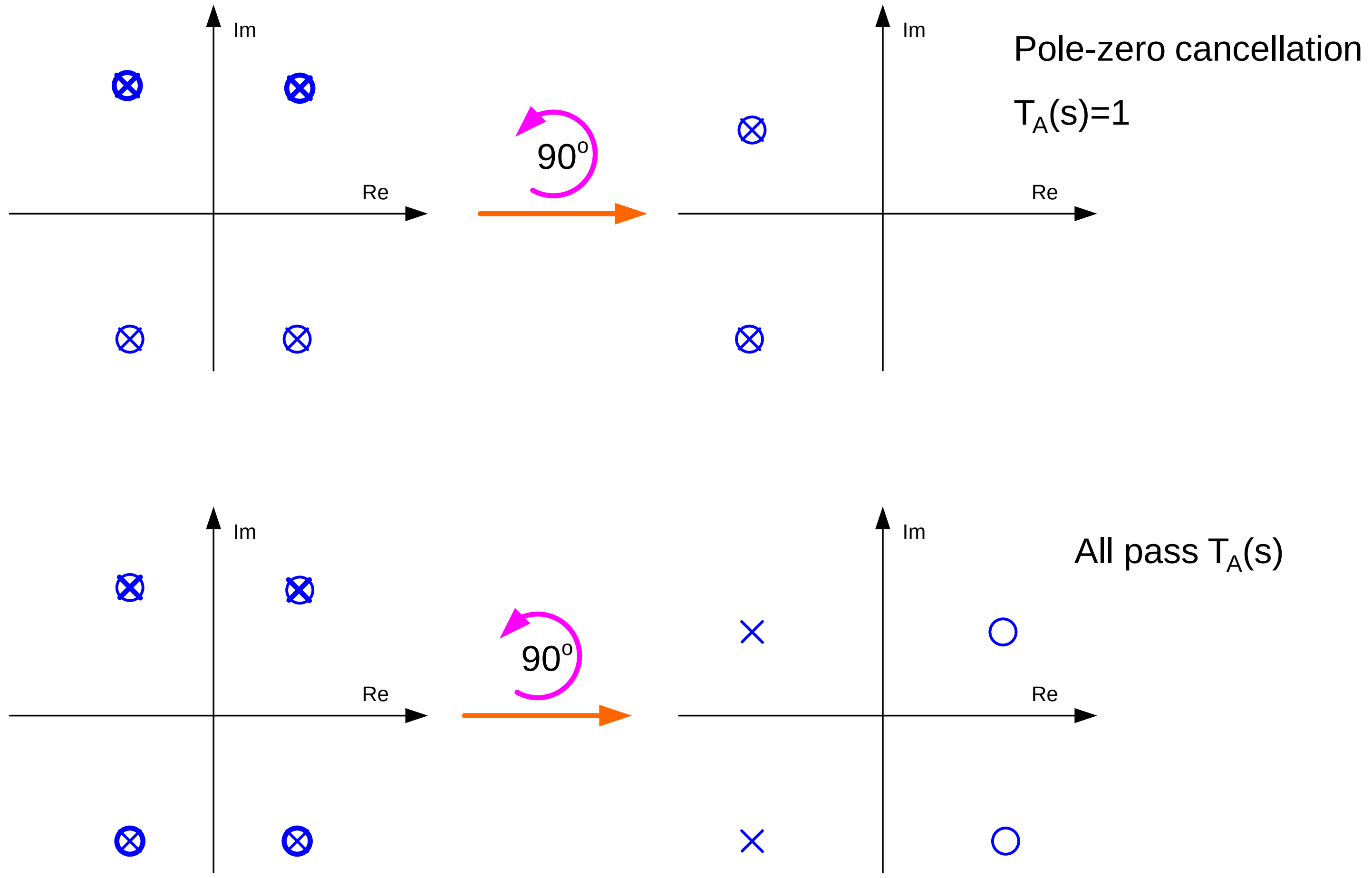
If inverse exists

$$T_{AM}(s) = \frac{H_0(s-jz_1)(s-jz_2)\bullet\ldots\bullet(s-jz_m)}{(s-jp_1)(s-jp_2)\bullet\ldots\bullet(s-jp_n)}$$





All pass functions (and factors)



- Must not allow cancellations to take place in $H_A(\omega^2)$ to obtain all-pass $T_A(s)$
- Must keep upper HP poles and lower HP zeros in $H_A(\omega^2)$ to obtain all-pass $T_A(s)$
- All-pass $T_A(s)$ is not minimum phase

End of Lecture 7

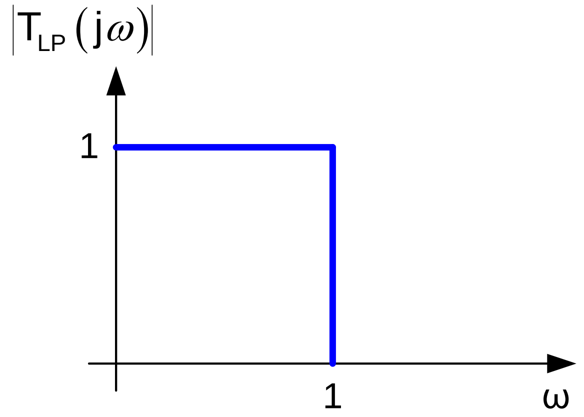
EE 508
Lecture 8

The Approximation Problem

The Approximation Problem

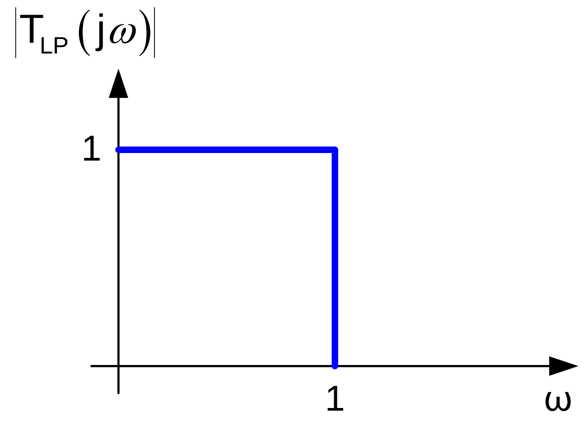
The goal in the approximation problem is simple, just want a function $T_A(s)$ or $H_A(z)$ that meets the filter requirements.

Will focus primarily on approximations of the standard normalized lowpass function



- Frequency scaling will be used to obtain other LP band edges
- Frequency transformations will be used to obtain HP, BP, and BR responses

The Approximation Problem



$$T_A(s) = ?$$

$T_A(s)$ is a rational fraction in s

$$T(s) = \frac{\sum_{i=0}^{m} a_i s^i}{\sum_{i=0}^{n} b_i s^i}$$

Rational fractions in s have no discontinuities in either magnitude or phase response

No natural metrics for $T_A(s)$ that relate to magnitude and phase characteristics (difficult to meaningfully compare $T_{A1}(s)$ and $T_{A2}(s)$)

Review from Last Time

Magnitude Squared Approximating Functions

$$T(s)=\frac{\sum_{i=0}^{m} a_i s^i}{\sum_{i=0}^{n} b_i s^i}$$

$$T(j\omega)=\frac{\left[F_1\left(\omega^2\right)\right]+j\left[\omega F_2\left(\omega^2\right)\right]}{\left[F_3\left(\omega^2\right)\right]+j\left[\omega F_4\left(\omega^2\right)\right]}$$

$$\left|T(j\omega)\right|=\sqrt{\frac{\left[F_1\left(\omega^2\right)\right]^2+\omega^2\left[F_2\left(\omega^2\right)\right]^2}{\left[F_3\left(\omega^2\right)\right]^2+\omega^2\left[F_4\left(\omega^2\right)\right]^2}}$$

Thus $\left|T(j\omega)\right|$ is an even function of ω

It follows that $\left|T(j\omega)\right|^2$ is a rational fraction in ω^2 with real coefficients

Since $\left|T(j\omega)\right|^2$ is a real variable, natural metrics exist for comparing approximating functions to $\left|T(j\omega)\right|^2$

Review from Last Time

Magnitude Squared Approximating Functions

$$T(s) = \frac{\sum_{i=0}^{m} a_i s^i}{\sum_{i=0}^{n} b_i s^i}$$

If a desired magnitude response is given, it is common to find a rational fraction in ω^2 with real coefficients, denoted as $H_A(\omega^2)$, that approximates the desired magnitude squared response and then obtain a function $T_A(s)$ that satisfies the relationship $\left|T_A(j\omega)\right|^2 = H_A(\omega^2)$

$H_A(\omega^2)$ is real so natural metrics exist for obtaining $H_A(\omega^2)$

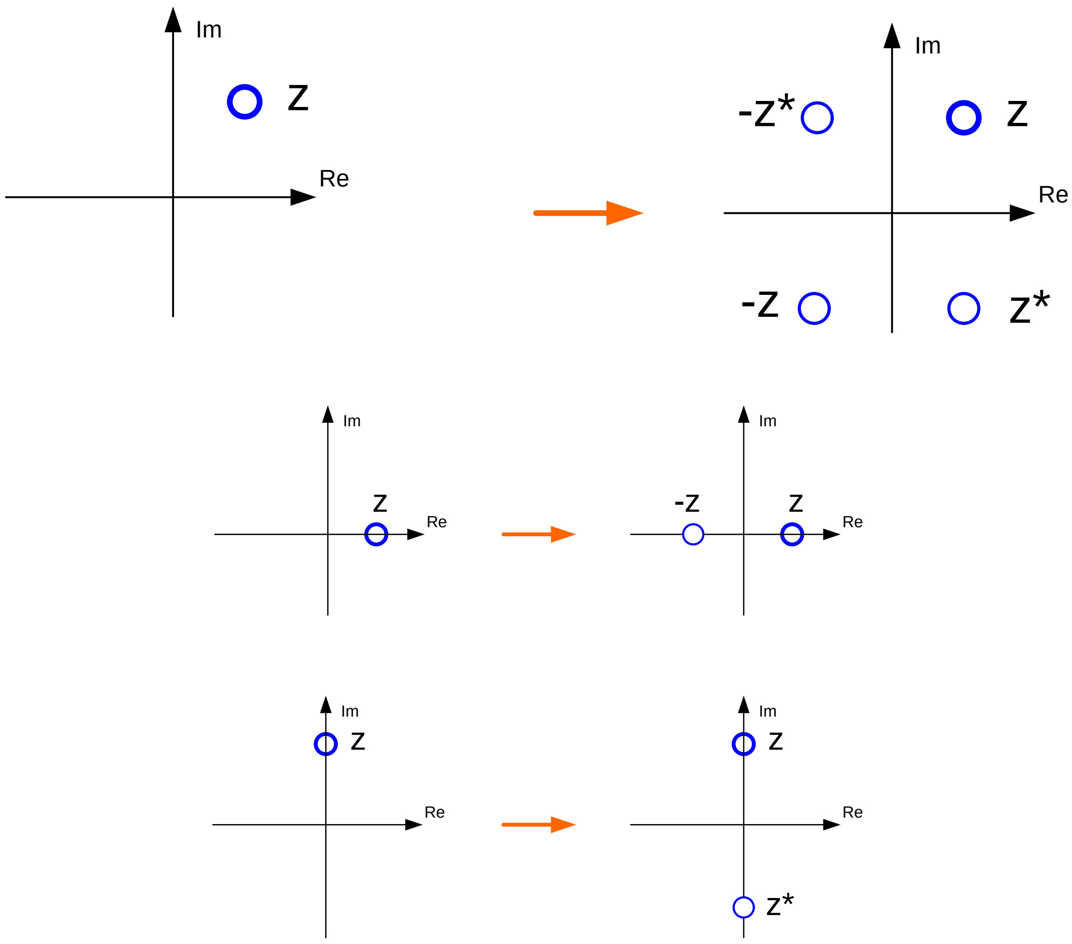
$$H_A(\omega^2) = \frac{\sum_{i=0}^{2l} c_i \omega^{2i}}{\sum_{i=0}^{2k} d_i \omega^{2i}}$$

Obtaining $T_A(s)$ from $H_A(\omega^2)$ is termed the inverse mapping problem

But how is $T_A(s)$ obtained from $H_A(\omega^2)$?

Review from Last Time

Observation: If z is a zero (pole) of $H_A(\omega^2)$, then –z, z*, and –z* are also zeros (poles) of $H_A(\omega^2)$



Thus, roots come as quadruples if off of the axis and as pairs if they lay on the axis

Review from Last Time

Theorem: If $H_A(\omega^2)$ is a rational fraction with real coefficients with no poles or zeros of odd multiplicity on the real axis, then there exists a real number H_0 such that the function

$$T_{AM}(s) = \frac{H_0(s-jz_1)(s-jz_2)\bullet\ldots\bullet(s-jz_m)}{(s-jp_1)(s-jp_2)\bullet\ldots\bullet(s-jp_n)}$$

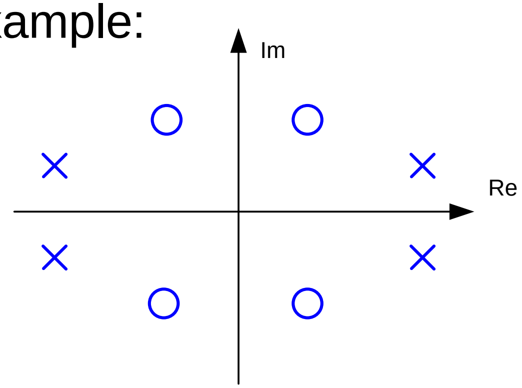
is a minimum phase rational fraction with real coefficents that satisfies the relationship

$$\left|T_{AM}(j\omega)\right| = \sqrt{H_A(\omega^2)}$$

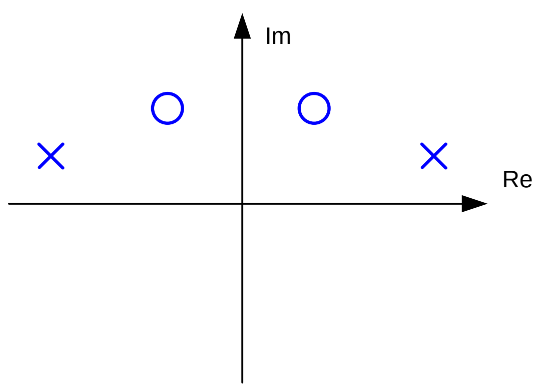
where $\{z_1, z_2, \ldots z_m\}$ are the upper half-plane zeros of $H_A(\omega^2)$ and exactly half of the real axis zeros,

and where where $\{p_1, p_2, \ldots p_n\}$ are the upper half-plane poles of $H_A(\omega^2)$ and exactly half of the real axis poles.

Example:



Roots of $H_A(\omega^2)$



Roots that Appear in $T_{AM}(s)$

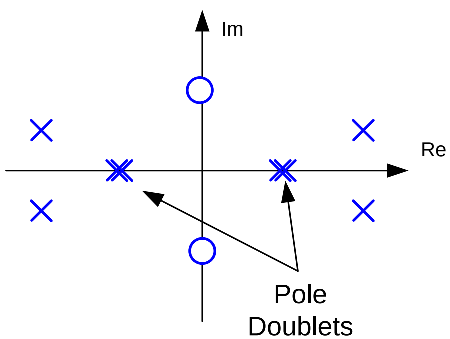
Review from Last Time

$$H_A\left(\omega^2\right)=\frac{H_0^2\left[(\omega\text{-}z_1)(\omega\text{-}z_2)\bullet\ldots\bullet(\omega\text{-}z_m)\right]\bullet\left[(\omega+z_1)(\omega+z_2)\bullet\ldots\bullet(\omega+z_m)\right]}{\left[(\omega\text{-}p_1)(\omega\text{-}p_2)\bullet\ldots\bullet(\omega\text{-}p_n)\right]\bullet\left[(\omega+p_1)(\omega+p_2)\bullet\ldots\bullet(\omega+p_n)\right]}$$

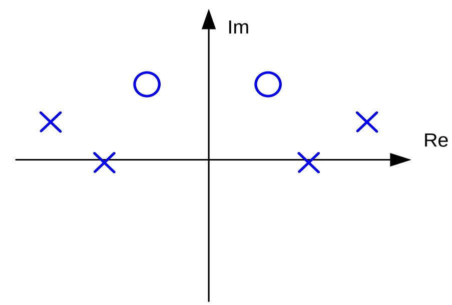
If inverse exists

$$T_{AM}(s)=\frac{H_0(s\text{-}jz_1)(s\text{-}jz_2)\bullet\ldots\bullet(s\text{-}jz_m)}{(s\text{-}jp_1)(s\text{-}jp_2)\bullet\ldots\bullet(s\text{-}jp_n)}$$

Example:

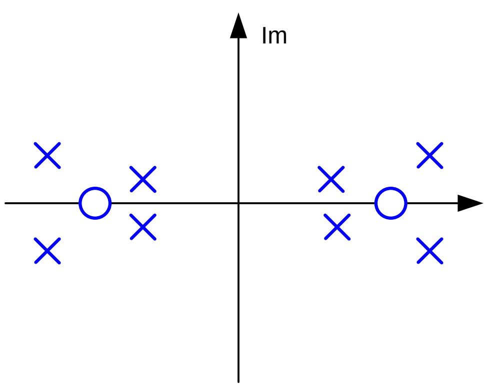


Roots of $H_A(\omega^2)$



Roots that appear in $T_{AM}(s)$

Example:



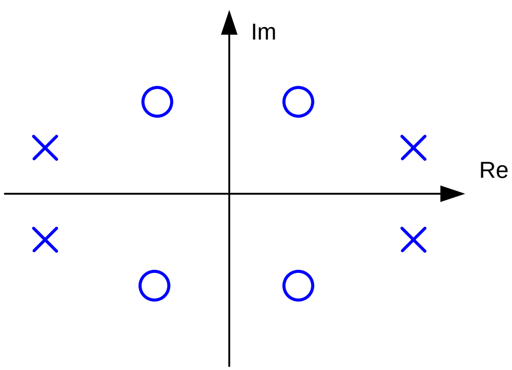
Inverse does not exist because zeros are of odd multiplicity on the real axis

Review from Last Time

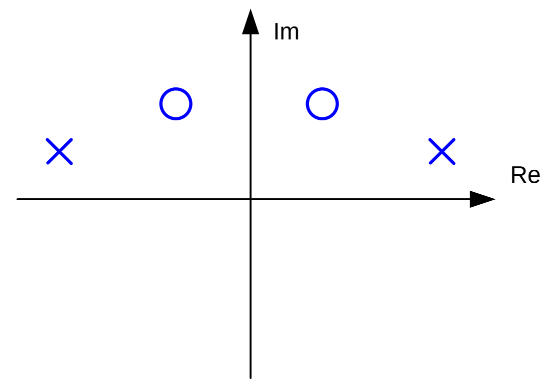
$$H_A\left(\omega^2\right)=\frac{H_0^2\left[(\omega-z_1)(\omega-z_2)\bullet\ldots\bullet(\omega-z_m)\right]\bullet\left[(\omega+z_1)(\omega+z_2)\bullet\ldots\bullet(\omega+z_m)\right]}{\left[(\omega-p_1)(\omega-p_2)\bullet\ldots\bullet(\omega-p_n)\right]\bullet\left[(\omega+p_1)(\omega+p_2)\bullet\ldots\bullet(\omega+p_n)\right]}$$

If inverse exists

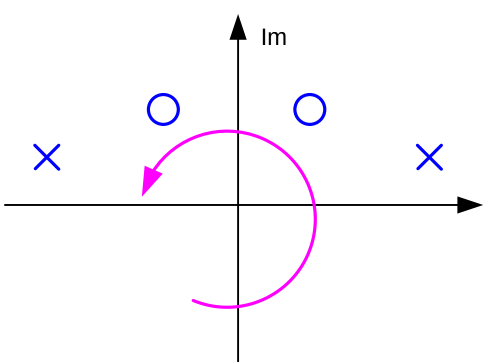
$$T_{AM}(s)=\frac{H_0(s-jz_1)(s-jz_2)\bullet\ldots\bullet(s-jz_m)}{(s-jp_1)(s-jp_2)\bullet\ldots\bullet(s-jp_n)}$$



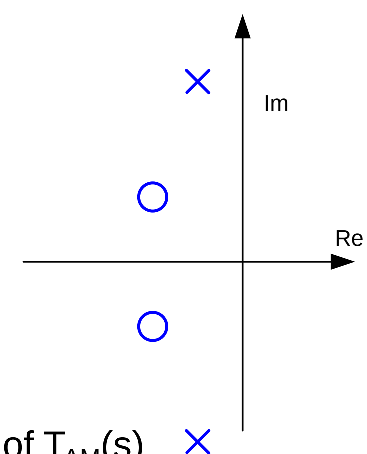
Roots of $H_A(\omega^2)$



Roots that appear in $T_{AM}(s)$



Rotate roots by 90°



Roots of $T_{AM}(s)$

Review from Last Time

$$H_A\left(\omega^2\right)=\frac{H_0^2\left[(\omega\text{-}z_1)(\omega\text{-}z_2)\bullet\ldots\bullet(\omega\text{-}z_m)\right]\bullet\left[(\omega+z_1)(\omega+z_2)\bullet\ldots\bullet(\omega+z_m)\right]}{\left[(\omega\text{-}p_1)(\omega\text{-}p_2)\bullet\ldots\bullet(\omega\text{-}p_n)\right]\bullet\left[(\omega+p_1)(\omega+p_2)\bullet\ldots\bullet(\omega+p_n)\right]}$$

If inverse exists

$$T_{AM}(s)=\frac{H_0(s\text{-}jz_1)(s\text{-}jz_2)\bullet\ldots\bullet(s\text{-}jz_m)}{(s\text{-}jp_1)(s\text{-}jp_2)\bullet\ldots\bullet(s\text{-}jp_n)}$$

Observations:

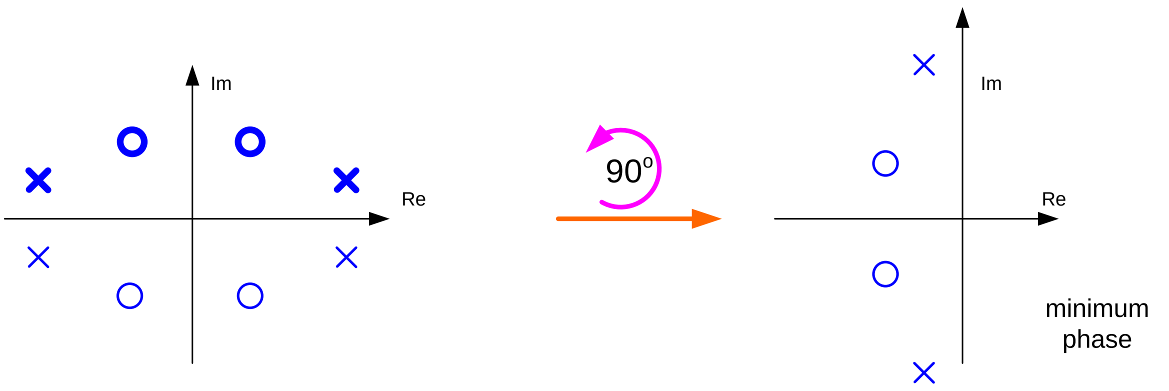
- Coefficients of $T_{AM}(s)$ are real
- If x is a root of $H_A(\omega^2)$, then jx is a root of $T_{AM}(s)$
- Multiplying a root by j is equivalent to rotating it by 90^o cc in the complex plane
- Roots of $T_{AM}(s)$ are obtained from roots of $H_A(\omega^2)$ by multiplying by j
- Roots of $T_{AM}(s)$ are upper half-plane roots and exactly half of real axis roots all rotated cc by 90^o
- If a root of $H_A(\omega^2)$ has odd multiplicity on the real axis, the inverse mapping does not exist
- Other (often many) inverse mappings exist but are not minimum phase
  (These can be obtained by reflecting any subset of the zeros or poles around the imaginary axis into the RHP)

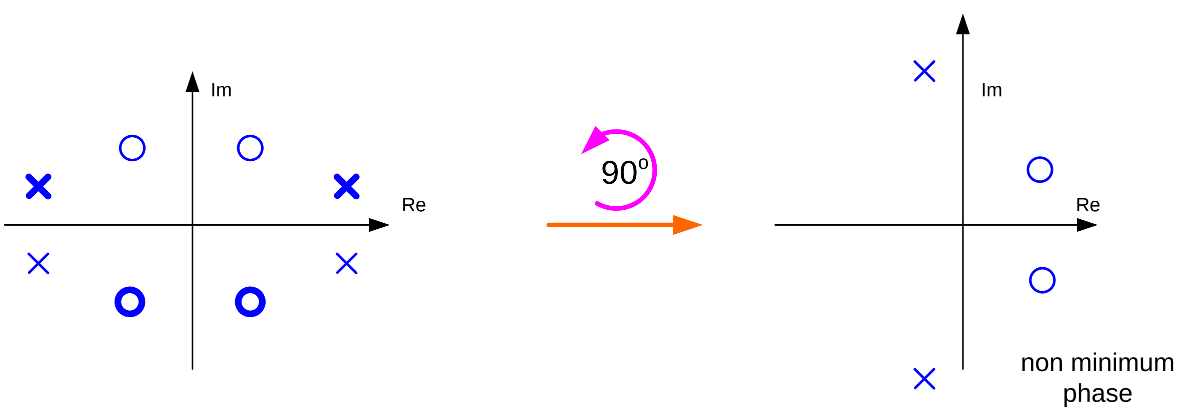
Review from Last Time

$$H_A\left(\omega^2\right)=\frac{H_0^2\left[(\omega\text{-}z_1)(\omega\text{-}z_2)\bullet\ldots\bullet(\omega\text{-}z_m)\right]\bullet\left[(\omega+z_1)(\omega+z_2)\bullet\ldots\bullet(\omega+z_m)\right]}{\left[(\omega\text{-}p_1)(\omega\text{-}p_2)\bullet\ldots\bullet(\omega\text{-}p_n)\right]\bullet\left[(\omega+p_1)(\omega+p_2)\bullet\ldots\bullet(\omega+p_n)\right]}$$

If inverse exists

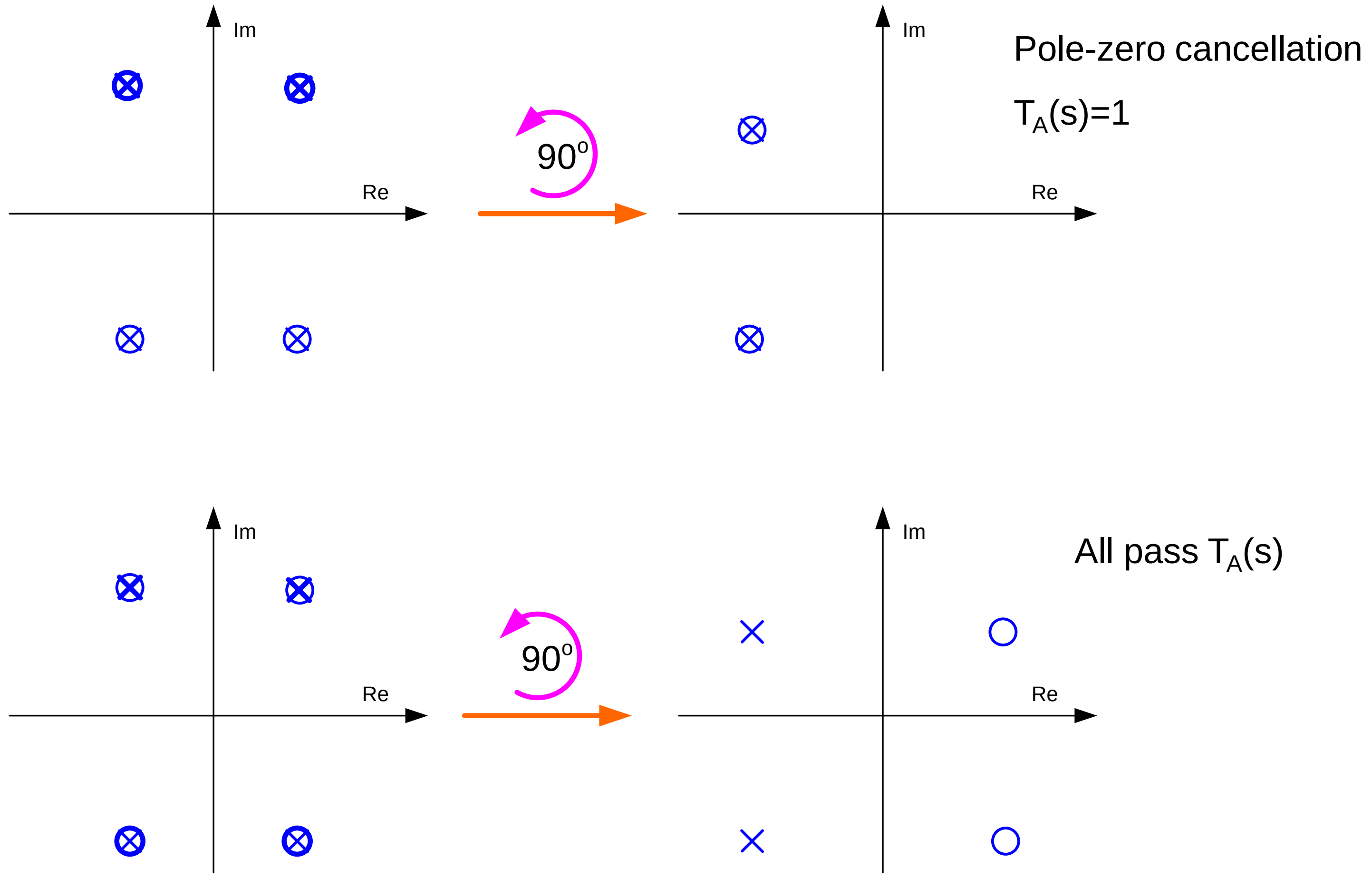
$$T_{AM}(s)=\frac{H_0(s\text{-}jz_1)(s\text{-}jz_2)\bullet\ldots\bullet(s\text{-}jz_m)}{(s\text{-}jp_1)(s\text{-}jp_2)\bullet\ldots\bullet(s\text{-}jp_n)}$$





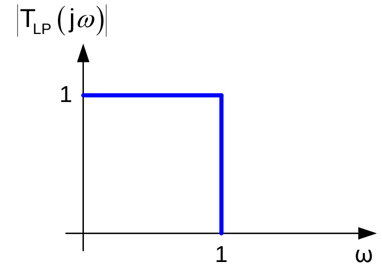
Review from Last Time

All pass functions (and factors)



- Must not allow cancellations to take place in $H_A(\omega^2)$ to obtain all-pass $T_A(s)$
- Must keep upper HP poles and lower HP zeros in $H_A(\omega^2)$ to obtain all-pass $T_A(s)$
- All-pass $T_A(s)$ is not minimum phase

The Approximation Problem

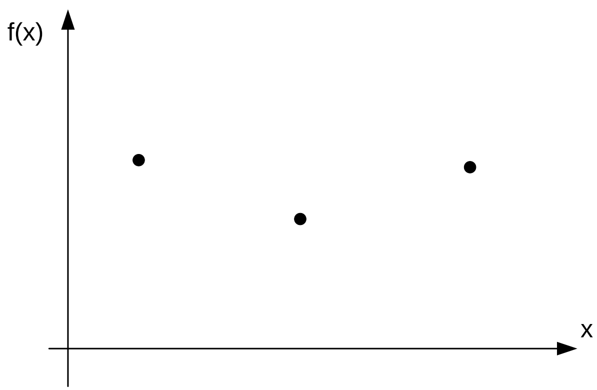


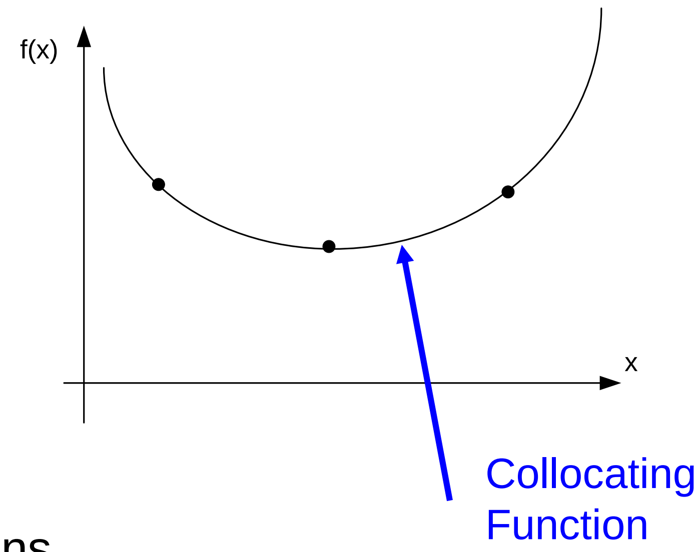
Approach we will follow:

- Magnitude Squared Approximating Functions $H_A(\omega^2)$
- Inverse Transform $H_A(\omega^2) \rightarrow T_A(s)$
- Collocation
- Least Squares
- Pade Approximations
- Other Analytical Optimization
- Numerical Optimization
- Canonical Approximations
  - $\rightarrow$ Butterworth (BW)
  - $\rightarrow$ Chebyschev (CC)
  - $\rightarrow$ Elliptic
  - $\rightarrow$ Thompson

Collocation

Collocation is the fitting of a function to a set of points (or measurements) so that the function agrees with the sample at each point in the set.





Often consider critically constrained functions

The function that is of interest for using collocation when addressing the approximation problem is $H_A(\omega^2)$

Collocation

Example: Collocation points $\{(x_1,y_1), (x_2,y_2),(x_3,y_3)\}$

Polynomial collocating function (critically constrained)

$$f(x) = a_0 + a_1x + a_2x^2$$

Unknowns: $\{a_1,a_2,a_3\}$

Set of equations:

$$y_1 = a_0 + a_1x_1 + a_2x_1^2$$
$$y_2 = a_0 + a_1x_2 + a_2x_2^2$$
$$y_3 = a_0 + a_1x_3 + a_2x_3^2$$

These equations are linear in the unknowns $\{a_1,a_2,a_3\}$

Can be expressed in matrix form

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{bmatrix} \bullet \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$$

$$\mathbf{Y} = \mathbf{X} \bullet \mathbf{A}$$

Solution:

$$\mathbf{A} = \mathbf{X}^{-1} \bullet \mathbf{Y}$$

Closed form solution exists when collocating to a polynomial

Collocation

Is it possible to get a closed-form solution when collocating to a rational fraction?

$$\{(x_1, y_1), (x_2, y_2) \ldots (x_k, y_k)\}$$

$$f(x) = \frac{a_0 + a_1 x + a_2 x^2 + \ldots + a_m x^m}{1 + b_1 x + b_2 x^2 + \ldots + b_n x^n}$$

where k=m+n+1

The rational fraction is nonlinear in x !

$$y_1\left(1 + b_1 x_1 + b_2 x_1^2 + \ldots + b_n x_1^n\right) = a_0 + a_1 x_1 + a_2 x_1^2 + \ldots + a_m x_1^n$$

This can be expressed as

$$y_1 = a_0 + a_1 x_1 + a_2 x_1^2 + \ldots + a_m x_1^n - b_1 x_1 y_1 - b_2 x_1^2 y_1 - \ldots - b_n x_1^n y_1$$

Note this equation is linear in the unknowns $\{a_0, a_1, \ldots a_m, b_1, b_2, \ldots b_n\}$

Collocation

Is it possible to get a closed-form solution when collocating to a rational fraction?

$$\{(x_1, y_1), (x_2, y_2) \ldots (x_k, y_k)\}$$

$$f(x) = \frac{a_0 + a_1 x + a_2 x^2 + \ldots + a_m x^m}{1 + b_1 x + b_2 x^2 + \ldots + b_n x^n}$$

where k=m+n+1

$$y_1 = a_0 + a_1 x_1 + a_2 x_1^2 + \ldots + a_m x_1^m - b_1 x_1 y_1 - b_2 x_1^2 y_1 - \ldots - b_n x_1^n y_1$$

$$y_2 = a_0 + a_1 x_2 + a_2 x_2^2 + \ldots + a_m x_2^m - b_1 x_2 y_2 - b_2 x_2^2 y_2 - \ldots - b_n x_2^n y_2$$

.

.

.

$$y_k = a_0 + a_1 x_k + a_2 x_k^2 + \ldots + a_m x_k^m - b_1 x_k y_k - b_2 x_k^2 y_k - \ldots - b_n x_k^n y_k$$

Collocation

Is it possible to get a closed-form solution when collocating to a rational fraction?

$$\{(x_1, y_1), (x_2, y_2) \ldots (x_k, y_k)\} \qquad f(x) = \frac{a_0 + a_1 x + a_2 x^2 + \ldots + a_m x^m}{1 + b_1 x + b_2 x^2 + \ldots + b_n x^n}$$

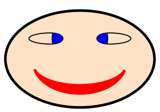
$$\begin{aligned}
y_1 &= a_0 + a_1 x_1 + a_2 x_1^2 + \ldots + a_m x_1^m - b_1 x_1 y_1 - b_2 x_1^2 y_1 - \ldots - b_n x_1^n y_1 \\
y_2 &= a_0 + a_1 x_2 + a_2 x_2^2 + \ldots + a_m x_2^m - b_1 x_2 y_2 - b_2 x_2^2 y_2 - \ldots - b_n x_2^n y_2 \\
&. \\
&. \\
&. \\
y_k &= a_0 + a_1 x_k + a_2 x_k^2 + \ldots + a_m x_k^m - b_1 x_k y_k - b_2 x_k^2 y_k - \ldots - b_n x_k^n y_k
\end{aligned}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \bullet \\ \bullet \\ y_k \end{bmatrix} = \begin{bmatrix} 1 \; x_1 \; x_1^2 \ldots x_1^m - x_1 y_1 - x_1^2 y_1 - \ldots - x_1^n y_1 \\ 1 \; x_2 \; x_2^2 \ldots x_2^m - x_2 y_2 - x_2^2 y_2 - \ldots - x_2^n y_2 \\ \bullet \\ \bullet \\ 1 \; x_k \; x_k^2 \ldots x_k^m - x_k y_k - x_k^2 y_k - \ldots - x_k^n y_k \end{bmatrix} \bullet \begin{bmatrix} a_0 \\ a_1 \\ .. \\ a_m \\ b_1 \\ b_2 \\ \ldots \\ b_n \end{bmatrix}$$

$$\mathbf{Y} = \mathbf{Z} \bullet \mathbf{C}$$

$$\mathbf{C} = \mathbf{Z}^{-1} \bullet \mathbf{Y}$$

Closed form solution when collocating to a rational fraction !



Collocation

Applying to $H_A\left(\omega^2\right)$

$$\left\{(\omega_1, y_1),(\omega_2, y_2)\dots(\omega_k, y_k)\right\}$$

$$H_A\left(\omega^2\right)=\frac{a_0+a_1\omega^2+a_2\omega^4+\dots+a_m\omega^{2m}}{1+b_1\omega^2+b_2\omega^4+\dots+b_n\omega^{2n}}$$

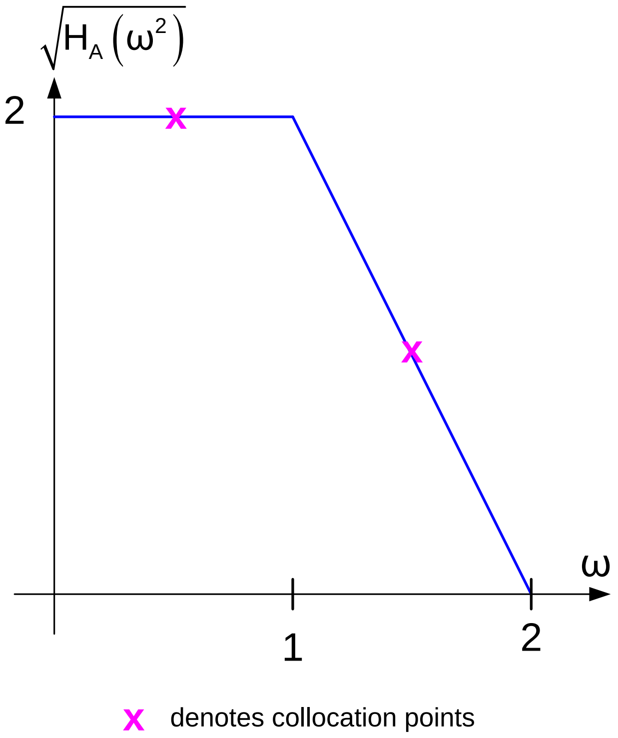
$$\begin{bmatrix} y_1 \\ y_2 \\ \bullet \\ \bullet \\ y_k \end{bmatrix} = \begin{bmatrix} 1 & \omega_1^2 & \omega_1^4 \dots \omega_1^{2m} - \omega_1^2 y_1 - \omega_1^4 y_1 - \dots - \omega_1^{2n} y_1 \\ 1 & \omega_2^2 & \omega_2^4 \dots \omega_2^{2m} - \omega_2^2 y_1 - \omega_2^4 y_1 - \dots - \omega_2^{2n} y_1 \\ \bullet & & \\ \bullet & & \\ 1 & \omega_k^2 & \omega_k^4 \dots \omega_k^{2m} - \omega_k^2 y_1 - \omega_k^4 y_1 - \dots - \omega_k^{2n} y_1 \end{bmatrix} \bullet \begin{bmatrix} a_0 \\ a_1 \\ .. \\ a_m \\ b_1 \\ b_2 \\ \dots \\ b_n \end{bmatrix}$$

$$\mathbf{Y}=\mathbf{Z}\bullet\mathbf{C}$$

$$\mathbf{C}=\mathbf{Z}^{-1}\bullet\mathbf{Y}$$

Collocation

Example:



$$H_A(\omega^2) = \frac{a_0}{1+b_1\omega^2}$$

$$\left.\begin{aligned} 4 &= \frac{a_0}{1+b_1\left(\tfrac{1}{2}\right)^2} \\ 1 &= \frac{a_0}{1+b_1\left(\tfrac{3}{2}\right)^2} \end{aligned}\right\} \Rightarrow H_A(\omega^2) = \frac{32/5}{1+\left(12/5\right)\omega^2}$$

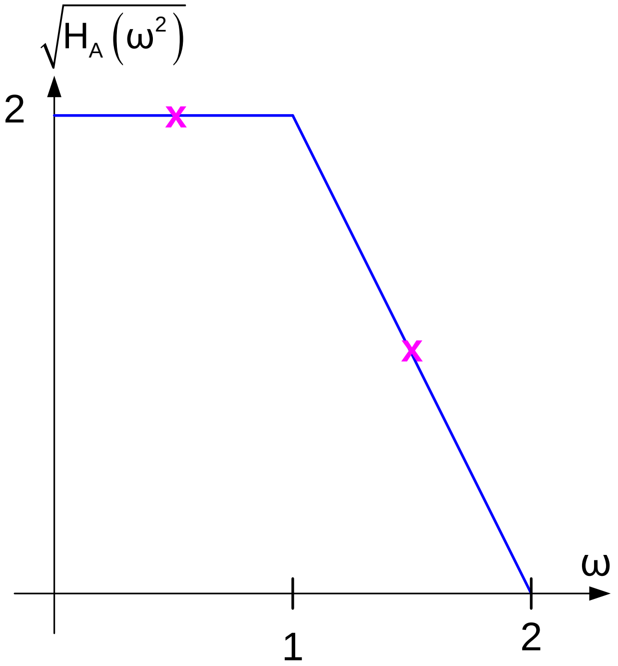
poles at $s = \pm j\sqrt{5/12}$

Collocation

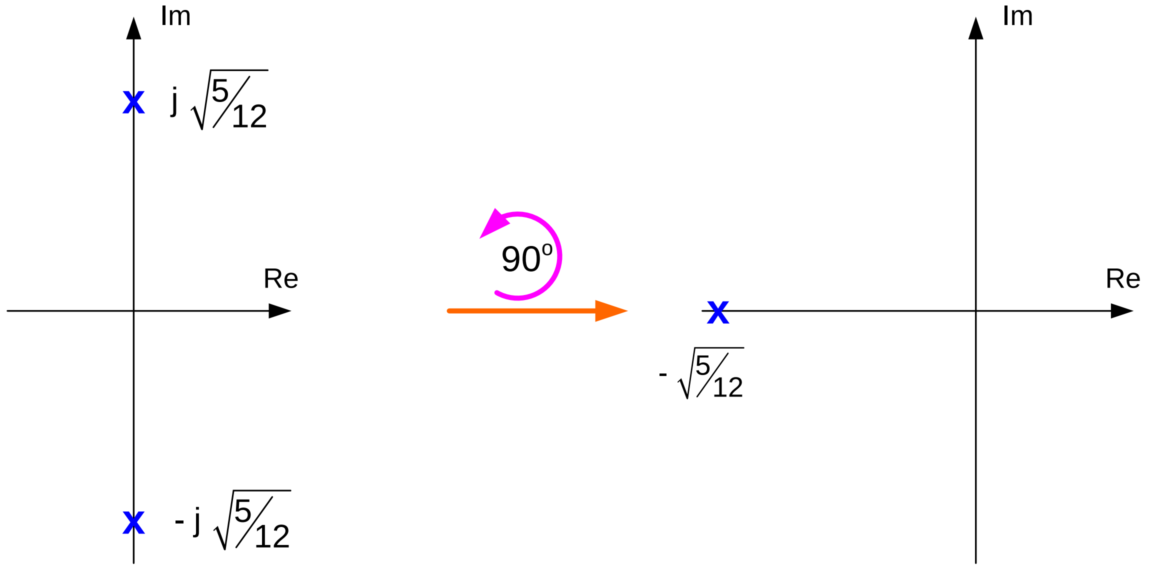
Example:

poles at

$$s = \pm j\sqrt{5/12}$$



x denotes collocation points



Roots of $H_A(\omega^2)$

Roots of $T_{AM}(s)$

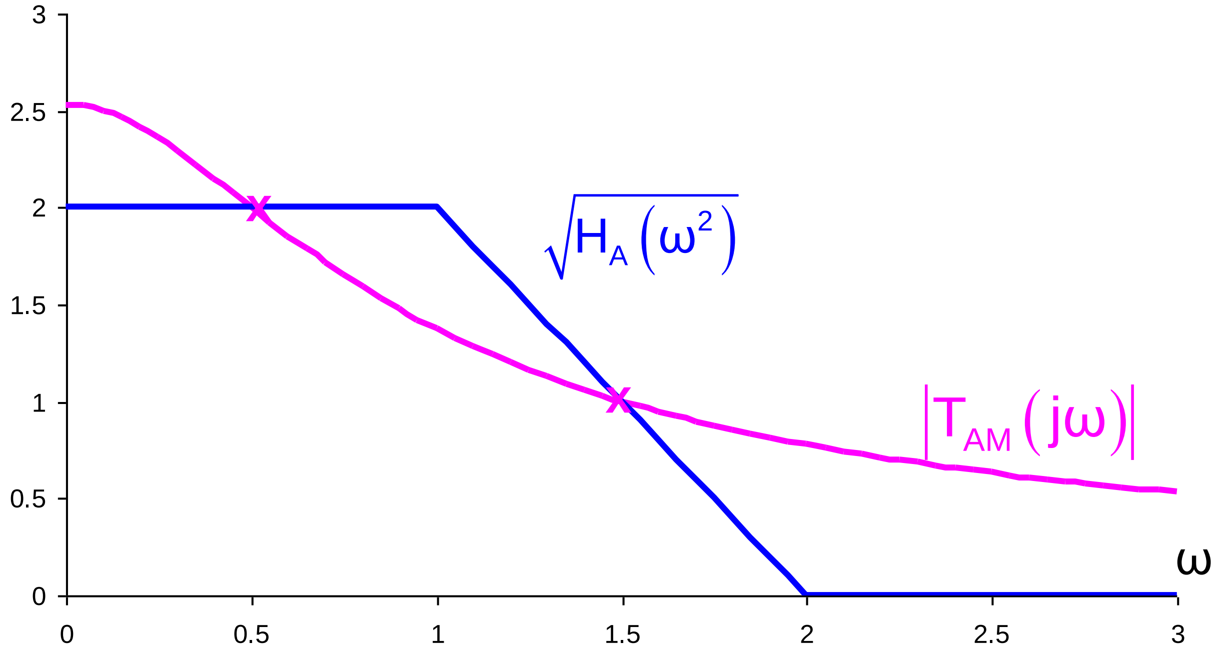
$$T_{AM}(s) = \frac{\sqrt{8/3}}{s + \sqrt{5/12}}$$

Collocation

Example:

$$H_A(\omega^2) = \frac{32/5}{1 + (12/5)\omega^2}$$

$$T_{AM}(s) = \frac{\sqrt{8/3}}{s + \sqrt{5/12}}$$

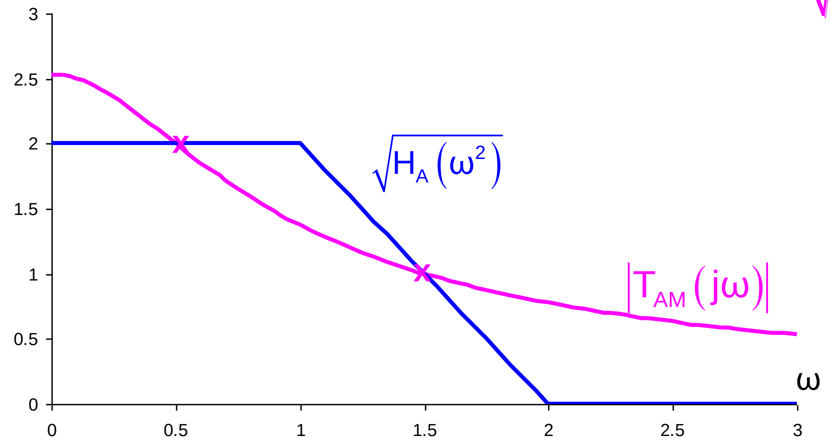


The approximation is reasonable but not too good

Collocation

Example:

$$H_A\left(\omega^2\right) = \frac{32/5}{1+\left(12/5\right)\omega^2} \qquad T_{AM}(s) = \frac{\sqrt{8/3}}{s+\sqrt{5/12}}$$



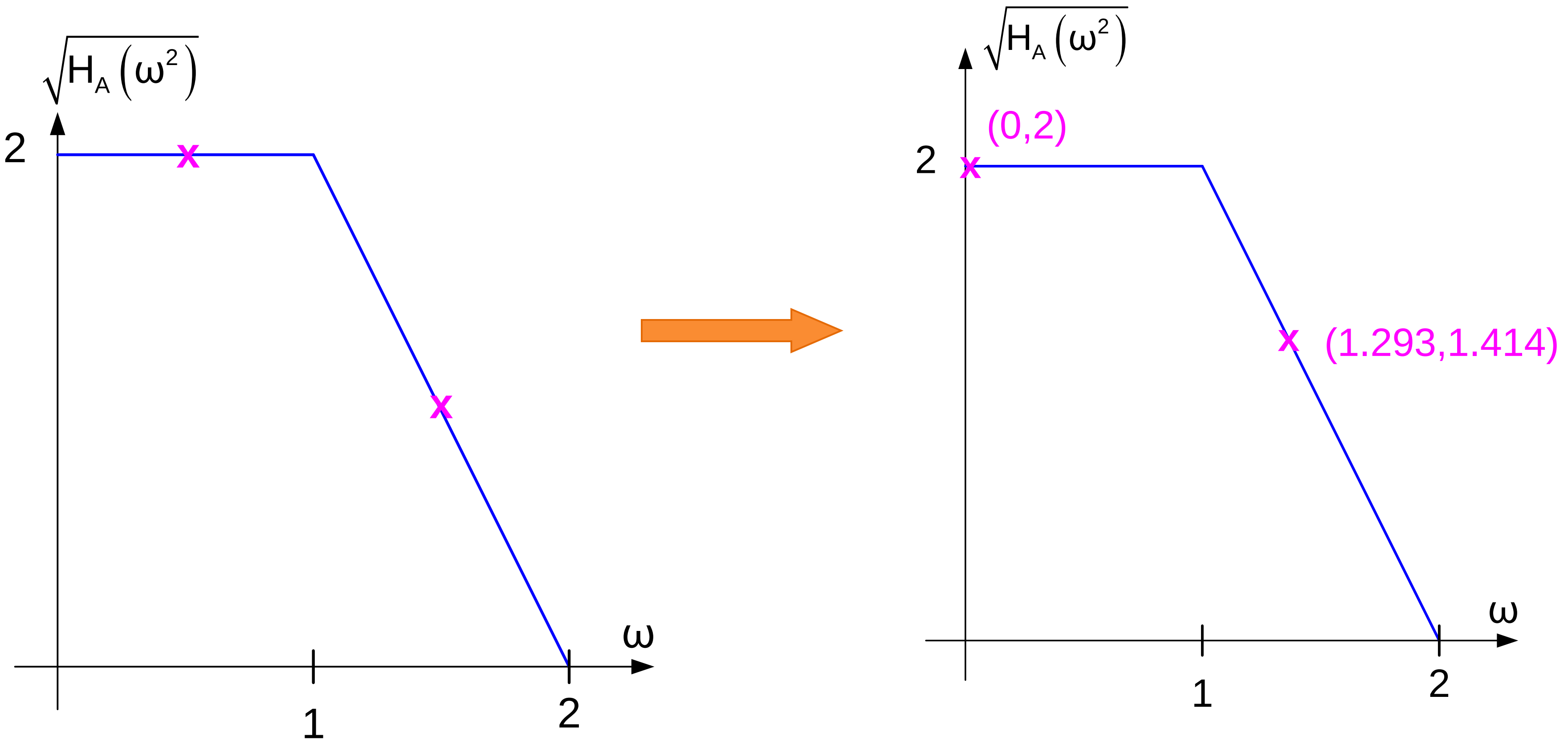
- The problem was critically constrained from a function viewpoint (two variables and two equations)

- Highly under-constrained as an approximation technique since the collocation points are also variables

Collocation

Example: same $H_A\left(\omega^2\right)$ but with different collocation points

$$H_A\left(\omega^2\right) = \frac{a_0}{1+b_1\omega^2}$$



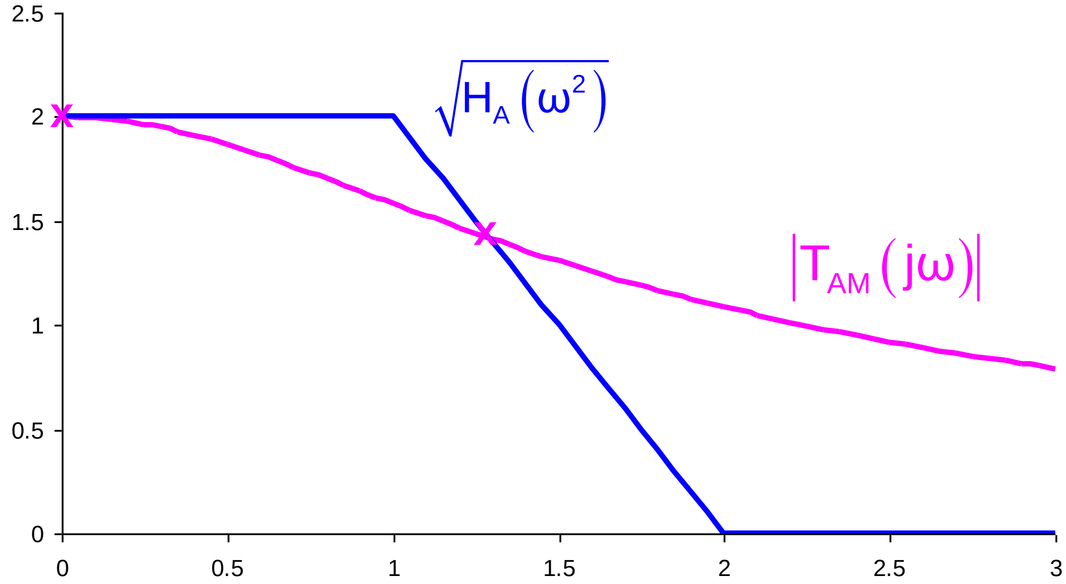
Collocation

Example: same $H_A(\omega^2)$ but with different collocation points

$$H_A(\omega^2) = \frac{a_0}{1+b_1\omega^2}$$

$$\left.\begin{aligned} 4 &= \frac{a_0}{1+b_1(0)^2} \\ 2 &= \frac{a_0}{1+b_1(1.293)^2} \end{aligned}\right\} \Rightarrow \quad H_A(\omega^2) = \frac{4}{1+(.598)\omega^2}$$

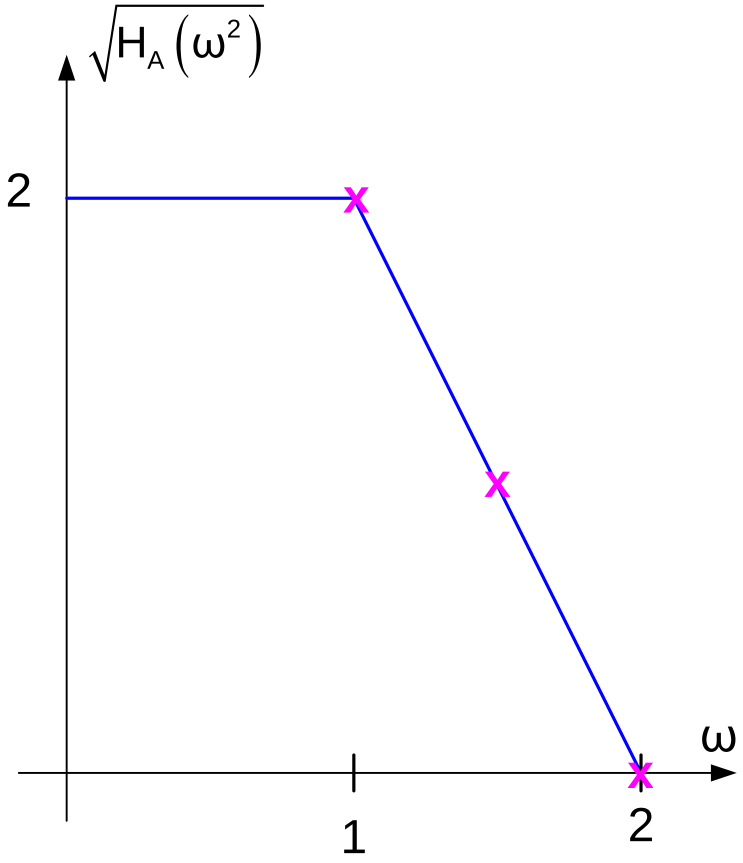
$$T_{AM}(s) = \frac{1.293}{s + 1.293}$$



Choice of collocation points plays a big role on the approximation

Collocation

Example: same $H_A\left(\omega^2\right)$ but with different collocation points and different approximating function



$$H_A\left(\omega^2\right) = \frac{a_0 + a_1\omega^2}{1+b_1\omega^2}$$

$$\left.\begin{aligned} 4 &= \frac{a_0 + a_1}{1+b_1} \\ 1 &= \frac{a_0 + a_1(3/2)^2}{1+b_1(3/2)^2} \\ 0 &= \frac{a_0 + a_1(4)}{1+b_1(4)} \end{aligned}\right\} \Rightarrow \quad H_A\left(\omega^2\right) = \frac{-80 + 20\omega^2}{1 + -16\omega^2}$$

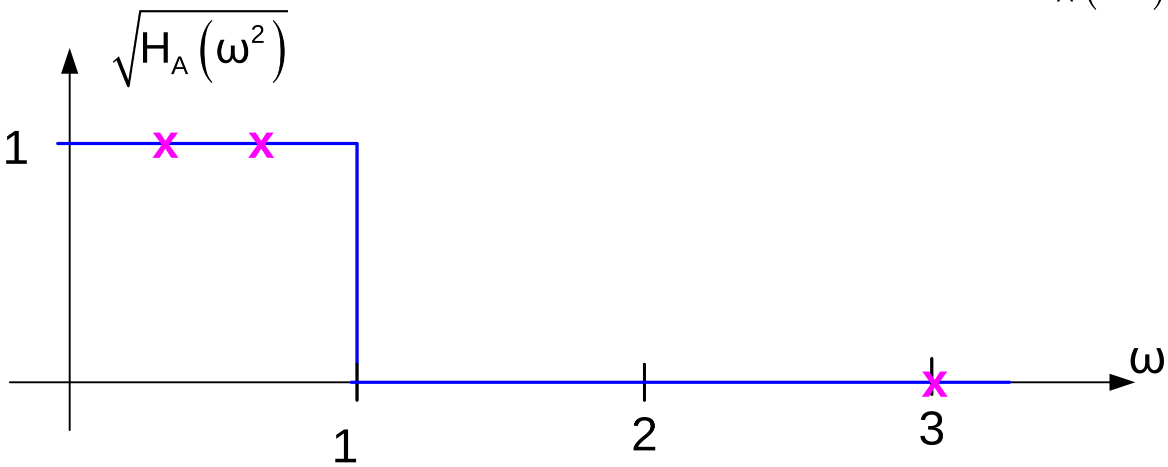
a_0=-80, a_1=20, b_1=-16

Inverse mapping does not exist because roots of odd multiplicity on real axis

Collocation

Example:

$$H_A\left(\omega^2\right) = \frac{a_0 + a_1\omega^2}{1+b_1\omega^2}$$



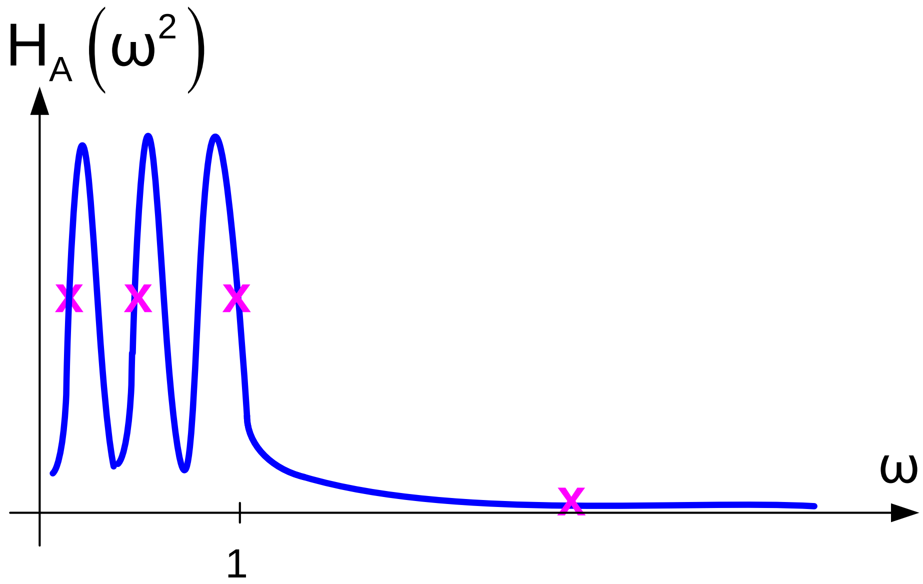
$$\left.\begin{aligned} 1 &= \frac{a_0 + a_1(1/9)}{1+b_1(1/9)} \\ 1 &= \frac{a_0 + a_1(4/9)}{1+b_1(4/9)} \\ 0 &= \frac{a_0 + a_1(9)}{1+b_1(9)} \end{aligned}\right\} \Rightarrow \quad H_A\left(\omega^2\right) = \frac{1+(-27/243)\omega^2}{1+(-27/243)\omega^2}$$

$a_0=1$, $a_1=-27/243$, $b_1=-27/243$

- This solution is equal to 1 at all frequencies except ω=3 where it is undefined
- Thus there is no solution with these collocation points

Collocation

Example:



In some situations, collocation causes a lot of ripple between the collocation points

Collocation Observations

Fitting an approximating function to a set of data or points (collocation points)

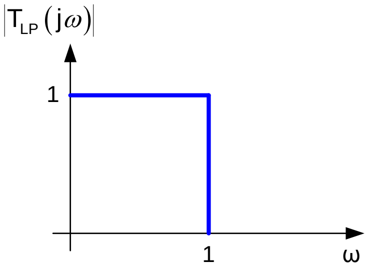
- Closed-form matrix solution for fitting to a rational fraction in ω^2
- Can be useful when somewhat nonstandard approximations are required
- Quite sensitive to collocation points
- Although function is critically constrained, since collocation points are variables, highly under constrained as an optimization approach
- Although fit will be perfect at collocation points, significant deviation can occur close to collocation points
- Inverse mapping to $T_A(s)$ may not exist
- Solution may not exist at specified collocation points

Collocation

What is the major contributor to the limitations observed with the collocation approach?

- Totally dependent upon the value of the desired response at a small but finite set of points (no consideration for anything else)
- Highly dependent upon value of approximating function at a single point or at a small number of points
- Highly dependent upon the collocation points

The Approximation Problem



Approach we will follow:

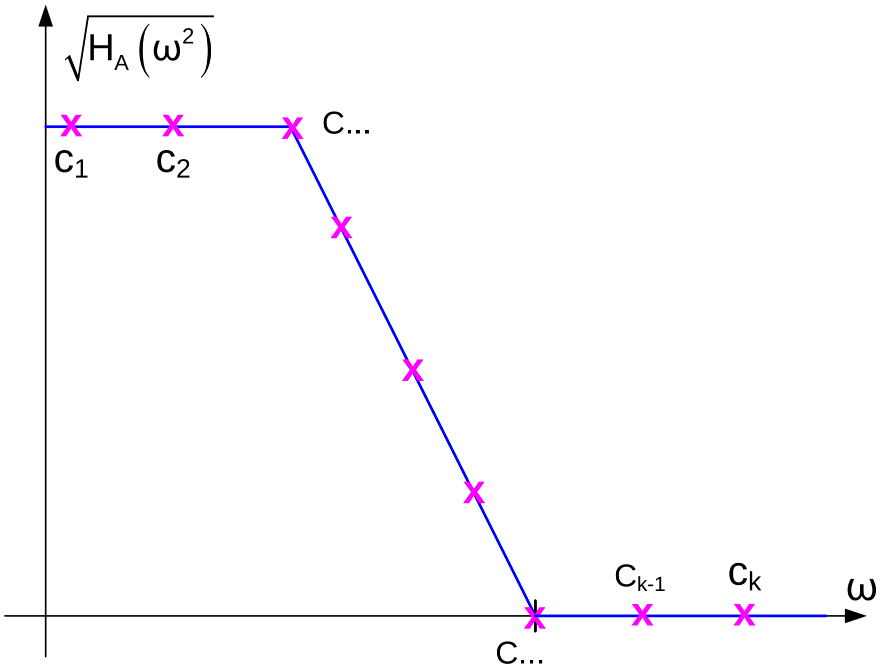
- Magnitude Squared Approximating Functions $H_A(\omega^2)$
- Inverse Transform $H_A(\omega^2) \rightarrow T_A(s)$
- Collocation
- Least Squares
- Pade Approximations
- Other Analytical Optimization
- Numerical Optimization
- Canonical Approximations
  - $\rightarrow$ Butterworth (BW)
  - $\rightarrow$ Chebyschev (CC)
  - $\rightarrow$ Elliptic
  - $\rightarrow$ Thompson

Least Squares Approximation

To minimize the heavy dependence on a small number of points, will consider many points thus creating an over-constrained system

$$H_A\left(\omega^2\right) = \frac{\sum_{i=0}^{m} a_i \omega^{2i}}{1+\sum_{i=1}^{n} b_i \omega^{2i}}$$

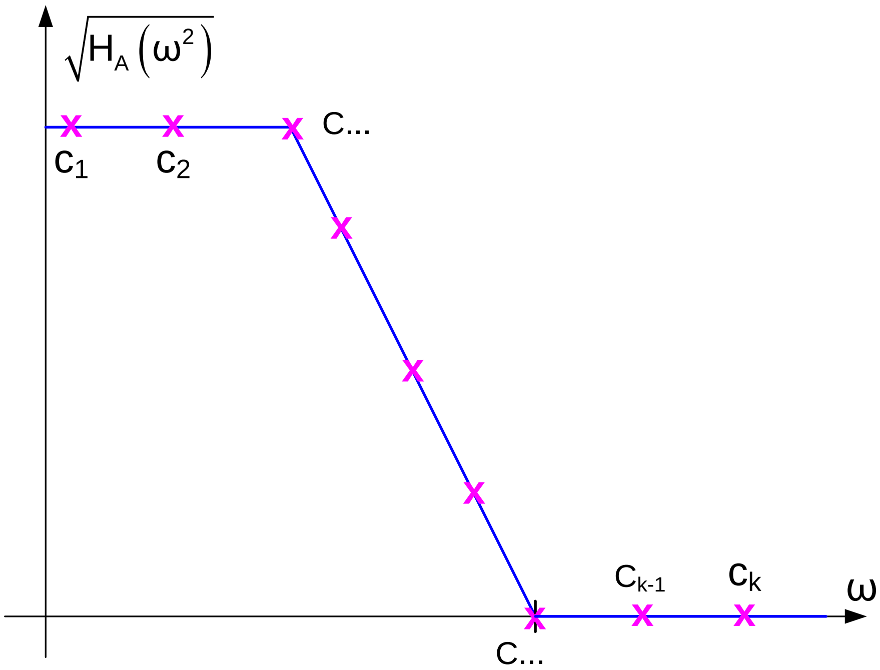
$k > m+n+1$



Approximating function can not be forced to go through all points
But, it can be “close” to all points in some sense

Least Squares Approximation

$$H_A\left(\omega^2\right) = \frac{\sum_{i=0}^{m} a_i \omega^{2i}}{1+\sum_{i=1}^{n} b_i \omega^{2i}}$$



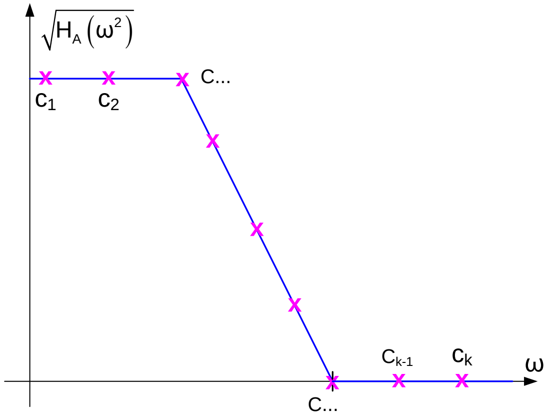
Define the error at point i by

$$\varepsilon_i = H_D\left(\omega_i\right) - H_A\left(\omega_i\right)$$

where $H_D(\omega_i)$ is the desired magnitude squared response at ω_i and where $H_A(\omega_i)$ is the magnitude squared response of the approximating function

Least Squares Approximation

$$H_A\left(\omega^2\right) = \frac{\sum_{i=0}^{m} a_i \omega^{2i}}{1+\sum_{i=1}^{n} b_i \omega^{2i}}$$



$$\varepsilon_i = H_D\left(\omega_i\right) - H_A\left(\omega_i\right)$$

Goal is to minimize some metrics associated with ε_i at a large number of points

Some possible cost functions

$$C_1 = \sum_{i=1}^{N} \left|\varepsilon_i\right| \qquad C_2 = \sum_{i=1}^{N} \varepsilon_i^2 \qquad C_3 = \sum_{i=1}^{N} w_i\, \varepsilon_i^2$$

w_i a weighting function

- Reduces emphasis on individual points
- Some much better than others from performance viewpoint
- Some much better than others from computation viewpoint

Least Squares Approximation

$$H_A\left(\omega^2\right) = \frac{\sum_{i=0}^{m} a_i \omega^{2i}}{1+\sum_{i=1}^{n} b_i \omega^{2i}}$$

$$\varepsilon_i = H_D\left(\omega_i\right) - H_A\left(\omega_i\right)$$

$$C_3 = \sum_{i=1}^{N} w_i \, \varepsilon_i^2$$

w_i a weighting function

Least Mean Square (LMS) based cost functions have minimums that can be analytically determined for some useful classes of approximating functions $H_A(\omega^2)$

Regression Analysis Review

Consider an nth order polynomial in x

$$F(x) = \sum_{k=0}^{n} a_k x^k$$

Consider N samples of a function $\tilde{F}(x)$

$$\hat{F}(x) = \left\langle \tilde{F}(x_i) \right\rangle_{i=1}^{N}$$

where the sampling coordinate variables are

$$X = \left\langle x_i \right\rangle_{i=1}^{N}$$

Define the summed square difference cost function as

$$C = \sum_{i=0}^{N} \left(F(x_i) - \tilde{F}(x_i) \right)^2$$

A standard regression analysis can be used to minimize C with respect to $\{a_0, a_1, \ldots a_n\}$

To do this, take the n+1 partials of C wrt the a_i variables

Regression Analysis Review

$$C = \sum_{i=0}^{N}\left(F(x_i) - \tilde{F}(x_i)\right)^2 \qquad F(x) = \sum_{k=0}^{n} a_k x^k$$

$$C = \sum_{i=0}^{N}\left(\sum_{k=0}^{n} a_k x_i^k - \tilde{F}(x_i)\right)^2$$

Taking the partial of C wrt each coefficient and setting to 0, we obtain the set of equations

$$\left.\begin{aligned}
\frac{\partial C}{\partial a_0} &= 2\sum_{i=0}^{N}\left(\sum_{k=0}^{n} a_k x_i^k - \tilde{F}(x_i)\right) = 0 \\
\frac{\partial C}{\partial a_1} &= 2\sum_{i=0}^{N} x_i^1\left(\sum_{k=0}^{n} a_k x_i^k - \tilde{F}(x_i)\right) = 0 \\
\frac{\partial C}{\partial a_2} &= 2\sum_{i=0}^{N} x_i^2\left(\sum_{k=0}^{n} a_k x_i^k - \tilde{F}(x_i)\right) = 0 \\
&\cdots \\
\frac{\partial C}{\partial a_n} &= 2\sum_{i=0}^{N} x_i^n\left(\sum_{k=0}^{n} a_k x_i^k - \tilde{F}(x_i)\right) = 0
\end{aligned}\right\}$$

This is linear in the a_ks.

$$X \bullet A = F$$

$$A = \begin{bmatrix} a_0 \\ a_1 \\ \ldots \\ a_n \end{bmatrix}$$

Solution is

$$A = X^{-1} \bullet F$$

Regression Analysis Review

A few details about regression analysis:

$$X \bullet A = F$$

$$A = X^{-1} \bullet F$$

$$X = \begin{bmatrix} N+1 & \sum_{i=0}^{N} X_i & \sum_{i=0}^{N} X_i^2 & \ldots & \sum_{i=0}^{N} X_i^n \\ \sum_{i=0}^{N} X_i & \sum_{i=0}^{N} X_i^2 & \ldots & & \sum_{i=0}^{N} X_i^{n+1} \\ \sum_{i=0}^{N} X_i^2 & \ldots. & & & \sum_{i=0}^{N} X_i^{n+2} \\ \ldots & & & & \\ \sum_{i=0}^{N} X_i^n & \sum_{i=0}^{N} X_i^n & \ldots & & \sum_{i=0}^{N} X_i^{2n} \end{bmatrix}$$

$$A = \begin{bmatrix} a_0 \\ a_1 \\ \ldots \\ a_n \end{bmatrix}$$

$$F = \begin{bmatrix} \sum_{i=0}^{N} \tilde{F}(x_i) \\ \sum_{i=0}^{N} x_i \tilde{F}(x_i) \\ \sum_{i=0}^{N} x_i^2 \tilde{F}(x_i) \\ \ldots \\ \sum_{i=0}^{N} x_i^n \tilde{F}(x_i) \end{bmatrix}$$

Regression Analysis Review

$$C = \sum_{i=0}^{N} \left(F(x_i) - \tilde{F}(x_i) \right)^2 \qquad F(x) = \sum_{k=0}^{n} a_k x^k$$

$$C = \sum_{i=0}^{N} \left(\sum_{k=0}^{n} a_k x_i^k - \tilde{F}(x_i) \right)^2$$

$$A = X^{-1} \bullet F$$

Observations about Regression Analysis:

- Closed form solution
- Requires inversion of a (n+1) dimensional square matrix
- Not highly sensitive to any single measurement
- Widely used for fitting a set of data to a polynomial model
- Points need not be uniformly distributed
- Adding weights does not complicate solution

End of Lecture 8

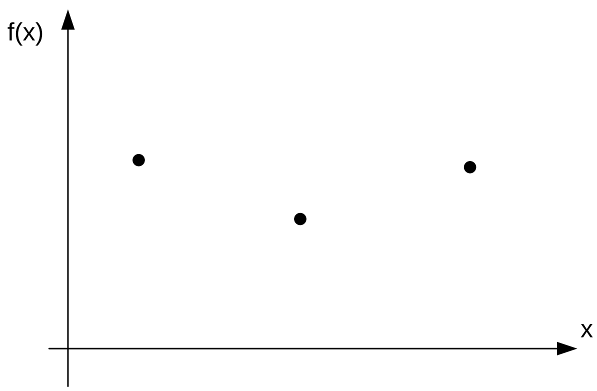
EE 508
Lecture 9

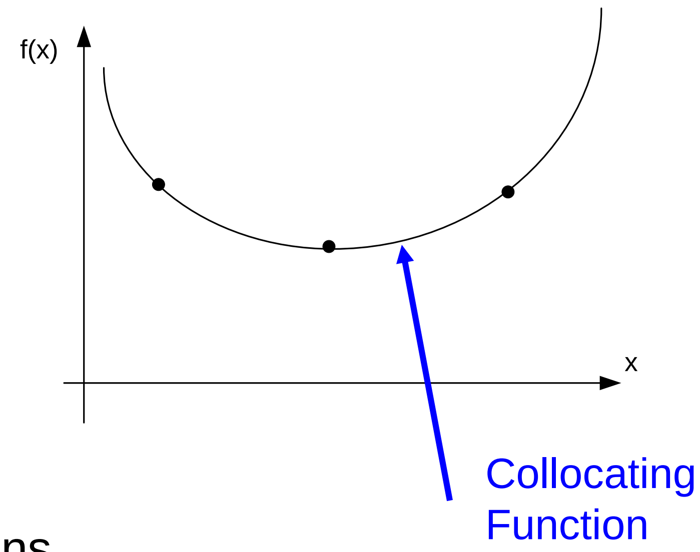
The Approximation Problem

Review from Last Time

Collocation

Collocation is the fitting of a function to a set of points (or measuremetns) so that the functin agrees wth the sample at each point in the set.





Often consider critically constrained functions

The function that is of interest for using collocation when addressing the approximation problem is $H_A(\omega^2)$

Review from Last Time

Collocation

Applying to $H_A\left(\omega^2\right)$

$$\left\{(\omega_1, y_1),(\omega_2, y_2)\ldots(\omega_k, y_k)\right\}$$

$$H_A\left(\omega^2\right)=\frac{a_0+a_1\omega^2+a_2\omega^4+\ldots+a_m\omega^{2m}}{1+b_1\omega^2+b_2\omega^4+\ldots+b_n\omega^{2n}}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \bullet \\ \bullet \\ y_k \end{bmatrix}=\begin{bmatrix} 1 & \omega_1^2 & \omega_1^4\ldots\omega_1^{2m} & -\omega_1^2 y_1 & -\omega_1^4 y_1 & -\ldots-\omega_1^{2n} y_1 \\ 1 & \omega_2^2 & \omega_2^4\ldots\omega_2^{2m} & -\omega_2^2 y_1 & -\omega_2^4 y_1 & -\ldots-\omega_2^{2n} y_1 \\ \bullet \\ \bullet \\ 1 & \omega_k^2 & \omega_k^4\ldots\omega_k^{2m} & -\omega_k^2 y_1 & -\omega_k^4 y_1 & -\ldots-\omega_k^{2n} y_1 \end{bmatrix}\bullet\begin{bmatrix} a_0 \\ a_1 \\ .. \\ a_m \\ b_1 \\ b_2 \\ \ldots \\ b_n \end{bmatrix}$$

$$\mathbf{Y}=\mathbf{Z}\bullet\mathbf{C}$$

$$\mathbf{C}=\mathbf{Z}^{-1}\bullet\mathbf{Y}$$

Collocation Observations

Fitting an approximating function to a set of data or points (collocation points)

- Closed-form matrix solution for fitting to a rational fraction in ω^2
- Can be useful when somewhat nonstandard approximations are required
- Quite sensitive to collocation points
- Although function is critically constrained, since collocation points are variables, highly under constrained as an optimization approach
- Although fit will be perfect at collocation points, significant deviation can occur close to collocation points
- Inverse mapping to $T_A(s)$ may not exist
- Solution may not exist at specified collocation points

Review from Last Time

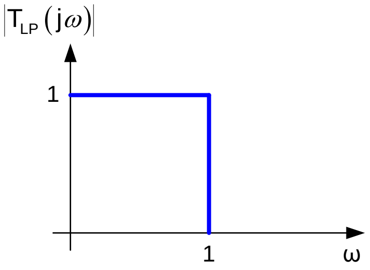
Collocation

What is the major contributor to the limitations observed with the collocation approach?

- Totally dependent upon the value of the desired response at a small but finite set of points (no consideration for anything else)
- Highly dependent upon value of approximating function at a single point or at a small number of points
- Highly dependent upon which points are chosen

Review from Last Time

The Approximation Problem



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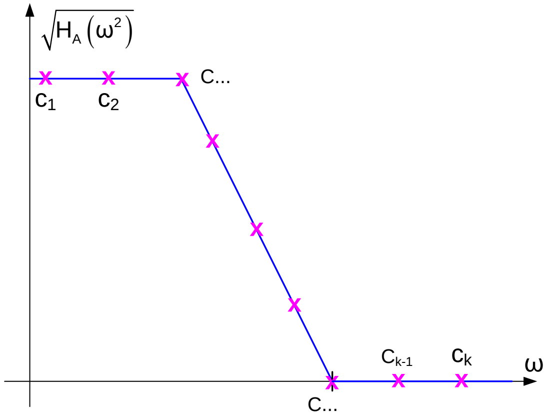
Review from Last Time

Least Squares Approximation

$$H_A(\omega^2) = \frac{\sum_{i=0}^{m} a_i \omega^{2i}}{1+\sum_{i=1}^{n} b_i \omega^{2i}}$$



$$\varepsilon_i = H_D(\omega_i) - H_A(\omega_i)$$

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Some possible cost functions

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Review from Last Time

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Define the summed square difference cost function as

$$C = \sum_{i=0}^{N} \left(F(x_i) - \tilde{F}(x_i) \right)^2$$

A standard regression analysis can be used to minimize C with respect to $\{a_0, a_1, \ldots a_n\}$

To do this, take the n+1 partials of C wrt the a_i variables

Regression Analysis Review

A few details about regression analysis:

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$$A = X^{-1} \bullet F$$

$$X = \begin{bmatrix} N+1 & \sum_{i=0}^{N} X_i & \sum_{i=0}^{N} X_i^2 & \dots & \sum_{i=0}^{N} X_i^n \\ \sum_{i=0}^{N} X_i & \sum_{i=0}^{N} X_i^2 & \dots & & \sum_{i=0}^{N} X_i^{n+1} \\ \sum_{i=0}^{N} X_i^2 & \dots\dots & & & \sum_{i=0}^{N} X_i^{n+2} \\ \dots & & & & \\ \sum_{i=0}^{N} X_i^n & \sum_{i=0}^{N} X_i^n & \dots & & \sum_{i=0}^{N} X_i^{2n} \end{bmatrix}$$

$$A = \begin{bmatrix} a_0 \\ a_1 \\ \dots \\ a_n \end{bmatrix}$$

$$F = \begin{bmatrix} \sum_{i=0}^{N} \tilde{F}(x_i) \\ \sum_{i=0}^{N} x_i \tilde{F}(x_i) \\ \sum_{i=0}^{N} x_i^2 \tilde{F}(x_i) \\ \dots \\ \sum_{i=0}^{N} x_i^n \tilde{F}(x_i) \end{bmatrix}$$

Review from Last Time

Regression Analysis Review

$$C = \sum_{i=0}^{N} \left(F(x_i) - \tilde{F}(x_i) \right)^2 \qquad F(x) = \sum_{k=0}^{n} a_k x^k$$

$$C = \sum_{i=0}^{N} \left(\sum_{k=0}^{n} a_k x_i^k - \tilde{F}(x_i) \right)^2$$

$$A = X^{-1} \bullet F$$

Observations about Regression Analysis:

- Closed form solution
- Requires inversion of a (n+1) dimensional square matrix
- Not highly sensitive to any single measurement
- Widely used for fitting a set of data to a polynomial model
- Points need not be uniformly distributed
- Adding weights does not complicate solution

Least Squares Approximations of Transfer Functions

$$T(s) = \frac{\sum_{i=0}^{m} a_i s^i}{\sum_{i=0}^{n} b_i s^i}$$

WLOG $b_0 = 1$

$$T(j\omega) = \frac{\left[\sum_{\substack{i=0\\ i\ \text{odd}}}^{m} (-1)^i a_i \omega^i\right] + \left[\sum_{\substack{i=0\\ i\ \text{even}}}^{m} (-1)^i a_i \omega^i\right] j}{\left[\sum_{\substack{i=0\\ i\ \text{odd}}}^{n} (-1)^i b_i \omega^i\right] + \left[\sum_{\substack{i=0\\ i\ \text{even}}}^{n} (-1)^i b_i \omega^i\right] j}$$

$$\left|T(j\omega)\right| = \sqrt{\frac{\left[\sum_{\substack{i=0\\ i\ \text{odd}}}^{m} (-1)^i a_i \omega^i\right]^2 + \left[\sum_{\substack{i=0\\ i\ \text{even}}}^{m} (-1)^i a_i \omega^i\right]^2}{\left[\sum_{\substack{i=0\\ i\ \text{odd}}}^{n} (-1)^i b_i \omega^i\right]^2 + \left[\sum_{\substack{i=0\\ i\ \text{even}}}^{n} (-1)^i b_i \omega^i\right]^2}}$$

$|T(j\omega)|$ is highly nonlinear in $\langle a_k \rangle$ and $\langle b_k \rangle$

Least Squares Approximations of Transfer Functions

$$T(s) = \frac{\sum_{i=0}^{m} a_i s^i}{\sum_{i=0}^{n} b_i s^i} \qquad \text{WLOG } b_0 = 1$$

$$\left|T(j\omega)\right| = \sqrt{\frac{\left[\sum_{\substack{i=0 \\ i\text{ odd}}}^{m} (-1)^i a_i \omega^i\right]^2 + \left[\sum_{\substack{i=0 \\ i\text{ even}}}^{m} (-1)^i a_i \omega^i\right]^2}{\left[\sum_{\substack{i=0 \\ i\text{ odd}}}^{n} (-1)^i b_i \omega^i\right]^2 + \left[\sum_{\substack{i=0 \\ i\text{ even}}}^{n} (-1)^i b_i \omega^i\right]^2}}$$

Consider the natural cost function

$$C = \sum_{k=1}^{N} \left(\left| T(j\omega_k) \right| - \tilde{T}(\omega_k) \right)^2$$

$$\left. \begin{array}{c} \dfrac{\partial C}{\partial a_k} \\[2ex] \dfrac{\partial C}{\partial b_k} \end{array} \right\}$$

both are highly nonlinear in $\langle a_k \rangle$ and $\langle b_k \rangle$

Closed form solution for optimal values of $\langle a_k \rangle$ and $\langle b_k \rangle$ does not exist

Least Squares Approximations of Transfer Functions

$$T(s) = \frac{\sum_{i=0}^{m} a_i s^i}{\sum_{i=0}^{n} b_i s^i} \qquad \text{WLOG } b_0 = 1$$

Consider

$$H_A\left(\omega^2\right) = \frac{\sum_{i=0}^{m} c_i \omega^{2i}}{\sum_{i=0}^{n} d_i \omega^{2i}}$$

Consider the cost function

$$C = \sum_{k=1}^{N} \left(H_A\left(\omega_k^2\right) - \tilde{H}\left(\omega_k^2\right) \right)^2$$

What about the sets of equations $\left\langle \frac{\partial C}{\partial c_k} \right\rangle_{k=1}^{m}$ and $\left\langle \frac{\partial C}{\partial d_k} \right\rangle_{k=1}^{n}$

Rewriting the cost function

$$C = \sum_{k=1}^{N} \left(\frac{\sum_{i=0}^{m} c_i \omega^{2i}}{\sum_{i=0}^{n} d_i \omega^{2i}} - \tilde{H}\left(\omega_k^2\right) \right)^2 \quad \longrightarrow \quad C = \sum_{k=1}^{N} \left(\frac{\sum_{i=0}^{m} c_i \omega^{2i} - \tilde{H}\left(\omega_k^2\right) \sum_{i=0}^{n} d_i \omega^{2i}}{\sum_{i=0}^{n} d_i \omega^{2i}} \right)^2$$

$\left\langle \frac{\partial C}{\partial c_k} \right\rangle_{k=1}^{m}$ is linear in $\langle c_k \rangle$ $\qquad$ $\left\langle \frac{\partial C}{\partial d_k} \right\rangle_{k=1}^{n}$ is highly nonlinear in $\langle d_k \rangle$

Closed form solution for optimal values of $\langle c_k \rangle$ and $\langle d_k \rangle$ does not exist

Least Squares Approximations of Transfer Functions

$$H_A(\omega^2) = \frac{\sum_{i=0}^{m} c_i \omega^{2i}}{\sum_{i=0}^{n} d_i \omega^{2i}}$$

$$C = \sum_{k=1}^{N} \left(H_A(\omega_k^2) - \tilde{H}(\omega_k^2) \right)^2$$

$$C = \sum_{k=1}^{N} \left(\frac{\sum_{i=0}^{m} c_i \omega^{2i} - \tilde{H}(\omega_k^2) \sum_{i=0}^{n} d_i \omega^{2i}}{\sum_{i=0}^{n} d_i \omega^{2i}} \right)^2$$

$\left\langle \frac{\partial C}{\partial c_k} \right\rangle_{k=1}^{m}$ is linear in $\langle c_k \rangle$ $\qquad$ $\left\langle \frac{\partial C}{\partial d_k} \right\rangle_{k=1}^{n}$ is highly nonlinear in $\langle d_k \rangle$

But

if $\langle d_k \rangle$ is fixed, optimal value of $\langle c_k \rangle$ can be easily obtained

equivalently,

if poles of $H_A(\omega^2)$ are fixed, optimal value of zeros of $H_A(\omega^2)$ can be easily obtained

Is this observation useful?

Least Squares Approximations of Transfer Functions

$$C=\sum_{k=1}^{N}\left(\frac{\sum_{i=0}^{m}c_i\omega^{2i}-\tilde{H}\left(\omega_k^2\right)\sum_{i=0}^{n}d_i\omega^{2i}}{\sum_{i=0}^{n}d_i\omega^{2i}}\right)^2$$

if poles of $H_A(\omega^2)$ are fixed, optimal value of zeros of $H_A(\omega^2)$ can be easily obtained

$$C=\sum_{k=1}^{N}\left(\frac{\sum_{i=0}^{m}c_i\omega^{2i}-\tilde{H}\left(\omega_k^2\right)\sum_{i=0}^{n}d_i\omega^{2i}}{\sum_{i=0}^{n}\hat{d}_i\omega^{2i}}\right)^2$$

if poles of $H_A(\omega^2)$ are fixed in denominator of C, the partials of C wrt both $<c_k>$ and $<d_k>$ are linear in $<c_k>$ and $<d_k>$

Are these observations useful?

- Several optimization approaches can be derived from these observations
- Some will provide a LMS optimization of $H_A(\omega^2)$
- No guarantee that inverse mapping exists
- Some may provide a good approximation even though not truly LMS
- Others may not be useful

Least Squares Approximations of Transfer Functions

$$C = \sum_{k=1}^{N} \left(\frac{\sum_{i=0}^{m} c_i \omega^{2i} - \tilde{H}\left(\omega_k^2\right) \sum_{i=0}^{n} d_i \omega^{2i}}{\sum_{i=0}^{n} d_i \omega^{2i}} \right)^2$$

Possible uses of these observations (four algorithms)

1. Guess poles and obtain optimal zero locations
2. Start with a “good” T(s) obtained by any means and improve by selecting optimal zeros
3. Guess poles and then update estimates of both poles and zeros, use new estimate of poles and again update both zeros and poles, continue until convergence or stop after fixed number of iterations
4. Guess poles and obtain optimal zeros. Then invert function and cost and obtain optimal zeros (which are actually poles). Then invert again and obtain optimal zeros. Process can be repeated. - Weighting may be necessary to de-emphasize stop-band values when working with the inverse function

$<c_k>$ $<d_k>$

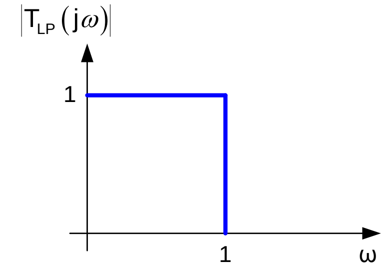
Least Squares Approximations of Transfer Functions

$$C = \sum_{k=1}^{N} \left(\frac{\sum_{i=0}^{m} c_i \omega^{2i} - \tilde{H}\left(\omega_k^2\right) \sum_{i=0}^{n} d_i \omega^{2i}}{\sum_{i=0}^{n} d_i \omega^{2i}} \right)^2$$

Comments/Observations about LMS approximations

1. **As with collocation, there is no guarantee that $T_A(s)$ can be obtained from $H_A(\omega^2)$**
2. **Closed-form analytical solutions exist for some useful mean square based cost functions**
3. **Any of the LMS cost functions discussed that have an analytical solution can have the terms weighted by a weight w_i. This weight will not change the functional form of the equations but will affect the fit**
4. **The best choice of sample frequencies is not obvious (both number and location)**
5. **The LMS cost function is not a natural indicator of filter performance**
6. **It is often used because more natural indicators are generally not mathematically tractable**
7. **The LMS approach may provide a good solution for some classes of applications but does not provide a universal solution**

The Approximation Problem



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Pade’ Approximations

Henri Eugène Padé (December 17, 1863 – July 9, 1953) was a French mathematician, who is now remembered mainly for his development of approximation techniques for functions using rational functions.

The Pade’ approximations were discussed in his doctoral dissertation in approximately 1890

(from Wikipedia)

Pade' Approximations

Consider the polynomial

$$T_D(s) = \sum_{i=0}^{\infty} c_i s^i$$

Define the rational fraction Rm,n(s) by

$$R_{m,n}(s) = \frac{\sum_{i=0}^{m} a_i s^i}{1 + \sum_{i=1}^{n} b_i s^i} = \frac{A(s)}{B(s)}$$

The rational fraction $R_{m,n}(s)$ is said to be a (m,n)th order Pade' approximation of $T_D(s)$ if $T_D(s)B(s)$ agrees with A(s) through the first m+n+1 powers of s

Note the Pade' approximation applies to any polynomial with the argument being either real, complex, or even an operator s

Can operate directly on functions in the s-domain

Pade' Approximations

Example

$$T_D(s) = 1 + s + \left(\frac{1}{2!}\right)s^2 + \left(\frac{1}{3!}\right)s^3 + \ldots$$

Determine $R_{2,3}(s)$

$$R_{2,3}(s) = \frac{a_0 + a_1 s + a_2 s^2}{1 + b_0 + b_1 s + b_2 s^2 + b_3 s^3} = \frac{A(s)}{B(s)}$$

setting

$$T_D(s)B(s) = A(s)$$

obtain

$$\left(1 + s + \left(\frac{1}{2!}\right)s^2 + \left(\frac{1}{3!}\right)s^3 + \ldots\right)\left(1 + b_1 s + b_2 s^2 + b_3 s^3\right) = a_0 + a_1 s + a_2 s^2$$

Pade' Approximations

Example

$$T_D(s) = 1 + s + \left(\frac{1}{2!}\right)s^2 + \left(\frac{1}{3!}\right)s^3 + \ldots$$

$$\left(1 + s + \left(\frac{1}{2!}\right)s^2 + \left(\frac{1}{3!}\right)s^3 + \ldots\right)\left(1 + b_1 s + b_2 s^2 + b_3 s^3\right) = a_0 + a_1 s + a_2 s^2$$

$$\left.\begin{aligned}
a_0 &= 1 \\
a_1 &= 1 + b_1 \\
a_2 &= b_1 + b_2 + \frac{1}{2!} \\
0 &= b_2 + b_3 + \frac{b_1}{2} + \frac{1}{6} \\
0 &= b_3 + \frac{b_2}{2} + \frac{b_1}{6} + \frac{1}{24} \\
0 &= \frac{b_3}{2} + \frac{b_2}{6} + \frac{b_1}{24} + \frac{1}{5!}
\end{aligned}\right\} \Rightarrow$$

$b_1 = -.6$

$b_2 = .15$

$b_3 = -.01666$

$a_0 = 1$

$a_1 = 0.4$

$a_2 = .05$

Pade' Approximations

Example

$$T(s) = \frac{1 + 0.4\,s + 0.05s^2}{1 - 0.6s + 0.15s^2 - 0.01\bar{6}\,s^3}$$

b_1=-.6

b_2=.15

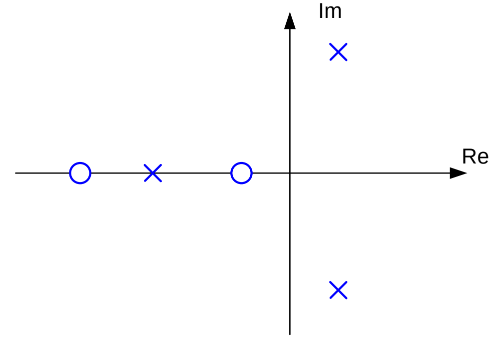
b_3=-.01666

a_0=1

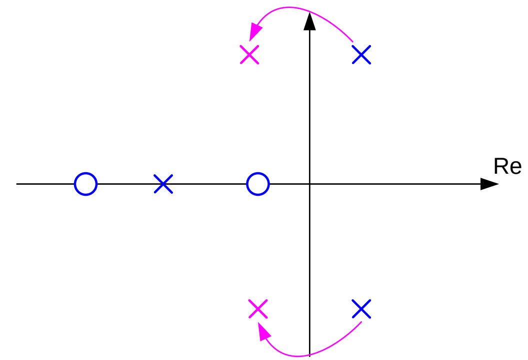
a_1=0.4

a_2=.05

T(s) has a pair of cc poles in the RHP and is thus unstable!



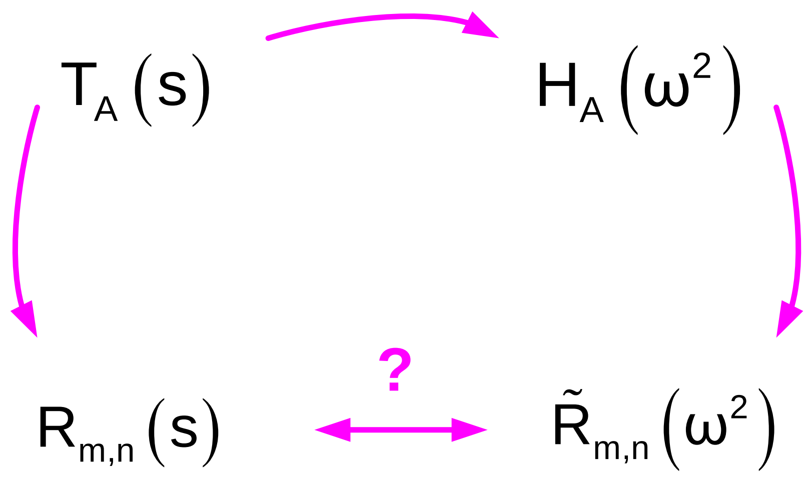
Poles can be reflected back into the LHP to obtain stability and maintain magnitude response



Pade' Approximations

If $T_A(s)$ is an all pole approximation, then the Pade' approximation of $1/T_A(s)$ is the reciprocal of the Pade' approximation of $T_A(s)$

Pade' approximations can be made for either $T_A(s)$ or $H_A(\omega^2)$.



Is it better to do Pade' approximations of $T_A(s)$ or $H_A(\omega^2)$?

What relationship, if any, exists between $R_{m,n}(s)$ and $\tilde{R}_{m,n}(s)$?

Pade’ Approximations

- Useful for order reduction of all-pole or all-zero approximations

- Can map an all-zero approximation to a realizable rational fraction in the s-domain

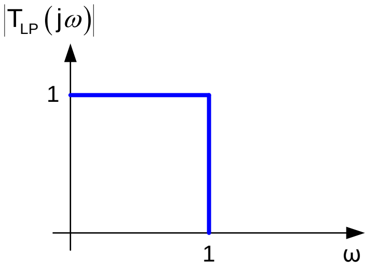
- Can extend concept to provide order reduction of higher-order rational fraction approximations

- Can always maintain stability or even minimum phase by reflecting any RHP roots back into the LHP

- Pade’ approximation is heuristic (no metrics associated with the approach)

- No guarantees about how good the approximations will be

The Approximation Problem



Approach we will follow:

- Magnitude Squared Approximating Functions $H_A(\omega^2)$
- Inverse Transform $H_A(\omega^2) \rightarrow T_A(s)$
- Collocation
- Least Squares
- Pade' Approximations
- Other Analytical Optimization
- Numerical Optimization
- Canonical Approximations
  - $\rightarrow$ Butterworth (BW)
  - $\rightarrow$ Chebyschev (CC)
  - $\rightarrow$ Elliptic
  - $\rightarrow$ Thompson

Other Analytical Approximations

- Numerous analytical strategies have been proposed over the years for realizing a filter
- Some focus on other characteristics (phase, time-domain response, group delay)
- Almost all based upon real function approximations
- Remember – inverse mapping must exist if a useful function T(s) is to be obtained

Approximations

- Magnitude Squared Approximating Functions – $H_A(\omega^2)$
- Inverse Transform - $H_A(\omega^2) \rightarrow T_A(s)$
- Collocation
- Least Squares Approximations
- Pade Approximations
- Other Analytical Optimizations
- Numerical Optimization
- Canonical Approximations
  - Butterworth
  - Chebyschev
  - Elliptic
  - Bessel
  - Thompson

Numerical Optimization

- Optimization algorithms can be used to obtain approximations in either the s-domain or the real domain
- The optimization problem often has a large number of degrees of freedom (m+n+1)

$$T(s)=\frac{\sum_{k=0}^{m} a_k s^k}{1+\sum_{k=0}^{n} b_k s^k}$$

- Need a good cost function to obtain good approximation
- Can work on either coefficient domain or root domain or other domains
- Rational fraction approximations inherently vulnerable to local minimums
- Can get very good results

End of Lecture 9

EE 508
Lecture 10

The Approximation Problem

Classical Approximations

Least Squares Approximations of Transfer Functions

$$C = \sum_{k=1}^{N} \left(\frac{\sum_{i=0}^{m} c_i \omega^{2i} - \tilde{H}\left(\omega_k^2\right) \sum_{i=0}^{n} d_i \omega^{2i}}{\sum_{i=0}^{n} d_i \omega^{2i}} \right)^2$$

Possible uses of these observations (four algorithms)

1. Guess poles and obtain optimal zero locations
2. Start with a "good" T(s) obtained by any means and improve by selecting optimal zeros
3. Guess poles and then update estimates of both poles and zeros, use new estimate of poles and again update both zeros and poles, continue until convergence or stop after fixed number of iterations
4. Guess poles and obtain optimal zeros. Then invert function and cost and obtain optimal zeros (which are actually poles). Then invert again and obtain optimal zeros. Process can be repeated. - Weighting may be necessary to de-emphasize stop-band values when working with the inverse function

$<c_k>$ $<d_k>$

Review from Last Time

Least Squares Approximations of Transfer Functions

$$C=\sum_{k=1}^{N}\left(\frac{\sum_{i=0}^{m}c_i\omega^{2i}-\tilde{H}\left(\omega_k^2\right)\sum_{i=0}^{n}d_i\omega^{2i}}{\sum_{i=0}^{n}d_i\omega^{2i}}\right)^2$$

Comments/Observations about LMS approximations

1. **As with collocation, there is no guarantee that $T_A(s)$ can be obtained from $H_A(\omega^2)$**
2. **Closed-form analytical solutions exist for some useful mean square based cost functions**
3. **Any of the LMS cost functions discussed that have an analytical solution can have the terms weighted by a weight w_i. This weight will not change the functional form of the equations but will affect the fit**
4. **The best choice of sample frequencies is not obvious (both number and location)**
5. **The LMS cost function is not a natural indicator of filter performance**
6. **It is often used because more natural indicators are generally not mathematically tractable**
7. **The LMS approach may provide a good solution for some classes of applications but does not provide a universal solution**

Pade' Approximations

Consider the polynomial

$$T_D(s)=\sum_{i=0}^{\infty} c_i s^i$$

Define the rational fraction Rm,n(s) by

$$R_{m,n}(s)=\frac{\sum_{i=0}^{m} a_i s^i}{1+\sum_{i=1}^{n} b_i s^i}=\frac{A(s)}{B(s)}$$

The rational fraction $R_{m,n}(s)$ is said to be a (m,n)th order Pade' approximation of $T_D(s)$ if $T_D(s)B(s)$ agrees with A(s) through the first m+n+1 powers of s

Note the Pade' approximation applies to any polynomial with the argument being either real, complex, or even an operator s

Can operate directly on functions in the s-domain

Pade' Approximations

- Useful for order reduction of all-pole or all-zero approximations
- Can map an all-zero approximation to a realizable rational fraction in the s-domain
- Can extend concept to provide order reduction of higher-order rational fraction approximations
- Can always maintain stability or even minimum phase by reflecting any RHP roots back into the LHP
- Pade' approximation is heuristic (no metrics associated with the approach)
- No guarantees about how good the approximations will be

Approximations

- Magnitude Squared Approximating Functions – $H_A(\omega^2)$
- Inverse Transform - $H_A(\omega^2) \rightarrow T_A(s)$
- Collocation
- Least Squares Approximations
- Pade Approximations
- Other Analytical Optimizations
- Numerical Optimization
- Canonical Approximations
  - Butterworth
  - Chebyschev
  - Elliptic
  - Bessel
  - Thompson

All special cases of analytical approximations

Approximations

- Magnitude Squared Approximating Functions – $H_A(\omega^2)$
- Inverse Transform - $H_A(\omega^2) \rightarrow T_A(s)$
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On the Theory of Filter Amplifiers.*

By S. Butterworth, M.Sc.
(*Admiralty Research Laboratory*).

THE orthodox theory of electrical wave filters has been admirably presented by Mr. M. Reed in recent numbers of *E.W. & W.E.* (p. 122, March, 1930 *et seq*), and it is not proposed in the present Paper to add to or to repeat any of that theory. In this work the problem of electrical filtering is attacked from a new angle in which use is made of systems of simple filter units separated by valves so that we combine in one amplifier the property of filtering with that of amplification. The simple units employed can, in the case of low pass filters, be so designed that they take up little more space than the anode resistance employed in the ordinary straight resistance capacity amplifier. The writer of filter circuit, the first condition is generally approximately fulfilled, but the second condition is usually either not obtained or is approximately arrived at by an empirical adjustment of the resistances of the elements.

The following theory was developed primarily in order to arrive at a logical scheme of design for low pass filters, but it will be shown that it is possible to make use of the theory for band pass, band stop, and high pass filters.

The theory of the general filter-circuit of the Campbell type including resistance is not attempted, but it is shown how to obtain the best results from a two element filter and then how to combine any number of elemen-

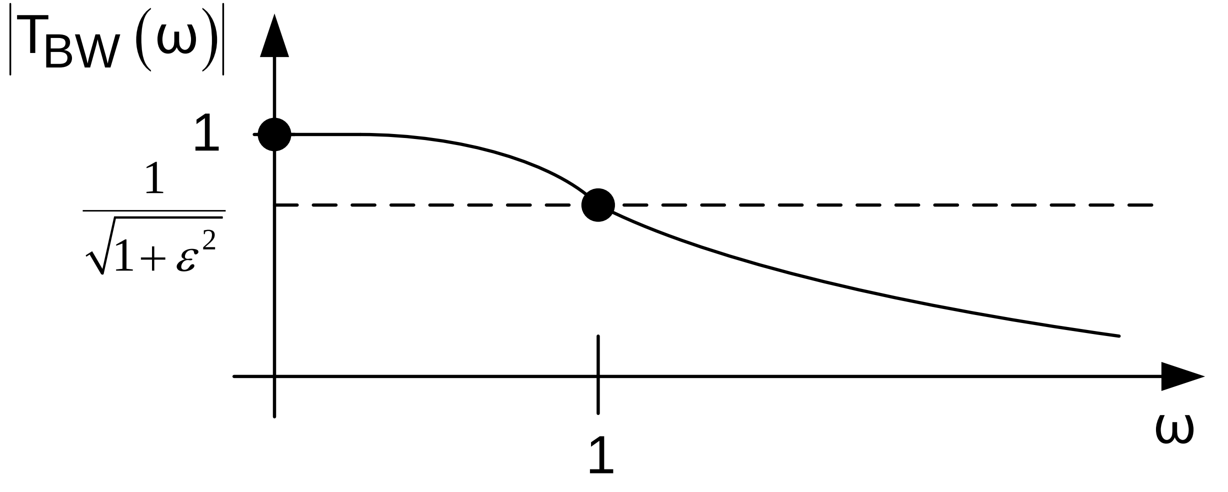
Butterworth Approximations

- Analytical formulation:
  - All pole approximation
  - Magnitude response is maximally flat at ω=0
  - Goes to 0 at ω=∞
  - Assumes value $\sqrt{\frac{1}{1+\varepsilon^2}}$ at ω=1
  - Assumes value of 1 at ω=0
  - Characterized by {n,ε}
- Emphasis almost entirely on performance at single frequency

"On the Theory of Filter Amplifiers", Wireless Engineer (also called Experimental Wireless and the Radio Engineer), Vol. 7, 1930, pp. 536-541.

Butterworth Approximations

- Analytical formulation:
  - Magnitude response is maximally flat at ω=0
  - Goes to 0 at ω=∞
  - Assumes value $\sqrt{\frac{1}{1+\varepsilon^2}}$ at ω=1
  - Assumes value of 1 at ω=0



Stephen Butterworth (1885-1958) was a British physicist who invented the Butterworth filter[1], a class of electrical circuits that are used to filter electrical signals.

Stephen Butterworth was born on 11 August 1885 in Rochdale, England (a town located about 10 miles north of the city of Manchester). He was the son of Alexander Butterworth, a postman, and Elizabeth (maiden name unknown).[2] He was the second of four children.[3] In 1904, he entered the University of Manchester, from which he received, in 1907, both a Bachelor of Science degree in physics (first class) and a teacher's certificate (first class). In 1908 he received a Master of Science degree in physics.[4] For the next 11 years he was a physics lecturer at the Manchester Municipal College of Technology. He subsequently worked for several years at the National Physical Laboratory, where he did theoretical and experimental work for the determination of standards of electrical inductance. In 1921 he joined the Admiralty's Research Laboratory. Unfortunately, the classified nature of his work prohibited the publication of much of his research there. Nevertheless, it is known that he worked in a wide range of fields; e.g., he determined the electromagnetic field around submarine cables carrying a.c. current,[5] and he investigated underwater explosions and the stability of torpedos. In 1939, he was a "Principal Scientific Officer" at the Admiralty Research Laboratory in the Admiralty's Scientific Research and Experiment Department.[6] During World War II, he investigated both magnetic mines and the degaussing of ships (as a means of protecting them from magnetic mines).

He was a first-rate applied mathematician. He often solved problems that others had regarded as insoluble. For his successes, he employed judicious approximations, penetrating physical insight, ingenious experiments, and skillful use of models. He was a quiet and unassuming man. Nevertheless, his knowledge and advice were widely sought and readily offered. He was respected by his colleagues and revered by his subordinates.

In 1942 he was awarded the Order of the British Empire.[7] In 1945 he retired from the Admiralty Research Laboratory. He died on 28 October 1958 at his home in Cowes on the Isle of Wight, England.[8][9

In *Wireless Engineer* (also called *Experimental Wireless and the Wireless Engineer*), vol. 7, 1930, pp. 536–541 - "On the Theory of Filter Amplifiers"-S. Butterworth

From: http://en.wikipedia.org/wiki/Butterworth_filter

Butterworth had a reputation for solving "impossible" mathematical problems. At the time filter design was largely by trial and error because of their mathematical complexity. His paper was far ahead of its time: the filter was not in common use for over 30 years after its publication. Butterworth stated that;

"An ideal electrical filter should not only completely reject the unwanted frequencies but should also have uniform sensitivity for the wanted frequencies."

Butterworth Approximation

$$H\left(\omega^2\right)=\frac{a_0}{\omega^{2n}+b_{n-1}\omega^{2n-2}+....b_1\omega^2+b_0}$$

$$H(1)=\frac{1}{1+\varepsilon^2}$$

$$H(0)=1 \longrightarrow a_0=b_0$$

Let $x=\omega^2$

$$H(x)=\frac{a_0}{x^n+b_{n-1}x^{n-1}+....b_1x+b_0}$$

$$\frac{\partial H}{\partial x}=-a_0\frac{nx^{n-1}+b_{n-1}(n-1)x^{n-2}+....b_1}{\left(x^n+b_{n-1}x^{n-1}+....b_1x+b_0\right)^2}$$

$$\left.\frac{\partial H}{\partial x}\right|_{x=0}=-a_0\left.\frac{nx^{n-1}+b_{n-1}(n-1)x^{n-2}+....b_1}{\left(x^n+b_{n-1}x^{n-1}+....b_1x+b_0\right)^2}\right|_{x=0}=0 \longrightarrow b_1=0$$

Butterworth Approximation

$$H(x)=\frac{a_0}{x^n+b_{n-1}x^{n-1}+....b_1x+b_0}$$

$$\frac{\partial^2 H}{\partial x^2}=-a_0\frac{\left(x^n+b_{n-1}x^{n-1}+....b_1x+b_0\right)^2\left(n(n-1)x^{n-2}+\left(b_{n-1}(n-1)(n-2)x^{n-2}\right)+...6b_3x+2b_2\right)-\left(nx^{n-1}+b_{n-1}(n-1)x^{n-2}+....b_1\right)^2 2\left(x^n+b_{n-1}x^{n-1}+....b_1x+b_0\right)}{\left(x^n+b_{n-1}x^{n-1}+....b_1x+b_0\right)^4}$$

$$\left.\frac{\partial^2 H}{\partial x^2}\right|_{x=0}=-\left.\frac{\left(x^n+b_{n-1}x^{n-1}+....b_1x+b_0\right)^2\left(n(n-1)x^{n-2}+\left(b_{n-1}(n-1)(n-2)x^{n-2}\right)+...6b_3x+2b_2\right)-\left(nx^{n-1}+b_{n-1}(n-1)x^{n-2}+....b_1\right)^2 2\left(x^n+b_{n-1}x^{n-1}+....b_1x+b_0\right)}{\left(x^n+b_{n-1}x^{n-1}+....b_1x+b_0\right)^4}\right|_{x=0}=0$$

$$\left.\frac{\partial^2 H}{\partial x^2}\right|_{x=0}=0 \longrightarrow 2b_0^2b_2-2b_0b_1^2=0$$

$$\longrightarrow b_2=0$$

Continuing this process obtain $b_3=0$, $b_4=0$, … $b_{n-1}=0$

Butterworth Approximation

$$H(x) = \frac{b_0}{x^n + b_0} \qquad H(1) = \frac{1}{1+\varepsilon^2}$$

$$\longrightarrow \qquad b_0 = \frac{1}{\varepsilon^2}$$

$$H(\omega) = \frac{1}{1+\varepsilon^2\omega^{2n}}$$

Roots of H(ω) are poles and are at

$$\omega = \varepsilon^{1/n}(-1)^{1/(2n)}$$

The 2n roots of -1 are uniformly spaced on a circle of radius 1

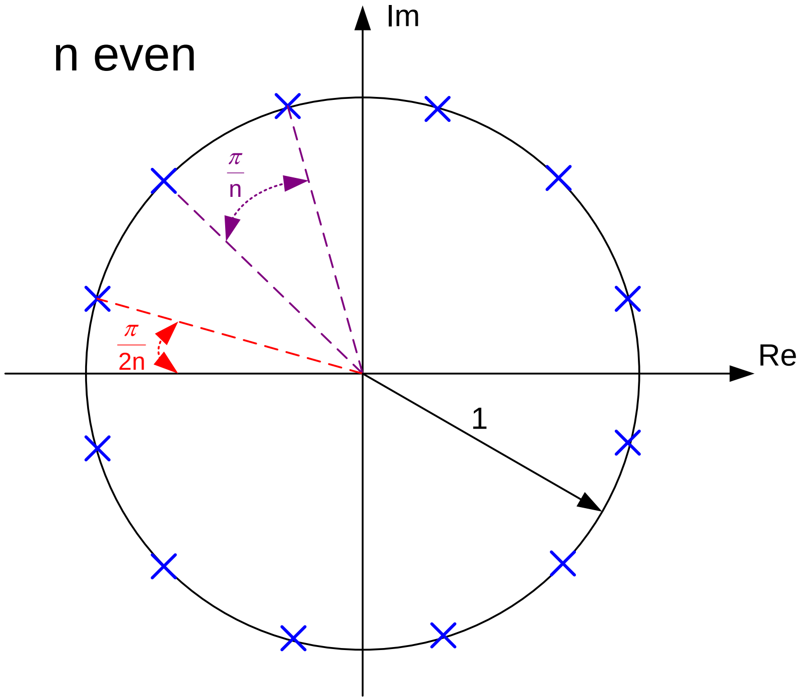
Butterworth Approximation

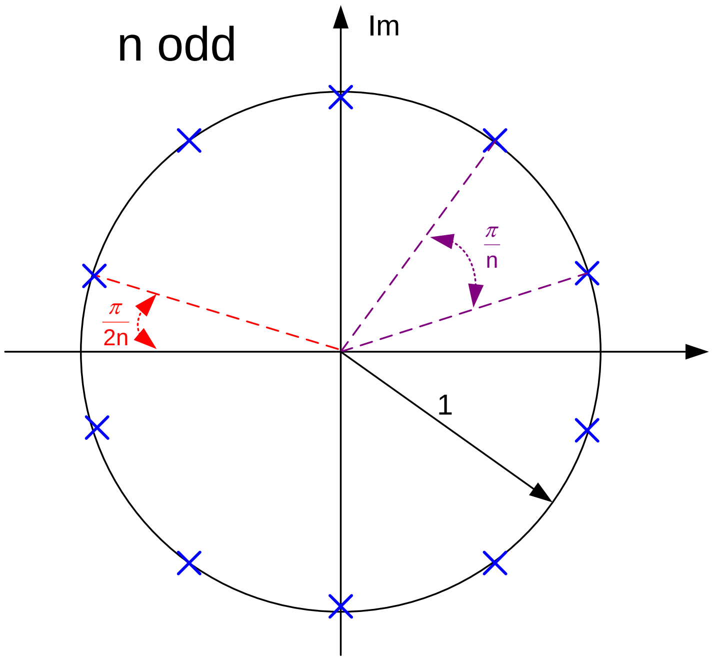
$$H(\omega) = \frac{1}{1+\varepsilon^2\omega^{2n}}$$

Roots of H(ω) are poles and are at

$$\omega = \varepsilon^{1/n}(-1)^{1/(2n)}$$

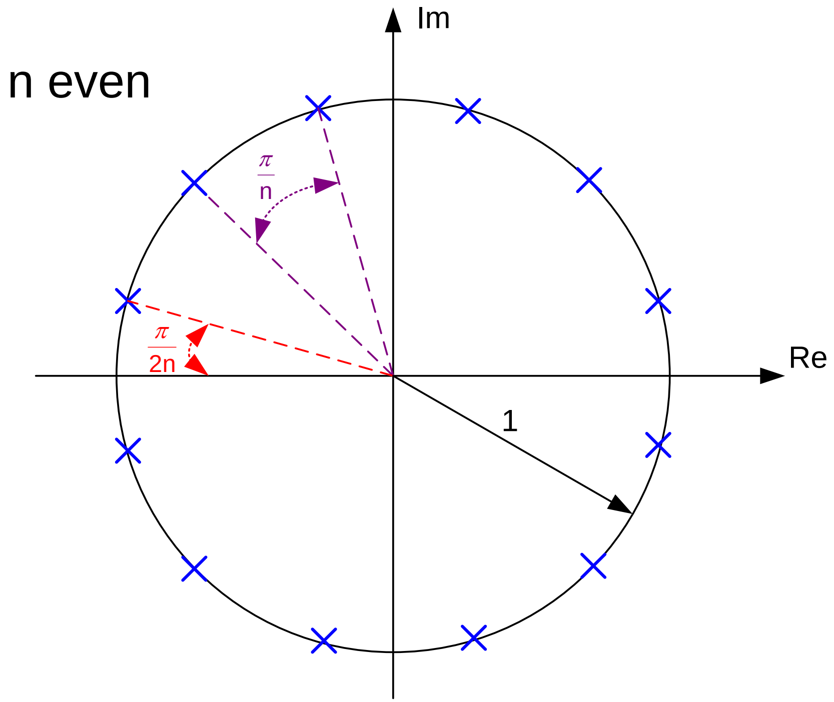
Poles of H(ω) are a scaled version of the roots of -1
Roots of -1 are uniformly spaced around a unit circle with symmetry around real and imaginary axis





Butterworth Approximation

Roots of -1 are uniformly spaced around a unit circle with symmetry around real axis and symmetry around the imaginary axis



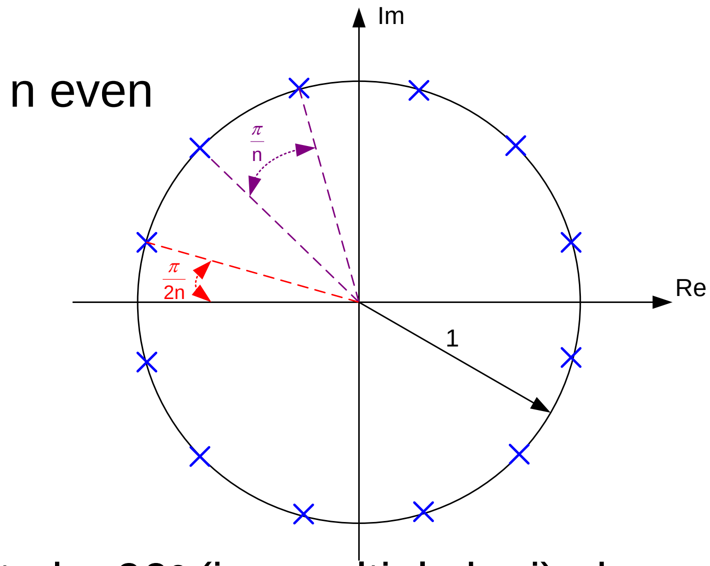
Analytical expression for roots of -1

$$\tilde{\omega}_k = -\cos\left(\left[1+2k\right]\frac{\pi}{2n}\right) \pm j\sin\left(\left[1+2k\right]\frac{\pi}{2n}\right)$$

k=0,1,…n-1

Butterworth Approximation

Roots of $T_{BW}(s)$



Take roots of H(ω), rotate by 90° (i.e. multiply by j) , keep those in LHP

$$p_k = j\varepsilon^{1/n}\left[-\cos\left(\left[1+2k\right]\frac{\pi}{2n}\right) \pm j\sin\left(\left[1+2k\right]\frac{\pi}{2n}\right)\right]$$

(actually denotes 2 poles for each index)

for n even

$$p_{k+1} = \varepsilon^{1/n}\left[-\sin\left(\left[1+2k\right]\frac{\pi}{2n}\right) \pm j\cos\left(\left[1+2k\right]\frac{\pi}{2n}\right)\right] \quad k=0,1,\ \dots\ \frac{n}{2}-1$$

(k=0 for poles closest to Im axis)

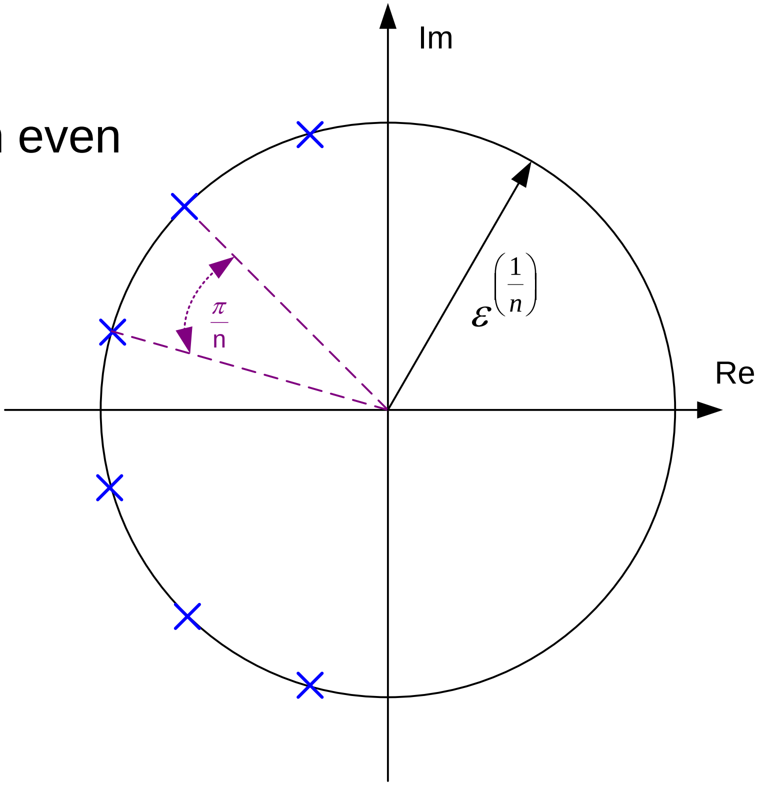
for n odd

$$p_n = \varepsilon^{1/n}\left[-1 + j0\right] \qquad p_k = \varepsilon^{1/n}\left[-\sin\left(\left[1+2k\right]\frac{\pi}{2n}\right) \pm j\cos\left(\left[1+2k\right]\frac{\pi}{2n}\right)\right] \quad k=0,\ \dots\ \frac{n-3}{2}$$

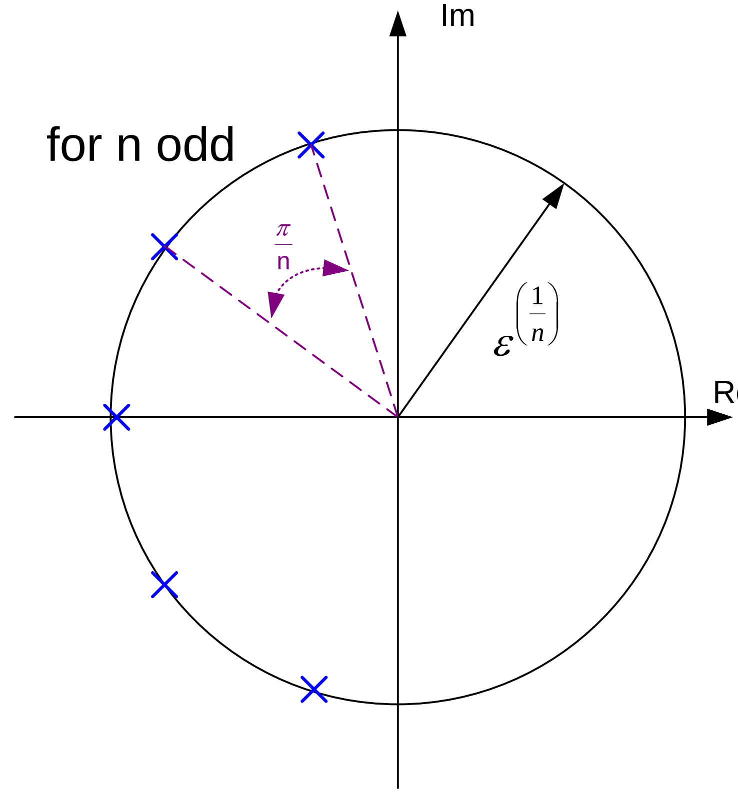
Butterworth Approximation

Poles of $T_{BW}(s)$

for n even



for n odd



$$p_{k+1} = \varepsilon^{1/n}\left[-\sin\left([1+2k]\frac{\pi}{2n}\right) \pm j\cos\left([1+2k]\frac{\pi}{2n}\right)\right]$$

k=0,1, ... $\frac{n}{2}-1$

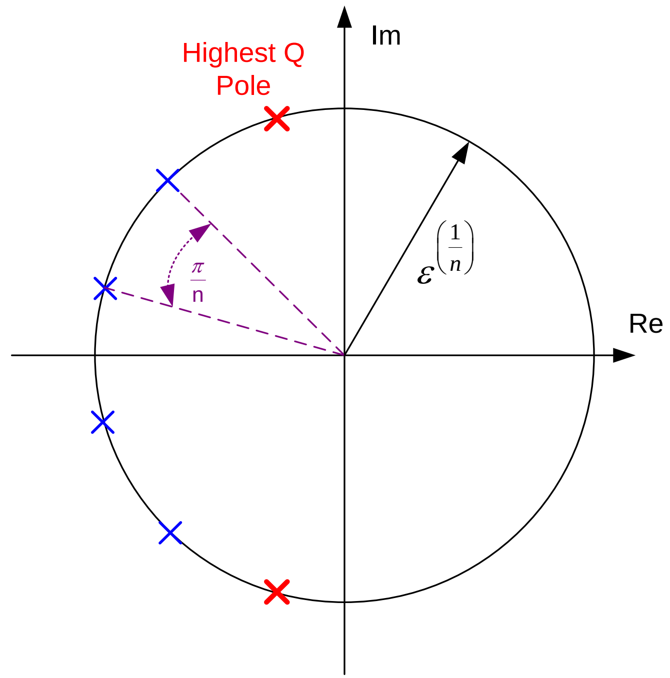
$$p_n = \varepsilon^{1/n}[-1+j0]$$

$$p_k = \varepsilon^{1/n}\left[-\sin\left([1+2k]\frac{\pi}{2n}\right) \pm j\cos\left([1+2k]\frac{\pi}{2n}\right)\right]$$

k=0, ... $\frac{\text{n-3}}{2}$

Butterworth Approximation

What is the Q of the highest Q pole for the BW approximation?



Highest Q pole corresponds to index k=0. Consider the Quadrant 2 high-Q pole

$$p_0 = \varepsilon^{1/n}\left[-\sin\left(\frac{\pi}{2n}\right) + j\cos\left(\frac{\pi}{2n}\right)\right] = \alpha + j\beta$$

But recall

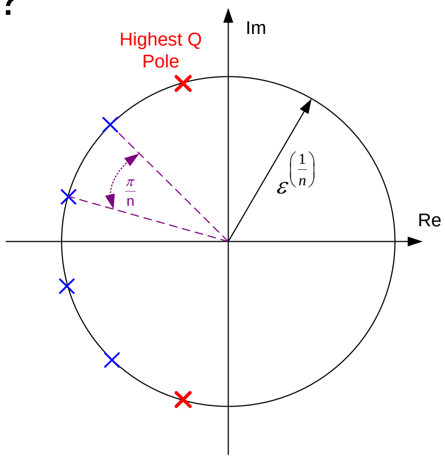
$$s^2 + s\left(\frac{\omega_o}{Q}\right) + \omega_o^2 = s^2 + s(-2\alpha) + \left(\alpha^2 + \beta^2\right)$$

thus

$$Q = \frac{\sqrt{\alpha^2 + \beta^2}}{-2\alpha}$$

Butterworth Approximation

What is the Q of the highest Q pole for the BW approximation?



$$p_0 = \varepsilon^{1/n}\left[-\sin\left(\frac{\pi}{2n}\right) + j\cos\left(\frac{\pi}{2n}\right)\right] = \alpha + j\beta$$

$$Q_{MAX} = \frac{\sqrt{\alpha^2 + \beta^2}}{-2\alpha}$$

$$Q_{MAX} = \frac{\varepsilon^{1/n}\sqrt{\sin^2\left(\frac{\pi}{2n}\right) + \cos^2\left(\frac{\pi}{2n}\right)}}{2\varepsilon^{1/n}\sin\left(\frac{\pi}{2n}\right)} = \frac{1}{2\sin\left(\frac{\pi}{2n}\right)}$$

$$Q_{MAX} = \frac{1}{2\sin\left(\frac{\pi}{2n}\right)}$$

Butterworth Approximation

What order can be used if goal is to keep the highest Q BW pole less than 10?

$$Q_{MAX} = \frac{1}{2\sin\left(\frac{\pi}{2n}\right)}$$

$$10 = \frac{1}{2\sin\left(\frac{\pi}{2n}\right)}$$

Solving for n, obtain n=31

What order can be used if goal is to keep the highest Q BW pole less than 2?

$$2 = \frac{1}{2\sin\left(\frac{\pi}{2n}\right)}$$

Solving for n, obtain n=6

Observe the pole Q of the BW approximation is quite low, even for high order BW approximations!

Butterworth Approximation

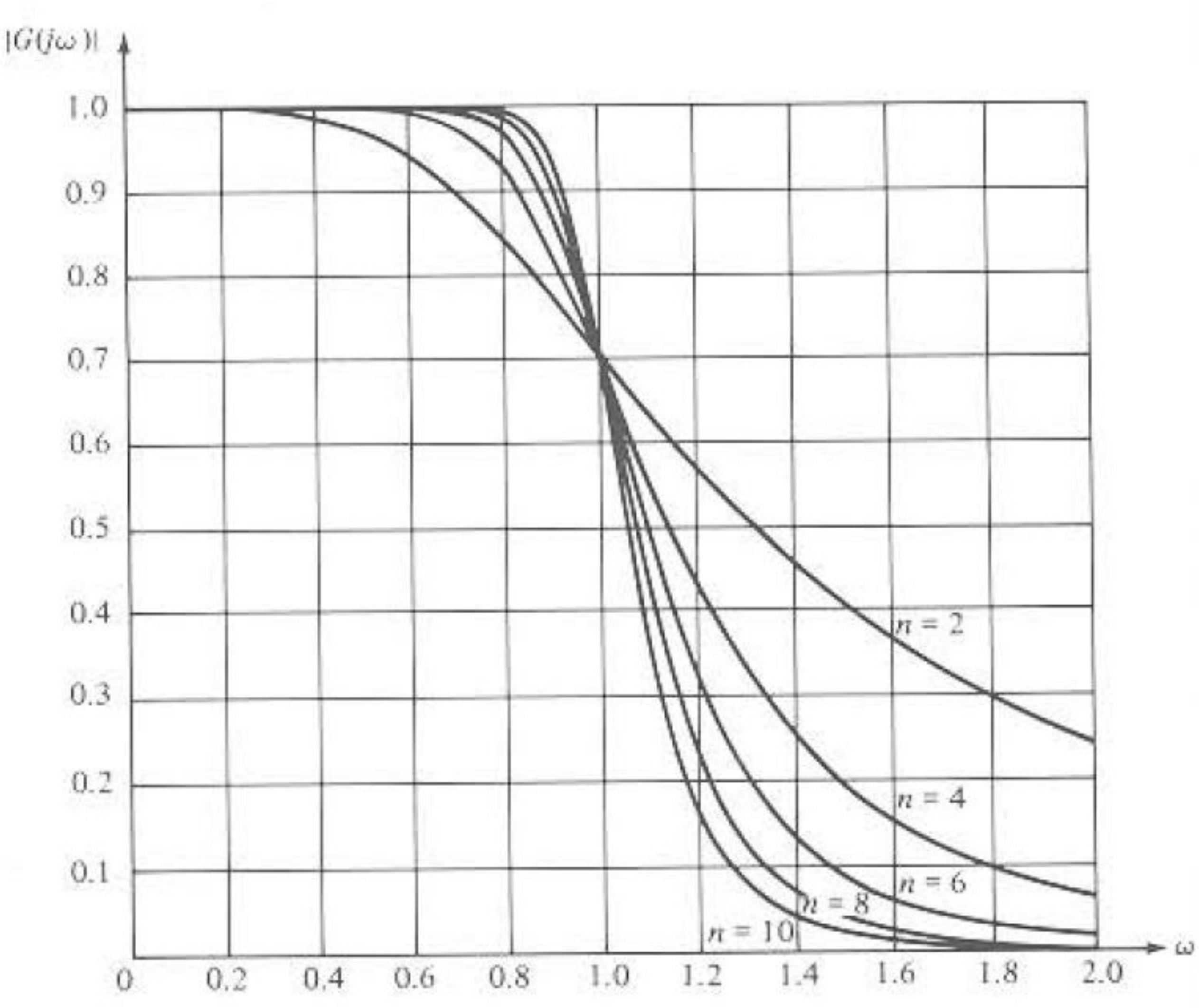


Fig. 17-3a Magnitude of the maximally flat approximation ($\varepsilon = 1$)

Order needs to be rather high to get steep transition

Figure from Passive and Acti
Network Analysis and
Synthesis, Budak

Butterworth Approximation

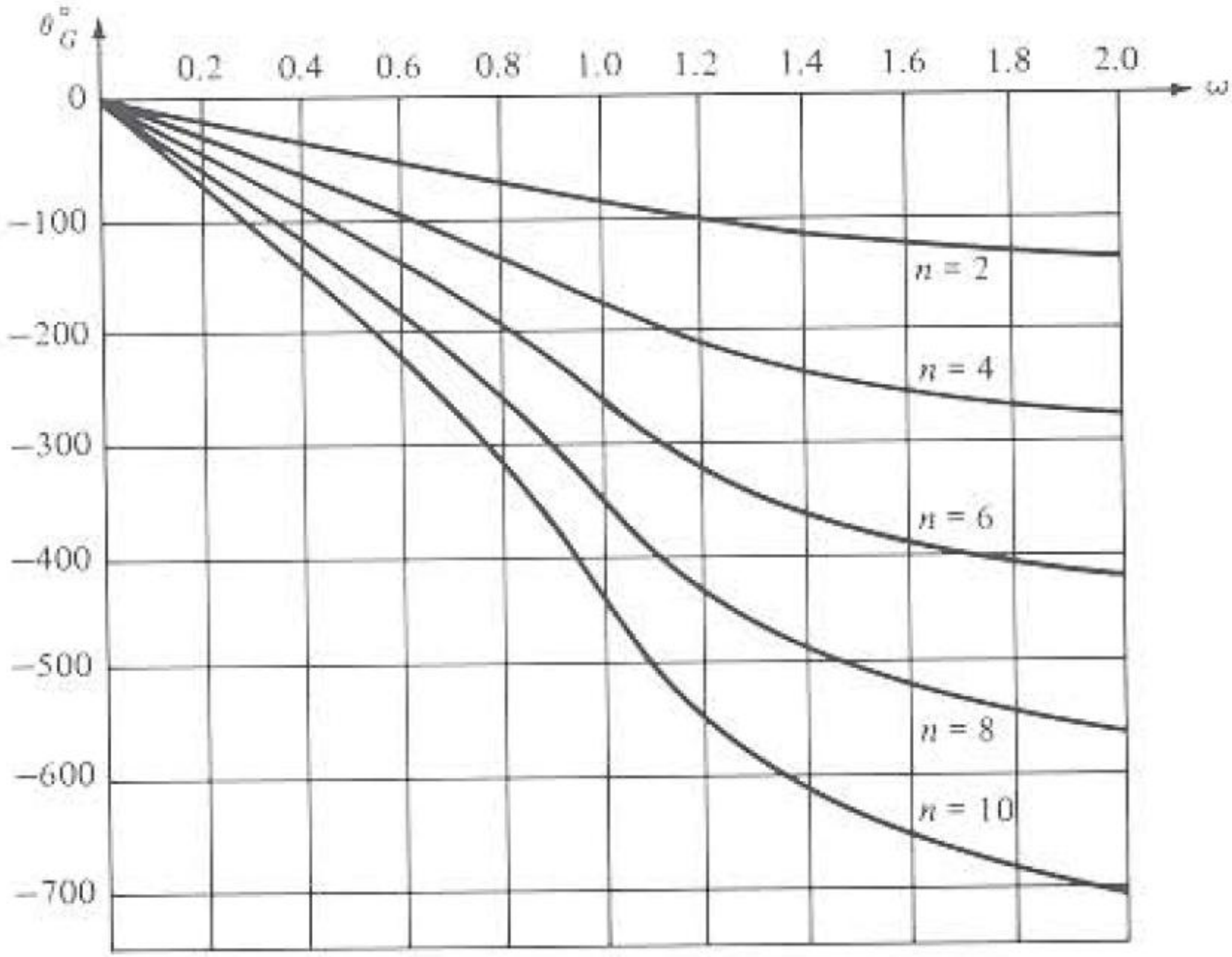


Fig. 17-3b Phase of the maximally flat approximation ($\varepsilon = 1$)

Figure from Passive and Activ
Network Analysis and
Synthesis, Budak

Phase is quite linear in passband (benefit unrelated to design requirements)

Butterworth Approximation

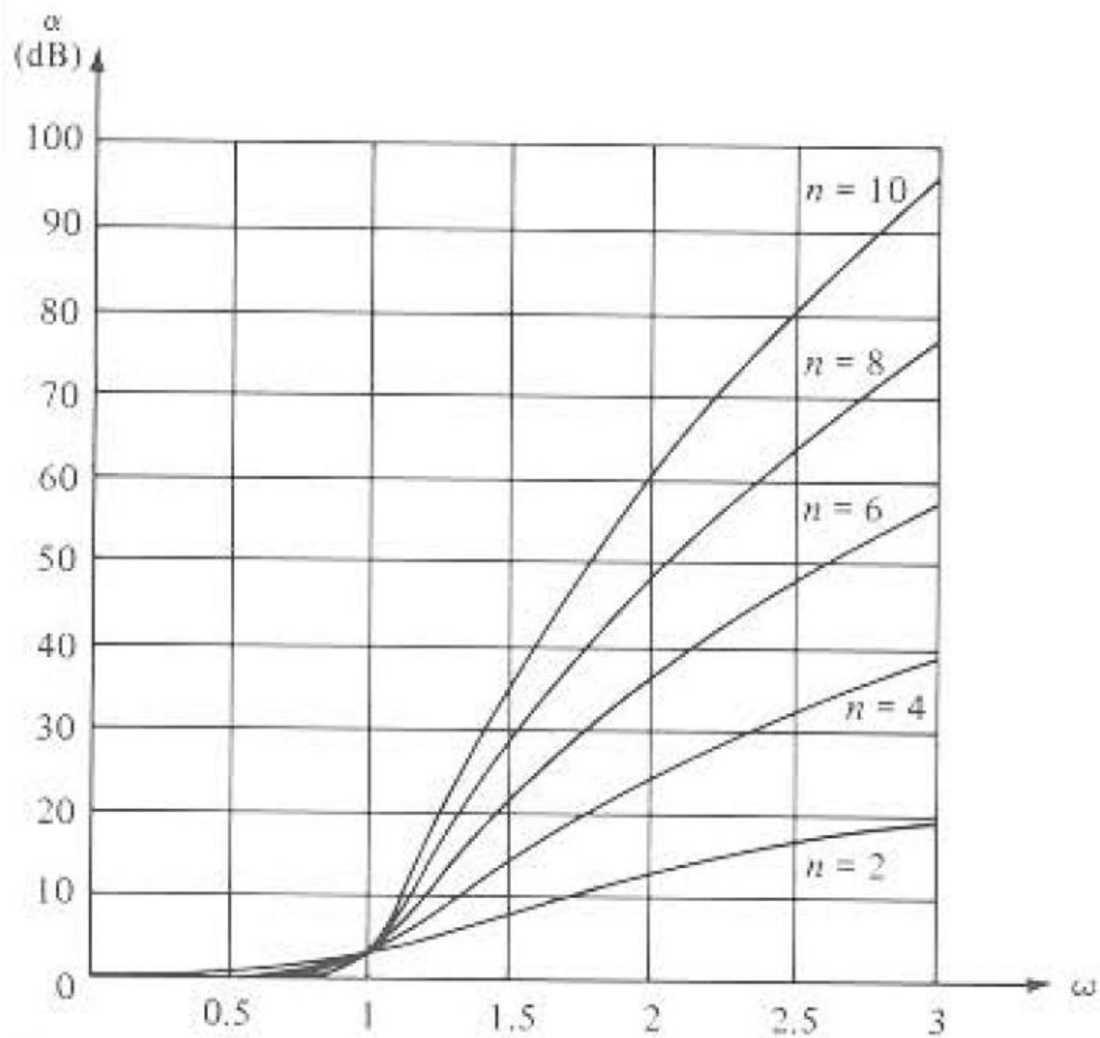


Figure from Passive and Active Network Analysis and Synthesis, Budak

Fig. 17-3c dB attenuation produced by the maximally flat approximation ($\varepsilon = 1$)

Attenuation in stopband is quite gradual

Butterworth Approximation

Table 17-1 Maximally flat (at $\omega = 0$) approximation $G_n(s) = \dfrac{1}{D_n(s)}$ (3.01-dB ripple)

$$D_1 = s + 1$$

$$D_2 = s^2 + 1.4142s + 1 = (s + 0.7071)^2 + 0.7071^2$$

$$D_3 = s^3 + 2.0000s^2 + 2.0000s + 1 = (s + 1.0000)[(s + 0.5000)^2 + 0.8660^2]$$

$$D_4 = s^4 + 2.6131s^3 + 3.4142s^2 + 2.6131s + 1$$
$$= [(s + 0.3827)^2 + 0.9239^2][(s + 0.9239)^2 + 0.3827^2]$$

$$D_5 = s^5 + 3.2361s^4 + 5.2361s^3 + 5.2361s^2 + 3.2361s + 1$$
$$= (s + 1.0000)[(s + 0.3090)^2 + 0.9511^2][(s + 0.8090)^2 + 0.5878^2]$$

$$D_6 = s^6 + 3.8637s^5 + 7.4641s^4 + 9.1416s^3 + 7.4641s^2 + 3.8637s + 1$$
$$= [(s + 0.2588)^2 + 0.9659^2][(s + 0.7071)^2 + 0.7071^2][(s + 0.9659)^2 + 0.2588^2]$$

$$D_7 = s^7 + 4.4940s^6 + 10.0978s^5 + 14.5918s^4 + 14.5918s^3 + 10.0978s^2 + 4.4940s + 1$$
$$= (s + 1.0000)[(s + 0.2225)^2 + 0.9749^2][(s + 0.6235)^2 + 0.7818)^2]$$
$$\times [(s + 0.9010)^2 + 0.4339^2]$$

$$D_8 = s^8 + 5.1258s^7 + 13.1371s^6 + 21.8462s^5 + 25.6884s^4 + 21.8462s^3 + 13.1371s^2$$
$$+ 5.1258s + 1$$
$$= [(s + 0.1951)^2 + 0.9808^2][(s + 0.5556)^2 + 0.8315^2][(s + 0.8315)^2 + 0.5556^2]$$
$$\times [(s + 0.9808)^2 + 0.1951^2]$$

$$D_9 = s^9 + 5.7588s^8 + 16.5817s^7 + 31.1634s^6 + 41.9864s^5 + 41.9864s^4 + 31.1634s^3$$
$$+ 16.5817s^2 + 5.7588s + 1$$
$$= (s + 1.0000)[(s + 0.1737)^2 + 0.9848^2][(s + 0.5000)^2 + 0.8660^2]$$
$$\times [(s + 0.7660)^2 + 0.6428^2][(s + 0.9397)^2 + 0.3420^2]$$

$$D_{10} = s^{10} + 6.3925s^9 + 20.4317s^8 + 42.8021s^7 + 64.8824s^6 + 74.2334s^5 + 64.8824s^4$$
$$+ 42.8021s^3 + 20.4317s^2 + 6.3925s + 1$$
$$= [(s + 0.1564)^2 + 0.9877^2][(s + 0.4540)^2 + 0.8910^2][(s + 0.7071)^2 + 0.7071^2]$$
$$\times [(s + 0.8910)^2 + 0.4540^2][(s + 0.9877)^2 + 0.1564^2]$$

Pole locations and denominator polynomial

Figure from Passive and Active Network Analysis and Synthesis, Budak

Butterworth’s vision was a bit different than what we presented but the results are completely attributable to Butterworth

From the seminal Butterworth paper:

> at first that we have perfect freedom in regard to the electrical constants of the elements and then these have been chosen with a view to obtaining the nearest approximation to the condition of uniform sensitivity in the “pass” region, and zero sensitivity in the “stop” region.

In the case of the low pass filter, if f_0 is the "cut off" frequency and f (xf_0) is any other frequency, the aim is to obtain a filter factor F, that is, the ratio of the output e.m.f. to the input e.m.f., of the form

$$F = (1 + x^m)^{-\frac{1}{2}} \quad \ldots \quad \ldots \quad \ldots \quad (1),$$

where m increases with the number of elements employed. It is clear that as m increases,

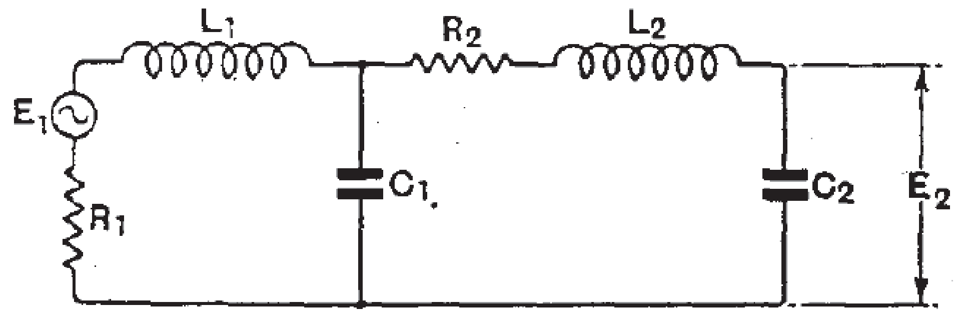


Fig. 2.

F will approximate more and more closely to the value unity when x is less than unity, and to zero when x is greater than unity.

Butterworth used a trig identity to factor (1) into a product of 4[th] order terms and then synthesized a circuit that realized each factor (no mention made of inverse mapping to T(s))

Butterworth Approximation

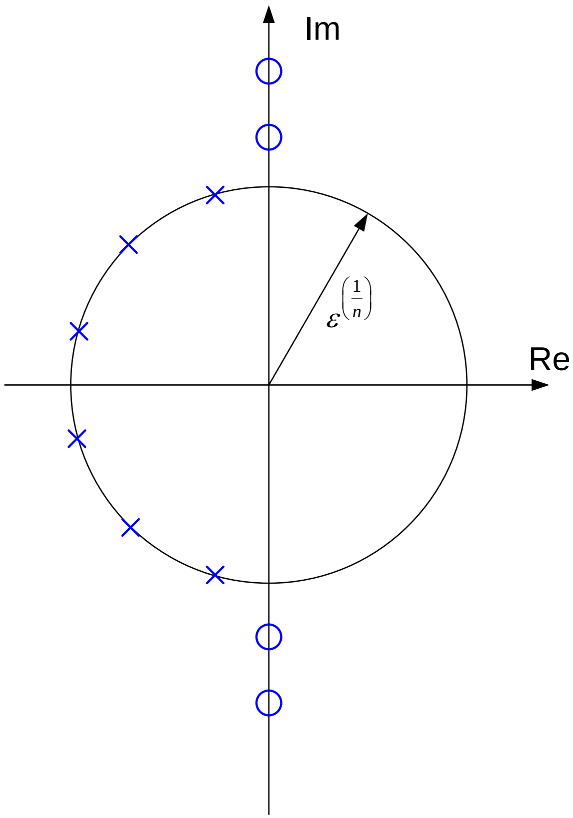
Summary

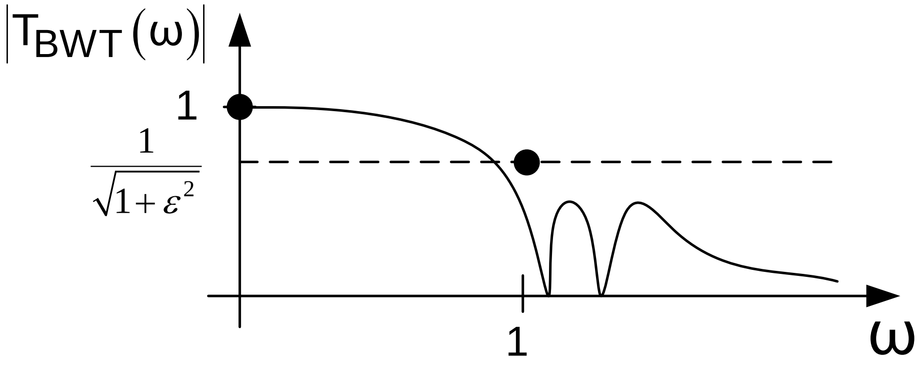
- Widely Used Analytical Approximation
- Characterized by {ε,n}
- Maximally flat at ω=0
- Almost all emphasis placed on characteristics at single frequency (ω=0)
- Transition not very steep (requires large order for steep transition)
- Pole Q is quite low
- Pass-band phase is quite linear (no emphasis was placed on phase!)
- Poles lie on a circle
- Simple closed-form analytical expressions for poles and |T(jω)|

Butterworth Approximation

What can be done to sharpen the transition of the BW approximation?

Add zeros on imaginary axis in stop band





- May need to readjust the poles to get good transition region
- Analytical expressions for poles may not be easy to obtain

End of Lecture 10

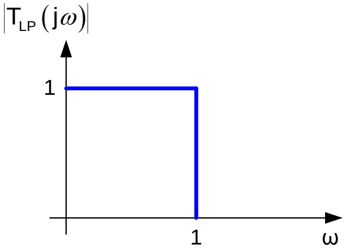
EE 508
Lecture 11

The Approximation Problem

Classical Approximations
– the Chebyschev and Elliptic Approximations

Review from Last Time

Butterworth Approximations



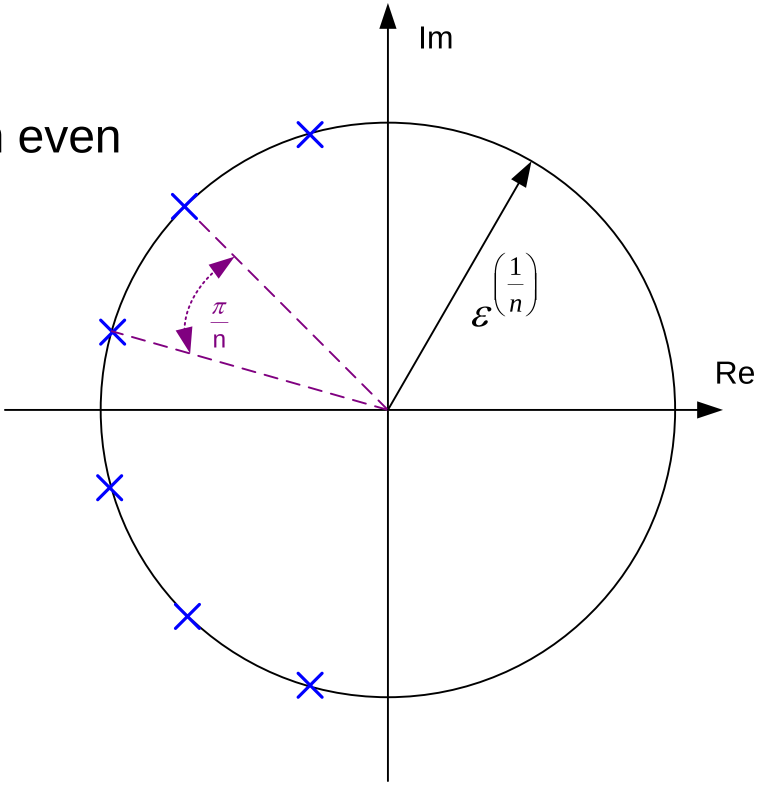
- Analytical formulation:
  - All pole approximation
  - Magnitude response is maximally flat at ω=0
  - Goes to 0 at ω=∞
  - Assumes value $\sqrt{\frac{1}{1+\varepsilon^2}}$ at ω=1
  - Assumes value of 1 at ω=0
  - Characterized by {n,ε}
- Emphasis almost entirely on performance at single frequency

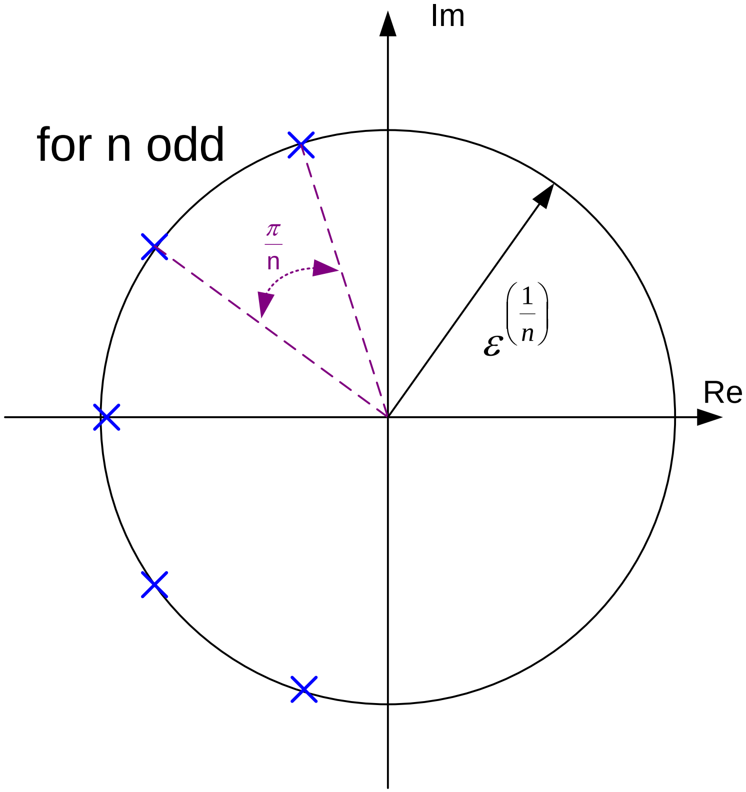
"On the Theory of Filter Amplifiers", Wireless Engineer (also called Experimental Wireless and the Radio Engineer), Vol. 7, 1930, pp. 536-541.

Review from Last Time

Butterworth Approximation

Poles of $T_{BW}(s)$





$$p_{k+1} = \varepsilon^{1/n}\left[-\sin\left(\left[1+2k\right]\frac{\pi}{2n}\right) \pm j\cos\left(\left[1+2k\right]\frac{\pi}{2n}\right)\right]$$

k=0,1, ... $\frac{n}{2}-1$

$$p_n = \varepsilon^{1/n}\left[-1+j0\right]$$

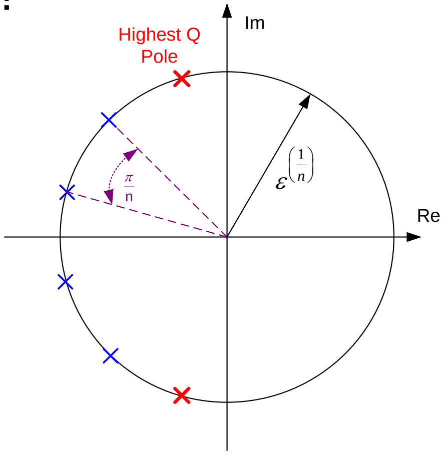
$$p_k = \varepsilon^{1/n}\left[-\sin\left(\left[1+2k\right]\frac{\pi}{2n}\right) \pm j\cos\left(\left[1+2k\right]\frac{\pi}{2n}\right)\right]$$

k=0, ... $\frac{n-3}{2}$

Review from Last Time

Butterworth Approximation

What is the Q of the highest Q pole for the BW approximation?



$$p_0 = \varepsilon^{1/n}\left[-\sin\left(\frac{\pi}{2n}\right) + j\cos\left(\frac{\pi}{2n}\right)\right] = \alpha + j\beta$$

$$Q_{MAX} = \frac{\sqrt{\alpha^2+\beta^2}}{-2\alpha}$$

$$Q_{MAX} = \frac{\varepsilon^{1/n}\sqrt{\sin^2\left(\frac{\pi}{2n}\right)+\cos^2\left(\frac{\pi}{2n}\right)}}{2\varepsilon^{1/n}\sin\left(\frac{\pi}{2n}\right)} = \frac{1}{2\sin\left(\frac{\pi}{2n}\right)}$$

$$Q_{MAX} = \frac{1}{2\sin\left(\frac{\pi}{2n}\right)}$$

Butterworth Approximation

Review from Last Time

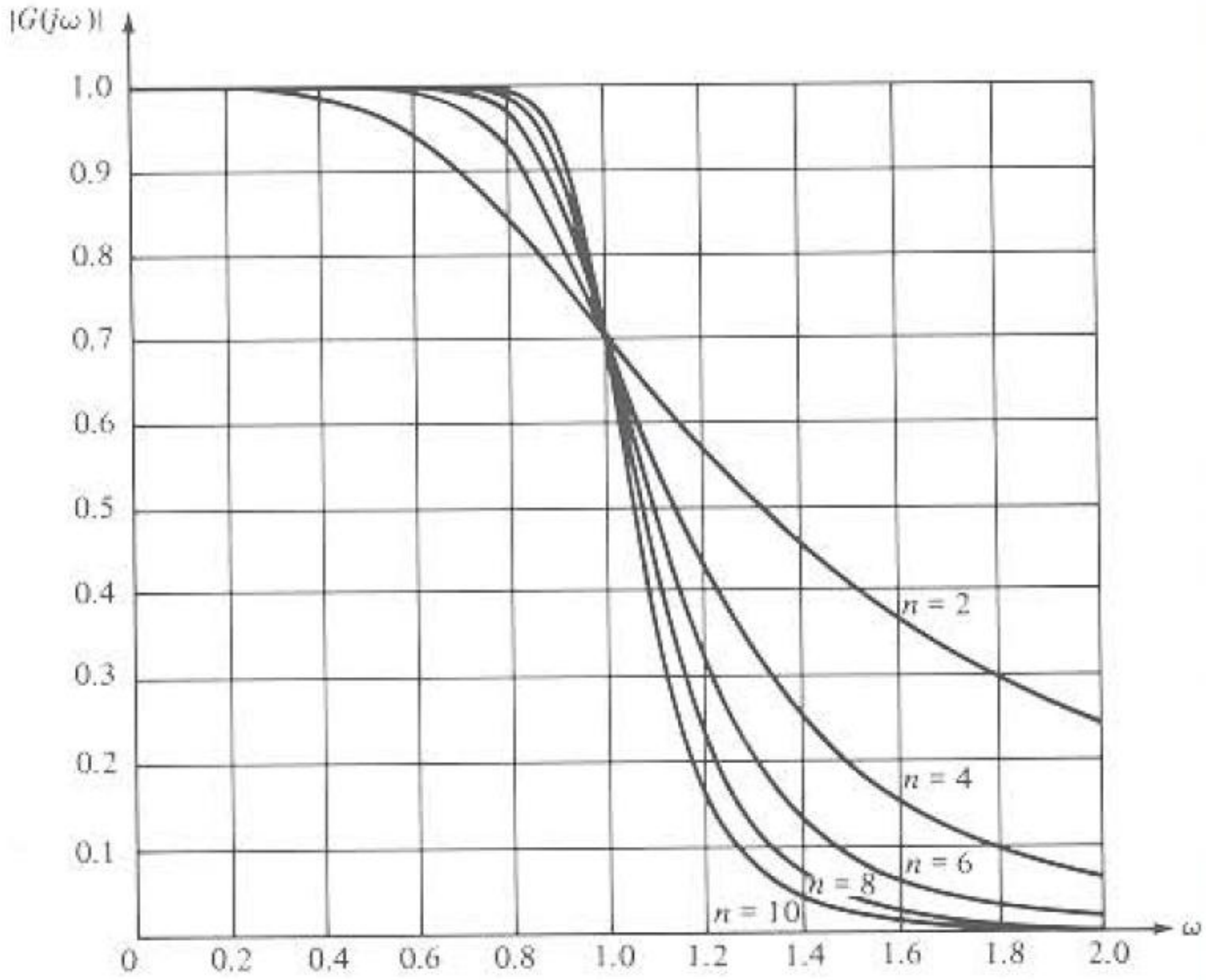


Fig. 17-3a Magnitude of the maximally flat approximation ($\varepsilon = 1$)

Order needs to be rather high to get steep transition

Figure from Passive and Acti
Network Analysis and
Synthesis, Budak

Butterworth Approximation

Review from Last Time

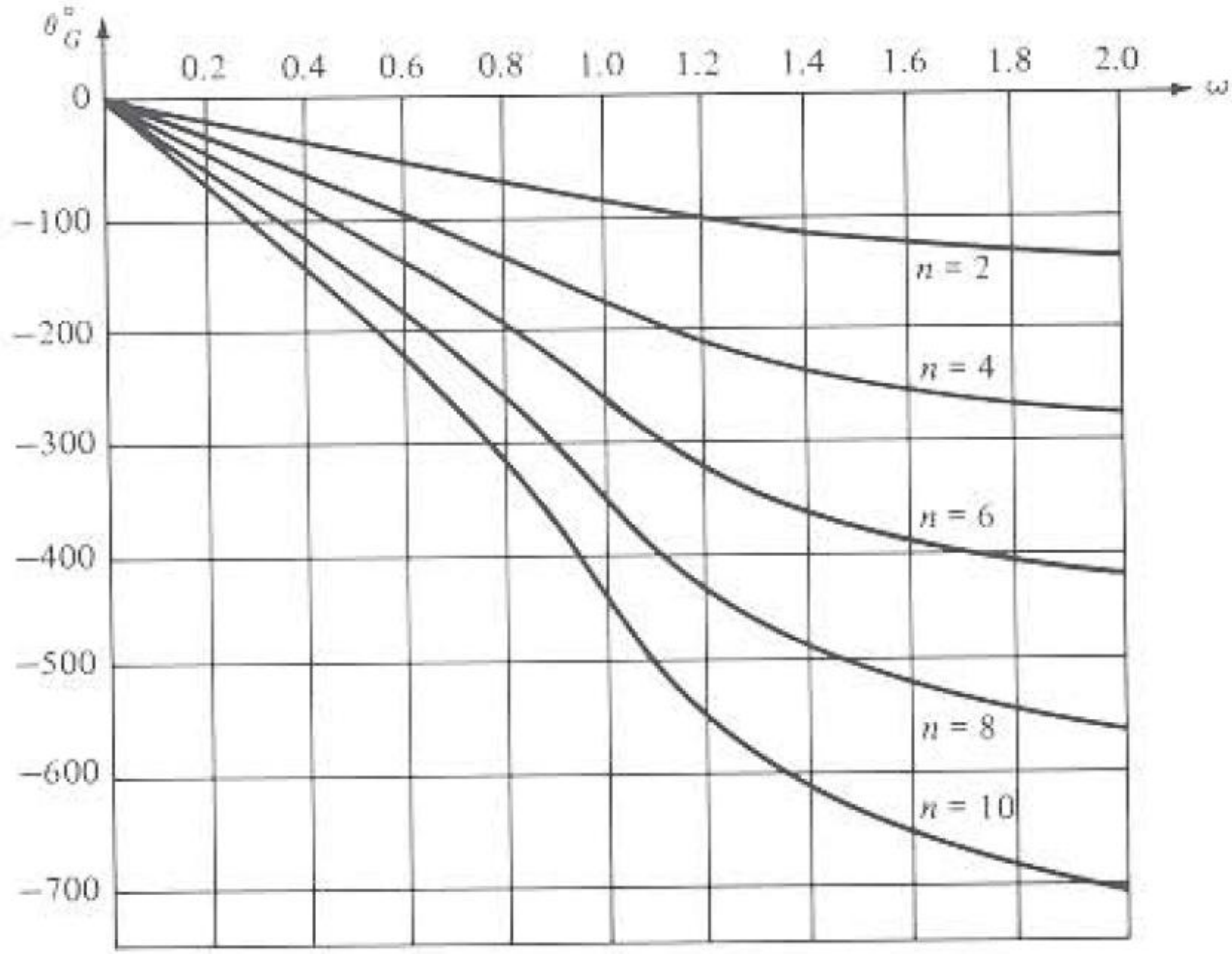


Fig. 17-3b Phase of the maximally flat approximation ($\varepsilon = 1$)

Figure from Passive and Active Network Analysis and Synthesis, Budak

Phase is quite linear in passband (benefit unrelated to design requirements)

Butterworth Approximation

Review from Last Time

Summary

- Widely Used Analytical Approximation
- Characterized by {ε,n}
- Maximally flat at ω=0
- Almost all emphasis placed on characteristics at single frequency (ω=0)
- Transition not very steep (requires large order for steep transition)
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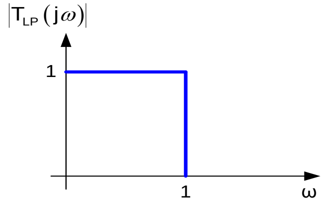
- Poles lie on a circle
- Simple closed-form analytical expressions for poles and |T(jω)|

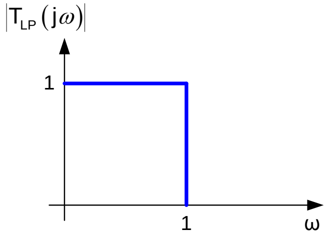
Approximations

- Magnitude Squared Approximating Functions – $H_A(\omega^2)$
- Inverse Transform - $H_A(\omega^2) \rightarrow T_A(s)$
- Collocation
- Least Squares Approximations
- Pade Approximations
- Other Analytical Optimizations
- Numerical Optimization
- Canonical Approximations
  - Butterworth
  - Chebyschev
  - Elliptic
  - Bessel
  - Thompson



Pafnuty Lvovich Chebyshev
Born May 16, 1821
Died December 8,1894
Nationality Russian
FieldsMathematician

Chebyshev Approximations

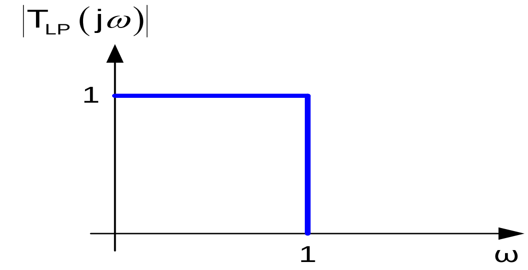


Type I Chebyshev Approximations

- Analytical formulation:
  - All pole approximation
  - Magnitude response bounded between 1 and $\sqrt{\frac{1}{1+\varepsilon^2}}$ in the pass band
  - Assumes the value of $\sqrt{\frac{1}{1+\varepsilon^2}}$ at ω=1
  - Goes to 0 at ω=∞
  - Assumes extreme values maximum no times in [0 1]
  - Characterized by {n,ε}
- Based upon Chebyshev Polynomials

Chebyshev polynomials were first presented in: P. L. Chebyshev (1854) "Théorie des mécanismes connus sous le nom parallelogrammes," *Mémoires des Savants étrangers présentes à l'Academie de Saint-Pétersbourg*, vol. 7, pages 539-586.

Chebyshev Approximations



Type II Chebyshev Approximations (not so common)

- Analytical formulation:
  - Magnitude response bounded between 0 and $\frac{\varepsilon}{\sqrt{1+\varepsilon^2}}$ in the stop band
  - Assumes the value of $\sqrt{\frac{1}{1+\varepsilon^2}}$ at ω=1
  - Value of 1 at ω=0
  - Assumes extreme values maximum times in [1 ∞]
  - Characterized by {n,ε}
- Based upon Chebyshev Polynomials

Chebyshev Approximations

Chebyshev Polynomials

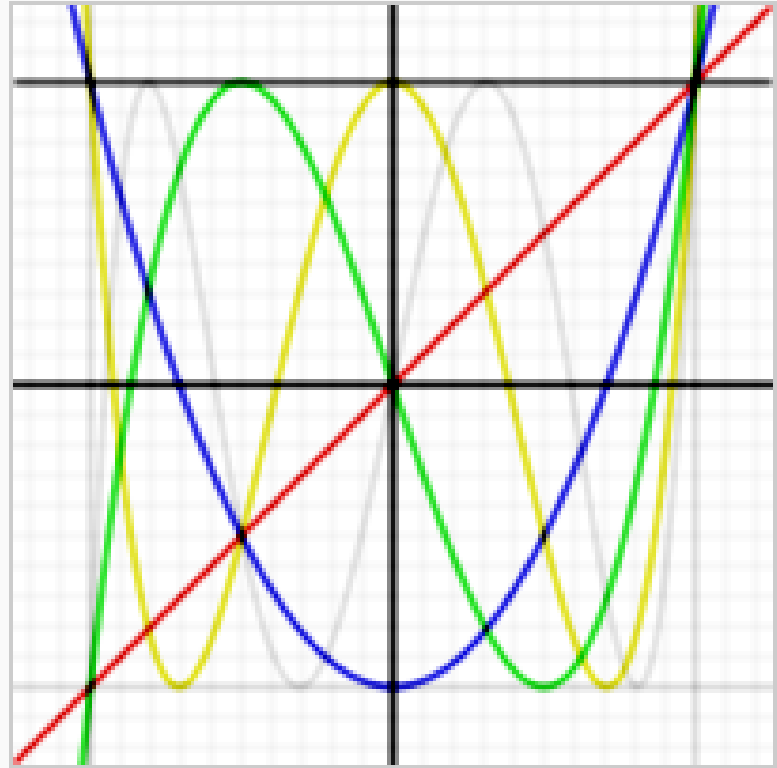
The Chebyshev polynomials are characterized by the property that the polynomial assumes the extremum values of 0 and 1 a maximum number of times in the interval [0,1] and go to ∞ for x large.

In polynomial form they can be expressed as

$$C_0(x)=1$$
$$C_1(x)=x$$
$$C_{n+1}(x)=2xC_n(x) - C_{n-1}(x)$$

Or, equivalently, in trigonometric form as

$$C_n(x)=\begin{cases} \cos\left(n\bullet arc\cos(x)\right) & x\in[-1,1] \\ \cosh\left(n\bullet arc\cos h(x)\right) & x\geq 1 \\ (-1)^n\cosh\left(n\bullet arc\cos h(-x)\right) & x\leq -1 \end{cases}$$



This image shows the first few Chebyshev polynomials of the first kind in the domain −1¼ < x < 1¼, −1¼ < y < 1¼; the flat T_0, and T_1, T_2, T_3, T_4 and T_5.

Figure from Wikipedia

Chebyshev Approximations

Chebyshev Polynomials

The first 9 CC polynomials:

$$C_0(x) = 1$$

$$C_1(x) = x$$

$$C_2(x) = 2x^2 - 1$$

$$C_3(x) = 4x^3 - 3x$$

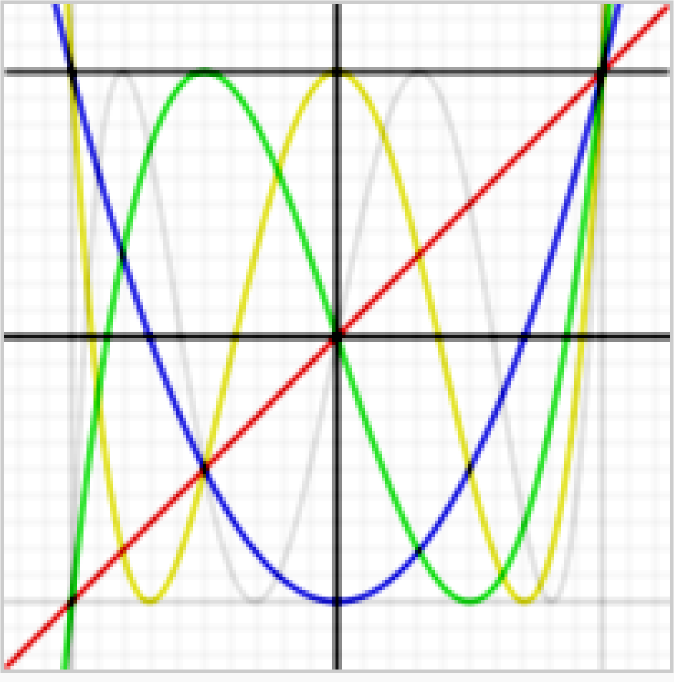
$$C_4(x) = 8x^4 - 8x^2 + 1$$

$$C_5(x) = 16x^5 - 20x^3 + 5x$$

$$C_6(x) = 32x^6 - 48x^4 + 18x^2 - 1$$

$$C_7(x) = 64x^7 - 112x^5 + 56x^3 - 7x$$

$$C_8(x) = 128x^8 - 256x^6 + 160x^4 - 32x^2 + 1$$



This image shows the first few Chebyshev polynomials of the first kind in the domain −1¼ < x < 1¼, −1¼ < y < 1¼; the flat T_0, and T_1, T_2, T_3, T_4 and T_5. Figure from Wikipedia

- Even-indexed polynomials are functions of x^2
- Odd-indexed polynomials are product of x and function of x^2
- Square of all polynomials are function of x^2 (i.e. an even function of x)

Chebyshev Approximations

Type 1

$$H_{BW}(\omega)=\frac{1}{1+\varepsilon^2\omega^{2n}}$$

Butterworth

$$H(\omega)=\frac{1}{1+\varepsilon^2 F_n\left(\omega^2\right)}$$

A General Form

Observation:

$F_n(\omega^2)$ close to 1 in the pass band and gets very large in stop-band

The square of the Chebyshev polynomials have this property

$$H_{CC}(\omega)=\frac{1}{1+\varepsilon^2 C_n^2(\omega)}$$

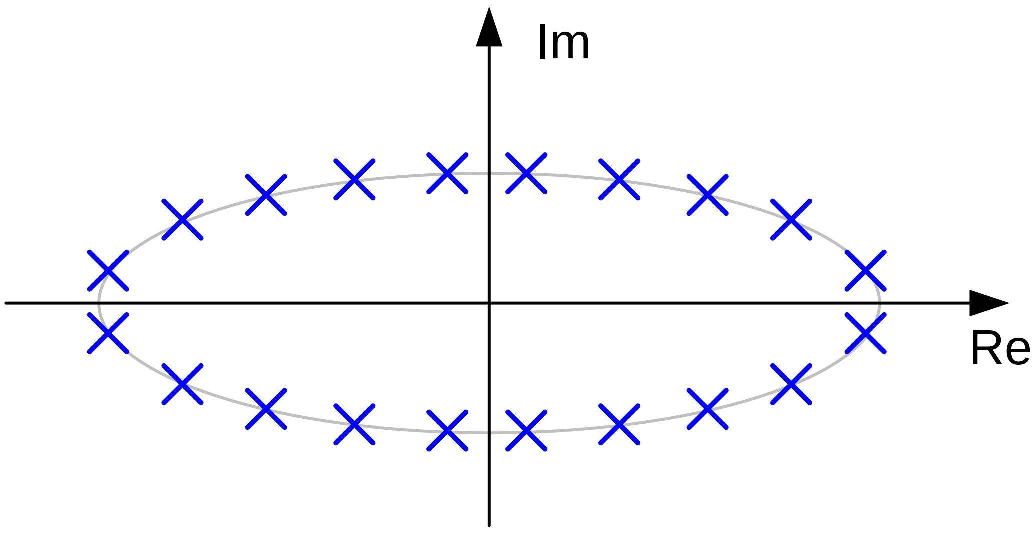
This is the magnitude squared approximating function of the Type 1 CC approximation

Chebyshev Approximations

Type 1

$$H_{CC}(\omega) = \frac{1}{1+\varepsilon^2 C_n^2(\omega)}$$

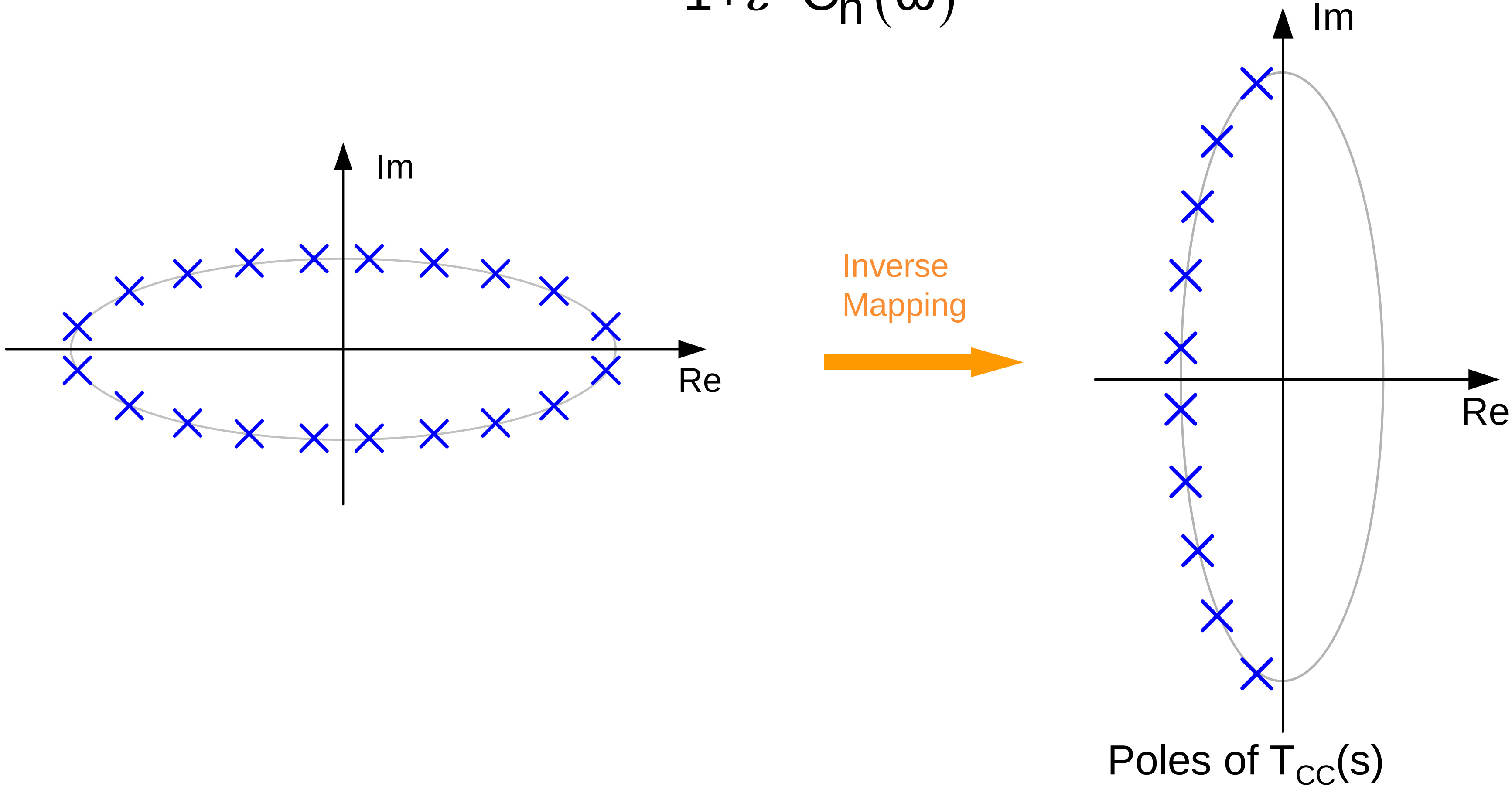
Poles of $H_{CC}(\omega)$ lie on an ellipse with none on the real axis



Chebyshev Approximations

Type 1

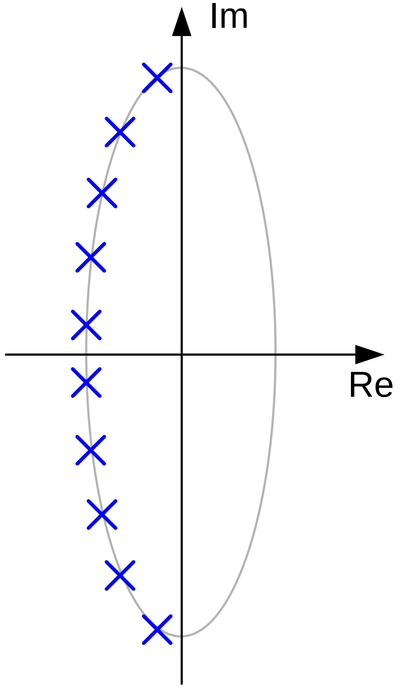
$$H_{CC}(\omega)=\frac{1}{1+\varepsilon^2 C_n^2(\omega)}$$



Chebyshev Approximations

Type 1

$$\left[\frac{\alpha_k}{\sinh\left[\frac{1}{n} arc\sinh\left(\frac{1}{\varepsilon}\right)\right]}\right]^2 + \left[\frac{\beta_k}{\cosh\left[\frac{1}{n} arc\sinh\left(\frac{1}{\varepsilon}\right)\right]}\right]^2 = 1$$

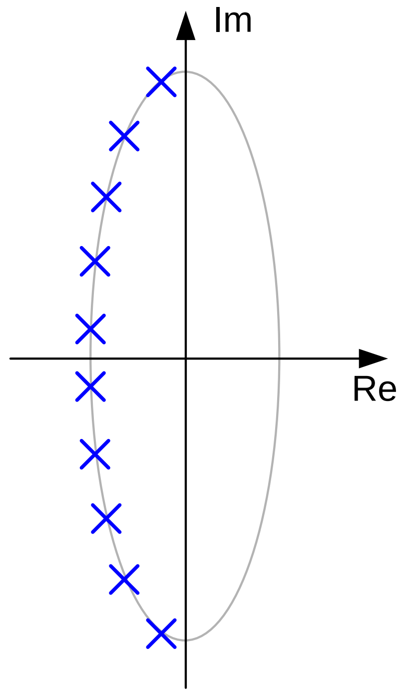


Ellipse Intersect Points for select n and ε

n	ε		Y int	X int
2	1		1.099	0.455
2	0.25		1.600	1.250
2	0.1		2.351	2.127
2	0.05		3.242	3.084
4	1		1.024	0.222
4	0.25		1.140	0.548
4	0.1		1.294	0.822
4	0.05		1.456	1.059
6	1		1.011	0.147
6	0.25		1.062	0.356
6	0.1		1.127	0.521
6	0.05		1.195	0.654
8	1		1.006	0.110
8	0.25		1.034	0.265
8	0.1		1.071	0.384
8	0.05		1.108	0.478

Chebyshev Approximations

Type 1



Poles of $T_{CC}(s)$

$$p_k = -\sin\left[\frac{\pi}{2n}(1+2k)\right]\sinh\left[\frac{1}{n}arc\sinh\left(\frac{1}{\varepsilon}\right)\right] \pm j\cos\left[\frac{\pi}{2n}(1+2k)\right]\cosh\left[\frac{1}{n}arc\sinh\left(\frac{1}{\varepsilon}\right)\right] \quad k=0\ldots.n-1$$

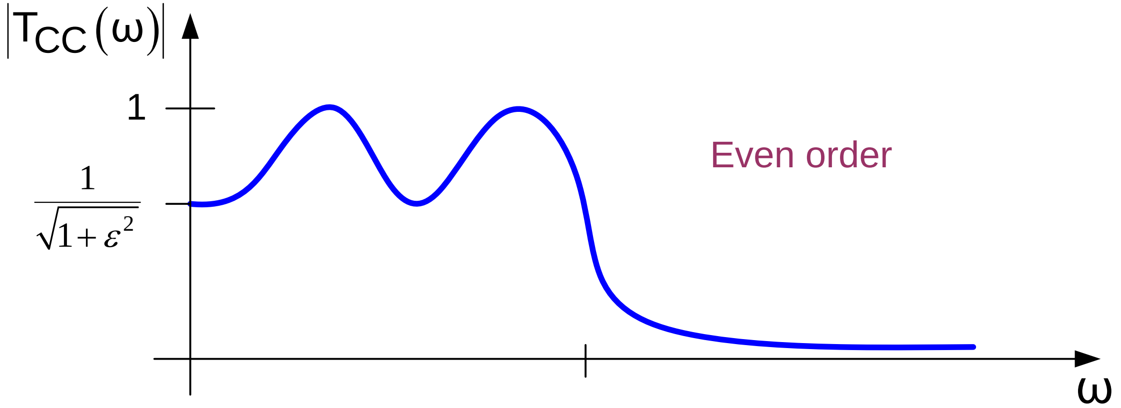
Properties of the ellipse

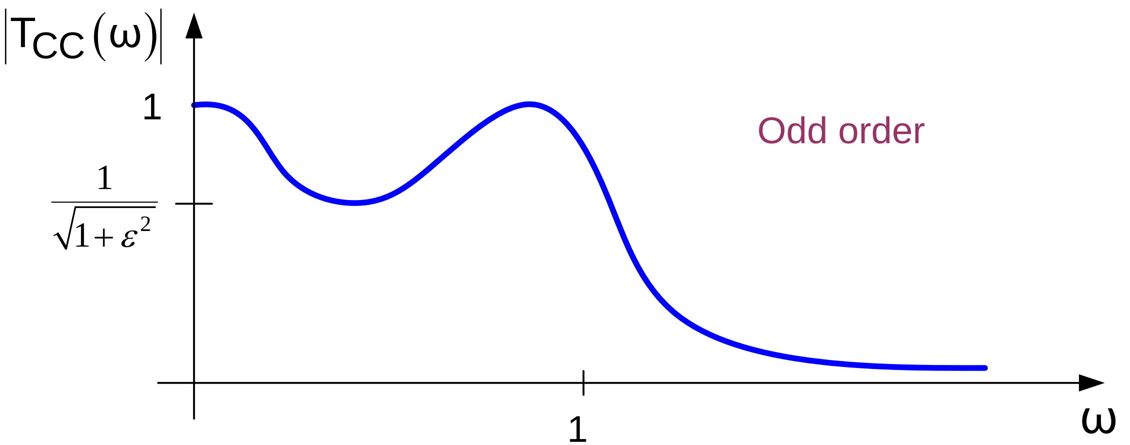
$$p_k = -\alpha_k \pm j\beta_k$$

$$\left[\frac{\alpha_k}{\sinh\left[\frac{1}{n}arc\sinh\left(\frac{1}{\varepsilon}\right)\right]}\right]^2 + \left[\frac{\beta_k}{\cosh\left[\frac{1}{n}arc\sinh\left(\frac{1}{\varepsilon}\right)\right]}\right]^2 = 1$$

Chebyshev Approximations

Type 1





- $|T_{CC}(0)|$ alternates between 1 and $\sqrt{\frac{1}{1+\varepsilon^2}}$ with index number
- Substantial pass band ripple
- Sharp transitions from pass band to stop band

Chebyshev Approximations

Type 1

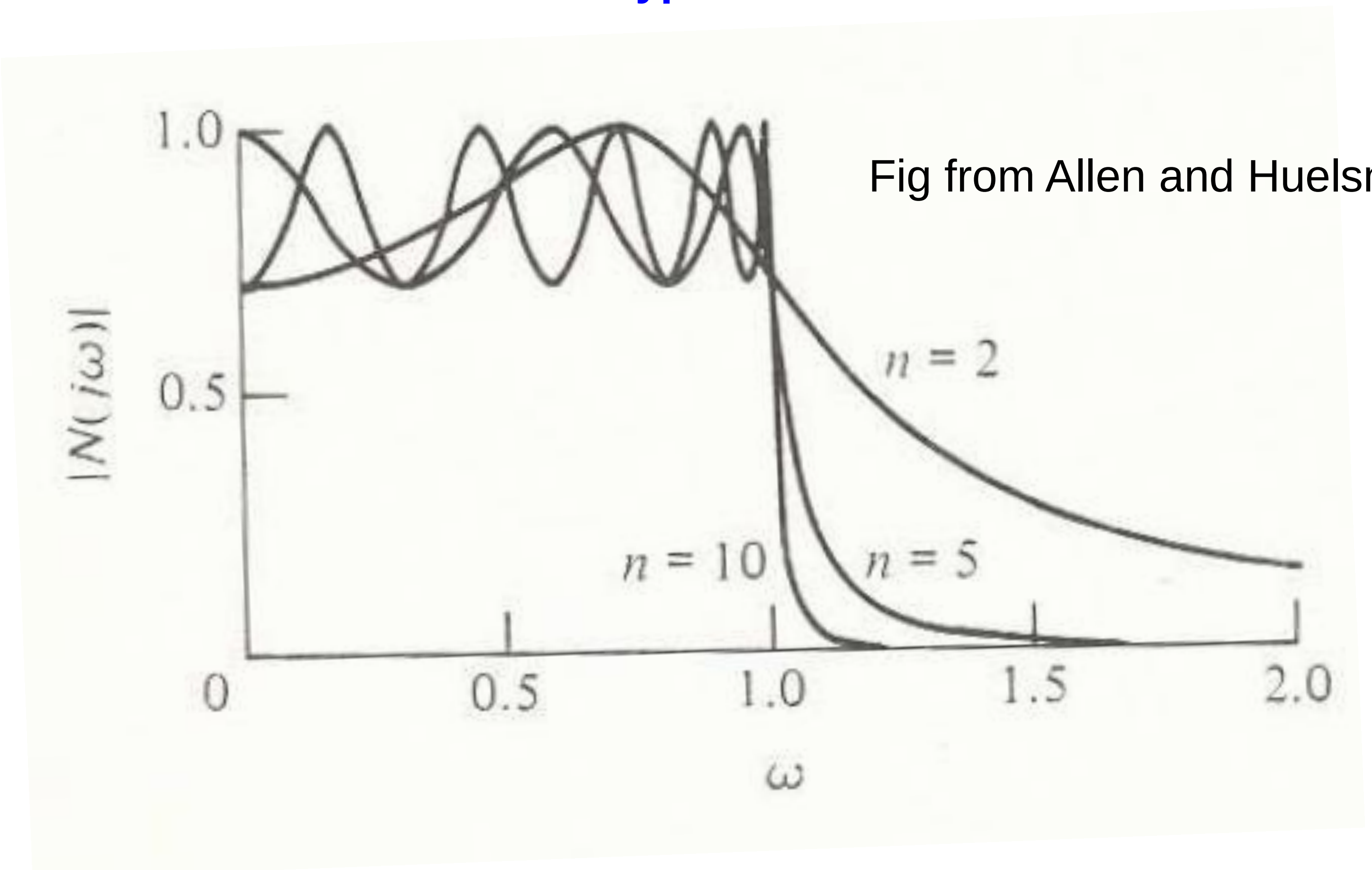


Fig from Allen and Huelsman

Sharp transitions from pass band to stop band

Chebyshev Approximations

Type 1

From Budak Text

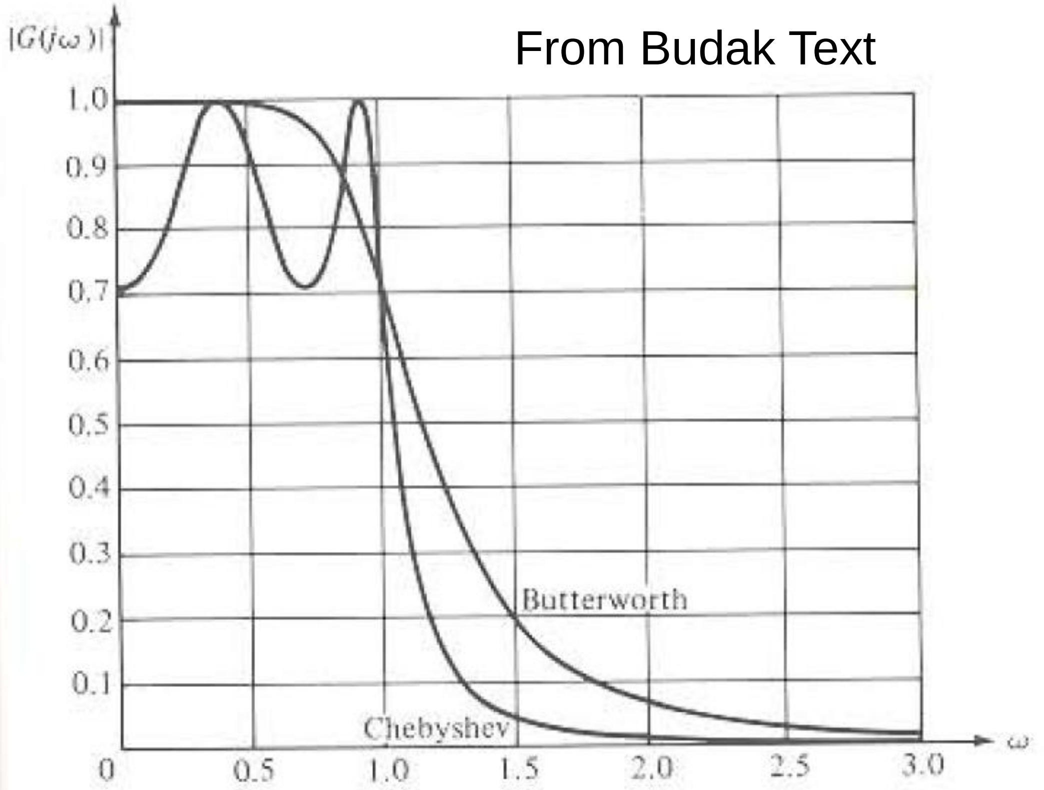


Fig. 17-6a Fourth-order Chebyshev and Butterworth magnitude characteristics

CC transition is much steeper than BW transition

Comparison of BW and CC Responses

- CC slope at band edge much steeper than that of BW

$$Slope_{CC}(\omega=1)=\left(\frac{-n}{2\sqrt{2}}\right)n=[Slope_{BW}(\omega=1)]$$

- Corresponding pole Q of CC much higher than that of BW
- Lower-order CC filter can often meet same band-edge transition as a given BW filter
- Both are widely used
- Cost of implementation of BW and CC for same order is about the same

Chebyshev Approximations

Type 1

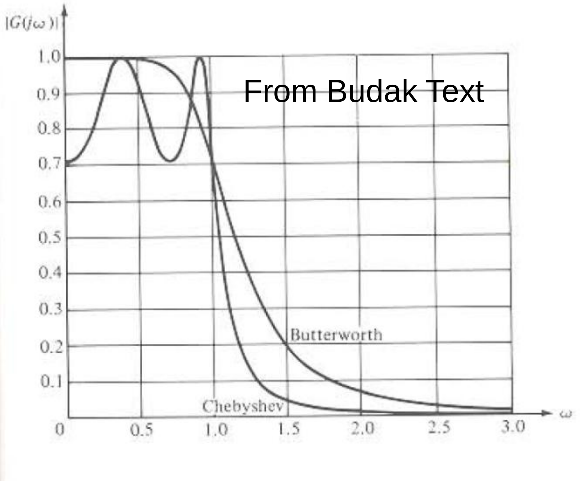


Fig. 17-6a Fourth-order Chebyshev and Butterworth magnitude characteristics

Analytically, it can be shown that, at the band-edge

$$\frac{d\left|T_{BW}(j\omega)\right|}{d\omega} = -n\frac{\varepsilon^2}{\left(1+\varepsilon^2\right)^{3/2}}$$

$$\frac{d\left|T_{CC}(j\omega)\right|}{d\omega} = -n^2\frac{\varepsilon^2}{\left(1+\varepsilon^2\right)^{3/2}}$$

CC slope is n times steeper than that of the BW slope

Chebyshev Approximations

Type 1

From Budak Text

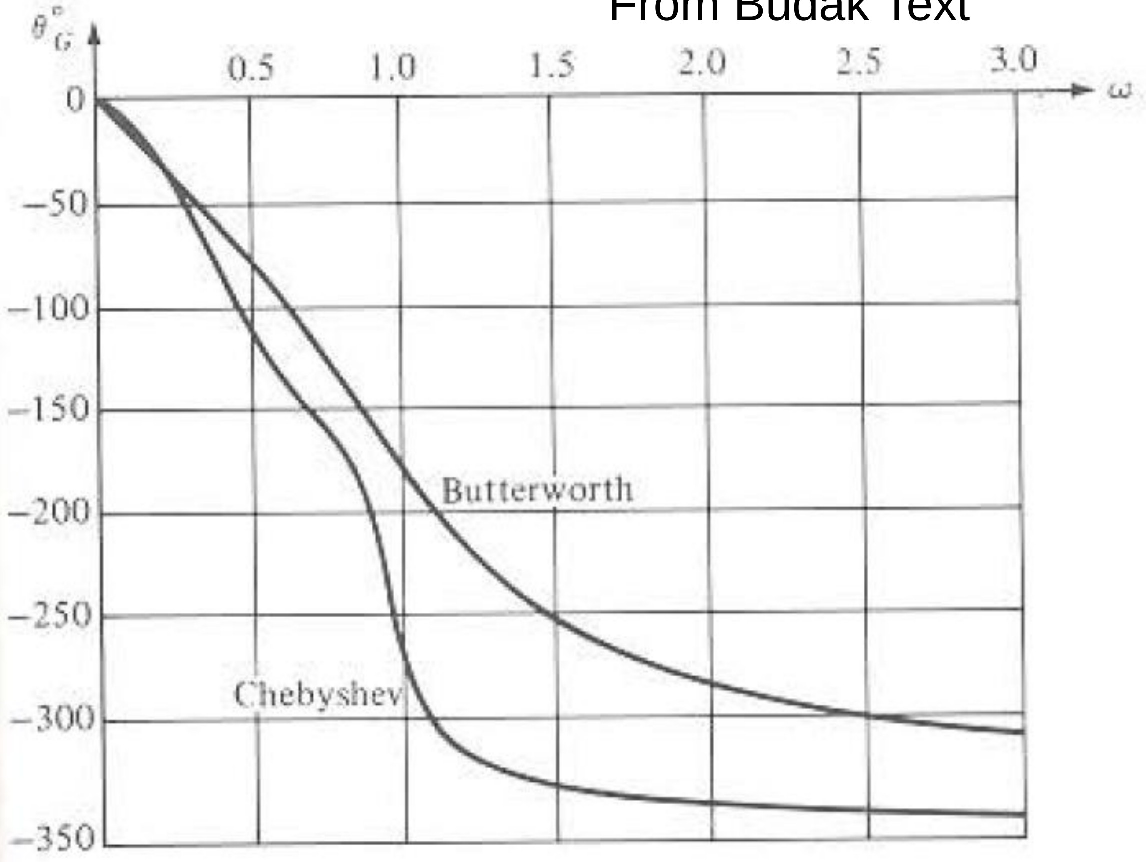
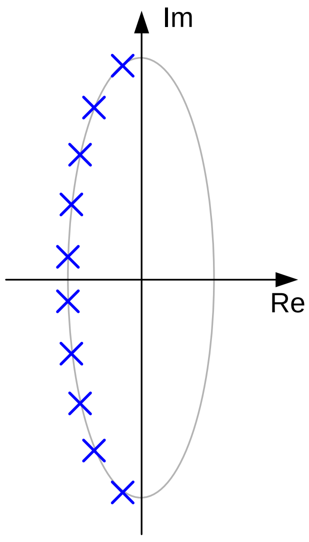


Fig. 17-6b Fourth-order Chebyshev and Butterworth phase characteristics

CC phase is much more nonlinear than BW phase

Chebyshev Approximations

Type 1



$$p_k = -\sin\left[\frac{\pi}{2n}(1+2k)\right]\sinh\left[\frac{1}{n}arc\sinh\left(\frac{1}{\varepsilon}\right)\right] \pm j\cos\left[\frac{\pi}{2n}(1+2k)\right]\cosh\left[\frac{1}{n}arc\sinh\left(\frac{1}{\varepsilon}\right)\right]$$

Maximum pole Q of CC approximation can be obtained by considering pole with index k=0

$$p_0 = -\sin\left[\frac{\pi}{2n}\right]\sinh\left[\frac{1}{n}arc\sinh\left(\frac{1}{\varepsilon}\right)\right] \pm j\cos\left[\frac{\pi}{2n}\right]\cosh\left[\frac{1}{n}arc\sinh\left(\frac{1}{\varepsilon}\right)\right]$$

$$p_0 = \alpha + j\beta$$

Recall

$$Q_{MAX} = \frac{\sqrt{\alpha^2+\beta^2}}{-2\alpha}$$

$$Q_{MAX,CC} = \left(\frac{1}{2\sin\left(\frac{\pi}{2n}\right)}\right)\sqrt{1+\left[\frac{\cos\left(\frac{\pi}{2n}\right)}{\sinh\left(\frac{1}{n}arc\sinh\left(\frac{1}{\varepsilon}\right)\right)}\right]^2}$$

Chebyshev Approximations

Type 1

Comparison of maximum pole Q of CC approximation with that of BW approximation

$$Q_{MAX,BW} = \frac{1}{2\sin\left(\frac{\pi}{2n}\right)} \qquad Q_{MAX,CC} = \left(\frac{1}{2\sin\left(\frac{\pi}{2n}\right)}\right)\sqrt{1+\left[\frac{\cos\left(\frac{\pi}{2n}\right)}{\sinh\left(\frac{1}{n}arc\sinh\left(\frac{1}{\varepsilon}\right)\right)}\right]^2}$$

$$Q_{MAX,CC} = Q_{MAX,BW}\sqrt{1+\left[\frac{\cos\left(\frac{\pi}{2n}\right)}{\sinh\left(\frac{1}{n}arc\sinh\left(\frac{1}{\varepsilon}\right)\right)}\right]^2}$$

Example – compare the Q's for n=10 and ε=1

$Q_{BW}=3.19$ $\qquad$ $Q_{CC}=35.9$

For large n, the CC filters have a very high pole Q !

Chebyshev Approximations

Type 2

$$H_{BW}(\omega)=\frac{1}{1+\varepsilon^2\omega^{2n}}$$

Butterworth

$$H(\omega)=\frac{1}{1+\varepsilon^2 F_n\left(\omega^2\right)}$$

A General Form

Another General Form

$$H(\omega)=\frac{1}{1+\frac{1}{\varepsilon^2 F_n\left(1/\omega^2\right)}}$$

$$H_{CC2}(\omega)=\frac{1}{1+\frac{1}{\varepsilon^2 C_n^2\left(1/\omega\right)}}$$

Note: The second general form is not limited to use of the Chebyshev polynomials

Chebyshev Approximations

Type 2

$$H_{CC2}(\omega)=\frac{1}{1+\frac{1}{\varepsilon^2 C_n^2\left(1/\omega\right)}}$$

- Equal-ripple in stop band
- Monotone in pass band
- Both poles and zeros present
- Poles of Type II CC are reciprocal of poles of Type I
- Zeros of Type II are inverse of the zeros of the CC Polynomials

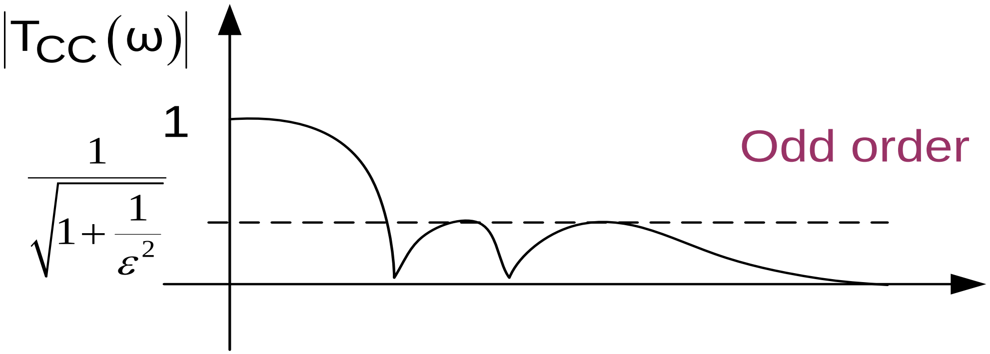
$$p_k=\frac{-1}{\sin\left[\frac{\pi}{2n}(1+2k)\right]\sinh\left[\frac{1}{n}arc\sinh\left(\frac{1}{\varepsilon}\right)\right]\pm j\cos\left[\frac{\pi}{2n}(1+2k)\right]\cosh\left[\frac{1}{n}arc\sinh\left(\frac{1}{\varepsilon}\right)\right]}$$

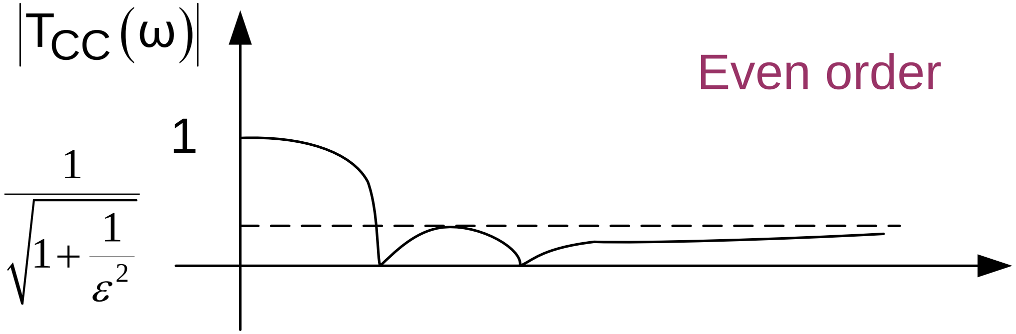
$$z_k=j\frac{1}{\cos\left(\frac{\pi}{2}\frac{(2k-1)}{n}\right)}$$

Chebyshev Approximations

Type 2

$$H_{CC2}(\omega) = \frac{1}{1 + \frac{1}{\varepsilon^2 C_n^2\left(1/\omega\right)}}$$





Chebyshev Approximations

Type 2

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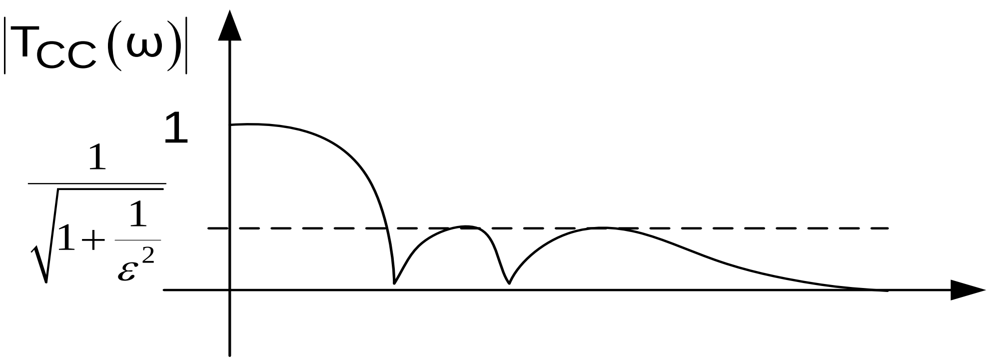
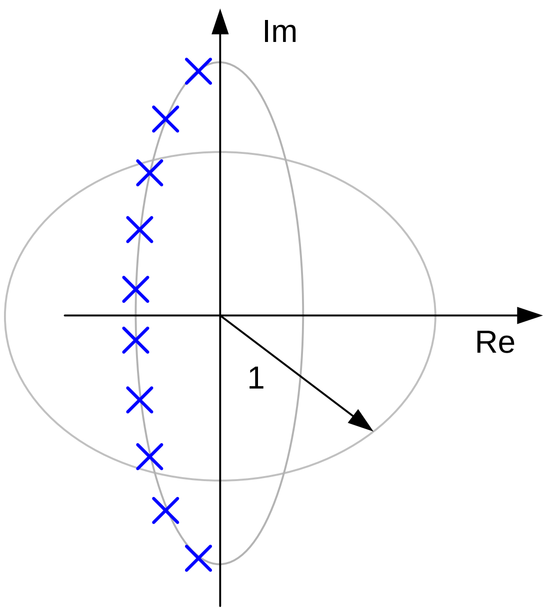


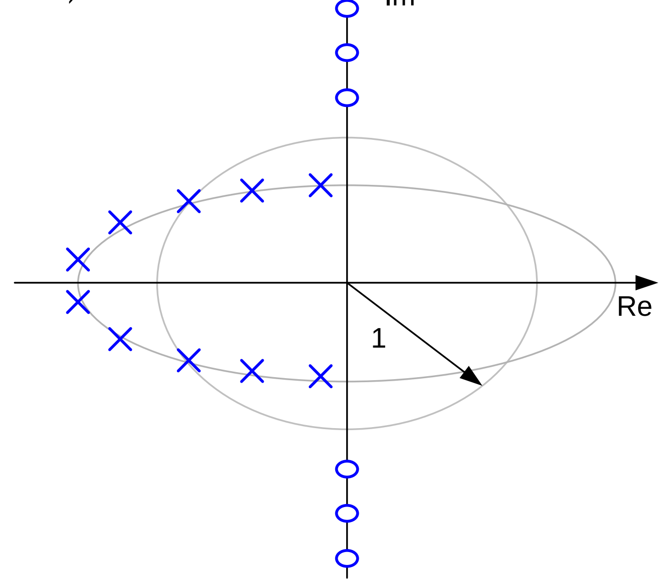
- Transition region not as steep as for Type 1
- Considerably less popular

Chebyshev Approximations

Type 2

$$H_{CC2}(\omega)=\frac{1}{1+\frac{1}{\varepsilon^2 C_n^2\left(1/\omega\right)}}$$





- Pole Q expressions identical since poles are reciprocals
- Maximum pole Q is just as high as for Type 1

Transitional BW-Chebyshev Approximations

$$H(\omega)=\frac{1}{1+\varepsilon^2 F_n\left(\omega^2\right)}$$

General Form

Define $F_{BWk}=\omega^{2k}$ $F_{CCk}=C_n^2(\omega)$

Consider:

$$H(\omega)=\frac{1}{1+\varepsilon^2 F_{BWk} F_{CC(n-k)}} \qquad 0 \le k \le n$$

$$H(\omega)=\frac{1}{1+\varepsilon^2\left[(\theta)F_{BWk}+(1-\theta)F_{CC(n-k)}\right]} \qquad 0 \le \theta \le 1$$

- Other transitional approximations are possible
- Transitional approximations have some of the properties of both “parents”

Transitional BW-CC filters

$$H_{ABW}\left(\omega^2\right)=\frac{1}{1+\varepsilon^2\omega^{2n}}$$

$$H_{ACC}\left(\omega^2\right)=\frac{1}{1+\varepsilon^2\left(C_n\left(\omega\right)\right)^2}$$

$$H_{ATRAN1}\left(\omega^2\right)=\frac{1}{1+\varepsilon^2\left(\omega^{2k}\right)C_{n-k}^2\left(\omega\right)}$$

$$0\le k\le n$$

$$H_{ATRAN2}\left(\omega^2\right)=\frac{1}{1+\varepsilon^2\left[\theta\omega^{2n}+\left(1-\theta\right)C_n^2\left(\omega\right)\right]}$$

$$0\le\theta\le 1$$

Other transitional BW-CC approximations exist as well

Transitional BW-CC filters

$$H_{ATRAN1}\left(\omega^2\right)=\frac{1}{1+\varepsilon^2\left(\omega^{2k}\right)C_{n-k}^2\left(\omega\right)}$$

$$H_{ATRAN2}\left(\omega^2\right)=\frac{1}{1+\varepsilon^2\left[\theta\omega^{2n}+\left(1-\theta\right)C_n^2\left(\omega\right)\right]}$$

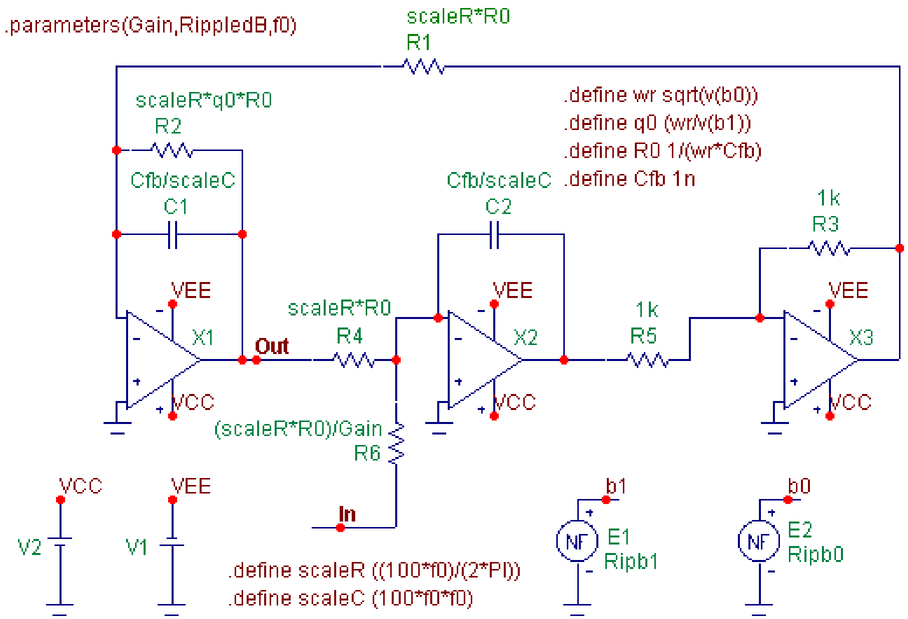
Transitional filters will exhibit flatness at ω=0, passband ripple, and intermediate slope characteristics at band-edge

Chebyshev Approximations

from Spectrum Software:

Chebyshev Filter Macro

Filters are a circuit element that seem to mesh perfectly with the macro capability of Micro-Cap. The macro capability is designed to produce components that can be varied through the use of parameters. Most filters consist of a basic structure whose component values can be modified through the use of well known equations. A macro component can be created that represents a specific filter's type, order, response, and implementation. The circuit below is the macro circuit for a low pass, 2nd order, Chebyshev filter with Tow-Thomas implementation.



- Note that this is introduced as a Chebyshev filter, the source correctly points out that it implements the CC filter in a specific filter topology
- It is important to not confuse the approximation from the architecture and this Tow-Thomas Structure can be used to implement either BW or CC functions only differing in the choice of the component values

End of Lecture 11

EE 508
Lecture 12

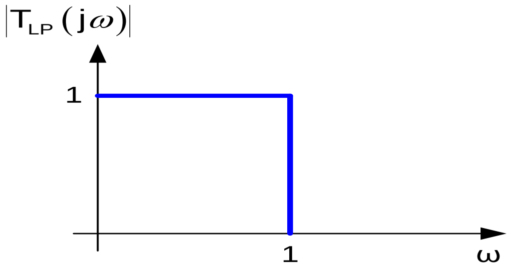
The Approximation Problem

Classical Approximating Functions

- Elliptic Approximations
- Thompson and Bessel Approximations

Review from Last Time

Chebyshev Approximations



Type II Chebyshev Approximations (not so common)

- Analytical formulation:
  - Magnitude response bounded between 0 and $\frac{\varepsilon}{\sqrt{1+\varepsilon^2}}$ in the stop band
  - Assumes the value of $\sqrt{\frac{1}{1+\varepsilon^2}}$ at ω=1
  - Value of 1 at ω=0
  - Assumes extreme values maximum times in [1 ∞]
  - Characterized by {n,ε}
- Based upon Chebyshev Polynomials

Review from Last Time

Chebyshev Approximations

Chebyshev Polynomials

The first 9 CC polynomials:

$$C_0(x) = 1$$

$$C_1(x) = x$$

$$C_2(x) = 2x^2 - 1$$

$$C_3(x) = 4x^3 - 3x$$

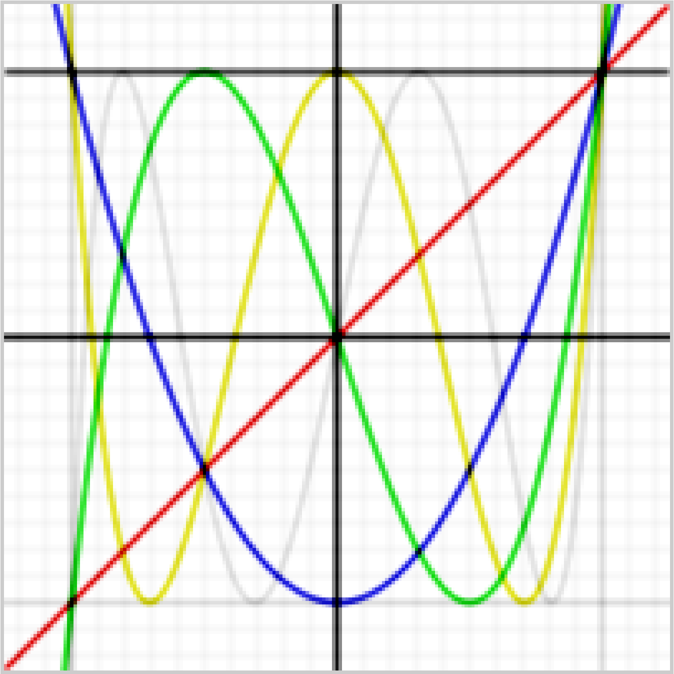
$$C_4(x) = 8x^4 - 8x^2 + 1$$

$$C_5(x) = 16x^5 - 20x^3 + 5x$$

$$C_6(x) = 32x^6 - 48x^4 + 18x^2 - 1$$

$$C_7(x) = 64x^7 - 112x^5 + 56x^3 - 7x$$

$$C_8(x) = 128x^8 - 256x^6 + 160x^4 - 32x^2 + 1$$



This image shows the first few Chebyshev polynomials of the first kind in the domain $-1\frac{1}{4} < x < 1\frac{1}{4}$, $-1\frac{1}{4} < y < 1\frac{1}{4}$; the flat T_0, and T_1, T_2, T_3, T_4 and T_5. Figure from Wikipedia

- Even-indexed polynomials are functions of x^2
- Odd-indexed polynomials are product of x and function of x^2
- Square of all polynomials are function of x^2 (i.e. an even function of x)

Review from Last Time

Chebyshev Approximations

Type 1

$$H_{BW}(\omega)=\frac{1}{1+\varepsilon^2\omega^{2n}}$$

Butterworth

$$H(\omega)=\frac{1}{1+\varepsilon^2 F_n\left(\omega^2\right)}$$

A General Form

Observation:

$F_n(\omega^2)$ close to 1 in the pass band and gets very large in stop-band

The square of the Chebyshev polynomials have this property

$$H_{CC}(\omega)=\frac{1}{1+\varepsilon^2 C_n^2(\omega)}$$

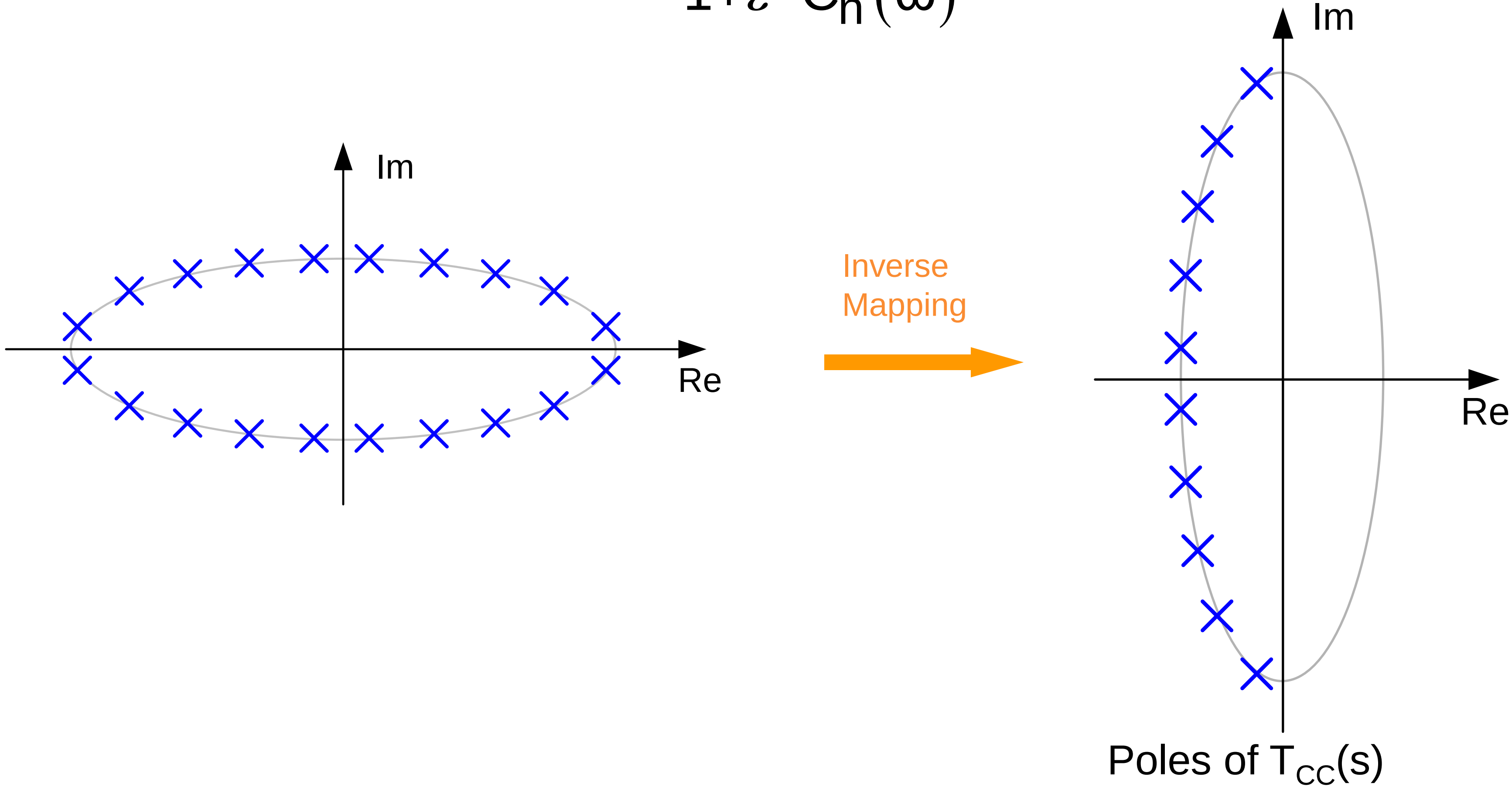
This is the magnitude squared approximating function of the Type 1 CC approximation

Review from Last Time

Chebyshev Approximations

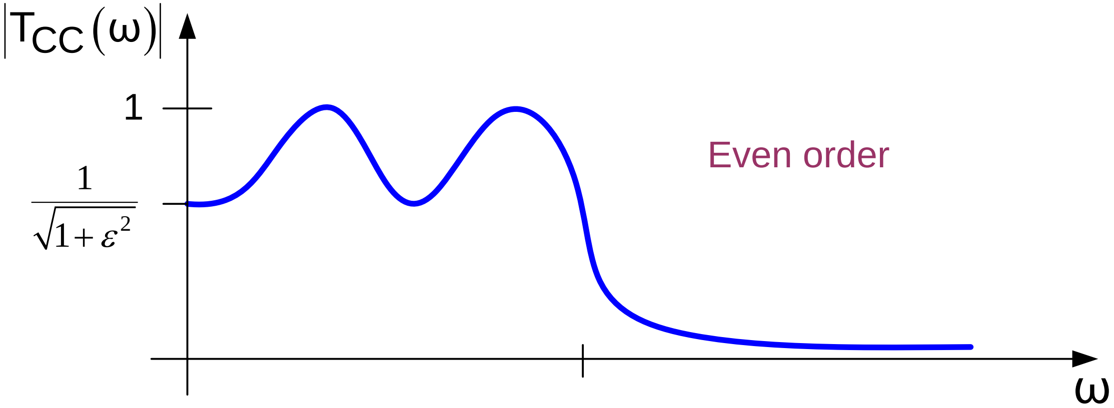
Type 1

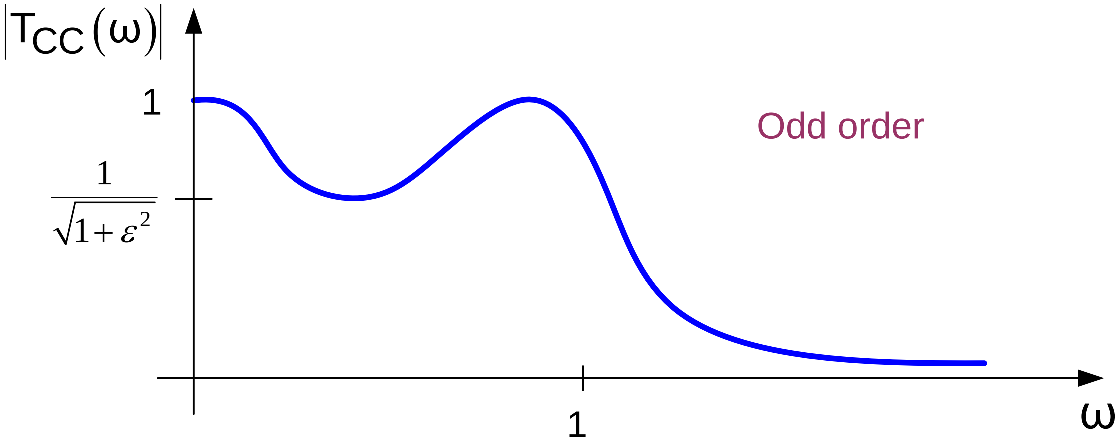
$$H_{CC}(\omega) = \frac{1}{1+\varepsilon^2 C_n^2(\omega)}$$



Chebyshev Approximations

Type 1





- $|T_{CC}(0)|$ alternates between 1 and $\sqrt{\frac{1}{1+\varepsilon^2}}$ with index number
- Substantial pass band ripple
- Sharp transitions from pass band to stop band

Comparison of BW and Type 1 CC Responses

- CC slope at band edge much steeper than that of BW

$$Slope_{CC}(\omega=1)=\left(\frac{-n}{2\sqrt{2}}\right)n=[Slope_{BW}(\omega=1)]$$

- Corresponding pole Q of CC much higher than that of BW
- Lower-order CC filter can often meet same band-edge transition as a given BW filter
- Both are widely used
- Cost of implementation of BW and CC for same order is about the same

Review from Last Time

Chebyshev Approximations

Type 2

$$H_{BW}(\omega)=\frac{1}{1+\varepsilon^2\omega^{2n}}$$

Butterworth

$$H(\omega)=\frac{1}{1+\varepsilon^2 F_n\left(\omega^2\right)}$$

A General Form

Another General Form

$$H(\omega)=\frac{1}{1+\frac{1}{\varepsilon^2 F_n\left(1/\omega^2\right)}}$$

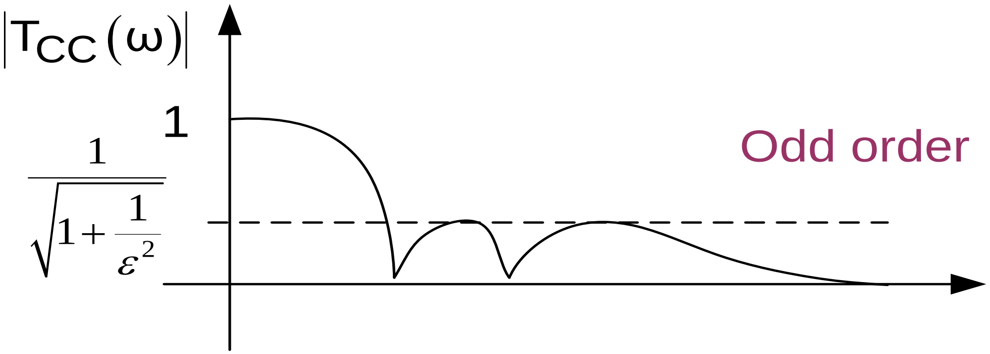
$$H_{CC2}(\omega)=\frac{1}{1+\frac{1}{\varepsilon^2 C_n^2\left(1/\omega\right)}}$$

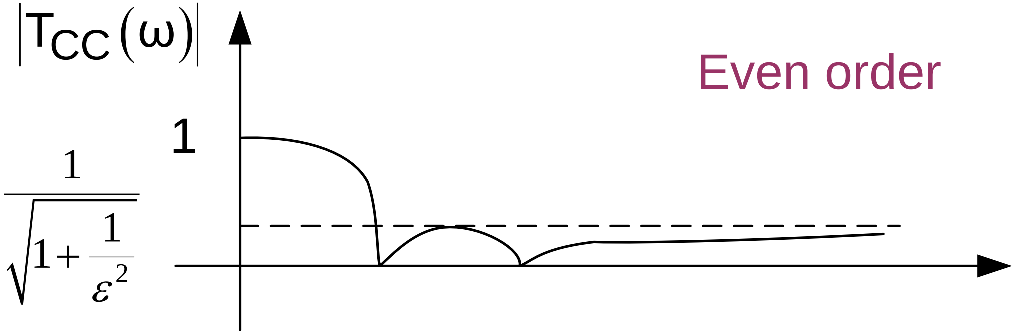
Review from Last Time

Chebyshev Approximations

Type 2

$$H_{CC2}(\omega)=\frac{1}{1+\frac{1}{\varepsilon^2 C_n^2\left(1/\omega\right)}}$$





Review from Last Time

Chebyshev Approximations

Type 2

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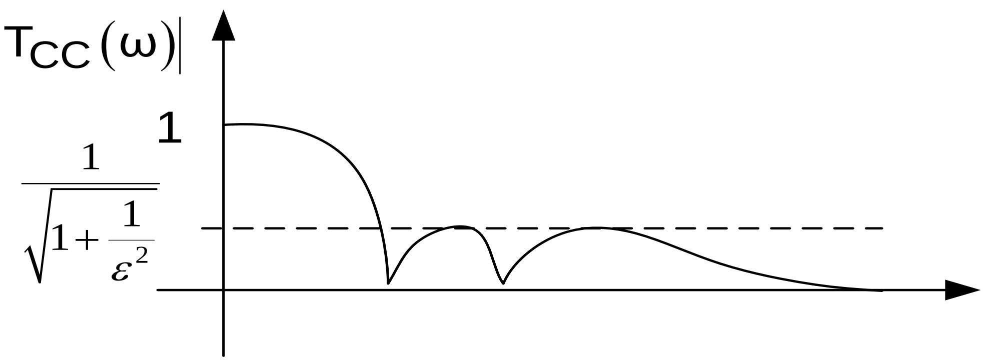

- Transition region not as steep as for Type 1
- Considerably less popular

Review from Last Time

Chebyshev Approximations

Type 2

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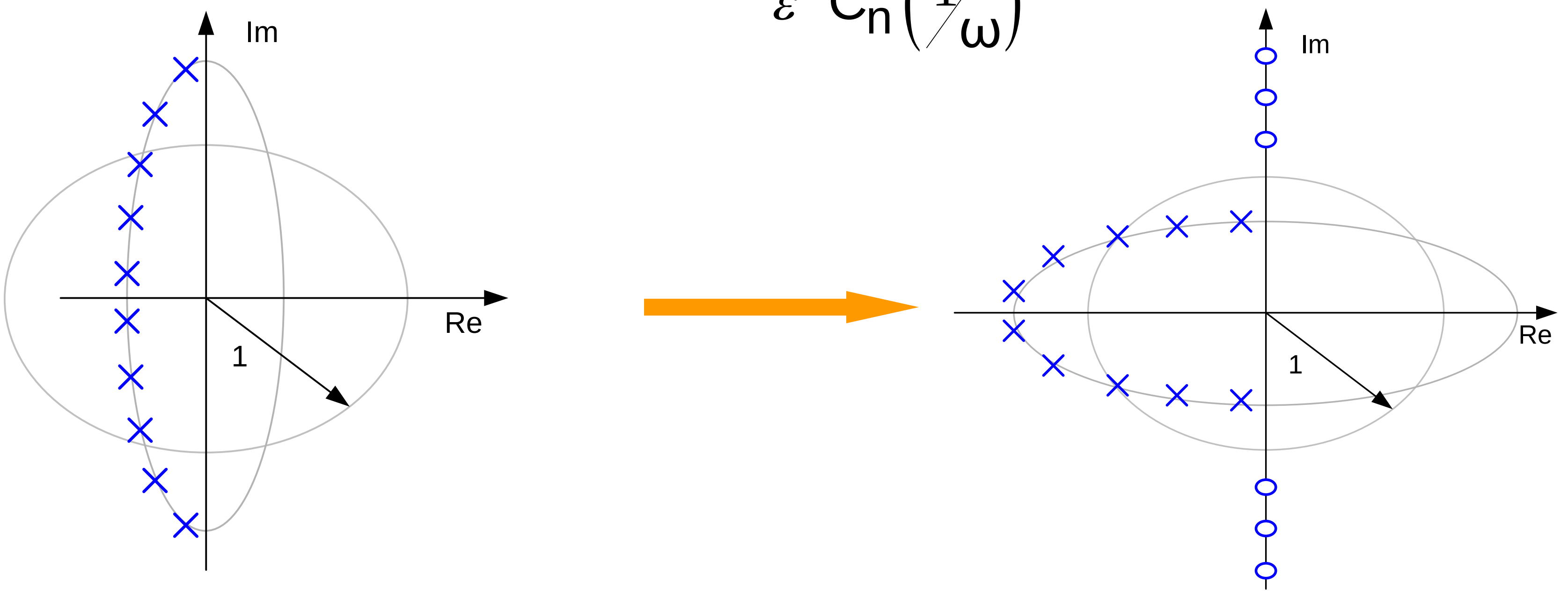


- Pole Q expressions identical since poles are reciprocals
- Maximum pole Q is just as high as for Type 1

Review from Last Time

Transitional BW-CC filters

$$H_{ATRAN1}\left(\omega^2\right)=\frac{1}{1+\varepsilon^2\left(\omega^{2k}\right)C_{n-k}^2\left(\omega\right)}$$

$$H_{ATRAN2}\left(\omega^2\right)=\frac{1}{1+\varepsilon^2\left[\theta\omega^{2n}+\left(1-\theta\right)C_{n}^2\left(\omega\right)\right]}$$

Transitional filters will exhibit flatness at ω=0, passband ripple, and intermediate slope characteristics at band-edge

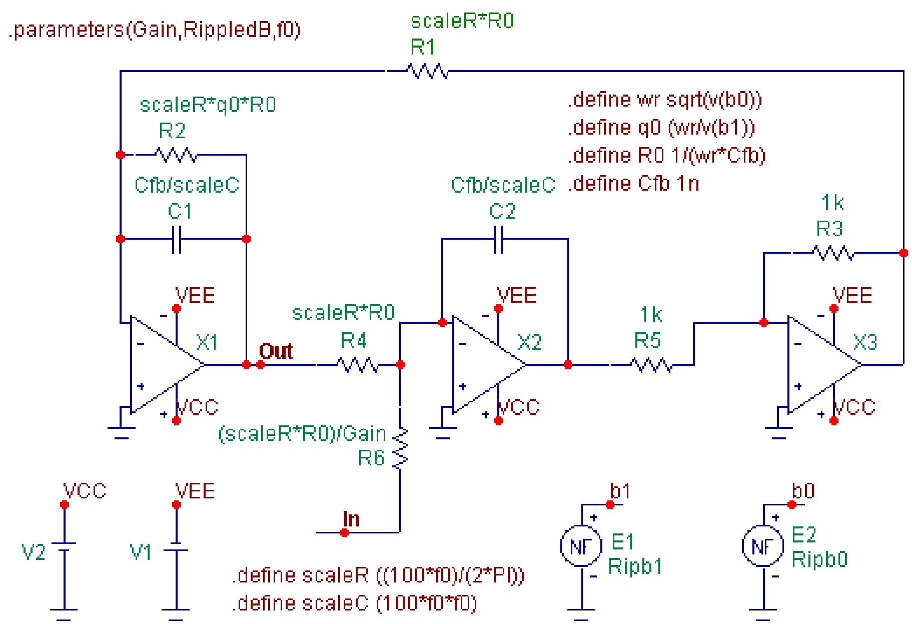
Chebyshev Approximations

from Spectrum Software:

Review from Last Time

Chebyshev Filter Macro

Filters are a circuit element that seem to mesh perfectly with the macro capability of Micro-Cap. The macro capability is designed to produce components that can be varied through the use of parameters. Most filters consist of a basic structure whose component values can be modified through the use of well known equations. A macro component can be created that represents a specific filter's type, order, response, and implementation. The circuit below is the macro circuit for a low pass, 2nd order, Chebyshev filter with Tow-Thomas implementation.



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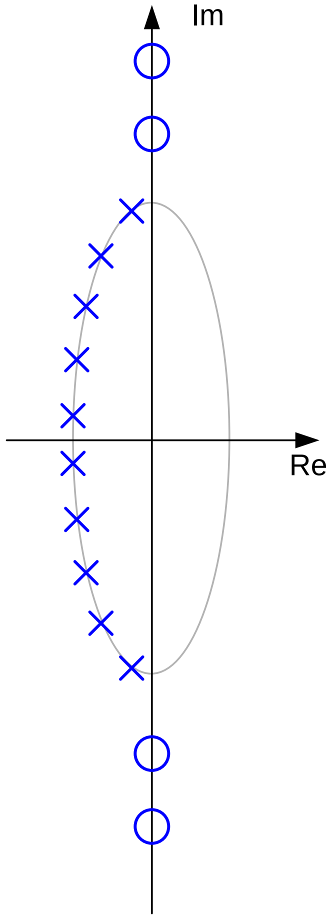
- It is important to not confuse the approximation from the architecture and this Tow-Thomas Structure can be used to implement either BW or CC functions only differing in the choice of the component values

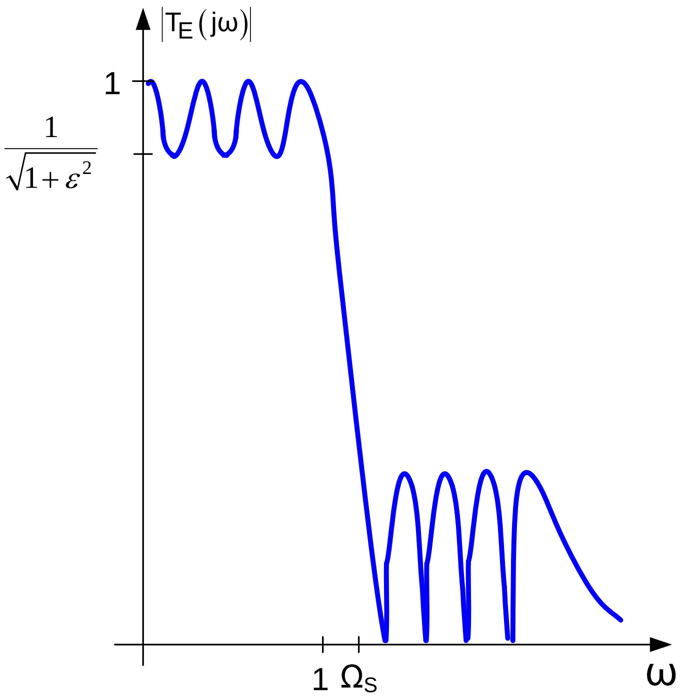
Approximations

- Magnitude Squared Approximating Functions – $H_A(\omega^2)$
- Inverse Transform - $H_A(\omega^2) \rightarrow T_A(s)$
- Collocation
- Least Squares Approximations
- Pade Approximations
- Other Analytical Optimizations
- Numerical Optimization
- Canonical Approximations
  - Butterworth
  - Chebyshev
  - Elliptic
  - Bessel
  - Thompson

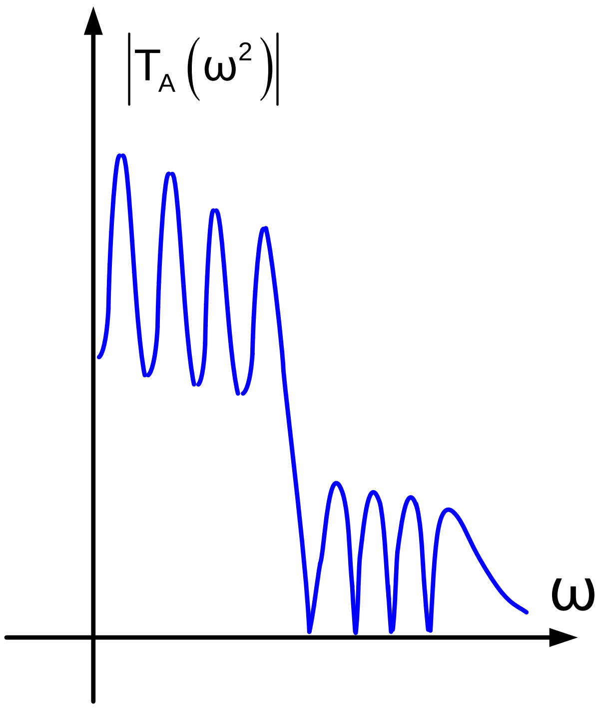
Elliptic Filters

Can be thought of as an extension of the CC approach by adding complex-conjugate zeros on the imaginary axis to increase the sharpness of the slope at the band edge





Concept



Actual effect of adding zeros

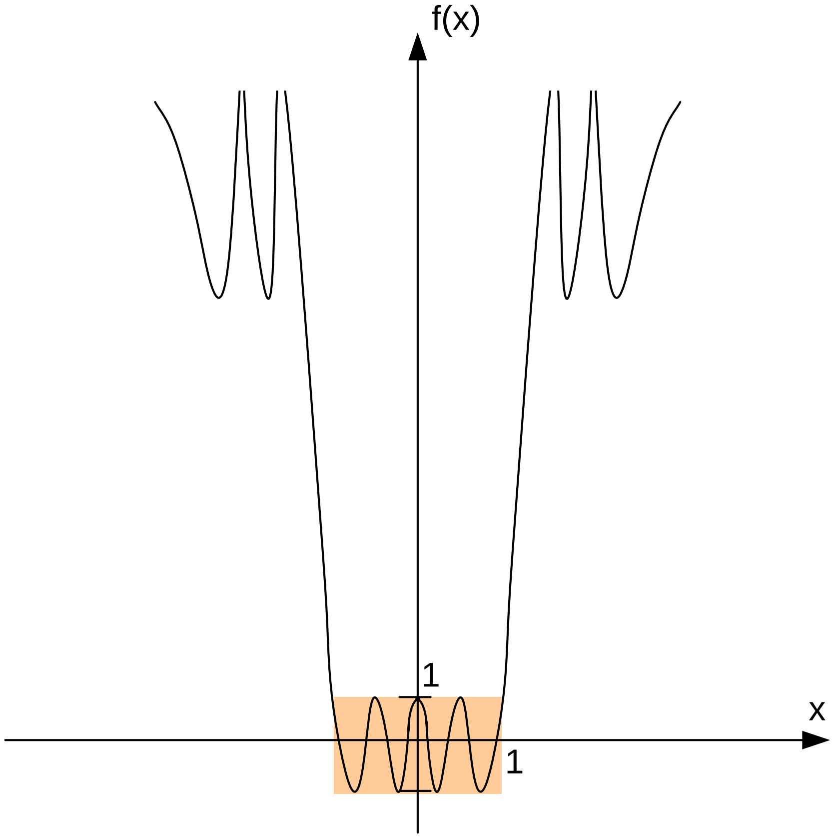
Elliptic Filters

- Basic idea comes from the concept of a Chebyschev Rational Fraction
- Sometimes termed Cauer filters

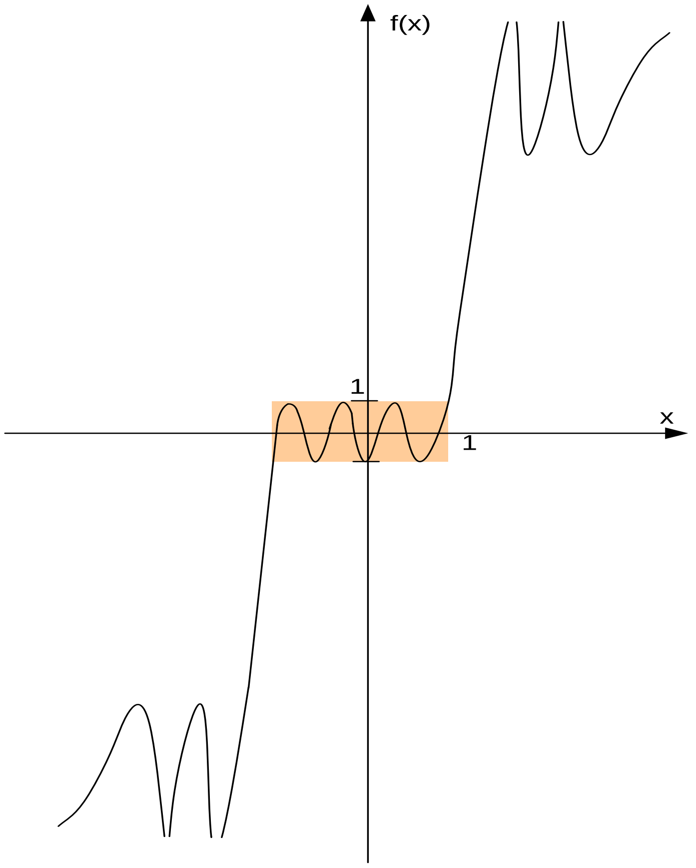
Chebyshev Rational Fraction

A Chebyshev Rational Fraction is a rational fraction that is equal ripple in [-1,1] and equal ripple in [-∞,-1] and [1,∞]

Chebyshev Rational Fractions



Even-order CC rational fraction



Odd-order CC rational fraction

Chebyshev Rational Fractions

Even-order CC rational fraction

$$C_{Rn}(x) = H\,\frac{\prod_{k=1}^{n/2}\left(x^2 - a_k\right)}{\prod_{k=1}^{n/2}\left(x^2 - b_k\right)}$$

Odd-order CC rational fraction

$$C_{Rn}(x) = H\,\frac{x\prod_{k=1}^{n/2}\left(x^2 - a_k\right)}{\prod_{k=1}^{n/2}\left(x^2 - b_k\right)}$$

Elliptic Filters

Magnitude-Squared Elliptic Approximating Function

$$H_E(\omega)=\frac{1}{1+\varepsilon^2 C_{Rn}^2(\omega)}$$

Inverse mapping to $T_E(s)$ exists

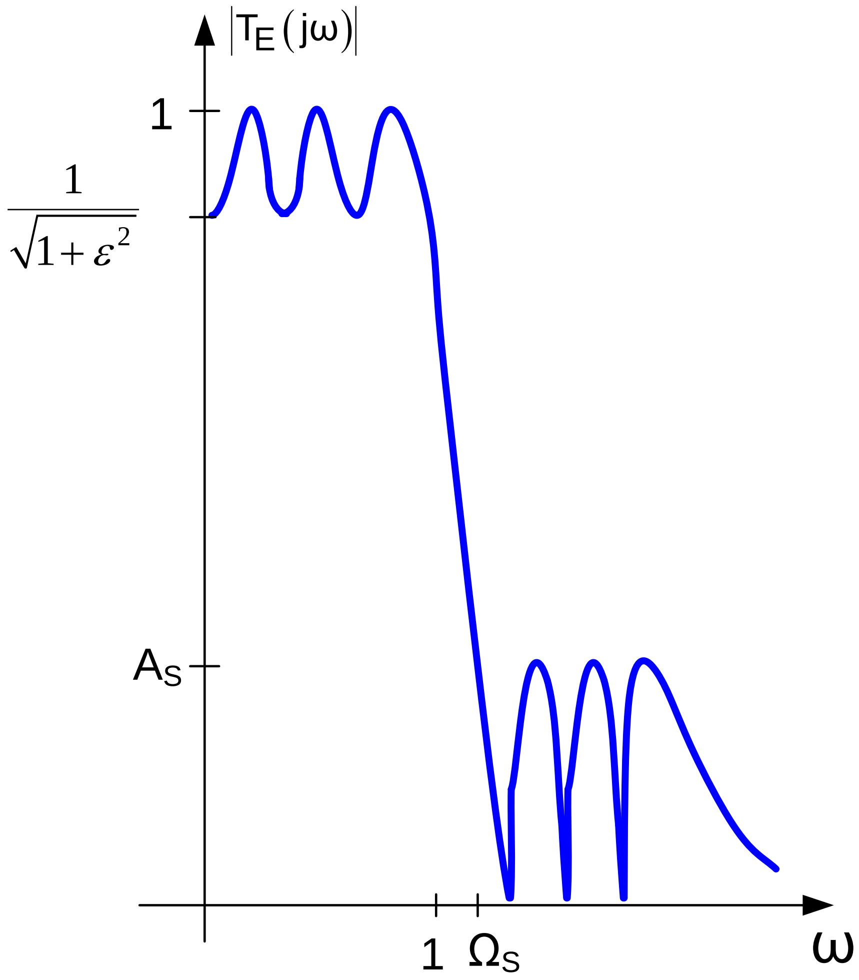
- For n even, n zeros on imaginary axis
- For n odd, n-1 zeros on imaginary axis

} Termed here "full order"

- Equal ripple in both pass band and stop band
- Analytical expression for poles and zeros not available
- Often choose to have less than n or n-1 zeros on imaginary axis

(No longer based upon CC rational fractions)

Elliptic Filters

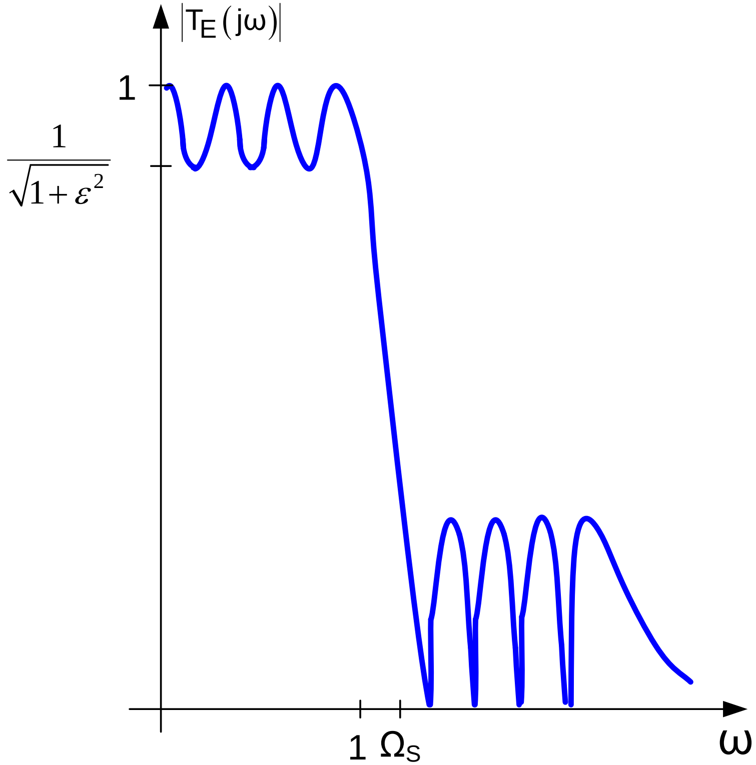


- If of full-order, response completely characterized by $\{n, \varepsilon, A_S . \Omega_S\}$

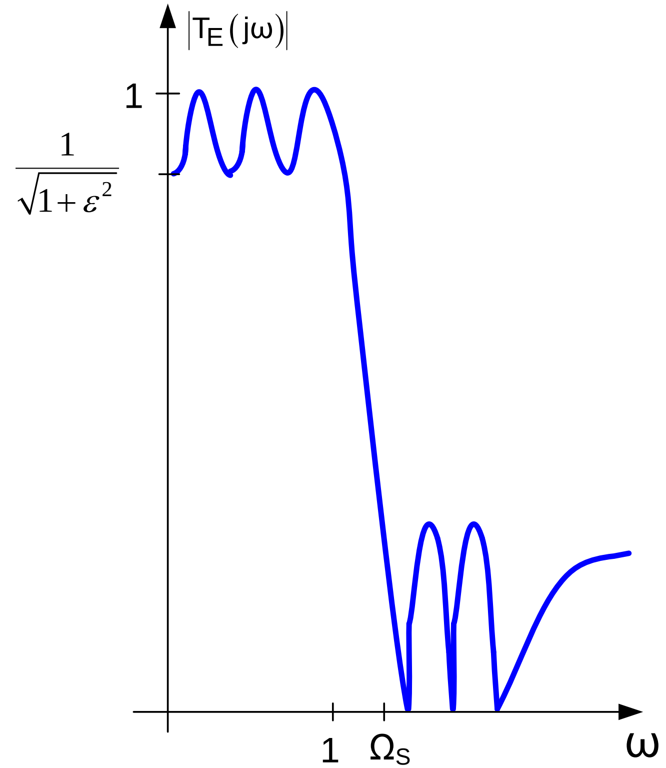
- Any 3 of these paramaters are independent

- Typically ε, Ω_S, and A_S are fixed by specifications (i.e. must determine n)

Elliptic Filters



n odd



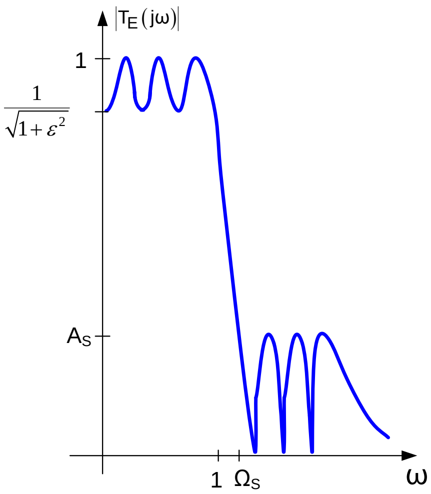
n even

For full-order elliptic approximations

- (n-1)/2 peaks in pass band
- (n-1)/2 peaks in stop band
- Maximum occurs at ω=0
- $|T(j\infty)|=0$

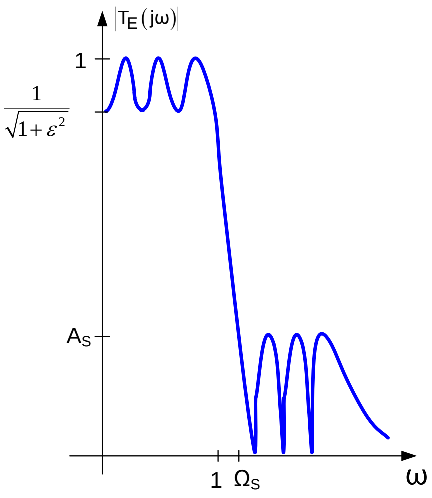
- n/2 peaks in pass band
- n/2 peaks in stop band
- $|T(j0)|=1/sq(1+\varepsilon^2)$
- $|T(j\infty)|=A_S$

Elliptic Filters



- Simple closed-form expressions for poles, zeros, and $|T_E(j\omega)|$ do not exist
- Simple closed form expressions for relationship between {n,ε,AS, and Ω_S} do not exist
- Simple expressions for max pole Q and slope at band edge do not exist
- Reduced-order elliptic approximations could be viewed as CC filters with zeros added to stop band
- General design tables not available though limited tables for specific characterization parameters do exist

Elliptic Filters



Observations about Elliptic Filters

- Elliptic filters have steeper transitions than CC1 filters
- Elliptic filters do not roll off as quickly in stop band as CC1 or even BW
- Highest Pole-Q of elliptic filters is larger than that of CC filters
- For a given transition requirement, order of elliptic filter typically less than that of CC filter
- Cost of implementing elliptic filter is comparable to that of CC filter if orders are the same
- Cost of implementing a given filter requirement is often less with the elliptic filters
- Often need computer to obtain elliptic approximating functions though limited tables are available
- Some authors refer to elliptic filters as Cauer filters

Canonical Approximating Functions

Butterworth
Chebyschev
Transitional BW-CC
Elliptic
Thompson
Bessel

Thompson and Bessel Approximating Functions are Two Different Names for the Same Approximation

Thompson and Bessel Approximations

- All-pole filters
- Maximally linear phase at $\omega=0$

Thompson and Bessel Approximations

Consider T(jω)

$$T(j\omega) = \frac{N(j\omega)}{D(j\omega)} = \frac{N_R(j\omega)+jN_{IM}(j\omega)}{D_R(j\omega)+jD_{IM}(j\omega)}$$

$$\text{phase} = \angle\big(T(j\omega)\big) = \tan^{-1}\left(\frac{N_I(j\omega)}{N_R(j\omega)}\right) - \tan^{-1}\left(\frac{D_I(j\omega)}{D_R(j\omega)}\right)$$

- Phase expressions are difficult to work with
- Will first consider group delay and frequency distortion

Linear Phase

Consider T(jω)

$$T(j\omega) = \frac{N(j\omega)}{D(j\omega)} = \frac{N_R(j\omega)+jN_{IM}(j\omega)}{D_R(j\omega)+jD_{IM}(j\omega)}$$

$$\angle\left(T(j\omega)\right) = \tan^{-1}\left(\frac{N_I(j\omega)}{N_R(j\omega)}\right) - \tan^{-1}\left(\frac{D_I(j\omega)}{D_R(j\omega)}\right)$$

Defn: A filter is said to have linear phase if the phase is given by the expression

$\angle\left(T(j\omega)\right) = \theta\omega$ where θ is a constant that is independent of ω

Distortion in Filters

Types of Distortion

Frequency Distortion

- Amplitude Distortion
- Phase Distortion

Nonlinear Distortion

Although the term "distortion" is used for these two basic classes, there is little in common between these two classes

Distortion in Filters

Frequency Distortion

- Amplitude Distortion

  A filter is said to have frequency (magnitude) distortion if the magnitude of the transfer function changes with frequency

- Phase Distortion

  A filter is said to have phase distortion if the phase of the transfer function is not equal to a constant times ω

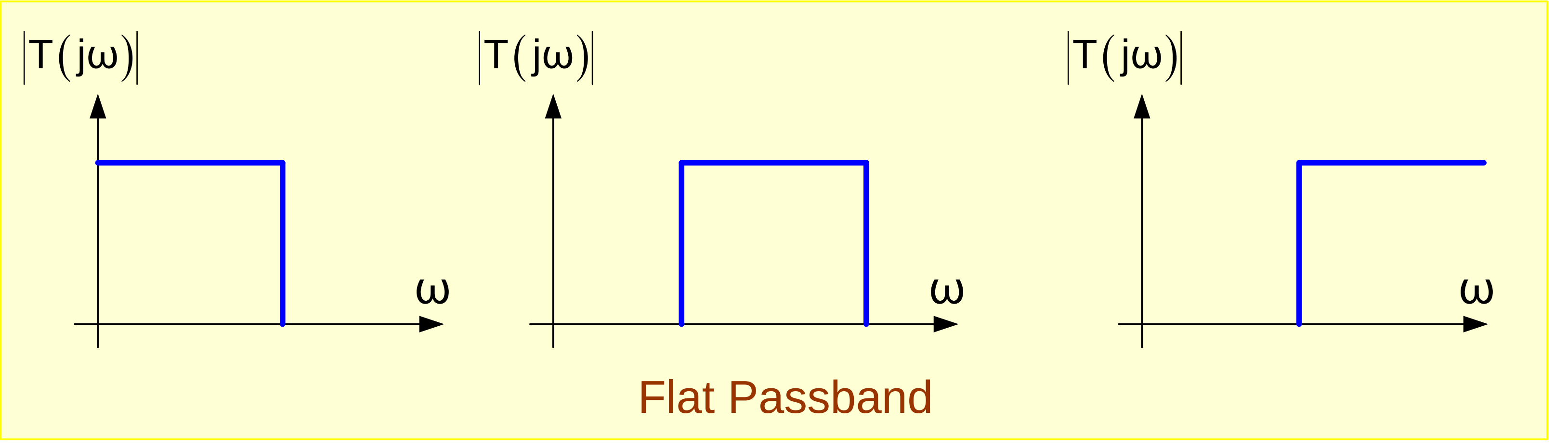
Nonlinear Distortion

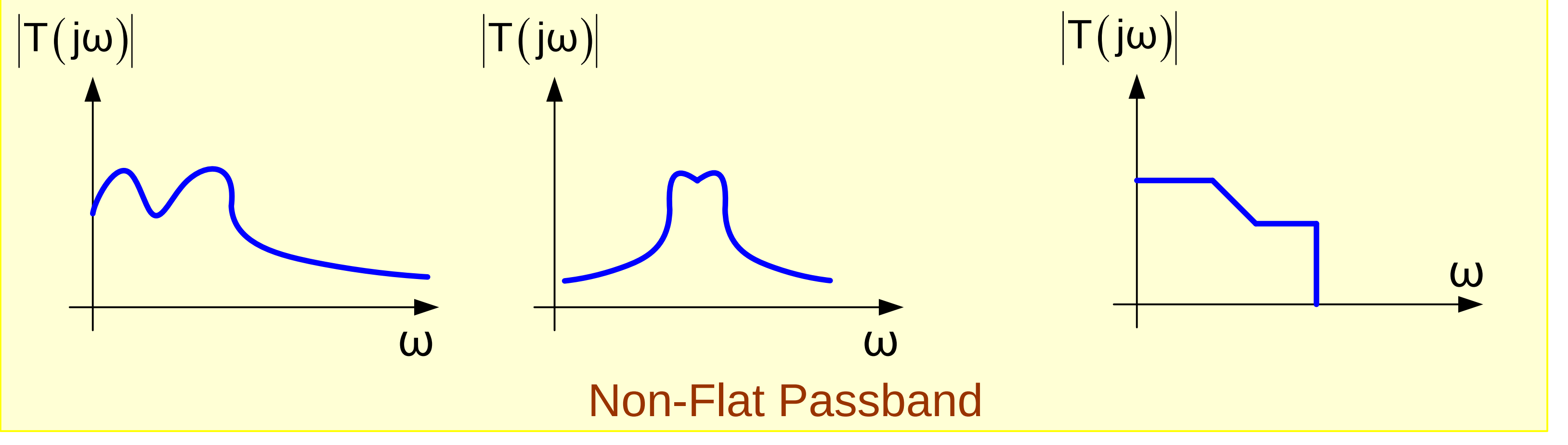
A filter is said to have nonlinear distortion if there is one or more spectral components in the output that are not present in the input

Distortion in Filters

- Phase and frequency distortion are concepts that apply to linear circuits
- If frequency distortion is present, the relative magnitude of the spectral components that are present will be different than the spectral components in the output
- If phase distortion is present, at least for some inputs, waveshape will not be preserved
- Nonlinear distortion does not exist in linear networks and is often used as a measure of the linearity of a filter.
- No magnitude distortion will be present in a specific output of a filter if all spectral components that are present in the input are in a flat passband
- No phase distortion will be present in a specific output of a filter if all spectral components that are present in the input are in a linear phase passband
- Linear phase can occur even when the magnitude in the passband is not flat
- Linear phase will still occur if the phase becomes nonlinear in the stopband

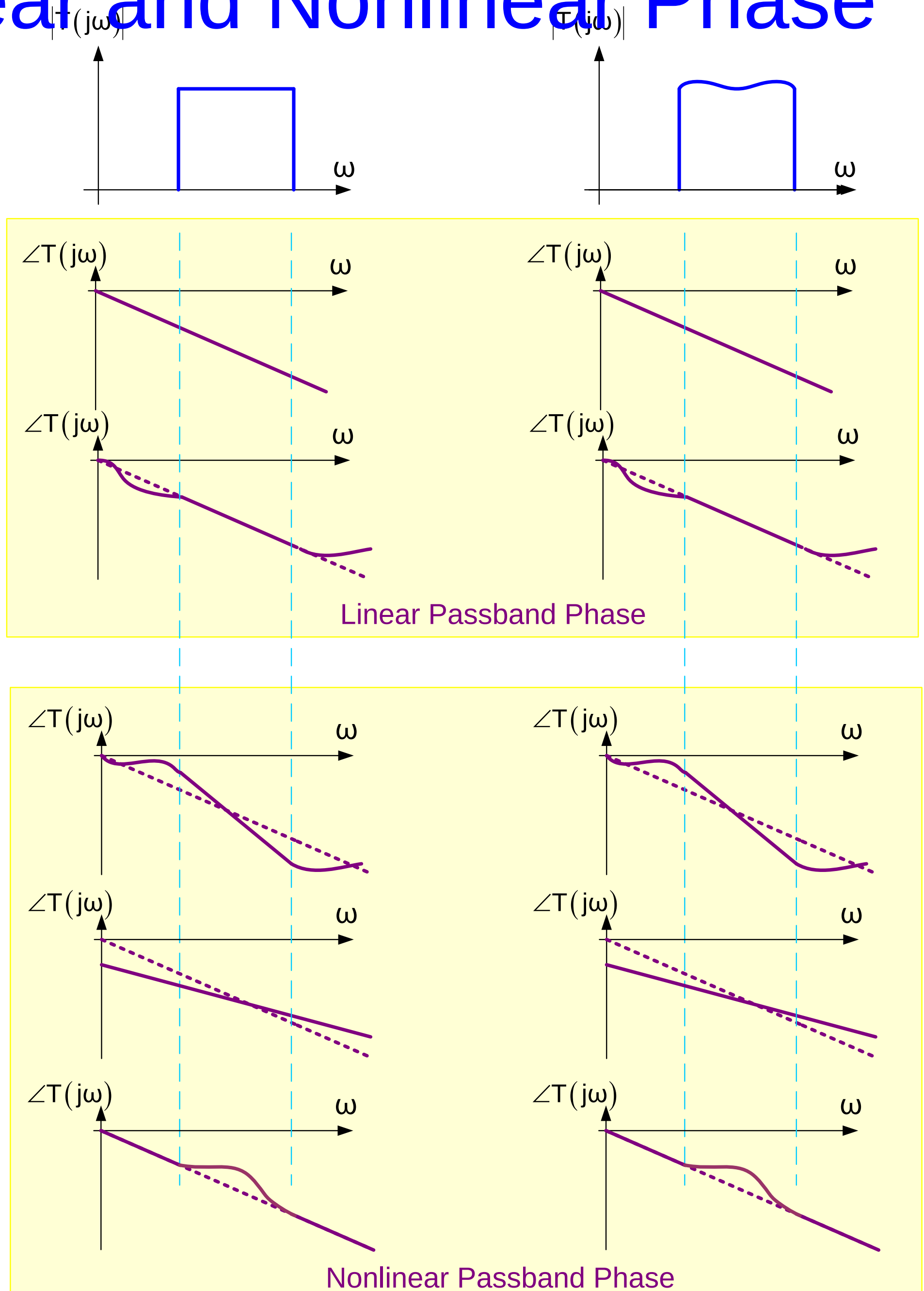
Filter Passband and Stopband





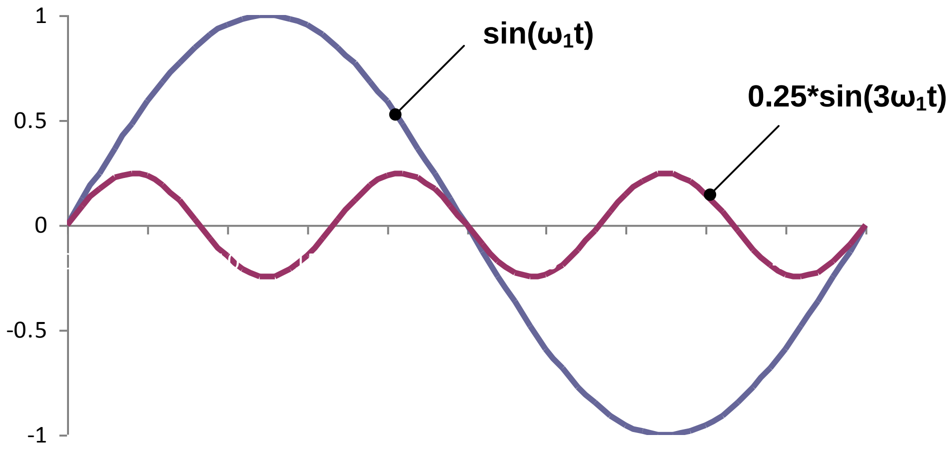
- Frequencies where gain is ideally 0 or very small is termed the stopband
- Frequencies where the gain is ideally not small is termed the passband
- Passband is often a continuous region in ω though could be split

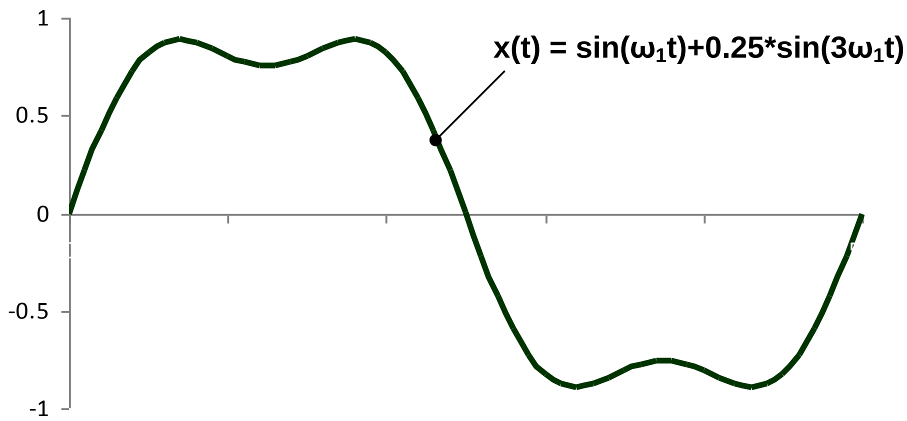
Linear and Nonlinear Phase

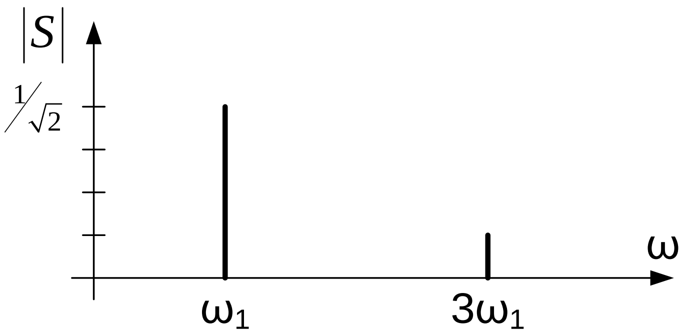


Preserving the Waveshape:

Example: Consider a signal $x(t)=\sin(\omega_1 t) + 0.25\sin(3\omega_1 t)$







Note the wave shape and spectral magnitude of x(t)

Preserving the waveshape

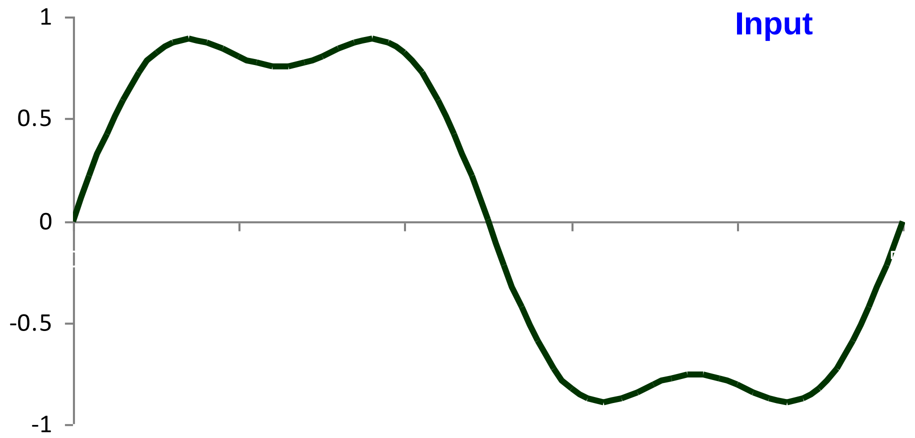
A filter has no frequency distortion for a given input if the output wave shape is preserved (i.e. the output wave shape is a magnitude scaled and possibly time-shifted version of the input)

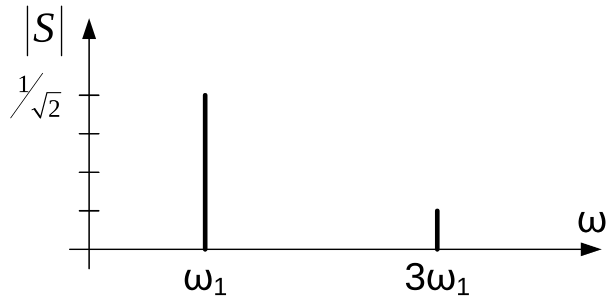
Mathematically, no frequency distortion for $V_{IN}(t)$ if

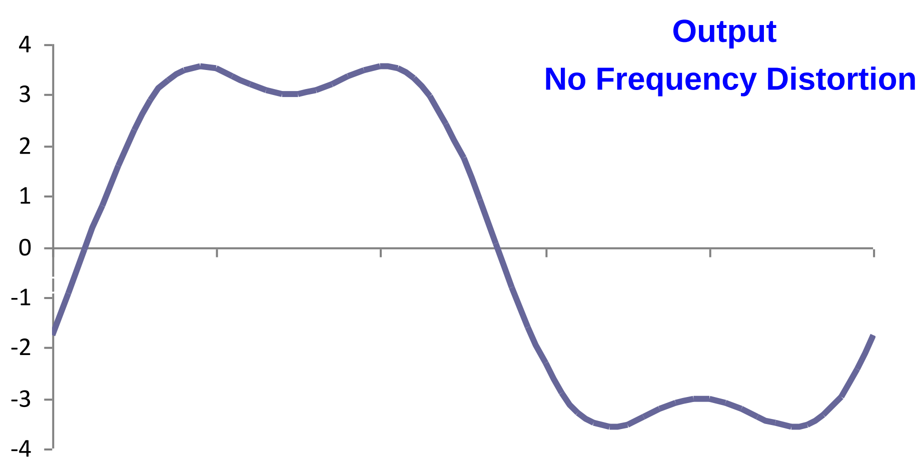
$$V_{OUT}(t)=KV_{IN}(t-t_{shift})$$

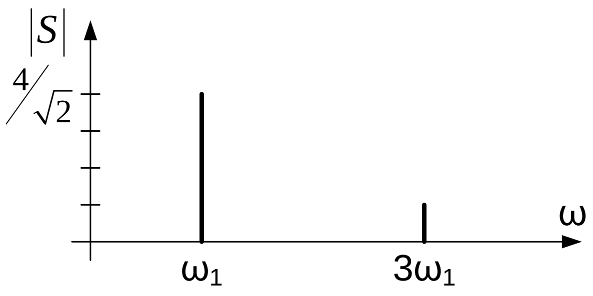
Could have frequency distortion for other inputs

Example of No Frequency Distortion

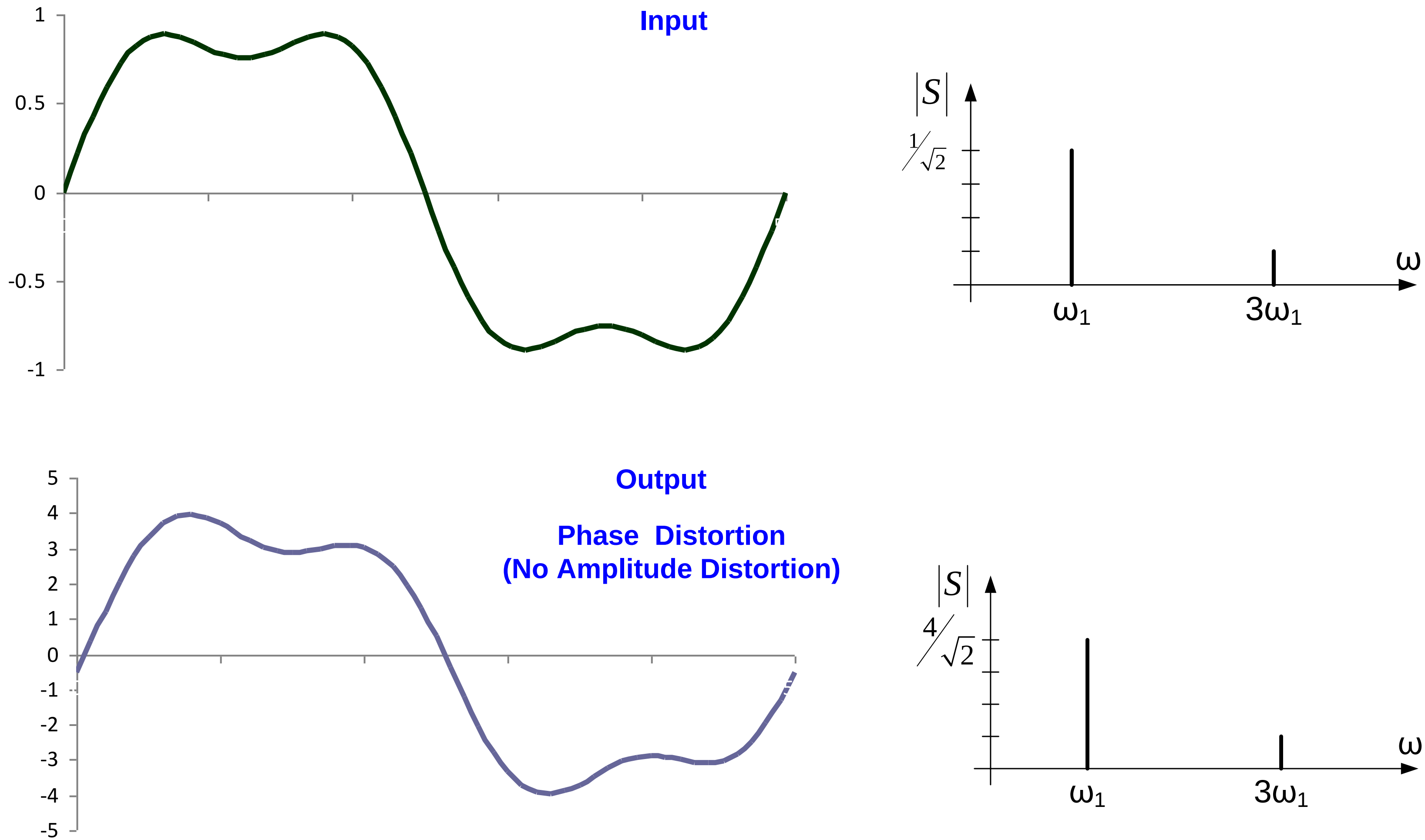






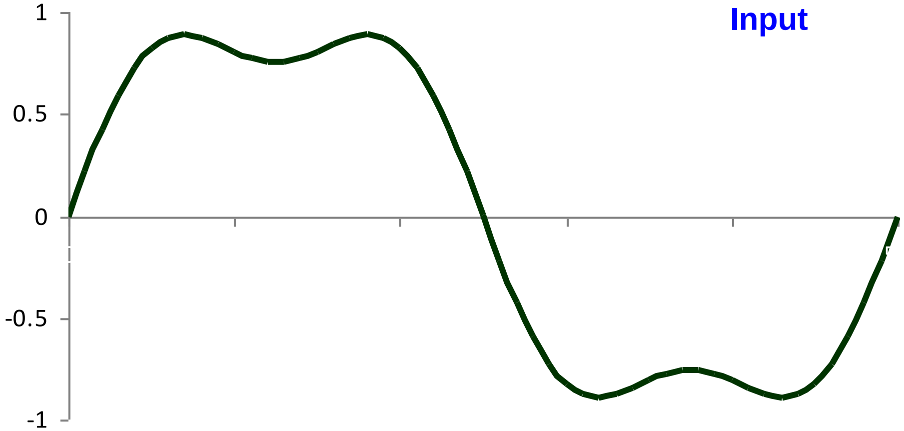


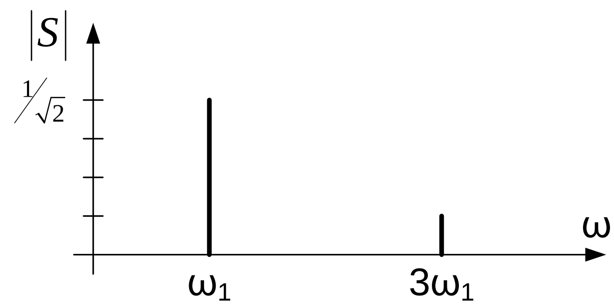
Example with Frequency Distortion



Example with Frequency Distortion

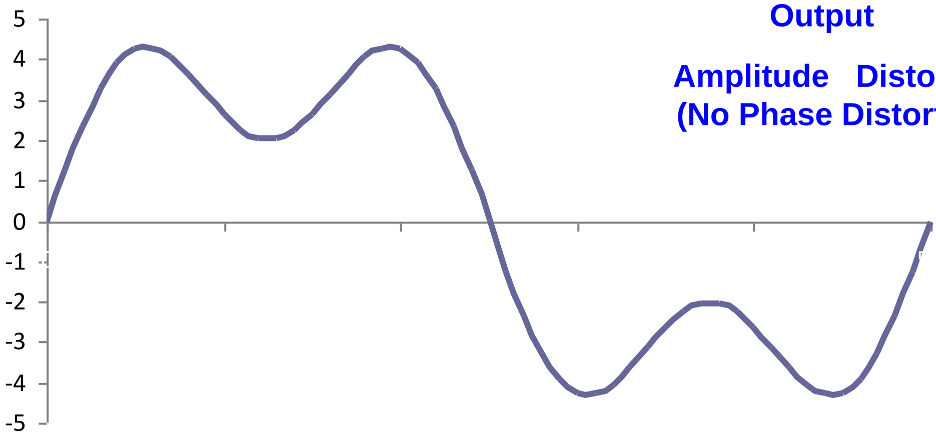
Input

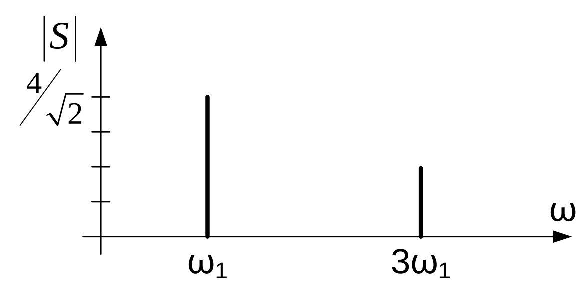




Output

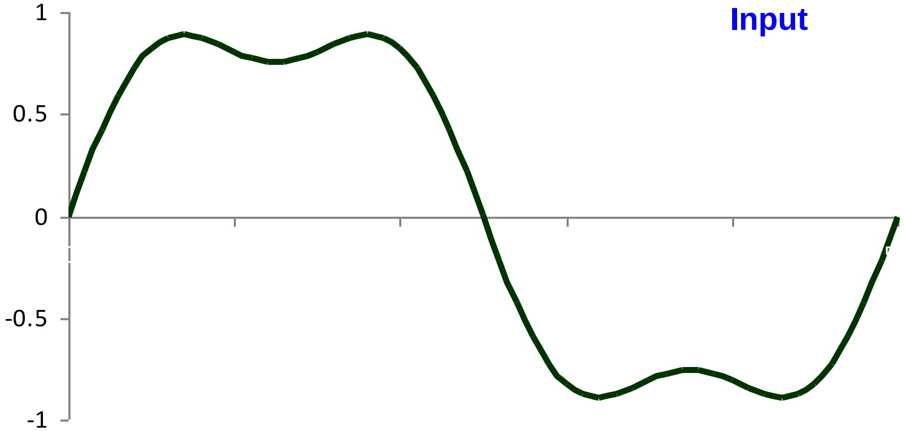
Amplitude Distortion (No Phase Distortion)

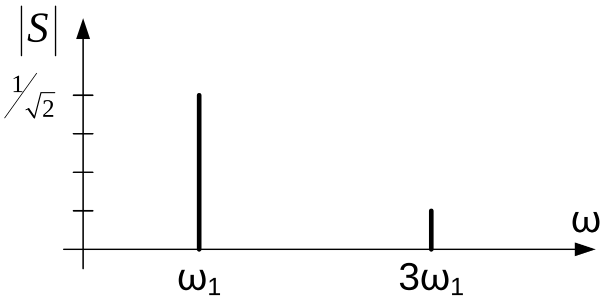




Example with Frequency Distortion

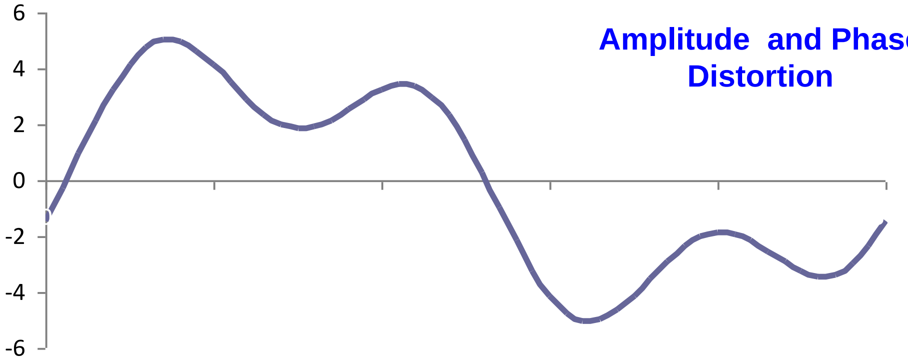
Input

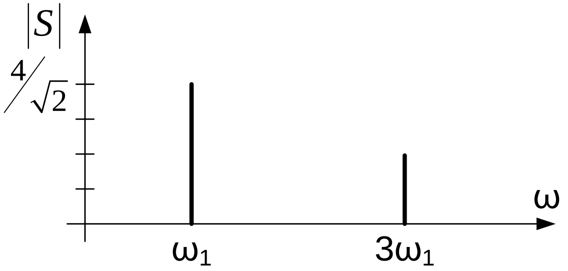




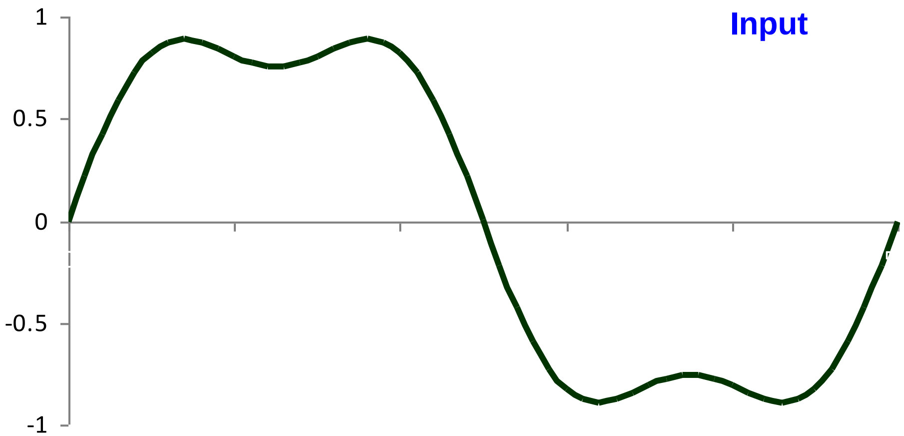
Output

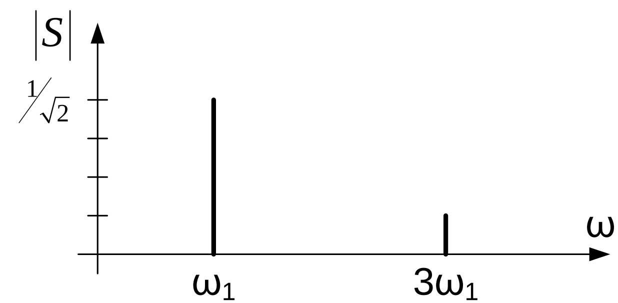
Amplitude and Phase Distortion

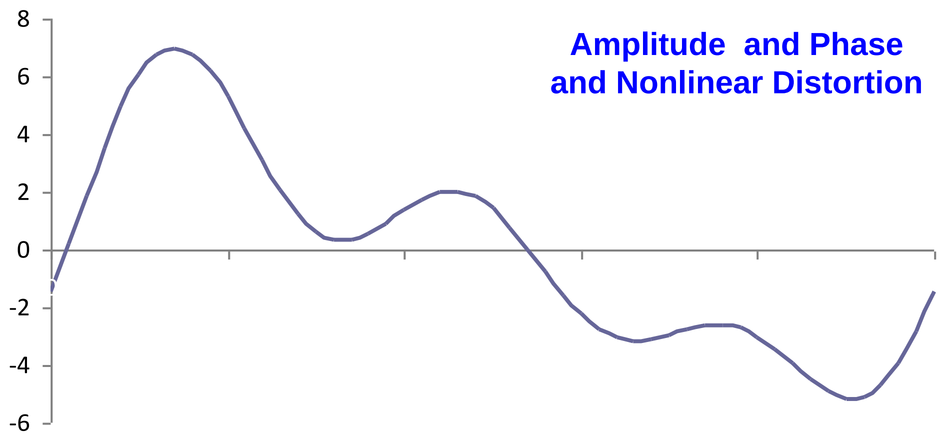


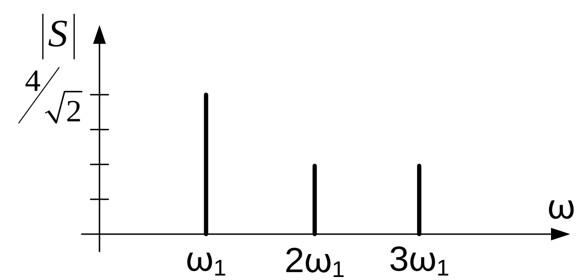


Example with Nonlinear Distortion and Frequency Distortion









Nonlinear distortion evidenced by presence of spectral components in output that are not in the input

Frequency Distortion

In most audio applications (and many other signal processing applications) there is little concern about phase distortion but some applications do require low phase distortion

In audio applications, any substantive magnitude distortion in the pass band is usually not acceptable

Any substantive nonlinear distortion in the pass band is unacceptable in most audio applications

End of Lecture 12

EE 508
Lecture 13

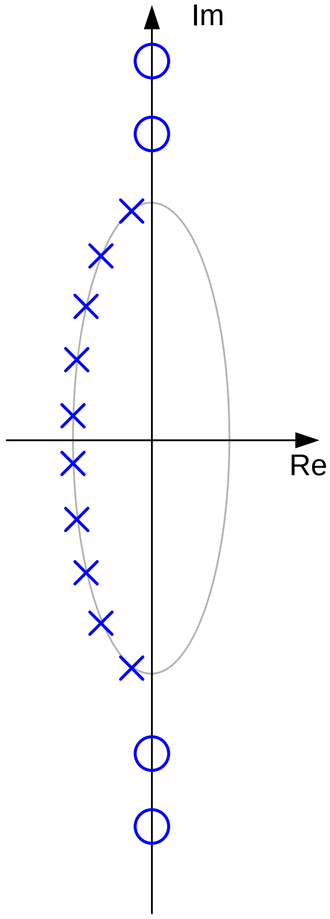
The Approximation Problem

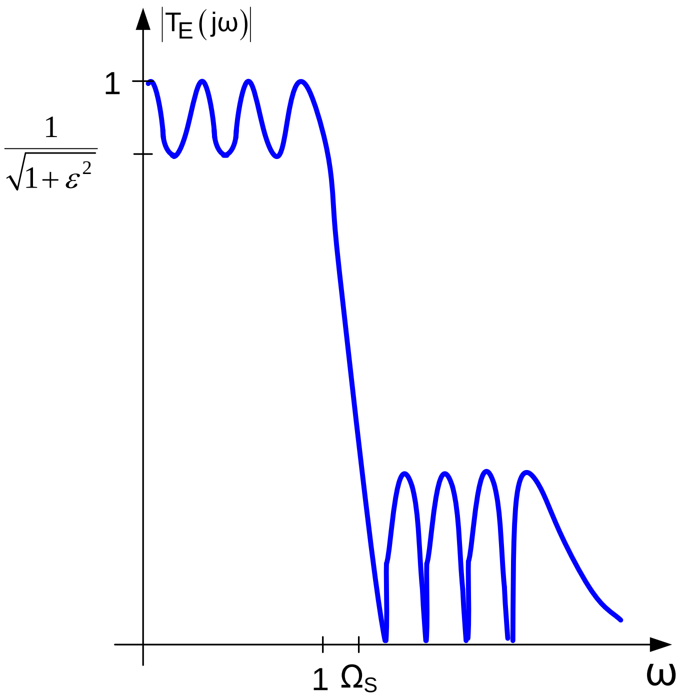
Classical Approximating Functions

- Thompson and Bessel Approximations

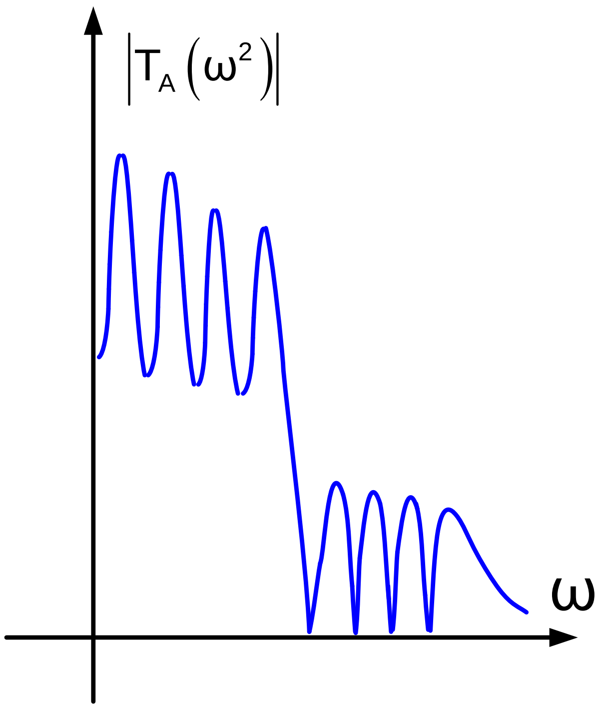
Elliptic Filters

Can be thought of as an extension of the CC approach by adding complex-conjugate zeros on the imaginary axis to increase the sharpness of the slope at the band edge





Concept



Actual effect of adding zeros

Elliptic Filters

Magnitude-Squared Elliptic Approximating Function

$$H_E(\omega)=\frac{1}{1+\varepsilon^2 C_{Rn}^2(\omega)}$$

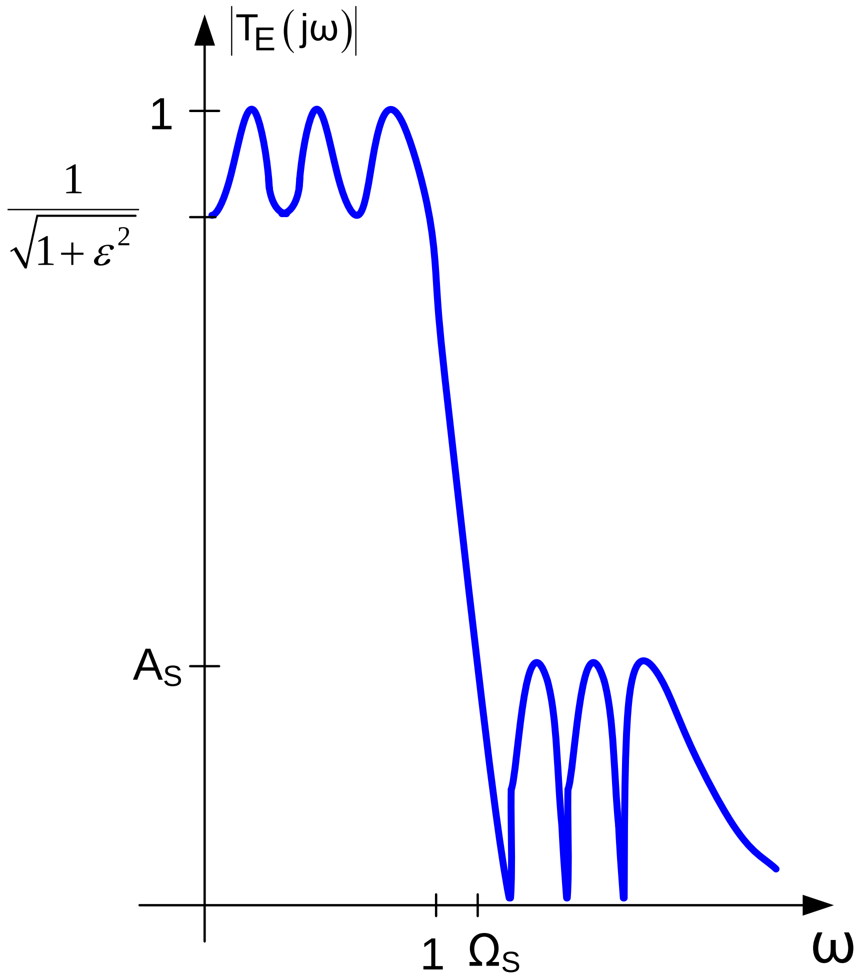
Inverse mapping to $T_E(s)$ exists

- For n even, n zeros on imaginary axis
- For n odd, n-1 zeros on imaginary axis
- Equal ripple in both pass band and stop band
- Analytical expression for poles and zeros not available
- Often choose to have less than n or n-1 zeros on imaginary axis

(No longer based upon CC rational fractions)

Termed here "full order"

Elliptic Filters

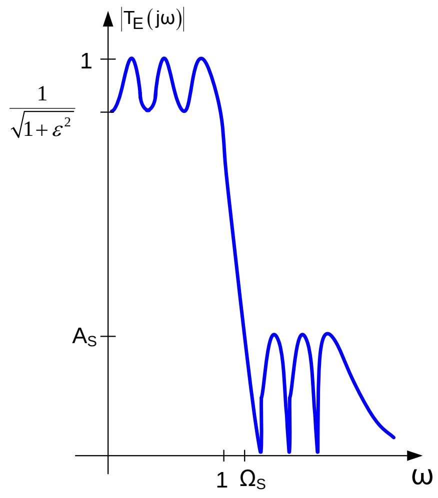


- If of full-order, response completely characterized by $\{n, \varepsilon, A_S . \Omega_S\}$

- Any 3 of these paramaters are independent

- Typically ε, Ω_S, and A_S are fixed by specifications (i.e. must determine n)

Elliptic Filters



Observations about Elliptic Filters

- Elliptic filters have steeper transitions than CC1 filters
- Elliptic filters do not roll off as quickly in stop band as CC1 or even BW
- Highest Pole-Q of elliptic filters is larger than that of CC filters
- For a given transition requirement, order of elliptic filter typically less than that of CC filter
- Cost of implementing elliptic filter is comparable to that of CC filter if orders are the same
- Cost of implementing a given filter requirement is often less with the elliptic filters
- Often need computer to obtain elliptic approximating functions though limited tables are available
- Some authors refer to elliptic filters as Cauer filters

Thompson and Bessel Approximations

- All-pole filters
- Maximally linear phase at $\omega=0$

Thompson and Bessel Approximations

Consider T(jω)

$$T(j\omega) = \frac{N(j\omega)}{D(j\omega)} = \frac{N_R(j\omega)+jN_{IM}(j\omega)}{D_R(j\omega)+jD_{IM}(j\omega)}$$

$$\text{phase} = \angle\big(T(j\omega)\big) = \tan^{-1}\left(\frac{N_I(j\omega)}{N_R(j\omega)}\right) - \tan^{-1}\left(\frac{D_I(j\omega)}{D_R(j\omega)}\right)$$

- Phase expressions are difficult to work with
- Will first consider group delay and frequency distortion

Review from Last Time

Linear Phase

Consider T(jω)

$$T(j\omega) = \frac{N(j\omega)}{D(j\omega)} = \frac{N_R(j\omega)+jN_{IM}(j\omega)}{D_R(j\omega)+jD_{IM}(j\omega)}$$

$$\angle\left(T(j\omega)\right) = \tan^{-1}\left(\frac{N_I(j\omega)}{N_R(j\omega)}\right) - \tan^{-1}\left(\frac{D_I(j\omega)}{D_R(j\omega)}\right)$$

Defn: A filter is said to have linear phase if the phase is given by the expression

$\angle\left(T(j\omega)\right) = \theta\omega$ where θ is a constant that is independent of ω

Review from Last Time

Preserving the waveshape

A filter has no frequency distortion for a given input if the output wave shape is preserved (i.e. the output wave shape is a magnitude scaled and possibly time-shifted version of the input)

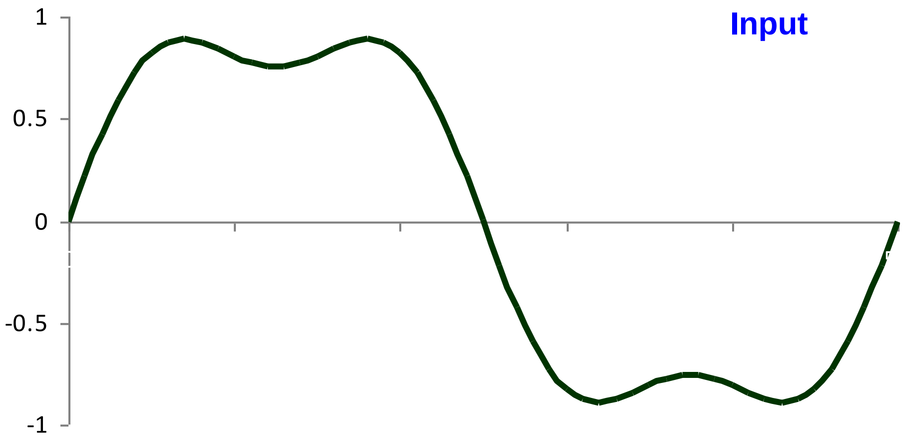
Mathematically, no frequency distortion for $V_{IN}(t)$ if

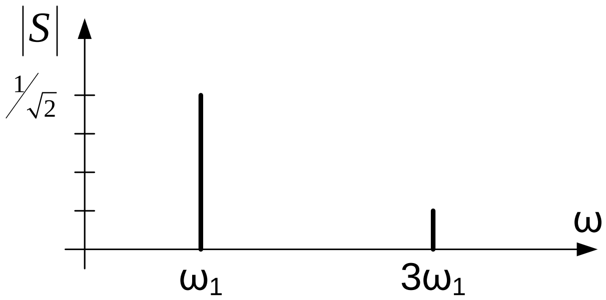
$$V_{OUT}(t)=KV_{IN}(t-t_{shift})$$

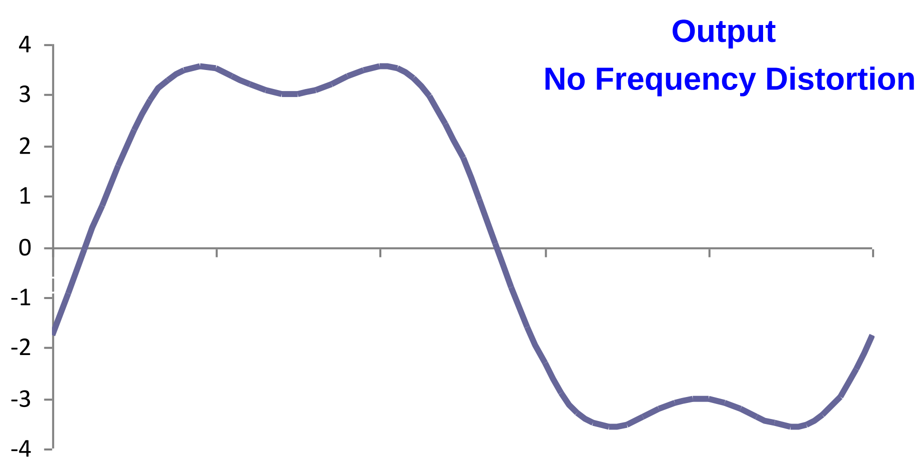
Could have frequency distortion for other inputs

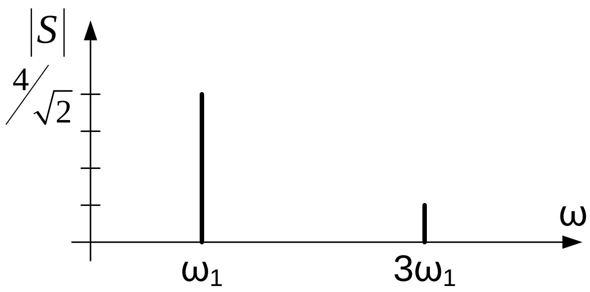
Review from Last Time

Example of No Frequency Distortion

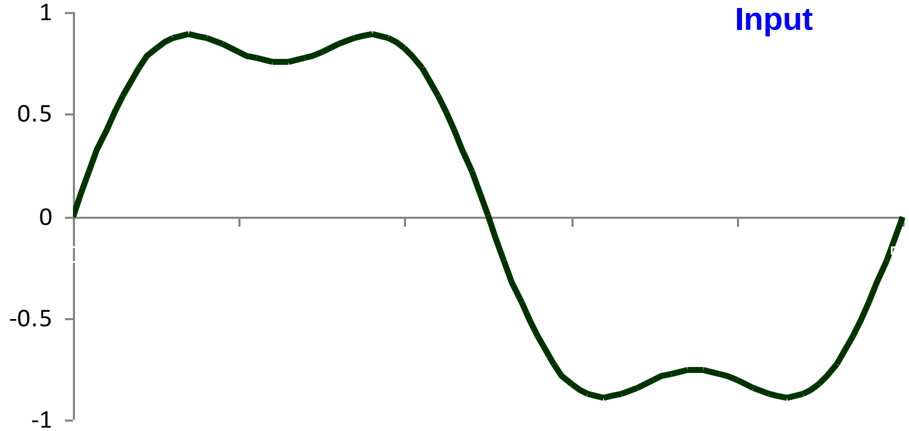


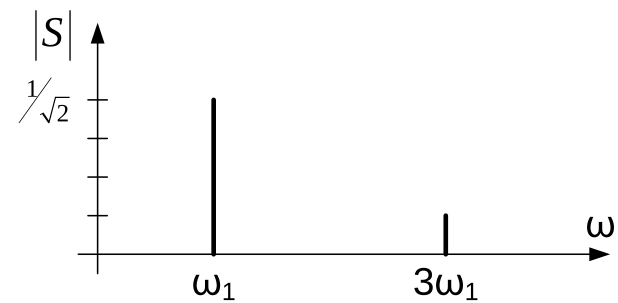


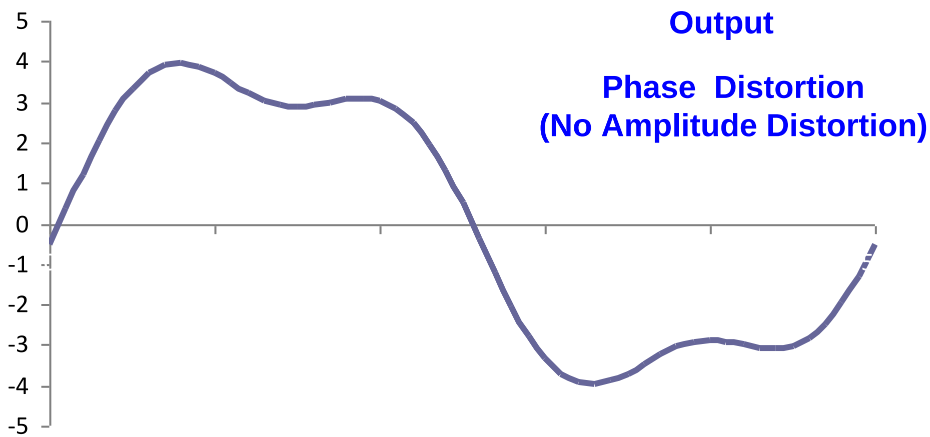


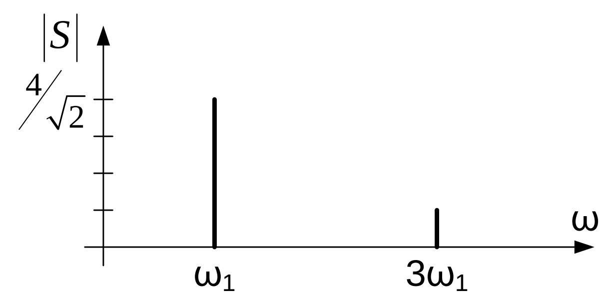


Example with Frequency Distortion









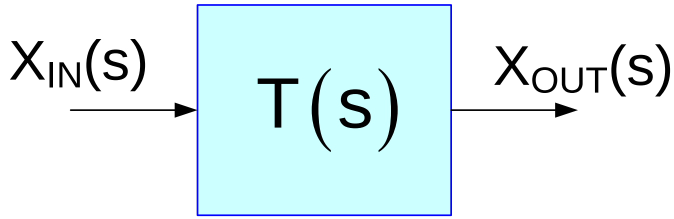
Preserving wave-shape in pass band

A filter is said to have linear passband phase if the phase in the passband of the filter is given by the expression $\angle\left(T\left(j\omega\right)\right)=\theta\omega$ where θ is a constant that is independent of ω

If a filter has linear passband phase in a flat passband, then the waveshape is preserved provided all spectral components of the input are in the passband and the output can be expressed as an amplitude scaled and time shifted version of the input by the expression

$$V_{OUT}(t)=KV_{IN}(t-t_{shift})$$

Preserving wave-shape in pass band



Example:

Consider a linear network with transfer function T(s)

Assume $$X_{in}(t) = A_1\sin(\omega_1 t+\theta_1) + A_2\sin(\omega_2 t+\theta_2)$$

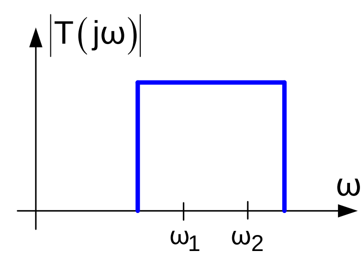
In the steady state

$$X_{OUT}(t) = A_1\left|T(j\omega_1)\right|\sin\left(\omega_1 t+\theta_1+\angle T(j\omega_1)\right) + A_2\left|T(j\omega_2)\right|\sin\left(\omega_2 t+\theta_2+\angle T(j\omega_2)\right)$$

Rewrite as:

$$X_{OUT}(t) = A_1\left|T(j\omega_1)\right|\sin\left(\omega_1\left[t+\frac{\angle T(j\omega_1)}{\omega_1}\right]+\theta_1\right) + A_2\left|T(j\omega_2)\right|\sin\left(\omega_2\left[t+\frac{\angle T(j\omega_2)}{\omega_2}\right]+\theta_2\right)$$

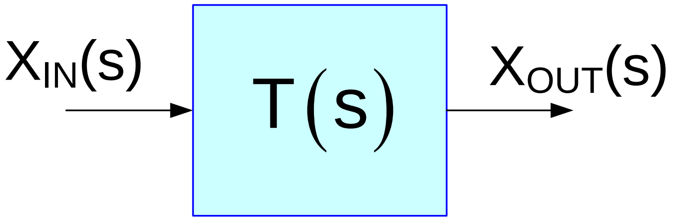
If ω_1 and ω_2 are in a flat passband, $\left|T(j\omega_1)\right| = \left|T(j\omega_2)\right|$



Can express as:

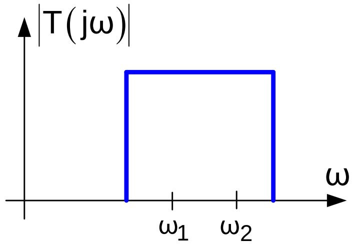
$$X_{OUT}(t) = \left|T(j\omega_1)\right|\left\{A_1\sin\left(\omega_1\left[t+\frac{\angle T(j\omega_1)}{\omega_1}\right]+\theta_1\right) + A_2\sin\left(\omega_2\left[t+\frac{\angle T(j\omega_2)}{\omega_2}\right]+\theta_2\right)\right\}$$

Preserving wave-shape in pass band



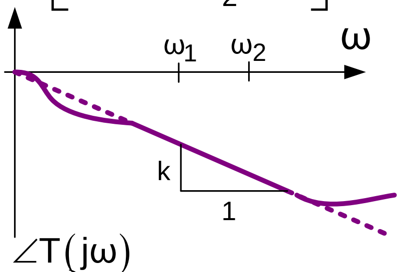
Example:

$$X_{in}(t) = A_1\sin(\omega_1 t+\theta_1) + A_2\sin(\omega_2 t+\theta_2)$$



If ω_1 and ω_2 are in a flat passband, $|T(j\omega_1)| = |T(j\omega_2)|$

$$X_{OUT}(t) = |T(j\omega_1)|\left\{A_1\sin\left(\omega_1\left[t+\frac{\angle T(j\omega_1)}{\omega_1}\right]+\theta_1\right) + A_2\sin\left(\omega_2\left[t+\frac{\angle T(j\omega_2)}{\omega_2}\right]+\theta_2\right)\right\}$$



If ω_1 and ω_2 are in a linear phase passband,

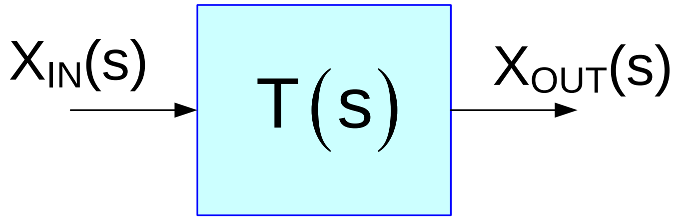
$$\angle T(j\omega_1) = k\omega_1 \quad \text{and} \quad \angle T(j\omega_2) = k\omega_2$$

$$X_{OUT}(t) = |T(j\omega_1)|\left\{A_1\sin\left(\omega_1\left[t+\frac{k\omega_1}{\omega_1}\right]+\theta_1\right) + A_2\sin\left(\omega_2\left[t+\frac{k\omega_2}{\omega_2}\right]+\theta_2\right)\right\}$$

$$X_{OUT}(t) = |T(j\omega_1)|\left\{A_1\sin(\omega_1[t+k]+\theta_1) + A_2\sin(\omega_2[t+k]+\theta_2)\right\}$$

$$X_{OUT}(t) = |T(j\omega_1)|\, x_{in}(t+k)$$

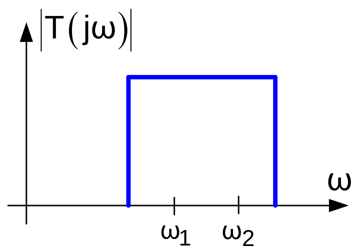
Preserving wave-shape in pass band



Example:

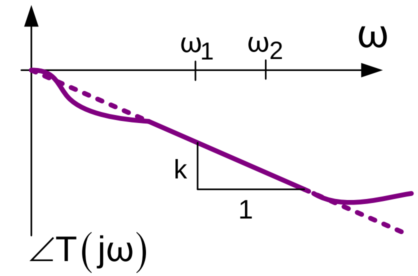
$$X_{in}(t) = A_1 \sin(\omega_1 t + \theta_1) + A_2 \sin(\omega_2 t + \theta_2)$$

$$|T(j\omega_1)| = |T(j\omega_2)|$$



$$\angle T(j\omega_1) = k\omega_1$$

$$\angle T(j\omega_2) = k\omega_2$$



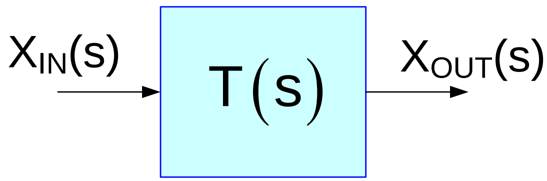
$$X_{OUT}(t) = |T(j\omega_1)| x_{in}(t+k)$$

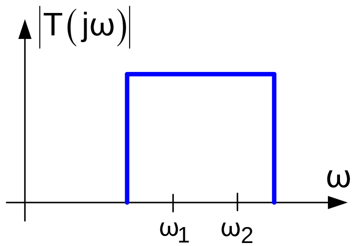
This is a magnitude scaled and time shifted version ot the input so waveshape is preserved

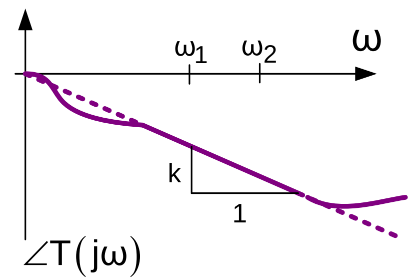
A weaker condition on the phase relationship will also preserve waveshape with two spectral components present

$$\frac{\angle T(j\omega_1)}{\angle T(j\omega_2)} = \frac{\omega_1}{\omega_2}$$

Amplitude (Magnitude) Distortion, Phase Distortion and Preserving wave-shape in pass band





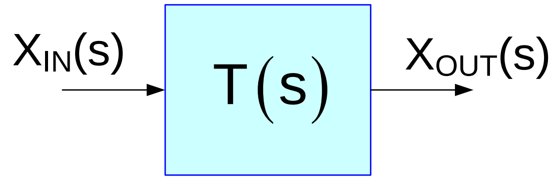


If ω_1 and ω_2 are any two spectral components of an input signal in which $\left|T(j\omega_1)\right| \neq \left|T(j\omega_2)\right|$ then the filter exhibits amplitude distortion for this input.

If ω_1 and ω_2 are any two spectral components of an input signal in which $\dfrac{\angle T(j\omega_1)}{\angle T(j\omega_2)} \neq \dfrac{\omega_1}{\omega_2}$ then the filter exhibits phase distortion for this input.

If ω_1 and ω_2 are any two spectral components of an input signal that exhibits either amplitude or phase distortion for these inputs, then the waveshape will not be preserved $X_{OUT}(t) \neq H \bullet x_{in}(t+k)$

Amplitude (Magnitude) Distortion, Phase Distortion and Preserving wave-shape in pass band



Amplitude and phase distortion are often of concern in filter applications requiring a flat passband and a flat zero-magnitude stop band

Amplitude distortion is usually of little concern in the stopband of a filter

Phase distortion is usually of little concern in the stopband of a filter

A filter with no amplitude distortion or phase distortion in the passband and a zero-magnitude stop band will exhibit waveform distortion for any input that has a frequency component in the passband and another frequency component in the stopband

It can be shown that the only way to avoid magnitude and phase distortion respectively for signals that have energy components in the interval $\omega_1 < \omega < \omega_2$ is to have constants k_1 and k_2 such that

$$\left.\begin{aligned} \left|T(j\omega)\right| &= k_1 \\ \angle T(j\omega) &= k_2\omega \end{aligned}\right\} \quad \text{for } \omega_1 < \omega < \omega_2$$

Group Delay

Defn: Group Delay is the negative of the phase derivative with respect to ω

$$\tau_G = -\frac{d\angle T(j\omega)}{d\omega}$$

Recall, by definition, the phase is linear iff $\angle T(j\omega) = k\omega$

If the phase is linear, $\tau_G = -\frac{d\angle T(j\omega)}{d\omega} = -\frac{d(k\omega)}{d\omega} = -k$

Thus, the phase is linear iff the group delay is constant

The group delay and the phase of a transfer function carry the same information

But, of what use is the group delay?

Group Delay

Example: Consider what is one of the simplest transfer functions

$$T(s)=\frac{1}{s+1}$$

$$T(j\omega)=\frac{1}{j\omega+1} \qquad \angle T(j\omega)= -\tan^{-1}\left(\frac{\omega}{1}\right) \qquad \tau_G=-\frac{d\angle T(j\omega)}{d\omega}$$

The phase of T(s) is analytically very complicated

$$\tau_G=-\frac{d\angle T(j\omega)}{d\omega}=-\frac{d\left(-\tan^{-1}\omega\right)}{d\omega}$$

Recall the identity

$$\frac{d\left(\tan^{-1}u\right)}{dx}=\left(\frac{1}{1+u^2}\right)\frac{du}{dx}$$

$$\tau_G=-\frac{d\left(-\tan^{-1}\omega\right)}{d\omega}=--\frac{1}{1+\omega^2}$$

Thus

$$\tau_G=\frac{1}{1+\omega^2}$$

Note that the group delay is a rational fraction in ω^2

Group Delay

But, of what use is the group delay?

The phase of almost all useful transfer functions are complicated functions involving Sums of arctan functions and these are difficult to work with analytically

Theorem: The group delay of any transfer function is a rational fraction in ω^2

From this theorem, it can be observed that the group delay is much more suited for analytical investigations than is the phase

Proof of Theorem:

(for notational convenience, will consider only all-pole transfer functions)

$$T(s) = \frac{1}{\sum_{k=0}^{n} a_k s^k}$$

Group Delay

Theorem: The group delay of any transfer function is a rational fraction in ω^2

Proof of Theorem:

$$T(s) = \frac{1}{\sum_{k=0}^{n} a_k s^k}$$

$$T(j\omega) = \frac{1}{\left(1 - a_2\omega^2 + a_4\omega^4 + \ldots\right) + j\omega\left(a_1 - a_3\omega^2 + a_5\omega^4 + \ldots\right)}$$

$$T(j\omega) = \frac{1}{F_1\left(\omega^2\right) + j\omega F_2\left(\omega^2\right)}$$

where F_1 and F_2 are even polynomials in ω

$$\angle T(j\omega) = -\tan^{-1}\left(\frac{\omega F_2\left(\omega^2\right)}{F_1\left(\omega^2\right)}\right)$$

Group Delay

Theorem: The group delay of any transfer function is a rational fraction in ω^2

Proof of Theorem: $$\angle T(j\omega) = -\tan^{-1}\left(\frac{\omega F_2(\omega^2)}{F_1(\omega^2)}\right)$$

but from identity $$\frac{d\left(\tan^{-1}u\right)}{dx} = \left(\frac{1}{1+u^2}\right)\frac{du}{dx}$$

$$\tau_G = -\frac{d\angle T(j\omega)}{d\omega} = -\frac{1}{1+\left[\frac{\omega F_2(\omega^2)}{F_1(\omega^2)}\right]^2} \bullet \frac{d\left[\frac{\omega F_2(\omega^2)}{F_1(\omega^2)}\right]}{d\omega}$$

Now consider the right-most term in the product

$$\frac{d\left[\frac{\omega F_2(\omega^2)}{F_1(\omega^2)}\right]}{d\omega} = \frac{F_1(\omega^2)\left[\frac{d\left(\omega F_2(\omega^2)\right)}{d\omega}\right] - \left(\omega F_2(\omega^2)\right)\frac{d\left(F_1(\omega^2)\right)}{d\omega}}{\left[F_1(\omega^2)\right]^2}$$

Group Delay

Theorem: The group delay of any transfer function is a rational fraction in ω^2

Proof of Theorem:

$$\angle T(j\omega) = -\tan^{-1}\left(\frac{\omega F_2(\omega^2)}{F_1(\omega^2)}\right)$$

$$\frac{d\left[\frac{\omega F_2(\omega^2)}{F_1(\omega^2)}\right]}{d\omega} = \frac{F_1(\omega^2)\left[\frac{d\left(\omega F_2(\omega^2)\right)}{d\omega}\right] - \left(\omega F_2(\omega^2)\right)\frac{d\left(F_1(\omega^2)\right)}{d\omega}}{\left[F_1(\omega^2)\right]^2}$$

Odd

Even

Even

Thus this term is an even rational fraction in ω

Group Delay

Theorem: The group delay of any transfer function is a rational fraction in ω^2

Proof of Theorem:

$$\tau_G = -\frac{d\angle T(j\omega)}{d\omega} = -\frac{1}{1+\left[\frac{\omega F_2(\omega^2)}{F_1(\omega^2)}\right]^2} \bullet \frac{d\left[\frac{\omega F_2(\omega^2)}{F_1(\omega^2)}\right]}{d\omega}$$

Even

It follows that τ_G is the product of rational fractions in ω^2 so it is also a rational fraction in ω^2

Although tedious, the results can be extended when there are zeros present in T(s) as well

Thompson and Bessel Approximations

- All-pole filters
- Maximally linear phase at ω=0

since $$\tau_G = -\frac{d\angle T(j\omega)}{d\omega}$$

These criteria can be equivalently expressed as

- All-pole filters
- Maximally constant group delay at ω=0
- $\tau_G = 1$ at ω=0

Thompson and Bessel Approximations

$$T_A(s) = \frac{1}{\sum_{k=0}^{n} a_k s^k}$$

Must find the coefficients a_0, a_1,… an to satisfy the constraints

$$T(j\omega) = \frac{1}{\left(1 - a_2\omega^2 + a_4\omega^4 + \ldots\right) + j\omega\left(a_1 - a_3\omega^2 + a_5\omega^4 + \ldots\right)}$$

Theorem: If $T(j\omega) = \dfrac{1}{x + jy}$ then τ_G is given by the expression

$$\tau_G = \frac{x\dfrac{dy}{d\omega} - y\dfrac{dx}{d\omega}}{x^2 + y^2}$$

This theorem is easy to prove using the identity given above,
proof will not be given here

Thompson and Bessel Approximations

$$T_A(s) = \frac{1}{\sum_{k=0}^{n} a_k s^k}$$

Must find the coefficients a_0, a_1,… an to satisfy the constraints

$$T(j\omega) = \frac{1}{\left(1 - a_2\omega^2 + a_4\omega^4 + ...\right) + j\omega\left(a_1 - a_3\omega^2 + a_5\omega^4 + ...\right)}$$

From this theorem, it follows that

$$\tau_G = \frac{a_1 + \omega^2\left(a_1a_2 - 3a_3\right) + \omega^4\left(5a_5 - 3a_1a_4 + a_2a_3\right) + ...}{1 + \omega^2\left(a_1^2 - 2a_2\right) + \omega^4\left(a_2^2 - 2a_1a_3 + 2a_4\right) + ...}$$

from the constraint $\tau_G = 1$ at ω=0, it follows that $a_1=1$

To make τ_G maximally constant at ω=0, want to match as many coefficients in the numerator and denominator as possible starting with the lowest powers of ω^2

from ω^2 terms $\qquad a_1a_2 - 3a_3 = a_1^2 - 2a_2$

from ω^4 terms $\qquad 5a_5 - 3a_1a_4 + a_2a_3 = a_2^2 - 2a_1a_3 + 2a_4$

….

Thompson and Bessel Approximations

$$T_A(s) = \frac{1}{\sum_{k=0}^{n} a_k s^k}$$

Must find the coefficients a_0, a_1,… an to satisfy the constraints

It can be shown that the a_ks are given by

$$a_k = \frac{(2n\text{-}k)!}{H2^{n-k}k!(n\text{-}k)!} \qquad \text{for } 1 \le k \le n\text{-}1$$

$$a_n = H$$

where

$$H = \frac{(2n)!}{2^n \; n!}$$

Thompson and Bessel Approximations

$$T_A(s) = \frac{1}{\sum_{k=0}^{n} a_k s^k}$$

Must find the coefficients a_0, a_1,… an to satisfy the constraints

Alternatively, if we define the recursive polynomial set by

$$B_1 = s+1$$

$$B_2 = s^2 + 3s + 3$$

...

$$B_k = (2k-1)B_{k-1} + s^2 B_{k-2}$$

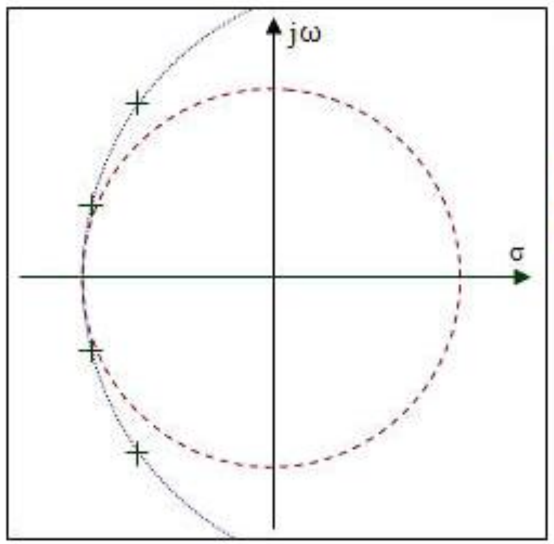
Then the n-th order Thompson approximation is given by

$$T_{An}(s) = \frac{B_n(0)}{B_n(s)}$$

Since the recursive set of polynomials are termed Bessel functions, this is often termed the Bessel approximation

Thompson and Bessel Approximations

$$T_{An}(s) = \frac{B_n(0)}{B_n(s)}$$



http://www.rfcafe.com/references/electrical/bessel-poles.htm

- Poles of Bessel Filters lie on circle
- Circle does not go through the origin
- Poles not uniformly space on circumference

Thompson and Bessel Approximations

$$T_{An}(s) = \frac{B_n(0)}{B_n(s)}$$

Observations:

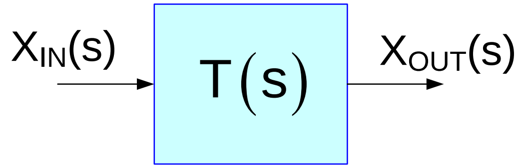
The Thompson approximation has relatively poor magnitude characteristic (at least if considered as an approximation to the standard lowpass function)

The normalized Thompson approximation has a group delay of 1 or a phase of ω at ω=0

Frequency scaling is used to denormalize the group delay or the phase to other values

Thompson and Bessel Approximations

Use of Bessel Filters:



Consider:

$$T(s) = e^{-sh}$$ (not realizable but can be approximated)

$$T(j\omega) = e^{-j\omega h}$$

$$T(j\omega) = \cos(-\omega h) + j\sin(-\omega h)$$

$$|T(j\omega)| = 1 \qquad \angle T(j\omega) = -h\omega$$

If $x_{IN}(t) = X_M \sin(\omega t + \theta)$

$$x_{OUT}(t) = X_M \sin(\omega t + \theta - h\omega)$$

$$x_{OUT}(t) = X_M \sin(\omega[t-h] + \theta)$$

This is simply a delayed version of the input

$$x_{OUT}(t) = x_{IN}(t-h)$$

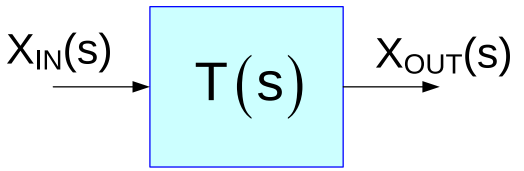
But

$$\tau_G = \frac{-d\angle T(j\omega)}{d\omega} = h \qquad x_{OUT}(t) = x_{IN}(t-\tau_G)$$

So, output is delayed version of input and the delay is the group delay

Thompson and Bessel Approximations

Use of Bessel Filters:



$$T(s) = e^{-sh}$$

$$|T(j\omega)| = 1 \qquad \angle T(j\omega) = -h\omega \qquad \tau_G = h$$

It is challenging to build filters with a constant delay

A filter with a constant group delay and unity magnitude introduces a constant delay

Bessel filters are filters that are used to approximate a constant delay

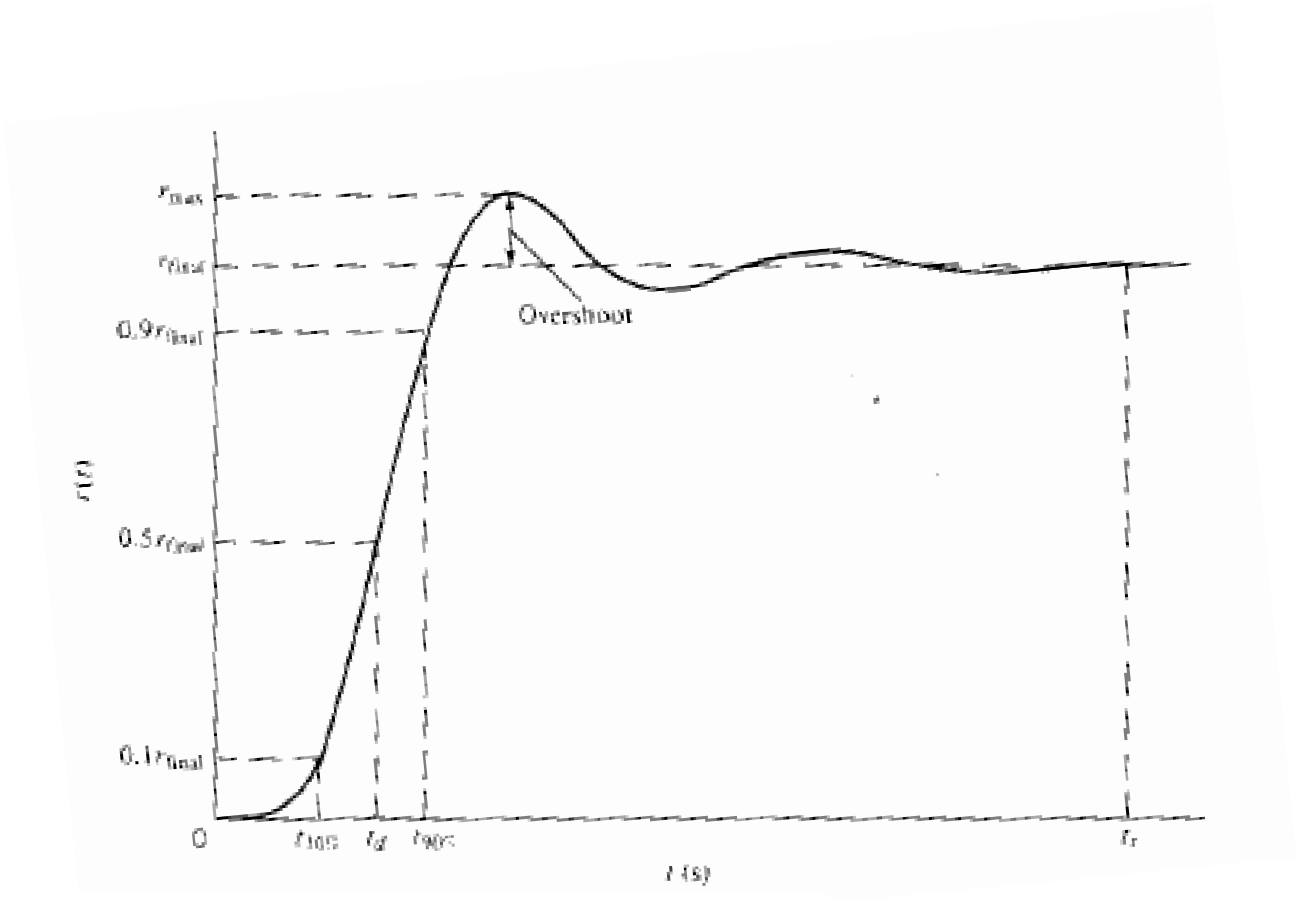
Bessel filters are attractive for introducing constant delays in digital systems

Some authors refer to Bessel filters as “Delay Filters”

An ideal delay filter would

- introduce a time-domain shift of a step input by the group delay
- introduce a time-domain shift each spectral component by the group delay
- introduce a time-domain shift of a square wave by the group delay

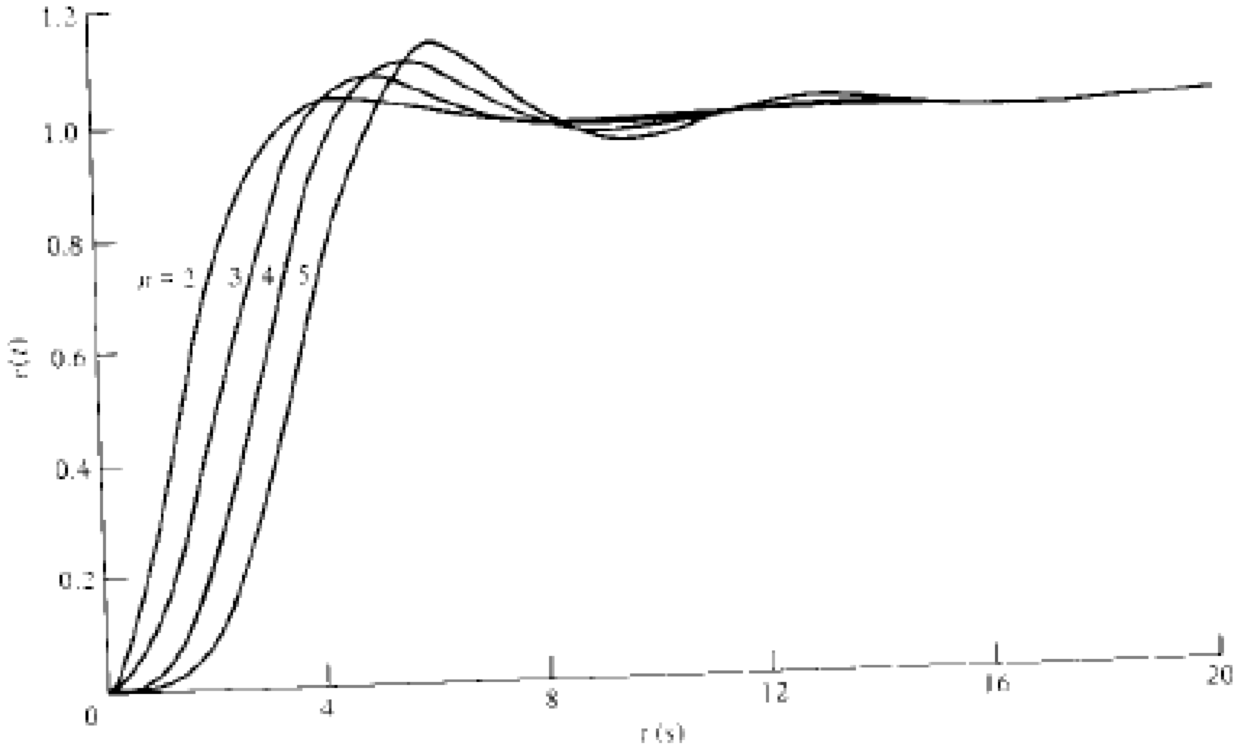
Thompson and Bessel Approximations



Characterization of the step response of a filter

From Introduction to the Theory and Design of Active Filters by Huelsman and Allen, p. 94-96

Thompson and Bessel Approximations

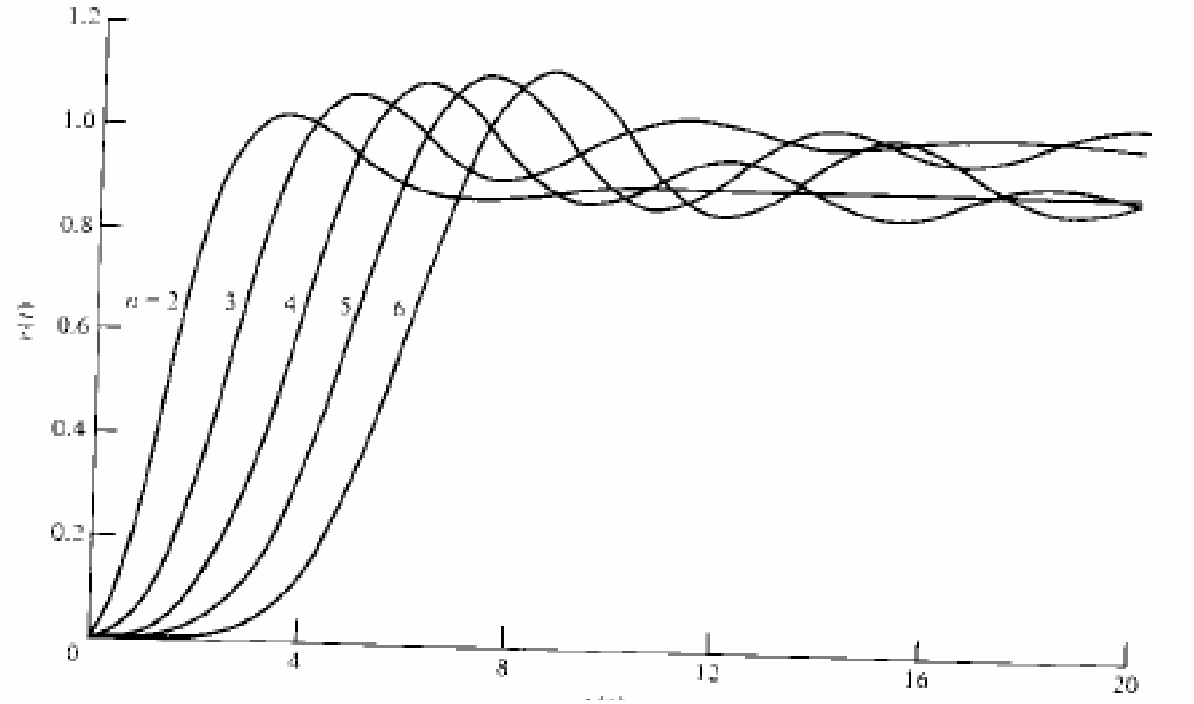


Step Response of Butterworth Filter

Delay is not constant
Overshoot present and increases with order
BW filters do not perform well as delay filters

From Introduction to the Theory and Design of Active Filters by Huelsman and Allen, p. 94-96

Thompson and Bessel Approximations

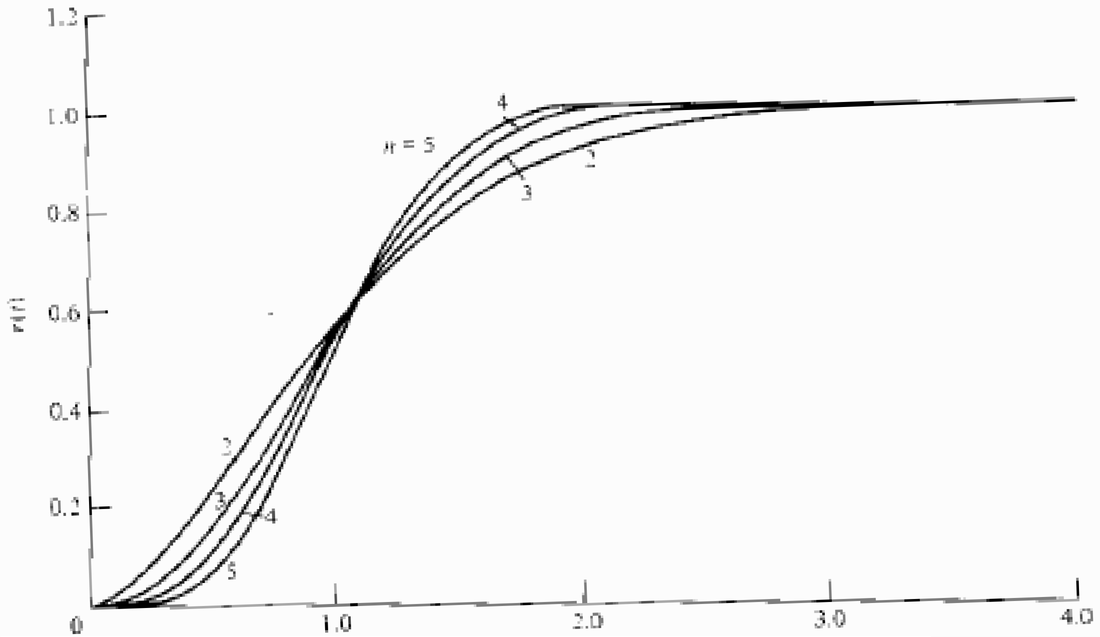


Step Response of Chebyschev Filter

Delay is not constant
Overshoot and ringing present and increases with order
CC filters do not perform well as delay filters

From Introduction to the Theory and Design of Active Filters by Huelsman and Allen, p. 94-96

Thompson and Bessel Approximations

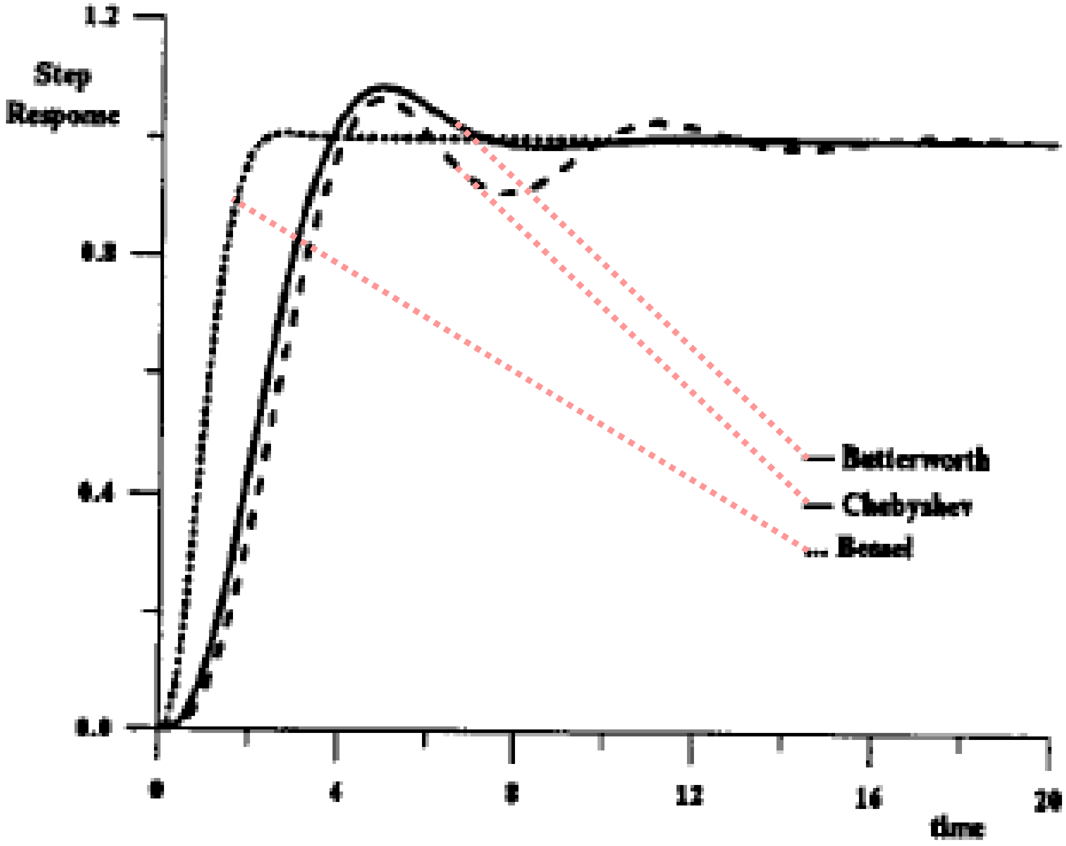


Step Response of Bessel Filters

- Delay becomes more constant as order increases
- No overshoot or ringing present
- Bessel filters widely used as delay filters
- Bessel filters often designed to achieve time-domain performance

From Introduction to the Theory and Design of Active Filters by Huelsman and Allen, p. 94-96

Thompson and Bessel Approximations



Comparison of Step Response of 3rd-order Bessel, BW and CC filters

From Continuous-Time Active Filter Design by Deliyannis, Sun and Fidler

Thompson and Bessel Approximations

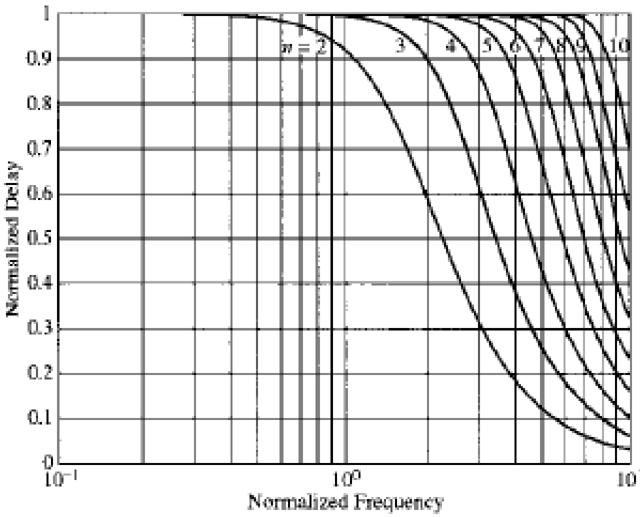


Figure 10.3 Delay of Bessel–Thomson filters of orders 2 through 10.

Harmonics in passband of Bessel Filter increase with n

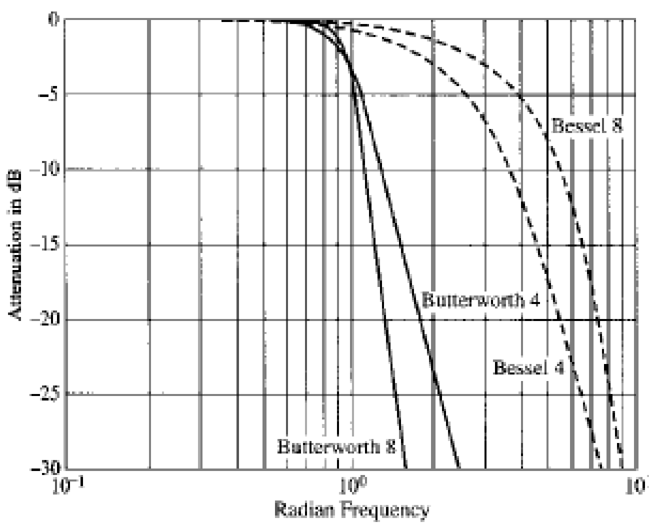
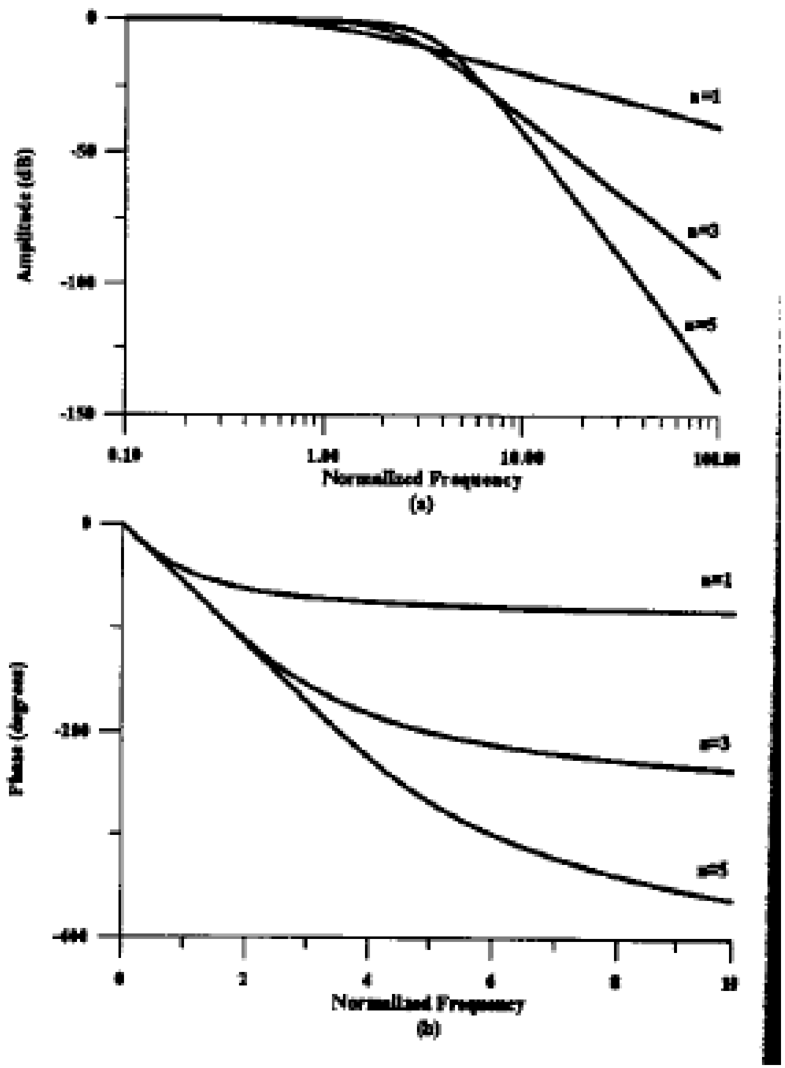


Figure 10.4 Comparison of Bessel–Thomson and Butterworth responses of orders 4 and 8.

Attenuation of amplitude for Bessel does not compare favorably wth BW, CC, or Eliptic filters

From Design of Analog Filters by Schaumann and Van Valkenburg, p405

Thompson and Bessel Approximations

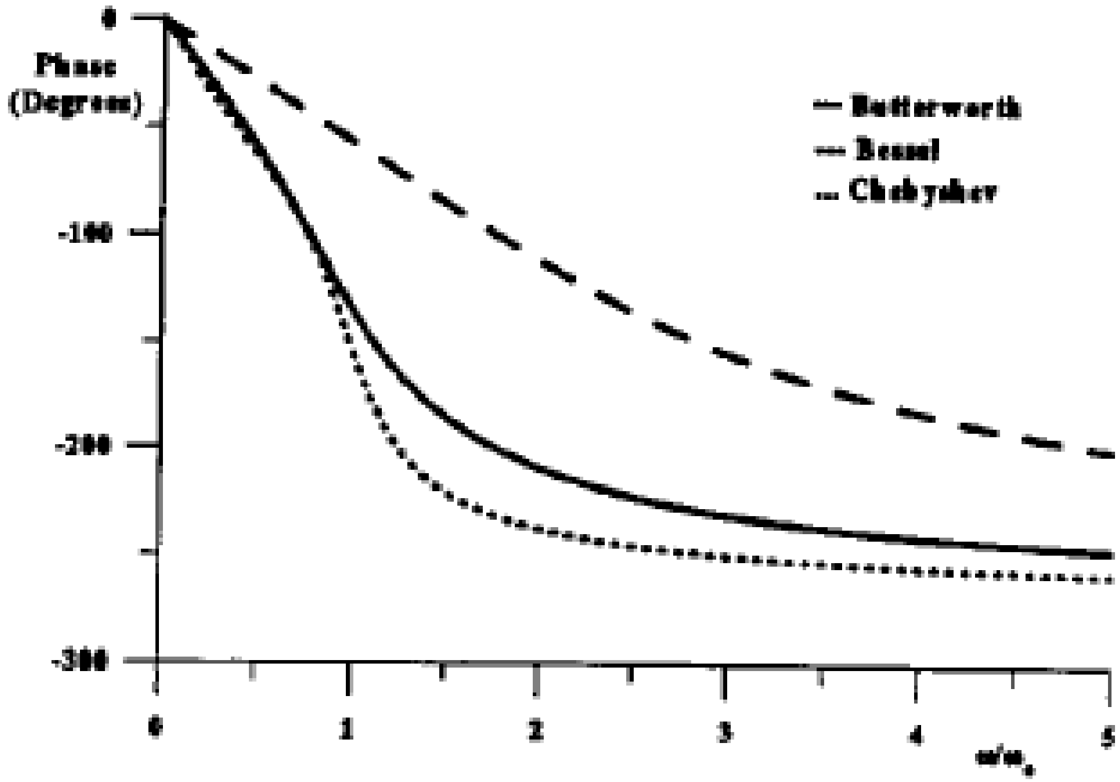


From Continuous-Time Active Filter Design by Deliyannis, Sun and Fidler

Magnitude of Bessel filters does not drop rapidly at band edge

Phase of Bessel filters becomes very linear in passband as order increases

Thompson and Bessel Approximations



Comparison of Phase Response of 3rd-order Bessel, BW and CC filters

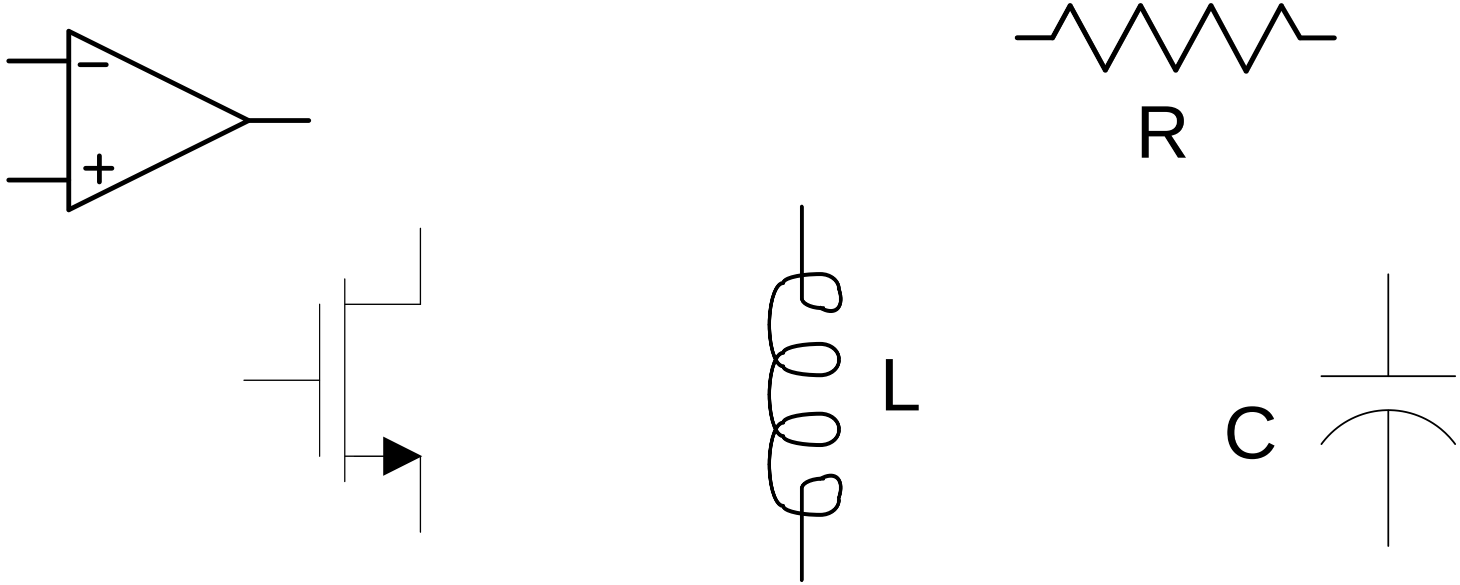
From Continuous-Time Active Filter Design by Deliyannis, Sun and Fidler

End of Lecture 13

EE 508
Lecture 14

Statistical Characterization of Filter Characteristics

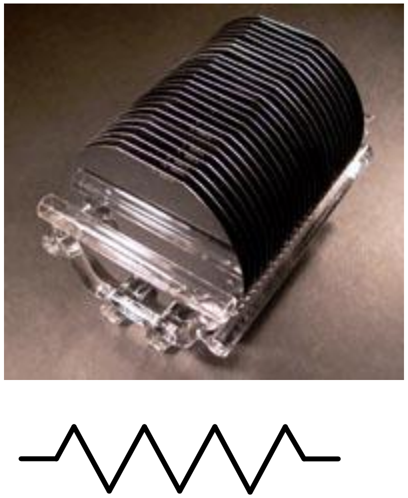
Components used to build filters are not precisely predictable



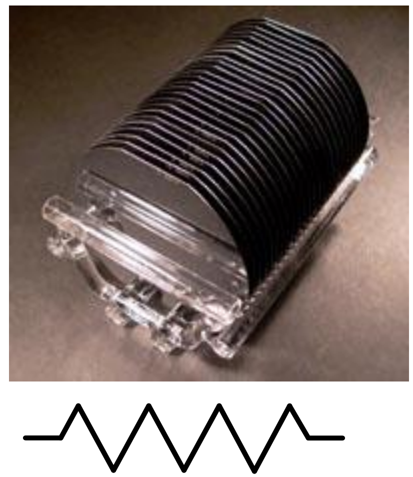
- Temperature Variations
- Manufacturing Variations
- Aging
- Model variations

- Different approaches are used to address each of these problems
- Manufacturing variations is one of the most challenging and will be the focus of this lecture

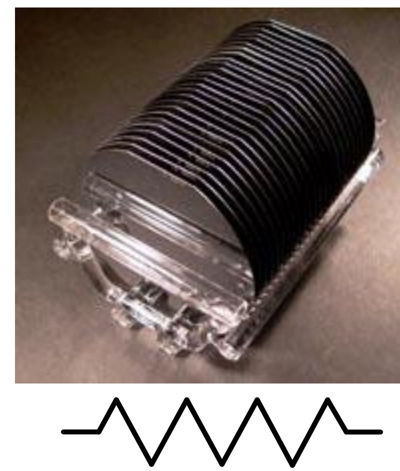
Wafers are processed in “batches” or “lots” of 20 to 40 wafers and variations occur over time or over location



$R(t_1)$



$R(t_2)$



$R(t_3)$

These variations are often the major contributor to process variability and can be in the $\pm 30\%$ range or larger

These variations often look like random variations

Within a batch, individual wafers are subjected to some variability during processing

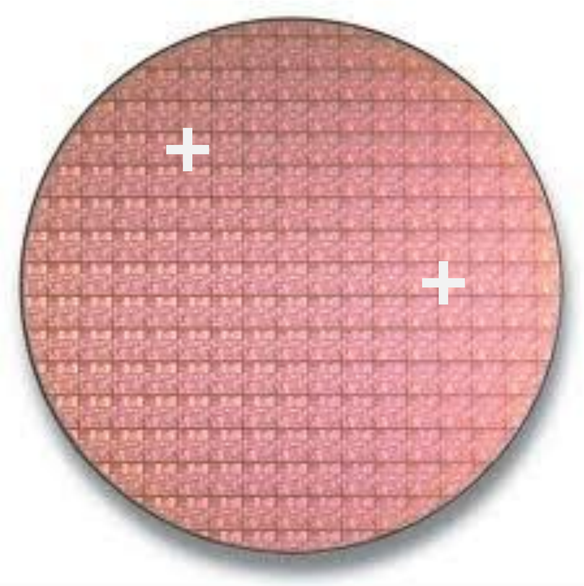
Temperature may vary with position of wafer in the boat during diffusion

Environment may vary with position of wafer in boat during diffusion or other processing steps

This variation causes characteristics of components to vary from wafer-to-wafer

These variations often look like random variations

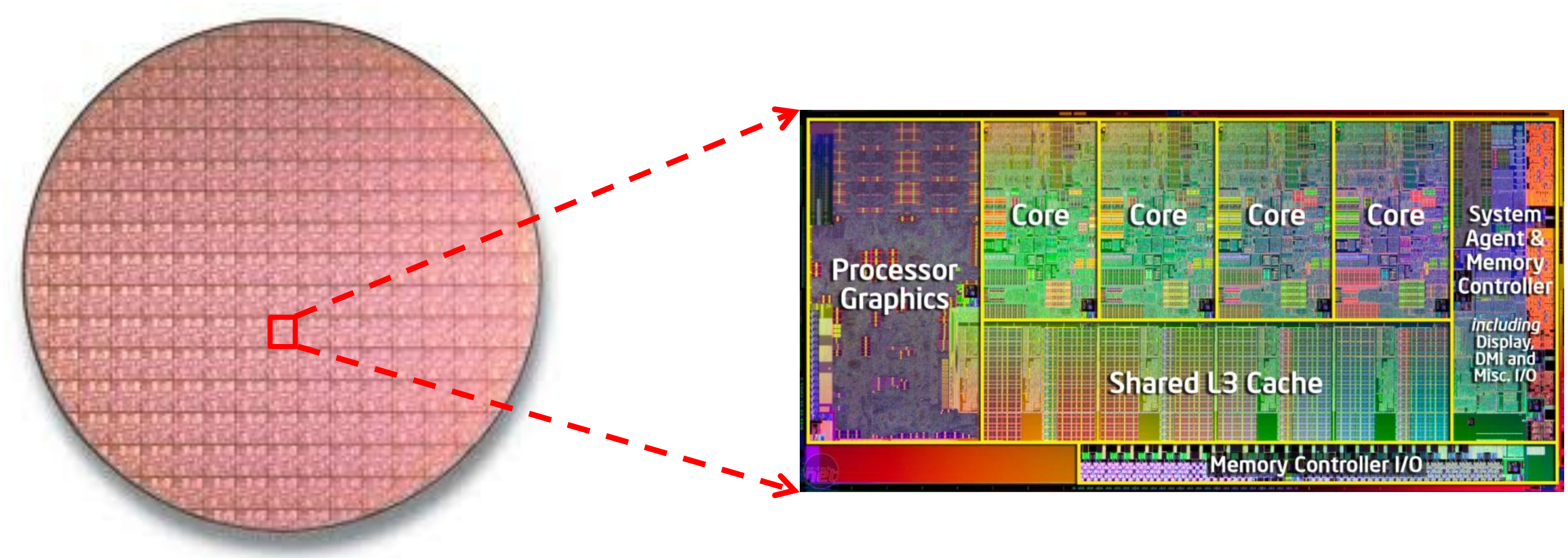
Environment may vary across individual wafers due to gradients in environmental variables during processing



This variation causes characteristics of components to vary from die to die on a wafer

These variations often look like random variations

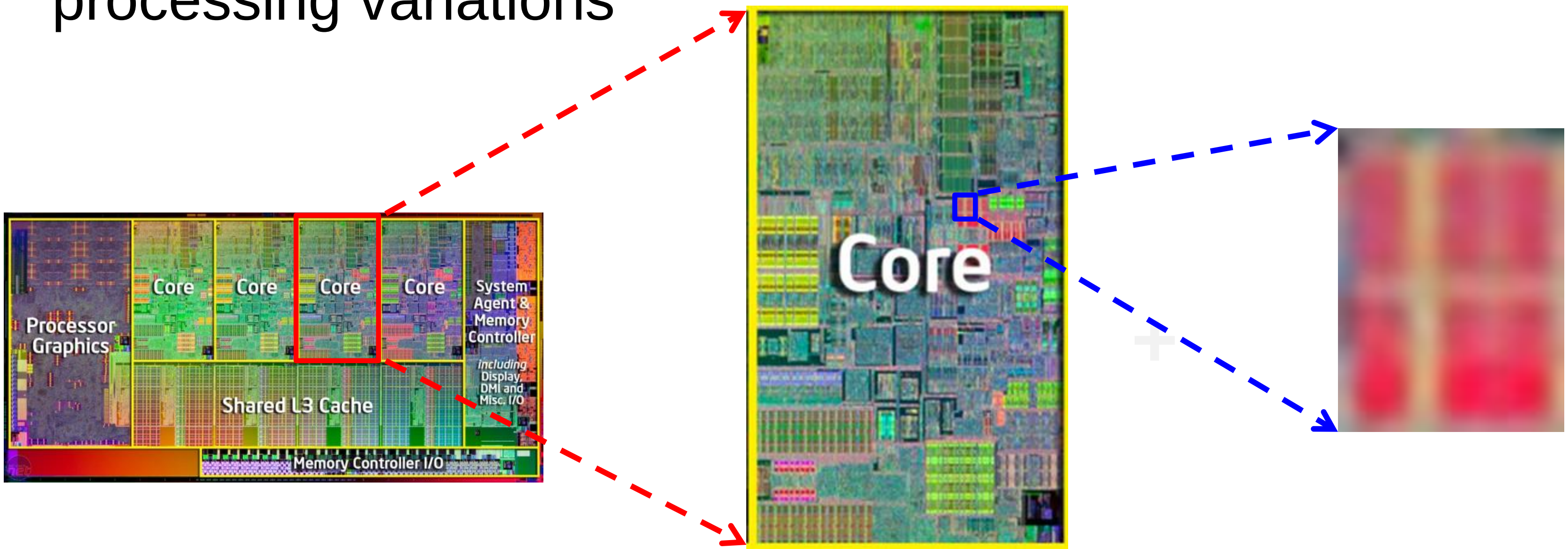
Smaller variations may occur across individual die due to gradients in environmental variables during processing



This variation causes characteristics of components to vary across a die

These variations often look like random variations

Even smaller variations may occur across individual closely placed devices due to local gradients and local random processing variations

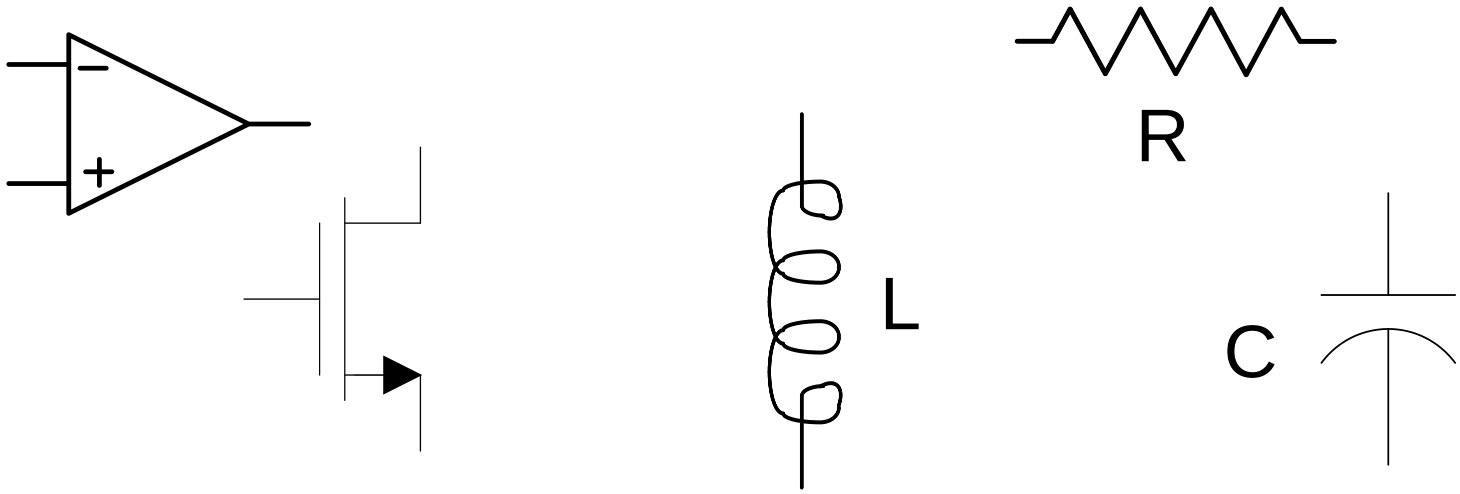


This variation introduces local gradients in device characteristics as well as local random variations

The direction and magnitude of the local gradients are random variables

The local random variations are also random variables

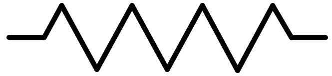
Effects of manufacturing variations on components



- A rigorous statistical analysis can be used to analytically predict how components vary and how component variations impact circuit performance

- Montecarlo simulations are often used to simulate effects of component variations
  - Requires minimal statistical knowledge to use MC simulations
  - Simulation times may be prohibitively long to get useful results
  - Gives little insight into specific source of problems
  - Must be sure to correctly include correlations in setup
- Often key statistical information is not readily available from the foundry

Modeling process variations in semiconductor processes



R

$$X = X_{NOM} + x_{RPROC} + x_{RWAFER} + x_{RDIE} + x_{RLGRAD} + x_{RLVAR}$$

X_{NOM} is the nominal value of the parameter (typically TT) and is a constant and part of the standard device model

x_{RPROC} is a random variable that changes from one “lot” of wafers to another

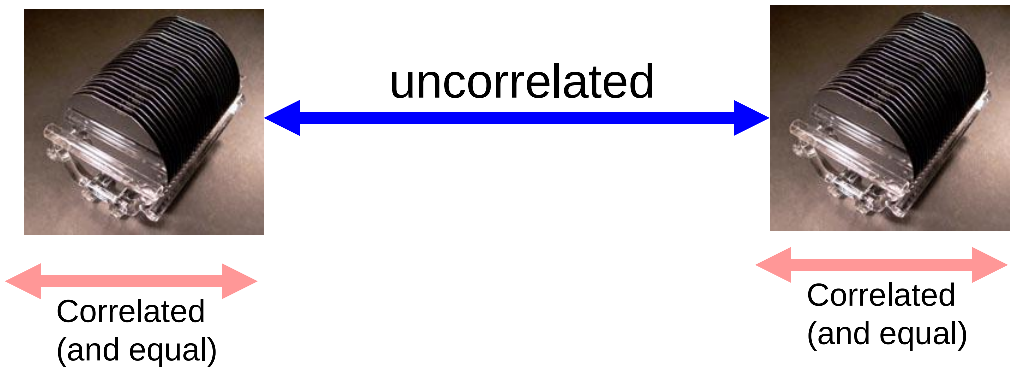
x_{RWAFER} is a random variable that changes from one wafer to another in a batch

x_{RDIE} is a random variable that changes from die to another on a wafer

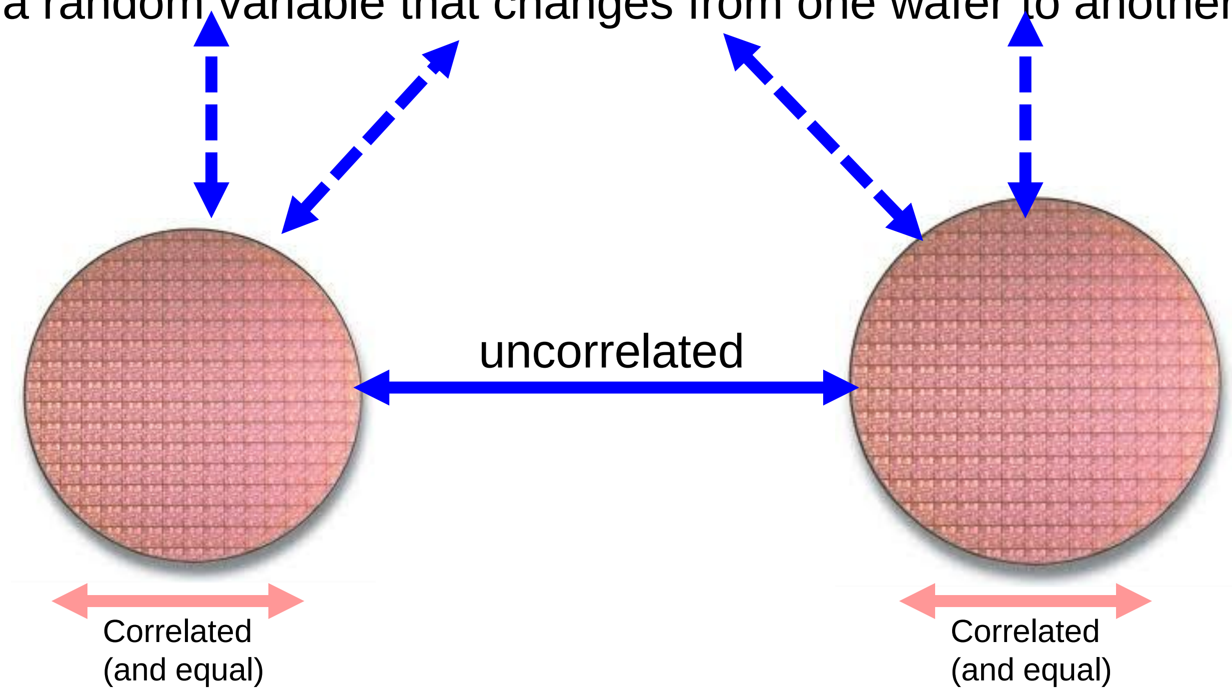
x_{RLGRAD} is a random variable that is comprised of a magnitude and direction which are themselves both random variables and characterizes very local variations on a die

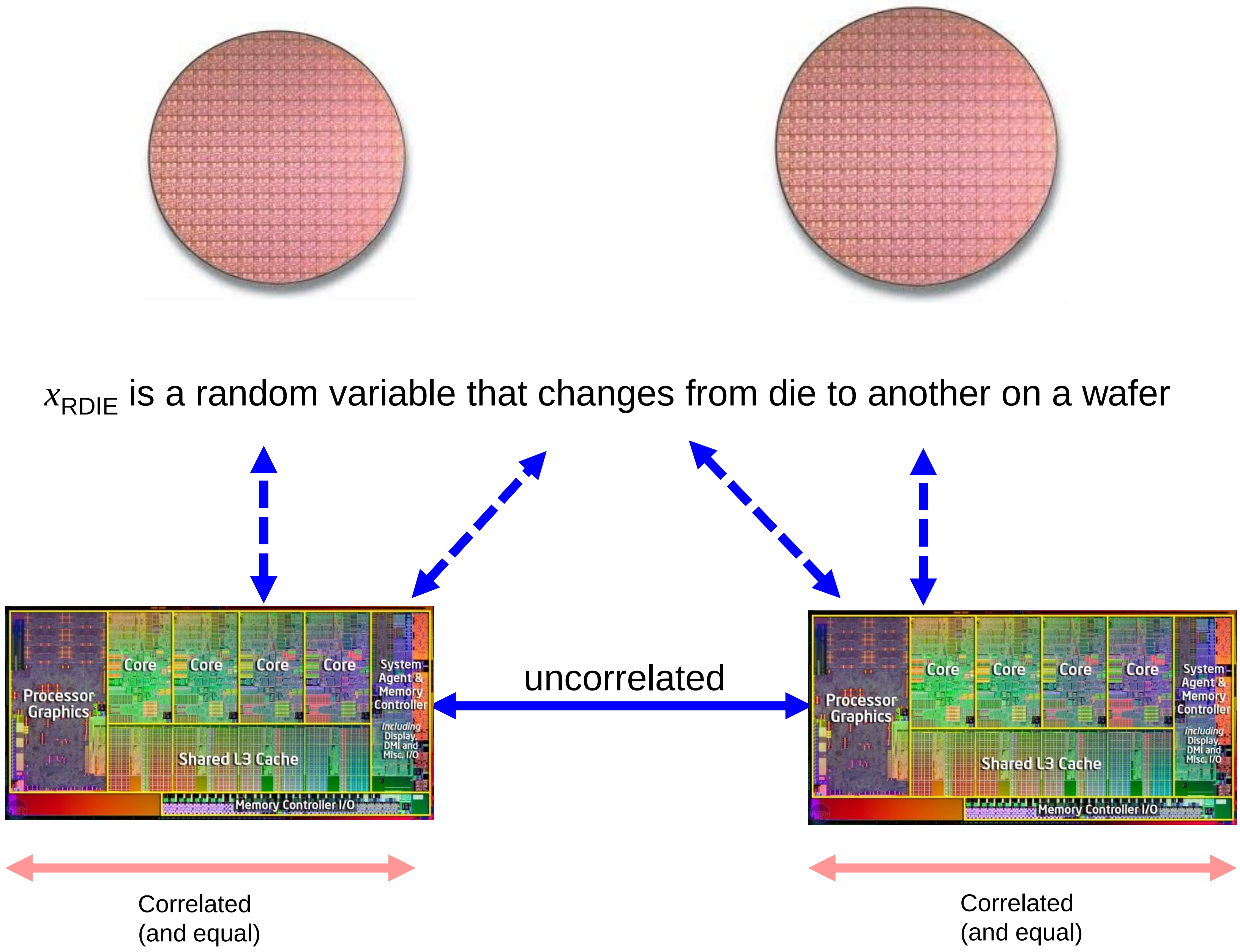
x_{RLVAR} is a random variable that characterizes very local variations on a die

x_{RPROC} is a random variable that changes from one “lot” of wafers to another

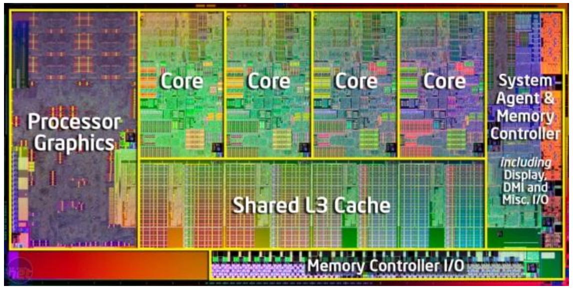


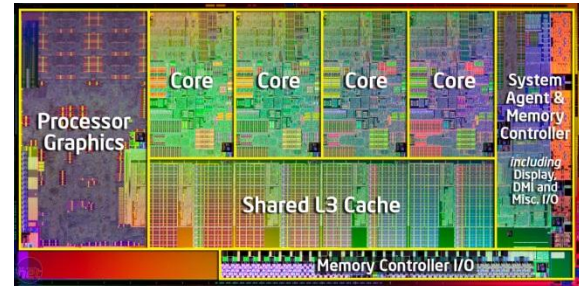
x_{RWAFER} is a random variable that changes from one wafer to another in a batch





x_{RDIE} is a random variable that changes from die to another on a wafer
uncorrelated
Core
Core
Core
Core
Processor Graphics
System Agent & Memory Controller
Including Display, DMI and Misc. I/O
Shared L3 Cache
Memory Controller I/O
Correlated (and equal)
Correlated (and equal)





x_{RLGRAD} is a random variable that is comprised of a magnitude and direction which are themselves both random variables and characterizes very local variations on a die

x_{RLVAR} is a random variable that characterizes very local variations on a die

uncorrelated

x_{RLGRAD} x_{RLVAR}

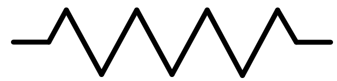
X_{RLGRAD} Correlated (and equal)

X_{RLVAR} Uncorrelated

X_{RLGRAD} Correlated (and equal)

X_{RLVAR} Uncorrelated

Modeling process variations in semiconductor processes



R

$$X = X_{NOM} + x_{RPROC} + x_{RWAFER} + x_{RDIE} + x_{RLGRAD} + x_{RLVAR}$$

x_{RPROC}, x_{RWAFER}, x_{RDIE}, x_{RLVAR} often assumed to be Gausian with zero mean

Magnitude of x_{RLGRAD} is usually assumed Gaussian with zero mean, direction is uniform from 0º to 360º

$$\sigma_{PROC} >> \sigma_{WAFER} >> \sigma_{DIE}$$

$$\sigma_{DIE} >> \sigma_{LVAR}$$

$$\sigma_{DIE} >> \sigma_{|GRAD|}$$

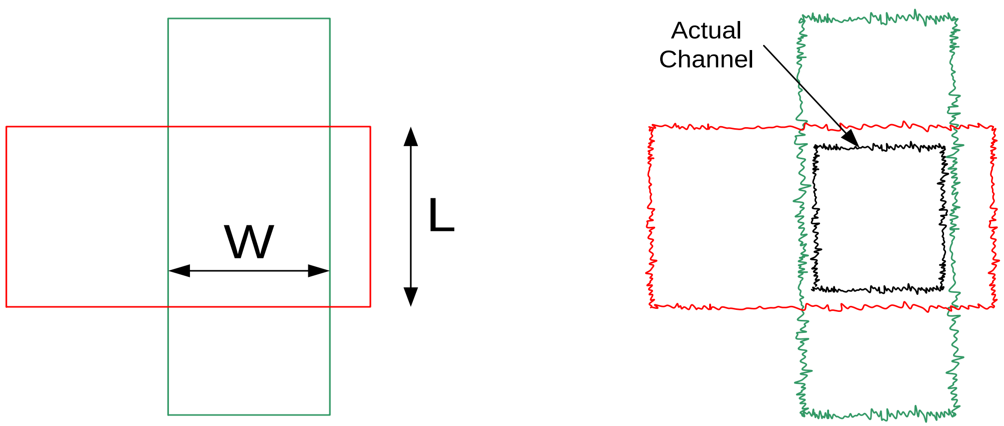
σ_{LVAR} Strongly dependent upon area and layout

$$\sigma_{LVAR} \sim \frac{1}{\sqrt{\text{Area}}}$$

$$\sigma_{LVAR} \sim \text{Perimeter}$$

Relative size between σ_{LVAR} and $\sigma_{|GRAD|}$ dependent upon A, P, and process

Effects of layout on local random variations



Drawn and Actual Features for MOS Transistor

Variations also occur vertically in both oxide thickness and doping levels/profiles and often these will dominate the lateral effects

Modeling process variations in semiconductor processes

- Statistics associated with value of dimensioned parameters (poles, GB, SR,R,C,transresistance gains, transconductance gains, … dominated by x_{RPROC})

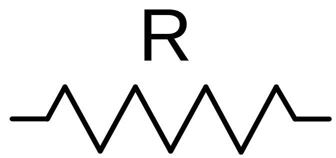
- Statistics associated with matching/sensitive dimensionless parameters such as voltage or current gains, component ratios, pole Q, … (almost always closely placed) dominated by x_{RLGRAD} and x_{RLVAR} (because locally x_{RPROC}, x_{RWAFER}, x_{RDIE} are all correlated and equal)

- Gradients are dominantly linear if spacing is not too large

- Special layout techniques using common centroid approaches can be used to eliminate (or dramatically reduce) linear gradient effects so, if employed, matching/sensitive parameters dominated by x_{RLVAR} but occasionally common centroid layouts become impractical or areas become too large so that gradients become nonlinear and in these cases gradient effects will still limit performance

- Higher-order gradient effects can be eliminated with layout approaches that cancel higher “moments” but area and effort may not be attractive

Be sure correct statistical information is available when doing a statistical analysis using either analytical or Montecarlo methods



- Some statistics associated with making many measurements over many devices over many lots of wafers

- Some statistics associated with many measurements in a particular process run

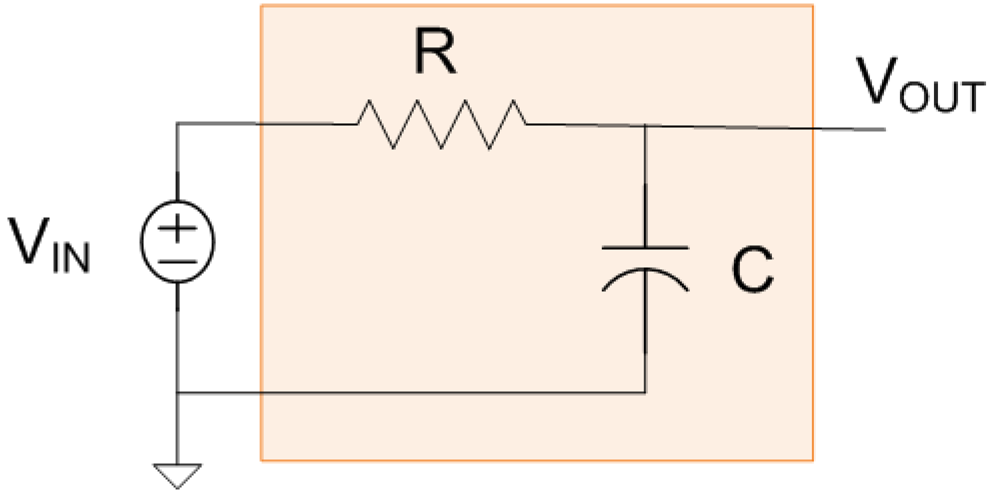
- Some statistics associated with making many measurements across a wafer

- Some statistics associated with making many measurements on closely-placed devices

- Some statistics associated with making many measurements on closely-placed devices that have common-centroid layouts

- Some statistics presented (particularly in literature or occasionally in PDK) with limited information about how data was gathered

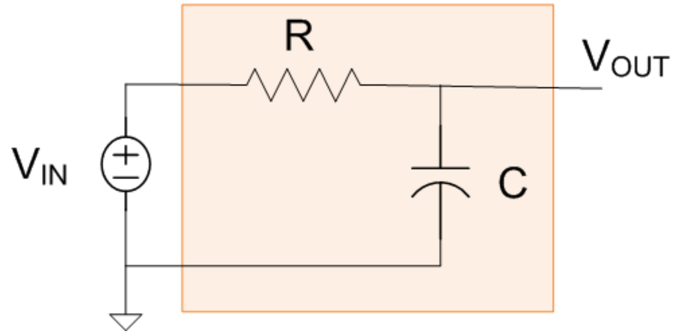
Statistical Modeling of dimensioned parameters - example



Determine the standard deviation of the pole frequency (or band edge) of the first-order passive filter.

Assume the process variables are zero mean with standard deviations given by

$$\sigma_{\frac{R_{PROC}}{R_{NOM}}} = 0.2 \qquad \sigma_{\frac{C_{PROC}}{C_{NOM}}} = 0.1$$



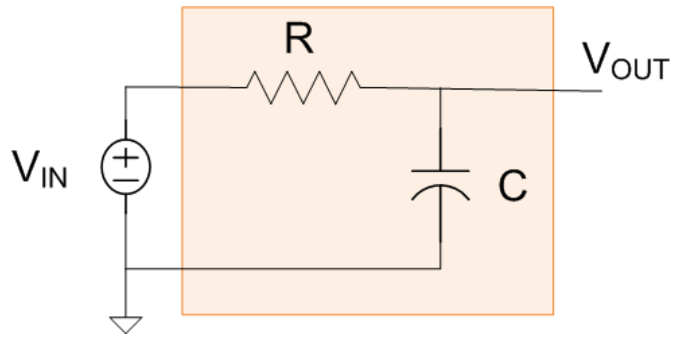
$$p = \frac{1}{RC}$$

Since R and C are random variables, the pole p is also a random variable

Theorem: The sum of uncorrelated Gaussian random variables is a multivariate Gaussian random variable

Theorem: If $X_1 \dots X_m$ are uncorrelated random variables with standard deviations $\sigma_1, \sigma_2, \dots \sigma_m$, and $a_1, a_2, \dots a_m$ are constants, then the standard deviation of the random variable $y = \sum_{i=1}^{m} a_i X_i$ is given by the expression

$$\sigma_y = \sqrt{\sum_{i=1}^{m} a_i^2 \sigma_i^2}$$



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$$p = \frac{1}{(\mathrm{R_{NOM}} + R_{RAN})(\mathrm{C_{NOM}} + C_{RAN})}$$

Unfortunately the pdf p which is the reciprocal of the product of Gaussian variables is very difficult to obtain

Observe can express p as

$$p = \frac{1}{(\mathrm{R_{NOM}} + R_{RAN})(\mathrm{C_{NOM}} + C_{RAN})} = \left(\frac{1}{\mathrm{R_{NOM}C_{NOM}}}\right)\left(\frac{1}{\left[1+\frac{R_{RAN}}{\mathrm{R_{NOM}}}\right]\left[1+\frac{C_{RAN}}{\mathrm{C_{NOM}}}\right]}\right)$$

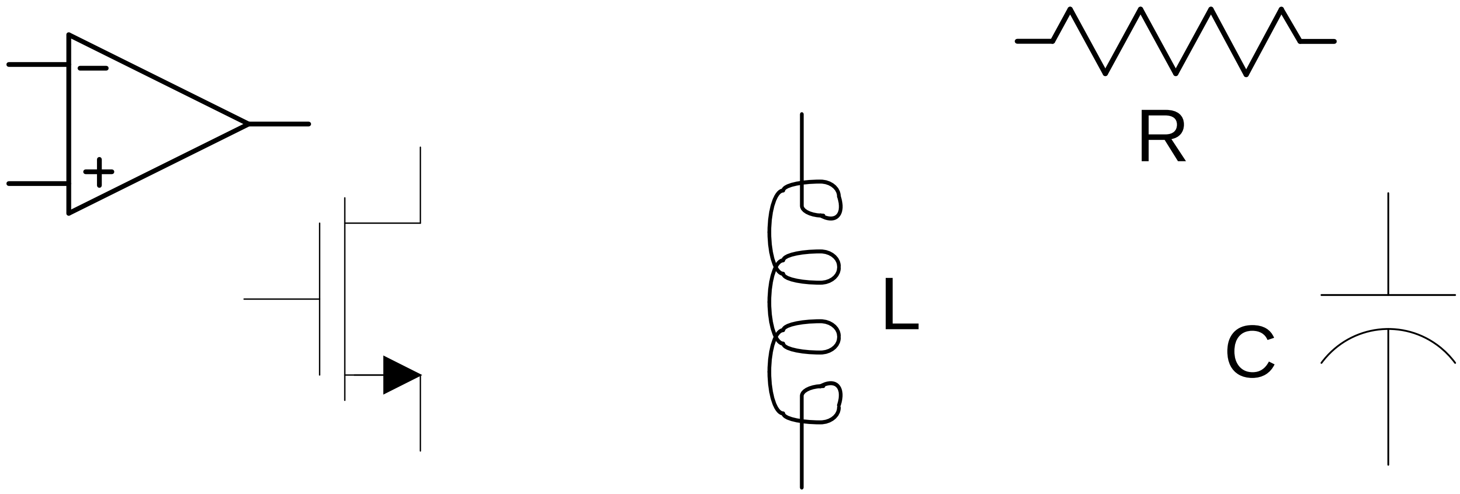
End of Lecture 14

EE 508
Lecture 15

Statistical Characterization of Filter Characteristics

Review from last lecture

Effects of manufacturing variations on components

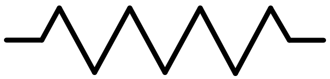


- A rigorous statistical analysis can be used to analytically predict how components vary and how component variations impact circuit performance

- Montecarlo simulations are often used to simulate effects of component variations
  - Requires minimal statistical knowledge to use MC simulations
  - Simulation times may be prohibitively long to get useful results
  - Gives little insight into specific source of problems
  - Must be sure to correctly include correlations in setup
- Often key statistical information is not readily available from the foundry

Review from last lecture

Modeling process variations in semiconductor processes



R

$$X = X_{NOM} + x_{RPROC} + x_{RWAFER} + x_{RDIE} + x_{RLGRAD} + x_{RLVAR}$$

x_{RPROC}, x_{RWAFER}, x_{RDIE}, x_{RLVAR} often assumed to be Gausian with zero mean

Magnitude of x_{RLGRAD} is usually assumed Gaussian with zero mean, direction is uniform from 0^o to 360^o

$$\sigma_{PROC} >> \sigma_{WAFER} >> \sigma_{DIE}$$

$$\sigma_{DIE} >> \sigma_{LVAR}$$

$$\sigma_{DIE} >> \sigma_{|GRAD|}$$

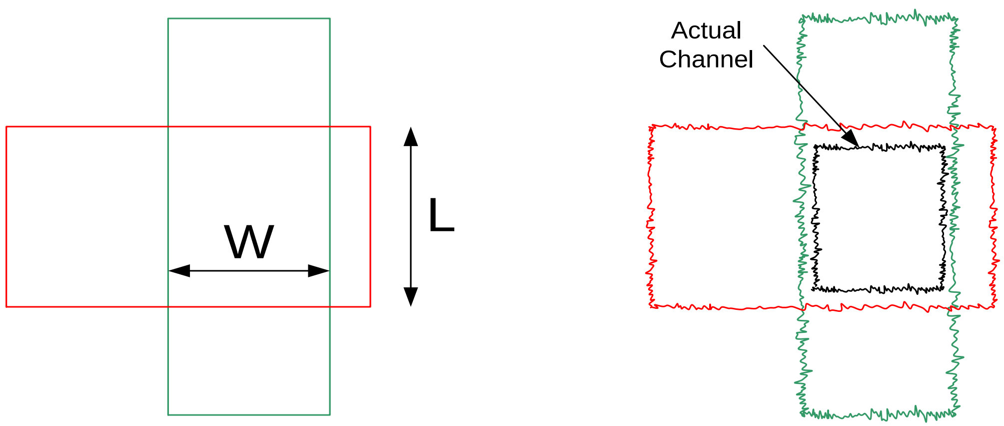
σ_{LVAR} Strongly dependent upon area and layout

$$\sigma_{LVAR} \sim \frac{1}{\sqrt{\text{Area}}}$$

$$\sigma_{LVAR} \sim \text{Perimeter}$$

Relative size between σ_{LVAR} and $\sigma_{|GRAD|}$ dependent upon A, P, and process

Effects of layout on local random variations



Drawn and Actual Features for MOS Transistor

Variations also occur vertically in both oxide thickness and doping levels/profiles and often these will dominate the lateral effects

Review from last lecture

Modeling process variations in semiconductor processes

- Statistics associated with value of dimensioned parameters (poles, GB, SR,R,C,transresistance gains, transconductance gains, … dominated by x_{RPROC})

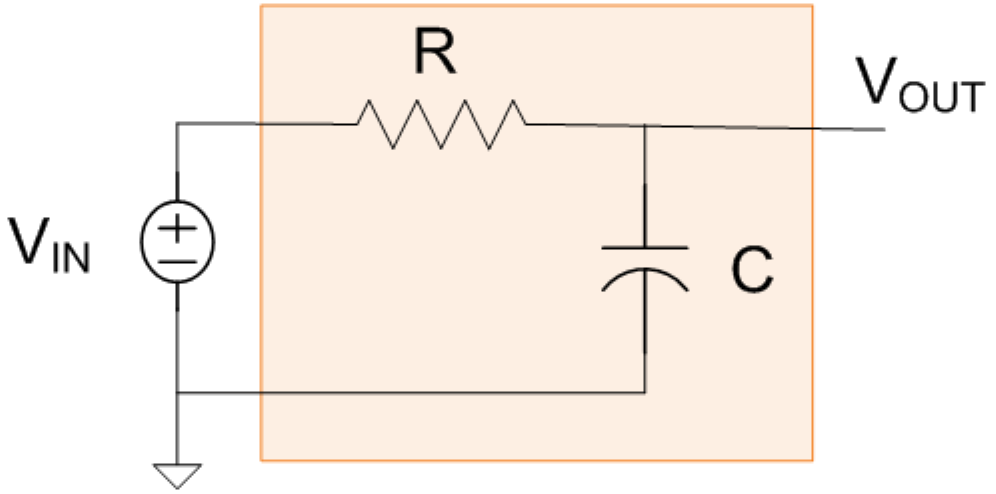
- Statistics associated with matching/sensitive dimensionless parameters such as voltage or current gains, component ratios, pole Q, … (almost always closely placed) dominated by x_{RLGRAD} and x_{RLVAR} (because locally x_{RPROC}, x_{RWAFER}, x_{RDIE} are all correlated and equal)

- Gradients are dominantly linear if spacing is not too large

- Special layout techniques using common centroid approaches can be used to eliminate (or dramatically reduce) linear gradient effects so, if employed, matching/sensitive parameters dominated by x_{RLVAR} but occasionally common centroid layouts become impractical or areas become too large so that gradients become nonlinear and in these cases gradient effects will still limit performance

- Higher-order gradient effects can be eliminated with layout approaches that cancel higher "moments" but area and effort may not be attractive

Statistical Modeling of dimensioned parameters - example



Determine the standard deviation of the pole frequency (or band edge) of the first-order passive filter.

Assume the process variables are zero mean with standard deviations given by

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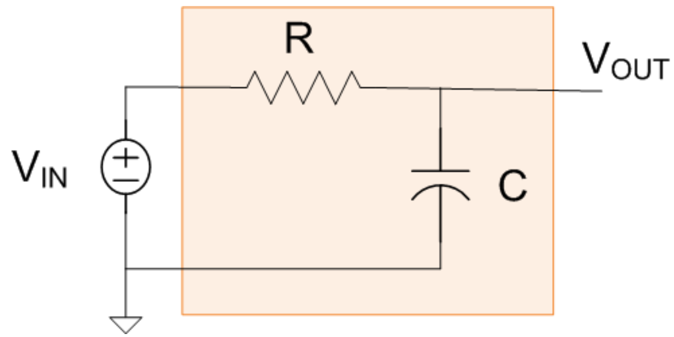
Review from last lecture



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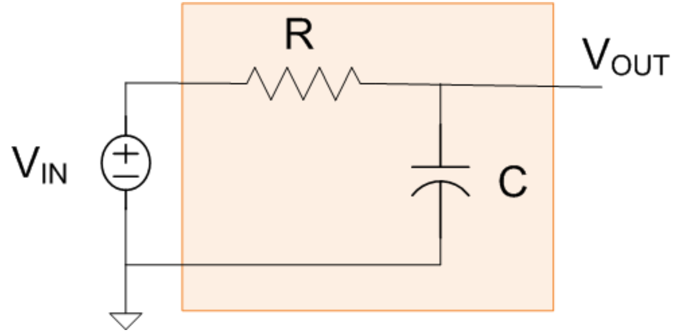
Review from last lecture



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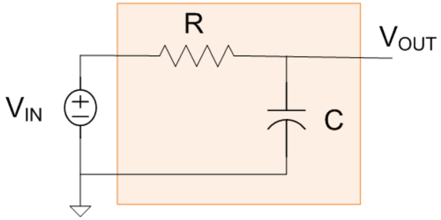
Unfortunately the pdf p which is the reciprocal of the product of Gaussian variables is very difficult to obtain

Observe can express p as

$$p = \frac{1}{(R_{NOM} + R_{RAN})(C_{NOM} + C_{RAN})} = \left(\frac{1}{R_{NOM}C_{NOM}}\right)\left(\frac{1}{\left[1+\frac{R_{RAN}}{R_{NOM}}\right]\left[1+\frac{C_{RAN}}{C_{NOM}}\right]}\right)$$



$$p = \frac{1}{RC}$$

$$p = \frac{1}{(R_{NOM} + R_{RAN})(C_{NOM} + C_{RAN})} = \left(\frac{1}{R_{NOM}C_{NOM}}\right)\left(\frac{1}{\left[1+\frac{R_{RAN}}{R_{NOM}}\right]\left[1+\frac{C_{RAN}}{C_{NOM}}\right]}\right)$$

But $R_{RAN} << R_{NOM}$ and $C_{RAN} << C_{NOM}$

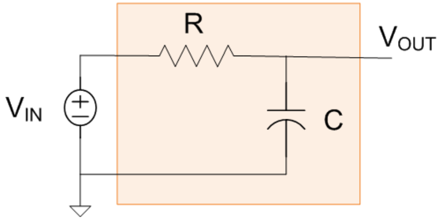
It thus follows from a truncated power series expansion of the two-variable fraction that

$$p \simeq \left(\frac{1}{R_{NOM}C_{NOM}}\right)\left(\left[1-\frac{R_{RAN}}{R_{NOM}}\right]\left[1-\frac{C_{RAN}}{C_{NOM}}\right]\right)$$

$$p \simeq \left(\frac{1}{R_{NOM}C_{NOM}}\right)\left(1-\frac{R_{RAN}}{R_{NOM}}-\frac{C_{RAN}}{C_{NOM}}\right)$$

These operations were used to linearize p in terms of the random variables !

Note that p is the sum of two Gaussian random variables that are assumed to be uncorrelated so p is also Gaussian



$$p = \frac{1}{RC}$$

$$p \simeq \left(\frac{1}{R_{NOM}C_{NOM}}\right)\left(1 - \frac{R_{RAN}}{R_{NOM}} - \frac{C_{RAN}}{C_{NOM}}\right)$$

It thus follows from the theorem that

$$\sigma_p \simeq \left(\frac{1}{R_{NOM}C_{NOM}}\right)\sqrt{\sigma^2_{\frac{R_{RAN}}{R_{NOM}}} + \sigma^2_{\frac{C_{RAN}}{C_{NOM}}}}$$

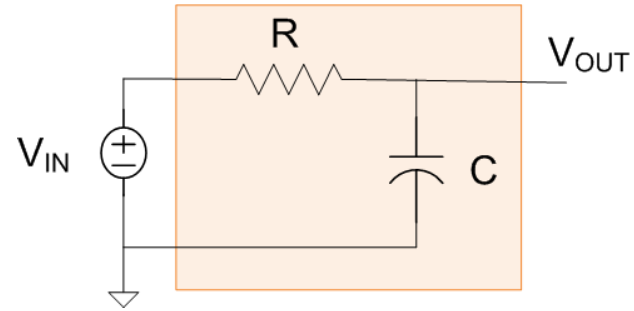
But the nominal value of the pole is $p_{NOM} \simeq \dfrac{1}{R_{NOM}C_{NOM}}$

It thus follows that

$$\sigma_{\frac{p}{p_{NOM}}} \simeq \sqrt{\sigma^2_{\frac{R_{RAN}}{R_{NOM}}} + \sigma^2_{\frac{C_{RAN}}{C_{NOM}}}}$$

Observe:

$$\frac{p}{p_{NOM}} \sim N\left(1, \sigma_{\frac{p}{p_{NOM}}}\right)$$



$$p = \frac{1}{RC}$$

$$\sigma_{\frac{p}{p_{NOM}}} \simeq \sqrt{\sigma^2_{\frac{R_{RAN}}{R_{NOM}}} + \sigma^2_{\frac{C_{RAN}}{C_{NOM}}}}$$

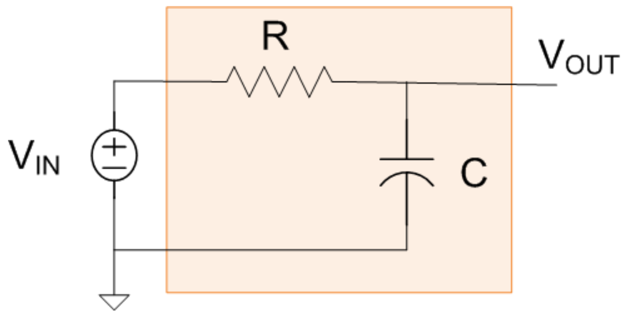
But R_{RAN} and C_{RAN} are approximately R_{PROC} and C_{PROC}

$$\sigma_{\frac{p}{p_{NOM}}} \simeq \sqrt{\sigma^2_{\frac{R_{PROC}}{R_{NOM}}} + \sigma^2_{\frac{C_{PROC}}{C_{NOM}}}}$$

recall

$$\sigma_{\frac{R_{PROC}}{R_{NOM}}} = 0.2 \qquad \sigma_{\frac{C_{PROC}}{C_{NOM}}} = 0.1$$

$$\sigma_{\frac{p}{p_{NOM}}} \simeq \sqrt{0.2^2 + 0.1^2} = 0.22$$



$$p = \frac{1}{RC}$$

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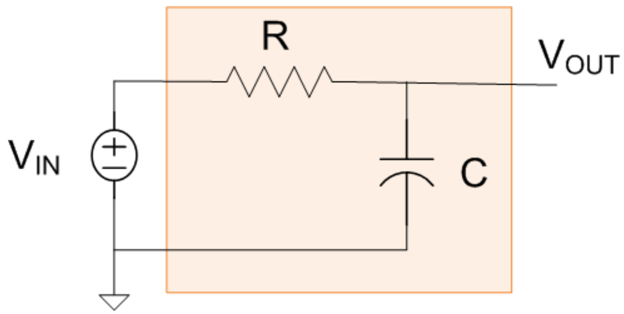
1. Determine the 3σ range in the pole location

2. Determine the percent of the process lots that will have a pole with mean that is within 10% of the nominal value

3. What can the designer do to tighten the band edge of this filter?



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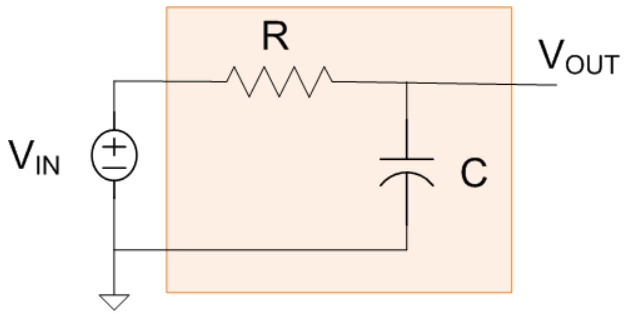
$$\sigma_{\frac{p}{p_{NOM}}} \simeq \sqrt{0.2^2 + 0.1^2} = 0.22$$

1. Determine the 3σ range in the pole location

The 3σ range is simply $$0.34 \leq \frac{p}{p_{NOM}} \leq 1.66$$

So, if the nominal pole location is 10KHz, the average value of the pole location from lot to lot will vary between 3.4KHz and 16.6KHz



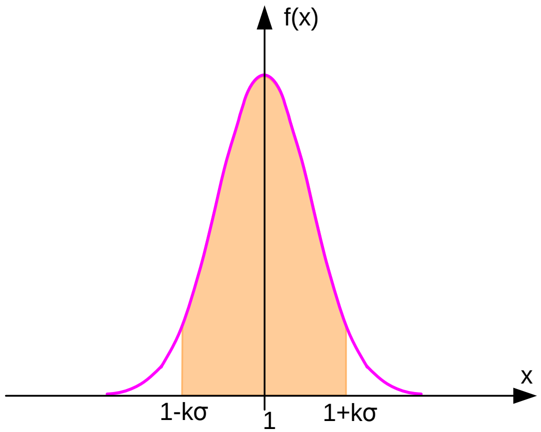
$$p = \frac{1}{RC}$$

$$\sigma_{\frac{p}{p_{NOM}}} \simeq \sqrt{0.2^2 + 0.1^2} = 0.22$$

2. Determine the percent of the process lots that will have a pole with mean that is within 10% of the nominal value

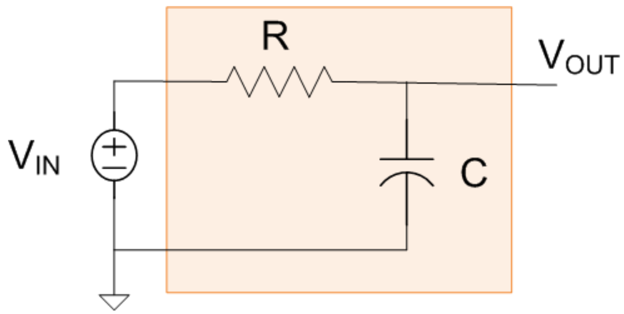
Observe a 10% window is $\left(\frac{.1}{.22}\right)\sigma_{\frac{p}{p_{NOM}}} = 0.45\sigma_{\frac{p}{p_{NOM}}}$

Recall $\frac{p}{p_{NOM}} \sim N\left(1, \sigma_{\frac{p}{p_{NOM}}}\right)$ For a kσ window the probability of being inside that window is the area under the pdf curve between 1– kσ and 1+kσ



Observe

$$\tilde{p} = \frac{\frac{p}{p_{NOM}} - 1}{\sigma_{\frac{p}{p_{NOM}}}} \sim N(0,1)$$



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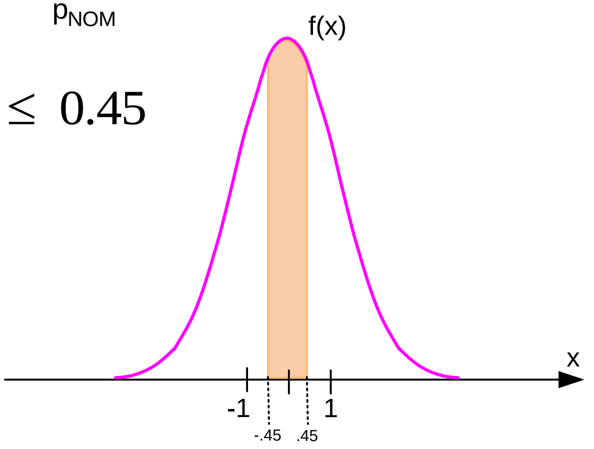
$$1 - 0.45\sigma_{\frac{p}{p_{NOM}}} \le p \le 1 - 0.45\sigma_{\frac{p}{p_{NOM}}} \Rightarrow -0.45 \le \tilde{p} \le 0.45$$

$$\tilde{p} = \frac{\frac{p}{p_{NOM}} - 1}{\sigma_{\frac{p}{p_{NOM}}}}$$

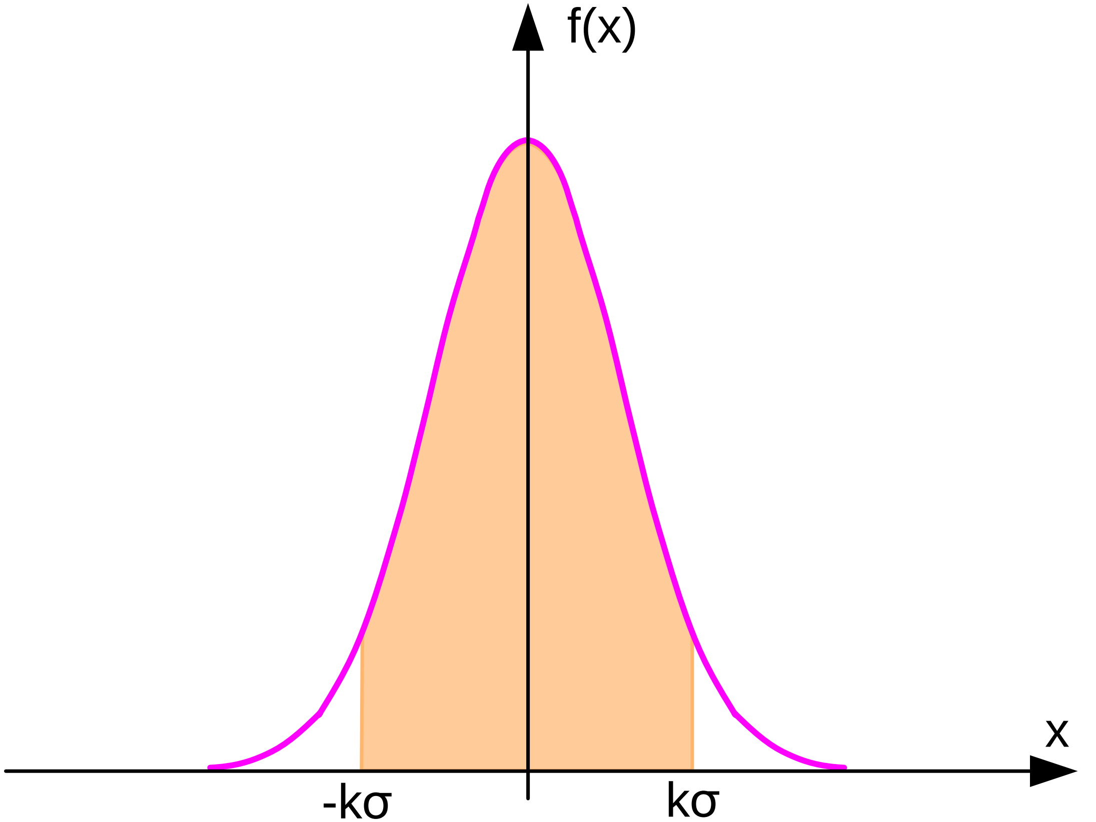


For a Gaussian variable, this area is given by

$$\theta_{prob} = 2F_{N(0,1)}(k) - 1 = 2F_{N(0,1)}(0.45) - 1$$

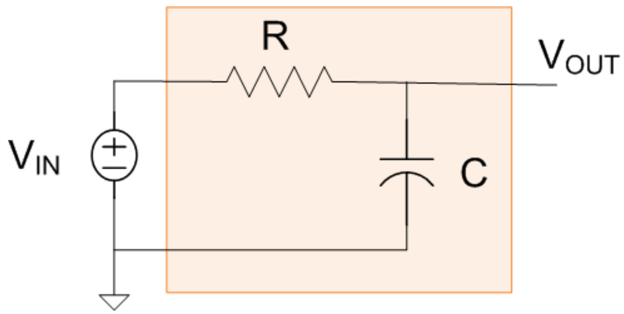
Offset Voltage Distribution

Pdf of zero-mean Gaussian distribution



Percent between:	±σ	68.3%
	±2σ	95.5%
	±3σ	99.73%

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91308	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997
4.0	0.99997	0.99997	0.99997	0.99997	0.99997	0.99997	0.99998	0.99998	0.99998	0.99998



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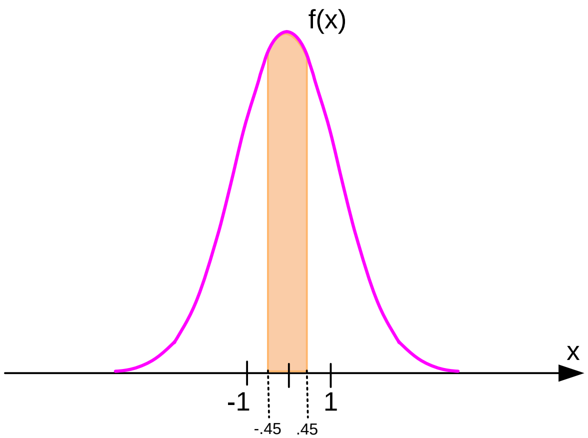
$$\sigma_{\frac{p}{p_{NOM}}} \simeq \sqrt{0.2^2 + 0.1^2} = 0.22$$

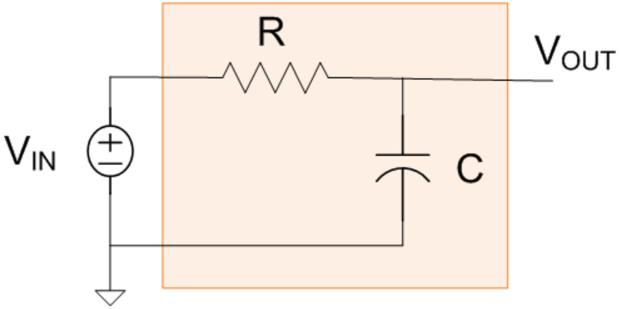
2. Determine the percent of the process lots that will have a pole with mean that is within 10% of the nominal value

$$\theta_{prob} = 2F_{N(0,1)}(0.45) - 1$$

$$\theta_{prob} = 2 \bullet .6736 - 1 = 0.347$$

Thus, approximately 35% of the wafer lots will have a pole within 10% of the nominal value





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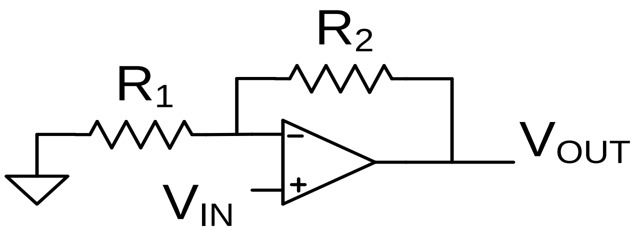
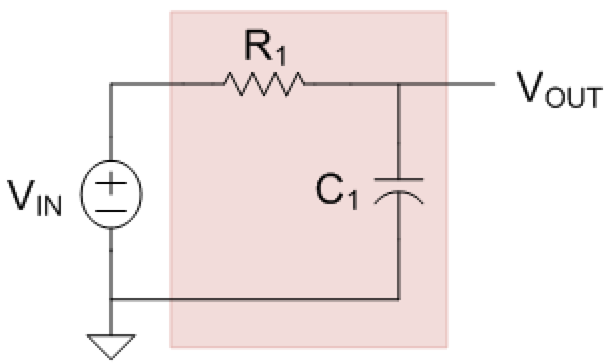
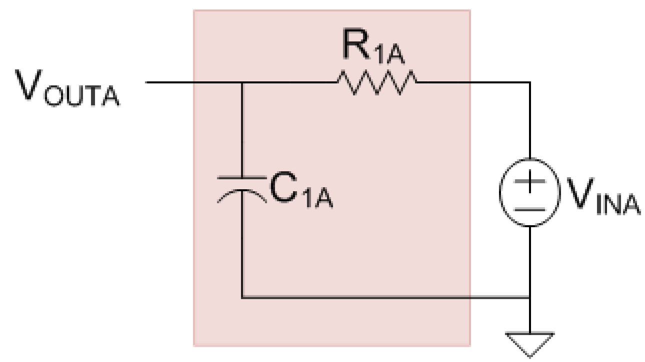
3. What can the designer do to tighten the band edge of this filter?

Statistical Modeling of Dimensionless Parameters



$$K = 1 + \frac{R_2}{R_1}$$



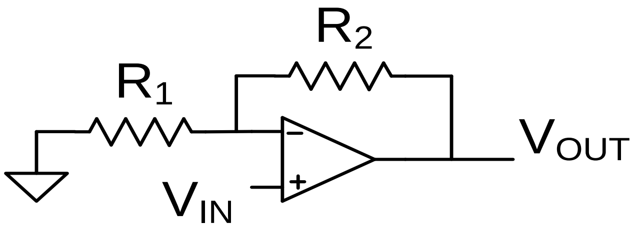


$$p_1 = \frac{1}{RC}$$

$$p_A = \frac{1}{R_{1A} C_{1A}}$$

$$\theta = \frac{p_A - p_1}{p_1}$$

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the standard deviation of the voltage gain K

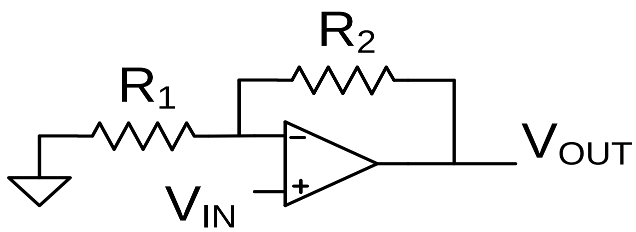
Determine the yield if the nominal gain is 10 $\pm 1\%$

Assume a common centroid layout of R_1 and R_2 has been used and the area of R_1 is 100u^2 and both resistors have the same resistance density and R_2 is comprised of K-1 copies of R_1 . Neglect variable edge effects in the layout

A_ρ=.01µm

$$\sigma_{\frac{R_{PROC}}{R_{NOM}}} = 0.2$$

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the standard deviation of the voltage gain K

$$K = 1 + \frac{R_{2N} + R_{2R}}{R_{1N} + R_{1N}}$$

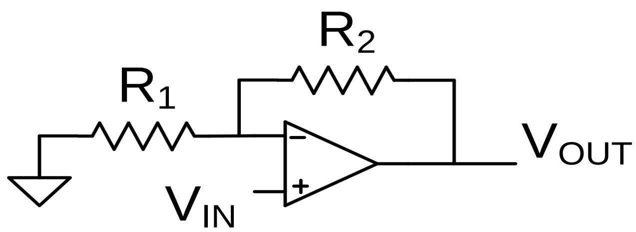
$$K \simeq 1 + \frac{R_{2N}}{R_{1N}}\left(1 + \frac{R_{2R}}{R_{2N}} - \frac{R_{1R}}{R_{1N}}\right)$$

$$K = 1 + \frac{R_{2N}\left(1 + \frac{R_{2R}}{R_{2N}}\right)}{R_{1N}\left(1 + \frac{R_{1R}}{R_{1N}}\right)}$$

$$K \simeq \left(1 + \frac{R_{2N}}{R_{1N}}\right) + \frac{R_{2N}}{R_{1N}}\left(\frac{R_{2R}}{R_{2N}} - \frac{R_{1R}}{R_{1N}}\right)$$

$$K \simeq 1 + \frac{R_{2N}}{R_{1N}}\left(1 + \frac{R_{2R}}{R_{2N}}\right)\left(1 - \frac{R_{1R}}{R_{1N}}\right)$$

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the standard deviation of the voltage gain K

$$K \simeq \left(1 + \frac{R_{2N}}{R_{1N}}\right) + \frac{R_{2N}}{R_{1N}}\left(\frac{R_{2R}}{R_{2N}} - \frac{R_{1R}}{R_{1N}}\right)$$

$$K \simeq K_N + (K_N - 1)\left(\frac{R_{2R}}{R_{2N}} - \frac{R_{1R}}{R_{1N}}\right)$$

$$R_{2R} \simeq R_{2RPROC} + R_{2RFRAD} + R_{2RLVAR}$$

$$R_{1R} \simeq R_{1RPROC} + R_{1RFRAD} + R_{1RLVAR}$$

$$K \simeq K_N + (K_N - 1)\left(\frac{R_{2RPROC} + R_{2RFRAD} + R_{2RLVAR}}{R_{2N}} - \frac{R_{1RPROC} + R_{1RFRAD} + R_{1RLVAR}}{R_{1N}}\right)$$

But R_{2RPROC} and R_{1RPROC} are correlated

$$R_{2RPROC} = (K_N - 1) R_{1RPROC}$$

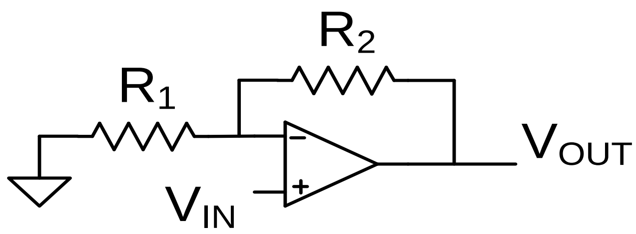
And, since a common centroid layout is used,

R_{2RGRAD} and R_{1RGRAD} are correlated

$$R_{2RGRAD} = (K_N - 1) R_{1RGRAD}$$

R_{2RLVAR} and R_{1RLVAR} are uncorrelated

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the standard deviation of the voltage gain K

$$K \simeq K_N + (K_N - 1)\left(\frac{R_{2RPROC} + R_{2RFRAD} + R_{2RLVAR}}{R_{2N}} - \frac{R_{2RPROC} + R_{2RFRAD} + R_{2RLVAR}}{R_{1N}}\right)$$

$$K \simeq K_N + (K_N - 1)\left(\frac{(K_N - 1)R_{1RPROC} + (K_N - 1)R_{1RPROC} + R_{2RLVAR}}{R_{2N}} - \frac{R_{1RPROC} + R_{1RFRAD} + R_{1RLVAR}}{R_{1N}}\right)$$

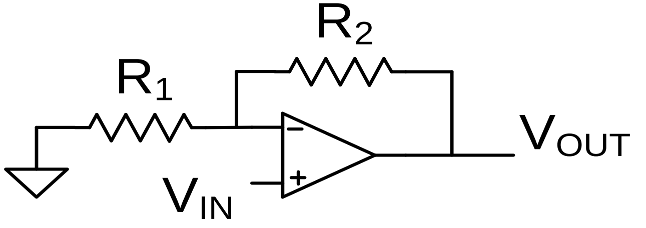
Since $R_{2N}=(K_N-1)R_{1N}$

$$K \simeq K_N + (K_N - 1)\left(\frac{(K_N - 1)R_{1RPROC} + (K_N - 1)R_{1RPROC}}{(K_N - 1)R_{1N}} + \frac{R_{2RLVAR}}{R_{2N}} - \frac{R_{1RPROC} + R_{1RFRAD} + R_{1RLVAR}}{R_{1N}}\right)$$

$$K \simeq K_N + (K_N - 1)\left(\left[\frac{(K_N - 1)R_{1RPROC} + (K_N - 1)R_{1RPROC}}{(K_N - 1)R_{1N}} - \frac{R_{1RPROC} + R_{1RFRAD}}{R_{1N}}\right] + \frac{R_{2RLVAR}}{R_{2N}} - \frac{R_{1RLVAR}}{R_{1N}}\right)$$

$$K \simeq K_N + (K_N - 1)\left(\frac{R_{2RLVAR}}{R_{2N}} - \frac{R_{1RLVAR}}{R_{1N}}\right)$$

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the standard deviation of the voltage gain K

$$K \simeq K_N + (K_N - 1)\left(\frac{R_{2RLVAR}}{R_{2N}} - \frac{R_{1RLVAR}}{R_{1N}}\right)$$

$$\sigma_K \simeq (K_N - 1)\sqrt{\sigma^2_{\frac{R_{2R}}{R_{2N}}} + \sigma^2_{\frac{R_{1R}}{R_{1N}}}}$$

$$\sigma_{\frac{K}{K_N}} \simeq \left(1 - \frac{1}{K_N}\right)\sqrt{\sigma^2_{\frac{R_{2R}}{R_{2N}}} + \sigma^2_{\frac{R_{1R}}{R_{1N}}}}$$

Theorem: If the perimeter variations and contact resistance are neglected, the standard deviation of the local random variations of a resistor of area A is given by the expression

$$\sigma_{\frac{R}{R_N}} = \frac{A_\rho}{\sqrt{A}}$$

Theorem: If the perimeter variations are neglected, the standard deviation of the local random variations of a capacitor of area A is given by the expression

$$\sigma_{\frac{C}{C_N}} = \frac{A_C}{\sqrt{A}}$$

Theorem: If the perimeter variations are neglected, the variance of the local random variations of the normalized threshold voltage of a rectangular MOS transistor of dimensions W and L is given by the expression

$$\sigma^2_{\frac{V_T}{V_{T_N}}} = \frac{A^2_{VTO}}{V^2_{T_N} WL} \qquad \text{or as} \qquad \sigma^2_{\frac{V_T}{V_{T_N}}} = \frac{A^2_{VT}}{WL}$$

Theorem: If the perimeter variations are neglected, the variance of the local random variations of the normalized C_{OX} of a rectangular MOS transistor of dimensions W and L is given by the expression

$$\sigma^2_{\frac{C_{OX}}{C_{OXN}}} = \frac{A^2_{COX}}{WL}$$

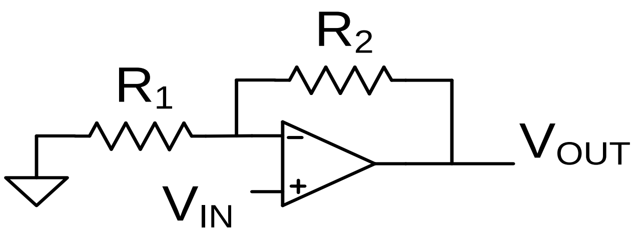
Theorem: If the perimeter variations are neglected, the variance of the local random variations of the normalized mobility of a rectangular MOS transistor of dimensions W and L is given by the expression

$$\sigma^2_{\frac{\mu_R}{\mu_N}} = \frac{A^2_{\mu}}{WL}$$

where the parameters A_X are all constants characteristic of the process (i.e. model parameters)

- The effects of edge roughness on the variance of resistors, capacitors, and transistors can readily be included but for most layouts is dominated by the area dependent variations
- There is some correlation between the model parameters of MOS transistors but they are often ignored to simplify calculations

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the standard deviation of the voltage gain K

$$K \simeq K_N + (K_N - 1)\left(\frac{R_{2RLVAR}}{R_{2N}} - \frac{R_{1RLVAR}}{R_{1N}}\right)$$

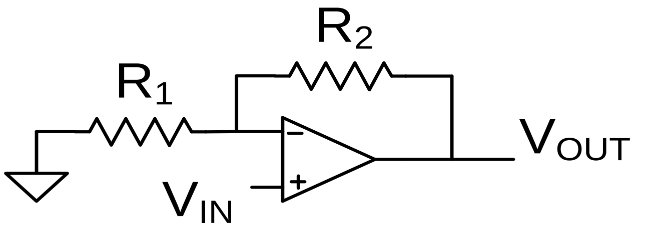
$$\sigma_K \simeq (K_N - 1)\sqrt{\sigma^2_{\frac{R_{2R}}{R_{2N}}} + \sigma^2_{\frac{R_{1R}}{R_{1N}}}}$$

$$\sigma_{\frac{R}{R_N}} = \frac{A_\rho}{\sqrt{A}}$$

$$\sigma_K \simeq (K_N - 1) A_\rho \sqrt{\frac{1}{A_{R2}} + \frac{1}{A_{R1}}}$$

$$\sigma_K \simeq (K_N - 1) A_\rho \sqrt{\frac{1}{(K_N - 1) A_{R1}} + \frac{1}{A_{R1}}}$$

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the standard deviation of the voltage gain K

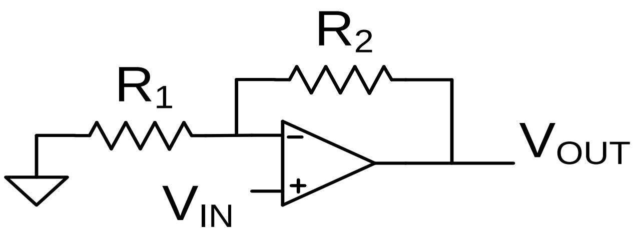
$$\sigma_K \simeq (K_N - 1) A_\rho \sqrt{\frac{1}{(K_N - 1) A_{R1}} + \frac{1}{A_{R1}}}$$

$$\sigma_K \simeq (K_N - 1) \frac{A_\rho}{\sqrt{A_{R1}}} \sqrt{\frac{1}{(K_N - 1)} + 1} = \frac{A_\rho}{\sqrt{A_{R1}}} (K_N - 1) \sqrt{\frac{K_N}{(K_N - 1)}}$$

$$\sigma_K \simeq \frac{A_\rho}{\sqrt{A_{R1}}} \sqrt{K_N (K_N - 1)}$$

$$\sigma_{\frac{K}{K_N}} \simeq \frac{A_\rho}{\sqrt{A_{R1}}} \sqrt{1 - \frac{1}{K_N}}$$

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the standard deviation of the voltage gain K

$$\sigma_K \simeq \frac{A_\rho}{\sqrt{A_{R1}}}\sqrt{K_N(K_N-1)}$$

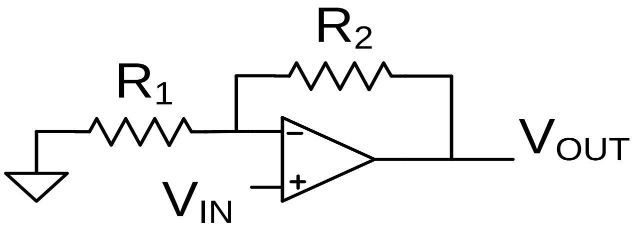
A_ρ=.01u A_{R1}=100u^2 $\sigma_{\frac{R_{PROC}}{R_{NOM}}} = 0.2$

$$\sigma_K \simeq \frac{.01}{10}\sqrt{K_N(K_N-1)} = .001\sqrt{K_N(K_N-1)}$$

$$\sigma_{\frac{K}{K_N}} \simeq .001\sqrt{1-\frac{1}{K_N}}$$

- The standard deviation can be improved by increasing area but a 4X increase in area is needed for a 2X reduction in sigma
- Note the standard deviation of the normalized gain is much smaller than the standard deviation of the process variations

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the standard deviation of the voltage gain K

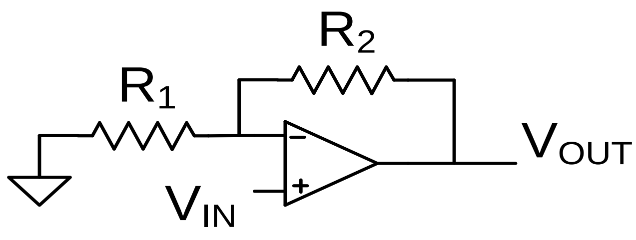
$$\sigma_{\frac{K}{K_N}} \simeq .001\sqrt{1 - \frac{1}{K_N}}$$

Determine the yield if the nominal gain is 10 $\pm 1\%$

$$\sigma_{\frac{K}{K_N}} \simeq .001\sqrt{1 - \frac{1}{10}} = .00095$$

$$\frac{K}{K_N} \sim N(1,\ 0.00095)$$

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the yield if the nominal gain is 10 $\pm 1\%$

$$\frac{K}{K_N} \sim N(1,\ 0.00095)$$

$$\frac{\frac{K}{K_N} - 1}{0.00095} \sim N(0,1)$$

$$9.9 < K < 10.1$$

$$.99 < \frac{K}{K_N} < 1.01$$

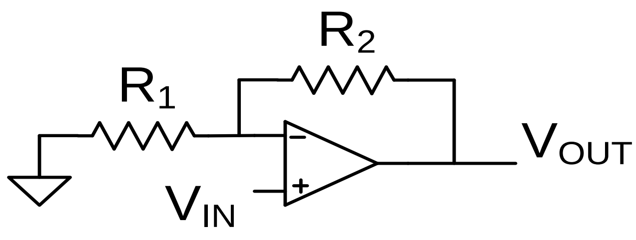
$$-10 < \frac{\frac{K}{K_N} - 1}{.00095} < 10$$

$$-.01 < \frac{K}{K_N} - 1 < .01$$

The gain yield is essentially 100%

Could substantially decrease area or increase gain accuracy if desired

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

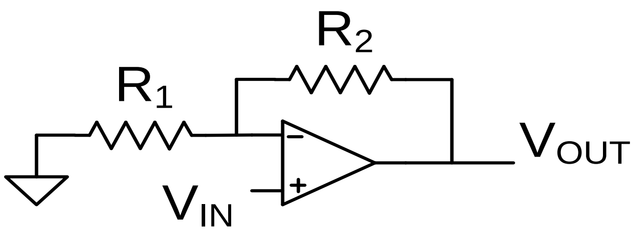
Determine the yield if the nominal gain is 10 $\pm 1\%$

Assume a common centroid layout of R_1 and R_2 has been used and the area of R_1 is $10u^2$ and both resistors have the same resistance density and R_2 is comprised of K-1 copies of R_1 . Neglect variable edge effects in the layout

A_ρ=.025µm

$$\sigma_{\frac{R_{PROC}}{R_{NOM}}} = 0.2$$

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the standard deviation of the voltage gain K

$$\sigma_K \simeq \frac{A_\rho}{\sqrt{A_{R1}}}\sqrt{K_N\left(K_N - 1\right)}$$

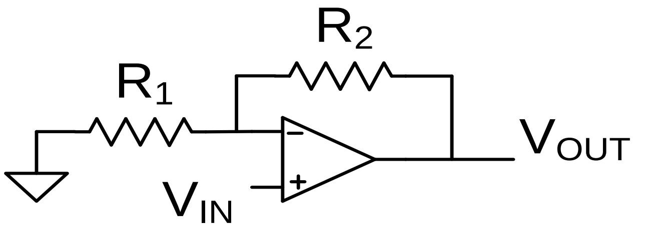
A_ρ=.025u A_{R1}=10u^2

$$\sigma_{\frac{R_{PROC}}{R_{NOM}}} = 0.2$$

$$\sigma_K \simeq \frac{.025}{\sqrt{10}}\sqrt{K_N\left(K_N - 1\right)} = .0079\sqrt{K_N\left(K_N - 1\right)}$$

$$\sigma_{\frac{K}{K_N}} \simeq .0079\sqrt{1 - \frac{1}{K_N}}$$

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the standard deviation of the voltage gain K

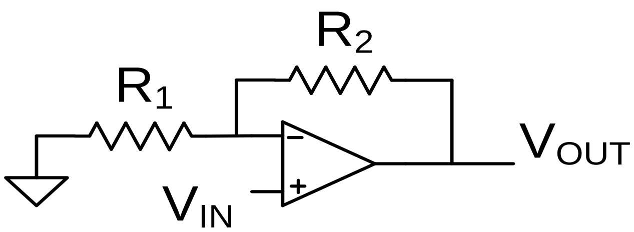
$$\sigma_{\frac{K}{K_N}} \simeq .0079\sqrt{1 - \frac{1}{K_N}}$$

Determine the yield if the nominal gain is 10 $\pm 1\%$

$$\sigma_{\frac{K}{K_N}} \simeq .0079\sqrt{1 - \frac{1}{10}} = .0075$$

$$\frac{K}{K_N} \sim N(1,\ 0.0075)$$

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the yield if the nominal gain is 10 $\pm 1\%$

$$\frac{K}{K_N} \sim N(1,\ 0.0075)$$

$$\frac{\frac{K}{K_N} - 1}{0.0075} \sim N(0,1)$$

$$9.9 < K < 10.1$$

$$-1.33 < \frac{\frac{K}{K_N} - 1}{.00095} < 1.33$$

$$.99 < \frac{K}{K_N} < 1.01$$

$$Y = 2F_{N(0,1)}(1.33) - 1 = 2^{*}.9082 - 1 = 0.8164$$

$$-.01 < \frac{K}{K_N} - 1 < .01$$

Dramatic drop from 100% yield to about 82% yield!

End of Lecture 15

EE 508
Lecture 16

Filter Transformations

Lowpass to Bandpass
Lowpass to Highpass
Lowpass to Band-reject

Review from Last Time

Theorem: If the perimeter variations and contact resistance are neglected, the standard deviation of the local random variations of a resistor of area A is given by the expression

$$\sigma_{\frac{R}{R_N}} = \frac{A_\rho}{\sqrt{A}}$$

Theorem: If the perimeter variations are neglected, the standard deviation of the local random variations of a capacitor of area A is given by the expression

$$\sigma_{\frac{C}{C_N}} = \frac{A_C}{\sqrt{A}}$$

Theorem: If the perimeter variations are neglected, the variance of the local random variations of the normalized threshold voltage of a rectangular MOS transistor of dimensions W and L is given by the expression

$$\sigma^2_{\frac{V_T}{V_{T_N}}} = \frac{A^2_{VTO}}{V^2_{T_N} WL}$$

or as

$$\sigma^2_{\frac{V_T}{V_{T_N}}} = \frac{A^2_{VT}}{WL}$$

Review from Last Time

Theorem: If the perimeter variations are neglected, the variance of the local random variations of the normalized C_{OX} of a rectangular MOS transistor of dimensions W and L is given by the expression

$$\sigma^2_{\frac{C_{OX}}{C_{OXN}}} = \frac{A^2_{COX}}{WL}$$

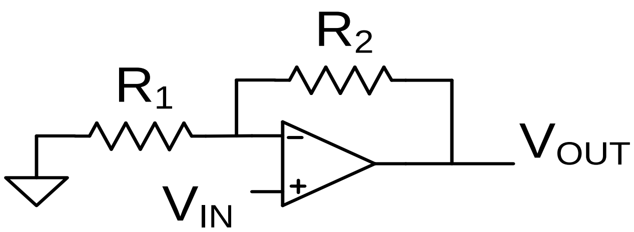
Theorem: If the perimeter variations are neglected, the variance of the local random variations of the normalized mobility of a rectangular MOS transistor of dimensions W and L is given by the expression

$$\sigma^2_{\frac{\mu_R}{\mu_N}} = \frac{A^2_{\mu}}{WL}$$

where the parameters A_X are all constants characteristic of the process (i.e. model parameters)

- The effects of edge roughness on the variance of resistors, capacitors, and transistors can readily be included but for most layouts is dominated by the area dependent variations
- There is some correlation between the model parameters of MOS transistors but they are often ignored to simplify calculations

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Assume common centroid layout
area of R_1 is $100u^2$ A_ρ=.01µm

Determine the yield if the nominal gain is 10 $\pm 1\%$

$$\frac{K}{K_N} \sim N(1,\ 0.00095)$$

$$\frac{\frac{K}{K_N} - 1}{0.00095} \sim N(0,1)$$

$$9.9 < K < 10.1$$

$$.99 < \frac{K}{K_N} < 1.01$$

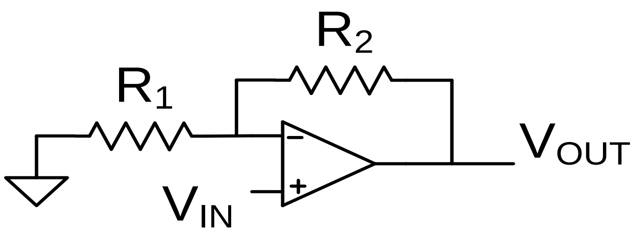
$$-.01 < \frac{K}{K_N} - 1 < .01$$

$$-10 < \frac{\frac{K}{K_N} - 1}{.00095} < 10$$

The gain yield is essentially 100%

Could substantially decrease area or increase gain accuracy if desired

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the yield if the nominal gain is 10 $\pm 1\%$

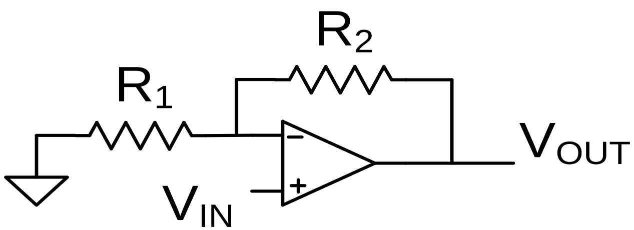
Assume a common centroid layout of R_1 and R_2 has been used and the area of R_1 is $10u^2$ and both resistors have the same resistance density and R_2 is comprised of K-1 copies of R_1 . Neglect variable edge effects in the layout

$A_\rho = .025\mu m$

$$\sigma_{\frac{R_{PROC}}{R_{NOM}}} = 0.2$$

Review from Last Time

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the standard deviation of the voltage gain K

$$\sigma_K \simeq \frac{A_\rho}{\sqrt{A_{R1}}}\sqrt{K_N(K_N-1)}$$

A_ρ=.025um A_{R1}=10um^2

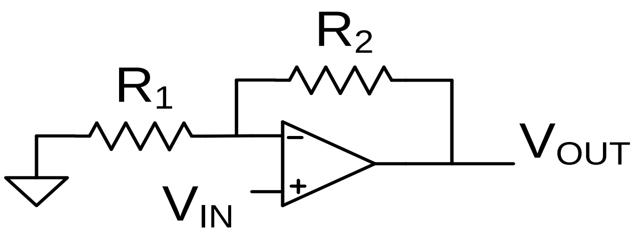
$$\sigma_{\frac{R_{PROC}}{R_{NOM}}} = 0.2$$

$$\sigma_K \simeq \frac{.025}{\sqrt{10}}\sqrt{K_N(K_N-1)} = .0079\sqrt{K_N(K_N-1)}$$

$$\sigma_{\frac{K}{K_N}} \simeq .0079\sqrt{1-\frac{1}{K_N}}$$

Review from Last Time

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the standard deviation of the voltage gain K

$$\sigma_{\frac{K}{K_N}} \simeq .0079\sqrt{1-\frac{1}{K_N}}$$

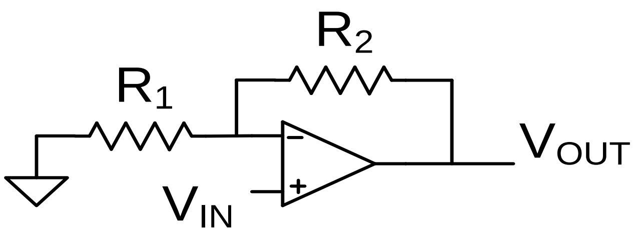
Determine the yield if the nominal gain is 10$\pm$1%

$$\sigma_{\frac{K}{K_N}} \simeq .0079\sqrt{1-\frac{1}{10}} = .0075$$

$$\frac{K}{K_N} \sim N(1,\ 0.0075)$$

Review from Last Time

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the yield if the nominal gain is 10 $\pm 1\%$

$$\frac{K}{K_N} \sim N(1,\ 0.0075)$$

$$9.9 < K < 10.1$$

$$.99 < \frac{K}{K_N} < 1.01$$

$$-.01 < \frac{K}{K_N} - 1 < .01$$

$$\frac{\frac{K}{K_N} - 1}{0.0075} \sim N(0,1)$$

$$-1.33 < \frac{\frac{K}{K_N} - 1}{.0075} < 1.33$$

$$Y = 2F_{N(0,1)}(1.33) - 1 = 2*.9082 - 1 = 0.8164$$

Dramatic drop from 100% yield to about 82% yield!

Filter Transformations

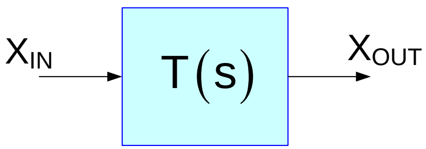
Lowpass to Bandpass **(LP to BP)**
Lowpass to Highpass **(LP to HP)**
Lowpass to Band-reject **(LP to BR)**

Approach will be to take advantage of the results obtained for the standard LP approximations

Will focus on flat passband and zero-gain stop-band transformations

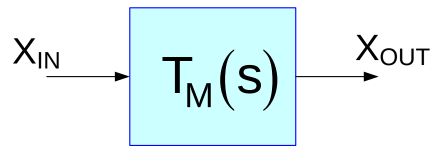
Will focus on transformations that map passband to passband and stopband to stopband

Filter Transformations



$$s \rightarrow f(s)$$

$$T_M(s) = T(f(s))$$

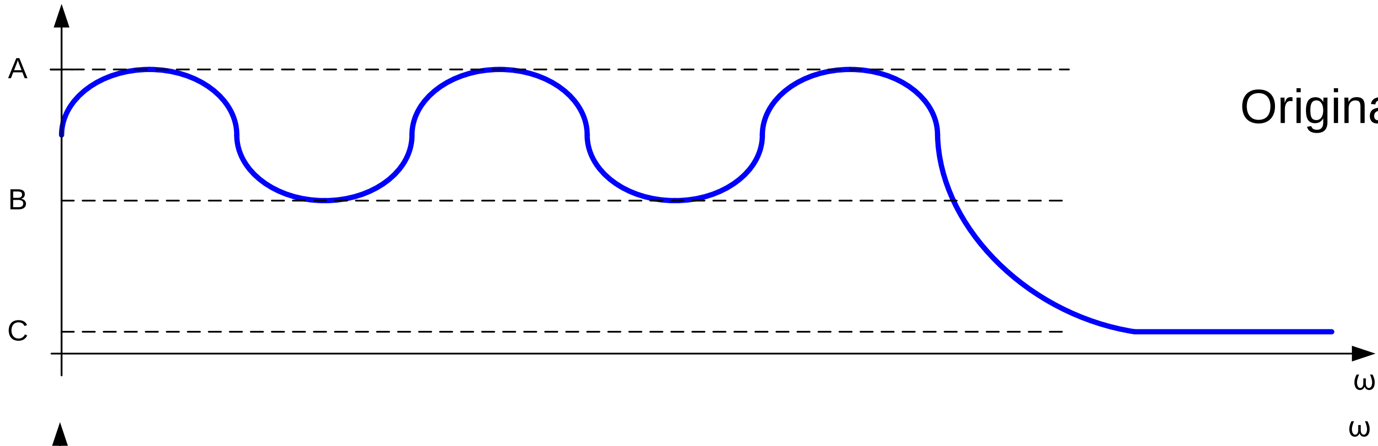


Claim:

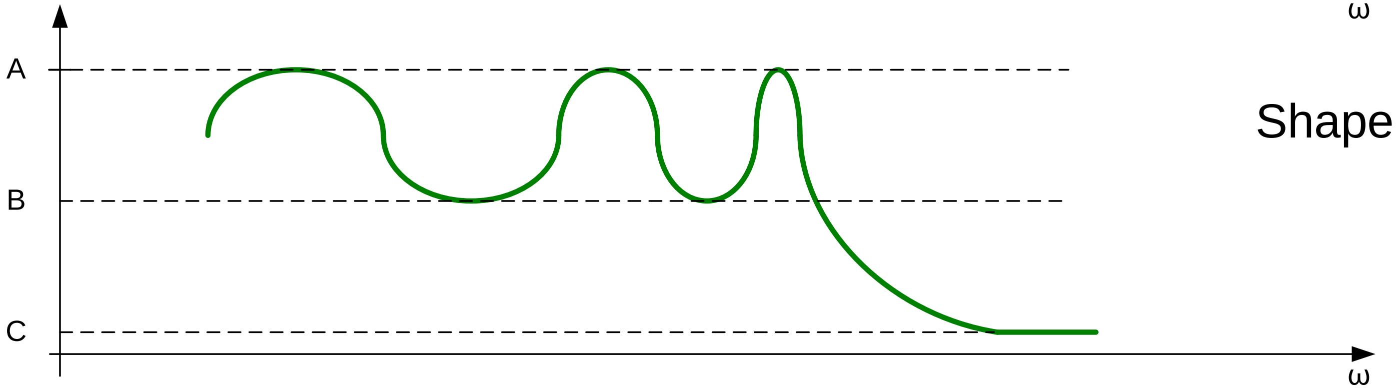
If the imaginary axis in the s-plane is mapped to the imaginary axis in the s-plane with a variable mapping function, the basic shape of the function T(s) will be preserved in the function F(T(s)) but the frequency axis may be warped and/or folded in the magnitude domain

Preserving basic shape, in this context, constitutes maintaining features in the magnitude response of F(T(s)) that are in T(s) including, but not limited to, the peak amplitude, number of ripples, peaks of ripples,

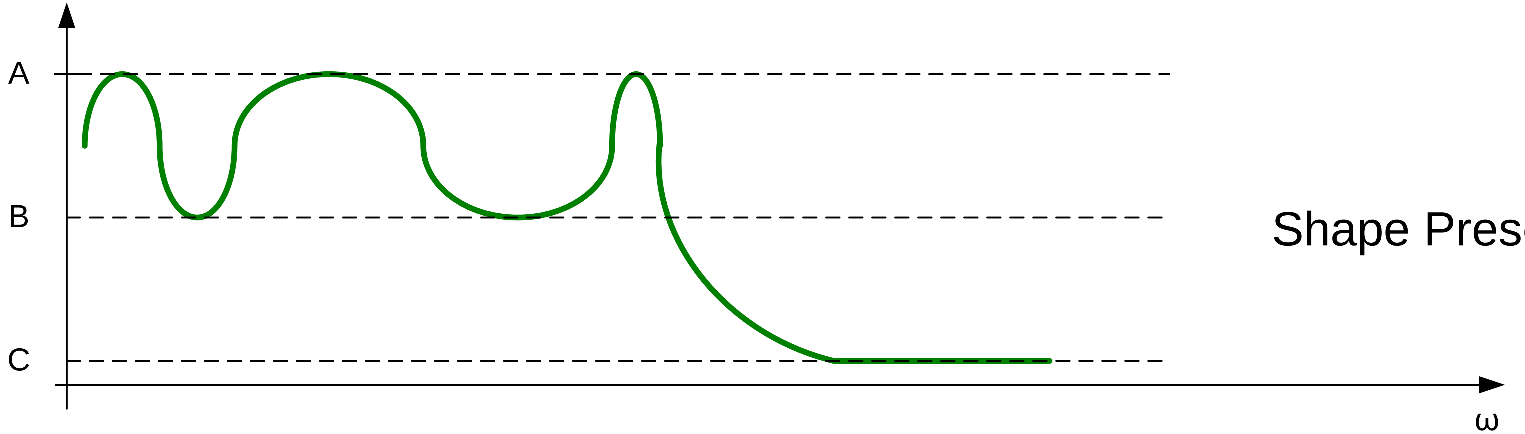
Example: Shape Preservation



Original Function

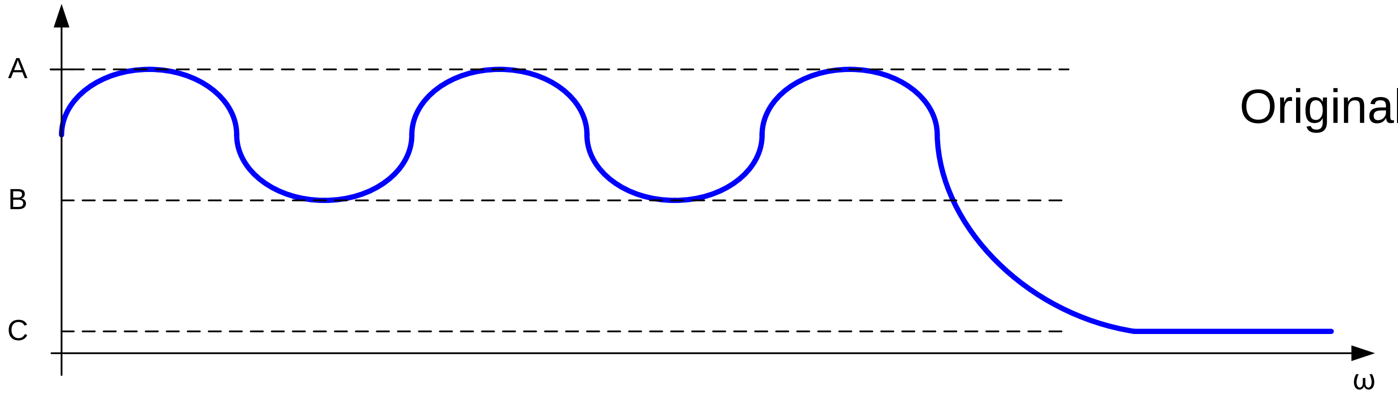


Shape Preserved

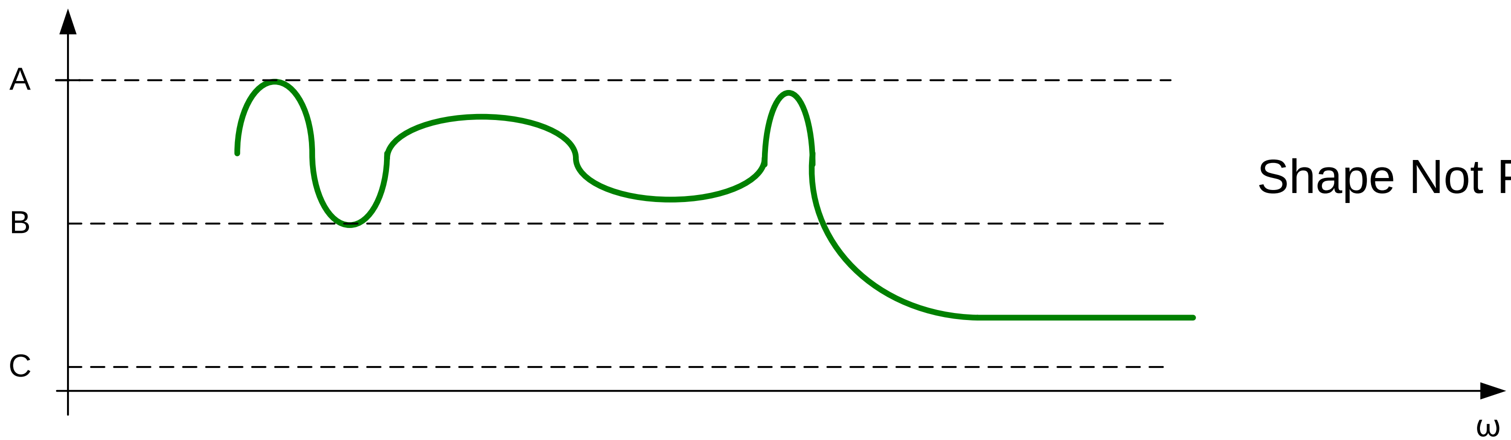


Shape Preserved

Example: Shape Preservation

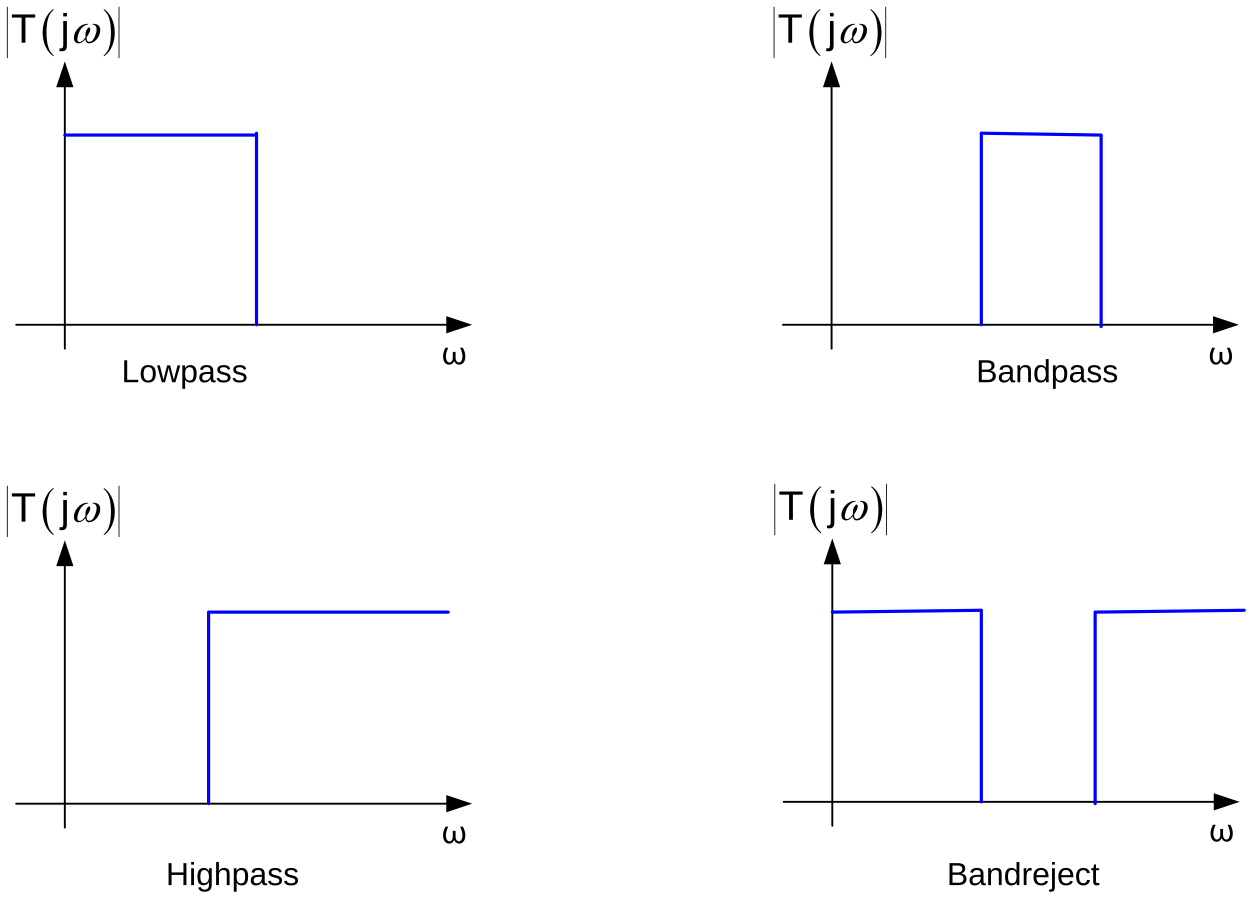


Original Function



Shape Not Preserved

Flat Passband/Stopband Filters



Filter Transformations

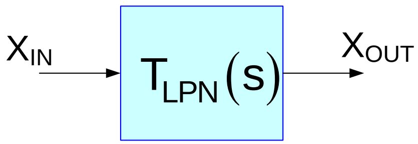
→ Lowpass to Bandpass **(LP to BP)**
Lowpass to Highpass **(LP to HP)**
Lowpass to Band-reject **(LP to BR)**

- Approach will be to take advantage of the results obtained for the standard LP approximations

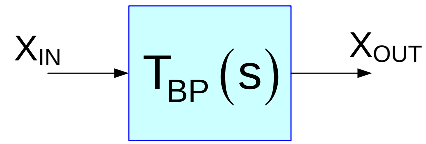
- Will focus on flat passband and zero-gain stop-band transformations

- Will focus on transformations that map passband to passband and stopband to stopband

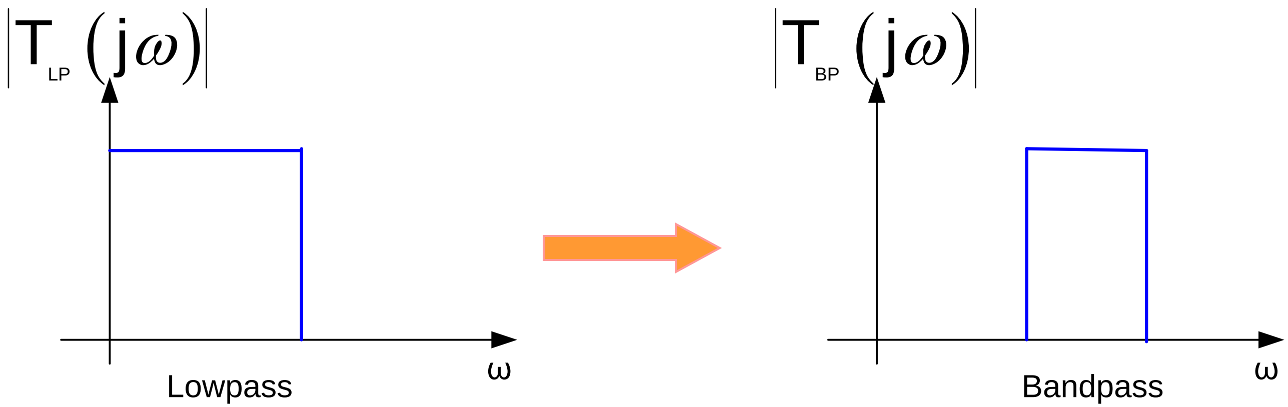
LP to BP Filter Transformations



$$s \rightarrow f(s)$$



$$T_{BP}(s) = T_{LPN}(f(s))$$



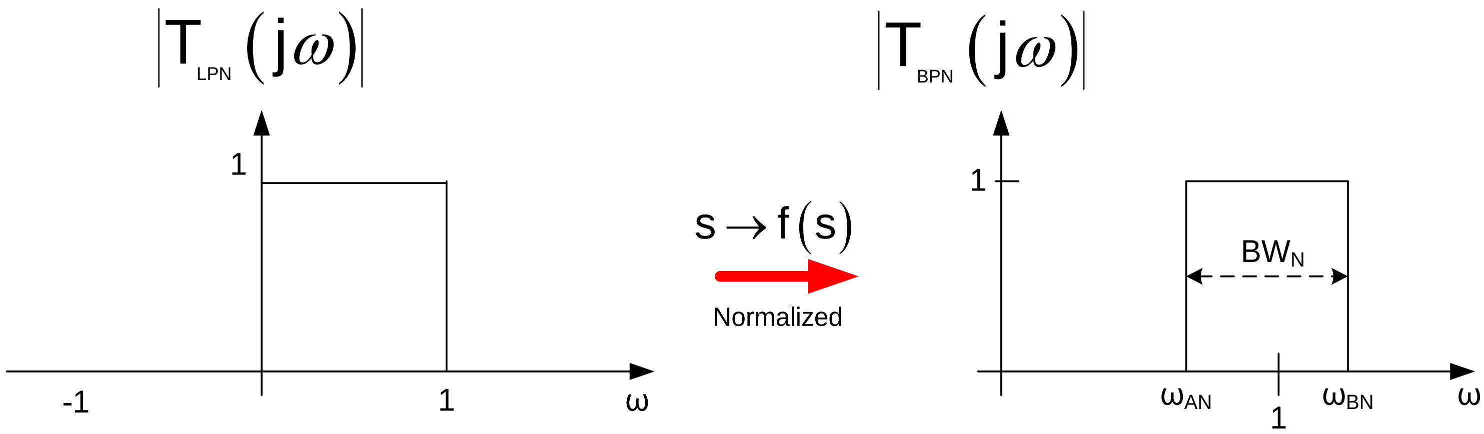
Will consider rational fraction mappings

$$f(s) = \frac{\sum_{i=0}^{m_T} a_{Ti} s^i}{\sum_{i=0}^{n_T} b_{Ti} s^i}$$

- Not all rational fraction mappings will map Im axis to the Im axis
- Not all rational fraction mappings will map passband to passband and stopband to stopband
- Consider only that subset of those mappings with these properties

LP to BP Transformation

Mapping Strategy: Consider first a mapping to a normalized BP approximation



$$BW_N = \omega_{BN} - \omega_{AN}$$

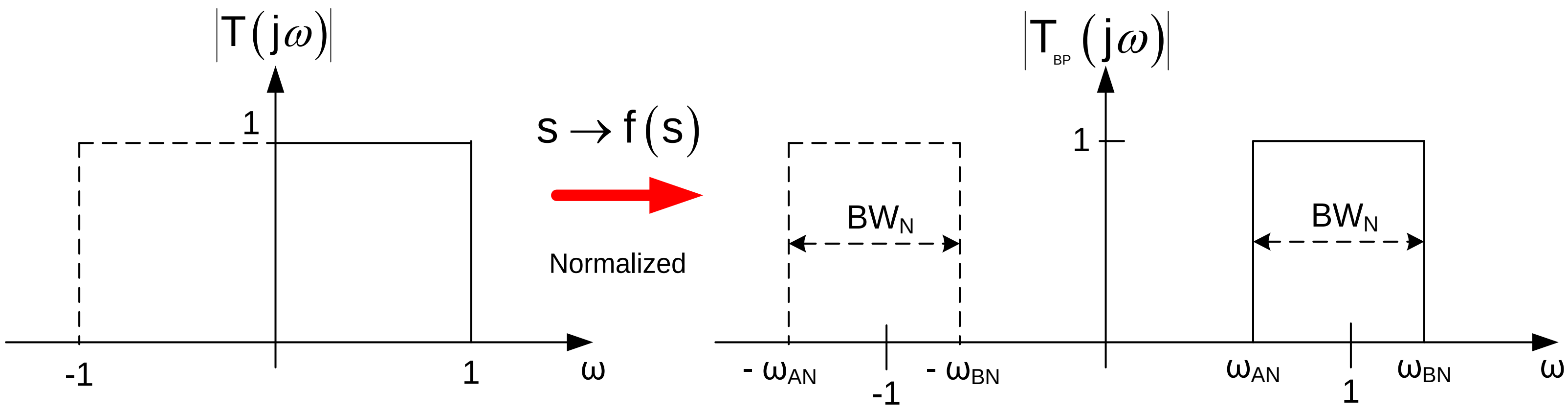
$$\sqrt{\omega_{AN}\omega_{BN}} = 1$$

LP to BP Transformation

Mapping Strategy: Consider first a mapping to a normalized BP approximation

A mapping from s → f(s) will map the entire imaginary axis in the frequency domain

Thus, must consider both positive and negative frequencies. Since $|T(j\omega)|$ is a function of ω^2, the magnitude response on the negative ω axis will be a mirror image of that on the positive ω axis

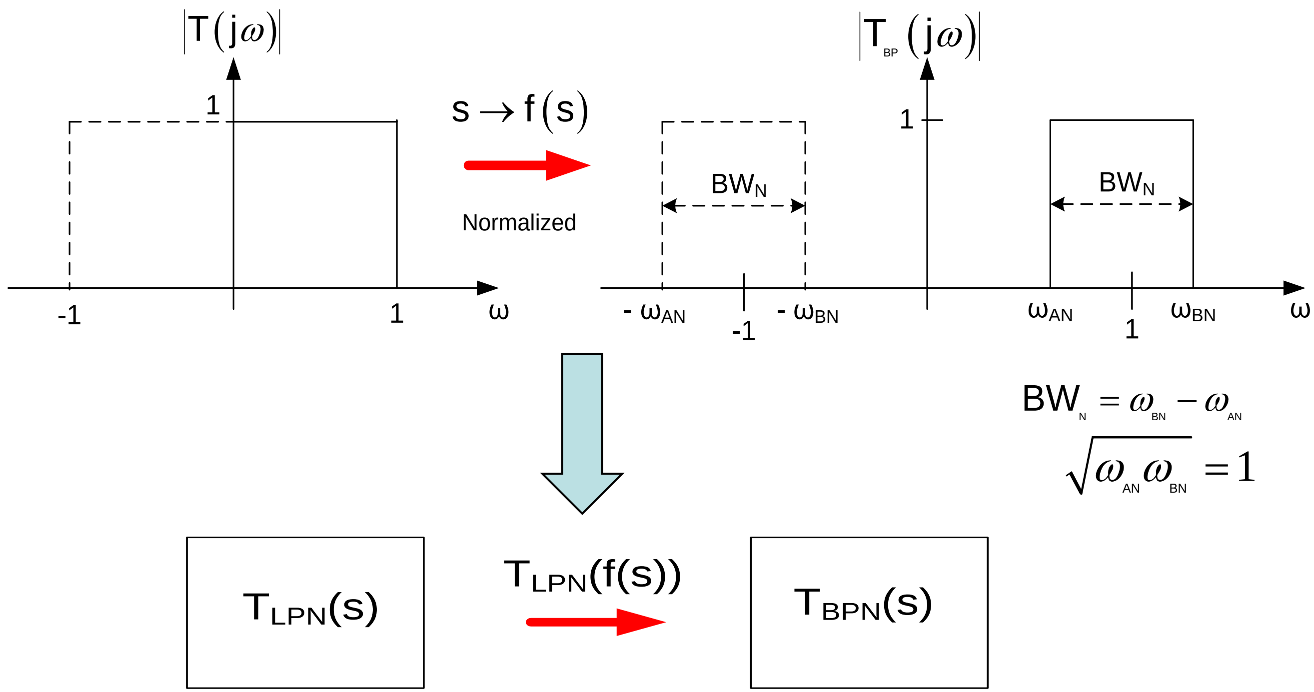


$$BW_N = \omega_{BN} - \omega_{AN}$$

$$\sqrt{\omega_{AN}\omega_{BN}} = 1$$

Standard LP to BP Transformation

Normalized LP to Normalized BP mapping Strategy:



$$BW_N = \omega_{BN} - \omega_{AN}$$

$$\sqrt{\omega_{AN}\omega_{BN}} = 1$$

Variable Mapping Strategy to Preserve Shape of LP function:

consider:

s-domain

map s=j0 to s= j1
map s=j1 to s=jω_{BN}
map s= –j1 to s= jω_{AN}

$T_{LPN}(f(s))$ ⟺

ω-domain

map ω=0 to ω=1
map ω=1 to ω=ω_{BN}
map ω= –1 to ω=ω_{AN}

This mapping will introduce 3 constraints

Standard LP to BP Transformation

Mapping Strategy: $T_{LPN}(s)$ $\xrightarrow{T_{LPN}(f(s))}$ $T_{BPN}(s)$

s-domain		ω-domain
map s=0 to s= j1	$T_{LPN}(f(s))$	map ω=0 to ω=1
map s=j1 to s=$j\omega_{BN}$	⟷	map ω=1 to ω=ω_{BN}
map s= –j1 to s= $j\omega_{AN}$		map ω= –1 to ω=ω_{AN}

Consider variable mapping

$$f(s) = \frac{a_{T2}s^2 + a_{T1}s + a_{T0}}{b_{T1}s + b_{T0}}$$

With this mapping, there are 5 D.O.F and 3 mathematical constraints and the additional constraints that the Im axis maps to the Im axis and maps PB to PB and SB to SB

Will now show that the following mapping will meet these constraints

$$f(s) = \frac{s^2+1}{s \bullet BW_N} \quad \text{or equivalently} \quad s \rightarrow \frac{s^2+1}{s \bullet BW_N}$$

This is the lowest-order mapping that will meet these constraints and it doubles the order of the approximation

Standard LP to BP Transformation

s-domain

map s=0 to s= j1
map s=j1 to s=$j\omega_{BN}$
map s= –j1 to s= $j\omega_{AN}$

$T_{LPN}(f(s))$

↔

ω-domain

map ω=0 to ω=1
map ω=1 to $\omega=\omega_{BN}$
map ω= –1 to $\omega=\omega_{AN}$

Verification of mapping Strategy:

$$s \rightarrow \frac{s^2+1}{s \bullet BW_N}$$

$$\left.\frac{s^2+1}{s \bullet BW_N}\right|_{j1} = 0 \qquad \Rightarrow \qquad 0 \rightarrow j1$$

$$\left.\frac{s^2+1}{s \bullet BW_N}\right|_{j\omega_{BN}} = \frac{1-\omega_{BN}^2}{j\omega_{BN}\left(\omega_{BN}-\omega_{AN}\right)} = j\frac{\omega_{BN}^2-1}{\omega_{BN}^2-\omega_{AN}\omega_{BN}} = j\frac{\omega_{BN}^2-1}{\omega_{BN}^2-1} = j \qquad \Rightarrow \qquad j1 \rightarrow j\omega_{BN}$$

$$\left.\frac{s^2+1}{s \bullet BW_N}\right|_{j\omega_{AN}} = \frac{1-\omega_{AN}^2}{j\omega_{AN}\left(\omega_{BN}-\omega_{AN}\right)} = j\frac{\omega_{AN}^2-1}{\omega_{AN}\omega_{BN}-\omega_{AN}^2} = j\frac{\omega_{AN}^2-1}{1-\omega_{AN}^2} = -j \qquad \Rightarrow \qquad -j1 \rightarrow j\omega_{AN}$$

Must still show that the Im axis maps to the Im axis and maps PB to PB and SB to SB

Standard LP to BP Transformation

s-domain

map s=0 to s= j1
map s=j1 to s=$j\omega_{BN}$
map s= –j1 to s= $j\omega_{AN}$

$T_{LPN}(f(s))$

ω-domain

map ω=0 to ω=1
map ω=1 to ω=ω_{BN}
map ω= –1 to ω=ω_{AN}

Verification of mapping Strategy:

$$s \rightarrow \frac{s^2+1}{s \bullet BW_N}$$

Image of Im axis:

$$j\omega = \frac{s^2+1}{s \bullet BW_N}$$

solving for s, obtain

$$s = \frac{j\omega \bullet BW_N \pm \sqrt{(BW_N \bullet j\omega)^2 - 4}}{2} = j\left(\frac{\omega \bullet BW_N \pm \sqrt{(BW_N \bullet \omega)^2 + 4}}{2}\right)$$

this has no real part so the imaginary axis maps to the imaginary axis

Can readily show this mapping maps PB to PB and SB to SB

The mapping $s \rightarrow \frac{s^2+1}{s \bullet BW_N}$ is termed the standard LP to BP transformation

Standard LP to BP Transformation

The standard LP to BP transformation $$s \rightarrow \frac{s^2+1}{s \bullet BW_N}$$

If we add a subscript to the LP variable for notational convenience, can express this mapping as

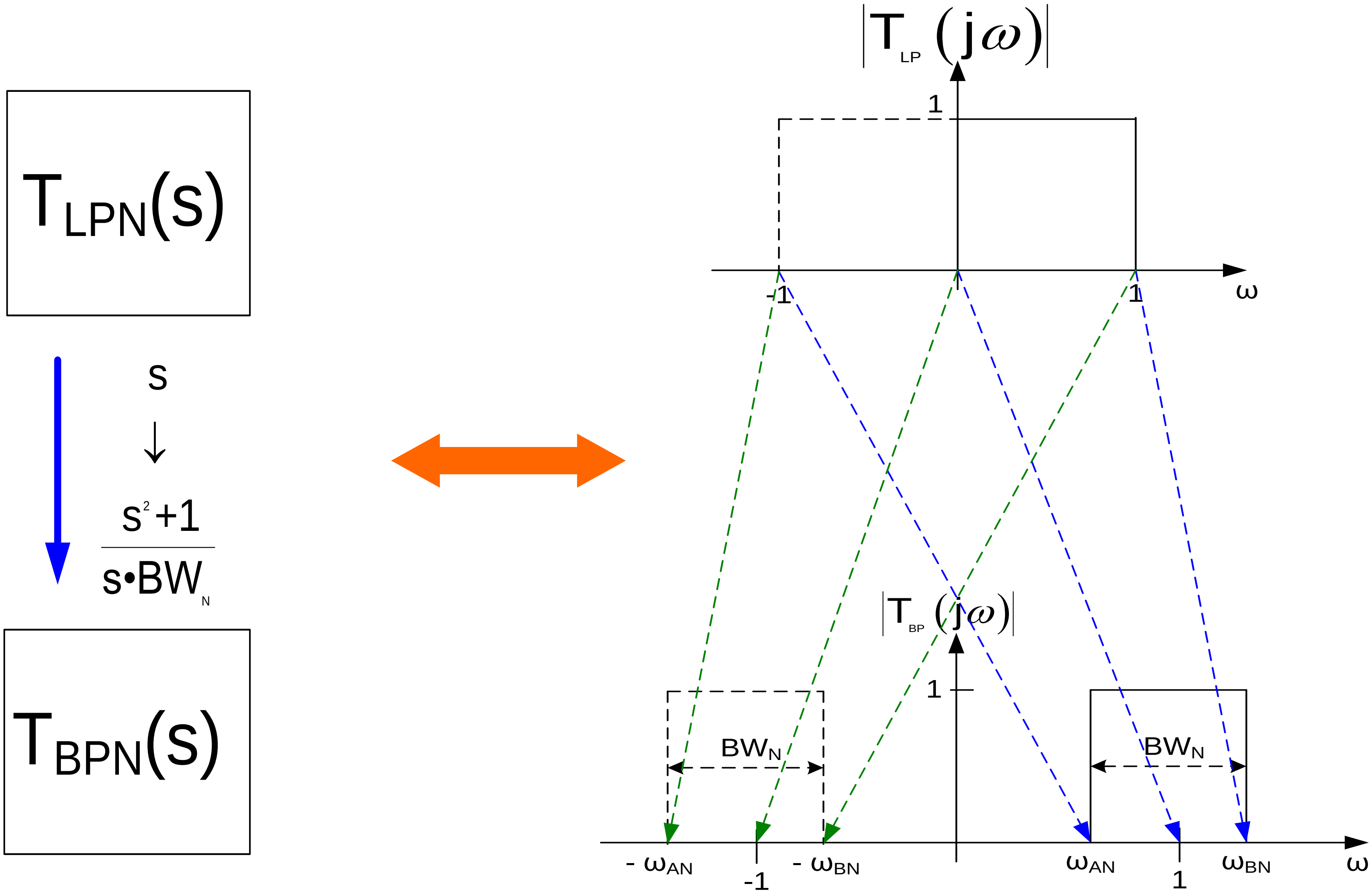
$$s_x = \frac{s^2+1}{s \bullet BW_N}$$

Question: Is this mapping dimensionally consistent ?

- The dimensions of the constant “1” in the numerator must be set so that this is dimensionally consistent

- The dimensions of BW_N must be set so that this is dimensionally consistent

Standard LP to BP Transformation



Standard LP to BP Transformation

Frequency and s-domain Mappings

(subscript variable in LP approximation for notational convenience)

$$T_{LPN}(s_X)$$

$$s_X \downarrow \frac{s^2+1}{s \bullet BW_N}$$

$$T_{BPN}(s)$$

$$s_X \rightarrow \frac{s^2+1}{s \bullet BW_N}$$

$$\omega_X \rightarrow \frac{\omega^2-1}{\omega \bullet BW_N}$$

solving for s or ω

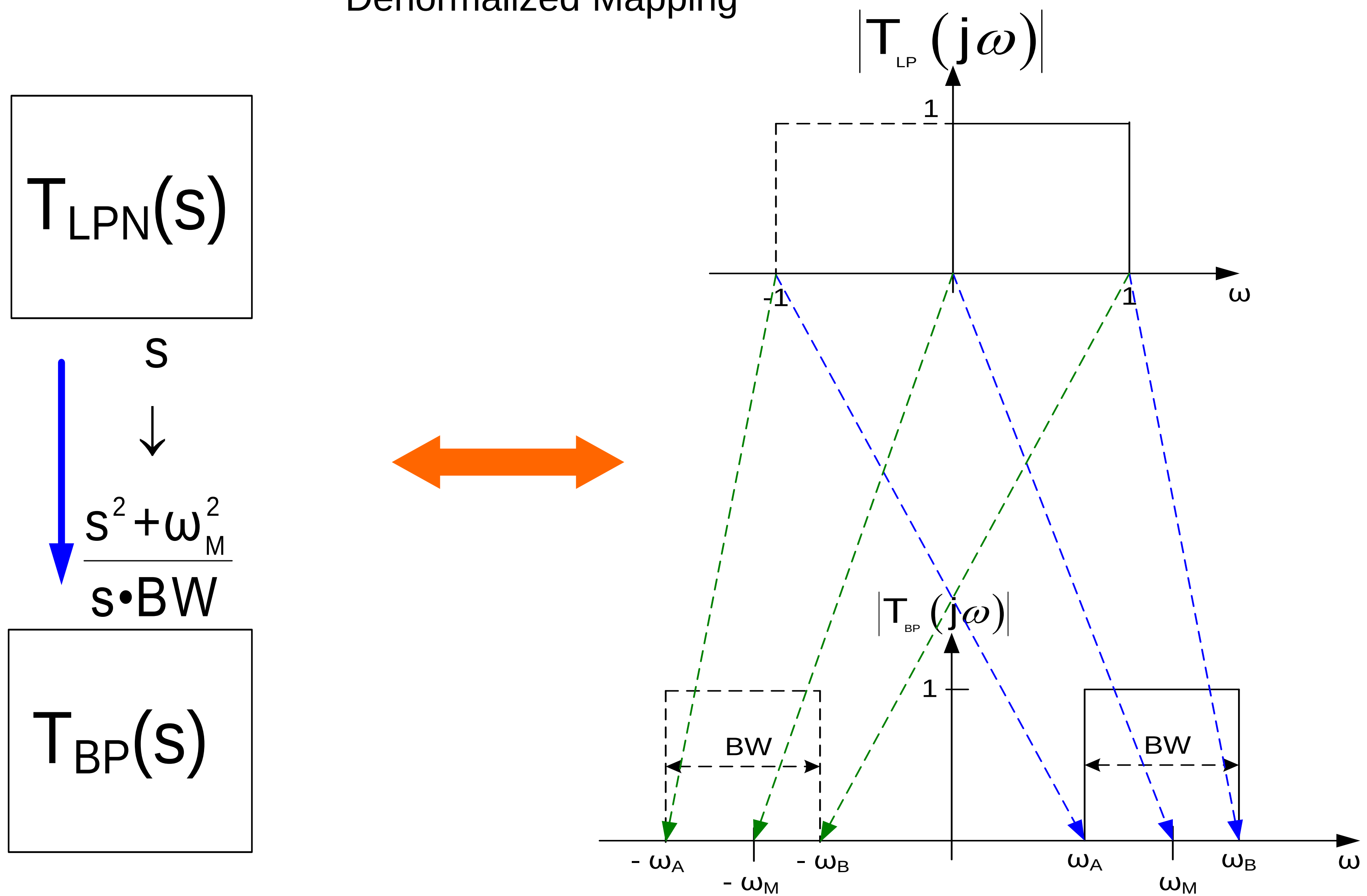
$$s \leftarrow \frac{s_X \bullet BW_N \pm \sqrt{(BW_N \bullet s_X)^2 - 4}}{2}$$

$$\omega \leftarrow \frac{\omega_X \bullet BW_N \pm \sqrt{(BW_N \bullet \omega_X)^2 + 4}}{2}$$

Exercise: Resolve the dimensional consistency in the last equation

Standard LP to BP Transformation

Denormalized Mapping



Standard LP to BP Transformation

Frequency and s-domain Mappings - Denormalized

(subscript variable in LP approximation for notational convenience)

$T_{LPN}(s_X)$

$$s_X \downarrow \frac{s^2+\omega_M^2}{s\bullet BW}$$

$T_{BP}(s)$

$$s_X \rightarrow \frac{s^2+\omega_M^2}{s\bullet BW}$$

$$\omega_X \rightarrow \frac{\omega^2-\omega_M^2}{\omega\bullet BW}$$

$$s \leftarrow \frac{s_X\bullet BW \pm \sqrt{\left(BW\bullet s_X\right)^2 - 4\omega_M^2}}{2}$$

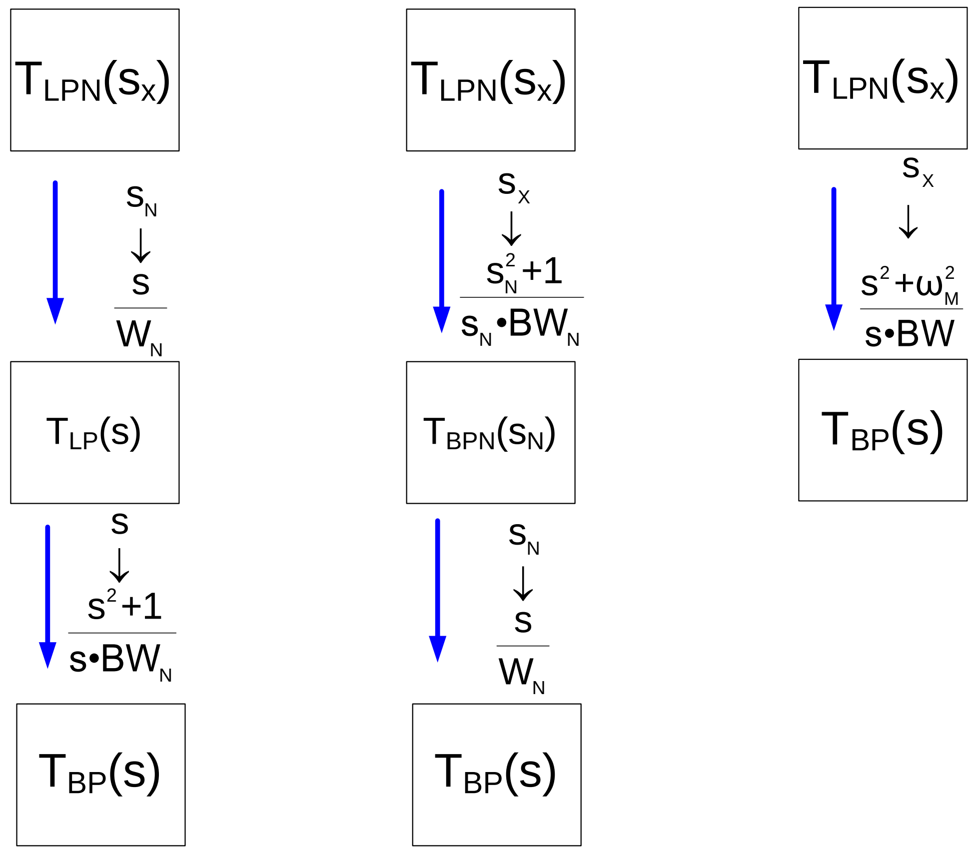
$$\omega \leftarrow \frac{\omega_X\bullet BW \pm \sqrt{\left(BW\bullet \omega_X\right)^2 + 4\omega_M^2}}{2}$$

Exercise: Resolve the dimensional consistency in the last equation

Standard LP to BP Transformation

Frequency and s-domain Mappings - Denormalized

(subscript variable in LP approximation for notational convenience)



All three approaches give same approximation

Which is most practical to use? Often none of them !

Standard LP to BP Transformation

Frequency and s-domain Mappings - Denormalized

(subscript variable in LP approximation for notational convenience)

$$T_{LPN}(s_X)$$

$$s_X \rightarrow \frac{s_N^2+1}{s_N \bullet BW_N}$$

$$T_{BPN}(s_N)$$

Often most practical to synthesize directly from the T_{BPN} and then do the frequency scaling of components at the circuit level rather than at the approximation level

Standard LP to BP Transformation

Frequency and s-domain Mappings

(subscript variable in LP approximation for notational convenience)

Poles and Zeros of the BP approximations

$$s_X \xrightarrow{f} \frac{s^2+1}{s \bullet BW_N} \quad \xrightarrow{\text{solving for } s} \quad s \xleftarrow{f^{-1}} \frac{s_X \bullet BW_N \pm \sqrt{(BW_N \bullet s_X)^2 - 4}}{2}$$

$$T_{BP}(s) = T_{LPN}(f(s))$$

$$T_{LPN}(p_X) = 0$$

$$T_{LPN}(f(p)) = 0$$

$$T_{BP}(p) = T_{LPN}(f(p)) = 0$$

Since this relationship maps the complex plane to the complex plane, it also maps the poles and zeros of the LP approximation to the poles and zeros of the BP approximation

Standard LP to BP Transformation

Pole Mappings

Claim: With a variable mapping transform, the variable mapping naturally defines the mapping of the poles of the transformed function

$$T_{LPN}(s_X)$$

$$s_X \downarrow \frac{s^2+1}{s \bullet BW_N}$$

$$T_{BPN}(s)$$

$$p_X \rightarrow \frac{p^2+1}{p \bullet BW_N}$$

$$p \leftarrow \frac{p_X \bullet BW_N \pm \sqrt{\left(BW_N \bullet p_X\right)^2 - 4}}{2}$$

Exercise: Resolve the dimensional consistency in the last equation

Standard LP to BP Transformation

Pole Mappings

$$p \leftarrow \frac{p_X \bullet BW_N \pm \sqrt{\left(BW_N \bullet p_X\right)^2 - 4}}{2}$$

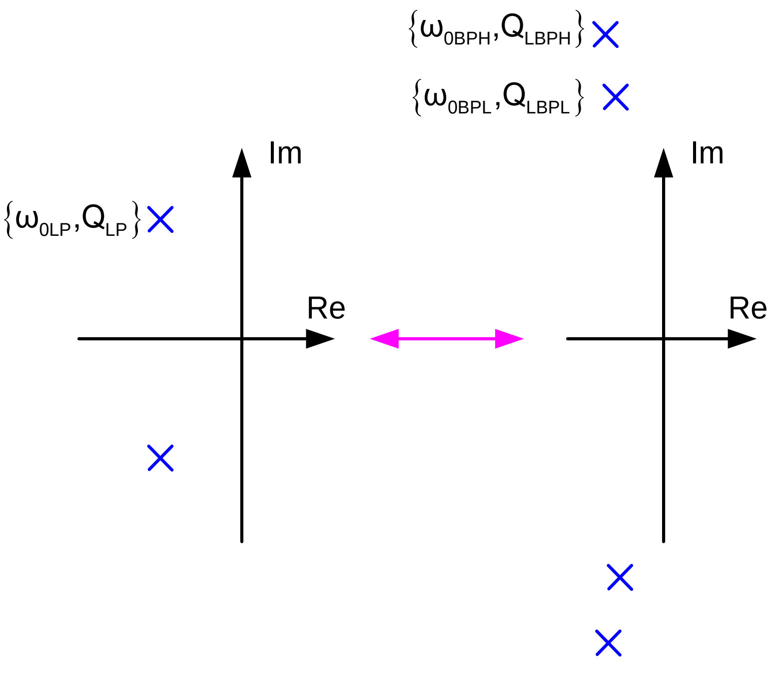
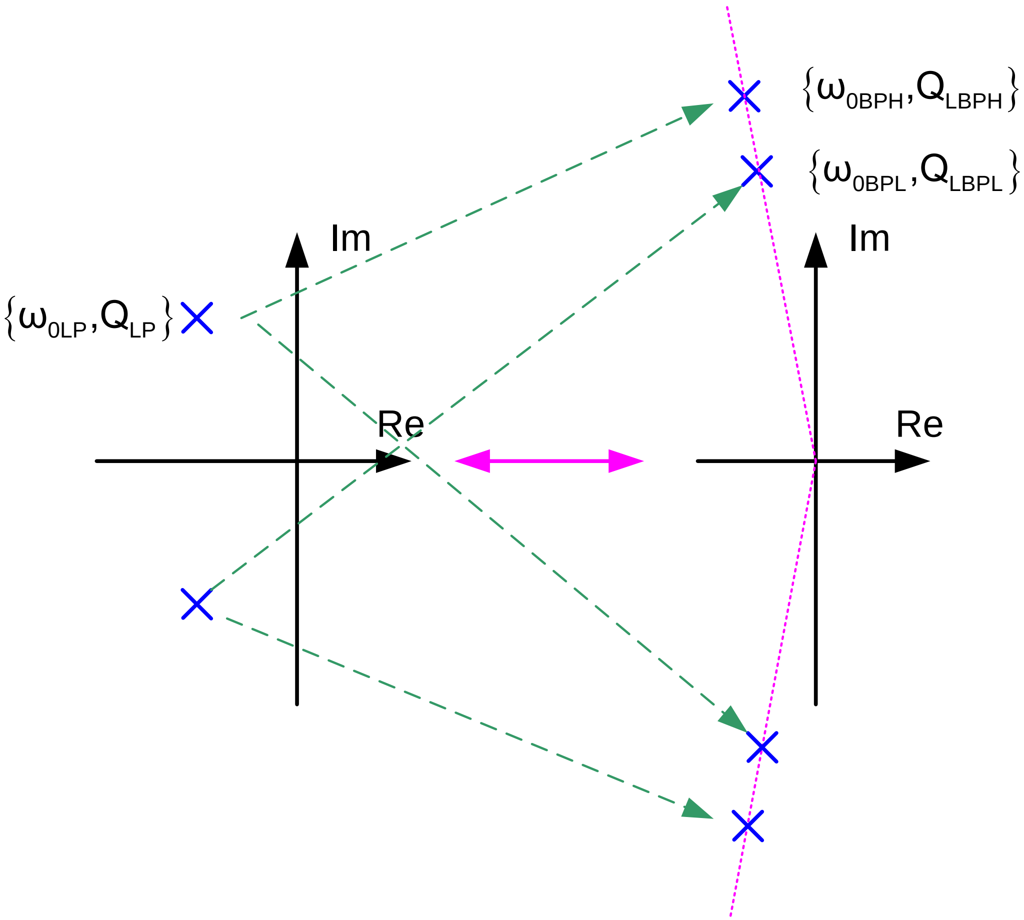


Image of the cc pole pair is the two pairs of poles

Standard LP to BP Transformation

Pole Mappings

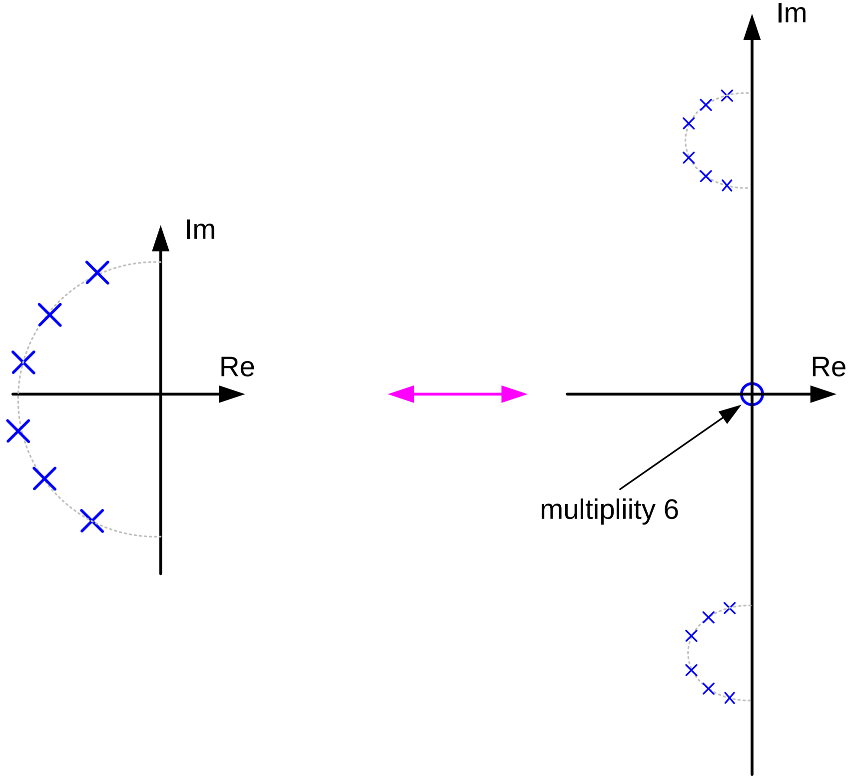


Can show that the upper hp pole maps to one upper hp pole and one lower hp pole as shown. Corresponding mapping of the lower hp pole is also shown

Standard LP to BP Transformation

Pole Mappings

$$p \leftarrow \frac{p_X \bullet BW_N \pm \sqrt{\left(BW_N \bullet p_X\right)^2 - 4}}{2}$$



Note doubling of poles, addition of zeros, and likely Q enhancement

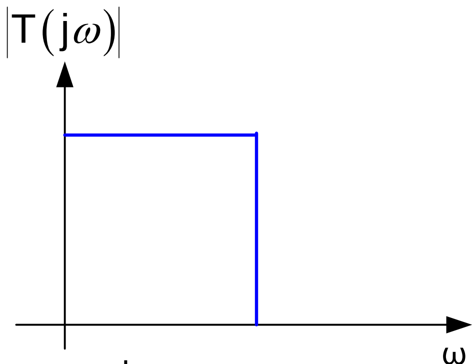
End of Lecture 16

EE 508
Lecture 17

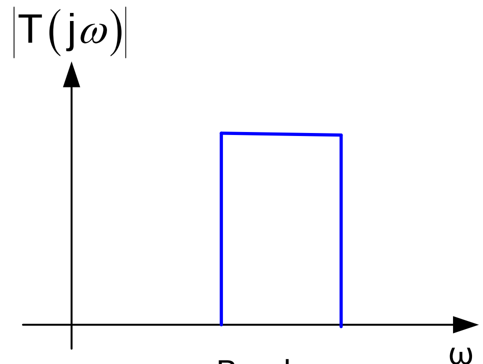
Filter Transformations

Lowpass to Bandpass
Lowpass to Highpass
Lowpass to Band-reject

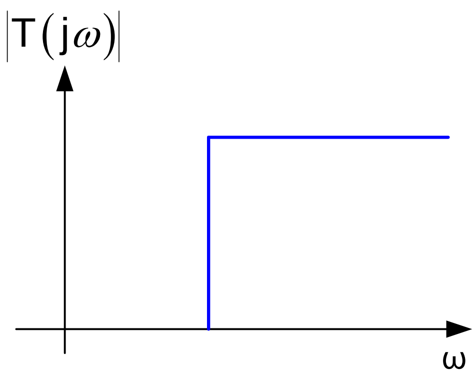
Flat Passband/Stopband Filters



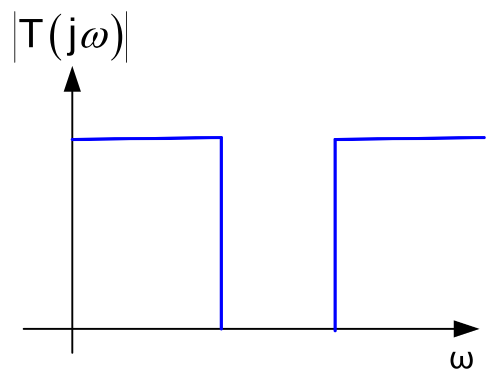
Lowpass



Bandpass



Highpass



Bandreject

Review from Last Time

Standard LP to BP Transformation

s-domain

map s=0 to s= j1
map s=j1 to s=$j\omega_{BN}$
map s= –j1 to s= $j\omega_{AN}$

$T_{LPN}(f(s))$

ω-domain

map ω=0 to ω=1
map ω=1 to $\omega=\omega_{BN}$
map ω= –1 to $\omega=\omega_{AN}$

Verification of mapping Strategy:

$$s \rightarrow \frac{s^2+1}{s \bullet BW_N}$$

Image of Im axis:

$$j\omega = \frac{s^2+1}{s \bullet BW_N}$$

solving for s, obtain

$$s = \frac{j\omega \bullet BW_N \pm \sqrt{(BW_N \bullet j\omega)^2 - 4}}{2} = j\left(\frac{\omega \bullet BW_N \pm \sqrt{(BW_N \bullet \omega)^2 + 4}}{2}\right)$$

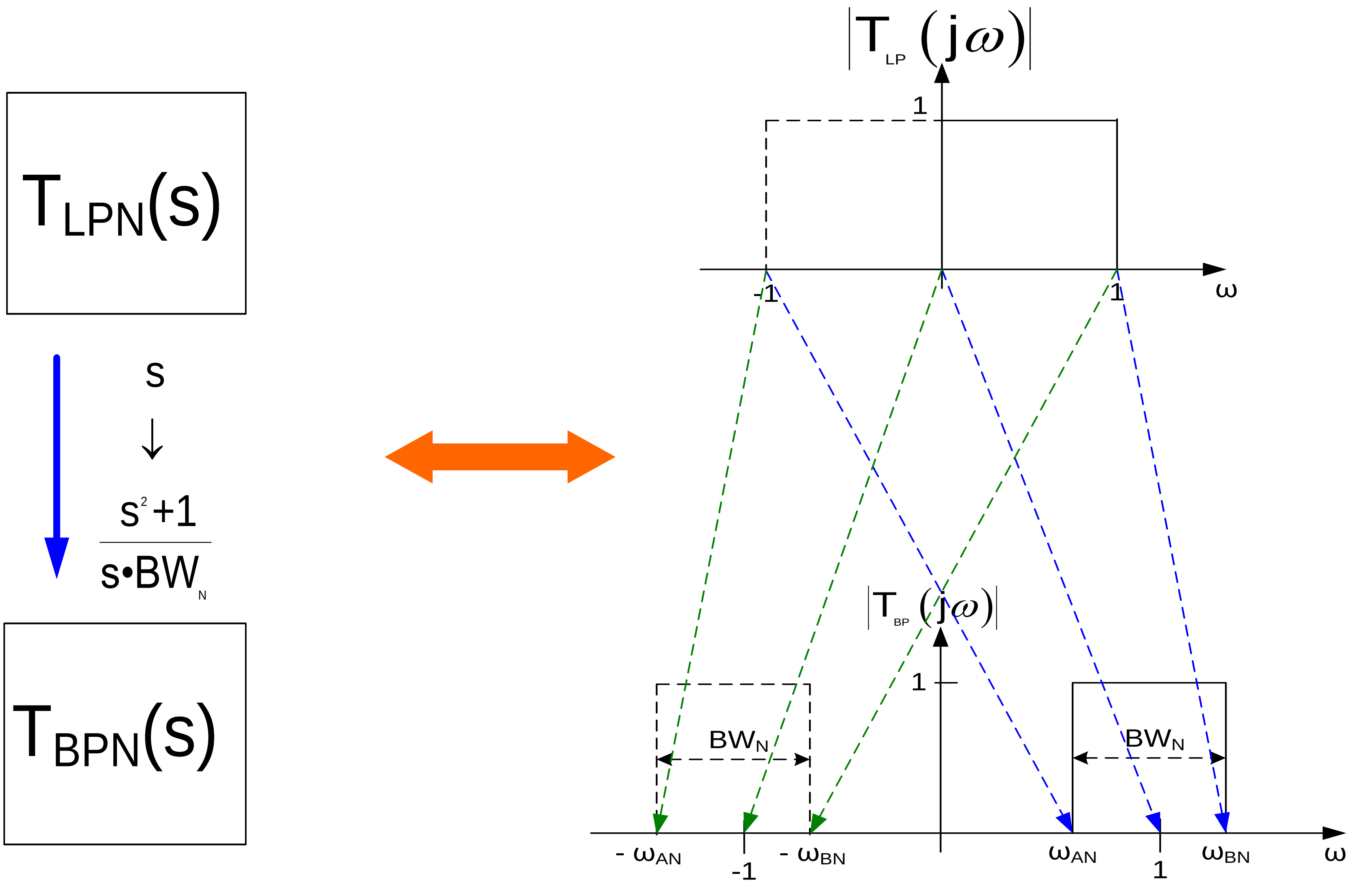
this has no real part so the imaginary axis maps to the imaginary axis

Can readily show this mapping maps PB to PB and SB to SB

The mapping $s \rightarrow \dfrac{s^2+1}{s \bullet BW_N}$ is termed the standard LP to BP transformation

Review from Last Time

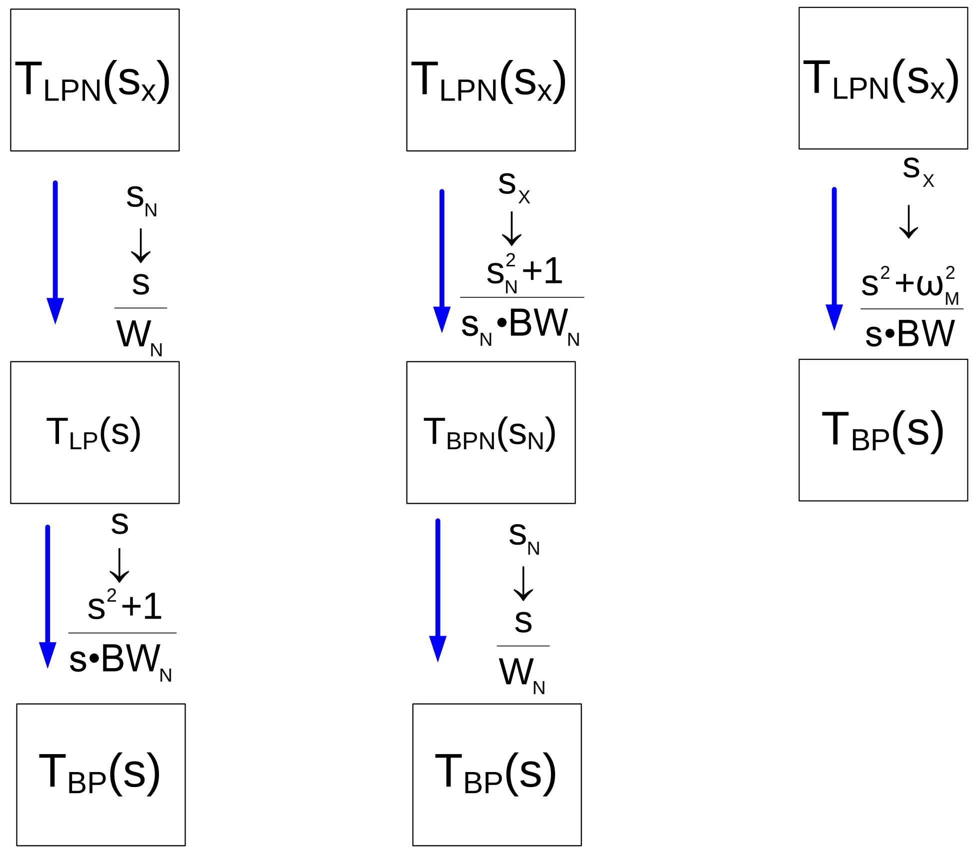
Standard LP to BP Transformation



Standard LP to BP Transformation

Frequency and s-domain Mappings - Denormalized

(subscript variable in LP approximation for notational convenience)



All three approaches give same approximation

Which is most practical to use? Often none of them !

Review from Last Time

Standard LP to BP Transformation

Frequency and s-domain Mappings - Denormalized

(subscript variable in LP approximation for notational convenience)

$$T_{LPN}(s_X)$$

$$s_X \rightarrow \frac{s_N^2+1}{s_N \bullet BW_N}$$

$$T_{BPN}(s_N)$$

Often most practical to synthesize directly from the T_{BPN} and then do the frequency scaling of components at the circuit level rather than at the approximation level

Review from Last Time

Standard LP to BP Transformation

Pole Mappings

$$p \leftarrow \frac{p_X \bullet BW_N \pm \sqrt{\left(BW_N \bullet p_X\right)^2 - 4}}{2}$$

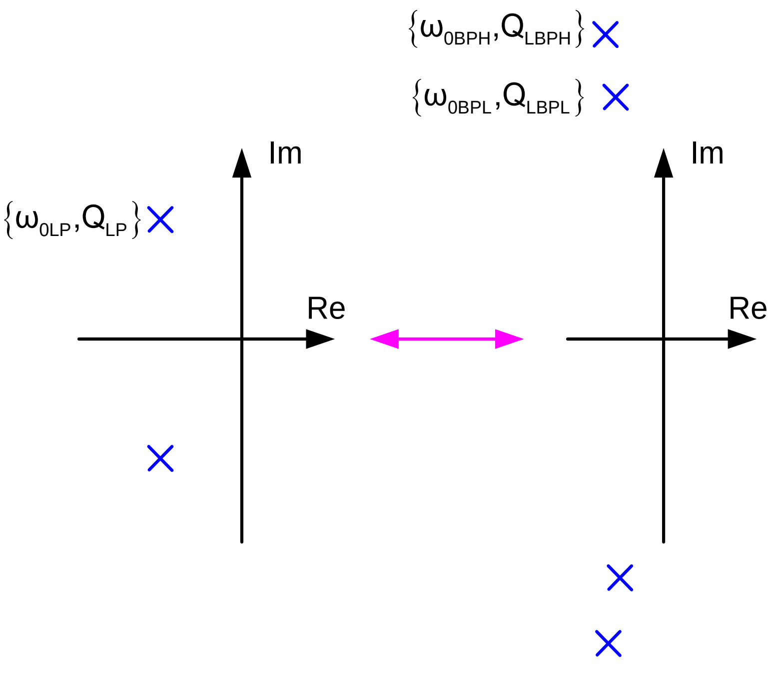
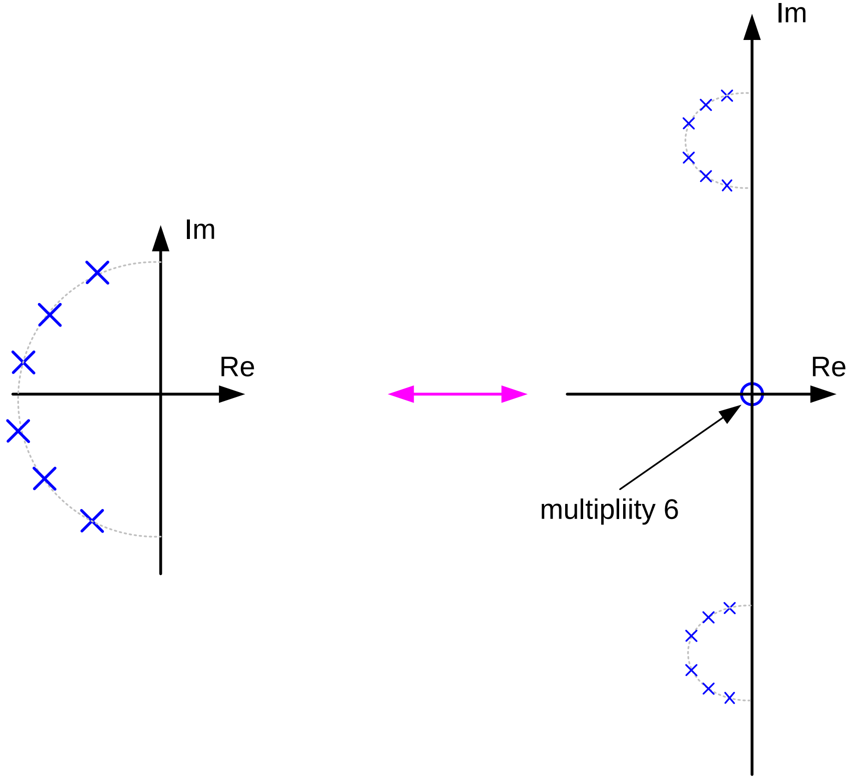


Image of the cc pole pair is the two pairs of poles

Standard LP to BP Transformation

Pole Mappings

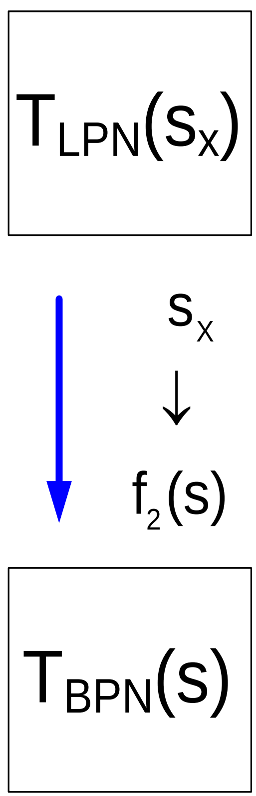
$$p \leftarrow \frac{p_X \bullet BW_N \pm \sqrt{\left(BW_N \bullet p_X\right)^2 - 4}}{2}$$



Note doubling of poles, addition of zeros, and likely Q enhancement

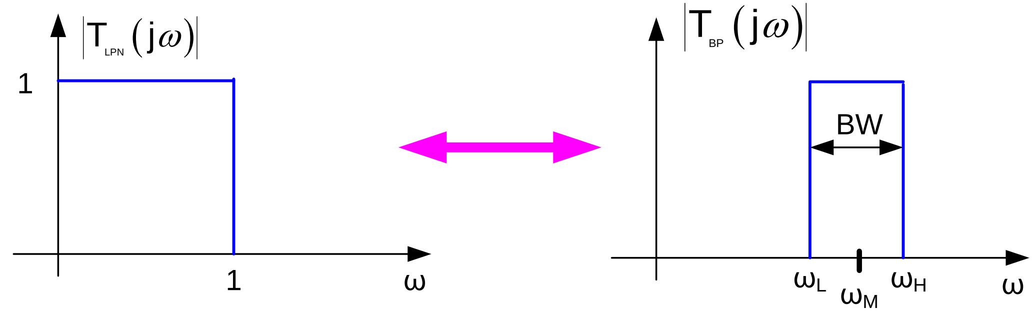
LP to BP Transformation

Claim: Other variable mapping transforms exist that satisfy the imaginary axis mapping properties needed to obtain the LP to BP transformation but are seldom, if ever, discussed. The Standard LP to BP transform Is by far the most popular and most authors treat it as if it is unique.



LP to BP Transformation

Pole Q of BP Approximations



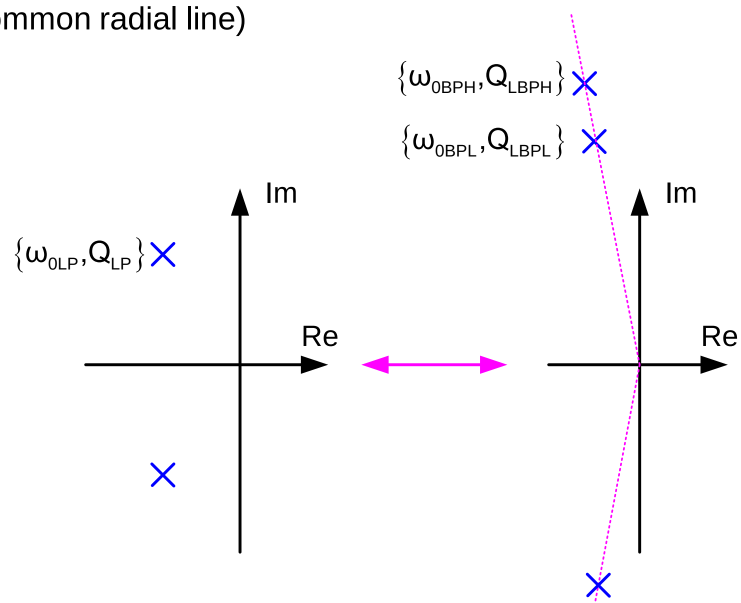
$$BW = \omega_H - \omega_L$$

$$\omega_M = \sqrt{\omega_H \omega_L}$$

Consider a pole in the LP approximation characterized by $\{\omega_{0LP}, Q_{LP}\}$

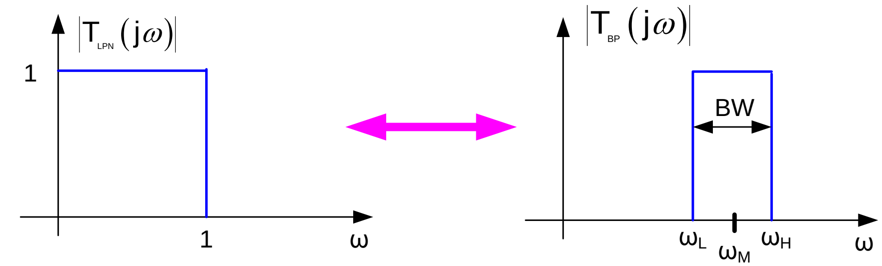
It can be shown that the corresponding BP poles have the same Q

(i.e. both bp poles lie on a common radial line)



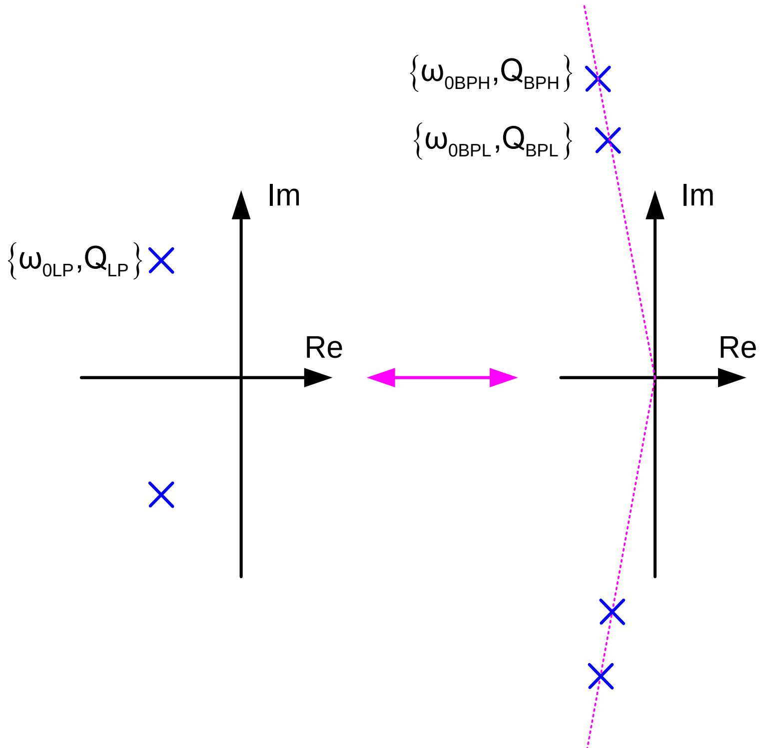
LP to BP Transformation

Pole Q of BP Approximations



$$BW = \omega_H - \omega_L$$

$$\omega_M = \sqrt{\omega_H \omega_L}$$



Define: $$\delta = \left(\frac{BW}{\omega_M}\right)\omega_{0LP}$$

It can be shown that

$$Q_{BPL} = Q_{BPH} = \frac{Q_{LP}}{\sqrt{2}}\sqrt{1+\frac{4}{\delta^2}+\sqrt{\left(1+\frac{4}{\delta^2}\right)^2-\frac{4}{\delta^2 Q_{2LP}^2}}}$$

For δ small, $$Q_{BP} \simeq \frac{2Q_{LP}}{\delta}$$

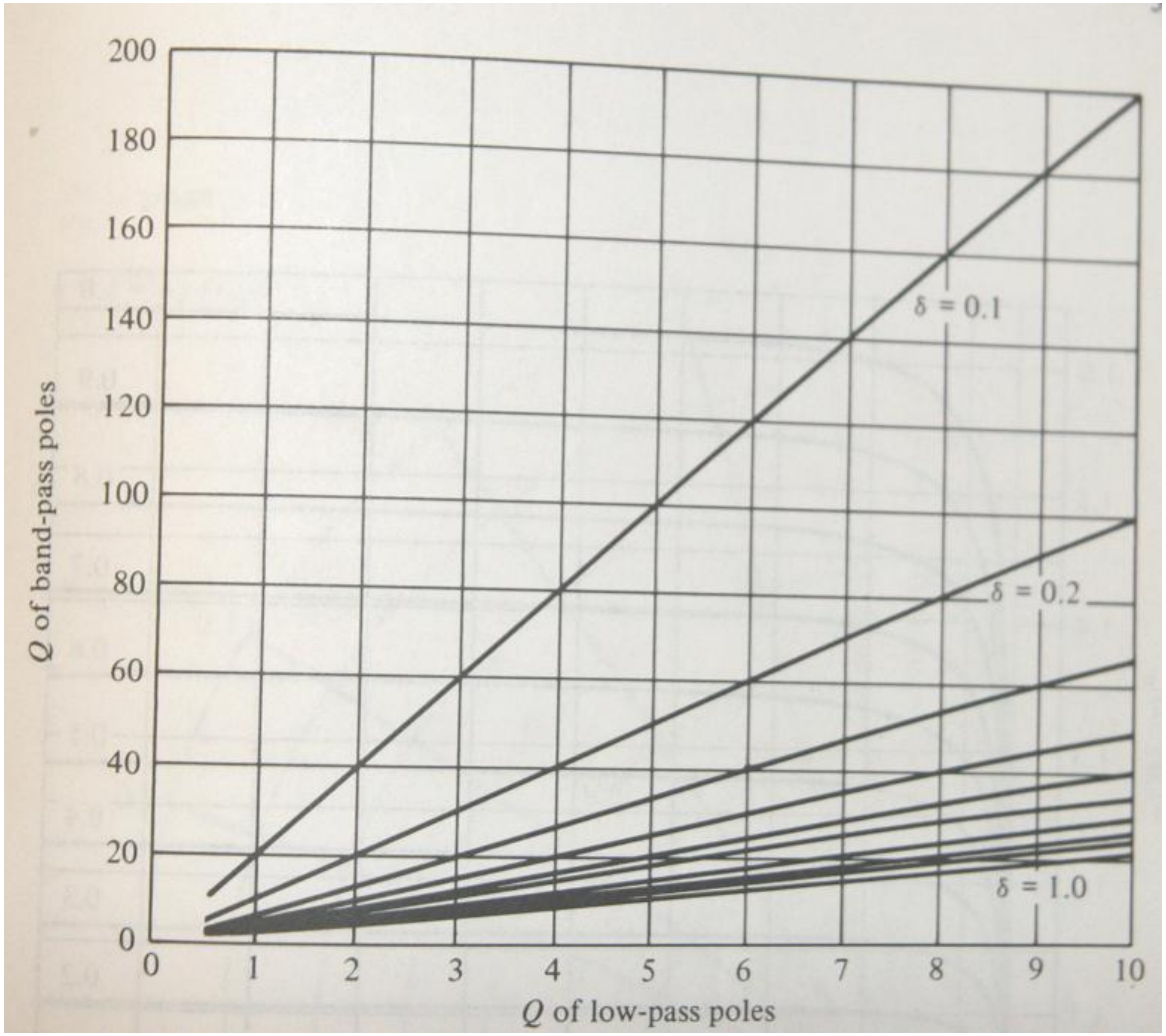
It can be shown that

$$\omega_{0BP} = \frac{\omega_M}{2}\left[\delta\frac{Q_{BP}}{Q_{LP}} \pm \sqrt{\left(\delta\frac{Q_{BP}}{Q_{LP}}\right)^2 - 4}\right]$$

Note for δ **small, Q_{BP} can get very large**

LP to BP Transformation

Pole Q of BP Approximations



$$\delta = \left(\frac{BW}{\omega_M}\right)\omega_{0LP}$$

$$Q_{BPL} = Q_{BPH} = \frac{Q_{LP}}{\sqrt{2}}\sqrt{1+\frac{4}{\delta^2}+\sqrt{\left(1+\frac{4}{\delta^2}\right)^2-\frac{4}{\delta^2 Q_{2LP}^2}}}$$

LP to BP Transformation

Pole locations vs Q_{LP} and δ

$$\delta = \left(\frac{BW}{\omega_M} \right) \omega_{0LP}$$

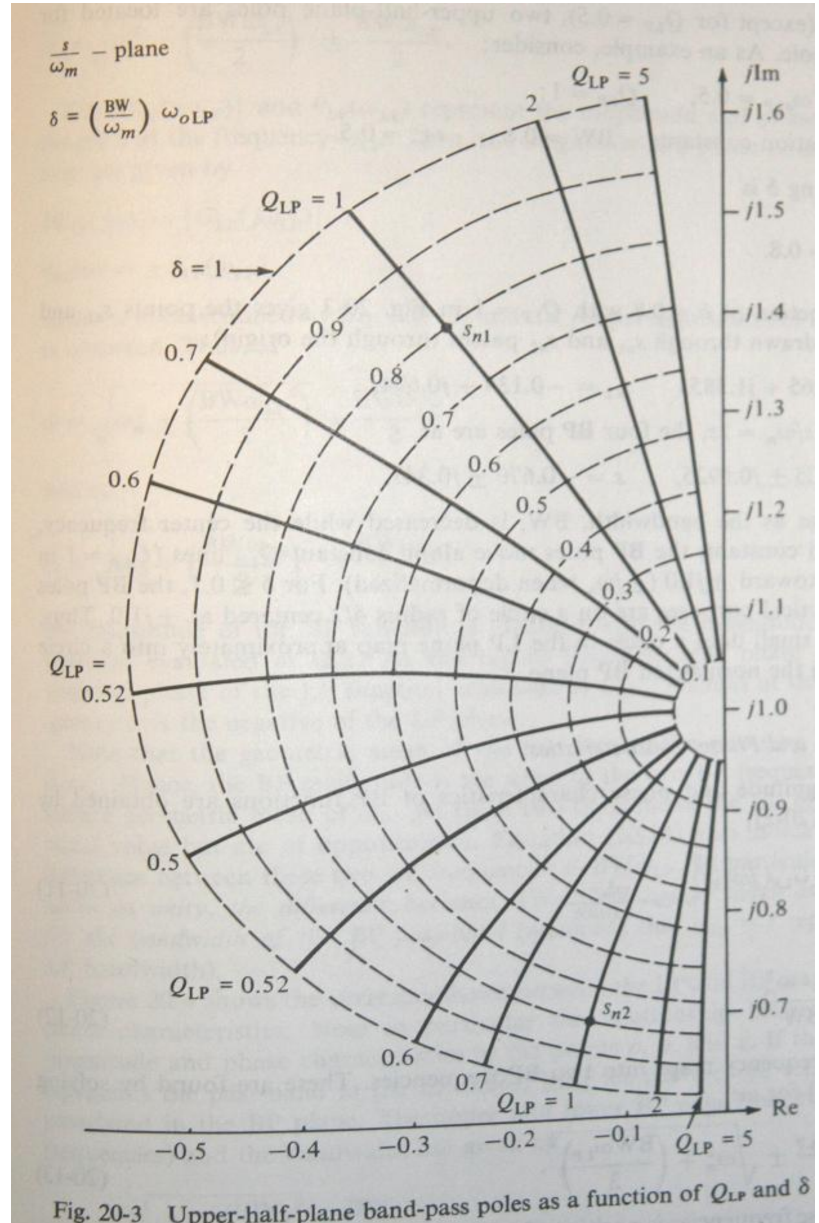
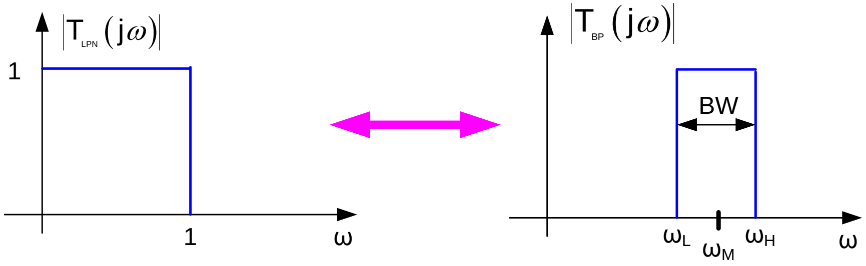


Fig. 20-3 Upper-half-plane band-pass poles as a function of Q_{LP} and δ

LP to BP Transformation



Classical BP Approximations

Butterworth
Chebyschev
Elliptic
Bessel

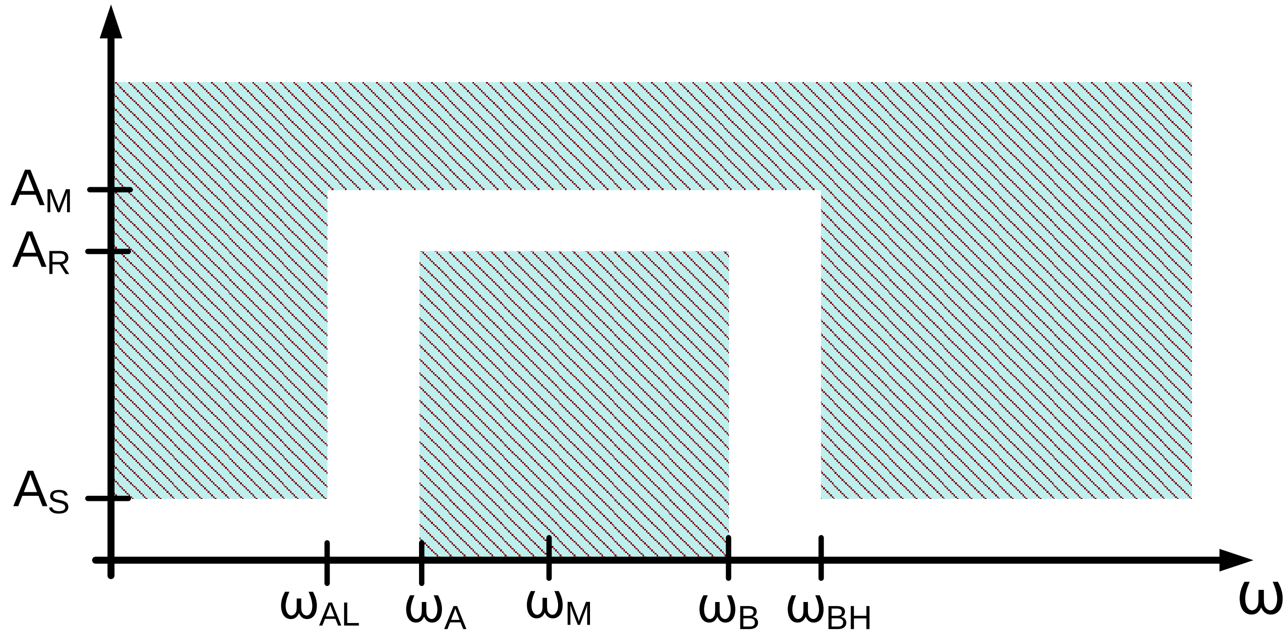
Obtained by the LP to BP transformation of the corresponding LP approximations

Standard LP to BP Transformation

$$s \rightarrow \frac{s^2+1}{s \bullet BW_N}$$

- Standard LP to BP transform is a variable mapping transform
- Maps jω axis to jω axis
- Maps LP poles to BP poles
- Preserves basic shape but warps frequency axis
- Doubles order
- Pole Q of resultant band-pass functions can be very large for narrow pass-band
- Sequencing of frequency scaling and transformation does not affect final function

Example 1: Obtain an approximation that meets the following specifications



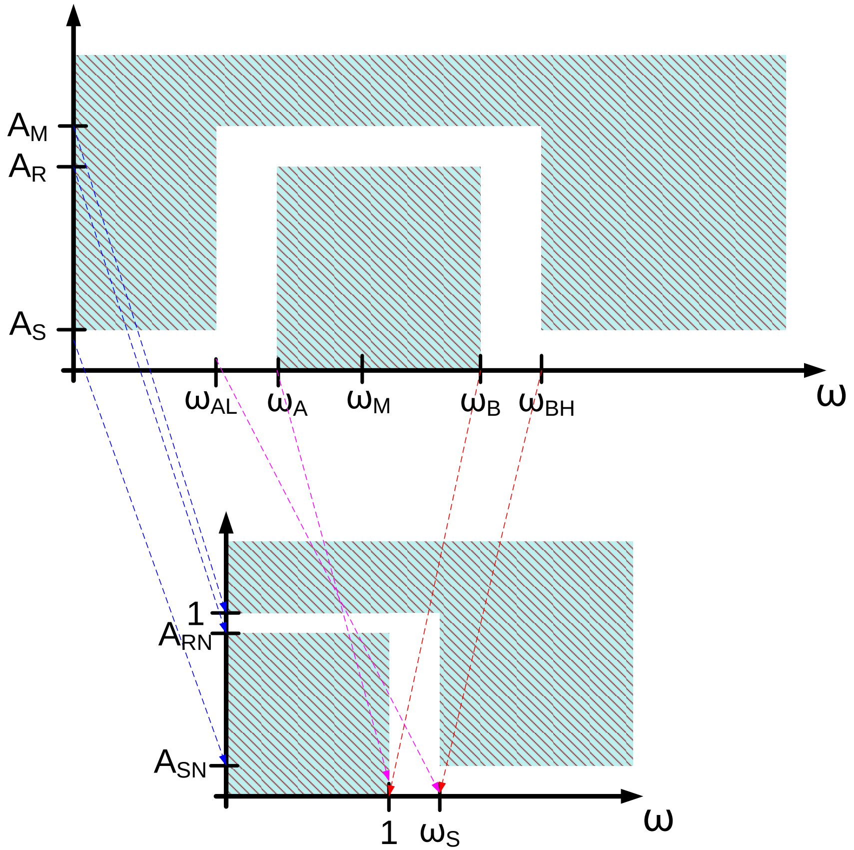
$$BW=\omega_B-\omega_A$$

$$\omega_M=\sqrt{\omega_B \bullet \omega_A}$$

Assume that ω_{AL}, ω_{BH} and ω_M satsify

$$\frac{\omega_M^2-\omega_{AL}^2}{\omega_{AL}\bullet BW}=\frac{\omega_{BH}^2-\omega_M^2}{\omega_{BH}\bullet BW}$$

Example 1: Obtain an approximation that meets the following specifications



$$A_{RN}=\frac{A_R}{A_M}$$

$$A_{SN}=\frac{A_S}{A_M}$$

$$\frac{1}{\sqrt{1+\varepsilon^2}}=\frac{A_R}{A_M}$$

$$\varepsilon=\sqrt{\left(\frac{A_M}{A_R}\right)^2-1}$$

$$\omega_S=\frac{\omega_M^2-\omega_{AL}^2}{\omega_{AL}\bullet BW}$$

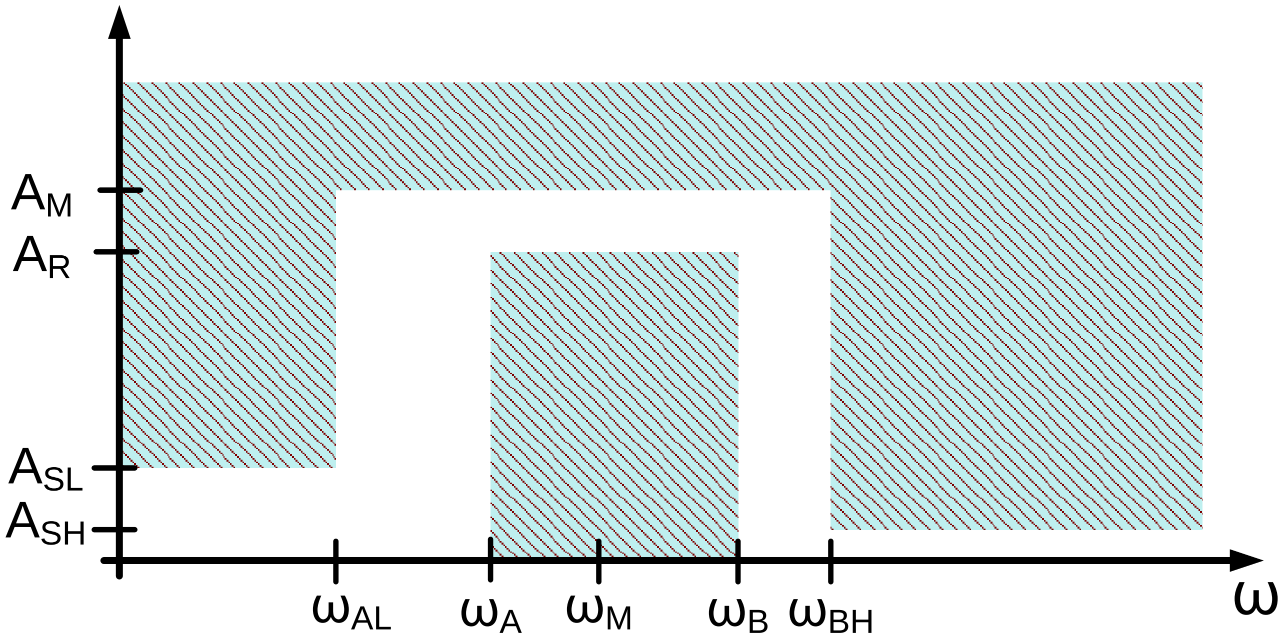
$$BW=\omega_B-\omega_A$$

$$\omega_M=\sqrt{\omega_B\bullet\omega_A}$$

$$\frac{\omega_M^2-\omega_{AL}^2}{\omega_{AL}\bullet BW}=\frac{\omega_{BH}^2-\omega_M^2}{\omega_{BH}\bullet BW}$$

(actually $-\omega_A$ and $-\omega_{AL}$ that map to 1 and ω_S respectively but show ω_A and ω_{AL} for convenience)

Example 2: Obtain an approximation that meets the following specifications



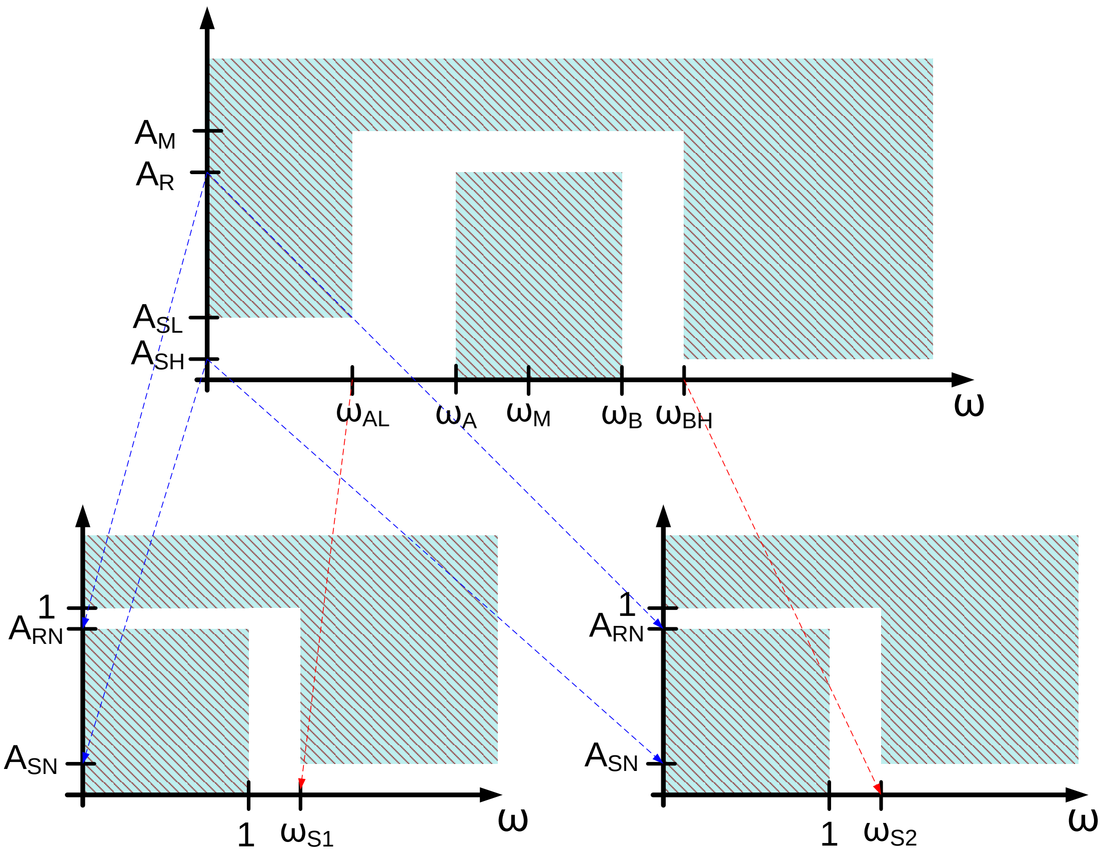
$$BW=\omega_B-\omega_A$$

$$\omega_M=\sqrt{\omega_B \bullet \omega_A}$$

In this example,

$$\frac{\omega_M^2-\omega_{AL}^2}{\omega_{AL}\bullet BW} \neq \frac{\omega_{BH}^2-\omega_M^2}{\omega_{BH}\bullet BW}$$

Example 2: Obtain an approximation that meets the following specifications



$$A_{RN} = \frac{A_R}{A_M}$$

$$\frac{1}{\sqrt{1+\varepsilon^2}} = \frac{A_R}{A_M}$$

$$A_{SN} = \min\left\{\frac{A_{SH}}{A_M}, \frac{A_{SL}}{A_M}\right\}$$

$$\varepsilon = \sqrt{\left(\frac{A_M}{A_R}\right)^2 - 1}$$

$$\omega_{S1} = \frac{\omega_M^2 - \omega_{AL}^2}{\omega_{AL} \bullet BW}$$

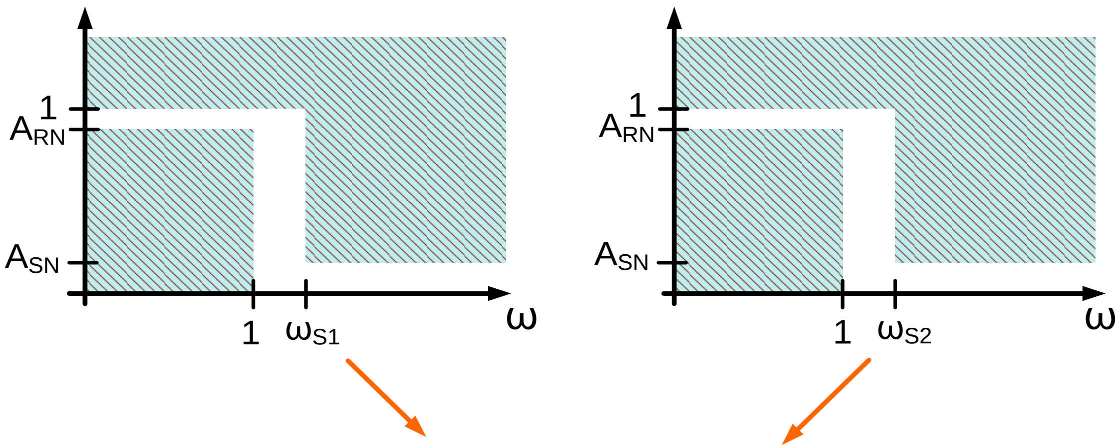
$$\omega_{S2} = \frac{\omega_{BH}^2 - \omega_M^2}{\omega_{BH} \bullet BW}$$

$$BW = \omega_B - \omega_A$$

$$\omega_M = \sqrt{\omega_B \bullet \omega_A}$$

$$\omega_{SN} = \min\{\omega_{S1}, \omega_{S2}\}$$

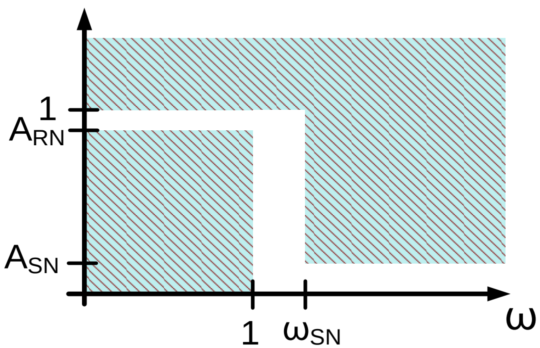
Example 2: Obtain an approximation that meets the following specifications



$$A_{RN}=\frac{A_R}{A_M}$$

$$\frac{1}{\sqrt{1+\varepsilon^2}}=\frac{A_R}{A_M}$$

$$A_{SN}=\min\left\{\frac{A_{SH}}{A_M},\frac{A_{SL}}{A_M}\right\}$$



$$\omega_{SN}=\min\{\omega_{S1},\omega_{S2}\}$$

$$\varepsilon=\sqrt{\left(\frac{A_M}{A_R}\right)^2-1}$$

$$\omega_{S1}=\frac{\omega_M^2-\omega_{AL}^2}{\omega_{AL}\bullet BW}$$

$$\omega_{S2}=\frac{\omega_{BH}^2-\omega_M^2}{\omega_{BH}\bullet BW}$$

$$BW=\omega_B-\omega_A$$

$$\omega_M=\sqrt{\omega_B\bullet\omega_A}$$

$$\omega_{SN}=\min\{\omega_{S1},\omega_{S2}\}$$

Filter Transformations

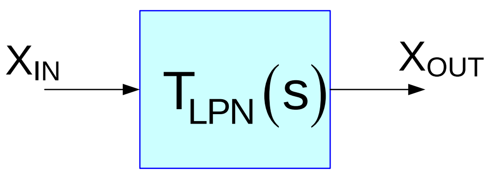
Lowpass to Bandpass **(LP to BP)**
Lowpass to Highpass **(LP to HP)**
→ Lowpass to Band-reject **(LP to BR)**

- Approach will be to take advantage of the results obtained for the standard LP approximations

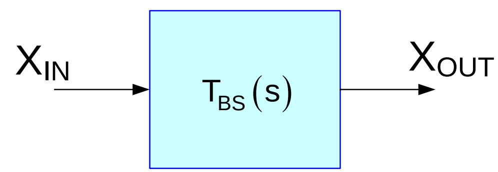
- Will focus on flat passband and zero-gain stop-band transformations

LP to BS Transformation

Strategy: As was done for the LP to BP approximations, will use a variable mapping strategy that maps the imaginary axis in the s-plane to the imaginary axis in the s-plane so the basic shape is preserved.



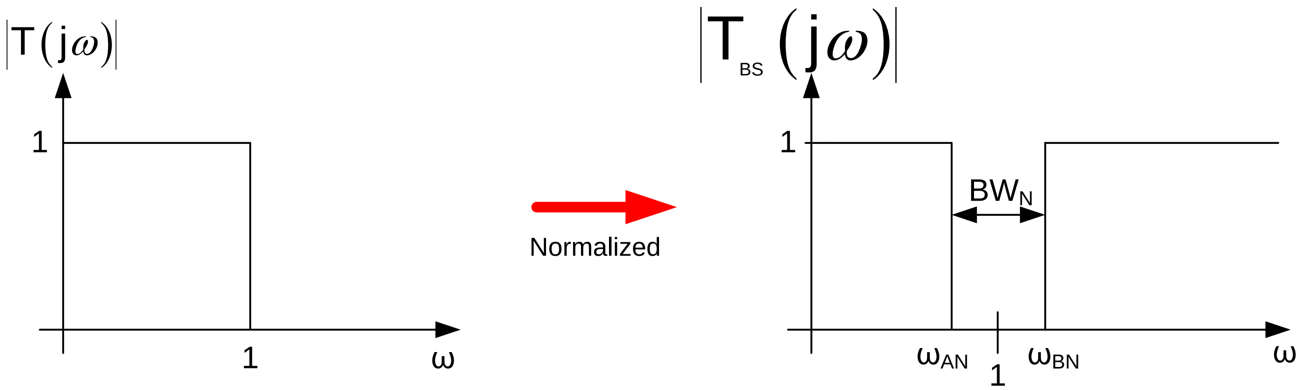
$$s \rightarrow f(s)$$



$$T_{BS}(s) = T_{LPN}(f(s))$$

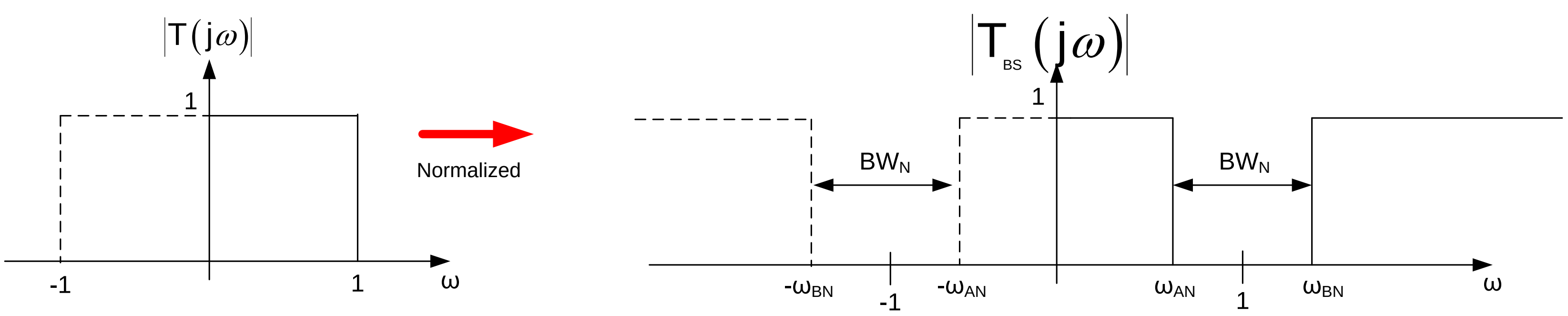
$$f(s) = \frac{\sum_{i=0}^{m_T} a_{Ti} s^i}{\sum_{i=0}^{n_T} b_{Ti} s^i}$$

LP to BS Transformation



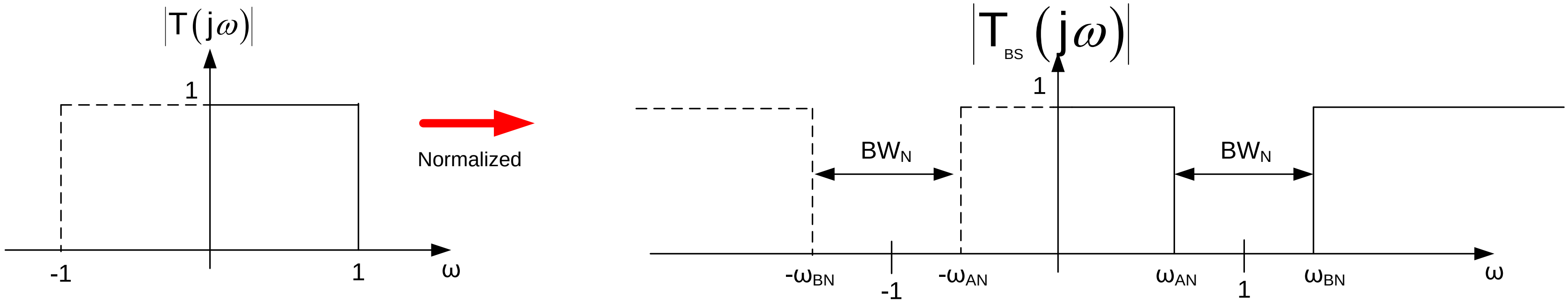
$$BW_N = \omega_{BN} - \omega_{AN}$$

$$\sqrt{\omega_{AN}\omega_{BN}} = 1$$



Standard LP to BS Transformation

Mapping Strategy:



Variable Mapping Strategy to Preserve Shape of LP function:

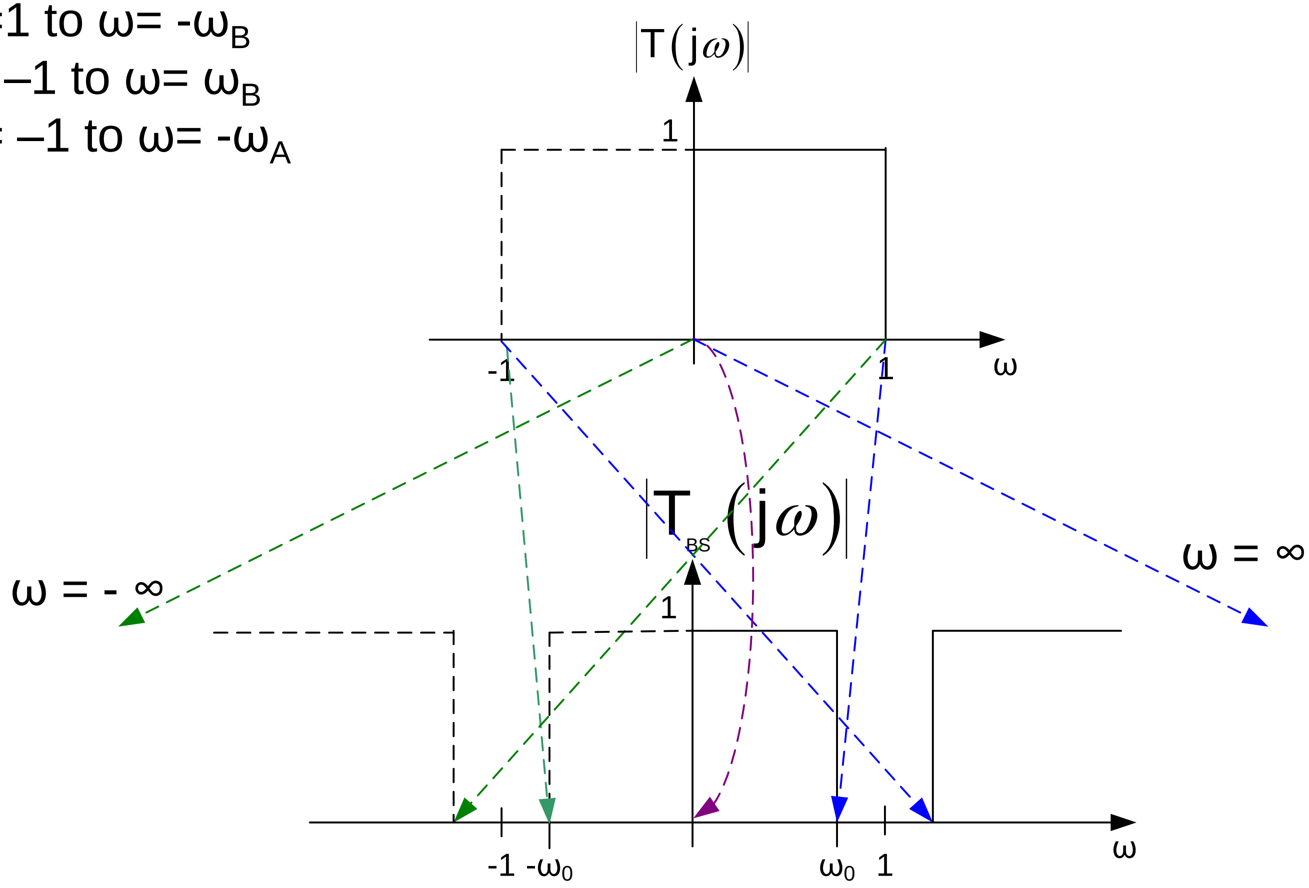
$F_N(s)$ should

map s=0 to s=± j∞
map s=0 to s= j0
map s=j1 to s=jω_A
map s=j1 to s=-jω_B
map s=-j1 to s=jω_B
map s=-j1 to s=-jω_A

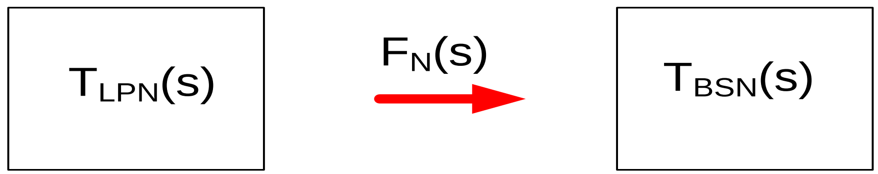
map ω=0 to ω = ±∞
map ω=0 to ω = 0
map ω=1 to ω = ω_A
map ω=1 to ω= -ω_B
map ω= –1 to ω= ω_B
map ω= –1 to ω= -ω_A

Standard LP to BS Transformation

map $\omega=0$ to $\omega = \pm\infty$
map $\omega=0$ to $\omega = 0$
map $\omega=1$ to $\omega = \omega_A$
map $\omega=1$ to $\omega= -\omega_B$
map $\omega= -1$ to $\omega= \omega_B$
map $\omega= -1$ to $\omega= -\omega_A$



Standard LP to BS Transformation



Mapping Strategy: consider variable mapping transform

$F_N(s)$ should

map s=0 to s=± j∞
map s=0 to s= j0
map s=j1 to s=$j\omega_A$
map s=j1 to s=$-j\omega_B$
map s=-j1 to s=$j\omega_B$
map s=-j1 to s=$-j\omega_A$

→

map ω=0 to ω = ±∞
map ω=0 to ω = 0
map ω=1 to ω = ω_A
map ω=1 to ω= $-\omega_B$
map ω= –1 to ω= ω_B
map ω= –1 to ω= $-\omega_A$

Consider variable mapping

$$T_{LPN}\left(F_N(s)\right) = T_{BSN}(s)\Big|_{s=\frac{s \bullet BW_N}{s^2+1}}$$

$$s \rightarrow \frac{s \bullet BW_N}{s^2+1}$$

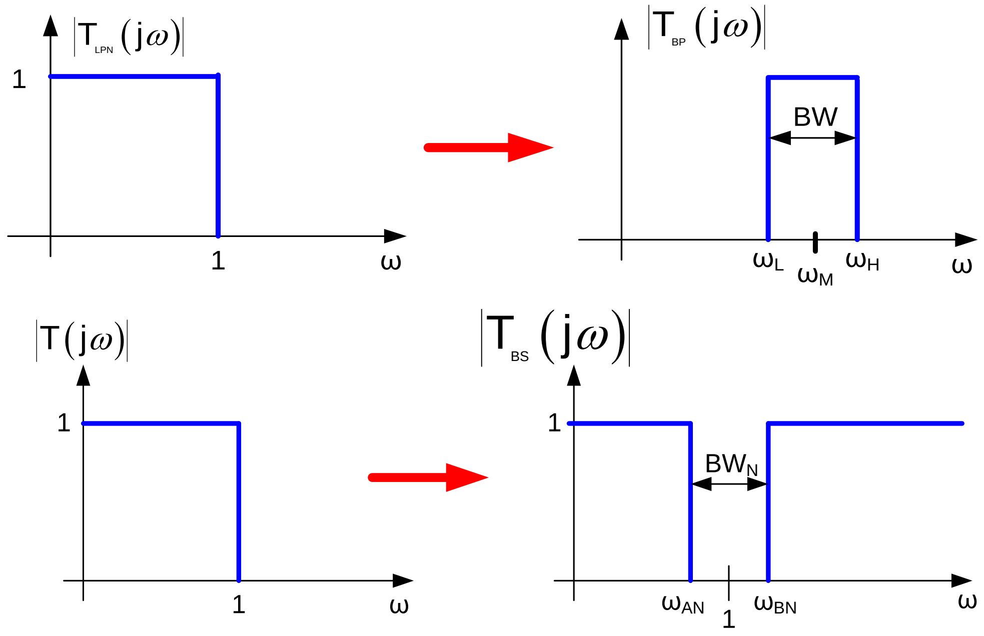
Comparison of Transforms

LP to BP

$$s \rightarrow \frac{s^2+1}{s \bullet BW_N}$$

LP to BS

$$s \rightarrow \frac{s \bullet BW_N}{s^2+1}$$



Standard LP to BS Transformation

Frequency and s-domain Mappings

(subscript variable in LP approximation for notational convenience)

$$T_{LPN}(s_X)$$

$$s_X \downarrow \frac{s \bullet BW_N}{s^2+1}$$

$$T_{BSN}(s)$$

$$s_X \rightarrow \frac{s \bullet BW_N}{s^2+1}$$

$$\omega_X \rightarrow \frac{\omega \bullet BW_N}{1\text{-}\omega^2}$$

$$s \leftarrow \frac{1}{2}\frac{BW_N}{s_X} \pm \frac{1}{2}\sqrt{\left(\frac{BW_N}{s_X}\right)^2 - 4}$$

$$\omega \leftarrow \frac{-1}{2}\frac{BW_N}{\omega_X} \pm \frac{1}{2}\sqrt{\left(\frac{BW_N}{\omega_X}\right) + 4}$$

Standard LP to BS Transformation

Un-normalized Frequency and s-domain Mappings

(subscript variable in LP approximation for notational convenience)

$T_{LPN}(s_X)$

$$s_X \downarrow \frac{s \bullet BW}{s^2 + \omega_M^2}$$

$T_{BS}(s)$

$$s_X \rightarrow \frac{s \bullet BW}{s^2 + \omega_M^2}$$

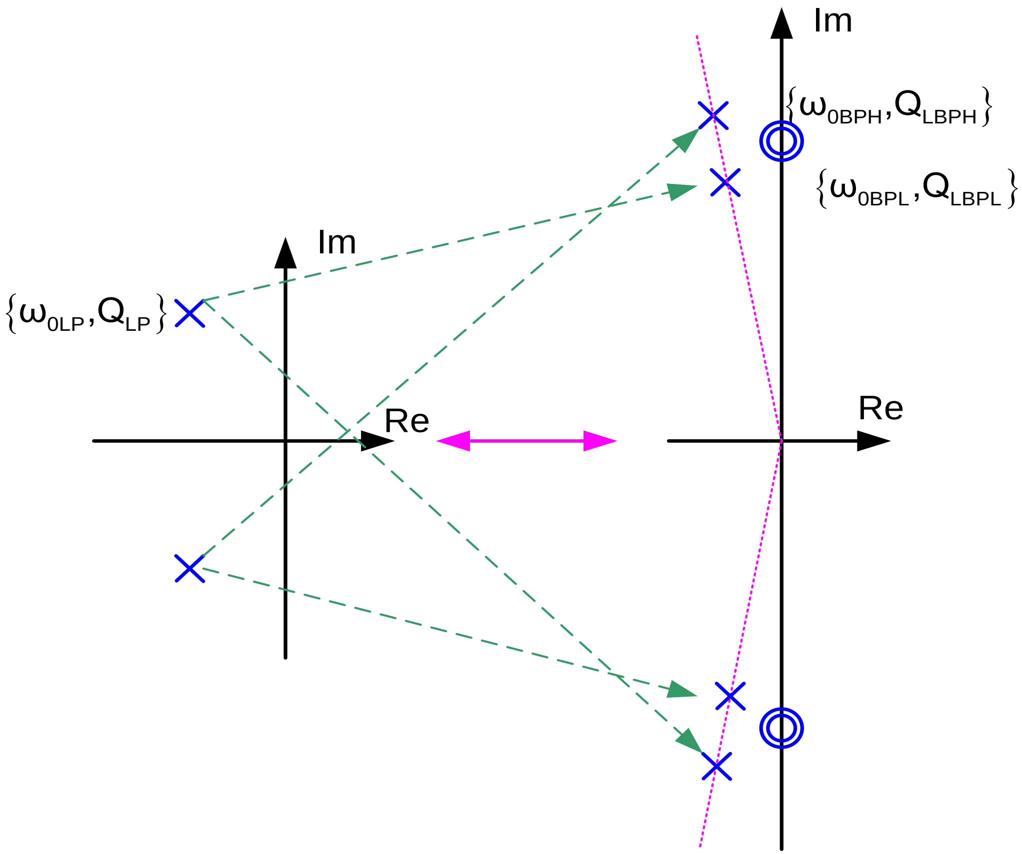
$$\omega_X \rightarrow \frac{\omega \bullet BW}{\omega_M^2 - \omega^2}$$

$$s \leftarrow \frac{1}{2}\frac{BW}{s_X} \pm \frac{1}{2}\sqrt{\left(\frac{BW}{s_X}\right)^2 - 4\omega_M^2}$$

$$\omega \leftarrow \frac{-1}{2}\frac{BW}{\omega_X} \pm \frac{1}{2}\sqrt{\left(\frac{BW}{\omega_X}\right) + 4\omega_M^2}$$

Standard LP to BS Transformation

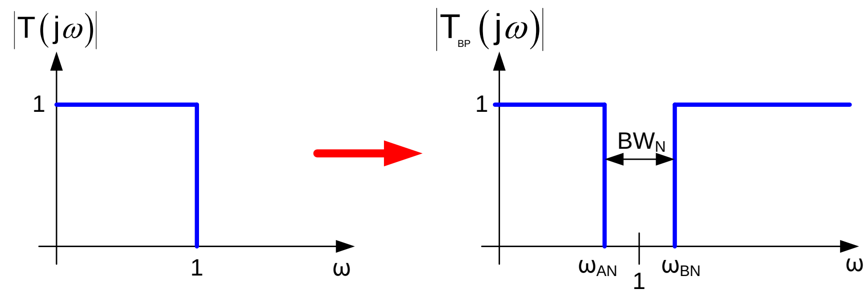
Pole Mappings

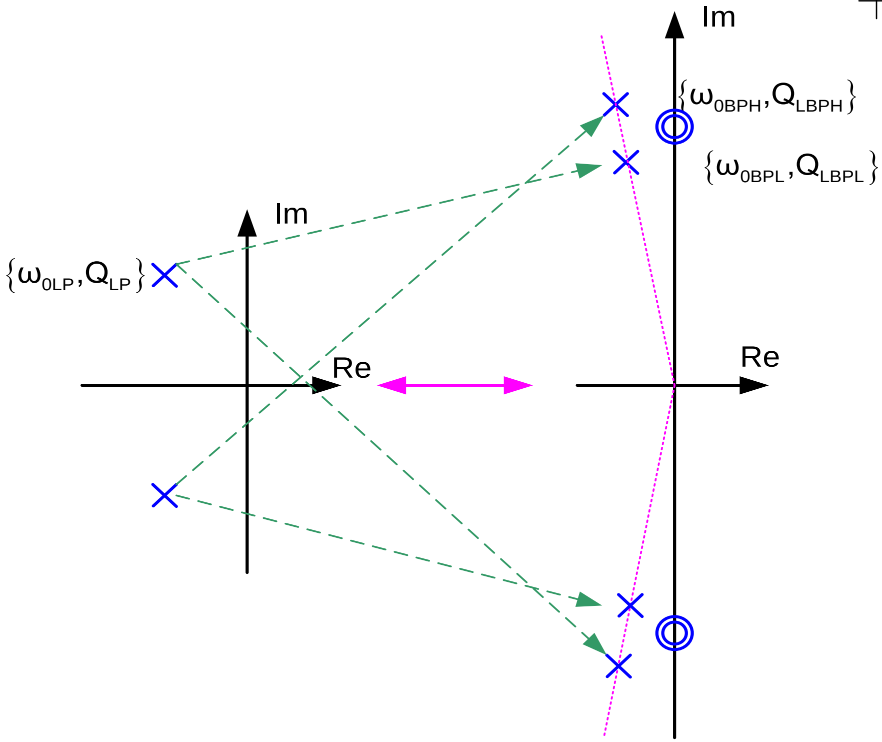


Can show that the upper hp pole maps to one upper hp pole and one lower hp pole as shown. Corresponding mapping of the lower hp pole is also shown

LP to BS Transformation

Pole Q of BS Approximations





$$BW = \omega_{BN} - \omega_{AN}$$

$$\omega_M = \sqrt{\omega_{AN}\omega_{BN}}$$

Define: $$\gamma = \left(\frac{BW}{\omega_M \omega_{0LP}}\right)$$

It can be shown that

$$Q_{BSL} = Q_{BSH} = \frac{Q_{LP}}{\sqrt{2}}\sqrt{1+\frac{4}{\gamma^2}+\sqrt{\left(1+\frac{4}{\gamma^2}\right)^2-\frac{4}{\gamma^2 Q_{LP}^2}}}$$

For γ small, $$Q_{BS} \simeq \frac{2Q_{LP}}{\gamma}$$

It can be shown that

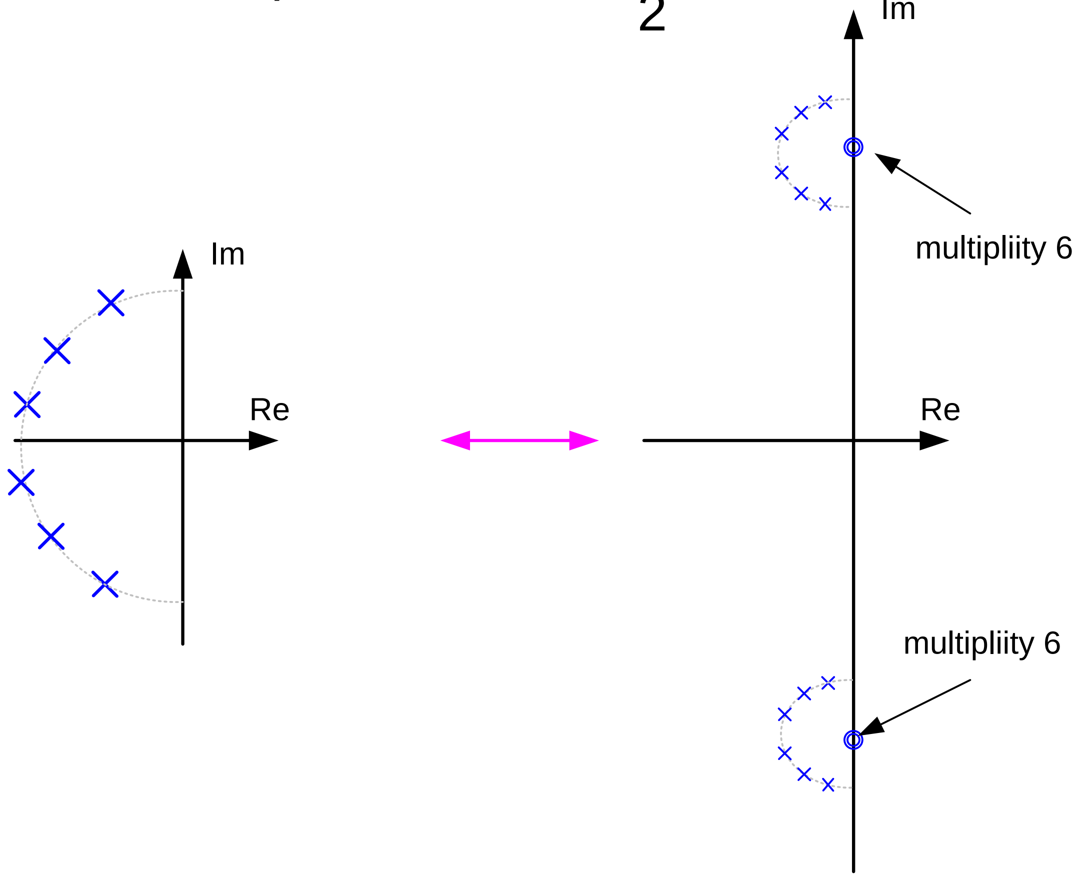
$$\omega_{0BS} = \frac{\omega_M}{2}\left[\gamma\frac{Q_{BS}}{Q_{LP}} \pm \sqrt{\left(\gamma\frac{Q_{BS}}{Q_{LP}}\right)^2-4}\right]$$

Note for γ **small, Q_{BS} can get very large**

Standard LP to BS Transformation

Pole Mappings

$$p \leftarrow \frac{\frac{BW_N}{p_X} \pm \sqrt{\left(\frac{BW_N}{p_X}\right)^2 - 4}}{2}$$



Note doubling of poles, addition of zeros, and likely Q enhancement

Standard LP to BS Transformation

$$s_X \rightarrow \frac{s \bullet BW}{s^2 + \omega_M^2}$$

- **Standard LP to BS transformation is a variable mapping transform**
- **Maps jω axis to jω axis in the s-plane**
- **Preserves basic shape of an approximation but warps frequency axis**
- **Order of BS approximation is double that of the LP Approximation**
- **Pole Q and ω_0 expressions are identical to those of the LP to BP transformation**
- **Pole Q of BS approximation can get very large for narrow BW**
- **Other variable transforms exist but the standard is by far the most popular**

Filter Transformations

Lowpass to Bandpass **(LP to BP)**

Lowpass to Highpass **(LP to HP)**

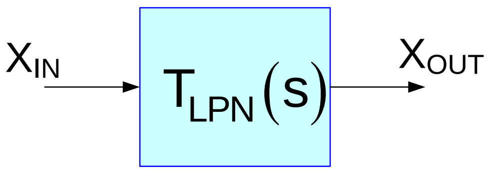
Lowpass to Band-reject **(LP to BR)**

- Approach will be to take advantage of the results obtained for the standard LP approximations

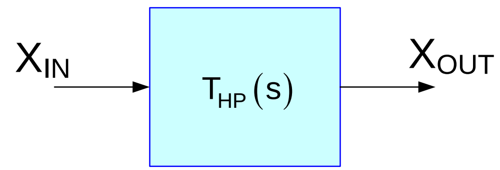
- Will focus on flat passband and zero-gain stop-band transformations

LP to HP Transformation

Strategy: As was done for the LP to BP approximations, will use a variable mapping strategy that maps the imaginary axis in the s-plane to the imaginary axis in the s-plane so the basic shape is preserved.



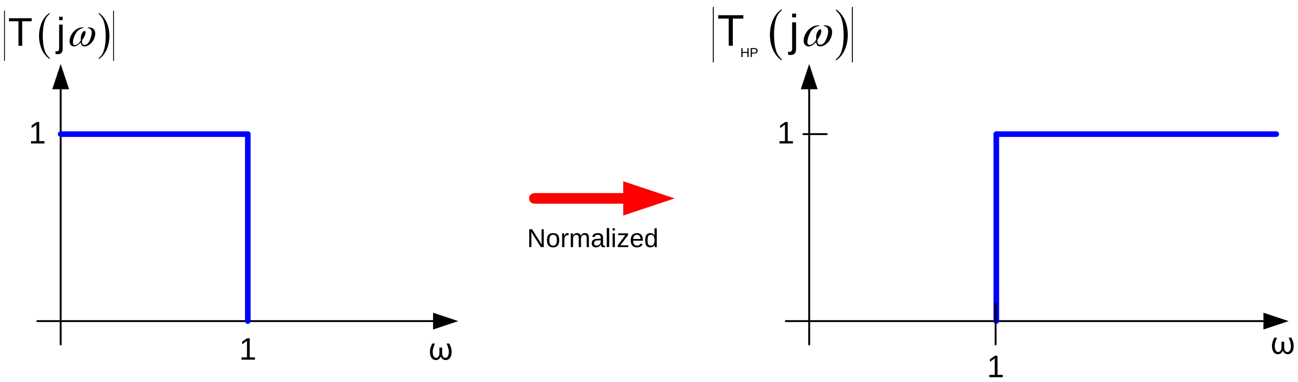
$$s \rightarrow f(s)$$

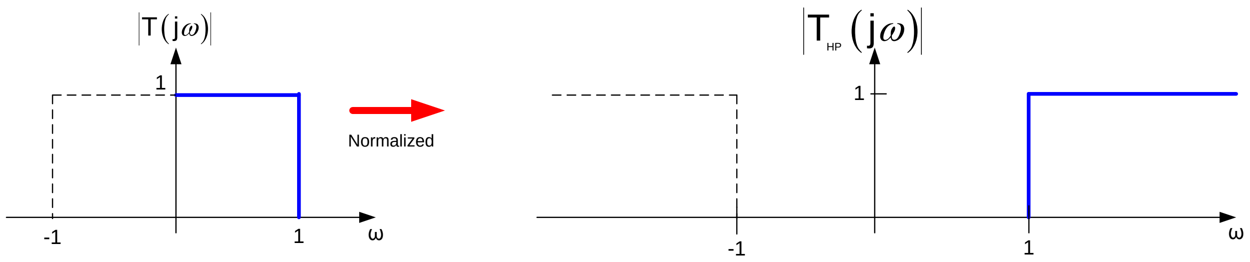


$$T_{HP}(s) = T_{LPN}(f(s))$$

$$f(s) = \frac{\sum_{i=0}^{m_T} a_{Ti} s^i}{\sum_{i=0}^{n_T} b_{Ti} s^i}$$

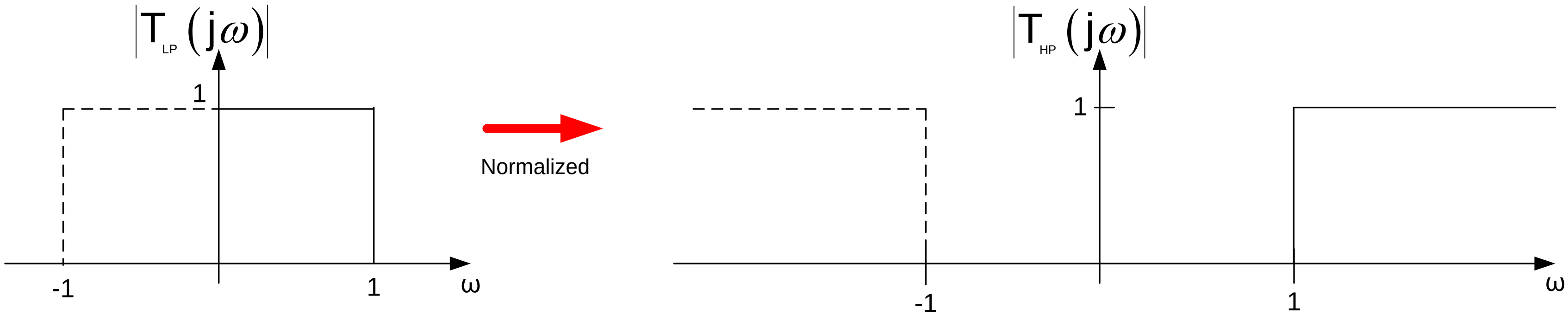
LP to HP Transformation





Standard LP to HP Transformation

Mapping Strategy:



Variable Mapping Strategy to Preserve Shape of LP function:

$F_N(s)$ should

map s=0 to s=± j∞
map s=j1 to s=-j1
map s= –j1 to s=j1

map ω=0 to ω=∞
map ω=1 to ω=-1
map ω= –1 to ω=1

Standard LP to HP Transformation

$T_{LPN}(s)$ $\xrightarrow{F_N(s)}$ $T_{HPN}(s)$

Mapping Strategy: consider variable mapping transform

$F_N(s)$ should

map s=0 to s=± j∞
map s=j1 to s=-j1
map s= –j1 to s=j1

→

map ω=0 to ω=∞
map ω=1 to ω=-1
map ω= –1 to ω=1

Consider variable mapping

$$T_{LPN}\left(F(s)\right)=T_{LPN}\left(s\right)\Big|_{s=\frac{1}{s}}$$

$$s \rightarrow \frac{1}{s}$$

Comparison of Transforms

LP to BP

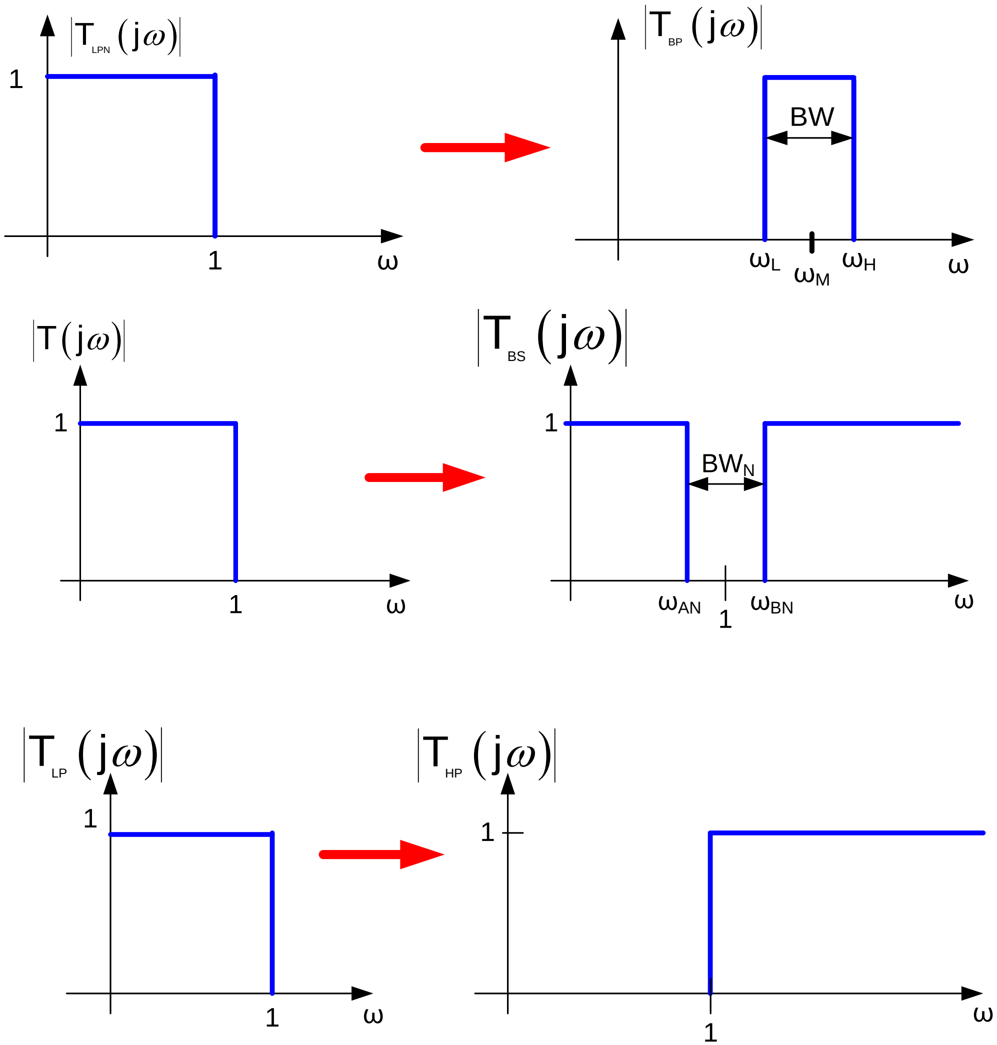
$$s \rightarrow \frac{s^2+1}{s \bullet BW_N}$$

LP to BS

$$s \rightarrow \frac{s \bullet BW_N}{s^2+1}$$

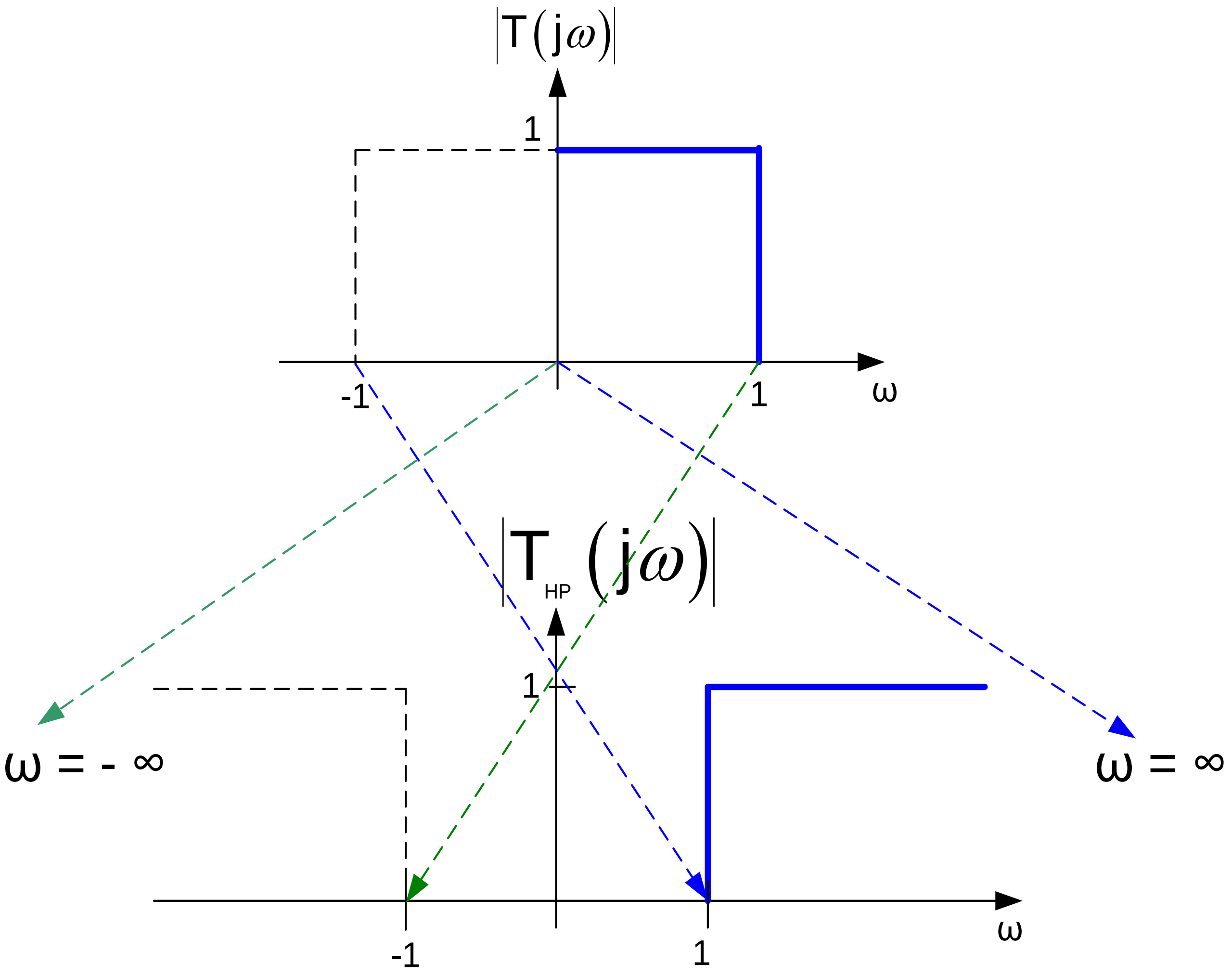
LP to HP

$$s \rightarrow \frac{1}{s}$$



LP to HP Transformation

(Normalized Transform)



Standard LP to HP Transformation

Frequency and s-domain Mappings

(subscript variable in LP approximation for notational convenience)

$$T_{LPN}(s_X)$$

$$s_X \downarrow \frac{1}{s}$$

$$T_{HPN}(s)$$

$$s_X \rightarrow \frac{1}{s}$$

$$\omega_X \rightarrow \frac{-1}{\omega}$$

$$s \leftarrow \frac{1}{s_X}$$

$$\omega \leftarrow \frac{-1}{\omega_X}$$

Standard LP to HP Transformation

Pole Mappings

Claim: With a variable mapping transform, the variable mapping naturally defines the mapping of the poles of the transformed function

$$T_{LPN}(s_X)$$

$$s_X \rightarrow \frac{1}{s}$$

$$T_{HPN}(s)$$

$$p_X \rightarrow \frac{1}{p}$$

$$p \leftarrow \frac{1}{p_X}$$

Standard LP to HP Transformation

Pole Mappings

$T_{LPN}(s_X)$

$$s_X \rightarrow \frac{1}{s}$$

$T_{HPN}(s)$

$$p \leftarrow \frac{1}{p_X}$$

If $p_X=\alpha+j\beta$ and $p_X=\alpha-j\beta$

$$p=\frac{1}{\alpha+j\beta}=\frac{\alpha-j\beta}{\alpha^2+\beta^2} \qquad p=\frac{1}{\alpha+j\beta}=\frac{\alpha+j\beta}{\alpha^2+\beta^2}$$

Standard LP to HP Transformation

Pole Mappings

$$T_{LPN}(s_X)$$

$$s_X \rightarrow \frac{1}{s}$$

$$T_{HPN}(s)$$

$$p \leftarrow \frac{1}{p_X}$$

If $p_X = \alpha + j\beta$ and $p_X = \alpha - j\beta$

$$p = \frac{1}{\alpha + j\beta} = \frac{\alpha - j\beta}{\alpha^2 + \beta^2} \qquad p = \frac{1}{\alpha + j\beta} = \frac{\alpha + j\beta}{\alpha^2 + \beta^2}$$

Highpass poles are scaled in magnitude but make identical angles with imaginary axis

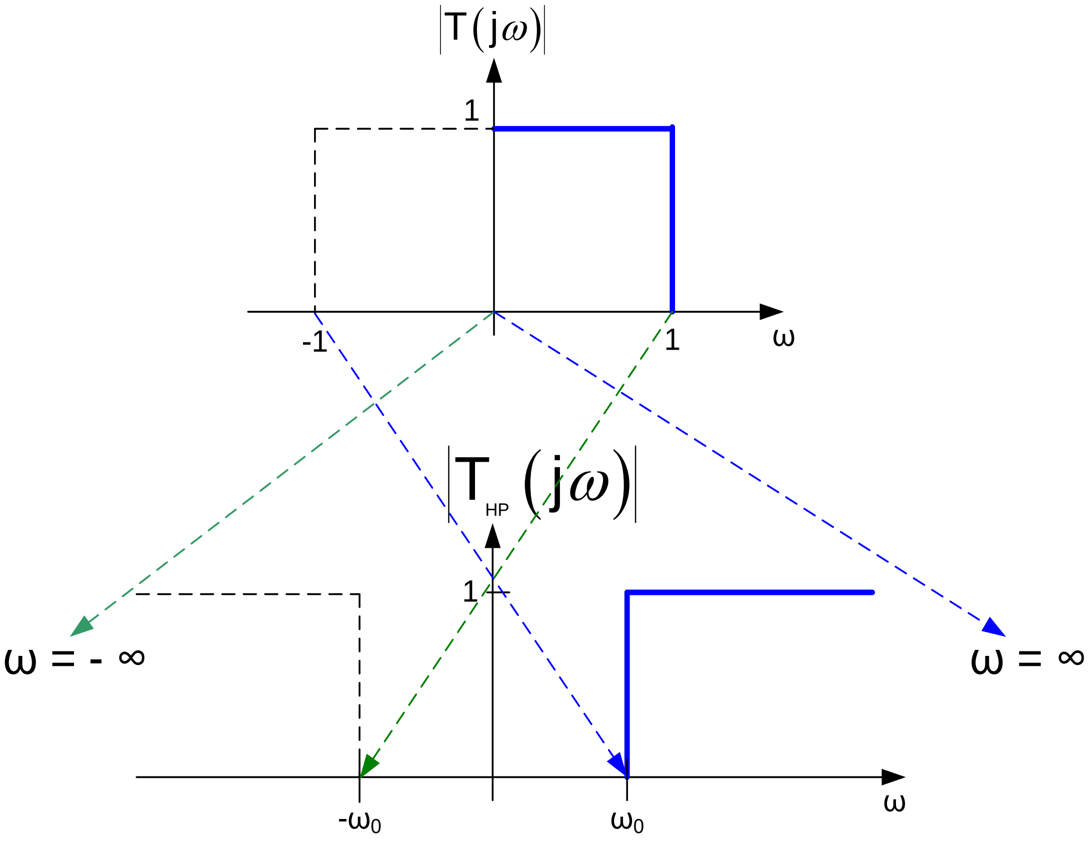
HP pole Q is same as LP pole Q

Order is preserved

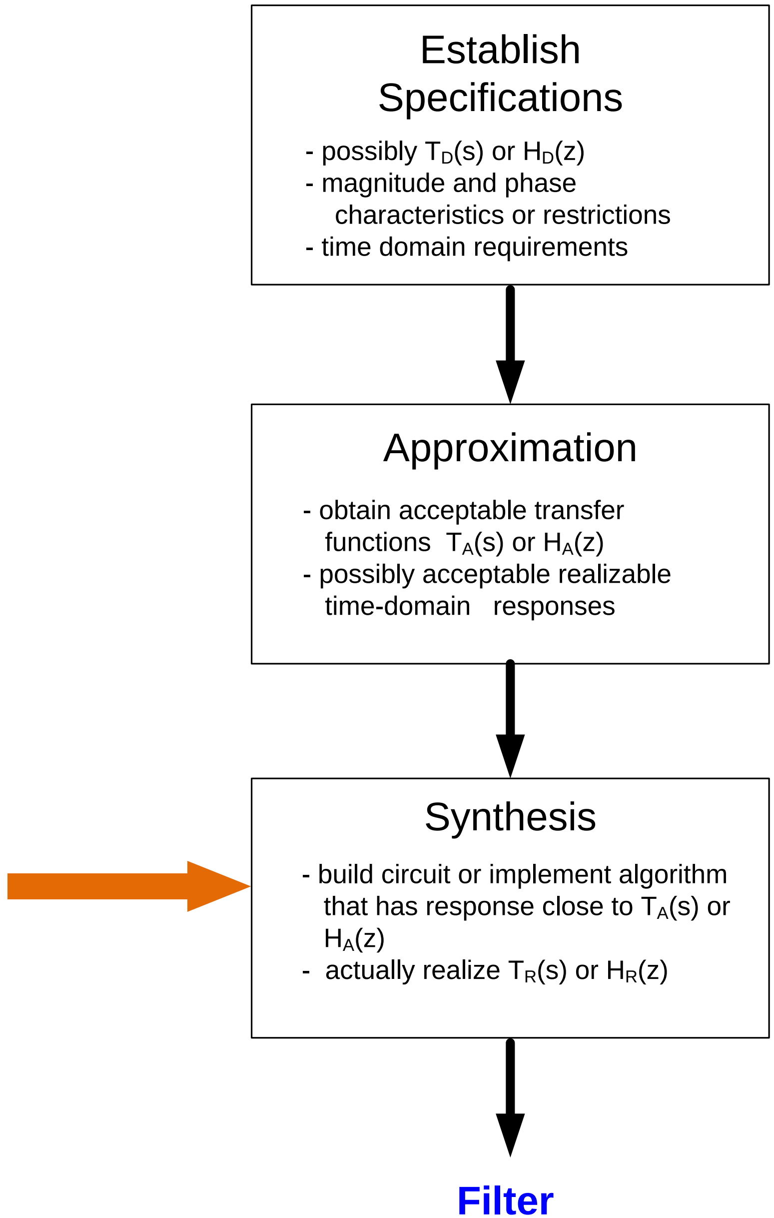
Standard LP to HP Transformation

(Un-normalized variable mapping transform)

$$s \rightarrow \frac{\omega_0}{s}$$



Filter Design Process



End of Lecture 17

EE 508
Lecture 18

Basic Biquadratic Active Filters

Second-order Bandpass
Second-order Lowpass
Effects of Op Amp on Filter Performance

Review from Last Time

Standard LP to BS Transformation

map ω=0 to $\omega = \pm\infty$
map ω=0 to ω = 0
map ω=1 to $\omega = \omega_A$
map ω=1 to $\omega = -\omega_B$
map ω= –1 to $\omega = \omega_B$
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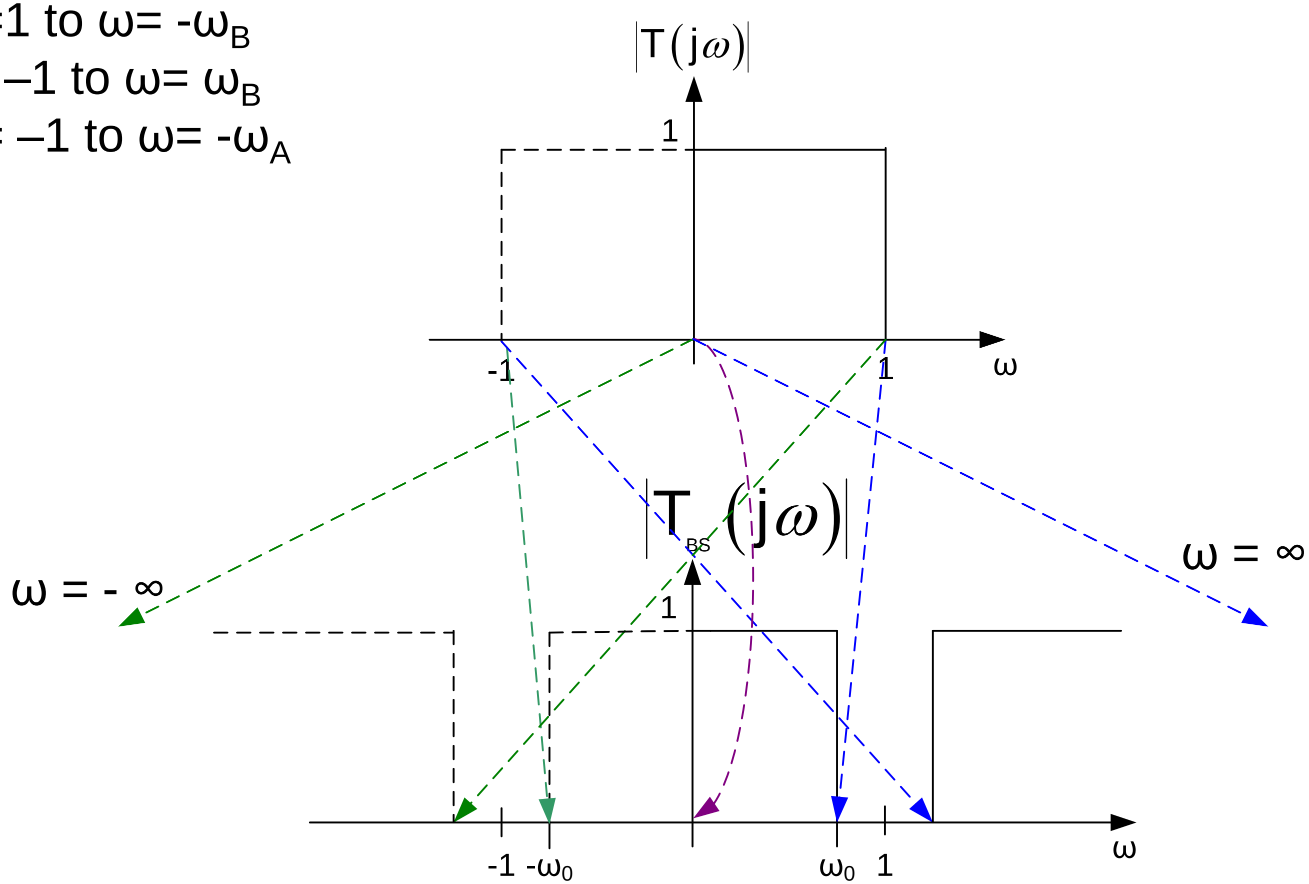
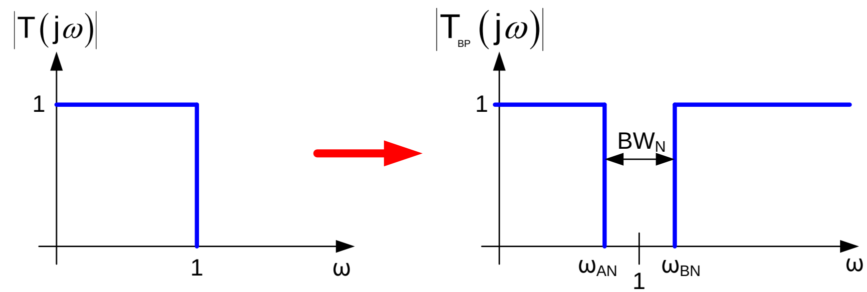
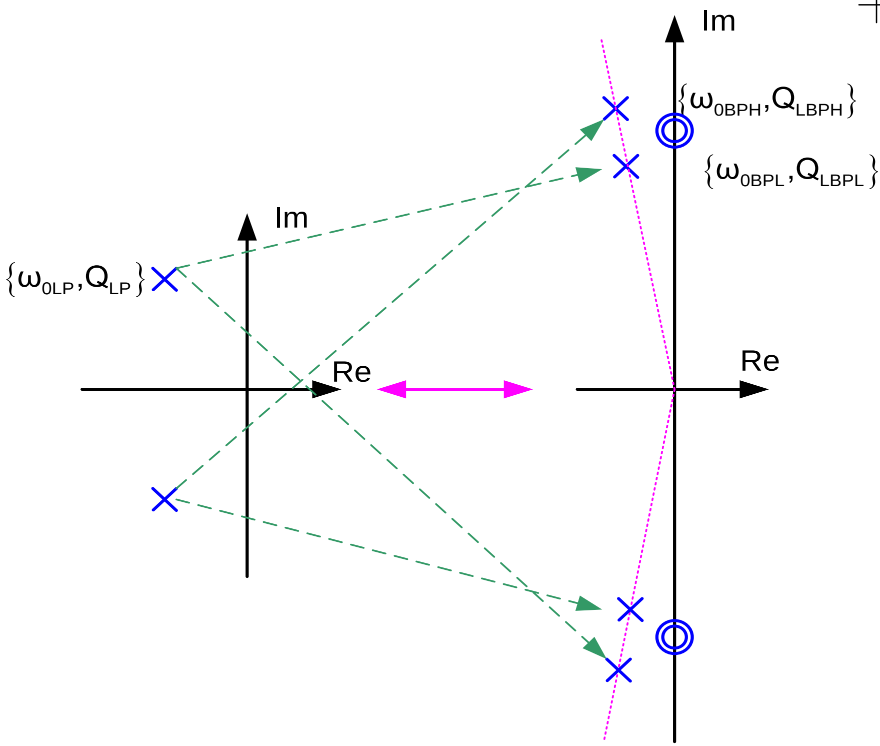


Review from Last Time

LP to BS Transformation

Pole Q of BS Approximations





$$BW = \omega_{BN} - \omega_{AN}$$

$$\omega_M = \sqrt{\omega_{AN}\omega_{BN}}$$

Define: $$\gamma = \left(\frac{BW}{\omega_M \omega_{0LP}}\right)$$

It can be shown that

$$Q_{BSL} = Q_{BSH} = \frac{Q_{LP}}{\sqrt{2}}\sqrt{1+\frac{4}{\gamma^2}+\sqrt{\left(1+\frac{4}{\gamma^2}\right)^2-\frac{4}{\gamma^2 Q_{LP}^2}}}$$

For γ small, $$Q_{BS} \simeq \frac{2Q_{LP}}{\gamma}$$

It can be shown that

$$\omega_{0BS} = \frac{\omega_M}{2}\left[\gamma\frac{Q_{BS}}{Q_{LP}} \pm \sqrt{\left(\gamma\frac{Q_{BS}}{Q_{LP}}\right)^2-4}\right]$$

Note for γ **small, Q_{BS} can get very large**

Review from Last Time

Standard LP to BS Transformation

$$s_X \rightarrow \frac{s \bullet BW}{s^2 + \omega_M^2}$$

- Standard LP to BS transformation is a variable mapping transform
- Maps jω axis to jω axis in the s-plane
- Preserves basic shape of an approximation but warps frequency axis
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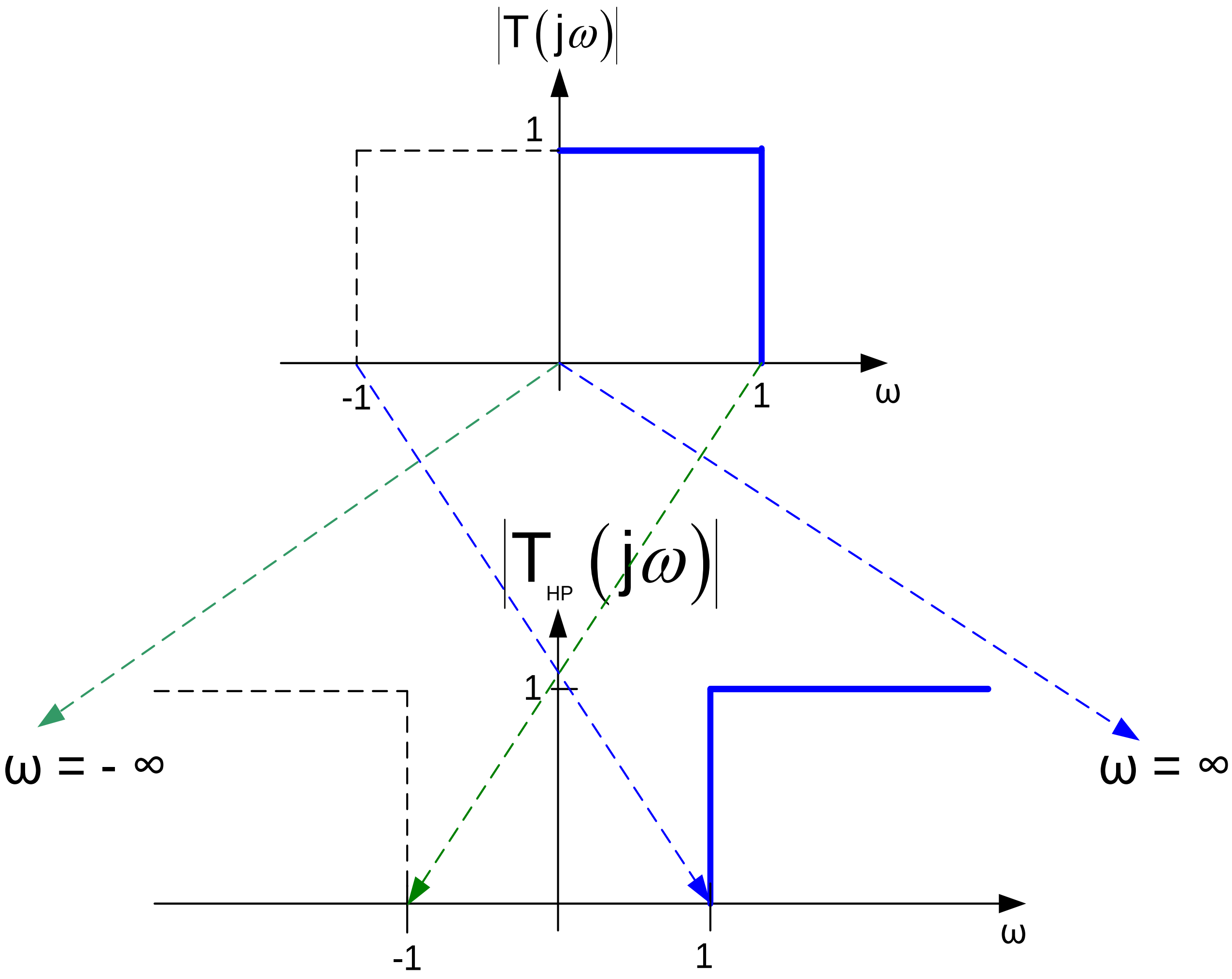
- Other variable transforms exist but the standard is by far the most popular

LP to HP Transformation

(Normalized Transform)

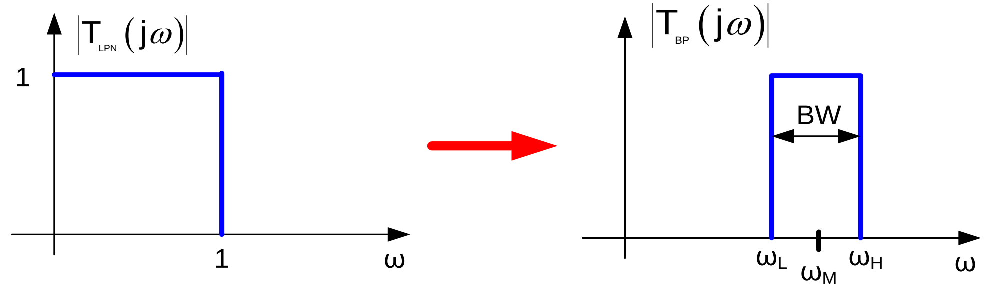


Review from Last Time

Comparison of Transforms

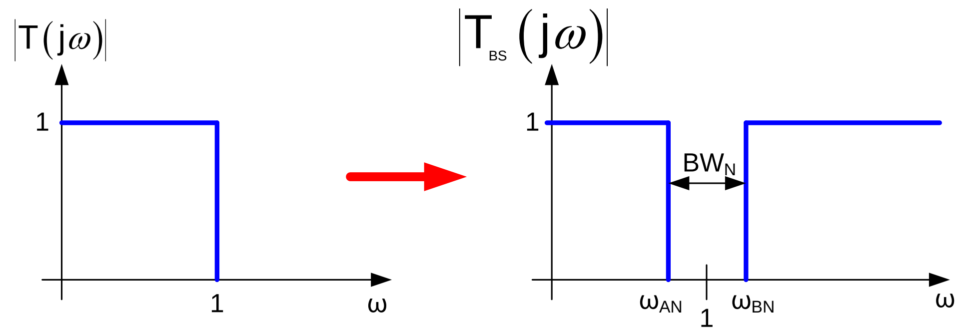
LP to BP

$$s \rightarrow \frac{s^2+1}{s \bullet BW_N}$$



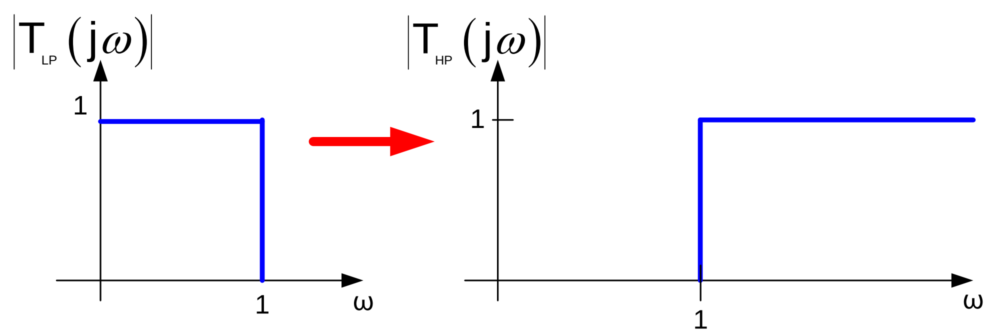
LP to BS

$$s \rightarrow \frac{s \bullet BW_N}{s^2+1}$$



LP to HP

$$s \rightarrow \frac{1}{s}$$



Filter Design Process

Establish Specifications

- possibly $T_D(s)$ or $H_D(z)$
- magnitude and phase characteristics or restrictions
- time domain requirements

Approximation

- obtain acceptable transfer functions $T_A(s)$ or $H_A(z)$
- possibly acceptable realizable time-domain responses

Synthesis

- build circuit or implement algorithm that has response close to $T_A(s)$ or $H_A(z)$
- actually realize $T_R(s)$ or $H_R(z)$

Filter

Filter Design/Synthesis Considerations

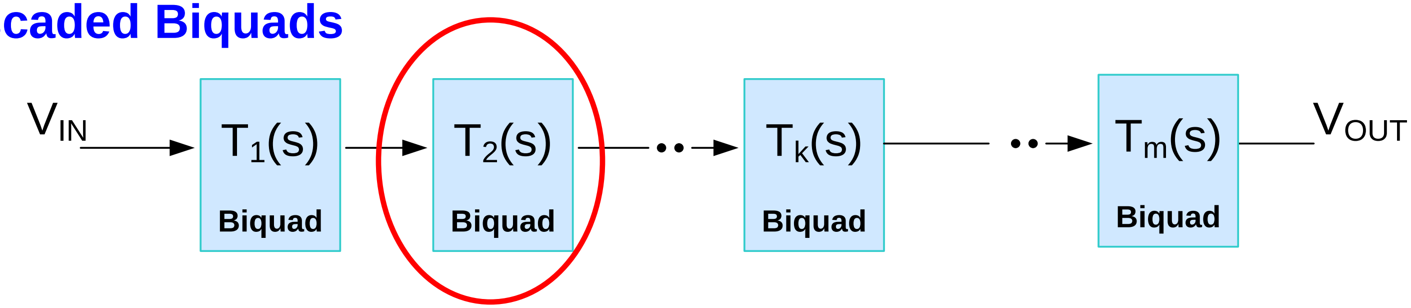
There are many different filter architectures that can realize a given transfer function

Considerable effort has been focused over the years on "inventing" these architectures and on determining which is best suited for a given application

Filter Design/Synthesis Considerations

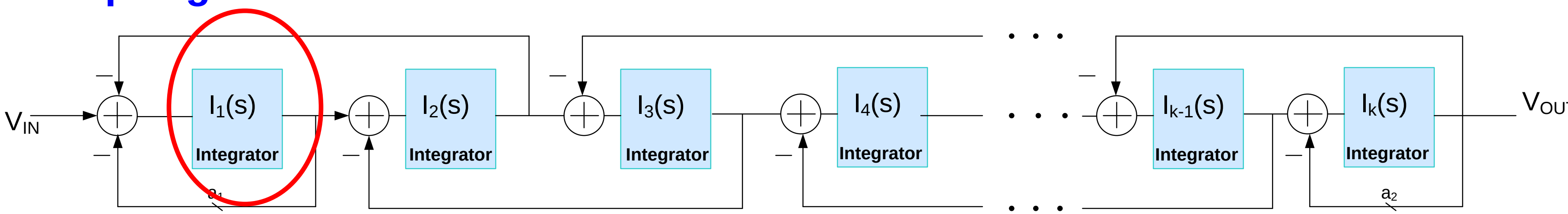
Most even-ordered designs today use one of the following three basic architectures

Cascaded Biquads

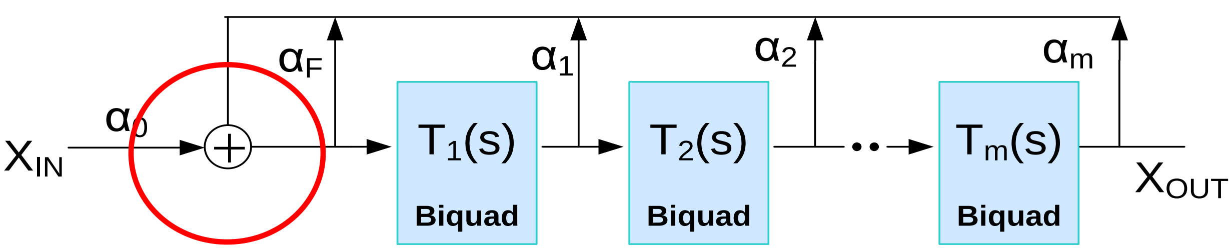


$$T(s) = T_1 T_2 \bullet\bullet T_m$$

Leapfrog



Multiple-loop Feedback (less popular)

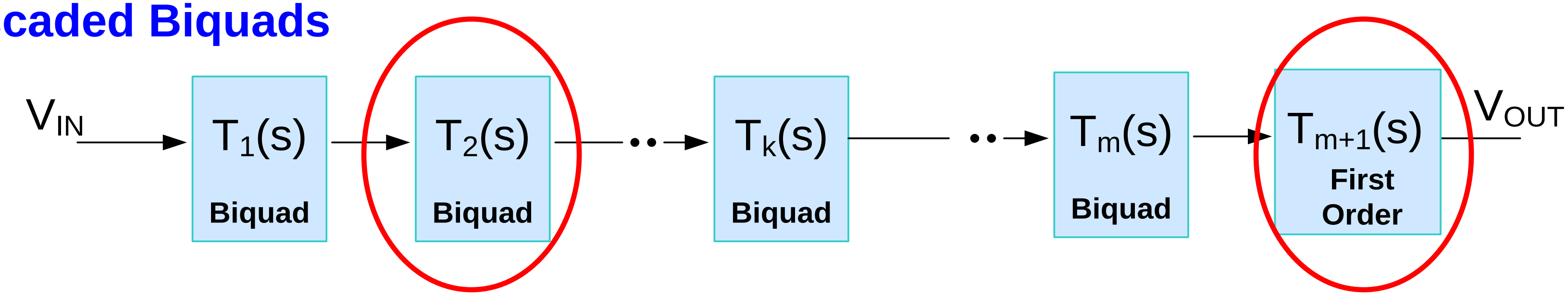


What's unique in all of these approaches?

Filter Design/Synthesis Considerations

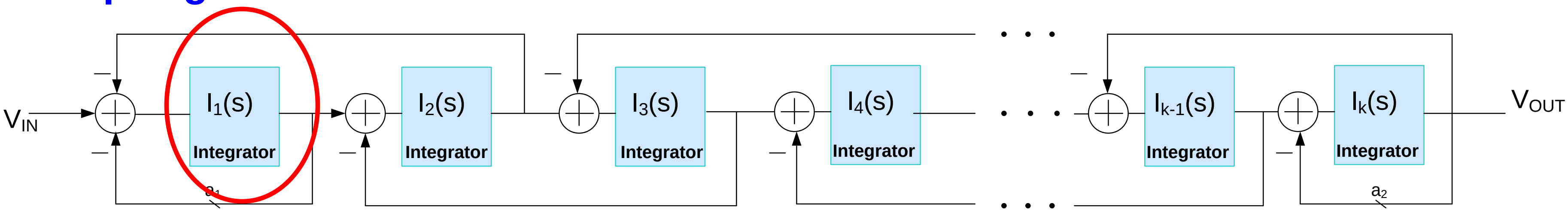
Most odd-ordered designs today use one of the following three basic architectures

Cascaded Biquads

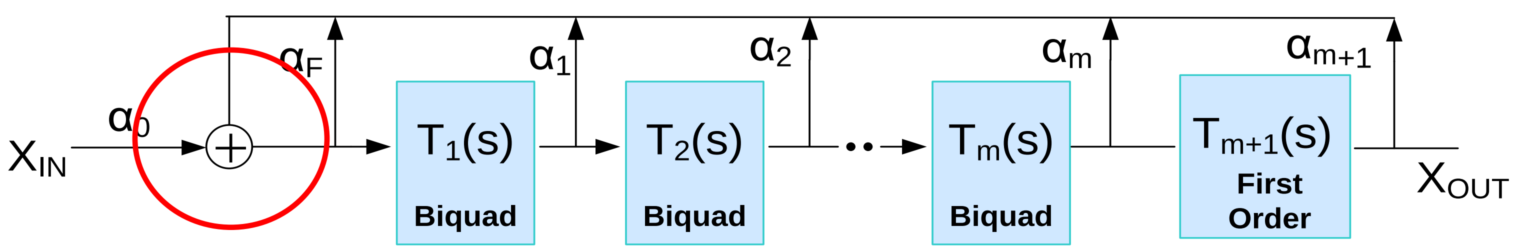


$$T(s) = T_1 T_2 \bullet\bullet T_m$$

Leapfrog



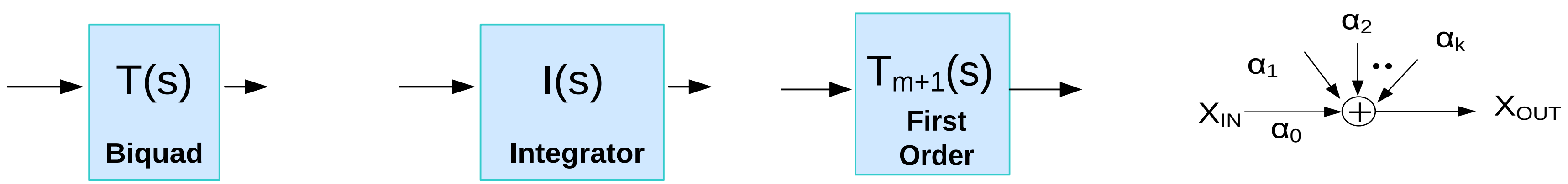
Multiple-loop Feedback (less popular)



What's unique in all of these approaches?

Filter Design/Synthesis Considerations

What's unique in all of these approaches?



$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0}$ $\quad I(s) = \frac{I_0}{s}$ $\quad T(s) = \frac{a_1 s + a_0}{s + b_0}$ $\quad X_{OUT} = \sum_{i=0}^{k} \alpha_i$

- Most effort on synthesis can focus on synthesizing these four blocks

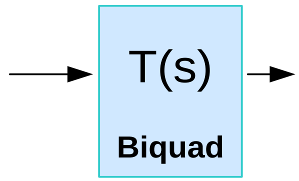
  (the summing function is often incorporated into the Biquad or Integrator)

  (the first-order block is much less challenging to design than the biquad)
- Some issues associated with their interconnections
- And, in many integrated structures, the biquads are made with integrators

  (thus, much filter design work simply focuses on the design of integrators)

Biquads

How many biquad filter functions are there?



$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0}$$

$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0} \qquad a_0 \neq 0,\ a_1 \neq 0,\ a_2 \neq 0$$

$$T(s) = \frac{a_0}{s^2 + b_1 s + b_0} \qquad a_0 \neq 0$$

$$T(s) = \frac{a_2 s^2 + a_0}{s^2 + b_1 s + b_0} \qquad a_0 \neq 0,\ a_2 \neq 0$$

$$T(s) = \frac{a_1 s}{s^2 + b_1 s + b_0} \qquad a_1 \neq 0$$

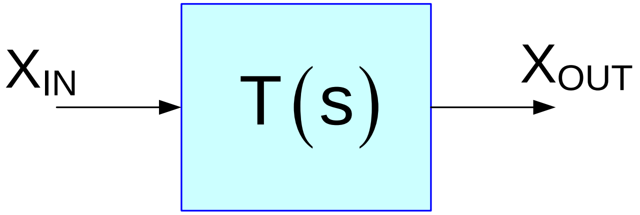
$$T(s) = \frac{a_1 s + a_0}{s^2 + b_1 s + b_0} \qquad a_0 \neq 0,\ a_1 \neq 0$$

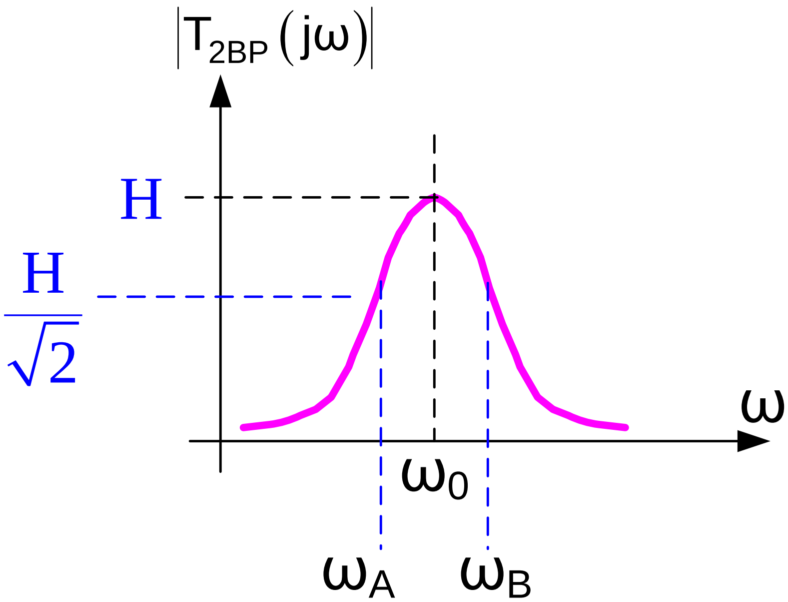
$$T(s) = \frac{a_2 s^2}{s^2 + b_1 s + b_0} \qquad a_2 \neq 0$$

$$T(s) = \frac{a_2 s^2 + a_1 s}{s^2 + b_1 s + b_0} \qquad a_2 \neq 0,\ a_1 \neq 0$$

Filter Design/Synthesis Considerations

Review: Second-order bandpass transfer function





$$\left|T_{2BP}(s)\right| = H\frac{s\left(\frac{\omega_0}{Q}\right)}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

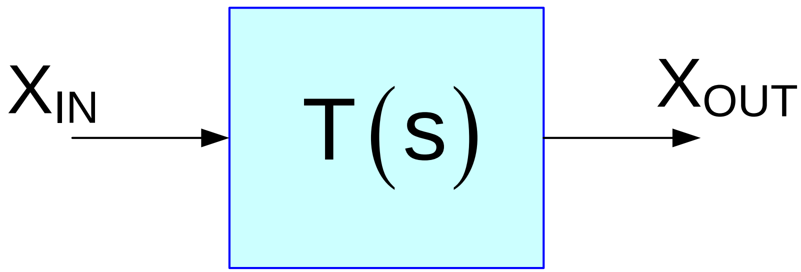
$$BW = \omega_B - \omega_A = \frac{\omega_0}{Q}$$

$$\omega_{PEAK} = \omega_0$$

Filter Design/Synthesis Considerations

There are many different filter architectures that can realize a given transfer function

Will first consider second-order Bandpass filter structures



$$|T(s)| = H \frac{s\left(\frac{\omega_0}{Q}\right)}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

$$BW = \omega_B - \omega_A = \frac{\omega_0}{Q}$$

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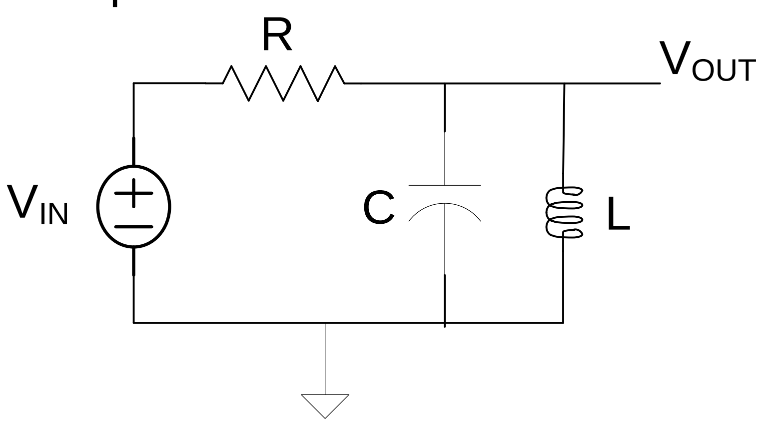
Filter Design/Synthesis Considerations

There are many different filter architectures that can realize a given transfer function

Will first consider second-order Bandpass filter structures

Example 1:



$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{1}{RC}\frac{s}{s^2 + s\left(\frac{1}{RC}\right) + \frac{1}{LC}}$$

Second-order Bandpass Filter

3 degrees of freedom

2 degrees of freedom for determining dimensionless transfer function

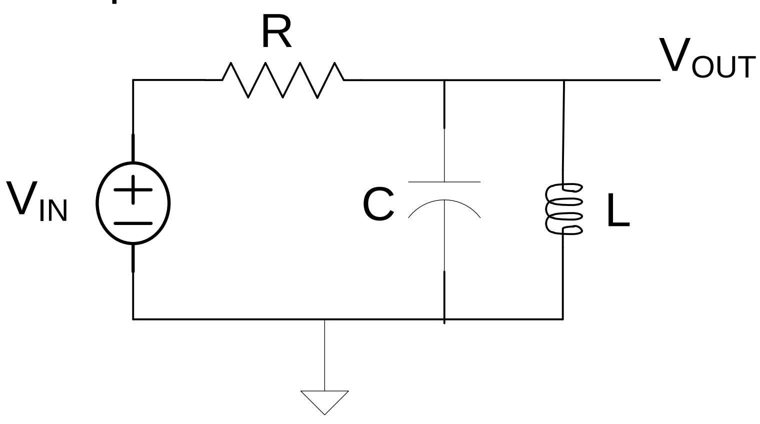
(impedance values scale)

$\omega_0 = ?$

Q = ?

BW = ?

Example 1:



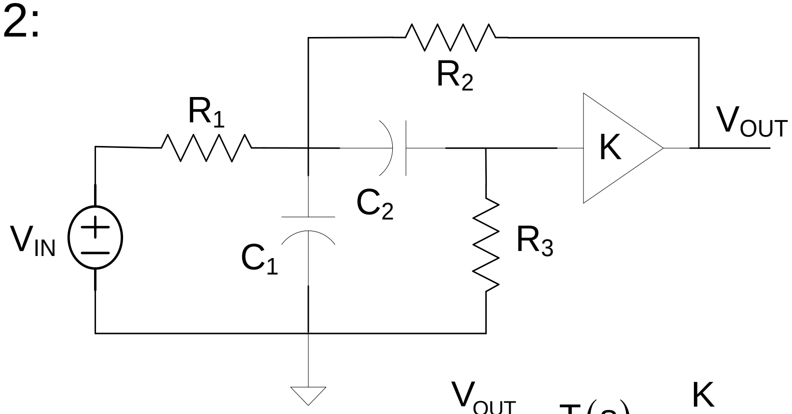
$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{1}{RC}\frac{s}{s^2+s\left(\frac{1}{RC}\right) + \frac{1}{LC}}$$

$$\omega_0 = \sqrt{\frac{1}{LC}} \qquad Q = R\sqrt{\frac{C}{L}} \qquad BW = \frac{1}{RC}$$

Can realize an arbitrary 2nd order bandpass function within a gain factor

Simple design process (sequential but not independent control of ω_0 and Q)

Example 2:



$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{K}{R_1C_1} \frac{s}{s^2+s\left(\frac{1}{R_1C_1}+\frac{1}{R_3C_1}+\frac{1}{R_3C_2}+\frac{1\text{-}K}{R_2C_1}\right) + \frac{R_1+R_2}{R_1R_2R_3C_1C_2}}$$

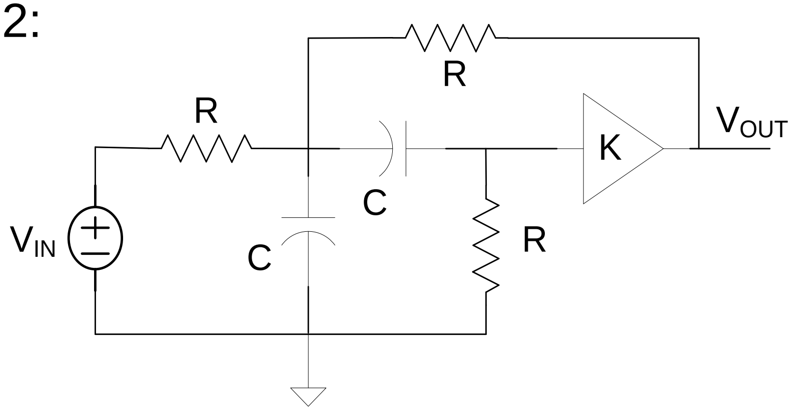
Second-order Bandpass Filter

6 degrees of freedom (effectively 5 because dimensionless)

Denote as a +KRC filter

$\omega_0 = ?$ Q = ? BW = ?

Example 2:



Equal R, Equal C Realization

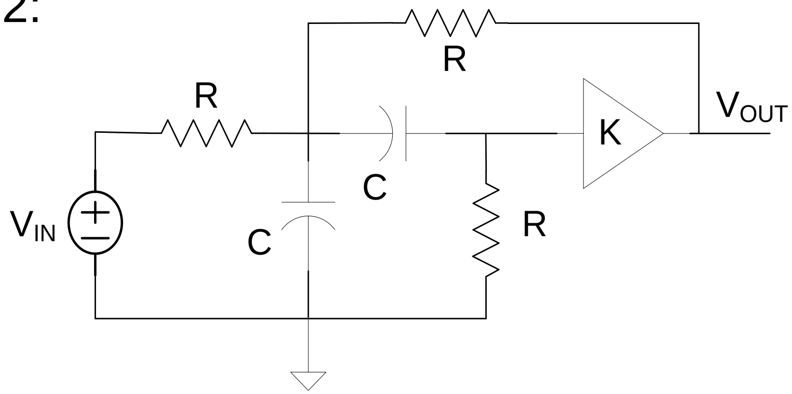
$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{K}{RC} \frac{s}{s^2 + s\left(\frac{4-K}{RC}\right) + \frac{2}{(RC)^2}}$$

$\omega_0 = ?$

Q = ?

BW = ?

Example 2:



Equal R, Equal C Realization

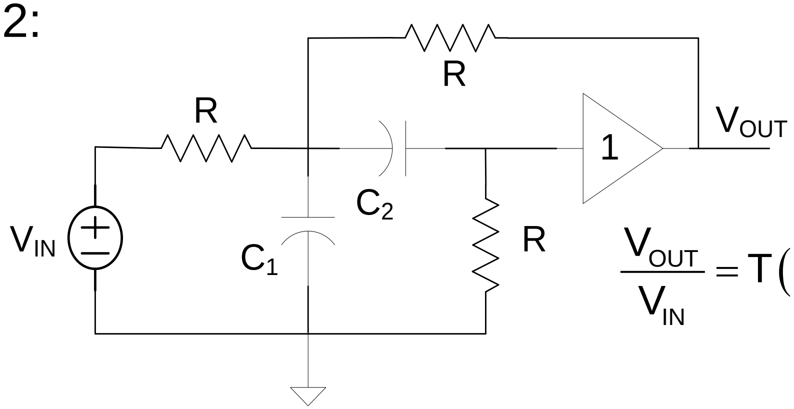
$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{K}{RC}\frac{s}{s^2 + s\left(\frac{4\text{-}K}{RC}\right) + \frac{2}{(RC)^2}}$$

$$\omega_0 = \frac{\sqrt{2}}{RC} \qquad Q = \frac{\sqrt{2}}{4\text{-}K} \qquad BW = \frac{4\text{-}K}{RC}$$

3 degrees of freedom (effectively 2 since dimensionless)

- Can satisfy arbitrary 2nd=order BP constraints within a gain factor with this circuit
- Very simple circuit structure
- Independent control of ω_0 and Q but requires tuning more than one component
- Can actually move poles in RHP by making K >4

Example 2:



Unity Gain, Equal R

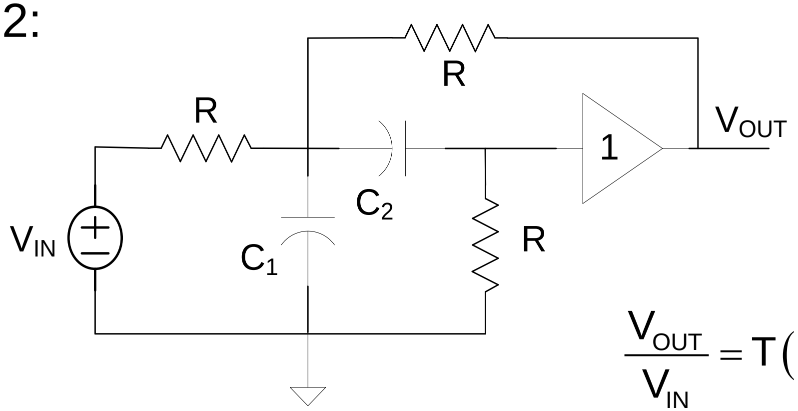
$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{1}{R\,C_1}\frac{s}{s^2 + s\left(\left[\frac{1}{R}\right]\left(\frac{2}{C_1} + \frac{1}{C_2}\right)\right) + \frac{2}{R^2 C_1 C_2}}$$

$\omega_0 = ?$

Q = ?

BW = ?

Example 2:



Unity Gain, Equal R

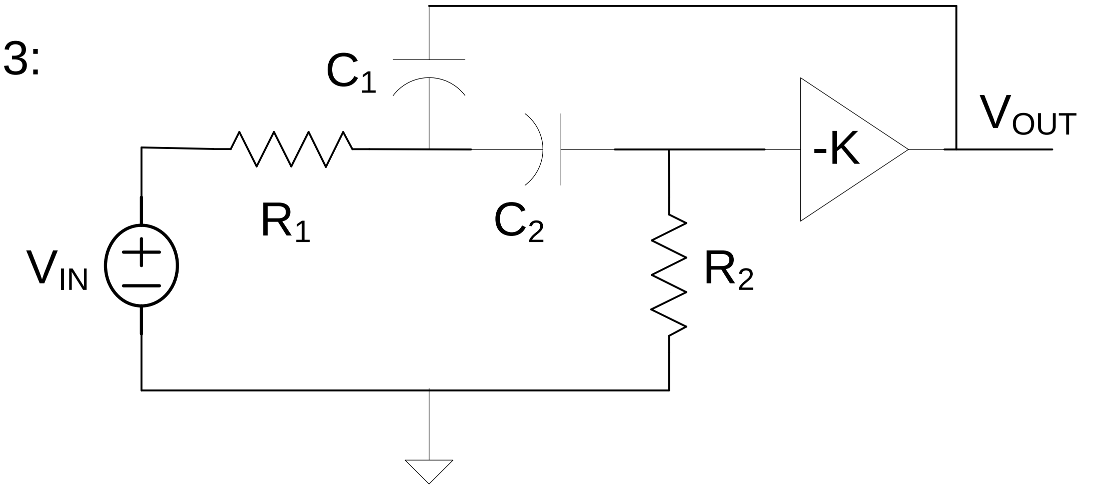
$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{1}{R\,C_1}\,\frac{s}{s^2 + s\left(\left[\frac{1}{R}\right]\left(\frac{2}{C_1} + \frac{1}{C_2}\right)\right) + \frac{2}{R^2 C_1 C_2}}$$

$$\omega_0 = \frac{\sqrt{2}}{R\sqrt{C_1 C_2}}$$

$$Q = \sqrt{2}\sqrt{\frac{C_2}{C_1}} + \frac{1}{\sqrt{2}}\sqrt{\frac{C_1}{C_2}}$$

$$BW = \left[\frac{1}{R}\right]\left(\frac{2}{C_1} + \frac{1}{C_2}\right)$$

Example 3:



$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{K}{(1+K)R_1C_1}\frac{s}{s^2+s\left(\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1}\right]\frac{1}{(1+K)}\right)+\frac{1}{(1+K)R_1R_2C_1C_2}}$$

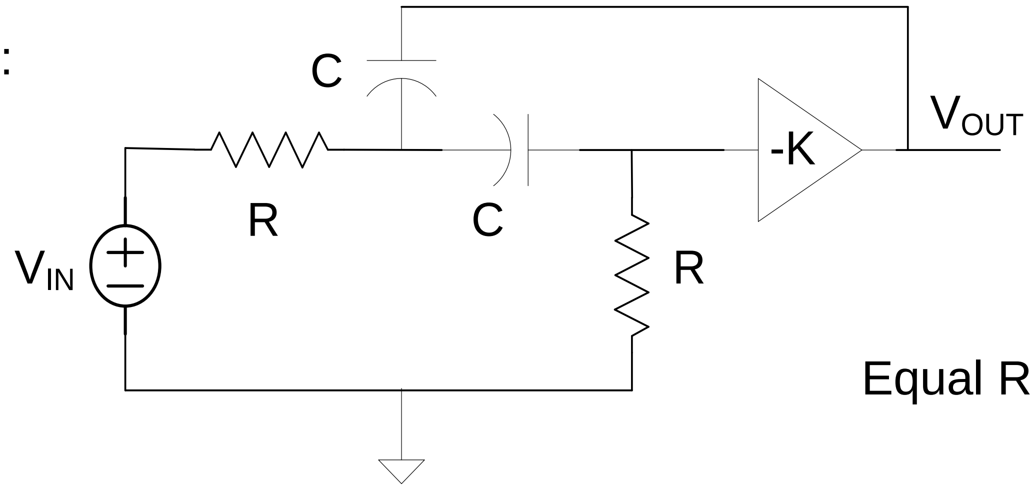
Second-order Bandpass Filter

5 degrees of freedom (4 effective since dimensionless)

Denote as a -KRC filter

$\omega_0 = ?$ Q = ? BW = ?

Example 3:



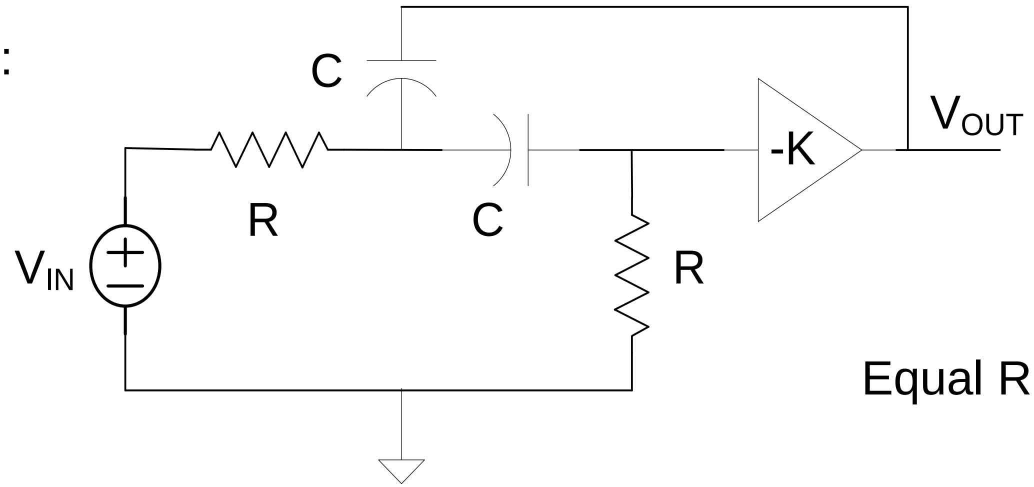
Equal R, Equal C Realization

$$\frac{V_{OUT}}{V_{IN}}=T(s)=-\frac{K}{(1+K)RC}\frac{s}{s^2+s\left(\left[\frac{3}{RC}\right]\frac{1}{(1+K)}\right)+\frac{1}{(1+K)(RC)^2}}$$

3 degrees of freedom

$\omega_0=?$ $\quad$ Q = ? $\quad$ BW = ?

Example 3:



Equal R, Equal C Realization

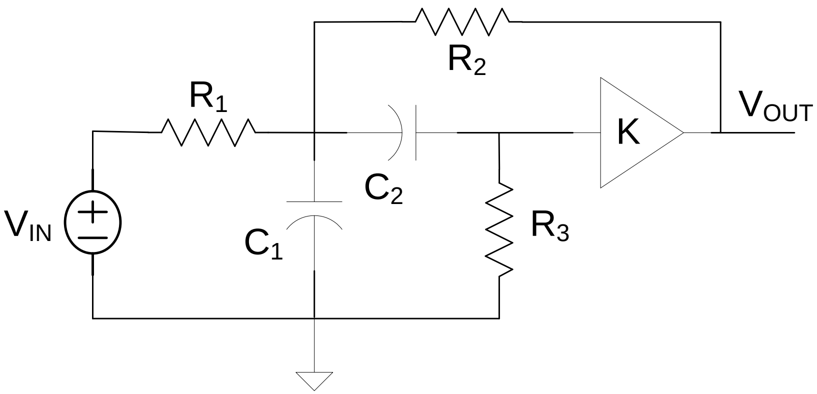
$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{K}{(1+K)RC}\frac{s}{s^2+s\left(\left[\frac{3}{RC}\right]\frac{1}{(1+K)}\right)+\frac{1}{(1+K)(RC)^2}}$$

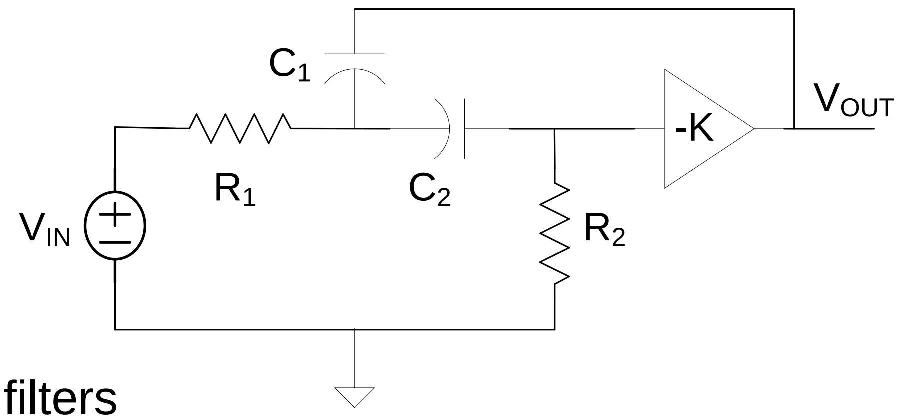
$$\omega_0 = \frac{1}{RC\sqrt{1+K}} \qquad Q = \frac{\sqrt{1+K}}{3} \qquad BW = \frac{3}{RC(1+K)}$$

3 degrees of freedom (2 effective since dimensionless)

- Can satisfy arbitrary 2nd=order BP constraints within a gain factor with this circuit
- Very simple circuit structure
- Simple design process (sequential but not independent control of ω_0 and Q, requires tuning of more than 1 component if Rs used)

Observation:





These are often termed Sallen and Key filters

Sallen and Key introduced a host of filter structures

Sallen and Key structures comprised of summers,
RC network, and finite gain amplifiers

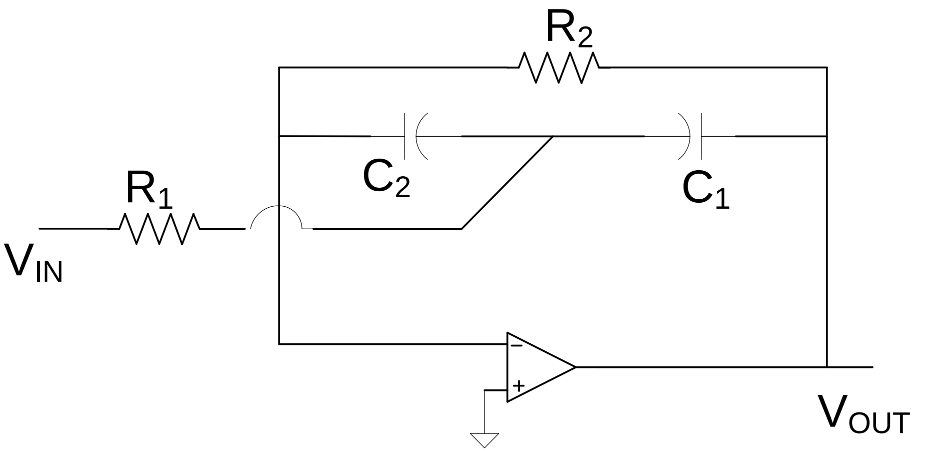
These filters were really ahead of their time and appeared long before practical implementations were available

IRE TRANSACTIONS—CIRCUIT THEORY *March* 1955

A Practical Method of Designing RC Active Filters*

R. P. SALLEN† AND E. L. KEY†

Example 4:



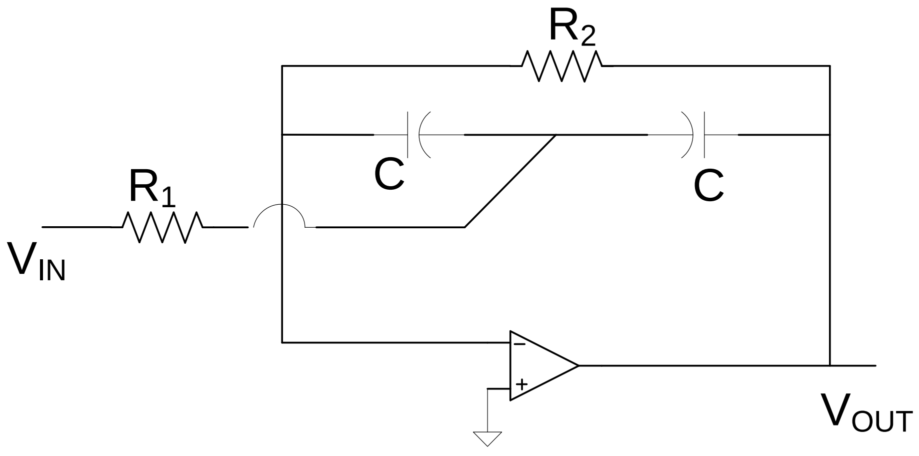
$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{1}{R_1C_1}\frac{s}{s^2+s\left(\frac{1}{R_2}\left[\frac{1}{C_1}+\frac{1}{C_2}\right]\right)+\frac{1}{R_1R_2C_1C_2}}$$

Second-order Bandpass Filter

4 degrees of freedom (3 effective since dimensionless)

Denote as a bridged T feedback structure

Example 4:



Equal C implementation

$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{1}{R_1 C} \frac{s}{s^2 + s\left(\frac{2}{CR_2}\right) + \frac{1}{R_1 R_2 C^2}}$$

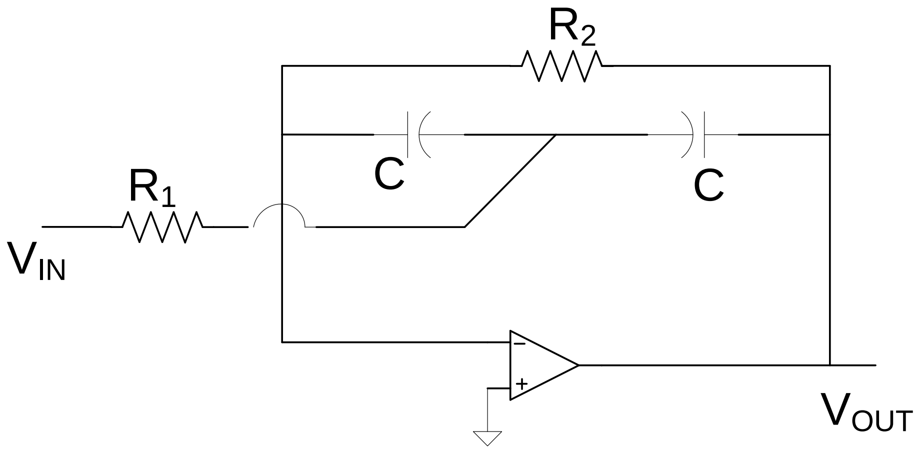
3 degrees of freedom (2 effective since dimensionless)

$\omega_0 = ?$

Q = ?

BW = ?

Example 4:



Equal C implementation

$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{1}{R_1 C}\frac{s}{s^2 + s\left(\frac{2}{CR_2}\right) + \frac{1}{R_1 R_2 C^2}}$$

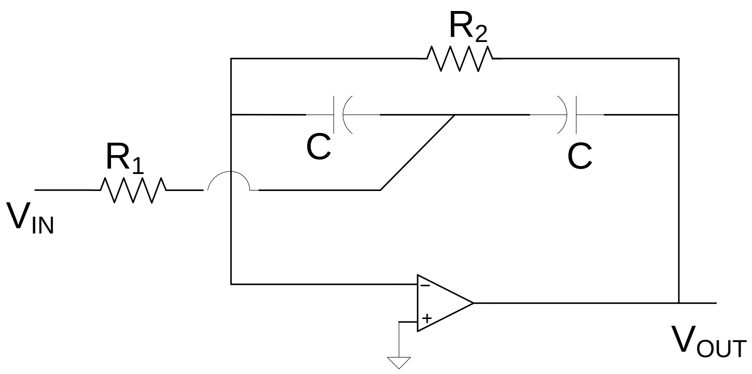
$$\omega_0 = \frac{1}{C\sqrt{R_1 R_2}} \qquad Q = \frac{1}{2}\sqrt{\frac{R_2}{R_1}} \qquad BW = \frac{2}{R_2 C}$$

Simple circuit structure

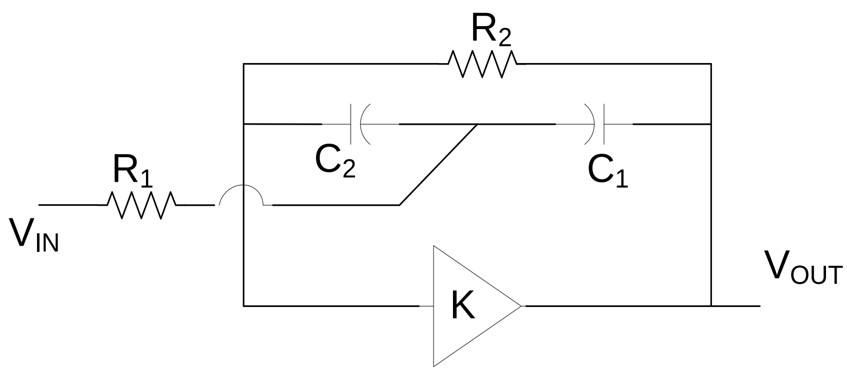
More tedious design/calibration process for ω_0 and Q (iterative if C is fixed)

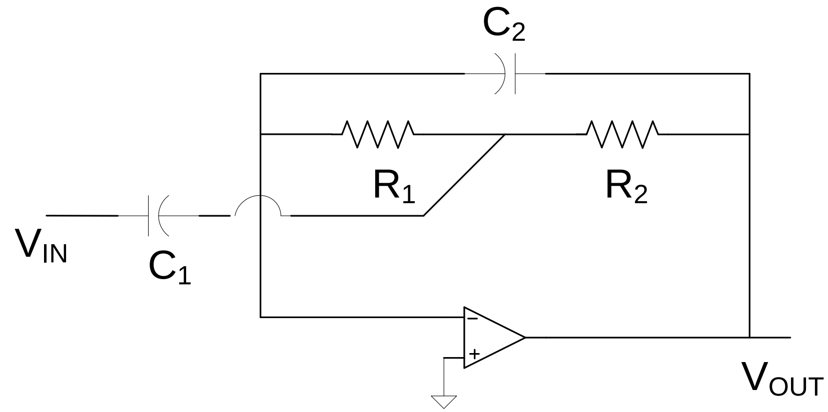
Resistor ratio is $4Q^2$

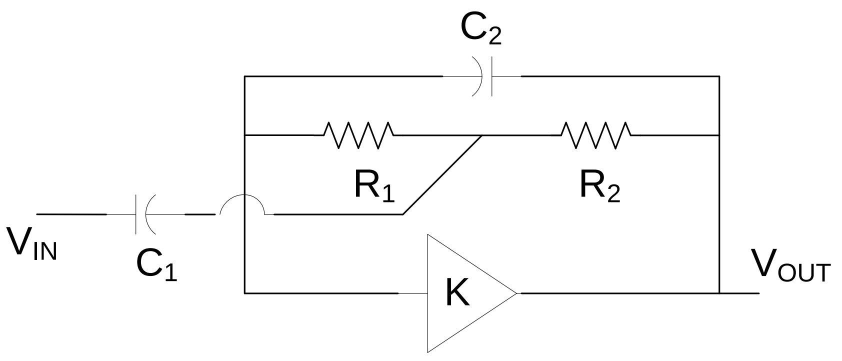
Example 4:



Some variants of the bridged-T feedback structure







Are there more 2nd order bandpass filter structures?

Yes, many other 2nd-order bandpass filter structures exist

But, if we ask the question differently

Are there more 2nd-order bandpass filter structures comprised of one amplifier and four passive components?

Yes, but not too many more

Are there more 2nd-order bandpass filter structures comprised of one amplifier, two capacitors, and three resistors?

Yes, but not too many more

Similar comments can be made about 2nd-order LP, BP, and BR

$$T(s)=\frac{H\omega_0^2}{s^2+s\left(\frac{\omega_0}{Q}\right)+\omega_0^2}$$

$$T(s)=\frac{Hs^2}{s^2+s\left(\frac{\omega_0}{Q}\right)+\omega_0^2}$$

$$T(s)=\frac{H\left(s^2+\omega_0^2\right)}{s^2+s\left(\frac{\omega_0}{Q}\right)+\omega_0^2}$$

Similar comments can be made about full 2nd-order biquadratic function

$$T(s)=H\frac{s^2+s\left(\frac{\omega_{0N}}{Q_N}\right)+\omega_{0N}^2}{s^2+s\left(\frac{\omega_0}{Q}\right)+\omega_0^2}$$

End of Lecture 18

EE 508
Lecture 19

Basic Biquadratic Active Filters

Sensitivity Functions

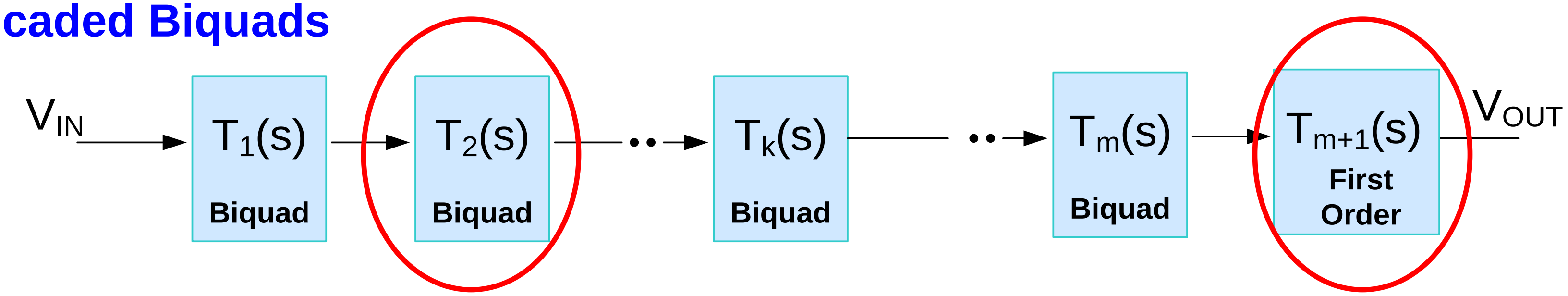
- Comparison of Filter Structures
- Performance Prediction
- Design Characterization

Review from last time

Filter Design/Synthesis Considerations

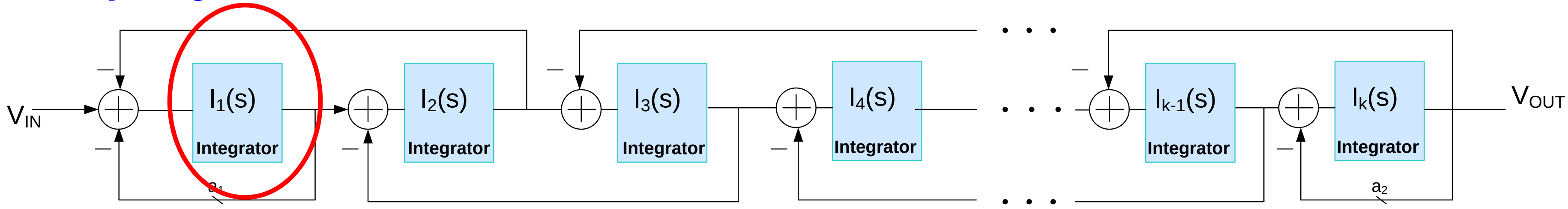
Most odd-ordered designs today use one of the following three basic architectures

Cascaded Biquads

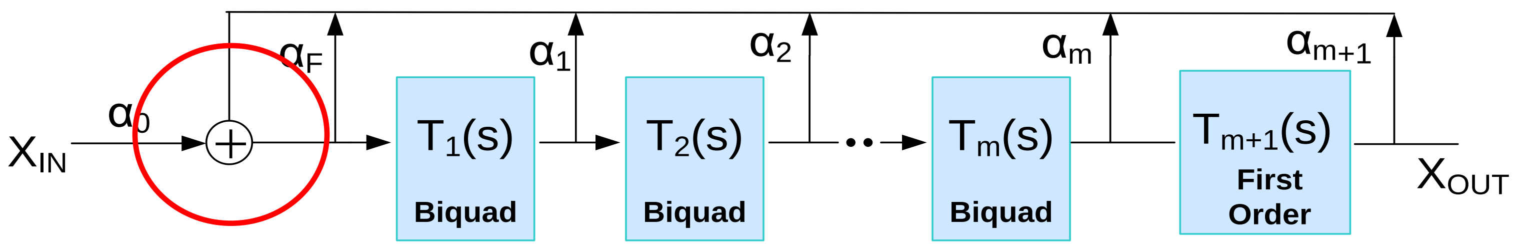


$$T(s) = T_1 T_2 \bullet\bullet T_m$$

Leapfrog



Multiple-loop Feedback (less popular)

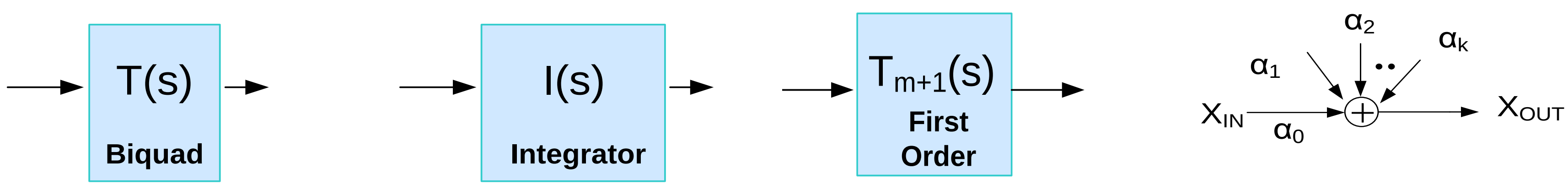


What's unique in all of these approaches?

Review from last time

Filter Design/Synthesis Considerations

What's unique in all of these approaches?



$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0}$$

$$I(s) = \frac{I_0}{s}$$

$$T(s) = \frac{a_1 s + a_0}{s + b_0}$$

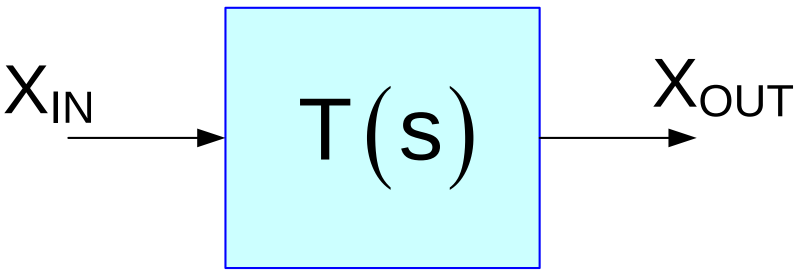
$$X_{OUT} = \sum_{i=0}^{k} \alpha_i$$

- Most effort on synthesis can focus on synthesizing these four blocks
  - (the summing function is often incorporated into the Biquad or Integrator)
  - (the first-order block is much less challenging to design than the biquad)
- Some issues associated with their interconnections
- And, in many integrated structures, the biquads are made with integrators
  - (thus, much filter design work simply focuses on the design of integrators)

Filter Design/Synthesis Considerations

There are many different filter architectures that can realize a given transfer function

Will first consider second-order Bandpass filter structures



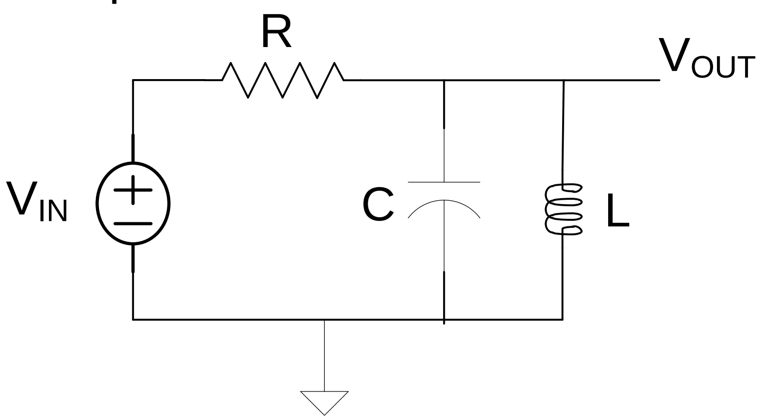
$$\left|T(s)\right| = H\frac{s\left(\frac{\omega_0}{Q}\right)}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

$$BW = \omega_B - \omega_A = \frac{\omega_0}{Q}$$

$$\omega_{PEAK} = \omega_0$$

Review from last time

Example 1:



$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{1}{RC}\,\frac{s}{s^2 + s\left(\frac{1}{RC}\right) + \frac{1}{LC}}$$

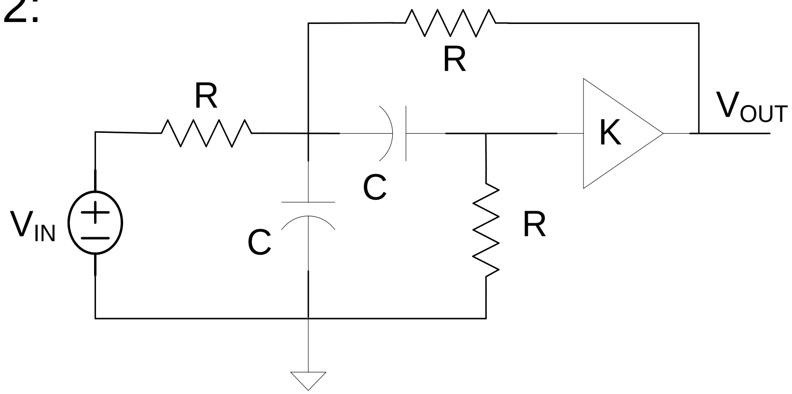
$$\omega_0 = \sqrt{\frac{1}{LC}} \qquad Q = R\sqrt{\frac{C}{L}} \qquad BW = \frac{1}{RC}$$

Can realize an arbitrary 2nd order bandpass function within a gain factor

Simple design process (sequential but not independent control of ω_0 and Q)

Review from last time

Example 2:



Equal R, Equal C Realization

$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{K}{RC}\frac{s}{s^2 + s\left(\frac{4\text{-}K}{RC}\right) + \frac{2}{(RC)^2}}$$

$$\omega_0 = \frac{\sqrt{2}}{RC} \qquad Q = \frac{\sqrt{2}}{4\text{-}K} \qquad BW = \frac{4\text{-}K}{RC}$$

3 degrees of freedom (effectively 2 since dimensionless)

Can satisfy arbitrary 2nd=order BP constraints within a gain factor with this circuit

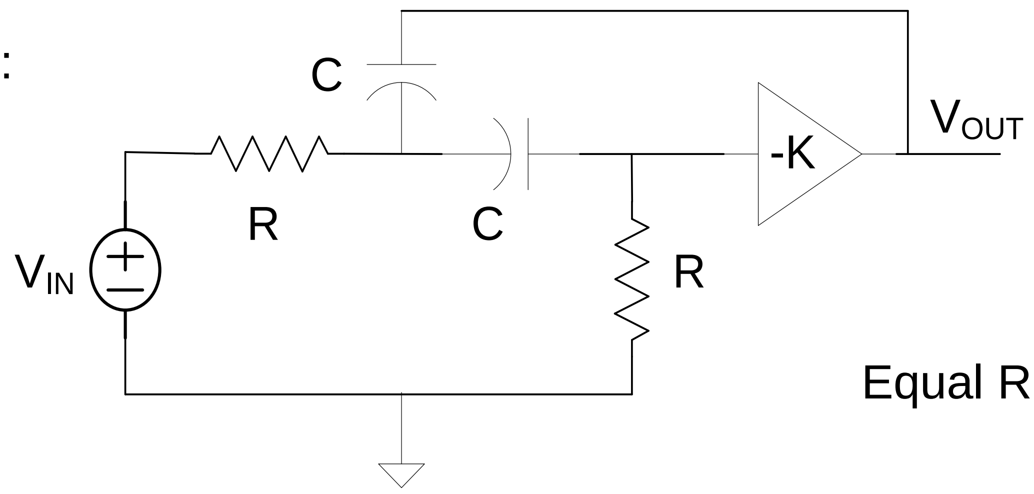
Very simple circuit structure

Independent control of ω_0 and Q but requires tuning more than one component

Can actually move poles in RHP by making K >4

Review from last time

Example 3:



Equal R, Equal C Realization

$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{K}{(1+K)RC}\frac{s}{s^2+s\left(\left[\frac{3}{RC}\right]\frac{1}{(1+K)}\right)+\frac{1}{(1+K)(RC)^2}}$$

$$\omega_0 = \frac{1}{RC\sqrt{1+K}} \qquad Q = \frac{\sqrt{1+K}}{3} \qquad BW = \frac{3}{RC(1+K)}$$

3 degrees of freedom (2 effective since dimensionless)

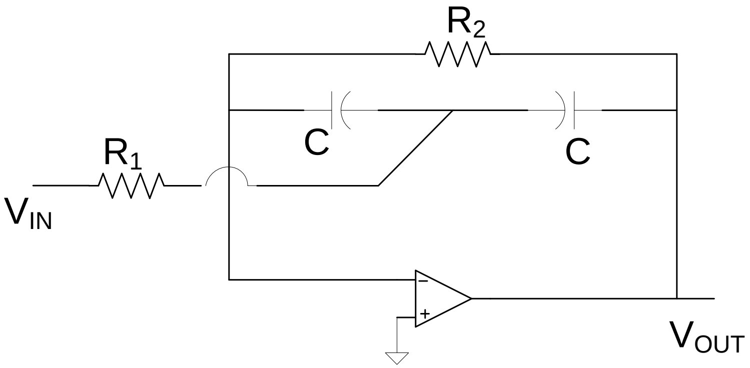
Can satisfy arbitrary 2nd=order BP constraints within a gain factor with this circuit

Very simple circuit structure

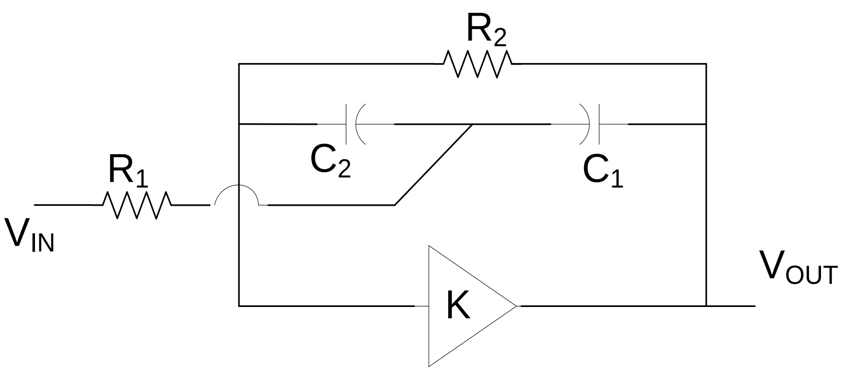
Simple design process (sequential but not independent control of ω_0 and Q, requires tuning of more than 1 component if Rs used)

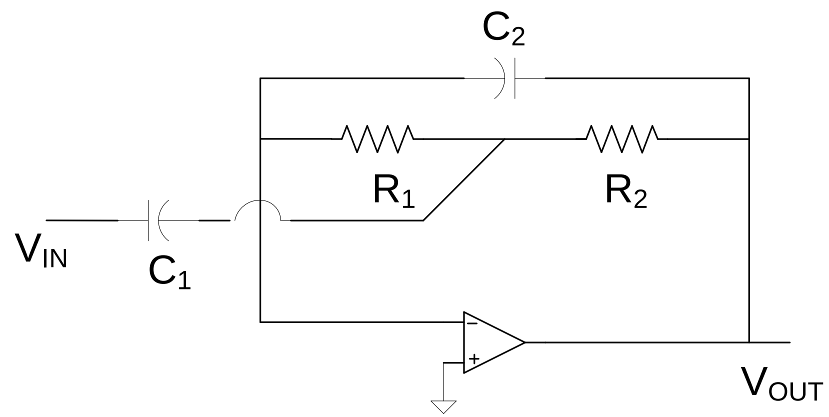
Review from last time

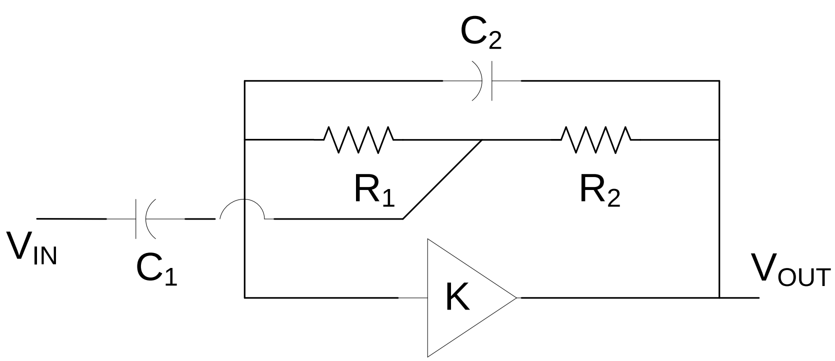
Example 4:



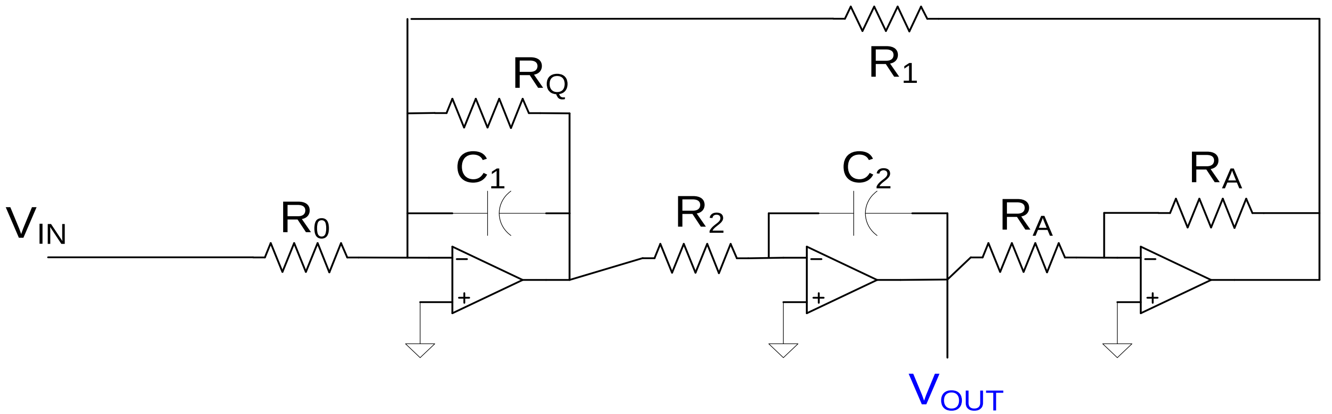
Some variants of the bridged-T feedback structure







Example 5:



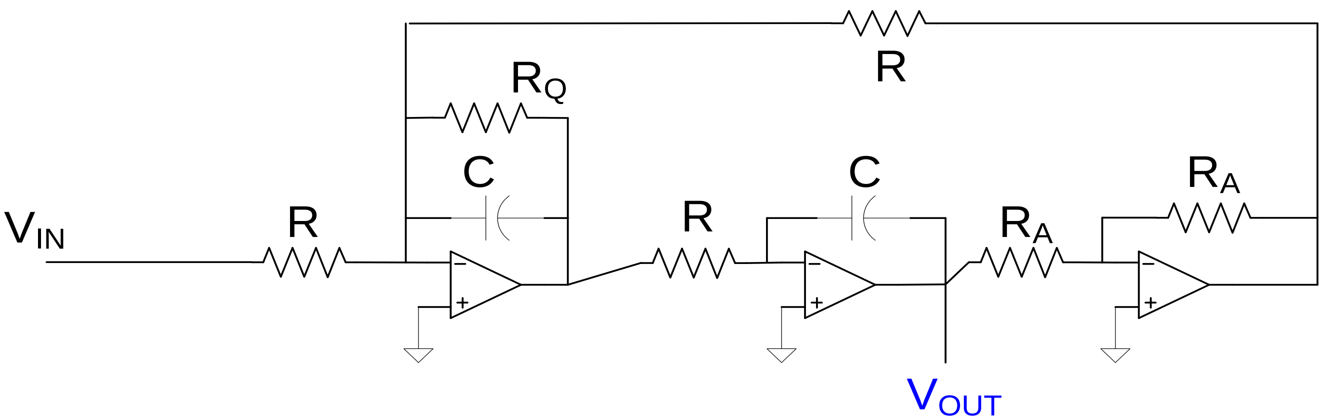
$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{1}{R_0 C_2} \frac{s}{s^2 + s\left(\frac{1}{R_Q C_2}\right) + \frac{1}{R_1 R_2 C_1 C_2}}$$

Second-order Bandpass Filter

8 degrees of freedom (effectively 7 since dimensionless)

Denote as a two-integrator-loop structure

Example 5:



Equal R Equal C
(except R_Q)

$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{1}{RC}\frac{s}{s^2+s\left(\left[\frac{R}{R_Q}\right]\frac{1}{RC}\right)+\frac{1}{(RC)^2}}$$

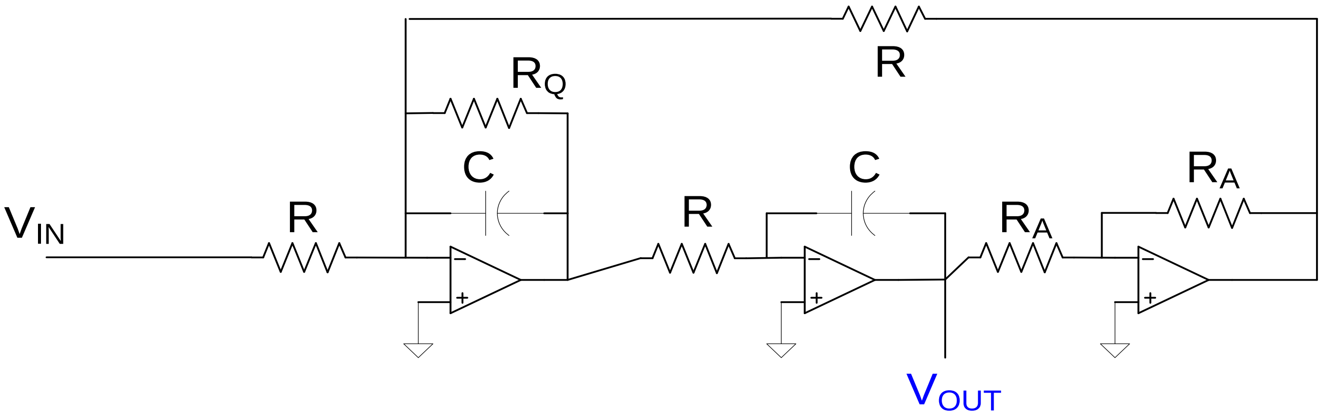
3 degrees of freedom (effectively 2 since dimensionless)

$\omega_0 = ?$

$Q = ?$

$BW = ?$

Example 5:



Equal R Equal C (except R_Q)

$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{1}{RC}\frac{s}{s^2+s\left(\left[\frac{R}{R_Q}\right]\frac{1}{RC}\right)+\frac{1}{(RC)^2}}$$

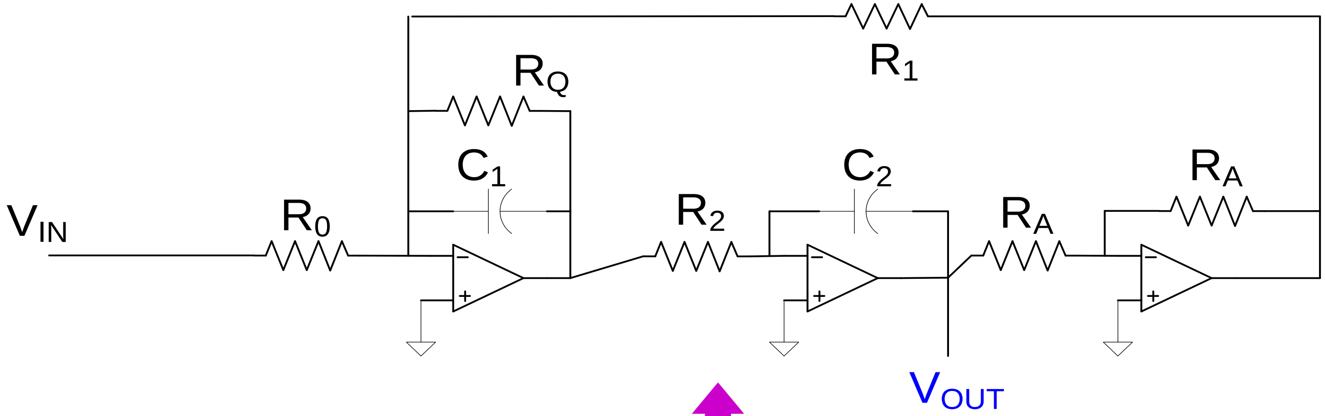
3 degrees of freedom (effectively 2 since dimensionless)

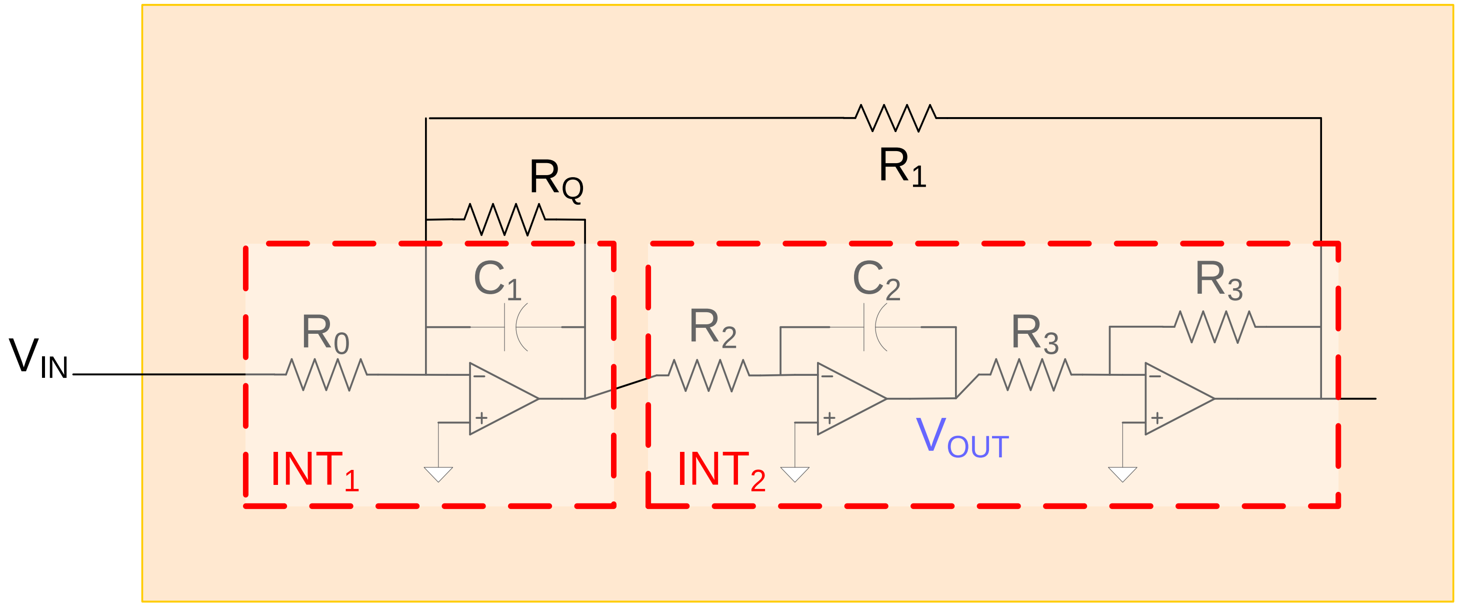
$$\omega_0 = \frac{1}{RC} \qquad Q = \frac{R_Q}{R} \qquad BW = \left[\frac{R}{R_Q}\right]\frac{1}{RC}$$

Simple design process (sequential but not independent control of ω_0 and Q with Rs, requires more tuning more than one R if Cs fixed)

Modest component spread even for large Q

Example 5:

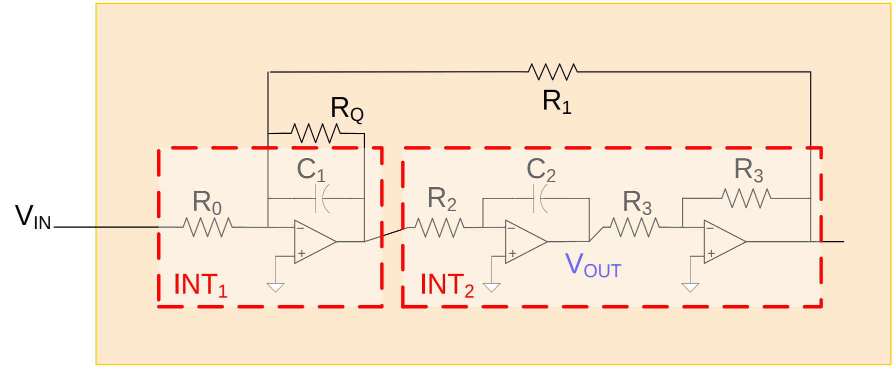


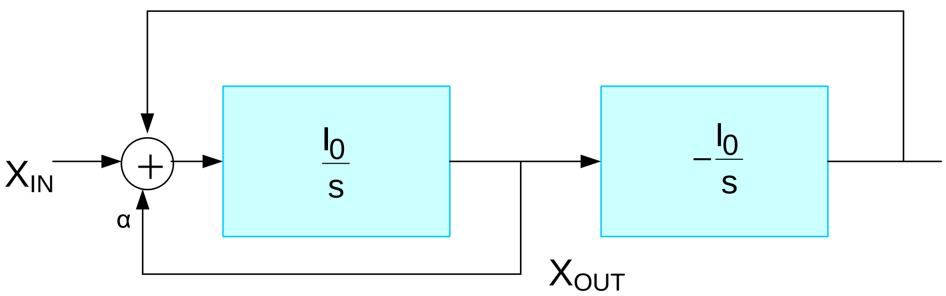


Two Integrator Loop Representation

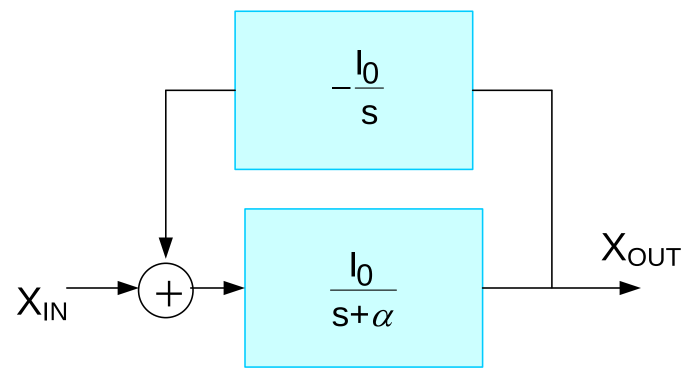
Example 5:

Two Integrator Loop Representation

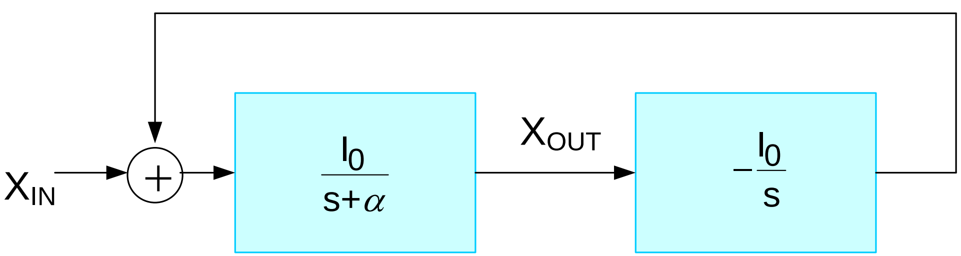




Inverting and Noninverting Integrator Loop

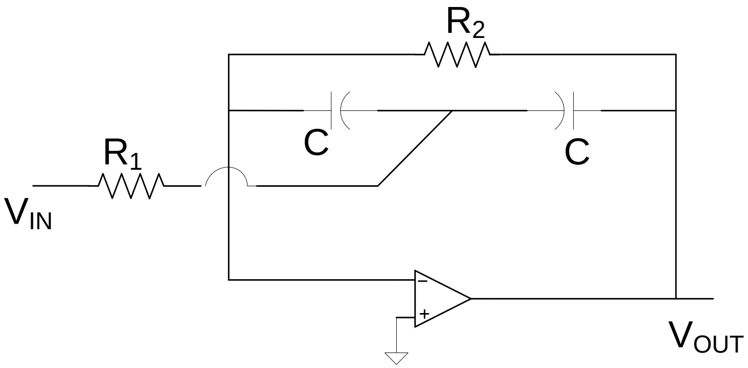


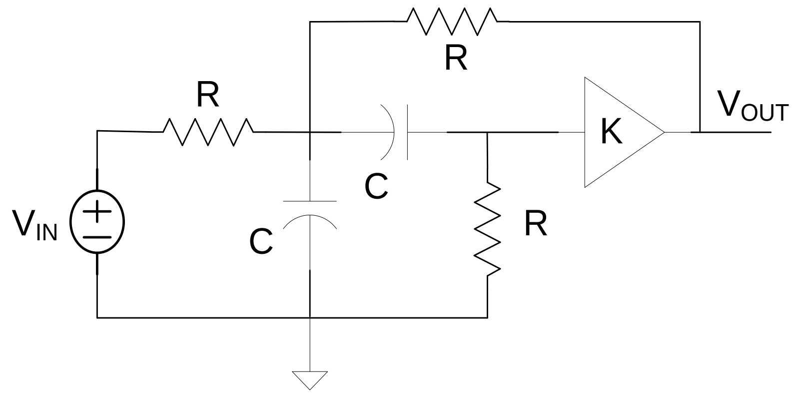
Integrator and Lossy Integrator Loop

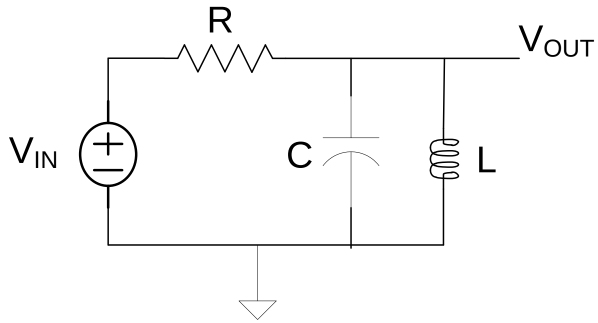


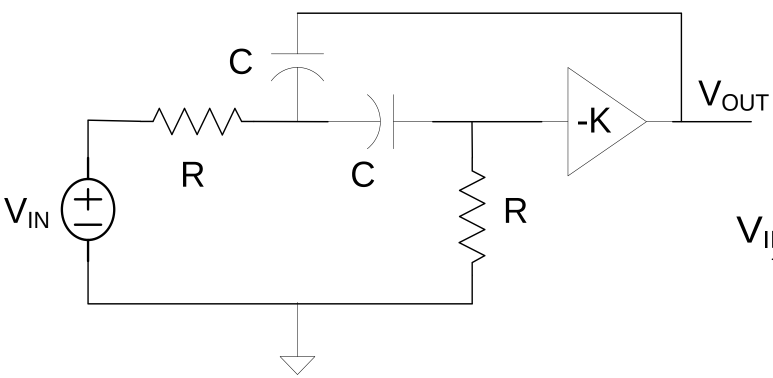
Integrator and Lossy Integrator Loop

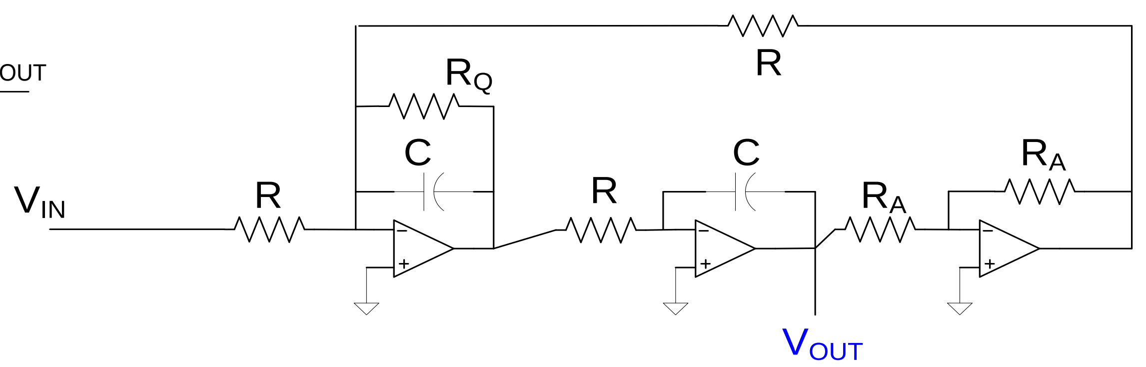
How does the performance of these bandpass filters compare?





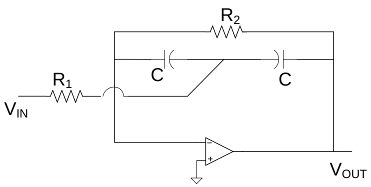


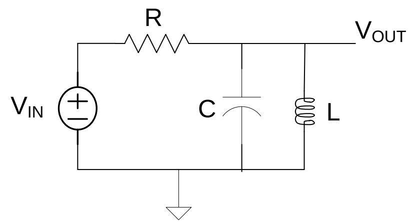


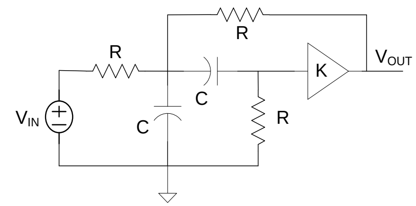


Ideally, all give same performance (within a gain factor)

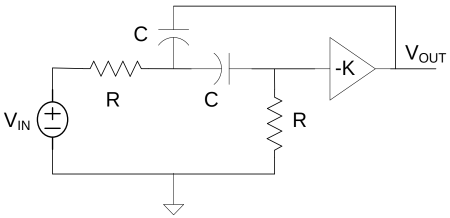
How does the performance of these bandpass filters compare?

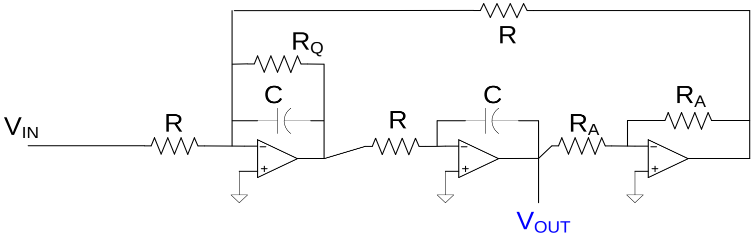






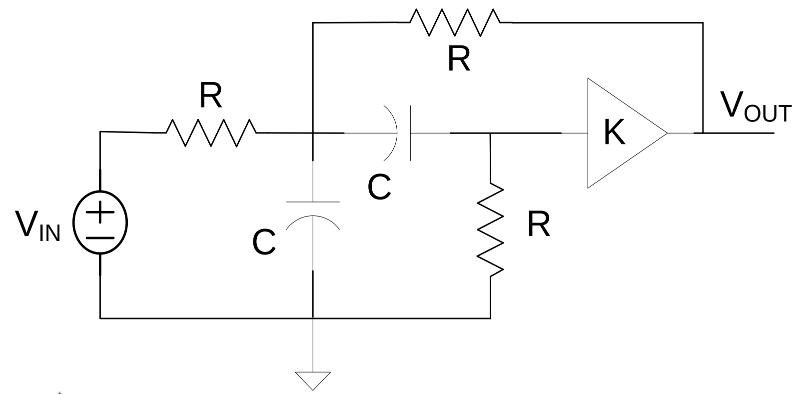
- Component Spread
- Number of Op Amps
- Is the performance strongly dependent upon how DOF are used?
- Ease of tunability/calibration (but practical structures often are not calibrated)
- Total capacitance or total resistance
- Power Dissipation
- Sensitivity
- Effects of Op Amps



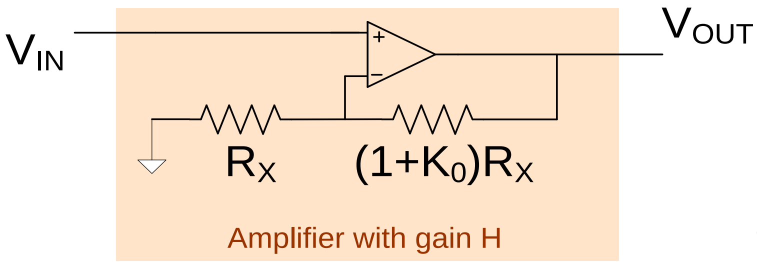


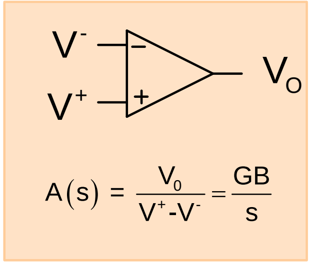
Consider effects of Op Amp on +KRC Bandpass with Equal R, Equal C

$$\omega_0 = \frac{\sqrt{2}}{RC} \qquad Q = \frac{\sqrt{2}}{4-K}$$



Assume K realized with standard Op Amp Circuit





$$K(s) = \frac{K_0}{1+\frac{K_0}{GB}s}$$

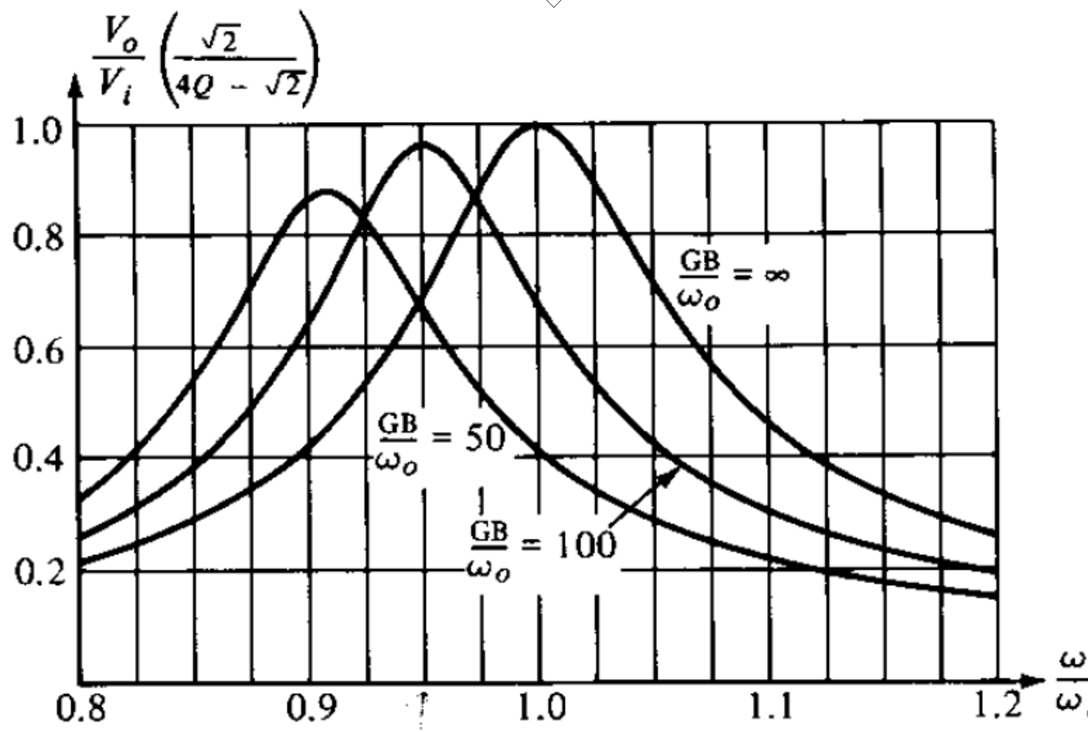
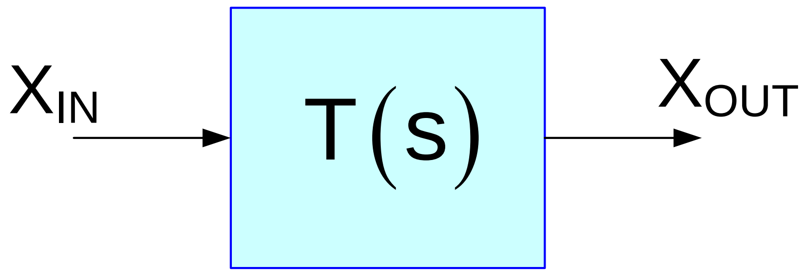


Fig. 11-4 Effect of GB on the magnitude curve for $Q = 10$

- Significant shift in peak frequency
- BW does not change very much
- Some drop in gain at peak frequency

Practically, GB/ω_0 must be must less than 100

Consider 2nd Order Lowpass Biquads

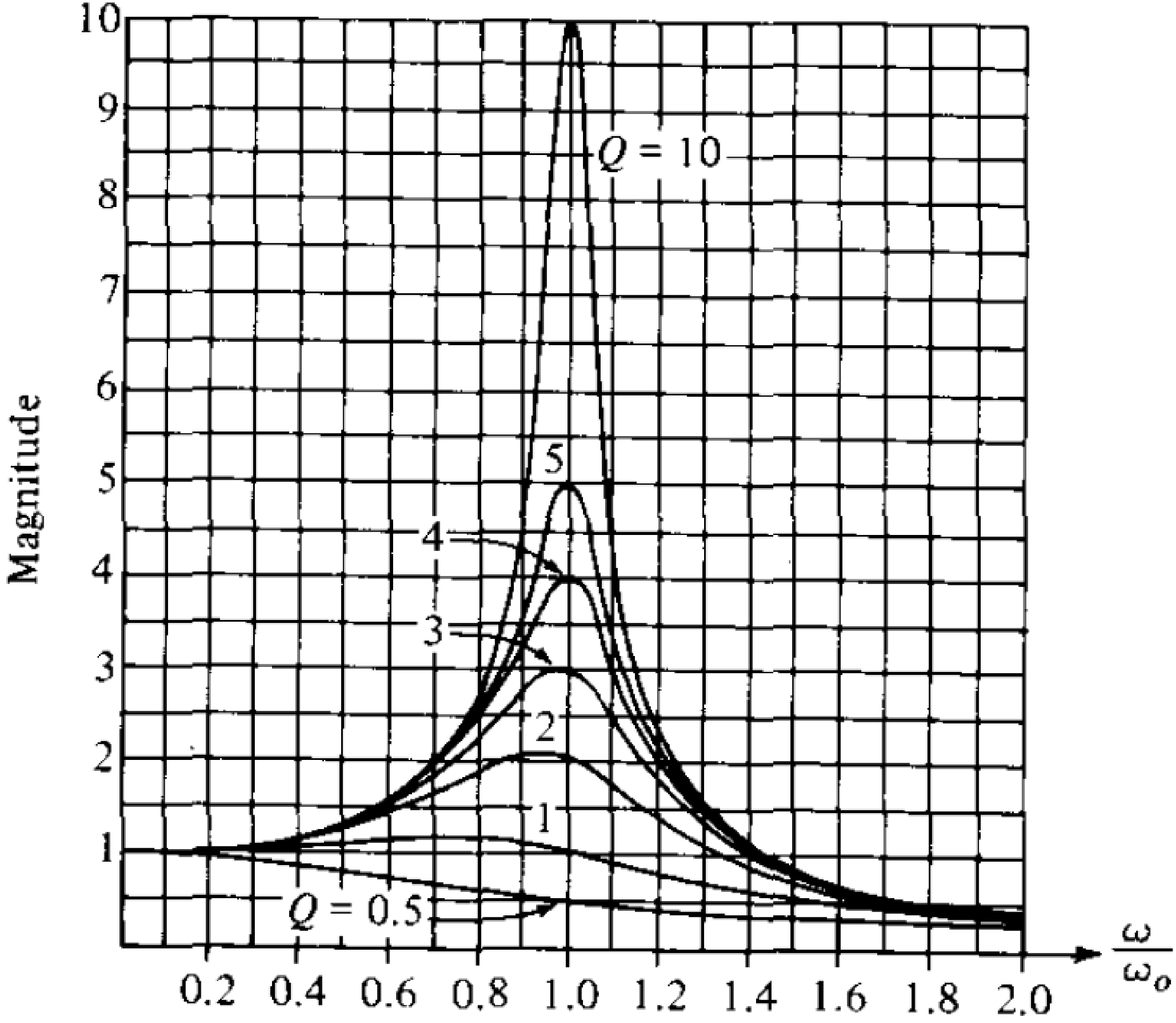


$$\left|T(s)\right| = H\frac{\omega_0^2}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

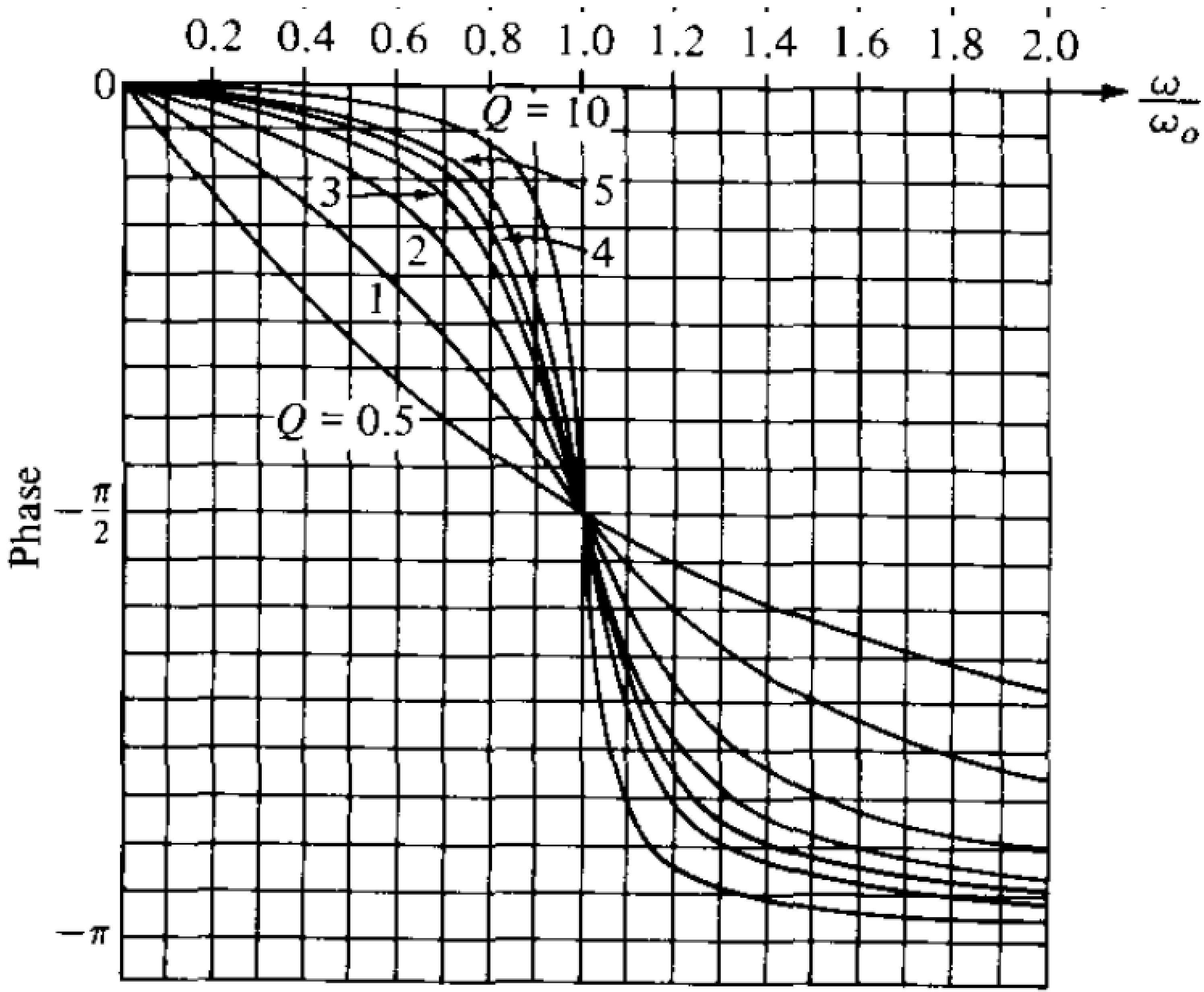
$$BW = \omega_B - \omega_A \neq \frac{\omega_0}{Q}$$

$$\omega_{PEAK} \neq \omega_0$$

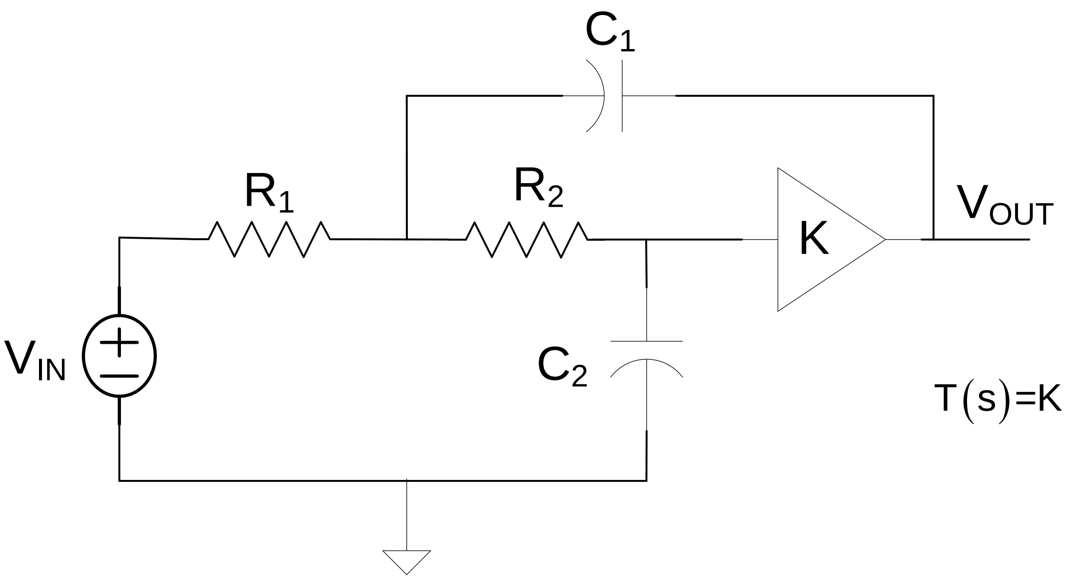
Consider 2nd Order Lowpass Biquads



Consider 2nd Order Lowpass Biquads

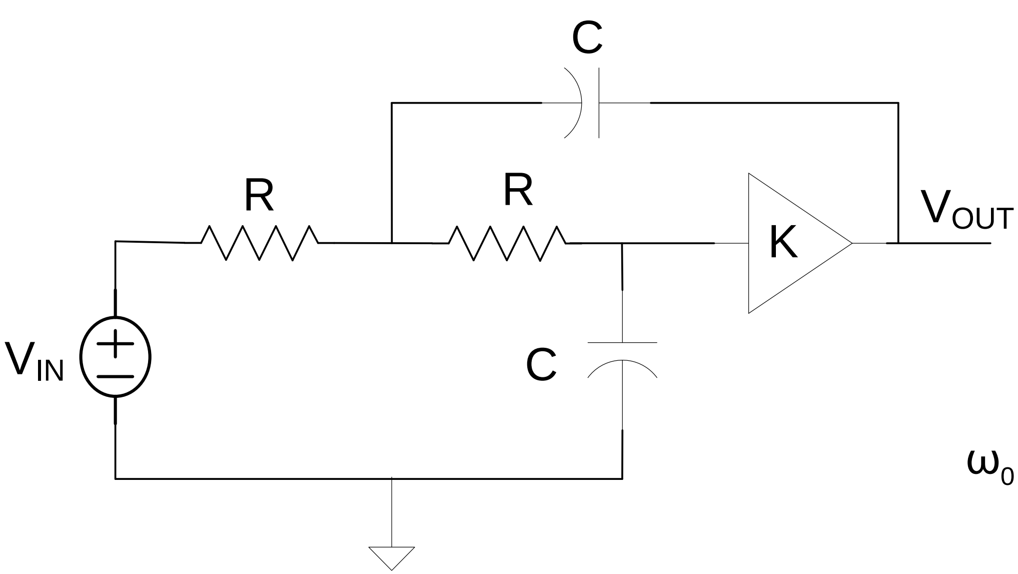


Example: 2nd Order +KRC Lowpass



$$T(s)=K\frac{\frac{1}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1-K}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}}$$

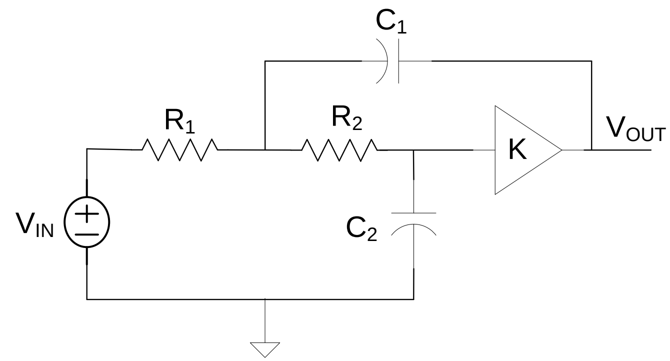
Equal R, Equal C

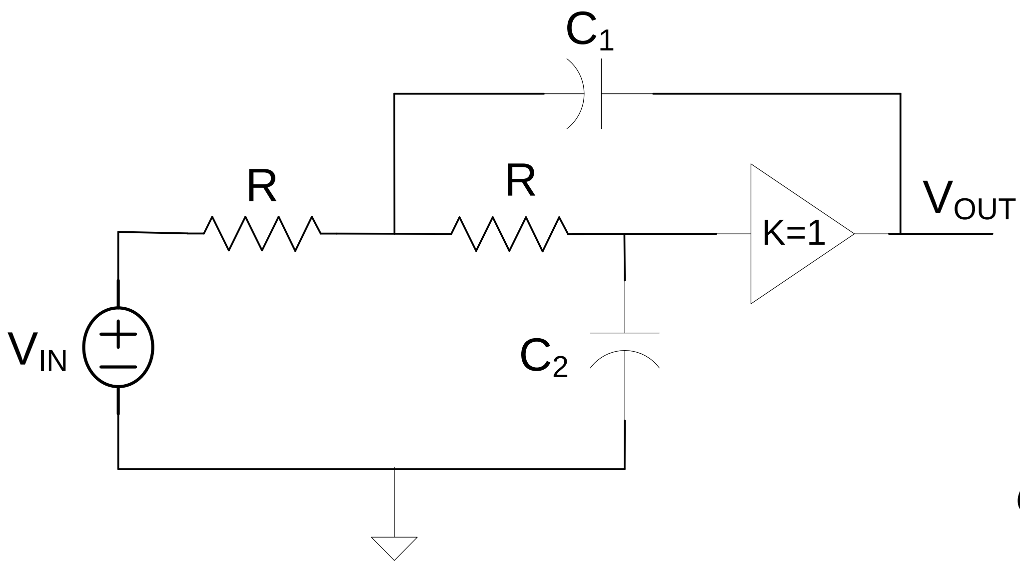


$$T(s)=K\frac{\frac{1}{R^2C^2}}{s^2+s\left[\frac{(3-K)}{RC}\right]+\frac{1}{R^2C^2}}$$

$$\omega_0=\frac{1}{RC}\qquad Q=\frac{1}{3-K}$$

Example: 2nd Order +KRC Lowpass

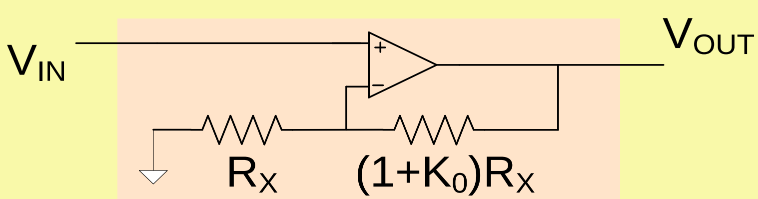




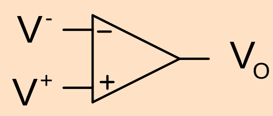
Equal R, K=1

$$T(s) = K\frac{\frac{1}{R^2C_1C_2}}{s^2 + s\left[\frac{2}{RC_1}\right] + \frac{1}{R^2C_1C_2}}$$

$$\omega_0 = \frac{1}{R\sqrt{C_1C_2}} \qquad Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}}$$



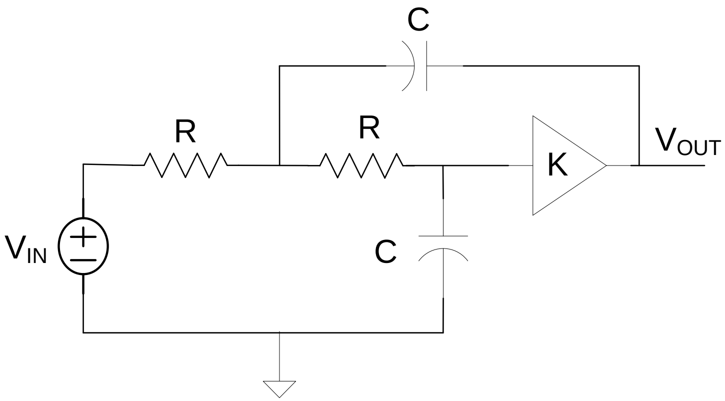
Amplifier with gain H



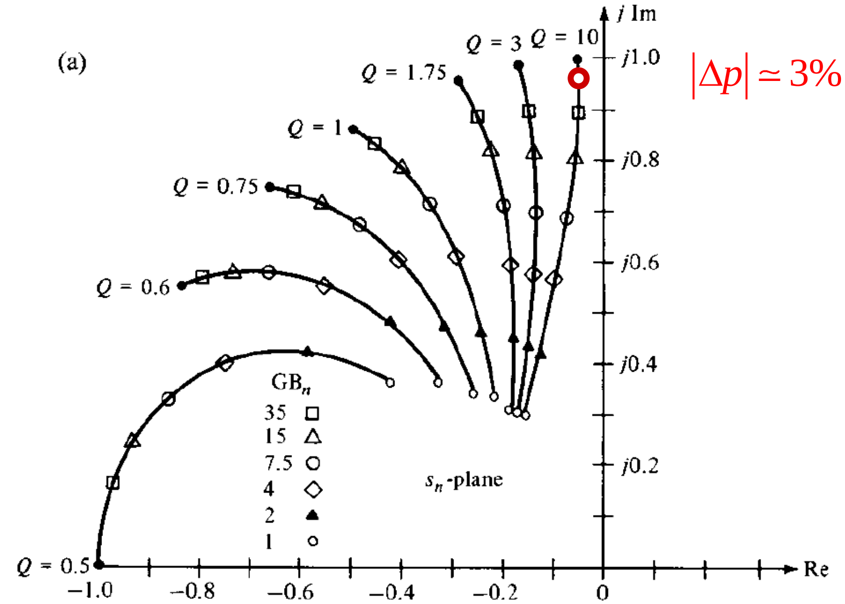
$$A(s) = \frac{V_0}{V^+ - V^-} = \frac{GB}{s}$$

$$K(s) = \frac{K_0}{1 + \frac{K_0}{GB}s}$$

Example: 2nd Order +KRC Lowpass

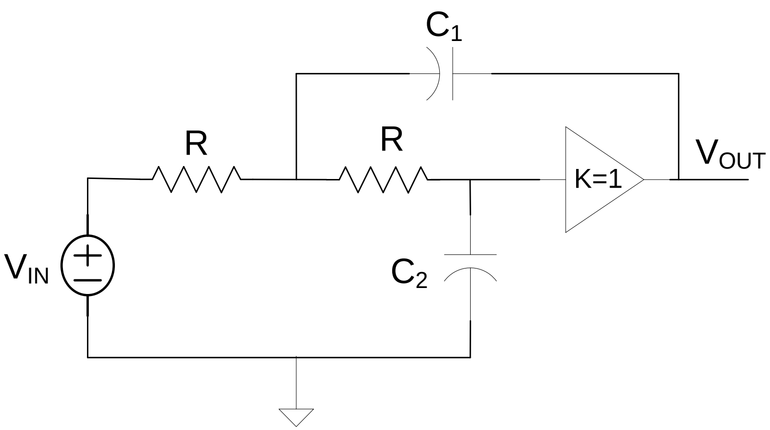


Equal R, Equal C

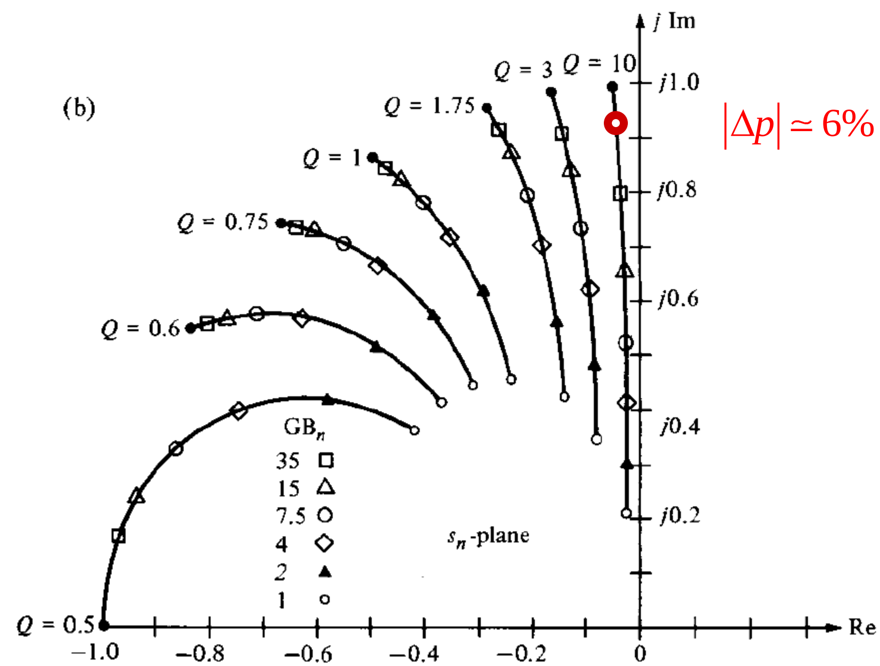


consider

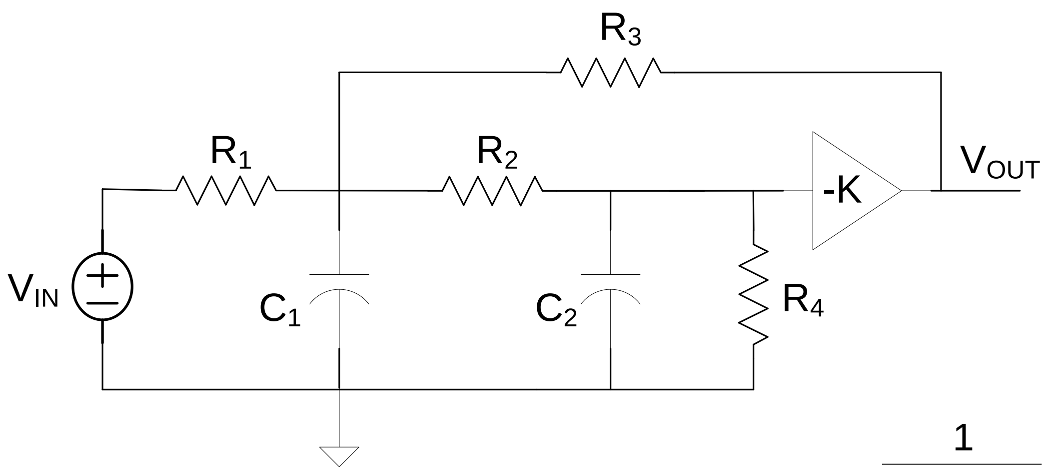
⊙ ⟷ $GB_n = \dfrac{GB}{\omega_0} = .01$



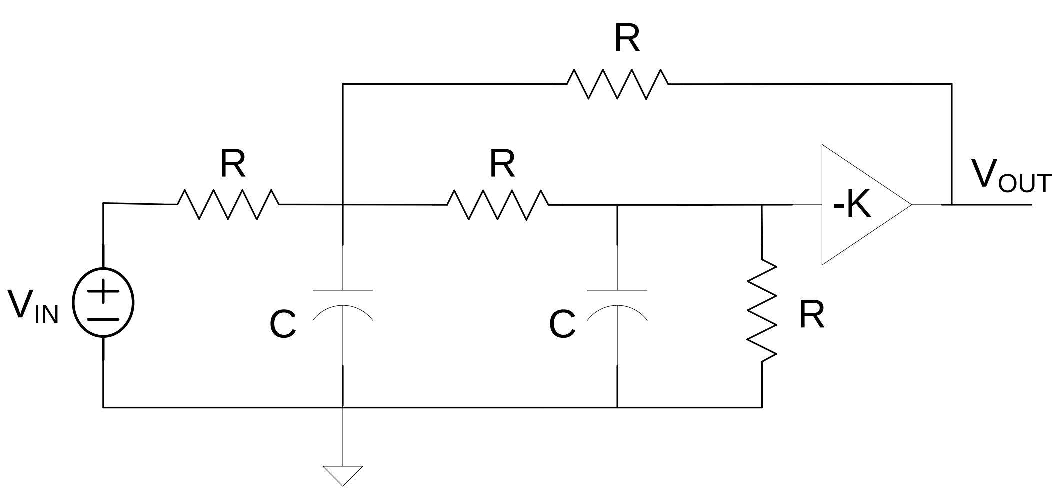
Equal R, K=1



Example: 2nd Order -KRC Lowpass



$$T(s) = -K\frac{\dfrac{1}{R_1R_2C_1C_2}}{s^2+s\left[\dfrac{1}{R_1C_1}\left(1+\dfrac{R_1}{R_3}\right)+\dfrac{1}{R_4C_2}+\dfrac{1}{R_2C_2}\left(1+\dfrac{C_2}{C_1}\right)\right]+\left[\dfrac{1+(R_1/R_3)(1+K)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}\right]}$$

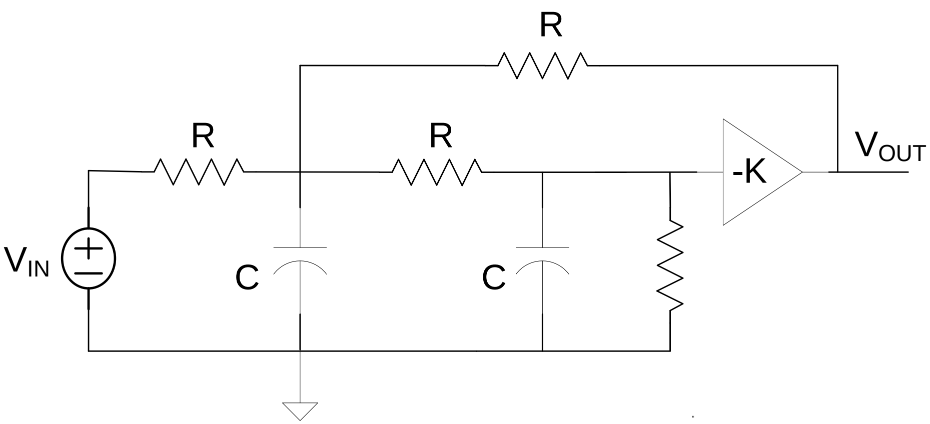


Equal R, Equal C

$$T(s) = -K\frac{\dfrac{1}{R^2C^2}}{s^2+s\left[\dfrac{5}{RC}\right]+\left[\dfrac{5+K}{R^2C^2}\right]}$$

$$\omega_0 = \frac{\sqrt{5+K}}{RC} \qquad Q = \frac{\sqrt{5+K}}{5}$$

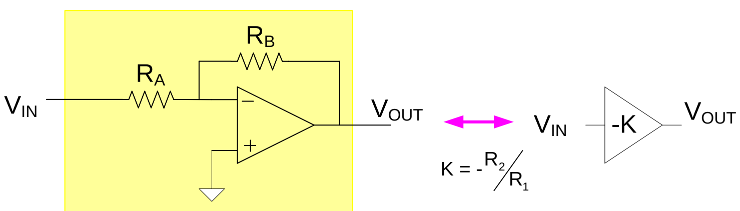
Example: 2nd Order -KRC Lowpass

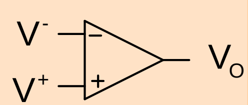


consider

$\odot \longleftrightarrow GB_n = \frac{GB}{\omega_0} = .01$

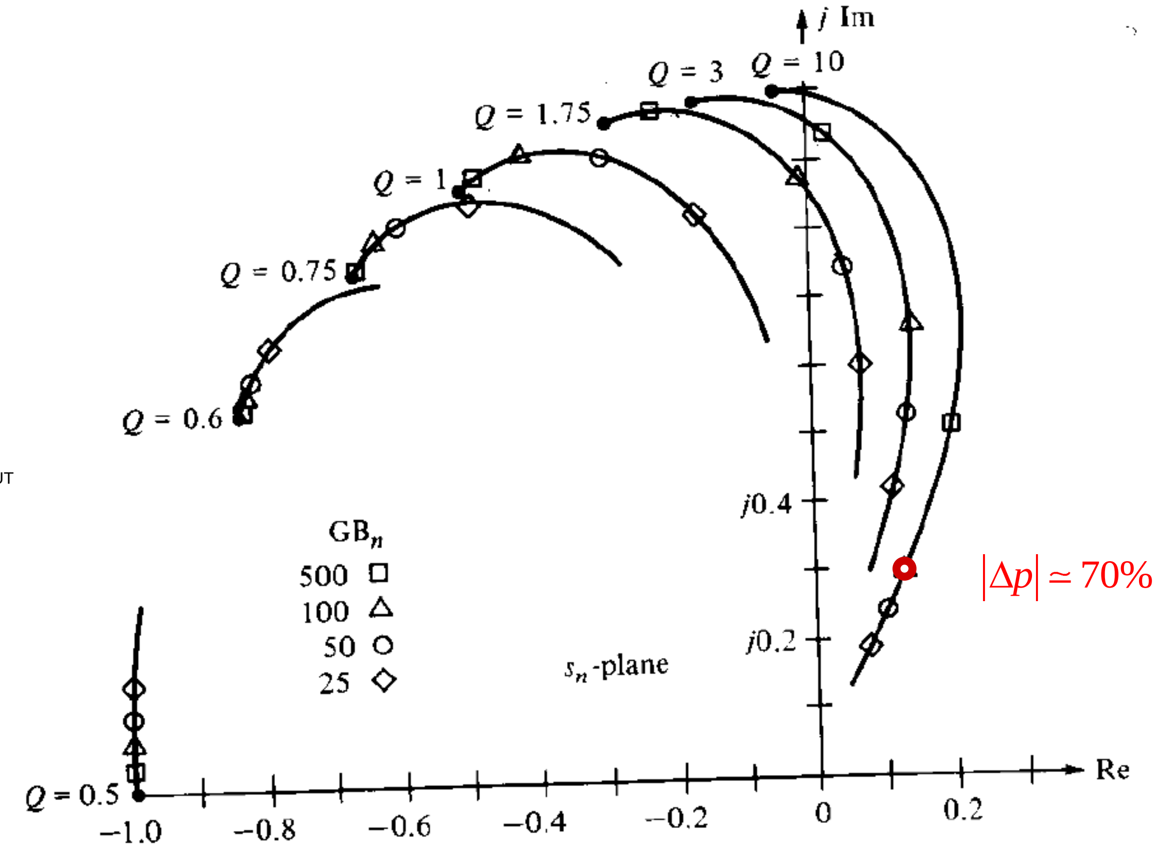
$$\omega_0 = \frac{\sqrt{5+K}}{RC} \qquad Q = \frac{\sqrt{5+K}}{5}$$





$$A(s) = \frac{V_0}{V^+ - V^-} = \frac{GB}{s}$$

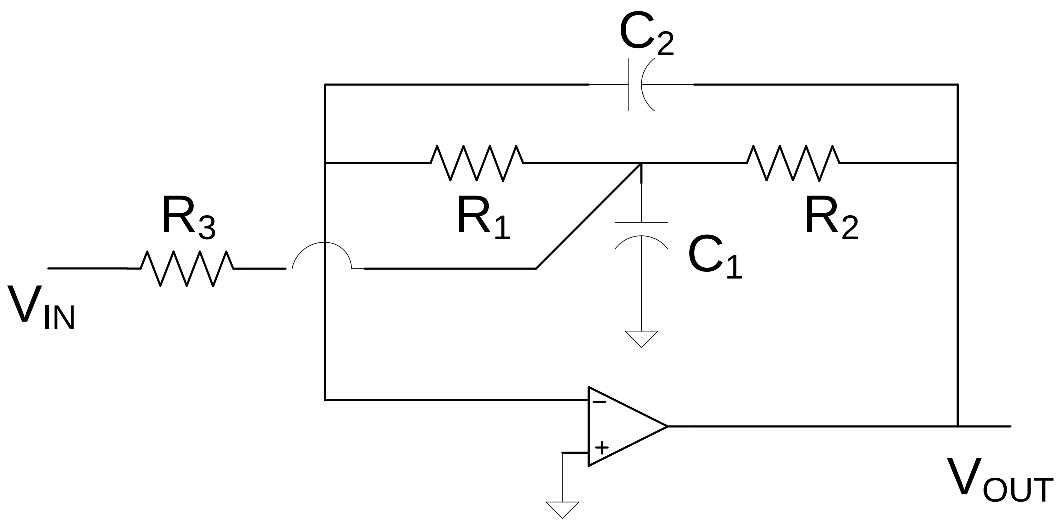
$$K(s) = -\frac{K_0}{1+\frac{(1+K_0)s}{GB}}$$



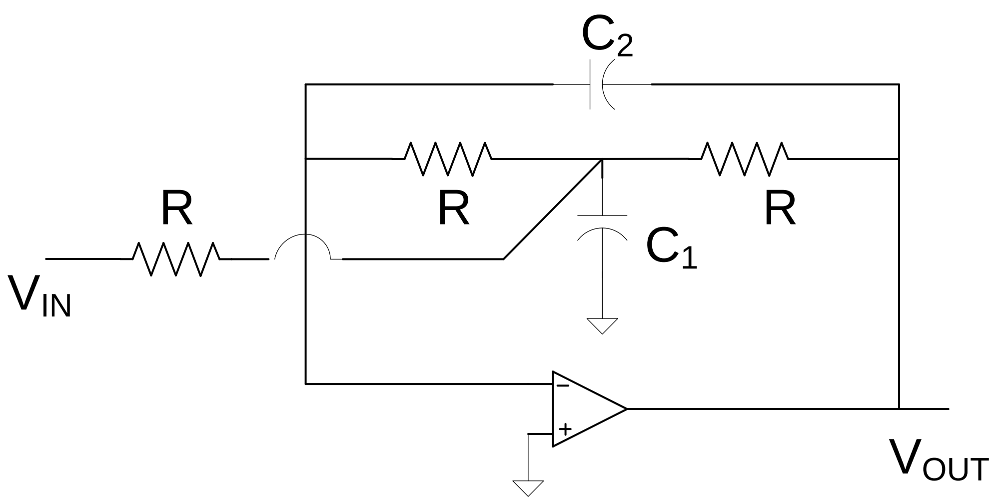
Poles "move" towards RHP as GB degrades

Even very large values of GB will cause instability

Example: 2nd Bridged-T FB Lowpass



$$T(s) = -\frac{\frac{1}{R_2R_3C_1C_2}}{s^2 + s\frac{1}{C_1}\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]+\frac{1}{R_1R_2C_1C_2}}$$

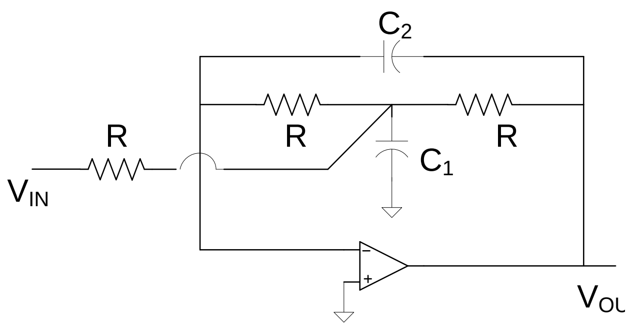


Equal R

$$T(s) = -\frac{\frac{1}{R^2C_1C_2}}{s^2 + s\left(\frac{3}{RC_1}\right)+\frac{1}{R^2C_1C_2}}$$

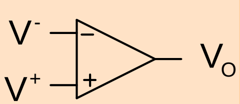
$$\omega_0 = \frac{1}{R\sqrt{C_1C_2}} \qquad Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}}$$

Example: 2nd Bridged-T FB Lowpass

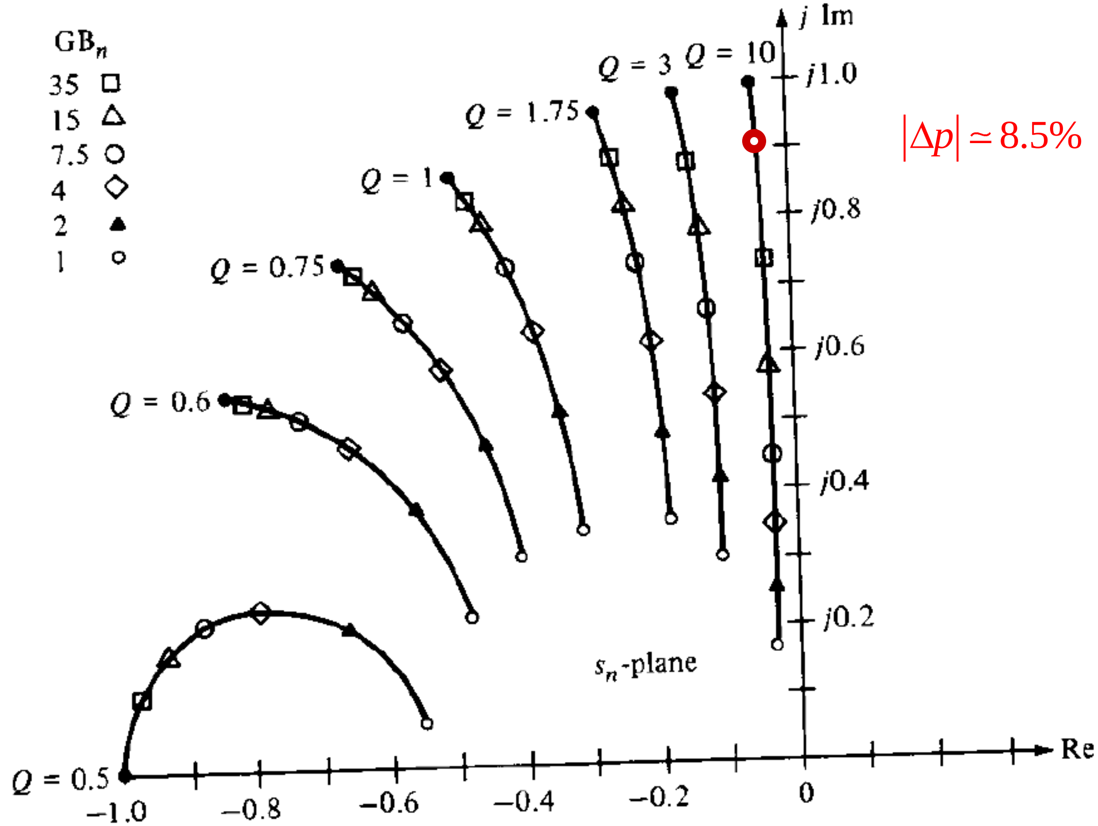


$$\omega_0 = \frac{1}{R\sqrt{C_1C_2}} \qquad Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}}$$

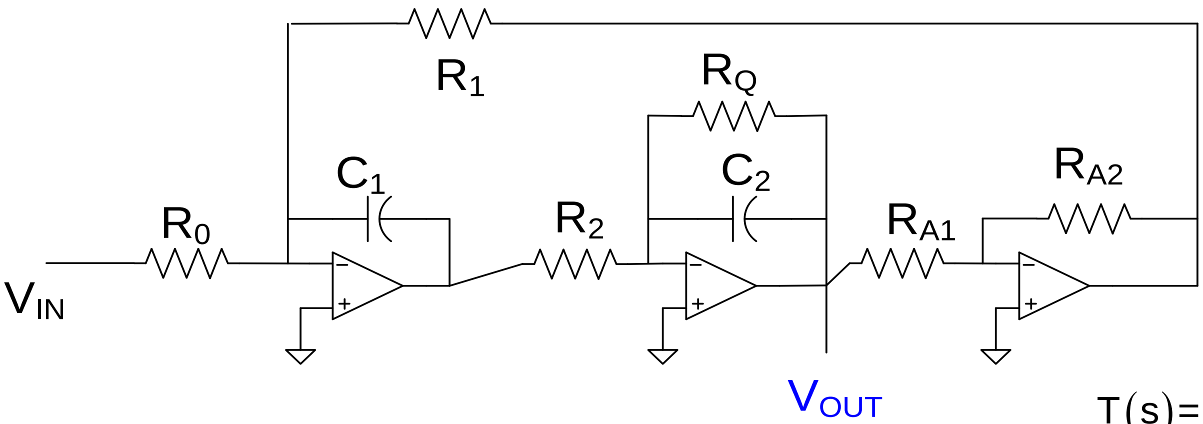
consider ⊙ ⟷ $GB_n = \frac{GB}{\omega_0} = .01$



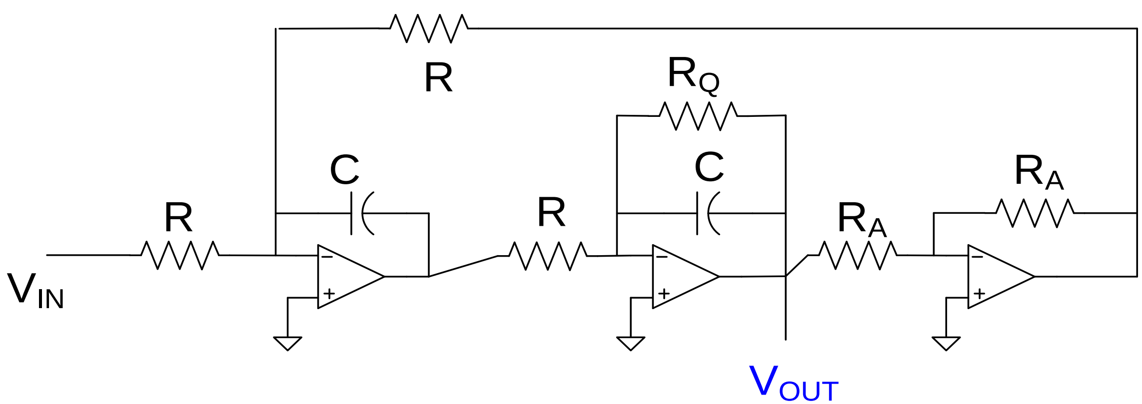
$$A(s) = \frac{V_0}{V^+ - V^-} = \frac{GB}{s}$$



Example: 2nd Two-Integrator-Loop Lowpass



$$T(s) = -\frac{\dfrac{1}{R_0R_2C_1C_2}}{s^2 + s\left(\dfrac{1}{C_2R_Q}\right) + \dfrac{R_{A2}/R_{A1}}{R_1R_2C_1C_2}}$$

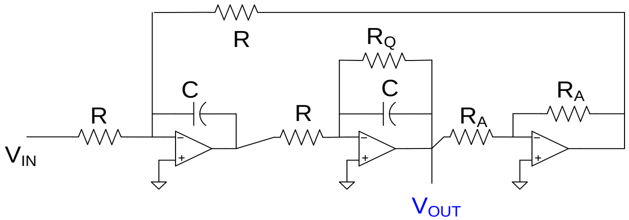


Equal R, Equal C
(except R_Q)

$$T(s) = -\frac{\dfrac{1}{R^2C^2}}{s^2 + s\left(\dfrac{1}{CR_Q}\right) + \dfrac{1}{R^2C^2}}$$

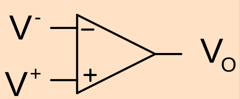
$$\omega_0 = \frac{1}{RC} \qquad Q = \frac{R_Q}{R}$$

Example: 2nd Two-Integrator-Loop Lowpass

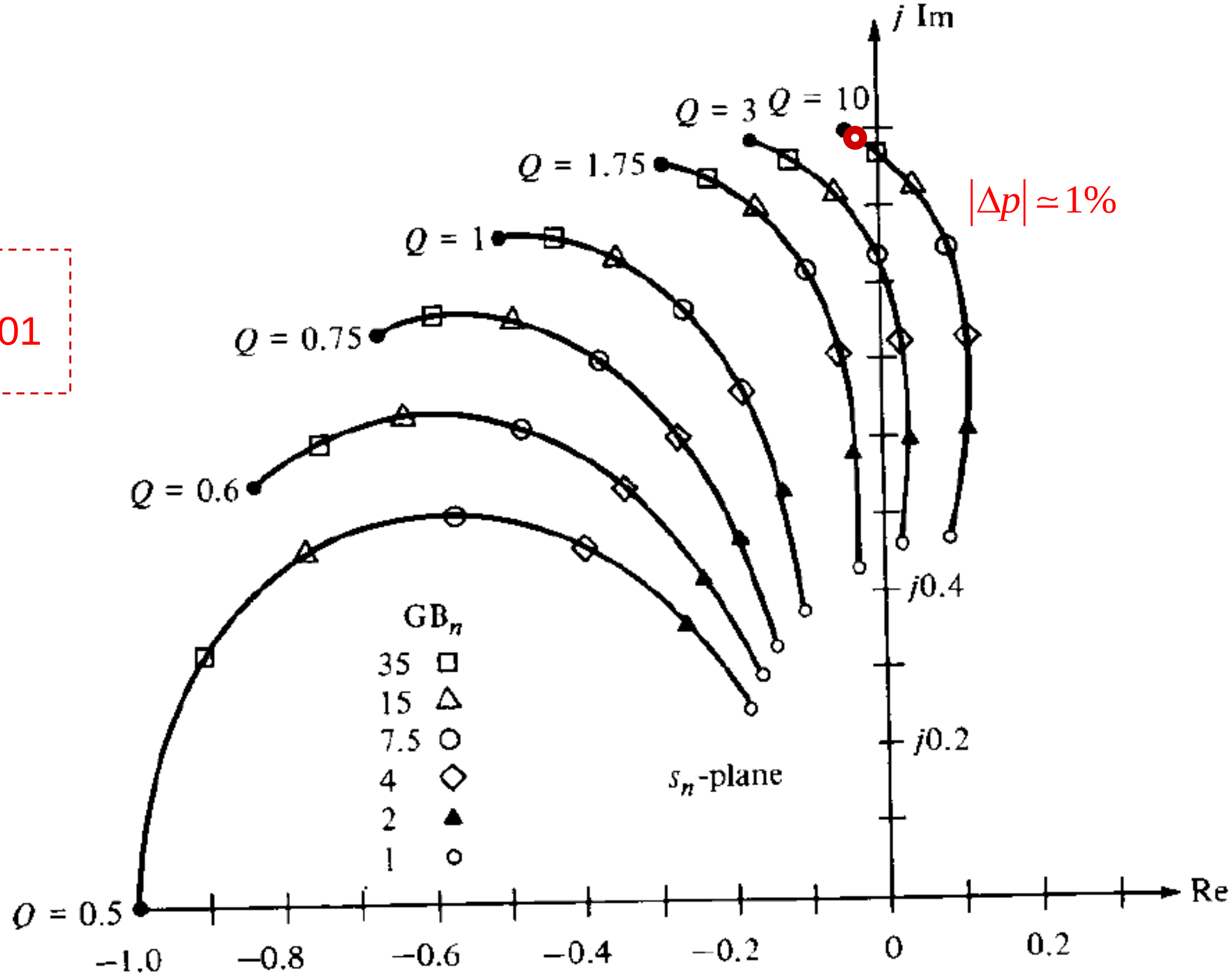


$$\omega_0 = \frac{1}{RC} \qquad Q = \frac{R_Q}{R}$$

consider ⊙ ⟷ $GB_n = \frac{GB}{\omega_0} = .01$

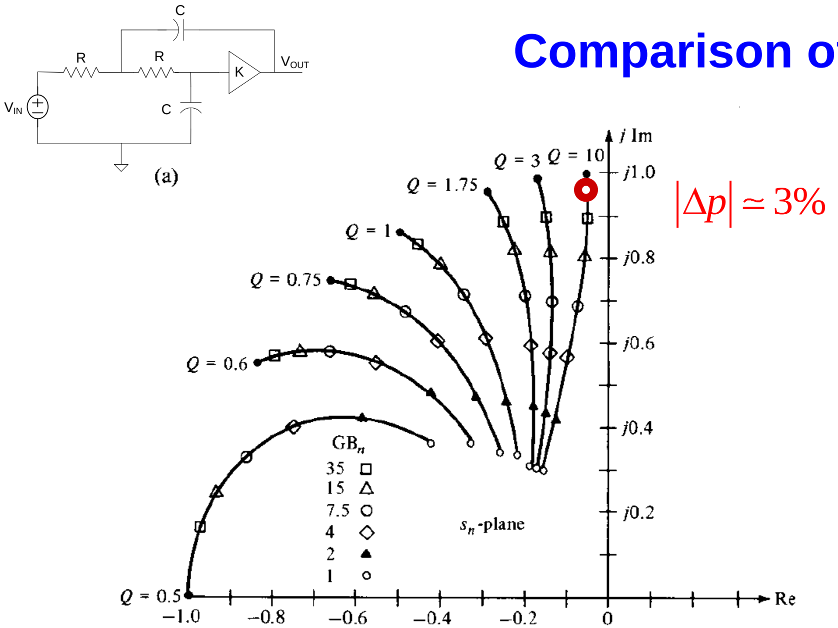


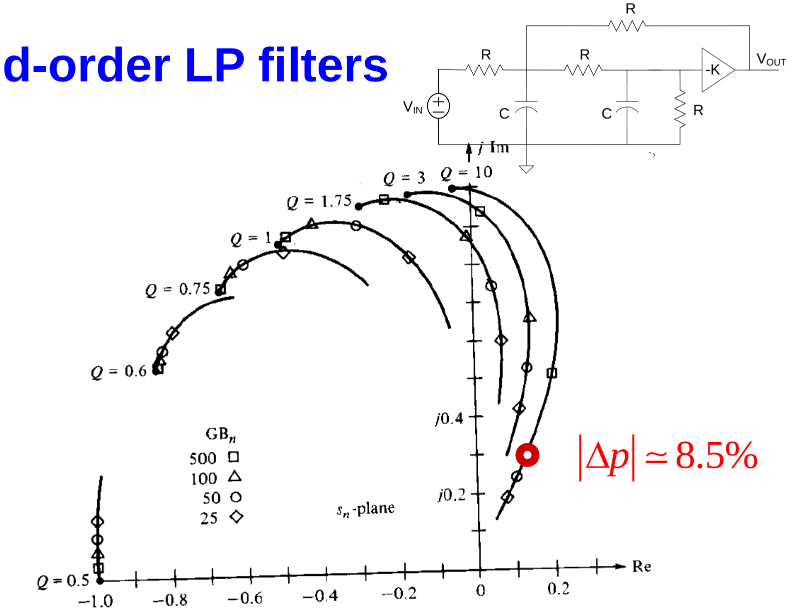
$$A(s) = \frac{V_0}{V^+ - V^-} = \frac{GB}{s}$$

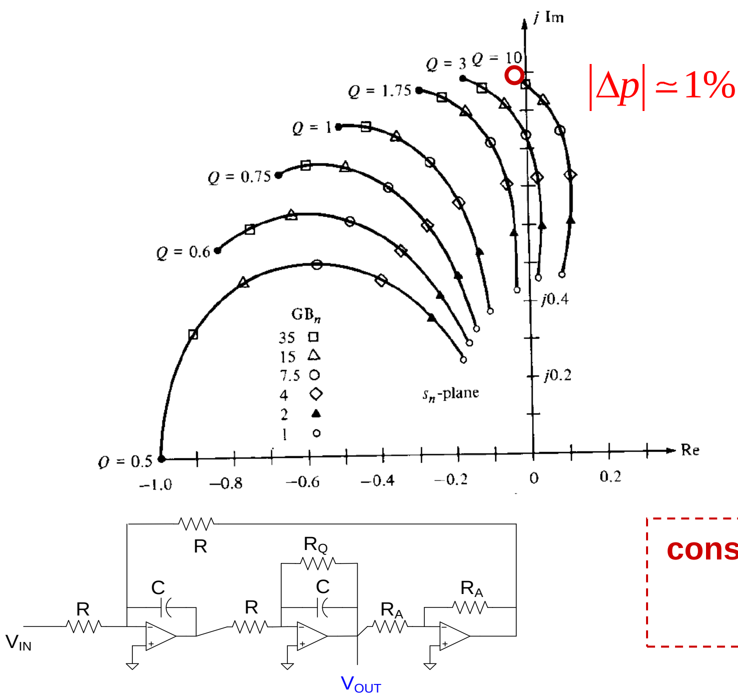


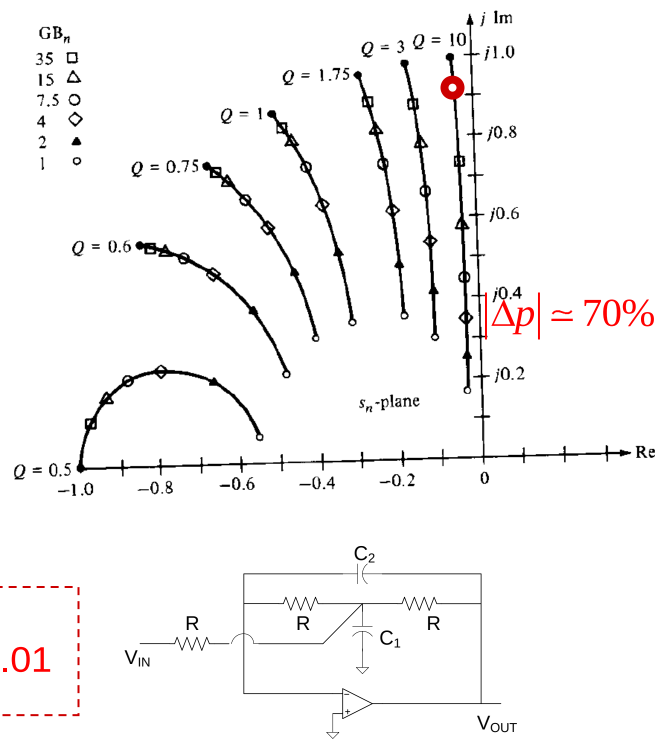
Poles “move” towards RHP as GB degrades

Comparison of 4 second-order LP filters









consider ● ⟷ $GB_n = \dfrac{GB}{\omega_0} = .01$

Some Observations

- Seemingly similar structures have dramatically different sensitivity to frequency response of the Op Amp
- Critical to have enough GB if filter is to perform as desired
- Performance strongly affected by both magnitude and direction of pole movement
- Some structures appear to be totally impractical – at least for larger Q
- Different use of the Degrees of Freedom produces significantly different results

Sensitivity analysis is useful for analytical characterization of the performance of a filter

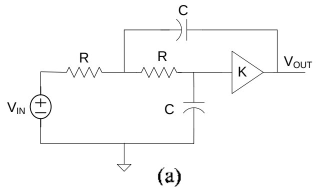
EE 508
Lecture 20

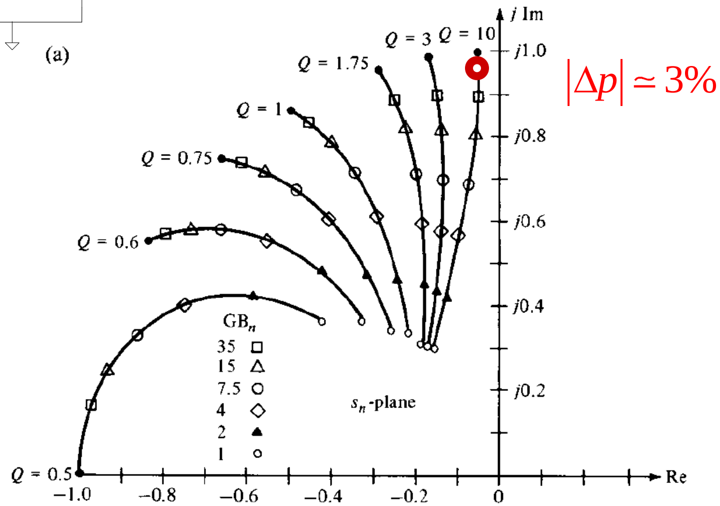
Sensitivity Functions

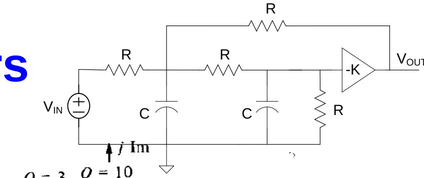
- Comparison of Filter Structures
- Performance Prediction
- Design Characterization

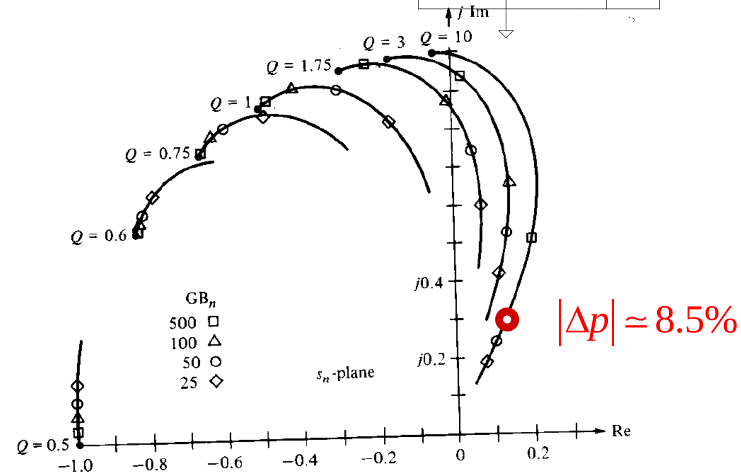
Comparison of 4 second-order LP filters

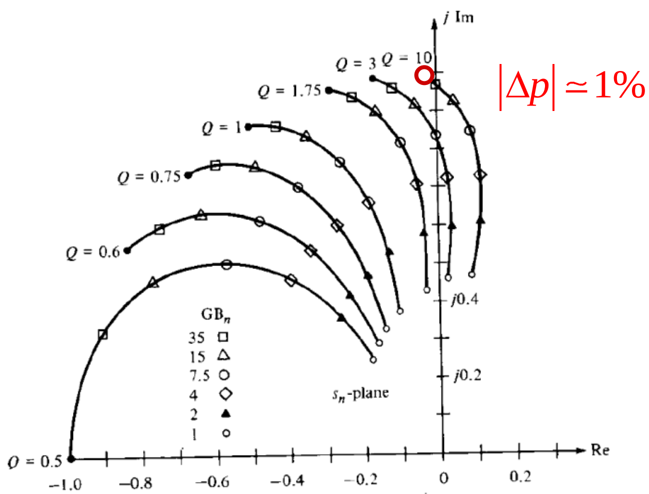
Review from last time

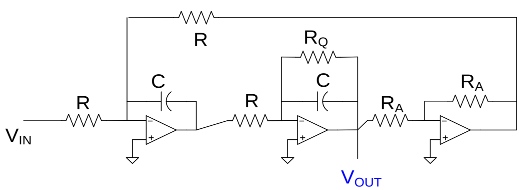


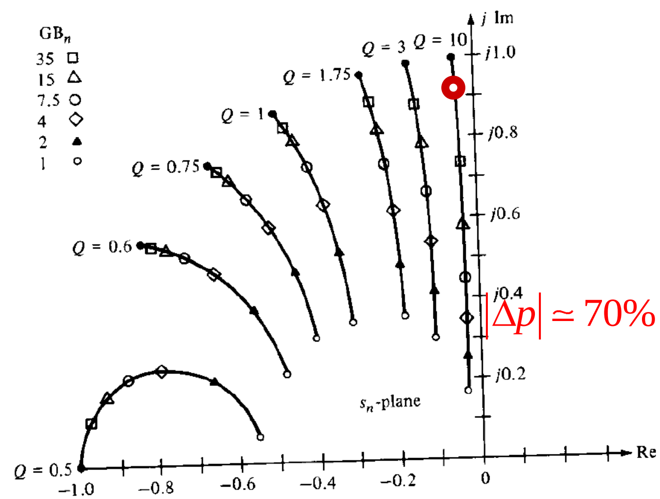


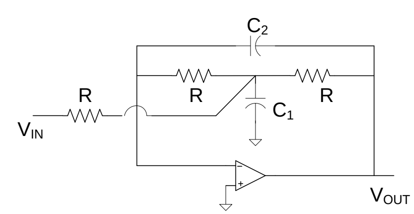






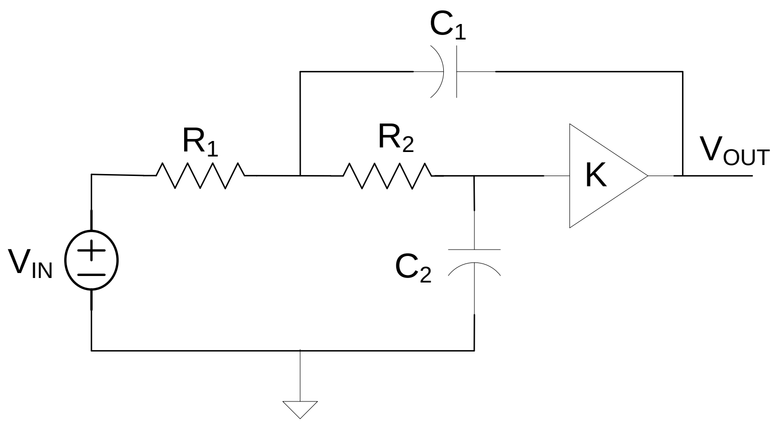


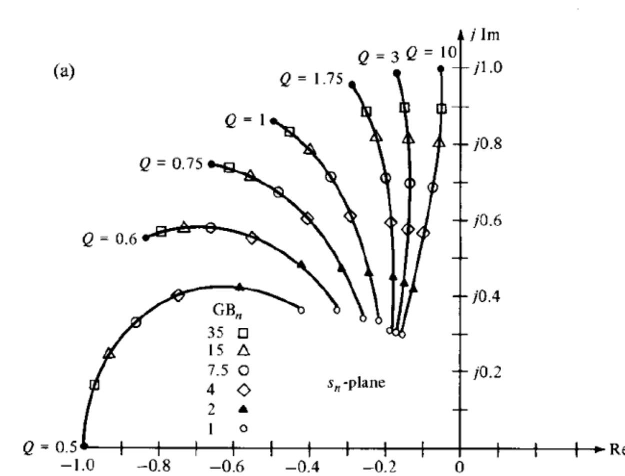




consider ⊙ ⟷ $GB_n = \frac{GB}{\omega_0} = .01$

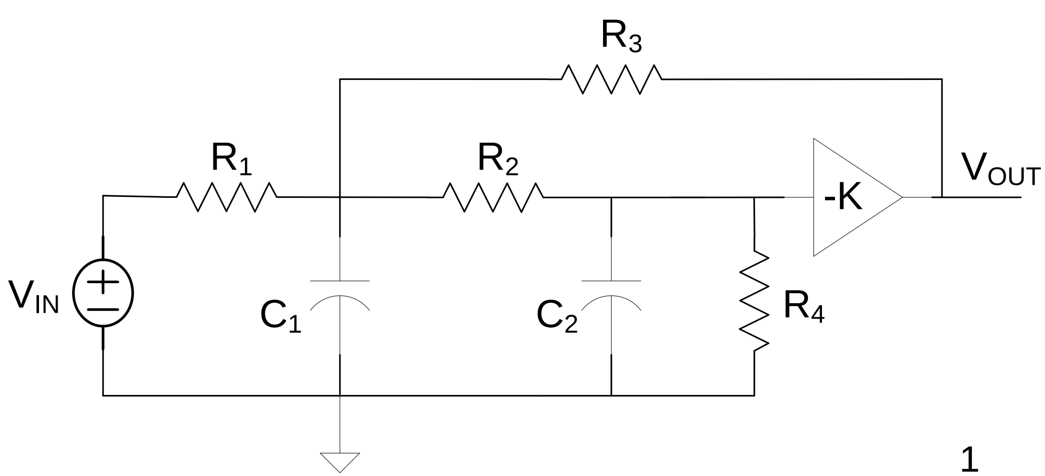
What causes the dramatic differences in performance between these two structures?
How can the performance of different structures be compared in general?

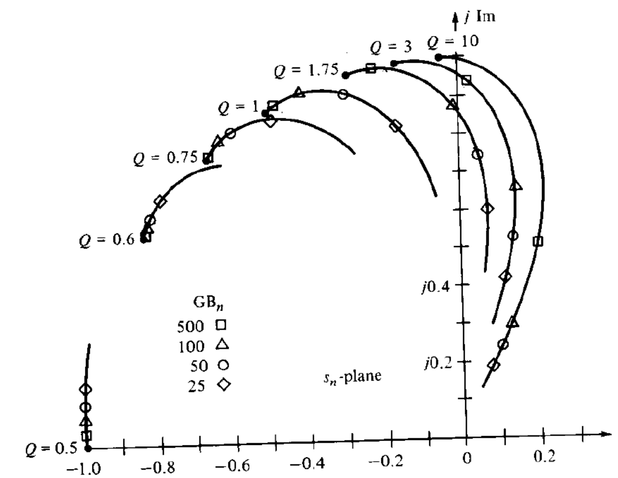




$$T(s)=K\frac{\frac{1}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1-K}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}}$$

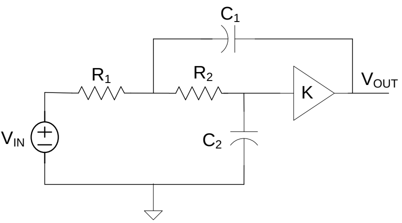
Equal R, Equal C, Q=10 Pole Locus vs GB_N

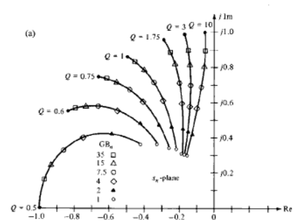




$$T(s)=-K\frac{\frac{1}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}\left(1+\frac{R_1}{R_3}\right)+\frac{1}{R_4C_2}+\frac{1}{R_2C_2}\left(1+\frac{C_2}{C_1}\right)\right]+\left[\frac{1+(R_1/R_3)(1+K)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}\right]}$$

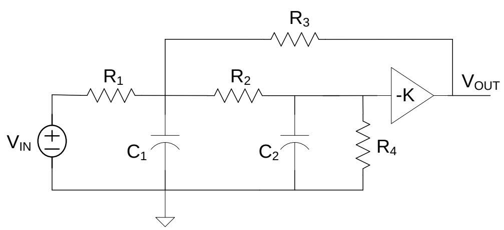
How can the performance of different structures be compared in general?

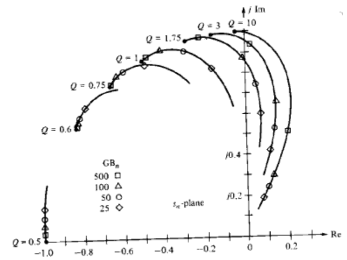




$$T(s)=K\frac{\frac{1}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1-K}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}}$$

$$Q=\frac{1}{\sqrt{\frac{R_2C_2}{R_1C_1}}+\sqrt{\frac{R_1C_2}{R_2C_1}}+(1-K)\sqrt{\frac{R_1C_1}{R_2C_2}}}\qquad \omega_0=\frac{1}{\sqrt{R_1R_2C_1C_2}}$$





$$T(s)=-K\frac{\frac{1}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}\left(1+\frac{R_1}{R_3}\right)+\frac{1}{R_4C_2}+\frac{1}{R_2C_2}\left(1+\frac{C_2}{C_1}\right)\right]+\left[\frac{1+(R_1/R_3)(1+K)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}\right]}$$

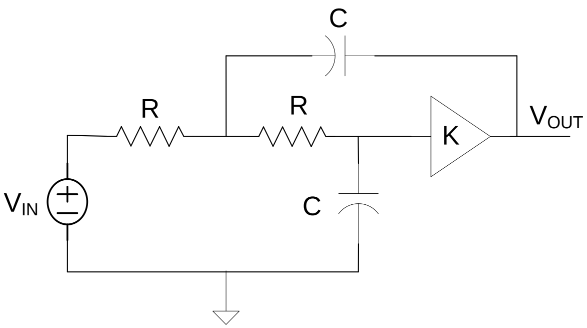
$$\omega_0=\sqrt{\frac{1+(R_1/R_3)(1+K)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}}$$

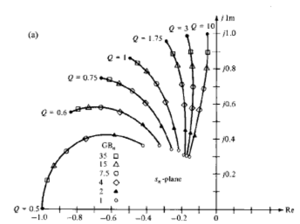
$$Q=\frac{\sqrt{\frac{1+(R_1/R_3)(1+K)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}}}{\frac{1}{R_1C_1}\left(1+\frac{R_1}{R_3}\right)+\frac{1}{R_4C_2}+\frac{1}{R_2C_2}\left(1+\frac{C_2}{C_1}\right)}$$

- **Equations for key performance parameters give little insight into the differences**
- **Expressions for key performance parameters quite complicated**

How can the performance of different structures be compared in general?

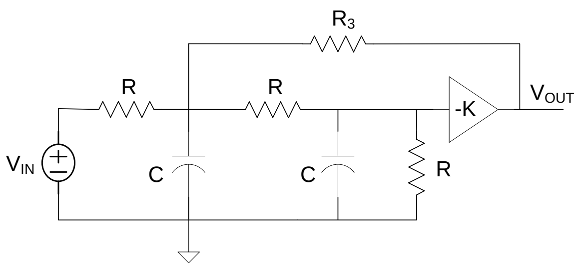
Equal R, Equal C implementations

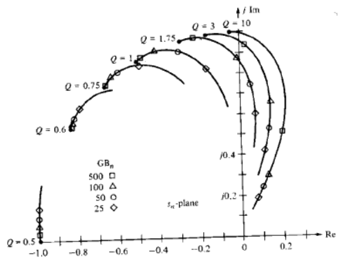




$$T(s)=K\frac{\frac{1}{R^2C^2}}{s^2+s\left[\frac{(3\text{-}K)}{RC}\right]+\frac{1}{R^2C^2}}$$

$$Q=\frac{1}{3\text{-}K} \qquad \omega_0=\frac{1}{RC}$$



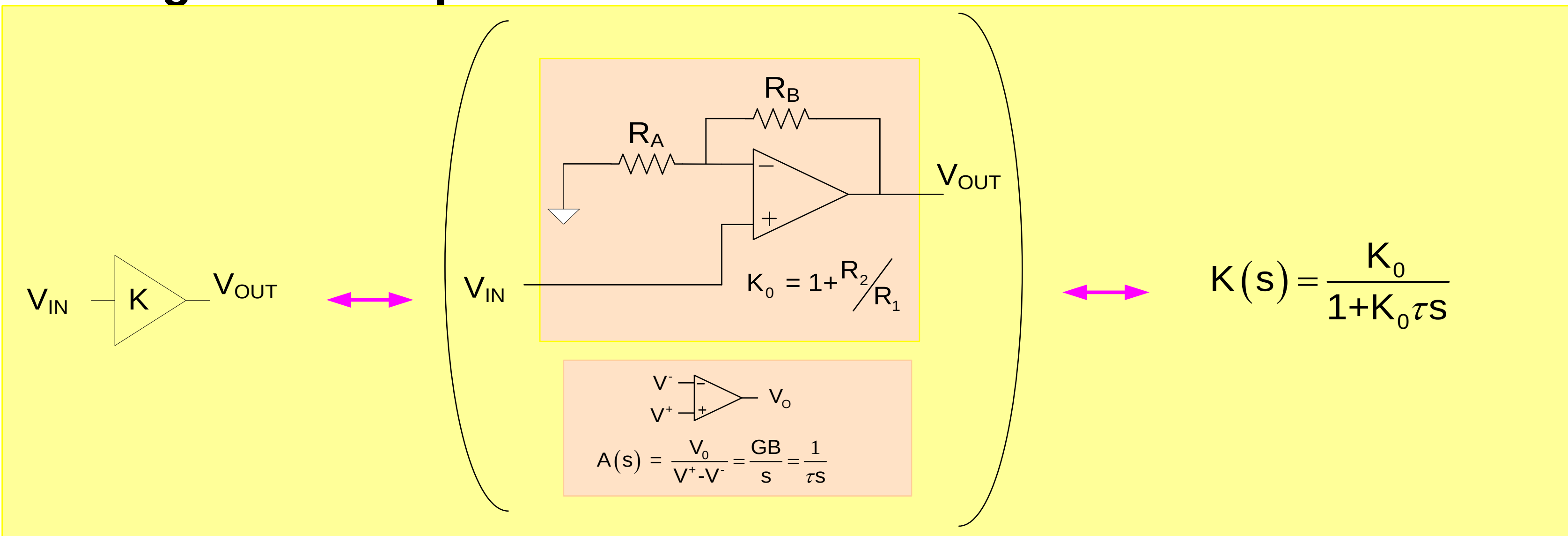


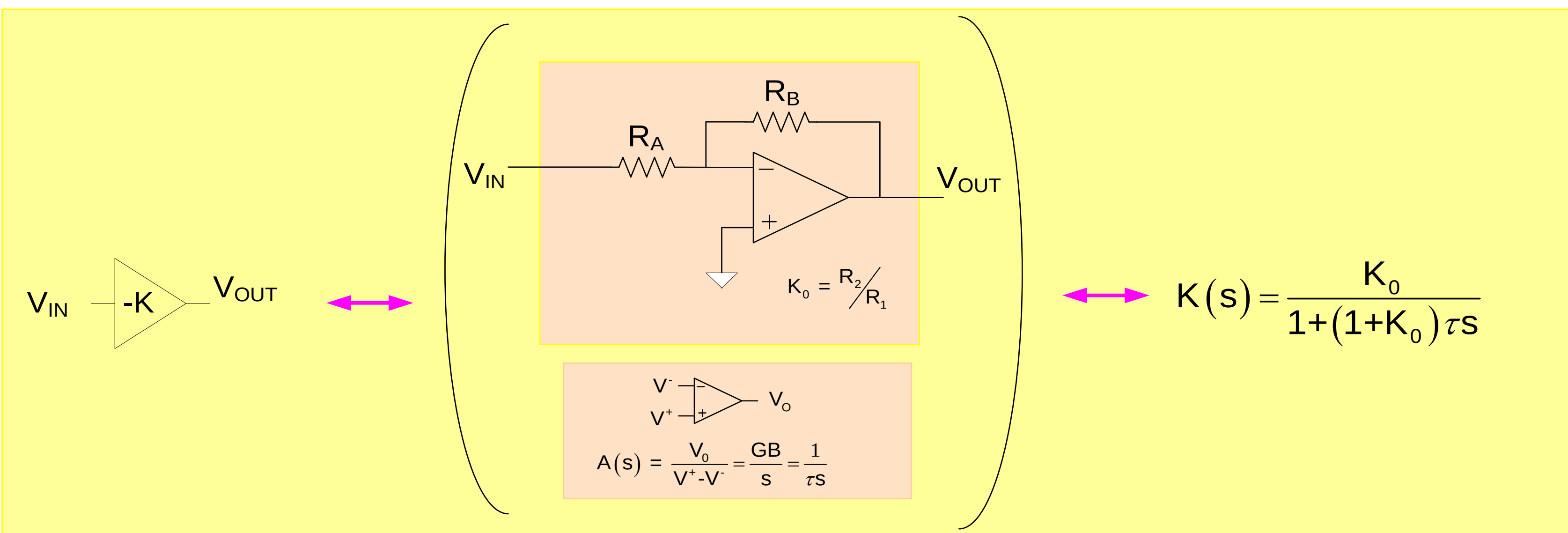
$$T(s)=-K\frac{\frac{1}{R^2C^2}}{s^2+s\left[\frac{5}{RC}\right]+\left[\frac{5+K}{R^2C^2}\right]}$$

$$Q=\frac{\sqrt{5+K}}{5} \qquad \omega_0=\frac{\sqrt{5+K}}{RC}$$

- **Analytical expressions for ω_0 and Q much simpler**
- **Equations for key performance parameters give little insight into the differences**
- **Effects of individual components is obscured in these expressions**
- **GB effects absent in this analytical formulation**

Modeling of the Amplifiers

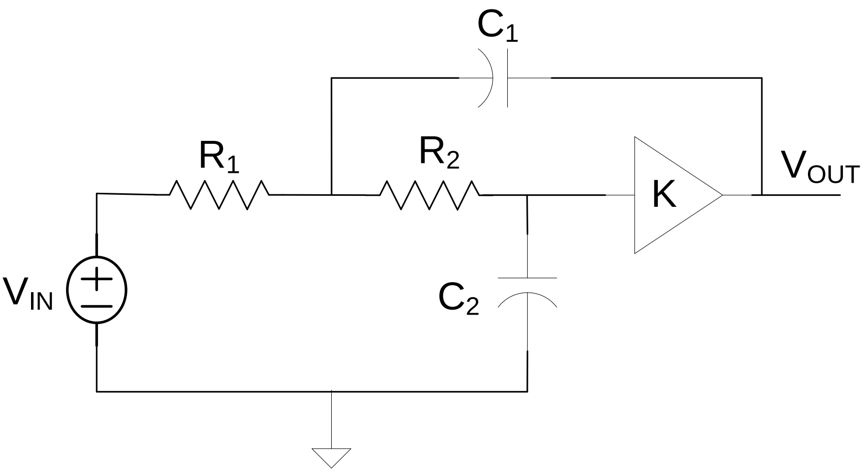




Different implementations of the amplifiers are possible

Have used the op amp time constant in these models $\tau = GB^{-1}$

GB effects in +KRC Lowpass Filter



$$T(s)=K\frac{\frac{1}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1-K}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}}$$

$$K(s)=\frac{K_0}{1+K_0\tau s}$$

$$T(s)=\frac{\frac{K_0}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{(1-K_0)}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}+K_0\tau s\left(s^2+s\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}\right)}$$

$$\omega_0=\frac{1}{\sqrt{R_1R_2C_1C_2}}\qquad Q=\frac{1}{\sqrt{\frac{R_2C_2}{R_1C_1}}+\sqrt{\frac{R_1C_2}{R_2C_1}}+(1-K)\sqrt{\frac{R_1C_1}{R_2C_2}}}$$

ω_0 and Q in these expressions are for ideal op amp

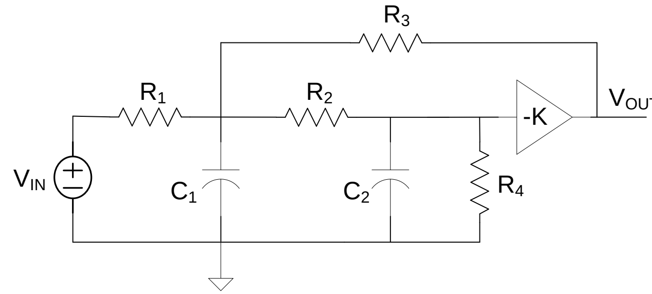
$$T(s)=\frac{K_0\omega_0^2}{s^2+s\left[\frac{\omega_0}{Q}\right]+\omega_0^2+K_0\tau s\left(s^2+s\left[\frac{\omega_0}{Q}\left(1+K_0Q\sqrt{\frac{R_1C_1}{R_2C_2}}\right)\right]+\omega_0^2\right)}$$

$$T(s)=\frac{\frac{K_0}{R_1R_2C_1C_2}}{D_I(s)+K_0\tau s\left(D_{RC0}(s)\right)}$$

DI(s) is the D(s) if the OA is ideal

$D_{RC0}(s)$ is the D(s) of RC circuit with K=0

GB effects in -KRC Lowpass Filter



$$T(s) = -K\frac{\dfrac{1}{R_1R_2C_1C_2}}{s^2+s\left[\dfrac{1}{R_1C_1}\left(1+\dfrac{R_1}{R_3}\right)+\dfrac{1}{R_4C_2}+\dfrac{1}{R_2C_2}\left(1+\dfrac{C_2}{C_1}\right)\right]+\left[\dfrac{1+(R_1/R_3)(1+K)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}\right]}$$

$$K(s)=\frac{K_0}{1+(1+K_0)\tau s}$$

$$Q=\frac{\sqrt{\dfrac{1+(R_1/R_3)(1+K)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}}}{\dfrac{1}{R_1C_1}\left(1+\dfrac{R_1}{R_3}\right)+\dfrac{1}{R_4C_2}+\dfrac{1}{R_2C_2}\left(1+\dfrac{C_2}{C_1}\right)}$$

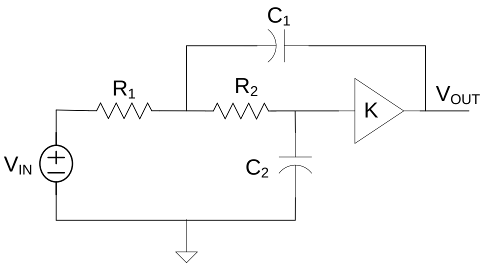
$$\omega_0=\sqrt{\frac{1+(R_1/R_3)(1+K)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}}$$

ω_0 and Q in these expressions are for ideal op amp

$$T(s) = -K_0\frac{\dfrac{1}{R_1R_2C_1C_2}}{\left(s^2+s\left[\dfrac{1}{R_1C_1}\left(1+\dfrac{R_1}{R_3}\right)+\dfrac{1}{R_4C_2}+\dfrac{1}{R_2C_2}\left(1+\dfrac{C_2}{C_1}\right)\right]+\left[\dfrac{1+(R_1/R_3)(1+K_0)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}\right]\right) +\tau s(1+K_0)\left(s^2+s\left[\dfrac{1}{R_1C_1}\left(1+\dfrac{R_1}{R_3}\right)+\dfrac{1}{R_4C_2}+\dfrac{1}{R_2C_2}\left(1+\dfrac{C_2}{C_1}\right)\right]+\left[\dfrac{1+(R_1/R_3)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}\right]\right)}$$

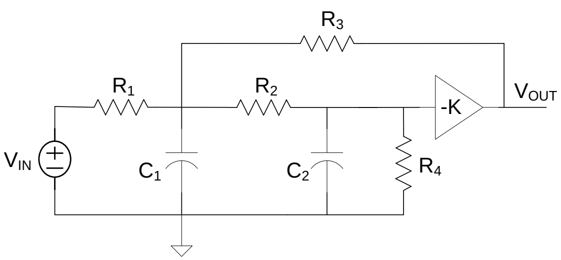
$$T(s)=\frac{\dfrac{-K_0}{R_1R_2C_1C_2}}{D_I(s)+(1+K_0)\tau s\left(D_{RC0}(s)\right)}$$

GB effects in KRC and -KRC Lowpass Filter



$$T(s)=\frac{K_0\omega_0^2}{s^2+s\left[\frac{\omega_0}{Q}\right]+\omega_0^2+K_0\tau s\left(s^2+s\left[\frac{\omega_0}{Q}\left(1+K_0Q\sqrt{\frac{R_1C_1}{R_2C_2}}\right)\right]+\omega_0^2\right)}$$

$$T(s)=\frac{\frac{K_0}{R_1R_2C_1C_2}}{D_I(s)+K_0\tau s\left(D_{RC0}(s)\right)}$$



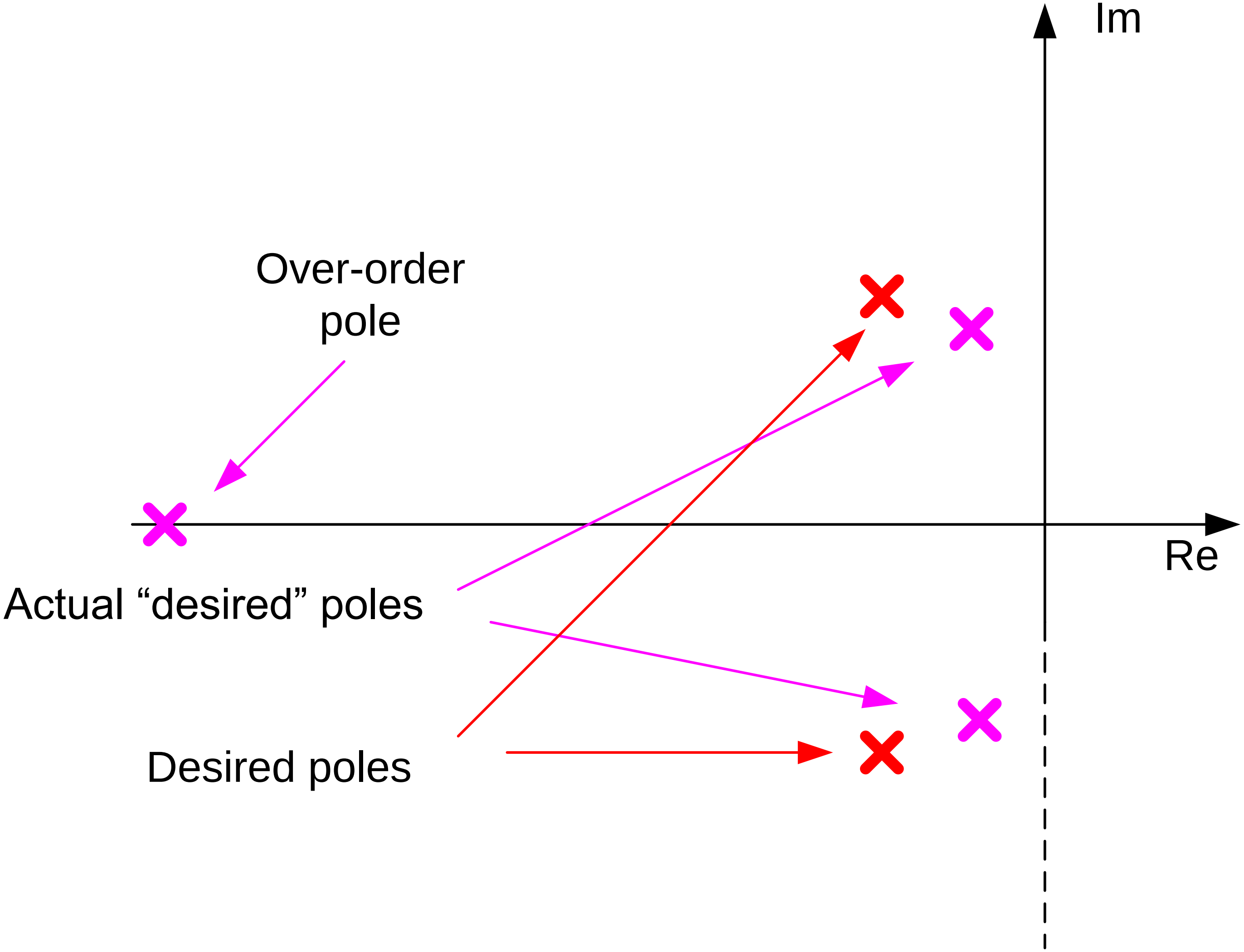
$$T(s)=-K_0\frac{\frac{1}{R_1R_2C_1C_2}}{\left(s^2+s\left[\frac{1}{R_1C_1}\left(1+\frac{R_1}{R_3}\right)+\frac{1}{R_4C_2}+\frac{1}{R_2C_2}\left(1+\frac{C_2}{C_1}\right)\right]+\left[\frac{1+(R_1/R_3)(1+K_0)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}\right]\right)+\tau s(1+K_0)\left(s^2+s\left[\frac{1}{R_1C_1}\left(1+\frac{R_1}{R_3}\right)+\frac{1}{R_4C_2}+\frac{1}{R_2C_2}\left(1+\frac{C_2}{C_1}\right)\right]+\left[\frac{1+(R_1/R_3)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}\right]\right)}$$

$$T(s)=\frac{\frac{-K_0}{R_1R_2C_1C_2}}{D_I(s)+(1+K_0)\tau s\left(D_{RC0}(s)\right)}$$

All linear performance effects can be obtained from this formulation

Op amp introduced an additional pole and moves the desired poles

Effects of GB on poles of KRC and -KRC Lowpass Filters



GB effects in KRC and -KRC Lowpass Filter

$$T(s)=\frac{K_0\omega_0^2}{s^2+s\left[\frac{\omega_0}{Q}\right]+\omega_0^2+K_0\tau s\left(s^2+s\left[\frac{\omega_0}{Q}\left(1+K_0Q\sqrt{\frac{R_1C_1}{R_2C_2}}\right)\right]+\omega_0^2\right)}$$

$$T(s)=-K_0\frac{\frac{1}{R_1R_2C_1C_2}}{\left(s^2+s\left[\frac{1}{R_1C_1}\left(1+\frac{R_1}{R_3}\right)+\frac{1}{R_4C_2}+\frac{1}{R_2C_2}\left(1+\frac{C_2}{C_1}\right)\right]+\left[\frac{1+(R_1/R_3)(1+K_0)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}\right]\right) +\tau s(1+K_0)\left(s^2+s\left[\frac{1}{R_1C_1}\left(1+\frac{R_1}{R_3}\right)+\frac{1}{R_4C_2}+\frac{1}{R_2C_2}\left(1+\frac{C_2}{C_1}\right)\right]+\left[\frac{1+(R_1/R_3)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}\right]\right)}$$

- **Analytical expressions for $\omega_{0,}$ Q, poles, zeros, and other key parameters are unwieldly in these circuits and as bad or worse in many other circuits**
- **Sensitivity metrics give considerable insight into how filters perform and widely used to assess relative performance**
- **Need sensitivity characterization of real numbers as well as complex quantities such as poles and zeros**
- **Since analytical expressions for key parameters are unwieldly in even simple circuits, obtaining expressions for the purpose of calculating sensitivity appears to be a formidable task !**

Sensitivity Characterization of Filter Structures

Let F be a filter characteristic of interest

F might be ω_0 or Q of a pole or zero, a band edge, a peak frequency, a BW, T(s), |T(jω)|, a coefficient in T(s), etc

Can express F in terms of all components and model parameters as

$$F=f(R_1, \ldots R_{k1}, C_1, \ldots C_{k2}, L_{l1},\ldots L_{lk3}, \tau_1, \ldots \tau_{k4}, W_1,\ldots W_{k5}, L_1, \ldots L_{k5},\ldots.)$$

$$F=f(x_1,x_2, \ldots x_k)$$

The differential dF of the multivariate function F can be expressed as

$$dF = \frac{\partial F}{\partial R_1}dR_1 + \frac{\partial F}{\partial R_2}dR_2 + \ldots. + \frac{\partial F}{\partial R_{k1}}dR_{k1}$$
$$+ \frac{\partial F}{\partial C_1}dC_1 + \frac{\partial F}{\partial C_2}dC_2 + \ldots. + \frac{\partial F}{\partial C_{k2}}dRC_{k2}$$
$$+ \ldots\ldots..$$

$$dF = \sum_{i=1}^{k} \frac{\partial F}{\partial x_i}dx_i$$

Define the standard sensitivity function as

$$S_x^f = \frac{\partial f}{\partial x} \bullet \frac{x}{f}$$

S_x^f Is widely used except when x or f assume extreme values of 0 or ∞

Define the derivative sensitivity function as

$$\mathfrak{s}_x^f = \frac{\partial f}{\partial x}$$

$\mathfrak{s}_x^f$ Is more useful when x or f ideally assume extreme values of 0 or ∞

Consider the normalized differential $\frac{dF}{F}$

$$\frac{dF}{F} \simeq \frac{\Delta F}{F}$$

This approximates the percent change in F due to changes in ALL components

$$\frac{dF}{F} = \frac{\sum_{i=1}^{k} \frac{\partial F}{\partial x_i} dx_i}{F} = \sum_{i=1}^{k} \frac{\partial F}{\partial x_i} \bullet \frac{dx_i}{F} \overset{\text{All } x_i \neq 0,\infty}{=} \sum_{i=1}^{k} \left(\frac{\partial F}{\partial x_i} \bullet \frac{x_i}{F} \right) \bullet \frac{dx_i}{x_i}$$

This can be expressed in terms of the standard sensitivity function as

$$\frac{dF}{F} \overset{\text{All } x_i \neq 0,\infty}{=} \sum_{i=1}^{k} \left(S_{x_i}^{f} \bullet \frac{dx_i}{x_i} \right)$$

This relates the percent change in F to the sensitivity function and the percent change in each component

Consider the normalized differential

$$\frac{dF}{F}=\sum_{i=1}^{k}\left(S_{x_i}^{f}\bullet\frac{dx_i}{x_i}\right)$$

This can be expressed as

$$\frac{dF}{F}=\left(\sum_{all\ resistors}S_{R_i}^{f}\bullet\frac{dR_i}{R_i}\right)+\left(\sum_{all\ capacitors}S_{C_i}^{f}\bullet\frac{dC_i}{C_i}\right)+\left(\sum_{all\ opamps}S_{\tau_i}^{f}\bullet\frac{d\tau_i}{\tau_i}\right)+\ldots$$

Often interested in $\frac{dF}{F}$ evaluated at the ideal (or nominal value)

If the nominal values are all not extreme (0 or ∞), then

$$\frac{dF}{F}=\sum_{i=1}^{k}\left(S_{x_i}^{f}\Big|_{\vec{X}_N}\bullet\frac{dx_i}{x_{iN}}\right)$$

The normalized differential – a different perspective

$$\frac{dF}{F} = \sum_{i=1}^{k} \left(S_{x_i}^{f} \Big|_{\vec{X}_N} \bullet \frac{dx_i}{x_{iN}} \right)$$

Consider the multivariate Taylors series expansion of F

$$F(\vec{X}) = F(\vec{X}_N) + \sum_{i=1}^{k} \frac{\partial F}{\partial x_i}\Big|_{\vec{X}_N} (x_i - x_{iN}) + \left[\frac{1}{2!} \sum_{i=1}^{k} \frac{\partial^2 F}{\partial x_i^2}\Big|_{\vec{X}_N} (x_i - x_{iN})^2 + \sum_{\substack{i=1 \\ j=1 \\ i \neq j}}^{k,k} \frac{\partial^2 F}{\partial x_i \partial x_j}\Big|_{\vec{X}_N} (x_i - x_{iN})(x_j - x_{jN}) \right] + \ldots..$$

$$F(\vec{X}) \simeq F(\vec{X}_N) + \sum_{i=1}^{k} \frac{\partial F}{\partial x_i}\Big|_{\vec{X}_N} (x_i - x_{iN}) +$$

$$F(\vec{X}) - F(\vec{X}_N) \simeq + \sum_{i=1}^{k} \frac{\partial F}{\partial x_i}\Big|_{\vec{X}_N} (x_i - x_{iN})$$

$$\Delta F(\vec{X}) \simeq \sum_{i=1}^{k} \frac{\partial F}{\partial x_i}\Big|_{\vec{X}_N} \Delta x_i$$

The normalized differential – a different perspective

$$\frac{dF}{F} = \sum_{i=1}^{k} \left(S^{f}_{x_i} \Big|_{\vec{X}_N} \bullet \frac{dx_i}{x_{iN}} \right)$$

Consider the multivariate Taylors series expansion of F

$$\Delta F(\vec{X}) \simeq \sum_{i=1}^{k} \frac{\partial F}{\partial x_i}\Big|_{\vec{X}_N} \Delta x_i$$

$$\frac{\Delta F(\vec{X})}{F} \simeq \sum_{i=1}^{k} \frac{\partial F}{\partial x_i}\Big|_{\vec{X}_N} \frac{\Delta x_i}{F} = \sum_{i=1}^{k} \frac{\partial F}{\partial x_i}\Big|_{\vec{X}_N} \frac{x_i}{x_i} \frac{\Delta x_i}{F} = \sum_{i=1}^{k} \left(\frac{\partial F}{\partial x_i}\Big|_{\vec{X}_N} \frac{x_i}{F} \right) \frac{\Delta x_i}{x_i}$$

$$\frac{\Delta F}{F} \simeq \sum_{i=1}^{k} \left(S^{f}_{x_i} \Big|_{\vec{X}_N} \right) \frac{\Delta x_i}{x_i}$$

Note this is essentially the same expression that was arrived at from the sensitivity analysis approach

$$\frac{dF}{F} = \sum_{i=1}^{k} \left(S_{x_i}^{f} \Big|_{\vec{X}_N} \bullet \frac{dx_i}{x_{iN}} \right)$$

Dependent only on components (not circuit structure)

Dependent on circuit structure (for some circuits, also not dependent on components)

The sensitivity functions are thus useful for comparing different circuit structures

The variability which is the product of the sensitivity function and the normalized component differential is more important for predicting circuit performance

Variability Formulation

$$V_{x_i}^{f} = S_{x_i}^{f}\Big|_{\vec{X}_N} \bullet \frac{dx_i}{x_{iN}}$$

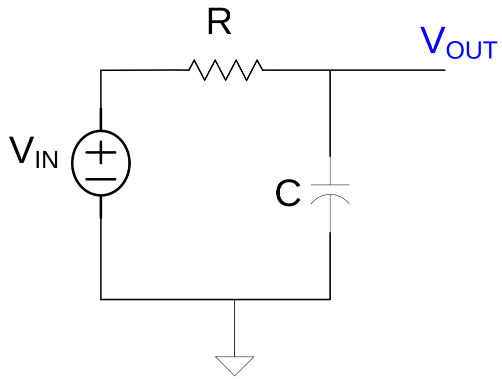
$$\frac{dF}{F} = \sum_{i=1}^{k} V_{x_i}^{f}\Big|_{\vec{X}_N}$$

Variability includes effects of both circuit structure and components on performance

If component variations are small, high sensitivities are acceptable

If component variations are large, low sensitivities are critical

Example



$$T(s) = \frac{1}{1+RCs} = \frac{\omega_0}{s+\omega_0}$$

If $\omega_0 = 1/RC$, determine $S_R^{\omega_0}$ and $S_C^{\omega_0}$

$$S_R^{\omega_0} = \frac{\partial \omega_0}{\partial R} \bullet \frac{R}{\omega_0}$$

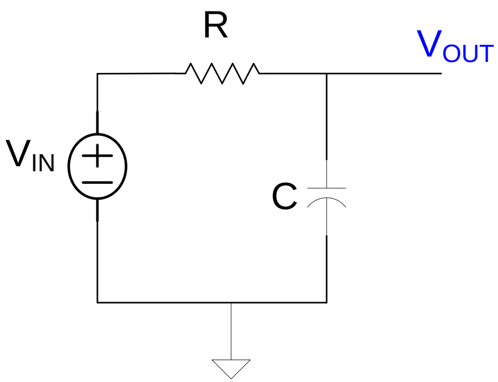
$$S_R^{\omega_0} = \left(\frac{-1}{R^2C}\right) \bullet \frac{R}{\omega_0}$$

$$S_R^{\omega_0} = -\frac{1}{R}\left(\frac{1}{RC}\right) \bullet \frac{R}{\omega_0} = -\frac{1}{R}(\omega_0) \bullet \frac{R}{\omega_0} = -1$$

Likewise

$$S_C^{\omega_0} = -1$$

Example



$$T(s) = \frac{1}{1+RCs} = \frac{\omega_0}{s+\omega_0}$$

$$\omega_0 = {}^1\!/_{RC}$$

$$S_R^{\omega_0} = -1 \qquad S_C^{\omega_0} = -1$$

$$\frac{d\omega_0}{\omega_0} = \sum_{i=1}^{k} V_{x_i}^{\omega_0}\Big|_{\vec{X}_N}$$

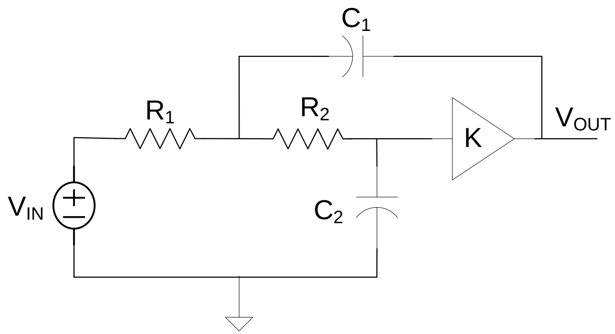
Thus a 1% increase in R will cause approximately a 1% decrease in ω_0

a 1% increase in C will cause approximately a 1% decrease in ω_0

a 1% increase in both C and R will cause approximately a 2% decrease in ω_0

Example

$$T(s)=K\frac{\frac{1}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1-K}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}}$$



$$Q=\frac{1}{\sqrt{\frac{R_2C_2}{R_1C_1}}+\sqrt{\frac{R_1C_2}{R_2C_1}}+(1-K)\sqrt{\frac{R_1C_1}{R_2C_2}}}\qquad \omega_0=\frac{1}{\sqrt{R_1R_2C_1C_2}}$$

Determine $S_{C_1}^{\omega_0}$ $S_{C_2}^{\omega_0}$ $S_{R_1}^{\omega_0}$ $S_{R_2}^{\omega_0}$

$$S_{C_1}^{\omega_0}=\frac{\partial\left[\frac{1}{\sqrt{R_1R_2C_1C_2}}\right]}{\partial C_1}\frac{C_1}{\omega_0}$$

$$S_{C_1}^{\omega_0}=\frac{1}{\sqrt{R_1R_2C_2}}\frac{\partial\left[\frac{1}{\sqrt{C_1}}\right]}{\partial C_1}\frac{C_1}{\omega_0}$$

$$S_{C_1}^{\omega_0}=\frac{1}{\sqrt{R_1R_2C_2}}\left(-\tfrac{1}{2}C_1^{-3/2}\right)\frac{C_1}{\omega_0}$$

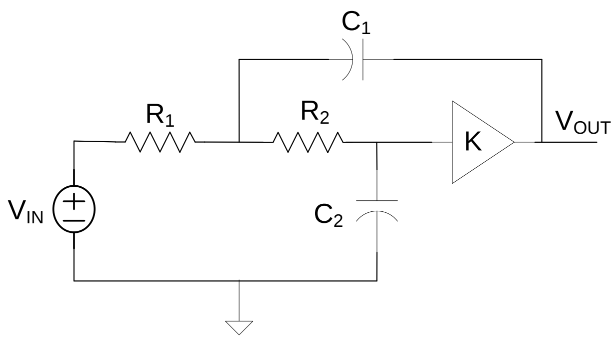
$$S_{C_1}^{\omega_0}=-\tfrac{1}{2}\frac{1}{\sqrt{R_1R_2C_2}}\left(\frac{1}{\sqrt{C_1}C_1}\right)\frac{C_1}{\omega_0}$$

$$S_{C_1}^{\omega_0}=-\tfrac{1}{2}\frac{1}{\sqrt{R_1R_2C_2C_1}}\left(\frac{1}{C_1}\right)\frac{C_1}{\omega_0}$$

$$S_{C_1}^{\omega_0}=-\tfrac{1}{2}\omega_0\left(\frac{1}{C_1}\right)\frac{C_1}{\omega_0}$$

$$S_{C_1}^{\omega_0}=-\tfrac{1}{2}$$

Example



$$T(s)=K\frac{\frac{1}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1-K}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}}$$

$$Q=\frac{1}{\sqrt{\frac{R_2C_2}{R_1C_1}}+\sqrt{\frac{R_1C_2}{R_2C_1}}+(1-K)\sqrt{\frac{R_1C_1}{R_2C_2}}}$$

$$\omega_0=\frac{1}{\sqrt{R_1R_2C_1C_2}}$$

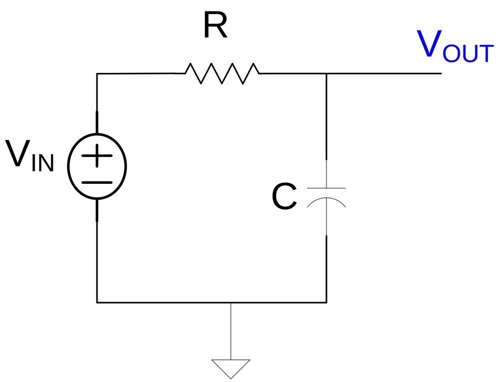
Determine $S_{C_1}^{\omega_0}$ $S_{C_2}^{\omega_0}$ $S_{R_1}^{\omega_0}$ $S_{R_2}^{\omega_0}$

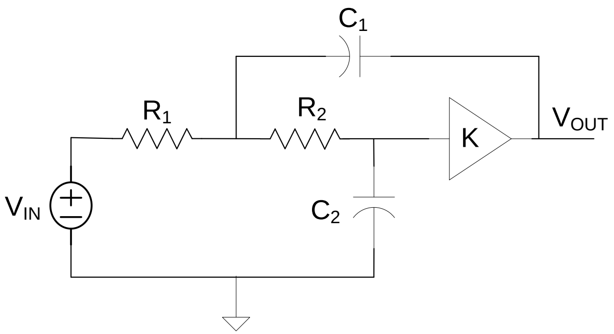
$$S_{C_1}^{\omega_0}=-1/2$$

Likewise

$$S_{C_2}^{\omega_0}=S_{R_1}^{\omega_0}=S_{R_2}^{\omega_0}=-1/2$$

Observation:





$$\omega_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}}$$

$$\omega_0 = \frac{1}{RC}$$

$$S_R^{\omega_0} = -1 \qquad S_C^{\omega_0} = -1$$

$$S_{R_1}^{\omega_0} = -\frac{1}{2} \qquad S_{C_1}^{\omega_0} = -\frac{1}{2}$$

$$S_{R_2}^{\omega_0} = -\frac{1}{2} \qquad S_{C_2}^{\omega_0} = -\frac{1}{2}$$

$$\sum_{All\ resistors} S_{R_i}^{\omega_0} = -1 \qquad \sum_{All\ capacitors} S_{C_i}^{\omega_0} = -1$$

$$\sum_{All\ resistors} S_{R_i}^{\omega_0} = -1 \qquad \sum_{All\ capacitors} S_{C_i}^{\omega_0} = -1$$

At this stage, this is just an observation about summed sensitivities but later will establish some fundamental properties of summed sensitivities

End of Lecture 20

EE 508
Lecture 21

Sensitivity Functions

- Comparison of Filter Structures
- Performance Prediction
- Design Characterization

Review from last time

Define the standard sensitivity function as

$$S_x^f = \frac{\partial f}{\partial x} \bullet \frac{x}{f}$$

S_x^f Is widely used except when x or f assume extreme values of 0 or ∞

Define the derivative sensitivity function as

$$\mathcal{S}_x^f = \frac{\partial f}{\partial x}$$

$\mathcal{S}_x^f$ Is more useful when x or f ideally assume extreme values of 0 or ∞

Review from last time

$$\frac{dF}{F} = \sum_{i=1}^{k} \left(S^{f}_{x_i}\Big|_{\vec{X}_N} \bullet \frac{dx_i}{x_{iN}} \right)$$

Dependent only on components (not circuit structure)

Dependent on circuit structure (for some circuits, also not dependent on components**)**

The sensitivity functions are thus useful for comparing different circuit structures

The variability which is the product of the sensitivity function and the normalized component differential is more important for predicting circuit performance

Variability Formulation

Review from last time

$$V_{x_i}^{f} = S_{x_i}^{f}\Big|_{\vec{X}_N} \bullet \frac{dx_i}{x_{iN}}$$

$$\frac{dF}{F} = \sum_{i=1}^{k} V_{x_i}^{f}\Big|_{\vec{X}_N}$$

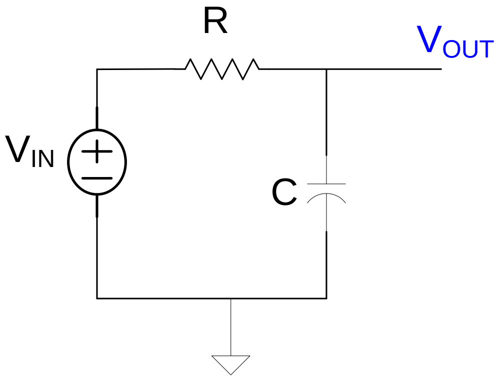
Variability includes effects of both circuit structure and components on performance

If component variations are small, high sensitivities are acceptable

If component variations are large, low sensitivities are critical

Observation:

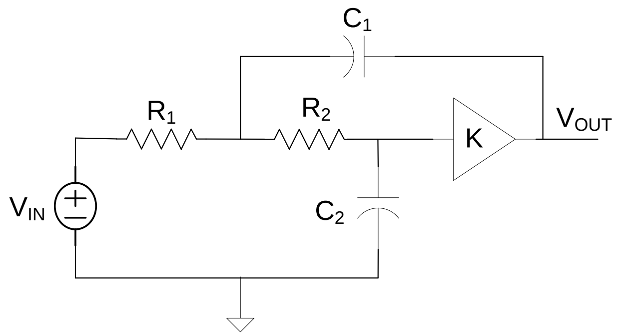
Review from last time



$$\omega_0 = \frac{1}{RC}$$

$$S_R^{\omega_0} = -1 \qquad S_C^{\omega_0} = -1$$

$$\sum_{All\ resistors} S_{R_i}^{\omega_0} = -1 \qquad \sum_{All\ capacitors} S_{C_i}^{\omega_0} = -1$$



$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

$$S_{R_1}^{\omega_0} = -\frac{1}{2} \qquad S_{C_1}^{\omega_0} = -\frac{1}{2}$$

$$S_{R_2}^{\omega_0} = -\frac{1}{2} \qquad S_{C_2}^{\omega_0} = -\frac{1}{2}$$

$$\sum_{All\ resistors} S_{R_i}^{\omega_0} = -1 \qquad \sum_{All\ capacitors} S_{C_i}^{\omega_0} = -1$$

At this stage, this is just an observation about summed sensitivities but later will establish some fundamental properties of summed sensitivities

Consider

$$\frac{dF}{F} = \left(\sum_{\textit{all resistors}} S_{R_i}^f \bullet \frac{dR_i}{R_i} \right) + \left(\sum_{\textit{all capacitors}} S_{C_i}^f \bullet \frac{dC_i}{C_i} \right) + \left(\sum_{\textit{all opamps}} S_{\tau_i}^f \bullet \frac{d\tau_i}{\tau_i} \right) + \ldots$$

The nominal value of the time constant of the op amps is 0 so this expression can not be evaluated at the ideal (nominal) value of GB=∞

Let {xi} be the components in a circuit whose nominal value is not 0

Let {yi} be the components in a circuit whose nominal value is 0

$$\frac{dF}{F} = \sum_{i=1}^{kx} \frac{\partial F}{\partial x_i} \bullet \frac{dx_i}{F} + \sum_{i=1}^{ky} \frac{\partial F}{\partial y_i} \bullet \frac{dy_i}{F} = \sum_{i=1}^{k} \left(\frac{\partial F}{\partial x_i} \bullet \frac{x_i}{F} \right) \bullet \frac{dx_i}{x_i} + \frac{1}{F} \sum_{i=1}^{ky} \frac{\partial F}{\partial y_i} dy_i$$

$$\frac{dF}{F} = \sum_{i=1}^{k} \left(S_{x_i}^f \Big|_{\vec{X}_N, \vec{Y}_N = 0} \bullet \frac{dx_i}{x_i} \right) + \frac{1}{F_N} \sum_{i=1}^{ky} \left(\mathcal{S}_{y_i}^f \Big|_{\vec{X}_N, \vec{Y}_N = 0} \bullet y_i \right)$$

This expression can be used for predicting the effects of all components in a circuit

Can set Y_N=0 before calculating $S_{x_i}^f$ **functions**

$$\frac{dF}{F}=\sum_{i=1}^{k}\left(S_{x_i}^{f}\Big|_{\vec{X}_N}\bullet\frac{dx_i}{x_i}\right)+\frac{1}{F_N}\sum_{i=1}^{ky}\left(\mathcal{S}_{y_i}^{f}\Big|_{\vec{Y}_N=0}\bullet y_i\right)$$

Low sensitivities in a circuit are often preferred but in some applications, low sensitivities would be totally unacceptable

Examples where low sensitivities are unacceptable are circuits where a charactristics F must be tunable or adjustable!

Some useful sensitivity theorems

$$S_x^{kf} = S_x^f$$

where k is a constant

$$S_x^{f^n} = n \bullet S_x^f$$

$$S_x^{1/f} = -S_x^f$$

$$S_x^{\sqrt{f}} = \frac{1}{2} S_x^f$$

$$S_x^{\prod_{i=1}^{k} f_i} = \sum_{i=1}^{k} S_x^{f_i}$$

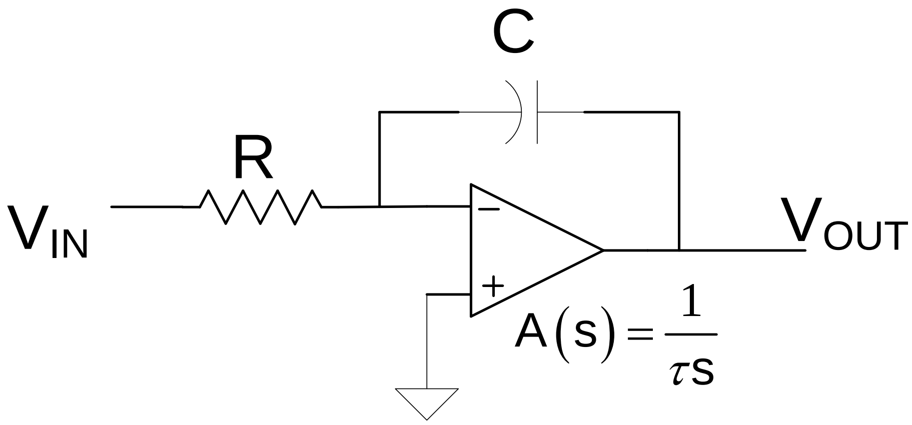
Some useful sensitivity theorems (cont)

$$S_x^{f/g} = S_x^f - S_x^g$$

$$S_x^{\sum_{i=1}^{k} f_i} = \frac{\sum_{i=1}^{k} f_i S_x^{f_i}}{\sum_{i=1}^{k} f_i}$$

$$S_{1/x}^{f} = -S_x^f$$

Example:



Ideally $$I(s) = -\frac{1}{RCs} = -\frac{I_0}{s}$$ I_0 termed the unity gain freq of integrator

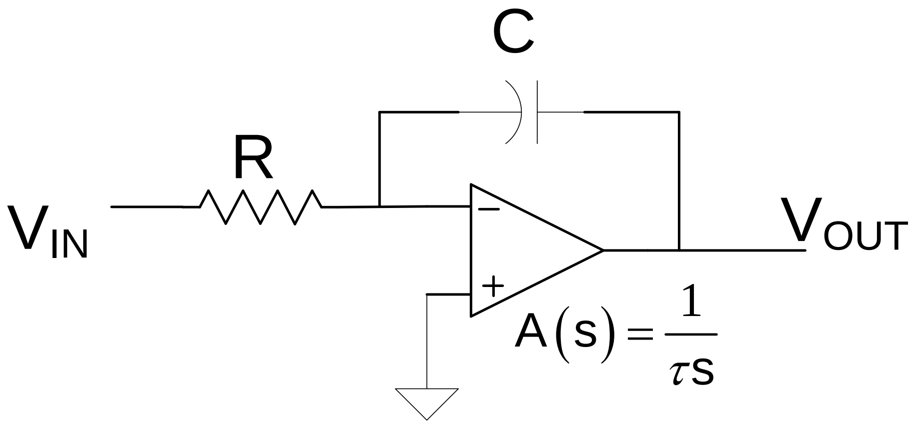
Assume ideally R=1K, C=3.18nF so that I_O=50KHz

Actually GB=600KHz, R=1.05K, and C=3.3nF

a) Determine an approximation to the actual unity gain frequency using a sensitivity analysis

b) Write an analytical expression for the actual unity gain frequency

Example:



Assume ideally R=1K, C=3.18nF so that I_O=50KHz

Actually GB=600KHz, R=1.05K, and C=3.3nF

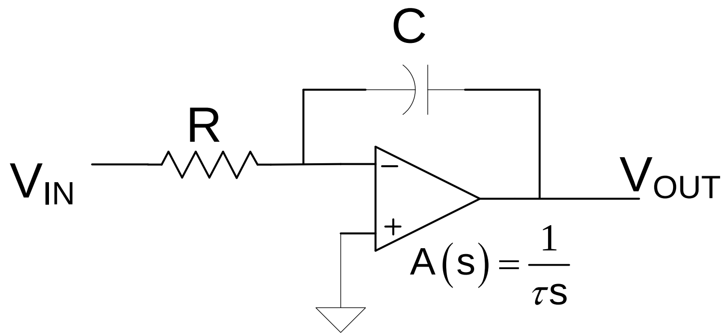
Observe

$$\frac{\Delta R}{R} = \frac{.05K}{1K} = .05$$

$$\frac{\Delta C}{C} = \frac{.12nF}{3.18nF} = .038$$

$$\frac{I_0}{GB} = \tau I_0 = \frac{50KHz}{600KHz} = .083$$

Example:



Ideally

$$I(s) = -\frac{1}{RCs} = -\frac{I_0}{s}$$

Define I_{0A} to be the actual unity gain frequency

Solution:

$$I_0 = \frac{1}{RC}$$

$$\frac{dF}{F} = \sum_{i=1}^{k}\left(S_{x_i}^{f}\Big|_{\vec{X}_N, \vec{Y}_N=0} \bullet \frac{dx_i}{x_i} \right) + \frac{1}{F_N}\sum_{i=1}^{ky}\left(\mathcal{S}_{y_i}^{f}\Big|_{\vec{X}_N, \vec{Y}_N=0} \bullet y_i \right)$$

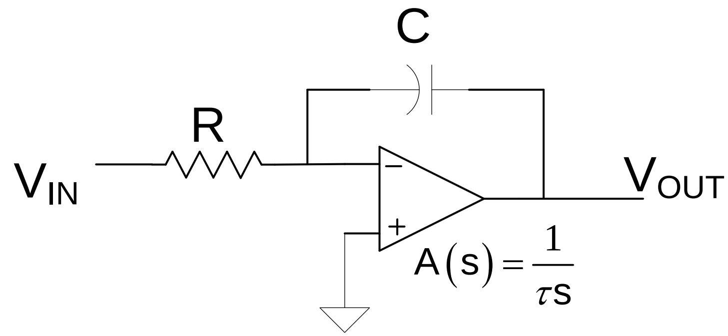
$$\frac{dI_{0A}}{I_{0A}} = \left[S_R^{I_{0A}}\Big|_{R_N, C_N, \tau=0} \right]\frac{dR}{R_N} + \left[S_C^{I_{0A}}\Big|_{R_N, C_N, \tau=0} \right]\frac{dC}{C_N} + \frac{1}{I_{0N}}\left(\mathcal{S}_{\tau}^{I_{0A}}\Big|_{\vec{X}_N, \vec{Y}_N=0} \bullet \tau \right)$$

$$S_R^{I_{0A}}\Big|_{R_N, C_N, \tau=0} = S_R^{I_0}\Big|_{R_N, C_N} \qquad S_C^{I_{0A}}\Big|_{R_N, C_N, \tau=0} = S_C^{I_0}\Big|_{R_N, C_N}$$

$$S_R^{I_0}\Big|_{R_N, C_N} = -1 \qquad S_C^{I_0}\Big|_{R_N, C_N} = -1$$

It remains to calculate $\mathcal{S}_{\tau}^{I_{0A}}\Big|_{\vec{X}_N, \vec{Y}_N=0}$

Example:



Ideally

$$I(s) = -\frac{1}{RCs} = -\frac{I_0}{s}$$

Solution:

Still need $S_{\tau}^{I_{0A}}\Big|_{\vec{X}_N,\vec{Y}_N=0}$

Define I_{0A} to be the actual unity gain frequency

$$I_A(s) = -\frac{1}{RCs+\tau s(1+RCs)}$$

$$\tau^2 I_{0A}^4 + I_{OA}^2 (RC+\tau)^2 = 1$$

$$I_A(j\omega) = -\frac{1}{-\tau\omega^2 + j(\omega RC+\tau\omega)}$$

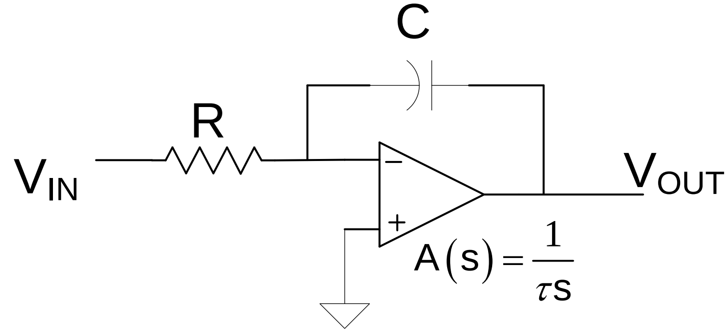
$S_{\tau}^{I_{0A}}\Big|_{\vec{X}_N,\vec{Y}_N=0}$ =?

$$\left|I_A(j\omega)\right|^2 = \frac{1}{\tau^2\omega^4 + \omega^2(RC+\tau)^2}$$

$$\left|I_A(j\omega)\right|^2 = \frac{1}{\tau^2\omega^4 + \omega^2(RC+\tau)^2} = 1$$

$$\frac{1}{\tau^2 I_{0A}^4 + I_{OA}^2 (RC+\tau)^2} = 1$$

Example:



Ideally

$$I(s) = -\frac{1}{RCs} = -\frac{I_0}{s}$$

Solution:

Still need $\left.\mathcal{S}_{\tau}^{I_{0A}}\right|_{\vec{X}_N,\vec{Y}_N=0}$

Define I_{0A} to be the actual unity gain frequency

$$\tau^2 I_{0A}^4 + I_{OA}^2 (RC+\tau)^2 = 1$$

$$\left.\mathcal{S}_{\tau}^{I_{0A}}\right|_{\vec{X}_N,\vec{Y}_N=0} = \left.\left(\frac{\partial I_{0A}}{\partial \tau}\right)\right|_{\vec{X}_N,\vec{Y}_N=0}$$

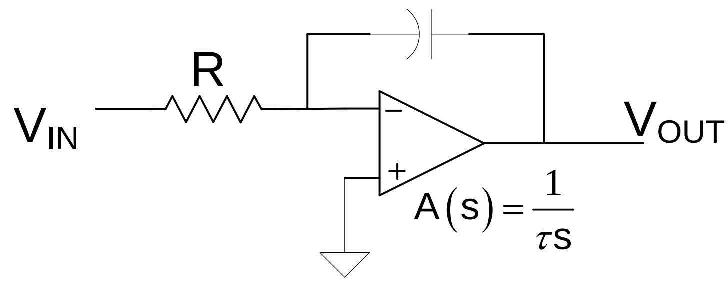
$$\tau^2 4 I_{0A}^3 \left(\frac{\partial I_{0A}}{\tau}\right) + 2\tau I_{0A}^4 + 2 I_{OA}^1 \left(\frac{\partial I_{0A}}{\tau}\right)(RC+\tau)^2 + 2(RC+\tau) I_{OA}^2 = 0$$

Evaluating at $\vec{X}_N, \vec{Y}_N = 0$

$$2 I_O^1 \left(\left.\frac{\partial I_{0A}}{\tau}\right|_{\vec{X}_N,\vec{Y}_N=0}\right)(RC)^2 + 2(RC) I_O^2 = 0$$

$$\left(\left.\frac{\partial I_{0A}}{\tau}\right|_{\vec{X}_N,\vec{Y}_N=0}\right) = \frac{-I_O}{RC} = \left.\mathcal{S}_{\tau}^{I_{0A}}\right|_{\vec{X}_N,\vec{Y}_N=0} = -I_O^2$$

Example:



Ideally

$$I(s) = -\frac{1}{RCs} = -\frac{I_0}{s}$$

Solution:

$$\frac{dI_{0A}}{I_{0A}} = \left[S_R^{I_{0A}} \Big|_{R_N, C_N, \tau=0} \right] \frac{dR}{R_N} + \left[S_C^{I_{0A}} \Big|_{R_N, C_N, \tau=0} \right] \frac{dC}{C_N} + \frac{1}{I_{0N}} \left(\mathcal{S}_\tau^{I_{0A}} \Big|_{\vec{X}_N, \vec{Y}_N=0} \bullet \tau \right)$$

$$S_R^{I_0} \Big|_{R_N, C_N} = S_C^{I_0} \Big|_{R_N, C_N} = -1 \qquad \mathcal{S}_\tau^{I_{0A}} \Big|_{\vec{X}_N, \vec{Y}_N=0} = -I_O^2$$

$$\frac{\Delta R}{R} = .05 \qquad \frac{\Delta C}{C} = .038 \qquad \tau I_0 = .083$$

$$\frac{dI_{0A}}{I_{0A}} = [-1].05 + [-1].038 + \frac{1}{I_{0N}} \left(-I_{0N}^2 \bullet \tau \right)$$

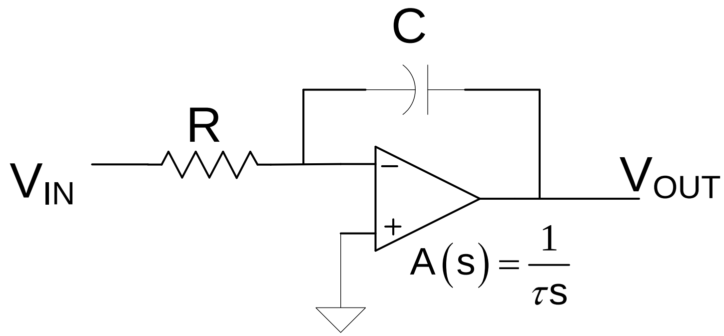
$$\frac{dI_{0A}}{I_{0A}} = [-1].05 + [-1].038 + (-.083)$$

Due to passives

$$\frac{dI_{0A}}{I_{0A}} = -.088 - .083$$

Due to actives

Example:



Ideally

$$I(s) = -\frac{1}{RCs} = -\frac{I_0}{s}$$

Solution:

$$\frac{dI_{0A}}{I_{0A}} = -.171$$

I_O=50KHz

$$I_{0A} \simeq 0.829 I_0 = 41.45\text{KHz}$$

Note that with the sensitivity analysis, it was not necessary to ever determine I_{0A}

a) Determine an approximation to the actual unity gain frequency using a sensitivity analysis

b) Write an analytical expression for the actual unity gain frequency

$$\tau^2 I_{0A}^4 + I_{OA}^2 (RC+\tau)^2 = 1$$

Must solve this quadratic for I_{0A}

Although in this simple example, it may have been easier to go directly to this expression, in more complicated circuits sensitivity analysis is much easier

How can sensitivity analysis be used to compare the performance of different circuits?

Circuits have many sensitivity functions

If two circuits have exactly the same number of sensitivity functions and all sensitivity functions in one circuit are lower than those in the other circuit, then the one with the lower sensitivities is a less sensitive circuit

But usually this does not happen !

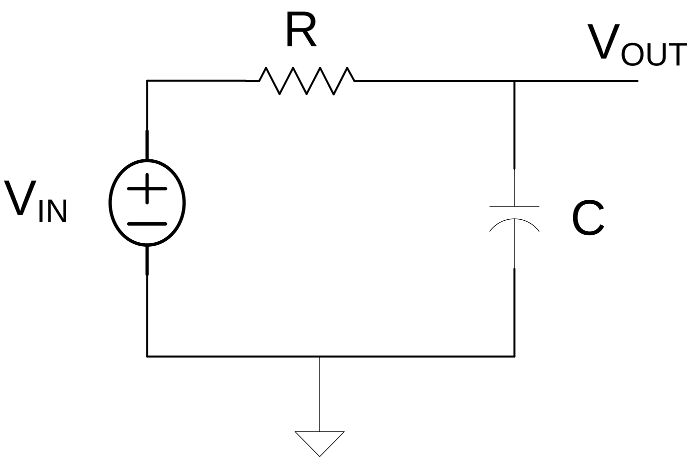
Designers would like a single metric for comparing two circuits !

$$\frac{dF}{F} = \sum_{i=1}^{k}\left(S_{x_i}^{f}\Big|_{\vec{X}_N} \bullet \frac{dx_i}{x_{iN}} \right)$$

Dependent on circuit structure
(for some circuits, also not dependent on components)

Dependent only on components
(not circuit structure)

Consider:

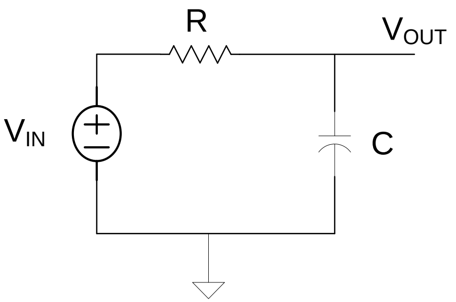


$$T(s) = \frac{1}{1+RCs}$$

$$T(s) = \frac{\omega_0}{s+\omega_0}$$

$$\omega_0 = \frac{1}{RC}$$

$$\omega_0 = \frac{1}{RC}$$



$$S_R^{\omega_0} = -1$$

$$S_C^{\omega_0} = -1$$

Dependent only on components
(not circuit structure)

$$\frac{d\omega_0}{\omega_0} = \sum_{i=1}^{2}\left(S_{x_i}^{\omega_0}\Big|_{\vec{X}_N} \bullet \frac{dx_i}{x_{iN}} \right)$$

$$\frac{d\omega_0}{\omega_0} = [-1] \bullet \frac{dR}{R_N} + [-1] \bullet \frac{dC}{C_N}$$

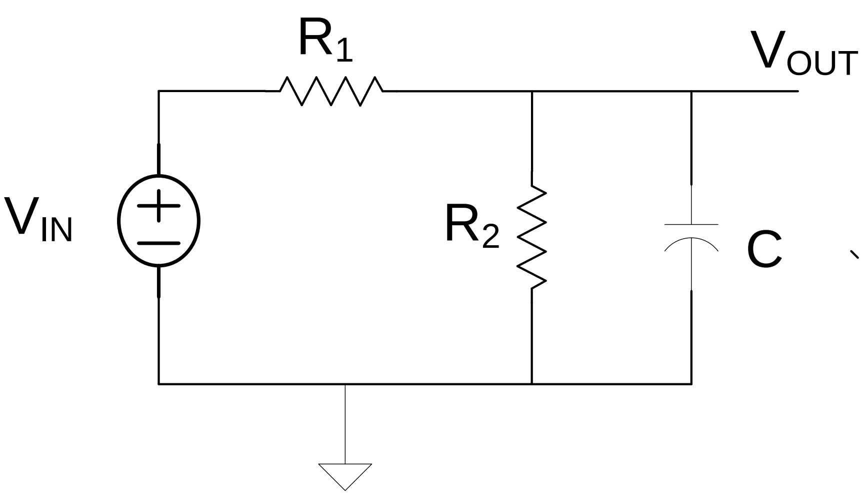
Dependent only on circuit structure

$$\frac{dF}{F} = \sum_{i=1}^{k} \left(S_{x_i}^{f}\Big|_{\vec{X}_N} \bullet \frac{dx_i}{x_{iN}} \right)$$

Dependent on circuit structure
(for some circuits, also not dependent on components)

Dependent only on components
(not circuit structure)

Consider now:



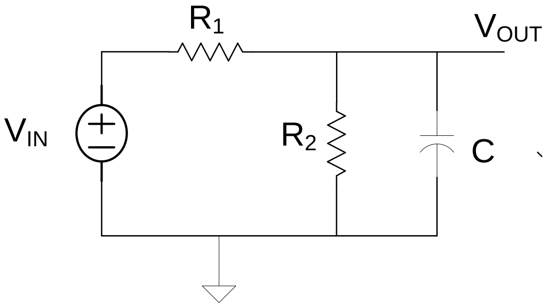
$$T(s) = \frac{\frac{R_2}{R_1+R_2}}{1+\left(\frac{R_1R_2}{R_1+R_2}C\right)s}$$

$$T(s) = \frac{R_2}{R_1+R_2} \bullet \frac{\omega_0}{s+\omega_0}$$

$$\omega_0 = \frac{R_1+R_2}{R_1R_2C}$$

$$S_{R_1}^{\omega_0} = ?$$

$$\omega_0 = \frac{R_1+R_2}{R_1R_2C}$$



$$\omega_0 = \frac{G_1+G_2}{C}$$

$$S_{R_1}^{\omega_0} = -S_{G_1}^{\omega_0}$$

$$S_{G_1}^{\omega_0} = S_{G_1}^{G_1+G_2}$$

$$S_{G_1}^{G_1+G_2} = \left(\frac{\partial(G_1+G_2)}{\partial G_1}\right)\frac{G_1}{G_1+G_2} = \frac{G_1}{G_1+G_2}$$

$$S_{R_1}^{\omega_0} = -\frac{R_2}{R_1+R_2}$$

Note this is dependent upon the components as well !
Actually dependent upon component ratio!

Theorem: If $f(x_1, ..x_m)$ can be expressed as $f = x_1^{\alpha_1} x_2^{\alpha_2} \dots x_m^{\alpha_m}$

where $\{\alpha_1, \alpha_2, \dots \alpha_m\}$ are real numbers, then $S^f_{x_i}$ is not dependent upon any of the variables in the set $\{x_1, ..x_m\}$

Proof:

$$S^f_{x_i} = S^{X_i^{\alpha_i}}_{x_i}$$

$$S^{X_i^{\alpha_i}}_{x_i} = \frac{\partial X_i^{\alpha_i}}{\partial x_i} \bullet \frac{x_i}{X_i^{\alpha_i}}$$

$$S^{X_i^{\alpha_i}}_{x_i} = \alpha_i X_i^{\alpha_i - 1} \bullet \frac{x_i}{X_i^{\alpha_i}}$$

$$S^{X_i^{\alpha_i}}_{x_i} = \alpha_i$$

$$S^f_{x_i} = \alpha_i$$

It is often the case that functions of interest are of the form expressed in the hypothesis of the theorem, and in these cases the previous claim is correct

Theorem: If $f(x_1, ..x_m)$ can be expressed as $$f = x_1^{\alpha_1} x_2^{\alpha_2} ... x_m^{\alpha_m}$$

where $\{\alpha_1, \alpha_2, \ldots \alpha_m\}$ are real numbers, then the sensitivity terms in

$$\frac{df}{f} = \sum_{i=1}^{k} \left(S_{x_i}^{f} \Big|_{\vec{X}_N} \bullet \frac{dx_i}{x_{iN}} \right)$$

are dependent only upon the circuit architecture and not dependent upon the components and and the right terms are dependent only upon the components and not dependent upon the architecture

This observation is useful for comparing the performance of two or more circuits where the function f shares this property

Metrics for Comparing Circuits

Summed Sensitivity

$$\rho_S = \sum_{i=1}^{m} S_{x_i}^{f}$$

Not very useful because sum can be small even when individual sensitivities are large

Schoeffler Sensitivity

$$\rho = \sum_{i=1}^{m} \left| S_{x_i}^{f} \right|$$

Strictly heuristic but does differentiate circuits with low sensitivities from those with high sensitivities

Metrics for Comparing Circuits

$$\rho = \sum_{i=1}^{m} \left| S_{x_i}^{f} \right|$$

Often will consider several distinct sensitivity functions to consider effects of different components

$$\rho_R = \sum_{All\ resistors} \left| S_{R_i}^{f} \right|$$

$$\rho_C = \sum_{All\ capacitors} \left| S_{C_i}^{f} \right|$$

$$\rho_{OA} = \sum_{All\ op\ amps} \left| \mathcal{S}_{\tau_i}^{f} \right|$$

Homogeniety (defn)

A function f is homogeneous of order m in the n variables $\{x_1, x_2, \ldots x_n\}$ if

$$f(\lambda x_1, \lambda x_2, \ldots \lambda x_n) = \lambda^m f(x_1, x_2, \ldots x_n)$$

Note: f may be comprised of more than n variables

Theorem: If a function f is homogeneous of order m in the n variables $\{x_1, x_2, \ldots x_n\}$ then

$$\sum_{i=1}^{n} S^{f}_{x_i} = m$$

Proof:

$$f(\lambda x_1, \lambda x_2, \ldots \lambda x_n) = \lambda^m f(x_1, x_2, \ldots x_n)$$

Differentiate WRT λ

$$\frac{\partial\left(f(\lambda x_1, \lambda x_2, \ldots \lambda x_n)\right)}{\partial \lambda} = m\lambda^{m-1} f(x_1, x_2, \ldots x_n)$$

$$\frac{\partial f}{\partial \lambda x_1} x_1 + \frac{\partial f}{\partial \lambda x_2} x_2 + \ldots + \frac{\partial f}{\partial \lambda x_n} x_n = m\lambda^{m-1} f(x_1, x_2, \ldots x_n)$$

$$\frac{\partial f}{\partial \lambda x_1} x_1 + \frac{\partial f}{\wp \lambda x_2} x_2 + \ldots + \frac{\partial f}{\partial \lambda x_n} x_n = m\lambda^{m-1} f\left(x_1, x_2, \ldots x_n\right)$$

Simplify notation

$$\frac{\partial f}{\partial \lambda x_1} x_1 + \frac{\partial f}{\partial \lambda x_2} x_2 + \ldots + \frac{\partial f}{\partial \lambda x_n} x_n = m\lambda^{m} f$$

Divide by f

$$\frac{\partial f}{\lambda x_1} \frac{x_1}{f} + \frac{\partial f}{\lambda x_2} \frac{x_2}{f} + \ldots + \frac{\partial f}{\lambda x_n} \frac{x_n}{f} = m\lambda^{m}$$

Since true for all λ, also true for λ=1, thus

$$\frac{\partial f}{x_1} \frac{x_1}{f} + \frac{\partial f}{x_2} \frac{x_2}{f} + \ldots + \frac{\partial f}{x_n} \frac{x_n}{f} = m$$

This can be expressed as

$$\sum_{i=1}^{n} S_{x_i}^{f} = m$$

Theorem: If a function f is homogeneous of order m in the n variables $\{x_1, x_2, \ldots x_n\}$ then

$$\sum_{i=1}^{n} S_{x_i}^{f} = m$$

$$f(\lambda x_1, \lambda x_2, \ldots \lambda x_n) = \lambda^m f(x_1, x_2, \ldots x_n)$$

The concept of homogeneity and this theorem were somewhat late to appear

Are there really any useful applications of this rather odd observation?

Let T(s) be a voltage or current transfer function

Observation: Impedance scaling does not change any of the following, provided Op Amps are ideal:

$T(s)$, $T(j\omega)$, $|T(j\omega)|$, ω_0, Q, p_k, z_k

So, consider impedance scaling by a parameter λ

$$R \rightarrow \lambda R$$
$$L \rightarrow \lambda L$$
$$C \rightarrow C/\lambda$$

For these <u>impedance</u> functions

$$f(\lambda x_1, \lambda x_2, \ldots \lambda x_n) = \lambda^0 f(x_1, x_2, \ldots x_n)$$

Thus, all of these functions are homogeneous of order m=0 in the impedances

Theorem: If all op amps in a filter are ideal, then ω_o, Q, BW, all band edges, and all poles and zeros are homogeneous of order 0 in the impedances.

Theorem: If all op amps in a filter are ideal and if T(s) is a dimensionless transfer function, T(s), T(jω), |T(jω)|, $\angle T\left(j\omega\right)$, are homogeneous of order 0 in the impedances

Theorem 1: If all op amps in a filter are ideal and if $T(s)$ is an impedance transfer function, $T(s)$ and $T(j\omega)$ are homogeneous of order 1 in the impedances

Theorem 2: If all op amps in a filter are ideal and if $T(s)$ is a conductance transfer function, $T(s)$ and $T(j\omega)$ are homogeneous of order -1 in the impedances

Corollary 1: If all op amps in an RC active filter are ideal and there are k_1 resistors and k_2 capacitors and if a function f is homogeneous of order 0 in the impedances, then

$$\sum_{i=1}^{k_1} S_{R_i}^{f} = \sum_{i=1}^{k_2} S_{C_i}^{f}$$

Corollary 2: If all op amps in an RC active filter are ideal and there are k_1 resistors and k_2 capacitors then

$$\sum_{i=1}^{k_1} S_{R_i}^{Q} = 0$$

$$\sum_{i=1}^{k_2} S_{C_i}^{Q} = 0$$

End of Lecture 21

EE 508
Lecture 22

Sensitivity Functions

- Root Sensitivity
- Bilinear Property of Filters
- Root Sensitivities

Review from last time

Homogeniety (defn)

A function f is homogeneous of order m in the n variables $\{x_1, x_2, \ldots x_n\}$ if

$$f(\lambda x_1, \lambda x_2, \ldots \lambda x_n) = \lambda^m f(x_1, x_2, \ldots x_n)$$

Note: f may be comprised of more than n variables

Review from last time

Theorem: If a function f is homogeneous of order m in the n variables (x_1, x_2, …x_n) then

$$\sum_{i=1}^{n} S^{f}_{x_i} = m$$

$$f(\lambda x_1, \lambda x_2, \ldots \lambda x_n) = \lambda^m f(x_1, x_2, \ldots x_n)$$

The concept of homogeneity and this theorem were somewhat late to appear

Are there really any useful applications of this rather odd observation?

Theorem: If all op amps in a filter are ideal, then ω_o, Q, BW, all band edges, and all poles and zeros are homogeneous of order 0 in the impedances.

Theorem: If all op amps in a filter are ideal and if T(s) is a dimensionless transfer function, T(s), $T(j\omega)$, $|T(j\omega)|$, $\angle T(j\omega)$, are homogeneous of order 0 in the impedances

Review from last time

Theorem 1: If all op amps in a filter are ideal and if T(s) is an impedance transfer function, T(s) and T(jω) are homogeneous of order 1 in the impedances

Theorem 2: If all op amps in a filter are ideal and if T(s) is a conductance transfer function, T(s) and T(jω) are homogeneous of order -1 in the impedances

Review from last time

Corollary 1: If all op amps in an RC active filter are ideal and there are k_1 resistors and k_2 capacitors and if a function f is homogeneous of order 0 in the impedances, then

$$\sum_{i=1}^{k_1} S_{R_i}^{f} = \sum_{i=1}^{k_2} S_{C_i}^{f}$$

Corollary 2: If all op amps in an RC active filter are ideal and there are k_1 resistors and k_2 capacitors then

$$\sum_{i=1}^{k_1} S_{R_i}^{Q} = 0$$

$$\sum_{i=1}^{k_2} S_{C_i}^{Q} = 0$$

Proof of Corollary 1:

Corollary 1: If all op amps in an RC active filter are ideal and there are k_1 resistors and k_2 capacitors and if a function f is homogeneous of order 0 in the impedances, then

$$\sum_{i=1}^{k_1} S_{R_i}^{f} = \sum_{i=1}^{k_2} S_{C_i}^{f}$$

Since f is homogenous of order zero in the impedances, z_1, z_2, … z_{k1+k2},

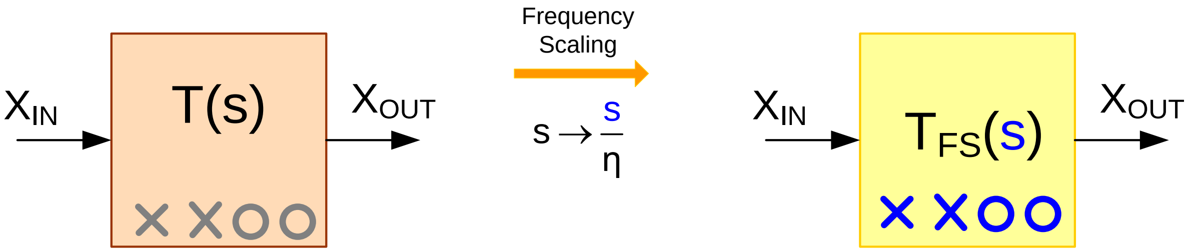
$$\sum_{i=1}^{k_1+k_2} S_{z_i}^{f} = 0$$

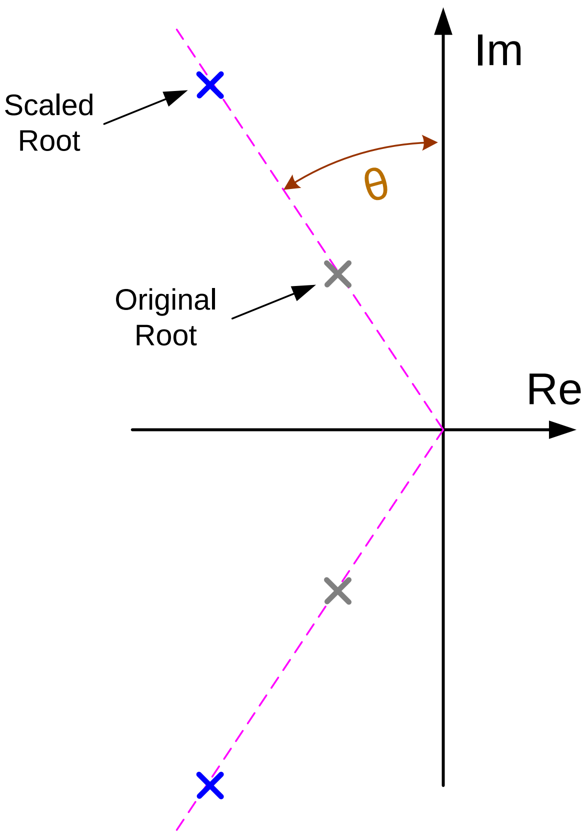
$$\therefore \quad \sum_{i=1}^{k_1} S_{R_i}^{f} + \sum_{i=1}^{k_2} S_{1/C_i}^{f} = 0$$

$$\therefore \quad \sum_{i=1}^{k_1} S_{R_i}^{f} - \sum_{i=1}^{k_2} S_{C_i}^{f} = 0$$

Proof of Corollary 2:

Recall:





Frequency Scaling: Scaling all frequency-dependent elements by a constant

$$L \to \eta L$$

$$C \to \eta C$$

Theorem: If all components are frequency scaled, roots (poles and zeros) will move along a constant Q locus

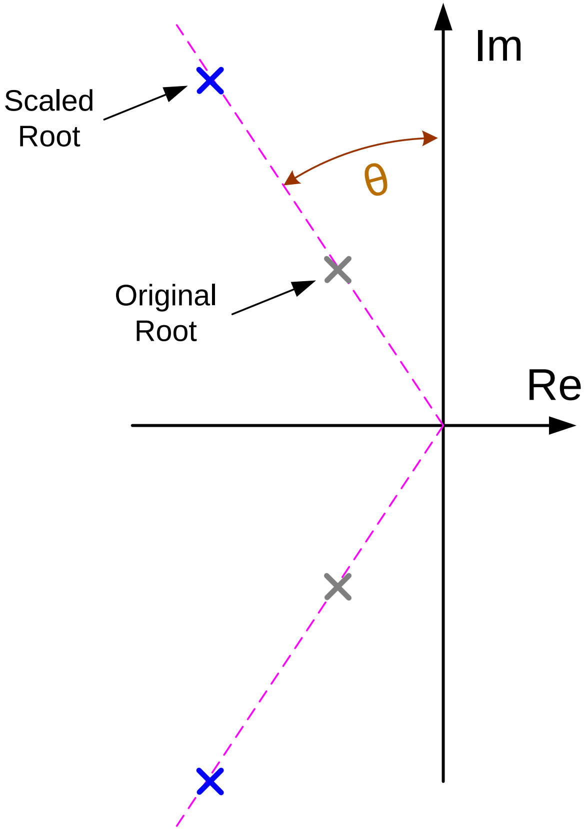
Proof:

$$T_{FS}(s) = T(s)\Big|_{s=\frac{s}{\eta}}$$

Proof of Corollary 2:

Recall:

Theorem: If all components are frequency scaled, roots (poles and zeros) will move along a constant Q locus



Proof:

$$T_{FS}(s) = T(s)\Big|_{s=\frac{s}{\eta}}$$

Let p be a pole (or zero) of T(s)

$$T(p)=0 \qquad \text{consider} \qquad p = \frac{p}{\eta}$$

$$T_{FS}(s) = T\left(\frac{s}{\eta}\right) = T(s)$$

Since true for any variable, substitute in p

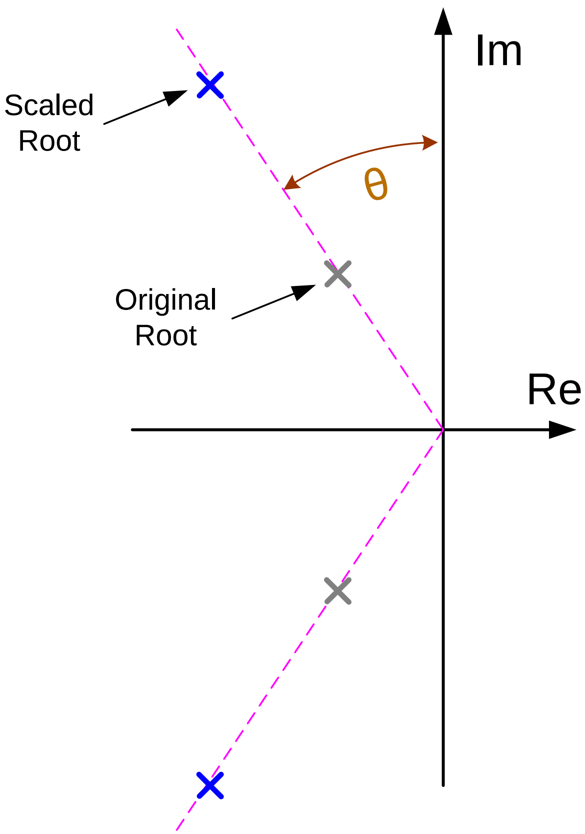
$$T_{FS}(p) = T\left(\frac{p}{\eta}\right) = T(p) = 0$$

Thus **p** is a pole (or zero) of $T_{FS}(s)$

Proof of Corollary 2:

Recall:

Theorem: If all components are frequency scaled, roots (poles and zeros) will move along a constant Q locus



Proof: Thus **p** is a pole (or zero) of $T_{FS}(s)$

$$p = \frac{p}{\eta}$$

$$p = p\eta$$

Express **p** in polar form

$$p = re^{j\beta}$$

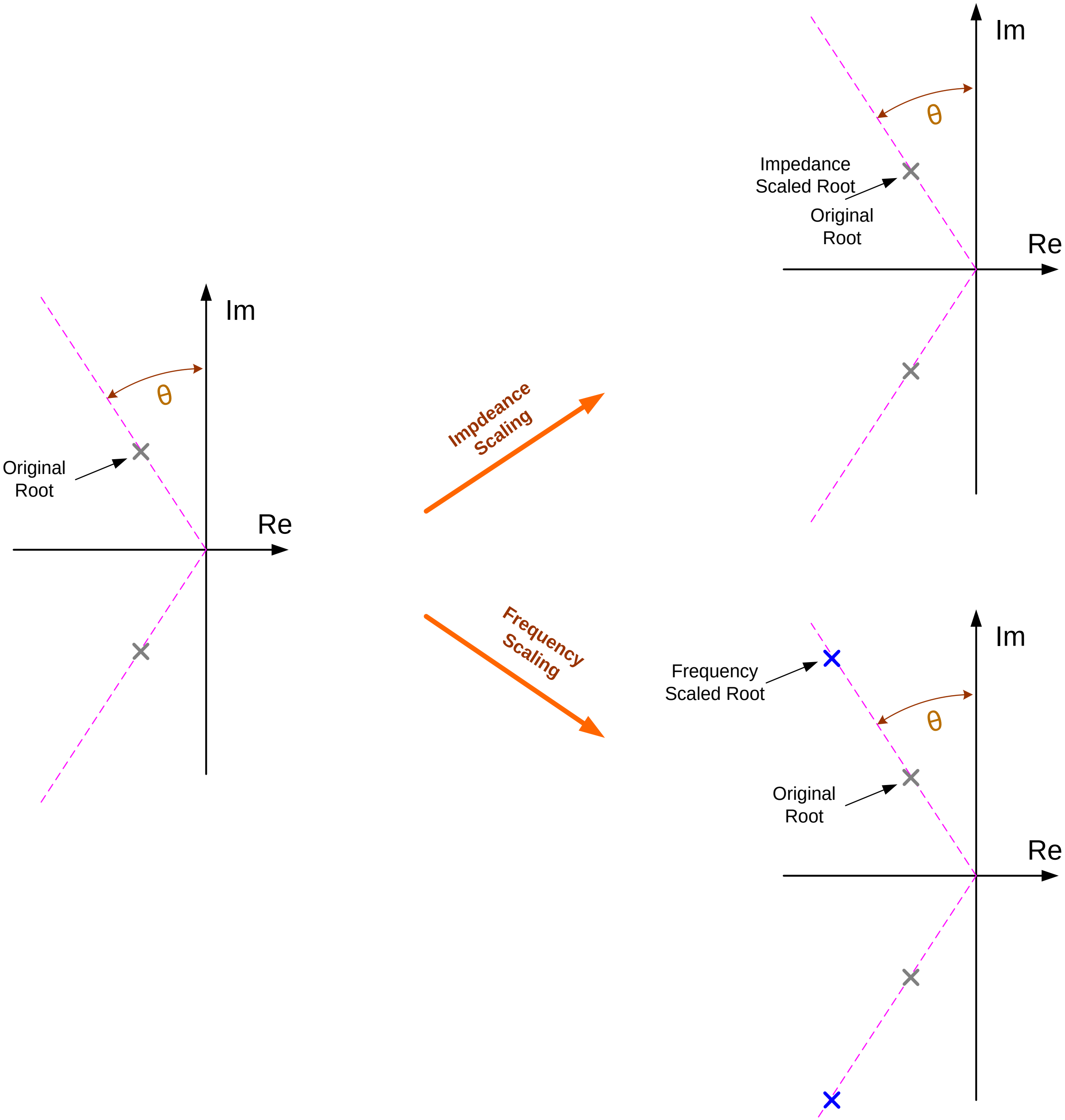
$$p = \eta p = \eta re^{j\beta}$$

Thus **p** and **p** have the same angle

Thus the scaled root has the same root Q

Proof of Corollary 2: Impedance and Frequency Scaling

Recall:



Proof of Corollary 2:

Corollary 2: If all op amps in an RC active filter are ideal and there are k_1 resistors and k_2 capacitors then $\sum_{i=1}^{k_2} S_{C_i}^{Q} = 0$ and $\sum_{i=1}^{k_1} S_{R_i}^{Q} = 0$

Since impedance scaling does not change pole (or zero) Q, the pole (or zero) Q must be homogeneous of order 0 in the impedances

(For more generality, assume k_3 inductors)

$$\sum_{i=1}^{k_1} S_{R_i}^{Q} + \sum_{i=1}^{k_2} S_{1/C_i}^{Q} + \sum_{i=1}^{k_3} S_{L_i}^{Q} = 0 \qquad (1)$$

Since frequency scaling does not change pole (or zero) Q, the pole (or zero) Q must be homogeneous of order 0 in the frequency scaling elements

$$\sum_{i=1}^{k_2} S_{C_i}^{Q} + \sum_{i=1}^{k_3} S_{L_i}^{Q} = 0 \qquad (2)$$

Proof of Corollary 2:

$$\sum_{i=1}^{k_1} S_{R_i}^{Q} + \sum_{i=1}^{k_2} S_{1/C_i}^{Q} + \sum_{i=1}^{k_3} S_{L_i}^{Q} = 0 \qquad (1)$$

$$\sum_{i=1}^{k_2} S_{C_i}^{Q} + \sum_{i=1}^{k_3} S_{L_i}^{Q} = 0 \qquad (2)$$

From theorem about sensitivity of reciprocals, can write (1) as

$$\sum_{i=1}^{k_1} S_{R_i}^{Q} - \sum_{i=1}^{k_2} S_{C_i}^{Q} + \sum_{i=1}^{k_3} S_{L_i}^{Q} = 0 \qquad (3)$$

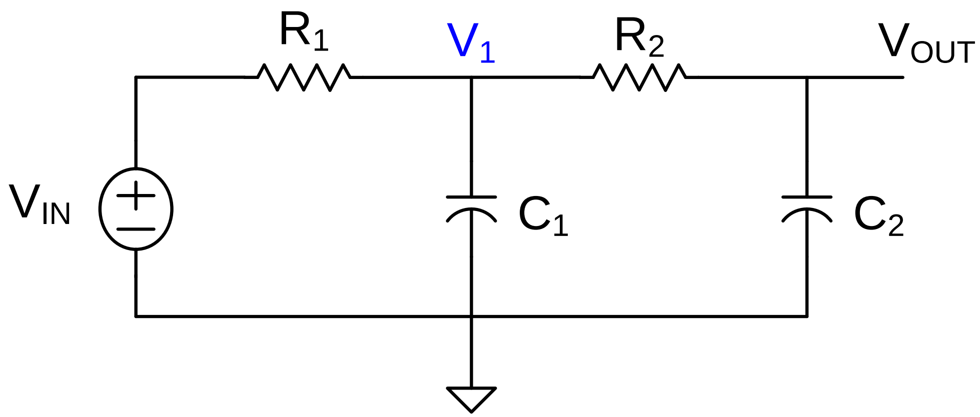
It follows from (2) and (3) that

$$\sum_{i=1}^{k_1} S_{R_i}^{Q} - 2\sum_{i=1}^{k_3} S_{L_i}^{Q} = 0 \qquad (4)$$

Since RC network, it follows from (4) and (2) that

$$\sum_{i=1}^{k_1} S_{R_i}^{Q} = 0 \qquad \sum_{i=1}^{k_2} S_{C_i}^{Q} = 0$$

Example



Determine the passive Q sensitivities

$$S_{R_1}^{Q} \quad S_{R_2}^{Q} \quad S_{C_1}^{Q} \quad S_{C_2}^{Q}$$

$$\left.\begin{aligned} V_{OUT}\left(sC_1+G_2\right) &= V_1 G_2 \\ V_1\left(sC_1+G_1+G_2\right) &= V_{IN}G_1 + V_{OUT}G_2 \end{aligned}\right\}$$

$$T(s) = \frac{1}{s^2\left(R_1R_2C_1C_2\right)+s\left(R_1C_1+R_1C_2+R_2C_2\right)+1}$$

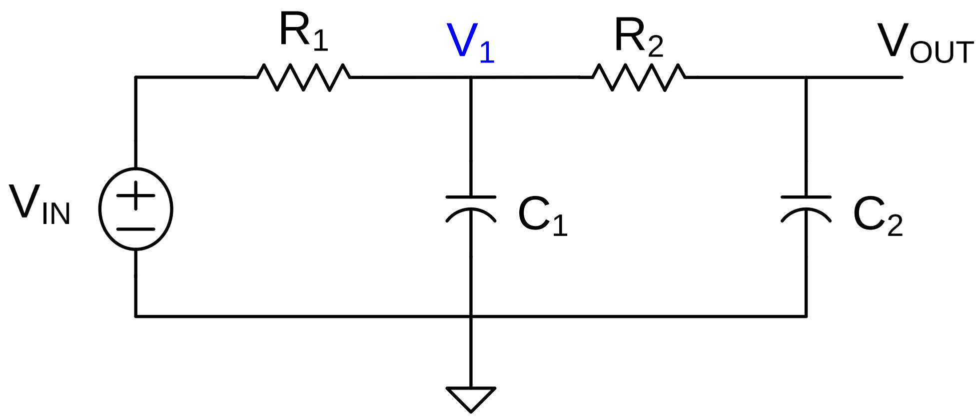
$$\omega_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}}$$

$$Q = \frac{\sqrt{R_1R_2C_1C_2}}{R_1C_1+R_1C_2+R_2C_2}$$

By the definition of sensitivity, it follows that

$$S_{R_1}^{Q} = \frac{\left(R_1C_1+R_1C_2+R_2C_1\right)\frac{1}{2}\left(R_1R_2C_1C_2\right)^{-1/2}R_2C_1C_2-\left(C_1+C_2\right)\left(R_1R_2C_1C_2\right)^{1/2}}{\left(R_1C_1+R_1C_2+R_2C_2\right)^2} \bullet \frac{R_1}{Q}$$

Example



Determine the passive Q sensitivities

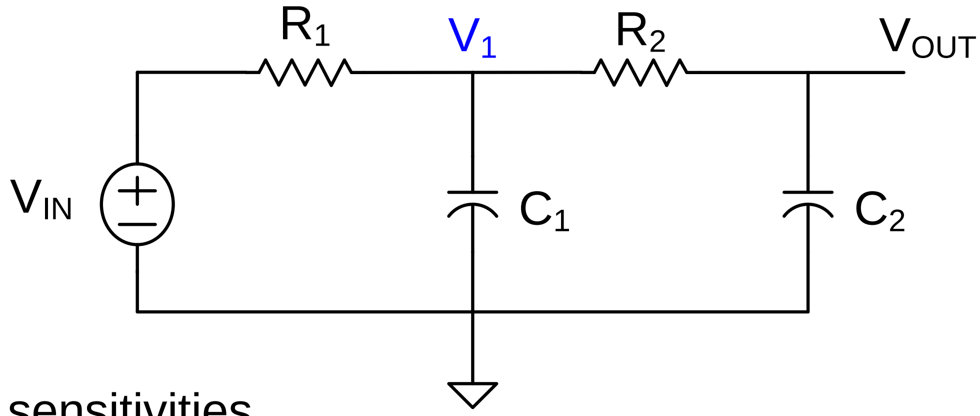
$$S_{R_1}^{Q} \quad S_{R_2}^{Q} \quad S_{C_1}^{Q} \quad S_{C_2}^{Q}$$

$$S_{R_1}^{Q} = \frac{(R_1C_1+R_1C_2+R_2C_1)\frac{1}{2}(R_1R_2C_1C_2)^{-1/2}R_2C_1C_2-(C_1+C_2)(R_1R_2C_1C_2)^{1/2}}{(R_1C_1+R_1C_2+R_2C_2)^2} \bullet \frac{R_1}{Q}$$

Following some tedious manipulations, this simplifies to

$$S_{R_1}^{Q} = \frac{1}{2} - \frac{R_1(C_1+C_2)}{R_1C_1+R_1C_2+R_2C_2}$$

Example



Determine the passive Q sensitivities

Following the same type of calculations, can obtain

$$S_{R_1}^{Q}=\frac{1}{2}-\frac{R_1\left(C_1+C_2\right)}{R_1C_1+R_1C_2+R_2C_2}\qquad S_{R_2}^{Q}=\frac{1}{2}-\frac{R_2C_2}{R_1C_1+R_1C_2+R_2C_2}$$

$$S_{C_1}^{Q}=\frac{1}{2}-\frac{R_1C_1}{R_1C_1+R_1C_2+R_2C_2}\qquad S_{C_2}^{Q}=\frac{1}{2}-\frac{C_2\left(R_1+R_2\right)}{R_1C_1+R_1C_2+R_2C_2}$$

Verify

$$\sum_{i=1}^{k_2}S_{C_i}^{Q}=0\qquad \sum_{i=1}^{k_1}S_{R_i}^{Q}=0$$

Could have saved considerable effort in calculations by using these theorems after $S_{R_1}^{Q}$ and $S_{C_1}^{Q}$ were calculated

Corollary 3: If all op amps in an RC active filter are ideal and there are k_1 resistors and k_2 capacitors and if p_k is any pole and z_h is any zero, then

$$\sum_{i=1}^{k_1} S_{R_i}^{p_k} = -1 \qquad \sum_{i=1}^{k_2} S_{C_i}^{p_k} = -1$$

and

$$\sum_{i=1}^{k_1} S_{R_i}^{z_h} = -1 \qquad \sum_{i=1}^{k_2} S_{C_i}^{z_h} = -1$$

Corollary 3: If all op amps in an RC active filter are ideal and there are k_1 resistors and k_2 capacitors and if p_k is any pole and z_h is any zero, then

$$\sum_{i=1}^{k_1} S_{R_i}^{p_k} = -1 \qquad \sum_{i=1}^{k_2} S_{C_i}^{p_k} = -1$$

and

$$\sum_{i=1}^{k_1} S_{R_i}^{z_h} = -1 \qquad \sum_{i=1}^{k_2} S_{C_i}^{z_h} = -1$$

Proof:

It was shown that scaling the frequency dependent elements by a factor η divides the pole (or zero) by η

Thus roots (poles and zeros) are homogeneous of order -1 in the frequency scaling elements

Proof:

Thus roots (poles and zeros) are homogeneous of order -1 in the frequency scaling elements

(For more generality, assume k_3 inductors)

$$\sum_{i=1}^{k_2} S_{C_i}^{p} + \sum_{i=1}^{k_3} S_{L_i}^{p} = -1 \qquad (1)$$

Since impedance scaling does not affects the poles, they are homogenous of order 0 in the impedances

$$\sum_{i=1}^{k_1} S_{R_i}^{p} + \sum_{i=1}^{k_2} S_{1/C_i}^{p} + \sum_{i=1}^{k_3} S_{L_i}^{p} = 0 \qquad (2)$$

Since there are no inductors in an active RC network, is follows from (1) that

$$\sum_{i=1}^{k_2} S_{C_i}^{p} = -1$$

And then from (2) and the theorem about sensitivity to reciprocals that

$$\sum_{i=1}^{k_1} S_{R_i}^{p} = -1$$

Corollary 4: If all op amps in an RC active filter are ideal and there are k_1 resistors and k_2 capacitors and if Z_{IN} is any input impedance of the network, then

$$\sum_{i=1}^{k_1} S_{R_i}^{Z_{IN}} - \sum_{i=1}^{k_2} S_{C_i}^{Z_{IN}} = 1$$

Claim: If op amps in the filters considered previously are not ideal but are modeled by a gain $A(s)=1/(\tau s)$, then all previous summed sensitivities developed for ideal op amps hold provided they are evaluated a the nominal value of $\tau=0$

Sensitivity Analysis

If a closed-form expression for a function f is obtained, a straightforward but tedious analysis can be used to obtain the sensitivity of the function to any components

$$S_x^f = \frac{\partial f}{\partial x} \bullet \frac{x}{f}$$

Consider:

$$T(s) = \frac{\sum_{i=0}^{m} a_i s^i}{\sum_{i=0}^{n} b_i s^i} = K \frac{\prod_{i=1}^{m} (s - z_i)}{\prod_{i=1}^{n} (s - p_i)}$$

Closed-form expressions for T(s), T(jω), |T(jω)|, $\angle T(j\omega)$, a_i, b_i, can be readily obtained

Sensitivity Analysis

If a closed-form expression for a function f is obtained, a straightforward but tedious analysis can be used to obtain the sensitivity of the function to any components

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Consider:

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Closed-form expressions for p_i, z_i, pole or zero Q, pole or zero ω_0, peak gain, ω_{3dB}, BW, … (generally the most critical and useful circuit characteristics) are difficult or impossible to obtain !

Bilinear Property of Electrical Networks

Theorem: Let x be any component or Op Amp time constant (1st order Op Amp model) of any linear active network employing a finite number of amplifiers and lumped passive components. Any transfer function of the network can be expressed in the form

$$T(s) = \frac{N_0(s) + xN_1(s)}{D_0(s) + xD_1(s)}$$

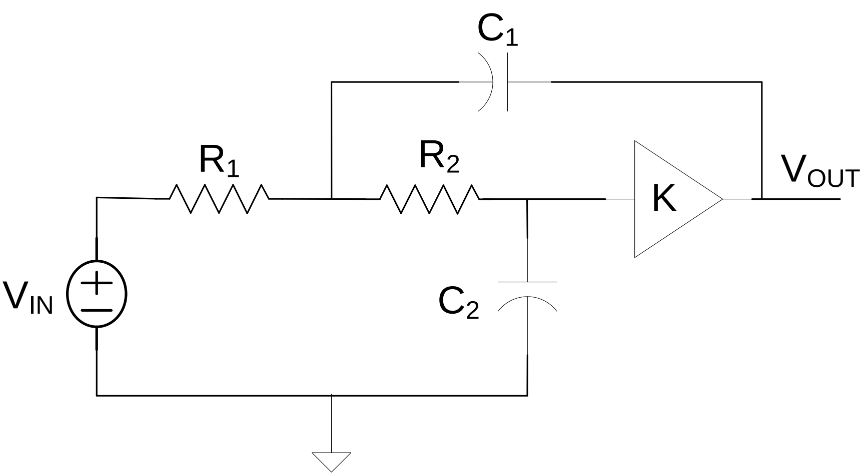
where N_0, N_1, D_0, and D_1 are polynomials in s that are not dependent upon x

A function that can be expressed as given above is said to be a bilinear function in the variable x and this is termed a bilateral property of electrical networks.

The bilinear relationship is useful for

1. Checking for possible errors in an analysis
2. Pole sensitivity analysis

Example of Bilinear Property : +KRC Lowpass Filter



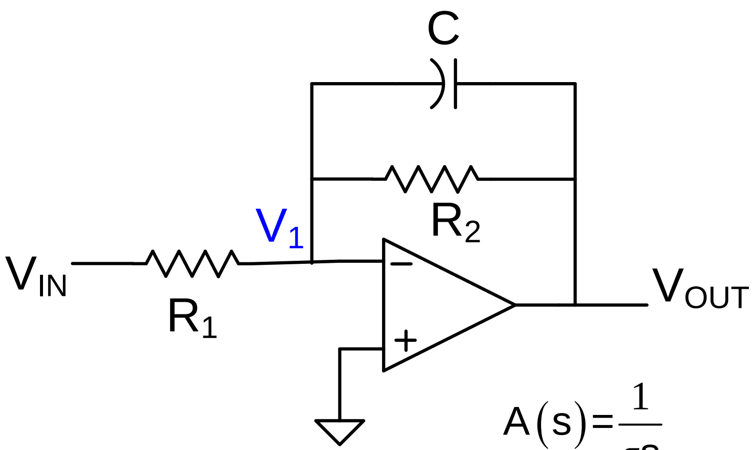
$$T(s)=\frac{\frac{K_0}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{(1-K_0)}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}+K_0\tau s\left(s^2+s\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}\right)}$$

Consider R_1

$$T(s)=\frac{\frac{K_0}{R_2C_1C_2}}{R_1s^2+s\left[\frac{1}{C_1}+R_1\frac{1}{R_2C_1}+R_1\frac{(1-K_0)}{R_2C_2}\right]+\frac{1}{R_2C_1C_2}+K_0\tau s\left(R_1s^2+s\left[\frac{1}{C_1}+R_1\frac{1}{R_2C_1}+R_1\frac{1}{R_2C_2}\right]+\frac{1}{R_2C_1C_2}\right)}$$

$$T(s)=\frac{\left[\frac{K_0}{R_2C_1C_2}\right]+R_1\bullet[0]}{\left[s\frac{1}{C_1}+\frac{1}{R_2C_1C_2}+K_0\tau s\left(s\frac{1}{C_1}\right)+\frac{1}{R_2C_1C_2}\right]+R_1\left[s^2+s\left[\frac{1}{R_2C_1}+\frac{(1-K_0)}{R_2C_2}\right]+K_0\tau s\left(s^2+s\left[\frac{1}{R_2C_1}+\frac{1}{R_2C_2}\right]\right)\right]}$$

Example of Bilinear Property



$$A(s)=\frac{1}{\tau s}$$

$$\left.\begin{aligned} & V_1\left(G_1+G_2+sC\right) = V_{IN}G_1+V_{OUT}\left(sC+G_2\right) \\ & V_{OUT} = -V_1\left(\frac{1}{\tau s}\right) \end{aligned}\right\}$$

$$T(s) = \frac{-R_2}{R_1+R_1R_2Cs+\tau s\left(sCR_1R_2+R_1+R_2\right)}$$

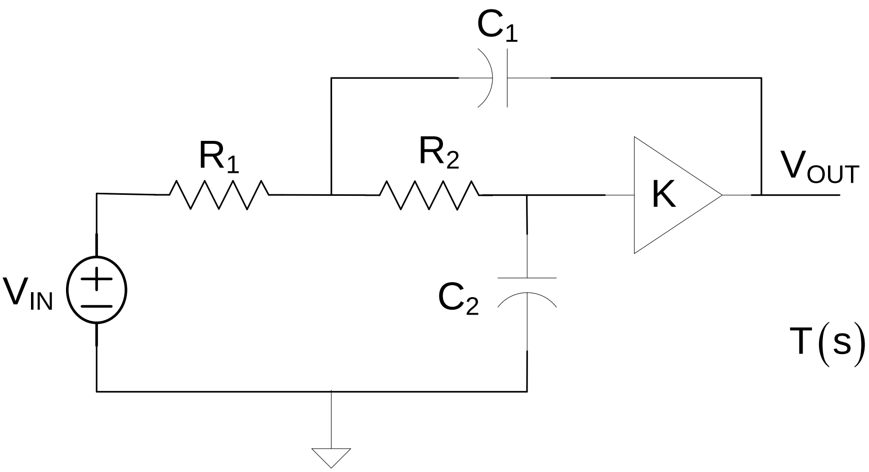
Consider R_1

$$T(s) = \frac{-R_2+0\bullet R_1}{\left[\tau sR_2\right]+R_1\left[1+R_2Cs+\tau s\left(sCR_2+1\right)\right]}$$

Consider τ

$$T(s) = \frac{-R_2+0\bullet \tau}{\left[R_1\left(1+R_2Cs\right)\right]+\tau\left[sR_2+sR_1\left(sCR_2+1\right)\right]}$$

Example of Bilinear Property : +KRC Lowpass Filter



Equal R Equal C

$$T(s)=\frac{\frac{K_0}{R^2C^2}}{s^2+s\left[\frac{(3\text{-}K_0)}{RC}\right]+\frac{1}{R^2C^2}+K_0\tau s\left(s^2+s\left[\frac{3}{RC}\right]+\frac{1}{R^2C^2}\right)}$$

$$T(s)=\frac{K_0}{R^2\left(C^2s^2+K_0\tau sC^2\right)+R\left(sC(3\text{-}K_0)+3K_0C\tau s^2\right)+(1+K_0\tau s)}$$

Can not eliminate the R^2 term

- Bilinear property only applies to individual components
- Bilinear property was established only for T(s)

Root Sensitivities

Consider expressing T(s) as a bilinear fraction in x

$$T(s)=\frac{N_0(s)+xN_1(s)}{D_0(s)+xD_1(s)}=\frac{N(s)}{D(s)}$$

Theorem: If z_i is any simple zero and/or p_i is any simple pole of T(s), then

$$S_x^{z_i}=\left(\frac{x}{z_i}\right)\left(\frac{-N_1(z_i)}{\frac{dN(z_i)}{dz_i}}\right) \quad \text{and} \quad S_x^{p_i}=\left(\frac{x}{p_i}\right)\left(\frac{-D_1(p_i)}{\frac{dD(p_i)}{dp_i}}\right)$$

Note: Do not need to find expressions for the poles or the zeros to fine the pole and zero sensitivities !

Root Sensitivities

Theorem: If p_i is any simple pole of T(s), then

$$S_x^{p_i} = \left(\frac{x}{p_i}\right)\left(\frac{-D_1(p_i)}{\frac{dD(p_i)}{dp_i}}\right)$$

Proof (similar argument for the zeros)

$$D(s) = D_0(s) + xD_1(s)$$

By definition of a pole,

$$D(p_i) = 0$$

$$\therefore \qquad D(p_i) = D_0(p_i) + xD_1(p_i)$$

Root Sensitivities

$$\therefore \qquad D(p_i) = D_0(p_i) + xD_1(p_i)$$

Differentiating this expression explicitly WRT x, we obtain

$$\frac{\partial D_0(p_i)}{\partial p_i}\frac{\partial p_i}{\partial x} + \left[x\frac{\partial D_1(p_i)}{\partial p_i}\frac{\partial p_i}{\partial x} + D_1(p_i)\right] = 0$$

Re-grouping, obtain

$$\frac{\partial p_i}{\partial x}\left[\frac{\partial D_0(p_i)}{\partial p_i} + x\frac{\partial D_1(p_i)}{\partial p_i}\right] = -D_1(p_i)$$

But term in brackets is derivative of $D(p_i)$, thus

$$\frac{\partial p_i}{\partial x} = -\frac{D_1(p_i)}{\left(\frac{\partial D(p_i)}{\partial p_i}\right)}$$

Root Sensitivities

$$\frac{\partial p_i}{\partial x} = -\frac{D_1(p_i)}{\left(\dfrac{\partial D(p_i)}{\partial p_i}\right)}$$

Finally, from the definition of sensitivity,

$$S_x^{p_i} = \frac{x}{p_i}\frac{\partial p_i}{\partial x} = -\left(\frac{x}{p_i}\right)\frac{D_1(p_i)}{\left(\dfrac{\partial D(p_i)}{\partial p_i}\right)}$$

Root Sensitivities

$$S_x^{p_i} = \frac{x}{p_i}\frac{\partial p_i}{\partial x} = -\left(\frac{x}{p_i}\right)\frac{D_1(p_i)}{\left(\frac{\partial D(p_i)}{\partial p_i}\right)}$$

Observation: Although the sensitivity expression is readily obtainable, direction information about the pole movement is obscured because the derivative is multiplied by the quantity p_i which is often complex. Usually will use either $\mathcal{S}_x^{p_i} = \frac{\partial p_i}{\partial x}$ or

$$\tilde{S}_x^{p_i} = \frac{x}{|p_i|}\frac{\partial p_i}{\partial x} = -\left(\frac{x}{|p_i|}\right)\frac{D_1(p_i)}{\left(\frac{\partial D(p_i)}{\partial p_i}\right)}$$

which preserve direction information when working with pole or zero sensitivity analysis.

Root Sensitivities

Summary: Pole (or zero) locations due to component variations can be approximated with simple analytical calculations without obtaining parametric expressions for the poles (or zeros).

$$p_i \simeq p_i\Big|_{\substack{\text{Ideal}\\\text{Components}}} + \Delta p_i$$

where

$$\Delta p_i \simeq \Delta x \bullet \mathcal{S}_x^{p_i}$$

$$\mathcal{S}_x^{p_i} = -\frac{D_1(p_i)}{\left.\left(\dfrac{\partial D(p_i)}{\partial p_i}\right)\right|_{p_{iN}}}$$

and

$$D(s) = D_0(s) + x \bullet D_1(s)$$

Alternately,

$$\Delta p_i \simeq \left(|p_i| \frac{\Delta x}{x}\right) \tilde{S}_x^{p_i}$$

End of Lecture 22

EE 508
Lecture 23

Sensitivity Functions

- Transfer Function Sensitivity
- Examples

Review from last time

Corollary 3: If all op amps in an RC active filter are ideal and there are k_1 resistors and k_2 capacitors and if p_k is any pole and z_h is any zero, then

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$$S_x^{p_i} = \frac{x}{p_i}\frac{\partial p_i}{\partial x} = -\left(\frac{x}{p_i}\right)\frac{D_1(p_i)}{\left(\frac{\partial D(p_i)}{\partial p_i}\right)}$$

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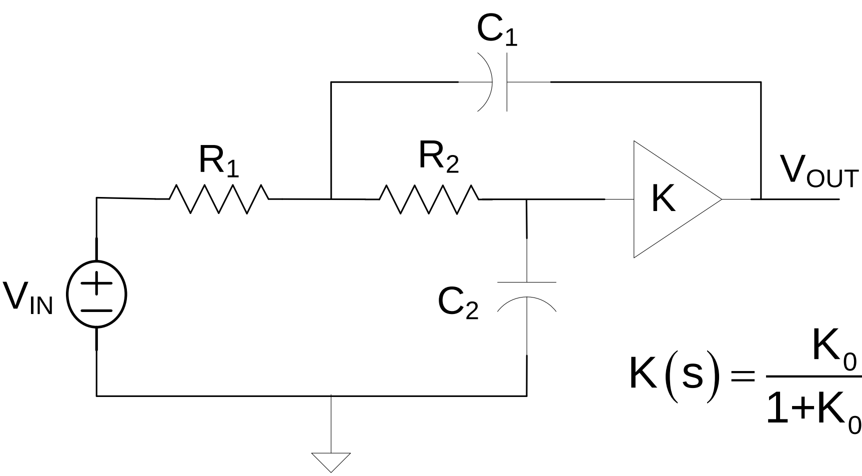
$$\tilde{S}_x^{p_i} = \frac{x}{|p_i|}\frac{\partial p_i}{\partial x} = -\left(\frac{x}{|p_i|}\right)\frac{D_1(p_i)}{\left(\frac{\partial D(p_i)}{\partial p_i}\right)}$$

which preserve direction information when working with pole or zero sensitivity analysis.

Example: Determine $\tilde{S}_{R_1}^{p_i}$ for the +KRC Lowpass Filter for equal R, equal C



$$T(s)=\frac{N_0(s)+xN_1(s)}{D_0(s)+xD_1(s)}$$

$$\tilde{S}_x^{p_i}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}=-\left(\frac{x}{|p_i|}\right)\frac{D_1(p_i)}{\left(\dfrac{\partial D(p_i)}{\partial p_i}\right)}$$

$$K(s)=\frac{K_0}{1+K_0\tau s}$$

$$T(s)=\frac{\dfrac{K_0}{R_1R_2C_1C_2}}{s^2+s\left[\dfrac{1}{R_1C_1}+\dfrac{1}{R_2C_1}+\dfrac{(1\text{-}K_0)}{R_2C_2}\right]+\dfrac{1}{R_1R_2C_1C_2}+K_0\tau s\left(s^2+s\left[\dfrac{1}{R_1C_1}+\dfrac{1}{R_2C_1}+\dfrac{1}{R_2C_2}\right]+\dfrac{1}{R_1R_2C_1C_2}\right)}$$

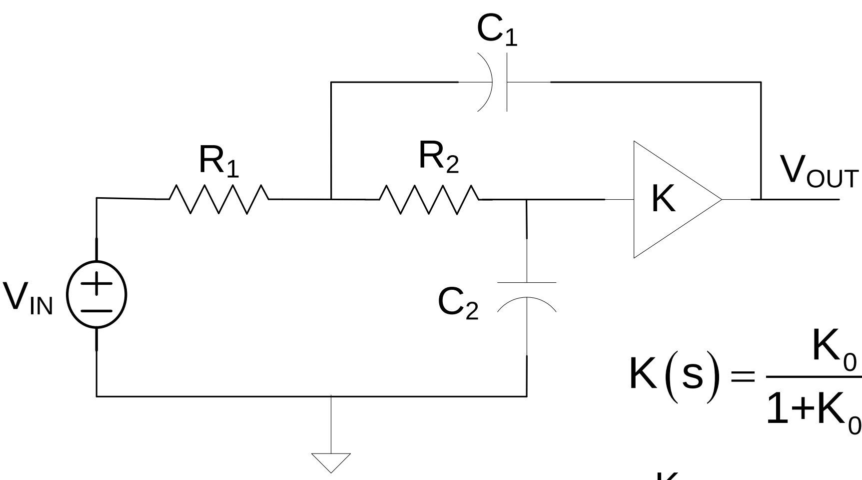
write in bilinear form

$$T(s)=\frac{\dfrac{K_0}{R_2C_1C_2}}{\left(s\dfrac{1}{C_1}+\dfrac{1}{R_2C_1C_2}+K_0\tau s\left(s\dfrac{1}{C_1}\right)+\dfrac{1}{R_2C_1C_2}\right)+R_1\left[s^2+s\left[\dfrac{1}{R_2C_1}+\dfrac{(1\text{-}K_0)}{R_2C_2}+K_0\tau s\left(s^2+s\left[\dfrac{1}{R_2C_1}+\dfrac{1}{R_2C_2}\right]\right)\right]\right]}$$

evaluate at τ=0

$$T(s)=\frac{\dfrac{K_0}{R_2C_1C_2}}{\left(s\dfrac{1}{C_1}+\dfrac{1}{R_2C_1C_2}+\dfrac{1}{R_2C_1C_2}\right)+R_1\left[s^2+s\left[\dfrac{1}{R_2C_1}+\dfrac{(1\text{-}K_0)}{R_2C_2}\right]\right]}$$

Example: Determine $\tilde{S}_{R_1}^{p_i}$ for the +KRC Lowpass Filter for equal R, equal C



$$T(s)=\frac{N_0(s)+xN_1(s)}{D_0(s)+xD_1(s)}$$

$$\tilde{S}_x^{p_i}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}=-\left(\frac{x}{|p_i|}\right)\frac{D_1(p_i)}{\left(\frac{\partial D(p_i)}{\partial p_i}\right)}$$

T(s)=

$$K(s)=\frac{K_0}{1+K_0\tau s}$$

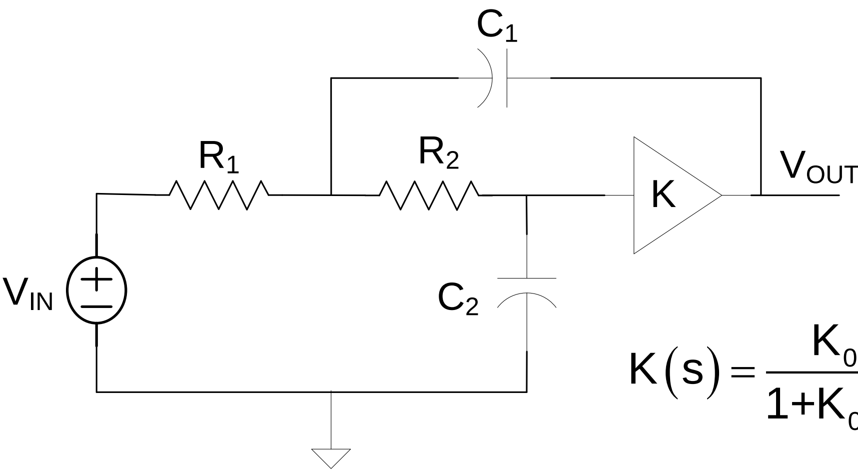
$$T(s)=\frac{\frac{K_0}{R_2C_1C_2}}{\left(s\frac{1}{C_1}+\frac{1}{R_2C_1C_2}+\frac{1}{R_2C_1C_2}\right)+R_1\left[s^2+s\left[\frac{1}{R_2C_1}+\frac{(1-K_0)}{R_2C_2}\right]\right]}$$

$$D_1(s)=s^2+s\left[\frac{1}{R_2C_1}+\frac{(1-K_0)}{R_2C_2}\right]$$

$$D(s)=\left(s\frac{1}{C_1}+\frac{1}{R_2C_1C_2}+\frac{1}{R_2C_1C_2}\right)+R_1\left[s^2+s\left[\frac{1}{R_2C_1}+\frac{(1-K_0)}{R_2C_2}\right]\right]=R_1\left(s^2+s\left[\frac{\omega_0}{Q}\right]+\omega_0^2\right)$$

$$\tilde{S}_{R_1}^{p}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}=-\left(\frac{1}{|p_i|}\right)\frac{p^2+p\left[\frac{1}{R_2C_1}+\frac{(1-K_0)}{R_2C_2}\right]}{\left(2p_i+\frac{\omega_0}{Q}\right)}$$

Example: Determine $\tilde{S}_{R_1}^{p_i}$ for the +KRC Lowpass Filter for equal R, equal C



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$$T(s)=\frac{N_0(s)+xN_1(s)}{D_0(s)+xD_1(s)}$$

$$\tilde{S}_x^{p_i}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}=-\left(\frac{x}{|p_i|}\right)\frac{D_1(p_i)}{\left(\frac{\partial D(p_i)}{\partial p_i}\right)}$$

$T(s)=$

$$T(s)=\frac{\frac{K_0}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{(1-K_0)}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}}$$

$$p^2+p\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{(1-K_0)}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}=0$$

$$p^2+p\left[\frac{1}{R_2C_1}+\frac{(1-K_0)}{R_2C_2}\right]=-\frac{1}{R_1R_2C_1C_2}-p\frac{1}{R_1C_1}$$

$$\tilde{S}_{R_1}^{p}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}=-\left(\frac{1}{|p_i|}\right)\frac{p^2+p\left[\frac{1}{R_2C_1}+\frac{(1-K_0)}{R_2C_2}\right]}{\left(2p_i+\frac{\omega_0}{Q}\right)}$$

$$\tilde{S}_{R_1}^{p}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}=\left(\frac{1}{|p_i|}\right)\frac{\frac{1}{R_1R_2C_1C_2}+p\frac{1}{R_1C_1}}{\left(2p_i+\frac{\omega_0}{Q}\right)}$$

$$\tilde{S}_{R_1}^{p}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}=\left(\frac{1}{\omega_0}\right)\frac{\omega_0^2+p\frac{1}{R_1C_1}}{\left(2p_i+\frac{\omega_0}{Q}\right)}$$

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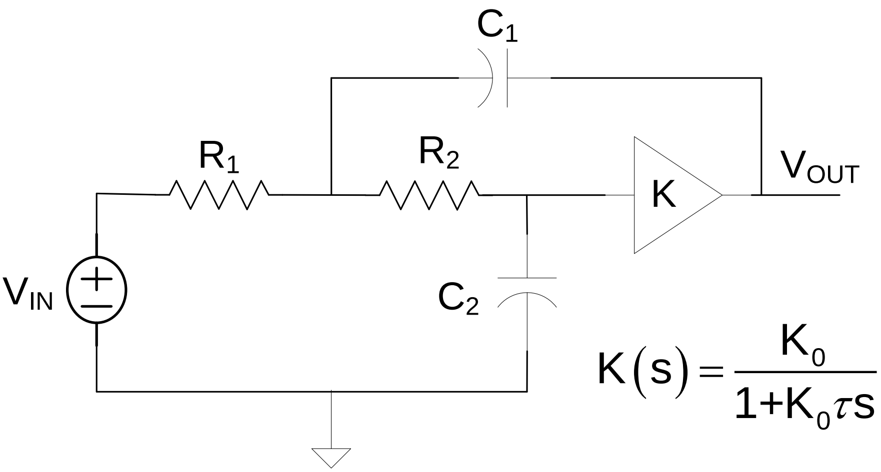


$$K(s) = \frac{K_0}{1+K_0\tau s}$$

$$\tilde{S}_x^{p_i} = \frac{x}{|p_i|}\frac{\partial p_i}{\partial x} = \left(\frac{1}{\omega_0}\right)\frac{\omega_0^2 + p\frac{1}{R_1C_1}}{\left(2p_i + \frac{\omega_0}{Q}\right)}$$

For equal R, equal C $\quad \omega_0 = \frac{1}{RC}$

$$\tilde{S}_{R_1}^{p} = \frac{x}{|p_i|}\frac{\partial p_i}{\partial x} = \left(\frac{1}{\omega_0}\right)\frac{\omega_0^2 + p\omega_0}{\left(2p_i + \frac{\omega_0}{Q}\right)}$$

$$\tilde{S}_{R_1}^{p} = \frac{x}{|p_i|}\frac{\partial p_i}{\partial x} = \frac{\omega_0 + p}{\left(2p + \frac{\omega_0}{Q}\right)}$$

$$\tilde{S}_{R_1}^{p} = \frac{\omega_0 - \frac{\omega_0}{2Q} \pm \frac{\omega_0}{2Q}\sqrt{1-4Q^2}}{\pm\frac{\omega_0}{Q}\sqrt{1-4Q^2}}$$

$$\tilde{S}_{R_1}^{p} = \frac{Q - \frac{1}{2} \pm \frac{1}{2}\sqrt{1-4Q^2}}{\pm\sqrt{1-4Q^2}}$$

Transfer Function Sensitivities

$$S_x^{T(s)}\Big|_{s=j\omega} = S_x^{T(j\omega)}$$

$$S_x^{T(j\omega)} = S_x^{|T(j\omega)|} + j\theta S_x^{\theta}$$

where $\theta = \angle T(j\omega)$

$$S_x^{|T(j\omega)|} = \mathrm{Re}\left(S_x^{T(j\omega)}\right)$$

$$S_x^{\theta} = \frac{1}{\theta}\,\mathrm{Im}\left(S_x^{T(j\omega)}\right)$$

Transfer Function Sensitivities

If T(s) is expressed as $T(s) = \dfrac{\sum_{i=0}^{m} a_i s^i}{\sum_{i=0}^{n} b_i s^i} = \dfrac{N(s)}{D(s)}$

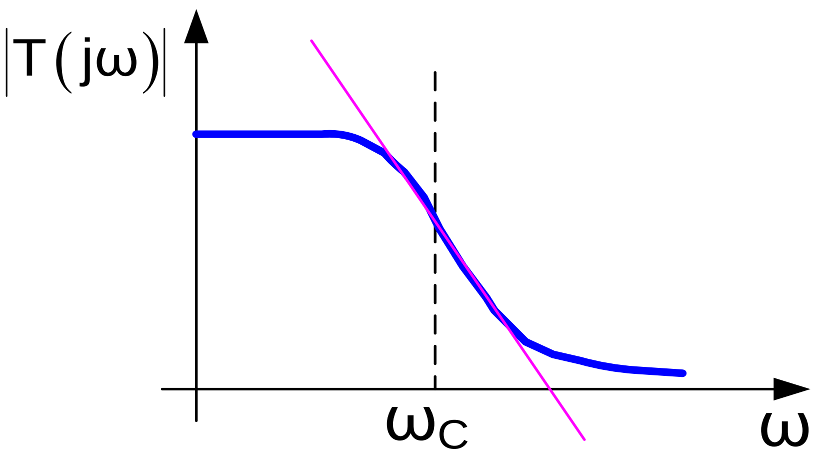
then $$S_x^{T(s)} = \frac{\sum_{i=0}^{m} a_i s^i S_x^{a_i}}{N(s)} - \frac{\sum_{i=0}^{n} b_i s^i S_x^{b_i}}{D(s)}$$

If T(s) is expressed as $T(s) = \dfrac{N_0(s) + xN_1(s)}{D_0(s) + xD_1(s)}$

$$S_x^{T(s)} = \frac{x\left[D_0(s)N_1(s) - N_0(s)D_1(s)\right]}{\left(N_0(s)+xN_1(s)\right)\left(D_0(s)+xD_1(s)\right)}$$

Band-edge Sensitivities

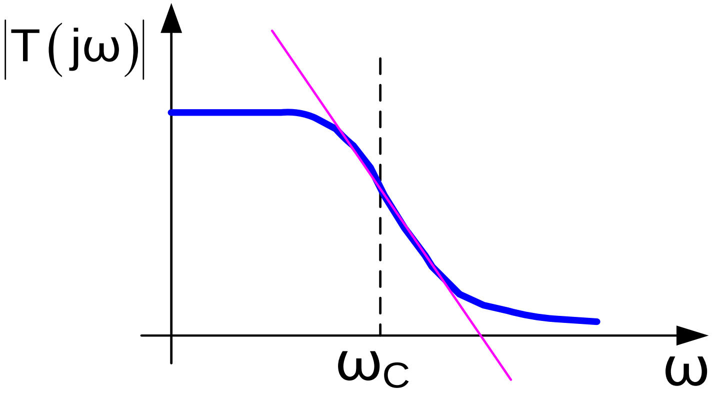
The band edge of a filter is often of interest. A closed-form expression for the band-edge of a filter may not be attainable and often the band-edges are distinct from the ω_0 of the poles. But the sensitivity of the band-edges to a parameter x is often of interest.



Want

$$S_x^{\omega_C} = \frac{\partial \omega_C}{\partial x} \bullet \frac{x}{\omega_C}$$

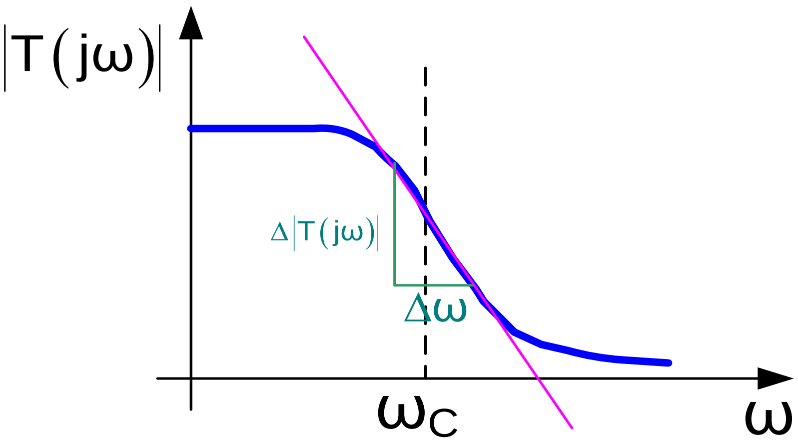
Band-edge Sensitivities



Theorem: The sensitivity of the band-edge of a filter is given by the expression

$$S_x^{\omega_C} = \frac{\left. S_x^{|T(j\omega)|} \right|_{\omega=\omega_C}}{\left. S_\omega^{|T(j\omega)|} \right|_{\omega=\omega_C}}$$

Band-edge Sensitivities



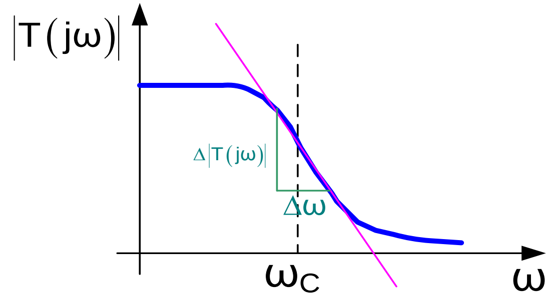
Proof:

Observe
$$\frac{\partial\left|T(j\omega)\right|}{\partial\omega} \simeq \frac{\Delta\left|T(j\omega)\right|}{\Delta\omega}$$

$$\frac{\partial\left|T(j\omega)\right|}{\partial\omega} \simeq \frac{\Delta\left|T(j\omega)\right|}{\Delta x} \bullet \frac{\Delta x}{\Delta\omega} \simeq \frac{\dfrac{\partial\left|T(j\omega)\right|}{\partial x}}{\dfrac{\partial\omega}{\partial x}}$$

Band-edge Sensitivities

$$\frac{\partial |T(j\omega)|}{\partial \omega} \simeq \frac{\Delta |T(j\omega)|}{\Delta x} \bullet \frac{\Delta x}{\Delta \omega} \simeq \frac{\dfrac{\partial |T(j\omega)|}{\partial x}}{\dfrac{\partial \omega}{\partial x}}$$



$$\frac{\partial \omega}{\partial x} \simeq \frac{\dfrac{\partial |T(j\omega)|}{\partial x}}{\dfrac{\partial |T(j\omega)|}{\partial \omega}}$$

$$\frac{\partial \omega}{\partial x} \simeq \frac{\dfrac{\partial |T(j\omega)|}{\partial x} \bullet \dfrac{x}{|T(j\omega)|}}{\dfrac{\partial |T(j\omega)|}{\partial \omega} \bullet \dfrac{\omega}{|T(j\omega)|}} \left(\frac{\omega}{x}\right)$$

$$\frac{\partial \omega}{\partial x} \bullet \left(\frac{x}{\omega}\right) \simeq \frac{\dfrac{\partial |T(j\omega)|}{\partial x} \bullet \dfrac{x}{|T(j\omega)|}}{\dfrac{\partial |T(j\omega)|}{\partial \omega} \bullet \dfrac{\omega}{|T(j\omega)|}}$$

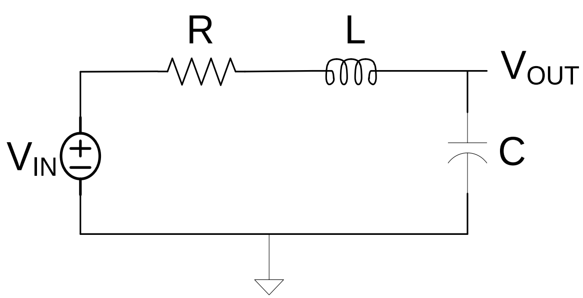
$$S_x^{\omega} = \frac{S_x^{|T(j\omega)|}}{S_\omega^{|T(j\omega)|}}$$

$$S_x^{\omega_C} = \frac{S_x^{|T(j\omega)|}\Big|_{\omega=\omega_C}}{S_\omega^{|T(j\omega)|}\Big|_{\omega=\omega_C}}$$

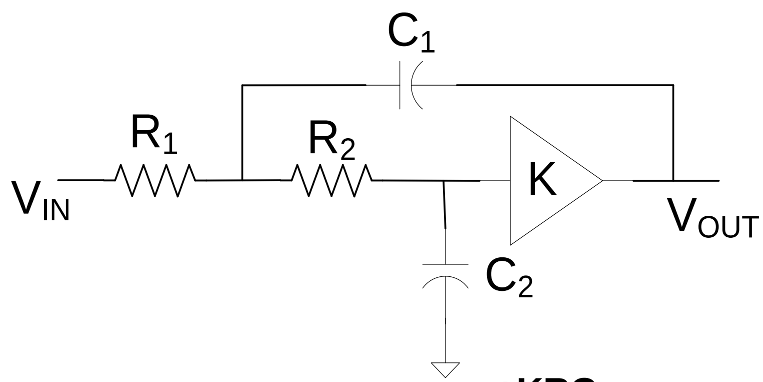
Sensitivity Comparisons

Consider 5 second-order lowpass filters

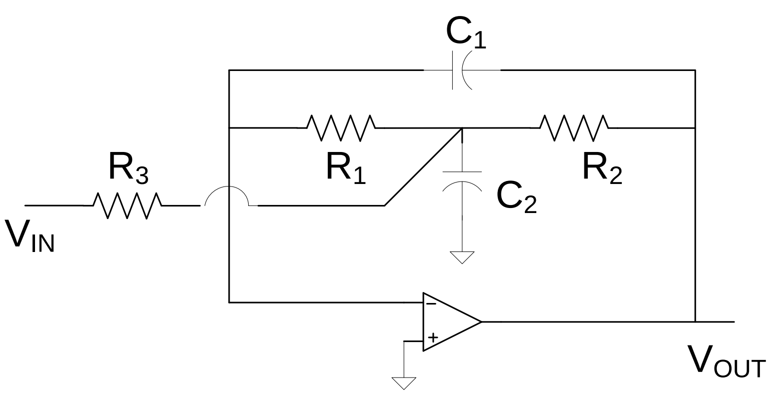
(all can realize same T(s) within a gain factor)



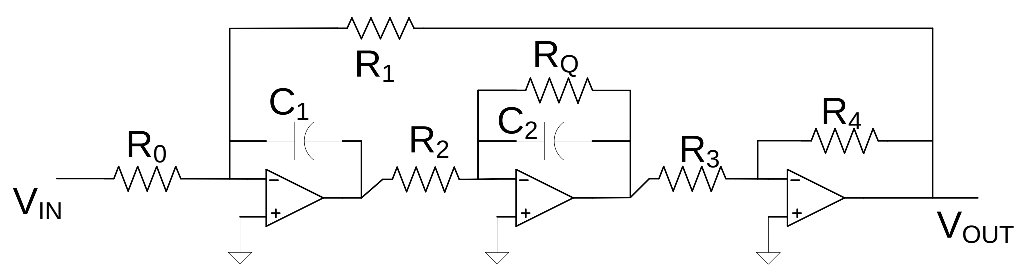
Passive RLC



+KRC



Bridged-T Feedback

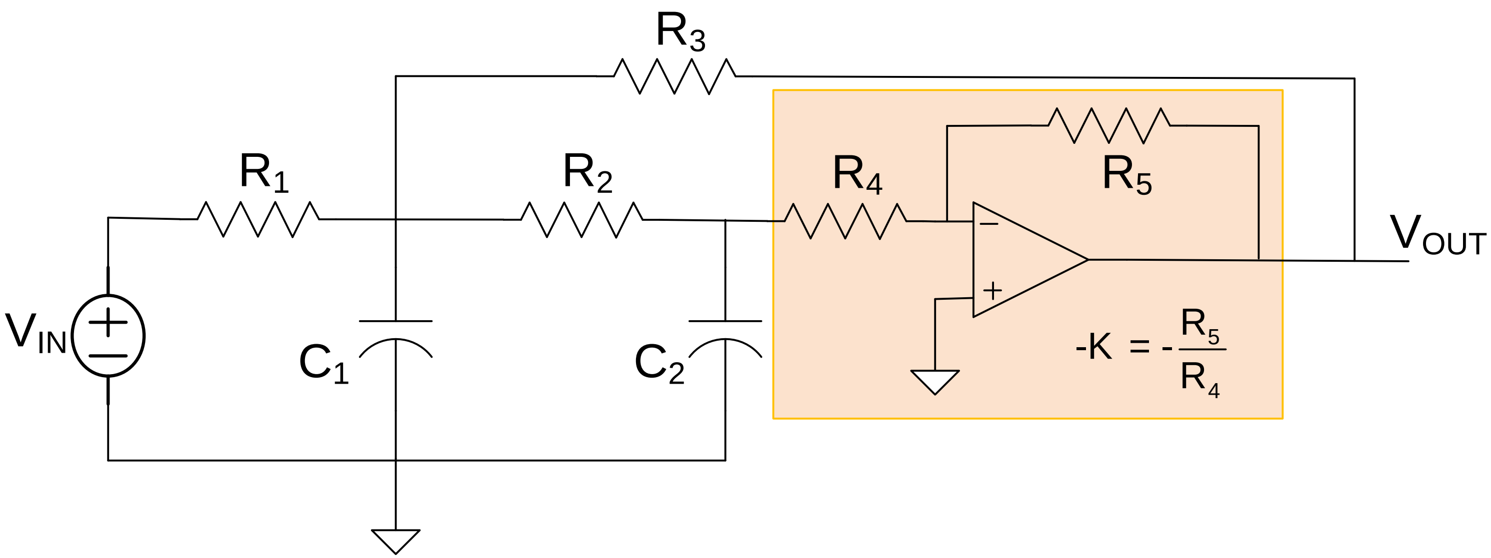


Two-Integrator Loop

Sensitivity Comparisons

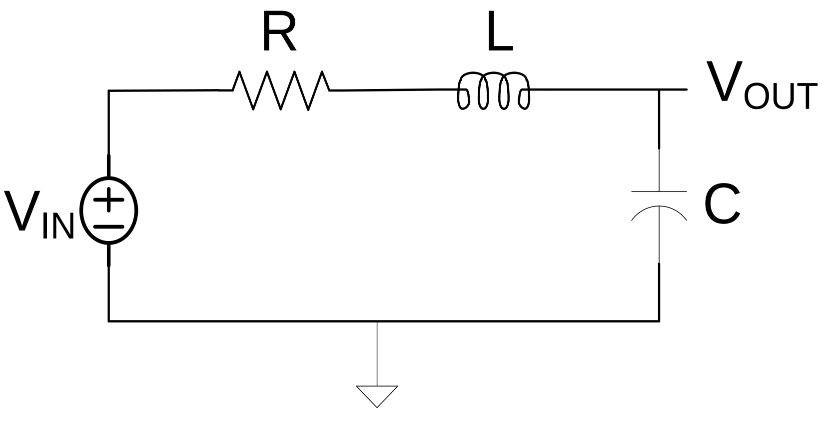
Consider 5 second-order lowpass filters

(all can realize same T(s) within a gain factor)



-KRC Lowpass

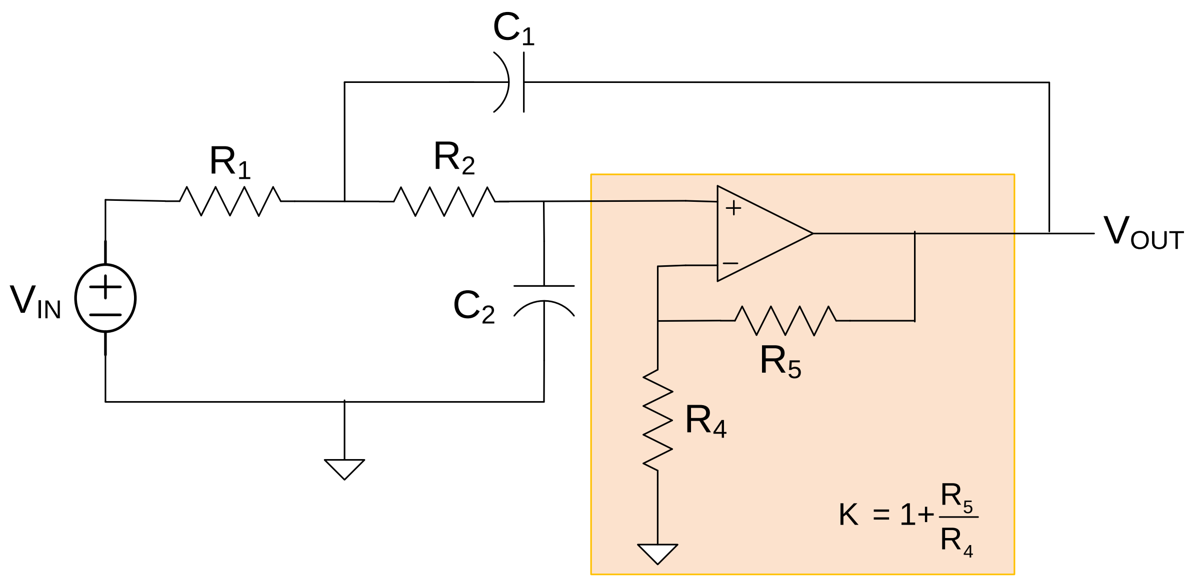
a) – Passive RLC



$$T(s) = \frac{V_{OUT}}{V_{IN}} = \frac{1/LC}{s^2 + s\frac{R}{L} + 1/LC}$$

$$\omega_0 = \sqrt{\frac{1}{LC}} \qquad Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

b) + KRC (a Sallen and Key filter)



$$T(s) = \frac{\dfrac{K}{R_1R_2C_1C_2}}{s^2 + s\left[\left(\dfrac{1}{\sqrt{R_1R_2C_1C_2}}\right)\left(\sqrt{\dfrac{R_1C_1}{R_2C_2}} + \sqrt{\dfrac{R_2C_2}{R_1C_1}} + \sqrt{\dfrac{R_1C_2}{R_2C_1}} - K\sqrt{\dfrac{R_1C_1}{R_2C_2}}\right)\right] + \dfrac{1}{R_1R_2C_1C_2}}$$

$$\omega_0 = \sqrt{\frac{1}{R_1R_2C_1C_2}}$$

$$Q = \frac{1}{\left(\sqrt{\dfrac{R_1C_1}{R_2C_2}} + \sqrt{\dfrac{R_2C_2}{R_1C_1}} + \sqrt{\dfrac{R_1C_2}{R_2C_1}} - K\sqrt{\dfrac{R_1C_1}{R_2C_2}}\right)}$$

Case b1 : Equal R, Equal C

$$R_1 = R_2 = R \qquad C_1 = C_2 = C$$

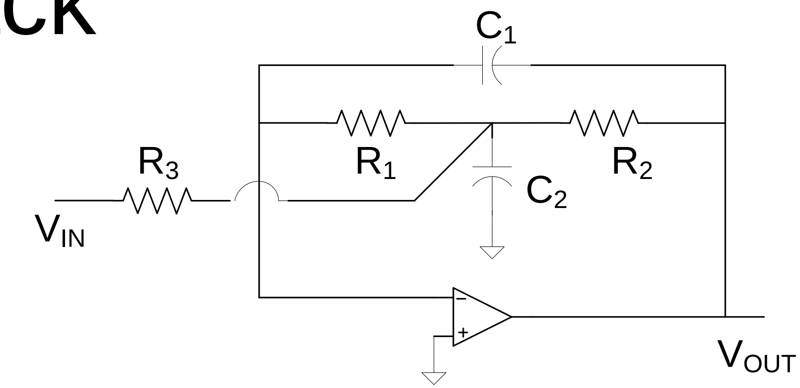
$$\omega_0 = \frac{1}{RC} \qquad K = 3 - \frac{1}{Q}$$

Case b2 : Equal R, K=1

$$R_1 = R_2 = R \qquad Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}}$$

$$T(s) = \frac{K\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

c) Bridged T Feedback



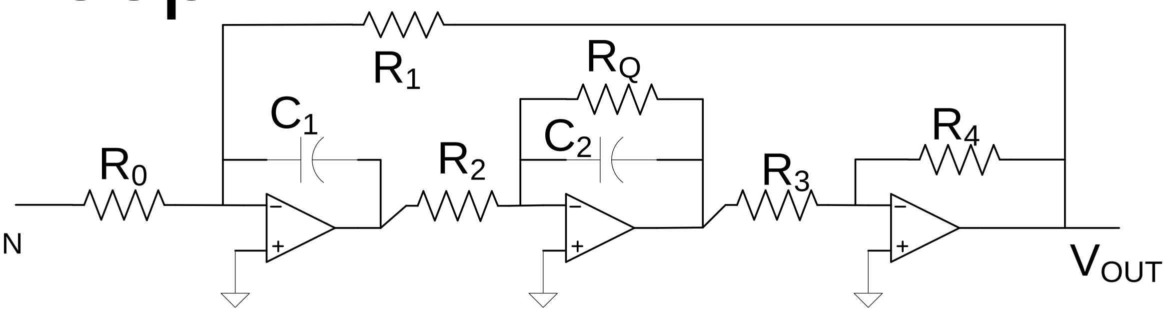
$$T(s) = \frac{\dfrac{1}{R_1R_3C_1C_2}}{s^2+s\left[\left(\sqrt{\dfrac{C_2}{C_1}}\right)\left(\dfrac{1}{\sqrt{R_1R_2C_1C_2}}\right)\left(\sqrt{\dfrac{R_1}{R_3}}+\sqrt{\dfrac{R_2}{R_1}}+\dfrac{\sqrt{R_1R_2}}{R_3}\right)\right]+\dfrac{1}{R_1R_2C_1C_2}}$$

$$\omega_0 = \sqrt{\frac{1}{R_1R_2C_1C_2}} \qquad Q = \frac{1}{\left(\sqrt{\dfrac{C_2}{C_1}}\right)\left(\sqrt{\dfrac{R_1}{R_3}}+\sqrt{\dfrac{R_2}{R_1}}+\dfrac{\sqrt{R_1R_2}}{R_3}\right)}$$

If $R_1=R_2=R_3=R$ and $C_2=9Q^2C_1$

$$T(s) = \frac{\dfrac{1}{9Q^2R^2C_1^2}}{s^2+s\left[\left(\dfrac{1}{3Q^2RC_1}\right)\right]+\dfrac{1}{9Q^2R^2C_1^2}}$$

d) 2 integrator loop



$$T(s) = -\frac{\frac{R_4}{R_3}\bullet\frac{1}{R_0R_2C_1C_2}}{s^2+s\left(\frac{1}{R_QC_2}\right)+\frac{R_4}{R_3}\bullet\frac{1}{R_0R_2C_1C_2}}$$

$$\omega_0 = \sqrt{\frac{R_4}{R_3}\bullet\frac{1}{R_0R_2C_1C_2}}$$

$$Q = \frac{R_Q}{\sqrt{R_0R_2}}\sqrt{\frac{C_2}{C_1}}$$

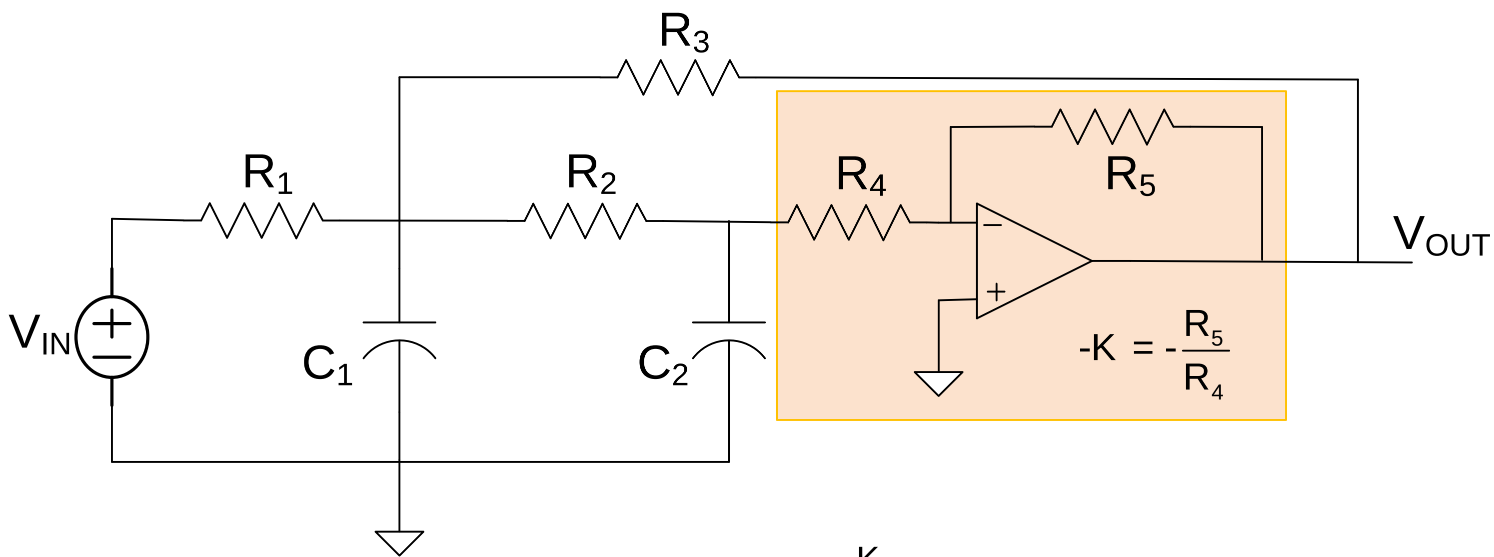
For: $R_0 = R_1 = R_2 = R$ $\quad C_1 = C_2 = C$ $\quad R_3 = R_4$

$$T(s) = -\frac{\frac{1}{R^2C^2}}{s^2+s\left(\frac{1}{R_QC}\right)+\frac{1}{R^2C^2}}$$

$$R_Q = QR$$

$$\omega_0 = \frac{1}{RC}$$

d) - KRC (a Sallen and Key filter)



$$T(s) = -\frac{\dfrac{K}{R_1R_2C_1C_2}}{s^2+s\left[\left(1+\dfrac{R_1}{R_3}\right)\left(\dfrac{1}{R_1C_1}\right)+\left(1+\dfrac{C_2}{C_1}\right)\left(\dfrac{1}{R_2C_2}\right)+\left(\dfrac{1}{R_4C_2}\right)\right]+\dfrac{1+(R_1/R_3)(1+K)+(R_1/R_4)(1+R_2/R_3+R_2/R_1)}{R_1R_2C_1C_2}}$$

$$\omega_0 = \sqrt{\frac{1+(R_1/R_3)(1+K)+(R_1/R_4)(1+R_2/R_3+R_2/R_1)}{R_1R_2C_1C_2}}$$

$$Q = \frac{\sqrt{\dfrac{1+(R_1/R_3)(1+K)+(R_1/R_4)(1+R_2/R_3+R_2/R_1)}{R_1R_2C_1C_2}}}{\left(1+\dfrac{R_1}{R_3}\right)\left(\dfrac{1}{R_1C_1}\right)+\left(1+\dfrac{C_2}{C_1}\right)\left(\dfrac{1}{R_2C_2}\right)+\left(\dfrac{1}{R_4C_2}\right)}$$

Often $R_1=R_2=R_3=R_4=R$, $C_1=C_2=C$

$$Q = \frac{\sqrt{5+K_0}}{5}$$

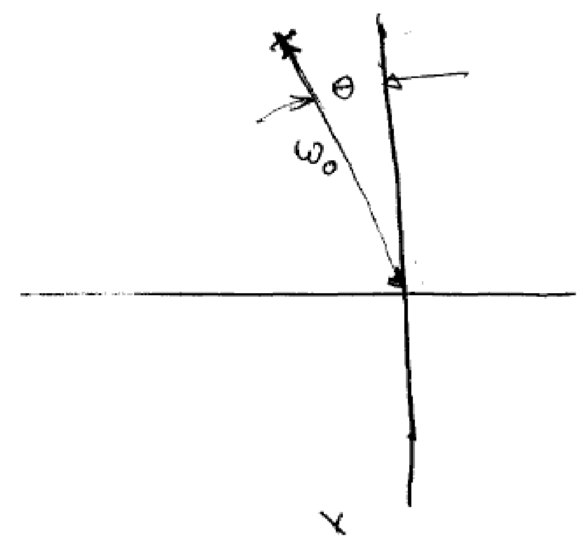
ω_0 and Q notation

$$s^2 + s\frac{\omega_0}{Q} + \omega_0^2$$

Poles $\omega_0 \left[-\frac{1}{2Q} \pm j\sqrt{1 - \frac{1}{4Q^2}} \right]$

$$|P| = \omega_0$$

$$\angle P \underset{Q > \frac{1}{2}}{=} \tan^{-1}\sqrt{4Q^2 - 1}$$



θ determined by Q

How do these five circuits compare?

a) From a passive sensitivity viewpoint?

- If Q is small
- If Q is large

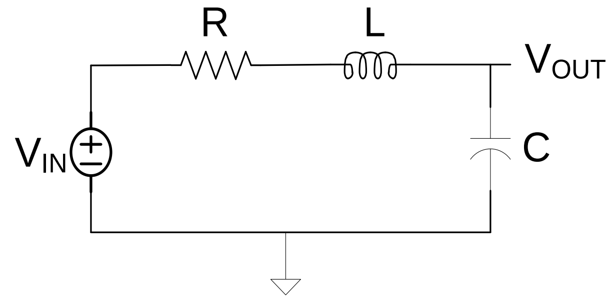
b) From an active sensitivity viewpoint?

- If Q is small
- If Q is large
- If $\tau\omega_0$ is large

Comparison: Calculate all ω_0 and Q sensitivities

Consider passive sensitivities first

a) – Passive RLC



$$S_R^{\omega_0} = 0$$

$$S_L^{\omega_0} = -\frac{1}{2}$$

$$S_C^{\omega_0} = -\frac{1}{2}$$

$$S_R^{Q} = -1$$

$$S_C^{Q} = -\frac{1}{2}$$

$$S_L^{Q} = \frac{1}{2}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

Case b1 : +KRC Equal R, Equal C

$$\omega_0 = \sqrt{\frac{1}{R_1R_2C_1C_2}}$$

$$Q = \frac{1}{\left(\sqrt{\frac{R_1C_1}{R_2C_2}}+\sqrt{\frac{R_2C_2}{R_1C_1}}+\sqrt{\frac{R_1C_2}{R_2C_1}}-K\sqrt{\frac{R_1C_1}{R_2C_2}}\right)}$$

$$S_{R_1}^{\omega_0} = S_{R_2}^{\omega_0} = S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -\frac{1}{2} \qquad S_K^{\omega_0} = 0$$

$$S_{R_1}^{Q} = Q - \frac{1}{2}$$

$$S_{R_2}^{Q} = -Q + \frac{1}{2}$$

$$S_{C_1}^{Q} = 2Q - \frac{1}{2}$$

$$S_{C_2}^{Q} = -2Q + \frac{1}{2}$$

$$S_K^{Q} = 3Q - 1$$

$$Q = \frac{1}{3-K}$$

$$\omega_0 = \frac{1}{RC}$$

If $Q_N = 10$, what happens to Q if

R_1 increases by 1% ?

10% ?

$\frac{\Delta R_1}{R_1} = .01$

$$\frac{\Delta Q}{Q} \approx S^{Q}_{R_1} \cdot \frac{\Delta R_1}{R_1} \approx (Q - 1/2)(.01) = .095$$

$\therefore$ Q changes by 9.5%

$\frac{\Delta R_1}{R_1} = 0.1$

$$\frac{\Delta Q}{Q} \approx S^{Q}_{R_1} \cdot \frac{\Delta R_1}{R_1} = (9.5)(.1) = .95$$

$\therefore$ Q changes by 95%

Actual: 10 → 11.04 for $\frac{\Delta R}{R} = .01$

10 → 105 for $\frac{\Delta R}{R} = 0.1$

Case b2 : +KRC Equal R, K=1

$$\omega_0 = \sqrt{\frac{1}{R_1 R_2 C_1 C_2}}$$

$$Q = \frac{1}{\left(\sqrt{\frac{R_1 C_1}{R_2 C_2}} + \sqrt{\frac{R_2 C_2}{R_1 C_1}} + \sqrt{\frac{R_1 C_2}{R_2 C_1}} - K\sqrt{\frac{R_1 C_1}{R_2 C_2}}\right)}$$

$$S_{R_1}^{\omega_0} = S_{R_2}^{\omega_0} = S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -\frac{1}{2}$$

$$S_K^{\omega_0} = 0$$

$$S_{R_1}^{Q} = 0$$

$$S_{R_2}^{Q} = 0$$

$$S_{C_1}^{Q} = \frac{1}{2}$$

$$S_{C_2}^{Q} = -\frac{1}{2}$$

$$S_K^{Q} = 2Q^2$$

$$\omega_0 = \frac{1}{RC}$$

$$Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}}$$

c) Bridged T Feedback

$$\omega_0 = \sqrt{\frac{1}{R_1R_2C_1C_2}}$$

$$Q = \frac{1}{\left(\sqrt{\frac{C_2}{C_1}}\right)\left(\sqrt{\frac{R_1}{R_3}}+\sqrt{\frac{R_2}{R_1}}+\frac{\sqrt{R_1R_2}}{R_3}\right)}$$

For $R_1=R_2=R_3=R$

$$S_{R_1}^{\omega_0} = S_{R_2}^{\omega_0} = S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -\frac{1}{2}$$

$$S_{R_3}^{\omega_0} = 0$$

$$S_{R_1}^{Q} = -\frac{1}{6}$$

$$S_{R_2}^{Q} = -\frac{1}{6}$$

$$S_{R_3}^{Q} = \frac{1}{3}$$

$$S_{C_1}^{Q} = -\frac{1}{2}$$

$$S_{C_2}^{Q} = \frac{1}{2}$$

$$\omega_0 = \frac{3Q}{RC_1}$$

$$Q = \frac{1}{3}\sqrt{\frac{C_1}{C_2}}$$

d) 2 integrator loop

$$\omega_0 = \sqrt{\frac{R_4}{R_3} \bullet \frac{1}{R_0 R_2 C_1 C_2}} \qquad Q = \frac{R_Q}{\sqrt{R_0 R_2}} \sqrt{\frac{C_2}{C_1}}$$

For: $R_0 = R_1 = R_2 = R$ $\quad C_1 = C_2 = C$ $\quad R_3 = R_4$

$$S_{R_1}^{\omega_0} = S_{R_2}^{\omega_0} = S_{R_3}^{\omega_0} = S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} - \frac{1}{2} \qquad S_{R_4}^{\omega_0} = \frac{1}{2}$$

$$S_{R_1}^{Q} = S_{R_2}^{Q} = S_{R_3}^{Q} = S_{C_1}^{Q} = -\frac{1}{2}$$

$$S_{R_4}^{Q} = S_{C_2}^{Q} = \frac{1}{2}$$

$$\omega_0 = \frac{1}{RC}$$

$$S_{R_Q}^{Q} = 1$$

$$Q = \frac{R_Q}{R}$$

$$S_{R_0}^{Q} = 0$$

d) -KRC passive sensitivities

$$\omega_0 = \sqrt{\frac{1+(R_1/R_3)(1+K)+(R_1/R_4)(1+R_2/R_3+R_2/R_1)}{R_1R_2C_1C_2}}$$

$$Q = \frac{\sqrt{\frac{1+(R_1/R_3)(1+K)+(R_1/R_4)(1+R_2/R_3+R_2/R_1)}{R_1R_2C_1C_2}}}{\left(1+\frac{R_1}{R_3}\right)\left(\frac{1}{R_1C_1}\right)+\left(1+\frac{C_2}{C_1}\right)\left(\frac{1}{R_2C_2}\right)+\left(\frac{1}{R_4C_2}\right)}$$

For $R_1=R_2=R_3=R_4=R$, $C_1=C_2=C$

$$Q = \frac{\sqrt{5+K_0}}{5} \qquad \omega_0 = \frac{\sqrt{5+K}}{R\,C}$$

$$S_{R_1}^{\omega_0} = -\frac{1}{25Q^2} \qquad S_{R_2}^{\omega_0} = -\frac{1}{2}+\frac{1}{25Q^2} \qquad S_{R_3}^{\omega_0} = -\frac{1}{2}+\frac{3}{50Q^2}$$

$$S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} - \frac{1}{2} \qquad S_{R_4}^{\omega_0} = -\frac{3}{50Q^2} \qquad S_{K}^{\omega_0} = \frac{1}{2}+\frac{1}{10Q^2}$$

$$S_{R_1}^{Q} = \frac{1}{5} - \frac{1}{25Q^2} \qquad S_{R_2}^{Q} = -\frac{1}{10}+\frac{1}{25Q^2} \qquad S_{R_3}^{Q} = -\frac{3}{10}+\frac{3}{50Q^2}$$

$$S_{R_4}^{Q} = \frac{1}{5} - \frac{3}{50Q^2} \qquad S_{C_2}^{Q} = -\frac{1}{10} \qquad S_{C_1}^{Q} = \frac{1}{10} \qquad S_{K}^{Q} = \frac{1}{2} - \frac{1}{10Q^2}$$

Passive Sensitivity Comparisons

	$\left\lvert S_x^{\omega_0} \right\rvert$	$\left\lvert S_x^{Q} \right\rvert$
Passive RLC	$\leq \frac{1}{2}$	**1,1/2**
+KRC		
Equal R, Equal C (K=3-1/Q)	**0,1/2**	**Q, 2Q, 3Q**
Equal R, K=1 ($C_1=4Q^2C_2$)	**0,1/2**	**0,1/2, $2Q^2$**
Bridged-T Feedback	**0,1/2**	**1/3,1/2, 1/6**
Two-Integrator Loop	**0,1/2**	**1,1/2, 0**
-KRC	**less than or equal to 1/2**	**less than or equal to 1/2**

Substantial Differences Between (or in) Architectures

How do active sensitivities compare ?

$$S^{\omega_0}_{t} = ? \qquad S^{Q}_{t} = ?$$

Recall $S^{f}_{x} = \frac{\partial f}{\partial x}\frac{x}{f}$

so $\frac{\Delta f}{f} \simeq \frac{\Delta x}{x} S^{f}_{x}$

but if x is ideally 0, not useful

$$\mathcal{S}^{f}_{x} = \frac{\partial f}{\partial x}$$

$$\frac{\Delta f}{f} \simeq \mathcal{S}^{f}_{x} \frac{\Delta x}{f}$$

Where we are at with sensitivity analysis:

Considered a group of five second-order filters

Passive Sensitivity Analysis

- Closed form expressions were obtained for ω_0 and Q
- Tedious but straightforward calculations provided passive sensitivities directly from the closed form expressions ???

Active Sensitivity Analysis

- Closed form expressions for ω_0 and Q are very difficult or impossible to obtain

If we consider higher-order filters

Passive Sensitivity Analysis

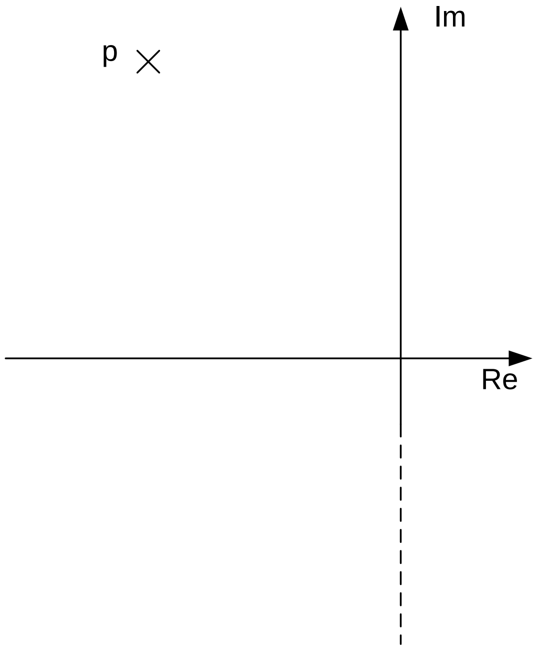
- Closed form expressions for ω_0 and Q are very difficult or impossible to obtain for many useful structures

Active Sensitivity Analysis

- Closed form expressions for ω_0 and Q are very difficult or impossible to obtain

Need some better method for obtaining sensitivities when closed-form expressions are difficult or impractical to obtain or manipulate !!

Relationship between pole sensitivities and ω_0 and Q sensitivities



$p = -\alpha + j\beta$

$D_2(s) = (s-p)(s-p^*)$

$D_2(s) = (s+\alpha-j\beta)(s+\alpha+j\beta)$

$D_2(s) = s^2 + s(2\alpha) + (\alpha^2 + \beta^2)$

Relationship between active pole sensitivities and ω_0 and Q sensitivities

Define $D(s)=D_0(s)+t\,D_1(s)$ (from bilinear form of T(s))

Recall: $$\mathcal{S}_\tau^p = \frac{-D_1(p)}{\left.\frac{\partial D(s)}{\partial s}\right|_{s=p,\tau=0}}$$

Theorem: $$\Delta p \simeq \tau \mathcal{S}_\tau^p$$

Theorem: $$\Delta\alpha \simeq \tau \,\mathrm{Re}\left(\mathcal{S}_\tau^p\right)$$
$$\Delta\beta \simeq \tau \,\mathrm{Im}\left(\mathcal{S}_\tau^p\right)$$

Theorem:

$$\frac{\Delta\omega_0}{\omega_0} \simeq \frac{1}{2Q}\frac{\Delta\alpha}{\omega_0} + \sqrt{1-\frac{1}{4Q^2}}\,\frac{\Delta\beta}{\omega_0}$$

$$\frac{\Delta Q}{Q} \simeq -2Q\left(1-\frac{1}{4Q^2}\right)\frac{\Delta\alpha}{\omega_0} + \sqrt{1-\frac{1}{4Q^2}}\,\frac{\Delta\beta}{\omega_0}$$

Claim: These theorems, with straightforward modification, also apply to other parameters (R, C, L, K, …) where, $D_0(s)$ and $D_1(s)$ will change since the parameter is different

Table 10-1 *KRC* Realization
(see Fig. 10-3b)

Equal-R, Equal-C

$$\omega_o = \frac{1}{RC}, \qquad Q = \frac{1}{3 - K_o}$$

$$\frac{V_o}{V_i} = \frac{\left(3 - \frac{1}{Q}\right)\omega_o^2}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2 + \frac{\left(3 - \frac{1}{Q}\right)}{\text{GB}}\, s(s^2 + s3\omega_o + \omega_o^2)} \qquad \left(\omega_o \ll \frac{\omega_a}{2Q}\right)$$

$$-\frac{\Delta\alpha}{\omega_o} \cong \frac{1}{2Q}\left(3 - \frac{1}{Q}\right)^2 \frac{\omega_o}{\text{GB}}, \qquad \frac{\Delta\beta}{\omega_o} \cong -\frac{1}{2}\left(3 - \frac{1}{Q}\right)^2 \frac{\left(1 - \frac{1}{2Q^2}\right)}{\sqrt{1 - \frac{1}{4Q^2}}} \frac{\omega_o}{\text{GB}}$$

$$\frac{\Delta\omega_o}{\omega_o} \cong -\frac{1}{2}\left(3 - \frac{1}{Q}\right)^2 \frac{\omega_o}{\text{GB}}, \qquad \frac{\Delta Q}{Q} \cong \frac{1}{2}\left(3 - \frac{1}{Q}\right)^2 \frac{\omega_o}{\text{GB}}$$

Unity-gain, Equal-R

$$\omega_o = \frac{1}{R\sqrt{C_1C_2}}, \qquad Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}}$$

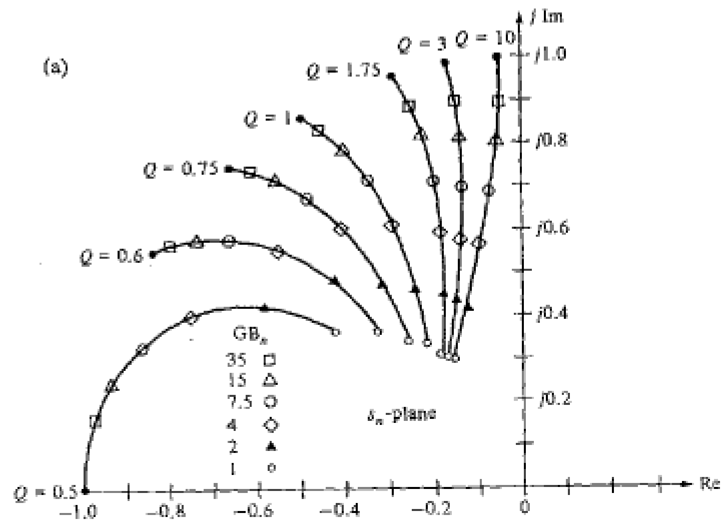
$$\frac{V_o}{V_i} = \frac{\omega_o^2}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2 + \frac{s}{\text{GB}}\left[s^2 + s\omega_o\left(2Q + \frac{1}{Q}\right) + \omega_o^2\right]} \qquad \left(\omega_o \ll \frac{\omega_a}{2Q}\right)$$

$$-\frac{\Delta\alpha}{\omega_o} \cong \frac{\omega_o}{\text{GB}}, \qquad \frac{\Delta\beta}{\omega_o} \cong -Q\frac{\left(1 - \frac{1}{2Q^2}\right)}{\sqrt{1 - \frac{1}{4Q^2}}}\frac{\omega_o}{\text{GB}}$$

$$\frac{\Delta\omega_o}{\omega_o} \cong -Q\frac{\omega_o}{\text{GB}}, \qquad \frac{\Delta Q}{Q} \cong Q\frac{\omega_o}{\text{GB}}$$

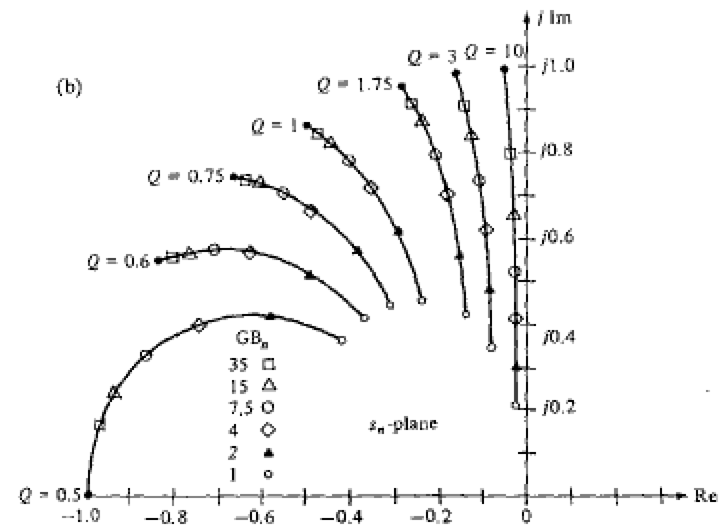
where

$$s_n = \frac{s}{\omega_o}, \qquad \text{GB}_n = \frac{\text{GB}}{\omega_o}.$$



◀Fig. 10-5a Plot of upper half-plane root of

$$s_n^3 + s_n^2\left(3+\frac{Q\,\mathrm{GB}_n}{3Q-1}\right) + s_n\left(1+\frac{\mathrm{GB}_n}{3Q-1}\right) + \frac{Q\,\mathrm{GB}_n}{3Q-1} = 0 \qquad \text{(Equal-}R\text{, equal-}C\text{)}$$



◀Fig. 10-5b Plot of upper half plane root of

$$s_n^3 + s_n^2\left(2Q+\frac{1}{Q}+\mathrm{GB}_n\right) + s_n\left(1+\frac{\mathrm{GB}_n}{Q}\right) + \mathrm{GB}_n = 0 \qquad \text{(Unity-gain, equal-}R\text{)}$$

c) Bridged-T structure

Table 10-3 Infinite-gain Realization
(see Fig. 10-10b)

Equal-R

$$\omega_p = \frac{1}{R\sqrt{C_1 C_2}}; \qquad Q = \frac{1}{3}\sqrt{\frac{C_1}{C_2}}$$

$$\frac{V_o}{V_i} = -\frac{\omega_p^2}{s^2 + s\dfrac{\omega_p}{Q} + \omega_p^2 + \dfrac{s}{\mathrm{GB}}\left[s^2 + s\omega_p\left(3Q + \dfrac{1}{Q}\right) + 2\omega_p^2\right]} \qquad \left(\omega_a \ll \frac{\omega_p}{2Q}\right)$$

$$-\frac{\Delta\alpha}{\omega_p} \simeq \frac{\omega_p}{\mathrm{GB}}, \qquad \frac{\Delta\beta}{\omega_p} \simeq -\frac{1}{2}\,\frac{3Q - \dfrac{1}{Q}}{\sqrt{1 - \dfrac{1}{4Q^2}}}\,\frac{\omega_p}{\mathrm{GB}}$$

$$\frac{\Delta\omega_p}{\omega_p} \simeq -\frac{3Q}{2}\,\frac{\omega_p}{\mathrm{GB}}, \qquad \frac{\Delta Q}{Q} \simeq \frac{Q}{2}\,\frac{\omega_p}{\mathrm{GB}}$$

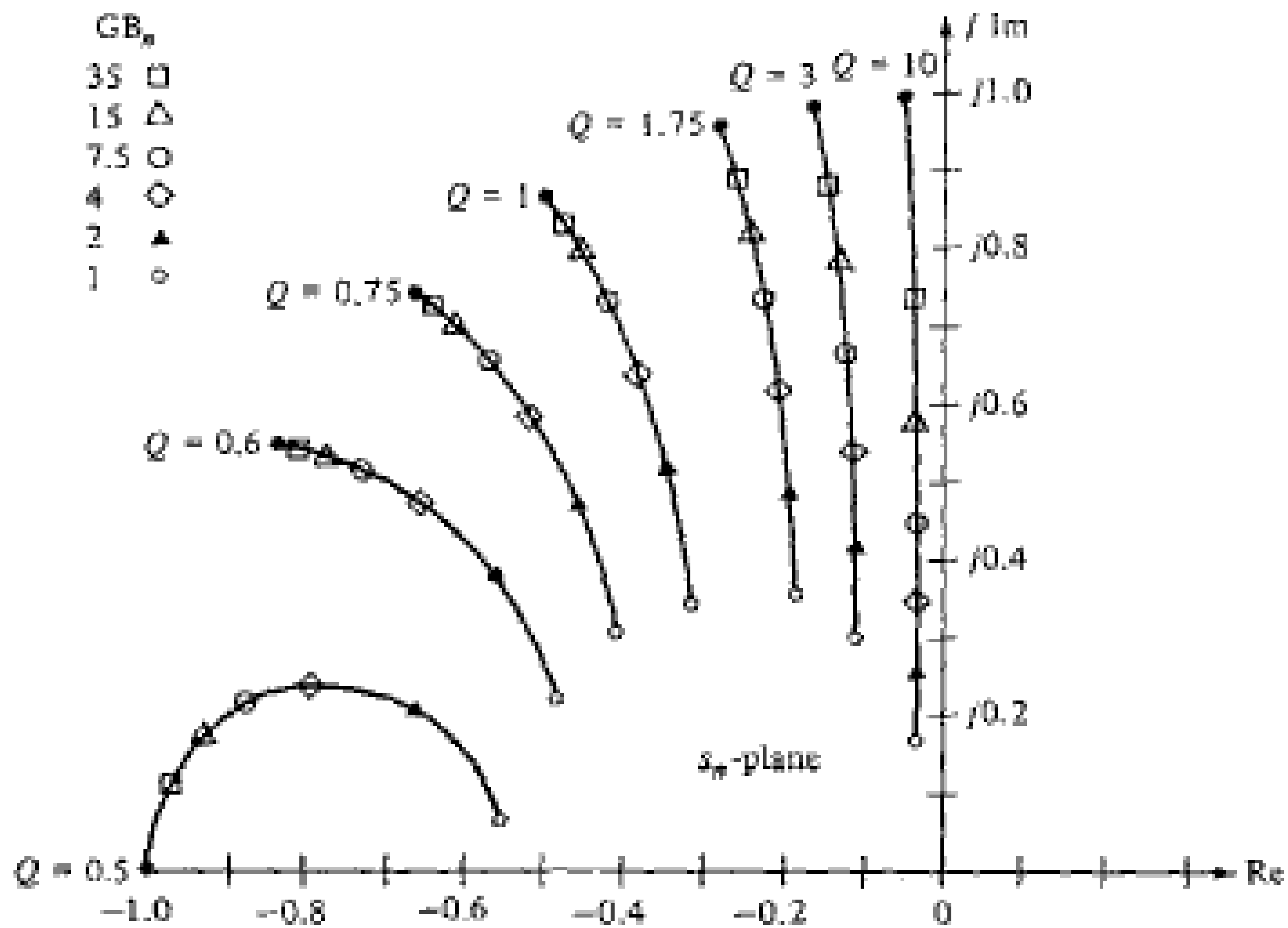


Fig. 10-12 Plot of upper half-plane root of

$$s_n^3 + s_n^2\left(3Q + \frac{1}{Q} + GB_n\right) + s_n\left(2 + \frac{GB_n}{Q}\right) + GB_n = 0$$

d) Two integrator loop architecture

Table 10-4 Three-Amplifier Realization
(see Fig. 10-16)

Equal-R (except R_Q) and Equal-C

$$\omega_o = \frac{1}{RC}, \qquad Q = \frac{R_Q}{R}$$

$$\frac{V_o}{V_i} \cong \frac{\omega_o^2\left(\frac{2}{\mathrm{GB}}s + 1\right)}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2 + \frac{1}{\mathrm{GB}}\left\{4s\left[s^2 + s\omega_o\left(\frac{1}{2} + \frac{1}{Q}\right) + \frac{\omega_o^2}{4Q}\right]\right\}} \qquad \left(\omega_o \ll \frac{\omega_a}{2Q}\right)$$

$$-\frac{\Delta\alpha}{\omega_o} \simeq 2\left(1 + \frac{1}{4Q}\right)\frac{\omega_o}{\mathrm{GB}}, \qquad \frac{\Delta\beta}{\omega_o} \cong -\frac{\left(1 - \frac{1}{Q} - \frac{1}{4Q^2}\right)}{\sqrt{1 - \frac{1}{4Q^2}}}\frac{\omega_o}{\mathrm{GB}}$$

$$\frac{\Delta\omega_o}{\omega_o} \simeq -\frac{\omega_o}{\mathrm{GB}}, \qquad \frac{\Delta Q}{Q} \simeq 4Q\frac{\omega_o}{\mathrm{GB}}$$

d) Two integrator loop architecture

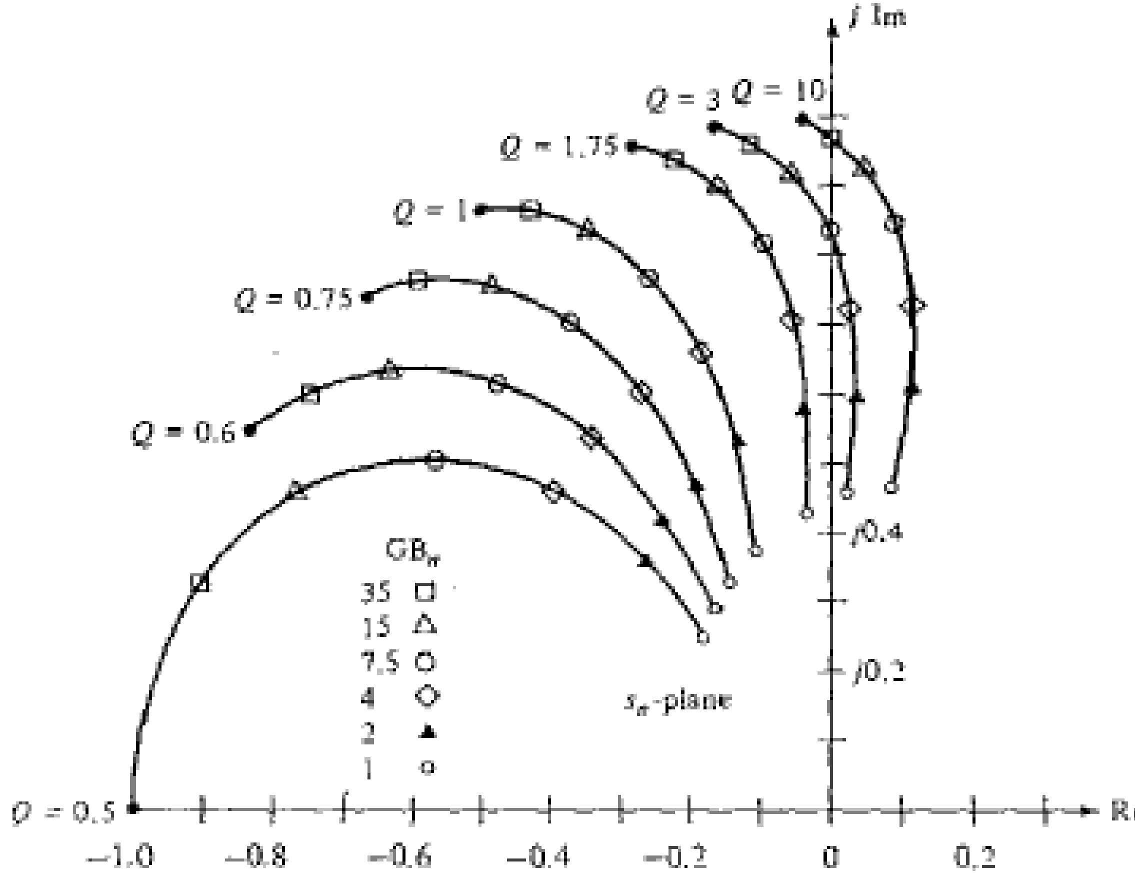


Fig. 10-17 Plot of upper half-plane root of

$$s_n^3 + s_n^2\left(\frac{1}{2} + \frac{1}{Q} + \frac{GB_n}{4}\right) + s_n\frac{1}{4Q}\left(1 + GB_n\right) + \frac{GB_n}{4} = 0$$

e) -KRC

Equal-R, Equal-C

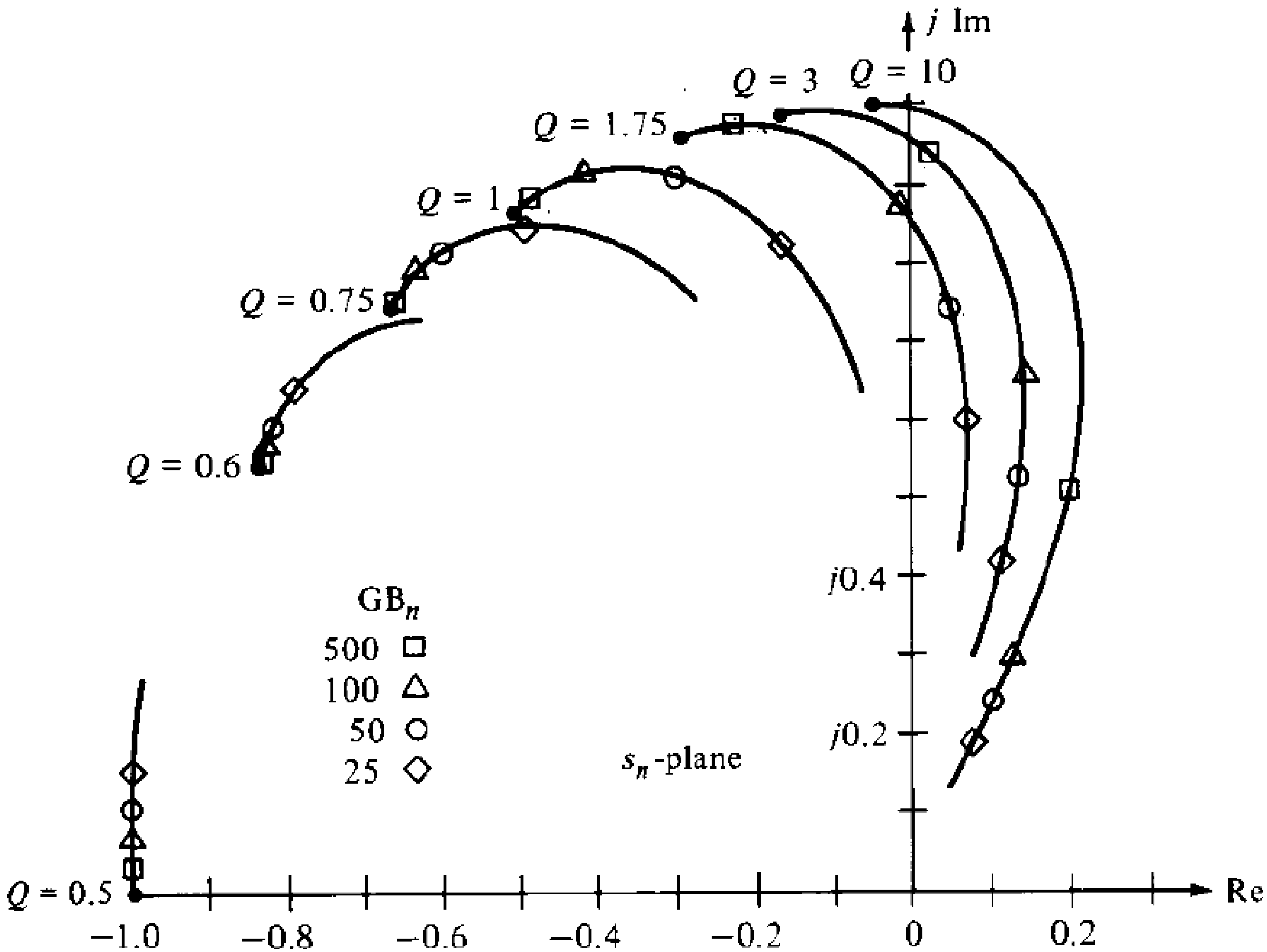
$$\omega_o = \frac{\sqrt{5 + K_0}}{RC}, \qquad Q = \frac{\sqrt{5 + K_0}}{5}$$

$$\frac{V_o}{V_i} = -\frac{\omega_o^2\left(1 - \frac{1}{5Q^2}\right)}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2 + \frac{s}{GB}\left[s^2(25Q^2 - 4) + s\omega_o\left(20Q - \frac{3}{Q}\right) + \left(2 - \frac{1}{5Q^2}\right)\omega_o^2\right]}$$

$$\left(\omega_a \ll \frac{\omega_o}{2Q}\right)$$

$$-\frac{\Delta\alpha}{\omega_o} \cong \frac{25Q^2}{2}\left(1 - \frac{1}{5Q^2}\right)\left(1 - \frac{6}{25Q^2}\right)\frac{\omega_o}{GB}, \qquad \frac{\Delta\beta}{\omega_o} \cong \frac{35Q}{4}\frac{\left(1 - \frac{1}{5Q^2}\right)\left(1 - \frac{6}{35Q^2}\right)}{\sqrt{1 - \frac{1}{4Q^2}}}\frac{\omega_o}{GB}$$

$$\frac{\Delta\omega_o}{\omega_o} \cong \frac{5Q}{2}\left(1 - \frac{1}{5Q^2}\right)\frac{\omega_o}{GB}, \qquad \frac{\Delta Q}{Q} \cong 25Q^3\left(1 - \frac{1}{5Q^2}\right)\left(1 - \frac{7}{5Q^2}\right)\frac{\omega_o}{GB}$$



j Im
Q = 10
Q = 3
Q = 1.75
Q = 1
Q = 0.75
Q = 0.6
Q = 0.5
GB$_n$
500
100
50
25
s_n-plane
j0.4
j0.2
Re
−1.0
−0.8
−0.6
−0.4
−0.2
0
0.2

Active Sensitivity Comparisons

	$\frac{\Delta\omega_0}{\omega_0}$	$\frac{\Delta Q}{Q}$
Passive RLC		
+KRC		
Equal R, Equal C (K=3-1/Q)	$-\frac{1}{2}\left(3-\frac{1}{Q}\right)^2 \tau\omega_0$	$-\frac{1}{2}\left(3-\frac{1}{Q}\right)^2 \tau\omega_0$
Equal R, K=1 ($C_1=4Q^2C_2$)	$-Q\tau\omega_0$	$Q\tau\omega_0$
Bridged-T Feedback	$-\frac{3}{2}Q\tau\omega_0$	$\frac{1}{2}Q\tau\omega_0$
Two-Integrator Loop	$-\tau\omega_0$	$4Q\tau\omega_0$
-KRC	$\frac{5}{2}Q\tau\omega_0$	$25Q^3\tau\omega_0$

Substantial Differences Between Architectures

Are these passive sensitivities acceptable?

	$\left\| S_x^{\omega_0} \right\|$	$\left\| S_x^{Q} \right\|$
Passive RLC	$\leq \frac{1}{2}$	**1,1/2**
+KRC		
Equal R, Equal C (K=3-1/Q)	**0,1/2**	**Q, 2Q, 3Q**
Equal R, K=1 ($C_1=4Q^2C_2$)	**0,1/2**	**0,1/2, $2Q^2$**
Bridged-T Feedback	**0,1/2**	**1/3,1/2, 1/6**
Two-Integrator Loop	**0,1/2**	**1,1/2, 0**
-KRC	**less than or equal to 1/2**	**less than or equal to 1/2**

Are these active sensitivities acceptable?

Active Sensitivity Comparisons

	$\frac{\Delta\omega_0}{\omega_0}$	$\frac{\Delta Q}{Q}$
Passive RLC		
+KRC		
Equal R, Equal C (K=3-1/Q)	$-\frac{1}{2}\left(3-\frac{1}{Q}\right)^2\tau\omega_0$	$-\frac{1}{2}\left(3-\frac{1}{Q}\right)^2\tau\omega_0$
Equal R, K=1 ($C_1=4Q^2C_2$)	$-Q\tau\omega_0$	$Q\tau\omega_0$
Bridged-T Feedback	$-\frac{3}{2}Q\tau\omega_0$	$\frac{1}{2}Q\tau\omega_0$
Two-Integrator Loop	$-\tau\omega_0$	$4Q\tau\omega_0$
-KRC	$\frac{5}{2}Q\tau\omega_0$	$25Q^3\tau\omega_0$

Are these sensitivities acceptable?

Passive Sensitivities:

$$\frac{\Delta\omega_0}{\omega_0} \simeq S_x^{\omega_0} \frac{\Delta x}{x}$$

In integrated circuits, Δ R/R and Δ C/C due to process variations can be K 30% or larger due to process variations

Many applications require $\Delta\omega_0/\omega_0 < .001$ or smaller and similar requirements on ΔQ/Q

Even if sensitivity is around ½ or 1, variability is often orders of magnitude too large

Active Sensitivities:

All are proportional to $\tau\omega_0$

Some architectures much more sensitive than others

Can reduce $\tau\omega_0$ by making GB large but this is at the expense of increased power and even if power is not of concern, process presents fundamental limits on how large GB can be made

End of Lecture 23

EE 508
Lecture 24

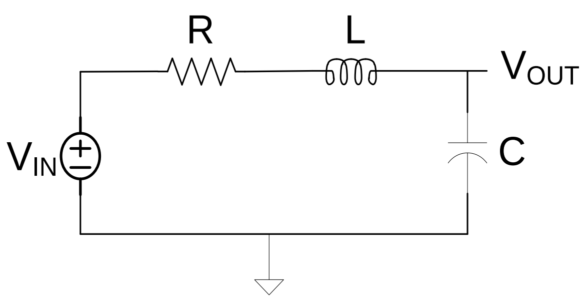
Sensitivity Functions

- Predistortion and Calibration

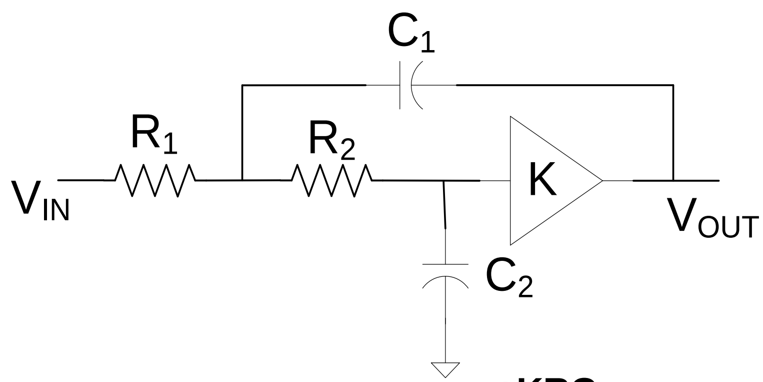
Sensitivity Comparisons

Consider 5 second-order lowpass filters

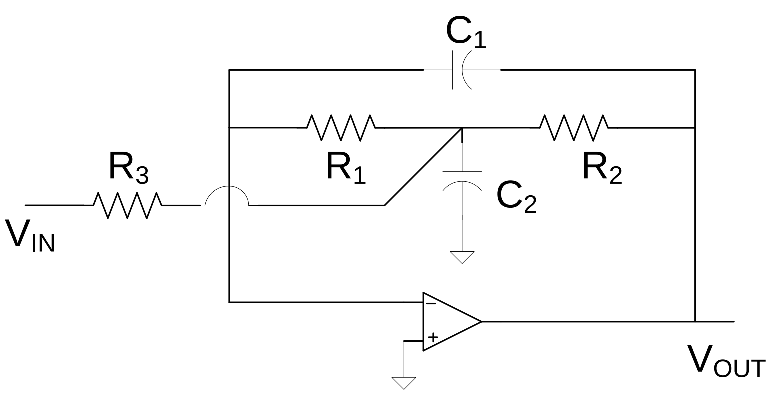
(all can realize same T(s) within a gain factor)



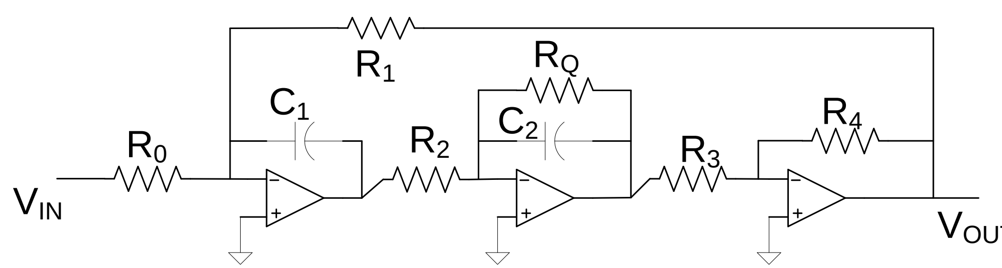
Passive RLC



+KRC



Bridged-T Feedback



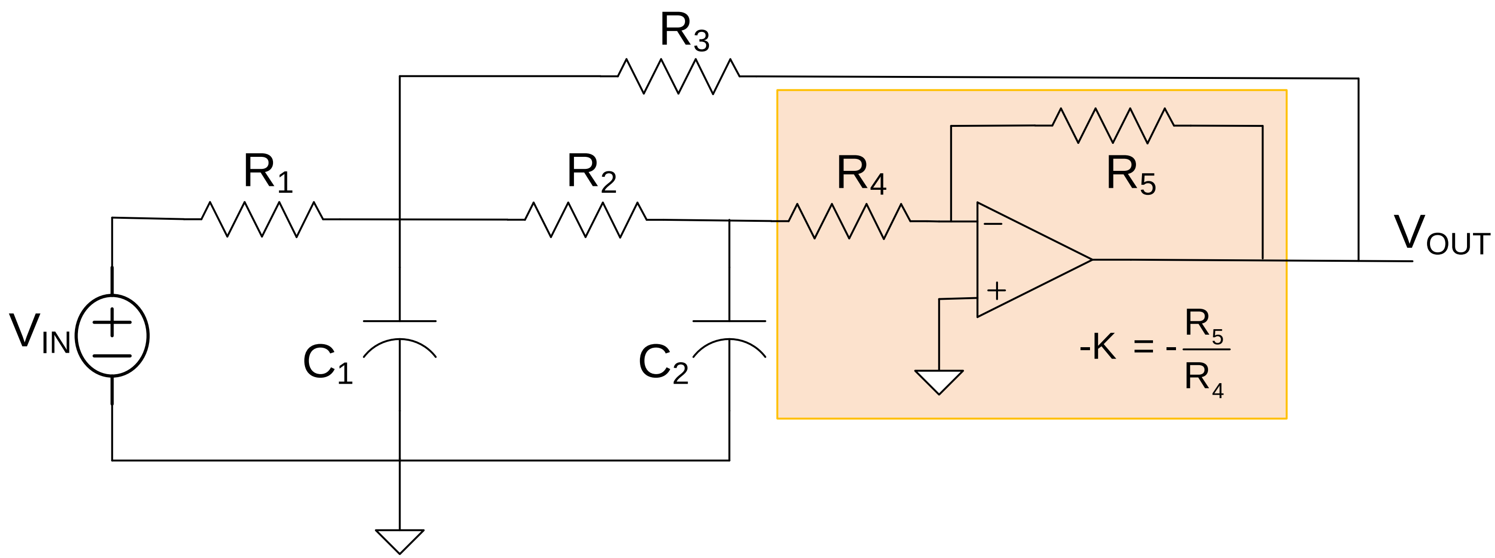
Two-Integrator Loop

Review from last time

Sensitivity Comparisons

Consider 5 second-order lowpass filters

(all can realize same T(s) within a gain factor)



-KRC Lowpass

Review from last time

Passive Sensitivity Comparisons

	$\left\| S_x^{\omega_0} \right\|$	$\left\| S_x^{Q} \right\|$
Passive RLC	$\leq \frac{1}{2}$	**1,1/2**
+KRC		
Equal R, Equal C (K=3-1/Q)	**0,1/2**	**Q, 2Q, 3Q**
Equal R, K=1 ($C_1=4Q^2C_2$)	**0,1/2**	**0,1/2, $2Q^2$**
Bridged-T Feedback	**0,1/2**	**1/3,1/2, 1/6**
Two-Integrator Loop	**0,1/2**	**1,1/2, 0**
-KRC	**less than or equal to 1/2**	**less than or equal to 1/2**

Substantial Differences Between (or in) Architectures

Review from last time

Where we are at with sensitivity analysis:

Considered a group of five second-order filters

Passive Sensitivity Analysis

- Closed form expressions were obtained for ω_0 and Q
- Tedious but straightforward calculations provided passive sensitivities directly from the closed form expressions ???

Active Sensitivity Analysis

- Closed form expressions for ω_0 and Q are very difficult or impossible to obtain

If we consider higher-order filters

Passive Sensitivity Analysis

- Closed form expressions for ω_0 and Q are very difficult or impossible to obtain for many useful structures

Active Sensitivity Analysis

- Closed form expressions for ω_0 and Q are very difficult or impossible to obtain

Need some better method for obtaining sensitivities when closed-form expressions are difficult or impractical to obtain or manipulate !!

Review from last time

Relationship between active pole sensitivities and ω_0 and Q sensitivities

Define $D(s)=D_0(s)+\tau D_1(s)$ (from bilinear form of T(s))

Recall: $$\mathcal{S}_\tau^p = \frac{-D_1(p)}{\left.\frac{\partial D(s)}{\partial s}\right|_{s=p,\tau=0}}$$

Theorem: $$\Delta p \simeq \tau \mathcal{S}_\tau^p$$

Theorem: $$\Delta\alpha \simeq \tau \operatorname{Re}\left(\mathcal{S}_\tau^p\right)$$
$$\Delta\beta \simeq \tau \operatorname{Im}\left(\mathcal{S}_\tau^p\right)$$

Theorem:

$$\frac{\Delta\omega_0}{\omega_0} \simeq \frac{1}{2Q}\frac{\Delta\alpha}{\omega_0} + \sqrt{1-\frac{1}{4Q^2}}\frac{\Delta\beta}{\omega_0}$$

$$\frac{\Delta Q}{Q} \simeq -2Q\left(1-\frac{1}{4Q^2}\right)\frac{\Delta\alpha}{\omega_0} + \sqrt{1-\frac{1}{4Q^2}}\frac{\Delta\beta}{\omega_0}$$

Claim: These theorems, with straightforward modification, also apply to other parameters (R, C, L, K, …) where, $D_0(s)$ and $D_1(s)$ will change since the parameter is different

Review from last time

Active Sensitivity Comparisons

	$\frac{\Delta\omega_0}{\omega_0}$	$\frac{\Delta Q}{Q}$
Passive RLC		
+KRC		
Equal R, Equal C (K=3-1/Q)	$-\frac{1}{2}\left(3-\frac{1}{Q}\right)^2\tau\omega_0$	$-\frac{1}{2}\left(3-\frac{1}{Q}\right)^2\tau\omega_0$
Equal R, K=1 ($C_1=4Q^2C_2$)	$-Q\tau\omega_0$	$Q\tau\omega_0$
Bridged-T Feedback	$-\frac{3}{2}Q\tau\omega_0$	$\frac{1}{2}Q\tau\omega_0$
Two-Integrator Loop	$-\tau\omega_0$	$4Q\tau\omega_0$
-KRC	$\frac{5}{2}Q\tau\omega_0$	$25Q^3\tau\omega_0$

Substantial Differences Between Architectures

Review from last time

Are these passive sensitivities acceptable?

	$\left\lvert S_x^{\omega_0} \right\rvert$	$\left\lvert S_x^{Q} \right\rvert$
Passive RLC	$\leq \frac{1}{2}$	**1,1/2**
+KRC		
Equal R, Equal C (K=3-1/Q)	**0,1/2**	**Q, 2Q, 3Q**
Equal R, K=1 ($C_1=4Q^2C_2$)	**0,1/2**	**0,1/2, $2Q^2$**
Bridged-T Feedback	**0,1/2**	**1/3,1/2, 1/6**
Two-Integrator Loop	**0,1/2**	**1,1/2, 0**
-KRC	**less than or equal to 1/2**	**less than or equal to 1/2**

Review from last time

Are these sensitivities acceptable?

Passive Sensitivities:

$$\frac{\Delta\omega_0}{\omega_0} \simeq S_x^{\omega_0} \frac{\Delta x}{x}$$

In integrated circuits, Δ R/R and Δ C/C due to process variations can be K 30% or larger due to process variations

Many applications require $\Delta\omega_0/\omega_0 < .001$ or smaller and similar requirements on $\Delta Q/Q$

Even if sensitivity is around ½ or 1, variability is often orders of magnitude too large

Active Sensitivities:

All are proportional to $\tau\omega_0$

Some architectures much more sensitive than others

Can reduce $\tau\omega_0$ by making GB large but this is at the expense of increased power and even if power is not of concern, process presents fundamental limits on how large GB can be made

What can be done to address these problems?

1. Predistortion

Design circuit so that after component shift, correct pole locations are obtained

Predistortion is generally used in integrated circuits to remove the bias associated with inadequate amplifier bandwidth

Tedious process after fabrication since depends on individual components

Temperature dependence may not track

Difficult to maintain over time and temperature

Over-ordering will adversely affect performance

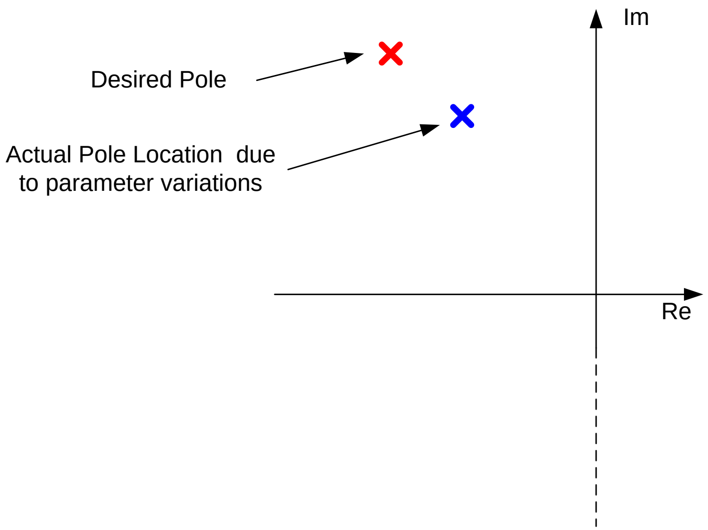
Seldom will predistortion alone be adequate to obtain acceptable performance

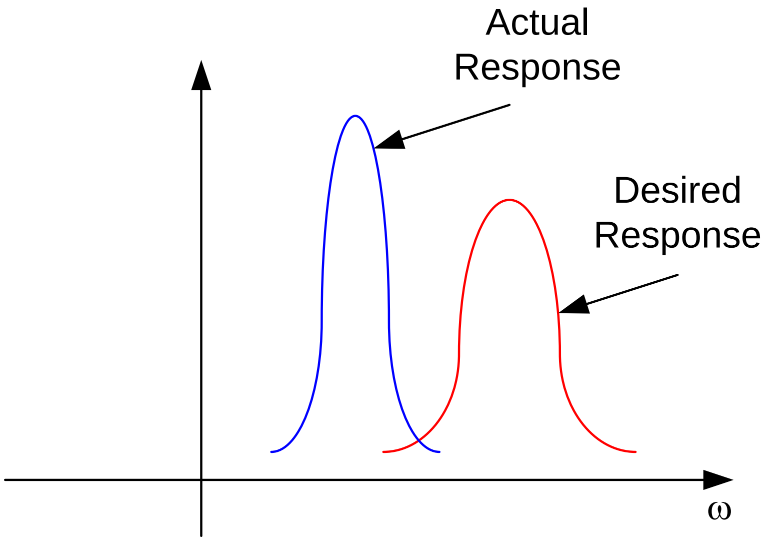
Bell Labs did to this in high-volume production (STAR Biquad)

What can be done to address these problems?

1. Predistortion

Design circuit so that <u>after</u> component shift, correct pole locations are obtained



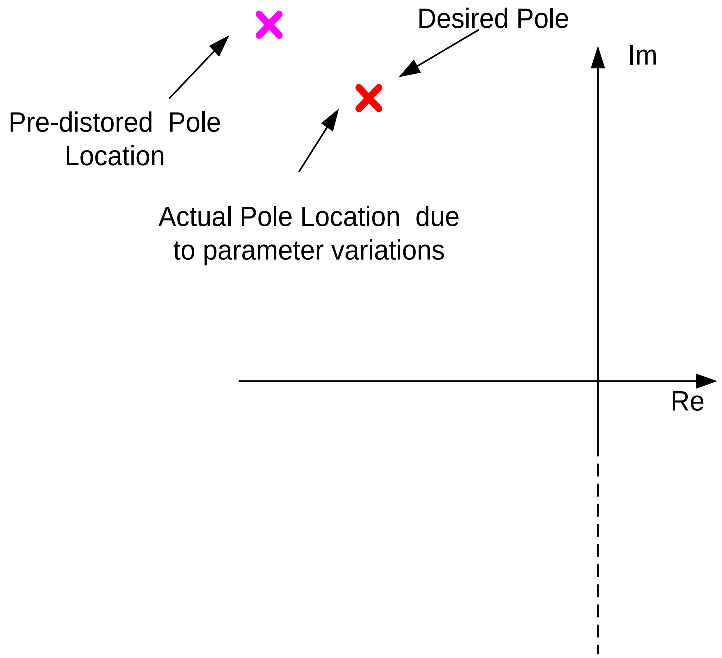


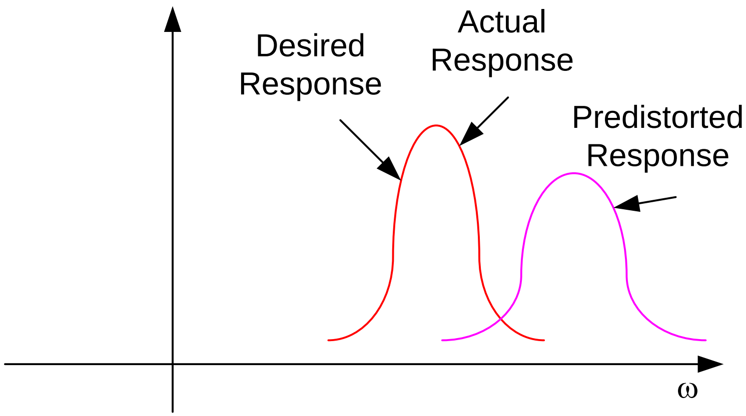
Pole shift due to parametric variations (e.g. inadequate GB)

What can be done to address these problems?

1. Predistortion

Design circuit so that after component shift, correct pole locations are obtained



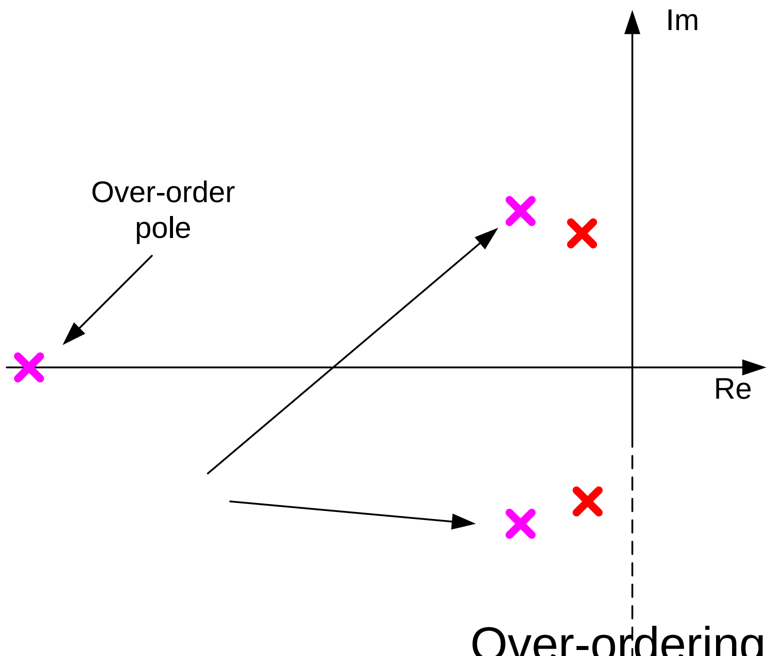


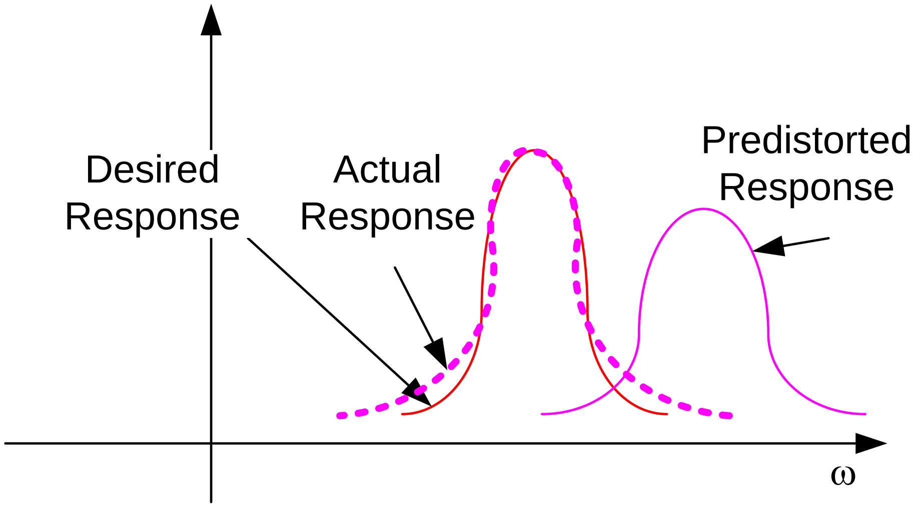
Pre-distortion concept

What can be done to address these problems?

1. Predistortion

Design circuit so that after component shift, correct pole locations are obtained





Over-ordering Limitations with Pre-distortion

Parasitic Pole Affects Response

Predistortion almost always done even if benefits only modest

Not effective is significant deviations exist before predistortion

What can be done to address these problems?

2. Trimming

a) Functional Trimming

- trim parameters of actual filter based upon measurements
- difficult to implement in many structures
- manageable for cascaded biquads

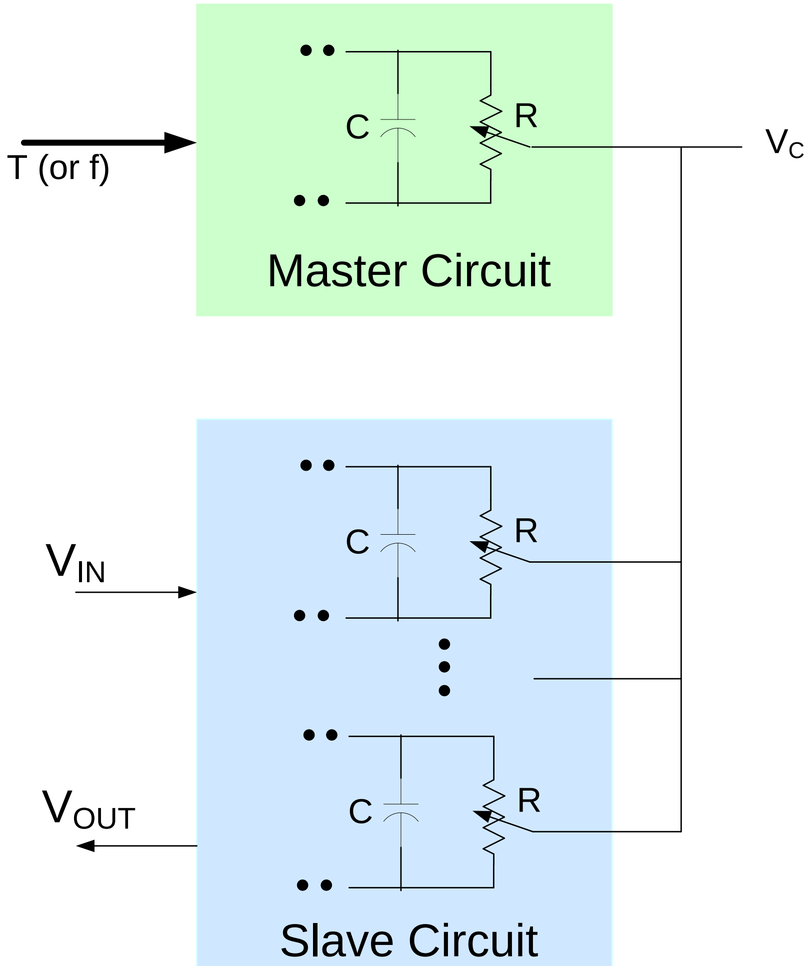
b) Deterministic Trimming (much preferred)

- Trim component values to their ideal value
  - Continuous-trims of resistors possible in some special processes
  - Continuous-trim of capacitors is more challenging
  - Link trimming of Rs or Cs is possible with either metal or switches
- If all components are ideal, the filter should also be ideal
  - R-trimming algorithms easy to implement
  - Limited to unidirectional trim
  - Trim generally done at wafer level for laser trimming, package for link trims
- Filter shifts occur due to stress in packaging and heat cycling

c) Master-slave reference control (depends upon matching in a process)

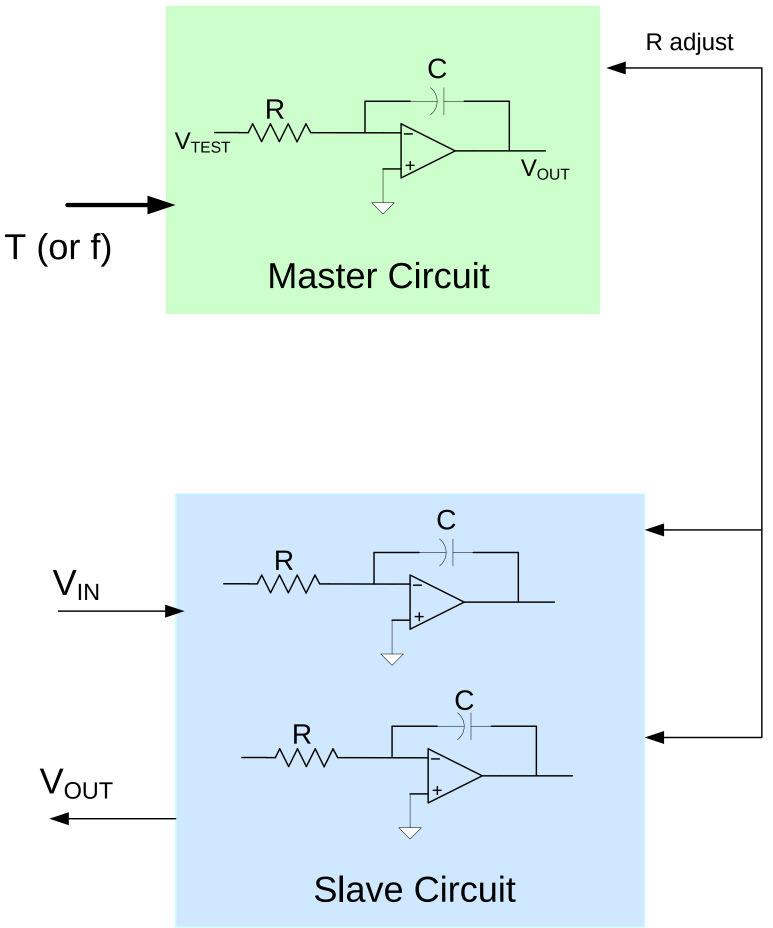
- Can be implemented in discrete or integrated structures
- Master typically frequency or period referenced
- Most effective in integrated form since good matching possible
- Widely used in integrated form

Master-slave Control (depends upon matching in a process)



- Automatically adjust R in the Master Circuit to match RC to T
- Rely on matching to match RC products in Slave Circuit to T
- Matching can be very good (1% or 0.1% or better)

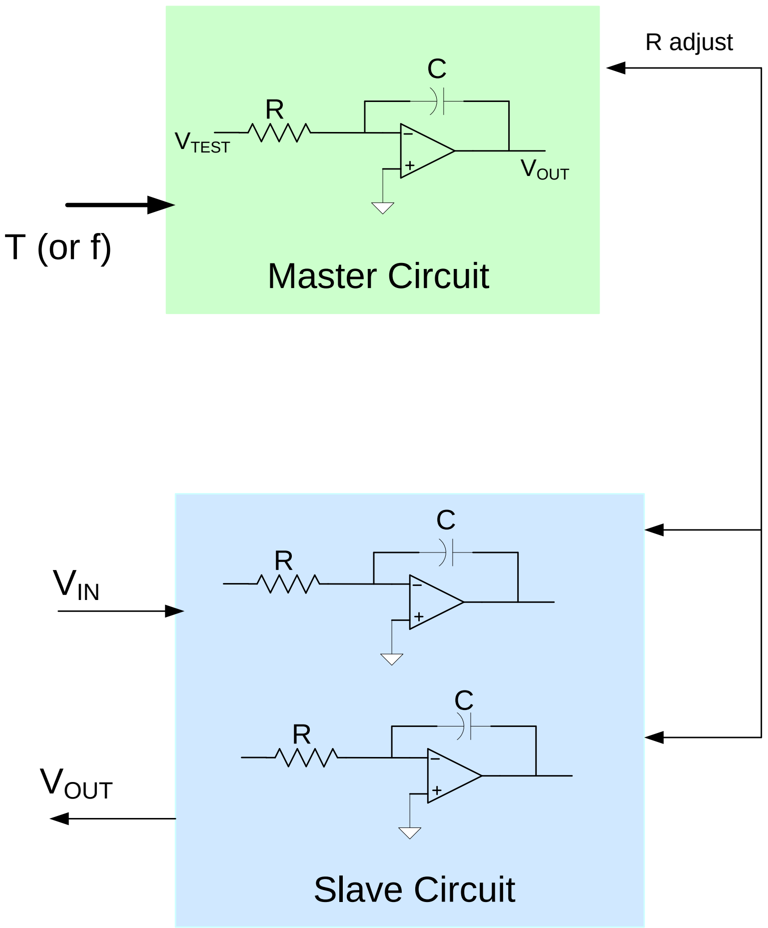
Master-slave Example:



$$T(s) = \frac{V_{OUT}}{V_{TEST}} = -\frac{1}{RCs}$$

- Key parameter of integrator is unity gain frequency I_0=1/RC
- Adjust R in Master Circuit so that I_0=1 at the input frequency f
- With matching, unity gain frequency of all integrators in Slave Circuit will also be 1

Master-slave Example:



$$T(s)=\frac{V_{OUT}}{V_{TEST}}=-\frac{1}{RCs}$$

$$T_{ACT}(s)=\frac{V_{OUT}}{V_{TEST}}=-\frac{1}{RCs+\tau\left(s+RCs^2\right)}$$

- Over-ordering will limit accuracy of master-slave approach even if unity gain frequency of master circuit is precisely obtained
- Technique is often used to maintain good control of effective RC products

What can be done to address these problems?

3. Select Appropriate Architecture

Helps a lot

Best architectures are not known

Performance of good architectures often not good enough

What can be done to address these problems?

4. Different Approach for Filter Implementation

- **Frequency Referenced Filters**
  Switched-Capacitor Filters
- **DSP- Based Filter Implementation**
- **Other Niche Methods**

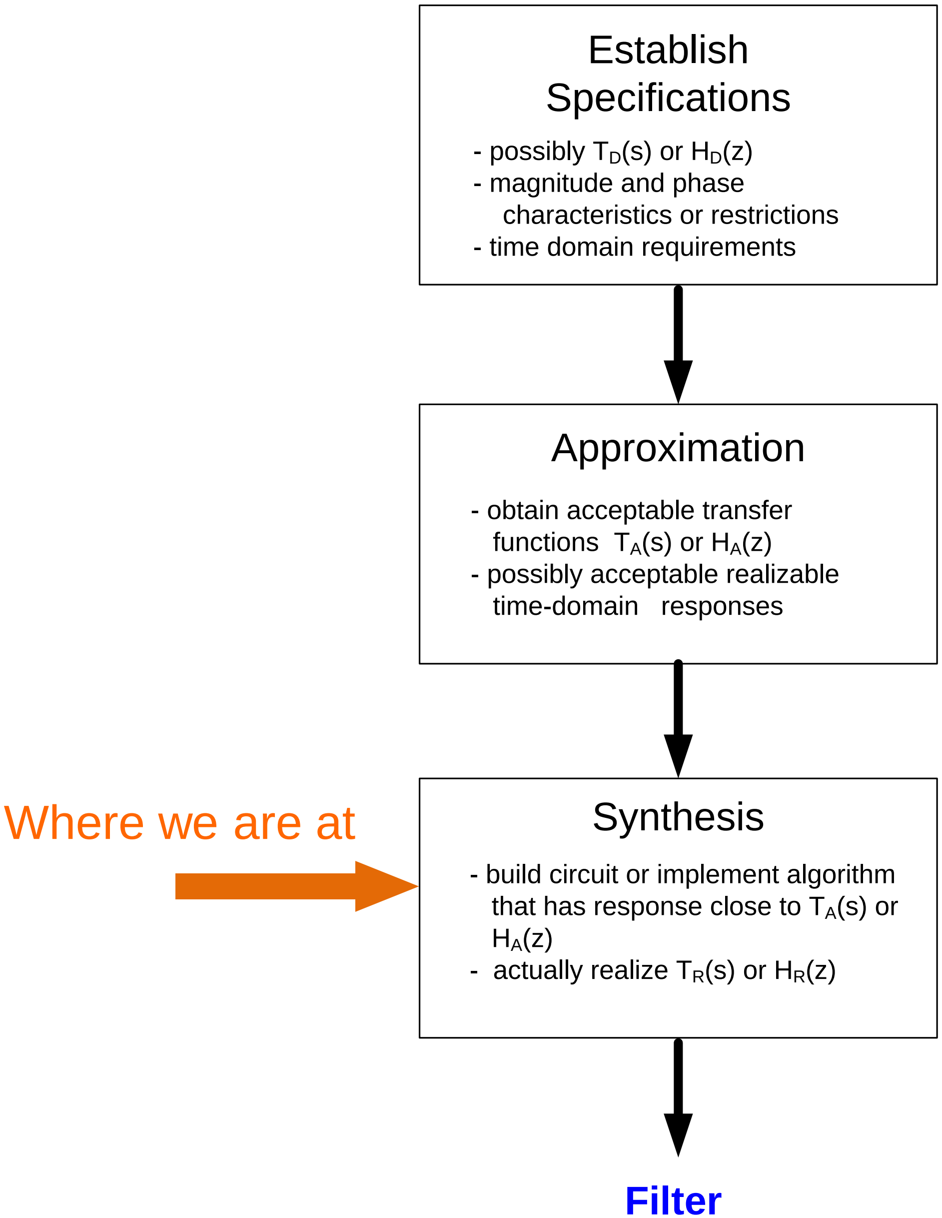
Summary of Sensitivity Observations

- Sensitivity varies substantially from one implementation to another

- Variability too high, even with low sensitivity, for more demanding applications

- Methods of managing high variability
  - Select good structures
  - Trimming
    - Functional
    - Deterministic
  - Predistortion
    - In particular, for active sensitivities
    - Useful but not a total solution
  - Frequency Referenced Techniques
    - Master-Slave Control
      - Depends upon matching
      - Can self-trim or self-compensate
    - Switched-Capacitor Filters
    - AD/digital filter/D/A
  - Alternate Design Approach
    - Other methods

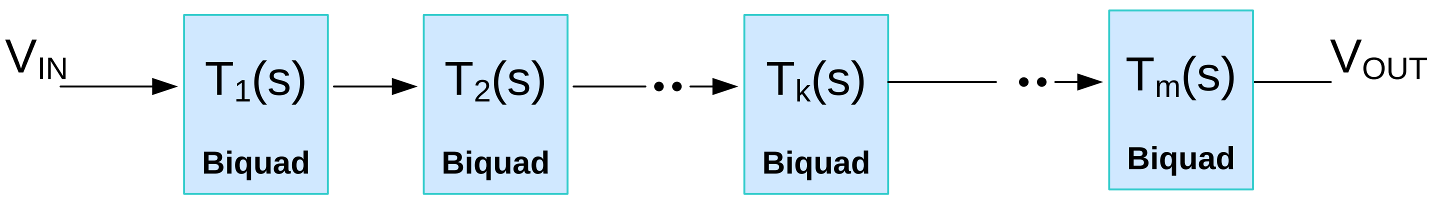
Filter Design Process



Filter Design/Synthesis Considerations

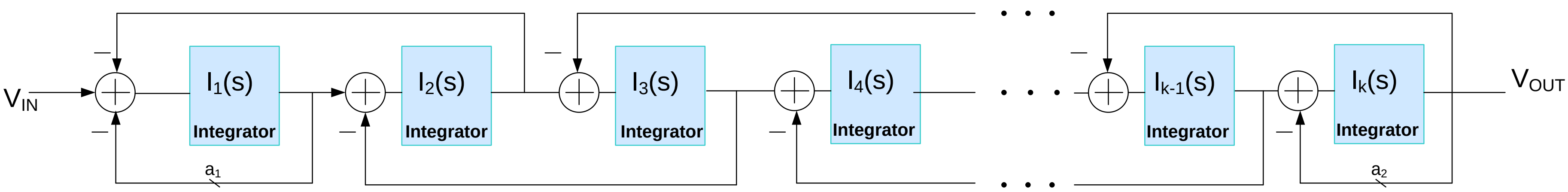
Most designs today use one of the following three basic architectures

Cascaded Biquads

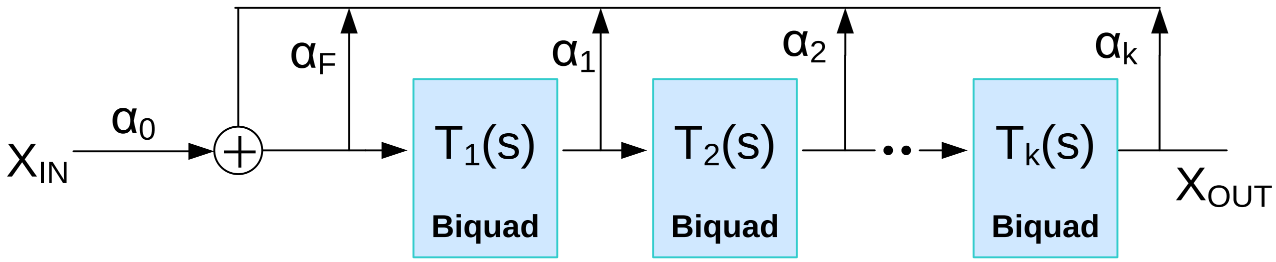


$$T(s) = T_1T_2 \bullet\bullet T_m$$

Leapfrog

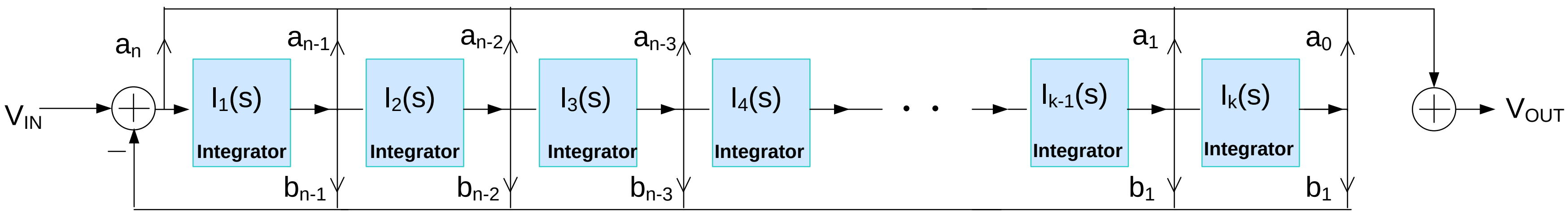


Multiple-loop Feedback – One type shown (less popular)



Filter Design/Synthesis Considerations

Multiple-loop Feedback – Another type



$$X = V_{IN} - X \bullet \sum_{k=1}^{n} b_{n-k}\left(\frac{I_0}{s}\right)^k$$

$$V_{OUT} = X \bullet \sum_{k=0}^{n} a_{n-k}\left(\frac{I_0}{s}\right)^k$$

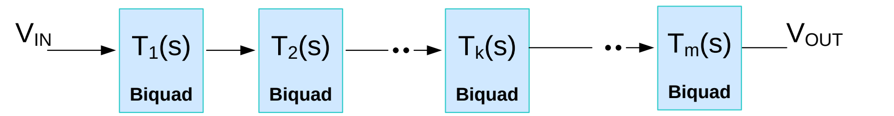
$$T(s) = \frac{\sum_{k=0}^{n} a_{n-k}\left(\frac{I_0}{s}\right)^k}{1+\sum_{k=1}^{n} b_{n-k}\left(\frac{I_0}{s}\right)^k}$$

$$T(s) = \frac{\sum_{k=0}^{n} a_{n-k} I_0^k s^{n-k}}{s^n + \sum_{k=1}^{n} b_{n-k} I_0^k s^{n-k}}$$

- Termed the direct synthesis method
- Directly implements the coefficients in the numerator and denominator
- Approach followed in the Analog Computers
- Not particularly attractive from an overall performance viewpoint

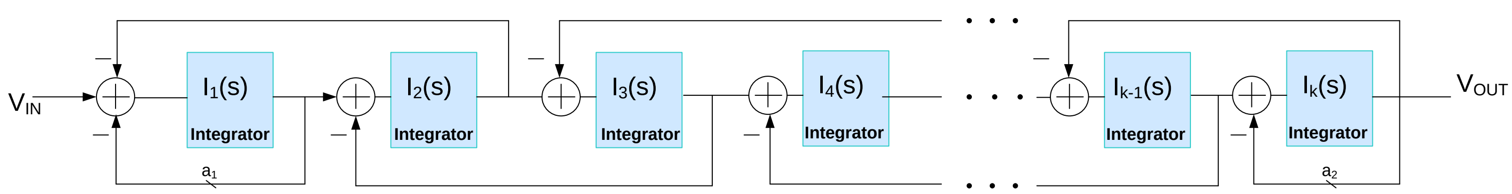
Filter Design/Synthesis Considerations

Cascaded Biquads

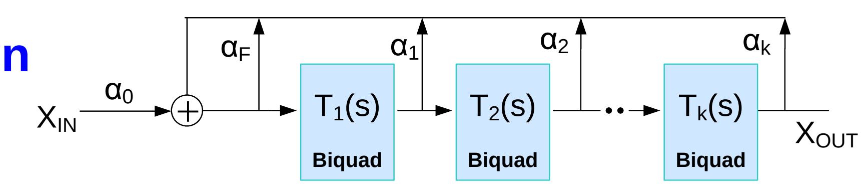


$$T(s) = T_1 T_2 \bullet\bullet T_m$$

Leapfrog



Multiple-loop Feedback – One type shown



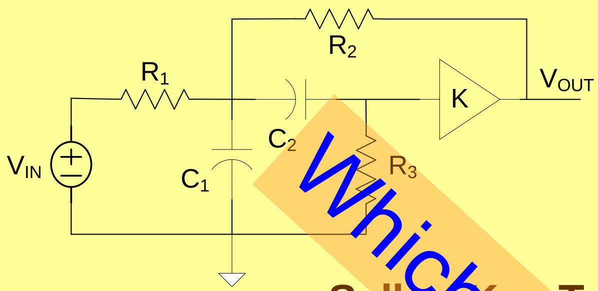
Will study details of all three types of architectures later

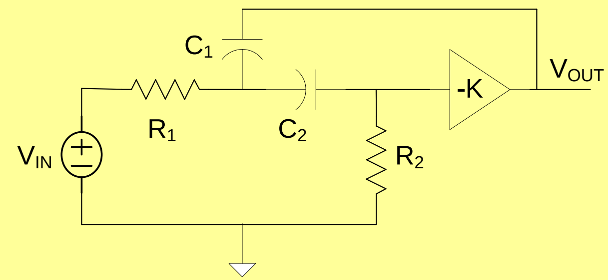
Observation: All filters are comprised of summers, biquads and integrators

Consider now the biquads

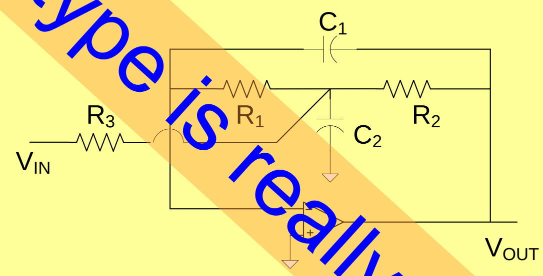
Biquad Filters Design Considerations

Several different Biquads were considered and other implementations exist

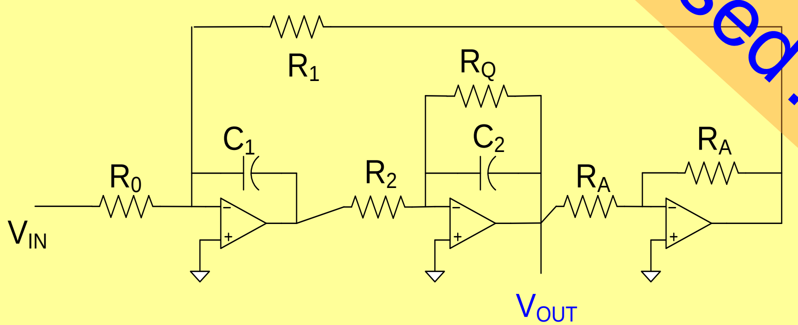




Sallen-Key Type (Dependent Sources)



Infinite Gain Amplifiers

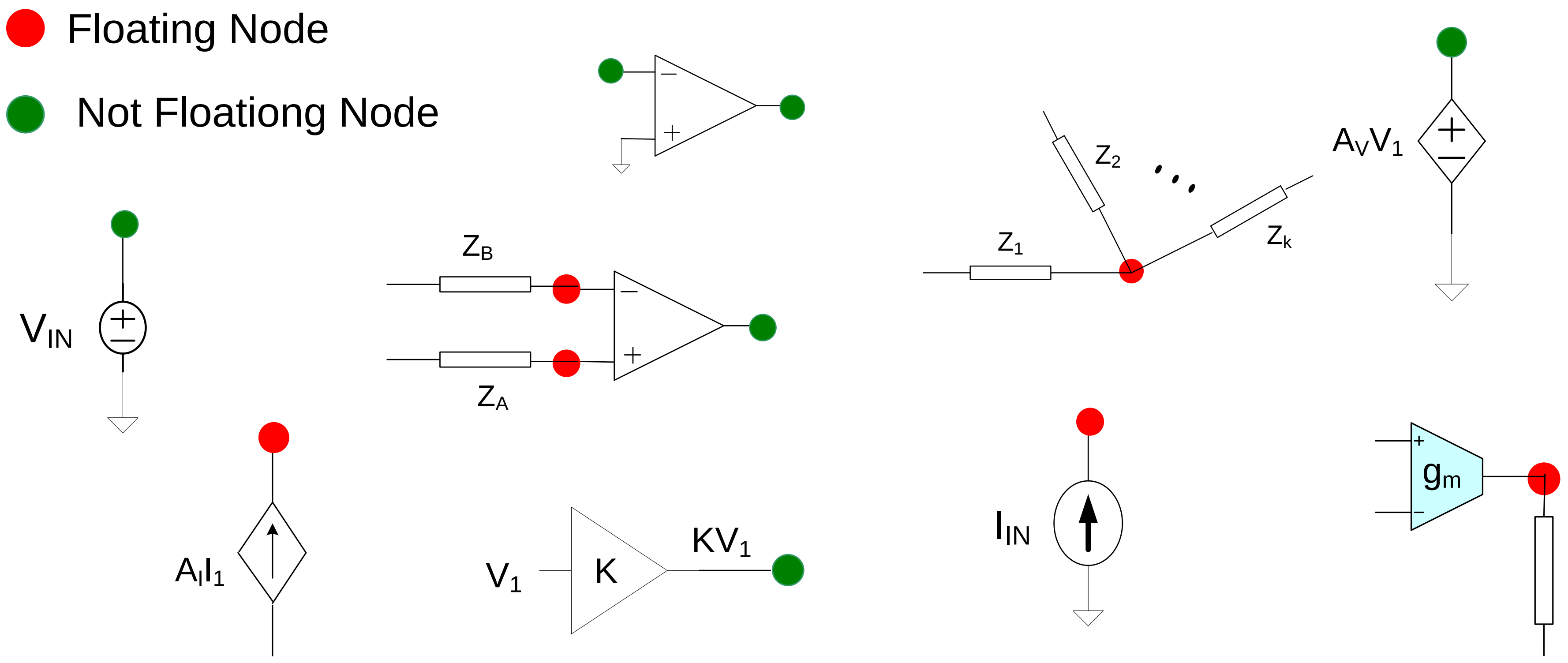


Integrator Based Structures

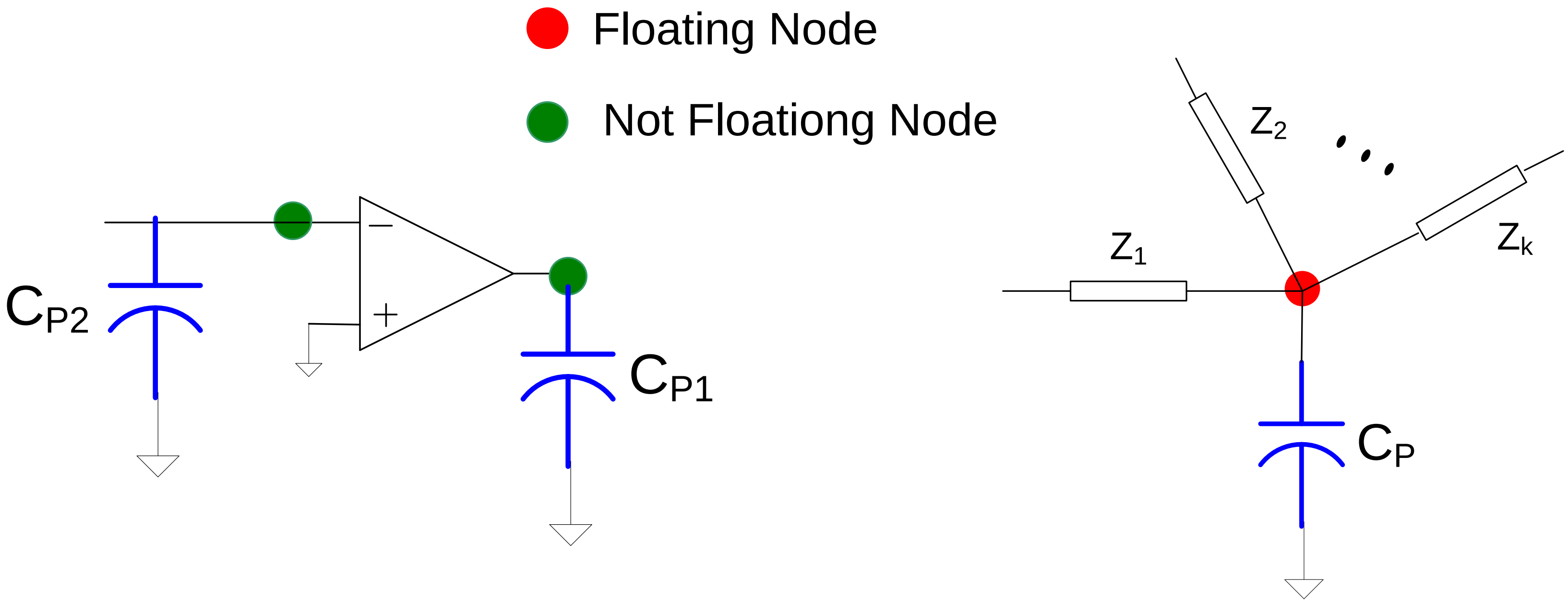
Which type is really used?

Floating Nodes

A node in a circuit is termed a floating node if it is not an output node of a ground-referenced voltage-output amplifier (dependent or independent), not connected to a ground-referenced voltage source, or not connected to a ground-referenced null-port



Parasitic Capacitances on Floating Nodes



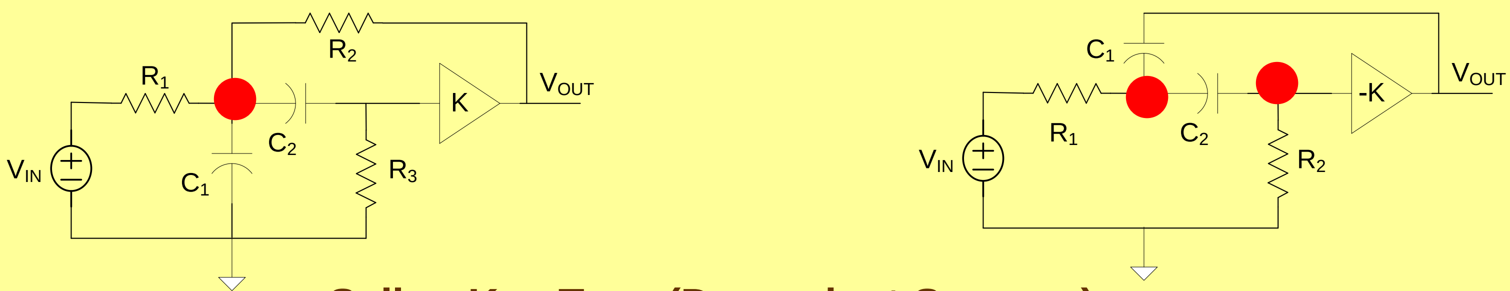
Parasitic capacitances ideally have no affect on filter when on a non-floating node but directly affect transfer function when they appear on a floating node

Parasitic capacitances are invariably large, nonlinear, and highly process dependent in integrated filters. Thus, it is difficult to build accurate integrated filters if floating nodes are present

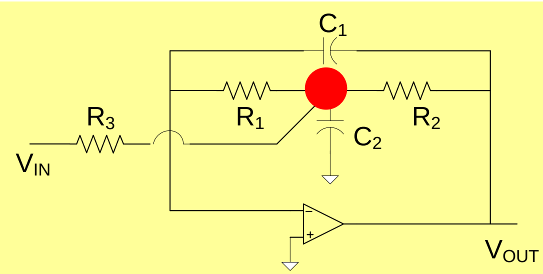
Generally avoid floating nodes, if possible, in integrated filters

Which type of Biquad is really used?

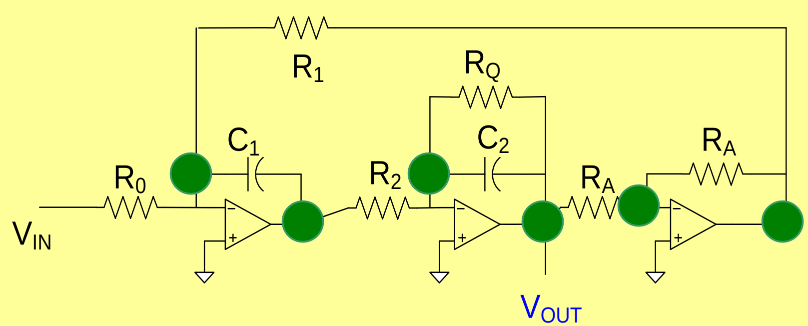
Not Floationg Node Floating Node



Sallen-Key Type (Dependent Sources)



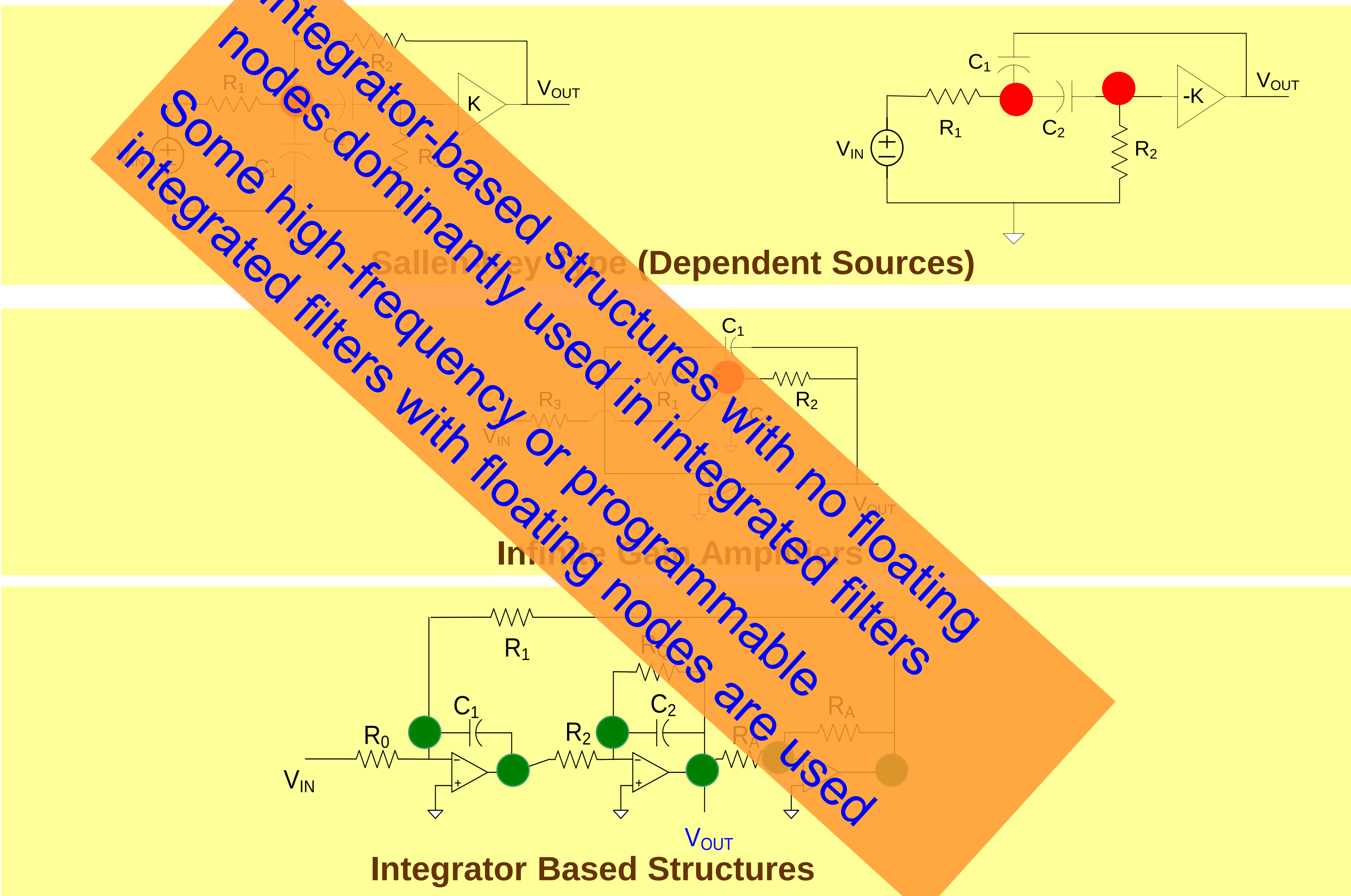
Infinite Gain Amplifiers



Integrator Based Structures

Which type of Biquad is really used?

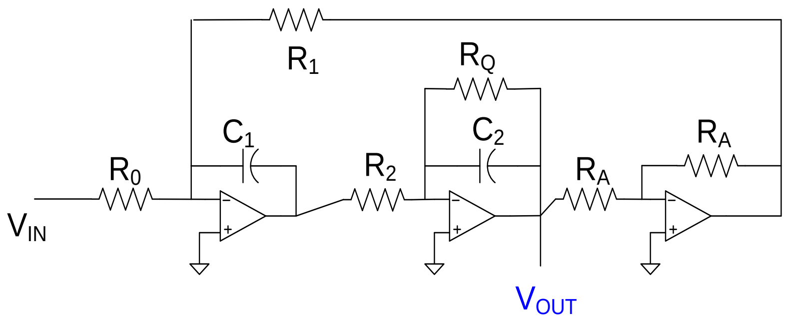
Not Floationg Node — Floating Node

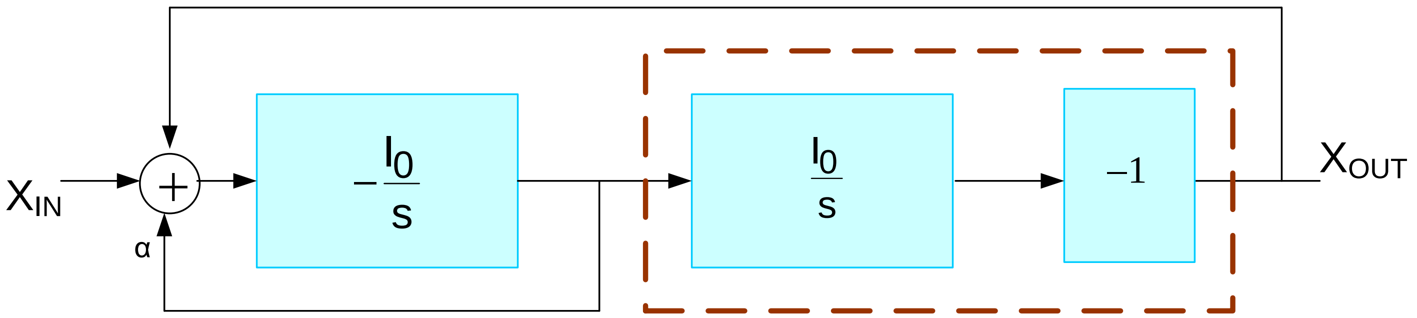


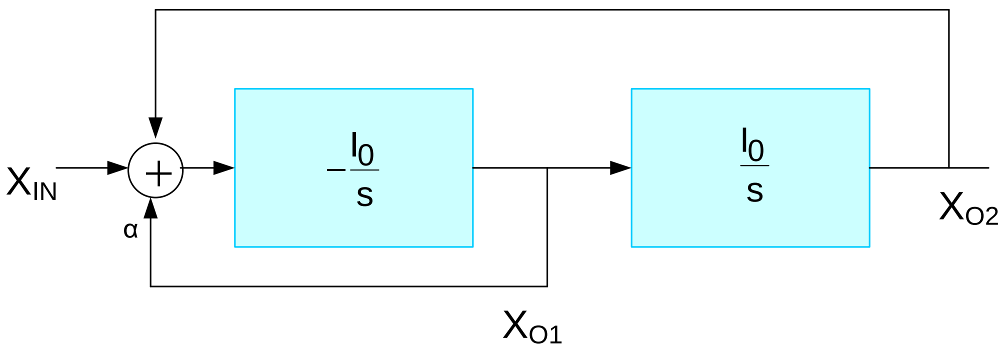
Integrator-based structures with no floating nodes dominantly used in integrated filters

Some high-frequency or programmable integrated filters with floating nodes are used

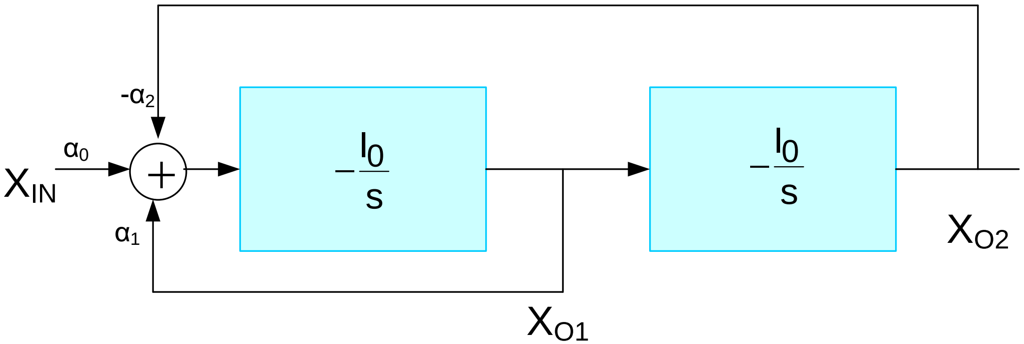
Integrator-based Biquads



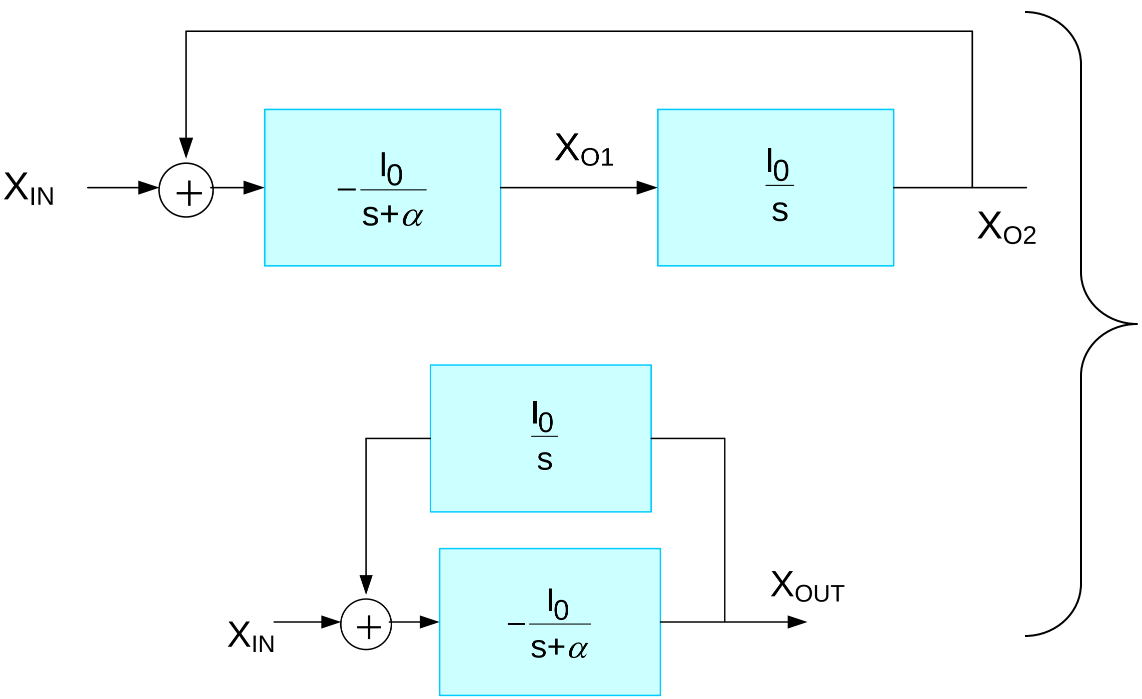




Integrator-based Biquads

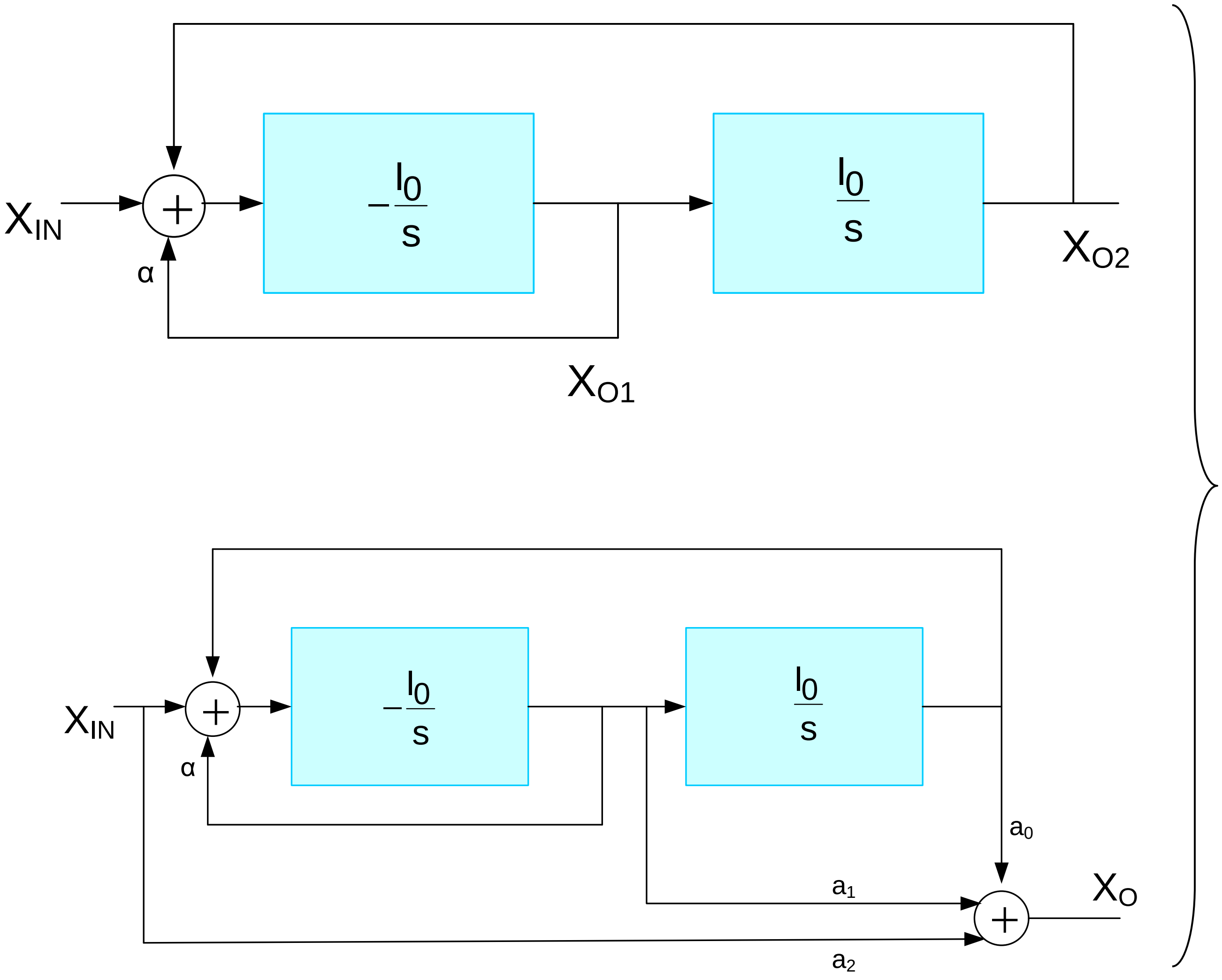


State Variable Biquad
(Alt KHN Biquad)



Integrator and lossy
integrator in a loop

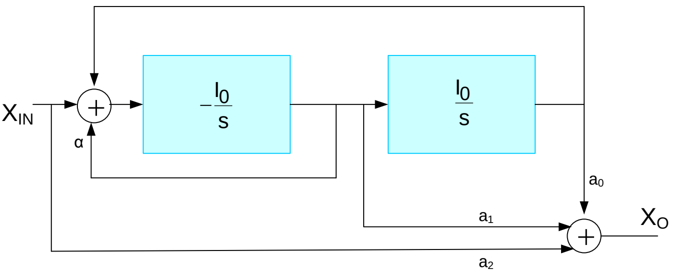
Integrator-based Biquads

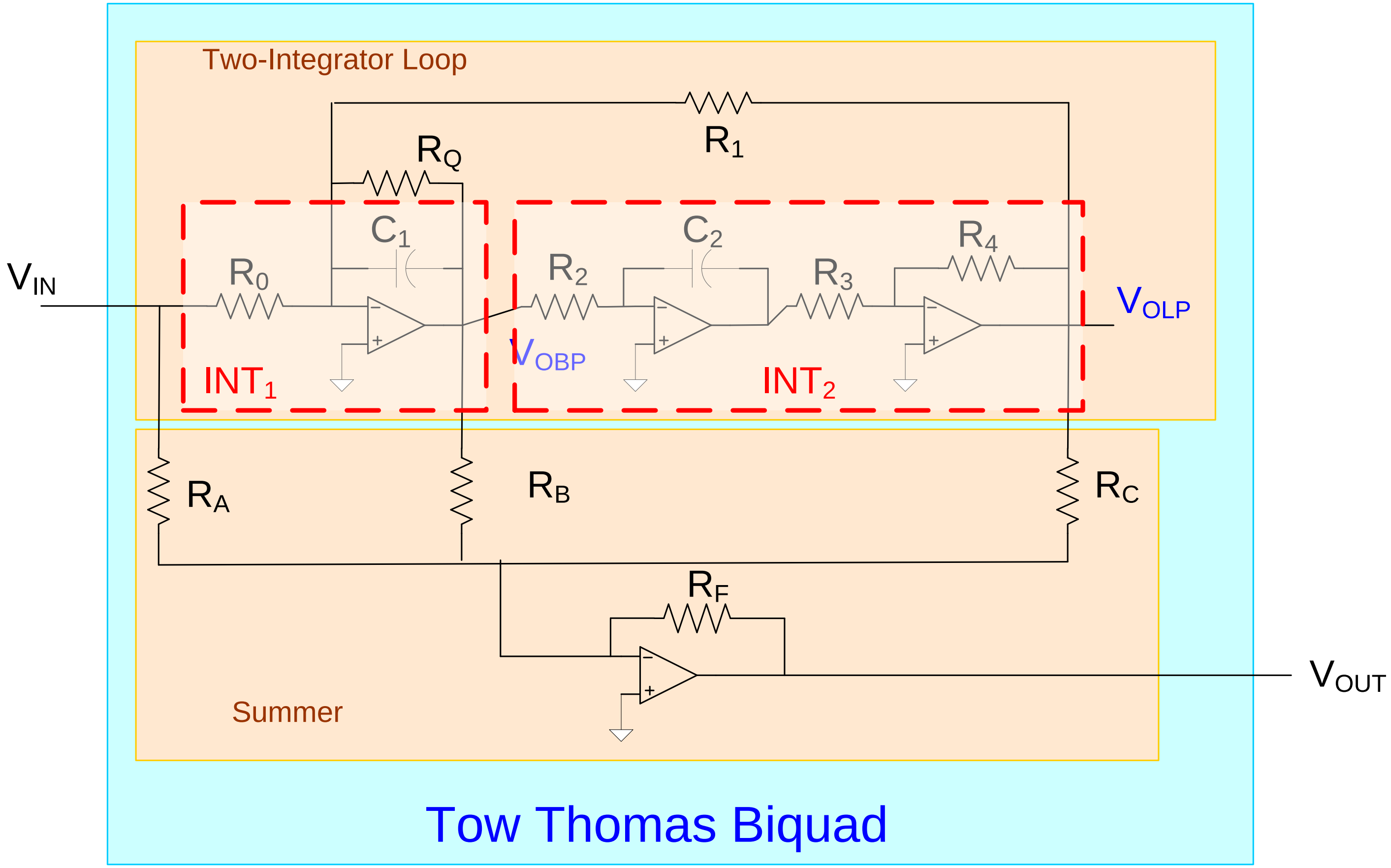


Tow-Thomas Biquad

With arbitrary zero locations

Integrator-based Biquads



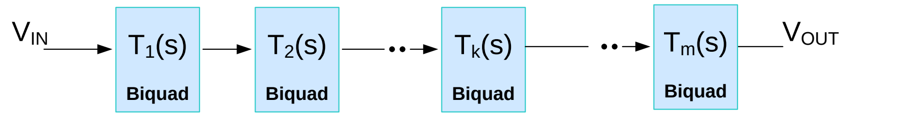


Integrator-based Biquads

- Integrator-based biquads all involve two integrators in a loop
- All integrator-based biquads discussed have no floating nodes
- Most biquads in integrated filters are based upon two integrator loop structures
- The summers are usually included as summing inputs on the integrators
- The loss can be combined with the integrator to form a lossy integrator
- Performance of the minor variants of the two integrator loop structures are comparable

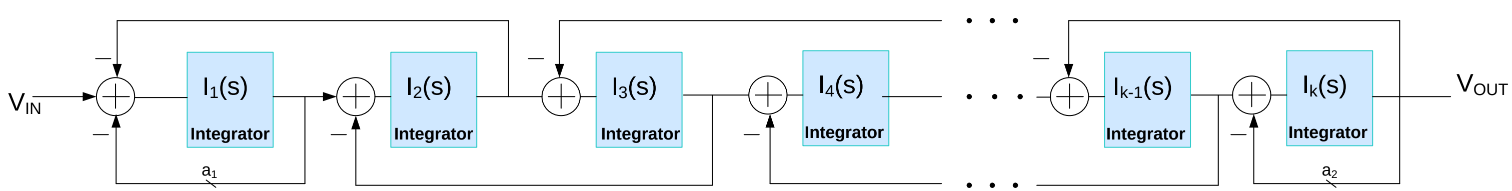
Filter Design/Synthesis Considerations

Cascaded Biquads

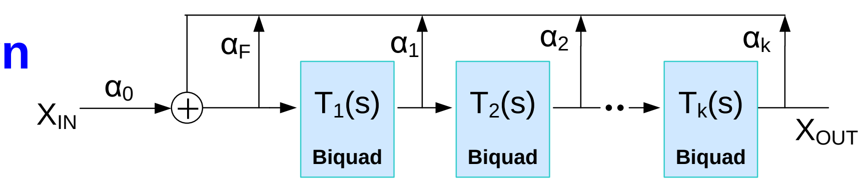


$$T(s) = T_1 T_2 \bullet\bullet T_m$$

Leapfrog



Multiple-loop Feedback – One type shown



Observation: All filters are comprised of summers, biquads and integrators

And biquads usually made with summers and integrators

Integrated filter design generally focused on design of integrators, summers, and amplifiers (Op Amps)

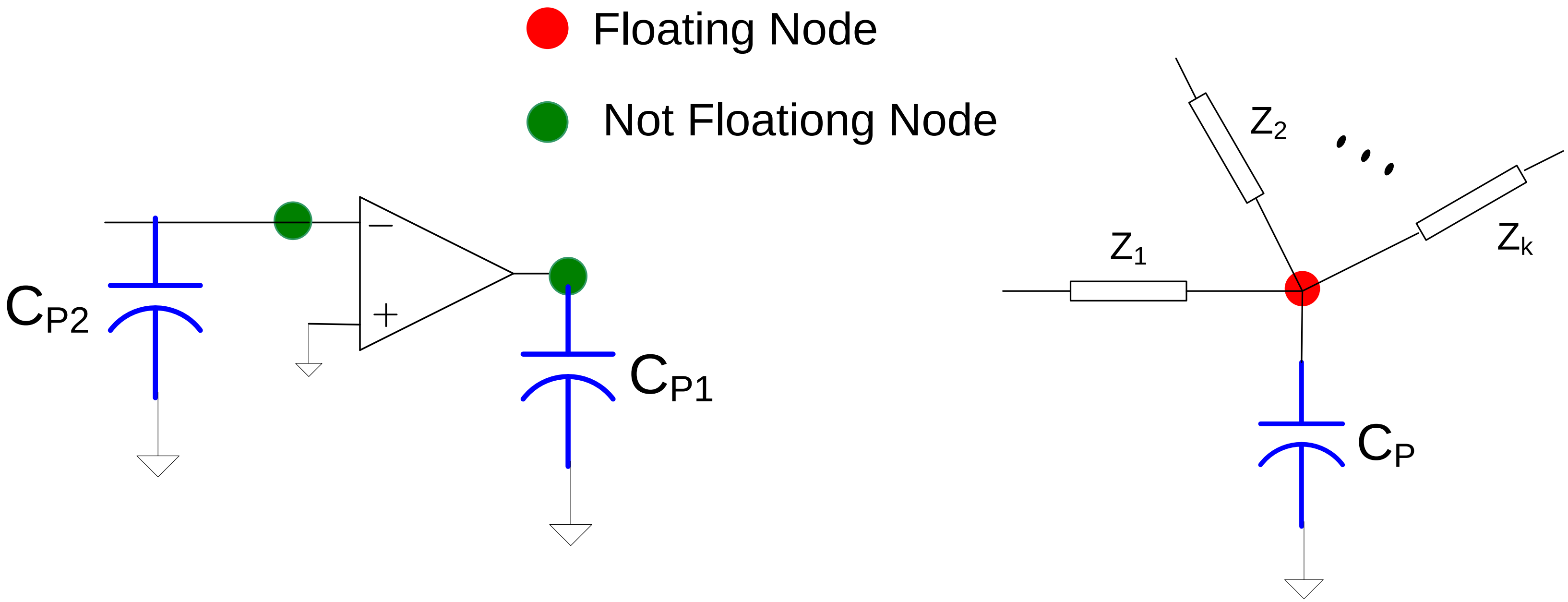
Will now focus on the design of integrators, summers, and op amps

End of Lecture 24

EE 508
Lecture 25

Integrator Design

Parasitic Capacitances on Floating Nodes



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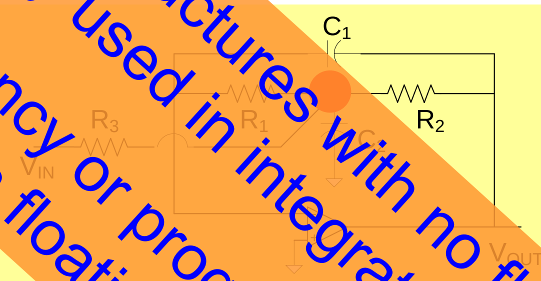
Generally avoid floating nodes, if possible, in integrated filters

Review from last time

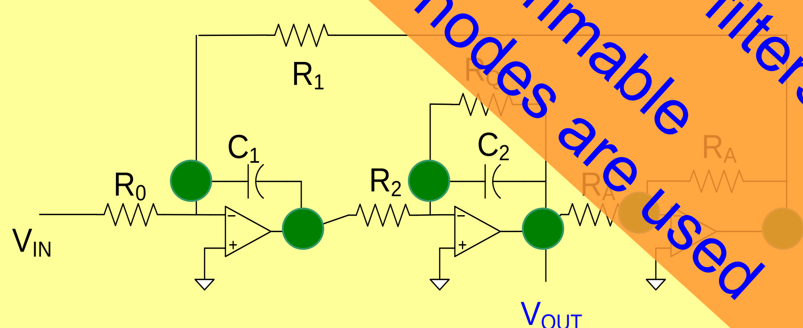
Which type of Biquad is really used?

Not Floationg Node — Floating Node

Sallen-Key type (Dependent Sources)



Infinite Gain Amplifiers



Integrator Based Structures

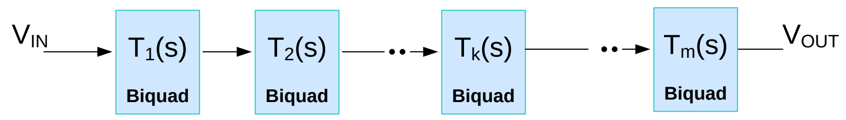
Integrator-based structures with no floating nodes dominantly used in integrated filters

Some high-frequency or programmable integrated filters with floating nodes are used

Review from last time

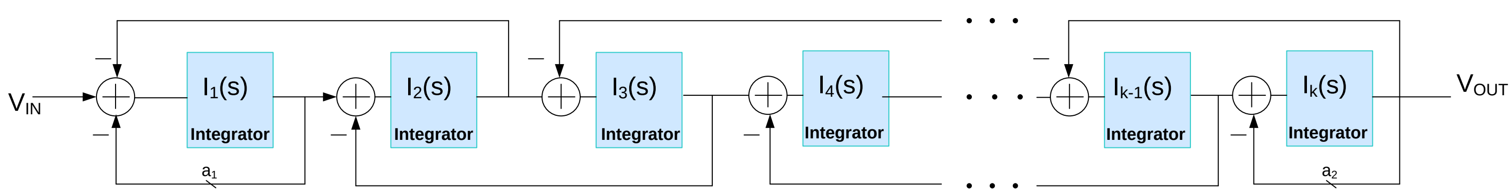
Filter Design/Synthesis Considerations

Cascaded Biquads

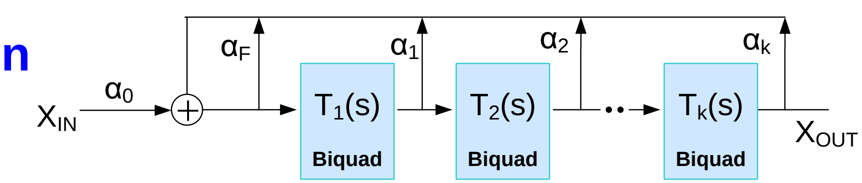


$$T(s) = T_1 T_2 \bullet\bullet T_m$$

Leapfrog



Multiple-loop Feedback – One type shown



Observation: All filters are comprised of summers, biquads and integrators

And biquads usually made with summers and integrators

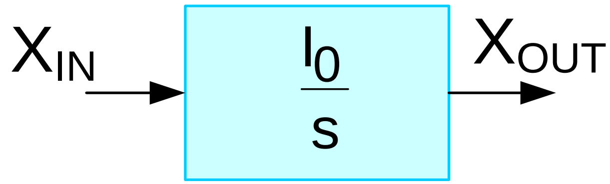
Integrated filter design generally focused on design of integrators, summers, and amplifiers (Op Amps)

Will now focus on the design of integrators, summers, and op amps

Basic Filter Building Blocks
(particularly for integrated filters)

- Integrators
- Summers
- Operational Amplifiers

Integrator Characteristics of Interest



$$I(s) = \frac{I_0}{s}$$

Properties of an ideal integrator:

$$|I(j\omega)| = \frac{I_0}{\omega}$$

Gain decreases with 1/ω

$$\angle I(j\omega) = -90^0$$

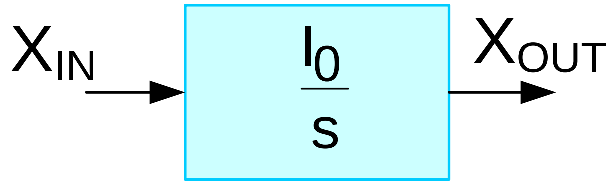
Phase is a constant -90º

$$|I(jI_0)| = 1$$

Unity Gain Frequency = 1

How important is it that an integrator have all 3 of these properties?

Integrator Characteristics of Interest

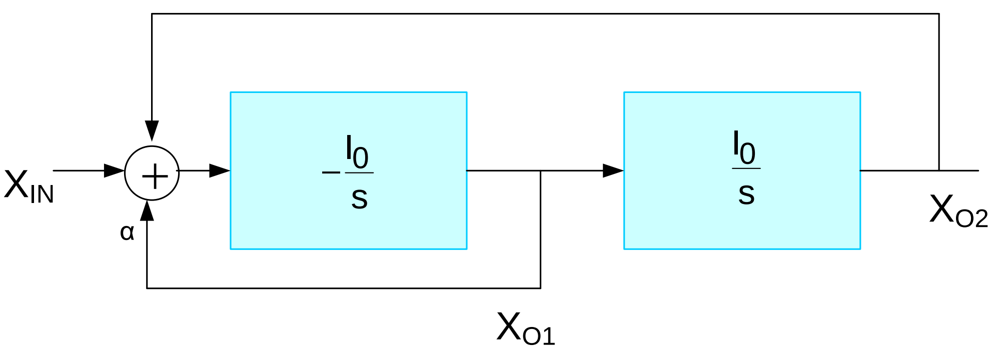


$$I(s) = \frac{I_0}{s}$$

$$\left|I(j\omega)\right| = \frac{I_0}{\omega} \qquad \angle I(j\omega) = -90^0 \qquad \left|I(jI_0)\right| = 1$$

How important is it that an integrator have all 3 of these properties?

Consider a filter example:



$$T(s) = \frac{-I_0^2}{s^2 + \alpha I_0 s + I_0^2}$$

$$Q = \frac{1}{\alpha} \qquad \omega_0 = I_0$$

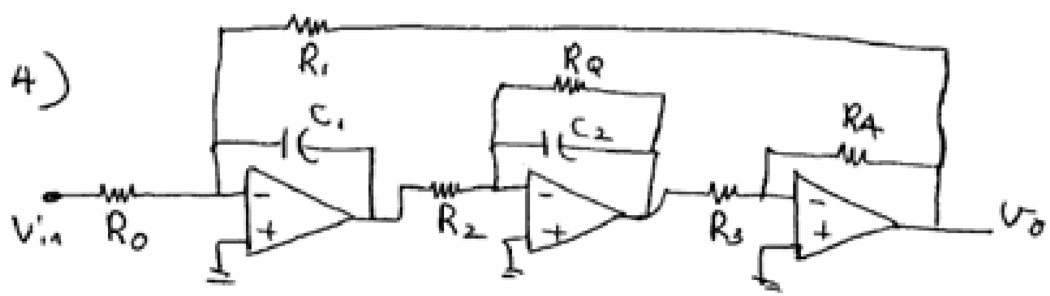
Band edges proportional to I_0

Phase critical to make Q expression valid

In many (most) applications it is critical that an integrator be very nearly ideal

(in the frequency range of interest)

4)



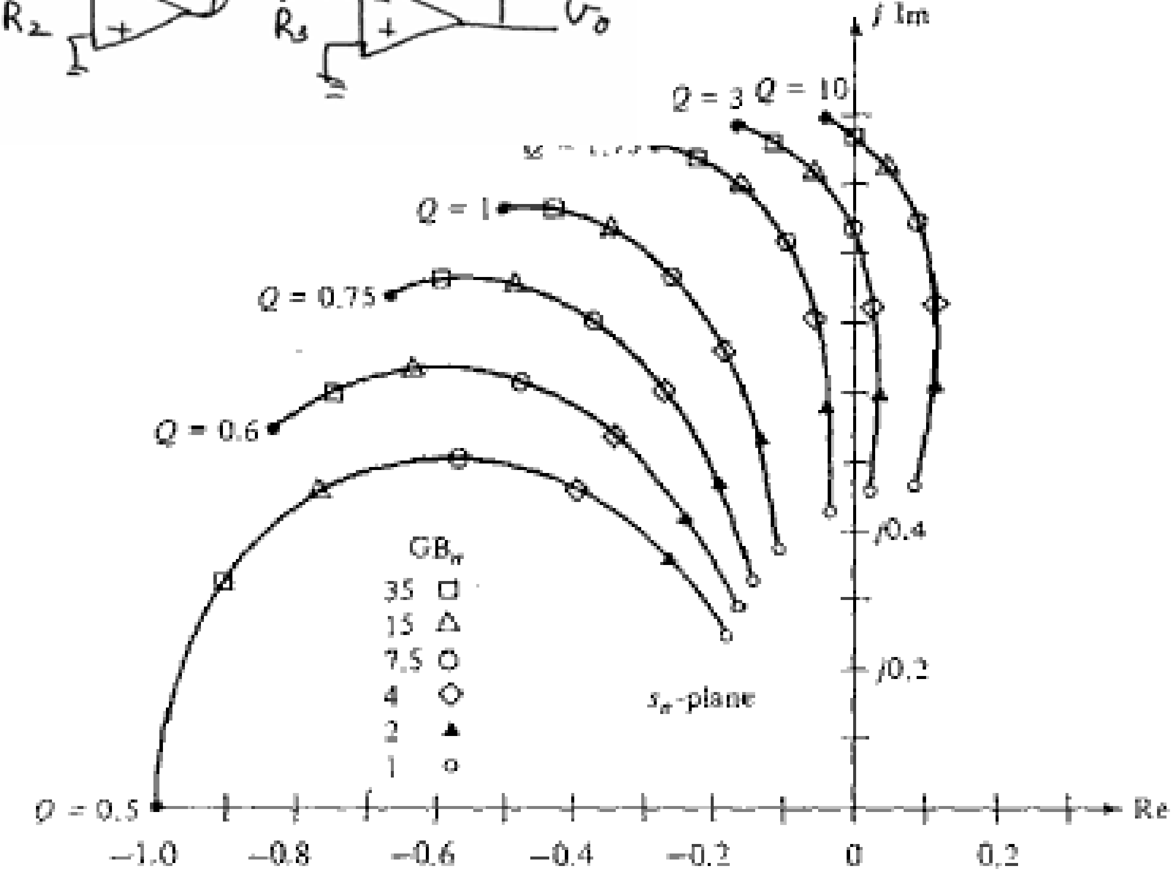
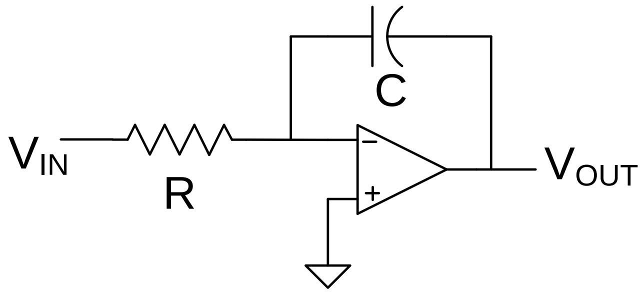


Fig. 10-17 Plot of upper half-plane root of

$$s_n^3 + s_n^2\left(\frac{1}{2} + \frac{1}{Q} + \frac{GB_n}{4}\right) + s_n\frac{1}{4Q}\left(1 + GB_n\right) + \frac{GB_n}{4} = 0$$

Some integrator structures

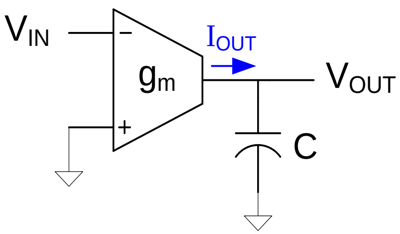


$$I(s) = -\frac{1}{RCs}$$

$$I_0 = \frac{1}{RC}$$

Inverting Active RC Integrator

Are there other integrator structures?



$$I_{OUT} = -g_m V_{IN}$$

$$V_{OUT} = I_{OUT}\frac{1}{sC}$$

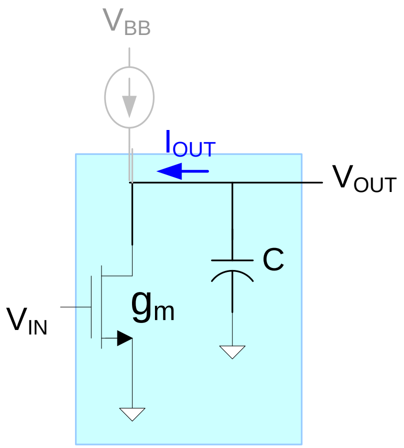
$$I(s) = -\frac{g_m}{sC}$$

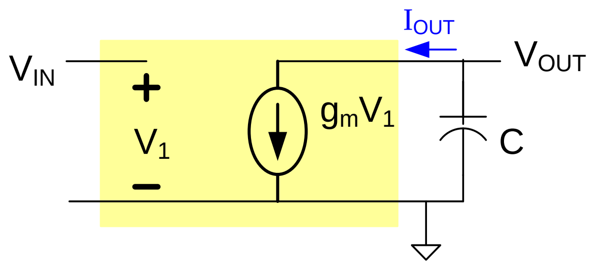
$$I_0 = \frac{g_m}{C}$$

Termed an OTA-C or a gm-C integrator

Some integrator structures

Are there other integrator structures?





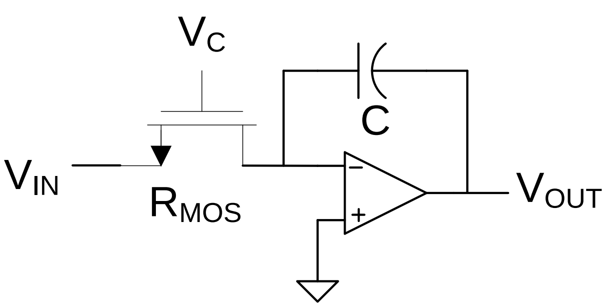
$$I_{OUT} = g_m V_{IN}$$

$$V_{OUT} = -I_{OUT}\frac{1}{sC}$$

$$I(s) = -\frac{g_m}{sC}$$

Termed a TA-C integrator

$$I_0 = \frac{g_m}{C}$$



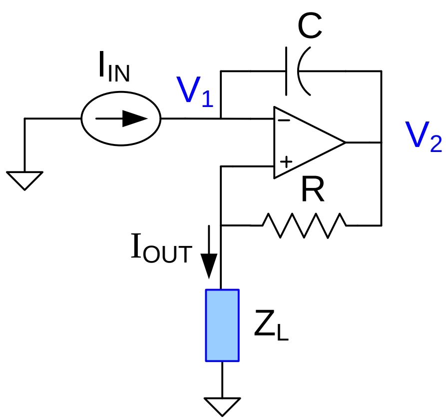
$$I(s) = -\frac{1}{sCR_{MOS}}$$

$$I_0 = -\frac{1}{R_{FET}C}$$

Termed MOSFET-C integrator

Some integrator structures

Are there other integrator structures?



$$\left.\begin{aligned} V_2 &= V_1 - I_{IN}\frac{1}{sC} \\ I_{OUT} &= \frac{V_2 - V_1}{R} \end{aligned}\right\}$$

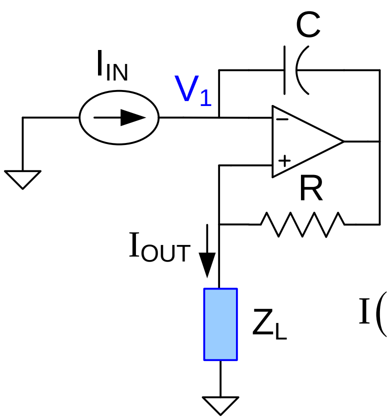
$$I(s) = \frac{I_{OUT}}{I_{IN}} = -\frac{1}{sRC}$$

$$I_0 = \frac{1}{RC}$$

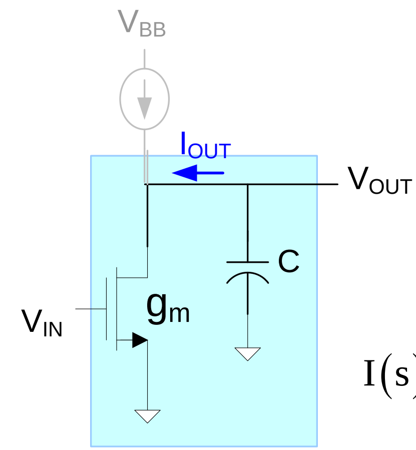
- Output current is independent of Z_L
- Thus output impedance is ∞ so provides current output

Termed active RC current-mode integrator

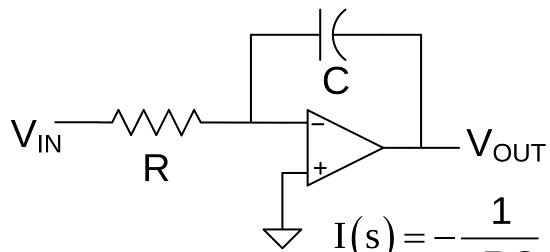
Some integrator structures



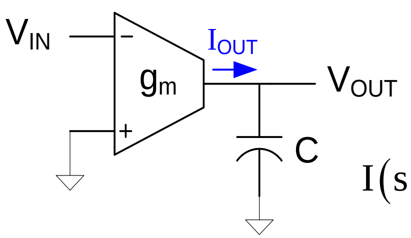
$$I(s) = -\frac{1}{sRC}$$



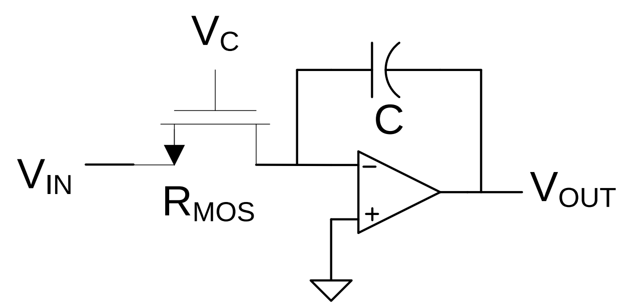
$$I(s) = -\frac{g_m}{sC}$$



$$I(s) = -\frac{1}{sRC}$$



$$I(s) = -\frac{g_m}{sC}$$

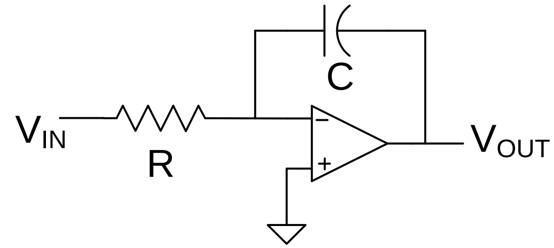


$$I(s) = -\frac{1}{sR_{MOS}C}$$

There are other useful integrator structures (some will be introduced later)

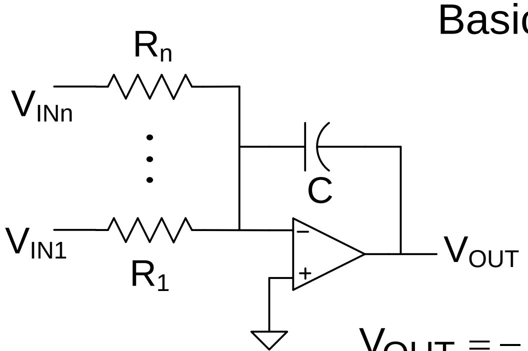
There are many different ways to build an inverting integrator

Integrator Functionality



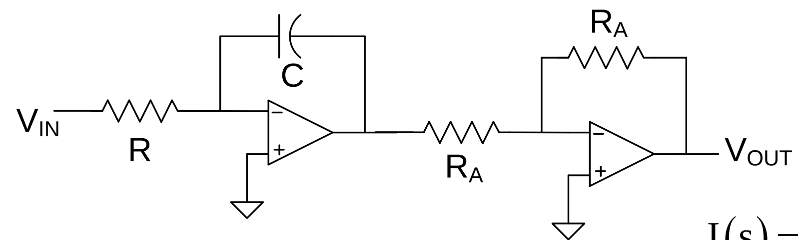
$$I(s) = -\frac{1}{sRC}$$

Basic Active RC Inverting Integrator



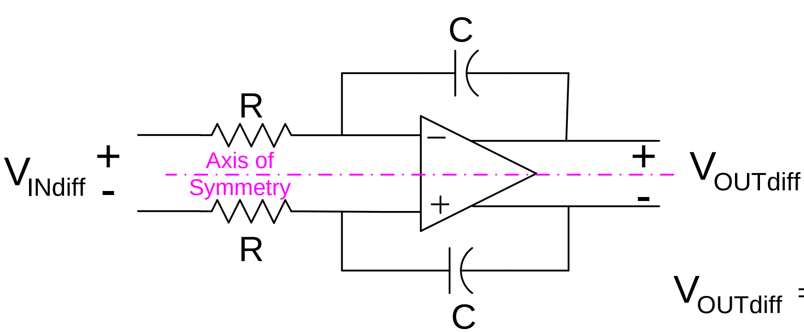
$$V_{OUT} = -\sum_{k=1}^{n} \frac{1}{CR_k s} V_{INk}$$

Summing Integrator



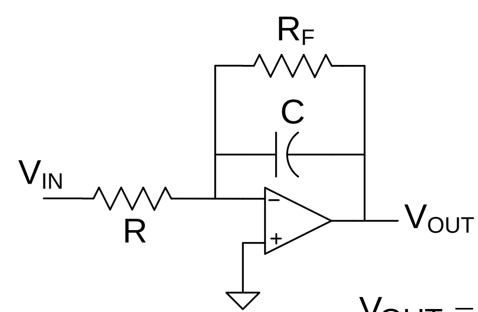
$$I(s) = \frac{1}{sRC}$$

Noninverting Integrator



$$V_{OUTdiff} = -\frac{1}{CRs} V_{INdiff}$$

Fully Differential Integrator

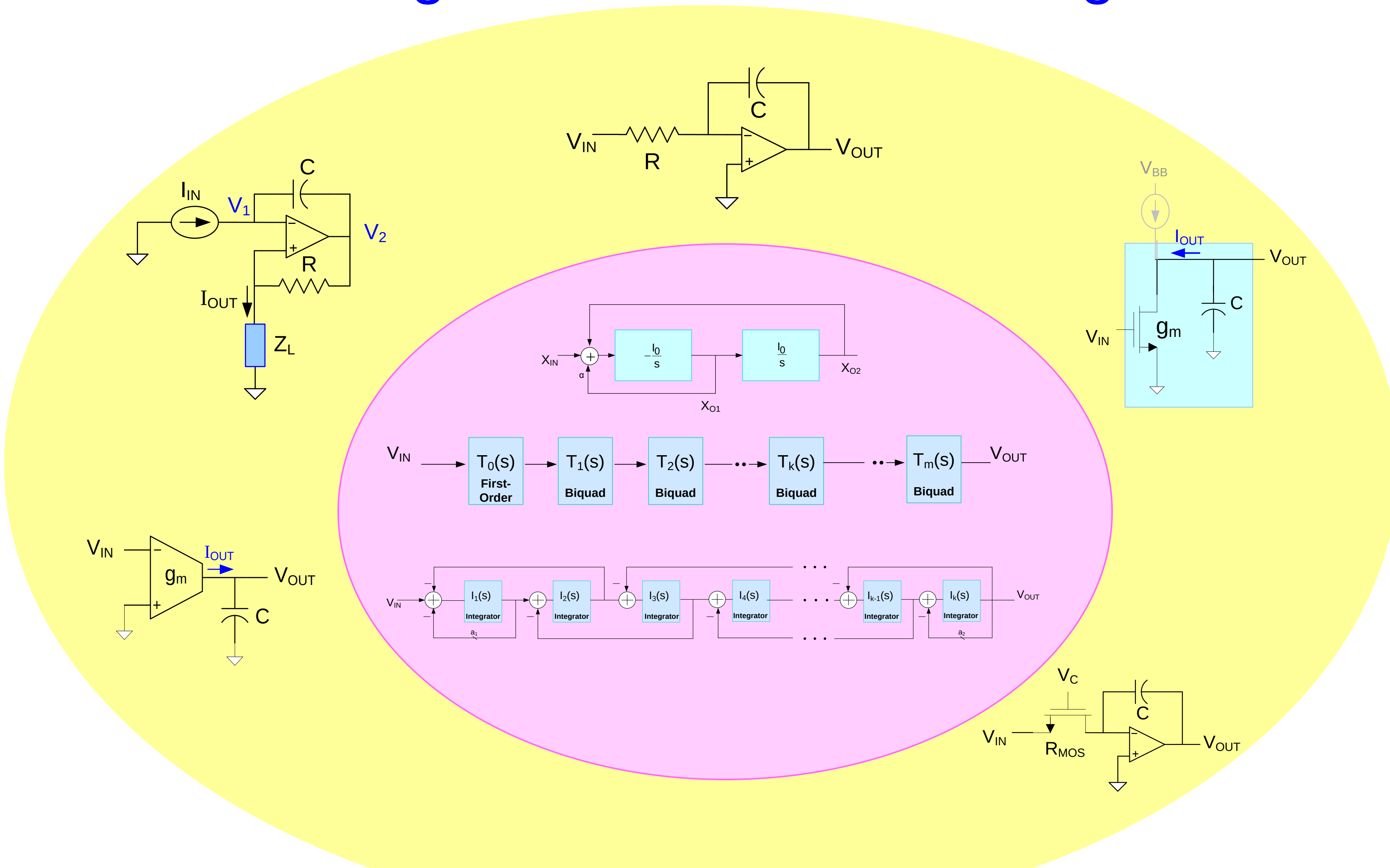


$$V_{OUT} = -\frac{R_F / R}{1+CR_F s} V_{IN}$$

Lossy Integrator

Many different types of functionality from basic inverting integrator
Same modifications exist for other integrator architectures

Integrator-Based Filter Design



Any of these different types of integrators can be used to build integrator-based filters

Are new integrators still being invented?

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Searching US Patent Collection...

Results of Search in US Patent Collection db for:
TTL/integrator: 419 patents.
*Hits **1** through **50** out of **419***

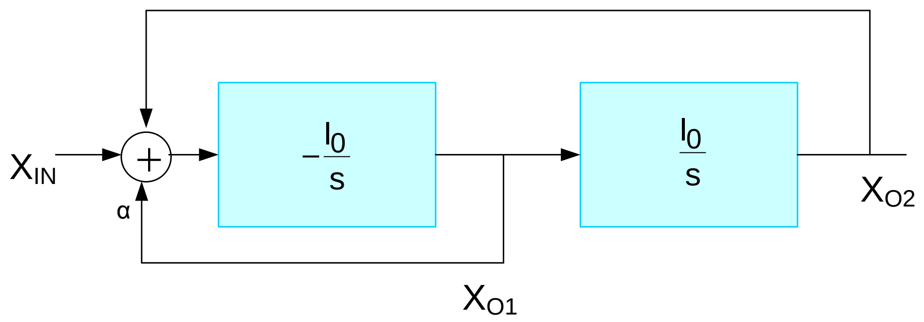
	PAT. NO.	Title
1	8,290,897	System integrator and method for mapping dynamic COBOL constructs to object instances for the automatic integration to object-oriented computing systems
2	8,283,966	Integrator circuit
3	8,275,307	Vehicle audio integrator
4	8,264,388	Frequency integrator with digital phase error message for phase-locked loop applications
5	8,258,990	Integrator, resonator, and oversampling A/D converter
6	8,253,473	Integrated circuit of an integrator with enhanced stability and related stabilization method
7	8,199,038	Active resistance-capacitor integrator and continuous-time sigma-delta modulator with gain control function
8	8,164,873	Integrator and circuit-breaker having an integrator
9	8,145,597	System integrator and method for mapping dynamic COBOL constructs to object instances for the automatic integration to object-oriented computing systems
10	8,129,972	Single integrator sensorless current mode control for a switching power converter
11	8,125,262	Low power and low noise switched capacitor integrator with flexible input common mode range
12	8,098,377	Electric gated integrator detection method and device thereof
13	8,081,098	Integrator, delta-sigma modulator, analog-to-digital converter and applications thereof
14	8,035,439	Multi-channel integrator
15	8,031,404	Fly's eye integrator, illuminator, lithographic apparatus and method
16	8,029,144	Color mixing rod integrator in a laser-based projector
17	8,028,304	Component integrator
18	8,013,657	Temperature compensated integrator
19	8,011,810	Light integrator for more than one lamp
20	7,997,740	Integrator unit
21	7,965,795	Prevention of integrator wind-up in PI type controllers
22	7,965,151	Pulse width modulator with two-way integrator
23	7,954,962	Laser image display, and optical integrator and laser light source package used in such laser image display
24	7,943,893	Illumination optical system and image projection device having a rod integrator uniformizing spatial energy distribution of diffused illumination beam
25	7,933,812	System integrator and commodity roll-up

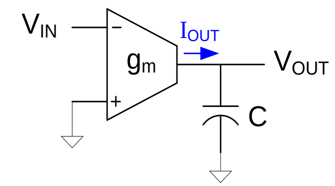
26 7,932,960 T Integrator array for HUD backlighting
27 7,911,256 T Dual integrator circuit for analog front end (AFE)
28 7,907,115 T Digitally synchronized integrator for noise rejection in system using PWM dimming signals to control brightness of cold cathode fluorescent lamp for backlighting liquid crystal display
29 7,905,631 T Illumination system having coherent light source and integrator rotatable transverse the illumination axis
30 7,884,662 T Multi-channel integrator
31 7,880,969 T Optical integrator for an illumination system of a microlithographic projection exposure apparatus
32 7,873,223 T Cognition integrator and language
33 7,834,963 T Optical integrator
34 7,830,197 T Adjustable integrator using a single capacitance
35 RE41,792 T Controllable integrator
36 7,788,309 T Interleaved comb and integrator filter structures
37 7,773,730 T Voice record integrator
38 7,729,577 T Waveguide-optical Kohler integrator utilizing geodesic lenses
39 7,726,819 T Structure for protecting a rod integrator having a light shield plate with an opening
40 7,724,063 T Integrator-based common-mode stabilization technique for pseudo-differential switched-capacitor circuits
41 7,714,634 T Pseudo-differential active RC integrator
42 7,706,072 T Optical integrator, illumination optical device, photolithograph, photolithography, and method for fabricating device
43 7,696,913 T Signal processing system using delta-sigma modulation having an internal stabilizer path with direct output-to-integrator connection
44 7,693,430 T Burst optical receiver with AC coupling and integrator feedback network
45 7,679,540 T Double sampling DAC and integrator
46 7,671,774 T Analog-to-digital converter with integrator circuit for overload recovery
47 7,658,497 T Rod integrator holder and projection type video display
48 7,629,917 T Integrator and cyclic AD converter using the same
49 7,619,550 T Delta-sigma AD converter apparatus using delta-sigma modulator circuit provided with reset circuit resetting integrator
50 7,611,246 T Projection display and optical integrator

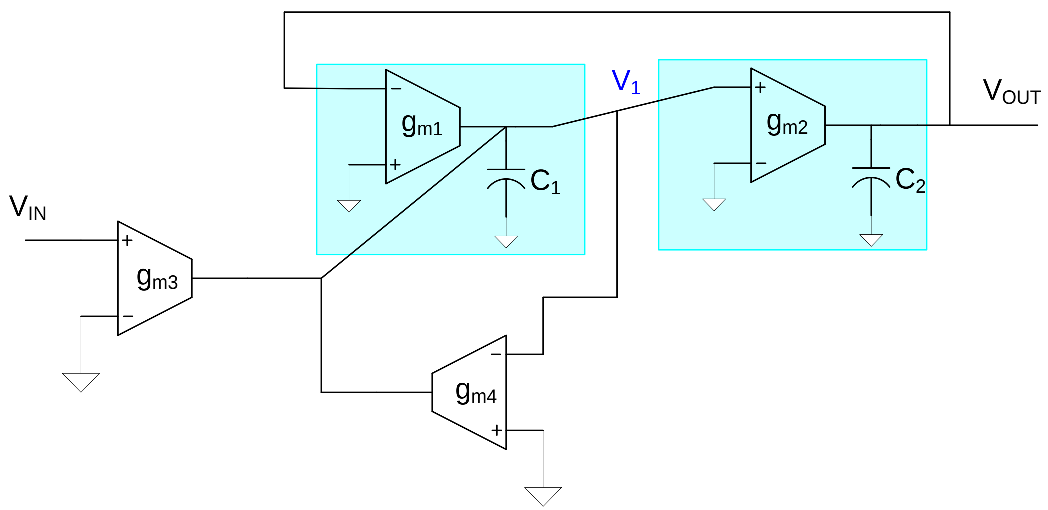
	PAT. NO.	Title
51	7,605,645	Transconductor, integrator, and filter circuit
52	7,599,631	Burst optical receiver with AC coupling and integrator feedback network
53	7,575,159	Point of sale integrator
54	7,570,032	Regulator with integrator in feedback signal
55	7,565,326	Dialect independent multi-dimensional integrator using a normalized language platform and secure controlled access
56	7,554,400	Integrator and error amplifier
57	7,543,945	Integrator module with a collimator and a compact light source and projection display having the same
58	7,532,145	High resolution and wide dynamic range integrator
59	7,528,818	Digitally synchronized integrator for noise rejection in system using PWM dimming signals to control brightness of light source
60	7,511,648	Integrating/SAR ADC and method with low integrator swing and low complexity
61	7,474,241	Delta-sigma modulator provided with a charge sharing integrator
62	7,471,456	Optical integrator, illumination optical device, exposure device, and exposure method
63	7,454,750	Integrator adaptor and proxy based composite application provisioning method and apparatus
64	7,447,049	Single ended flyback power supply controllers with integrator to integrate the difference between feedback signal a reference signal
65	7,423,729	Method of monitoring the light integrator of a photolithography system
66	7,417,485	Differential energy difference integrator
67	7,415,716	Component integrator
68	7,411,534	Analog-to-digital converter (ADC) having integrator dither injection and quantizer output compensation
69	7,411,198	Integrator circuitry for single channel radiation detector
70	7,395,090	Personal portable integrator for music player and mobile phone
71	7,385,426	Low current offset integrator with signal independent low input capacitance buffer circuit
72	7,379,160	Optical integrator, illumination optical device, exposure apparatus, and exposure method
73	7,352,510	Light-pipe integrator for uniform irradiance and intensity
74	7,345,285	Spectra acquisition system with threshold adaptation integrator
75	7,333,626	Arbitrary coverage angle sound integrator

76 7,324,654 T Arbitrary coverage angle sound integrator
77 7,324,025 T Non-integer interpolation using cascaded integrator-comb filter
78 7,315,268 T Integrator current matching
79 7,304,592 T Method of adding a dither signal in output to the last integrator of a sigma-delta converter and relative sigma-delta converter
80 7,280,405 T Integrator-based current sensing circuit for reading memory cells
81 7,262,056 T Enhancing intermolecular integration of nucleic acids using integrator complexes
82 7,243,844 T Point of sale integrator
83 7,242,333 T Alternate sampling integrator
84 7,205,849 T Phase locked loop including an integrator-free loop filter
85 7,187,948 T Personal portable integrator for music player and mobile phone
86 7,182,468 T Dual lamp illumination system using multiple integrator rods
87 7,180,357 T Operational amplifier integrator
88 7,170,959 T Tailored response cascaded integrator comb digital filter and methodology for parallel integrator processing
89 7,155,470 T Variable gain integrator
90 7,152,981 T Projection illumination system with tunnel integrator and field lens
91 7,152,084 T Parallelized infinite impulse response (IIR) and integrator filters
92 7,150,968 T Bridging INtegrator-2 (Bin2) nucleic acid molecules and proteins and uses therefor
93 7,138,848 T Switched capacitor integrator system
94 7,130,764 T Robust DSP integrator for accelerometer signals
95 7,102,844 T Dual direction integrator for constant velocity control for an actuator using sampled back EMF control
96 7,102,548 T Cascaded integrator comb filter with arbitrary integer decimation value and scaling for unity gain
97 7,098,845 T Apparatus for generating an integrator timing reference from a local oscillator signal
98 7,098,827 T Integrator circuit
99 7,098,718 T Tunable current-mode integrator for low-frequency filters
100 7,087,881 T Solid state image pickup device including an integrator with a variable reference potential

Example – OTA-C Tow Thomas Biquad







$$V_{OUT} s C_2 = g_{m2} V_1$$

$$V_1 s C_1 = -g_{m1} V_{OUT} + g_{m3} V_{IN} - g_{m4} V_1$$

$$\frac{V_{OUT}}{V_{IN}} = \frac{g_{m3} g_{m2}}{\left(s^2 C_1 C_2 + s g_{m4} C_2 + g_{m1} g_{m2}\right)}$$

Assume $g_{m1}=g_{m2}=g_m$, $C_1=C_2=C$

$$\frac{V_{OUT}}{V_{IN}} = \frac{\left(\frac{g_{m3}}{g_m}\right)\frac{g_m^2}{C^2}}{\left(s^2 + s\left(\frac{g_{m4}}{g_m}\right)\frac{g_m}{C} + \frac{g_m^2}{C^2}\right)}$$

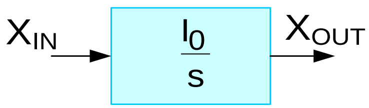
express as

$$\frac{V_{OUT}}{V_{IN}} = \frac{\left(\frac{g_{m3}}{g_m}\right)\omega_0^2}{\left(s^2 + s\frac{\omega_0}{Q} + \omega_0^2\right)}$$

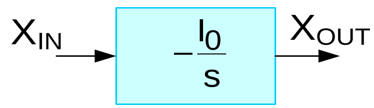
where

$$\omega_0 = \frac{g_m}{C} \qquad Q = \frac{g_m}{g_{m4}}$$

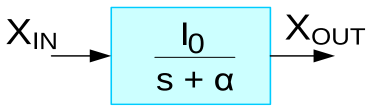
Basic Integrator Functionality



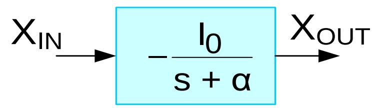
Noninverting



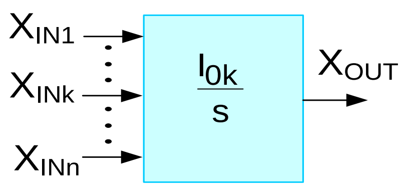
Inverting



Lossy Noninverting

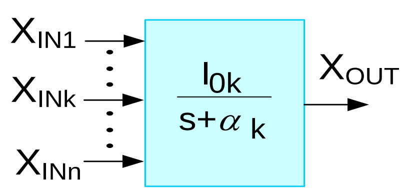


Lossy Inverting



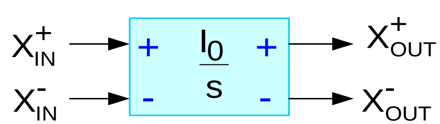
$$X_{OUT} = \sum_{k=1}^{n} \frac{\pm I_{0k}}{s}$$

Summing (Multiple-Input) Inverting/Noninverting



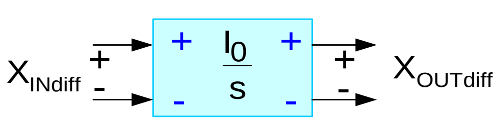
$$X_{OUT} = \sum_{k=1}^{n} \frac{\pm I_{0k}}{s+\alpha_k}$$

Summing (Multiple-Input) Lossy Inverting/Noninverting



$$X_{OUT}^{+} - X_{OUT}^{+} = \frac{I_0}{s}\left(X_{IN}^{+} - X_{IN}^{+}\right)$$

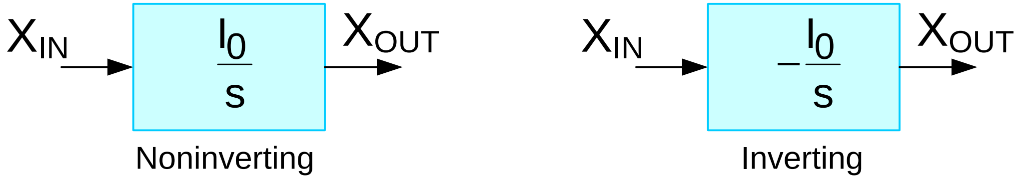
Balanced Differential



$$X_{OUTdiff} = \frac{I_0}{s} X_{INdiff}$$

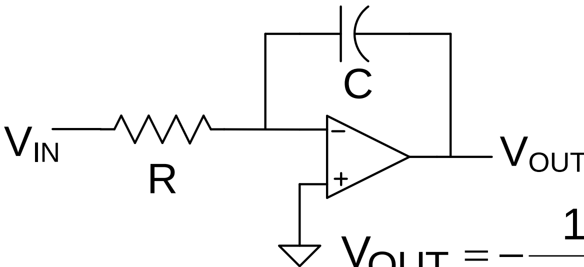
Fully Differential

Basic Integrator Functionality



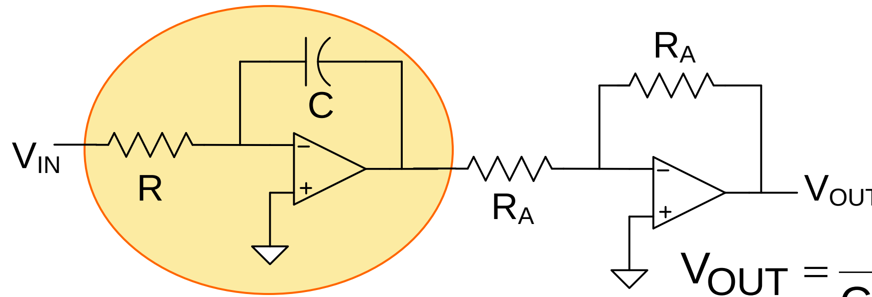
- An inverting/noninverting integrator pair define a family of integrators
- All integrator functional types can usually be obtained from the inverting/noninverting integrator pair
- Suffices to focus primarily on the design of the inverting/noninverting integrator pair since properties of class primarily determined by properties of integrator pair

Example – Basic Op-Amp Feedback Integrator



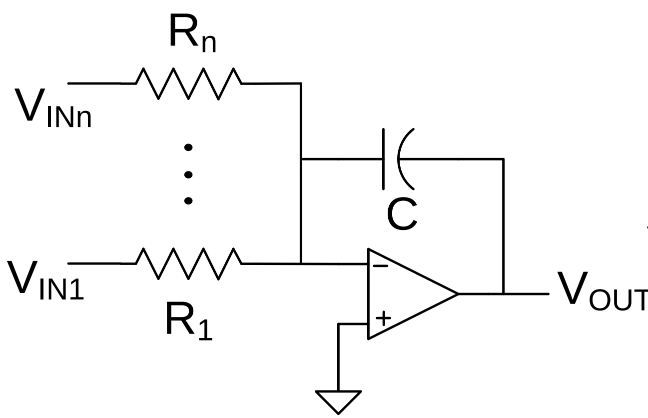
$$V_{OUT} = -\frac{1}{CRs} V_{IN}$$

Inverting Integrator of Family



$$V_{OUT} = \frac{1}{CRs} V_{IN}$$

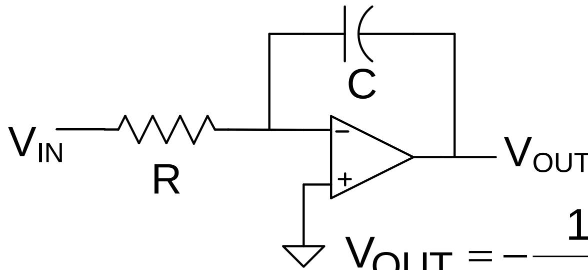
Noninverting Integrator



$$V_{OUT} = -\sum_{k=1}^{n} \frac{1}{CR_k s} V_{INk}$$

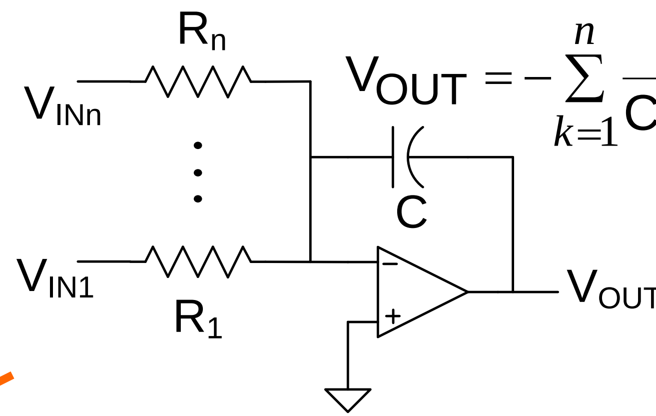
Summing Inverting Integrator

Example – Basic Op-Amp Feedback Integrator



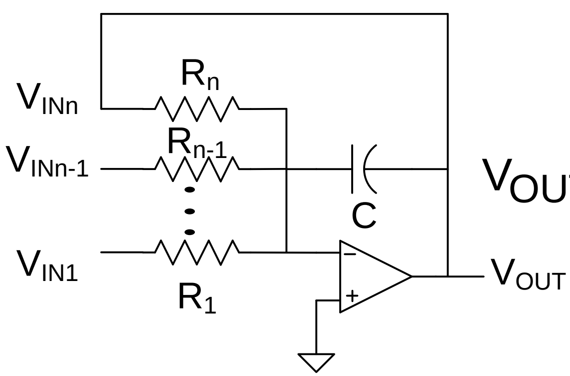
$$V_{OUT} = -\frac{1}{CRs} V_{IN}$$

Inverting Integrator of Family

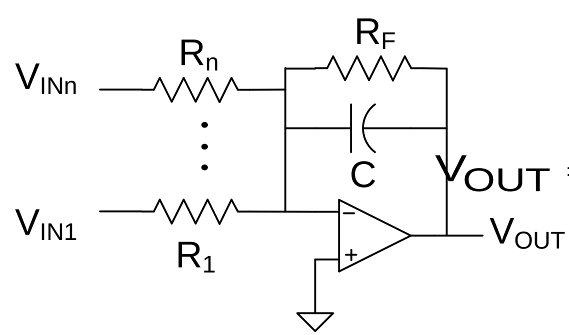


$$V_{OUT} = -\sum_{k=1}^{n} \frac{1}{CR_k s} V_{INk}$$

Summing Inverting Integrator



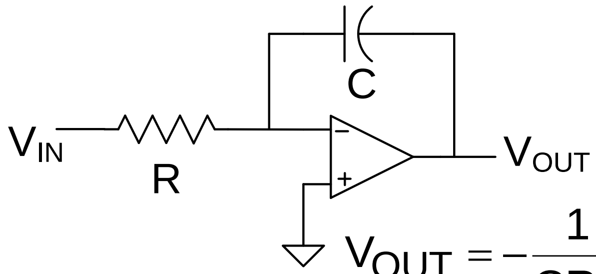
$$V_{OUT} = -\frac{\sum_{k=1}^{n-1} \frac{R_n}{R_k} V_{INk}}{1+CR_n s}$$



$$V_{OUT} = -\frac{\sum_{k=1}^{n} \frac{R_F}{R_k} V_{INk}}{1+CR_F s}$$

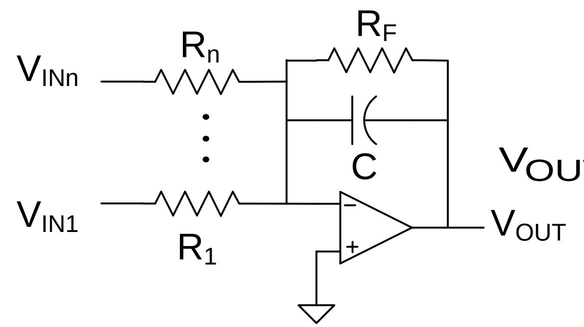
Lossy Summing Inverting Integrator

Example – Basic Op-Amp Feedback Integrator



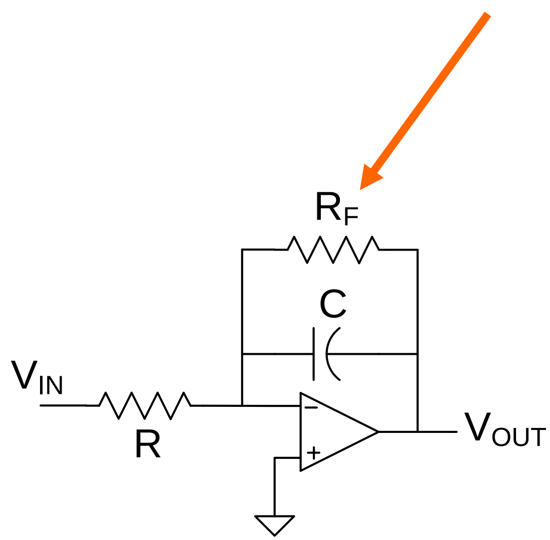
$$V_{OUT} = -\frac{1}{CRs} V_{IN}$$

Inverting Integrator of Family



$$V_{OUT} = -\frac{\sum_{k=1}^{n} \frac{R_F}{R_k} V_{INk}}{1+CR_F s}$$

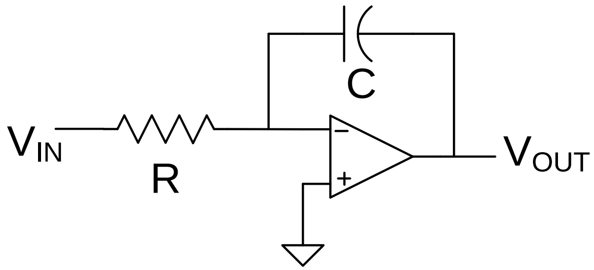
Lossy Summing Inverting Integrator



$$V_{OUT} = -\frac{R_F/R}{1+CR_F s} V_{IN}$$

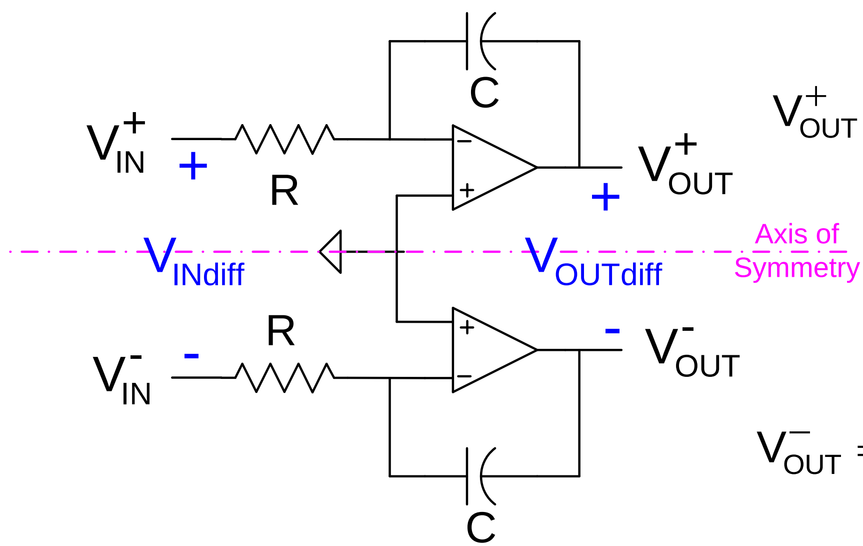
Lossy Inverting Integrator

Example – Basic Op-Amp Feedback Integrator



$$V_{OUT} = -\frac{1}{CRs} V_{IN}$$

Inverting Integrator of Family



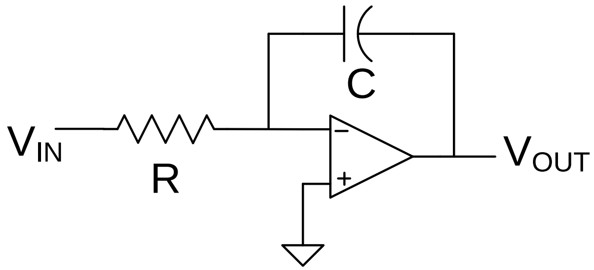
$$V_{OUT}^{+} = -\frac{1}{CRs} V_{IN}^{+}$$

$$V_{OUTdiff} = -\frac{1}{CRs} V_{INdiff}$$

$$V_{OUT}^{-} = -\frac{1}{CRs} V_{IN}^{-}$$

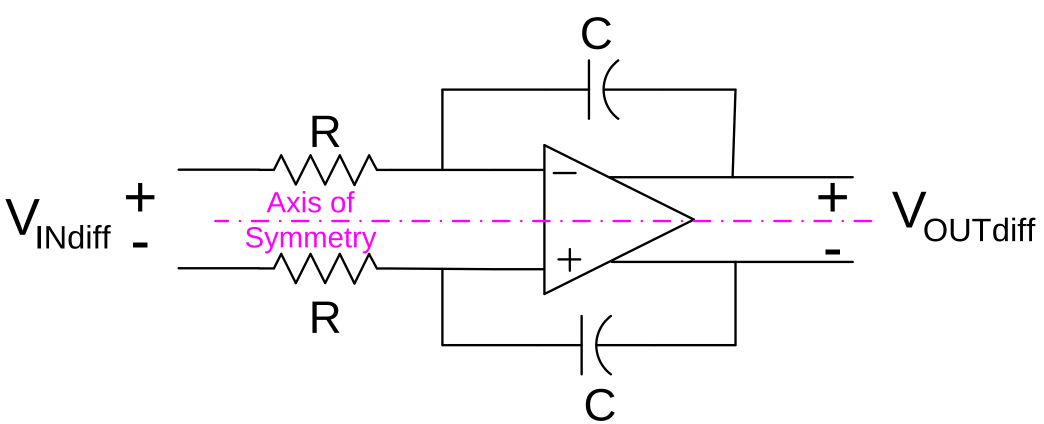
Balanced Differential Inverting Integrator

Example – Basic Op-Amp Feedback Integrator



$$V_{OUT} = -\frac{1}{CRs} V_{IN}$$

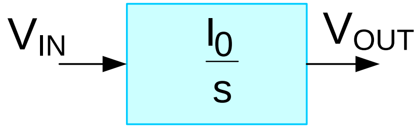
Inverting Integrator of Family



$$V_{OUTdiff} = -\frac{1}{CRs} V_{INdiff}$$

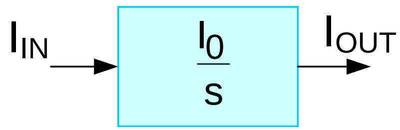
Fully Differential Inverting Integrator

Integrator Types



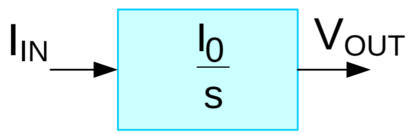
Voltage Mode

$$V_{OUT} = \frac{I_o}{s} V_{IN}$$



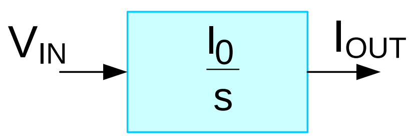
Current Mode

$$I_{OUT} = \frac{I_o}{s} I_{IN}$$



Transresistance Mode

$$V_{OUT} = \frac{I_o}{s} I_{IN}$$



Transconductance Mode

$$I_{OUT} = \frac{I_o}{s} V_{IN}$$

Will consider first the Voltage Mode type of integrators

Voltage Mode Integrators

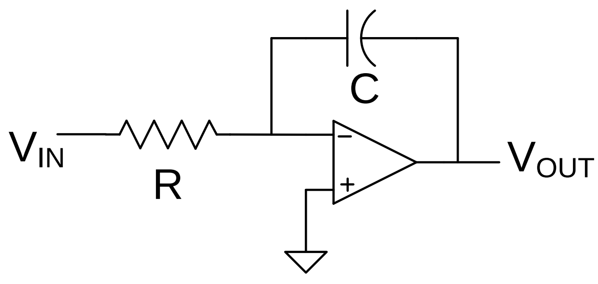
- Active RC (Feedback-based)
- MOSFET-C (Feedback-based)
- OTA-C
- TA-C

Sometimes termed “current mode” (OTA-C, TA-C)

- Switched Capacitor
- Switched Resistor

Will discuss later (Switched Capacitor, Switched Resistor)

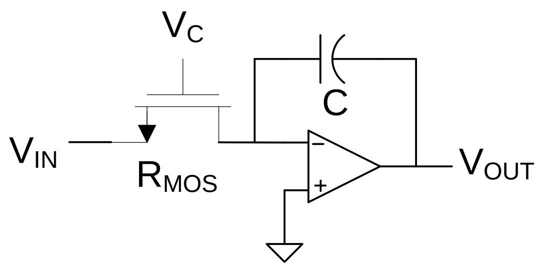
Active RC Voltage Mode Integrator



$$V_{OUT} = -\frac{1}{CRs} V_{IN}$$

- Limited to low frequencies because of Op Amp limitations
- No good resistors for monolithic implementations
    - Area for passive resistors is too large at low frequencies
        - Some recent work by Haibo Fei shows promise for some audio frequency applications
- Capacitor area too large at low frequencies for monolithic implementatins
- Active devices are highly temperature dependent, proc. dependent, and nonlinear
- No practical tuning or trimming scheme for integrated applications with passive resistors

MOSFET-C Voltage Mode Integrator

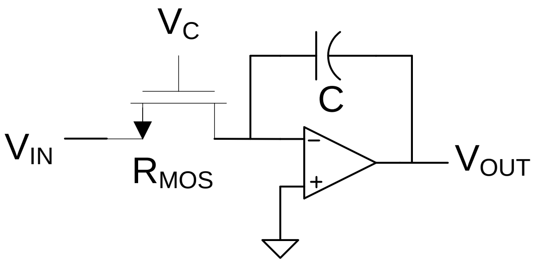


$$V_{OUT} = -\frac{1}{CR_{MOS}s} V_{IN}$$

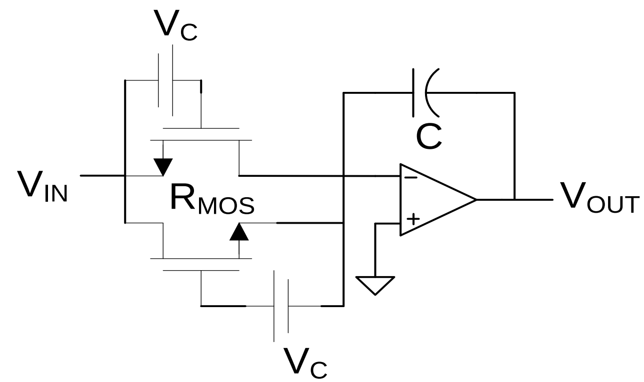
- Limited to low frequencies because of Op Amp limitations
- Area for R_{MOS} is manageable !
- Active devices are highly temperature dependent, process dependent
- Potential for tuning with V_C
- Highly Nonlinear (can be partially compensated with cross-coupled input

A Solution without a Problem

MOSFET-C Voltage Mode Integrator



$$V_{OUT} = -\frac{1}{C\,R_{MOS}s}V_{IN}$$

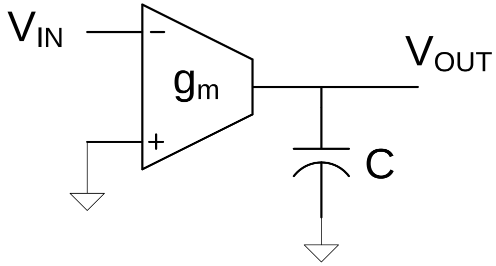


$$V_{OUT} = -\frac{1}{C\,R_{MOS}s}V_{IN}$$

- Improved Linearity
- Some challenges for implementing V_C

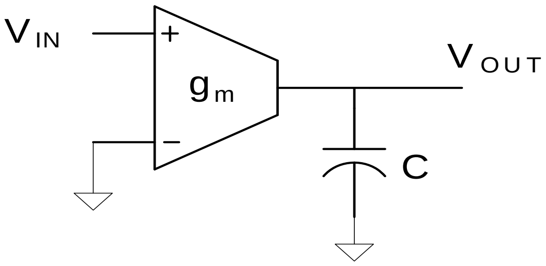
Still A Solution without a Problem

OTA-C Voltage Mode Integrator



$$V_{OUT} = -\frac{g_m}{sC} V_{IN}$$

Inverting



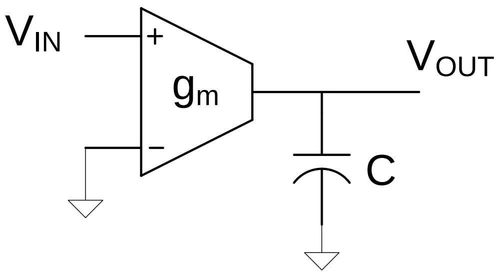
$$V_{OUT} = \frac{g_m}{sC} V_{IN}$$

Noninverting

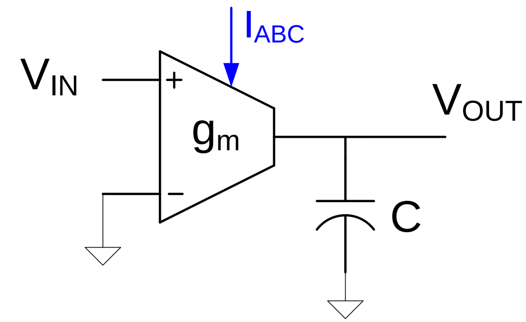
- Requires only two components
- Inverting and Noninverting structures of same complexity
- Good high-frequency performance
- Small area
- Linearity is limited (no feedback in integrator)
- Susceptible to process and temperature variations
- Tuning control can be readily added

Widely used in high frequency applications

OTA-C Voltage Mode Integrator



$$V_{OUT} = \frac{g_m}{sC} V_{IN}$$

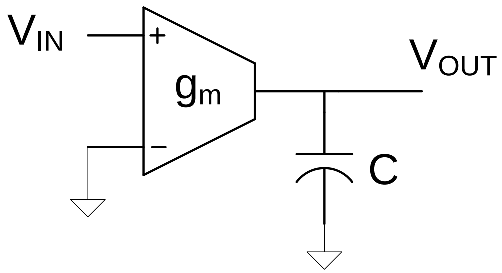


$$V_{OUT} = \frac{g_m}{sC} V_{IN}$$

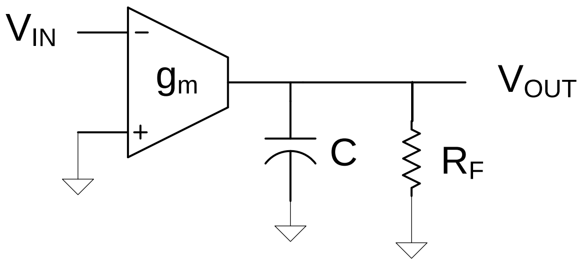
$$g_m = f\left(I_{ABC}\right)$$

Programmable Integrator

OTA-C Voltage Mode Integrator



$$V_{OUT} = \frac{g_m}{sC} V_{IN}$$



$$\frac{V_{OUT}(s)}{V_{IN}(s)} = \frac{g_m R_F}{1+s(R_F C)}$$

Lossy Integrator

But R_F is typically too large for integrated applications

End of Lecture 25

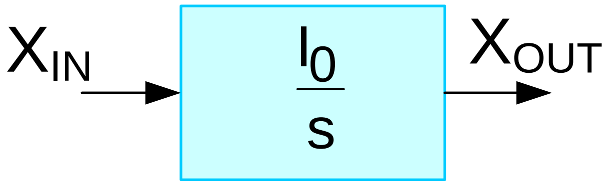
EE 508
Lecture 26

Integrator Design

TA-C Integrators
Other Integrator Structures

Review from last time

Integrator Characteristics of Interest



$$I(s) = \frac{I_0}{s}$$

Properties of an ideal integrator:

$$\left|I(j\omega)\right| = \frac{I_0}{\omega}$$

Gain decreases with 1/ω

$$\angle I(j\omega) = -90^0$$

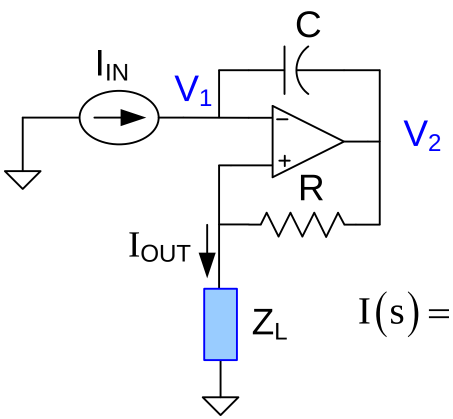
Phase is a constant -90°

$$\left|I(jI_0)\right| = 1$$

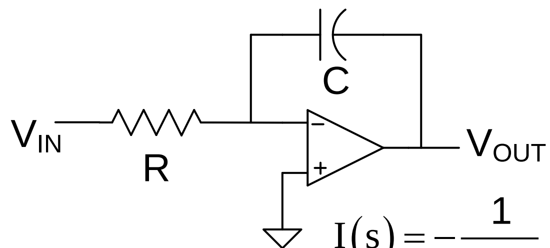
Unity Gain Frequency = 1

How important is it that an integrator have all 3 of these properties?

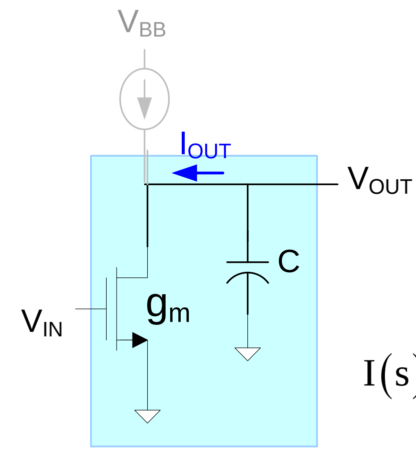
Some integrator structures



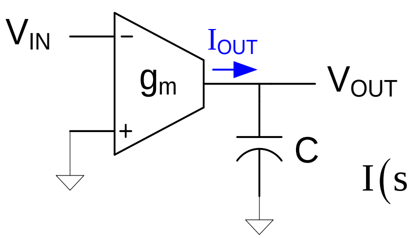
$$I(s) = -\frac{1}{sRC}$$



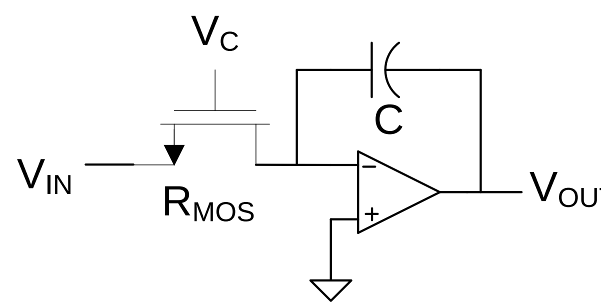
$$I(s) = -\frac{1}{sRC}$$



$$I(s) = -\frac{g_m}{sC}$$



$$I(s) = -\frac{g_m}{sC}$$



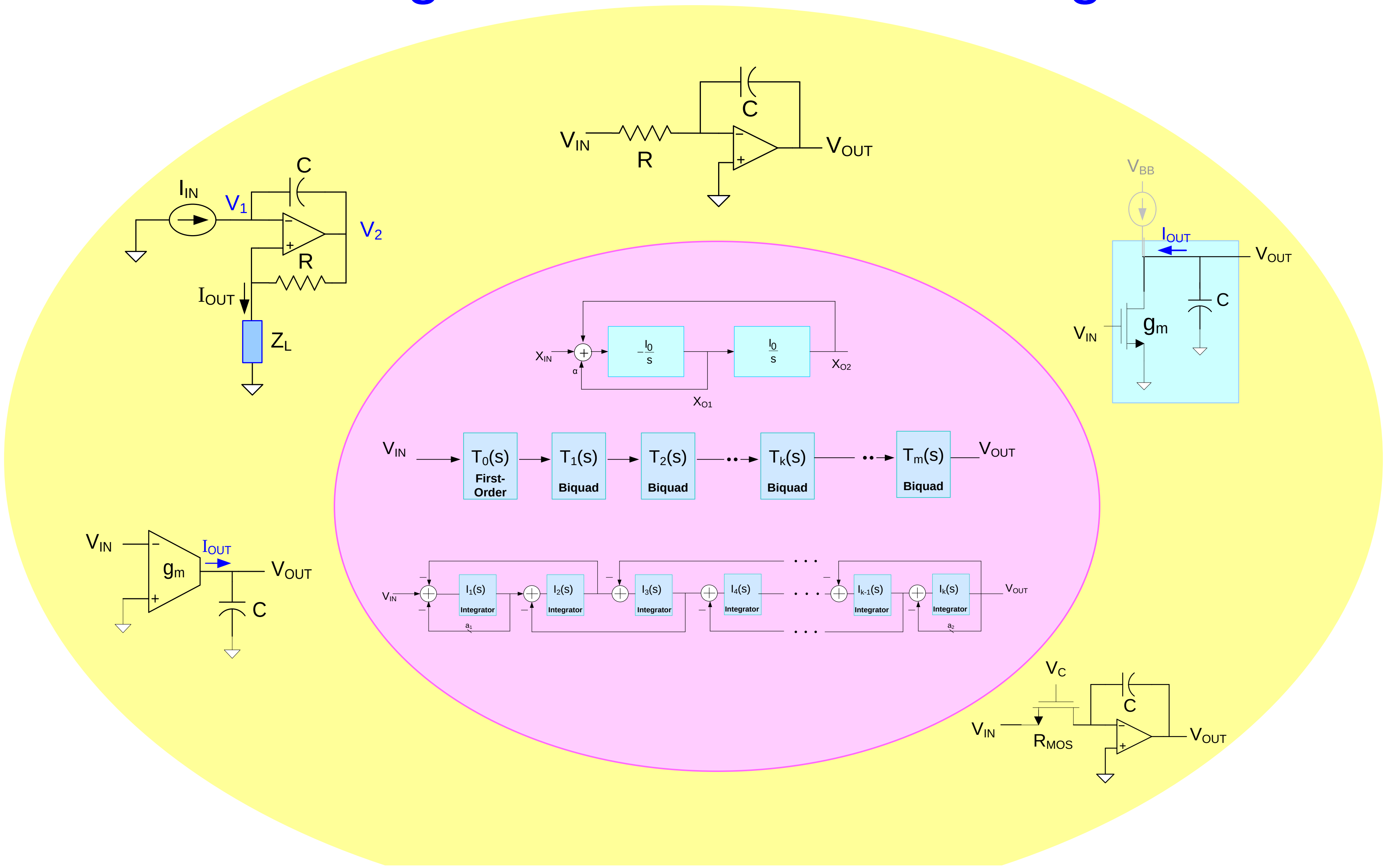
$$I(s) = -\frac{1}{sR_{MOS}C}$$

There are other useful integrator structures (some will be introduced later)

There are many different ways to build an inverting integrator

Review from last time

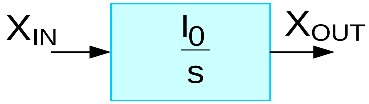
Integrator-Based Filter Design



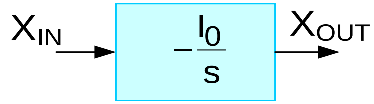
Any of these different types of integrators can be used to build integrator-based filters

Review from last time

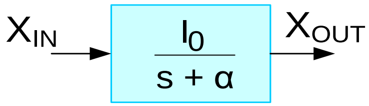
Basic Integrator Functionality



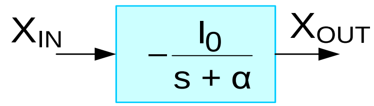
Noninverting



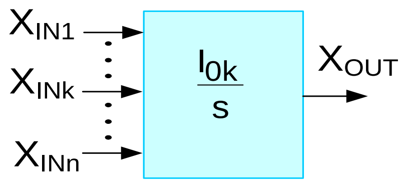
Inverting



Lossy Noninverting

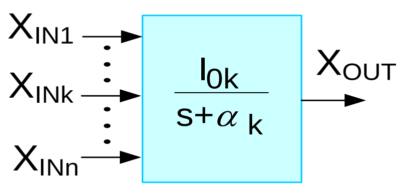


Lossy Inverting



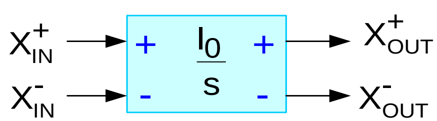
$$X_{OUT} = \sum_{k=1}^{n} \frac{\pm I_{Ok}}{s}$$

Summing (Multiple-Input) Inverting/Noninverting



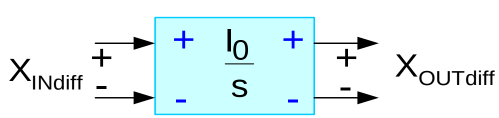
$$X_{OUT} = \sum_{k=1}^{n} \frac{\pm I_{Ok}}{s+\alpha_k}$$

Summing (Multiple-Input) Lossy Inverting/Noninverting



$$X_{OUT}^{+} - X_{OUT}^{+} = \frac{I_0}{s}\left(X_{IN}^{+} - X_{IN}^{+}\right)$$

Balanced Differential

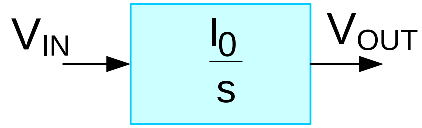


$$X_{OUTdiff} = \frac{I_0}{s} X_{INdiff}$$

Fully Differential

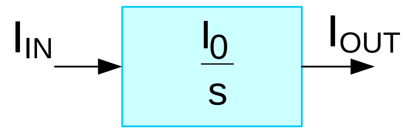
Review from last time

Integrator Types



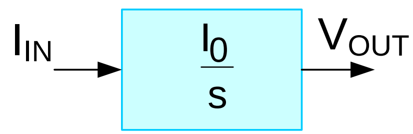
Voltage Mode

$$V_{OUT} = \frac{I_o}{s} V_{IN}$$



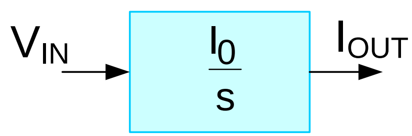
Current Mode

$$I_{OUT} = \frac{I_o}{s} I_{IN}$$



Transresistance Mode

$$V_{OUT} = \frac{I_o}{s} I_{IN}$$



Transconductance Mode

$$I_{OUT} = \frac{I_o}{s} V_{IN}$$

Will consider first the Voltage Mode type of integrators

Voltage Mode Integrators

- Active RC (Feedback-based)
- MOSFET-C (Feedback-based)
- OTA-C
- TA-C

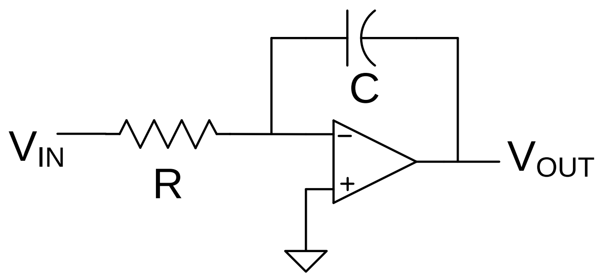
  } Sometimes termed "current mode"

- Switched Capacitor
- Switched Resistor

  } Will discuss later

- Other Structures

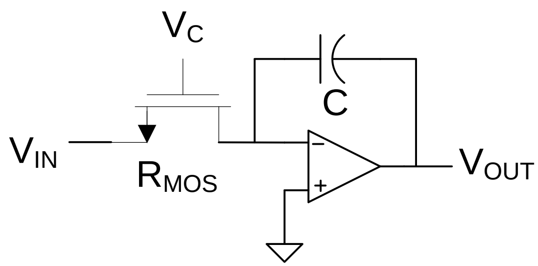
Active RC Voltage Mode Integrator



$$V_{OUT} = -\frac{1}{CRs} V_{IN}$$

- Limited to low frequencies because of Op Amp limitations
- No good resistors for monolithic implementations
  - Area for passive resistors is too large at low frequencies
    - Some recent work by Haibo Fei shows promise for some audio frequency applications
- Capacitor area too large at low frequencies for monolithic implementatins
- Active devices are highly temperature dependent, proc. dependent, and nonlinear
- No practical tuning or trimming scheme for integrated applications with passive resistors

MOSFET-C Voltage Mode Integrator

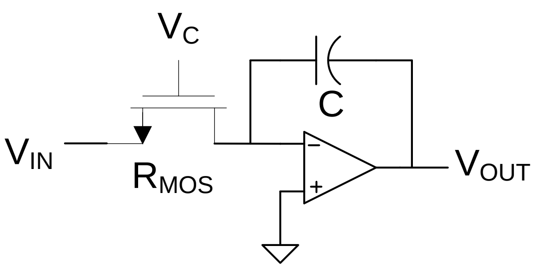


$$V_{OUT} = -\frac{1}{CR_{MOS}s} V_{IN}$$

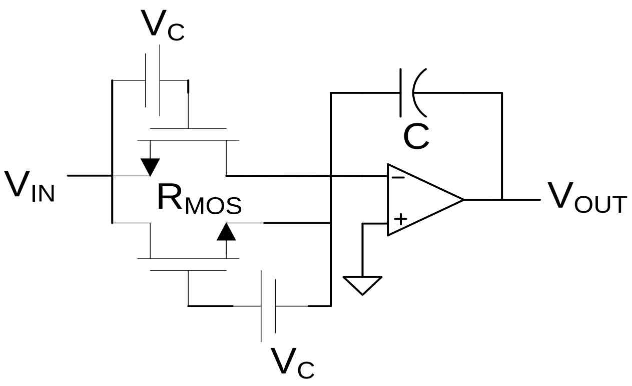
- Limited to low frequencies because of Op Amp limitations
- Area for R_{MOS} is manageable !
- Active devices are highly temperature dependent, process dependent
- Potential for tuning with V_C
- Highly Nonlinear (can be partially compensated with cross-coupled input

A Solution without a Problem

MOSFET-C Voltage Mode Integrator



$$V_{OUT} = -\frac{1}{C\,R_{MOS}s}V_{IN}$$

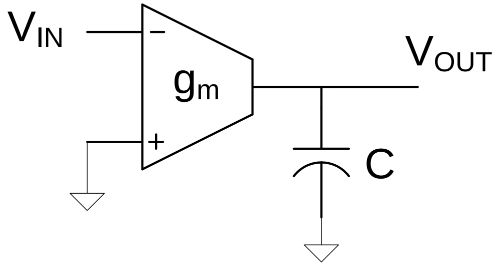


$$V_{OUT} = -\frac{1}{C\,R_{MOS}s}V_{IN}$$

- Improved Linearity
- Some challenges for implementing V_C

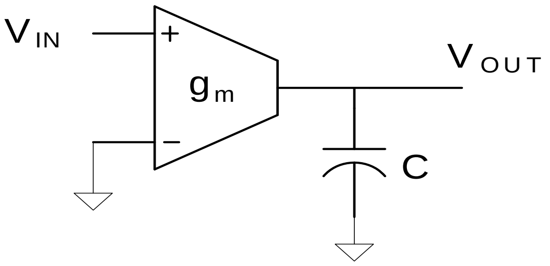
Still A Solution without a Problem

OTA-C Voltage Mode Integrator



$$V_{OUT} = -\frac{g_m}{sC}V_{IN}$$

Inverting



$$V_{OUT} = \frac{g_m}{sC}V_{IN}$$

Noninverting

- Requires only two components
- Inverting and Noninverting structures of same complexity
- Good high-frequency performance
- Small area
- Linearity is limited (no feedback in integrator)
- Susceptible to process and temperature variations
- Tuning control can be readily added

Widely used in high frequency applications

OTA-C Voltage Mode Integrator

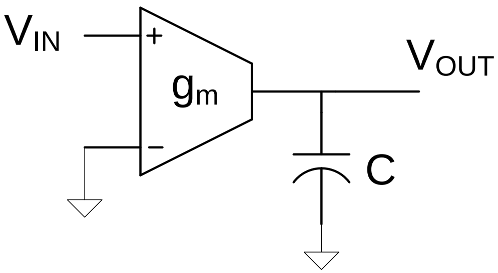
$$V_{OUT} = \frac{g_m}{sC} V_{IN}$$

$$V_{OUT} = \frac{g_m}{sC} V_{IN}$$

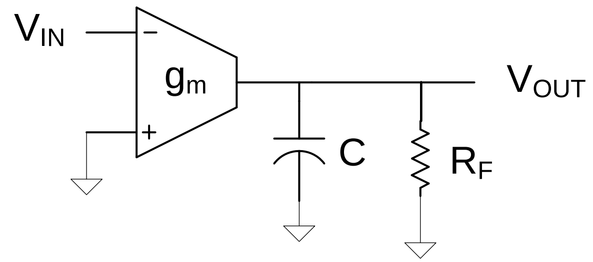
$$g_m = f\left(I_{ABC}\right)$$

Programmable Integrator

OTA-C Voltage Mode Integrator



$$V_{OUT} = \frac{g_m}{sC} V_{IN}$$

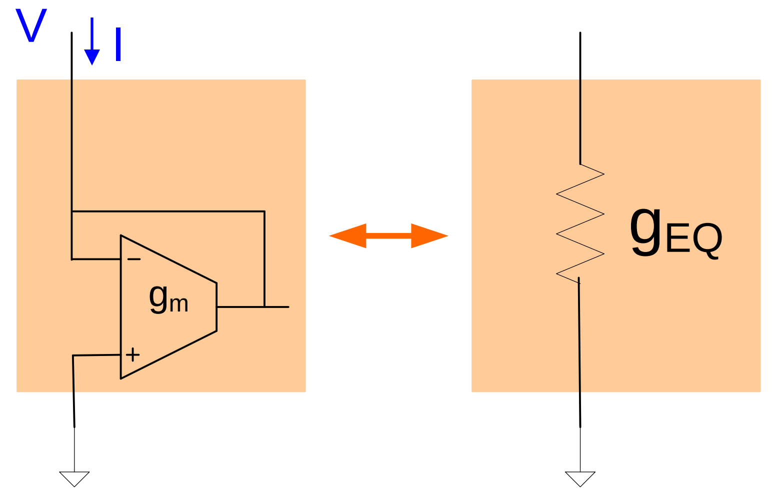


$$\frac{V_{OUT}(s)}{V_{IN}(s)} = \frac{g_m R_F}{1+s(R_F C)}$$

Lossy Integrator

But R_F is typically too large for integrated applications

OTA-C Voltage Mode Integrator



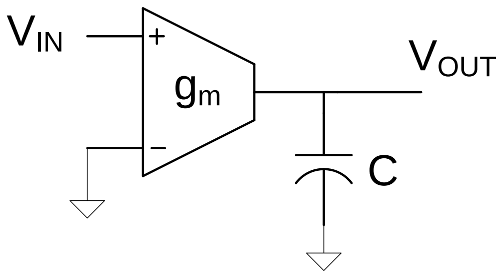
$$I = - -g_m V$$

$$g_{EQ} = \frac{I}{V}$$

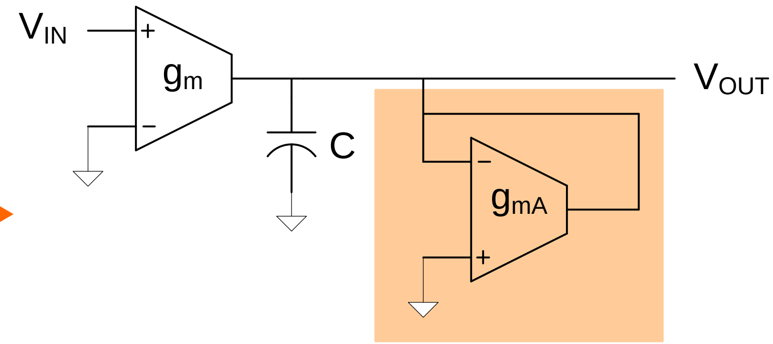
$$g_{EQ} = g_m$$

OTA is generally much smaller than a resistor

OTA-C Voltage Mode Integrator



$$V_{OUT} = \frac{g_m}{sC} V_{IN}$$

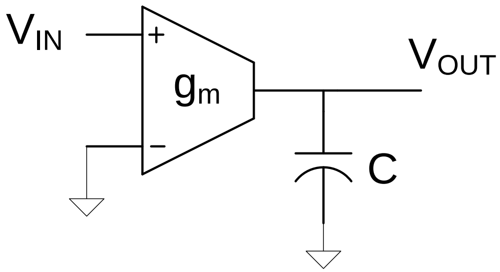


$$\frac{V_{OUT}(s)}{V_{IN}(s)} = \frac{g_m/g_{mA}}{1+s(C/g_{mA})}$$

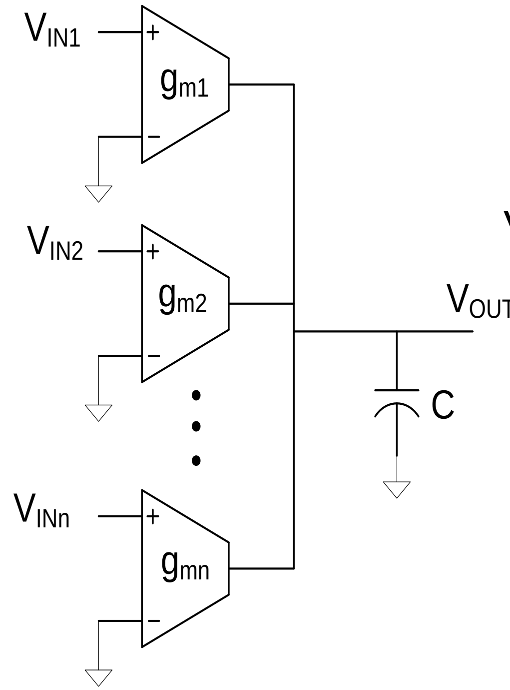
Lossy Integrator

- Practical implementation
- Both OTAs can be readily programmable

OTA-C Voltage Mode Integrator



$$V_{OUT} = \frac{g_m}{sC} V_{IN}$$

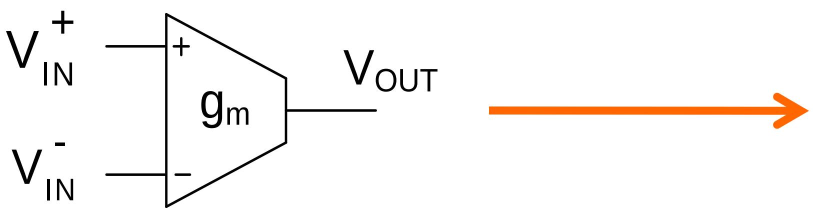


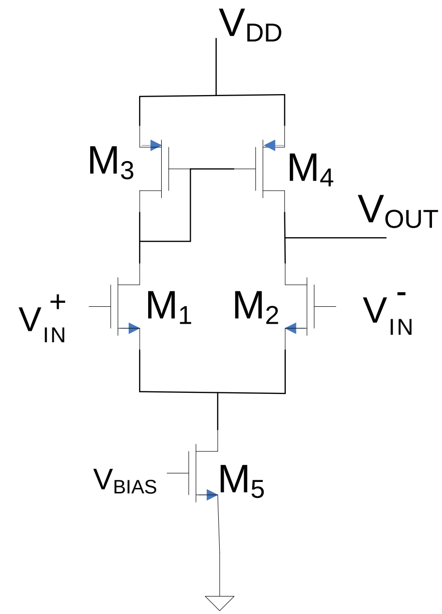
$$V_{OUT} = \frac{\sum_{k=1}^{n} g_{mk} V_{INk}}{sC}$$

Summing Integrator

- Inverting and noninverting functions can be combined in single summer
- All transconductance gains can be programmable

OTA Architecture





Mid-complexity OTA

- M_1 and M_2 matched
- M_2 and M_4 matched
- Define M to be the gain of the current mirror formed with M_2 and M_4
- g_m programmable with V_{BIAS}

$$g_m = \frac{g_{m1}}{2}(1+M)$$

Often M=1

$$g_m = g_{m1}$$

Other OTAs exist, considerable effort expended over past two decades on OTA design

Voltage Mode Integrators

- Active RC (Feedback-based)
- MOSFET-C (Feedback-based)
- OTA-C
- TA-C

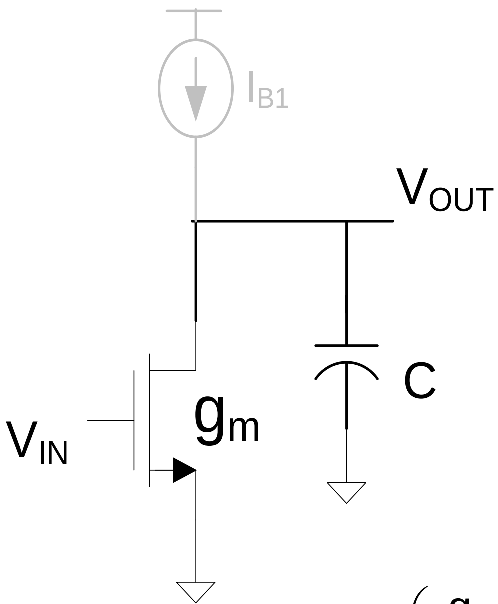
Sometimes termed "current mode" (OTA-C, TA-C)

- Switched Capacitor
- Switched Resistor

Will discuss later (Switched Capacitor, Switched Resistor)

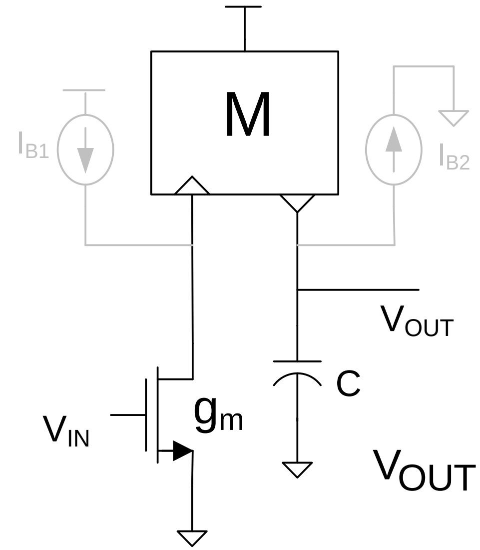
- Other Structures

TA-C Voltage Mode Integrator



$$V_{OUT} = \left(\frac{-g_m}{sC}\right) V_{IN}$$

Inverting Integrator



$$V_{OUT} = \left(\frac{g_m M}{sC}\right) V_{IN}$$

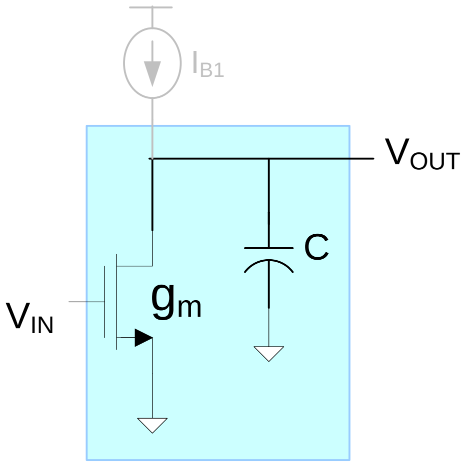
Typically M=1

Noninverting Integrator

- Can operate at very high frequencies
- Low device count circuit
- Simplicity is important for operating at very high frequencies
- I_0 is process and temperature dependent
- Linearity is limited

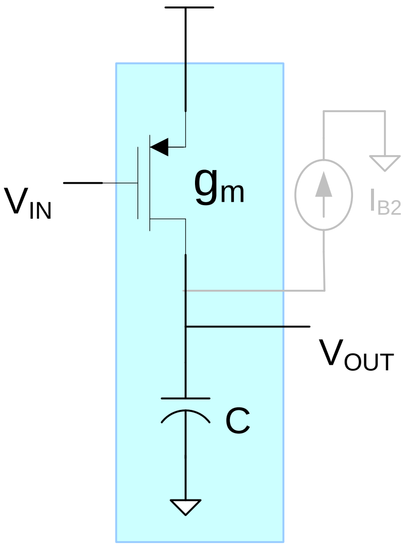
TA-C Voltage Mode Integrator

Some other perspectives



n-channel input

$$V_{OUT} = \left(\frac{-g_m}{sC}\right) V_{IN}$$

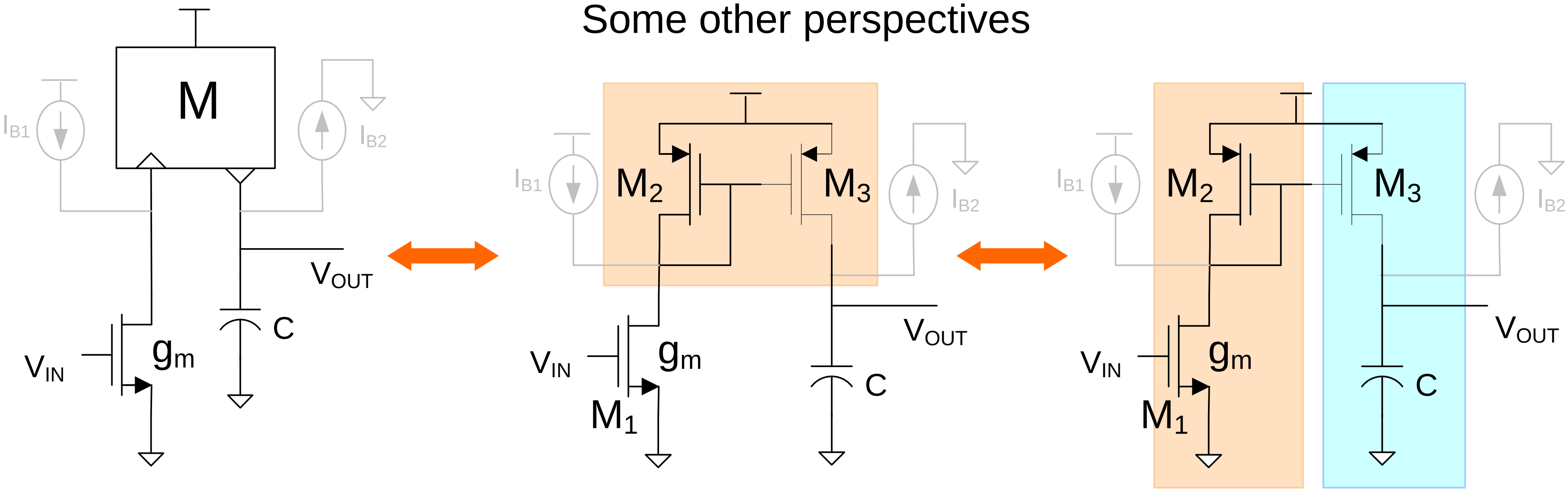


p-channel input

Inverting Integrators

TA-C Voltage Mode Integrator

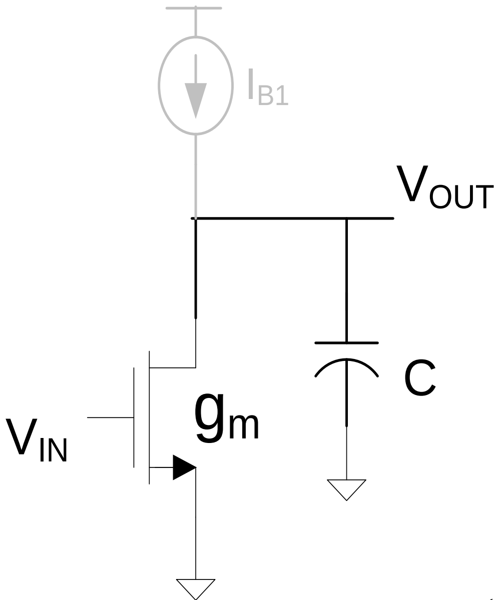
Some other perspectives



Noninverting Integrator

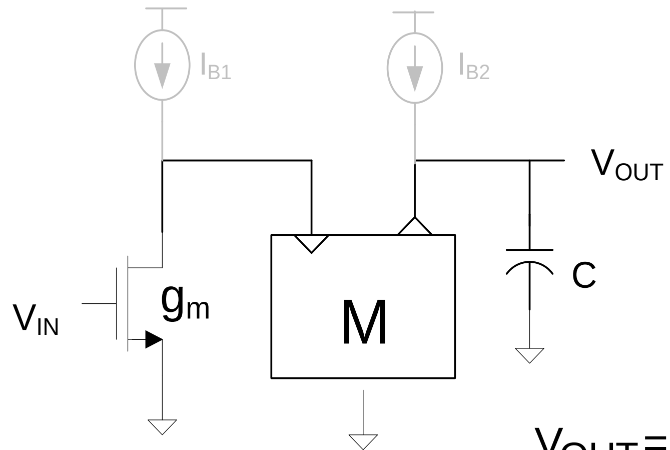
Can be viewed either as n-channel input with current mirror or as low-gain inverter driving a p-channel input inverting integrator

TA-C Voltage Mode Integrator



$$V_{OUT} = \left(\frac{-g_m}{sC}\right) V_{IN}$$

Inverting Integrator

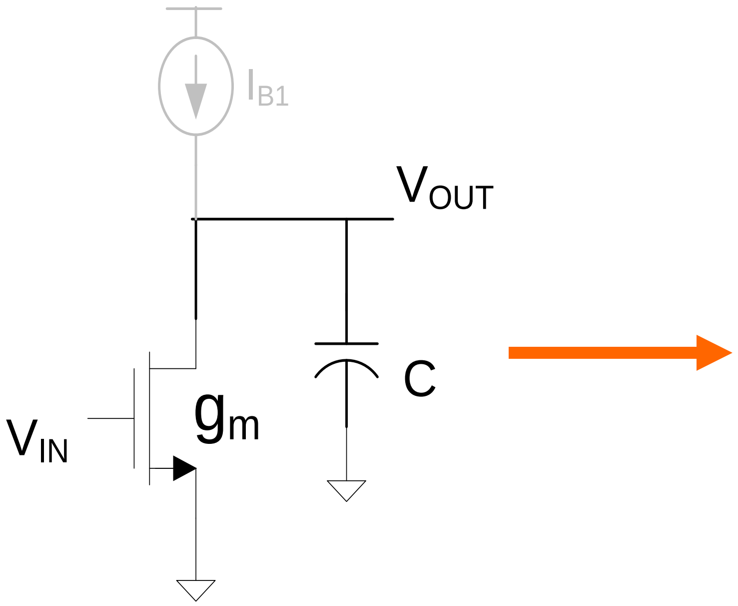


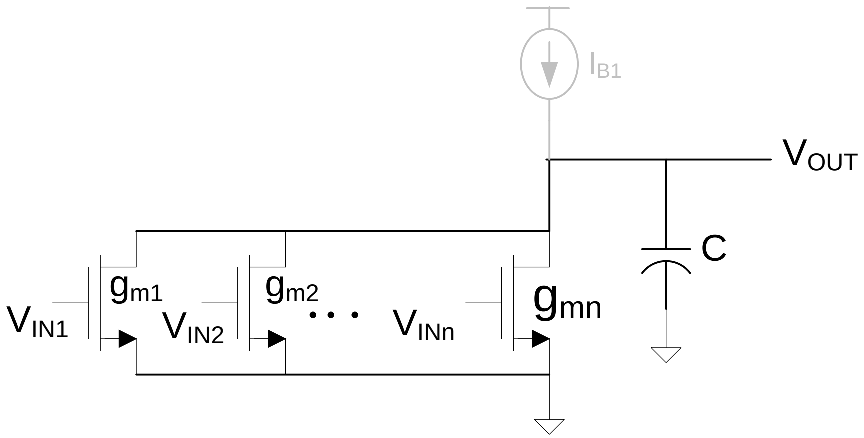
$$V_{OUT} = \left(\frac{g_m M}{sC}\right) V_{IN}$$

Typically M=1

Alternate noninverting Integrator

TA-C Voltage Mode Integrator





Summing Inverting Integrator

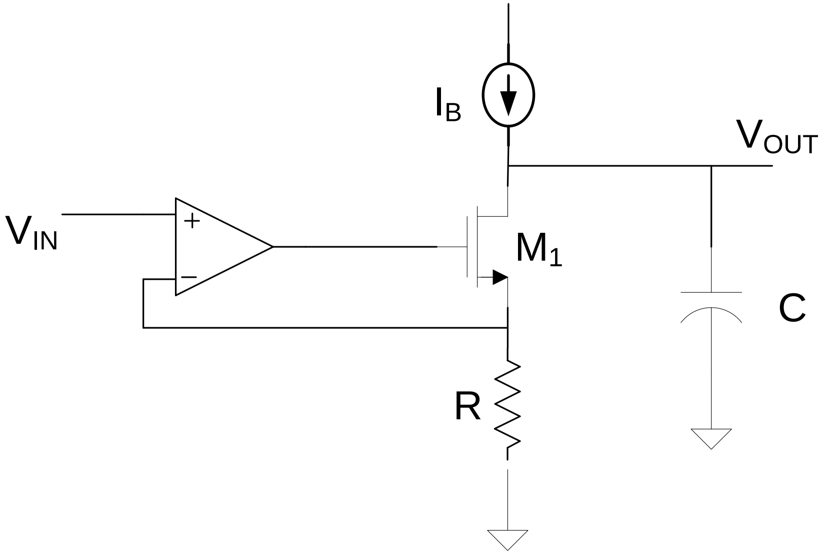
Voltage Mode Integrators

- Active RC (Feedback-based)
- MOSFET-C (Feedback-based)
- OTA-C
- TA-C

  } Sometimes termed "current mode"
- Switched Capacitor
- Switched Resistor

  } Will discuss later
- Other Structures

Another Voltage Mode Integrator

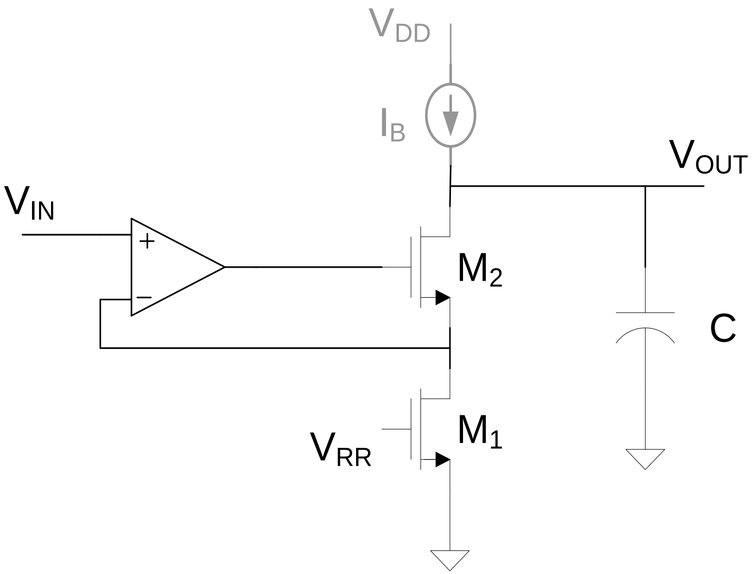


Inverting Integrator

$$V_{OUT} = \left(\frac{-1}{sRC}\right) V_{IN}$$

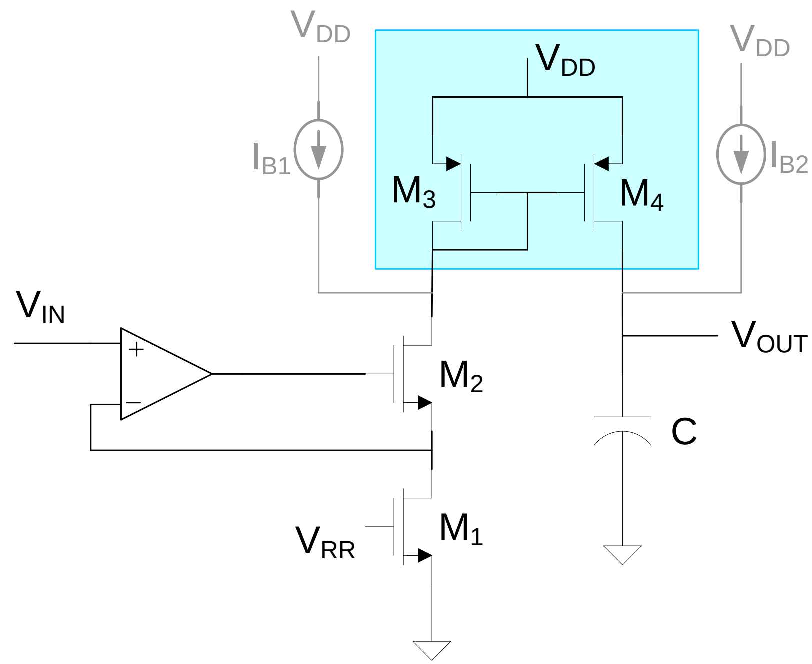
- **Infinite input impedance (in contrast to basic Active RC Integrator)**
- **Both R and C have one terminal grounded**
- **Requires integrated process**
- **Accuracy limited by process and temperature**
- **Size limitations same as basic Active RC Integrator**
- **Limited to lower frequencies because of Op Amp**
- **Good linearity**

Another Voltage Mode Integrator



Inverting Integrator

$$V_{OUT} = \left(\frac{-1}{sR_{FET}C} \right) V_{IN}$$

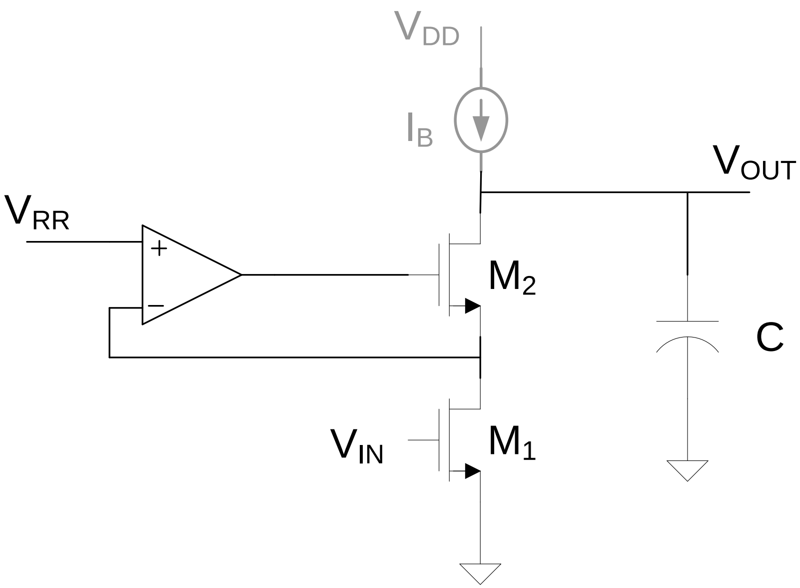


Noninverting Integrator

$$V_{OUT} = \left(\frac{1}{sR_{FET}C} \right) V_{IN}$$

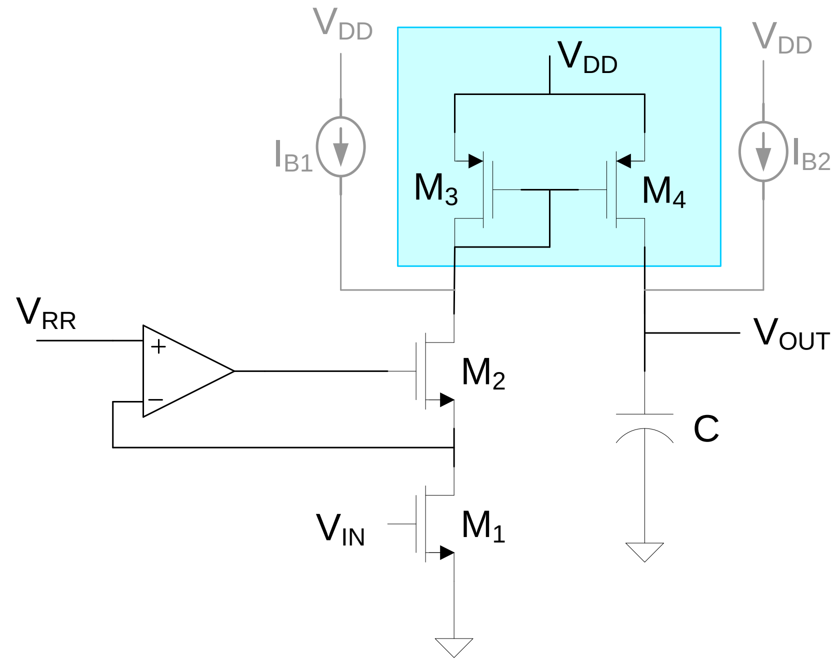
- M_1 in triode region
- Reduces Area Concerns but Loss of Linearity
- I_0 is programmable with V_{RR}
- Accurate control of I_B critical

Regulated Cascode Voltage Mode Integrator



Inverting Integrator

$$V_{OUT} = \left(\frac{-g_{mT}}{sC} \right) V_{IN}$$



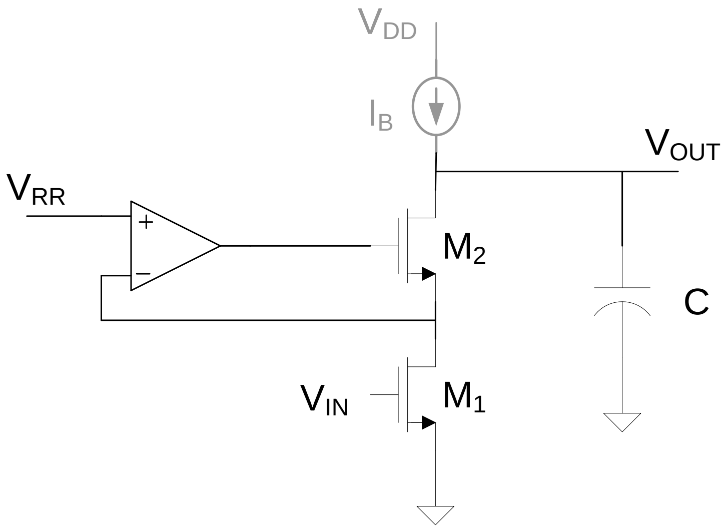
Noninverting Integrator

$$V_{OUT} = \left(\frac{g_{mT}}{sC} \right) V_{IN}$$

g_{MT} is triode region transconductance of M_1

- **M_1 operating in triode region**
- **R_{FET} programmable with V_{RR}**
- **Very good linearity properties**
- **Input impedance still infinite**

Regulated Cascode Voltage Mode Integrator



$$V_{OUT} = \left(\frac{-g_{mT}}{sC} \right) V_{IN}$$

Linearity Properties:

Assuming square-law triode model

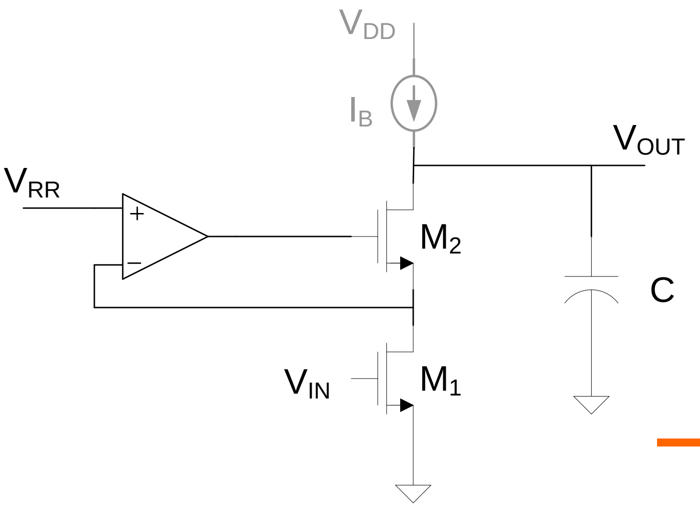
$$I_{D1} = \frac{\mu C_{OX} W}{L} \left(V_{GS} - V_T - \frac{V_{RR}}{2} \right) V_{RR}$$

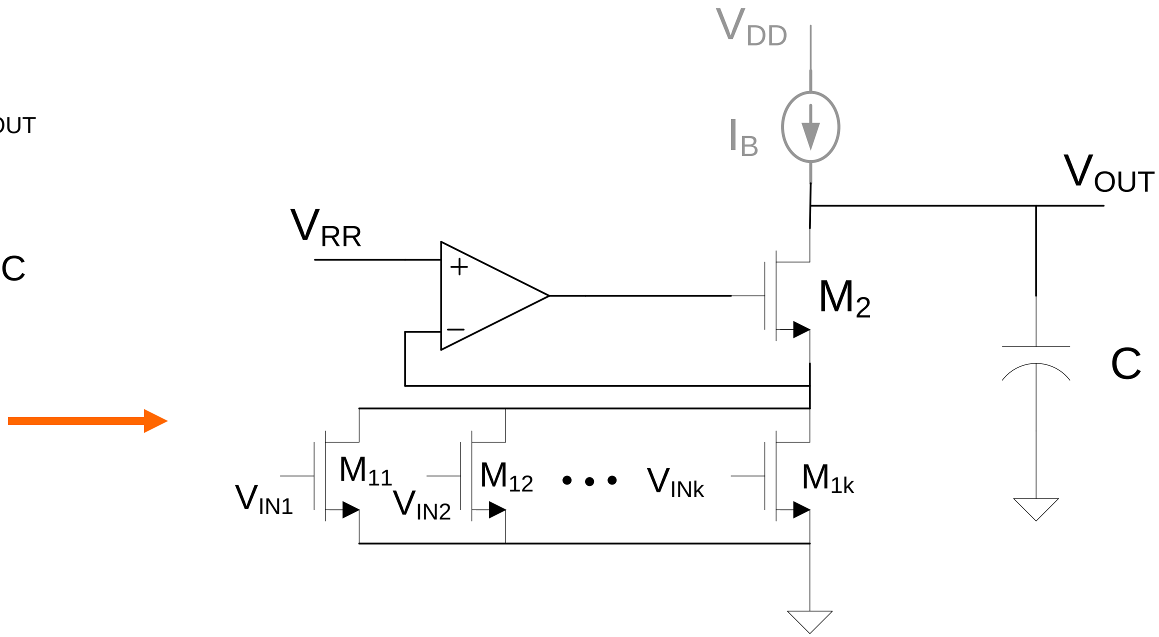
$$I_{D1} = \left[\frac{\mu C_{OX} W}{L} V_{RR} \right] V_{IN} + \left[\frac{\mu C_{OX} W}{L} \left(V_T + \frac{V_{RR}}{2} \right) V_{RR} \right]$$

Note linear dependence on V_{IN}

$$g_{mT} = \left[\frac{L}{\mu C_{OX} W \, V_{RR}} \right]$$

Regulated Cascode Voltage Mode Integrator



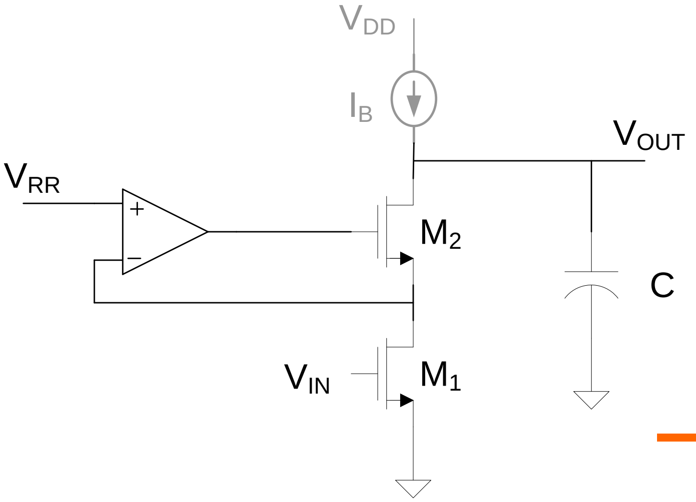


Inverting Integrator

$$V_{OUT} = \left(\frac{-1}{sR_{FET}C} \right) V_{IN}$$

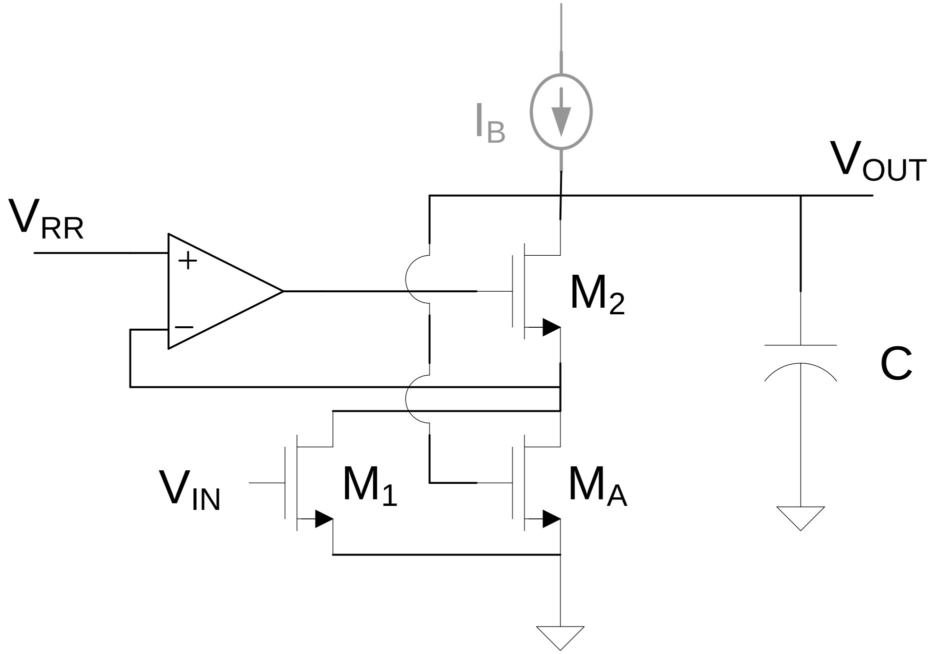
- **Multiple inputs require single additional transistor**
- **Accurate ratioing of gains practical**
- **Can also sum currents on C**

Regulated Cascode Voltage Mode Integrator



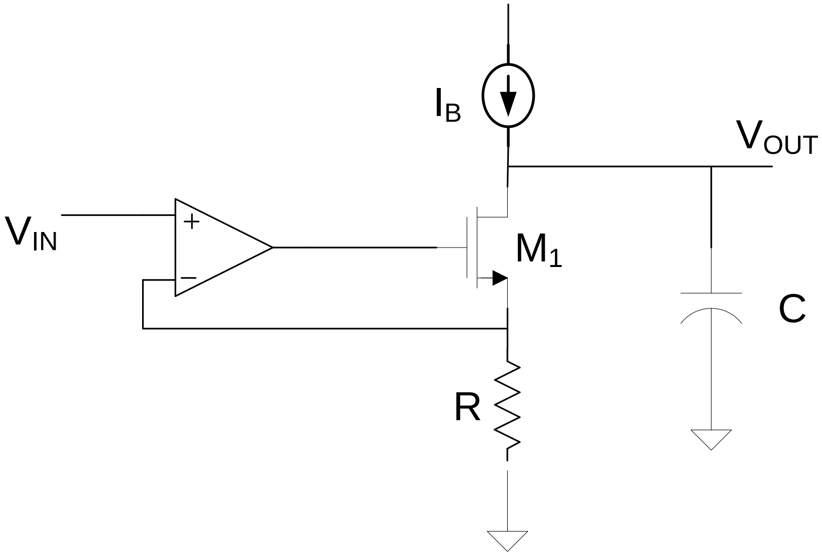
Inverting Integrator

$$V_{OUT} = \left(\frac{-1}{sR_{FET}C}\right)V_{IN}$$



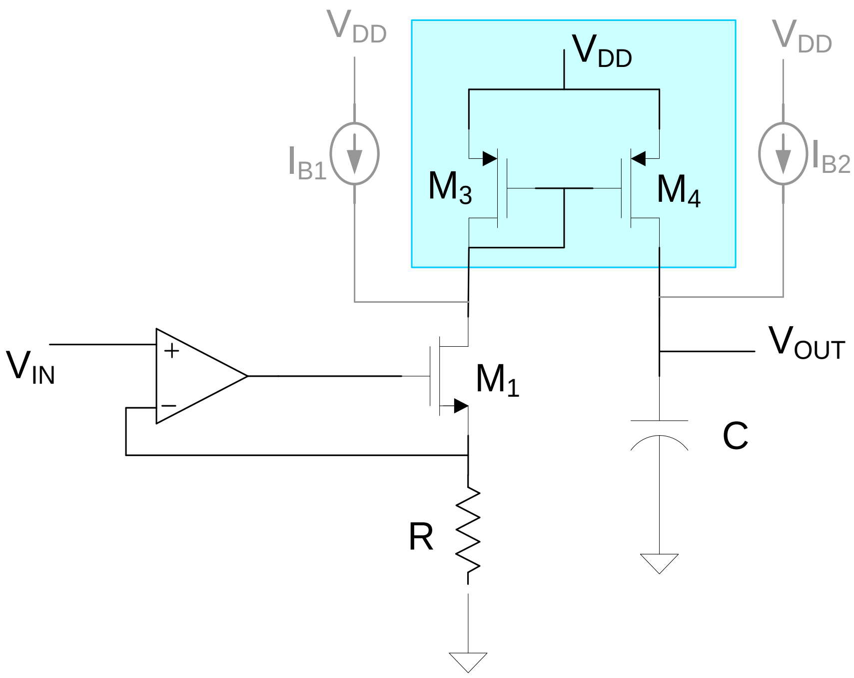
Inverting Lossy Integrator

Another Voltage Mode Integrator



Inverting Integrator

$$V_{OUT} = \left(\frac{-1}{sRC}\right) V_{IN}$$



Noninverting Integrator

$$V_{OUT} = \left(\frac{1}{sRC}\right) V_{IN}$$

Voltage Mode Integrators

- Active RC (Feedback-based)
- MOSFET-C (Feedback-based)
- OTA-C
- TA-C

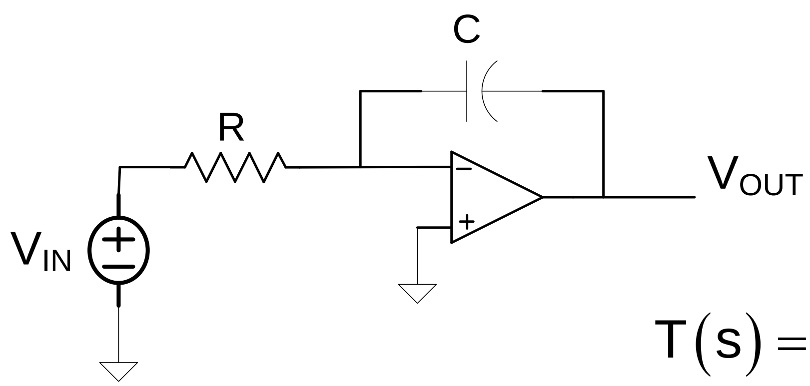
Sometimes termed "current mode"

- Switched Capacitor
- Switched Resistor

Will discuss later

- Other Structures

Consider the Basic Integrator



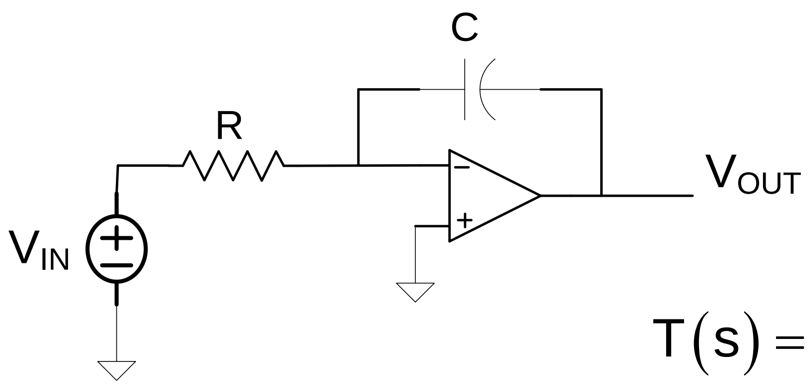
$$T(s) = -\frac{1}{RCs}$$

$$I_0 = \frac{1}{RC}$$

Key performance of integrator (and integrator-based filter) is determined by the integrator time constant I_0

Precision of time constants of a filter invariably determined by precision of I_0

Consider the Basic Integrator



$$T(s) = -\frac{1}{RCs}$$

$$I_0 = \frac{1}{RC}$$

1. Accuracy of R and C difficult to accurately control – particularly in integrated applications (often 2 or 3 orders of magnitude to variable)

2. Size of R and C unacceptably large if I_0 is in audio frequency range (2 or 3 orders of magnitude too large)

3. Amplifier GB limits performance

Incredible Challenge to Building Filters on Silicon!

Challenges for Integration of Active Filters

- Passive Component Variability
- Passive Component Size
- Op Amp Limitations

Historical Perspective

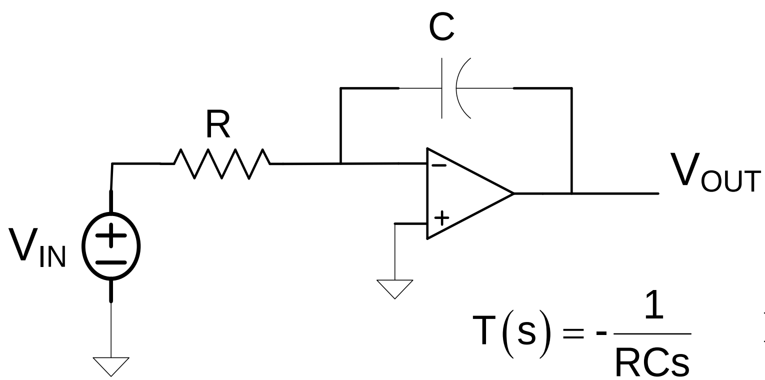
Filters were widely viewed as one of the most fundamental applications of integrated circuit technology

Considerable effort was expended on developing methods to build integrated filters but these three issues were viewed for years as a fundamental roadblock

Practical solution required finding SIMULTANEOUS solutions to three problems which were each 2 or 3 orders of magnitude problematic

This problem was not solved from the invention of the integrated circuit in 1959 up until the late 1970s

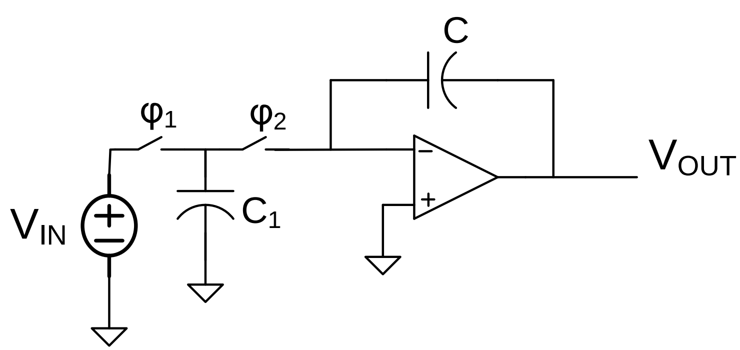
Switched-Capacitor Circuits

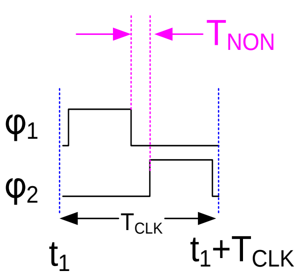


$$T(s) = -\frac{1}{RCs} \qquad I_0 = \frac{1}{RC}$$

Consider:

$$V_{IN} = V_M \sin(2\pi f_{SIG} t + \theta)$$





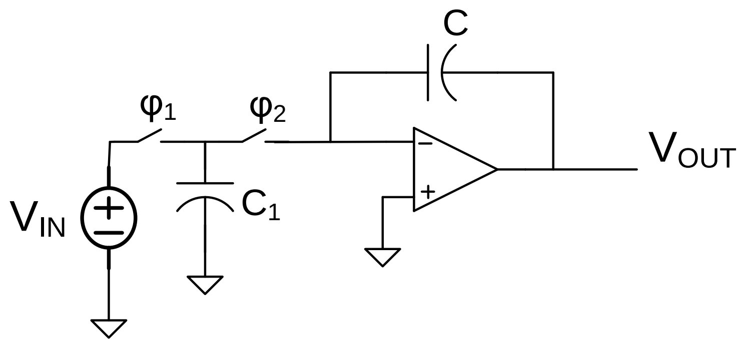
Φ_1 and Φ_2 are periodic signals
"clocks" shown for one period

Assume $T_{CLK} << T_{SIG}$

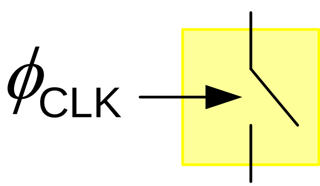
Φ_1 and Φ_2 are complimentary non-overlapping clocks

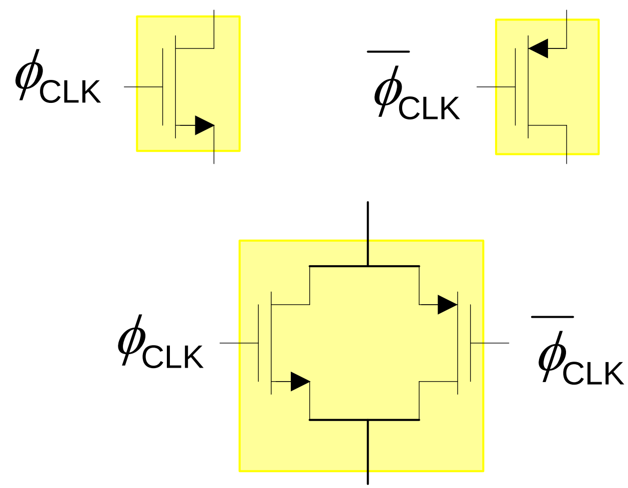
Termed a Switched-Capacitor circuit

Switched-Capacitor Circuits



How are the switches made?

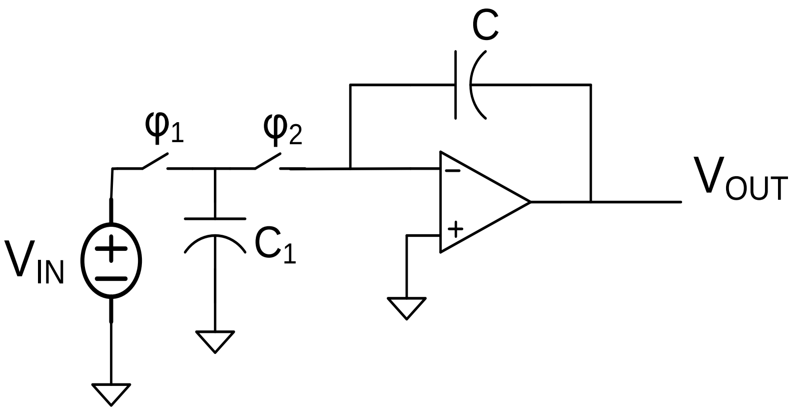




- Often single transistor
- Occasionally complimentary transistors
- On rare occasion more complicated
- Area overhead for switches small, clock routing a little more of concern
- Sizing of devices is important
- Clocking of switches may be important

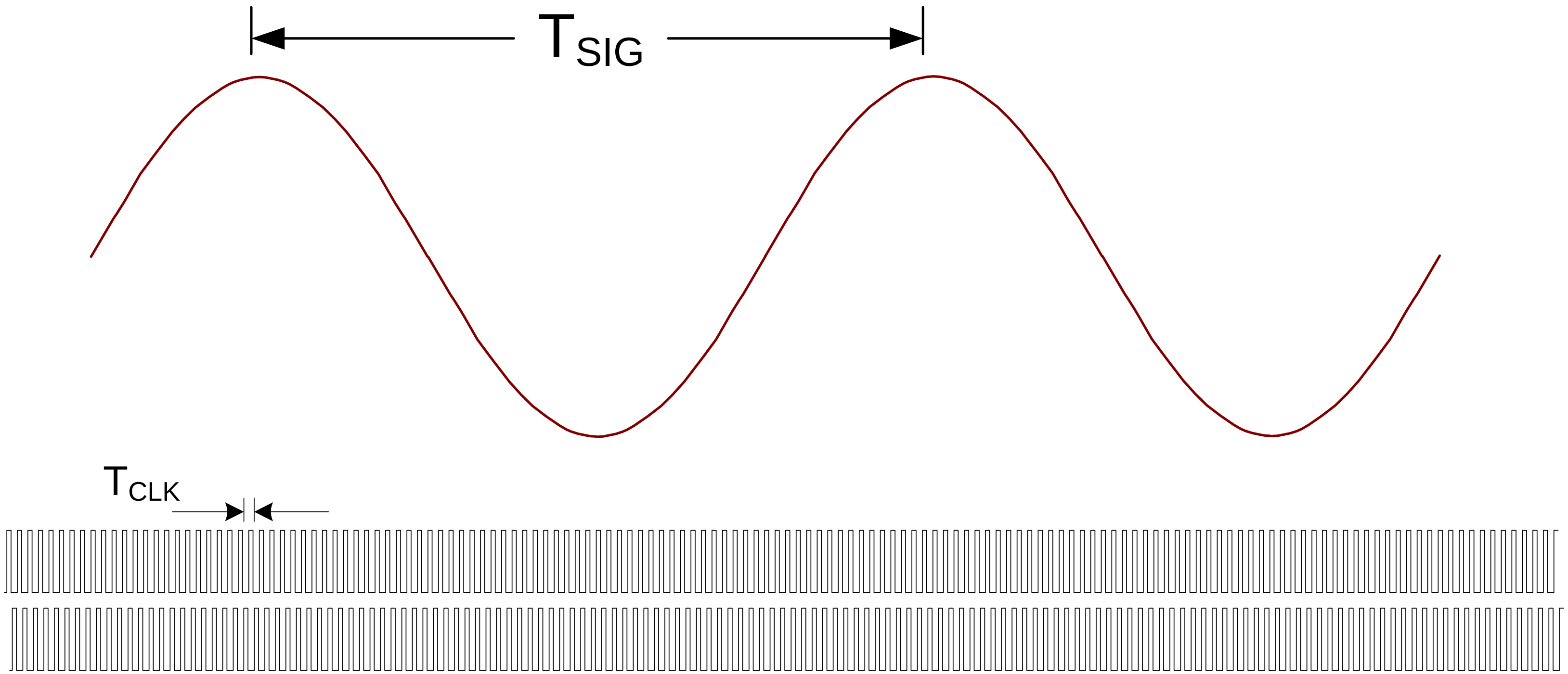
Although originating in SC filters, switched charge redistribution circuits widely used in other non-filtering applications

Consider the Switched-Capacitor Circuit



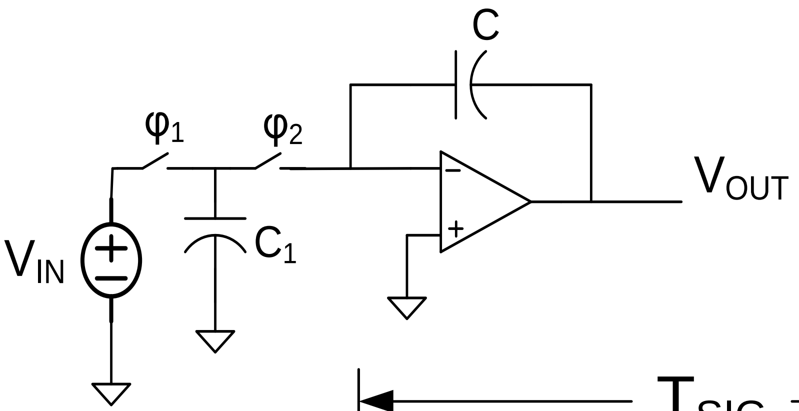
Assume $T_{CLK} << T_{SIG}$

Φ_1 and Φ_2 are complimenary nonoverlapping clocks



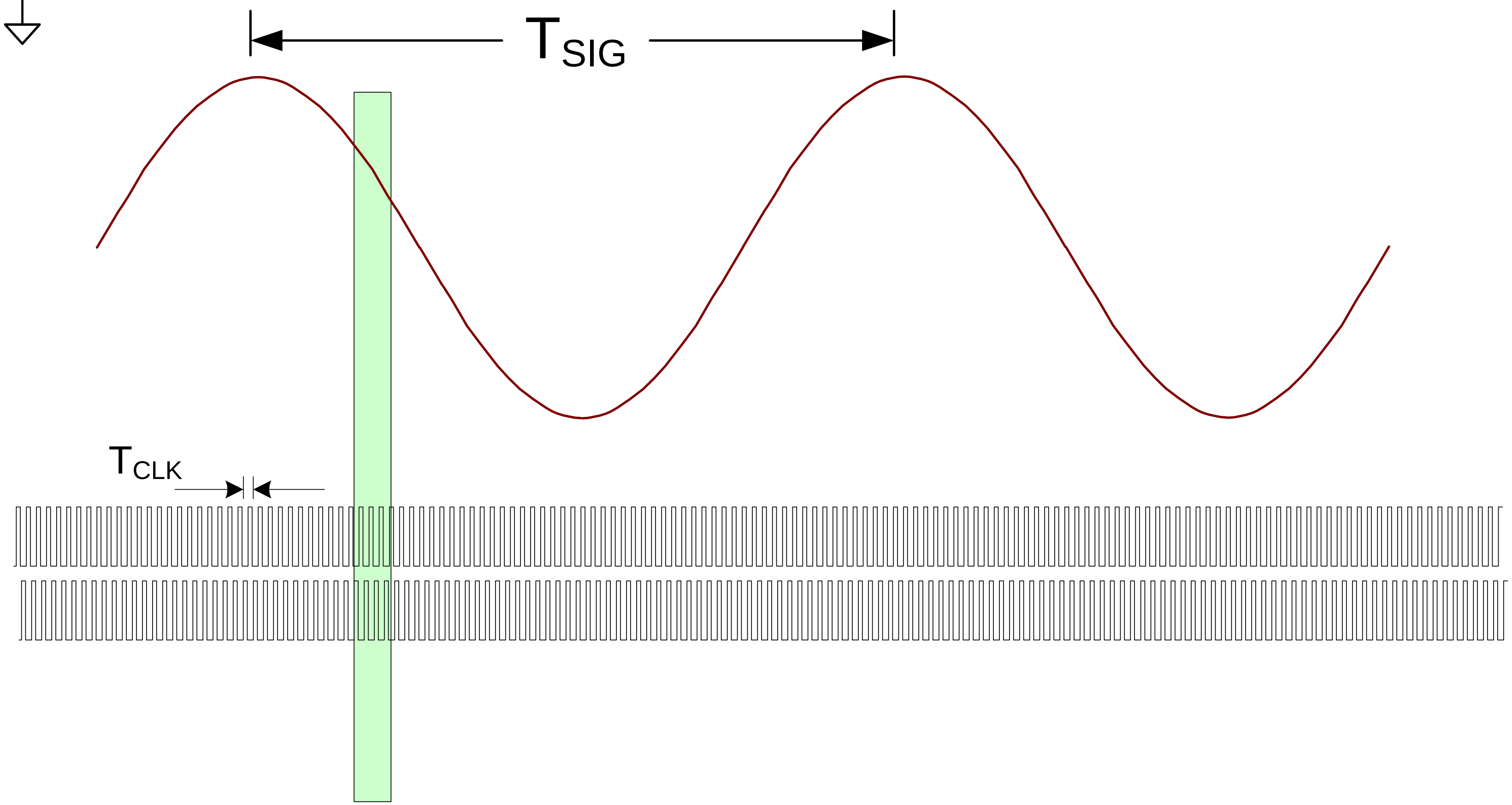
Lets now zoom in on the clock period

Consider the Switched-Capacitor Circuit

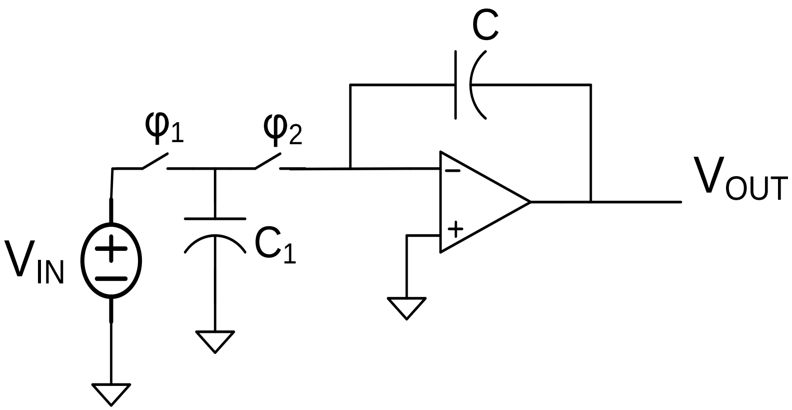


Assume $T_{CLK} << T_{SIG}$

Φ_1 and Φ_2 are complimenary nonoverlapping clocks

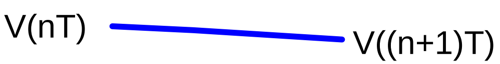


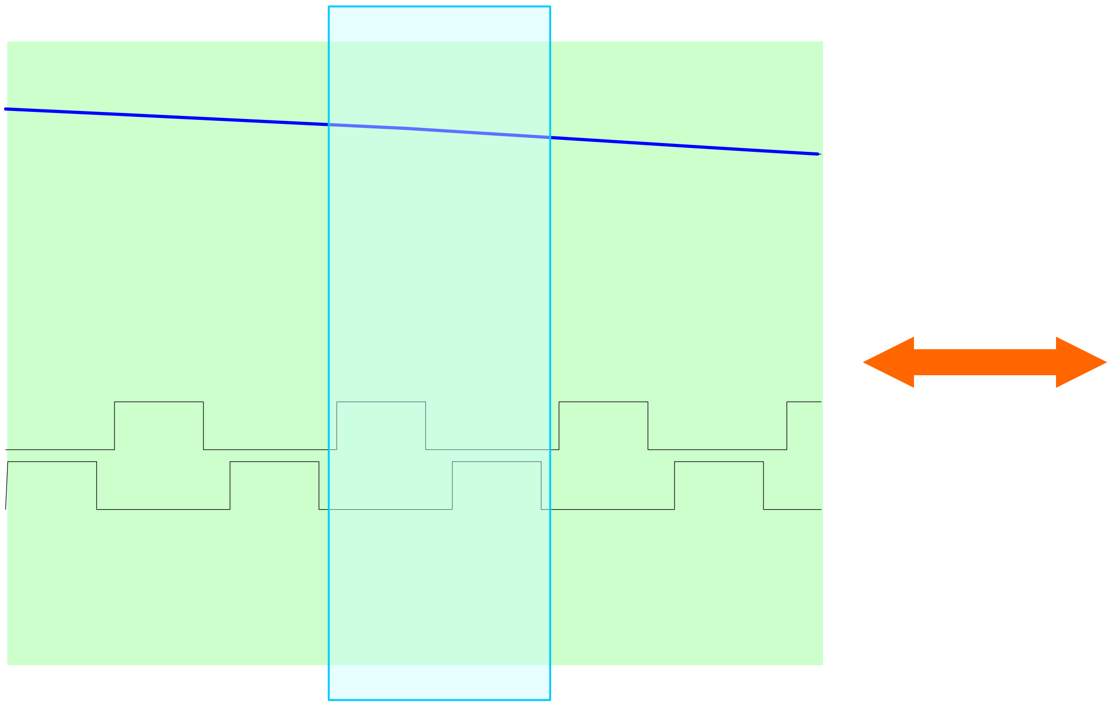
Consider the Switched-Capacitor Circuit

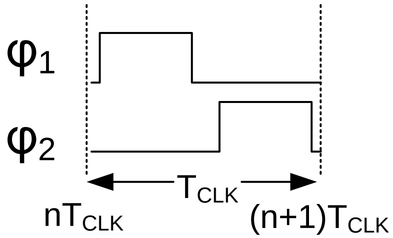


Assume $T_{CLK} << T_{SIG}$

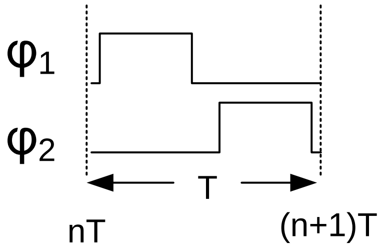
Φ_1 and Φ_2 are complimenary nonoverlapping clocks



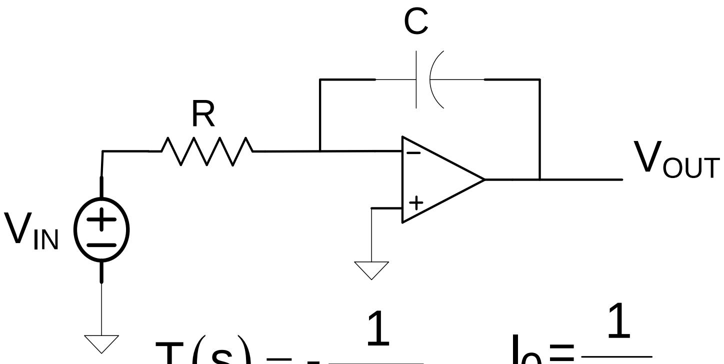




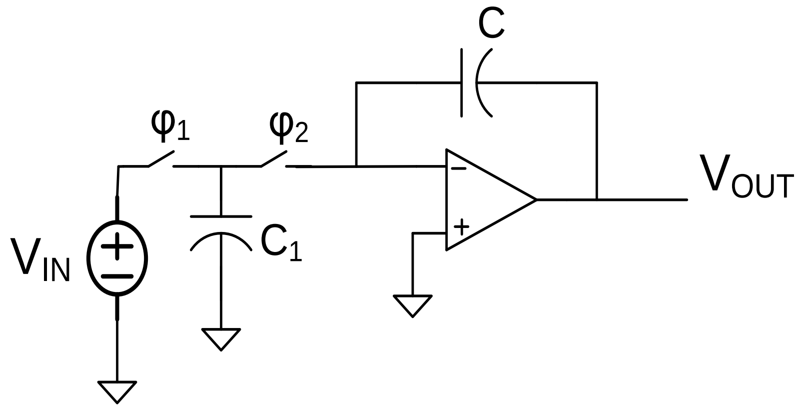
Define $T=T_{CLK}$



Compare the performance of the following two circuits

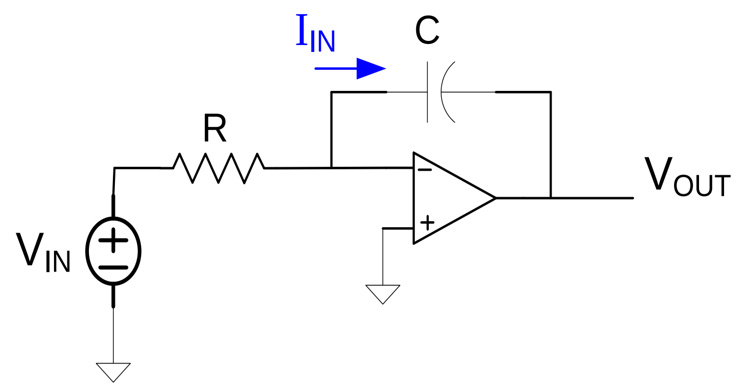


$$T(s) = -\frac{1}{RCs} \qquad I_0 = \frac{1}{RC}$$



Consider the charge transferred to the feedback capacitor for both circuits in an interval of length T_{CLK} at time t_1

For the RC circuit:



$$Q_{RC} = \int_{t_1}^{t_1+T_{CLK}} I_{in}(t)\,dt$$

$$Q_{RC} = \int_{t_1}^{t_1+T_{CLK}} \frac{V_{in}(t)}{R}\,dt$$

Since V_{in} changes slowly

$$Q_{RC} \simeq \int_{t_1}^{t_1+T_{CLK}} \frac{V_{in}(t_1)}{R}\,dt$$

$$Q_{RC} \simeq \left[\frac{V_{in}(t_1)}{R}\right] \int_{t_1}^{t_1+T_{CLK}} 1\,dt$$

$$Q_{RC} \simeq \left[\frac{V_{in}(t_1)}{R}\right] T_{CLK}$$

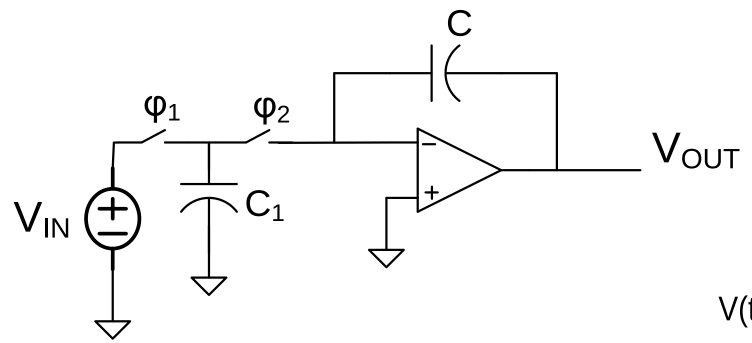
Consider the charge transferred to the feedback capacitor for both circuits in an interval of length T_{CLK} at time t_1

For the RC circuit:

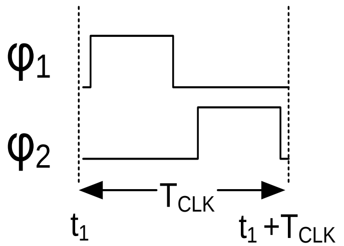
$$Q_{RC} \simeq \left[\frac{V_{in}(t_1)}{R} \right] T_{CLK}$$

Observe that a resistor "transfers" charge proportional to V_{in} in a short interval of T_{CLK}

For the SC circuit



$$Q_{C1} = C_1 V_{in}\left(t_1 + \frac{T_{CLK}}{2} - \varepsilon\right)$$



Since $V_{in}(t)$ is slowly varying

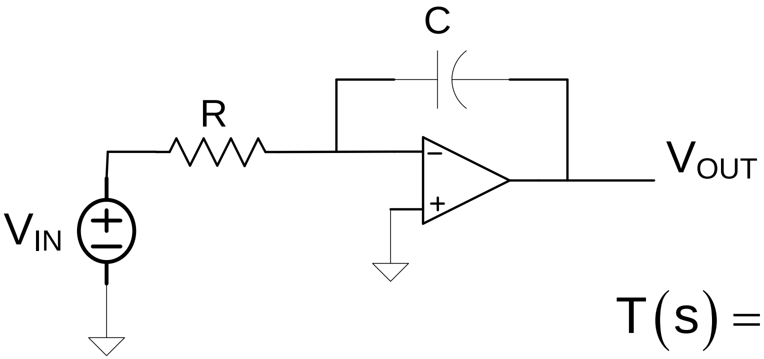
$$Q_{C1} \simeq C_1 V_{in}(t_1)$$

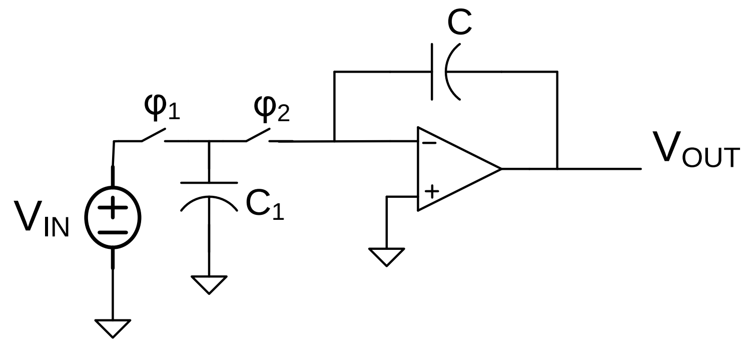
But this is the charge that will be transferred to C during phase Φ_2

$$Q_{SC} \simeq C_1 V_{in}(t_1)$$

Observe that the SC circuit also transfers charge proportional to V_{in} in short intervals of length T_{CLK}

This is precisely what a resistor does so the switched capacitor behaves as a resistor





$$T(s) = -\frac{1}{RCs}$$

$$I_0 = \frac{1}{RC}$$

Comparing the two circuits

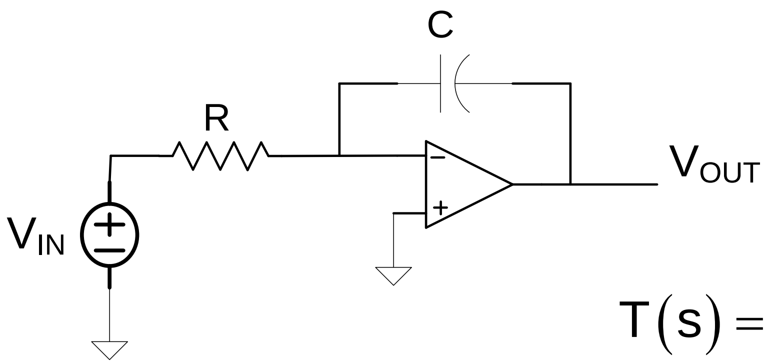
$$Q_{RC} \simeq \left[\frac{V_{in}(t_1)}{R}\right] T_{CLK}$$

$$Q_{SC} \simeq C_1 V_{in}(t_1)$$

Equating charges since both proportional to $V_{in}(t_1)$

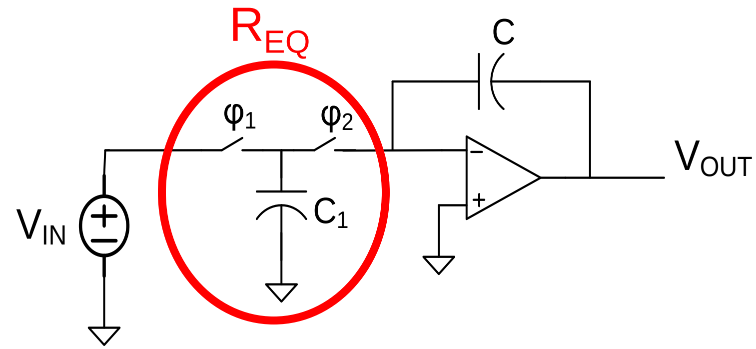
$$C_1 \simeq \left[\frac{1}{R}\right] T_{CLK}$$

$$R_{EQ} \simeq \frac{1}{f_{CLK} C_1}$$



$$T(s) = -\frac{1}{RCs}$$

$$I_0 = \frac{1}{RC}$$



$$R_{EQ} \simeq \frac{1}{f_{CLK} C_1}$$

Observe that a switched-capacitor behaves as a resistor!

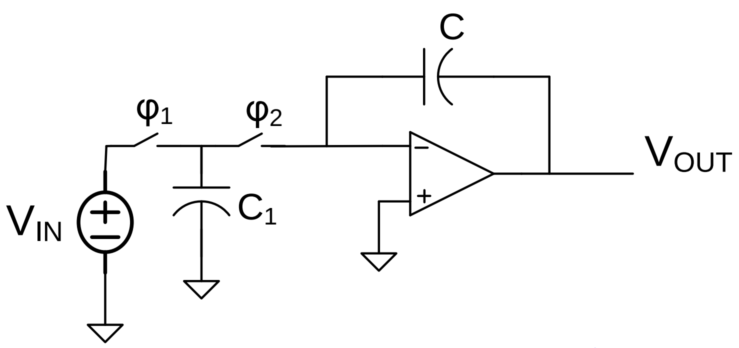
This is an interesting observation that was made by Maxwell over 100 years ago but in and of itself was of almost no consequence

Observation by Maxwell was forgotton and rediscovered several times over the years but remained of no conseqeunce

Note that large resistors require small capacitors !

This offers potential for overcoming one of the critical challenges for Implementing integrators on silicon at audio frequencies!

Consider again the SC integrator



$$R_{EQ} \simeq \frac{1}{f_{CLK} C_1}$$

$$T_{SC}(s) \simeq \frac{-1}{R_{EQ} C s}$$

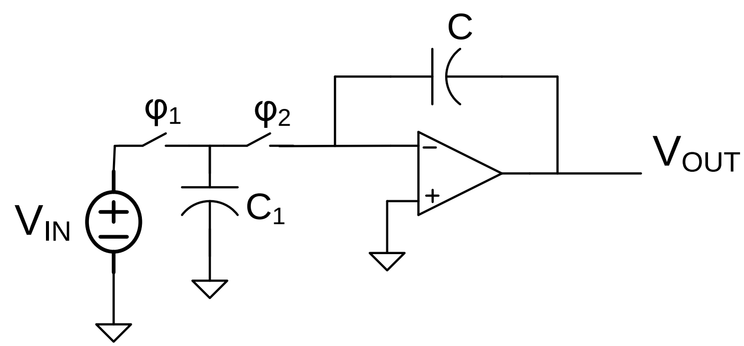
$$I_{0eq} = \frac{1}{R_{EQ} C}$$

$$I_{0eq} = \frac{1}{R_{EQ} C} = \frac{C_1 f_{CLK}}{C}$$

$$I_{0eq} = \left[\frac{C_1}{C}\right] f_{CLK}$$

This is a frequency referenced filter!

The SC integrator



$$T_{SC}(s) \simeq \frac{-1}{R_{EQ}Cs}$$

$$R_{EQ} \simeq \frac{1}{f_{CLK}C_1}$$

$$I_{0eq} = \left[\frac{C_1}{C}\right] f_{CLK}$$

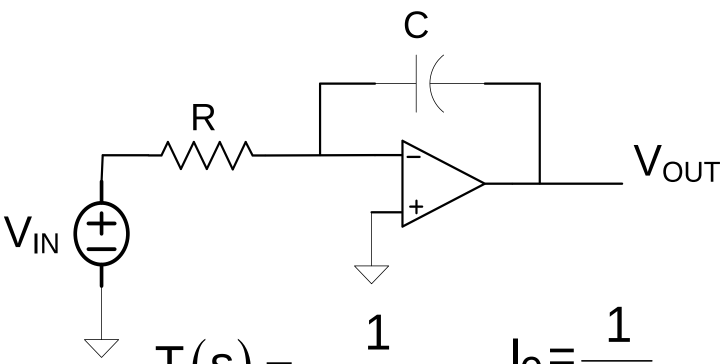
The expressions $S_{C}^{I_0}$ and $S_{C_1}^{I_0}$ have the same magnitude as for the RC integrator 

On-chip capacitor values CAN be highly correlated with proper selection and layout

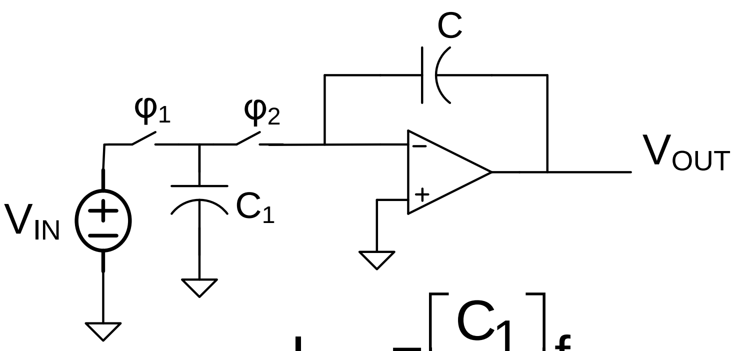
- The ratio of capacitors CAN be accurately controlled in IC processes
  (1% to .01% is achievable with careful layout)
- f_{CLK} CAN be VERY accurately controlled with a c low cost crystal (1 part in 10^6 or better)
- Variability of I_{0eq} is very small

The SC integrator CAN dramatically reduce the second main concern for building integrated integrators

The SC integrator



$$T(s) = -\frac{1}{RCs} \qquad I_0 = \frac{1}{RC}$$



$$I_{0eq} = \left[\frac{C_1}{C}\right] f_{CLK}$$

1. **Accuracy of R and C difficult to accurately control (often 2 or 3 orders of magnitude to variable)**
2. **Area of R and C too large in audio frequency range (2 or 3 orders of magnitude too large)**
3. **Amplifier GB limits performance**

1. **Accuracy of cap ratio and f_{CLK} very good**
2. **Area of C1 and C not too large**
3. **Amplifier GB limits performance less**

Two of these properties were discovered independently by Gray. Broderson and Hosticka at Berkeley and by Copeland of Carelton

[1] J. T. Caves, M. A. Copeland, C. F. Rahim, and S. D. Rosenbaum, "Sampled analog filtering using switched capacitors as resistor equivalents," *IEEE J. Solid-State Circuits*, vol. SC-12, pp. 592–599, Dec. 1977.

77 citations

[2] B. J. Hosticka, R. W. Brodersen, and P. R. Gray, "MOS sampled data recursive filters using switched capacitor integrators," *IEEE J. Solid-State Circuits*, vol. SC-12, pp. 600–608, Dec. 1977.

108 citations

Seminal source of SC concept received few citations!

But cited as a key contribution when Brodersen and Gray elected to NAE

EE Times BETA News & Analysis

Switched-Capacitor Filters Beat Active Filters at Their Own Game

Charles Yager and Carlos Laber

6/29/2000 12:00 AM EDT

Switched capacitor filters are growing increasingly popular because they have many advantages over active filters. Switched capacitor filters don't require external precision capacitors like active filters do. Their cutoff frequencies have a typical accuracy of ±0.3% and they are less sensitive to temperature changes. These characteristics allow consistent, repeatable filter designs.

Another distinct advantage of switched capacitor filters is that their cutoff frequency can be adjusted by changing the clock frequency. Switched capacitor filters offer higher integration at a lower system cost. Center frequencies of up to 150-kHz with Q values up to 20 are achievable.

Switched Capacitor Filters

The realization that a switched-capacitor was equivalent to a resistor was of little consequence

The realization that a small switched capacitor was equivalent to a resistor was of little consequence

The realization that a switched capacitor was dependent upon frequency was of little consequence

The realization that RC time constants could be accurately controlled with a small amount of area in silicon was of considerable consequence

The experimental validation and the efforts to convince industry that the SC techniques offered practical solutions was the MAJOR contribution !!

Basic Building Blocks in Both Cascaded Biquads and Multiple Feedback Structures

- **Developed from observations from feedback implementations**

1. Integrators
2. Summers
3. Op Amps (inc OTAs)
4. Switches

}

- First-order filter blocks
- Biquads

- **Same building blocks used in open-loop applications as well**

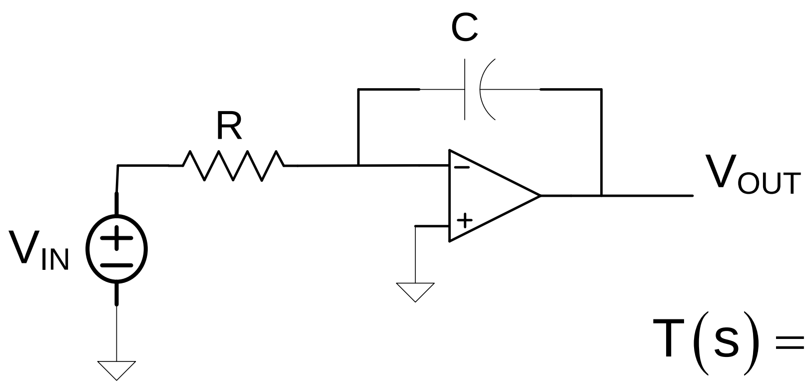
End of Lecture 26

EE 508
Lecture 27

Integrator Design

Switched Capacitor Integrators

Consider the Basic Integrator



$$T(s) = -\frac{1}{RCs}$$

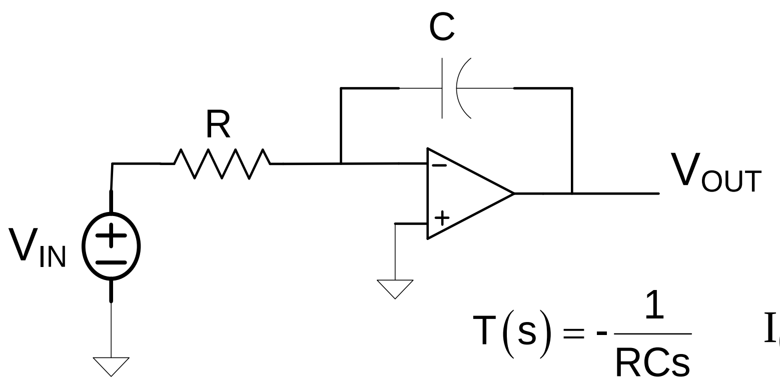
$$I_0 = \frac{1}{RC}$$

1. Accuracy of R and C difficult to accurately control – particularly in integrated applications (often 2 or 3 orders of magnitude to variable)
2. Size of R and C unacceptably large if I_0 is in audio frequency range (2 or 3 orders of magnitude too large)
3. Amplifier GB limits performance

Incredible Challenge to Building Filters on Silicon!

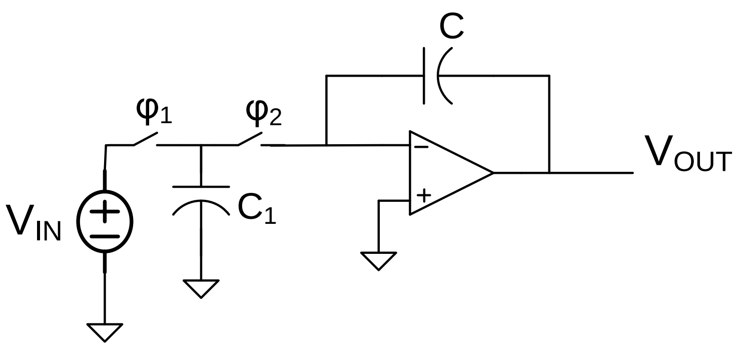
Review from last time

Switched-Capacitor Circuits

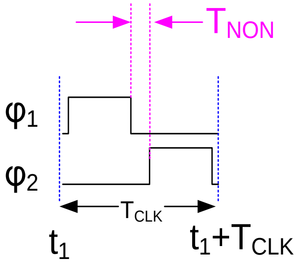


$$T(s) = -\frac{1}{RCs} \qquad I_0 = \frac{1}{RC}$$

Consider:



$$V_{IN} = V_M \sin(2\pi f_{SIG} t + \theta)$$



Assume $T_{CLK} << T_{SIG}$

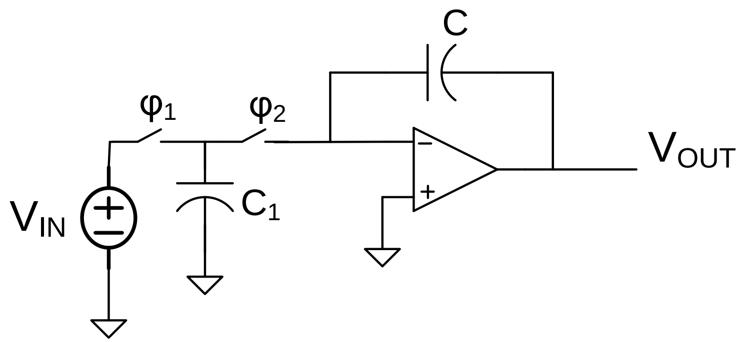
Φ_1 and Φ_2 are complimentary non-overlapping clocks

Φ_1 and Φ_2 are periodic signals "clocks" shown for one period

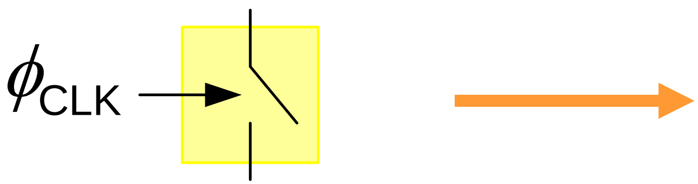
Termed a Switched-Capacitor circuit

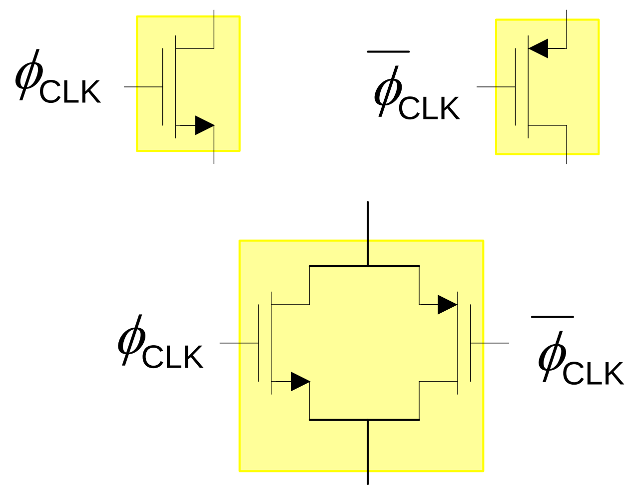
Review from last time

Switched-Capacitor Circuits



How are the switches made?



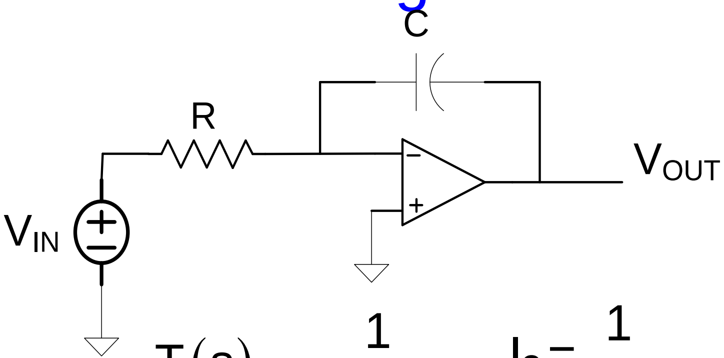


- Often single transistor
- Occasionally complimentary transistors
- On rare occasion more complicated
- Area overhead for switches small, clock routing a little more of concern
- Sizing of devices is important
- Clocking of switches may be important

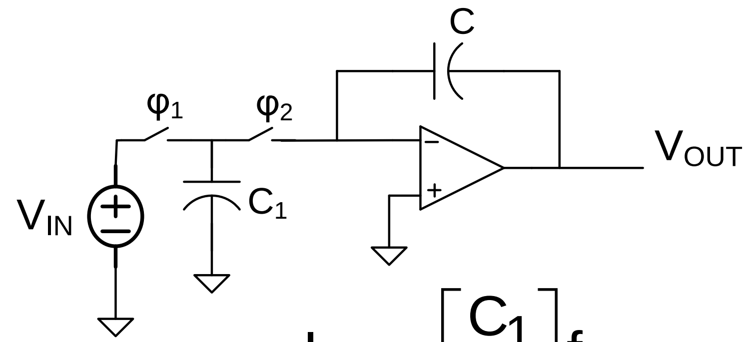
Although originating in SC filters, switched charge redistribution circuits widely used in other non-filtering applications

Review from last time

The SC integrator



$$T(s) = -\frac{1}{RCs} \qquad I_0 = \frac{1}{RC}$$



$$I_{0eq} = \left[\frac{C_1}{C}\right] f_{CLK}$$

1. **Accuracy of R and C difficult to accurately control (often 2 or 3 orders of magnitude to variable)**
2. **Area of R and C too large in audio frequency range (2 or 3 orders of magnitude too large)**
3. **Amplifier GB limits performance**

1. **Accuracy of cap ratio and f_{CLK} very good**
2. **Area of C1 and C not too large**
3. **Amplifier GB limits performance less**

Two of these properties were discovered independently by Gray. Broderson and Hosticka at Berkeley and by Copeland of Carelton

[1] J. T. Caves, M. A. Copeland, C. F. Rahim, and S. D. Rosenbaum, "Sampled analog filtering using switched capacitors as resistor equivalents," *IEEE J. Solid-State Circuits*, vol. SC-12, pp. 592–599, Dec. 1977.

77 citations

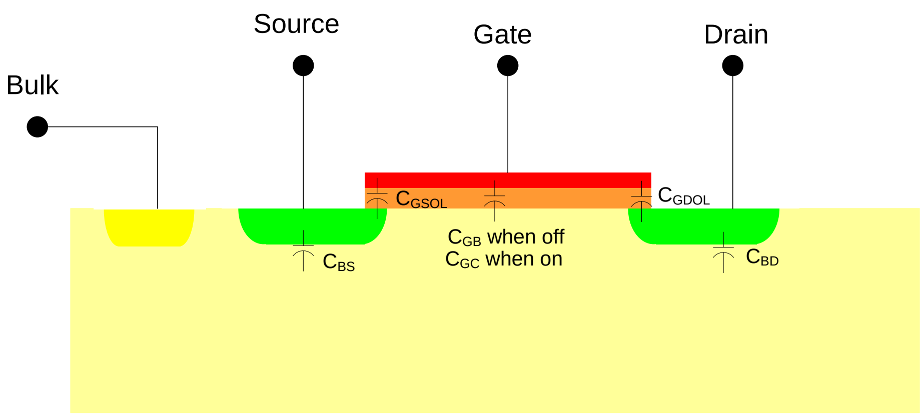
[2] B. J. Hosticka, R. W. Brodersen, and P. R. Gray, "MOS sampled data recursive filters using switched capacitor integrators," *IEEE J. Solid-State Circuits*, vol. SC-12, pp. 600–608, Dec. 1977.

108 citations

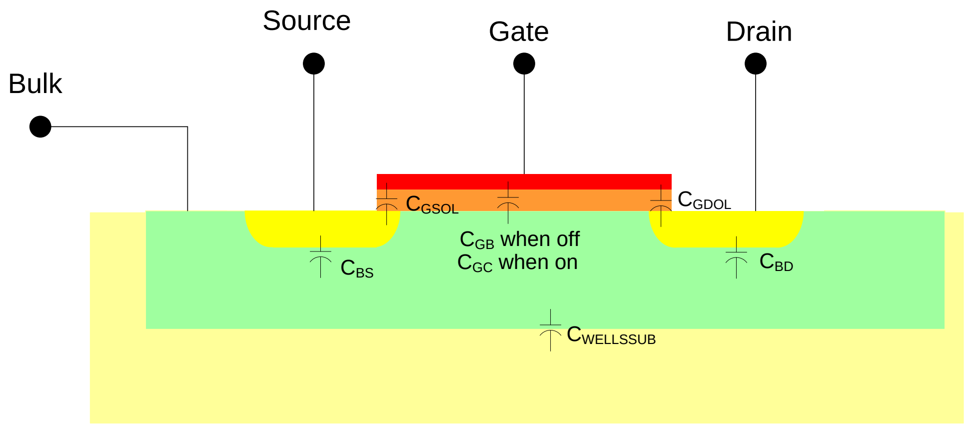
Seminal source of SC concept received few citations!

But cited as a key contribution when Brodersen and Gray elected to NAE

Parasitic Capacitors in MOS Transistors

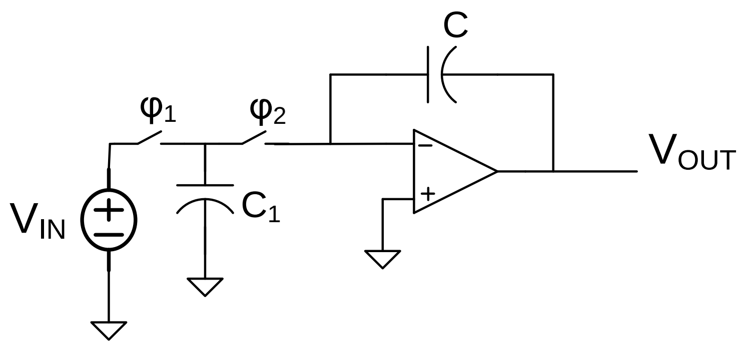


n-channel MOSFET



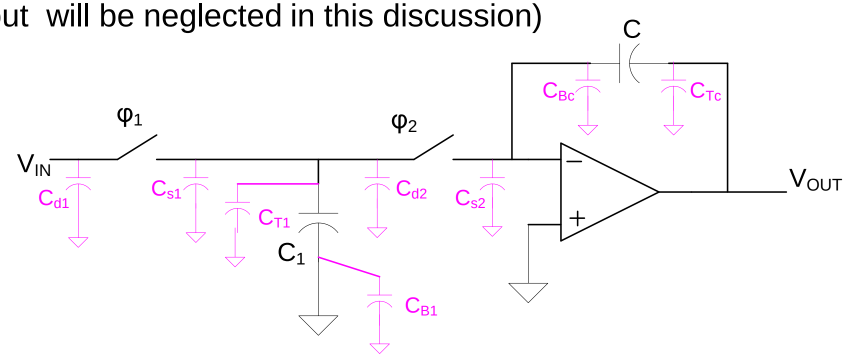
p-channel MOSFET

The SC integrator



$$I_{0eq}=\left[\frac{C_1}{C}\right]f_{CLK}$$

Observe this circuit has considerable parasitics (gate parasitics cause offset in this circuit and some signal-dependent distortion but will be neglected in this discussion)



$$C_{1EQ}=C_1+C_{s1}+C_{d2}+C_{T1}$$

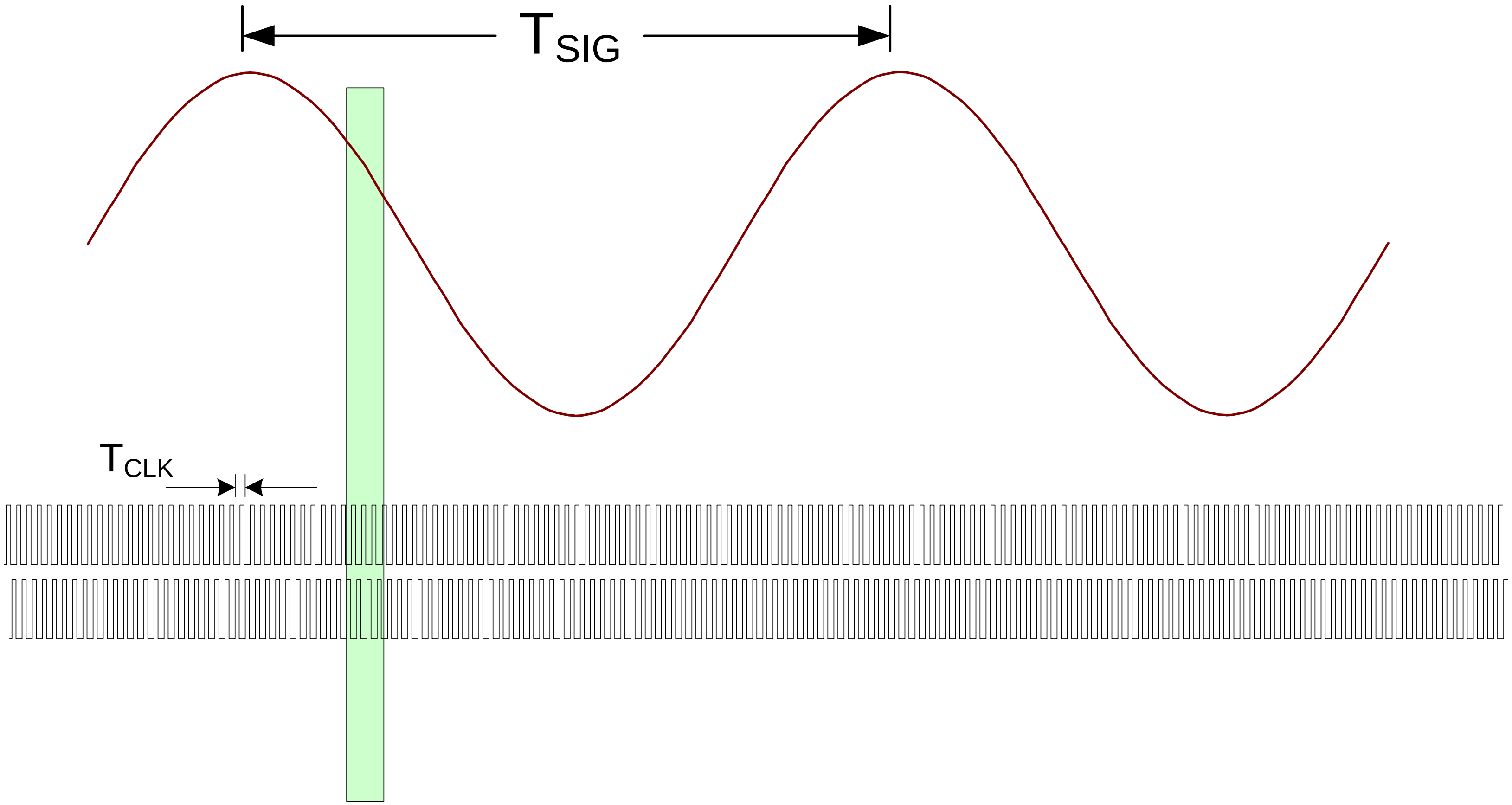
Parasitic capacitors $C_{s1}+C_{d2}+C_{T1}$ difficult to accurately match

- Parasitic capacitors of THIS SC integrator limit performance
- Other SC integrators (discussed later) offer same benefits but are not affected by parasitic capacitors

Switched-Capacitor Filter Issues

What if T_{CLK} is not much-much smaller than T_{SIG}?

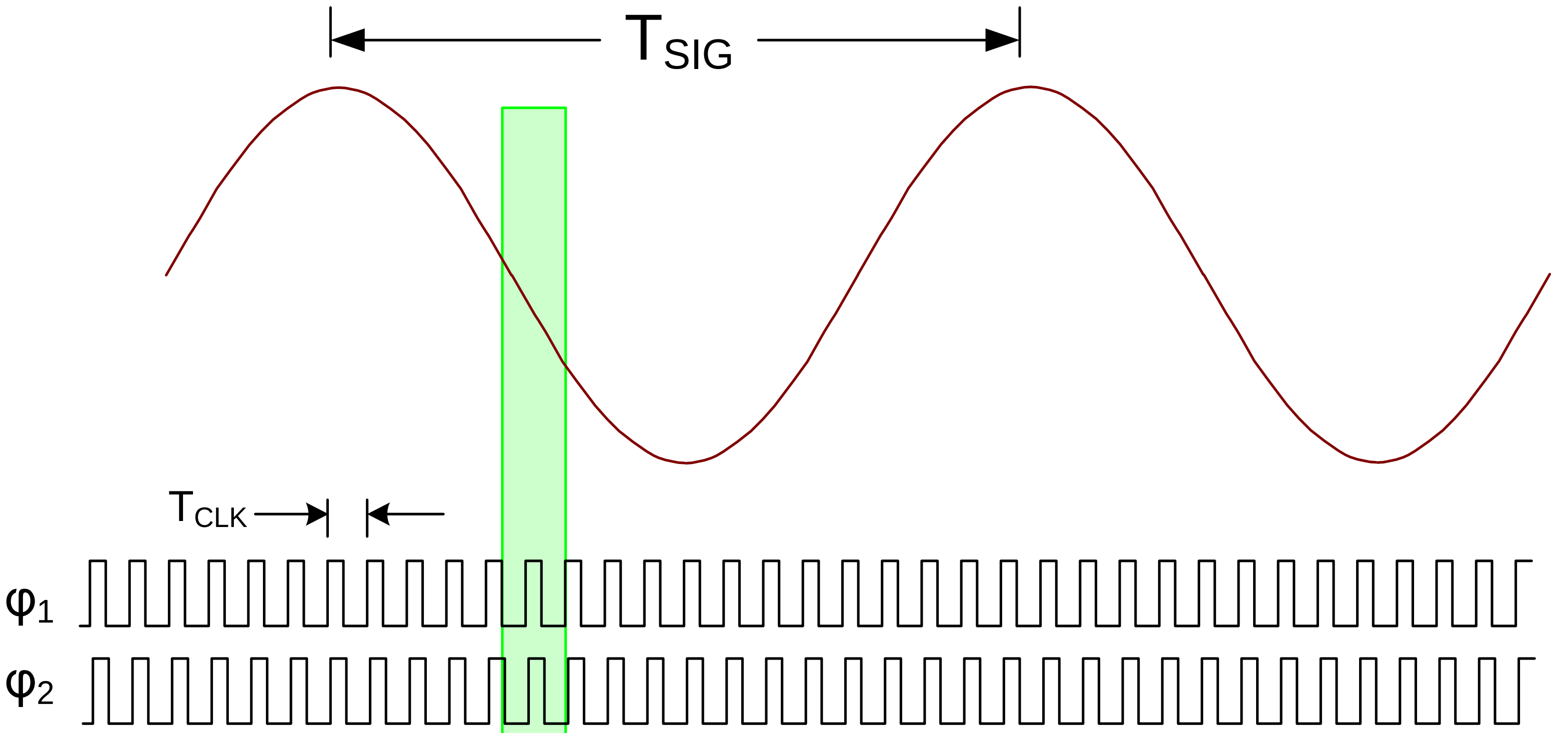
For $T_{CLK} << T_{SIG}$



Switched-Capacitor Filter Issues

What if T_{CLK} is not much-much smaller than T_{SIG}?

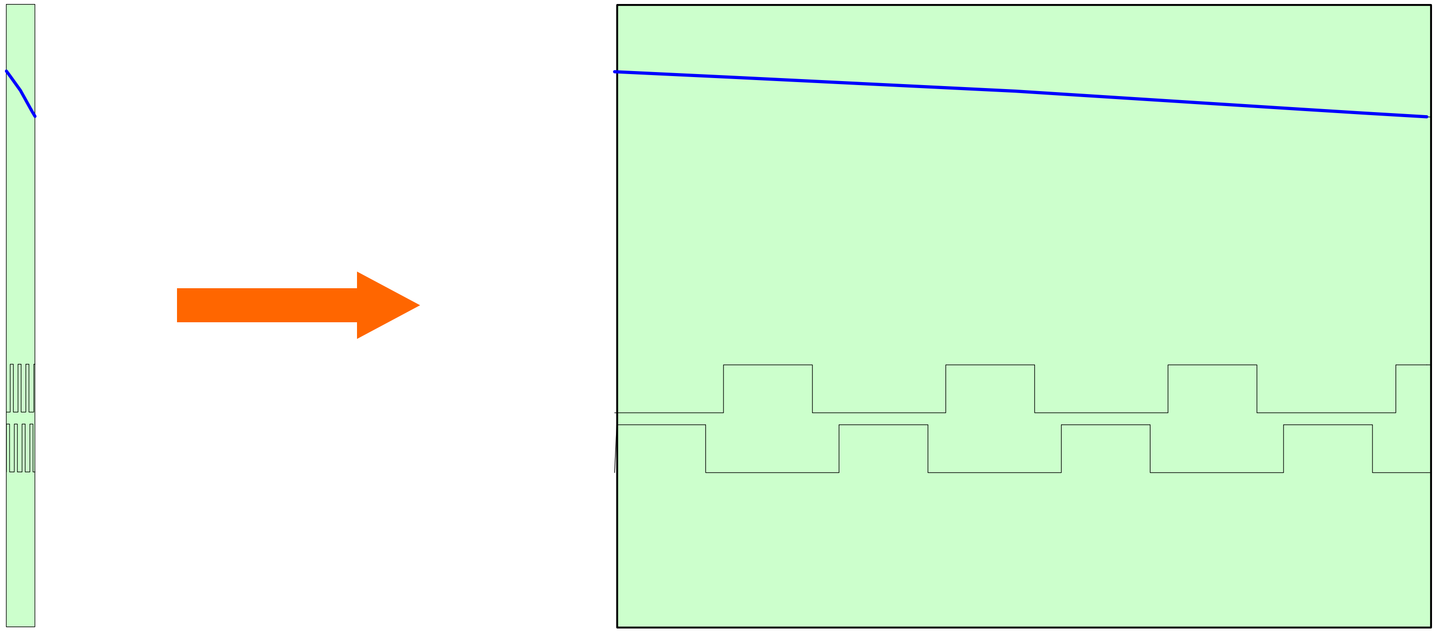
For $T_{CLK} < T_{SIG}$



Switched-Capacitor Filter Issues

What if T_{CLK} is not much-much smaller than T_{SIG}?

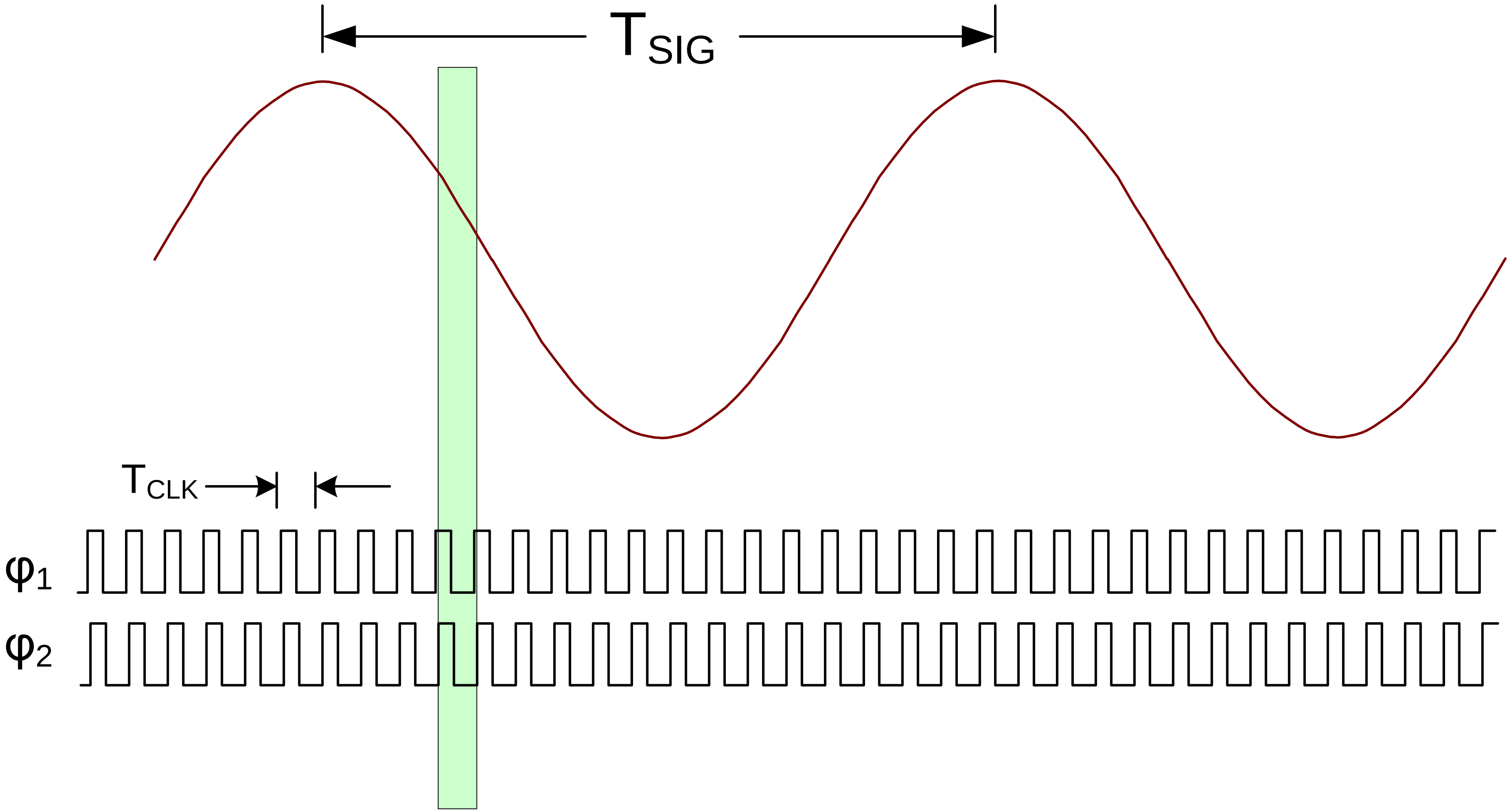
For $T_{CLK} << T_{SIG}$



Switched-Capacitor Filter Issues

What if T_{CLK} is not much-much smaller than T_{SIG}?

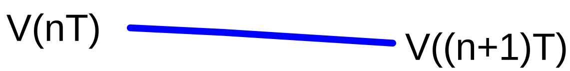
For $T_{CLK} < T_{SIG}$

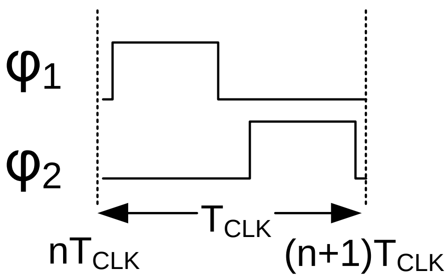


Switched-Capacitor Filter Issues

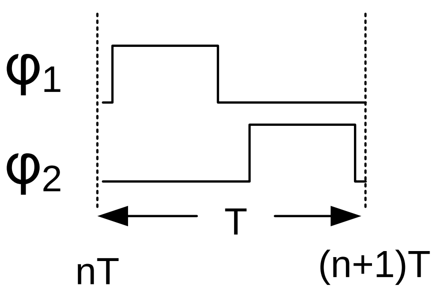
What if T_{CLK} is not much-much smaller than T_{SIG}?

For $T_{CLK} << T_{SIG}$

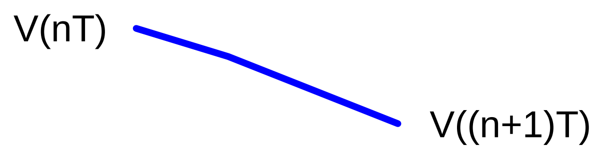


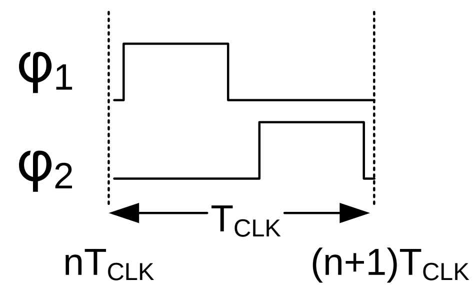


Define $T = T_{CLK}$

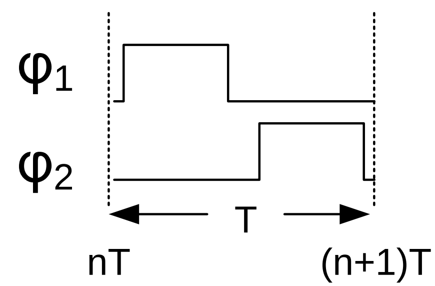


For $T_{CLK} < T_{SIG}$





Define $T = T_{CLK}$

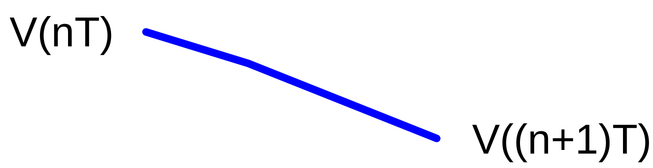


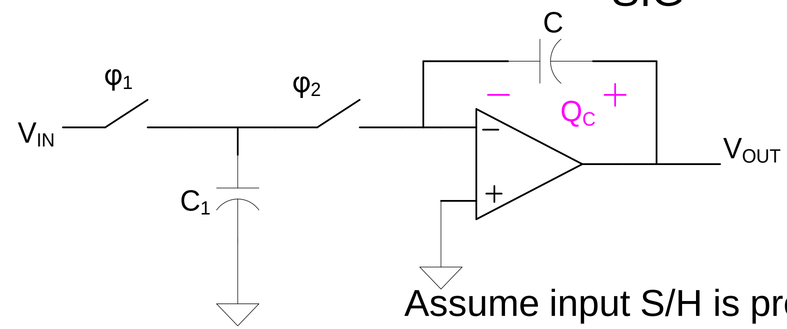
Considerable change in V(t) in clock period

Switched-Capacitor Filter Issues

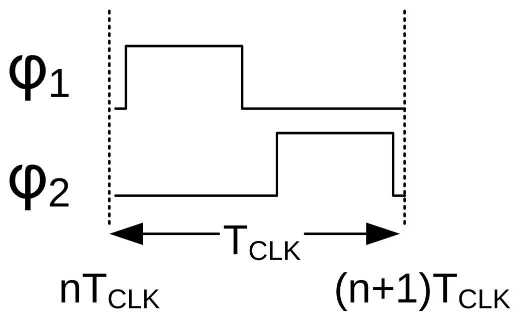
What if T_{CLK} is not much-much smaller than T_{SIG}?

For $T_{CLK} < T_{SIG}$

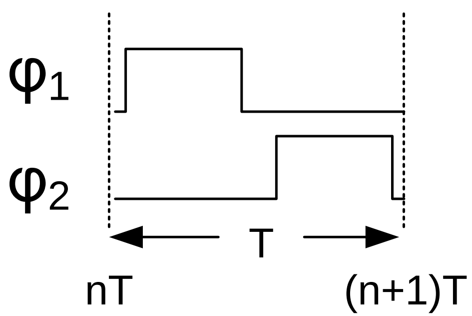




Assume input S/H is present



Define $T = T_{CLK}$



$$V_0(nT+T) = V_0(nT) + \frac{\Delta Q_C}{C}$$

but $-Q_c$ is the charge on C_1 at the time φ_1 opens

$$-\Delta Q_c \simeq C_1 V_{IN}(nT+T/2)$$

$$\therefore V_{OUT}(nT+T) = V_{OUT}(nT) - (C_1/C)V_{IN}(nT+T/2)$$

Due to input S/H, V_{IN} constant over periods of length T thus, assume $V_{IN}(nT+T/2) \simeq V_{IN}(nT)$

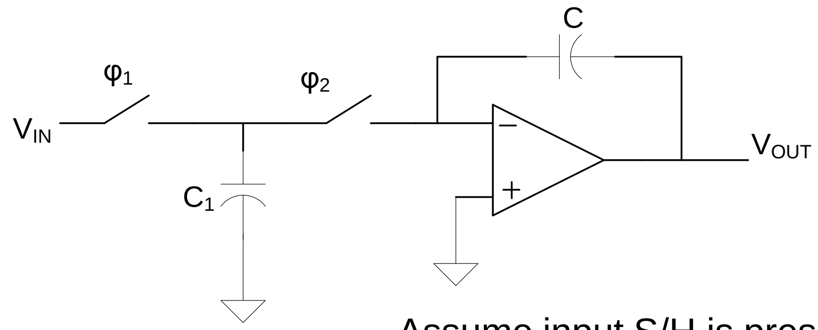
So obtain

$$V_{OUT}(nT+T) = V_{OUT}(nT) - (C_1/C)V_{IN}(nT)$$

How does this analysis differ from what we did earlier?

Switched-Capacitor Filter Issues

What if T_{CLK} is not much-much smaller than T_{SIG}?



Assume input S/H is present

$$V_{OUT}(nT+T)=V_{OUT}(nT)-(C_1/C)V_{IN}(nT)$$

for any T_{CLK}, characterized in time domain by difference equation

This can be characterized in the discrete-time frequency domain by transfer function obtained by taking z-transform of the difference equation

$$zV_{OUT}(z)=V_{OUT}(z)-(C_1/C)V_{IN}(z)$$

$$H(z)=-\frac{C_1/C}{z-1}$$

This is a standard integrator transfer function in the z-domain (but not unique)

Note pole at z=1

Switched-Capacitor Filter Issues

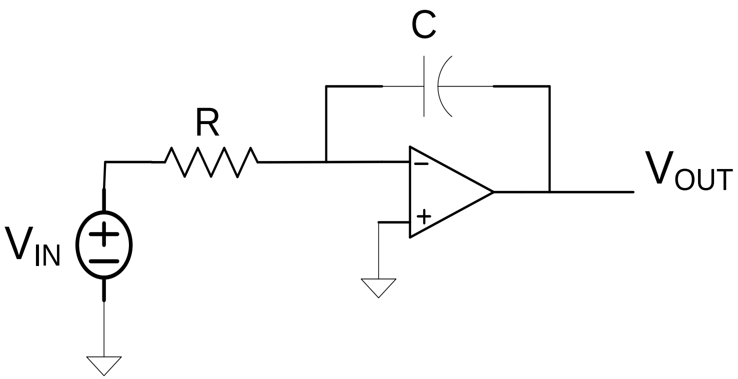
What if T_{CLK} is not much-much smaller than T_{SIG}?

Claim: The transfer function of any Switched-Capacitor Filter is a rational fraction in z with all coefficients in both the numerator and denominator determined totally by capacitor ratios

$$H(z) = \frac{\sum_{i=0}^{m} a_i z^i}{\sum_{i=0}^{n} b_i z^i}$$

What is really required for building a filter that has high-performance features?

Consider an integrator:



Frequency domain:

Transfer function

$$T(s) = -\frac{1}{RCs}$$

Time domain:

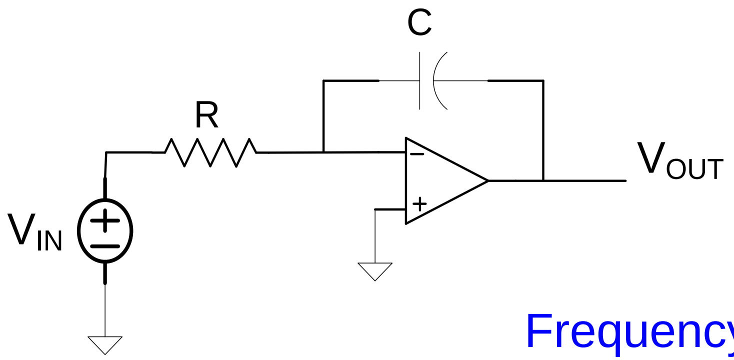
Differential Equation

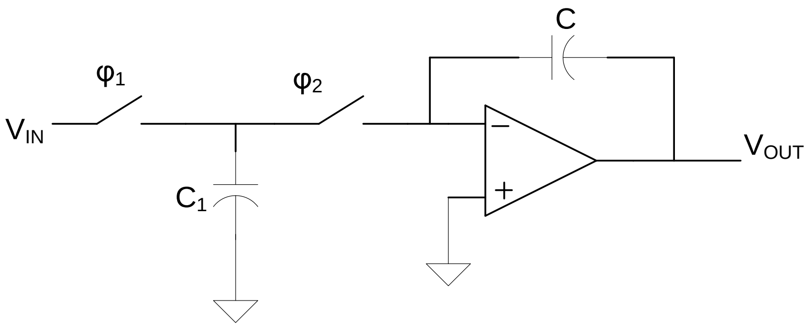
$$V_{OUT}(t) = V_{OUT}(t_0) - \frac{1}{RC}\int_{t_0}^{t} V_{IN}(\tau)\, d\tau$$

- Accurate control of polynomial coefficients in transfer function or accurate control of coefficients in the differential equation
- Absence of over-ordering terms due to parasitics

What is really required for building a filter that has high-performance features?

Consider continuous-time and discrete-time integrators:





Frequency domain:

Transfer function

$$T(s) = -\frac{1}{RCs}$$

$$H(z) = -\frac{C_1/C}{z-1}$$

Time domain:

Differential Equation

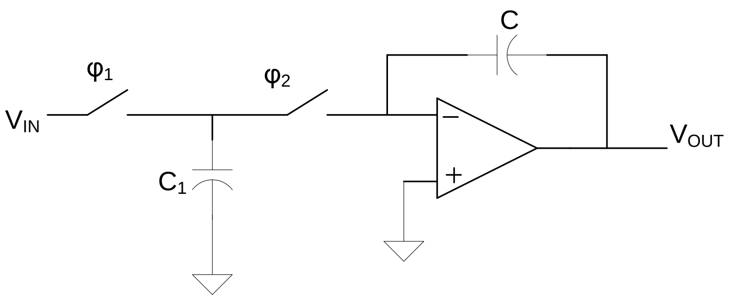
$$V_{OUT}(t) = V_{OUT}(t_0) - \frac{1}{RC}\int_{t_0}^{t} V_{IN}(\tau)\, d\tau$$

Difference Equation

$$V_{OUT}(nT+T) = V_{OUT}(nT) - (C_1/C)V_{IN}(nT)$$

- Accurate control of polynomial coefficients in transfer function or accurate control of coefficients in the differential/difference equation needed for good filter performance
- Absence of over-ordering terms due to parasitics

Switched-Capacitor Filter Issues



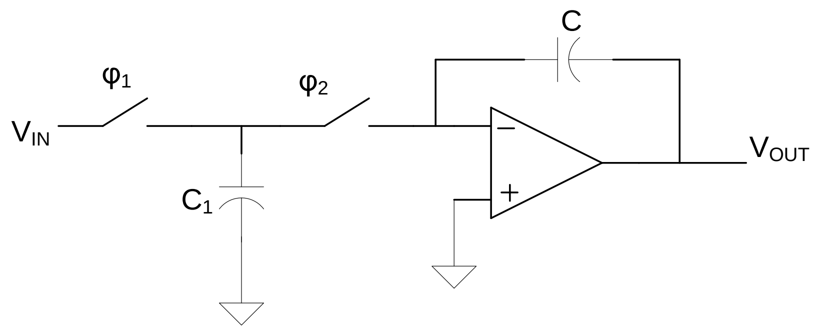
Transfer function of any SC filter of form:

$$H(z) = \frac{\sum_{i=0}^{m} a_i z^i}{\sum_{i=0}^{n} b_i z^i}$$

Switched-capacitor circuits have potential for good accuracy and attractive area irrespective of how T_{CLK} relates to T_{SIG}

But good layout techniques and appropriate area need to be allocated to realize this potential !

Switched-Capacitor Integrators

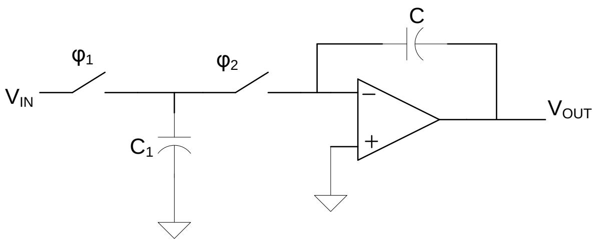


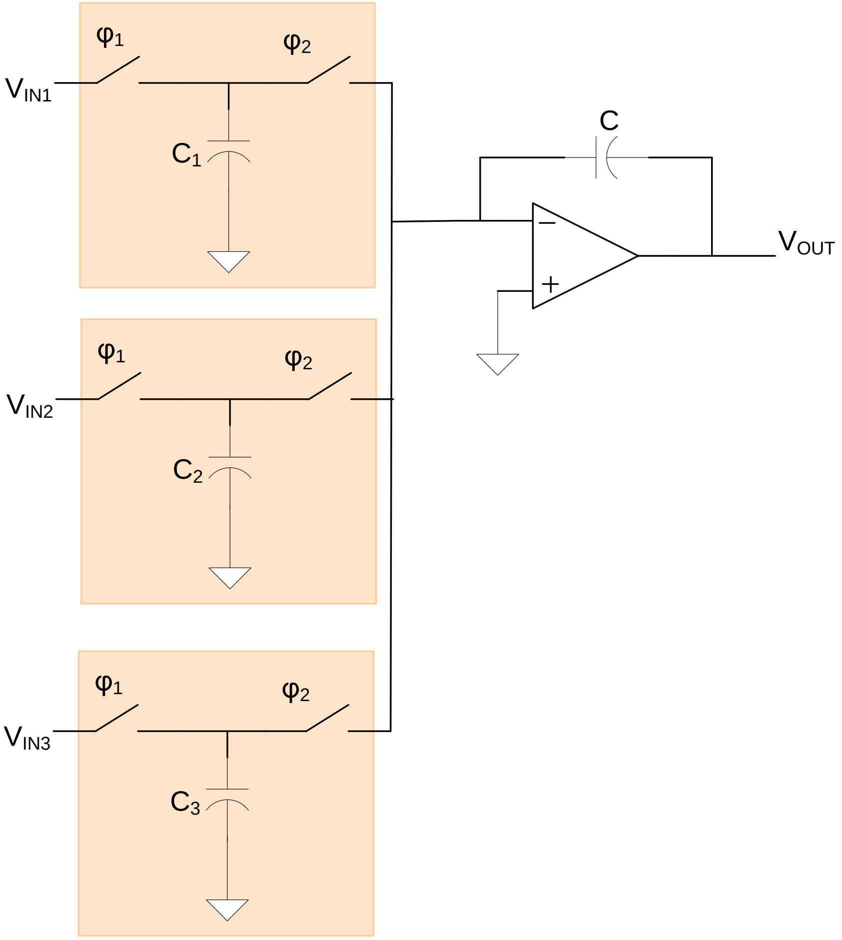
$$V_{OUT}(nT+T)=V_{OUT}(nT)-(C_1/C)V_{IN}(nT)$$

$$H(z)=-\frac{C_1/C}{z-1}$$

Sensitive to parasitic capacitances

Switched-Capacitor Integrators

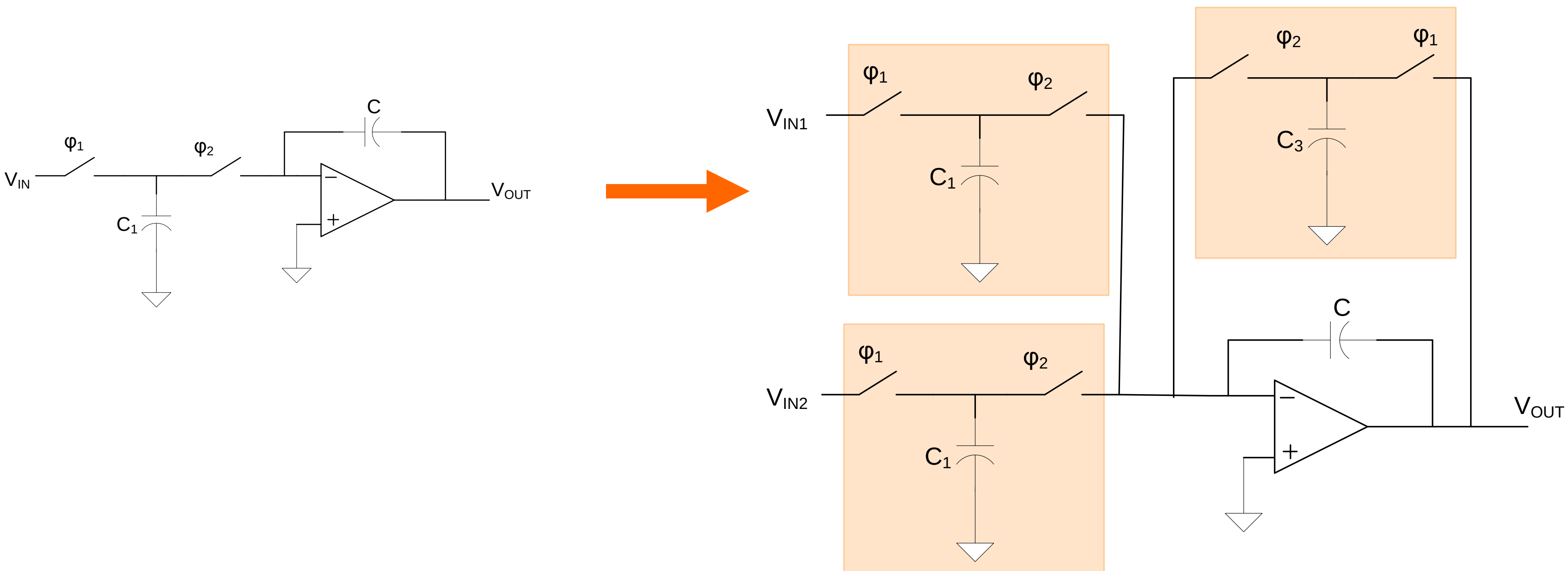




Summing Inputs

Sensitive to parasitic capacitances

Switched-Capacitor Integrators

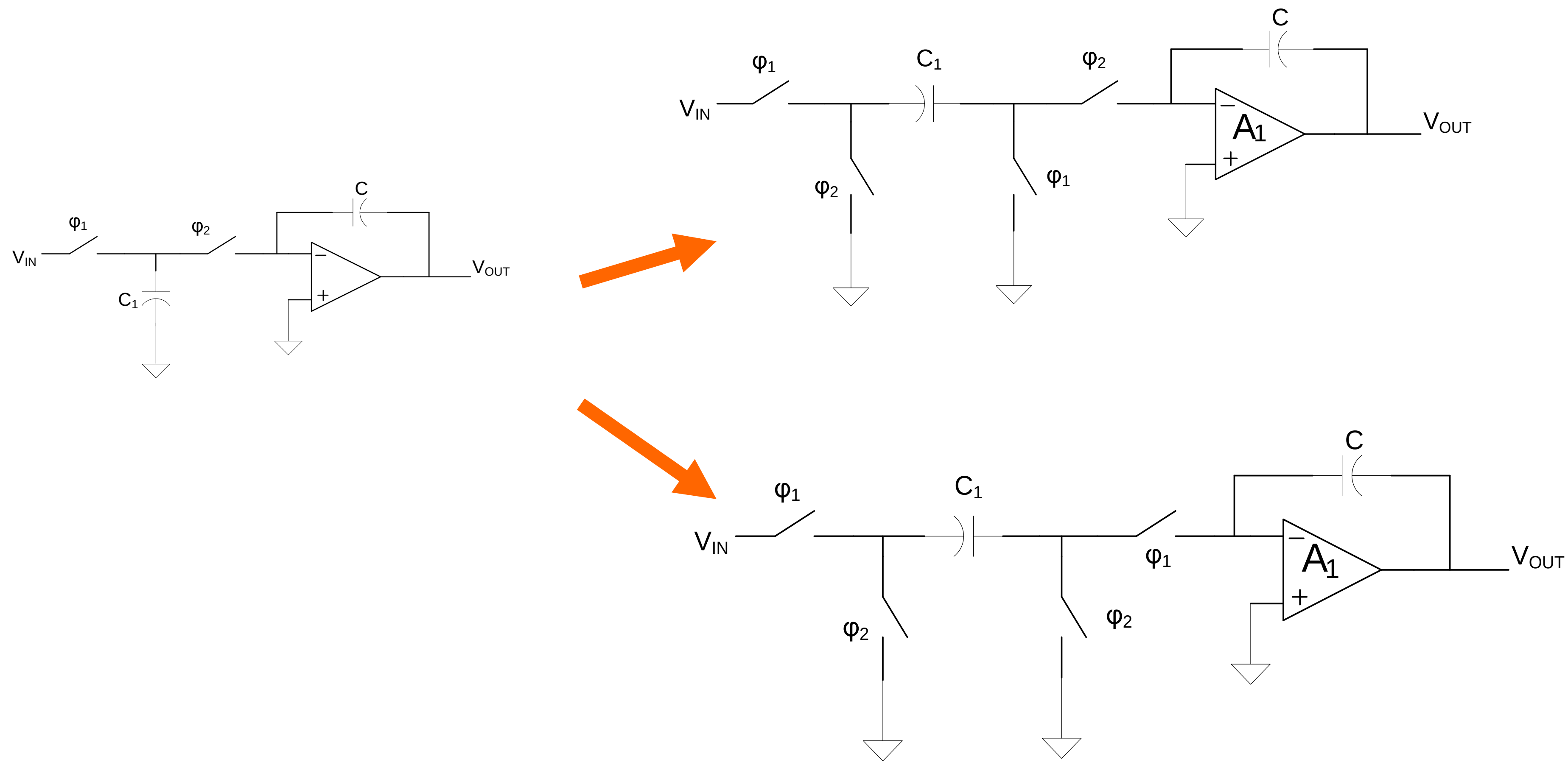


Summing Inputs and Lossy

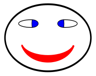
Sensitive to parasitic capacitances

Switched-Capacitor Integrators

Consider the following two SC circuits

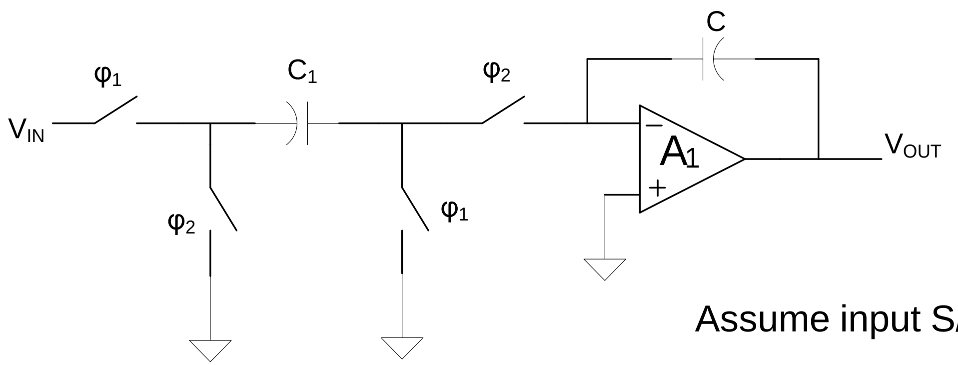


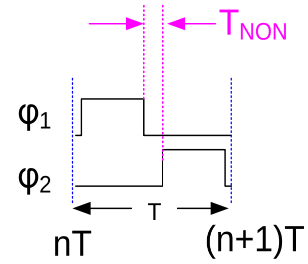
Still have two capacitors but twice as many switches ! 

But switches can be pretty small ! 

Switched-Capacitor Integrators

Consider the first SC circuit





Assume input S/H

During phase Φ_1, capacitor V_{IN} is charged up to $V_{IN}(nT)$

During phase Φ_2, this charge is transferred to C and increasing V_{OUT}

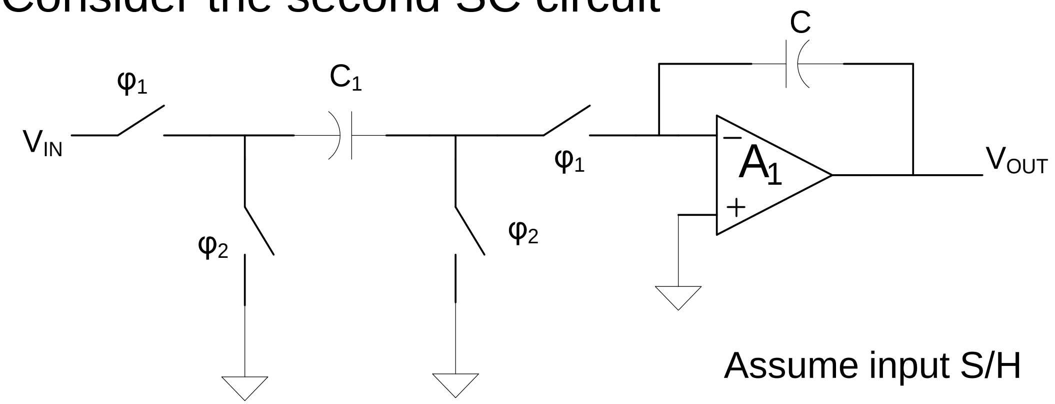
$$V_{OUT}(nT+T) = V_{OUT}(nT) + \frac{C_1}{C} V_{IN}(nT)$$

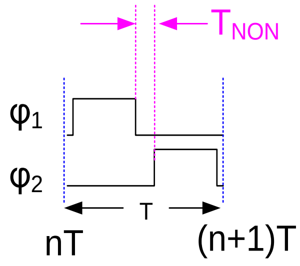
$$H(z) = \frac{C_1/C}{z-1}$$

Serves as a non-inverting integrator

Switched-Capacitor Integrators

Consider the second SC circuit





Assume input S/H

Prior to the start of phase Φ_1, the capacitor C_1 was discharged by Φ_2

During phase Φ_1, capacitor V_{IN} charges up to $V_{IN}(nT)$

While charge is flowing into C_1, it is also flowing into C thus decreasing V_{OUT}

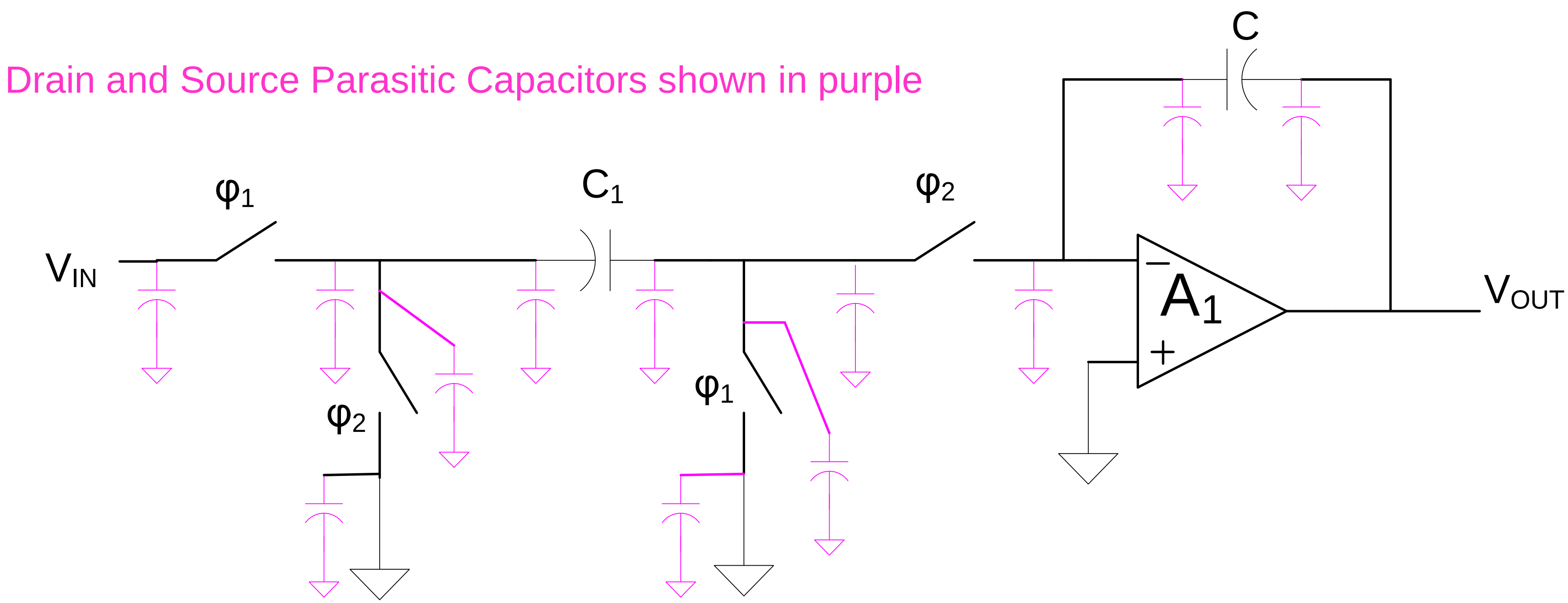
$$V_{OUT}(nT+T) = V_{OUT}(nT) - \frac{C_1}{C} V_{IN}(nT+T)$$

$$H(z) = \frac{z\, {}^{C_1}\!/\!_{C}}{z-1}$$

Since $\left|z\right|_{z=e^{j\omega T}} = 1$

Serves as an inverting integrator

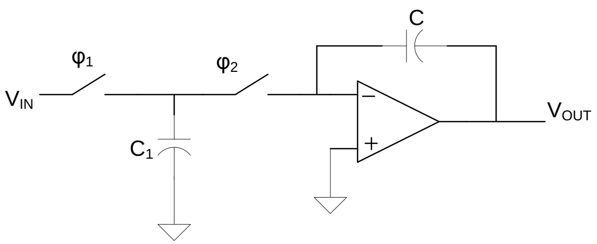
Switched-Capacitor Integrators

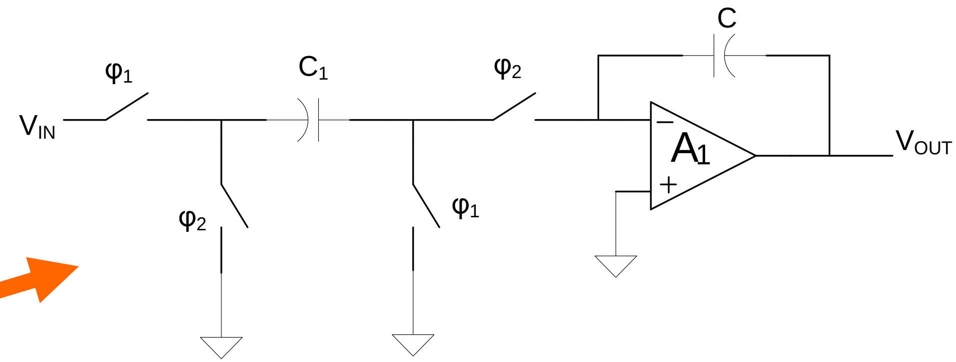


Stray-Insensitive Properties

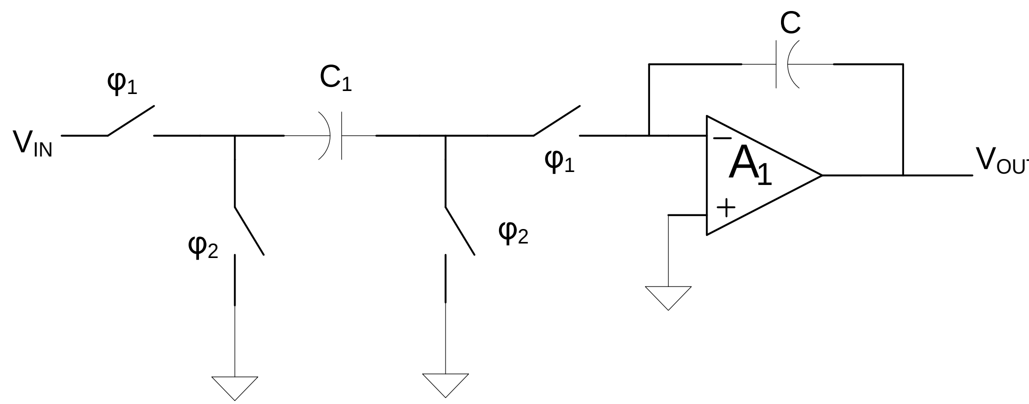
C_{GD} does not affect gain of integrator
$C_{GCHANNEL}$ doe not affect gain of integrator if switch not too fast

Switched-Capacitor Integrators





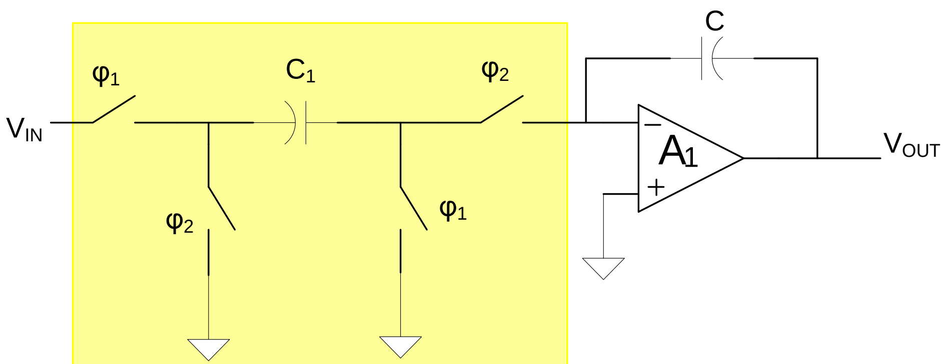
Stray-Insensitive Noninverting

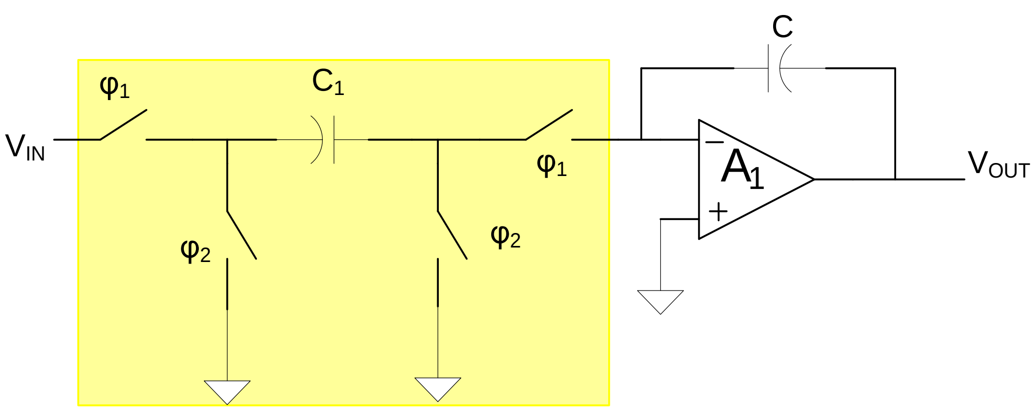


Stray-Insensitive Inverting

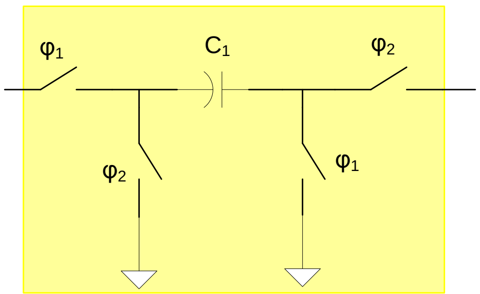
Switched-Capacitor Integrators

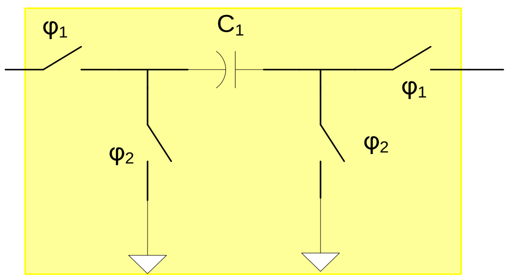
Stray-Insensitive SC Integrators





"Resistor Blocks"

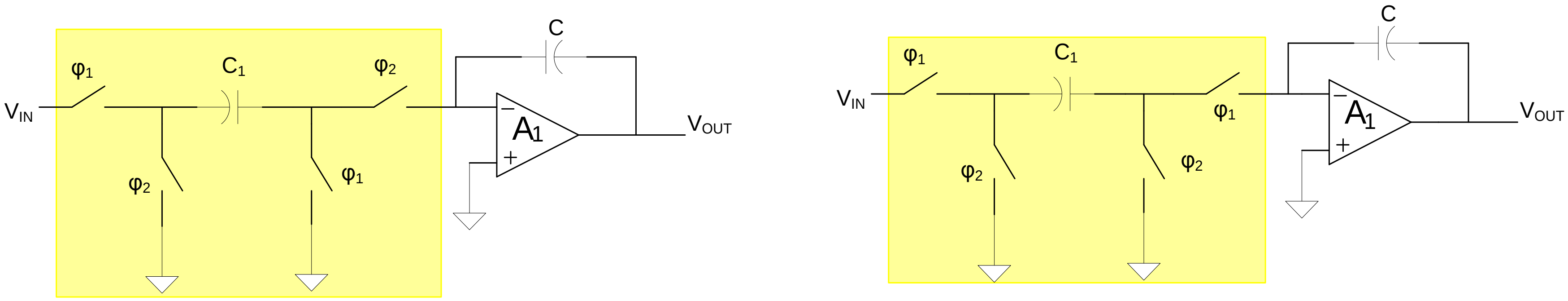




- Resistor blocks can be repeated and combined to provide summing inverting or noninverting inputs
- Resistor block can be placed in FB path to form lossy SC integrator

Switched-Capacitor Integrators

Stray-Insensitive SC Integrators

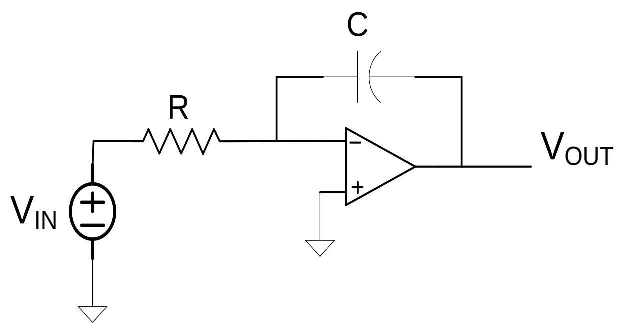


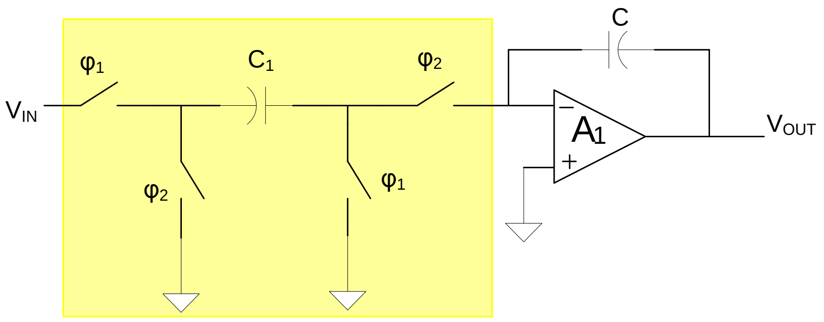
Many different SC filter structures have been proposed

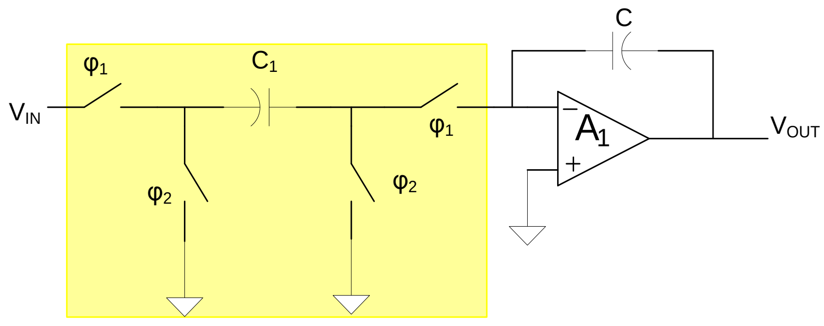
But most that are actually used are based upon these two circuits with the summing inputs or loss added as needed

Switched-Capacitor Integrators

Effects of Op Amp limitations

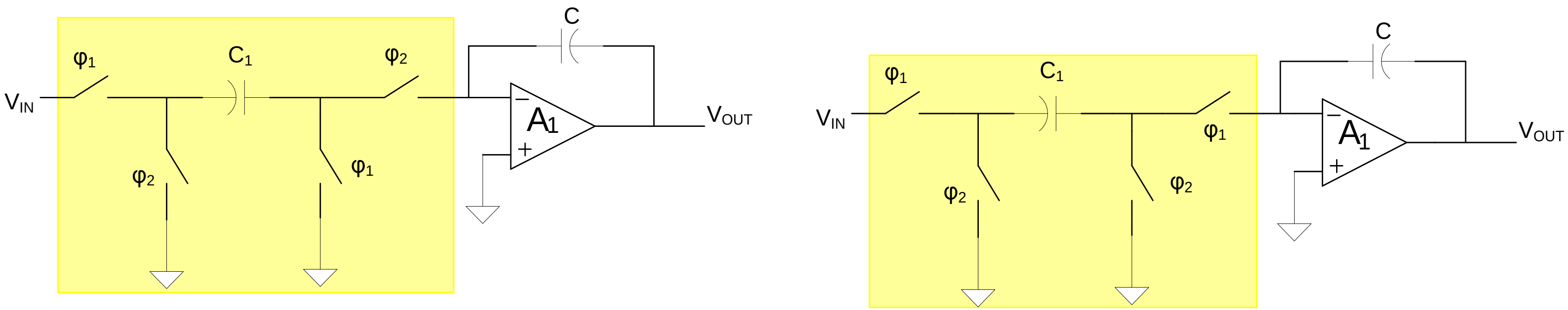






Can be shown that for a given band-edge, the GB requirements for the SC circuit are more relaxed than what is required for the corresponding Active RC integrator

Switched-Capacitor Integrators



Switched-capacitor filters are characterized in the z-domain

SC filters have continuous-amplitude inputs but outputs valid only at discrete times

Digital filters implemented with ADC/DAC approach have discrete amplitude and discrete time

What effects does the discrete-time property of a SC filter have on the filter performance?

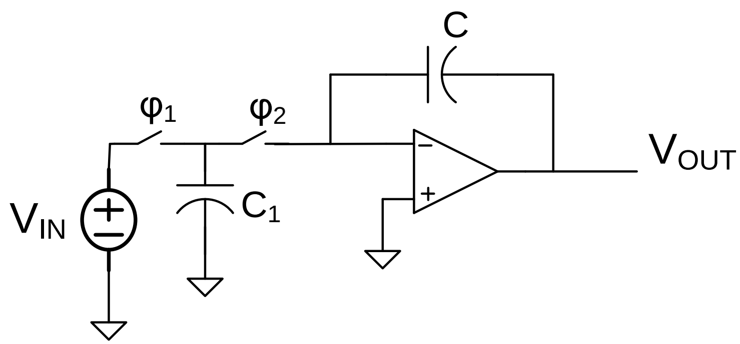
End of Lecture 27

EE 508
Lecture 28

Integrator Design

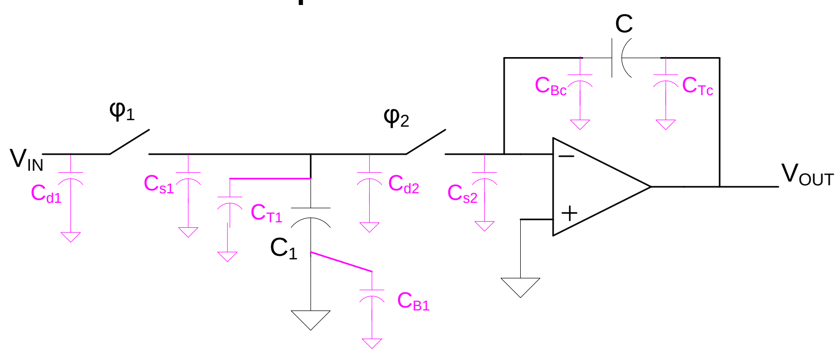
Alaising in SC Circuits
Elimination of redundant switches
Switched Resistor Integrators

The SC integrator



$$I_{0eq} = \left[\frac{C_1}{C}\right] f_{CLK}$$

Observe this circuit has considerable parasitics



$$C_{1EQ} = C_1 + C_{s1} + C_{d2} + C_{T1}$$

Parasitic capacitors $C_{s1}+C_{d2}+C_{T1}$ difficult to accurately match

- Parasitic capacitors of THIS SC integrator limit performance
- Other SC integrators (discussed later) offer same benefits but are not affected by parasitic capacitors

Switched-Capacitor Filter Issues

What if T_{CLK} is not much-much smaller than T_{SIG}?

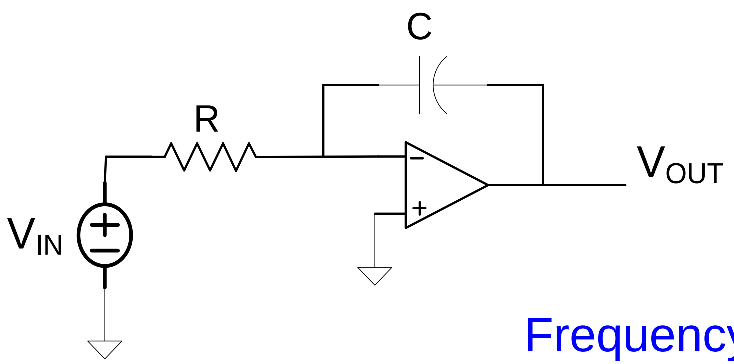
Claim: The transfer function of any Switched-Capacitor Filter is a rational fraction in z with all coefficients in both the numerator and denominator determined totally by capacitor ratios

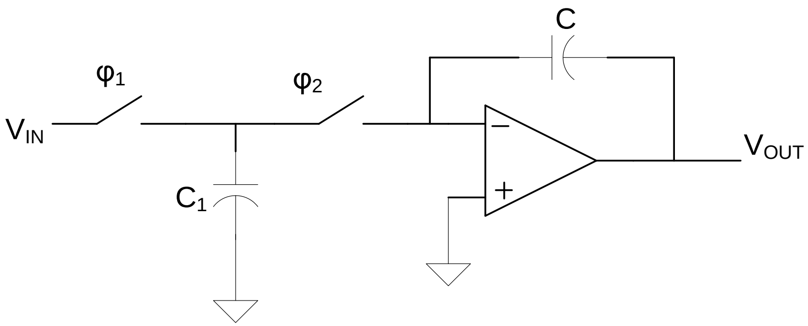
$$H(z) = \frac{\sum_{i=0}^{m} a_i z^i}{\sum_{i=0}^{n} b_i z^i}$$

Review from last time

What is really required for building a filter that has high-performance features?

Consider continuous-time and discrete-time integrators:





Frequency domain:

Transfer function

$$T(s) = -\frac{1}{RCs}$$

$$H(z) = -\frac{C_1/C}{z-1}$$

Time domain:

Differential Equation

$$V_{OUT}(t) = V_{OUT}(t_0) - \frac{1}{RC}\int_{t_0}^{t} V_{IN}(\tau)\,d\tau$$

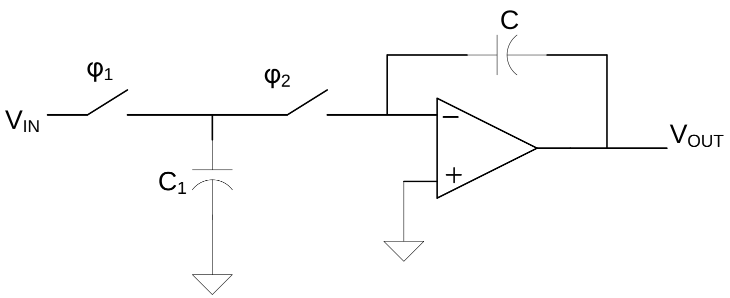
Difference Equation

$$V_{OUT}(nT+T) = V_{OUT}(nT) - (C_1/C)V_{IN}(nT)$$

- Accurate control of polynomial coefficients in transfer function or accurate control of coefficients in the differential/difference equation needed for good filter performance
- Absence of over-ordering terms due to parasitics

Review from last time

Switched-Capacitor Filter Issues



Transfer function of any SC filter of form:

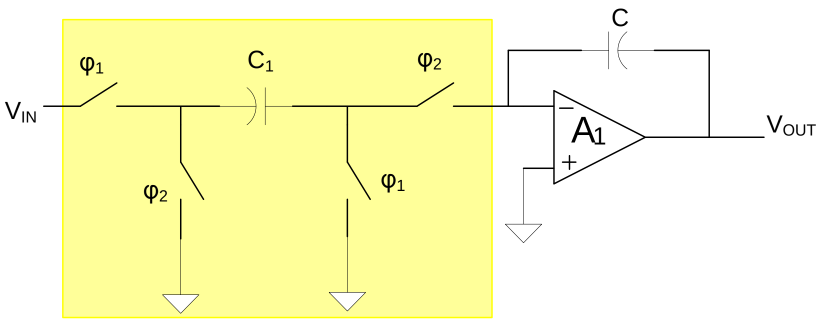
$$H(z) = \frac{\sum_{i=0}^{m} a_i z^i}{\sum_{i=0}^{n} b_i z^i}$$

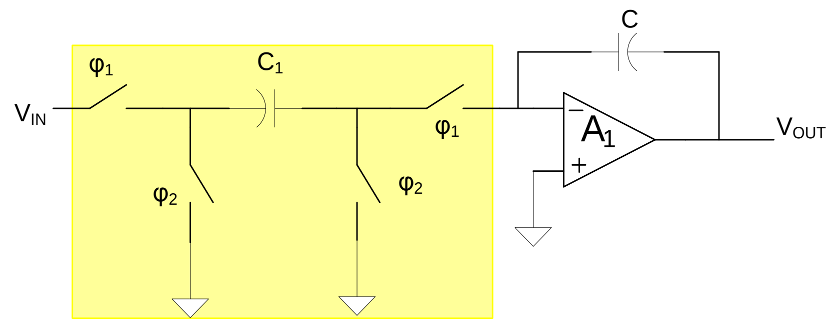
Switched-capacitor circuits have potential for good accuracy and attractive area irrespective of how T_{CLK} relates to T_{SIG}

But good layout techniques and appropriate area need to be allocated to realize this potential !

Switched-Capacitor Integrators

Stray-Insensitive SC Integrators





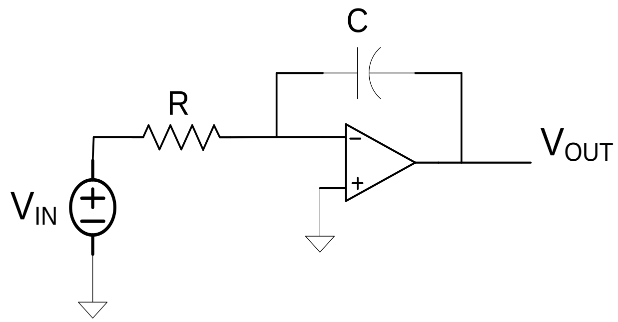
Many different SC filter structures have been proposed

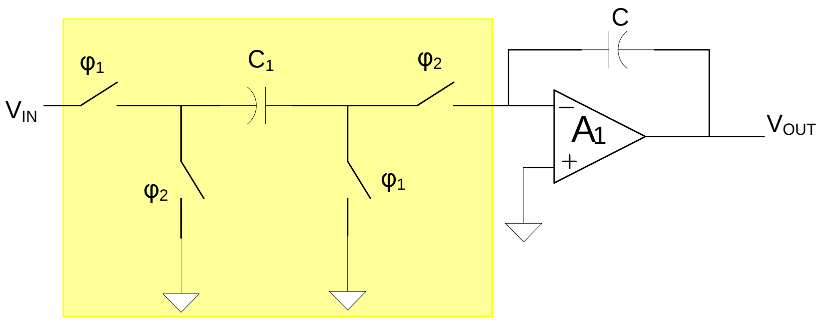
But most that are actually used are based upon these two circuits with the summing inputs or loss added as needed

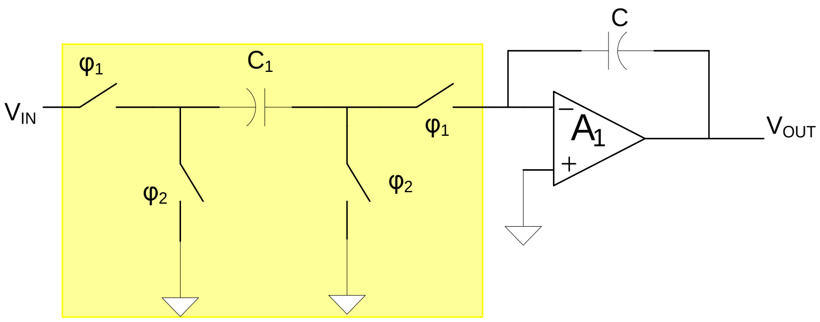
Review from last time

Switched-Capacitor Integrators

Effects of Op Amp limitations



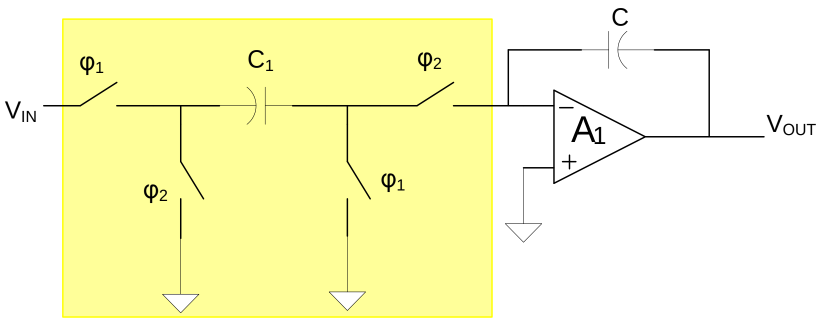


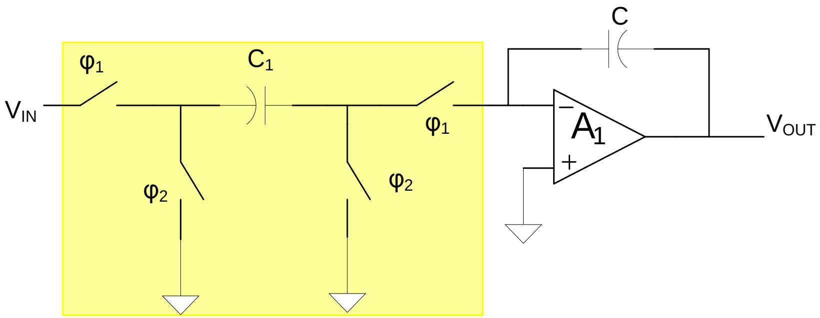


Can be shown that for a given band-edge, the GB requirements for the SC circuit are more relaxed than what is required for the corresponding Active RC integrator

Review from last time

Switched-Capacitor Integrators





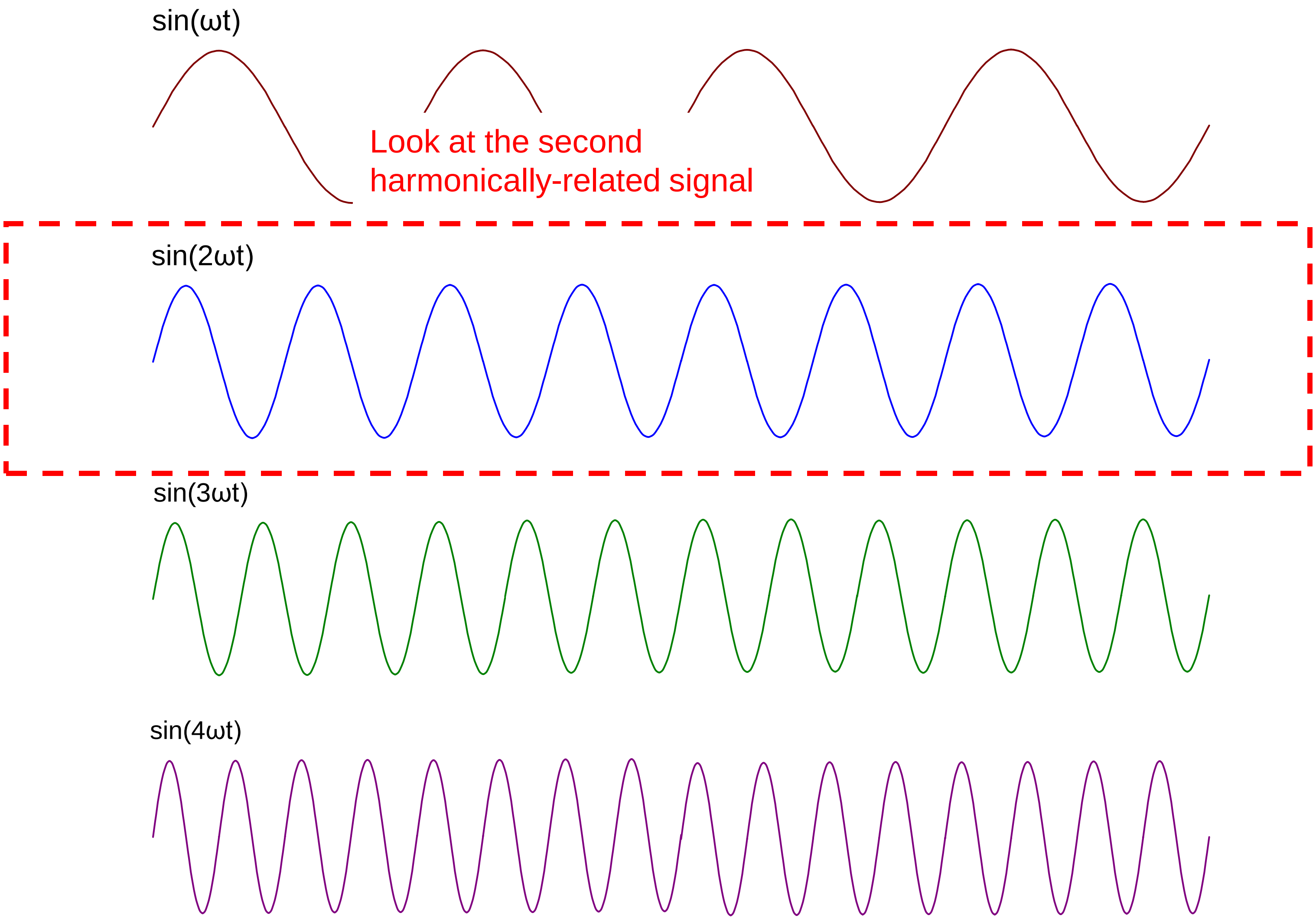
Switched-capacitor filters are characterized in the z-domain

SC filters have continuous-amplitude inputs but outputs valid only at discrete times

Digital filters implemented with ADC/DAC approach have discrete amplitude and discrete time

What effects does the discrete-time property of a SC filter have on the filter performance?

Consider a signal and harmonically-related signals



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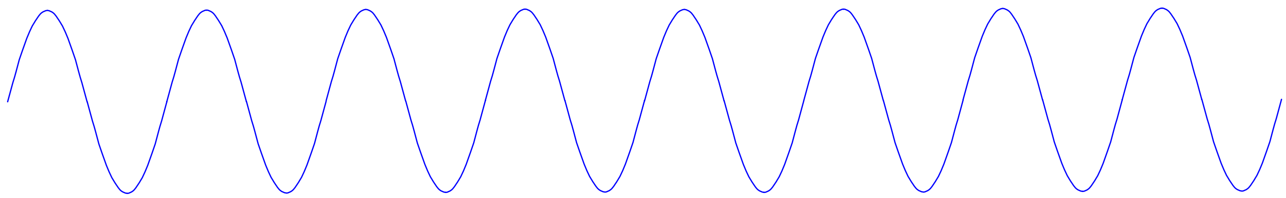
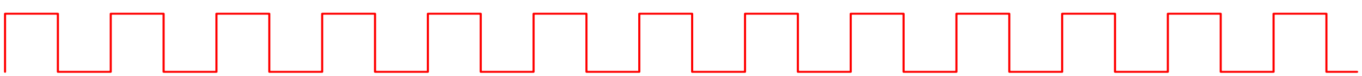
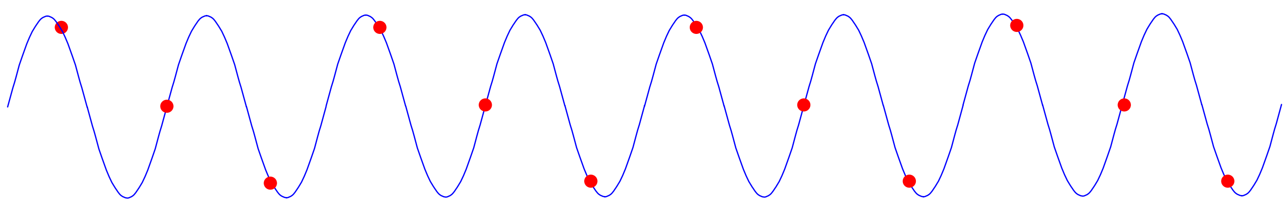


Sample with rising edges on the following clock (this could be Φ_1 for a SC filter)





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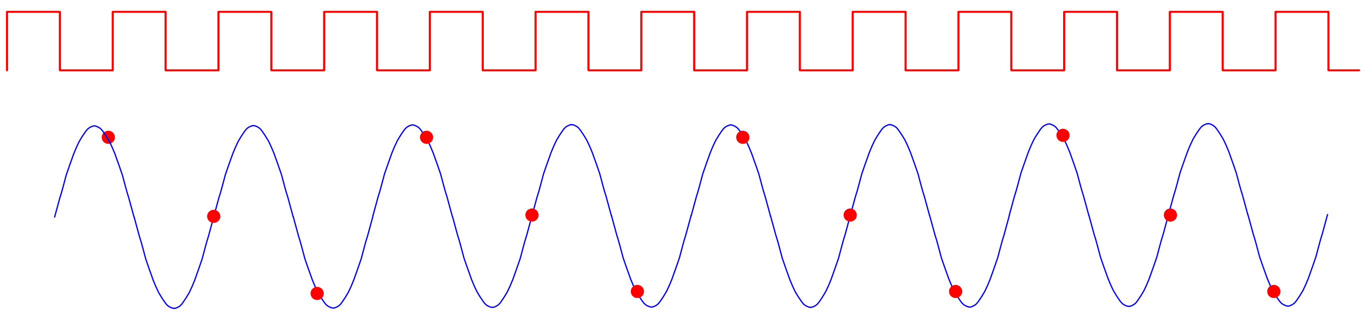
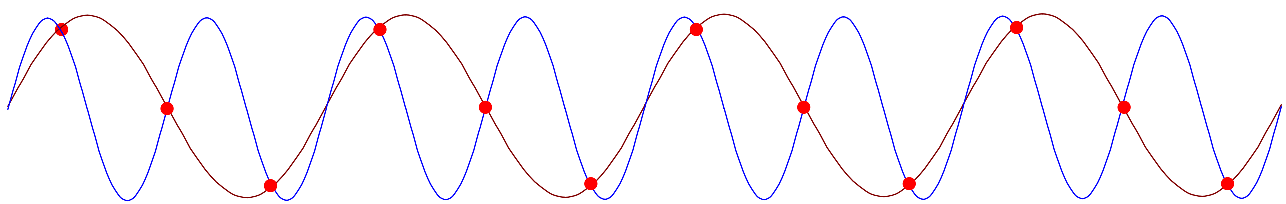


Now overlay the fundamental frequency signal on this sampled waveform

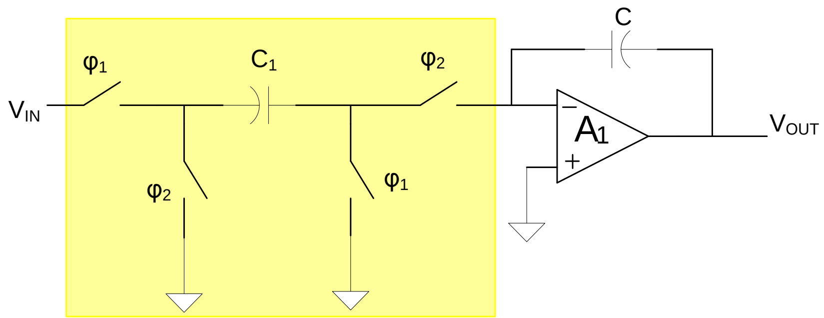


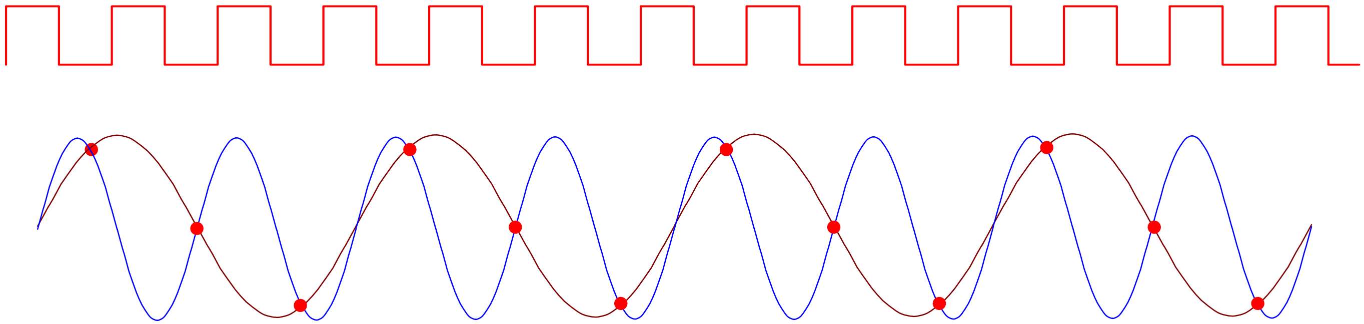
At these sample points, the samples of the two signals are indistinguishable

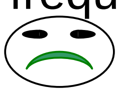
A similar observation will be observed if any of the other harmonically related signals are overlayed

A switched-capacitor filter can not distinguish between a fundamental and the harmonics if the ratio of the clock frequency to the signal frequency is too low

Consider a signal and harmonically-related signals

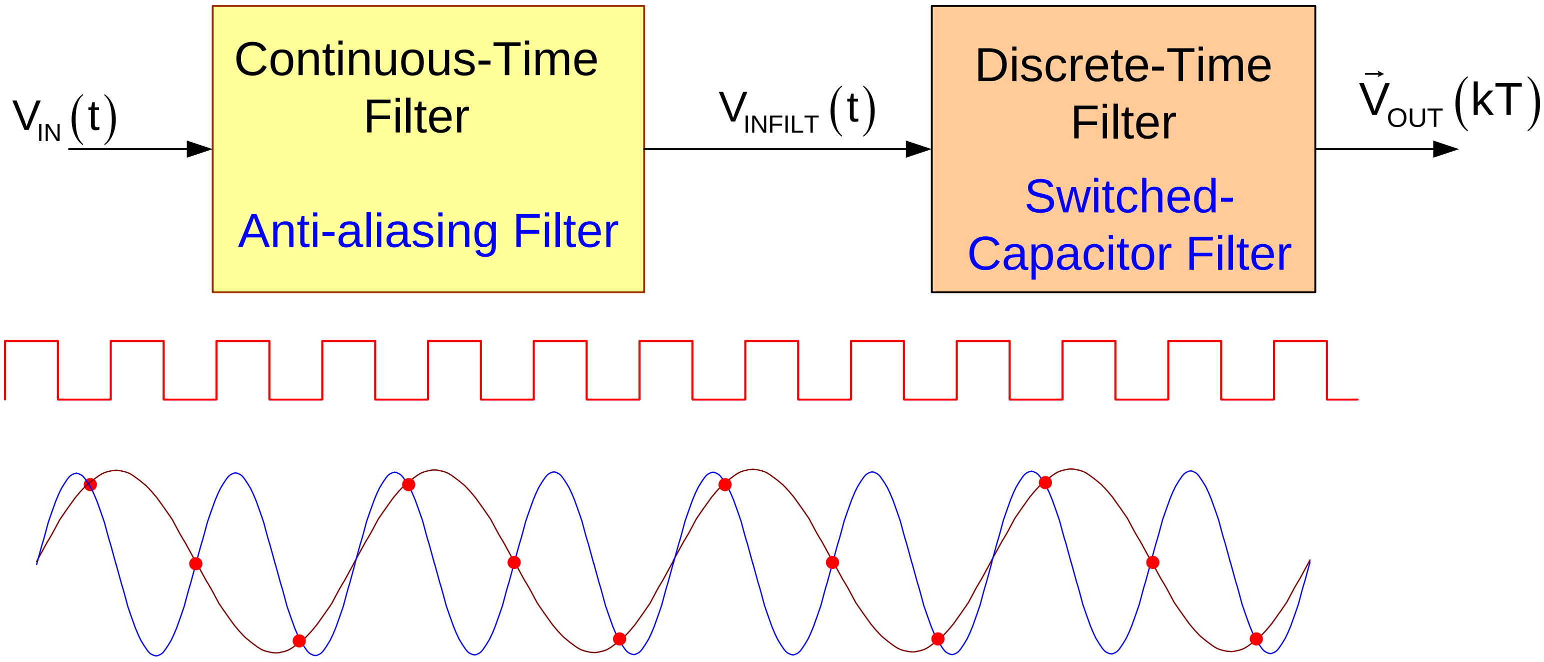




This aliases high frequency inputs (signals, noise, or even distortion) down to lower frequencies where it is indistinguishable from the lower frequency inputs 

How can this problem be resolved?

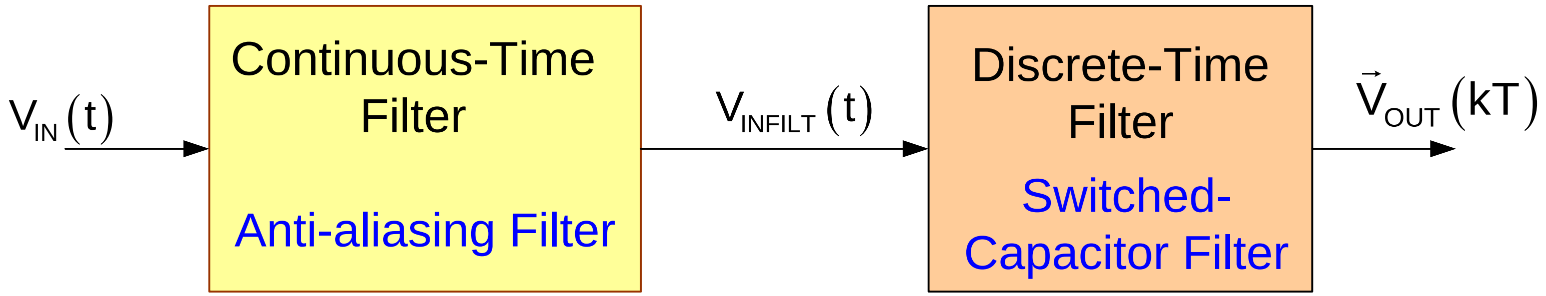
Anti-aliasing filter often required to limit frequency content at input to SC filters

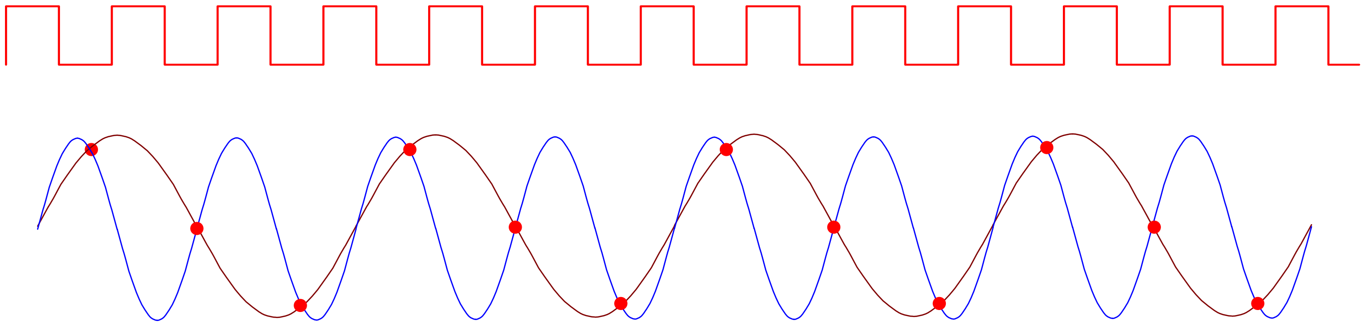


Does this completely negate the benefits of the SC filter?

- Anti-aliasing filter not needed if input is already band limited
- Anti-aliasing filter often continuous-time and occasionally off-chip
- Linearity requirements of anti-aliasing filter in passband are high
- Good passband linearity can be practically attained
- Transition sharpness and accuracy typically very relaxed in the anti-aliasing filter
- Passive first-order anti-aliasing filter often adequate

Anti-aliasing filter often required to limit frequency content at input to SC filters





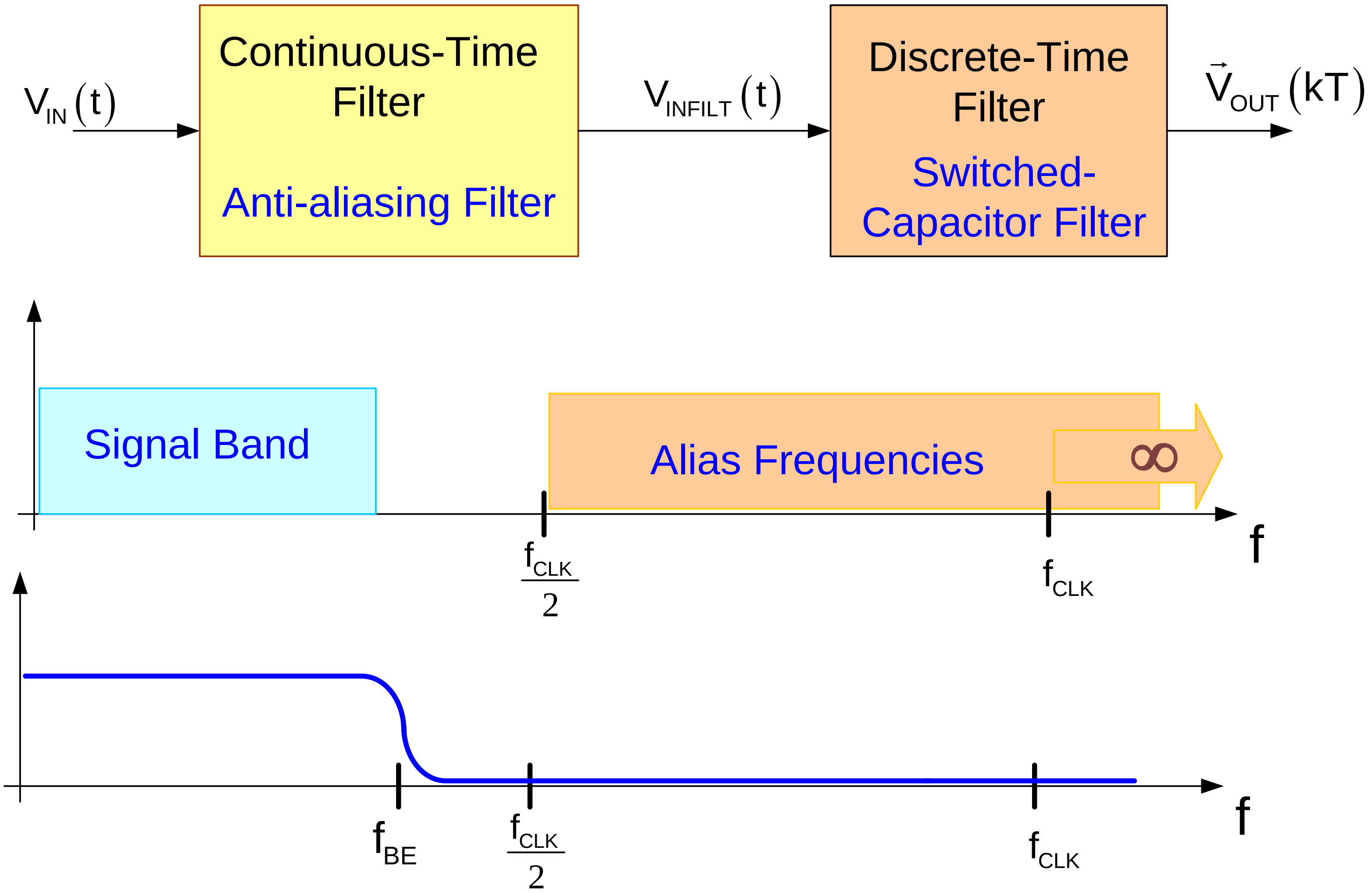
What are the band-edge requirements for the anti-aliasing filter?

Band edge of filter should limit all signals (and noise) at frequencies that are not wanted

What are SC clock requirements?

f_{CLK} must be at least twice the frequency of the signals that are to be passed by the SC filter

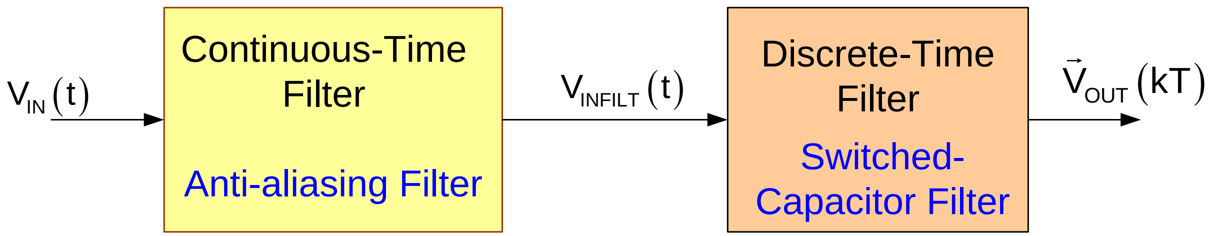
Anti-aliasing filter often required to limit frequency content at input to SC filters

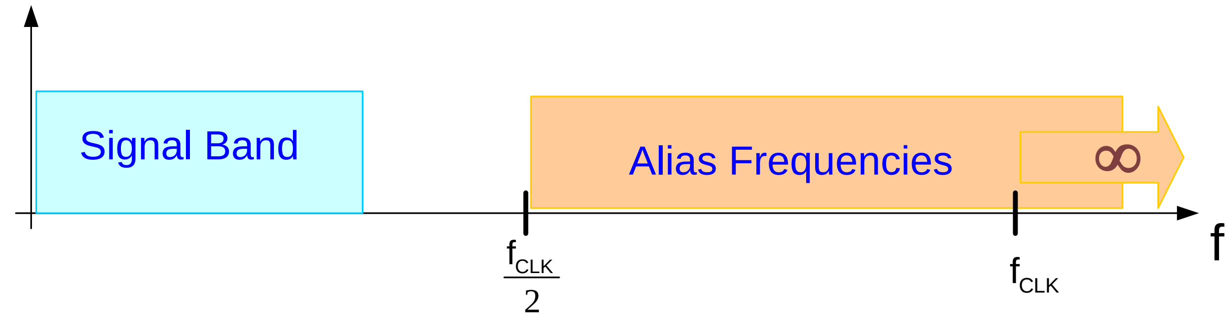


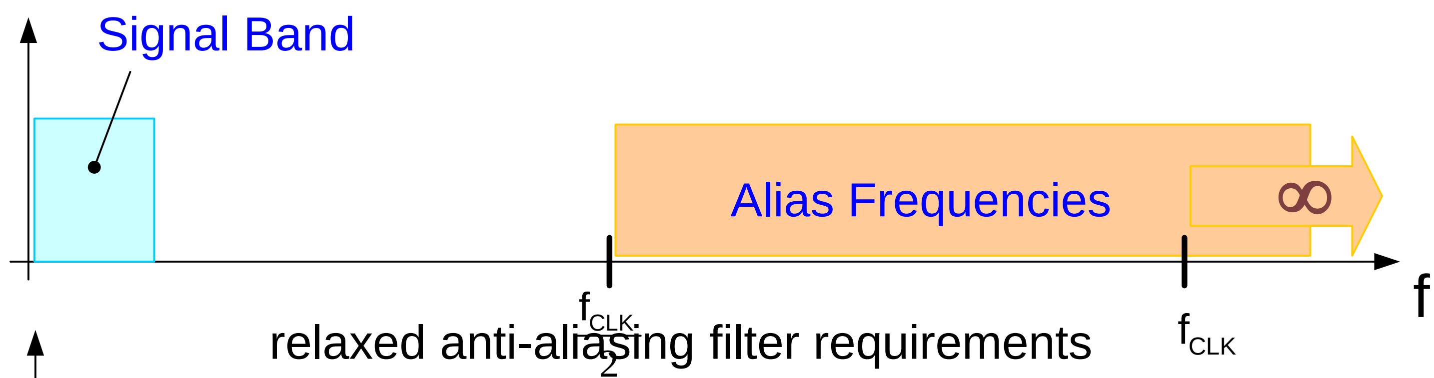
Antialiasing Filter

Must only attenuate at frequencies where energy is above an unacceptable level in the alias band

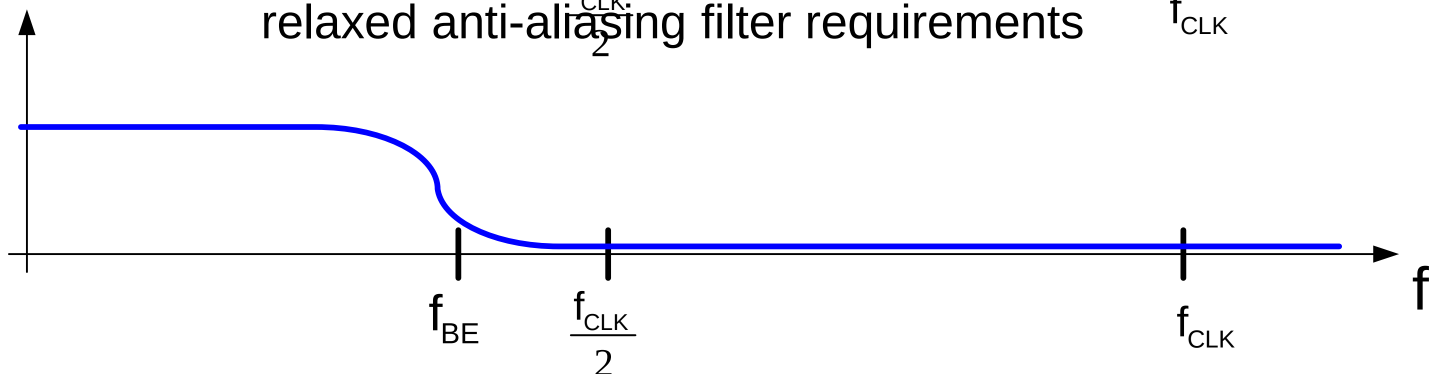
Anti-aliasing filter often required to limit frequency content at input to SC filters



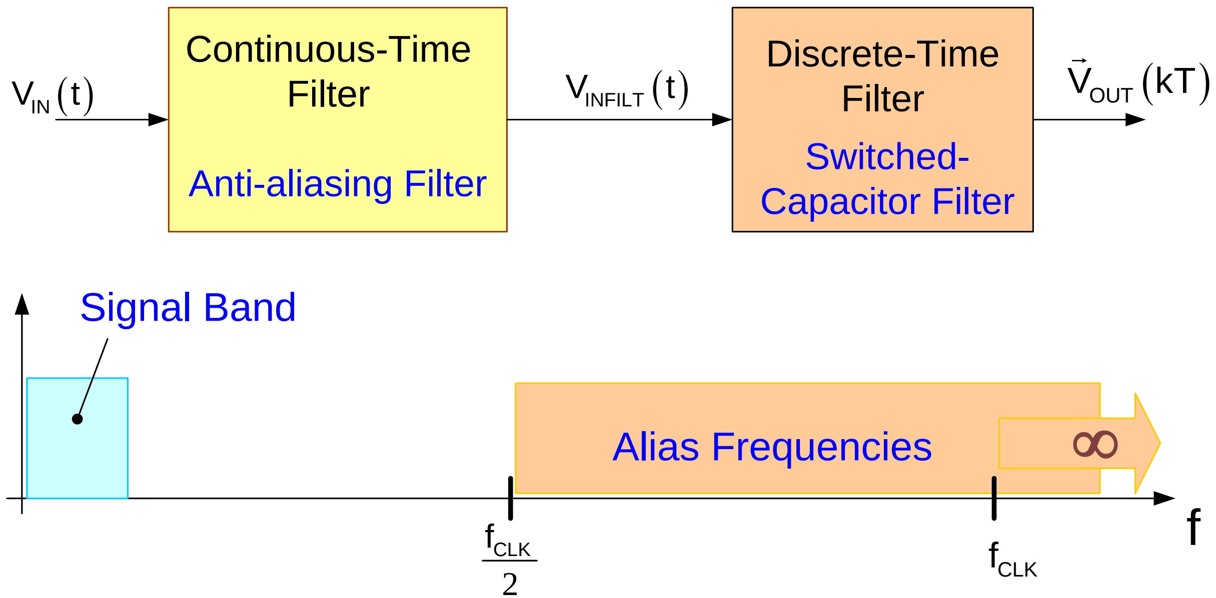




relaxed anti-aliasing filter requirements



Anti-aliasing filter often required to limit frequency content at input to SC filters



Why not just make the clock frequency >> signal band edge ?

Recall in the continuous-time RC-SC counterparts

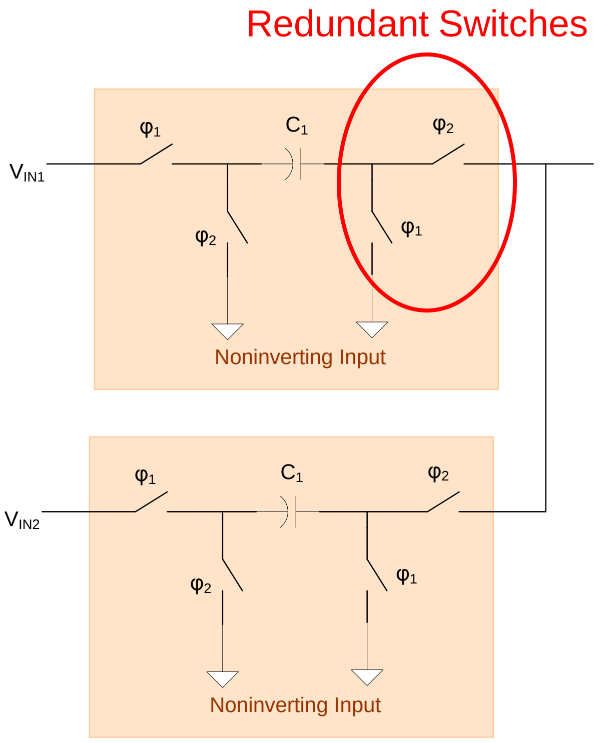
$$f_{POLES} \sim \frac{1}{RC} \simeq f_{CLK} \, {}^{C_1}\!/_{C}$$

Since f_{POLES} will be in the signal band (that is why we are building a filter) large f_{CLK} will require large capacitor ratios if $f_{CLK} >> f_{POLES}$

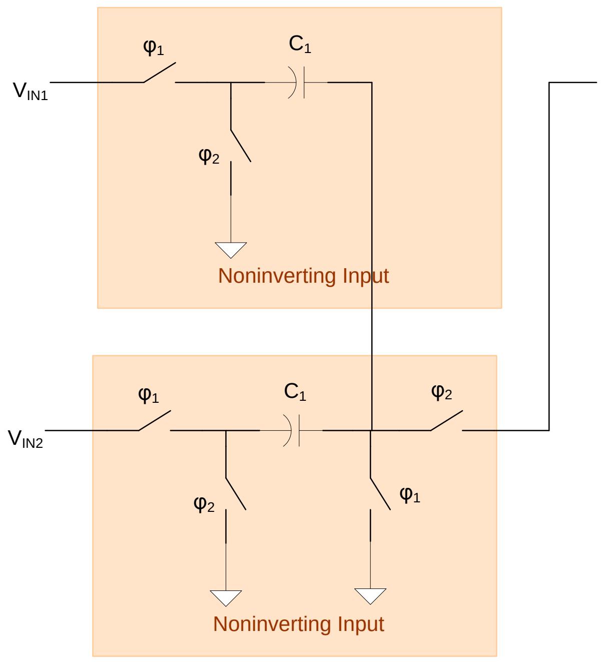
- Large capacitor ratios not attractive on silicon (area and matching issues)
- High f_{CLK} creates need for high GB in the op amps (area,power, and noise increase)

Often f_{CLK}/f_{POLES} in the 10:1 range proves useful (20:1 to 5:1 typical)

Elimination of Redundant Switches



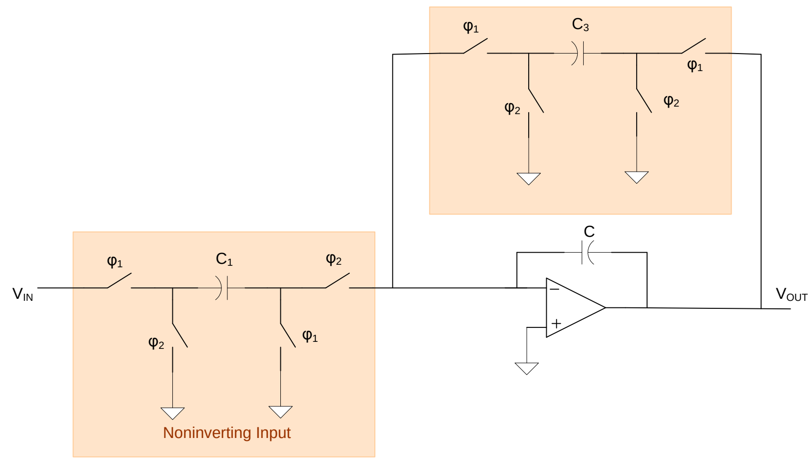
Switched-Capacitor Input
with Redundant Switches

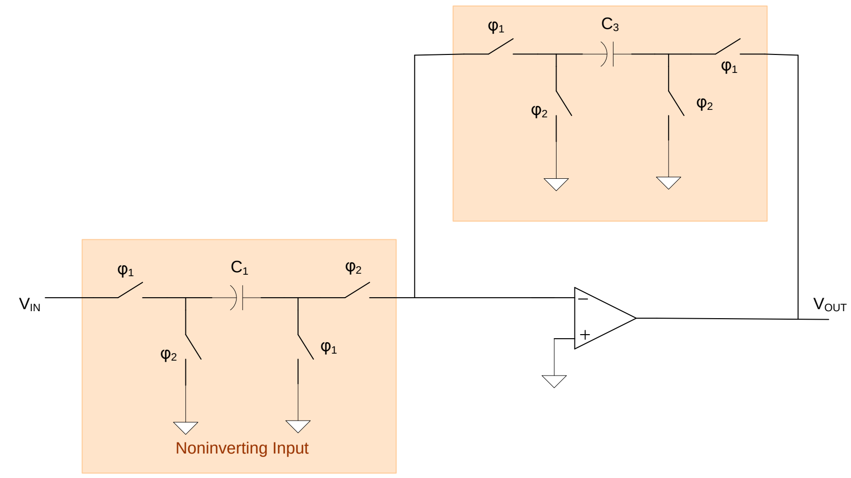


Switched-Capacitor Input with
Redundant Switches Removed

Although developed from the concept of SC-resistor equivalence, SC circuits often have no Resistor-Capacitor equivalents

Elimination of the Integration Capacitor



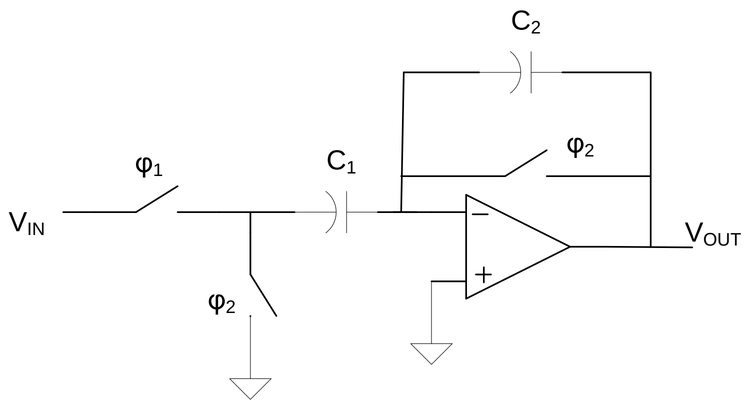


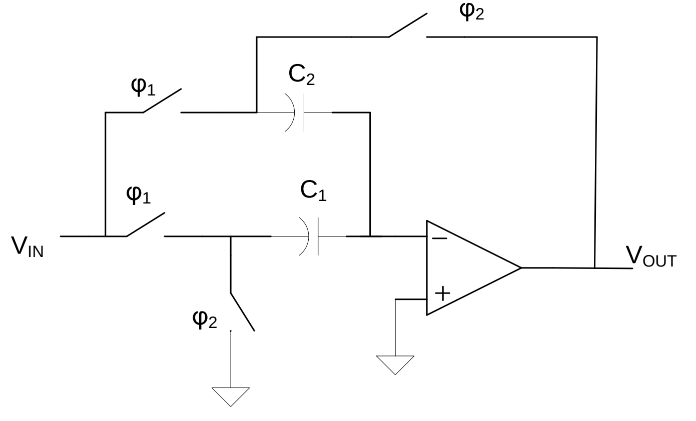
What happens if the integration capacitor is eliminated?

Serves as a SC amplifier with gain of $A_V=C_1/C_2$

SC amplifiers and SC summing amplifiers are widely used in filter and non-filter applications

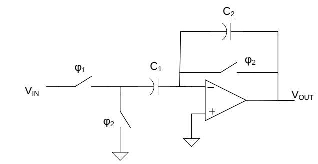
Switched Capacitor Amplifiers





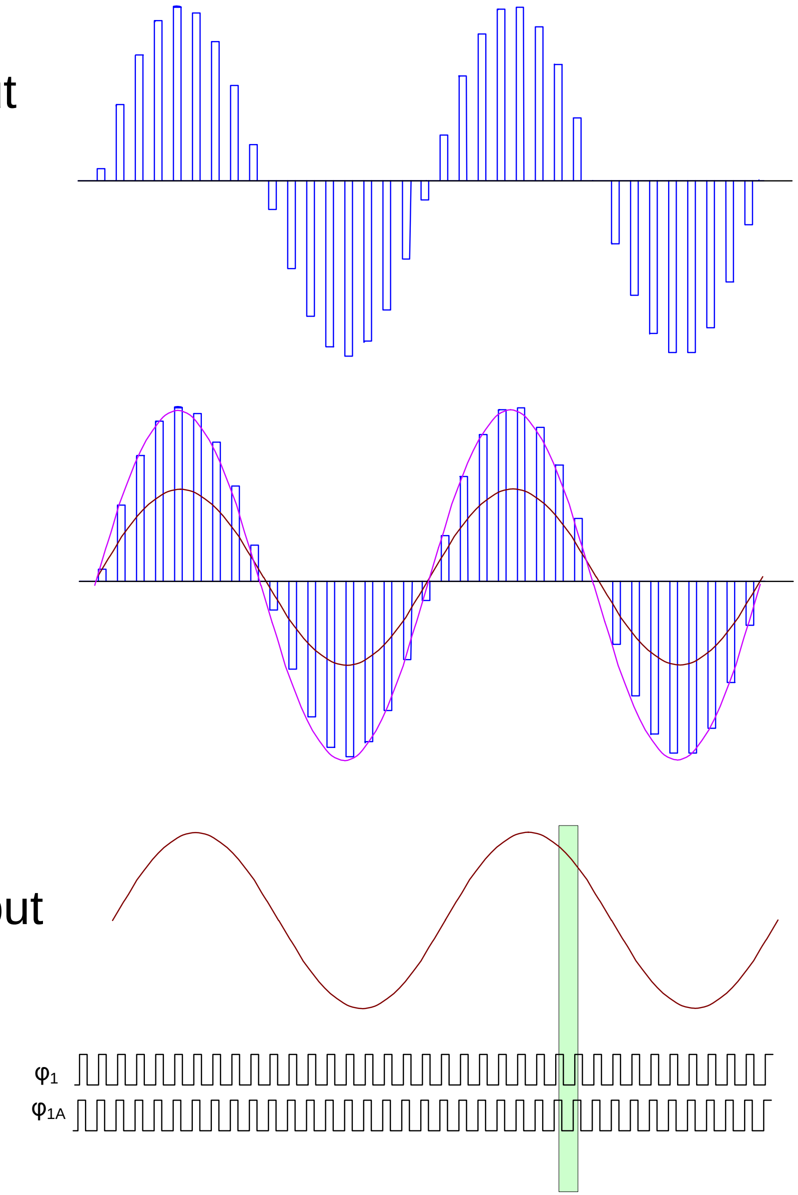
- Summing, Differencing, Inverting, and Noninverting SC Amplifiers Widely Used
- Significant reduction in switches from what we started with by eliminating C in SC integrator
- Must be stray insensitive in most applications
- Outputs valid only during one phase

Switched Capacitor Amplifiers



Output

Input



Voltage Mode Integrators

- Active RC (Feedback-based)
- MOSFET-C (Feedback-based)
- OTA-C
- TA-C

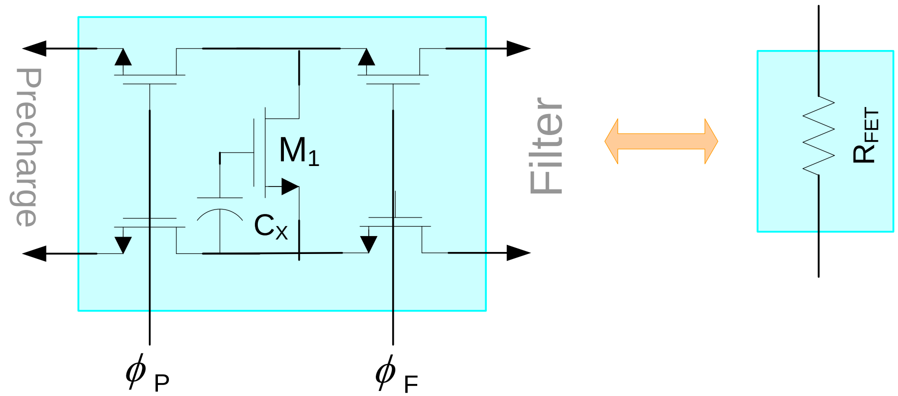
  Sometimes termed “current mode”

- Switched Capacitor
- Switched Resistor

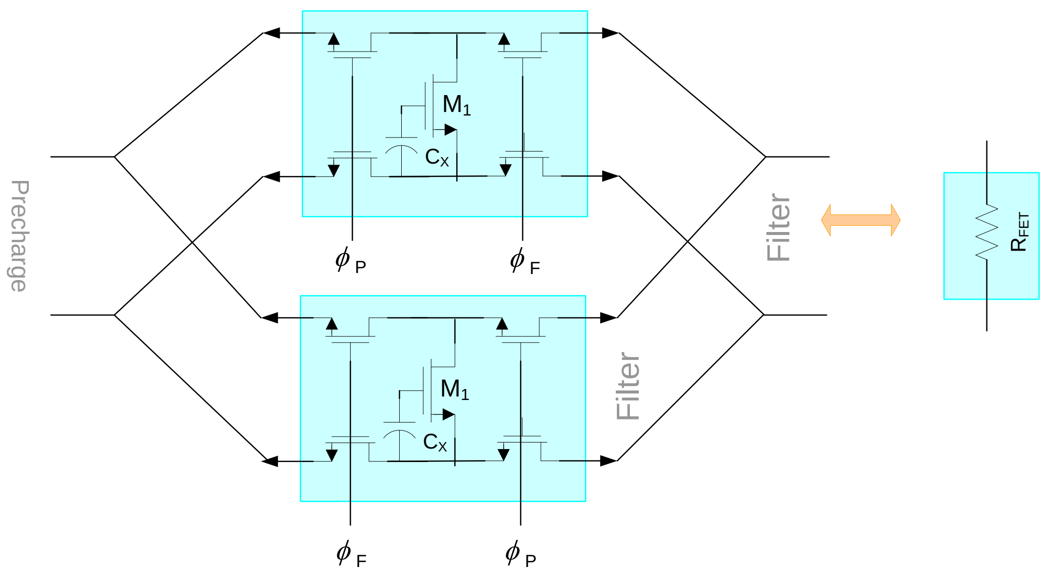
  Will discuss later

- Other Structures

Switched-Resistor Voltage Mode Integrators



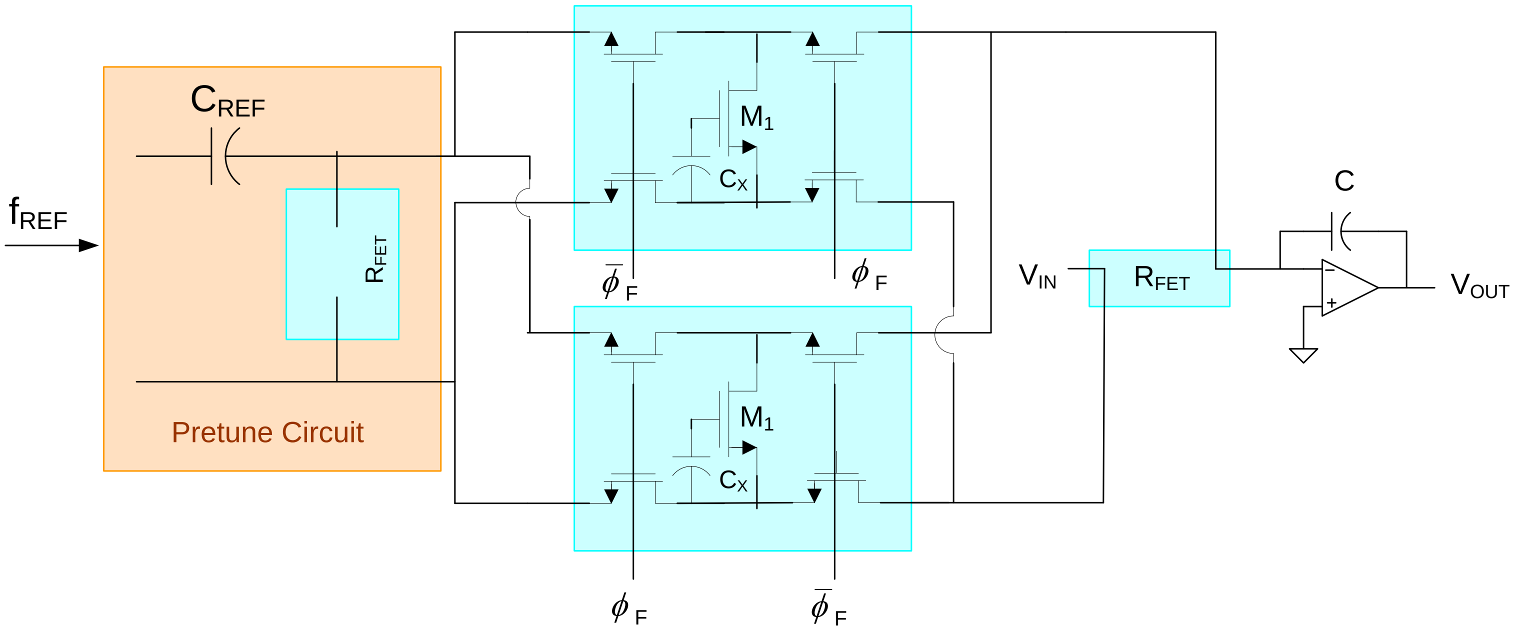
Observe that if a triode-region MOS device is switched between a precharge circuit and a filter circuit (or integrator) and V_{GS} is held constant, It will behave as a resistor while in the filter circuit



$$R_{FET} \simeq \frac{L}{\mu C_{OX} W (V_{GS} - V_T)}$$

Observe that if two such circuits are switched between a precharge circuit and a filter circuit (or integrator) and V_{GS} is held constant, It will behave as a resistor in the filter circuit at all times

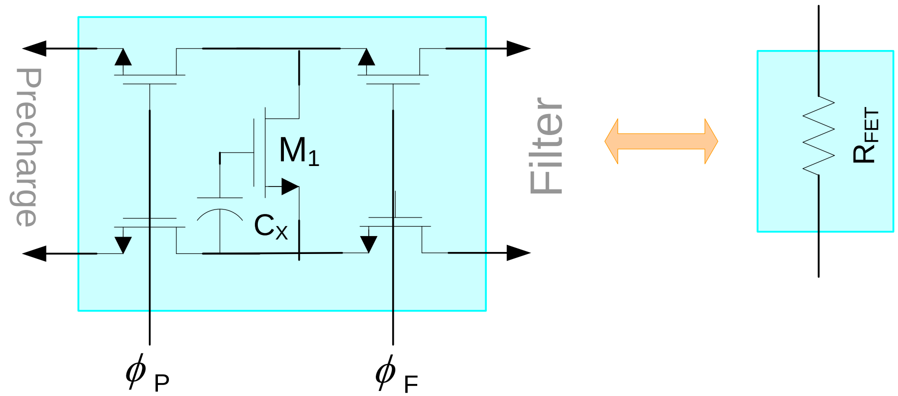
Switched-Resistor Voltage Mode Integrators



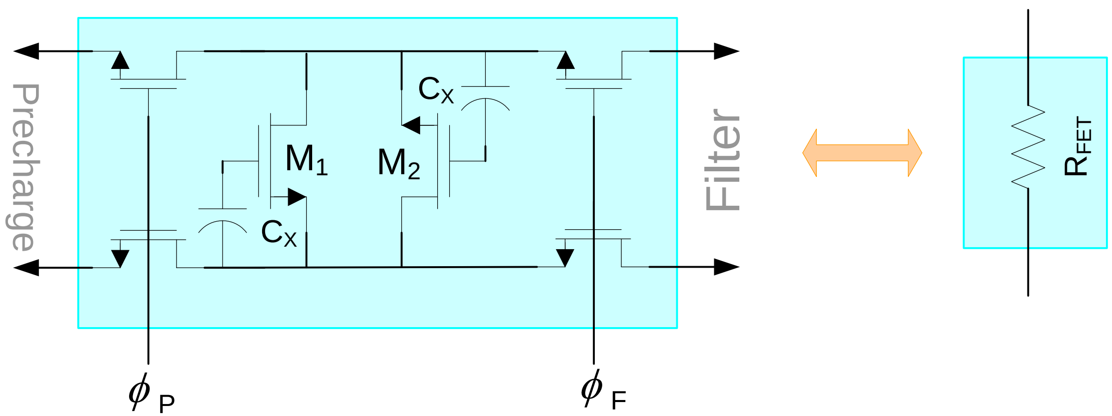
Switched-resistor integrator

- Clock frequency need only be fast enough to prevent droop on C_X
- Minor overlap or non-overlap of clock plays minimal role in integrator performance
- Switched-resistors can be used for integrator resistor or to replace all resistors in any filter
- Pretune circuit can accurately establish $R_{FET}C_{REF}$ product proportional to f_{REF}
- $R_{FET}C$ product is given by $R_{FET}C = R_{FET}C\frac{C_{REF}}{C_{REF}} = [R_{FET}C_{REF}]\bullet\left[\frac{C}{C_{REF}}\right]$ and is thus accurately controlled

Switched-Resistor Voltage Mode Integrators

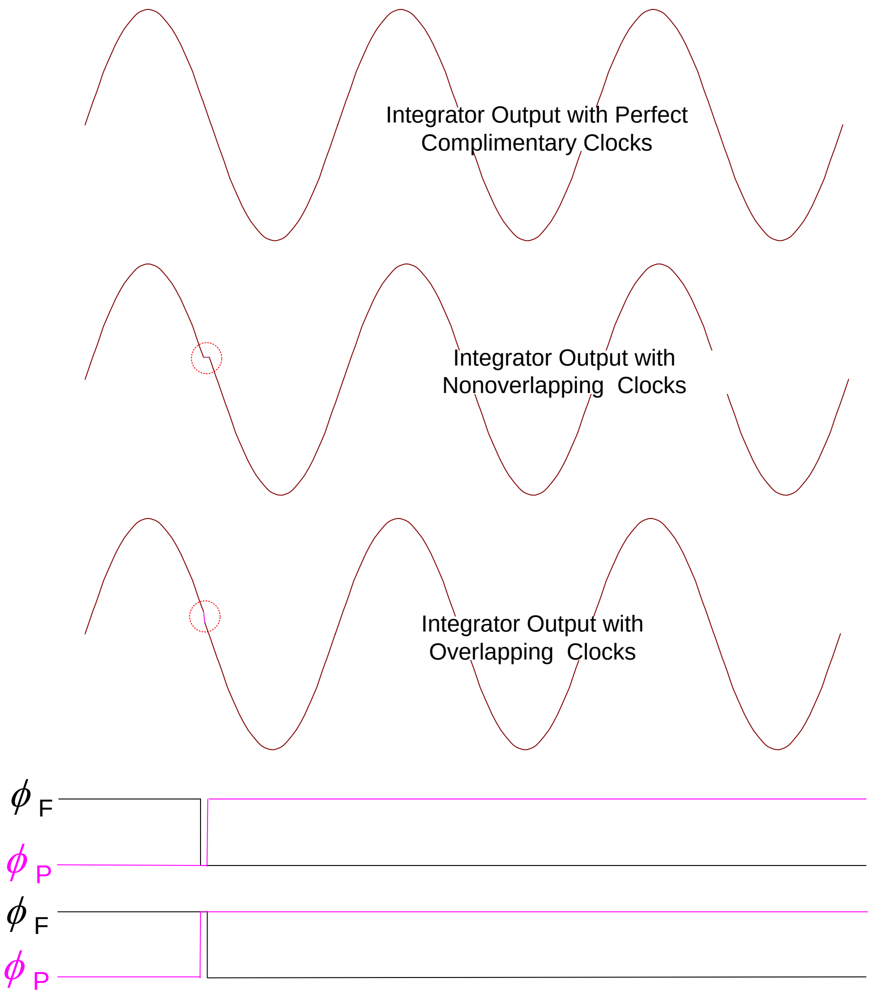


There are some modest nonlinearities in this MOSFET when operating in the triode region



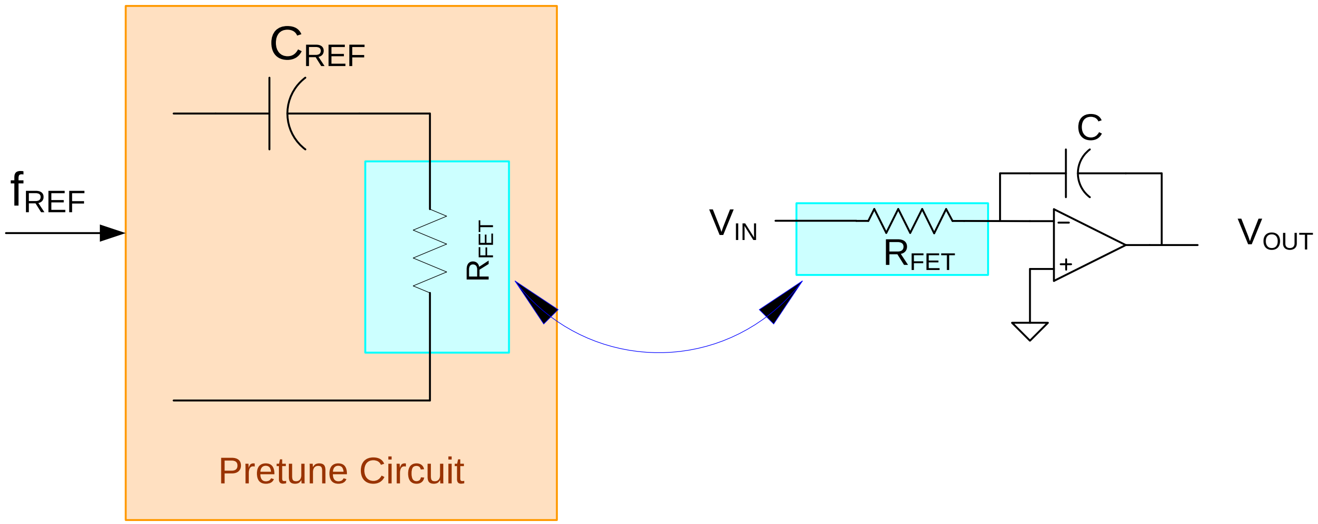
- Significant improvement in linearity by cross-coupling a pair of triode region resistors
- Perfectly cancels nonlinearities if square law model is valid for M_1 and M_2
- Only modest additional complexity in the Precharge circuit

Switched-Resistor Voltage Mode Integrators



- Aberrations are very small, occur very infrequently, and are further filtered
- Play almost no role on performance of integrator or filter

Switched-Resistor Voltage Mode Integrators



Switched-resistor integrator

- Accurate CR_{FET} products is possible
- Area reduced compared to Active RC structure because R_{FET} small
- Single pretune circuit can be used to “calibrate” large number of resistors
- Clock frequency not fast and not critical (but accuracy of f_{REF} is important)
- Since resistors are memoryless elements, no transients associated with switching
- Since filter is a feedback structure, speed limited by BW of op amp

EE 508
Lecture 29

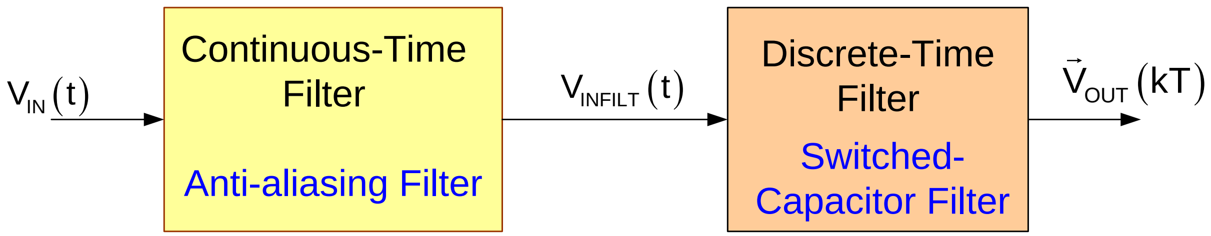
Integrator Design

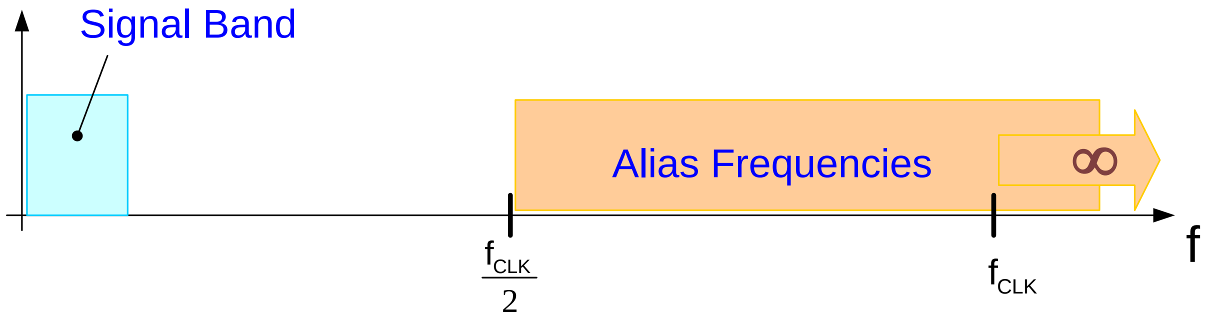
Metrics for comparing integrators

Current-Mode Integrators

Review from last time

Anti-aliasing filter often required to limit frequency content at input to SC filters





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Review from last time

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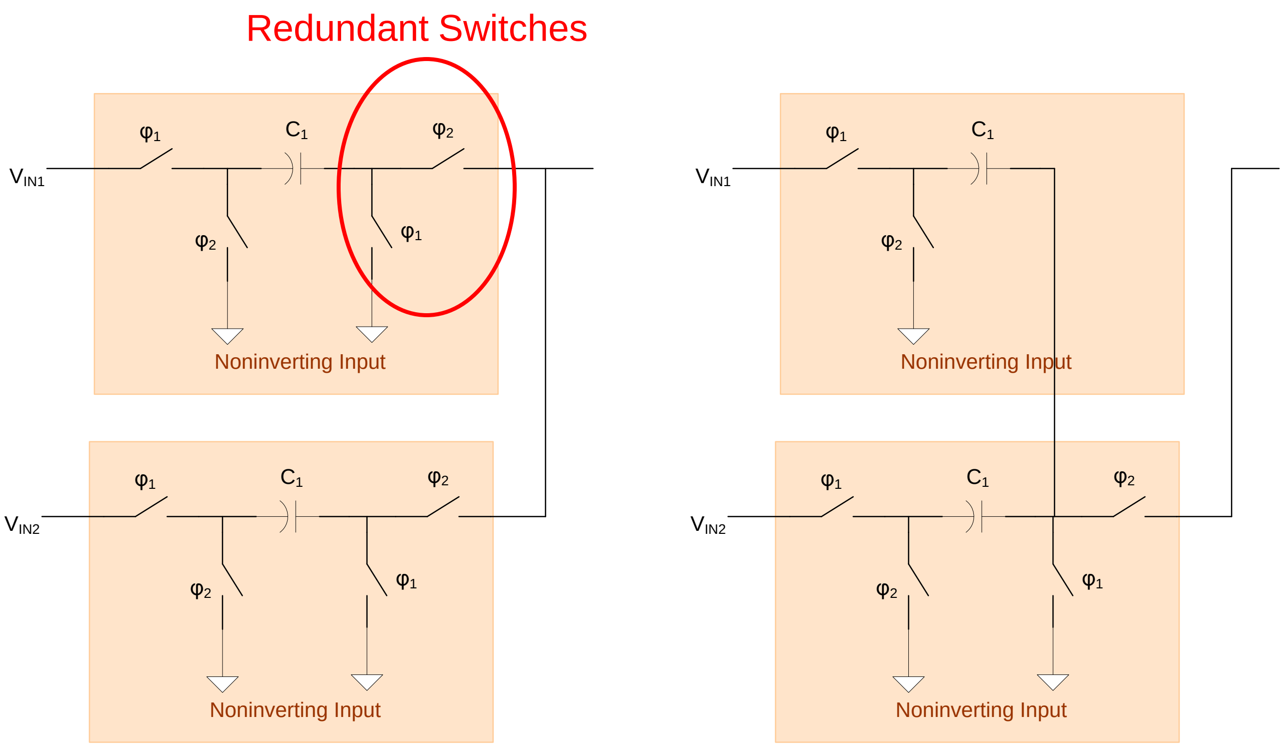

Switched-Capacitor Input with Redundant Switches

Switched-Capacitor Input with Redundant Switches Removed

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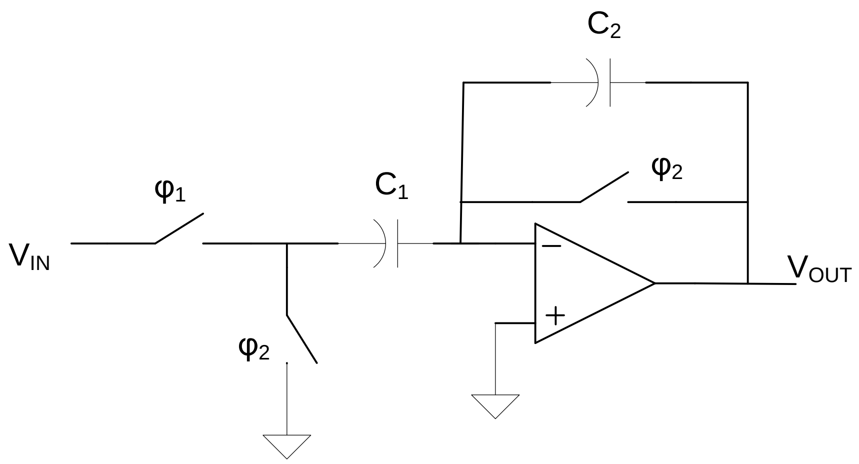
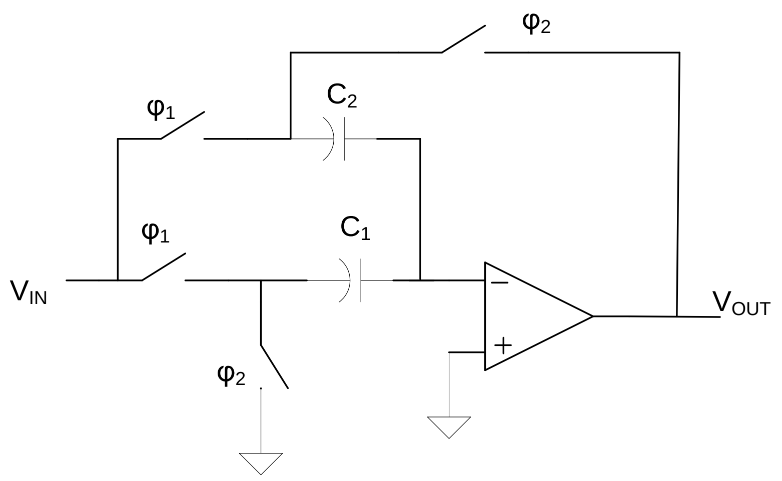
Review from last time

Switched Capacitor Amplifiers





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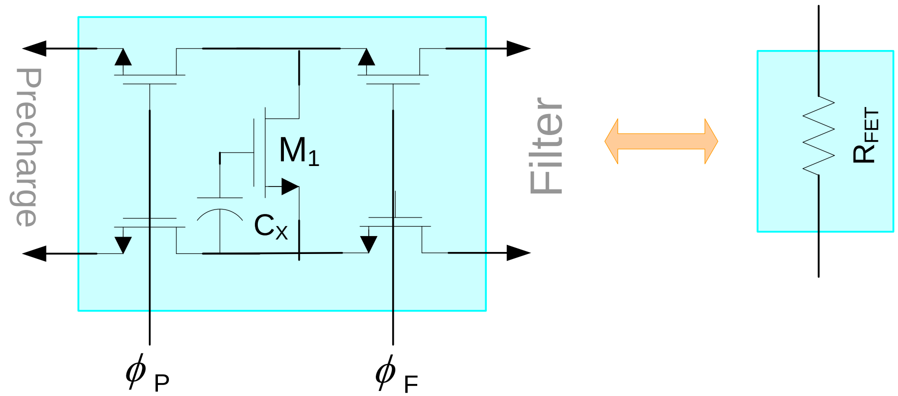
Review from last time

Switched-Resistor Voltage Mode Integrators



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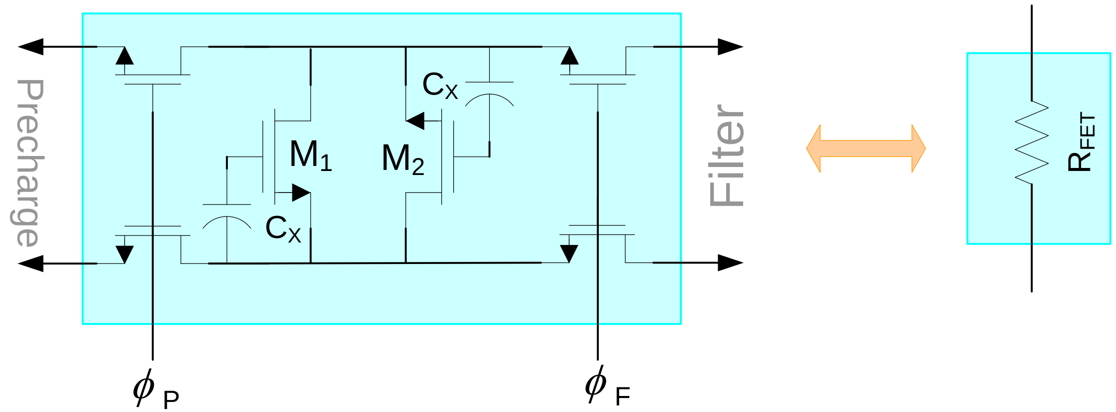
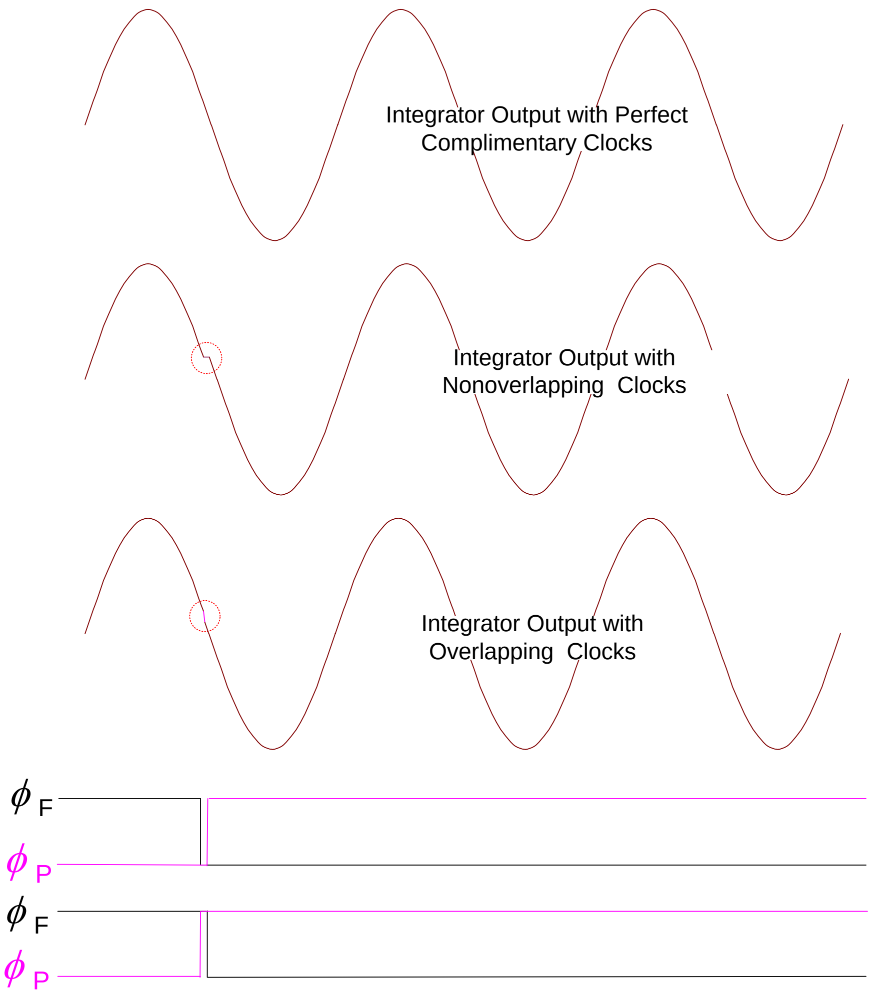


- Significant improvement in linearity by cross-coupling a pair of triode region resistors
- Perfectly cancels nonlinearities if square law model is valid for M_1 and M_2
- Only modest additional complexity in the Precharge circuit

Review from last time

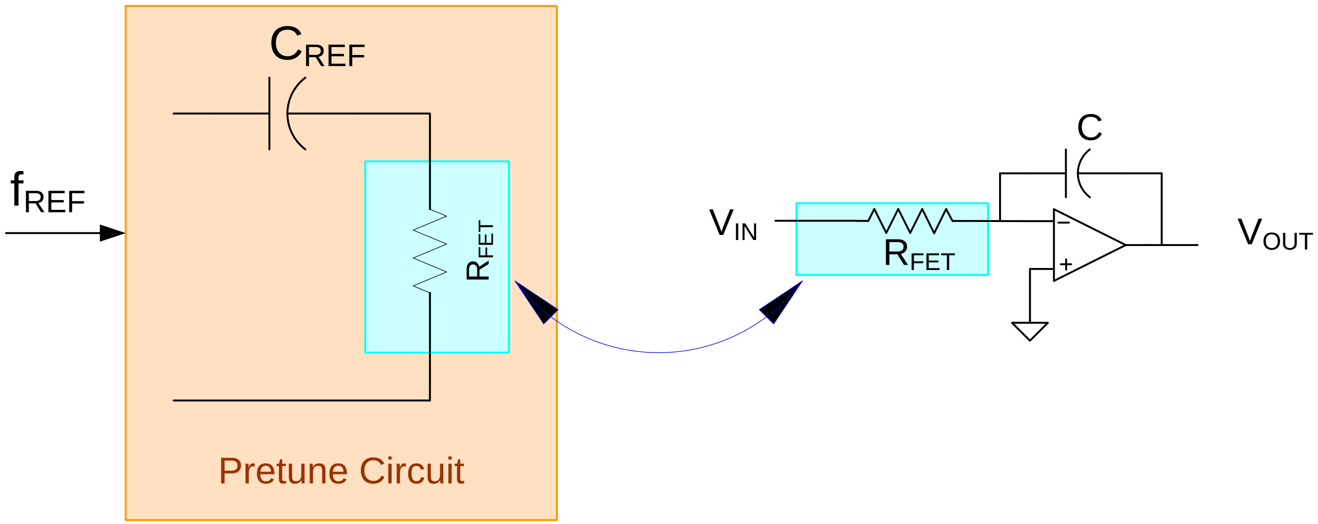
Switched-Resistor Voltage Mode Integrators



- Aberrations are very small, occur very infrequently, and are further filtered
- Play almost no role on performance of integrator or filter

Review from last time

Switched-Resistor Voltage Mode Integrators



Switched-resistor integrator

- Accurate CR_{FET} products is possible
- Area reduced compared to Active RC structure because R_{FET} small
- Single pretune circuit can be used to "calibrate" large number of resistors
- Clock frequency not fast and not critical (but accuracy of f_{REF} is important)
- Since resistors are memoryless elements, no transients associated with switching
- Since filter is a feedback structure, speed limited by BW of op amp

Voltage Mode Integrators

- Active RC (Feedback-based)
- MOSFET-C (Feedback-based)
- OTA-C
- TA-C

} Sometimes termed "current mode"

- Switched Capacitor
- Switched Resistor

} Will discuss later

- Other Structures

Have introduced a basic voltage-mode integrators in each of these approaches

All of these structures have applications where they are useful

Performance of all are limited by variability or Op Amp BW

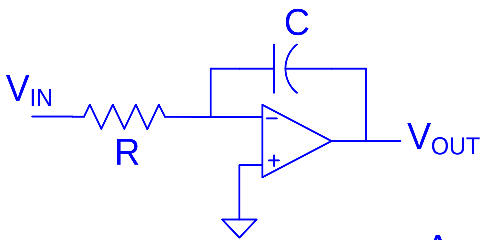
How can integrator performance be improved?

- Better op amps
- Better Integrator Architectures

How can the performance of integrator structures be compared?

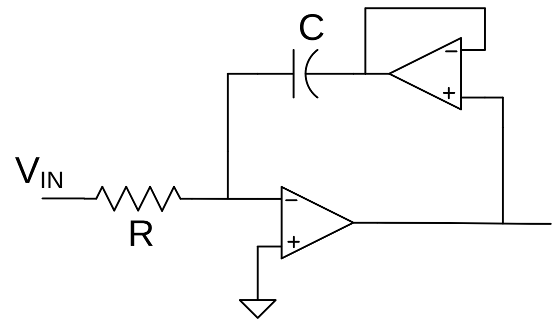
Need metric for comparing integrator performance

Are there other integrators in the basic classes that have been considered?



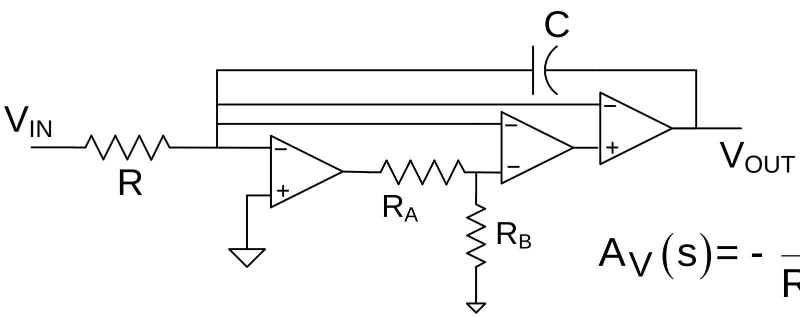
$$A_V(s) = -\frac{1}{RCs}$$

Miller Inverting



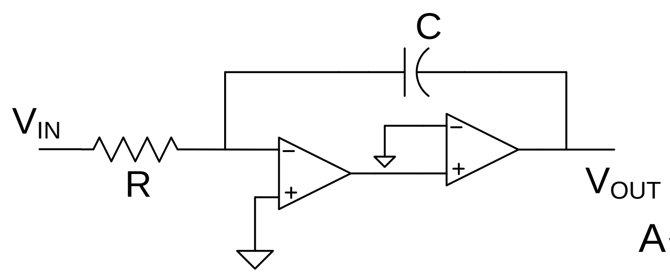
$$A_V(s) = -\frac{1}{RCs}$$

High-Q Inverting



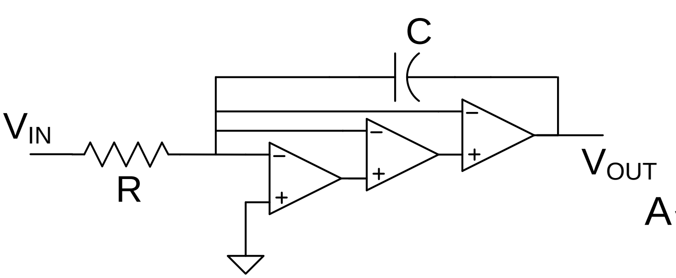
$$A_V(s) = -\frac{1}{RCs}$$

Zero Second Derivative Inverting



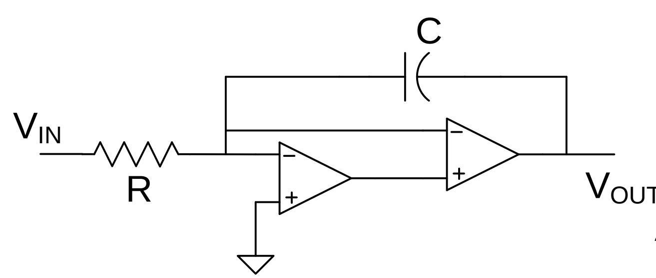
$$A_V(s) = -\frac{1}{RCs}$$

Cascaded Inverting



$$A_V(s) = -\frac{1}{RCs}$$

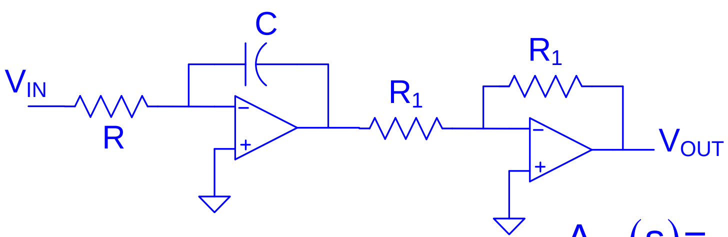
Zero Second Derivative Inverting



$$A_V(s) = -\frac{1}{RCs}$$

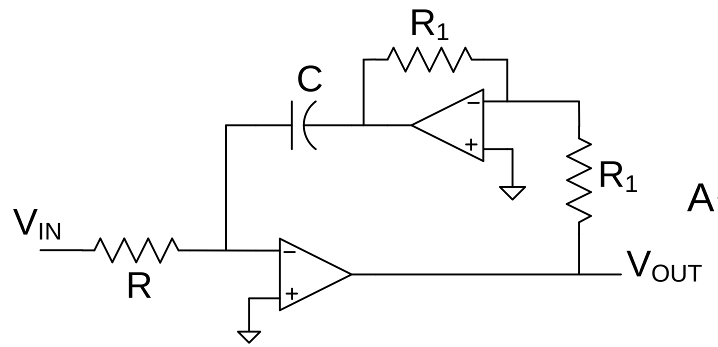
Zero Sensitivity Inverting

Are there other integrators in the basic classes that have been considered?



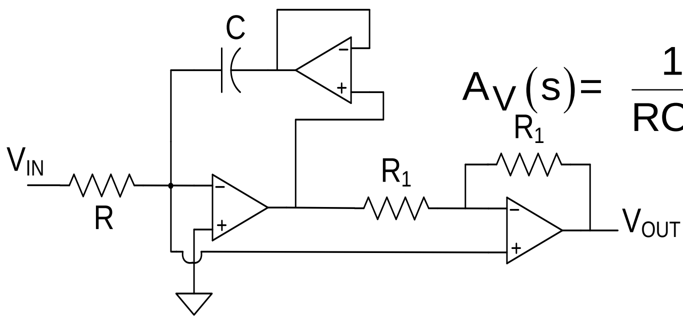
Miller Noninverting

$$A_V(s) = \frac{1}{RCs}$$



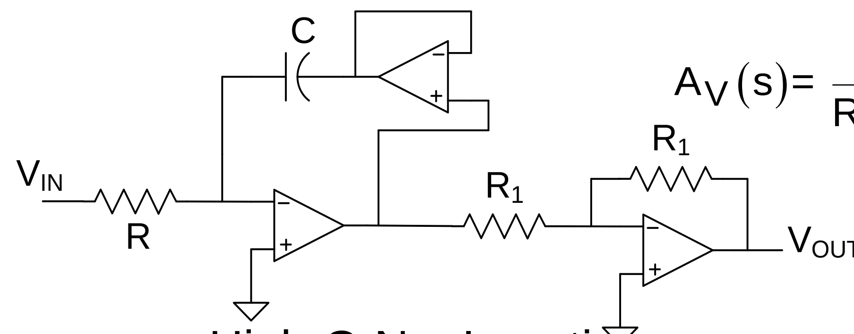
$$A_V(s) = \frac{1}{RCs}$$

Phase Lead Integrator



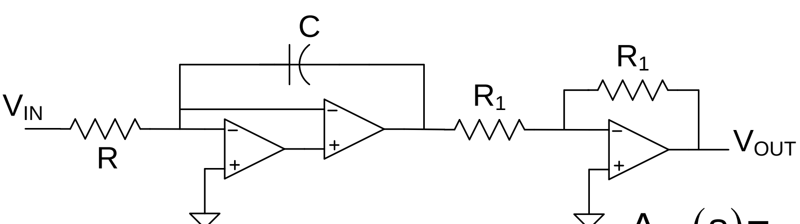
$$A_V(s) = \frac{1}{RCs}$$

Modified High-Q Noninverting



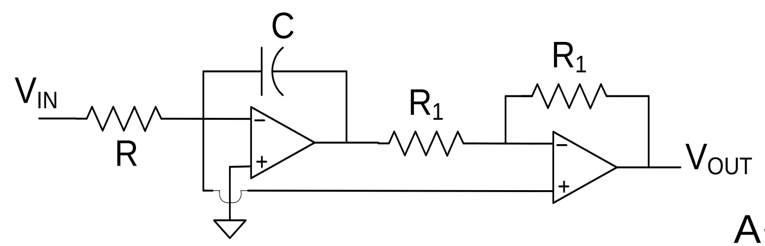
$$A_V(s) = \frac{1}{RCs}$$

High-Q NonInverting



$$A_V(s) = \frac{1}{RCs}$$

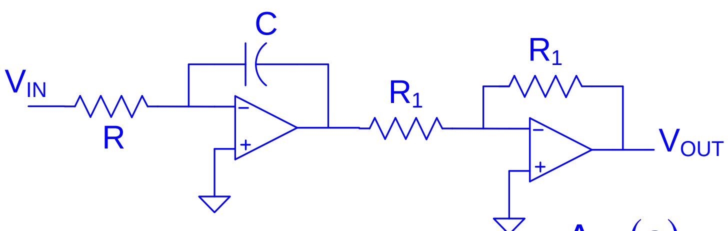
Zero Sensitivity Noninverting



$$A_V(s) = \frac{1}{RCs}$$

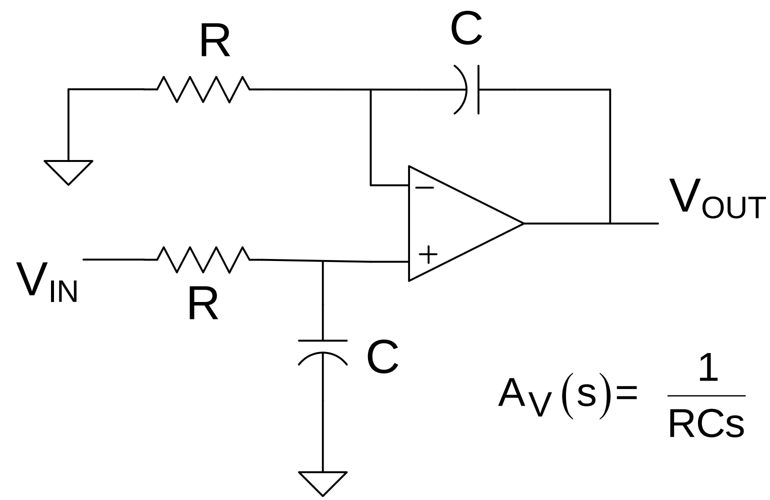
Modified Miller Noninverting

Are there other integrators in the basic classes that have been considered?



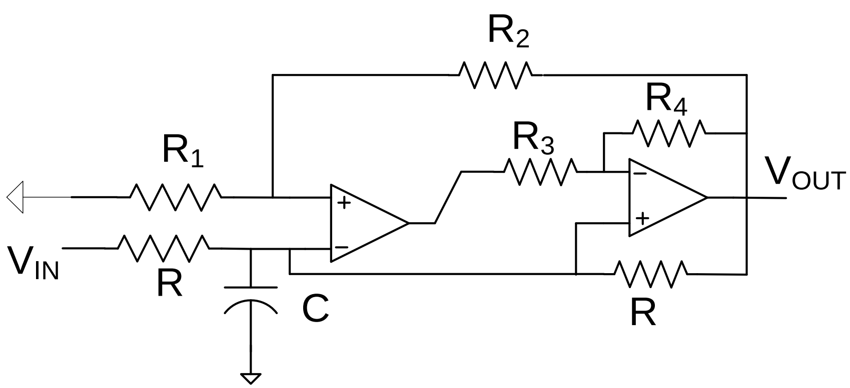
Miller Noninverting

$$A_V(s) = \frac{1}{RCs}$$



$$A_V(s) = \frac{1}{RCs}$$

Balanced Time Constant Noninverting



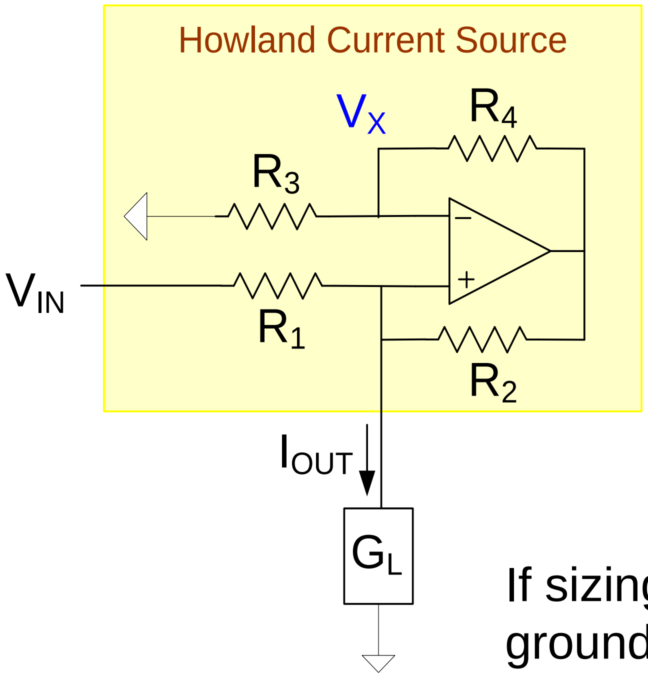
Zero Sensitivity Noninverting

$$A_V(s) = \frac{2}{RCs}$$

If R_1=R_2 and R_3=R_4

De Boo Integrator

Consider the Howland Current Source



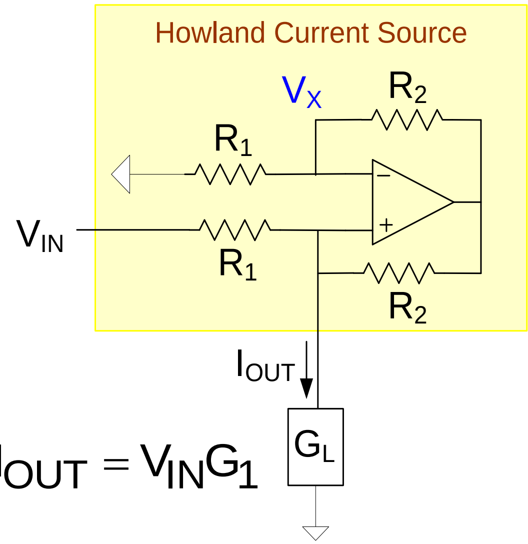
$$I_{OUT} = V_{IN}G_1 + \left[V_X \left(\frac{G_2 G_3}{G_4} - G_1 \right) \right]$$

If resistors sized so that $G_1 = \frac{G_2 G_3}{G_4}$

$$I_{OUT} = V_{IN}G_1$$

If sizing constraints are satisfied, behaves as a grounded constant-current source

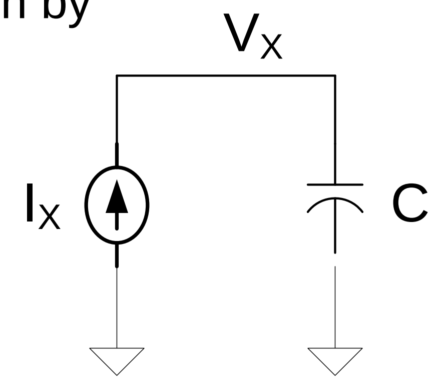
DeBoo Integrator



$$I_{OUT} = V_{IN}G_1$$

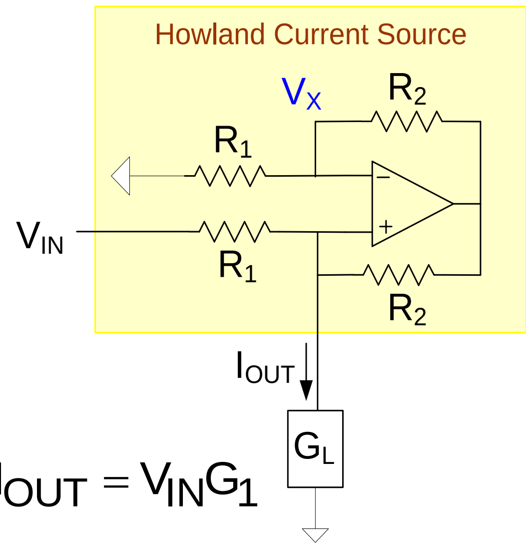
Observe that if a current source drives a grounded capacitor, then the nodal voltage on the capacitor is given by

$$V_X = I_X \frac{1}{sC}$$

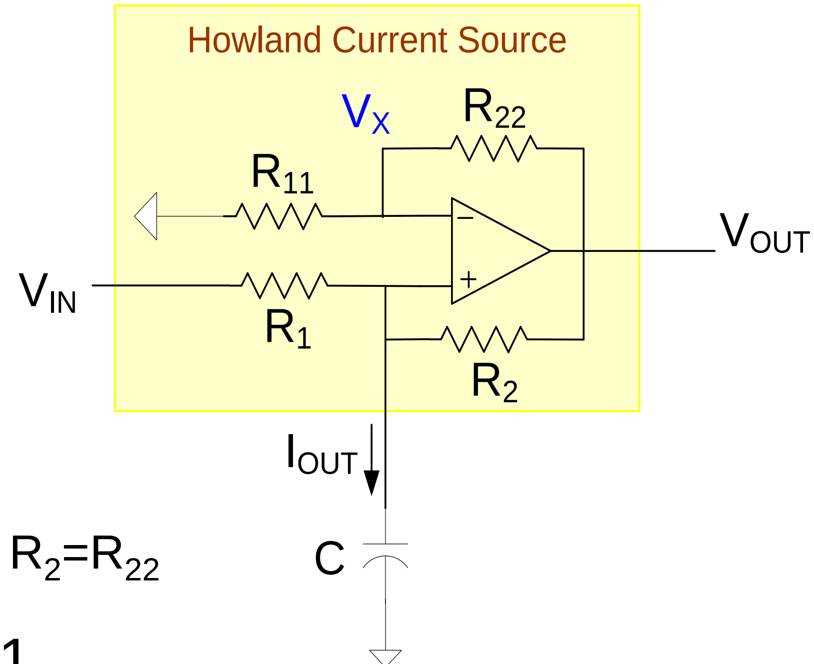


Thus, if we could make I_X proportional to V_{IN}, the voltage on the capacitor would be a weighted Integral of V_{IN}

De Boo Integrator



$$I_{OUT} = V_{IN}G_1$$



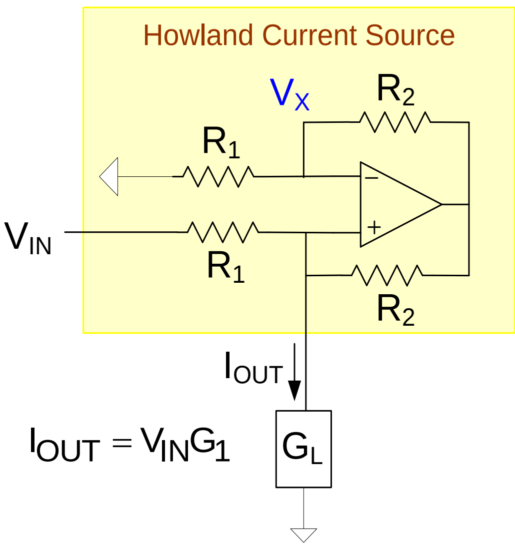
If $R_1=R_{11}$ and $R_2=R_{22}$

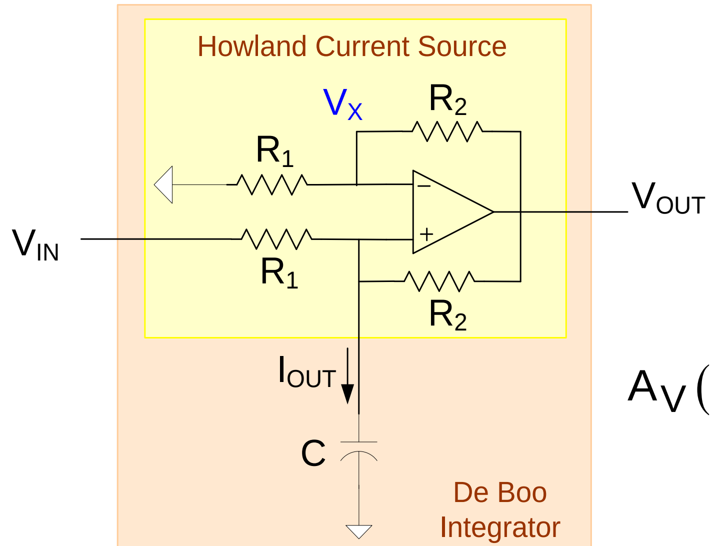
$$V_X = \frac{V_{IN}}{R_1}\frac{1}{sC}$$

$$V_{OUT} = V_X\left(1+\frac{R_{22}}{R_{11}}\right) = \frac{V_{IN}}{R_1}\frac{1}{sC}\left(1+\frac{R_{22}}{R_{11}}\right)$$

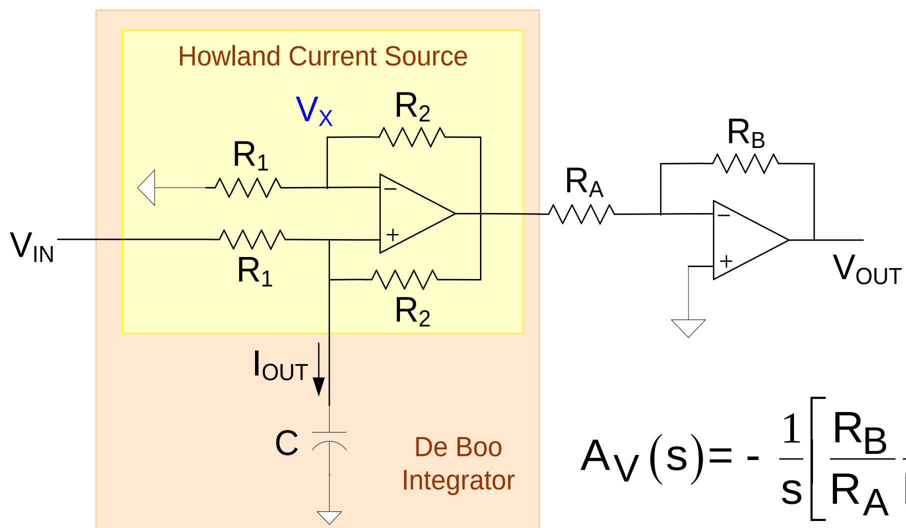
$$A_V(s) = \frac{1}{s}\left[\frac{1}{R_1C}\left(1+\frac{R_{22}}{R_{11}}\right)\right]$$

De Boo Integrator





$$A_V(s) = \frac{1}{s}\left[\frac{1}{R_1C}\left(1+\frac{R_{22}}{R_{11}}\right)\right]$$



$$A_V(s) = -\frac{1}{s}\left[\frac{R_B}{R_A}\frac{1}{R_1C}\left(1+\frac{R_{22}}{R_{11}}\right)\right]$$

Many different integrator architectures that ideally provide the same gain

Similar observations can be made for other classes of integrators

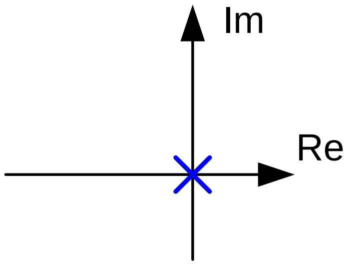
How can the performance of an integrator be characterized and how can integrators be compared?

How can the performance of an integrator be characterized and how can integrators be compared?

Consider Ideal Integrator Gain Function

$$A_V(s) = \frac{I_0}{s} \qquad A_V(j\omega) = \frac{I_0}{j\omega}$$

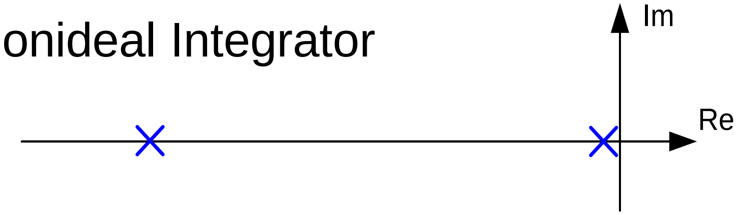
Ideal Integrator



Consider a nonideal integrator Gain Function

$$A_V(s) = \frac{\alpha I_{01}}{s+\alpha} A_{OO}(s)$$

Nonideal Integrator



Key characteristics of an ideal integrator:

- Magnitude of the gain at I_0=1
- Phase of integrator always 90^o
- Gain decreases with 1/ω

Are any of these properties more critical than others?

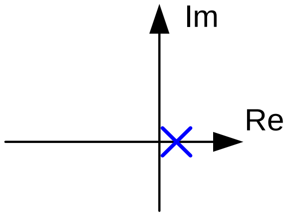
In many applications:

Key property of ideal integrator is a phase shift of 90^o at frequencies around I_0!

How can the performance of an integrator be characterized and how can integrators be compared?

Is stability of an integrator of concern?

Ideal Integrator



- Ideal integrator is not stable
- Integrator function is inherently ill-conditioned
- Integrator is almost never used open-loop
- Stability of integrator not of concern, stability of filter using integrator is of concern
- Some integrators may cause unstable filters, others may result in stable filters

How can the performance of an integrator be characterized and how can integrators be compared?

Express $A_V(j\omega)$ as

$$A_V(j\omega)=\frac{\pm 1}{R(\omega)+jX(\omega)}$$

where $R(\omega)$ and $X(\omega)$ are real and represent the real and imaginary parts of the denominator respectively

$$\text{Phase}=-\tan^{-1}\left(\frac{X(\omega)}{R(\omega)}\right)$$

Ideally $R(\omega)=0$

Definition: The Integrator Q factor is the ratio of the imaginary part of the denominator to the real part of the denominator

$$Q_{INT}=\left(\frac{X(\omega)}{R(\omega)}\right)$$

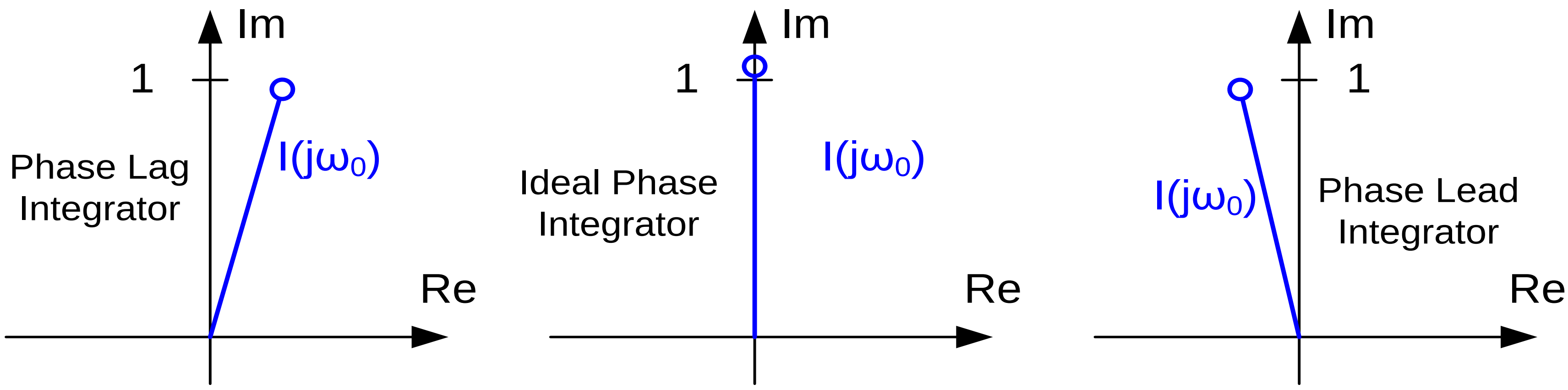
Typically most interested in Q_{INT} at the nominal unity gain frequency of the integrator

How can the performance of an integrator be characterized and how can integrators be compared?

Express $A_V(j\omega)$ as

$$I_V(j\omega) = \frac{\pm 1}{R(\omega) + jX(\omega)}$$

Lead/Lag Characteristics for Inverting Integrators

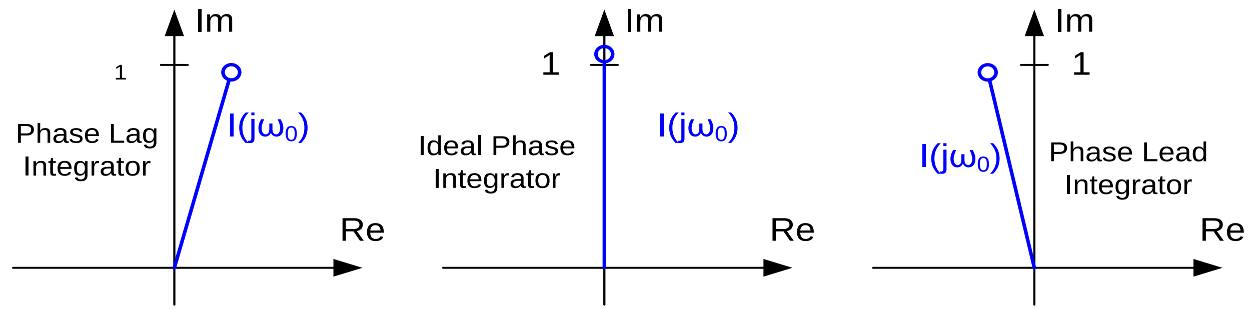


For Phase Lag Integrators, $R(\omega)$ is negative
For Phase Lead integrators, $R(\omega)$ is positive

How can the performance of an integrator be characterized and how can integrators be compared?

Lead/Lag Characteristics for Inverting Integrators

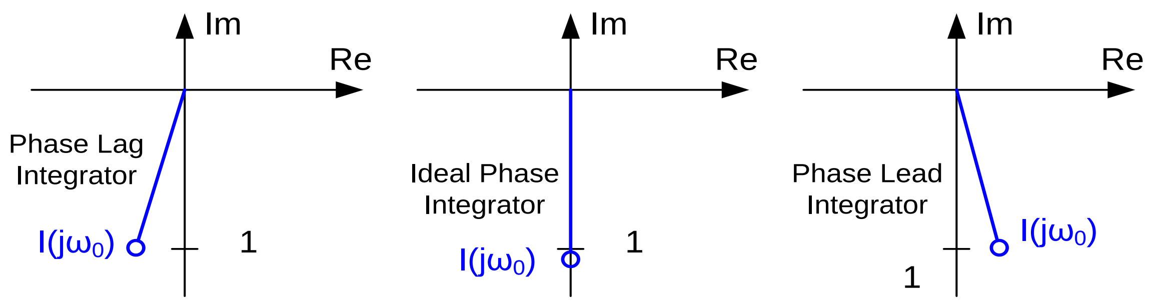
$$I_V(j\omega)=\frac{-1}{R(\omega)+jX(\omega)}$$



For Phase Lag Integrators, R(ω) and X(ω) have opposite signs. For Phase Lead integrators, R(ω) and X(ω) have the same sign. Phase shift ideally 90º

Lead/Lag Characteristics for Noninverting Integrators

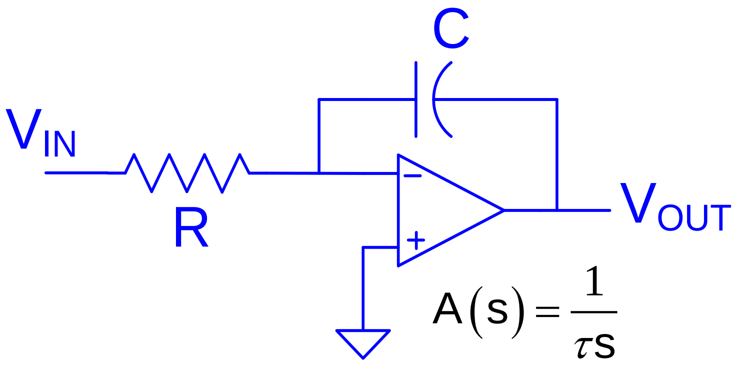
$$I_V(j\omega)=\frac{1}{R(\omega)+jX(\omega)}$$



For Phase Lag Integrators, R(ω) and X(ω) have opposite signs. For Phase Lead integrators, R(ω) and X(ω) have the same sign. Phase shift ideally 270º

Integrator Q Factor

Consider Miller Inverting Integrator



$$A_V(s)=\frac{-1}{RCs+\tau s(1+RCs)}$$

$$A_V(s)=\frac{-1}{RCj\omega+\tau j\omega(1+RCj\omega)}$$

$$A_V(s)=\frac{-1}{-\tau\omega^2RC+j(\omega[RC+\tau])}$$

Normalizing by ω_n= ωRC and τ_n = τ/RC = I_{0n}/GB

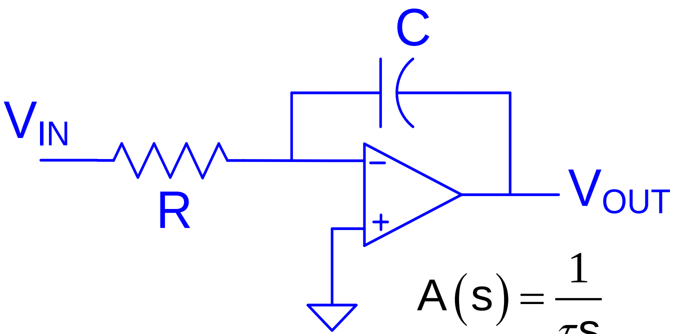
$$A_V(s)=\frac{-1}{-\tau_n\omega_n{}^2+j(\omega_n[1+\tau_n])}$$

Observe this integrator has excess phase shift (more than 90º in the denominator) at all frequencies

Integrator Q Factor

$$A_V(j\omega)=\frac{\pm 1}{R(\omega)+jX(\omega)} \qquad Q_{INT}=\left(\frac{X(\omega)}{R(\omega)}\right)$$

Consider Miller Inverting Integrator

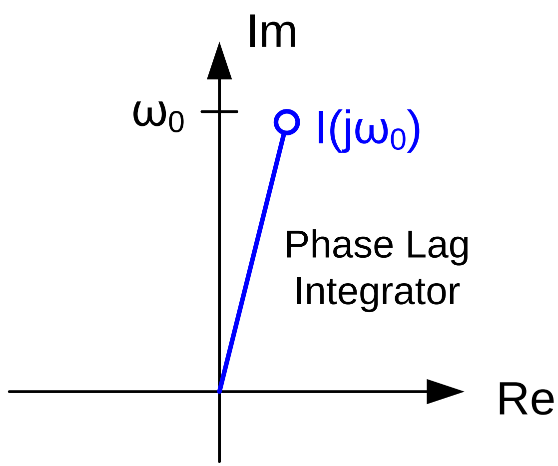


$$A(s)=\frac{1}{\tau s}$$

$$A_V(s)=\frac{-1}{-\tau_n\omega_n^2+j\left(\omega_n[1+\tau_n]\right)}$$

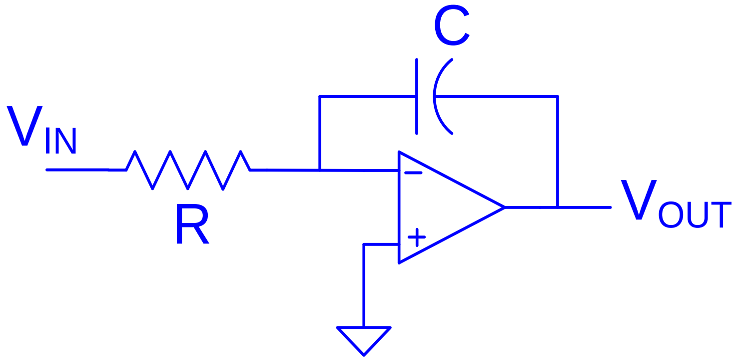
$$Q_{INT}=-\frac{\omega_n[1+\tau_n]}{\tau_n\omega_n^2}\simeq-\frac{1}{\tau_n\omega_n}=-\frac{GB}{\omega}=-A$$

Since the phase is less than 90º, the Miller Inverting Integrator is a Phase Lag Integrator and Q_{INT} is negative



Integrator Pole Locations

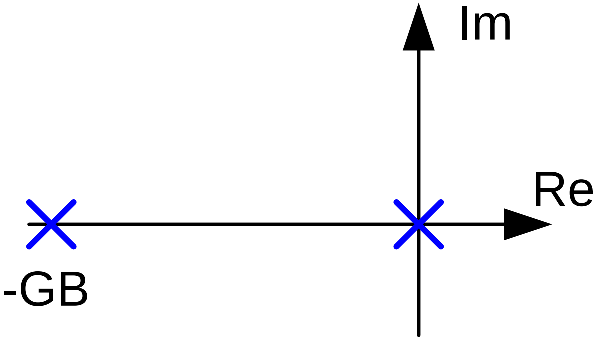
Consider Miller Inverting Integrator



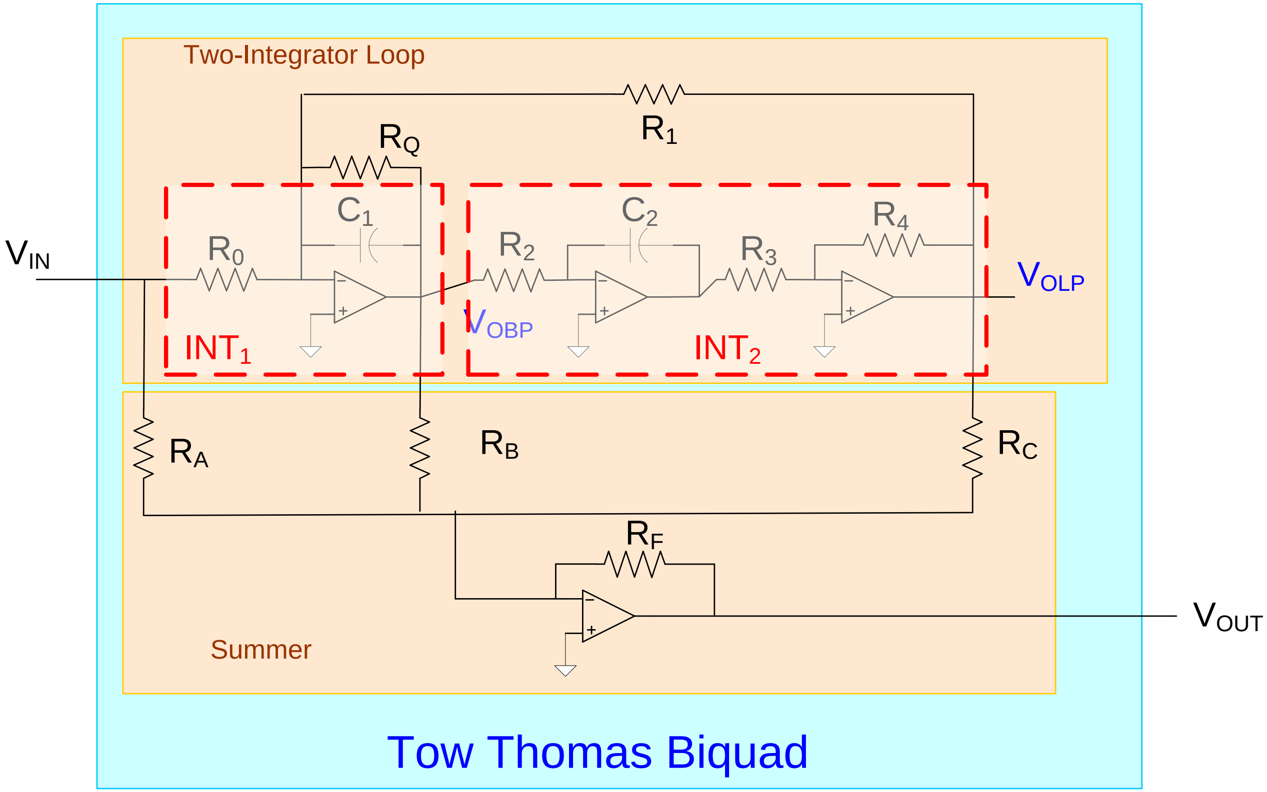
$$A_V(s) = \frac{-1}{RCs + \tau s(1+RCs)}$$

$$I_0 = 1/(RC)$$

Poles at s=0 and $s = -I_0(1+GB/I_0) \simeq -GB$



Is the integrator Q factors simply a metric or does it have some other significance?

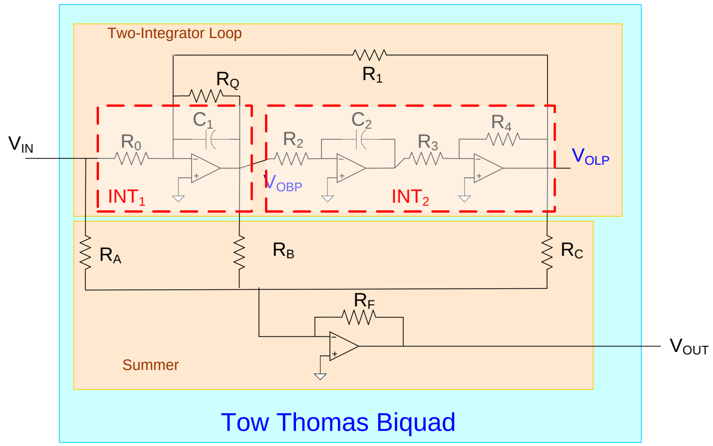


It can be shown that the pole Q for the TT Biquad can be approximated by

$$Q_P \cong \frac{1}{\frac{1}{Q_{PN}} + \frac{1}{Q_{INT1}} + \frac{1}{Q_{INT2}}}$$

where Q_{INT1} and Q_{INT2} are evaluated at $\omega=\omega_o$

Is the integrator Q factors simply a metric or does it have some other significance?



It can be shown that the pole Q for the TT Biquad can be approximated by

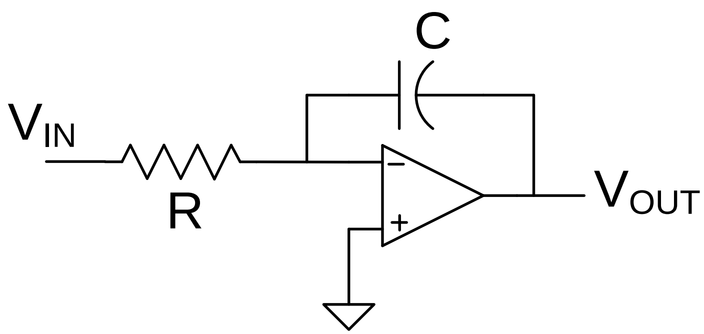
$$Q_P \cong \frac{1}{\frac{1}{Q_{PN}} + \frac{1}{Q_{INT1}(\omega_0)} + \frac{1}{Q_{INT2}(\omega_0)}}$$

Similar expressions for other second-order biquads

Observe that the integrator Q factors adversely affect the pole Q of the filter

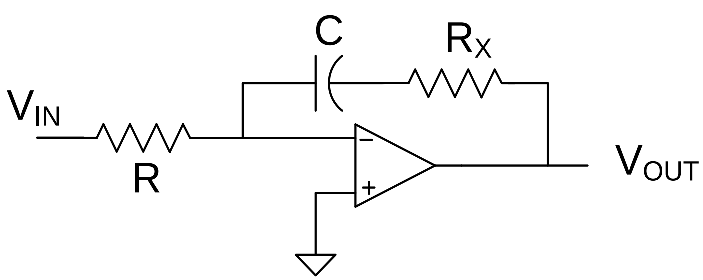
Observe that if Q_{INT1} and Q_{INT2} are of opposite signs and equal magnitudes, nonideal effects of integrator can cancel

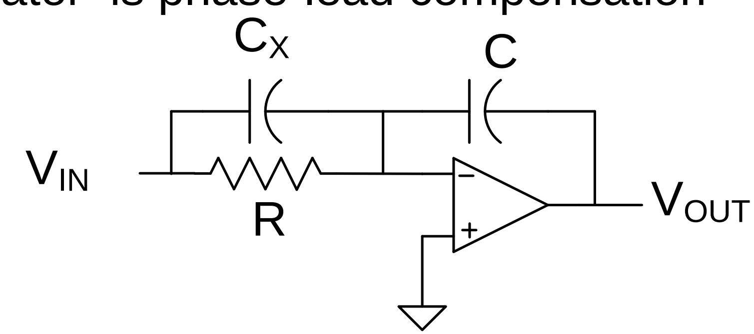
What can be done to correct the phase problems of an integrator?



$$A_V(s)=\frac{-1}{RCs+\tau s(1+RCs)}$$

One thing that can help the Miller Integrator is phase-lead compensation





$$A_V(s)=\frac{-(1+R_XCs)}{RCs+\tau s(1+[R+R_X]Cs)}$$

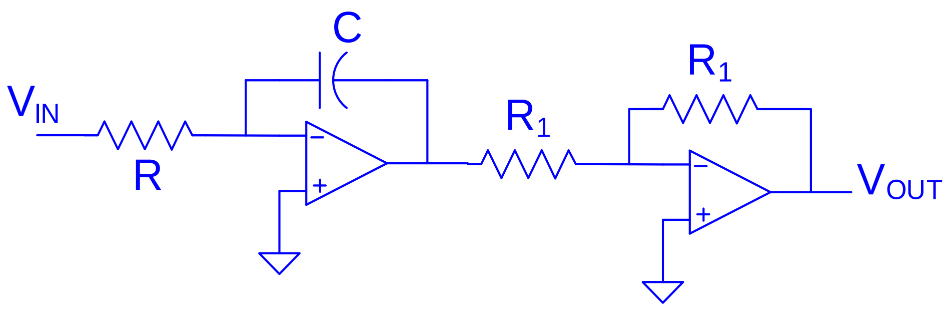
$$A_V(s)=\frac{-(1+RC_Xs)}{RCs+\tau s(1+R[C+C_X]s)}$$

R_x and C_x will add phase-lead by introduction of a zero

R_X and C_X will be small components

Integrator Q Factor

Consider Miller Noninverting Integrator



$$A_V(s)=\frac{-1}{RCs+\tau s(1+RCs)}\bullet\frac{-1}{1+2\tau s}$$

$$A_V(j\omega)=\frac{1}{-3\tau RC\omega^2+j[\omega RC(1+2\tau\omega)]}$$

$$A_V(j\omega)\simeq\frac{1}{-3\tau RC\omega^2+j\omega RC}$$

Observe this integrator has excess phase shift (more than 90º in the denominator) at all frequencies

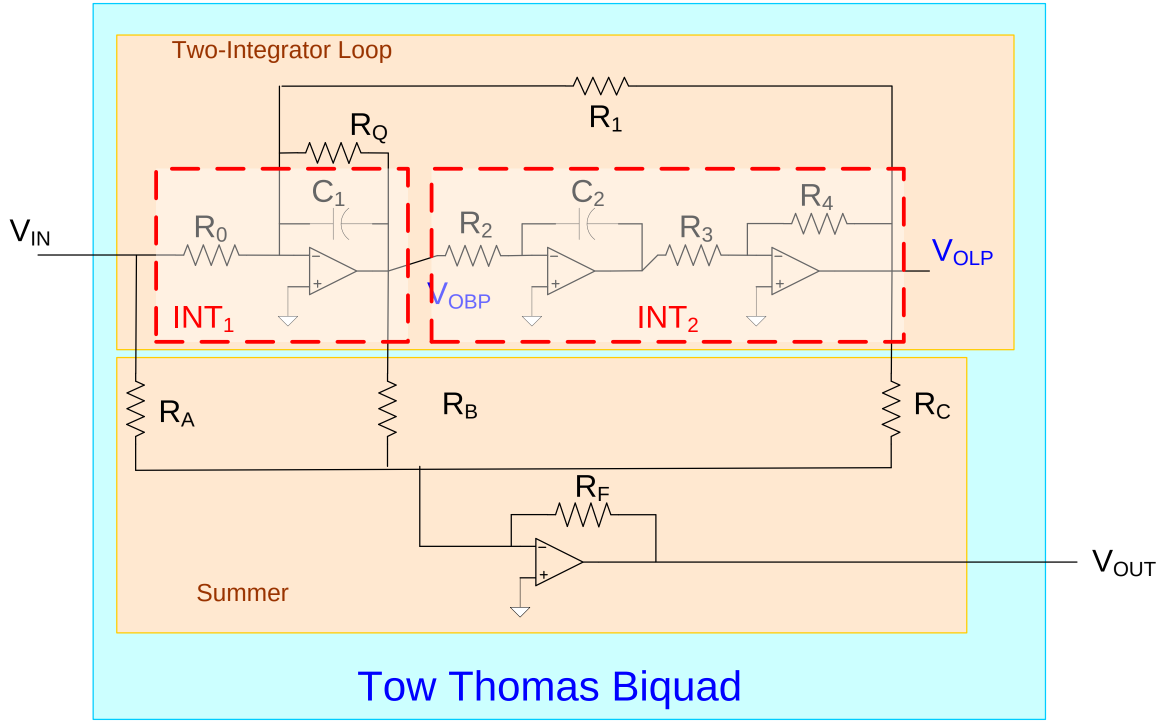
$$Q_{INT}\simeq\frac{\omega RC}{-3\tau RC\omega^2}=\frac{-1}{3\tau\omega}$$

$$Q_{INT}\simeq\frac{-1}{3}\left(\frac{GB}{\omega}\right)=-\frac{1}{3}\left|A(j\omega)\right|$$

Note: The Miller Noninverting Integrator has a modestly poorer Q_{INT} than the Miller Inverting Integrator

Example:

If f_0=10KHz, GB=1MHz, Q_{NOM}=10, estimate the pole Q for the Tow-Thomas Biquad if the Miller Integrator and the Miller Noninverting Integrators are used. Also determine the relative degradation in performance due to each of the integrators.



$$Q_P \cong \frac{1}{\frac{1}{Q_{PN}} + \frac{1}{Q_{INT1}} + \frac{1}{Q_{INT2}}}$$

Example:

If f_0=10KHz, GB=1MHz, Q_{NOM}=10, estimate the pole Q for the Tow-Thomas Biquad if the Miller Integrator and the Miller Noninverting Integrators are used. Also determine the relative degradation in performance due to each of the integrators.

$$Q_P \cong \frac{1}{\frac{1}{Q_{PN}}+\frac{1}{Q_{INT1}}+\frac{1}{Q_{INT2}}}$$

$$Q_{INT1} = \text{-A=}-\frac{GB}{\omega} = -\frac{1MHz}{10KHz} = -100$$

$$Q_{INT2} \simeq -\frac{1}{3}\left|A(j\omega)\right| = \frac{-1}{3}\left(\frac{GB}{\omega}\right) = -\frac{1MHz}{3 \bullet 10KHz} = -33$$

$$Q_P \cong \frac{1}{\frac{1}{10}-.01-.033} = 17.5$$

Note the nonideal integrators cause about a 75% shift in Q_P

Note that 3 times as much of the shift is due to the noninverting integrator as is due to the inverting integrator!

Similar effects of the integrators will be seen on other filter structures

Example:

If f_0=10KHz, GB=1MHz, Q_{NOM}=10, estimate the pole Q for the Tow-Thomas Biquad if the Miller Integrator and the Miller Noninverting Integrators are used. Also determine the relative degradation in performance due to each of the integrators.

$$Q_P \cong \frac{1}{\frac{1}{Q_{PN}}+\frac{1}{Q_{INT1}}+\frac{1}{Q_{INT2}}}$$

$$Q_{INT1} = \text{-A} = -\frac{GB}{\omega} = -\frac{1MHz}{10KHz} = -100$$

$$Q_{INT2} \simeq -\frac{1}{3}\left|A(j\omega)\right| = \frac{-1}{3}\left(\frac{GB}{\omega}\right) = -\frac{1MHz}{3\bullet 10KHz} = -33$$

$$Q_P \cong \frac{1}{\frac{1}{10}-.01-.033} = 17.5$$

How can the problem be solved?

1. Compensate Integrator
2. Use better integrators
3. Use phase-lead and phase/lag pairs

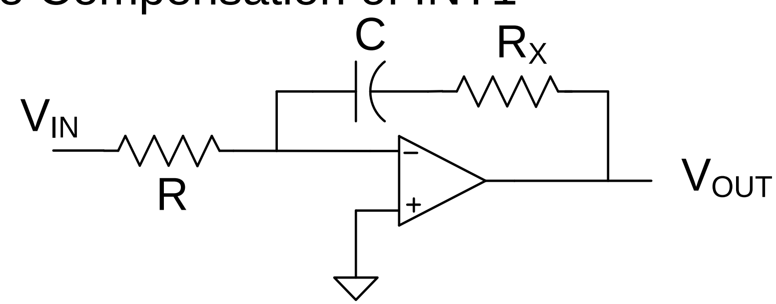
Example:

If f_0=10KHz, GB=1MHz, Q_{NOM}=10, estimate the pole Q for the Tow-Thomas Biquad if the Miller Integrator and the Miller Noninverting Integrators are used. Also determine the relative degradation in performance due to each of the integrators.

$$Q_P \cong \frac{1}{\frac{1}{Q_{PN}}+\frac{1}{Q_{INT1}}+\frac{1}{Q_{INT2}}} \qquad Q_{INT1} = -100 \qquad Q_{INT2} \simeq -33$$

How can the problem be solved?

Phase Compensation of INT1



$$A_V(s)=\frac{-(1+R_XCs)}{RCs+\tau s(1+[R+R_X]Cs)}$$

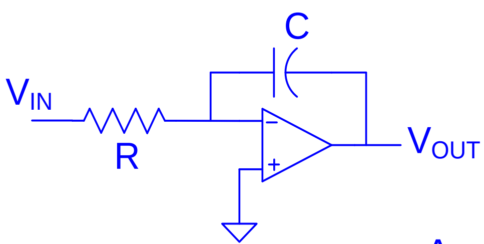
$$Q_{INT1}=\frac{-GB/\omega}{1-GB\bullet CR_X}$$

Pick R_x so that Q_{INT1}=33 at ω=1/(RC)

Solving, obtain CR_x=4/GB

Useful for hand calibration but not practical for volume production because of variability in components

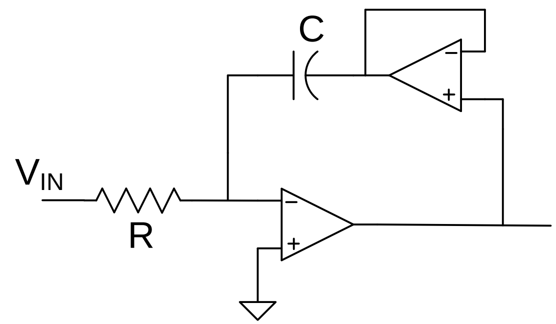
What are the integrator Q factors for other integrators that have been considered?



$Q_{INT} = -A$

$$A_V(s) = -\frac{1}{RCs}$$

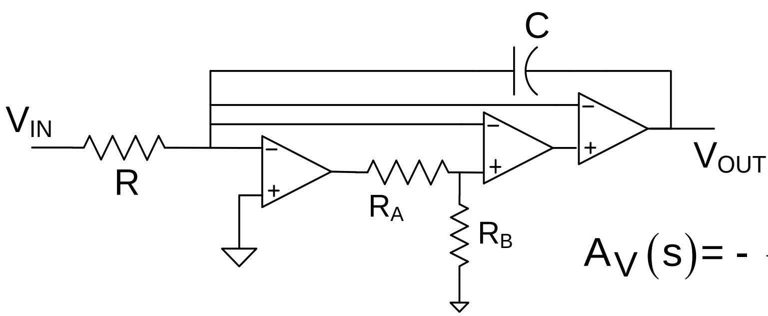
Miller Inverting



$$A_V(s) = -\frac{1}{RCs}$$

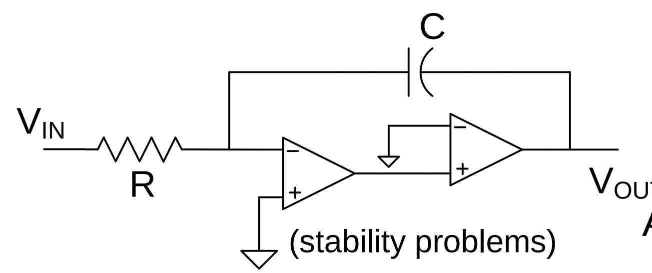
$Q_{INT} = -A^2$

High-Q Inverting



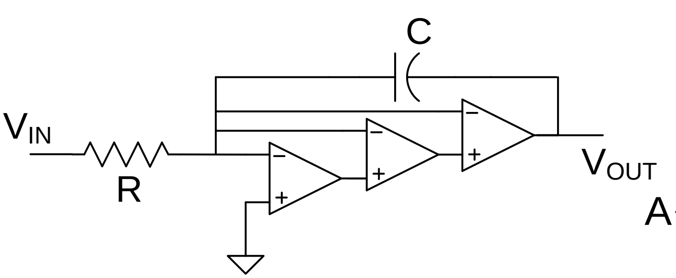
$$A_V(s) = -\frac{1}{RCs}$$

Zero Second Derivative Inverting



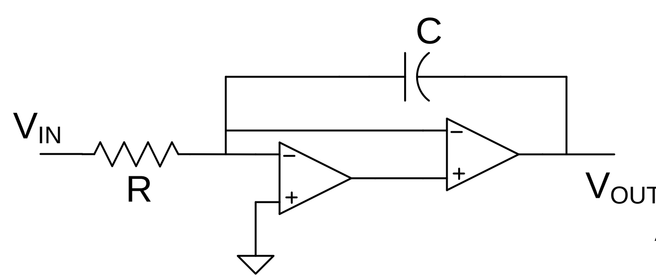
$$A_V(s) = -\frac{1}{RCs}$$

Cascaded Inverting



$$A_V(s) = -\frac{1}{RCs}$$

Zero Second Derivative Inverting

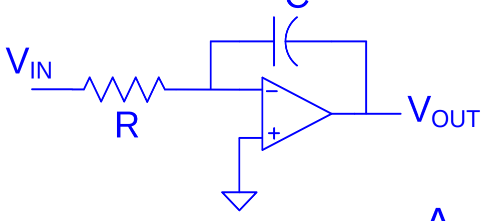


$Q_{INT} = -A^2$

$$A_V(s) = -\frac{1}{RCs}$$

Zero Sensitivity Inverting

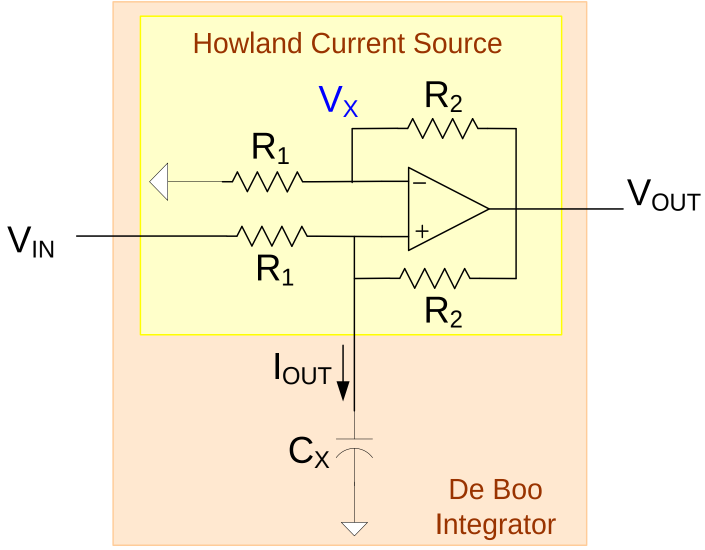
What are the integrator Q factors for other integrators that have been considered?



Miller Inverting

$$Q_{INT} = -A$$

$$A_V(s) = -\frac{1}{RCs}$$



$$A_V(s) = \frac{1+\frac{R_2}{R_1}}{R_1Cs+\tau_1 s\left(1+\frac{R_2}{R_1}\right)\left(1+\frac{R_1}{R_2}+sCR_1\right)}$$

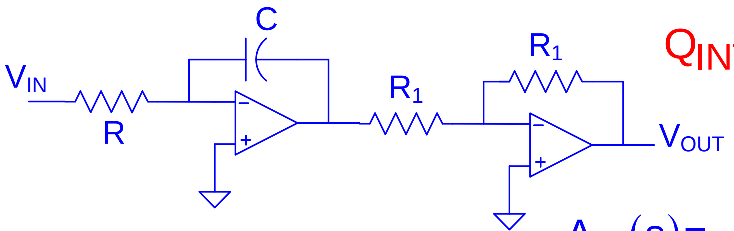
$$Q_{INT} = -\frac{A}{1+\frac{R_2}{R_1}}$$

If $R_1=R_2=R$

$$A_V(s) = \frac{2}{RCs}$$

$$Q_{INT} = -\frac{A}{2}$$

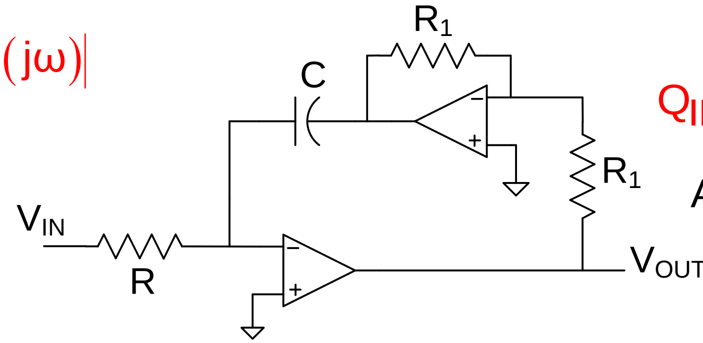
What are the integrator Q factors for other integrators that have been considered?



$$Q_{INT} \simeq -\frac{1}{3}|A(j\omega)|$$

$$A_V(s) = \frac{1}{RCs}$$

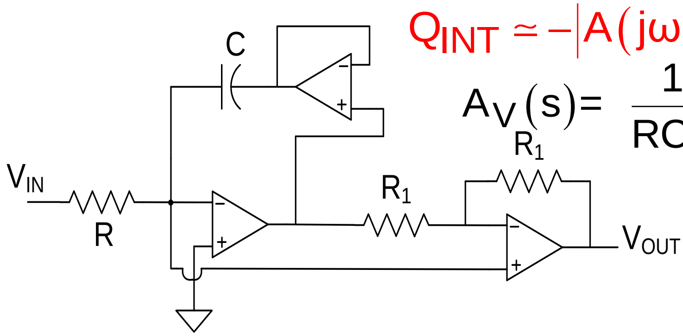
Miller Noninverting



$$Q_{INT} \simeq |A(j\omega)|$$

$$A_V(s) = \frac{1}{RCs}$$

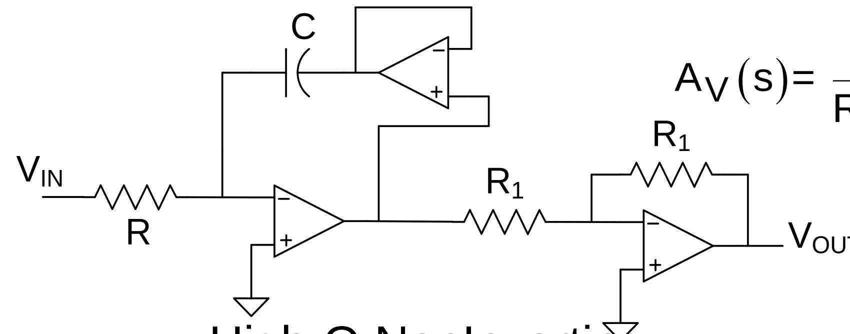
Phase Lead Integrator



$$Q_{INT} \simeq -|A(j\omega)|^3$$

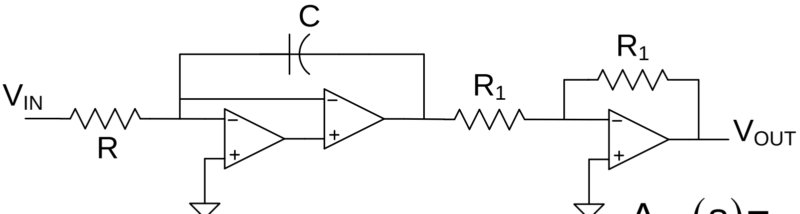
$$A_V(s) = \frac{1}{RCs}$$

Modified High-Q Noninverting



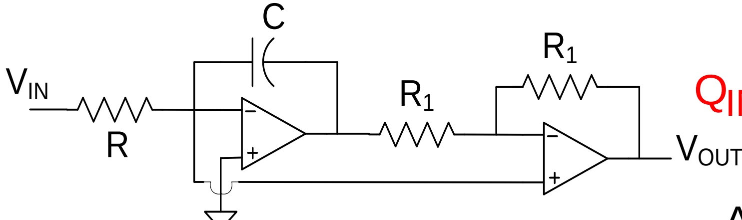
$$A_V(s) = \frac{1}{RCs}$$

High-Q NonInverting



$$A_V(s) = \frac{1}{RCs}$$

Zero Sensitivity Noninverting



$$Q_{INT} \simeq -|A(j\omega)|$$

$$A_V(s) = \frac{1}{RCs}$$

Modified Miller Noninverting

Improving Integrator Performance:

1. Compensate Integrator
2. Use better integrators
3. Use phase-lead and phase/lag pairs

- These methods all provide some improvements in integrator performance
- But both magnitude and phase of an integrator are important so focusing only on integrator Q factor only may only improve performance to a certain level
- In higher-order integrator-based filters, the linearity in $1/\omega$ of the integrator gain is also important. The integrator magnitude and Q factor at ω_0 ignore the frequency nonlinearity that may occur in the $1/\omega$ dependence
- There is little in the literature on improving the performance of integrated integrators within a basic class. At high frequencies where the active device performance degrades, particularly in finer-feature processes, there may be some benefits that can be derived from architectural modifications along the line of those discussed in this lecture

EE 508
Lecture 30

Integrator Design

Current-Mode Integrators

s-domian to z-domain mappings

How can integrator performance be improved?

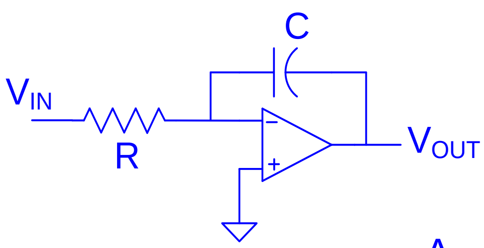
- Better op amps
- Better Integrator Architectures

How can the performance of integrator structures be compared?

Need metric for comparing integrator performance

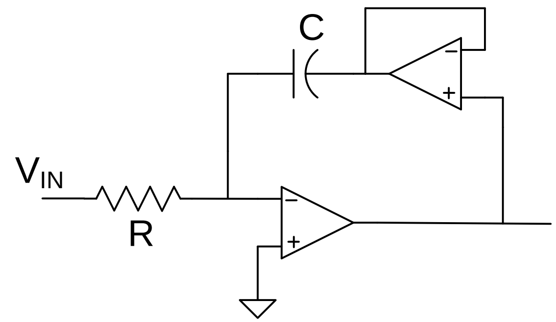
Review from last time

Are there other integrators in the basic classes that have been considered?



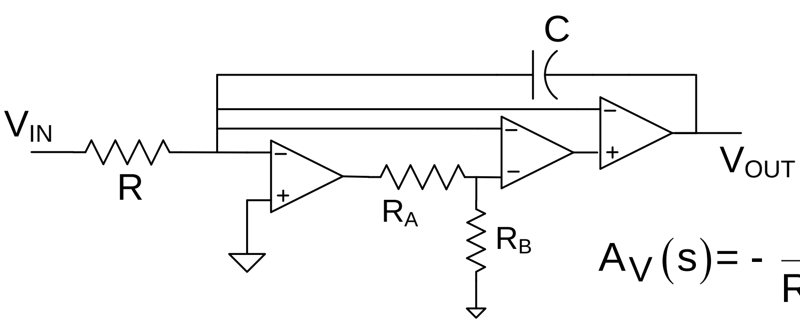
$$A_V(s) = -\frac{1}{RCs}$$

Miller Inverting



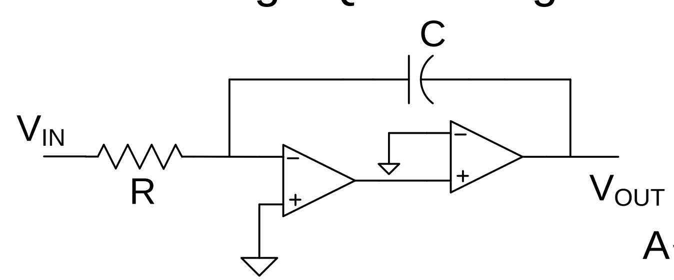
$$A_V(s) = -\frac{1}{RCs}$$

High-Q Inverting



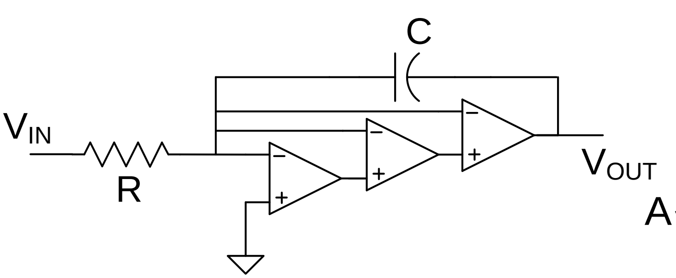
$$A_V(s) = -\frac{1}{RCs}$$

Zero Second Derivative Inverting



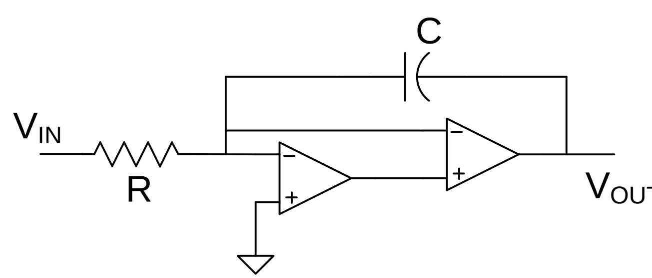
$$A_V(s) = -\frac{1}{RCs}$$

Cascaded Inverting



$$A_V(s) = -\frac{1}{RCs}$$

Zero Second Derivative Inverting



$$A_V(s) = -\frac{1}{RCs}$$

Zero Sensitivity Inverting

Review from last time

How can the performance of an integrator be characterized and how can integrators be compared?

Express $A_V(j\omega)$ as

$$A_V(j\omega)=\frac{\pm 1}{R(\omega)+jX(\omega)}$$

where R(ω) and X(ω) are real and represent the real and imaginary parts of the denominator respectively

$$\text{Phase}=-\tan^{-1}\left(\frac{X(\omega)}{R(\omega)}\right)$$

Ideally R(ω)=0

Definition: The Integrator Q factor is the ratio of the imaginary part of the denominator to the real part of the denominator

$$Q_{INT}=\left(\frac{X(\omega)}{R(\omega)}\right)$$

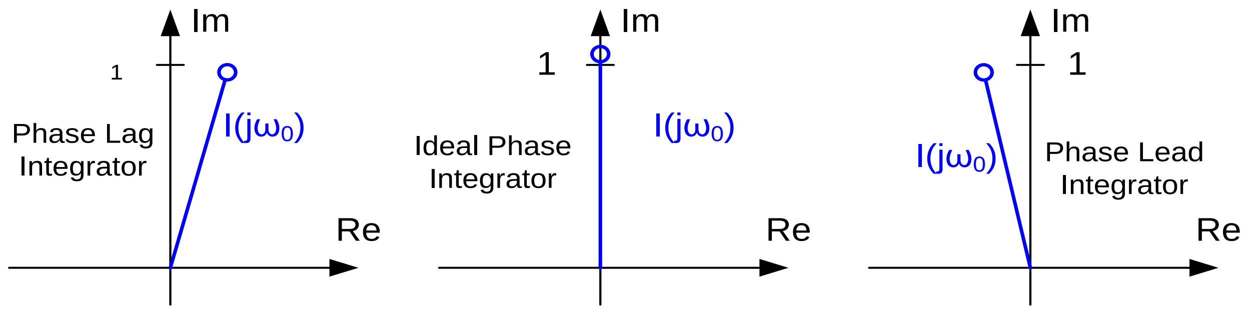
Typically most interested in Q_{INT} at the nominal unity gain frequency of the integrator

Review from last time

How can the performance of an integrator be characterized and how can integrators be compared?

Lead/Lag Characteristics for Inverting Integrators

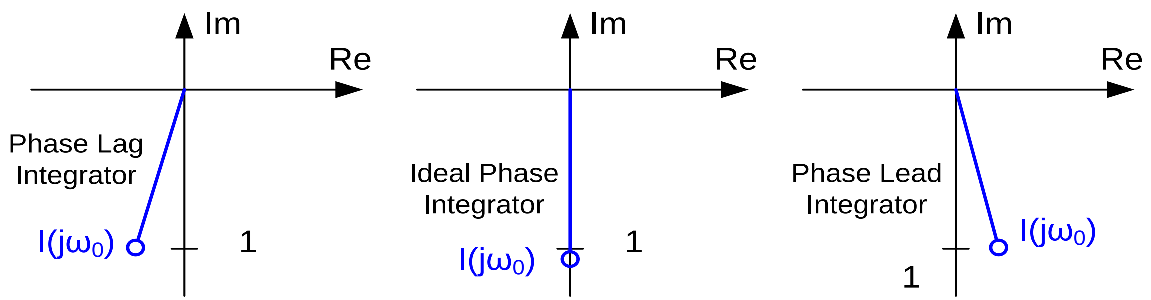
$$I_V(j\omega)=\frac{-1}{R(\omega)+jX(\omega)}$$



For Phase Lag Integrators, $R(\omega)$ and $X(\omega)$ have opposite signs. For Phase Lead integrators, $R(\omega)$ and $X(\omega)$ have the same sign. Phase shift ideally 90°

Lead/Lag Characteristics for Noninverting Integrators

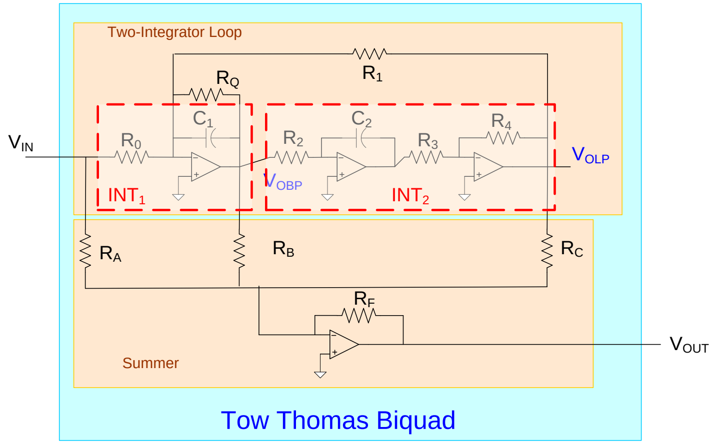
$$I_V(j\omega)=\frac{1}{R(\omega)+jX(\omega)}$$



For Phase Lag Integrators, $R(\omega)$ and $X(\omega)$ have opposite signs. For Phase Lead integrators, $R(\omega)$ and $X(\omega)$ have the same sign. Phase shift ideally 270°

Review from last time

Is the integrator Q factors simply a metric or does it have some other significance?



It can be shown that the pole Q for the TT Biquad can be approximated by

$$Q_P \cong \frac{1}{\frac{1}{Q_{PN}} + \frac{1}{Q_{INT1}(\omega_0)} + \frac{1}{Q_{INT2}(\omega_0)}}$$

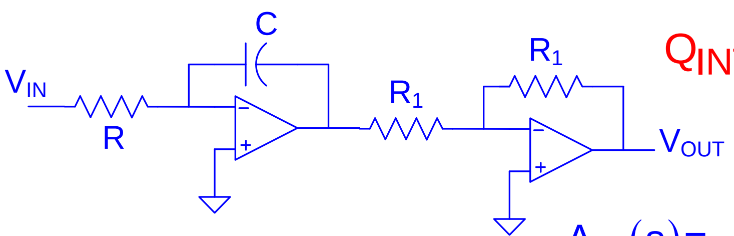
Similar expressions for other second-order biquads

Observe that the integrator Q factors adversely affect the pole Q of the filter

Observe that if Q_{INT1} and Q_{INT2} are of opposite signs and equal magnitudes, nonideal effects of integrator can cancel

Review from last time

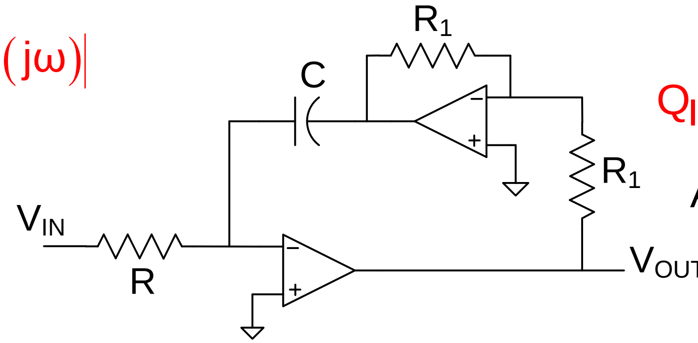
What are the integrator Q factors for other integrators that have been considered?



Miller Noninverting

$$Q_{INT} \simeq -\frac{1}{3}\left|A(j\omega)\right|$$

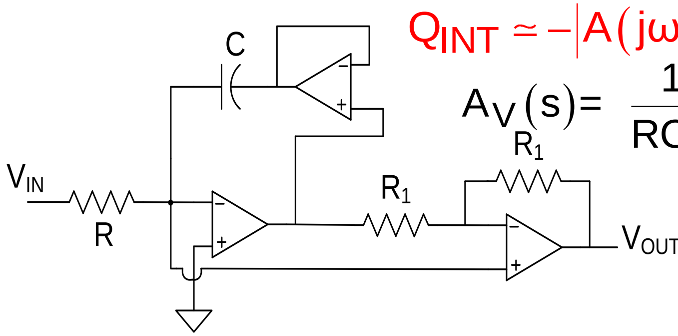
$$A_V(s) = \frac{1}{RCs}$$



Phase Lead Integrator

$$Q_{INT} \simeq \left|A(j\omega)\right|$$

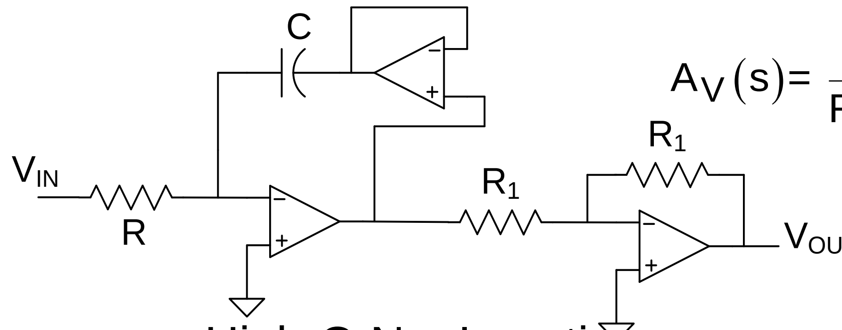
$$A_V(s) = \frac{1}{RCs}$$



Modified High-Q Noninverting

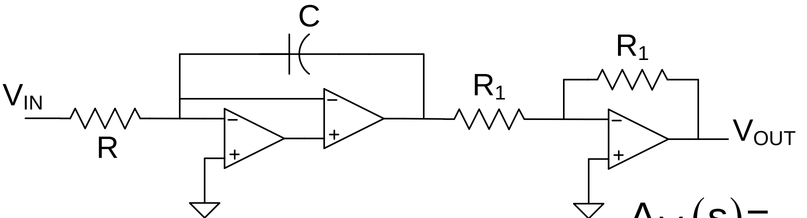
$$Q_{INT} \simeq -\left|A(j\omega)\right|^3$$

$$A_V(s) = \frac{1}{RCs}$$



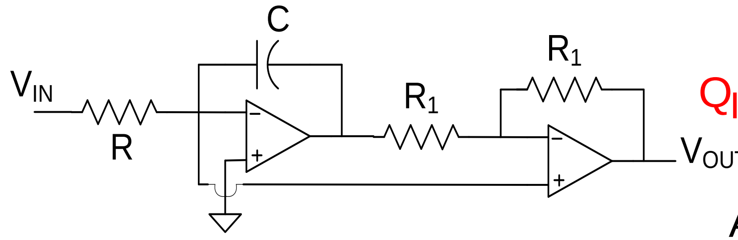
High-Q NonInverting

$$A_V(s) = \frac{1}{RCs}$$



Zero Sensitivity Noninverting

$$A_V(s) = \frac{1}{RCs}$$

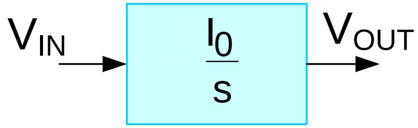


Modified Miller Noninverting

$$Q_{INT} \simeq -\left|A(j\omega)\right|$$

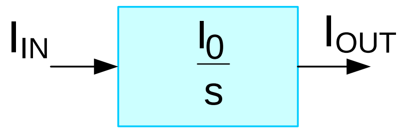
$$A_V(s) = \frac{1}{RCs}$$

Integrator Types



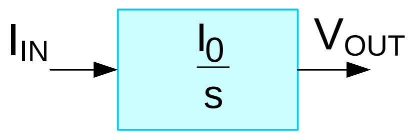
Voltage Mode

$$V_{OUT} = \frac{I_o}{s} V_{IN}$$



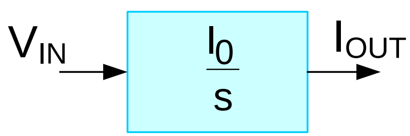
Current Mode

$$I_{OUT} = \frac{I_o}{s} I_{IN}$$



Transresistance Mode

$$V_{OUT} = \frac{I_o}{s} I_{IN}$$

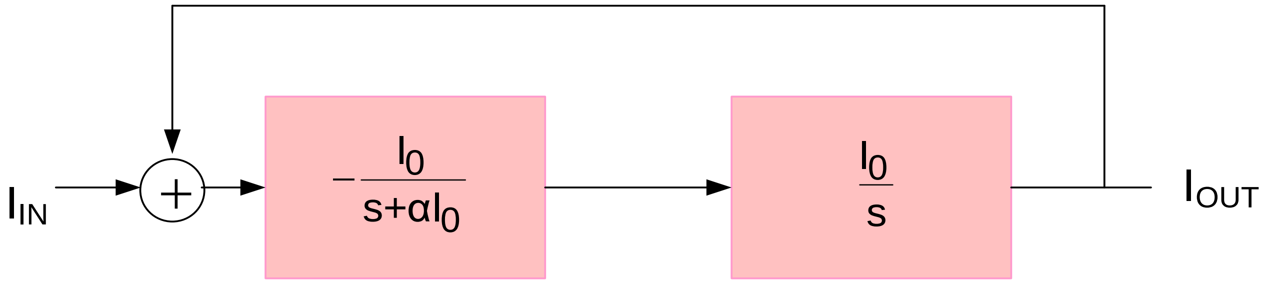


Transconductance Mode

$$I_{OUT} = \frac{I_o}{s} V_{IN}$$

Selected Current Mode, Transresistance Mode, and Transconductance Mode Integrators

Current-Mode Filters

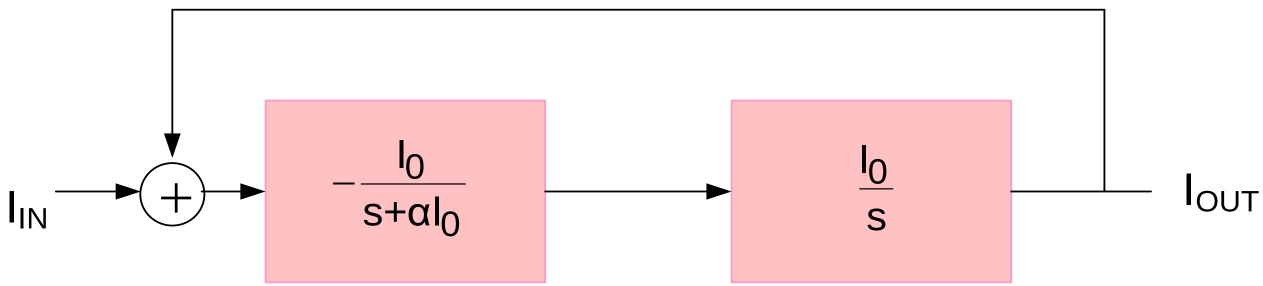


$$T(s)=\frac{I_{OUT}}{I_{IN}}=\frac{-I_0^2}{s^2+\alpha I_0 s+I_0^2}$$

Basic Concepts of Benefits of Current-Mode Filters:

- Large voltage swings difficult to maintain in integrated processes because of linearity concerns
- Large voltage swings slow a circuit down because of time required to charge capacitors
- Voltage swings can be very small when currents change
- Current swings are not inherently limited in integrated circuits (only voltage swings)
- With low voltage swings, current-mode circuits should dissipate little power

Current-Mode Filters



$$T(s)=\frac{I_{OUT}}{I_{IN}}=\frac{-I_0^2}{s^2+\alpha I_0 s+I_0^2}$$

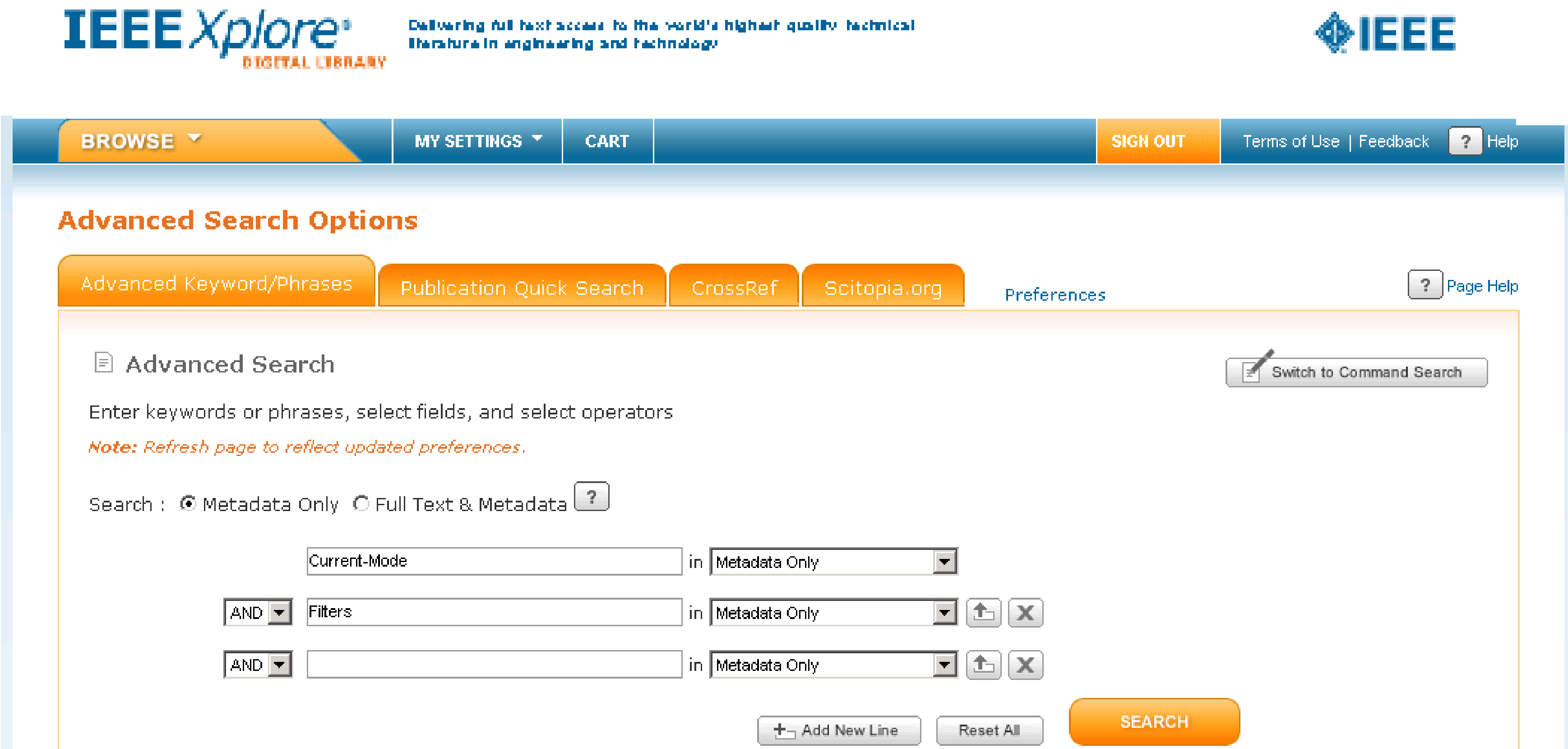
Concept of Current-Mode Filters is Somewhat Recent:

Key paper that generated interest in current-mode filters (ISCAS 1989):

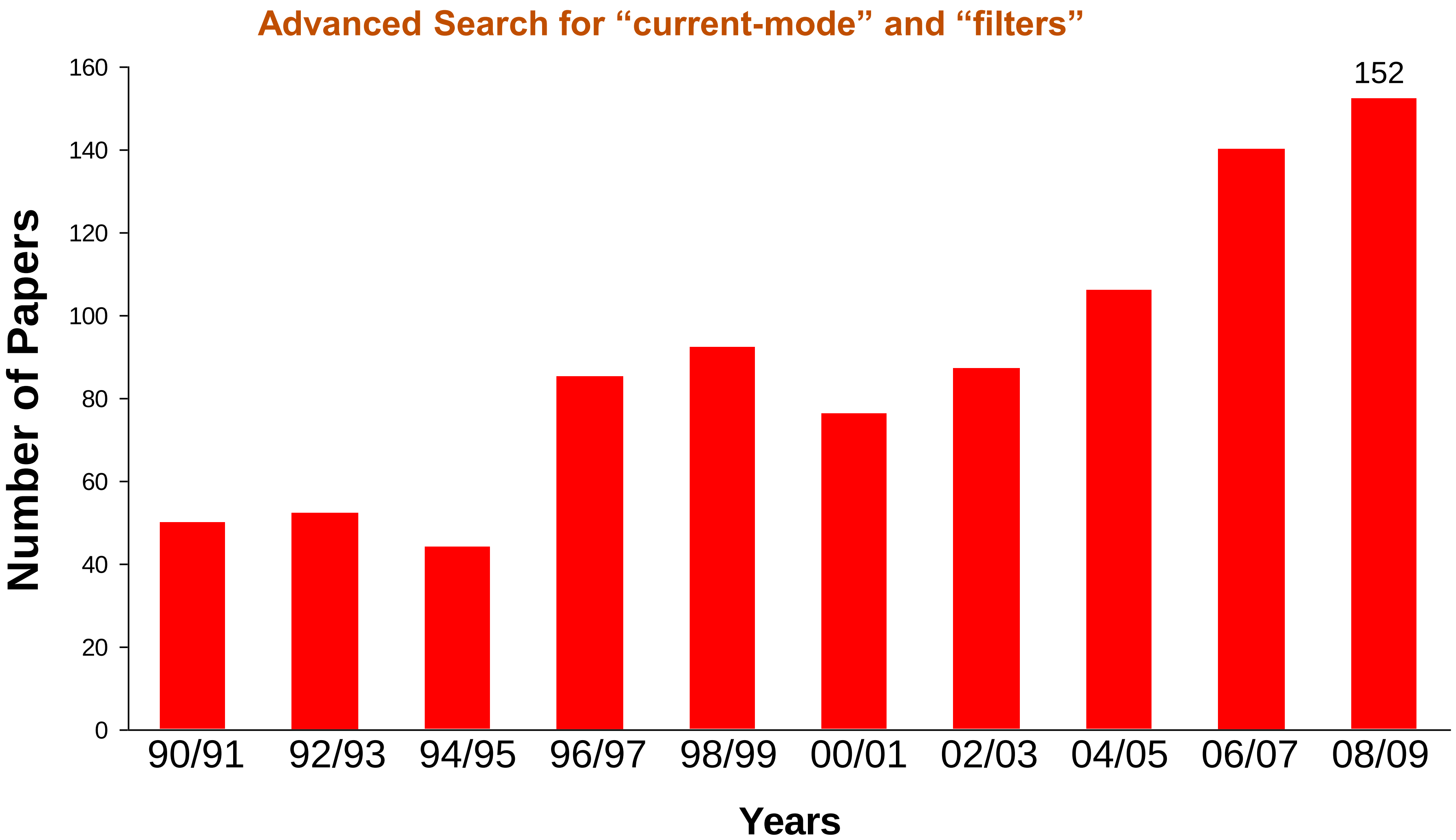
Switched currents-a new technique for analog sampled-data signal processing
JB Hughes, NC Bird... - Circuits and Systems, 1989 ..., 2002 - ieeexplore.ieee.org
NTRODUCTION The enormous complexity available in state-of-the-art CMOS processing has made possible the integration of complete systems, including both digital and analog signal processing functions, within the same chip Through the last decade, the **switched** capacitor technique **...**
Cited by 151 - Related articles

This paper is one of the most significant contributions that has ever come from ISCAS

Current-Mode Filters



Current-Mode Filters



1872-1989 – total of 19 references

Search done on Nov 7, 2010

Current-Mode Filters

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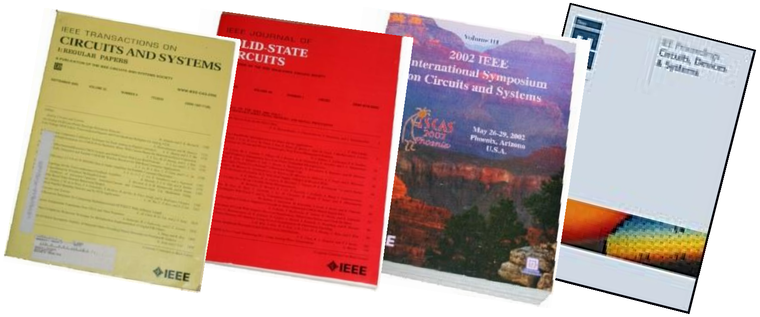

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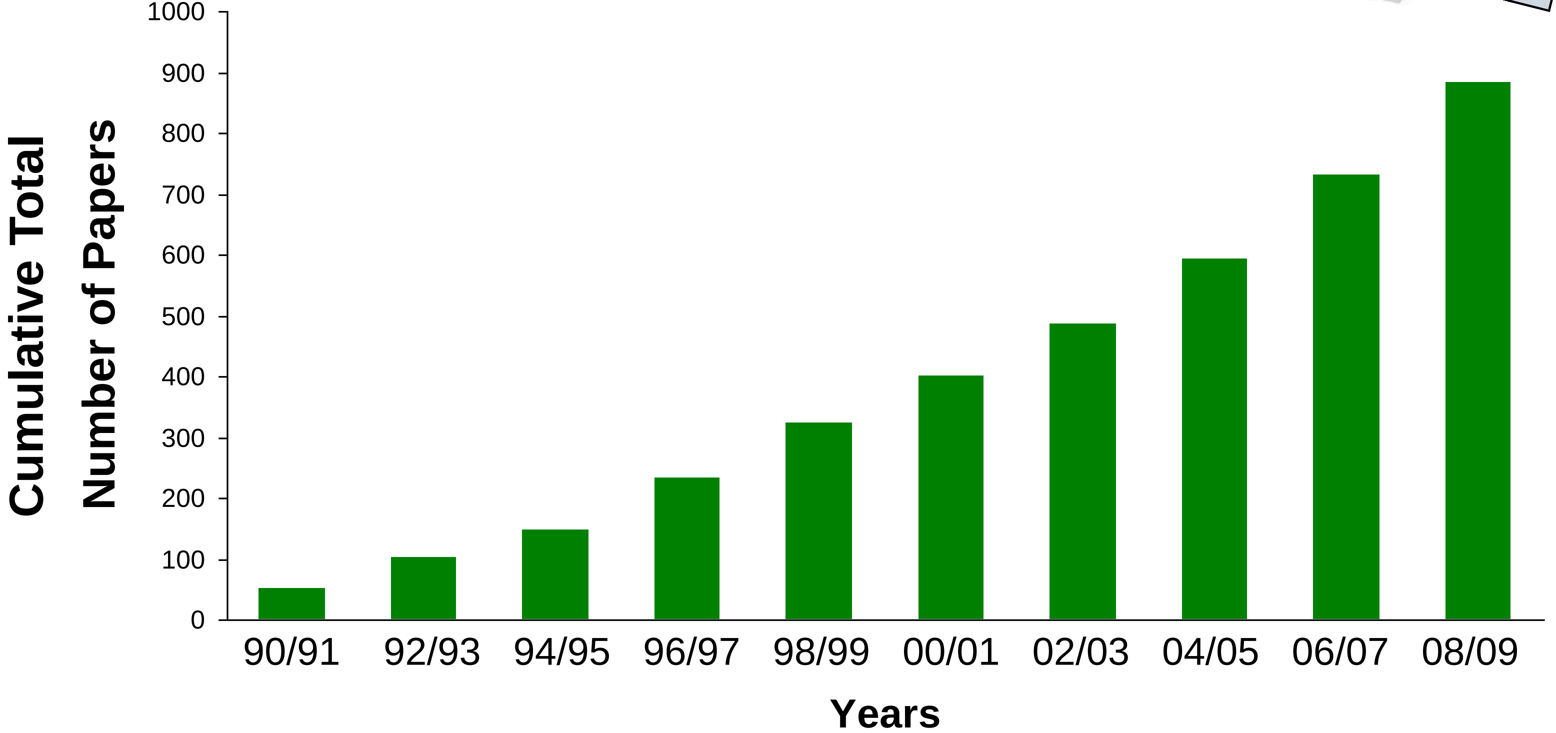

Steady growth in research in the area since 1990 and publication rate is growing with time !!

Current-Mode Filters

The Conventional Wisdom:


Fully differential current-mode third-order Butterworth VHF G_m-C filter in 0.18 μm CMOS

Download Citation Email Print

Hwang, Y.-S.; Chen, J.-J.; Lai, J.-H.; Sheu, P.-W.;
Dept. of Electron. Eng., Nat. Taipei Univ. of Technol., Taiwan

This paper appears in: Circuits, Devices and Systems, IEE Proceedings
Issue Date: Dec. 2006
Volume: 153 Issue:6
On page(s): 552 - 558

Proc. IEE Dec 2006:

1 Introduction

Current-mode circuits have been proven to offer advantages over their voltage-mode counterparts [1, 2]. They possess wider bandwidth, greater linearity and wider dynamic range. Since the dynamic range of the analogue circuits using low-voltage power supplies will be low, this problem can be solved by employing current-mode operation.

Proc. SICE-ICASE, Oct. 2006

1. INTRODUCTION

It is well known that current-mode circuits have been receiving significant attention owing to its advantage over the voltage-mode counterpart, particularly for higher frequency of operation and simpler filtering structure [1].


Current-controlled Current-mode Biquadratic Filter with two inputs and three outputs Using Multiple-Output FTFNs

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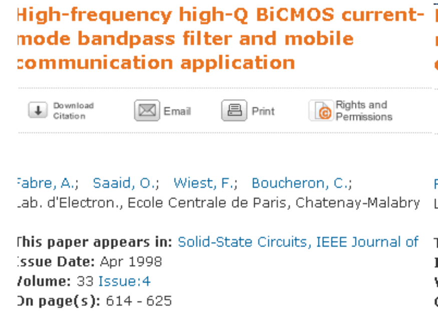
Hirunporm, J.; Pukkalanun, T.; Tangsrirat, W.;
Fac. of Eng., King Mongkut's Inst. of Technol., Bangkok

This paper appears in: SICE-ICASE, 2006. International Joint Conference
Issue Date: 18-21 Oct. 2006
On page(s): 5691 - 5694

Current-Mode Filters

The Conventional Wisdom:


High-frequency high-Q BiCMOS current-mode bandpass filter and mobile communication application

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Fabre, A.; Saaid, O.; Wiest, F.; Boucheron, C.;
Lab. d'Electron., Ecole Centrale de Paris, Chatenay-Malabry

This paper appears in: Solid-State Circuits, IEEE Journal of
Issue Date: Apr 1998
Volume: 33 Issue:4
On page(s): 614 - 625

JSC April 1998:

"… current-mode functions exhibit higher frequency potential, simpler architectures, and lower supply voltage capabilities than their voltage-mode counterparts."


Current-mode continuous-time filters: two design approaches

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Ramirez-Angulo, J. ; Robinson, M. ; Sanchez-Sinecio, E. ;
Dept. of Electr. & Comput. Eng., New Mexico State Univ., Las Cruces, NM

This paper appears in: Circuits and Systems II: Analog and Digital Signal Processing, IEEE Transactions on
Issue Date : Jun 1992
Volume : 39 , Issue:6
On page(s): 337 - 341

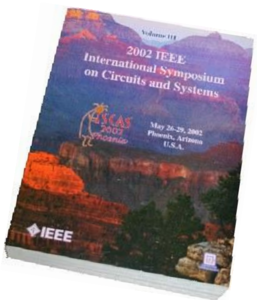
CAS June 1992

"Current-mode signal processing is a very attractive approach due to the simplicity in implementing operations such as … and the potential to operate at higher signal bandwidths than their voltage mode analogues" … "Some voltage-mode filter architectures using transconductance amplifiers and capacitors (TAC) have the drawback that …"

Current-Mode Filters

The Conventional Wisdom:

ISCAS 1993:



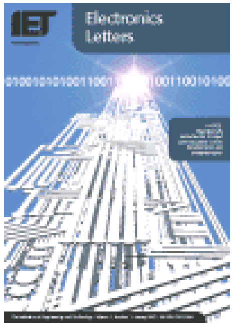
"In this paper we propose a fully balanced high frequency current-mode integrator for low voltage high frequency filters. Our use of the term current mode comes from the use of current amplifiers as the basic building block for signal processing circuits. This fully differential integrator offers significant improvement even over recently introduced circuit with respect to accuracy, high frequency response, linearity and power supply requirements. Furthermore, it is well suited to standard digital based CMOS processes."

3V high-frequency current-mode filters
Smith, S.L.; Sanchez-Sinencio, E.;
Circuits and Systems, 1993., ISCAS '93, 1993 IEEE International Symposium on
Digital Object Identifier: 10.1109/ISCAS.1993.394009
Publication Year: 1993 , Page(s): 1459 - 1462 vol.2

Current-Mode Filters

The Conventional Wisdom:

Two key publications where benefits of the current-mode circuits are often referenced:



All current-mode frequency selective circuits **GW Roberts**, AS **Sedra** - Electronics Letters, June 1989 - pp. 759-761 Cited by 161

"To make greatest use of the available transistor bandwidth f_T, and operate at low voltage supply levels, it has become apparent that analogue signal processing can greatly benefit from processing current signals rather than voltage signals. Besides this, it is well known by electronic circuit designers that the mathematical operations of adding, subtracting or multiplying signals represented by currents are simpler to perform than when they are represented by voltages. This also means that the resulting circuits are simpler and require less silicon area."

Current-Mode Filters

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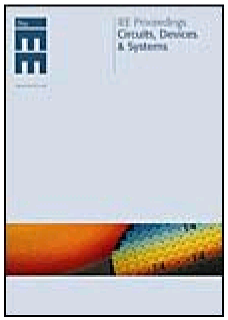


Recent developments in **current** conveyors and **current**-mode circuits **B Wilson** - Circuits, Devices and Systems, IEE Proceedings G, pp. 63-77, Apr. 1990 Cited by 203

“The **use** of current rather than voltage as the active parameter can result in higher usable gain, accuracy and bandwidth due to reduced voltage excursion at sensitive nodes. A current-mode approach is not just restricted to current processing, but also offers certain important advantages when interfaced to voltage-mode circuits.”

Current-Mode Filters

The Conventional Wisdom:

– Current-Mode circuits operate at higher-frequencies than voltage-mode counterparts

– Current-Mode circuits operate at lower supply voltages and lower power levels than voltage-mode counterparts

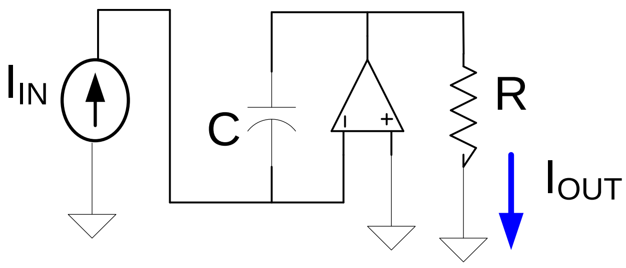
– Current-Mode circuits are simpler than voltage-mode counterparts

– Current-Mode circuits offer better linearity than voltage-mode counterparts

This represents four really significant benefits of current-mode circuits!

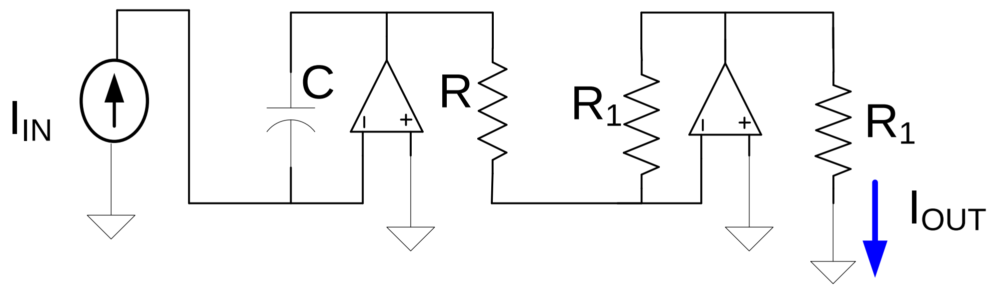
Some Current-Mode Integrators

Active RC



$$I_{OUT} = \left(\frac{-1}{RCs}\right) I_{IN}$$

Inverting



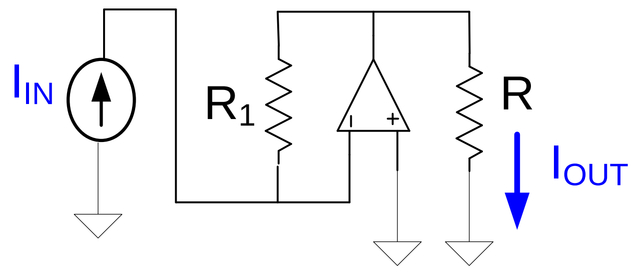
$$I_{OUT} = \left(\frac{1}{RCs}\right) I_{IN}$$

Noninverting

- Summing inputs really easy to obtain
- Loss is easy to add
- Same component count as voltage-mode integrators
- Some argue that since only interested in currents, can operate at lower voltages

Some Current-Mode Integrators

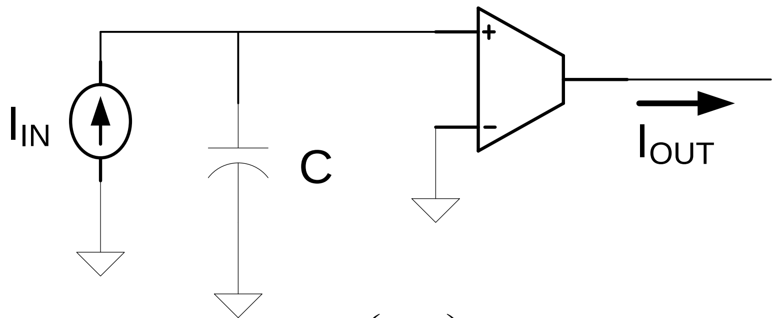
Current-Mode Inverting Amplifier



$$I_{OUT} = \left(-\frac{R_1}{R}\right) I_{IN}$$

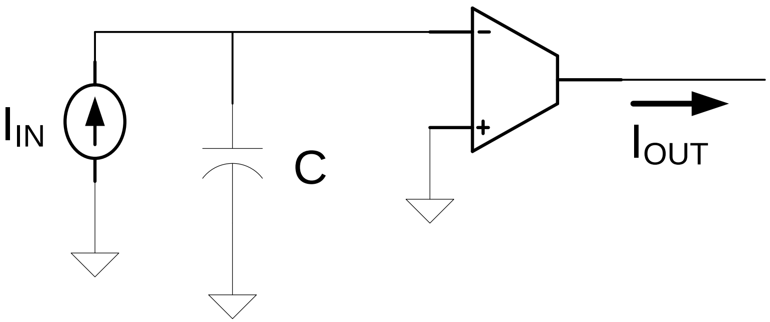
Some Current-Mode Integrators

OTA-C



$$I_{OUT} = \left(\frac{g_m}{Cs}\right) I_{IN}$$

Noninverting



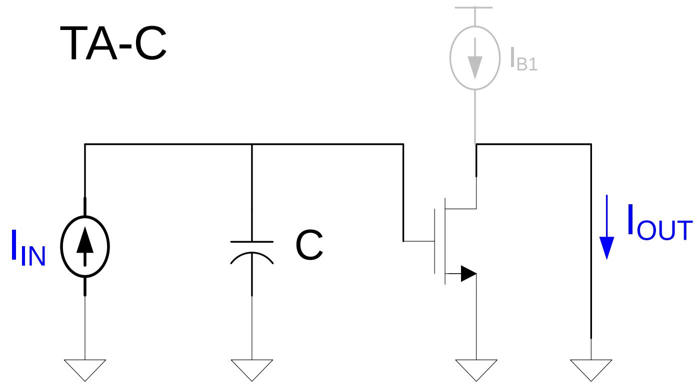
$$I_{OUT} = \left(\frac{-g_m}{Cs}\right) I_{IN}$$

Inverting

- Summing inputs really easy to obtain
- Loss is easy to add
- Same component count as voltage-mode integrators
- Many argue that since only interested in currents, can operate at lower voltages and higher frequencies

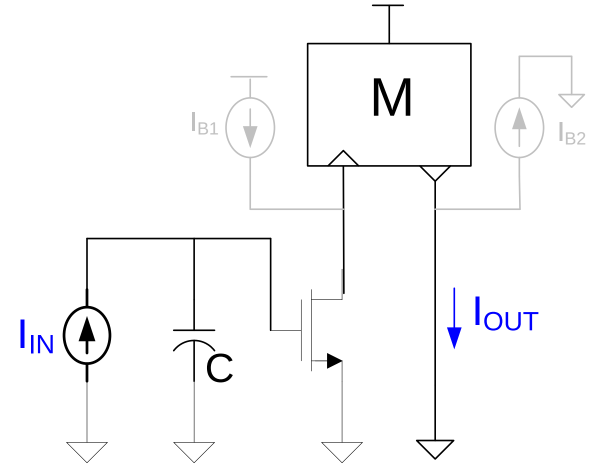
Some Current-Mode Integrators

TA-C



$$I_{OUT} = \left(\frac{-g_m}{Cs}\right) I_{IN}$$

Inverting



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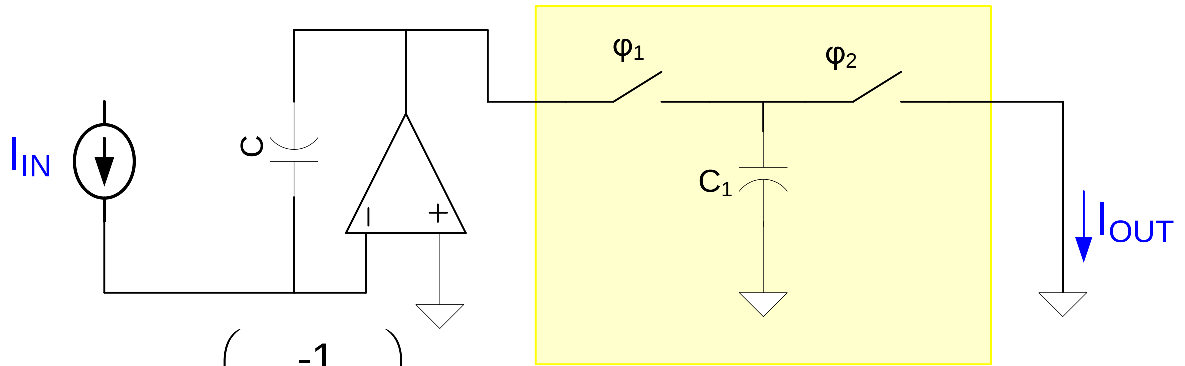
Some Current-Mode Integrators

Switched-C

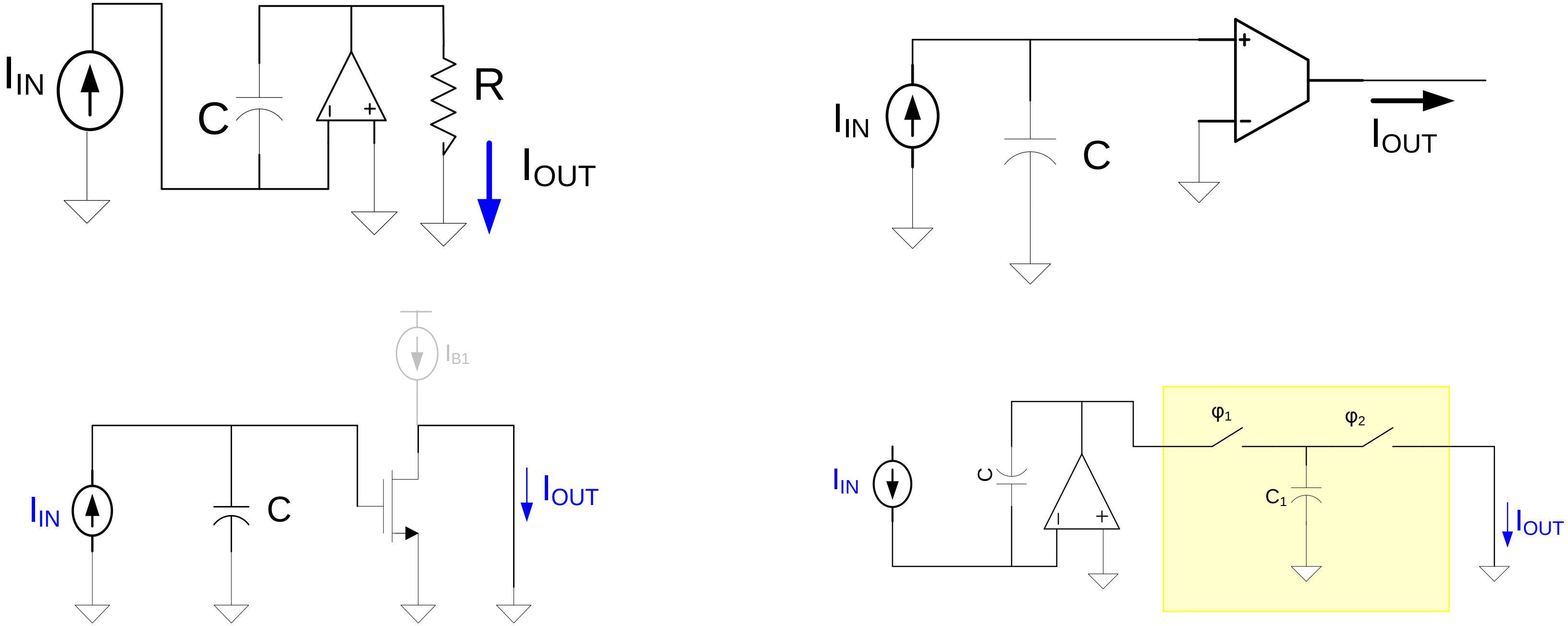


$$I_{OUT} = \left(\frac{-1}{R_{EQ}Cs} \right) I_{IN}$$

Inverting

- Noninverting input easy to obtain
- Summing inputs really easy to obtain
- Loss is easy to add
- Stray insensitive structures readily available
- Less component count than voltage-mode integrators because summing input requires no additional inputs
- SC current-mode integrators have not received much attention in the literature (likely because few have observed the equivalence noted above)

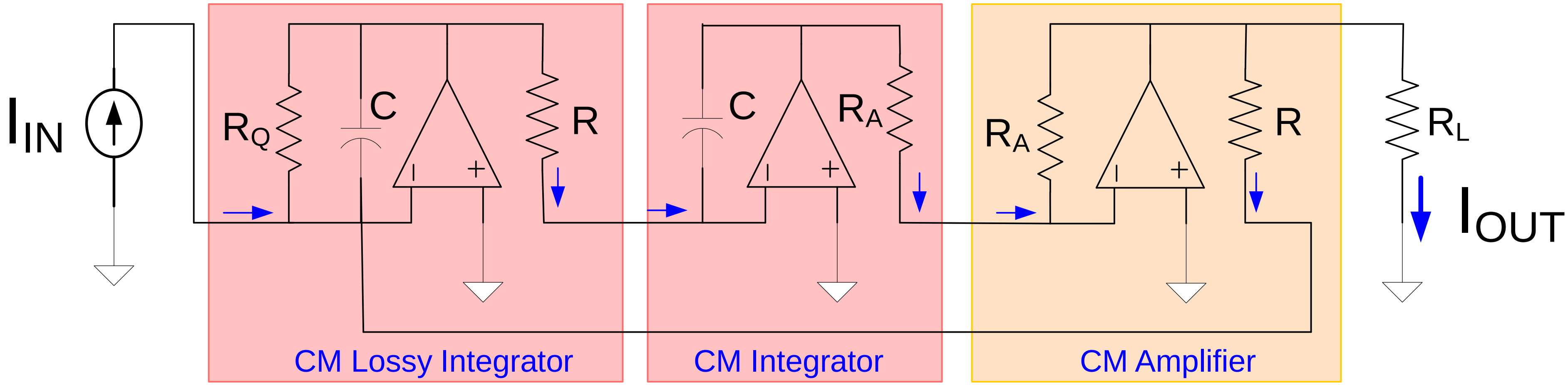
Current-Mode Integrators



The other basic types of voltage-mode integrators also have current-mode counterparts

- Switched-resistor
- MOSFET-C
- “Other”

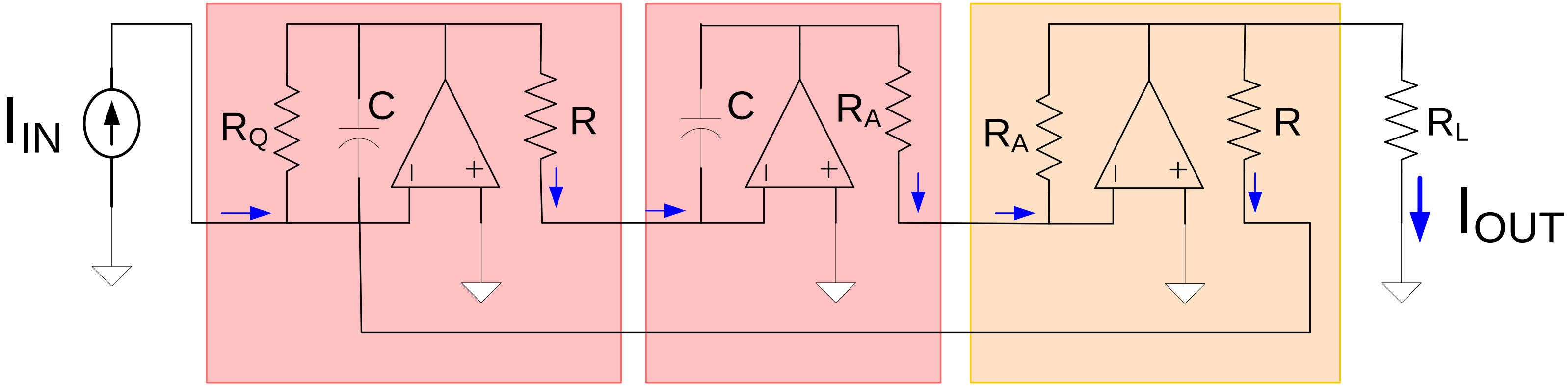
Current-Mode Two Integrator Loop

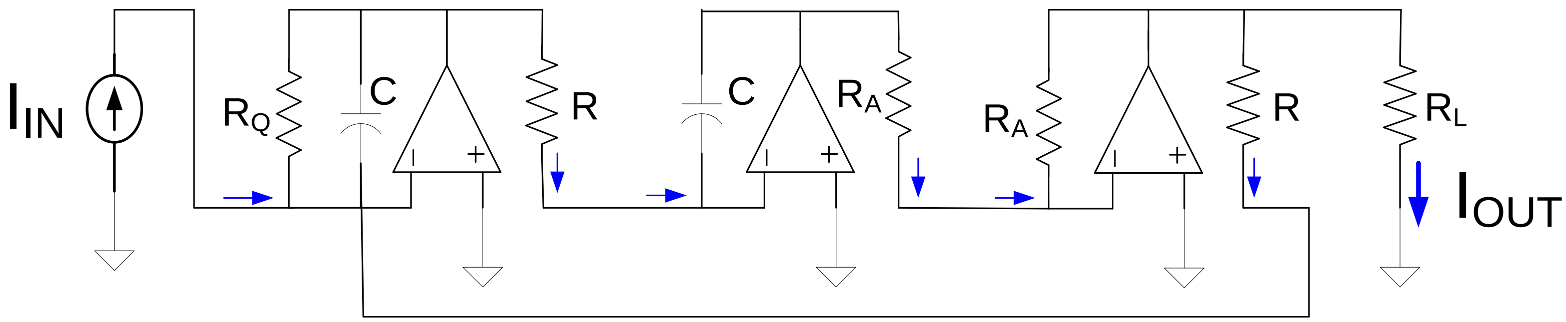


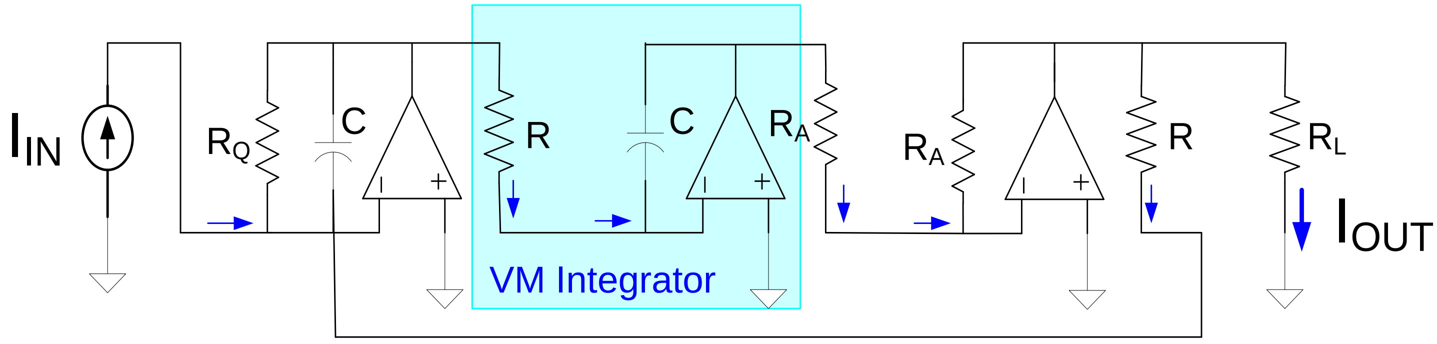
- Straightforward implementation of the two-integrator loop
- Simple structure

Current-Mode Two Integrator Loop

An Observation:







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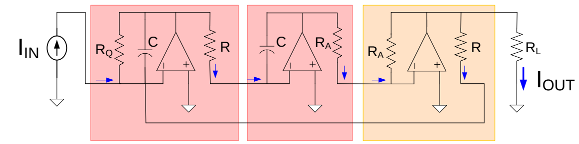

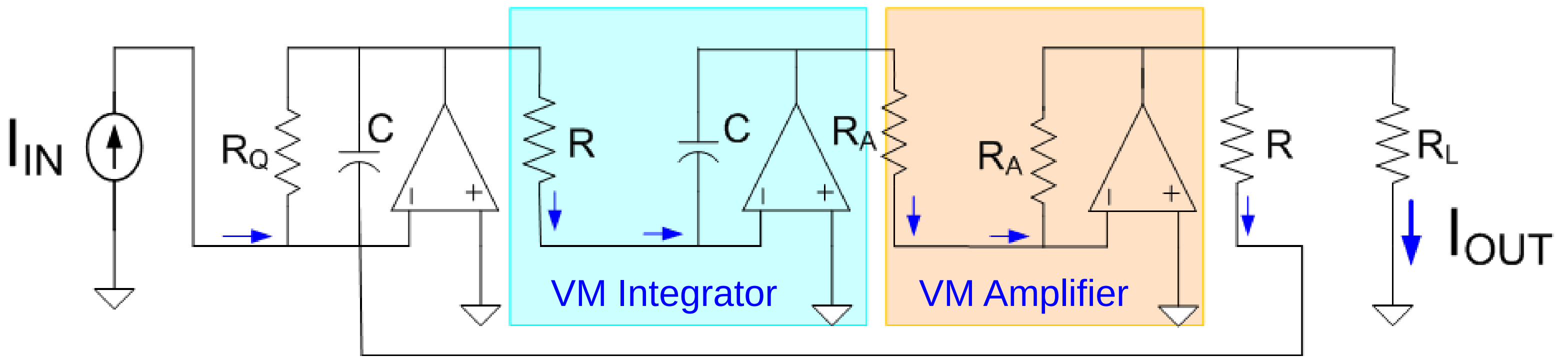

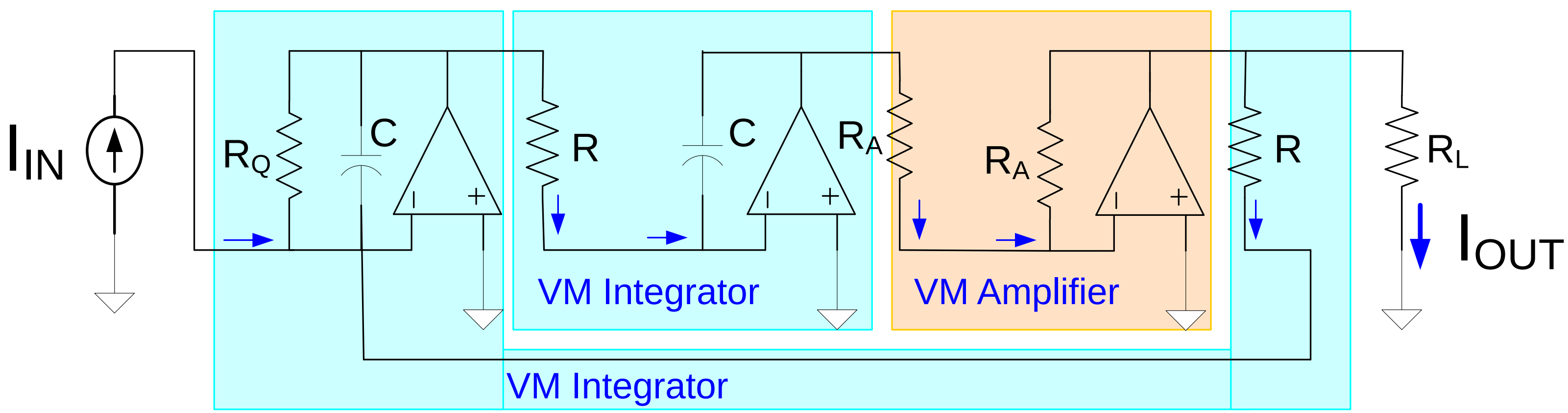

This circuit is identical to another one with two voltage-mode integrators and a voltage-mode amplifier !

Observation

- Many papers have appeared that tout the performance advantages of current-mode circuits
- In all of the current-mode papers that this instructor has seen, no attempt is made to provide a quantitative comparison of the key performance features of current-mode circuits with voltage-mode counterparts
- All justifications of the advantages of the current-mode circuits this instructor has seen are based upon qualitative statements

Observations (cont.)

- It appears easy to get papers published that have the term "current-mode" in the title
- Over 900 papers have been published in IEEE forums alone !
- Some of the "current-mode" filters published perform better than other "voltage-mode" filters that have been published
- We are still waiting for even one author to quantitatively show that current-mode filters offer even one of the claimed four advantages over their voltage-mode counterparts

Will return to a discussion of Current-Mode filters later

Switched-Current Filters

Basic idea introduced by Hughes and Bird at ISCAS 1989

Switched currents-a new technique for analog sampled-data signal processing
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NTRODUCTION The enormous complexity available in state-of-the-art CMOS processing has made possible the integration of complete systems, including both digital and analog signal processing functions, within the same chip Through the last decade, the **switched** capacitor technique **...**
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Technique introduced directly in the z-domain

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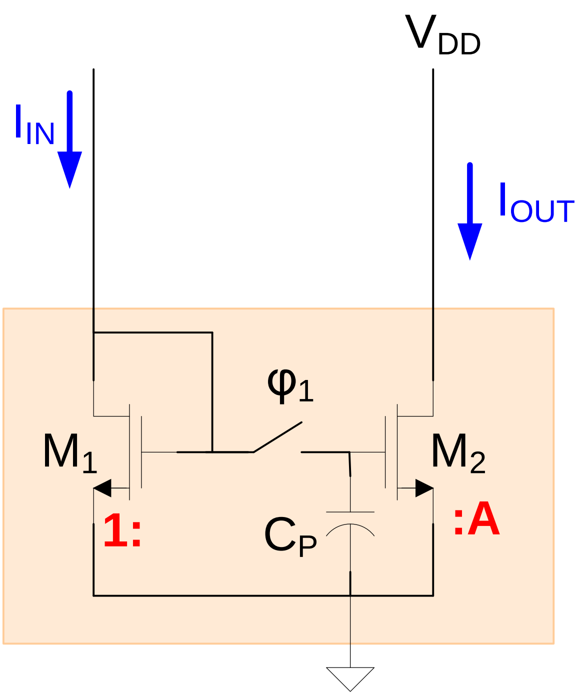

$$I_{OUT} = \begin{cases} AI_{IN}(t) & \text{for } \varphi_1 \text{ closed} \\ AI_{IN}(T_{SW}) & \text{for } \varphi_1 \text{ open} \end{cases}$$

If Φ_1 is a periodic signal and if I_{IN} is also appropriately clocked, the input/output currents of this circuit can be represented with the difference equation

$$I_{OUT}(nT) = AI_{IN}(nT-T)$$

This switched mirror becomes a delay element

"Gain" A is that of a current mirror

A can be accurately controlled

Circuit is small and very fast

Concept can be extended to implement arbitrary difference equation

Difference equation characterizes filter H(z)

Need only current mirrors and switches

Truly a "current-mode" circuit

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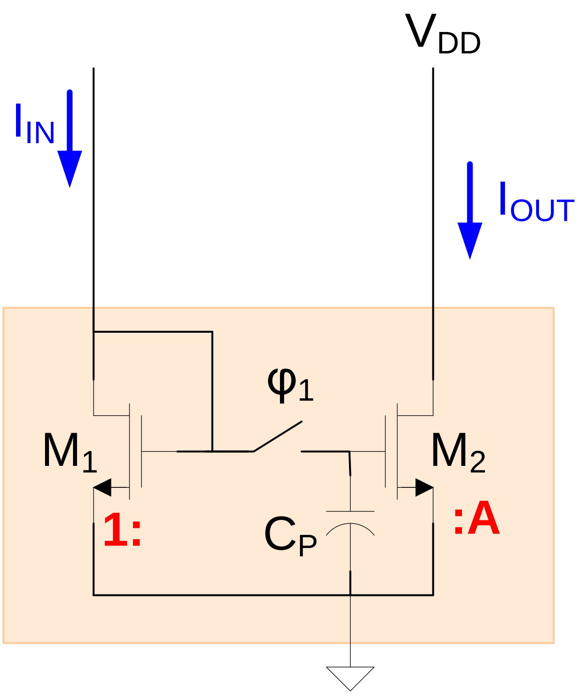


$$I_{OUT}(nT) = AI_{IN}(nT-T)$$

Cp is parasitic gate capacitance on M_2

Very low power dissipation

Potential to operate at very low voltages

Potential for accuracy of a SC circuit at both low and high frequencies but without the Op Amp and large C ratios

Neither capacitor or resistor values needed to do filtering!

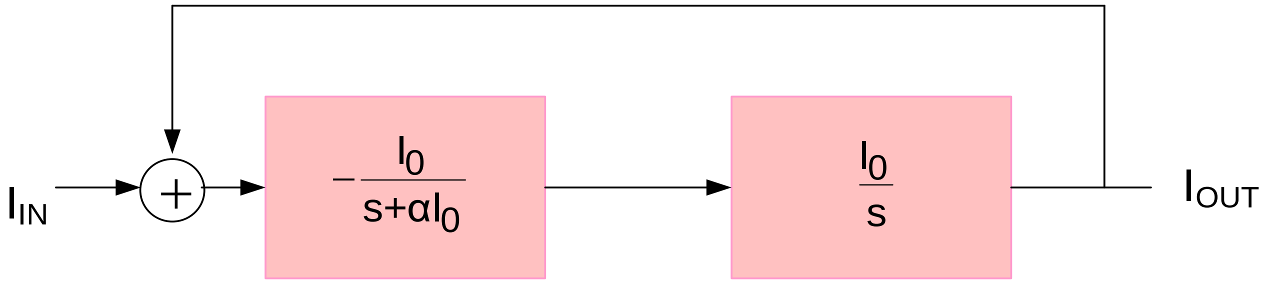
A completely new approach to designing filters that offers potential for overcoming most of the problems plaguing filter designers for decades !

Before developing Switch-Current concept, need to review background information in s to z domain transformations

EE 508
Lecture 31

Switched Current Filters

Current-Mode Filters



$$T(s)=\frac{I_{OUT}}{I_{IN}}=\frac{-I_0^2}{s^2+\alpha I_0 s+I_0^2}$$

Basic Concepts of Benefits of Current-Mode Filters:

- Large voltage swings difficult to maintain in integrated processes because of linearity concerns
- Large voltage swings slow a circuit down because of time required to charge capacitors
- Voltage swings can be very small when currents change
- Current swings are not inherently limited in integrated circuits (only voltage swings)
- With low voltage swings, current-mode circuits should dissipate little power

Review from last time

Current-Mode Filters

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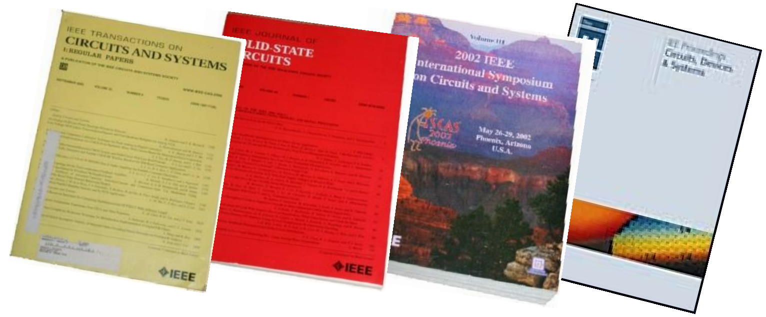

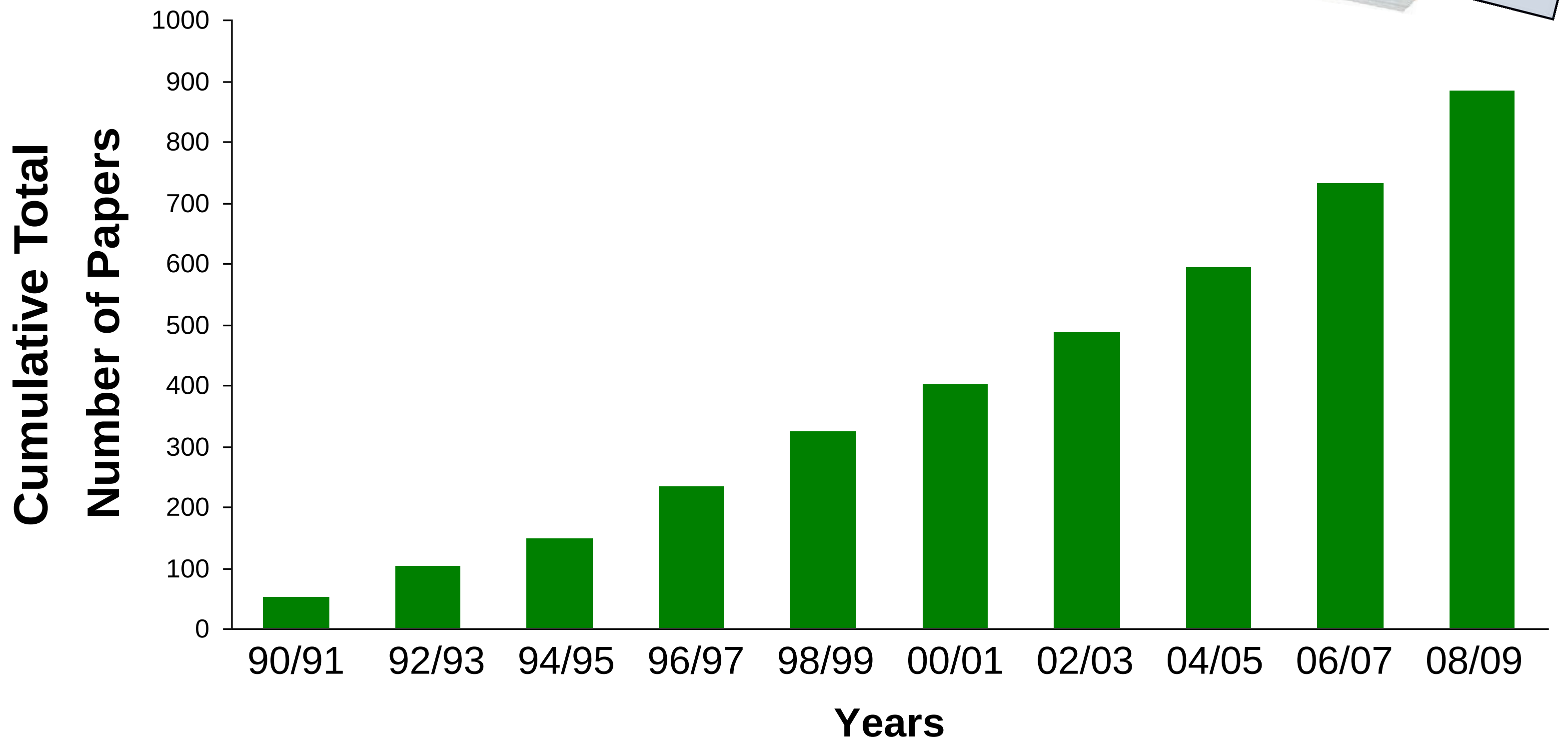

Steady growth in research in the area since 1990 and publication rate is growing with time !!

Current-Mode Filters

The Conventional Wisdom:

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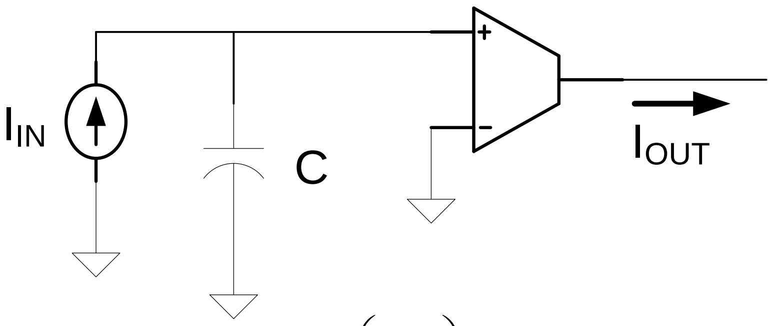
Review from last time

Some Current-Mode Integrators

OTA-C



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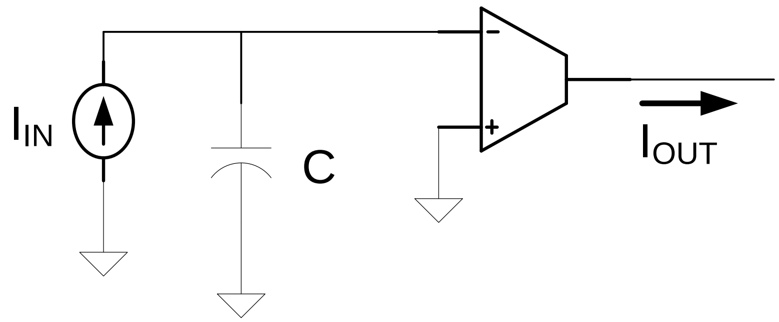

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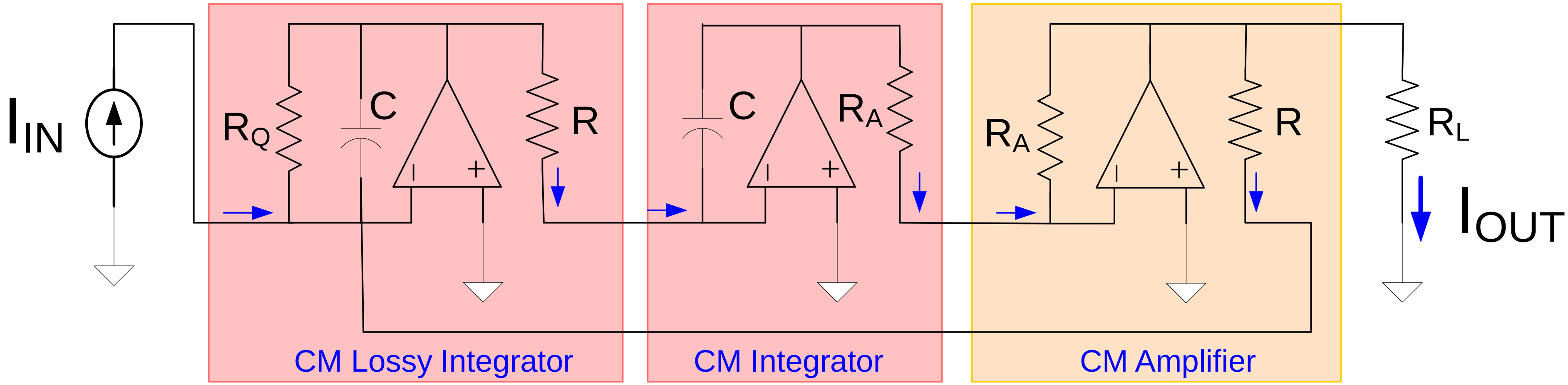
Review from last time

Current-Mode Two Integrator Loop



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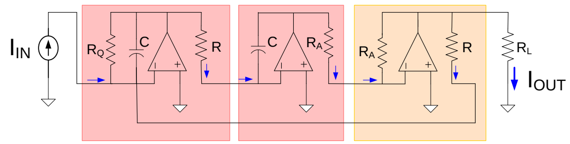

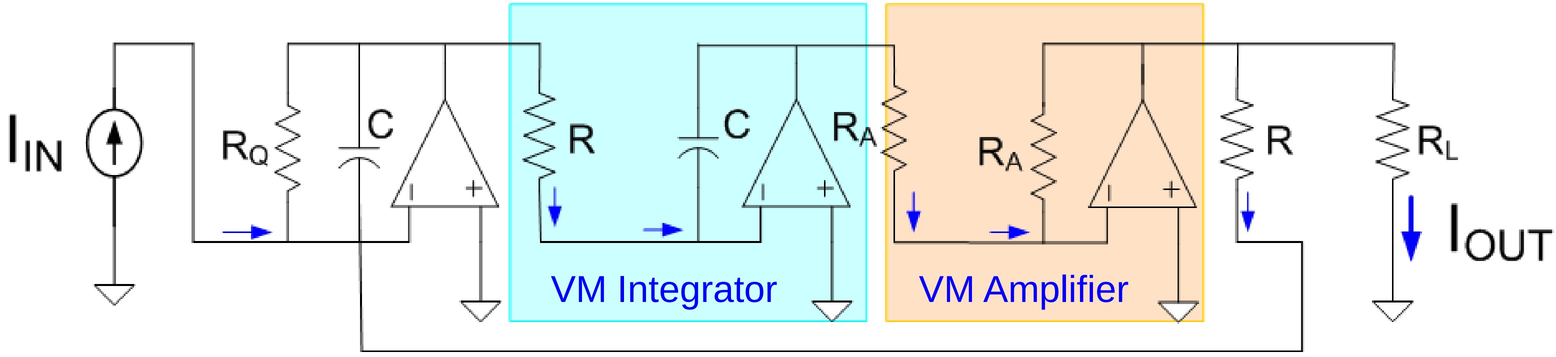

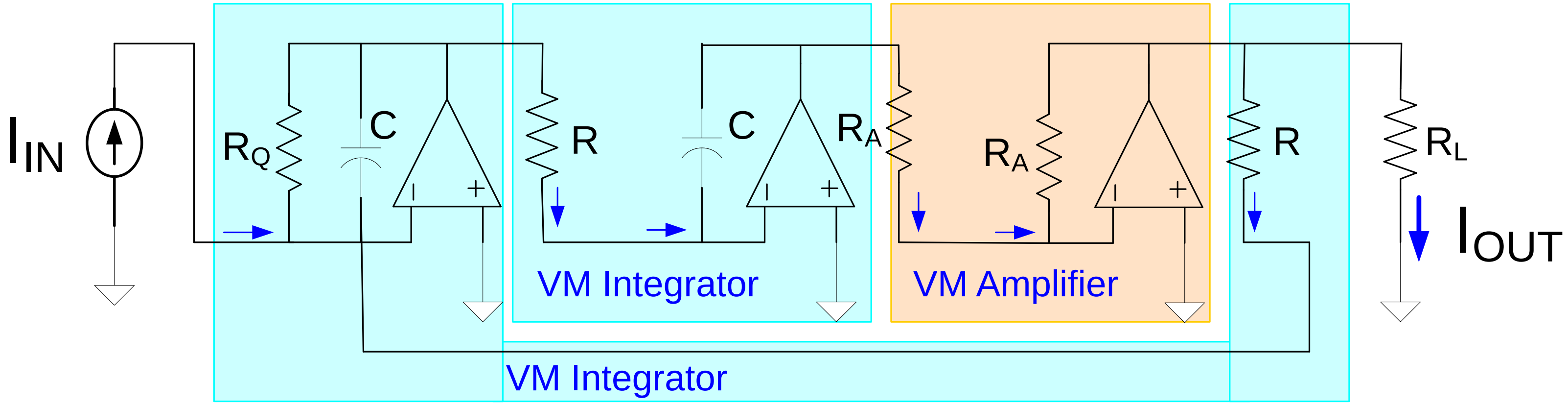

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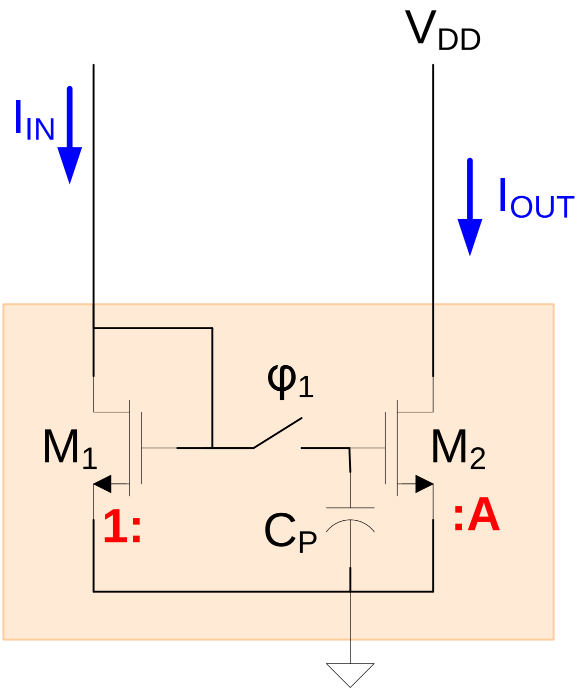

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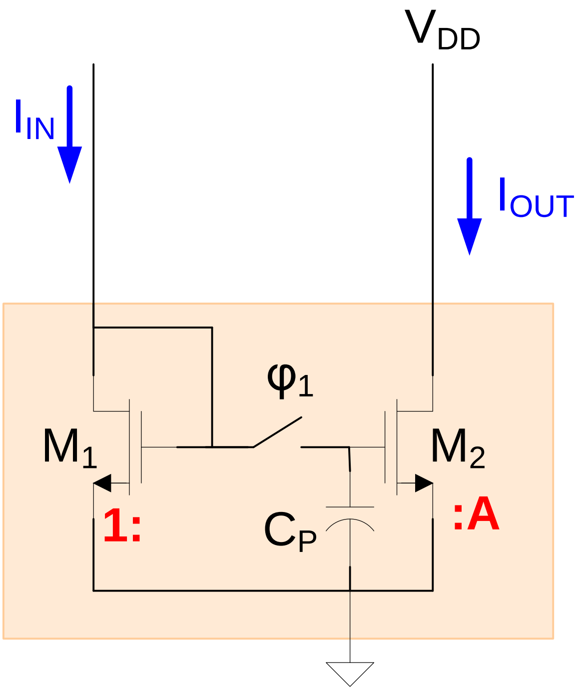

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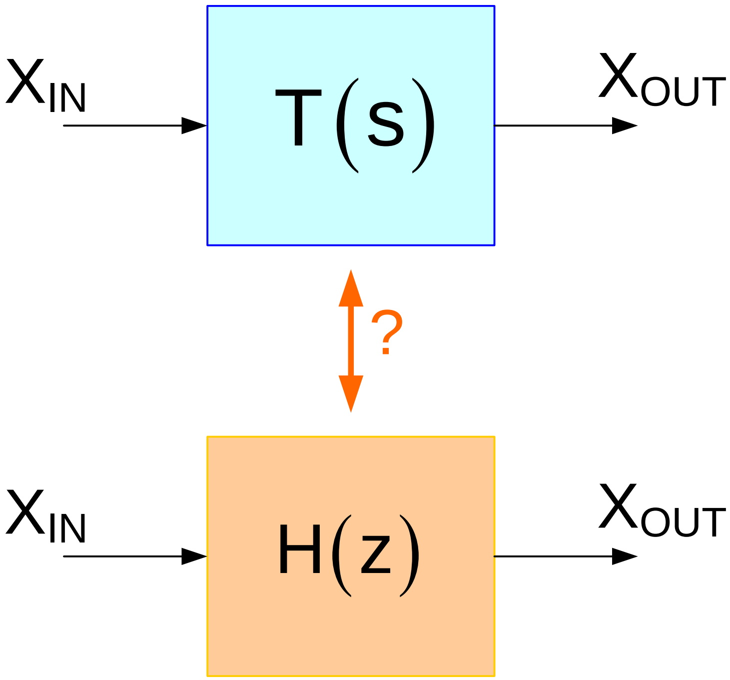
Before developing Switch-Current concept, need to review background information in s to z domain transformations

s-domain to z-domain transformations

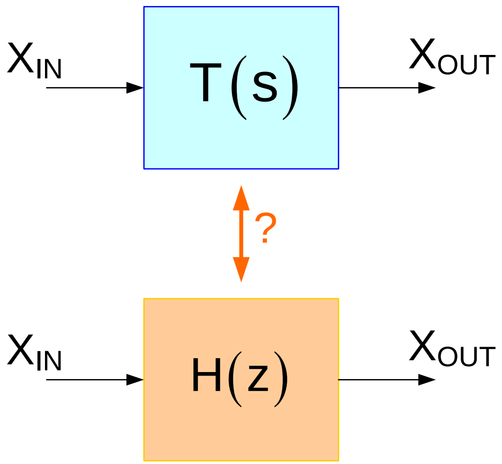


For a given T(s) would like to obtain a function H(z) or for a given H(z) would like to obtain a T(s) such that preserves the magnitude and phase response

Mathematically, would like to obtain the relationship:

$$T(s)\big|_{s=j\omega} = H(z)\big|_{z=e^{j\omega T}}$$

s-domain to z-domain transformations



want: $$T(s)\Big|_{s=j\omega} = H(z)\Big|_{z=e^{j\omega T}}$$

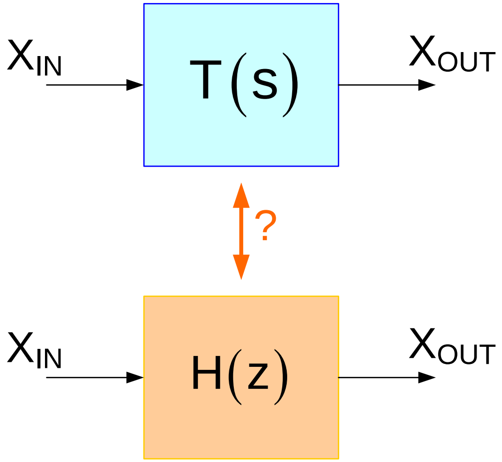
equivalently, want:

$$T(s) = H(z)\Big|_{z=e^{sT}}$$

But if this were to happen, T(s) would not be a rational fraction in s with real coeff.

Thus, it is impossible to obtain this mapping between T(s) and H(z)

s-domain to z-domain transformations



goal: $$T(s) = H(z)\Big|_{z=e^{sT}}$$

If can't achieve this goal, would like to map imaginary axis to unit circle and map stable filters to stable filters

consider: $$z = e^{sT}$$

Case 1:

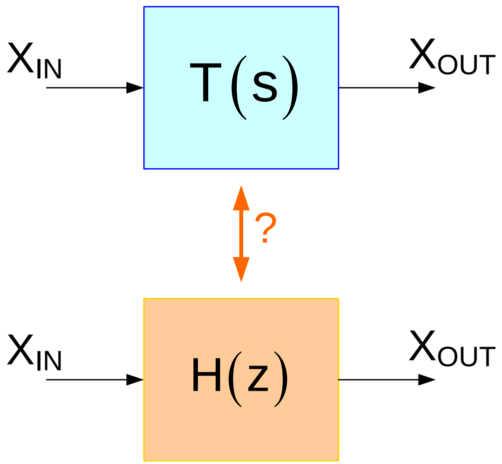
$$z = e^{sT} \simeq \sum_{i=0}^{\infty} \frac{1}{i!}(sT)^i$$

$$z = \sum_{i=0}^{\infty} \frac{1}{i!}(sT)^i \simeq 1 + sT$$

$$s = \frac{z - 1}{T}$$

Termed the Forward Euler transformation

s-domain to z-domain transformations

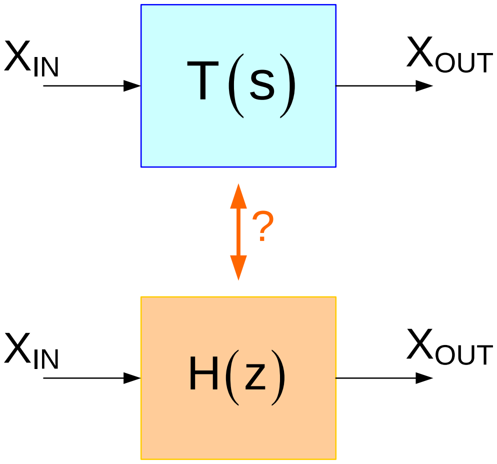


$$s = \frac{z - 1}{T}$$

Forward Euler transformation

- Doesn't map imaginary axis in s-plane to unit circle in z-plane
- Doesn't <u>guarantee</u> stable filter will map to stable filter
- But mapping may give stable filter with good frequency response

s-domain to z-domain transformations



consider: $z = e^{sT}$

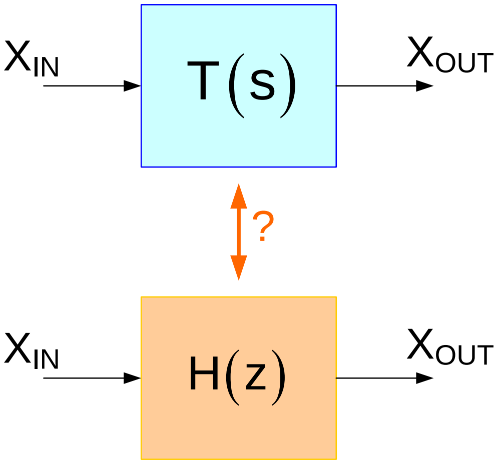
Case 2: $z = e^{sT} = \frac{1}{e^{-sT}} = \frac{1}{\sum_{i=0}^{\infty} \frac{1}{i!}(-sT)^i} \simeq \frac{1}{1-sT}$

$$z \simeq \frac{1}{1-sT}$$

$$s = \left(\frac{1}{T}\right)\frac{z-1}{z}$$

Termed the Backward Euler transformation

s-domain to z-domain transformations



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Backward Euler transformation

- Doesn't map imaginary axis in s-plane to unit circle in z-plane
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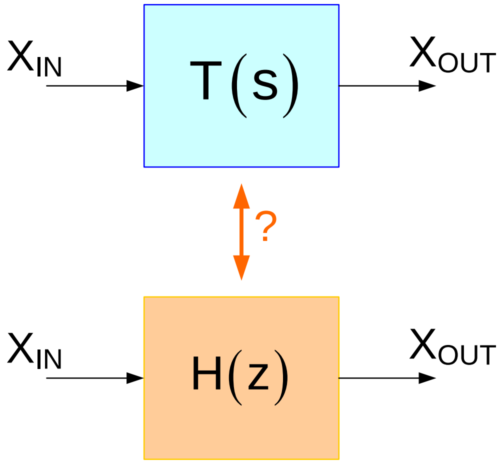
s-domain to z-domain transformations



consider: $z = e^{sT}$

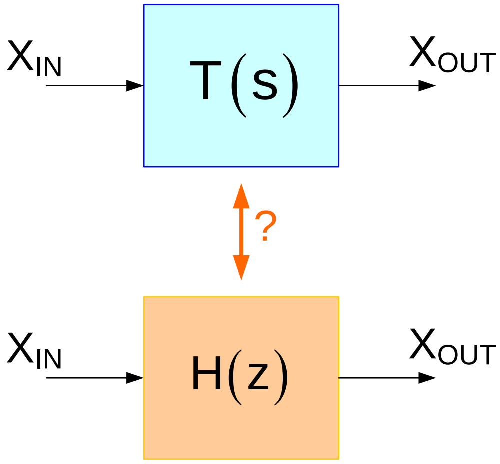
Case 3: $$z = e^{sT} = \frac{e^{s\frac{T}{2}}1}{e^{-s\frac{T}{2}}} = \frac{\sum_{i=0}^{\infty}\frac{1}{i!}\left(s\frac{T}{2}\right)^i}{\sum_{i=0}^{\infty}\frac{1}{i!}\left(-s\frac{T}{2}\right)^i} \simeq \frac{1+s\frac{T}{2}}{1-s\frac{T}{2}}$$

solving for s, obtain

$$s = \frac{2}{T} \bullet \frac{z-1}{z+1}$$

Termed the Bilinear z transformation

s-domain to z-domain transformations

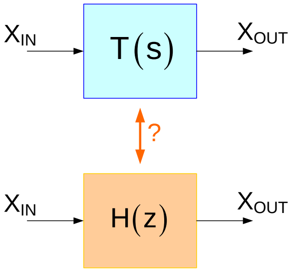


$$s = \frac{2}{T} \bullet \frac{z-1}{z+1}$$

Bilinear z transformation

- Maps imaginary axis in s-plane to unit circle in z-plane (preserves shape, distorts frequency axis)
- Does guarantee stable filter will map to stable filter
- Bilinear z transformation is widely used

s-domain to z-domain transformations



consider: $z = e^{sT}$

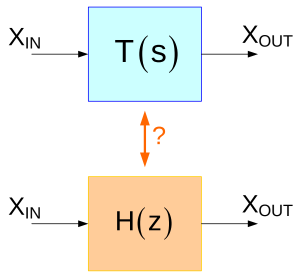
Three Popular Transformations

$$s = \frac{z-1}{T}$$ Forward Euler $$s = \frac{1-z^{-1}}{Tz^{-1}}$$

$$s = \frac{z-1}{Tz}$$ Backward Euler $$s = \frac{1-z^{-1}}{T}$$

$$s = \frac{2}{T} \bullet \frac{z-1}{z+1}$$ Bilinear z transform $$s = \frac{2}{T} \bullet \frac{1-z^{-1}}{1+z^{-1}}$$

s-domain to z-domain transformations

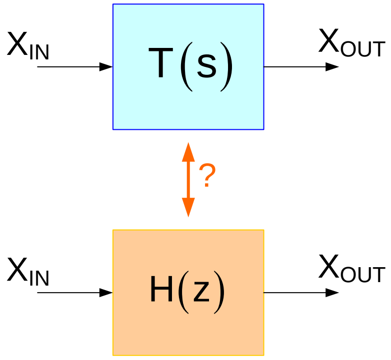


Three Popular Transformations

$s = \frac{z-1}{T}$	Forward Euler	$s = \frac{1-z^{-1}}{Tz^{-1}}$
$s = \frac{z-1}{Tz}$	Backward Euler	$s = \frac{1-z^{-1}}{T}$
$s = \frac{2}{T} \bullet \frac{z-1}{z+1}$	Bilinear z transform	$s = \frac{2}{T} \bullet \frac{1-z^{-1}}{1+z^{-1}}$

- Transformations of standard approximations in s-domain are the corresponding transformations in the z-domain
- Transformations are not unique
- Transformations cause warping of the imaginary axis and may cause change in basic shape
- Transformations do not necessarily guarantee stability
- These transformations preserve order

z-domain integrators



$$T(s) = \frac{I_0}{s}$$

Some z-domain integrators

$$H(z) = \begin{cases} \dfrac{TI_0}{z-1} & \text{Forward Euler} \\ \dfrac{I_0 Tz}{z-1} & \text{Backward Euler} \\ \dfrac{TI_0}{2}\left(\dfrac{z+1}{z-1}\right) & \text{Bilinear z} \end{cases}$$

Three Popular Transformations

$s = \dfrac{z-1}{T}$	Forward Euler	$s = \dfrac{1-z^{-1}}{Tz^{-1}}$
$s = \dfrac{z-1}{Tz}$	Backward Euler	$s = \dfrac{1-z^{-1}}{T}$
$s = \dfrac{2}{T} \bullet \dfrac{z-1}{z+1}$	Bilinear z transform	$s = \dfrac{2}{T} \bullet \dfrac{1-z^{-1}}{1+z^{-1}}$

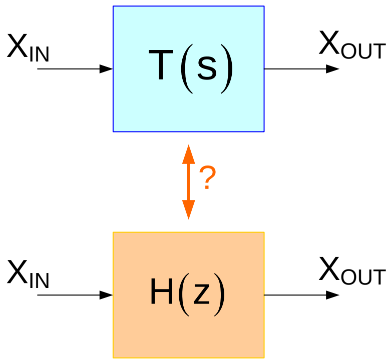
Corresponding difference equations:

$$V_{OUT}(nT+T) = TI_0 V_{IN}(nT) + V_{OUT}(nT) \qquad \text{Forward Euler}$$

$$V_{OUT}(nT+T) = I_0 T V_{IN}(nT+T) + V_{OUT}(nT) \qquad \text{Backward Euler}$$

$$V_{OUT}(nT+T) = \frac{TI_0}{2}\left(V_{IN}(nT+T) + V_{IN}(nT)\right) + V_{OUT}(nT) \qquad \text{Bilinear z}$$

z-domain lossy integrators



$$T(s) = \frac{I_0}{s+\alpha}$$

Three Popular Transformations

$s = \frac{z-1}{T}$	Forward Euler	$s = \frac{1-z^{-1}}{Tz^{-1}}$
$s = \frac{z-1}{Tz}$	Backward Euler	$s = \frac{1-z^{-1}}{T}$
$s = \frac{2}{T}\bullet\frac{z-1}{z+1}$	Bilinear z transform	$s = \frac{2}{T}\bullet\frac{1-z^{-1}}{1+z^{-1}}$

Some z-domain lossy integrators

$$H(z) = \begin{cases} \frac{TI_0}{z-1+\alpha T} & \text{Forward Euler} \\ \frac{I_0 Tz}{z(1+\alpha T)-1} & \text{Backward Euler} \\ \frac{TI_0}{2}\left(\frac{z+1}{z\left(1+\frac{\alpha T}{2}\right)+\left(\frac{\alpha T}{2}-1\right)}\right) & \text{Bilinear z} \end{cases}$$

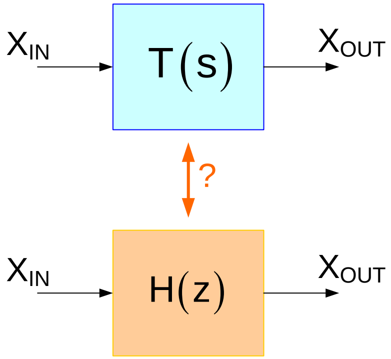
Corresponding difference equations:

$$V_{OUT}(nT+T) = TI_0 V_{IN}(nT) + [1-\alpha T]V_{OUT}(nT) \qquad \text{Forward Euler}$$

$$(1+\alpha T)V_{OUT}(nT+T) = I_0 T V_{IN}(nT+T) + V_{OUT}(nT) \qquad \text{Backward Euler}$$

$$\left(1+\frac{\alpha T}{2}\right)V_{OUT}(nT+T) = \frac{TI_0}{2}\left(V_{IN}(nT+T)+V_{IN}(nT)\right) + \left[1-\frac{\alpha T}{2}\right]V_{OUT}(nT) \qquad \text{Bilinear z}$$

z-domain lossy integrators



Some z-domain lossy integrators

$$T(s) = \frac{I_0}{s+\alpha}$$

	Functional Form	
$\frac{TI_0}{z-1+\alpha T}$	$\frac{G}{z-H}$	Forward Euler
$\frac{I_0 Tz}{z(1+\alpha T)-1}$	$\frac{Gz}{zH-1}$	Backward Euler
$\frac{TI_0}{2}\left(\frac{z+1}{z\left(1+\frac{\alpha T}{2}\right)+\left(\frac{\alpha T}{2}-1\right)}\right)$	$G\left(\frac{z+1}{z-H}\right)$	Bilinear z

$$H(z) = \begin{cases} \frac{TI_0}{z-1+\alpha T} \\ \frac{I_0 Tz}{z(1+\alpha T)-1} \\ \frac{TI_0}{2}\left(\frac{z+1}{z\left(1+\frac{\alpha T}{2}\right)+\left(\frac{\alpha T}{2}-1\right)}\right) \end{cases}$$

Corresponding difference equations:

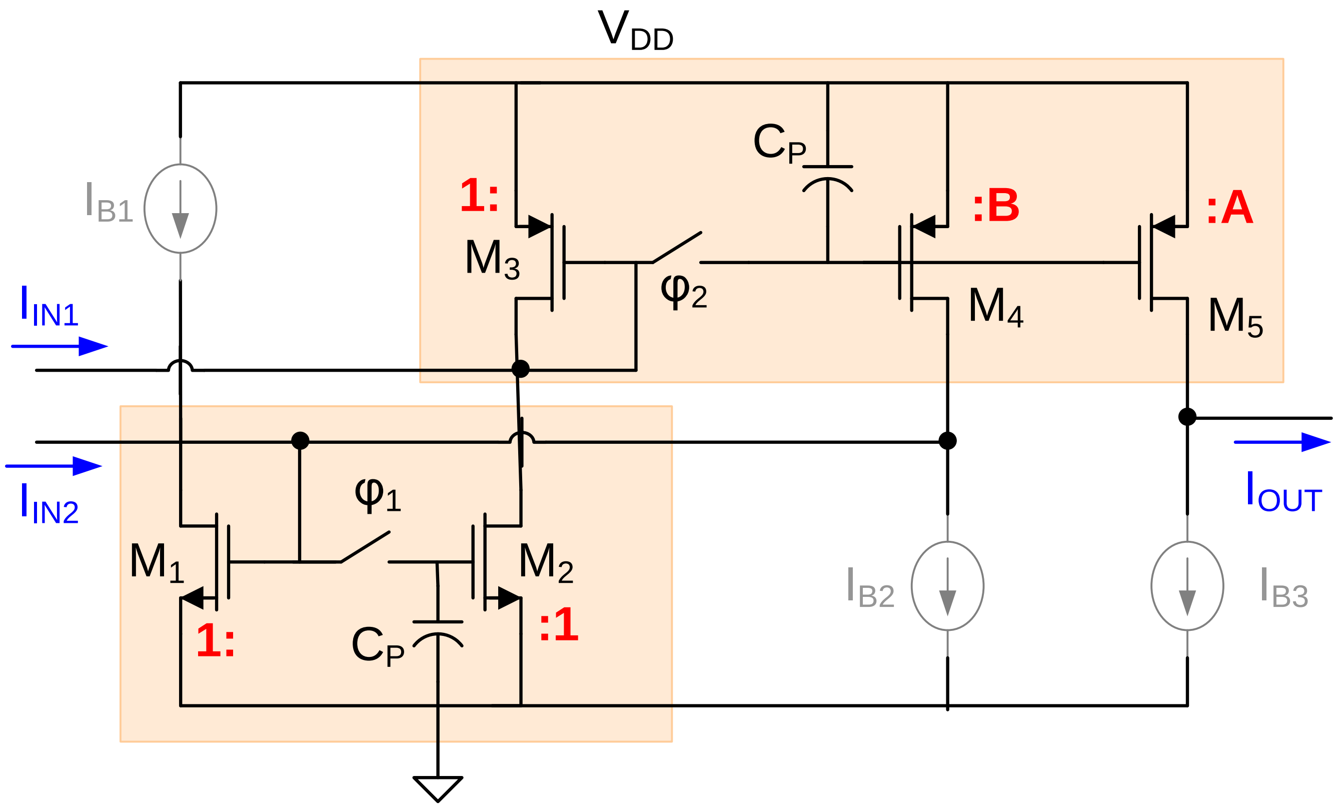
$$V_{OUT}(nT+T) = GV_{IN}(nT) + HV_{OUT}(nT) \quad \text{Forward Euler}$$

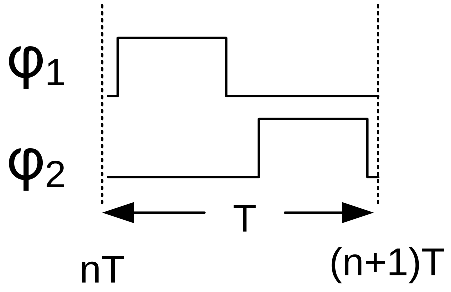
$$HV_{OUT}(nT+T) = GV_{IN}(nT+T) + V_{OUT}(nT) \quad \text{Backward Euler}$$

$$V_{OUT}(nT+T) = G\left(V_{IN}(nT+T) + V_{IN}(nT)\right) + HV_{OUT}(nT) \quad \text{Bilinear z}$$

Switched-Current Integrator

Consider this circuit



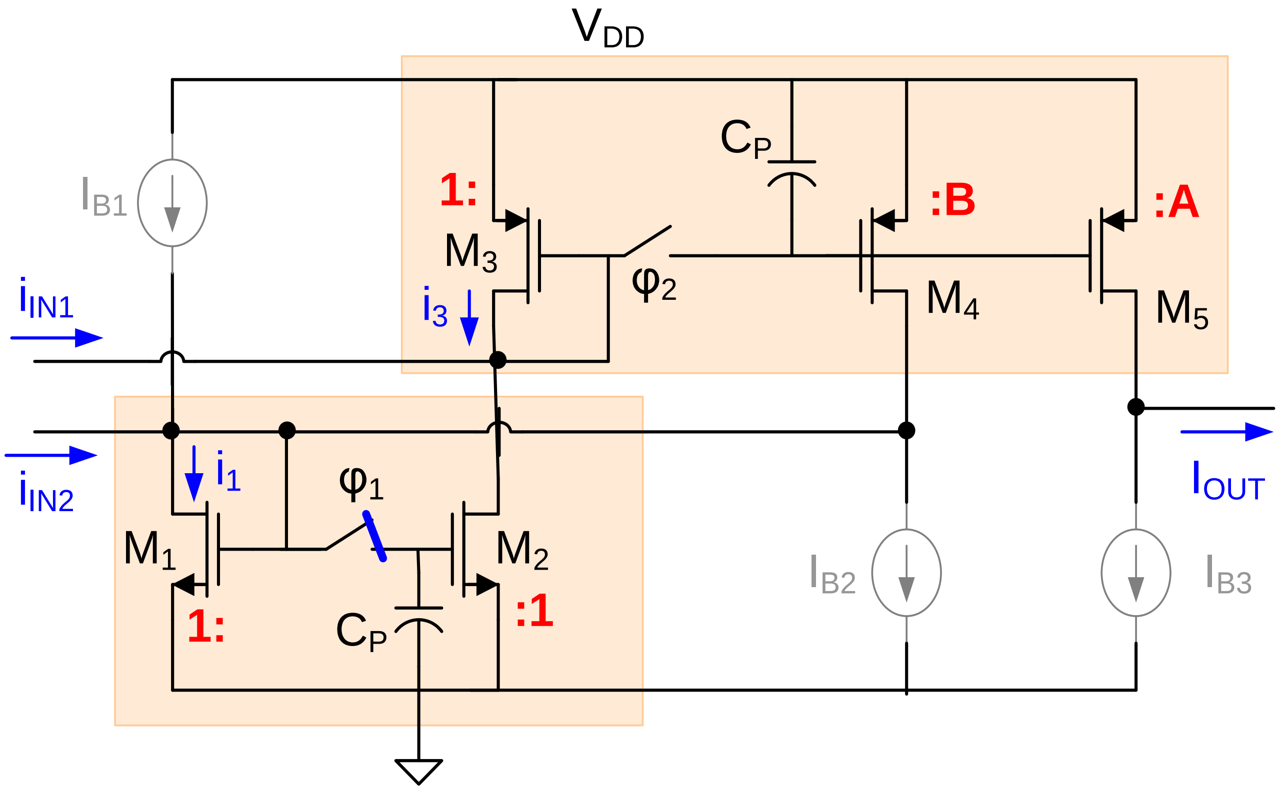


- Clocks complimentary, nonoverlapping
- Phase not critical

Assume inputs change only during phase Φ_2

(may be outputs from other like stages)

Switched-Current Integrator



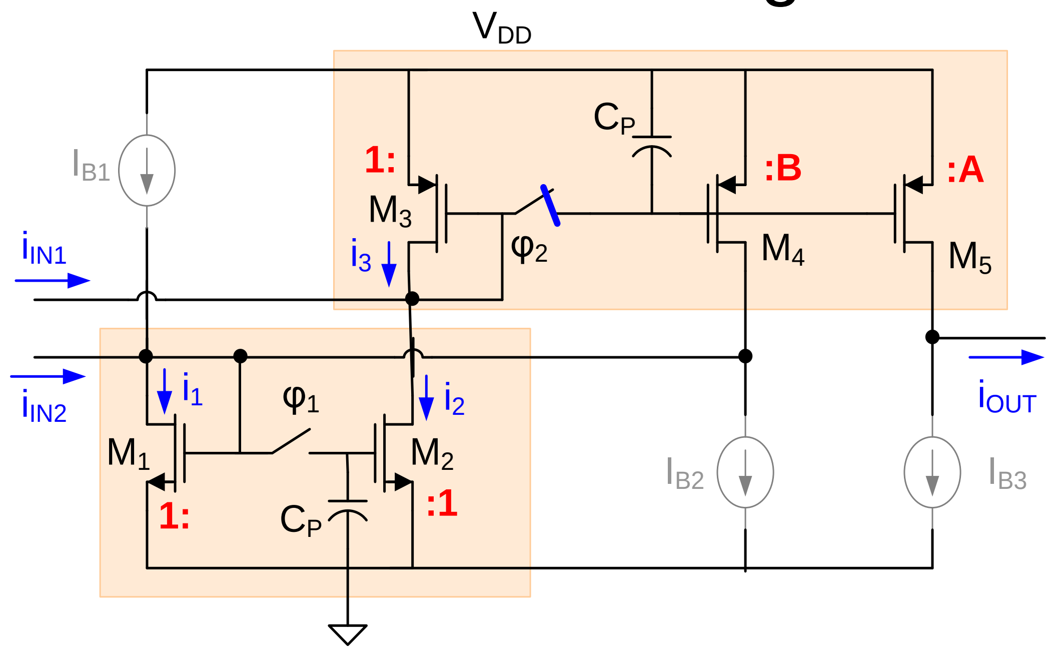
Consider Φ_1 closed, Φ_2 open (nT-T < t < nT-T/2)

$$i_1(t) = Bi_3(nT-T) + i_{iN2}(t)$$

Since current does not change during this interval

$$i_1(nT-T) = Bi_3(nT-T) + i_{iN2}(nT-T)$$

Switched-Current Integrator

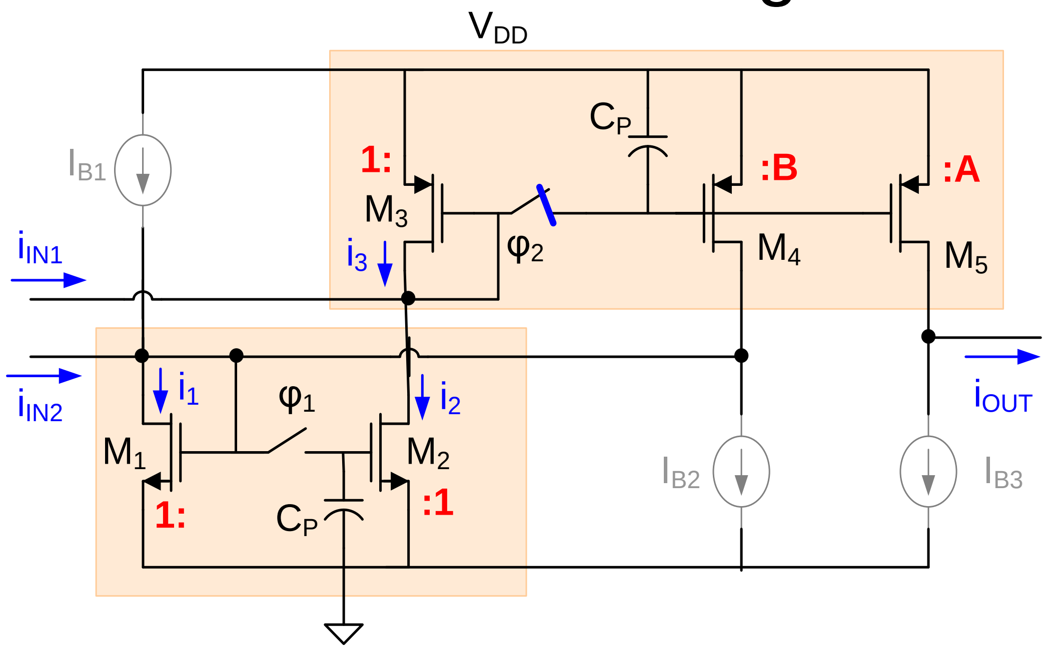


Consider Φ_2 closed, Φ_1 open (nT-T/2 < t < nT)

$$
\left.
\begin{aligned}
&i_2(t) = i_1(nT-T) \\
&i_2(t) = i_3(t) + i_{IN1}(t) \\
&i_{OUT}(t) = Ai_3(t) \\
&i_1(nT-T) = Bi_3(nT-T) + i_{iN2}(nT-T) \quad \text{(from first phase)}
\end{aligned}
\right\}
$$

$$\left(\frac{1}{A}\right) i_{OUT}(t) + i_{IN1}(t) = \frac{B}{A} i_{OUT}(nT-T) + i_{IN2}(nT-T)$$

Switched-Current Integrator



Consider Φ_2 closed, Φ_1 open (nT-T/2 < t < nT)

$$\left(\frac{1}{A}\right) i_{OUT}(t) + i_{IN1}(t) = \frac{B}{A} i_{OUT}(nT-T) + i_{IN2}(nT-T)$$

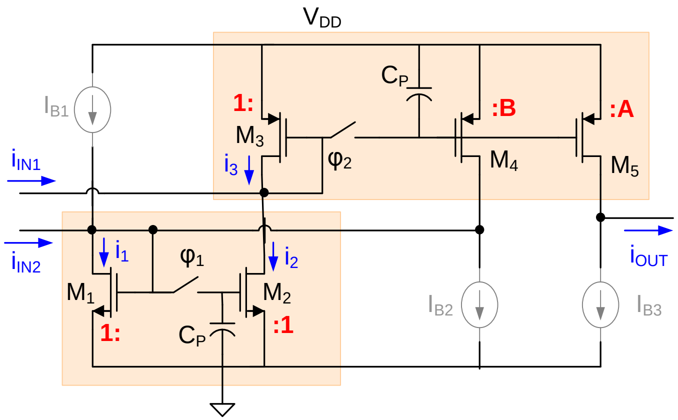
Evaluating at t=nT, we have

$$\left(\frac{1}{A}\right) i_{OUT}(nT) + i_{IN1}(nT) = \frac{B}{A} i_{OUT}(nT-T) + i_{IN2}(nT-T)$$

Taking z-transform, obtain

$$I_{OUT}(z) = \left(\frac{Az^{-1}}{1-Bz^{-1}}\right) I_{IN2}(z) - \left(\frac{A}{1-Bz^{-1}}\right) I_{IN1}(z)$$

Switched-Current Integrator



Recall lossy integrators:

$$H(z) = \begin{cases} \dfrac{Gz^{-1}}{1 - Hz^{-1}} & \text{Forward Euler} \\ \dfrac{G}{1 - Hz^{-1}} & \text{Backward Euler} \\ G\left(\dfrac{1 + z^{-1}}{1 - Hz^{-1}}\right) & \text{Bilinear z} \end{cases}$$

For H=1 becomes lossless

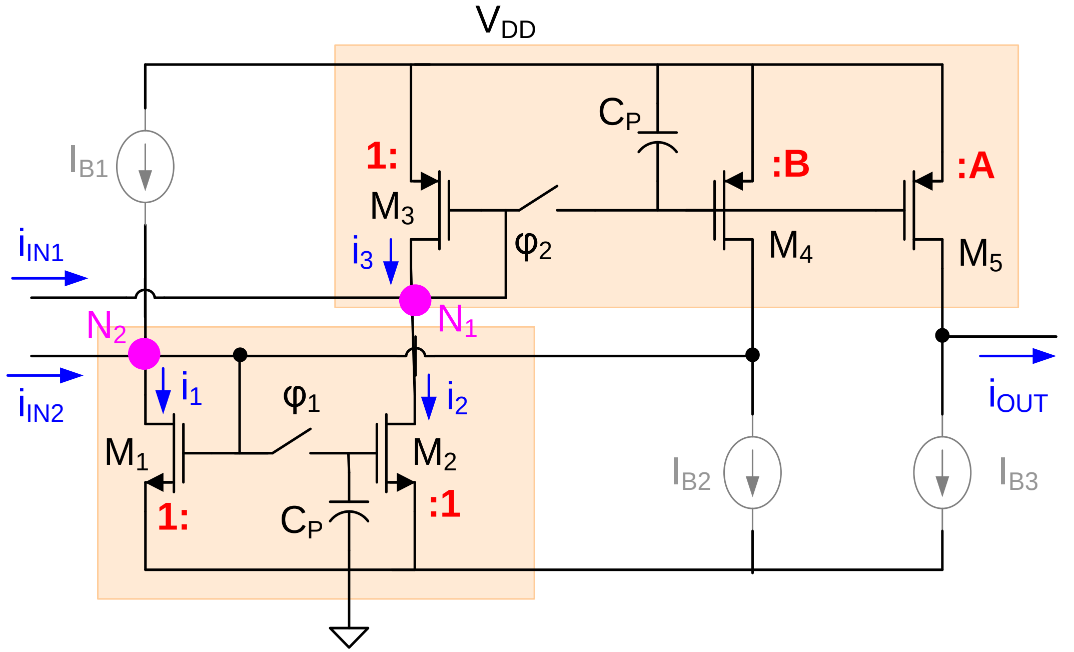
$$I_{OUT}(z) = \left(\frac{Az^{-1}}{1-Bz^{-1}}\right) I_{IN2}(z) - \left(\frac{A}{1-Bz^{-1}}\right) I_{IN1}(z)$$

If $I_{IN1}=0$, becomes Forward Euler integrator

If $I_{N2}=0$, becomes Backward Euler integrator

If $I_{N1} = -I_{IN2}$, becomes Bilinear Integrator

Switched-Current Integrator

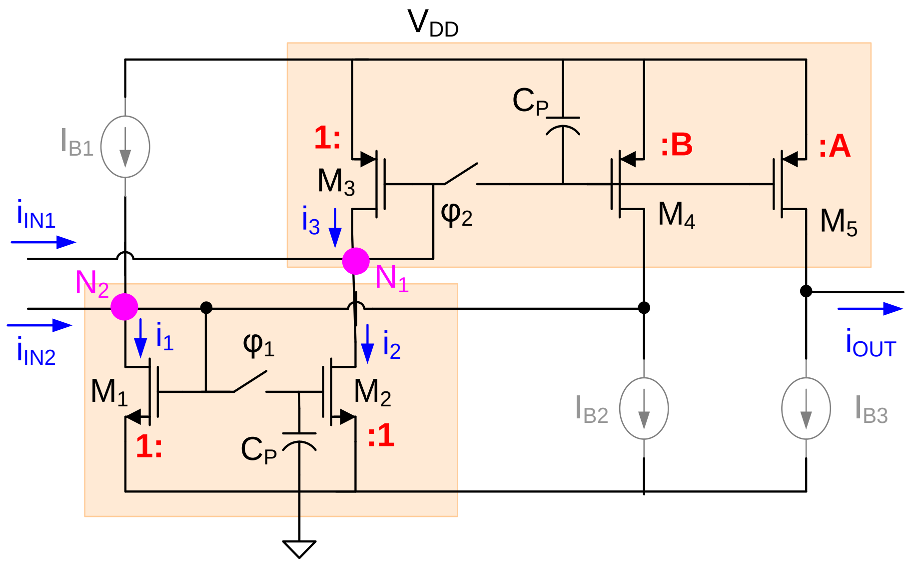


$$I_{OUT}(z) = \left(\frac{Az^{-1}}{1-Bz^{-1}}\right) I_{IN2}(z) - \left(\frac{A}{1-Bz^{-1}}\right) I_{IN1}(z)$$

- Summing inputs can be provided by summing currents on N_1 or N_2 or both
- Multiple outputs can be provided by adding outputs to upper mirror
- Amount of loss determined by mirror gain B

Switched-Current Integrator

Sensitivity Analysis



Consider Forward Euler

$$I_{OUT}(z) = \left(\frac{Az^{-1}}{1-Bz^{-1}}\right) I_{IN2}(z)$$

$$H(z) = \frac{TI_0}{z-1+\alpha T}$$

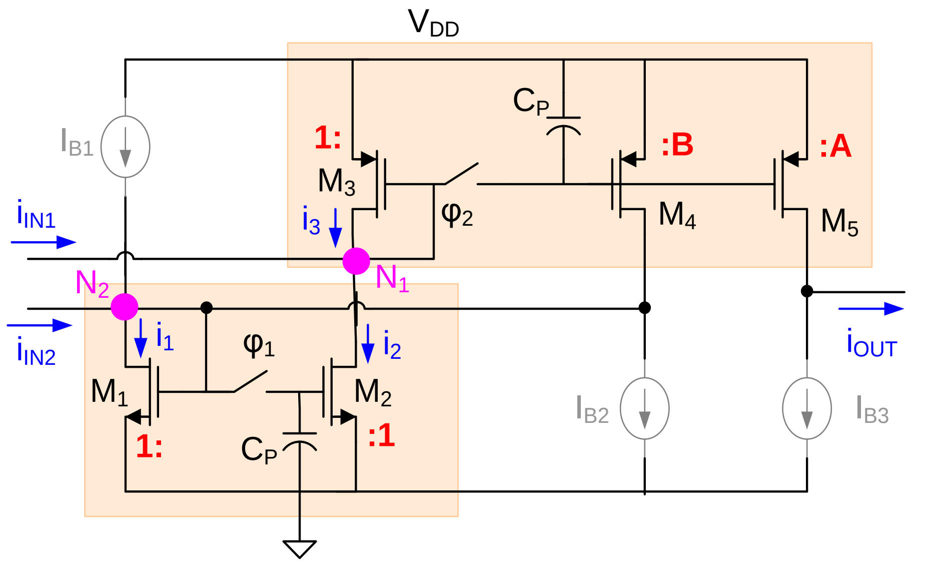
$$I_0 = \frac{A}{T} \qquad \frac{1-B}{T} = \alpha$$

$$S_A^{I_0} = 1 \qquad S_B^{\alpha} = \frac{-B}{1-B}$$

For low loss integrator (e.g. ideal integrator), the sensitivity of α is very large!

Switched-Current Integrator

Sensitivity Analysis



Consider Bilinear z

$$I_{OUT}(z) = A\left(\frac{z^{-1}+1}{1-Bz^{-1}}\right) I_{IN}(z)$$

$$H(z) = \frac{TI_0}{2}\left(\frac{z+1}{z\left(1+\frac{\alpha T}{2}\right)+\left(\frac{\alpha T}{2}-1\right)}\right)$$

$$I_0 = A\frac{2}{T(1+B)}$$

$$\alpha = \frac{2}{T}\frac{1-B}{1+B}$$

$$S_A^{I_0} = 1$$

$$S_B^{\alpha} = \frac{-B}{(1-B)(1+B)}$$

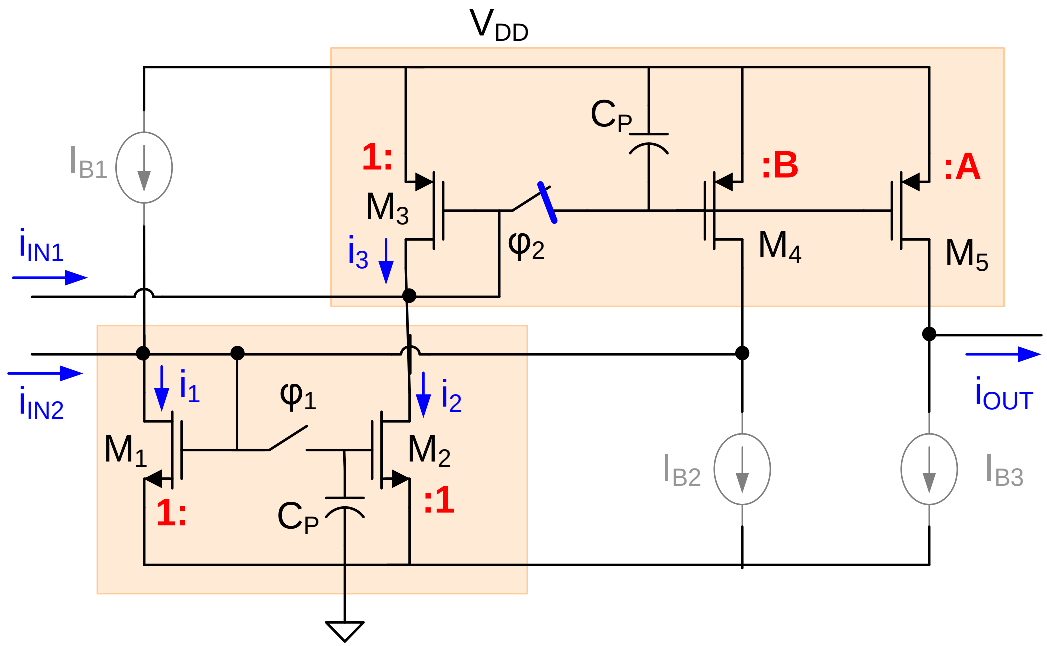
For low loss integrator (e.g. ideal integrator), the sensitivity of α is very large!

What about the sensitivity to the gain of the lower current mirror?

Switched-Current Integrator

Define A_1 to be the gain of the lower mirror

Sensitivity to A_1?



Consider Φ_2 closed, Φ_1 open (nT-T/2 < t < nT)

$$i_2(t) = A_1\, i_1(nT-T)$$

$$i_2(t) = i_3(t) + i_{IN1}(t)$$

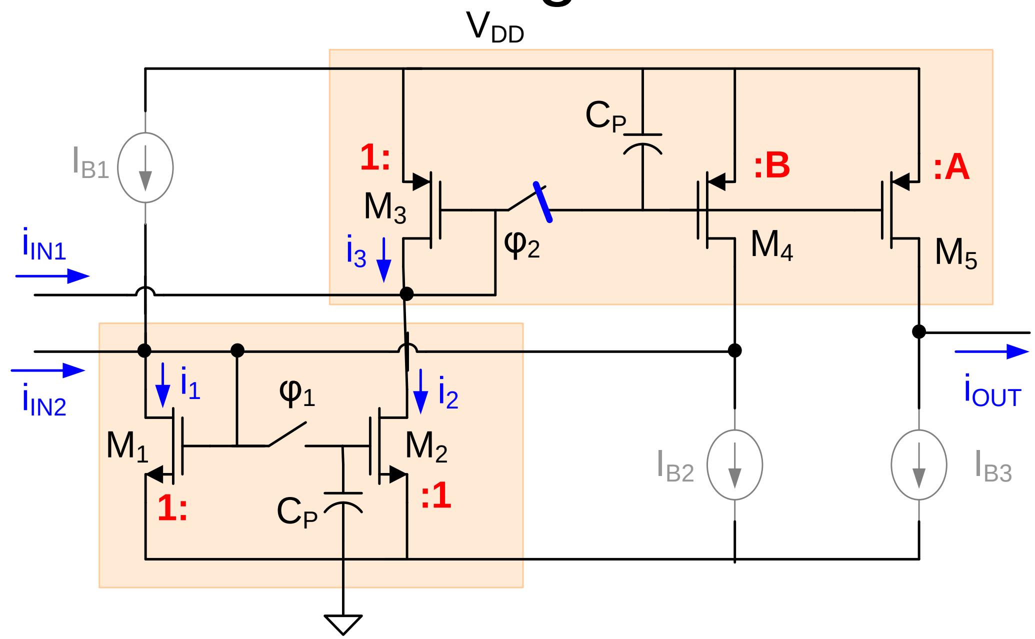
$$i_{OUT}(t) = A i_3(t)$$

$$i_1(nT-T) = B i_3(nT-T) + i_{IN2}(nT-T) \quad \text{(from first phase)}$$

$$\left(\frac{1}{A}\right) i_{OUT}(t) + i_{IN1}(t) = \frac{A_1 B}{A} i_{OUT}(nT-T) + A_1 i_{IN2}(nT-T)$$

Switched-Current Integrator

Define A_1 to be the gain of the lower mirror



Sensitivity to A_1?

$$\left(\frac{1}{A}\right)i_{OUT}(nT)+i_{IN1}(nT)=\frac{A_1B}{A}i_{OUT}(nT-T)+A_1 i_{IN2}(nT-T)$$

Taking z-transform, obtain

$$I_{OUT}(z)=\left(\frac{A_1Az^{-1}}{1-BA_1z^{-1}}\right)I_{IN2}(z)-\left(\frac{A}{1-BA_1z^{-1}}\right)I_{IN1}(z)$$

Consider Forward Euler

$$\frac{1-BA_1}{T}=\alpha \qquad S_B^{\alpha}=\frac{-BA_1}{1-BA_1} \qquad S_{A_1}^{\alpha}=\frac{-BA_1}{1-BA_1}$$

Sensitivity to A_1 is also large for low-loss or lossless integrator

EE 508
Lecture 32

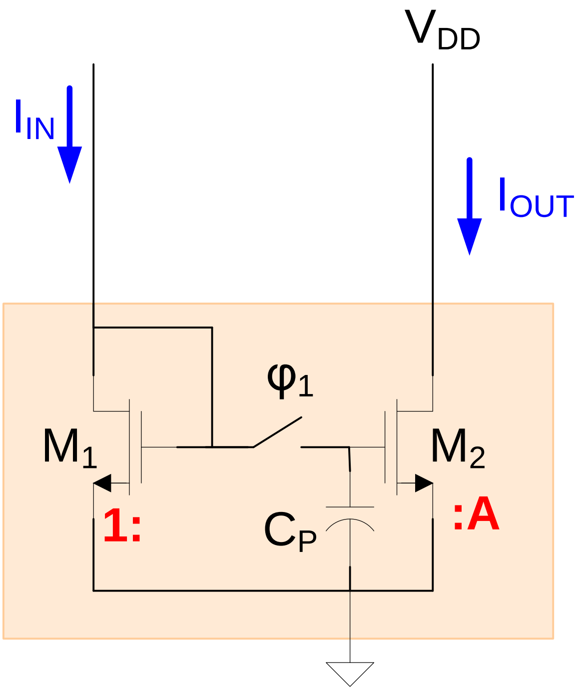
Switched Current Filters
Leapfrog Networks

Review from last time

Switched-Current Filters

Basic idea introduced by Hughes and Bird at ISCAS 1989

$$I_{OUT}(nT) = AI_{IN}(nT-T)$$



Cp is parasitic gate capacitance on M_2

Very low power dissipation

Potential to operate at very low voltages

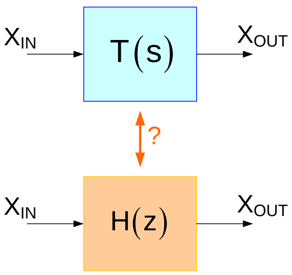
Potential for accuracy of a SC circuit at both low and high frequencies but without the Op Amp and large C ratios

Neither capacitor or resistor values needed to do filtering!

A completely new approach to designing filters that offers potential for overcoming most of the problems plaguing filter designers for decades !

Before developing Switch-Current concept, need to review background information in s to z domain transformations

s-domain to z-domain transformations



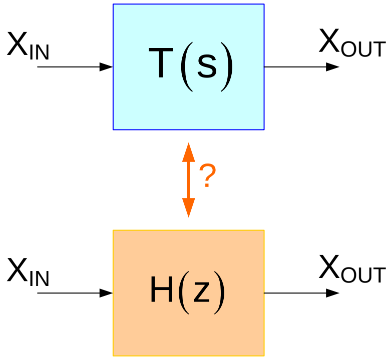
Three Popular Transformations

$s = \frac{z-1}{T}$	Forward Euler	$s = \frac{1-z^{-1}}{Tz^{-1}}$
$s = \frac{z-1}{Tz}$	Backward Euler	$s = \frac{1-z^{-1}}{T}$
$s = \frac{2}{T} \bullet \frac{z-1}{z+1}$	Bilinear z transform	$s = \frac{2}{T} \bullet \frac{1-z^{-1}}{1+z^{-1}}$

- Transformations of standard approximations in s-domain are the corresponding transformations in the z-domain
- Transformations are not unique
- Transformations cause warping of the imaginary axis and may cause change in basic shape
- Transformations do not necessarily guarantee stability
- These transformations preserve order

Review from last time

z-domain lossy integrators



$$T(s) = \frac{I_0}{s+\alpha}$$

Three Popular Transformations

$s = \frac{z-1}{T}$	Forward Euler	$s = \frac{1-z^{-1}}{Tz^{-1}}$
$s = \frac{z-1}{Tz}$	Backward Euler	$s = \frac{1-z^{-1}}{T}$
$s = \frac{2}{T} \bullet \frac{z-1}{z+1}$	Bilinear z transform	$s = \frac{2}{T} \bullet \frac{1-z^{-1}}{1+z^{-1}}$

Some z-domain lossy integrators

$$H(z) = \begin{cases} \dfrac{TI_0}{z-1+\alpha T} & \text{Forward Euler} \\ \dfrac{I_0 Tz}{z(1+\alpha T)-1} & \text{Backward Euler} \\ \dfrac{TI_0}{2}\left(\dfrac{z+1}{z\left(1+\dfrac{\alpha T}{2}\right)+\left(\dfrac{\alpha T}{2}-1\right)}\right) & \text{Bilinear z} \end{cases}$$

Corresponding difference equations:

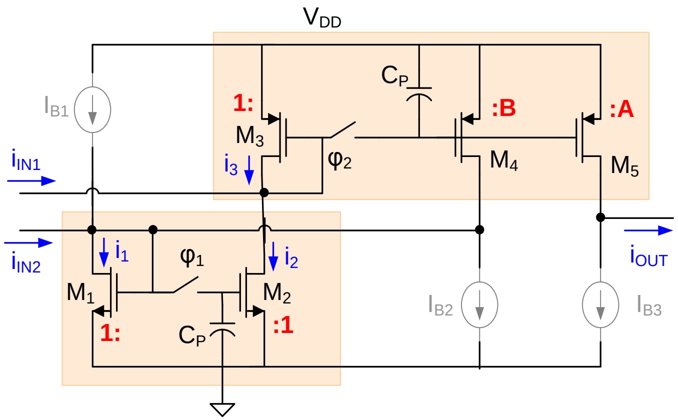
$$V_{OUT}(nT+T) = TI_0 V_{IN}(nT) + [1-\alpha T] V_{OUT}(nT) \qquad \text{Forward Euler}$$

$$(1+\alpha T) V_{OUT}(nT+T) = I_0 T V_{IN}(nT+T) + V_{OUT}(nT) \qquad \text{Backward Euler}$$

$$\left(1+\frac{\alpha T}{2}\right) V_{OUT}(nT+T) = \frac{TI_0}{2}\left(V_{IN}(nT+T)+V_{IN}(nT)\right) + \left[1-\frac{\alpha T}{2}\right] V_{OUT}(nT) \qquad \text{Bilinear z}$$

Review from last time

Switched-Current Integrator



Recall lossy integrators:

$$H(z) = \begin{cases} \dfrac{Gz^{-1}}{1-Hz^{-1}} & \text{Forward Euler} \\ \dfrac{G}{1-Hz^{-1}} & \text{Backward Euler} \\ G\left(\dfrac{1+z^{-1}}{1-Hz^{-1}}\right) & \text{Bilinear z} \end{cases}$$

For H=1 becomes lossless

$$I_{OUT}(z) = \left(\frac{Az^{-1}}{1-Bz^{-1}}\right) I_{IN2}(z) - \left(\frac{A}{1-Bz^{-1}}\right) I_{IN1}(z)$$

If $I_{IN1}=0$, becomes Forward Euler integrator

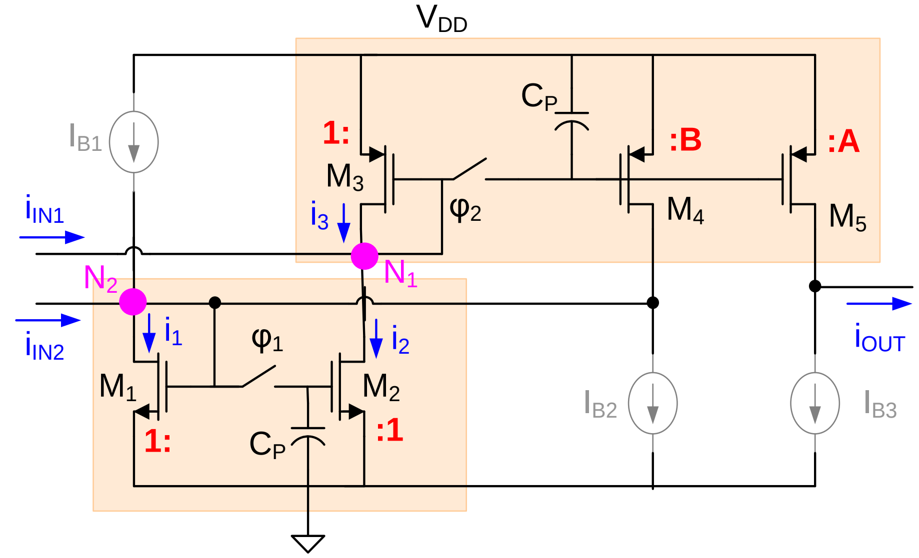
If $I_{N2}=0$, becomes Backward Euler integrator

If $I_{N1}= - I_{IN2}$, becomes Bilinear Integrator

Review from last time

Switched-Current Integrator

Sensitivity Analysis



Consider Forward Euler

$$I_{OUT}(z) = \left(\frac{Az^{-1}}{1-Bz^{-1}}\right) I_{IN2}(z) \qquad H(z) = \frac{TI_0}{z-1+\alpha T}$$

$$I_0 = \frac{A}{T} \qquad \frac{1-B}{T} = \alpha$$

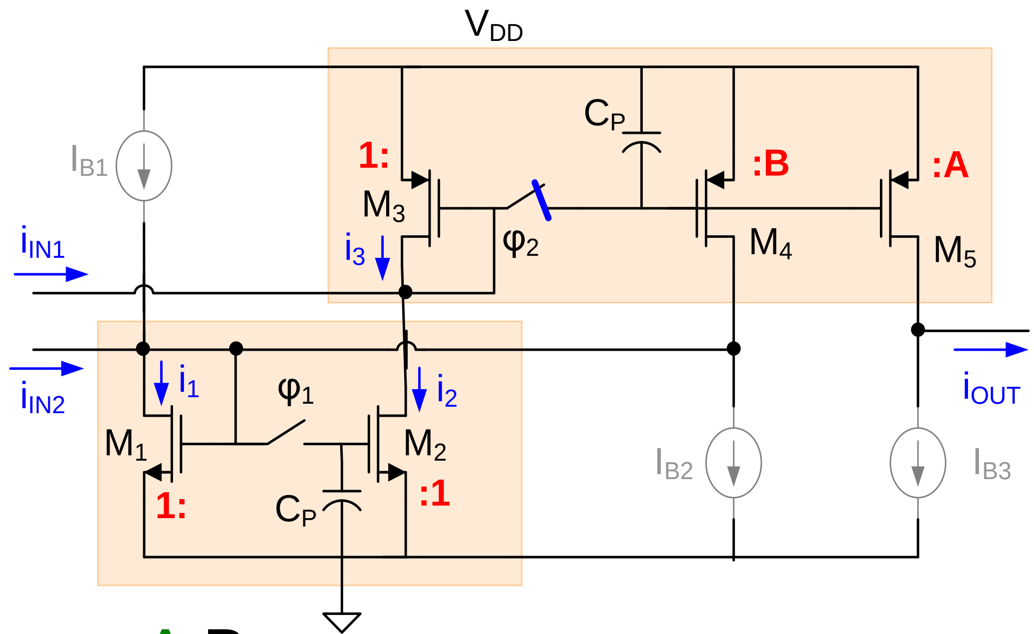
$$S_A^{I_0} = 1 \qquad S_B^{\alpha} = \frac{-TB}{1-B}$$

For low loss integrator (e.g. ideal integrator), the sensitivity of α is very large!

Review from last time

Switched-Current Integrator

Define A_1 to be the gain of the lower mirror



Sensitivity to A_1?

$$\left(\frac{1}{A}\right)i_{OUT}(nT) + i_{IN1}(nT) = \frac{A_1 B}{A} i_{OUT}(nT-T) + A_1 i_{IN2}(nT-T)$$

Taking z-transform, obtain

$$I_{OUT}(z) = \left(\frac{A_1 A z^{-1}}{1-BA_1 z^{-1}}\right) I_{IN2}(z) - \left(\frac{A}{1-BA_1 z^{-1}}\right) I_{IN1}(z)$$

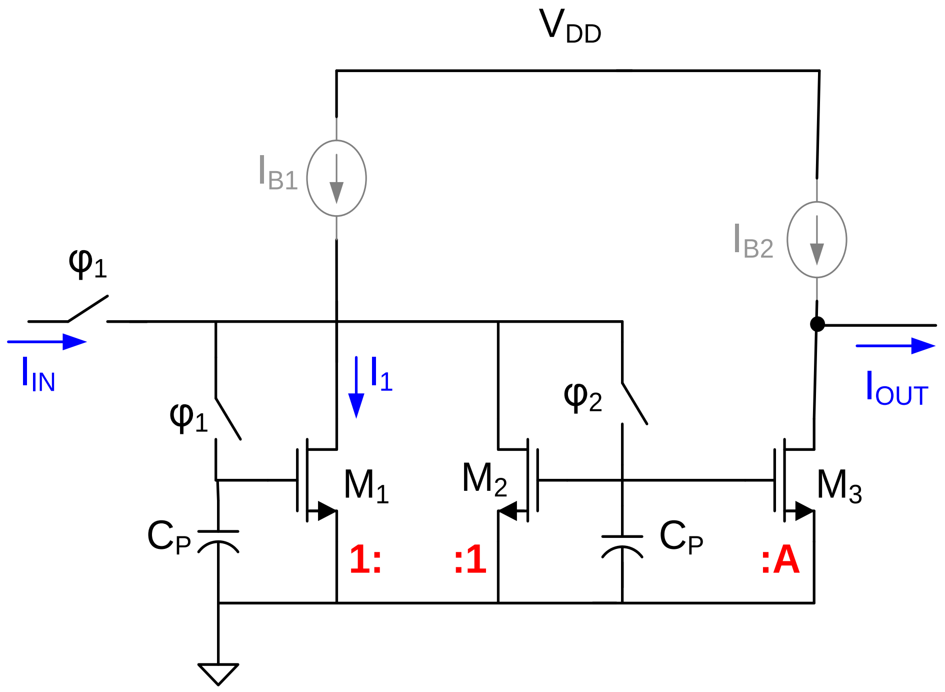
Consider Forward Euler

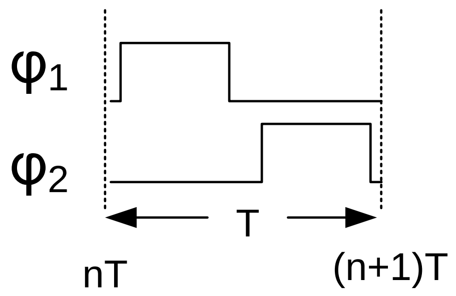
$$\frac{1-BA_1}{T} = \alpha \qquad S_B^{\alpha} = \frac{-BA_1}{1-BA_1} \qquad S_{A_1}^{\alpha} = \frac{-BA_1}{1-BA_1}$$

Sensitivity to A_1 is also large for low-loss or lossless integrator

Switched-Current Integrator

Consider another circuit



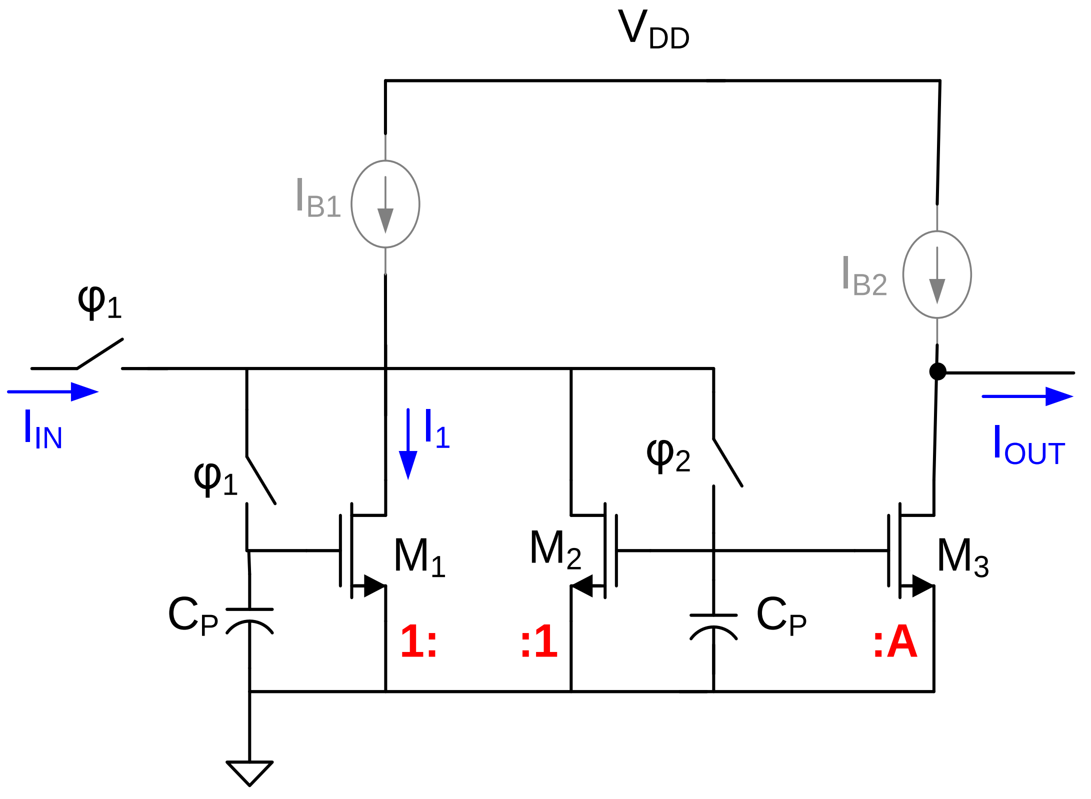


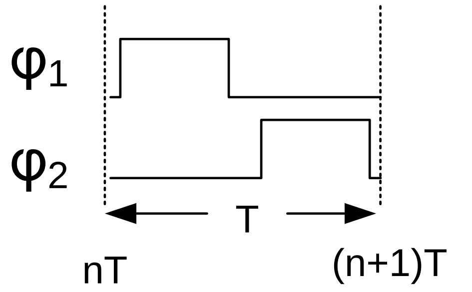
Consider Φ_1 closed, Φ_2 open (nT-T < t < nT-T/2)

$$i_1(t) = \frac{1}{A} i_{OUT}(nT-T) + i_{iN}(t)$$

$$i_1(nT-T) = \frac{1}{A} i_{OUT}(nT-T) + i_{iN}(nT-T) \qquad (1)$$

Switched-Current Integrator





Consider Φ_2 closed, Φ_1 open (nT-T/2 < t < nT)

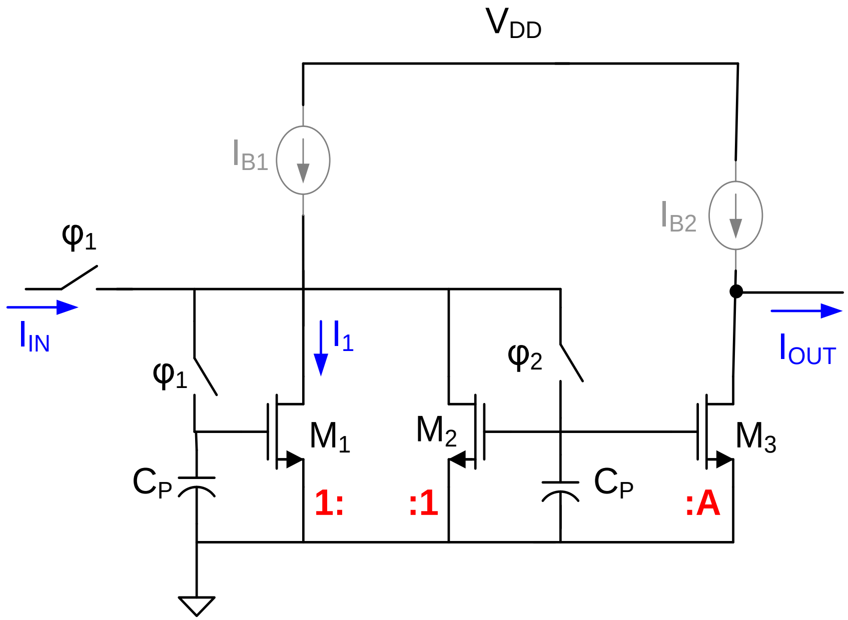
$$i_{OUT}(t) = Ai_1(nT-T)$$

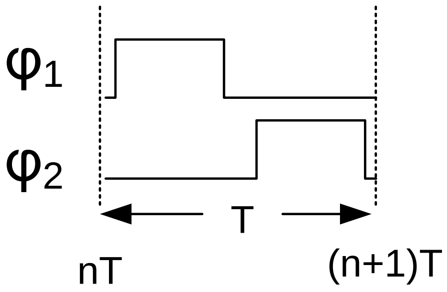
$$i_{OUT}(nT) = Ai_1(nT-T) \qquad (2)$$

combining (1) and (2), obtain

$$i_{OUT}(nT) = A \bullet \frac{1}{A} i_{OUT}(nT-T) + Ai_{iN}(nT-T)$$

Switched-Current Integrator





$$i_{OUT}(nT) = A \bullet \frac{1}{A} i_{OUT}(nT-T) + A i_{iN}(nT-T)$$

$$i_{OUT}(nT) = i_{OUT}(nT-T) + A i_{iN}(nT-T)$$

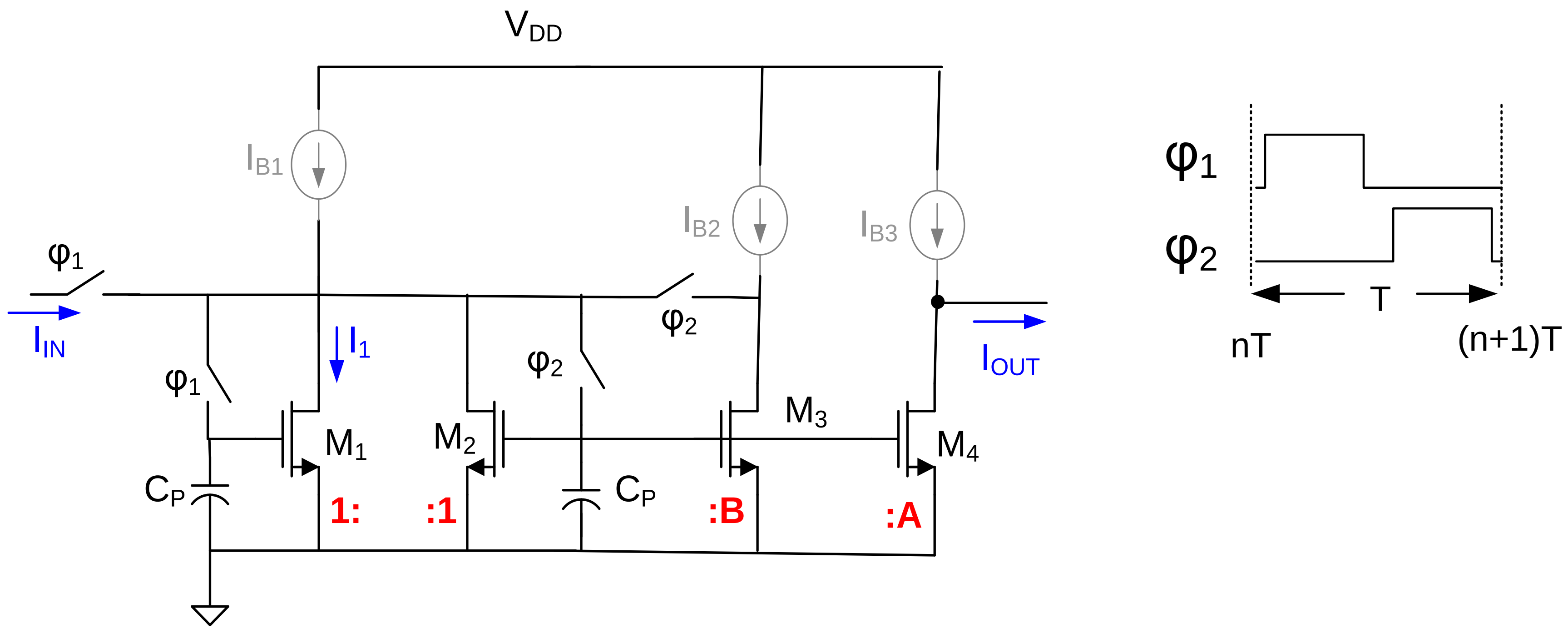
Taking z-transform, obtain

$$I_{OUT}(z) = \left(\frac{Az^{-1}}{1-z^{-1}}\right) I_{IN}(z)$$

Forward Euler Integrator

- Lossless Integrator (no matching required!)
- Matching of M_1 and M_2 not required
- Gain A does not affect coefficient of z^{-1} in the denominator

Switched-Current Integrator

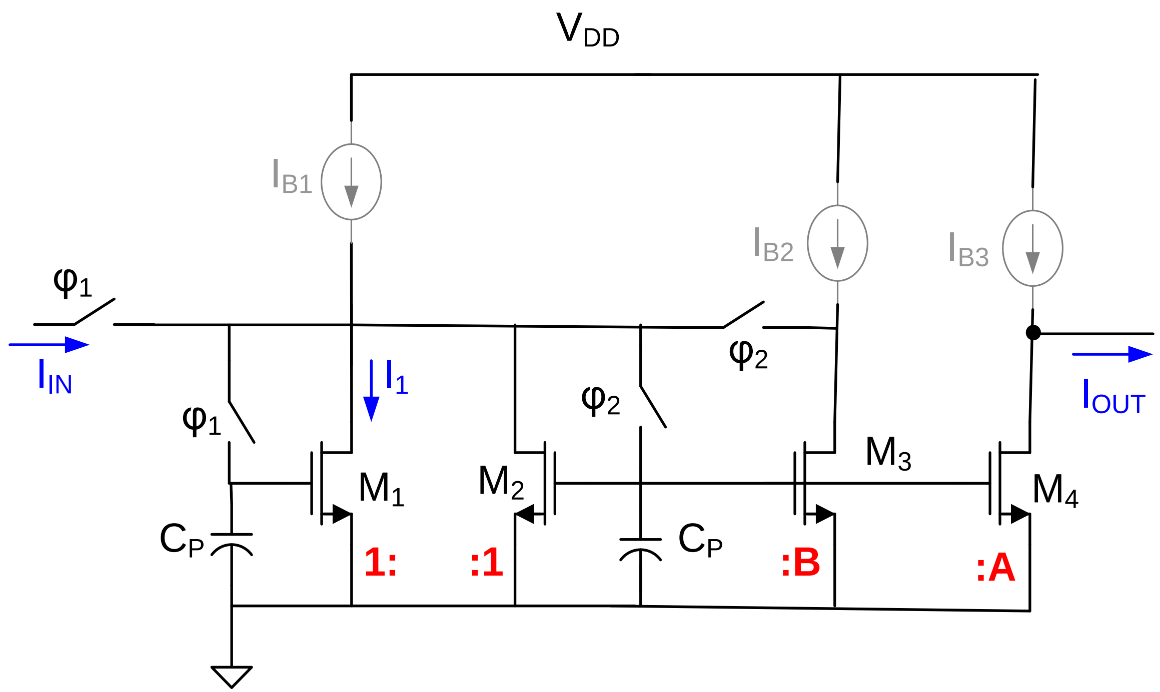


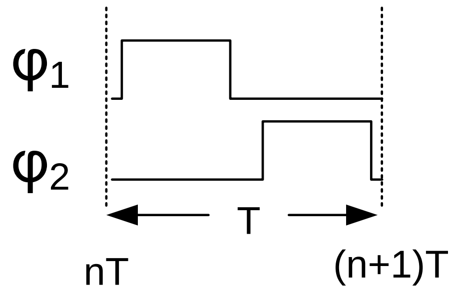
Consider Φ_1 closed, Φ_2 open $(nT\text{-}T < t < nT\text{-}T/2)$

$$i_1(t) = \frac{1}{A} i_{OUT}(nT-T) + i_{iN}(t)$$

$$i_1(nT-T) = \frac{1}{A} i_{OUT}(nT-T) + i_{iN}(nT-T) \qquad (1)$$

Switched-Current Integrator





Consider Φ_2 closed, Φ_1 open (nT-T/2 < t < nT)

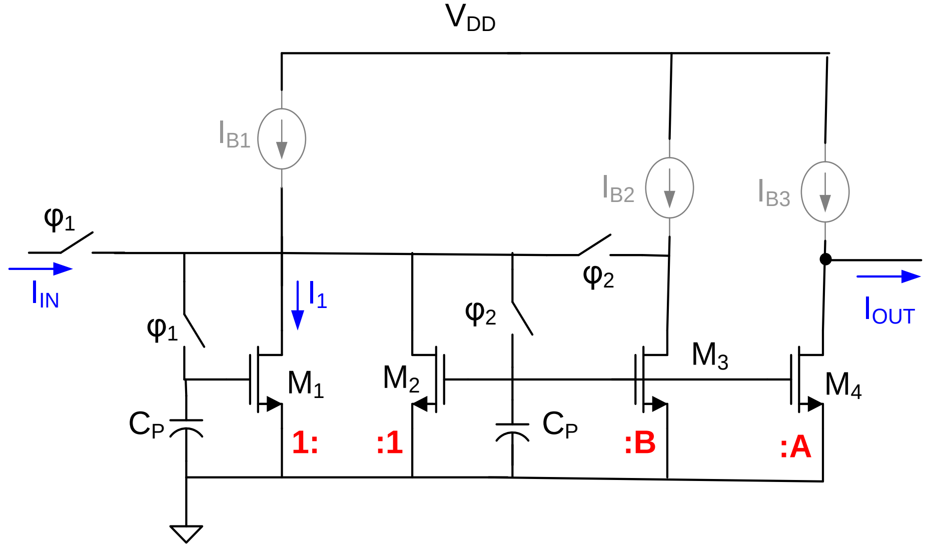
$$i_{OUT}(t) = A\left(i_1(nT-T) - \frac{B}{A} i_{OUT}(t)\right)$$

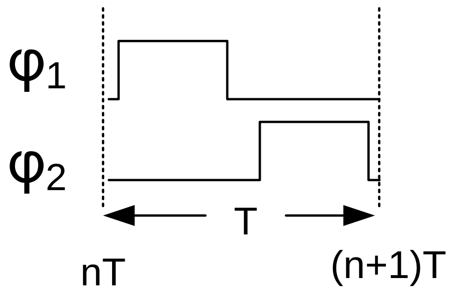
$$i_{OUT}(nT) = A\left(i_1(nT-T) - \frac{B}{A} i_{OUT}(nT)\right) \qquad (2)$$

combining (1) and (2), obtain

$$i_{OUT}(nT) = i_{OUT}(nT-T) - Bi_{OUT}(nT) + Ai_{IN}(nT-T)$$

Switched-Current Integrator





$$i_{OUT}(nT) = i_{OUT}(nT-T) - Bi_{OUT}(nT) + Ai_{IN}(nT-T)$$

Taking z-transform, obtain

$$I_{OUT}(z) = \left(\frac{Gz^{-1}}{1-Hz^{-1}}\right) I_{IN}(z)$$

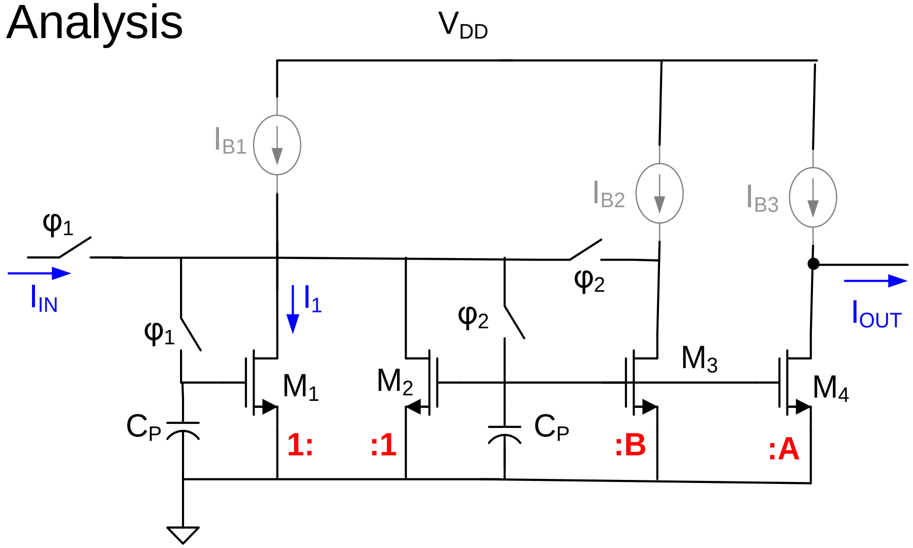
Forward Euler Integrator (Lossy)

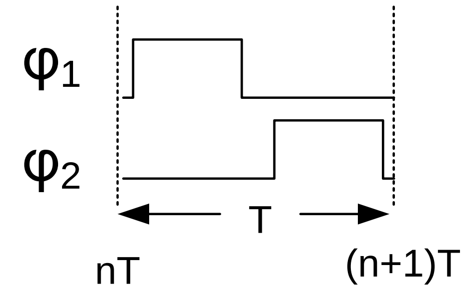
where $G = \dfrac{A}{1+B}$ $\quad H = \dfrac{1}{1+B}$

- Lossy Integrator
- Matching of M_1 and M_2 not required
- Gain A does not affect coefficient of z^{-1} in the denominator

Switched-Current Integrator

Sensitivity Analysis





$$I_{OUT}(z) = \left(\frac{Gz^{-1}}{1 - Hz^{-1}} \right) I_{IN}(z)$$

$$G = \frac{A}{1+B}$$

$$H = \frac{1}{1+B}$$

$$H(z) = \frac{TI_0}{z - 1 + \alpha T}$$

It can be shown that

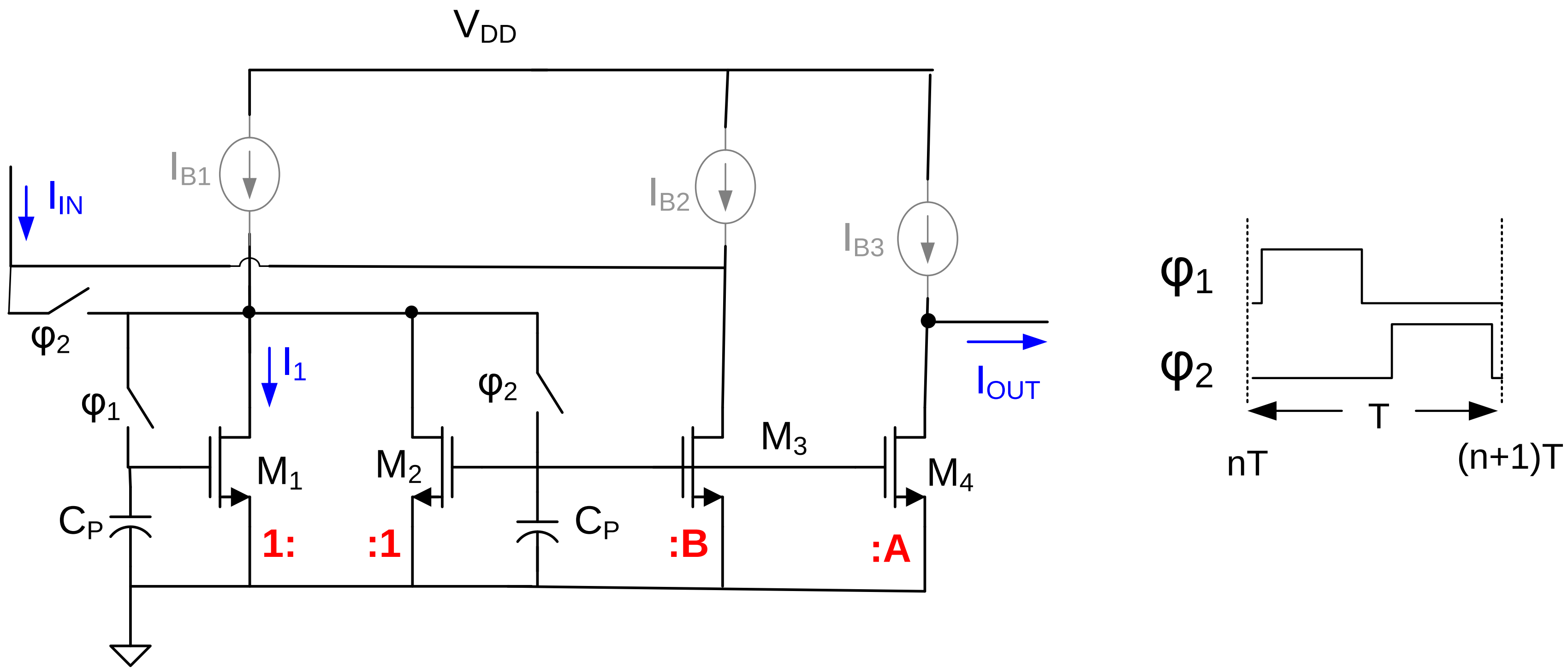
$$\alpha = \frac{1}{T}\left(\frac{B}{B+1} \right)$$

$$S_B^{\alpha} = \frac{T}{1+B}$$

For small loss, B is small and so is the sensitivity

Switched-Current Integrator

Another structure



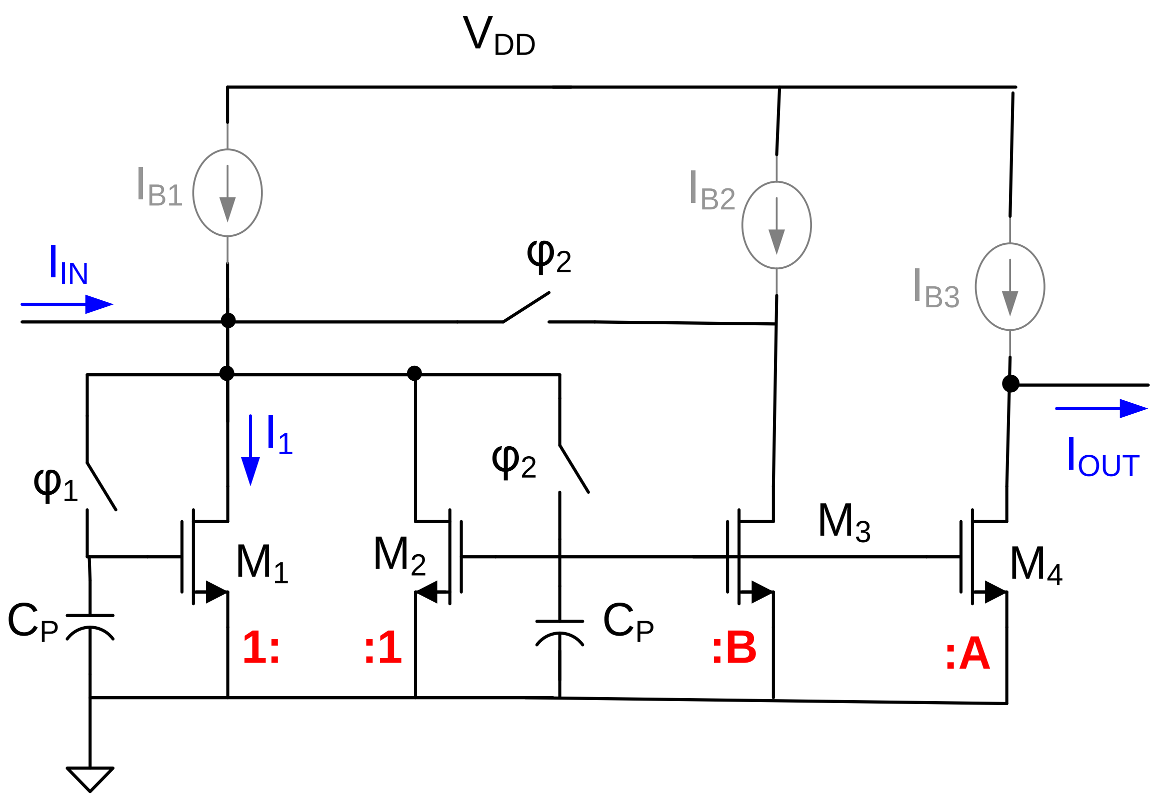
$$I_{OUT}(z) = \left(\frac{-G}{1-Hz^{-1}}\right) I_{IN}(z)$$

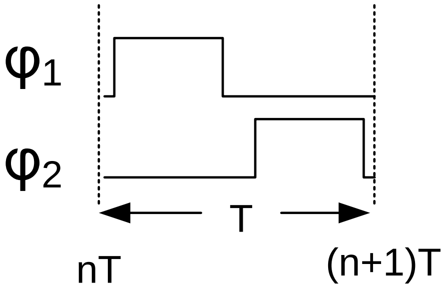
Backward Euler Lossy Inverting

$$G = \frac{A}{1+B} \qquad H = \frac{1}{1+B}$$

Switched-Current Integrator

Another structure

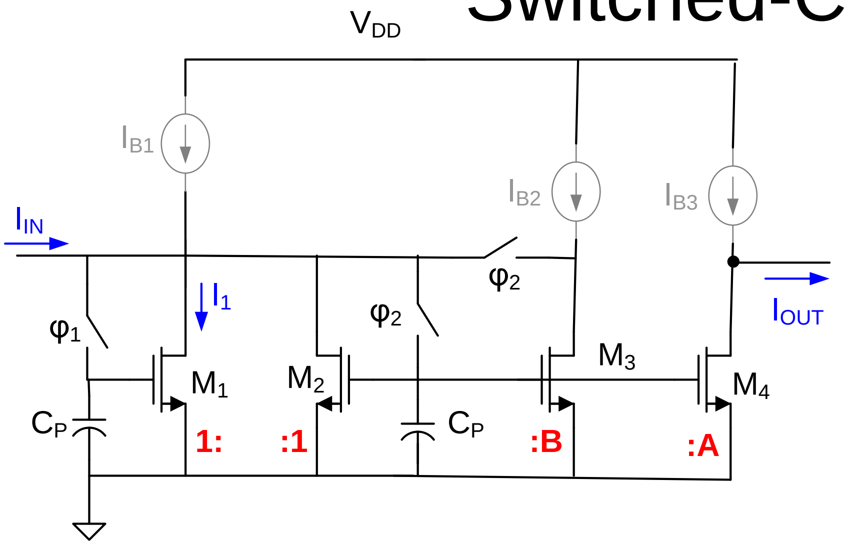


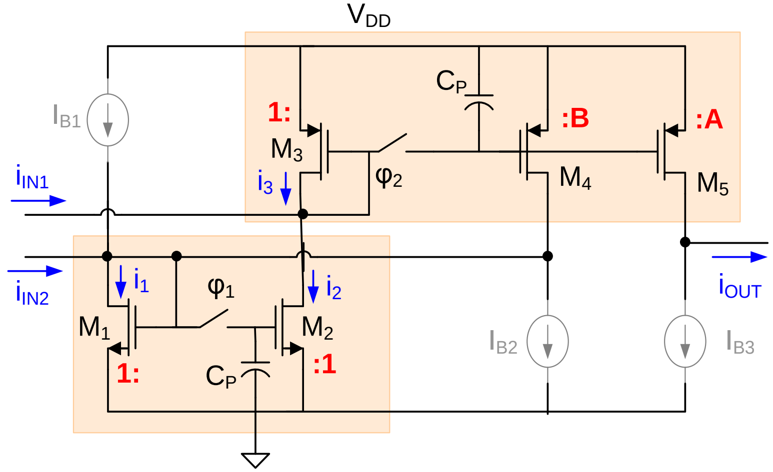


$$I_{OUT}(z) = -G\left(\frac{1-z^{-1}}{1-Hz^{-1}}\right) I_{IN}(z)$$

$$G = \frac{A}{1+B} \qquad H = \frac{1}{1+B}$$

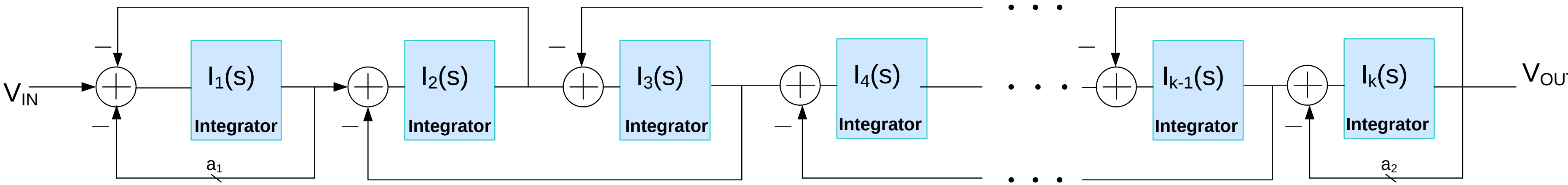
Switched-Current Filters





- Switched-current filters is an entirely different approach to designing filters with potential for overcoming many of the major problems facing the filter designer
- Other switched-current filter and integrator blocks have been proposed
- Integrators can be combined to form filter structures
- Single-ended and fully differential structures are readily formed
- Design of Switched-Current Filters is straightforward
- Beyond Hughes, a few others have looked at switched-current filters
- Hughes demonstrated experimentally modest performance with this technique
- Hughes was a world-class researcher and filter expert
- Hughes spent the better part of a decade trying to perfect the switched-current approach but performance remained modest when he retired
- Limited use of switched-current filters today
- Idea is really unique and there is bound to be some major useful applications of the basic concepts embodies in the switched-current filters!

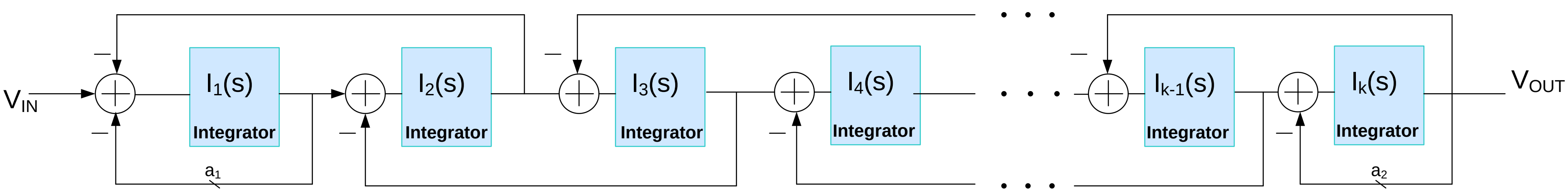
Leapfrog Filters



Introduced by Girling and Good, Wireless World, 1970

This structure has some very attractive properties and is widely used though the real benefits and limitations of the structure are often not articulated

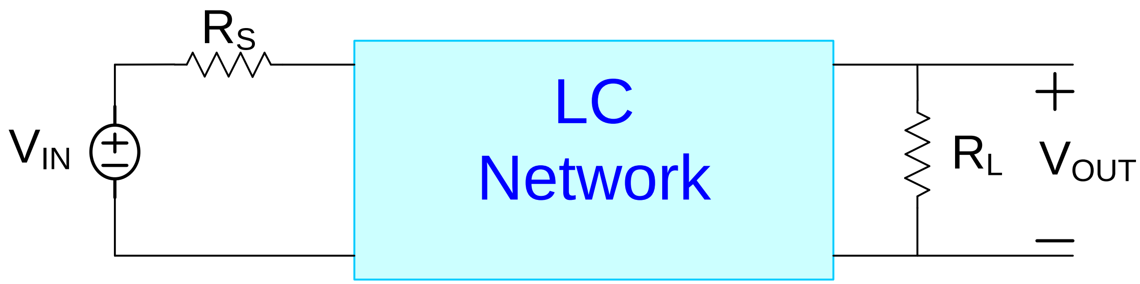
Leapfrog Filters



Observation: This structure appears to be dramatically different than anything else ever reported and it is not intuitive why this structure would serve as a filter, much less, have some unique and very attractive properties

To understand how the structure arose, why it has attractive properties, and to identify limitations, some mathematical background is necessary

Background Information for Leapfrog Filters



Assume the impedance R_S is fixed

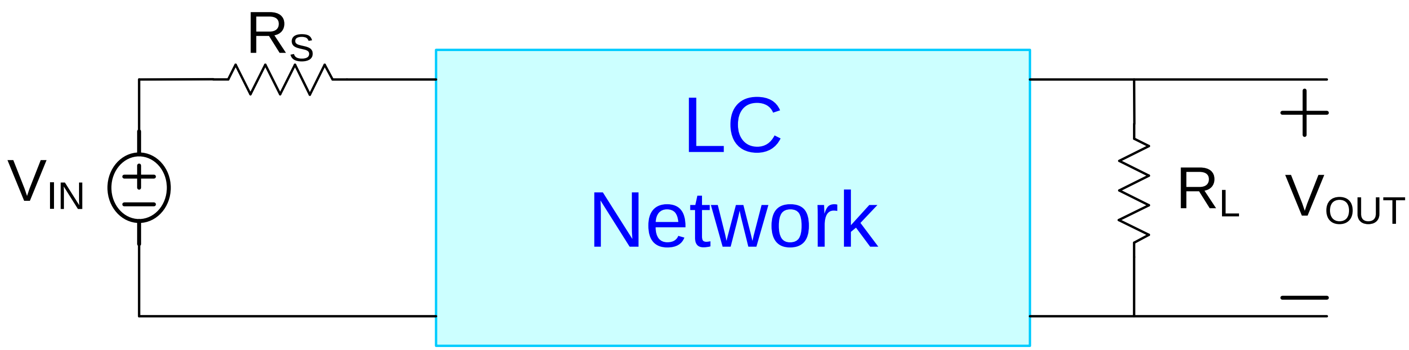
Theorem 1: If the LC network delivers maximum power to the load at a frequency ω, then

$$S_x^{|T(j\omega)|} = 0$$

for any circuit element in the system except for $x = R_L$

This theorem will follow after we prove the following theorem:

Background Information for Leapfrog Filters



Theorem 2: If the LC network delivers maximum power to the load at a frequency ω, then

$$S_{x}^{P_L(\omega)} = 0$$

where P(ω) is the power delivered to the load at input frequency ω and where x is any circuit element in the system except for $x = R_L$

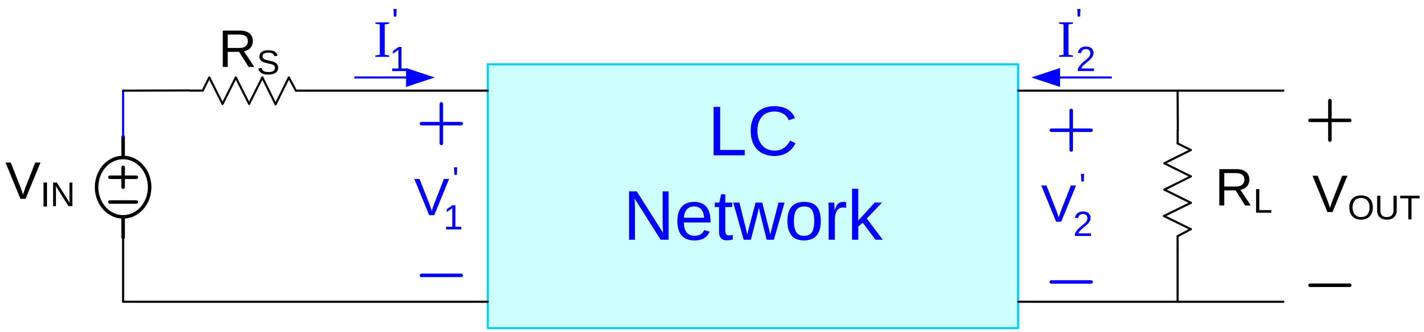
Note: There is no guarantee that there will be any frequencies where maximum power is transferred to the load and whether this does occur depends strongly on the LC circuit structure and the load R_L.

Proof of Theorem 2:

First, we will define the input impedance Z_{11}

Proof of Theorem 2:

Define the port phasors as $\{V_1^{'}, I_1^{'}, V_2^{'}, I_2^{'}\}$



$$Z_{11} = \frac{V_1^{'}}{I_1^{'}}$$

(input impedance to the LC Network)

this can be expressed as

$$Z_{11} = R_1 + jX_1$$

(R_1 and X_1 are real functions of ω and depend on R_L)

Since the LC network is lossless (dissipates no power) we have

$$P_L = \text{Re}\left(V_1^{'} \bullet I_1^{'*}\right)$$

$$P_L = \text{Re}\left(\left[\frac{R_1 + jX_1}{R_S + R_1 + jX_1} V_{in}\right] \bullet \left[\frac{V_{in}}{R_S + R_1 + jX_1}\right]^*\right)$$

$$P_L = |V_{in}|^2 \text{Re}\left(\frac{R_1 + jX_1}{(R_S + R_1)^2 + X_1^2}\right) = |V_{in}|^2 \frac{R_1}{(R_S + R_1)^2 + X_1^2}$$

Proof of Theorem 2:

$$P_L = |V_{in}|^2 \frac{R_1}{(R_S + R_1)^2 + X_1^2}$$

To maximize power delivered to a fixed load at a frequency ω, must have

$$\frac{\partial P_L}{\partial R_1} = 0 \qquad \frac{\partial P_L}{\partial X_1} = 0$$

$$\frac{\partial P_L}{R_1} = |V_{in}|^2 \left[\frac{\left((R_S + R_1)^2 + X_1^2\right) - R_1(2)(R_S + R_1)}{\left((R_S + R_1)^2 + X_1^2\right)^2} \right]$$

$$\frac{\partial P_L}{R_1} = |V_{in}|^2 \left[\frac{\left(R_S^2 + 2R_1R_S + R_S^2 + X_1^2 - 2R_1R_S - 2R_1^2\right)}{\left((R_S + R_1)^2 + X_1^2\right)^2} \right] = |V_{in}|^2 \left[\frac{\left(2\left(R_S^2 - R_1^2\right) + X_1^2\right)}{\left((R_S + R_1)^2 + X_1^2\right)^2} \right]$$

$$\frac{\partial P_L}{\partial R_1} = 0 \longrightarrow 2\left(R_S^2 - R_1^2\right) + X_1^2 = 0$$

$$\frac{\partial P_L}{X_1} = |V_{in}|^2 \left[\frac{-R_1(2X_1)}{\left((R_S + R_1)^2 + X_1^2\right)^2} \right]$$

$$\frac{\partial P_L}{\partial X_1} = 0 \longrightarrow X_1 = 0$$

$$\longrightarrow \quad X_1 = 0 \qquad (1)$$

$$R_1 = R_S \qquad (2)$$

Proof of Theorem 2:

$$X_1 = 0 \qquad (1) \qquad\qquad R_1 = R_S \qquad (2)$$

$$P_L = |V_{in}|^2 \frac{R_1}{(R_S + R_1)^2 + X_1^2}$$

Now let x be any element in the LC network

$$\frac{\partial P_L}{\partial x} = \frac{\partial P_L}{\partial R_1}\frac{\partial R_1}{\partial x} + \frac{\partial P_L}{\partial X_1}\frac{\partial X_1}{\partial x}$$

$$\frac{\partial P_L}{\partial x} = \left[|V_{in}|^2 \left[\frac{\left(2\left(R_S^2 - R_1^2\right) + X_1^2\right)}{\left((R_S + R_1)^2 + X_1^2\right)^2} \right]\right] \frac{\partial R_1}{\partial x} + \left[|V_{in}|^2 \left[\frac{-R_1(2X_1)}{\left((R_S + R_1)^2 + X_1^2\right)^2} \right]\right] \frac{\partial X_1}{\partial x}$$

It thus follows from (1) and (2) that at maximum power transfer, the two coefficients in this expression vanish, thus

$$\frac{\partial P_L}{\partial x} = \left[|V_{in}|^2 \left[\frac{0}{\left((R_S + R_1)^2 + X_1^2\right)^2} \right]\right] \frac{\partial R_1}{\partial x} + \left[|V_{in}|^2 \left[\frac{0}{\left((R_S + R_1)^2 + X_1^2\right)^2} \right]\right] \frac{\partial X_1}{\partial x} = 0$$

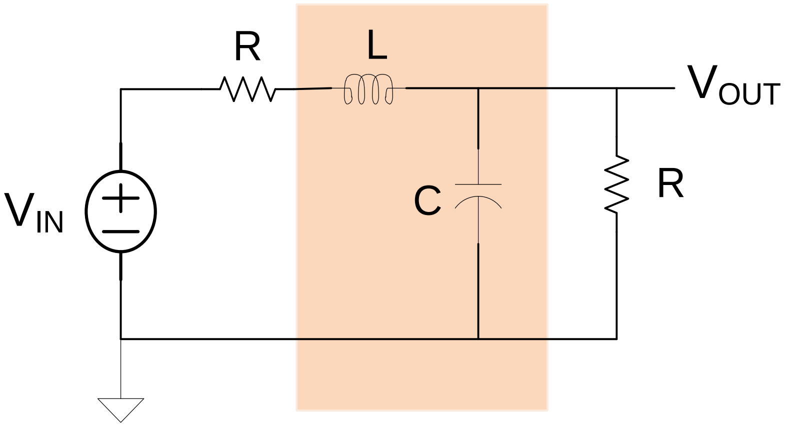
thus

$$S_x^{P_L} = \frac{\partial P_L}{\partial x}\frac{x}{P_L} = 0 \qquad \blacksquare$$

Question: Can we also make the claim that $S_{R_L}^{P_L(\omega)} = 0$ at any frequency where maximum power is transferred to the load?

Yes! Note that the previous analysis is based upon characterizing R_1 and X which are functions of k reactive components, $\{x_1, \quad x_k\}$ and $R_{L.}$

The following circuit has maximum power transfer at dc and it can be easily analytically shown that the sensitivity of P to L, C, and R_L is 0 at dc.



Proof of Theorem 1: $S_x^{|T(j\omega)|} = ?$

$$P_L = \mathrm{Re}\left(V_{out} \bullet \left(\frac{V_{out}}{R_L} \right)^* \right)$$

$$P_L = \mathrm{Re}\left(V_{in} T(j\omega) \bullet \left(\frac{V_{in} T(j\omega)}{R_L} \right)^* \right)$$

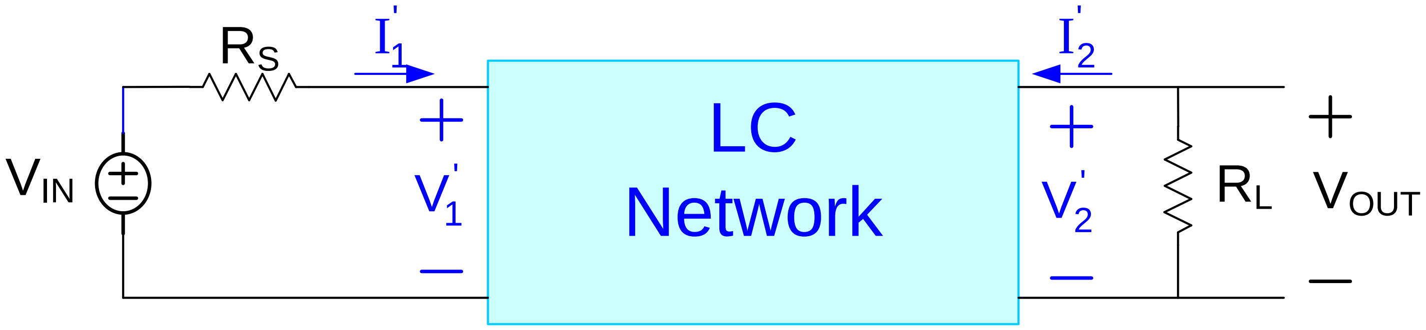
$$P_L = \left(\frac{|V_{in}|^2}{R_L} \right) \bullet |T(j\omega)|^2$$

Recall the following two sensitivity relationships

$$S_x^{kf} = S_x^f \qquad\qquad S_x^{f^2} = 2 \bullet S_x^f$$

It thus follows that

$$S_x^{P_L} = 2 \bullet S_x^{|T(j\omega)|} \quad \xrightarrow{S_x^{P_L} = 0} \quad S_x^{|T(j\omega)|} = 0 \quad \blacksquare$$



Lemma: If maximum power is transferred to the load in the doubly-terminated LC network at a frequency ω, then

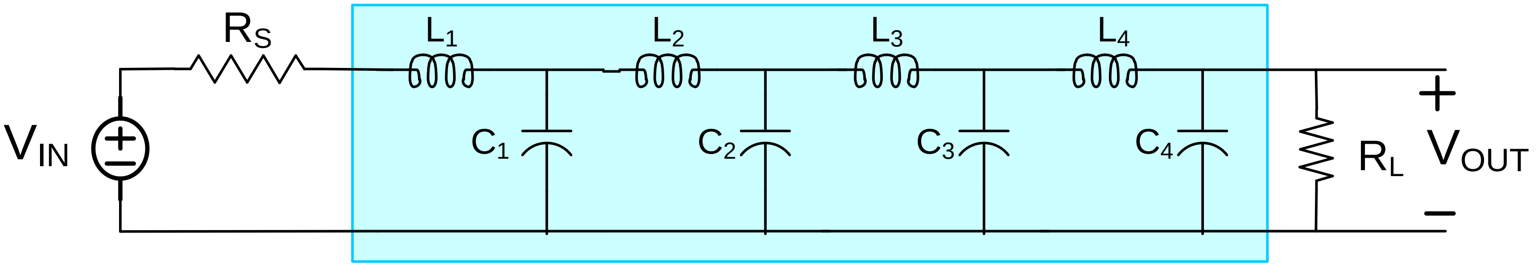
$$\left.\frac{V_2'(j\omega)}{I_2'(j\omega)}\right|_{V_{in}=0} = R_L$$

This lemma indicates that the impedance of the LC network loaded with R_S facing R_L is equal to R_L at frequencies where maximum-power is transferred to R_L

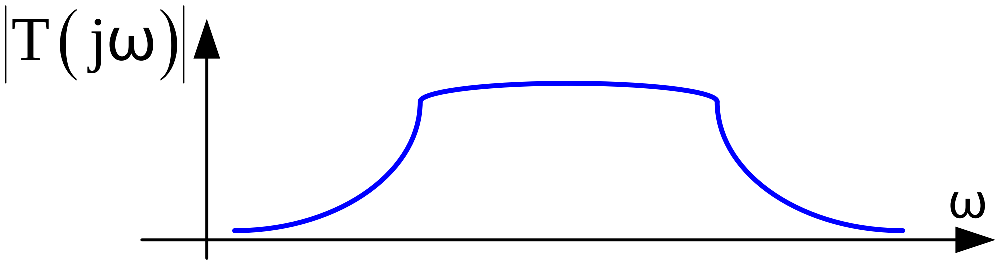
The proof follows directly by considering the Thevenin-equivalent circuit facing R_L

Implications of Theorem 1

Many passive LC filter such as that shown below exist that have near maximum power transfer in the passband

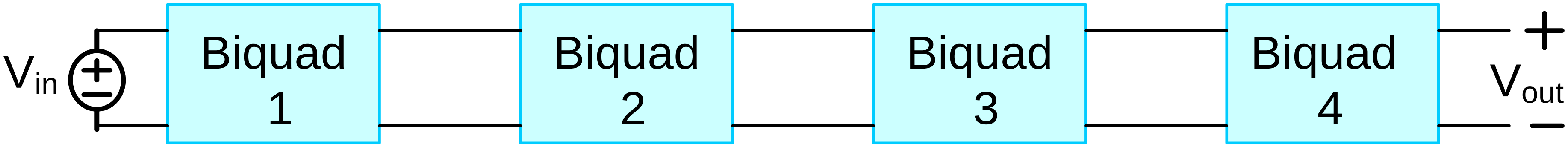


If a component in the LC network changes a little, there is little change in the passband gain characteristics (depicted as bandpass)

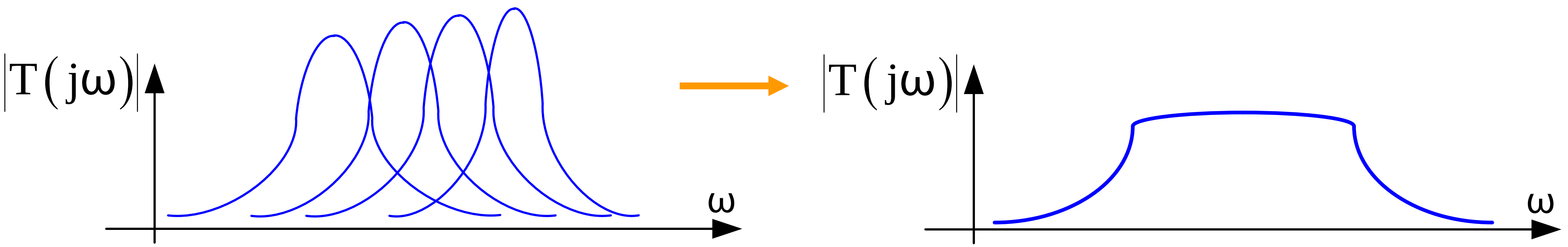


$$S_x^{|T(j\omega)|} \simeq 0$$ in passband

Implications of Theorem 1

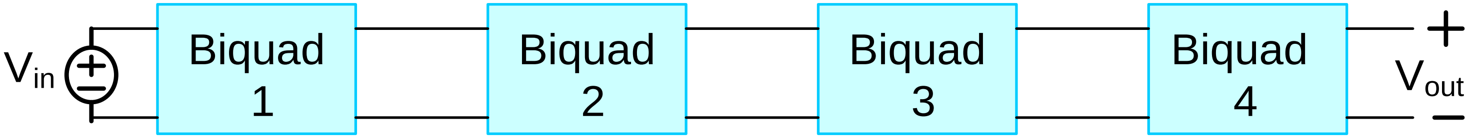


Cascaded Biquad has a response that is the product of the individual second-order transfer functions

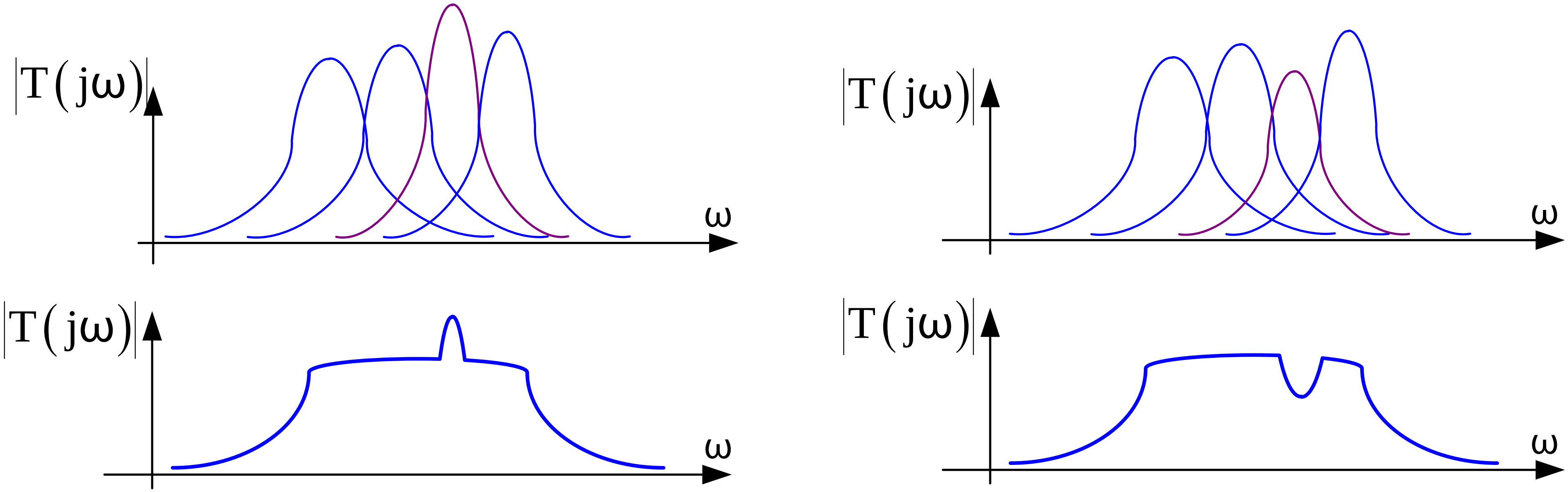


If a component in a biquad changes a little, there is often a large change in the passband gain characteristics (depicted as bandpass)

Implications of Theorem 1

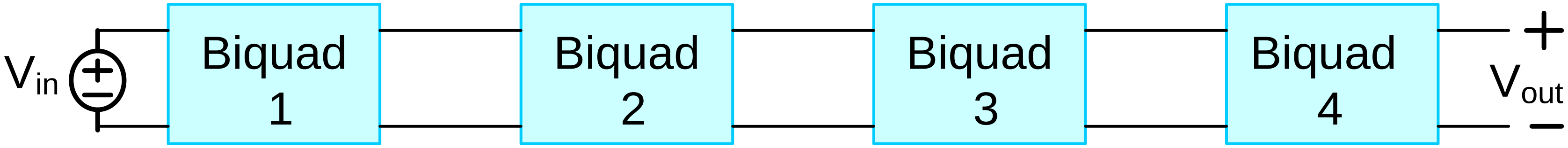


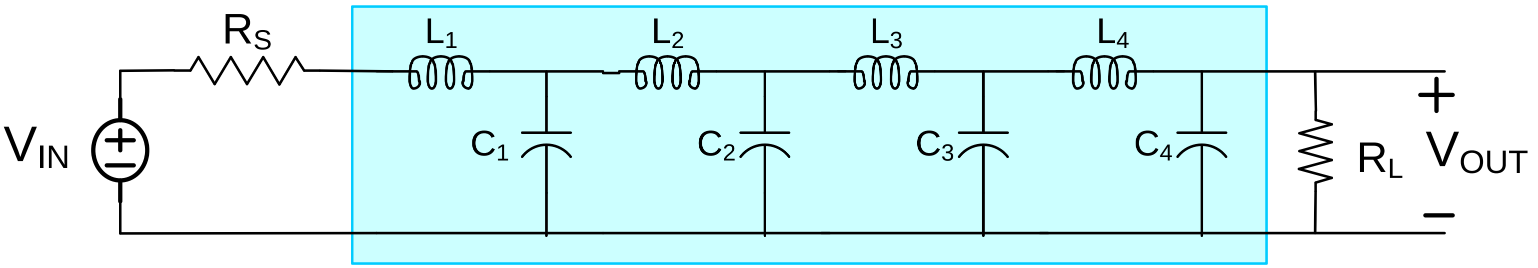
If a component in a biquad changes a little, there is often a large change in the passband gain characteristics (depicted as bandpass)



$$S_x^{|T(j\omega)|} \neq 0$$ in passband

Implications of Theorem 1

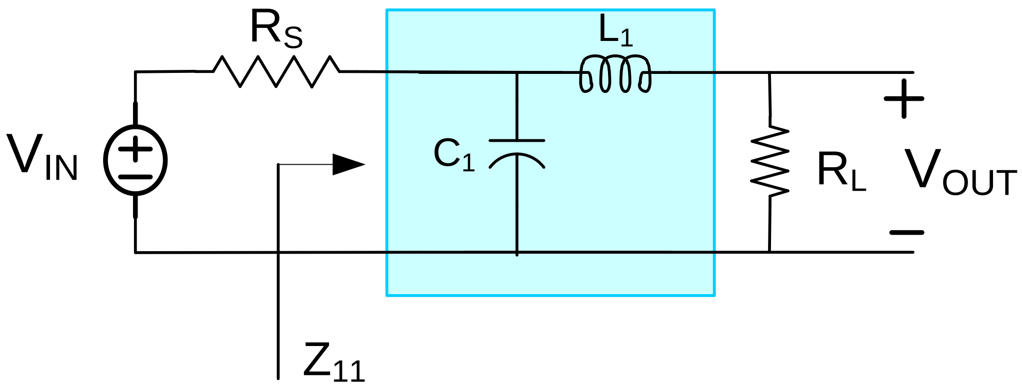




Good doubly-terminated LC networks often much less sensitive to most component values in the passband than are cascaded biquads !

This is a major advantage of the LC networks but can not be applied practically used in most integrated applications or even in pc-board based designs

Example: Determine at what frequencies maximum-power transfer to the load will occur and what value of R_L is needed for this to happen



Recall at maximum-power transfer, Z_{11} is real and equal to R_S

$$Z_{11} = \frac{R_L + sL}{s^2LC + sR_LC + 1}$$

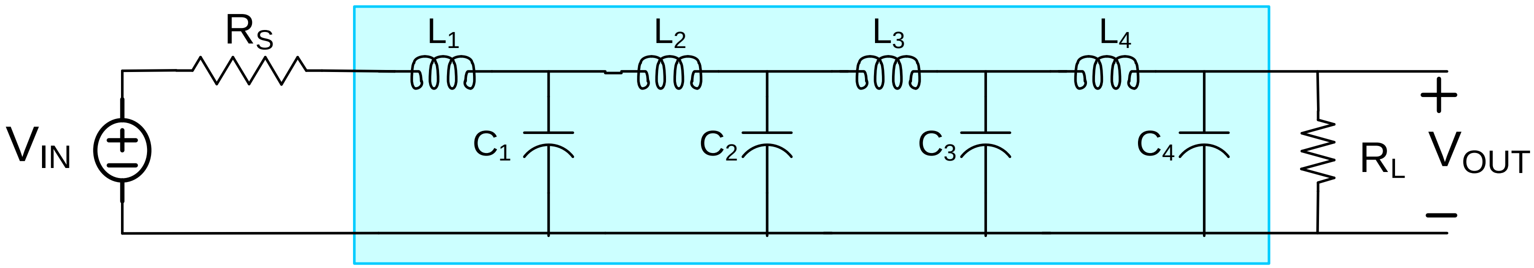
$$Z_{11}(j\omega) = \left(\frac{R_L}{\left(1-\omega^2LC\right)^2 + \omega^2R_L^2C}\right) + j\left(\frac{\omega L - \omega^2R_L^2C - \omega^3L^2C}{\left(1-\omega^2LC\right)^2 + \omega^2R_L^2C}\right)$$

$\text{Im}\left(Z_{11}(j\omega)\right) = 0$ only at ω=0 and one other positive value of ω

To get maximum power transfer at ω=0, must have $R_L=R_S$

Appears not to have maximum power transfer at other frequency where $\text{Im}\left(Z_{11}(j\omega)\right) \neq 0$

Consider again the doubly-terminated circuit that has multiple passband frequencies where maximum power transfer to the load occurs

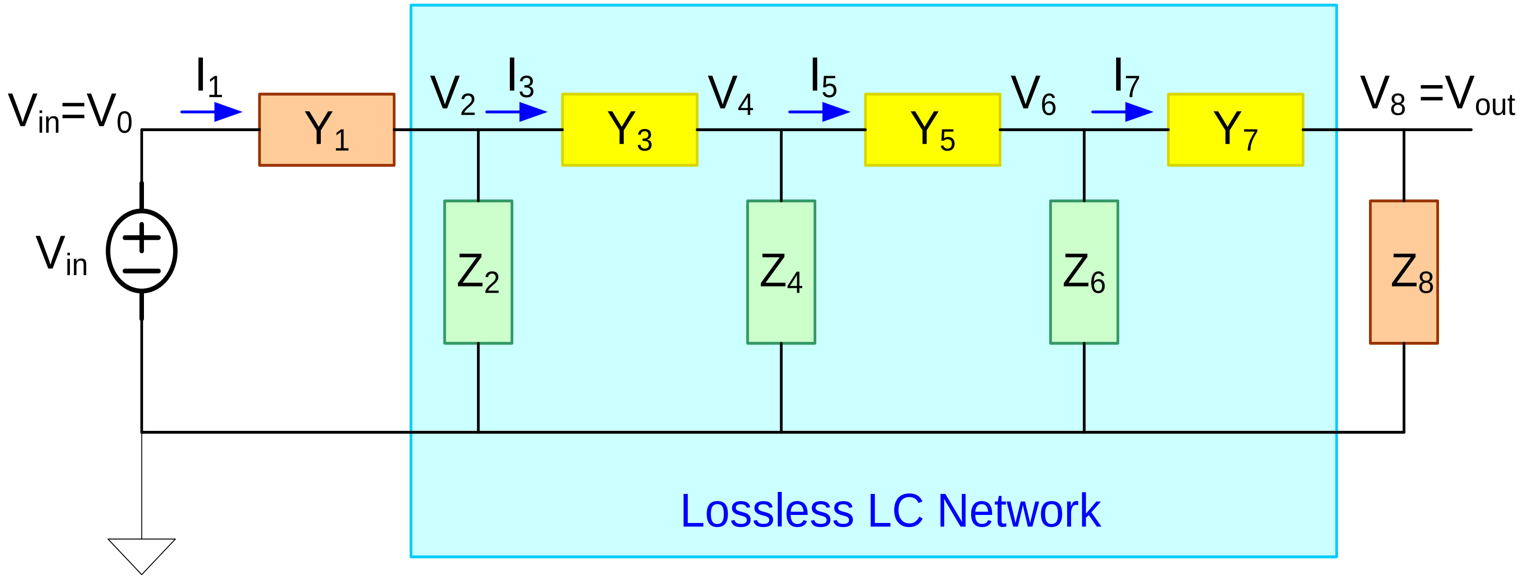


Observe that this structure is completely characterized by a set of equations that characterize the network

All sensitivity properties are inherently determined by this set of equations

Any circuit that has the same set of equations will have the same sensitivity properties

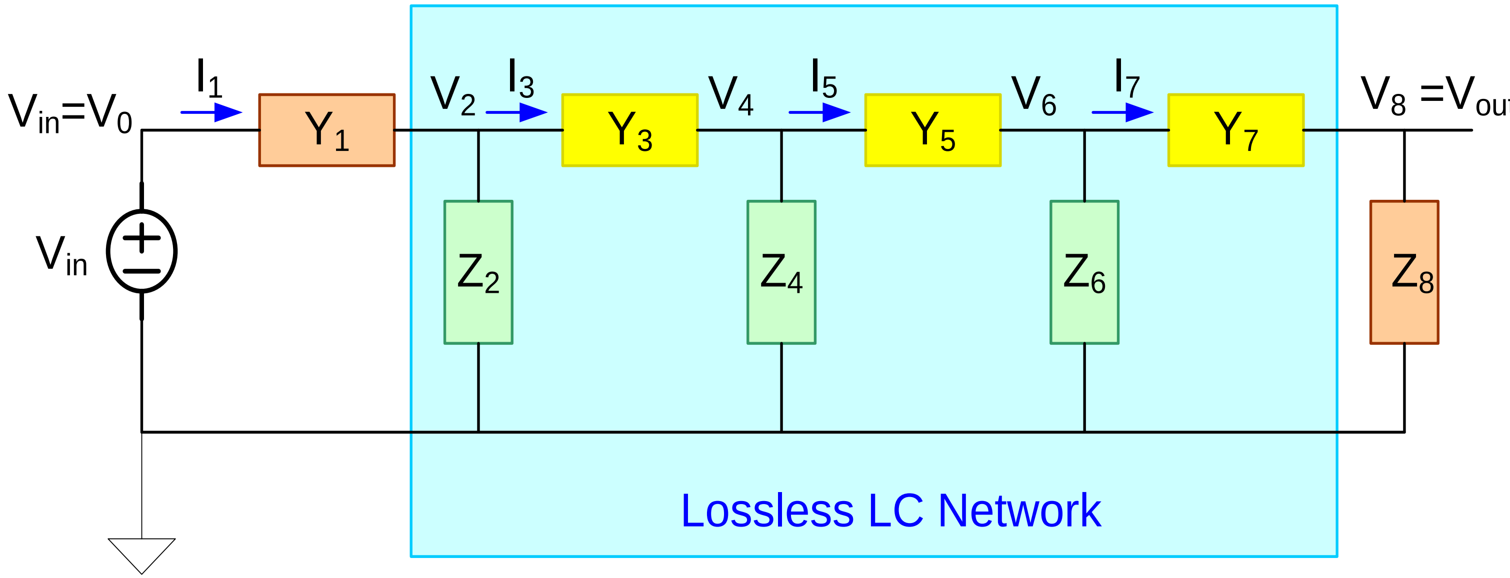
Doubly-terminated Ladder Network with Low Passband Sensitivities



For components in the LC Network observe

$$Y_k = \frac{1}{sL_k} \qquad Z_k = \frac{1}{sC_k}$$

Doubly-terminated Ladder Network with Low Passband Sensitivities



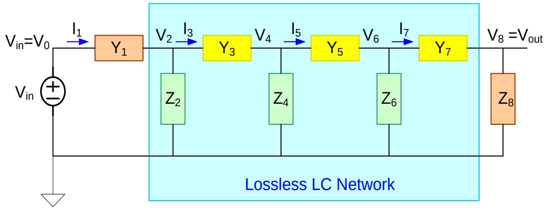
$$
\left.
\begin{aligned}
I_1 &= (V_0 - V_2)Y_1 \\
V_2 &= (I_1 - I_3)Z_2 \\
I_3 &= (V_2 - V_4)Y_3 \\
V_4 &= (I_3 - I_5)Z_4 \\
I_5 &= (V_4 - V_6)Y_5 \\
V_6 &= (I_5 - I_7)Z_6 \\
I_7 &= (V_6 - V_8)Y_7 \\
V_8 &= I_7 Z_8
\end{aligned}
\right\}
$$

Complete set of independent equations that characterize this filter

Solution of this set of equations is tedious

All sensitivity properties of this circuit are inherently embedded in these equations!

Consider now only the set of equations and disassociate them from the circuit from where they came



$$\left.\begin{aligned}
I_1 &= (V_0 - V_2)Y_1 \\
V_2 &= (I_1 - I_3)Z_2 \\
I_3 &= (V_2 - V_4)Y_3 \\
V_4 &= (I_3 - I_5)Z_4 \\
I_5 &= (V_4 - V_6)Y_5 \\
V_6 &= (I_5 - I_7)Z_6 \\
I_7 &= (V_6 - V_8)Y_7 \\
V_8 &= I_7 Z_8
\end{aligned}\right\}$$

Make the associations

$$I_1 = V_1'$$
$$I_3 = V_3'$$
$$I_5 = V_5'$$
$$I_7 = V_7'$$

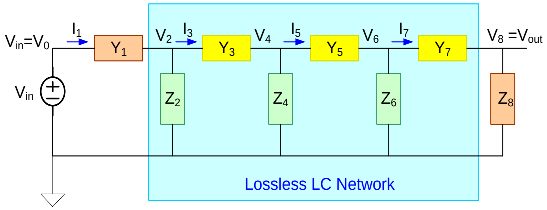
Rewrite the equations as

$$\left.\begin{aligned}
V_1' &= (V_0 - V_2)Y_1 \\
V_2 &= (V_1' - V_3')Z_2 \\
V_3' &= (V_2 - V_4)Y_3 \\
V_4 &= (V_3' - V_5')Z_4 \\
V_5' &= (V_4 - V_6)Y_5 \\
V_6 &= (V_5' - V_7')Z_6 \\
V_7' &= (V_6 - V_8)Y_7 \\
V_8 &= V_7' Z_8
\end{aligned}\right\}$$

This association is nothing more than a renaming of variables so all sensitivities WRT Y's and Z's will remain unchanged!

Consider now only the set of equations and disassociate them from the circuit from where they came

$$
\left.
\begin{aligned}
V_1' &= (V_0 - V_2)Y_1 \\
V_2 &= (V_1' - V_3')Z_2 \\
V_3' &= (V_2 - V_4)Y_3 \\
V_4 &= (V_3' - V_5')Z_4 \\
V_5' &= (V_4 - V_6)Y_5 \\
V_6 &= (V_5' - V_7')Z_6 \\
V_7' &= (V_6 - V_8)Y_7 \\
V_8 &= V_7' Z_8
\end{aligned}
\right\}
$$



For the LC filter, recall

$$Y_k = \frac{1}{sL_k} \qquad Z_k = \frac{1}{sC_k}$$

And the source and load termination relationships were

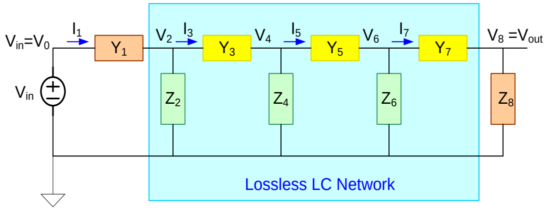
$$Y_1 = \frac{1}{R_1} \qquad Z_8 = R_8$$

These can be written as

$$
\left.
\begin{aligned}
V_1' &= (V_0 - V_2)\frac{1}{R_1} & V_5' &= (V_4 - V_6)\frac{1}{sL_5} \\
V_2 &= (V_1' - V_3')\frac{1}{sC_2} & V_6 &= (V_5' - V_7')\frac{1}{sC_6} \\
V_3' &= (V_2 - V_4)\frac{1}{sL_3} & V_7' &= (V_6 - V_8)\frac{1}{sL_7} \\
V_4 &= (V_3' - V_5')\frac{1}{sC_4} & V_8 &= V_7' R_8
\end{aligned}
\right\}
$$

Observe that in the new parameter domain the equations all look like integrator functions if the primed and unprimed variables are all voltages !

Consider now only the set of equations and disassociate them from the circuit from where they came



$$V_1^{'} = (V_0 - V_2)\frac{1}{R_1}$$

$$V_2 = (V_1^{'} - V_3^{'})\frac{1}{sC_2}$$

$$V_3^{'} = (V_2 - V_4)\frac{1}{sL_3}$$

$$V_4 = (V_3^{'} - V_5^{'})\frac{1}{sC_4}$$

$$V_5^{'} = (V_4 - V_6)\frac{1}{sL_5}$$

$$V_6 = (V_5^{'} - V_7^{'})\frac{1}{sC_6}$$

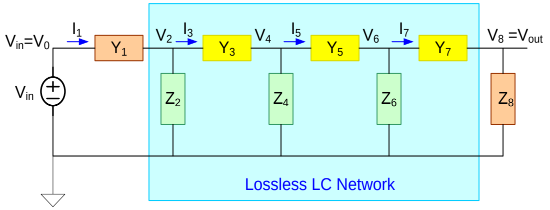
$$V_7^{'} = (V_6 - V_8)\frac{1}{sL_7}$$

$$V_8 = V_7^{'}R_8$$

Observe that in the new parameter domain the equations all look like integrator functions if the primed and unprimed variables are all voltages !

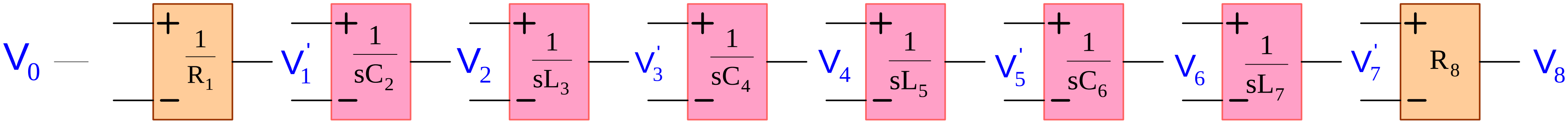
If any circuit is characterized by these equations, the sensitivities to the integrator gains will be identical to the sensitiviies of the original circuit to the Ls and Cs !

Consider now only the set of equations and disassociate them from the circuit from where they came



$$\left.\begin{aligned}
V_1' &= (V_0 - V_2)\frac{1}{R_1} & V_5' &= (V_4 - V_6)\frac{1}{sL_5} \\
V_2 &= (V_1' - V_3')\frac{1}{sC_2} & V_6 &= (V_5' - V_7')\frac{1}{sC_6} \\
V_3' &= (V_2 - V_4)\frac{1}{sL_3} & V_7' &= (V_6 - V_8)\frac{1}{sL_7} \\
V_4 &= (V_3' - V_5')\frac{1}{sC_4} & V_8 &= V_7' R_8
\end{aligned}\right\}$$

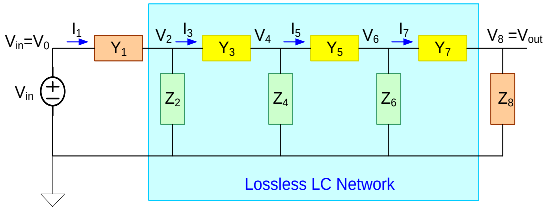
Each equation corresponds to either an integrator or summer with the output voltage output variables and the gain indicated (don't worry about the units)



$$V_0 = V_{in}$$

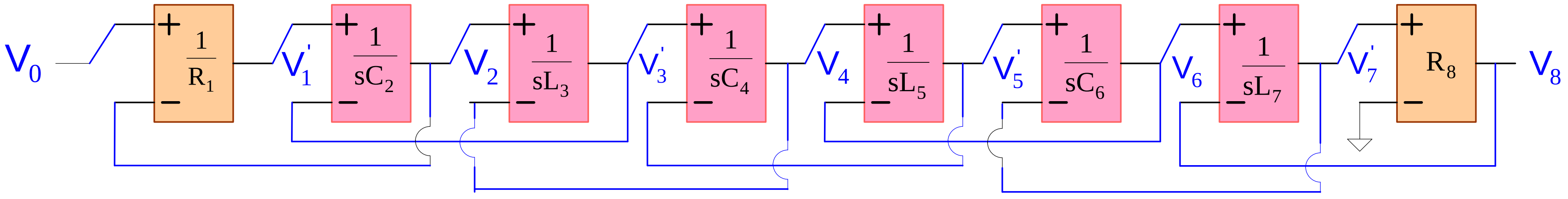
$$V_8 = V_{out}$$

Consider now only the set of equations and disassociate them from the circuit from where they came

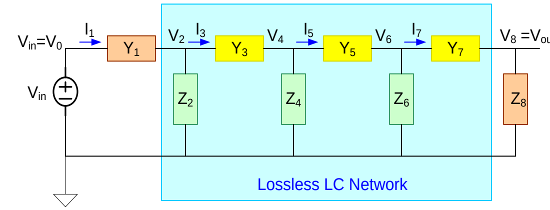


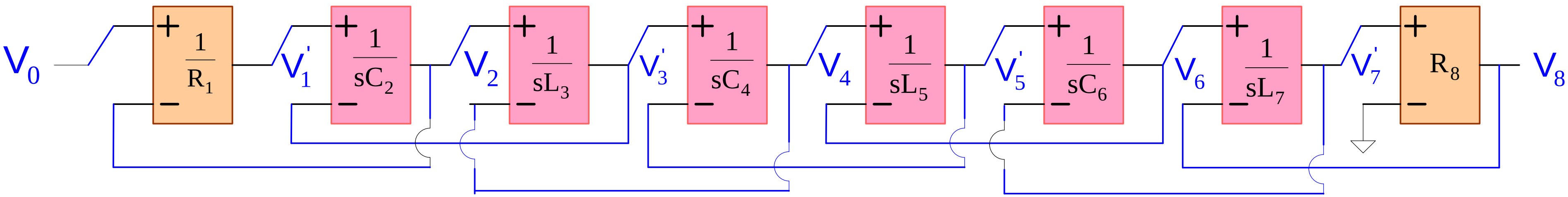
$$
\left.
\begin{aligned}
V_1' &= (V_0 - V_2)\frac{1}{R_1} & V_5' &= (V_4 - V_6)\frac{1}{sL_5} \\
V_2 &= (V_1' - V_3')\frac{1}{sC_2} & V_6 &= (V_5' - V_7')\frac{1}{sC_6} \\
V_3' &= (V_2 - V_4)\frac{1}{sL_3} & V_7' &= (V_6 - V_8)\frac{1}{sL_7} \\
V_4 &= (V_3' - V_5')\frac{1}{sC_4} & V_8 &= V_7' R_8
\end{aligned}
\right\}
$$

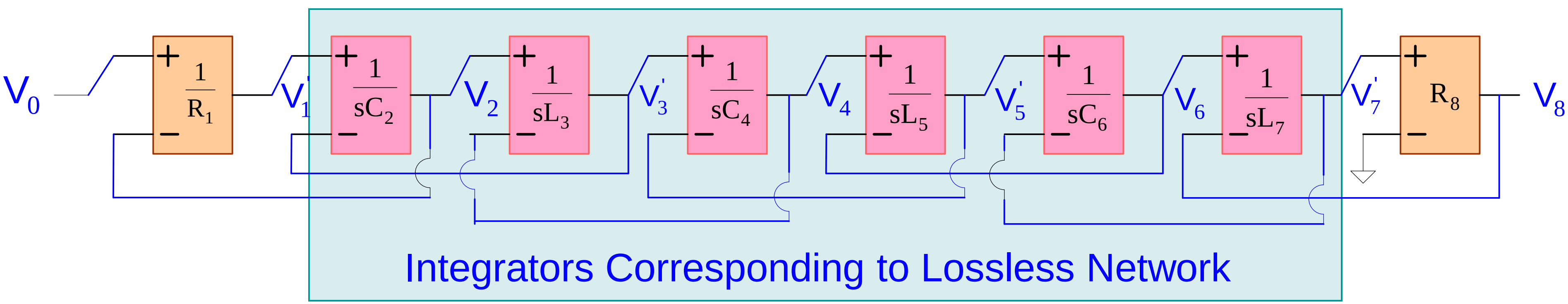
The interconnections that complete each equation can now be added



Consider now only the set of equations and disassociate them from the circuit from where they came



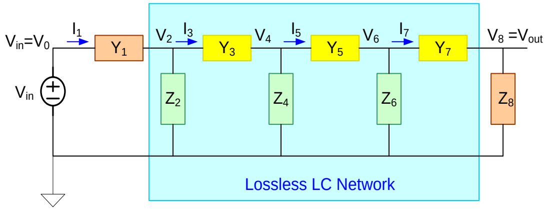


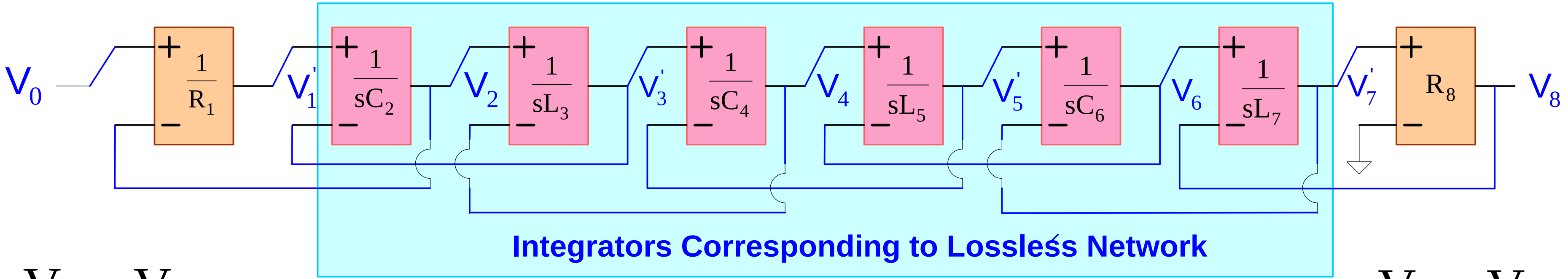


$$V_0 = V_{in}$$

$$V_8 = V_{out}$$

The Leapfrog Configuration



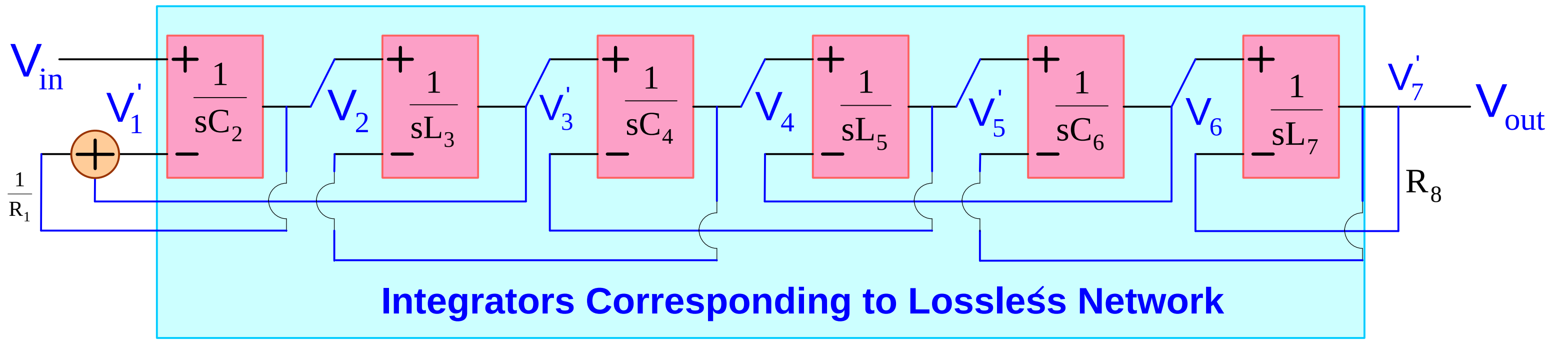


$$V_0 = V_{in}$$

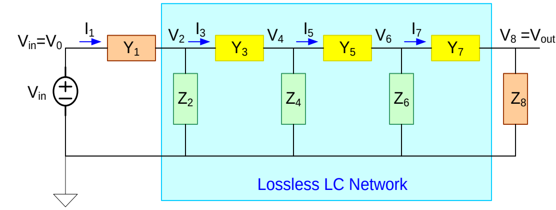
$$V_8 = V_{out}$$

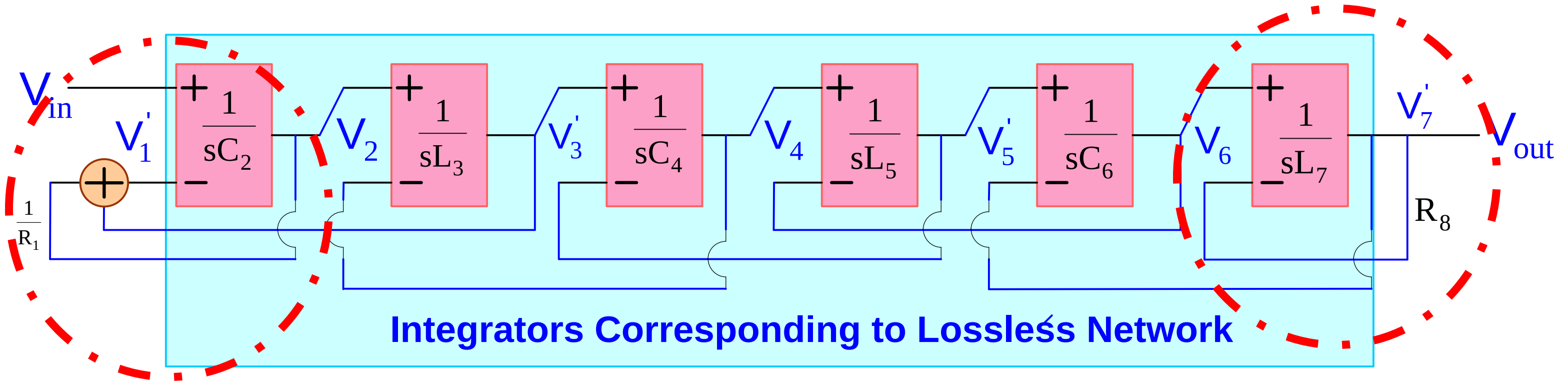
Input summing and weighting can occur at input to the first integrator

The difference between V_8 and V'_7 is only a scale factor that does not affect shape, and the weighting on the Vin input also does not affect shape, thus



The Leapfrog Configuration

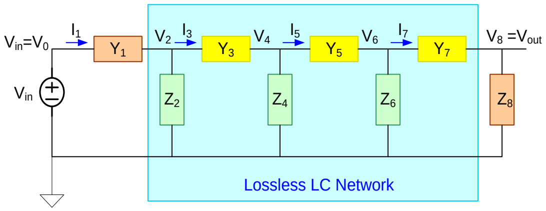


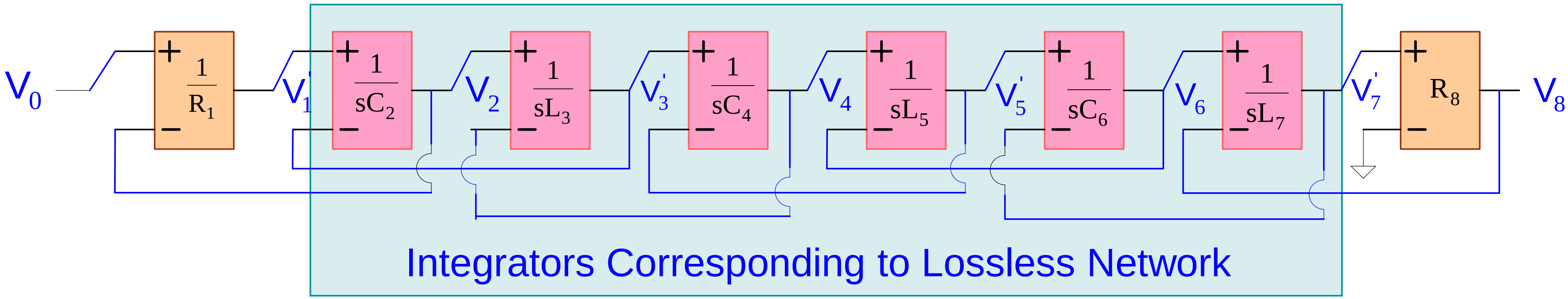


The terminations on both sides have local feedback around an integrator which can be alternately viewed as a lossy integrator

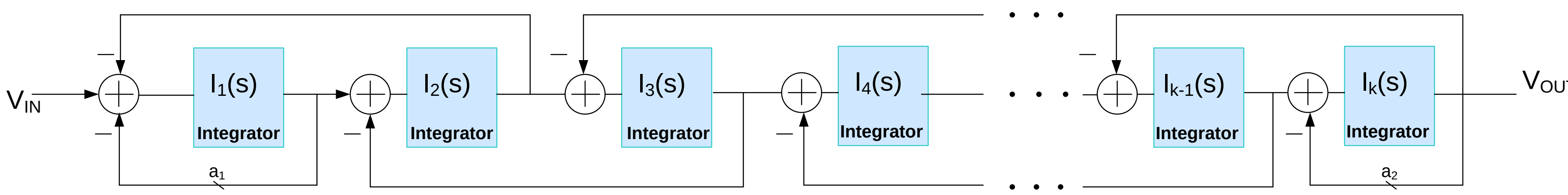
Could redraw the structure as a cascade of internal lossless integrators with terminations that are lossy integrators but since there are so many different ways to implement the integrators and summers, we will not attempt to make that association in the block diagram form but in most practical applications a lossy integrator is often used on the input or the output or both

The Leapfrog Configuration



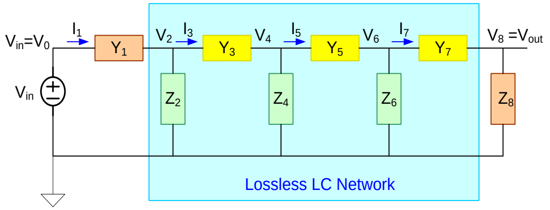


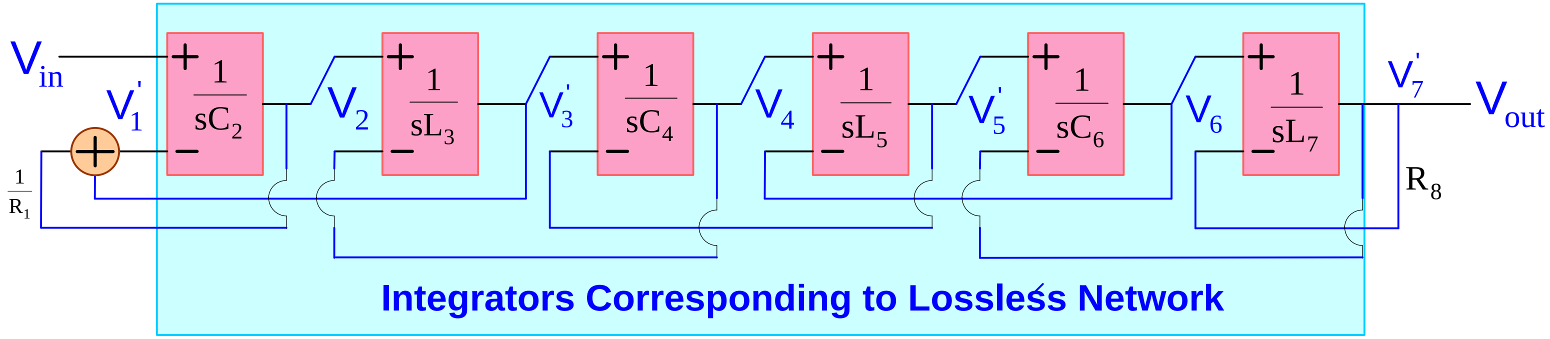
In the general case, this can be redrawn as shown below



Note the first and last integrators become lossy because of the local feedback

The Leapfrog Configuration





The passive prototype filter from which the leapfrog was designed has all shunt capacitors and all series inductors and is thus lowpass.

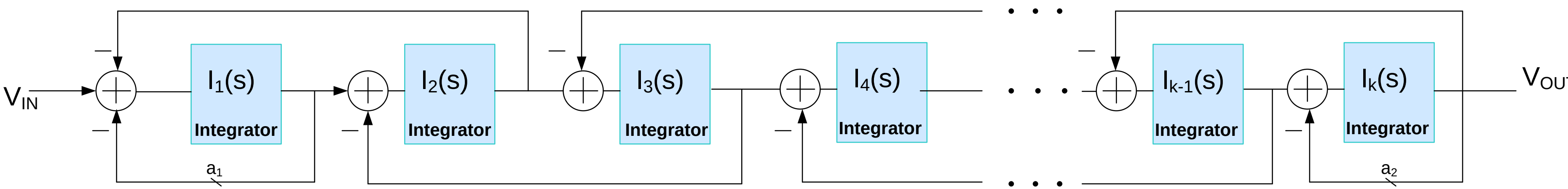
The resultant leapfrog filter has the same transfer function and is thus lowpass

EE 508
Lecture 33

Leapfrog Networks

Review from last time

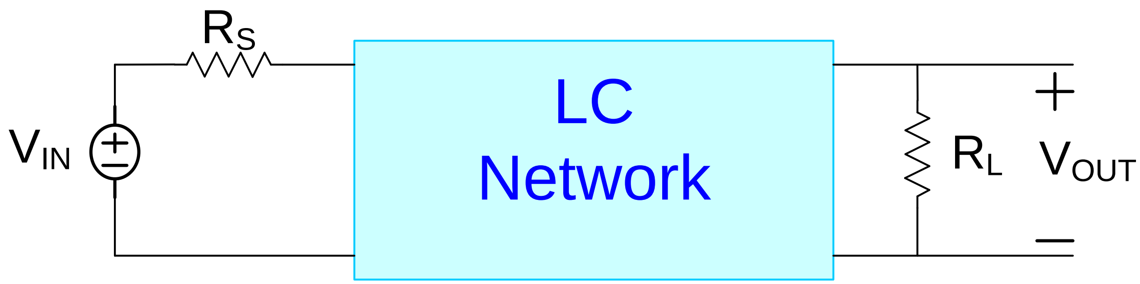
Leapfrog Filters



Introduced by Girling and Good, Wireless World, 1970

This structure has some very attractive properties and is widely used though the real benefits and limitations of the structure are often not articulated

Background Information for Leapfrog Filters



Assume the impedance R_S is fixed

Theorem 1: If the LC network delivers maximum power to the load at a frequency ω, then

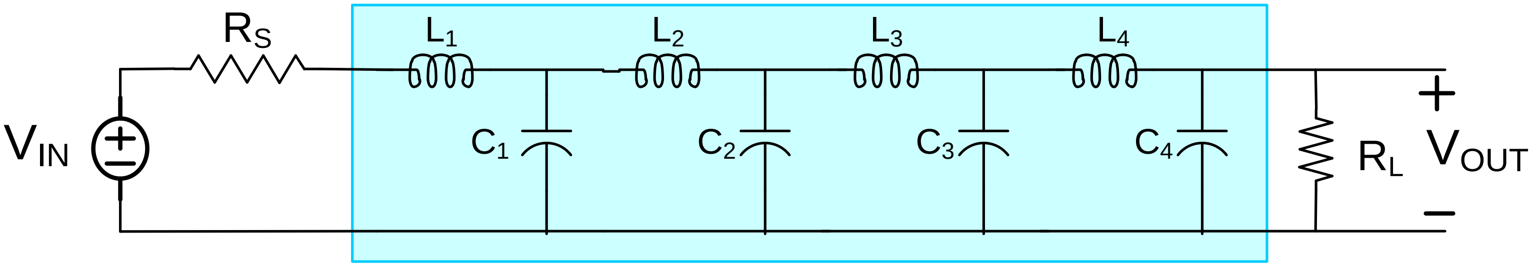
$$S_x^{|T(j\omega)|} = 0$$

for any circuit element in the system except for $x = R_L$

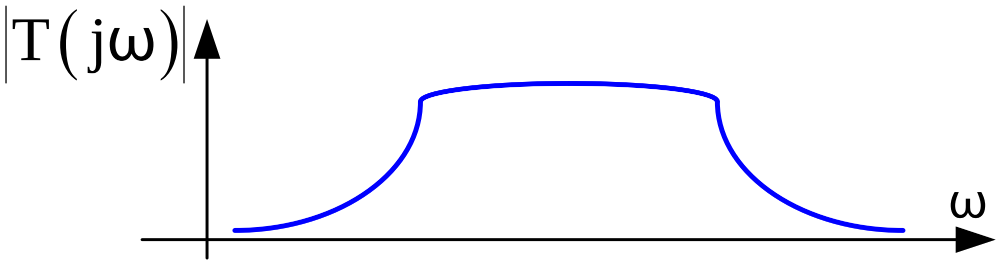
This theorem will follow after we prove the following theorem:

Implications of Theorem 1

Many passive LC filter such as that shown below exist that have near maximum power transfer in the passband



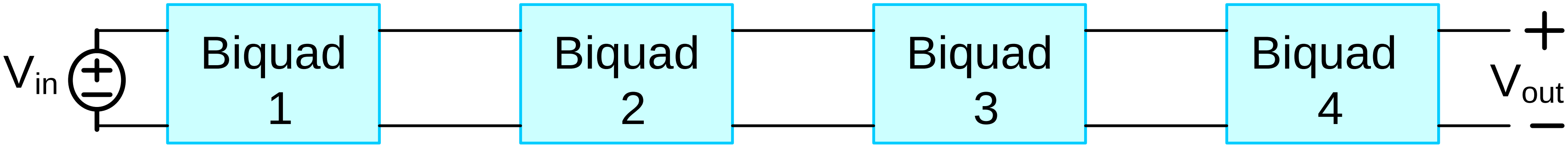
If a component in the LC network changes a little, there is little change in the passband gain characteristics (depicted as bandpass)



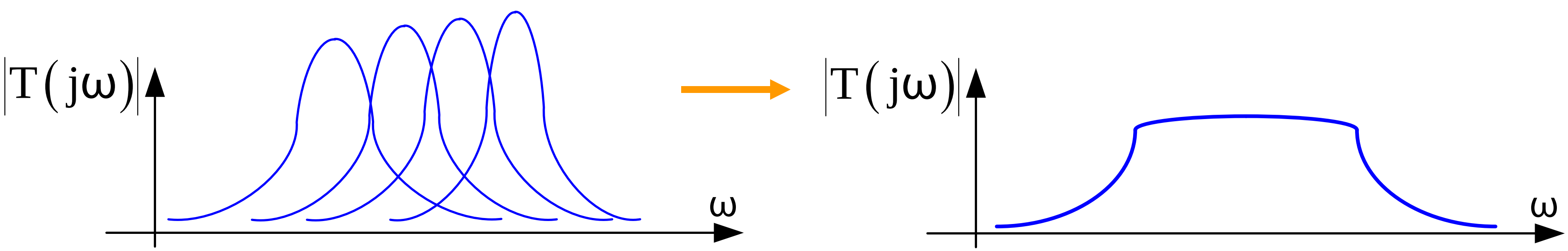
$$S_x^{|T(j\omega)|} \simeq 0$$ in passband

Review from last time

Implications of Theorem 1

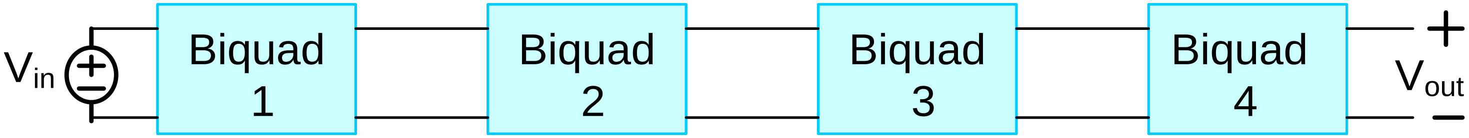


Cascaded Biquad has a response that is the product of the individual second-order transfer functions

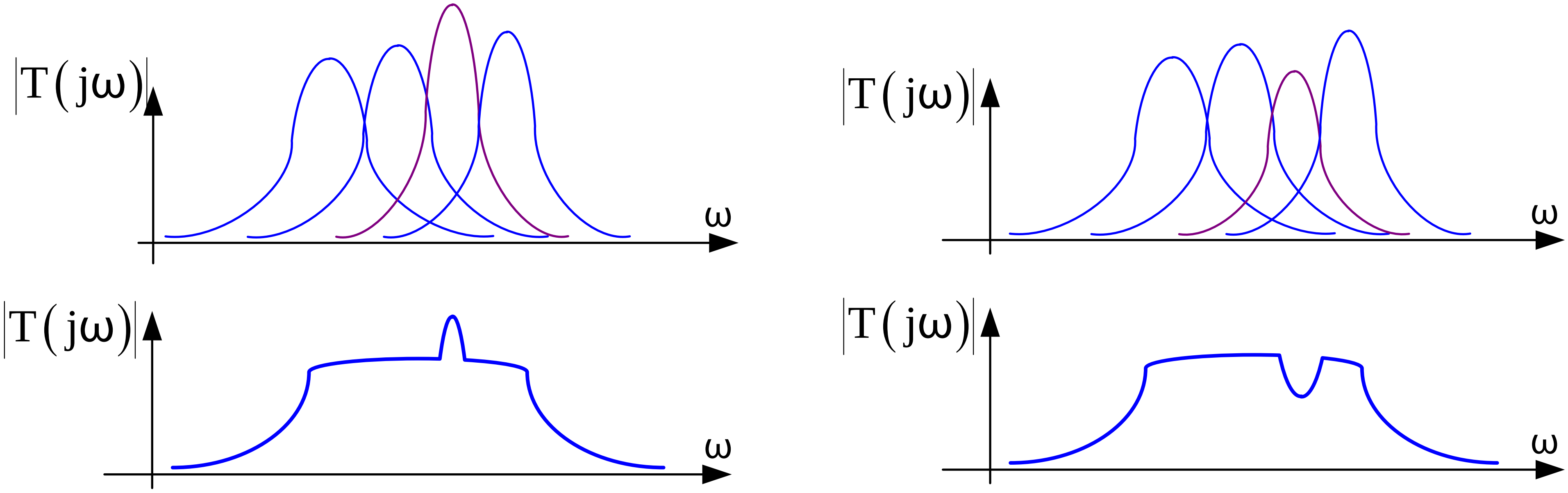


If a component in a biquad changes a little, there is often a large change in the passband gain characteristics (depicted as bandpass)

Implications of Theorem 1



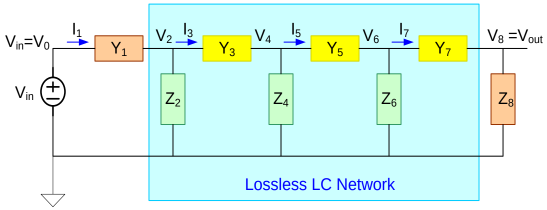
If a component in a biquad changes a little, there is often a large change in the passband gain characteristics (depicted as bandpass)



$$S_x^{|T(j\omega)|} \neq 0$$ in passband

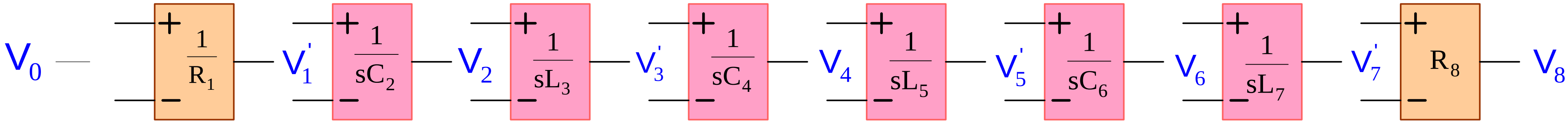
Review from last time

Consider now only the set of equations and disassociate them from the circuit from where they came



$$\left.\begin{aligned}
V_1' &= (V_0 - V_2)\frac{1}{R_1} & V_5' &= (V_4 - V_6)\frac{1}{sL_5}\\
V_2 &= (V_1' - V_3')\frac{1}{sC_2} & V_6 &= (V_5' - V_7')\frac{1}{sC_6}\\
V_3' &= (V_2 - V_4)\frac{1}{sL_3} & V_7' &= (V_6 - V_8)\frac{1}{sL_7}\\
V_4 &= (V_3' - V_5')\frac{1}{sC_4} & V_8 &= V_7' R_8
\end{aligned}\right\}$$

Each equation corresponds to either an integrator or summer with the output voltage output variables and the gain indicated (don’t worry about the units)

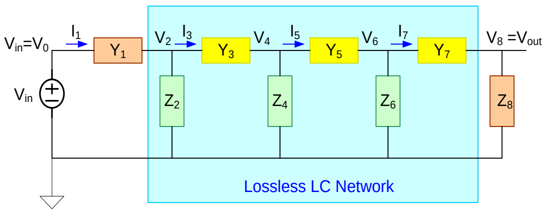


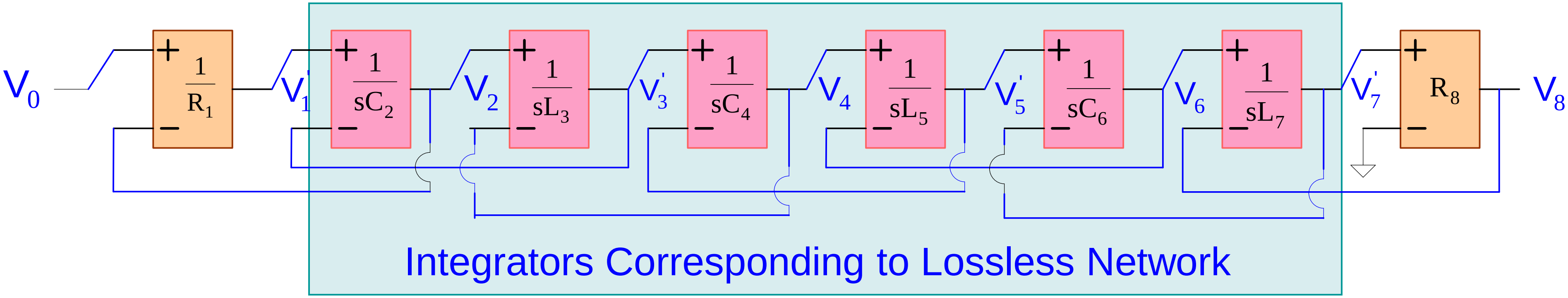
$$V_0 = V_{in}$$

$$V_8 = V_{out}$$

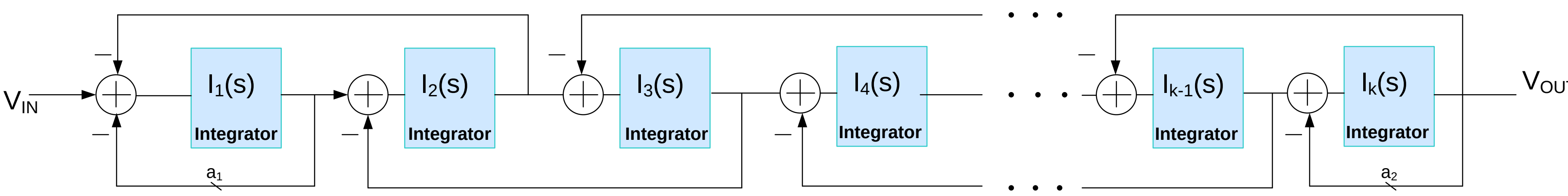
Review from last time

The Leapfrog Configuration





In the general case, this can be redrawn as shown below



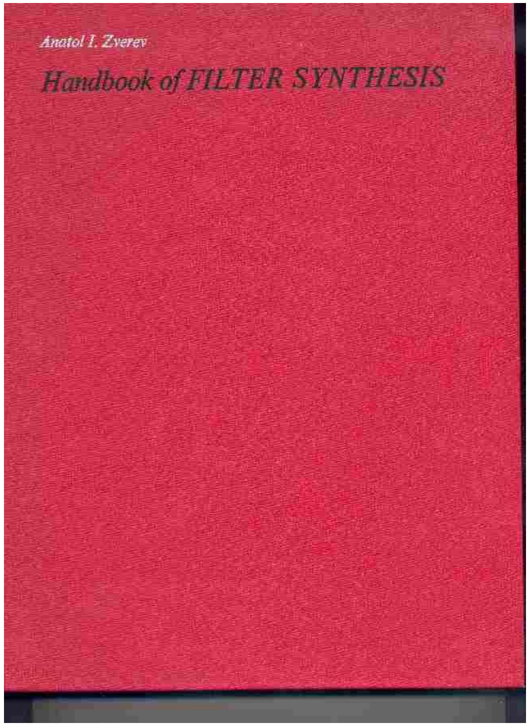
Note the first and last integrators become lossy because of the local feedback

The Passive Prototypes with Maximum Power Transfer in Passband

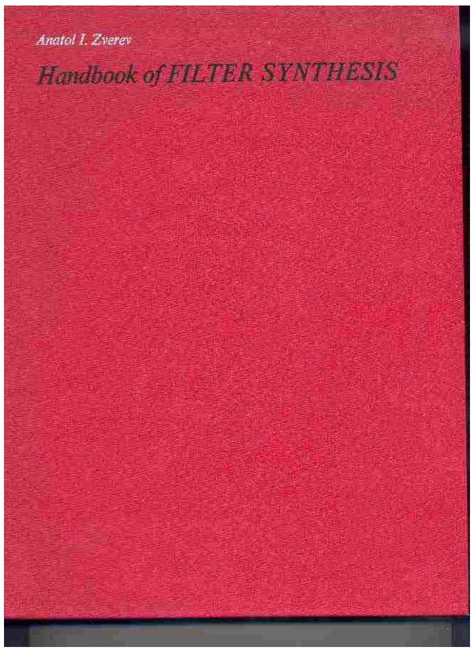
Doubly-terminated LC filters with near maximum power transfer in the passband were developed from the 30's to the 60's

Seldom discussed in current texts but older texts and occasionally software tools provide the passive structures needed to synthesize leapfrog networks

One good book is that by Zverev

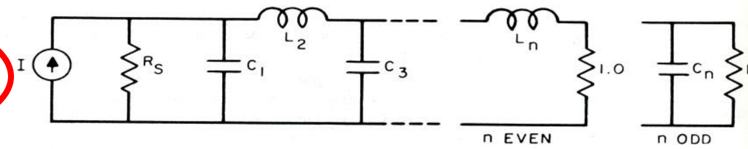


The Passive Prototypes with Maximum Power Transfer in Passband

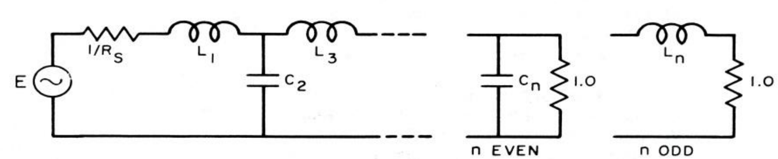


LOW PASS ELEMENT VALUES

BUTTERWORTH RESPONSE



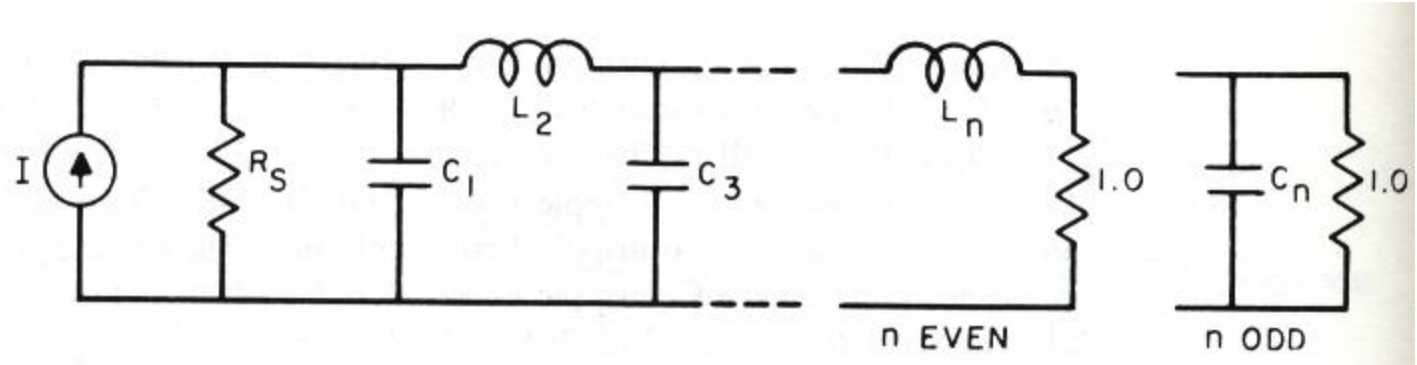
n	R_s	C_1	L_2	C_3	L_4
2	1.0000	1.4142	1.4142		
	1.1111	1.0353	1.8352		
	1.2500	0.8485	2.1213		
	1.4286	0.6971	2.4387		
	1.6667	0.5657	2.8284		
	2.0000	0.4483	3.3461		
	2.5000	0.3419	4.0951		
	3.3333	0.2447	5.3126		
	5.0000	0.1557	7.7067		
	10.0000	0.0743	14.8138		
	INF.	1.4142	0.7071		
3	1.0000	1.0000	2.0000	1.0000	
	0.9000	0.8082	1.6332	1.5994	
	0.8000	0.8442	1.3840	1.9259	
	0.7000	0.9152	1.1652	2.2774	
	0.6000	1.0225	0.9650	2.7024	
	0.5000	1.1811	0.7789	3.2612	
	0.4000	1.4254	0.6042	4.0642	
	0.3000	1.8380	0.4396	5.3634	
	0.2000	2.6687	0.2842	7.9102	
	0.1000	5.1672	0.1377	15.4554	
	INF.	1.5000	1.3333	0.5000	
4	1.0000	0.7654	1.8478	1.8478	0.7654
	1.1111	0.4657	1.5924	1.7439	1.4690
	1.2500	0.3882	1.6946	1.5110	1.8109
	1.4286	0.3251	1.8618	1.2913	2.1752
	1.6667	0.2690	2.1029	1.0824	2.6131
	2.0000	0.2175	2.4524	0.8826	3.1868
	2.5000	0.1692	2.9858	0.6911	4.0094
	3.3333	0.1237	3.8826	0.5072	5.3381
	5.0000	0.0804	5.6835	0.3307	7.9397
	10.0000	0.0392	11.0942	0.1616	15.6421
	INF.	1.5307	1.5772	1.0824	0.3827
n	$1/R_s$	L_1	C_2	L_3	C_4



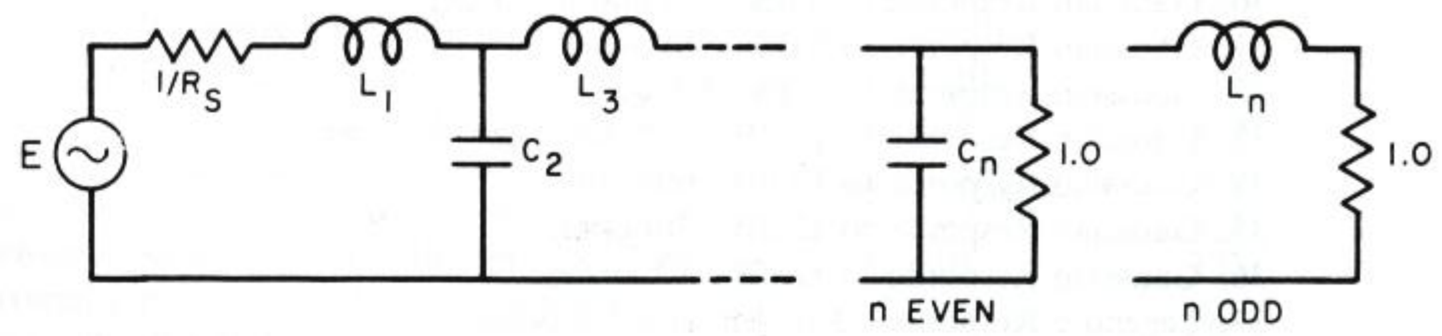
Must start with correct filter type:
(e.g. BW, CC, Cauer)

The Passive Prototypes with Maximum Power Transfer in Passband

The Butterworth Low-Pass Filters



Leading element is a shunt capacitor (appear from top to bottom in table)



Leading element is a series capacitor (appear from bottom to top in table)

Can do Thevenin-Norton Transformations

The Passive Prototypes with Maximum Power Transfer in Passband

Normalized so R_L=1

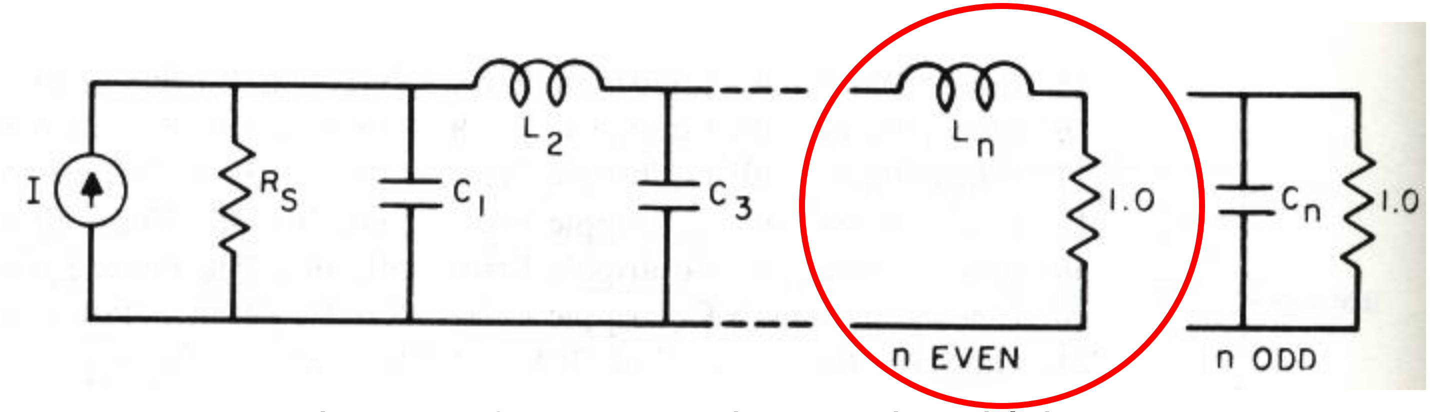
n	R_s	C_1	L_2	C_3	L_4
2	1.0000	1.4142	1.4142		
	1.1111	1.0353	1.8352		
	1.2500	0.8485	2.1213		
	1.4286	0.6971	2.4387		
	1.6667	0.5657	2.8284		
	2.0000	0.4483	3.3461		
	2.5000	0.3419	4.0951		
	3.3333	0.2447	5.3126		
	5.0000	0.1557	7.7067		
	10.0000	0.0743	14.8138		
	INF.	1.4142	0.7071		
3	1.0000	1.0000	2.0000	1.0000	
	0.9000	0.8082	1.6332	1.5994	
	0.8000	0.8442	1.3840	1.9259	
	0.7000	0.9152	1.1652	2.2774	
	0.6000	1.0225	0.9650	2.7024	
	0.5000	1.1811	0.7789	3.2612	
	0.4000	1.4254	0.6042	4.0642	
	0.3000	1.8380	0.4396	5.3634	
	0.2000	2.6687	0.2842	7.9102	
	0.1000	5.1672	0.1377	15.4554	
	INF.	1.5000	1.3333	0.5000	
4	1.0000	0.7654	1.8478	1.8478	0.7654
	1.1111	0.4657	1.5924	1.7439	1.4690
	1.2500	0.3882	1.6946	1.5110	1.8109
	1.4286	0.3251	1.8618	1.2913	2.1752
	1.6667	0.2690	2.1029	1.0824	2.6131
	2.0000	0.2175	2.4524	0.8826	3.1868
	2.5000	0.1692	2.9858	0.6911	4.0094
	3.3333	0.1237	3.8826	0.5072	5.3381
	5.0000	0.0804	5.6835	0.3307	7.9397
	10.0000	0.0392	11.0942	0.1616	15.6421
	INF.	1.5307	1.5772	1.0824	0.3827
n	$1/R_s$	L_1	C_2	L_3	C_4

n	R_s	C_1	L_2	C_3	L_4	C_5	L_6	C_7
	1.0000	0.6180	1.6180	2.0000	1.6180	0.6180		
	0.9000	0.4416	1.0265	1.9095	1.7562	1.3887		
	0.8000	0.4698	0.8660	2.0605	1.5443	1.7380		
	0.7000	0.5173	0.7313	2.2849	1.3326	2.1083		
	0.6000	0.5860	0.6094	2.5998	1.1255	2.5524		
5	0.5000	0.6857	0.4955	3.0510	0.9237	3.1331		
	0.4000	0.8378	0.3877	3.7357	0.7274	3.9648		
	0.3000	1.0937	0.2848	4.8835	0.5367	5.3073		
	0.2000	1.6077	0.1861	7.1849	0.3518	7.9345		
	0.1000	3.1522	0.0912	14.0945	0.1727	15.7103		
	INF.	1.5451	1.6944	1.3820	0.8944	0.3090		
	1.0000	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176	
	1.1111	0.2890	1.0403	1.3217	2.0539	1.7443	1.3347	
	1.2500	0.2445	1.1163	1.1257	2.2389	1.5498	1.6881	
	1.4286	0.2072	1.2363	0.9567	2.4991	1.3464	2.0618	
	1.6667	0.1732	1.4071	0.8011	2.8580	1.1431	2.5092	
6	2.0000	0.1412	1.6531	0.6542	3.3687	0.9423	3.0938	
	2.5000	0.1108	2.0275	0.5139	4.1408	0.7450	3.9305	
	3.3333	0.0816	2.6559	0.3788	5.4325	0.5517	5.2804	
	5.0000	0.0535	3.9170	0.2484	8.0201	0.3628	7.9216	
	10.0000	0.0263	7.7053	0.1222	15.7855	0.1788	15.7375	
	INF.	1.5529	1.7593	1.5529	1.2016	0.7579	0.2588	
	1.0000	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450
	0.9000	0.2985	0.7111	1.4043	1.4891	2.1249	1.7268	1.2961
	0.8000	0.3215	0.6057	1.5174	1.2777	2.3338	1.5461	1.6520
	0.7000	0.3571	0.5154	1.6883	1.0910	2.6177	1.3498	2.0277
	0.6000	0.4075	0.4322	1.9284	0.9170	3.0050	1.1503	2.4771
7	0.5000	0.4799	0.3536	2.2726	0.7512	3.5532	0.9513	3.0640
	0.4000	0.5899	0.2782	2.7950	0.5917	4.3799	0.7542	3.9037
	0.3000	0.7745	0.2055	3.6706	0.4373	5.7612	0.5600	5.2583
	0.2000	1.1448	0.1350	5.4267	0.2874	8.5263	0.3692	7.9079
	0.1000	2.2571	0.0665	10.7004	0.1417	16.8222	0.1823	15.7480
	INF.	1.5576	1.7988	1.6588	1.3972	1.0550	0.6560	0.2225
n	$1/R_s$	L_1	C_2	L_3	C_4	L_5	C_6	L_7

Example:

Design a 6th-order BW lowpass Leapfrog filter with a leading capacitor, with equal source and load terminations, and with a 3dB band edge of 4KHz.

Start with the normalized BW lowpass filter



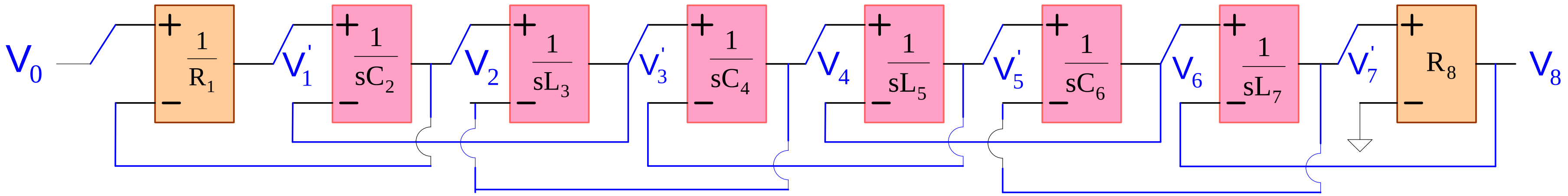
(appear from top to bottom in table)

Do Norton to Thevenin transformation at input

n	R_s	C_1	L_2	C_3	L_4	C_5	L_6	C_7
5	1.0000	0.6180	1.6180	2.0000	1.6180	0.6180		
	0.9000	0.4416	1.0265	1.9095	1.7562	1.3887		
	0.8000	0.4698	0.8660	2.0605	1.5443	1.7380		
	0.7000	0.5173	0.7313	2.2849	1.3326	2.1083		
	0.6000	0.5860	0.6094	2.5998	1.1255	2.5524		
	0.5000	0.6857	0.4955	3.0510	0.9237	3.1331		
	0.4000	0.8378	0.3877	3.7357	0.7274	3.9648		
	0.3000	1.0937	0.2848	4.8835	0.5367	5.3073		
	0.2000	1.6077	0.1861	7.1849	0.3518	7.9345		
	0.1000	3.1522	0.0912	14.0945	0.1727	15.7103		
	INF.	1.5451	1.6944	1.3820	0.8944	0.3090		
6	1.0000	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176	
	1.1111	0.2890	1.0403	1.3217	2.0539	1.7443	1.3347	
	1.2500	0.2445	1.1163	1.1257	2.2389	1.5498	1.6881	
	1.4286	0.2072	1.2363	0.9567	2.4991	1.3464	2.0618	
	1.6667	0.1732	1.4071	0.8011	2.8580	1.1431	2.5092	
	2.0000	0.1412	1.6531	0.6542	3.3687	0.9423	3.0938	
	2.5000	0.1108	2.0275	0.5139	4.1408	0.7450	3.9305	
	3.3333	0.0816	2.6559	0.3788	5.4325	0.5517	5.2804	
	5.0000	0.0535	3.9170	0.2484	8.0201	0.3628	7.9216	
	10.0000	0.0263	7.7053	0.1222	15.7855	0.1788	15.7375	
	INF.	1.5529	1.7593	1.5529	1.2016	0.7579	0.2588	
7	1.0000	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450
	0.9000	0.2985	0.7111	1.4043	1.4891	2.1249	1.7268	1.2961
	0.8000	0.3215	0.6057	1.5174	1.2777	2.3338	1.5461	1.6520
	0.7000	0.3571	0.5154	1.6883	1.0910	2.6177	1.3498	2.0277
	0.6000	0.4075	0.4322	1.9284	0.9170	3.0050	1.1503	2.4771
	0.5000	0.4799	0.3536	2.2726	0.7512	3.5532	0.9513	3.0640
	0.4000	0.5899	0.2782	2.7950	0.5917	4.3799	0.7542	3.9037
	0.3000	0.7745	0.2055	3.6706	0.4373	5.7612	0.5600	5.2583
	0.2000	1.1448	0.1350	5.4267	0.2874	8.5263	0.3692	7.9079
	0.1000	2.2571	0.0665	10.7004	0.1417	16.8222	0.1823	15.7480
	INF.	1.5576	1.7988	1.6588	1.3972	1.0550	0.6560	0.2225
n	$1/R_s$	L_1	C_2	L_3	C_4	L_5	C_6	L_7

$R_s=1$, $C_1=.5176$, $L_2=1.414$, $C_3=1.939$, $L_4=1.9319$, $C_5=1.4142$, $L_6=0.5176$

Note index differs by 1 from that used for Leapfrog formulation



Labeled voltages are single-ended voltages at “+” inputs to the integrators

Changing the index notation:

R_1=1, C_2=.5176, L_3=1.414, C_4=1.939, L_5=1.9319, C_6=1.4142, L_7=0.5176

Implement in the technology of choice

Combine loss on input and output integrators to eliminate two stages

Do frequency denormalization to obtain band-edge at 4KHz

Do impedance scaling to obtain acceptable component values

Bandpass Leapfrog Structures

Consider lowpass to bandpass transformations

Un-normalized

$$s_n \rightarrow \frac{s^2 + \omega_0^2}{sBW}$$

$$\frac{1}{s_n} \rightarrow \frac{sBW}{s^2 + \omega_0^2}$$

$$\frac{1}{s_n + \alpha} \rightarrow \frac{sBW}{s^2 + s\alpha BW + \omega_0^2}$$

Normalized

$$s_n \rightarrow \frac{s^2 + 1}{sBW_n}$$

$$\frac{1}{s_n} \rightarrow \frac{sBW_n}{s^2 + 1}$$

$$\frac{1}{s_n + \alpha} \rightarrow \frac{sBW_n}{s^2 + s\alpha BW_n + 1}$$

Bandpass Leapfrog Structures

Bandpass Leapfrog Structure obtained by replacing integrators by the corresponding transformed block

Zero sensitivity to parameters in the transformed blocks will be retained at the image frequencies of the bandpass filter

$$s_n \rightarrow \frac{s^2 + \omega_0^2}{sBW}$$

$$\frac{1}{s_n} \rightarrow \frac{sBW}{s^2 + \omega_0^2}$$

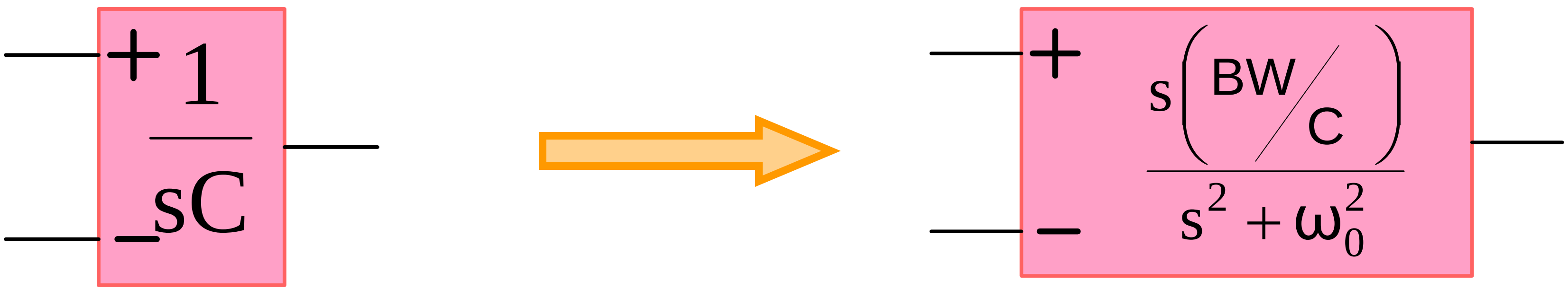
Integrators map to bandpass biquads with infinite Q

$$\frac{1}{s_n + \alpha} \rightarrow \frac{sBW}{s^2 + s\alpha BW + \omega_0^2}$$

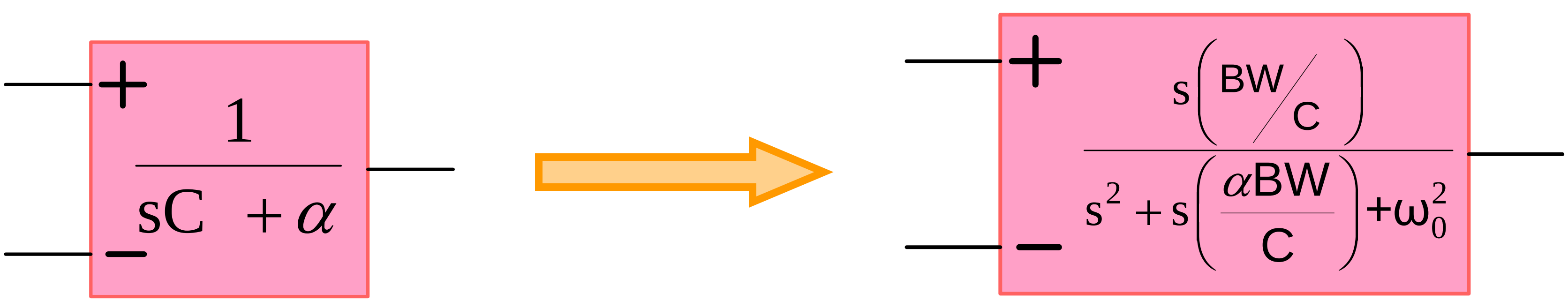
Lossy integrators map to bandpass biquads with finite Q

Bandpass Leapfrog Structures

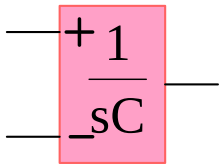
$$\frac{1}{s_n} \rightarrow \frac{sBW}{s^2 + \omega_0^2}$$



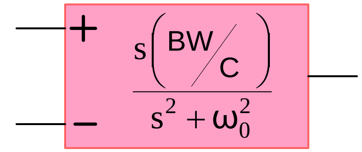
$$\frac{1}{s_n + \alpha} \rightarrow \frac{sBW}{s^2 + s\alpha BW + \omega_0^2}$$

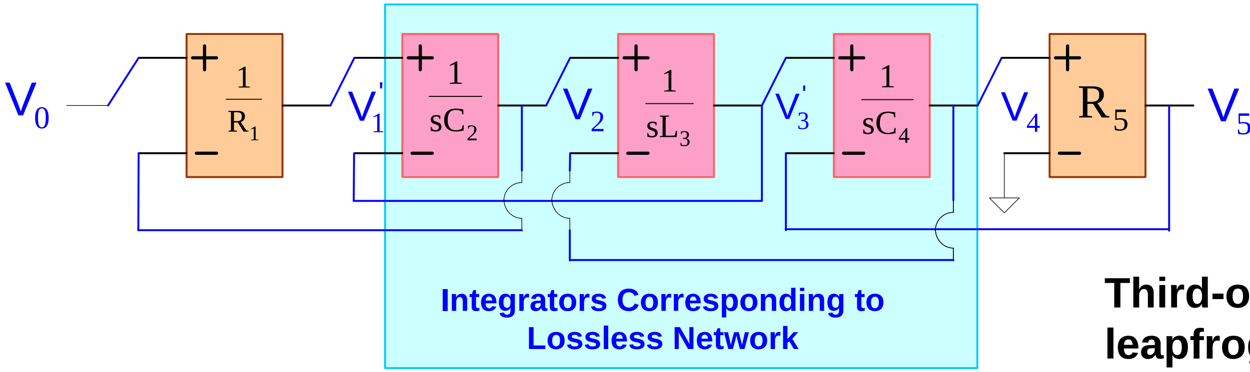


Bandpass Leapfrog Structures

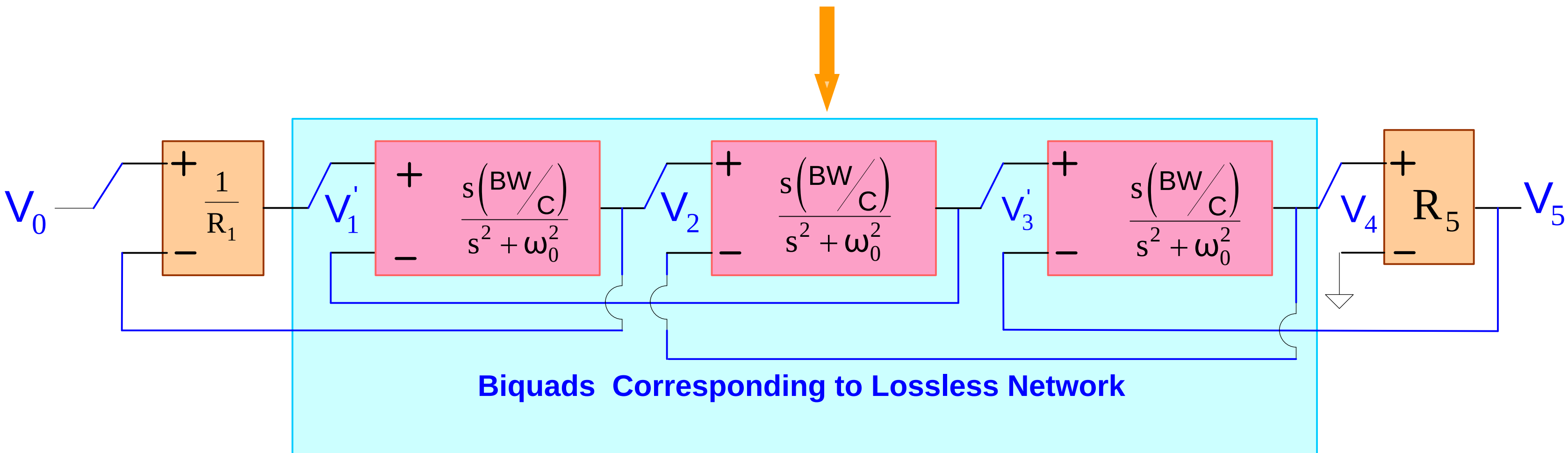


$$\frac{1}{s_n} \rightarrow \frac{sBW}{s^2+\omega_0^2}$$



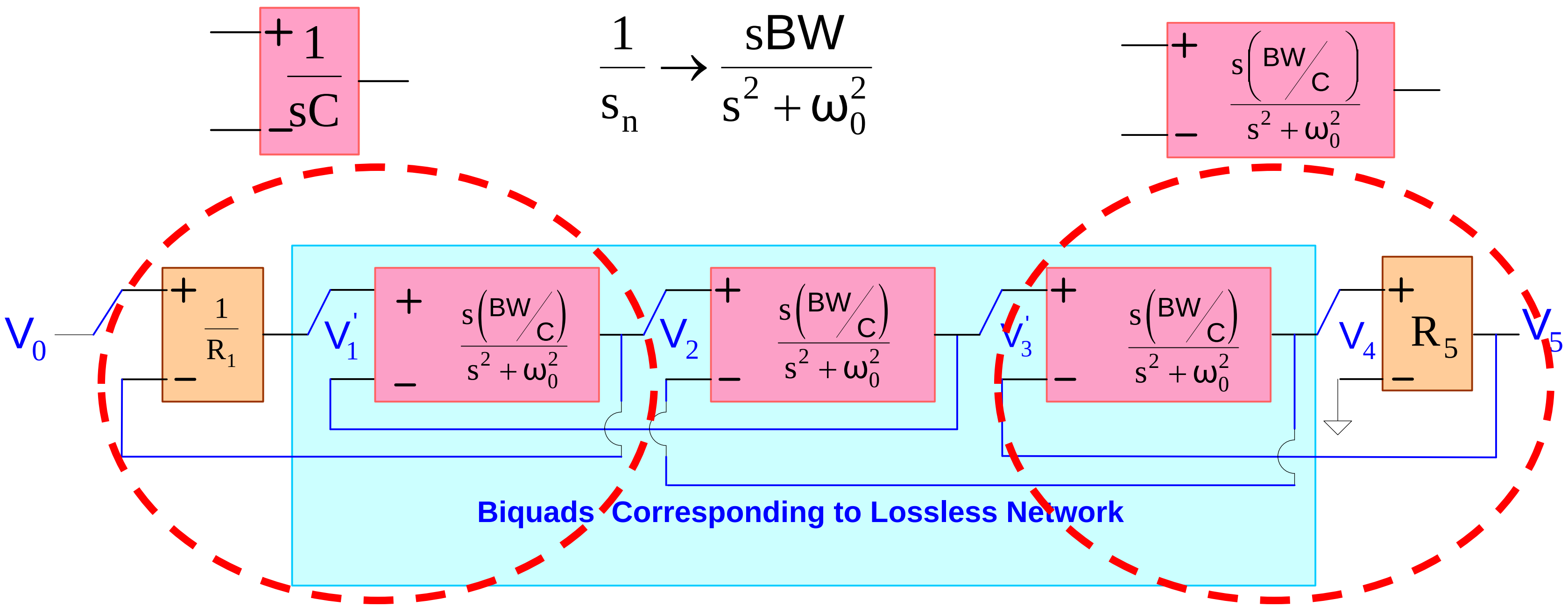


Third-order lowpass leapfrog filter



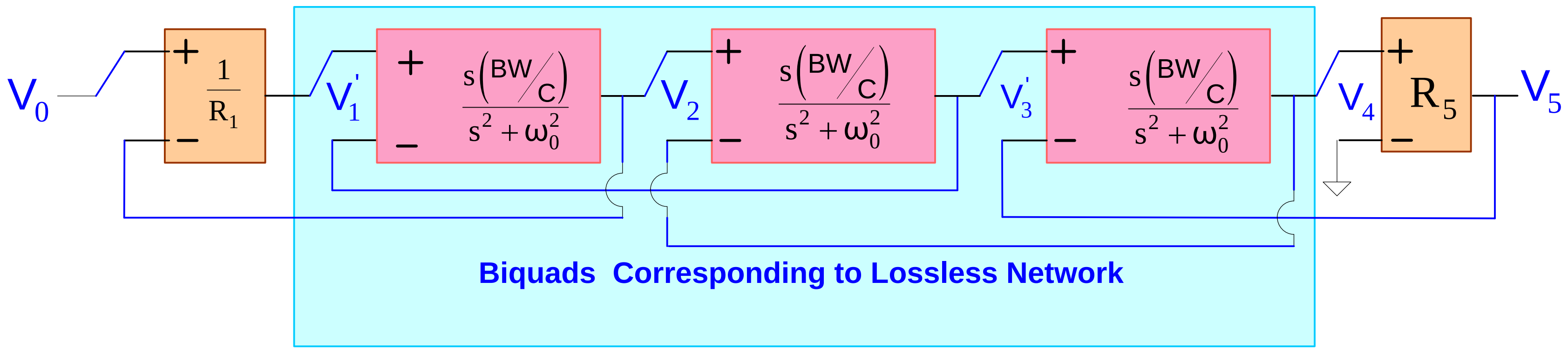
Sixth-order bandpass leapfrog filter

Bandpass Leapfrog Structures



"Loss" at input and/or output can usually be incorporated into finite-Q terminating biquads instead of requiring additional voltage amplifiers

Bandpass Leapfrog Structures

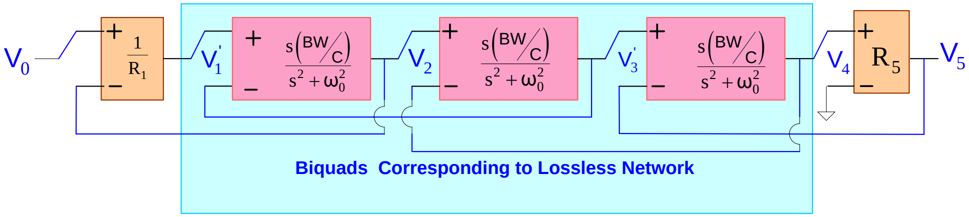


- The bandpass biquads can be implemented with various architectures and the architecture does not ideally affect the passband sensitivity of the filter
- Integrator-based biquads are often used in integrated applications
- Note the lossless biquads are infinite Q structures !

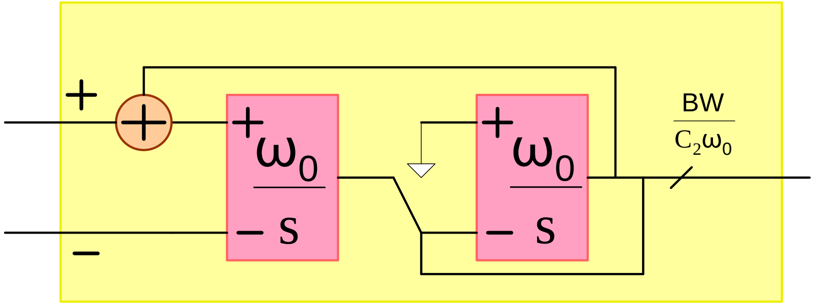
Is it easy or practical to implement infinite Q biquads?

Are there stability concerns about the infinite Q biquads?

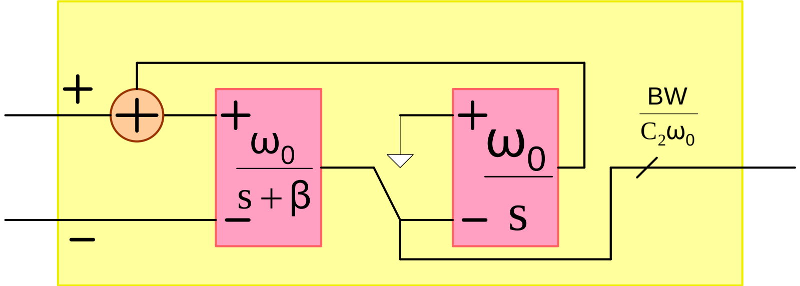
Bandpass Leapfrog Structures



Integrator-based biquads



$$T(s) = \frac{s\left(BW/C\right)}{s^2 + \omega_0^2}$$



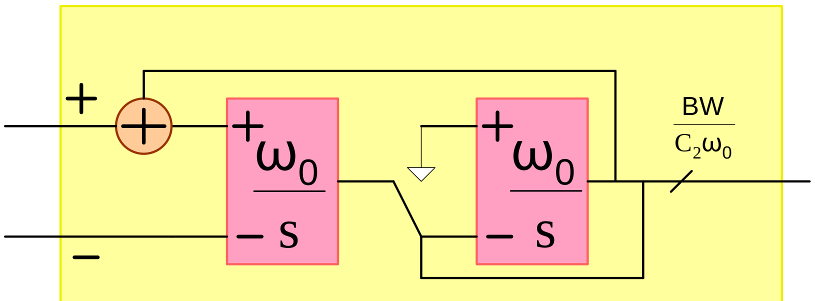
$$T(s) = \frac{s\left(BW/C\right)}{s^2 + s\alpha BW + \omega_0^2}$$

Bandpass Leapfrog Structures

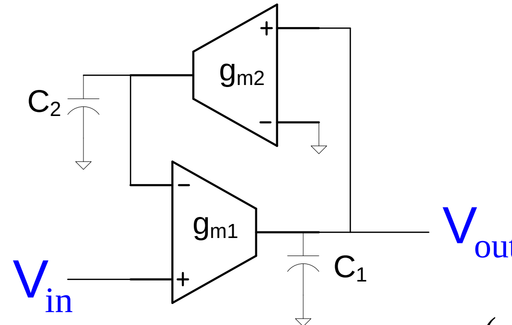
Integrator-based biquads

OTA-C Implementations (Concept)

Infinite Q bandpass biquad



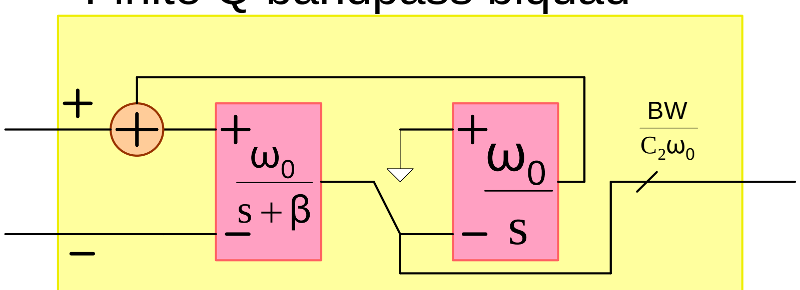
$$T(s) = \frac{s(BW/C)}{s^2 + \omega_0^2}$$



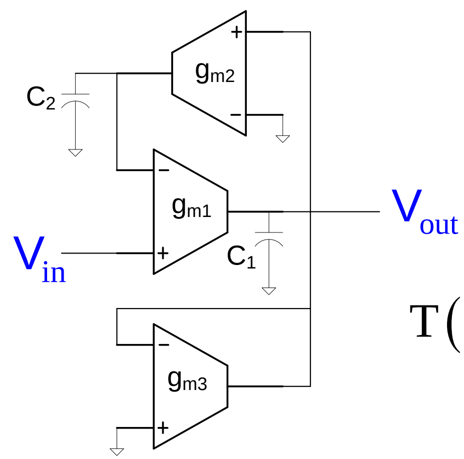
(Not Differential)

$$T(s) = \frac{s\left(\frac{g_{m1}}{C_1}\right)}{s^2 + \frac{g_{m1}g_{m2}}{C_1C_2}}$$

Finite Q bandpass biquad



$$T(s) = \frac{s(BW/C)}{s^2 + s\alpha BW + \omega_0^2}$$



(Not Differential)

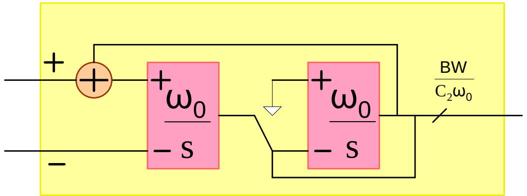
$$T(s) = \frac{s\left(\frac{g_{m1}}{C_1}\right)}{s^2 + s\frac{g_{m3}}{C_1} + \frac{g_{m1}g_{m2}}{C_1C_2}}$$

Bandpass Leapfrog Structures

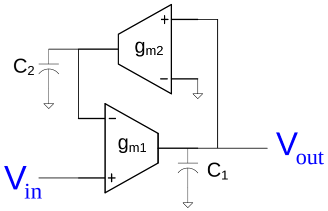
Integrator-based biquads

OTA-C Implementations

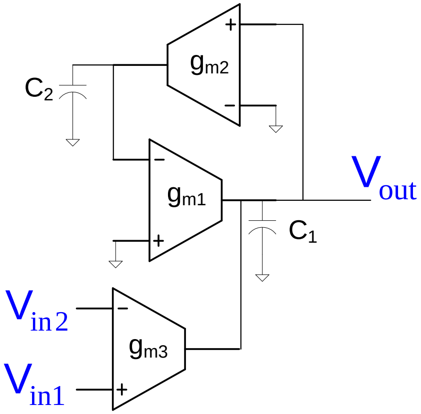
Infinite Q bandpass biquad



$$T(s)=\frac{s\left(BW/C\right)}{s^2+\omega_0^2}$$



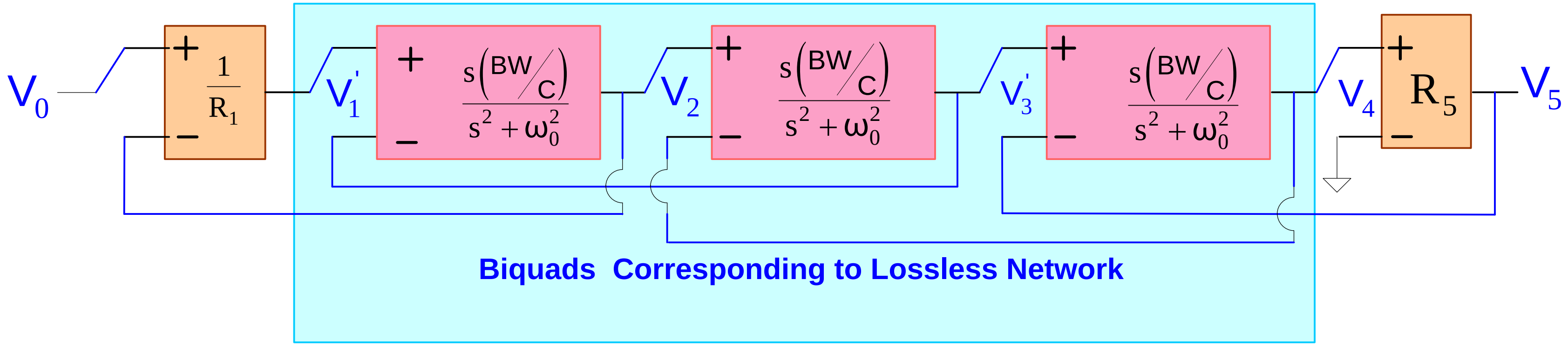
$$T(s)=\frac{s\left(\frac{g_{m1}}{C_1}\right)}{s^2+\frac{g_{m1}g_{m2}}{C_1C_2}}$$



$$V_{OUT}(s)=\frac{s\left(\frac{g_{m3}}{C_1}\right)\left[V_{in1}-V_{in2}\right]}{s^2+\frac{g_{m1}g_{m2}}{C_1C_2}}$$

Multiple inputs can be added to lossy integrator too!

Bandpass Leapfrog Structures



Note the lossless biquads are infinite Q structures !

Is it easy or practical to implement infinite Q biquads?

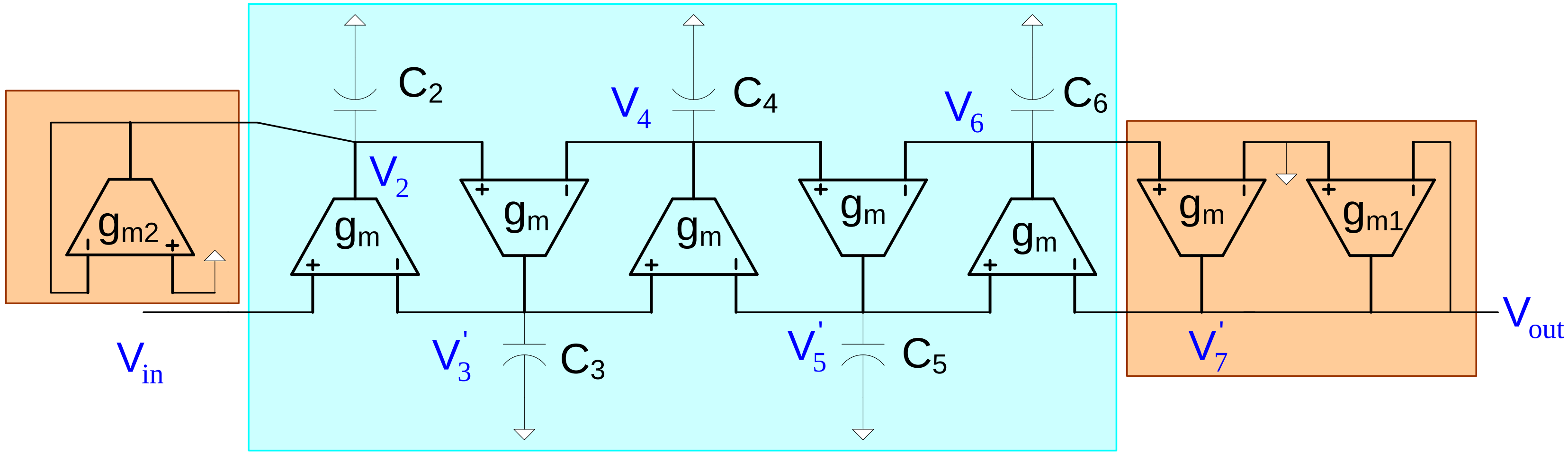
Yes – have shown by example in g_m-C family and also easy in other families

Are there stability concerns about the infinite Q biquads?

Stability of overall leapfrog structure of concern, not stability of individual biquads
Overall leapfrog structure is robust with low passband sensitivities !

Leapfrog Implementations

Fifth-order Lowpass Leapfrog with OTAs



$$V_1' = \frac{1}{R_1}\left(V_{in} - V_2\right)$$

$$V_4 = \frac{g_m/C_4}{s}\left(V_3' - V_5'\right)$$

$$V_7' = \left(g_m/g_{m1}\right)V_6$$

$$V_2 = \frac{g_m/C_2}{s}\left(V_1' - V_3'\right)$$

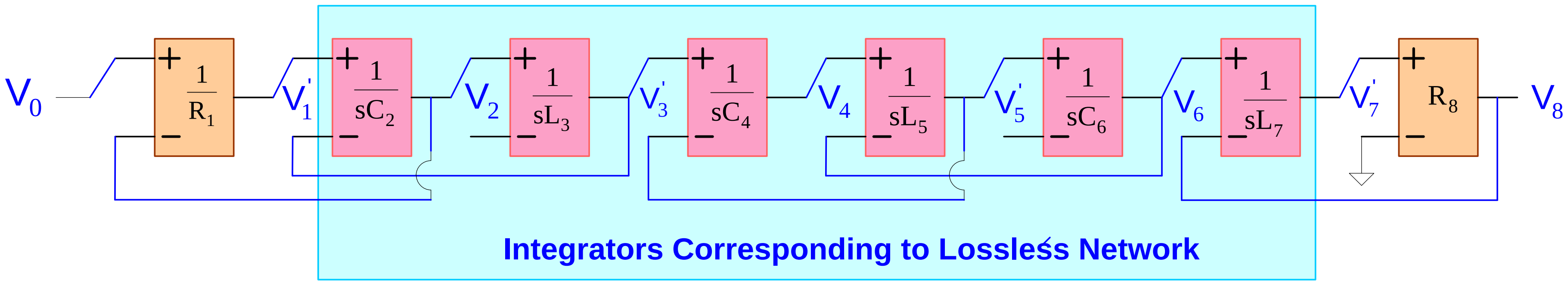
$$V_5' = \frac{g_m/C_5}{s}\left(V_4 - V_6\right)$$

$$V_3' = \frac{g_m/C_3}{s}\left(V_2 - V_4\right)$$

$$V_6 = \frac{g_m/C_6}{s}\left(V_5 - V_7'\right)$$

Practically can either fix g_ms and vary capacitors or fix capacitors and vary g_m's

Some leapfrog properties



What can be said about sensitivities of parameters such as band edges of leapfrog filters? If these sensitivities are not at or near 0, are they at least very small?

No! Nothing can be said about these sensitivities and they are not necessarily any smaller than what one may have for other structures such as cascaded biquads

Instead of having components (such as L's or C's) in the image of the lossless ladder network there are circuits such as integrators or biquads with more than one characterization parameters. Are the sensitivities of $|T(j\omega)|$ to these components also 0 at frequencies where the "parent" passive filter are 0?

Yes! The following theorem addresses this issue in the case of integrators

Theorem: If f(u) is a function of a variable u where $u=x_1x_2$, then

$$S_u^f = S_{x_1}^f = S_{x_2}^f$$

Note: Although the results are the same as for the sensitivity of kf, in this case both x_1 and x_2 are variables whereas in the former case k is a constant.

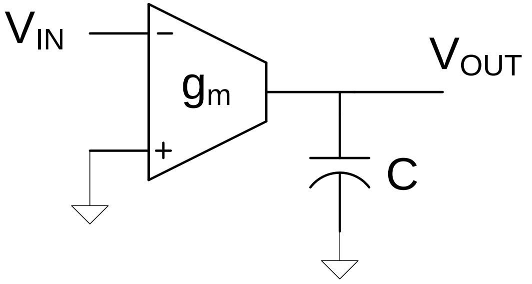
As a consequence, if the unity gain frequency of an integrator which may be expressed (for example) as 1/RC, the transfer function magnitude sensitivity to both R and C vanish at frequencies where the sensitivity to I_0 vanishes

EE 508

Lecture 34

Transconductor Design

Transconductor Design



Transconductor-based filters depend directly on the g_m of the transconductor

Feedback is not used to make the filter performance insensitive to the transconductance gain

Linearity and spectral performance of the filter strongly dependent upon the linearity of the transconductor

Often can not justify elegant linearization strategies in the transconductors because of speed, area, and power penalties

Seminal Work on the OTA

RCA

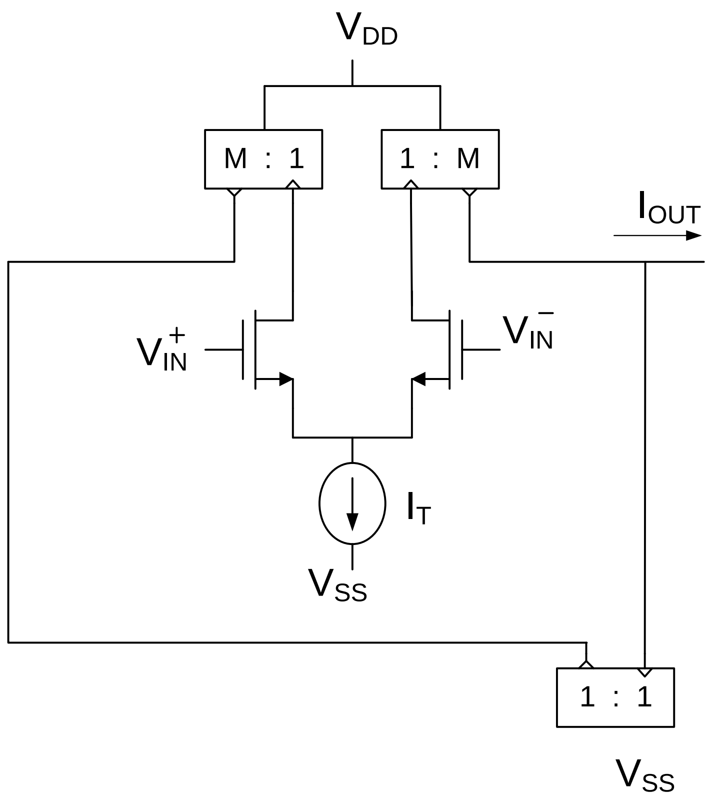
OTA Obsoletes Op Amp

by C.F. Wheatley
H.A. Wittlinger

From:

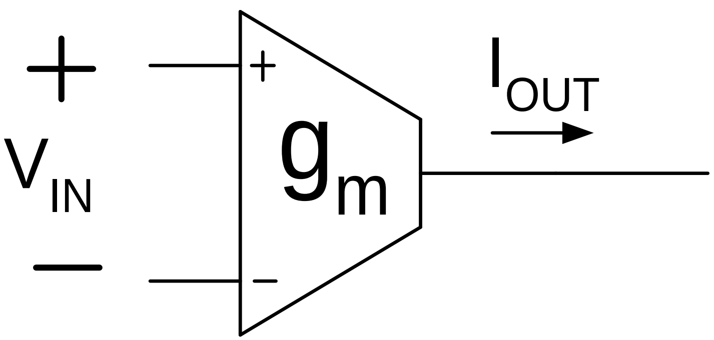
1969 N.E.C. PROCEEDINGS
December 1969

Current Mirror Op Amp W/O CMFB



$$g_{mEQ} = Mg_{m1}$$

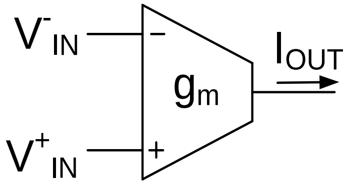
Often termed an OTA

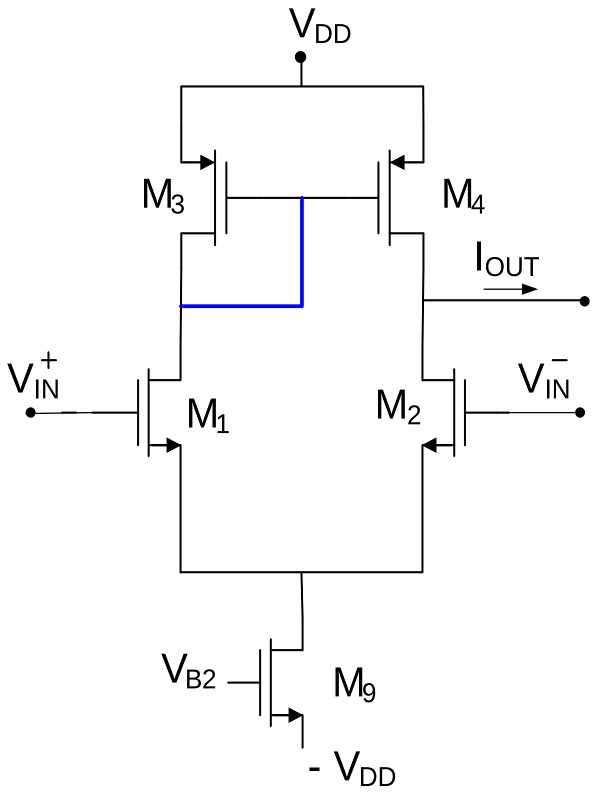


Introduced by Wheatley and Whitlinger in 1969

$$I_{OUT} = g_m V_{IN}$$

Basic OTA based upon differential pair

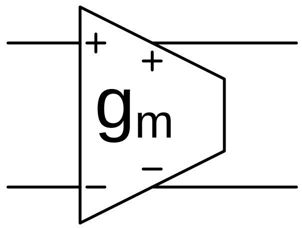


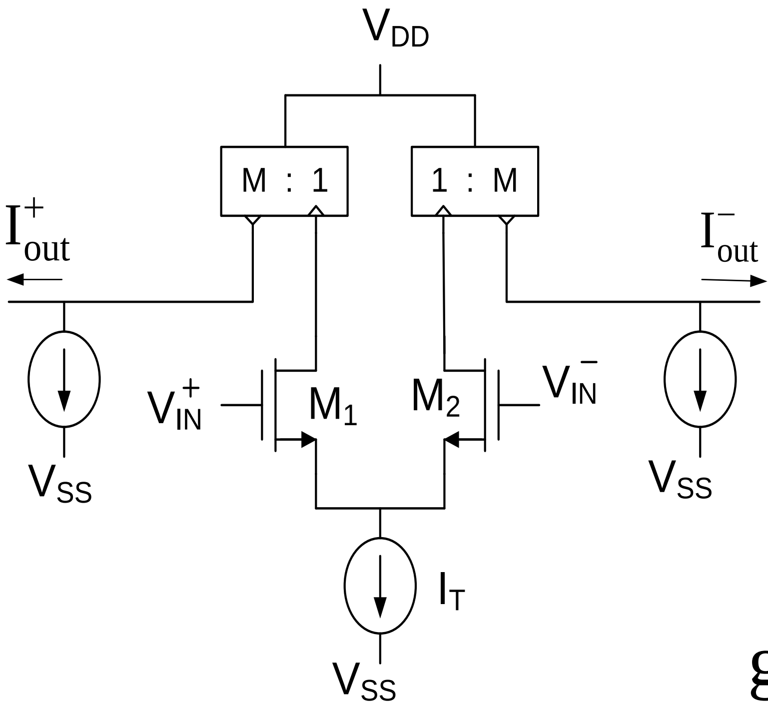


$$g_m = g_{m1}$$

Assume M_1 and M_2 matched,
M_3 and M_4 matched

Differential output OTA based upon differential pair

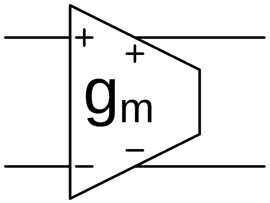


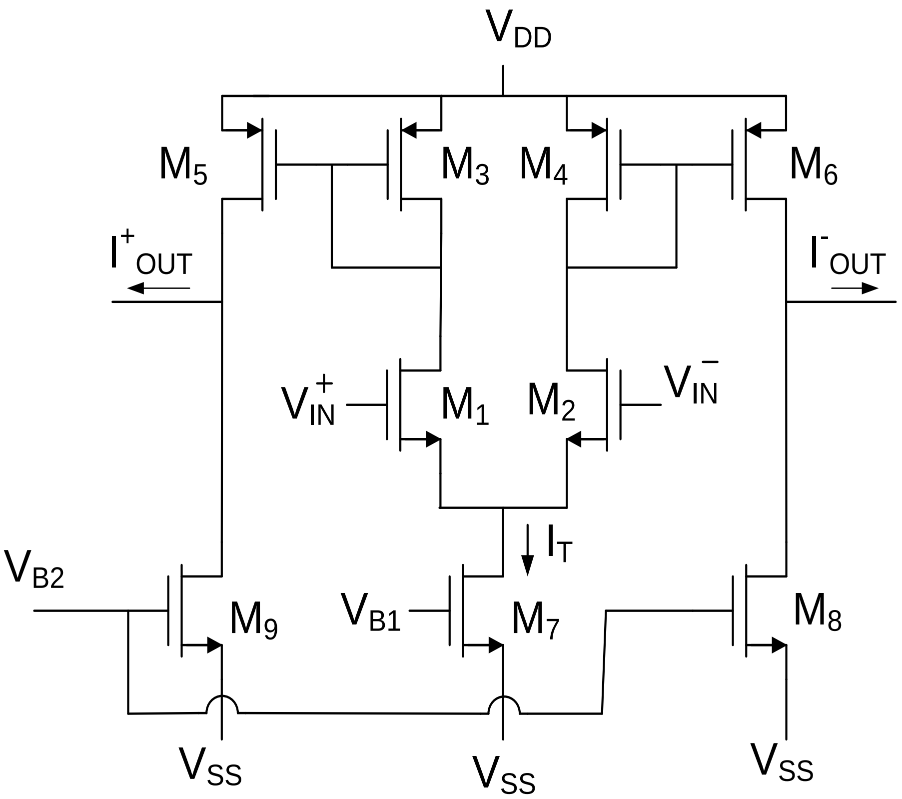


$$g_m = \frac{g_{m1}}{2} M$$

CMFB needed for the two output biasing current sources

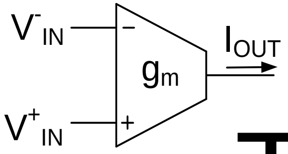
Differential output OTA based upon differential pair



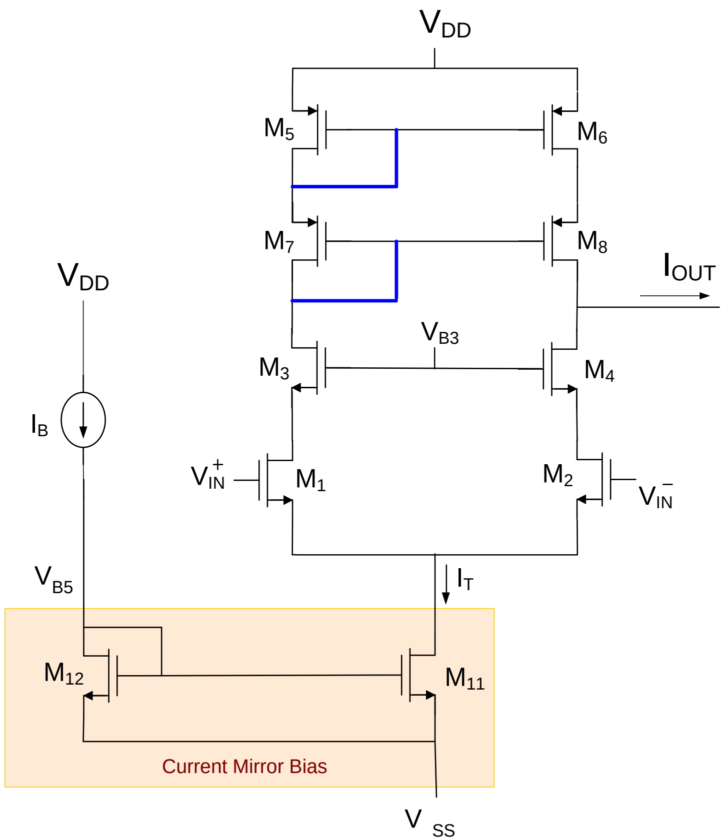


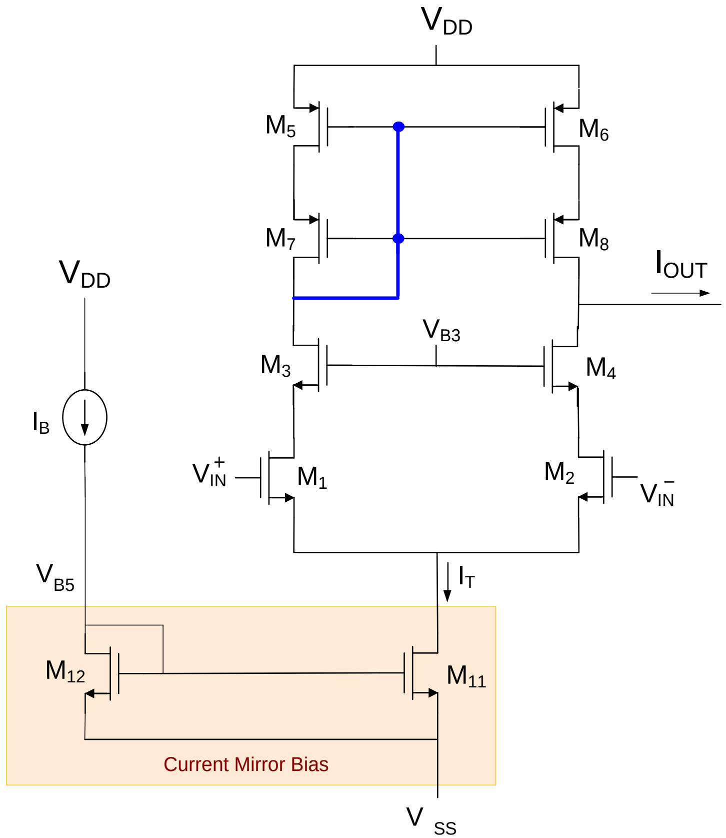
$$g_m = \frac{g_{m1}}{2} M$$

CMFB needed for the two output biasing current sources



Telescopic Cascode OTA



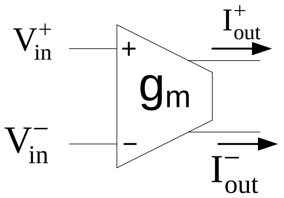


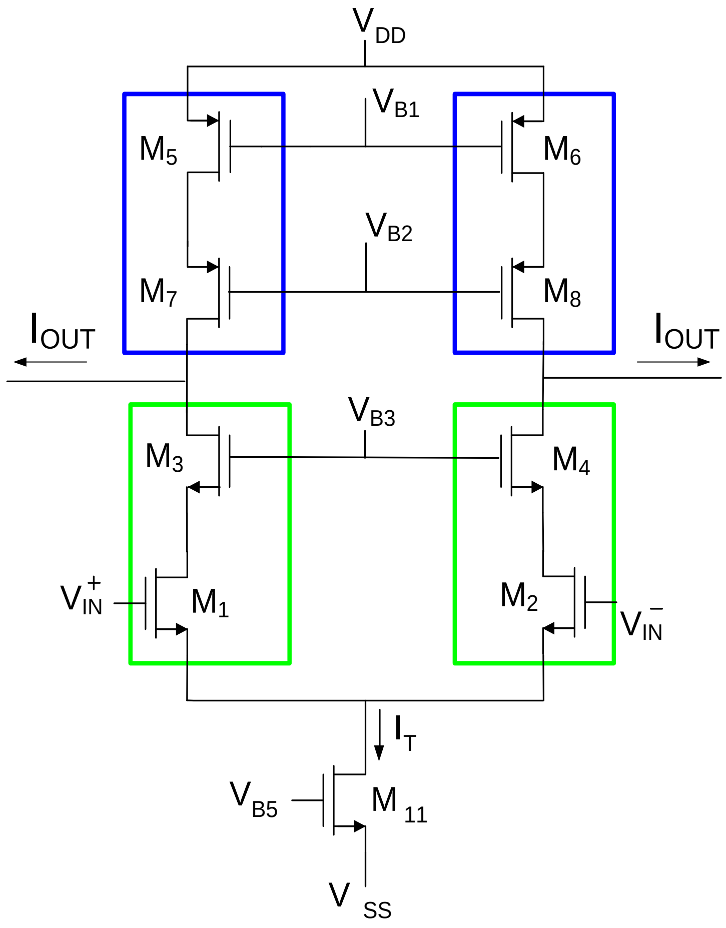
Standard p-channel Cascode Mirror

Wide-Swing p-channel Cascode Mirror

- Current-Mirror p-channel Bias to Eliminate CMFB
- Only single-ended output available

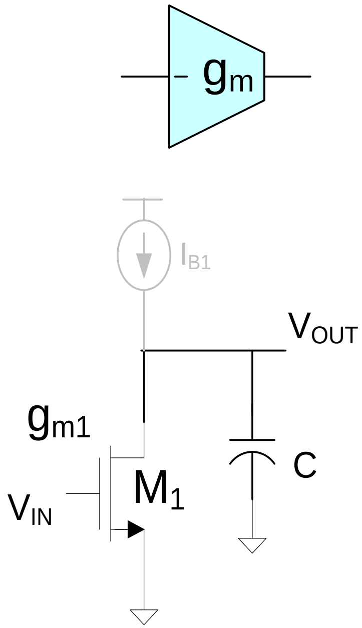
Telescopic Cascode OTA



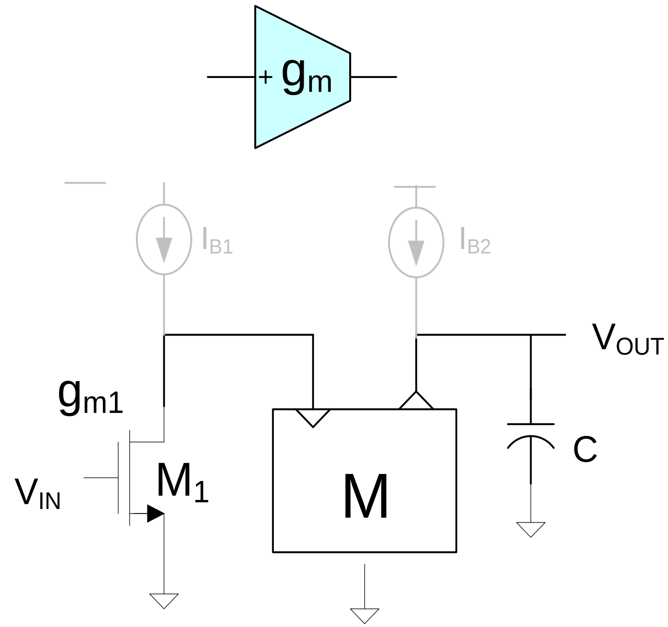


CMFB needed

Single-ended High-Frequency TA

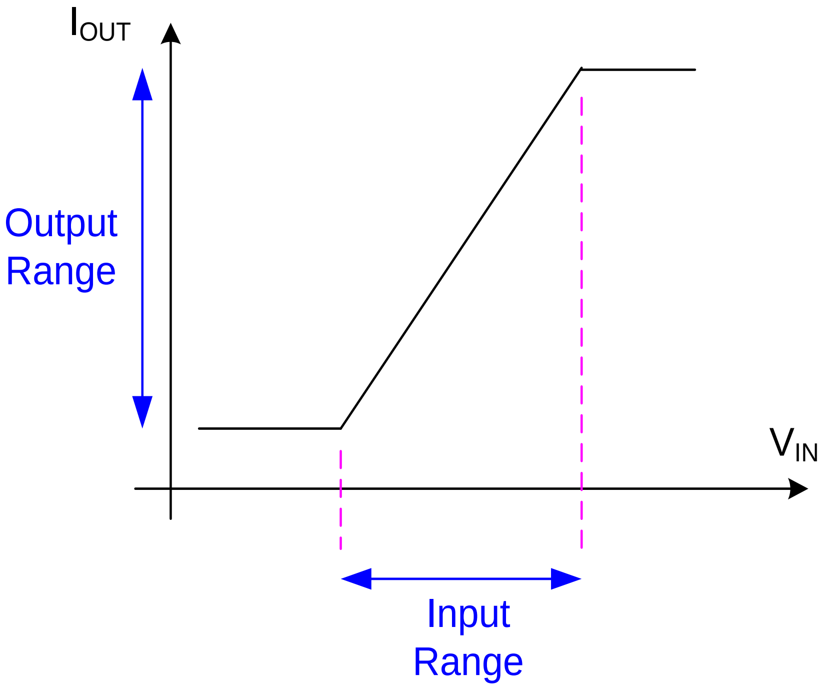


$$g_m = -g_{m1}$$



$$g_m = Mg_{m1}$$

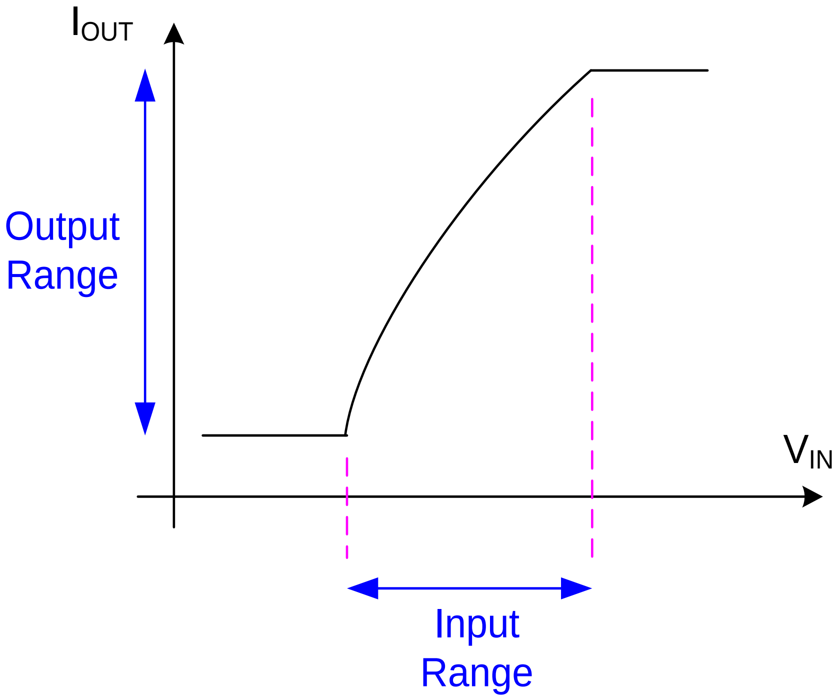
Signal Swing and Linearity



Ideal Scenario:

Completely Linear over Input and Output Range

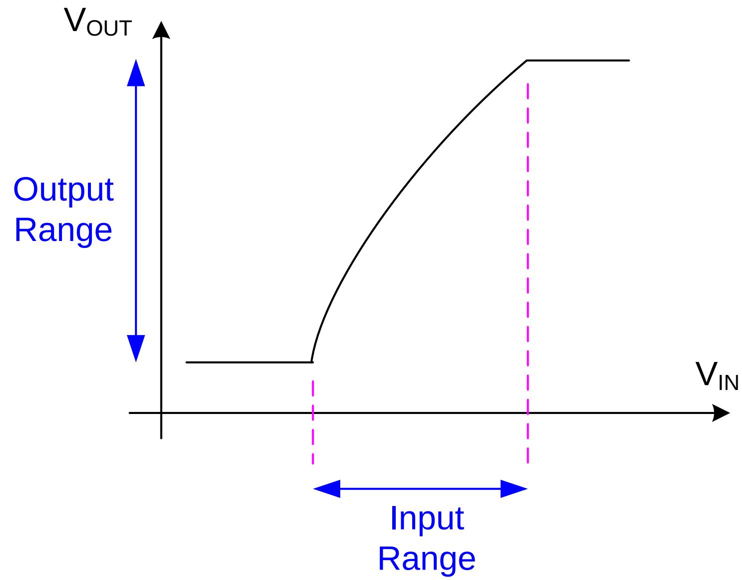
Signal Swing and Linearity

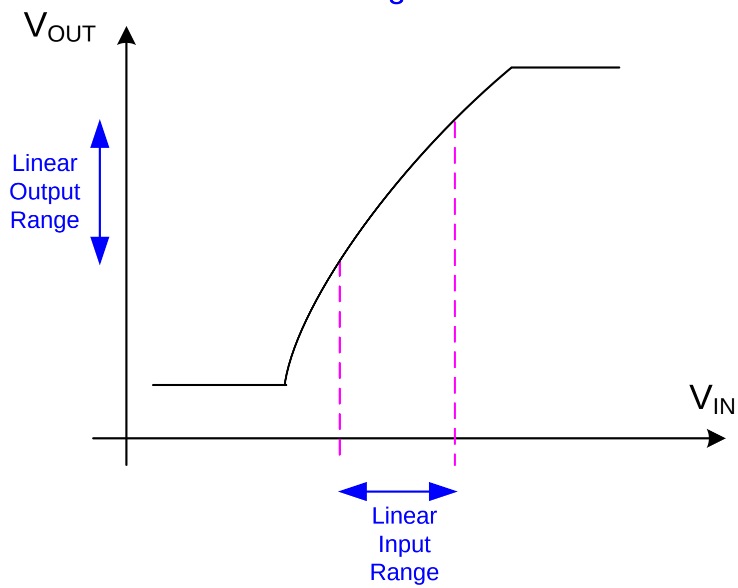


Realistic Scenario:

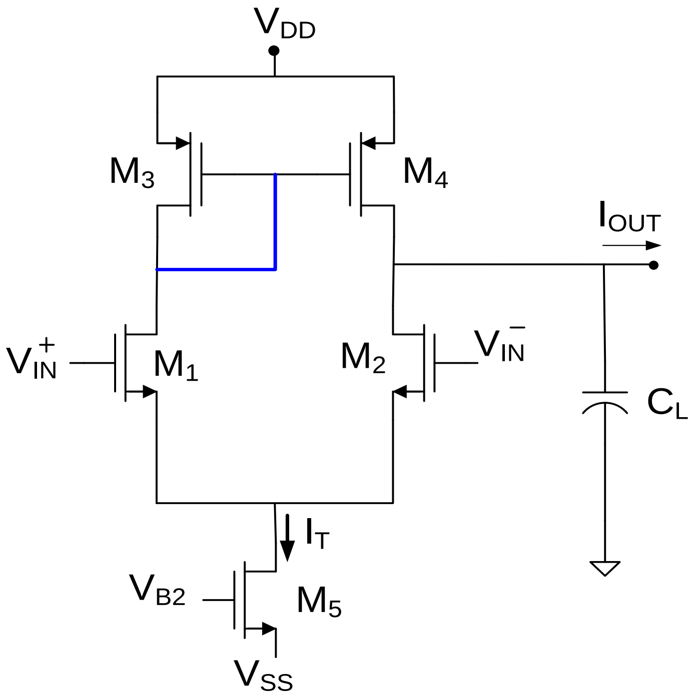
- Modest Nonlinearity throughout Input Range
- But operation will be quite linear over subset of this range

Signal Swing and Linearity



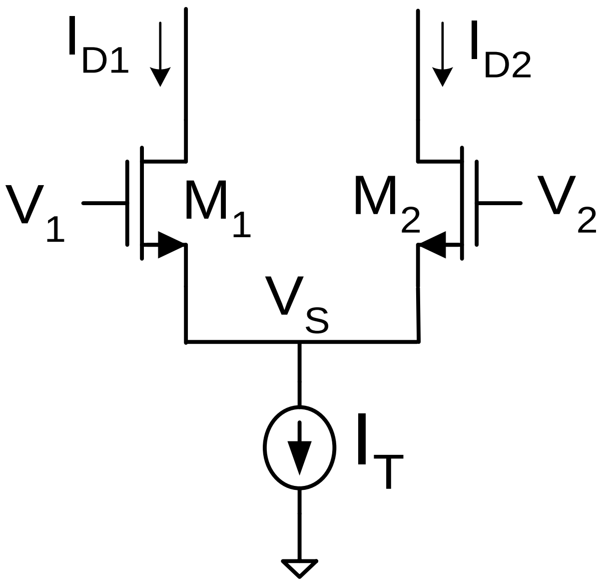


Linearity of Amplifiers

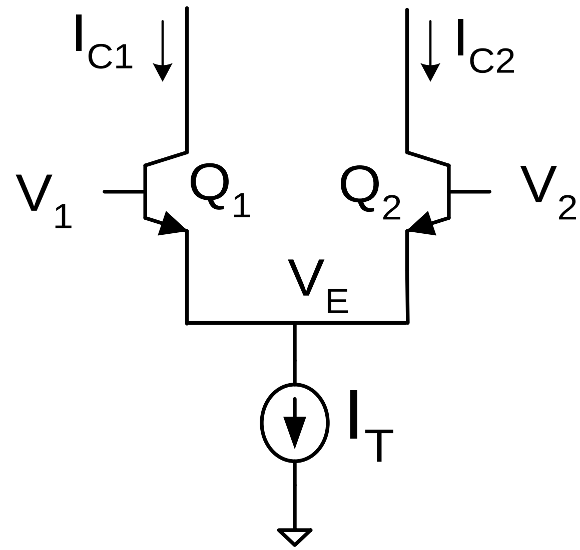


Strongly dependent upon linearity of transconductance of differential pair

Differential Input Pairs

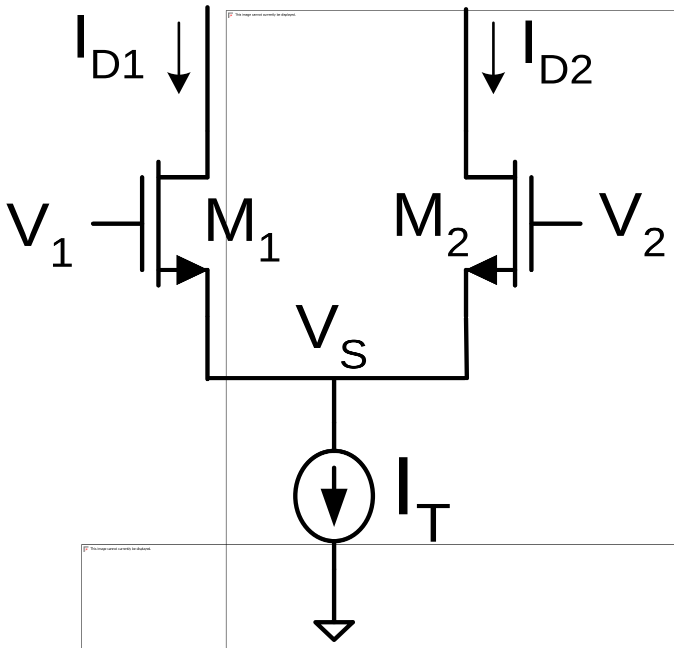


MOS Differential Pair



Bipolar Differential Pair

MOS Differential Pair



$$I_{D1} = \frac{\mu C_{OX} W}{2L}(V_1 - V_S - V_T)^2$$

$$I_{D2} = \frac{\mu C_{OX} W}{2L}(V_2 - V_S - V_T)^2$$

$$I_{D1} + I_{D2} = I_T$$

$$\sqrt{I_{D1}}\sqrt{\frac{2L}{\mu C_{OX} W}} = V_1 - V_S - V_T$$

$$\sqrt{I_{D2}}\sqrt{\frac{2L}{\mu C_{OX} W}} = V_2 - V_S - V_T$$

$$V_d = V_2 - V_1$$

$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_T - I_{D1}} - \sqrt{I_{D1}}\right)$$

$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}\right)$$

MOS Differential Pair

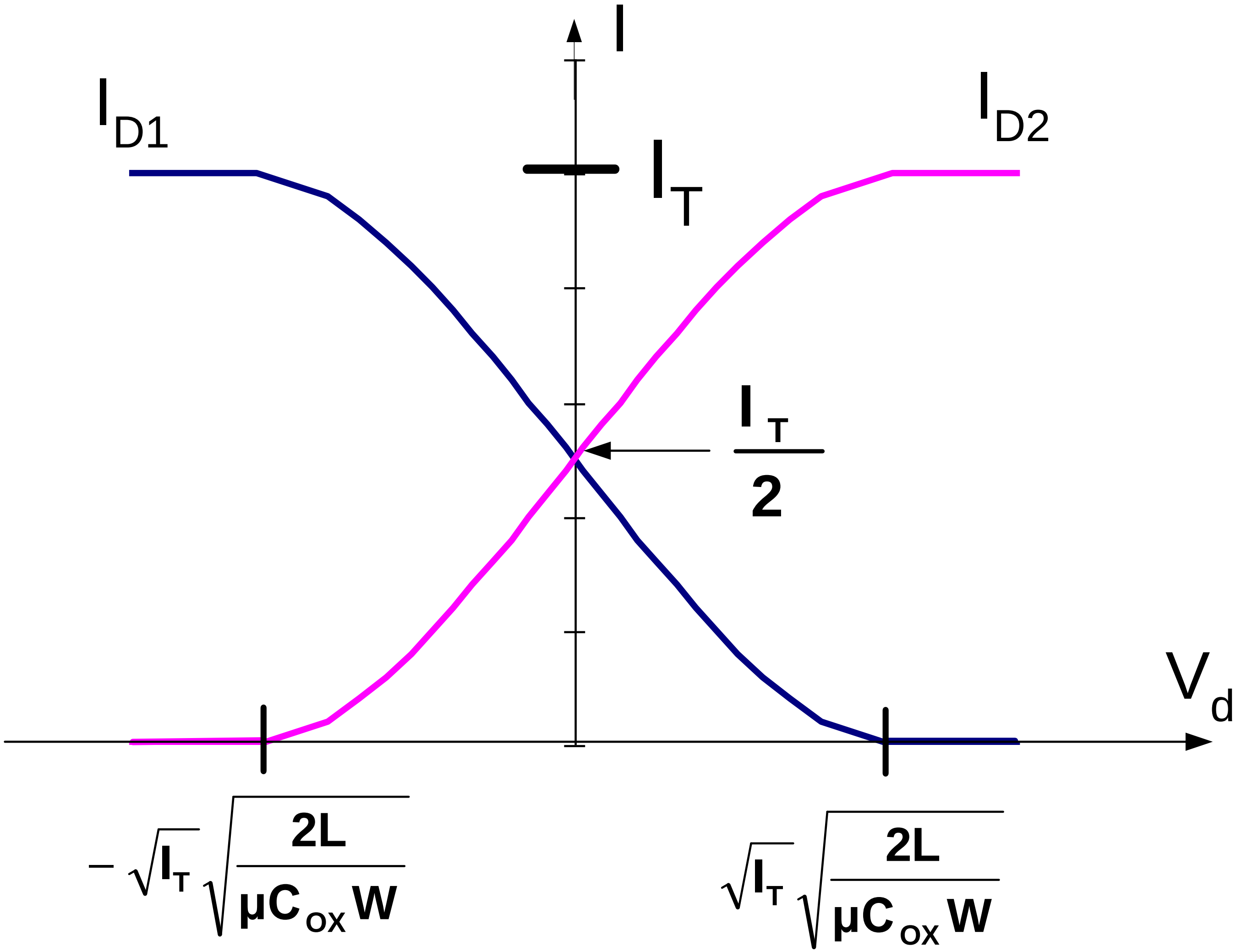
$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_T - I_{D1}} - \sqrt{I_{D1}}\right)$$

$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}\right)$$

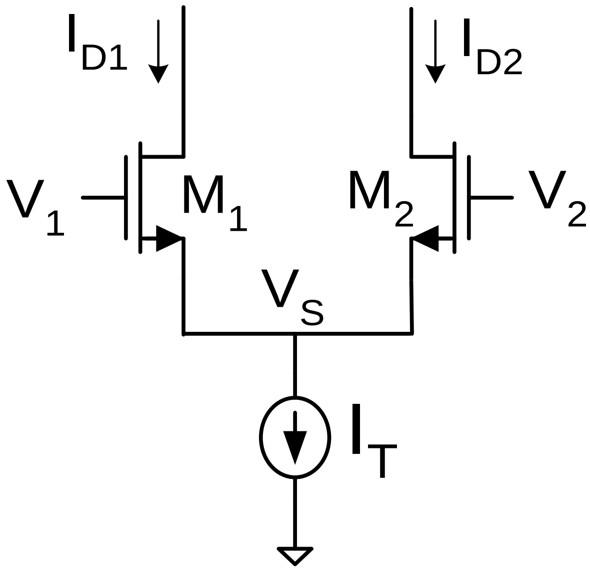
What values of V_d will cause all of the current to be steered to the left or the right ?

$$V_{dx} = \pm\sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_T}\right)$$

$$V_d = \sqrt{\frac{2L}{\mu C_{ox} W}}\left(\sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}\right)$$



Q-point Calculations



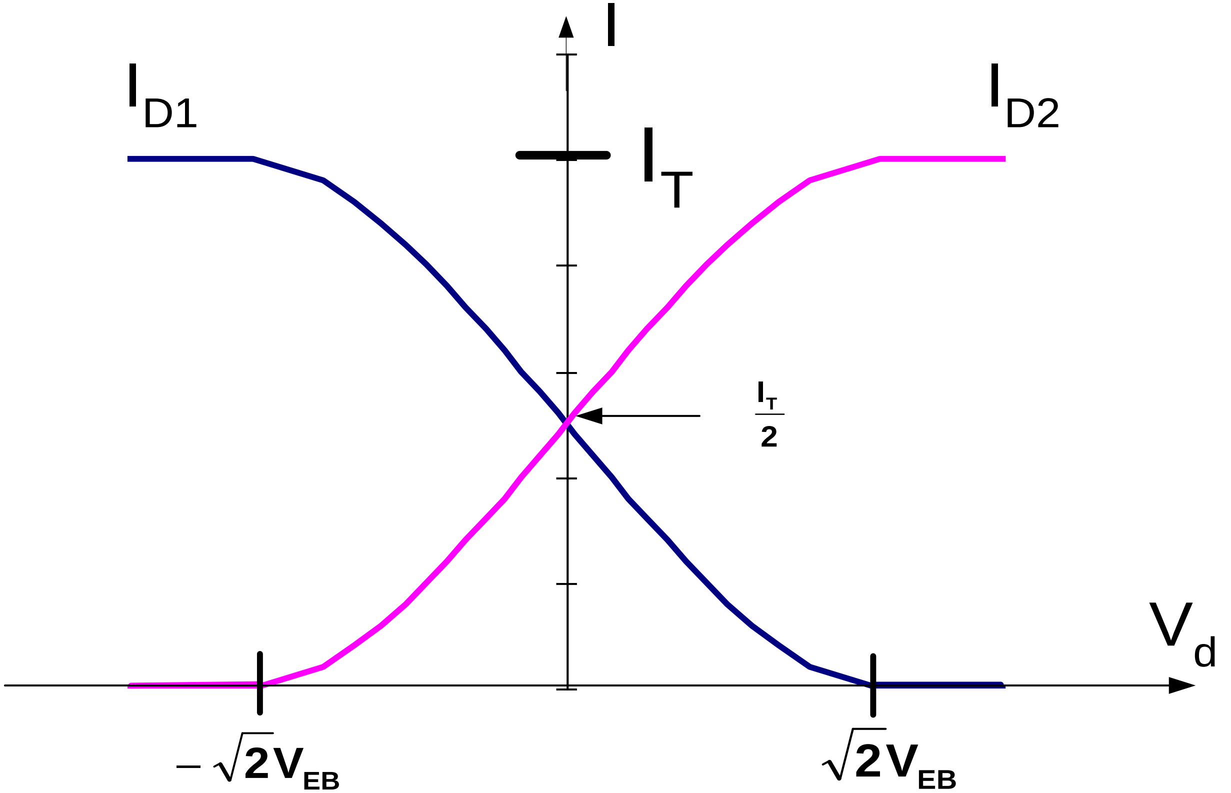
$$\frac{I_T}{2} = \frac{\mu C_{OX} W}{2L}\left(V_{EB}\right)^2$$

$$V_{EB} = \sqrt{I_T}\sqrt{\frac{L}{\mu C_{OX} W}}$$

Observe !!

$$V_{dx} = \pm\sqrt{2}V_{EB}$$

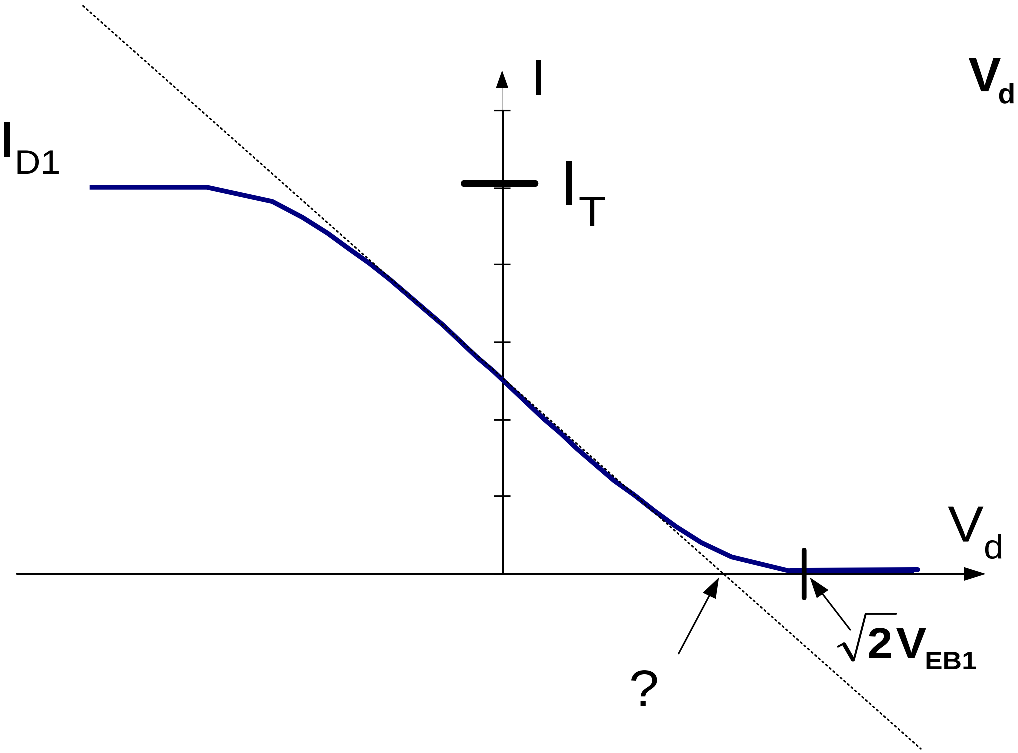
$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}\right)$$



V_{EB} affects linearity

How linear is the amplifier ?

How linear is the amplifier ?



$$V_d = \sqrt{\frac{2L}{\mu C_{ox} W}}\left(\sqrt{I_T - I_{D1}} - \sqrt{I_{D1}}\right)$$

Consider the fit line:

$$I = mV_d + h$$

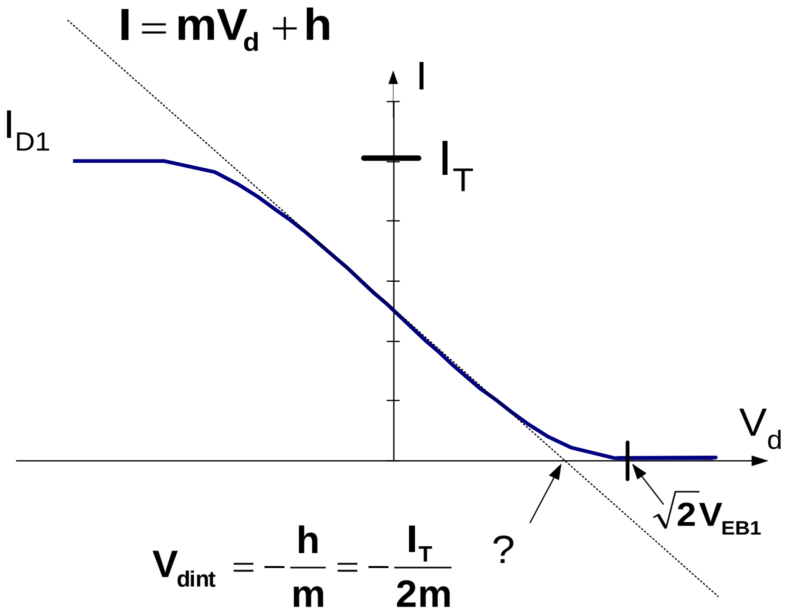
When V_d=0, I=I_T/2, thus

$$h = \frac{I_T}{2}$$

$$V_{dint} = -\frac{h}{m} = -\frac{I_T}{2m}$$

$$m = \left.\frac{\partial I_{D1}}{\partial V_d}\right|_{Q-pt}$$

How linear is the amplifier ?



$$V_{dint} = -\frac{h}{m} = -\frac{I_T}{2m}$$

$$m = \left.\frac{\partial I_{D1}}{\partial V_d}\right|_{Q-pt}$$

$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_T - I_{D1}} - \sqrt{I_{D1}}\right)$$

$$\frac{\partial V_d}{\partial I_{D1}} = \sqrt{\frac{2L}{\mu C_{OX} W}}\left.\left(\frac{1}{2}(I_T - I_{D1})^{-1/2}(-1) - \frac{1}{2}(I_{D1})^{-1/2}\right)\right|_{Q-point}$$

$$\frac{\partial V_d}{\partial I_{D1}} = -2\sqrt{\frac{L}{\mu C_{OX} W}}\sqrt{\frac{1}{I_T}}$$

$$\sqrt{\frac{L}{\mu C_{OX} W}} = \frac{V_{EB1}}{\sqrt{I_T}}$$

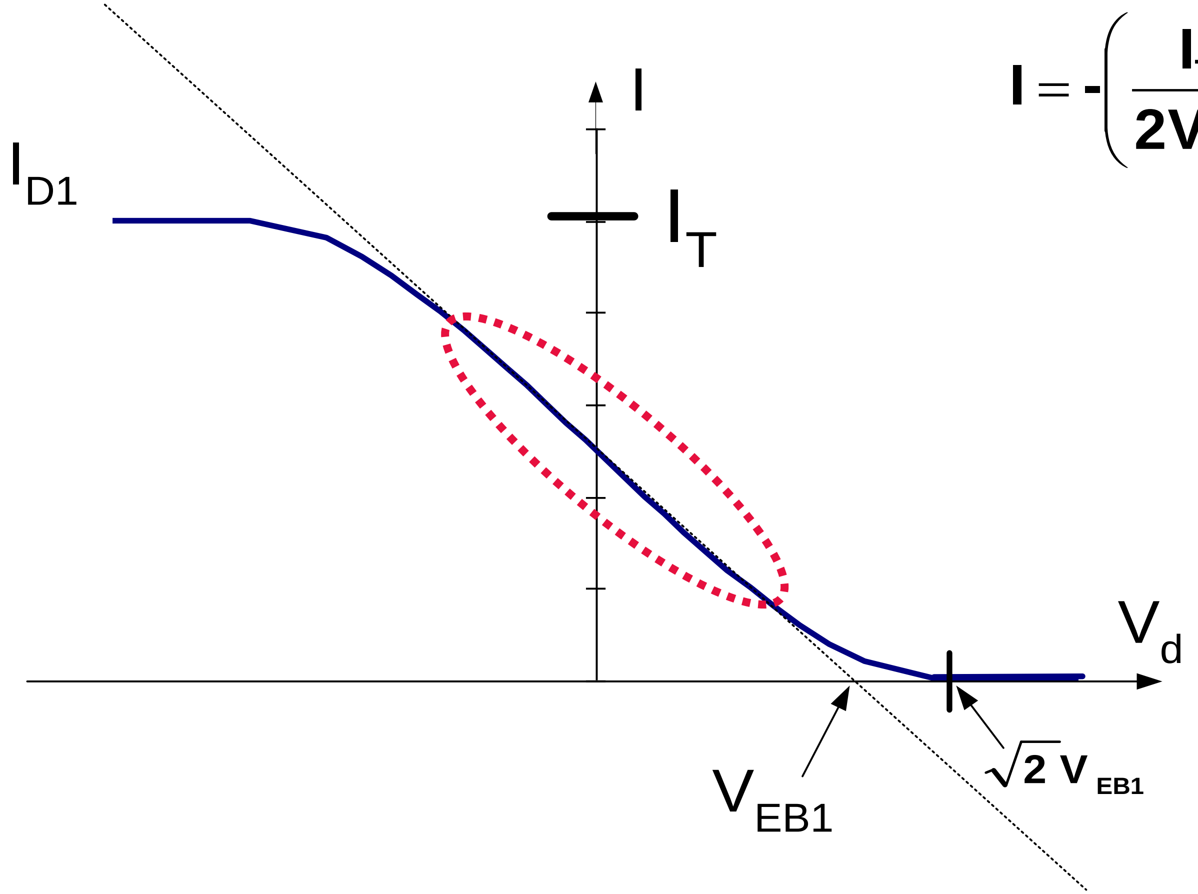
$$\frac{\partial V_d}{\partial I_{D1}} = -2\frac{V_{EB1}}{I_T}$$

$$m = \left.\frac{\partial I_{D1}}{\partial V_d}\right|_{Q-pt} = -\frac{I_T}{2V_{EB1}}$$

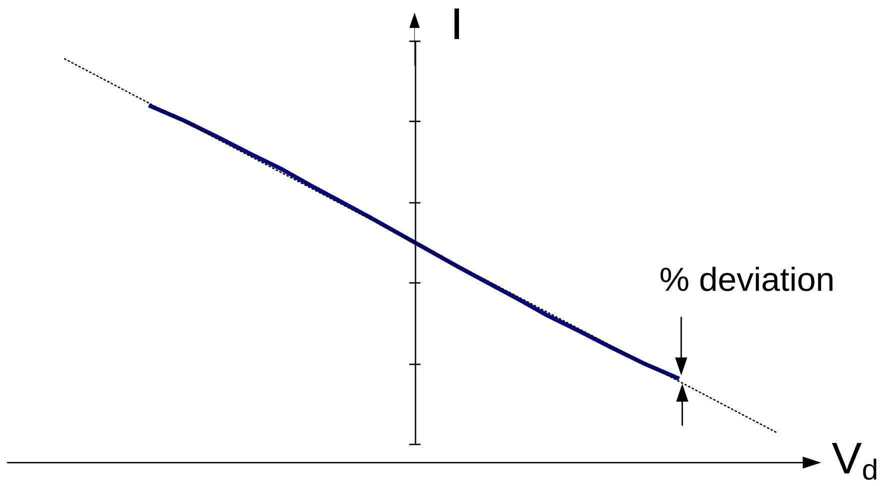
How linear is the amplifier ?

$$V_{dint} = -\frac{h}{m} = -\frac{I_T}{2m} = V_{EB1}$$

$$I = -\left(\frac{I_T}{2V_{EB1}}\right)V_d + \frac{I_T}{2}$$



How linear is the amplifier ?

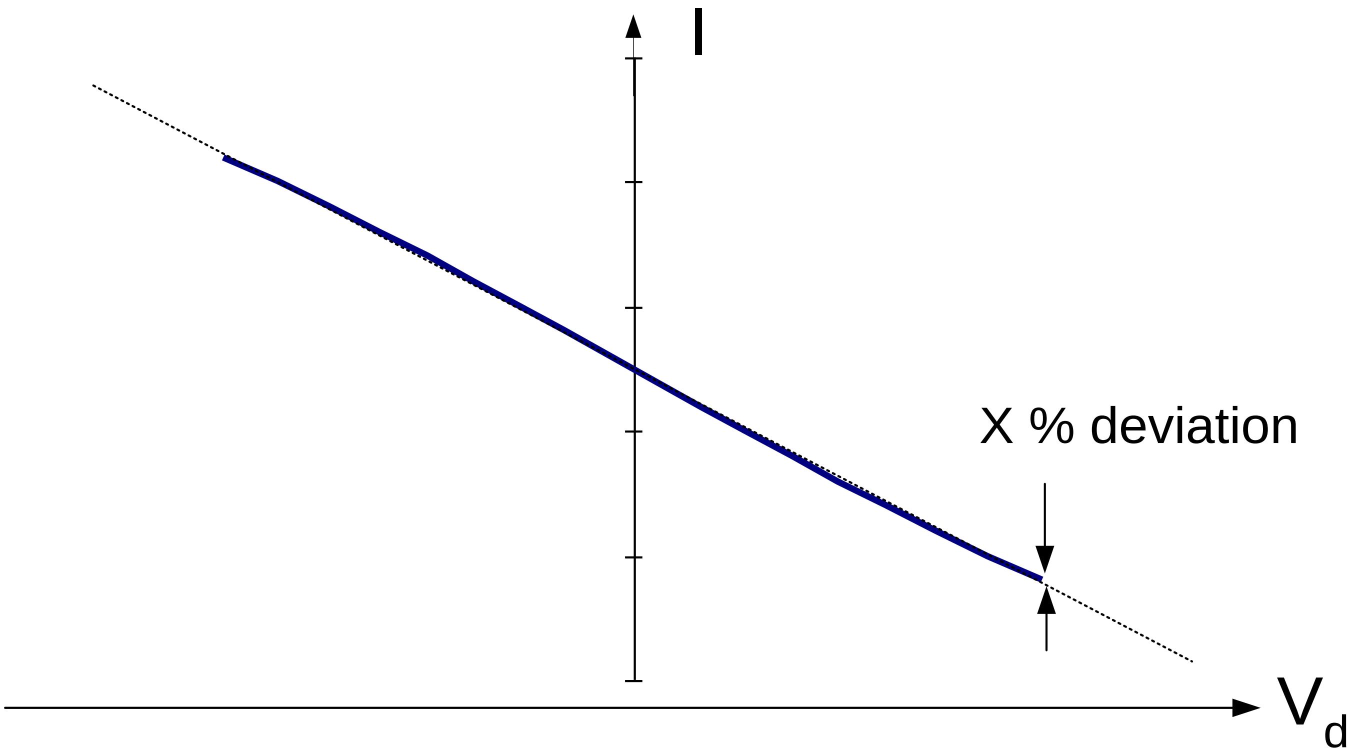


It can be shown that the deviation from the line in % is given by

$$\theta = 100\% \left(1 - \sqrt{1 - \frac{\left(V_d / V_{EB} \right)^2}{4}} \right)$$

Vd/VEB	θ	Vd/VEB	θ	Vd/VEB	θ
0.02	0.005	0.22	0.607	0.42	2.23
0.04	0.020	0.24	0.723	0.44	2.45
0.06	0.045	0.26	0.849	0.46	2.68
0.08	0.080	0.28	0.985	0.48	2.92
0.1	0.125	0.3	1.13	0.5	3.18
0.12	0.180	0.32	1.29	0.52	3.44
0.14	0.245	0.34	1.46	0.54	3.71
0.16	0.321	0.36	1.63	0.56	4.00
0.18	0.406	0.38	1.82	0.58	4.30
0.2	0.501	0.4	2.02	0.6	4.61

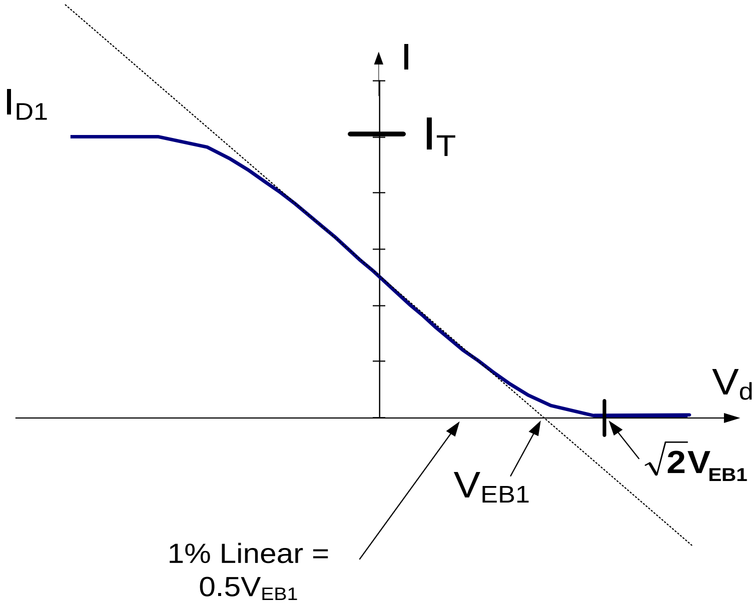
How linear is the amplifier ?



A 1% deviation from the straight line occurs at

$V_d \cong 0.3V_{EB}$ and a 0.1% variation occurs at $V_d \cong \frac{V_{EB}}{10}$

What swings on drain currents are typical when using the differential pair in an amplifier?



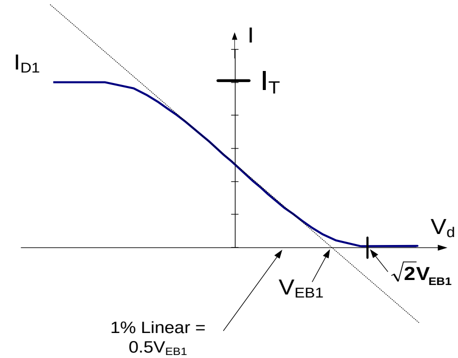
Assume the differential amplifier is the input stage to an op amp with gain Av and signal swing V_{OUTpp}

The differential swing at the input is thus

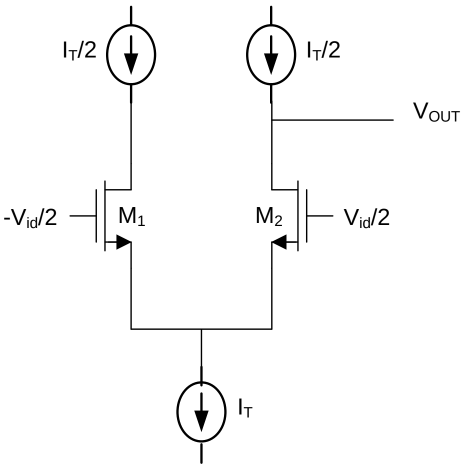
$$V_{INpp} = \frac{V_{OUTpp}}{A_V}$$

What swings on drain currents are typical when using the differential pair in an amplifier?

$$V_{INpp} = \frac{V_{OUTpp}}{A_V}$$



If the amplifier is the simple differential amplifier with current source loads



$$A_V = -\frac{g_{m1}}{2g_0} = \frac{2I_{DQ}/V_{EB1}}{2\lambda I_{DQ}}$$

$$A_V = -\frac{1}{\lambda V_{EB1}}$$

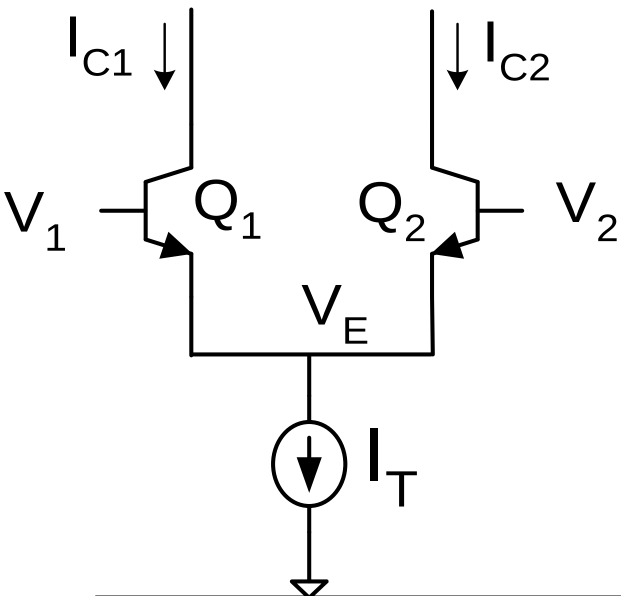
$$V_{INpp} = (\lambda V_{OUTpp})V_{EB1}$$

If $\lambda = .01V^{-1}$ and $V_{OUTpp} = 5V$,

$$V_{INpp} = 0.05V_{EB1}$$

This results in a very small nonlinearity and a very small change in current
When used in two-stage structure, even much smaller!

Bipolar Differential Pair



$$\left.\begin{aligned} I_{C1} &= J_S A_{E1} e^{\frac{V_1 - V_E}{V_t}} \\ I_{C2} &= J_S A_{E2} e^{\frac{V_2 - V_E}{V_t}} \\ I_{C1} + I_{C2} &= I_T \end{aligned}\right\}$$

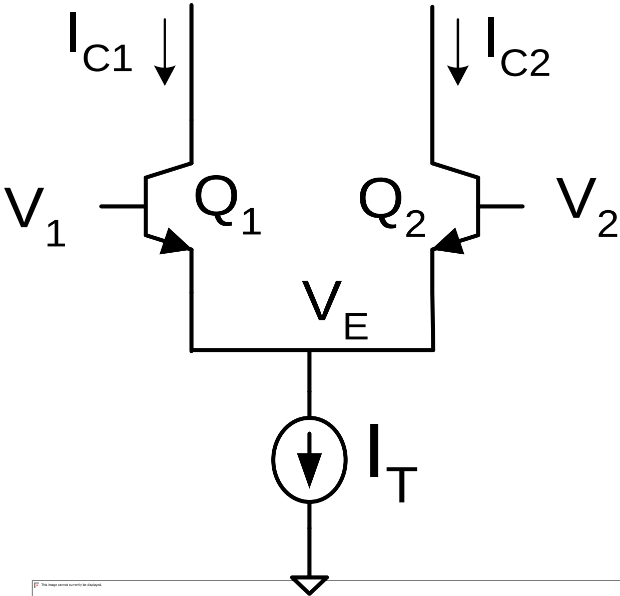
$$V_1 = V_E + V_t \ln\left(\frac{I_{C1}}{J_S A_{E1}}\right)$$

$$V_2 = V_E + V_t \ln\left(\frac{I_{C2}}{J_S A_{E2}}\right)$$

$$V_d = V_2 - V_1$$

$$V_d = V_t\left(\ln\left(\frac{I_{C2}}{J_S A_{E2}}\right) - \ln\left(\frac{I_{C1}}{J_S A_{E1}}\right)\right) \overset{A_{E1}=A_{E2}}{=} V_t \ln\left(\frac{I_{C2}}{I_{C1}}\right)$$

Bipolar Differential Pair



$$V_d = V_t\left(\ln\left(\frac{I_{C2}}{J_S A_{E2}}\right) - \ln\left(\frac{I_{C1}}{J_S A_{E1}}\right)\right) \overset{A_{E1}=A_{E2}}{=} V_t \ln\left(\frac{I_{C2}}{I_{C1}}\right)$$

$$V_d = V_t \ln\left(\frac{I_T - I_{C1}}{I_{C1}}\right)$$

$$V_d = V_t \ln\left(\frac{I_{C2}}{I_T - I_{C2}}\right)$$

$$V_d = V_2 - V_1$$

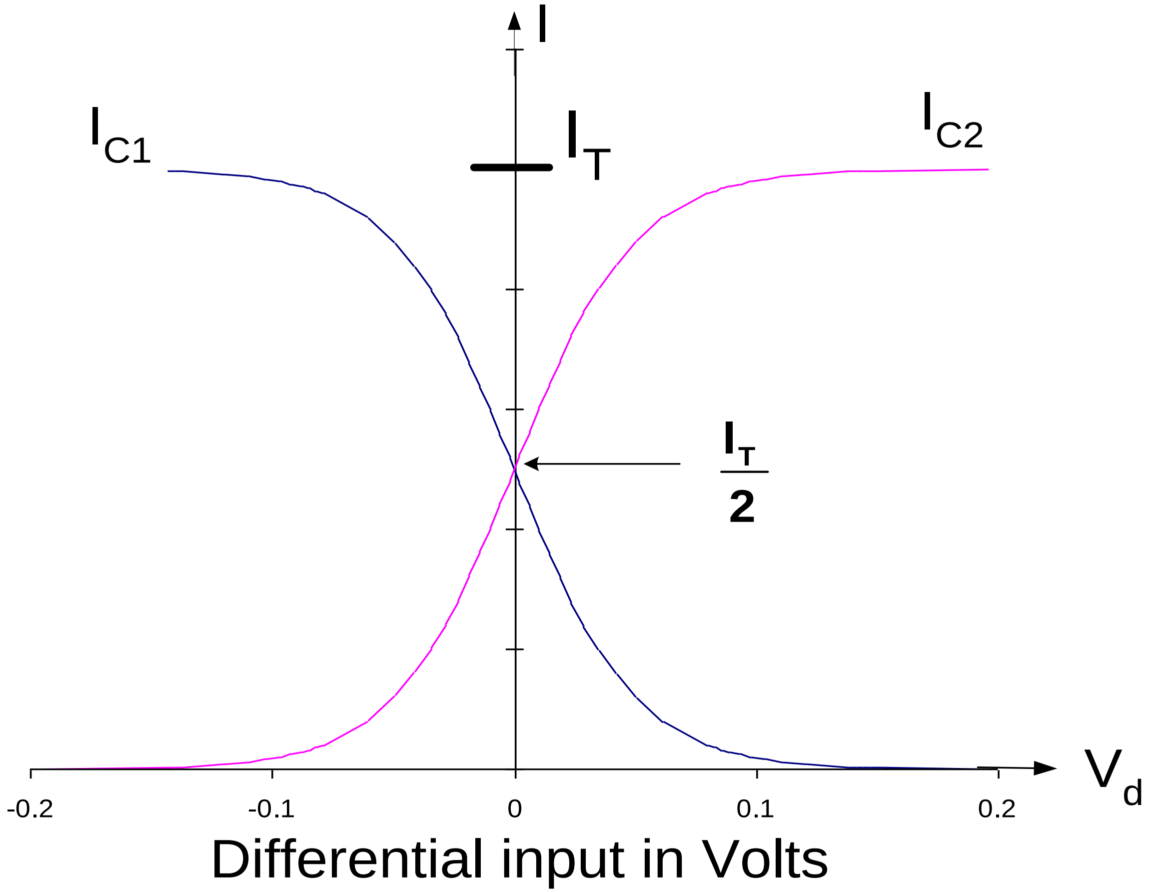
At $I_{C1}=I_{C2}=I_T/2$, $V_d=0$

As I_{C1} approaches 0, V_d approaches infinity

As I_{C1} approaches I_T, V_d approaches minus infinity

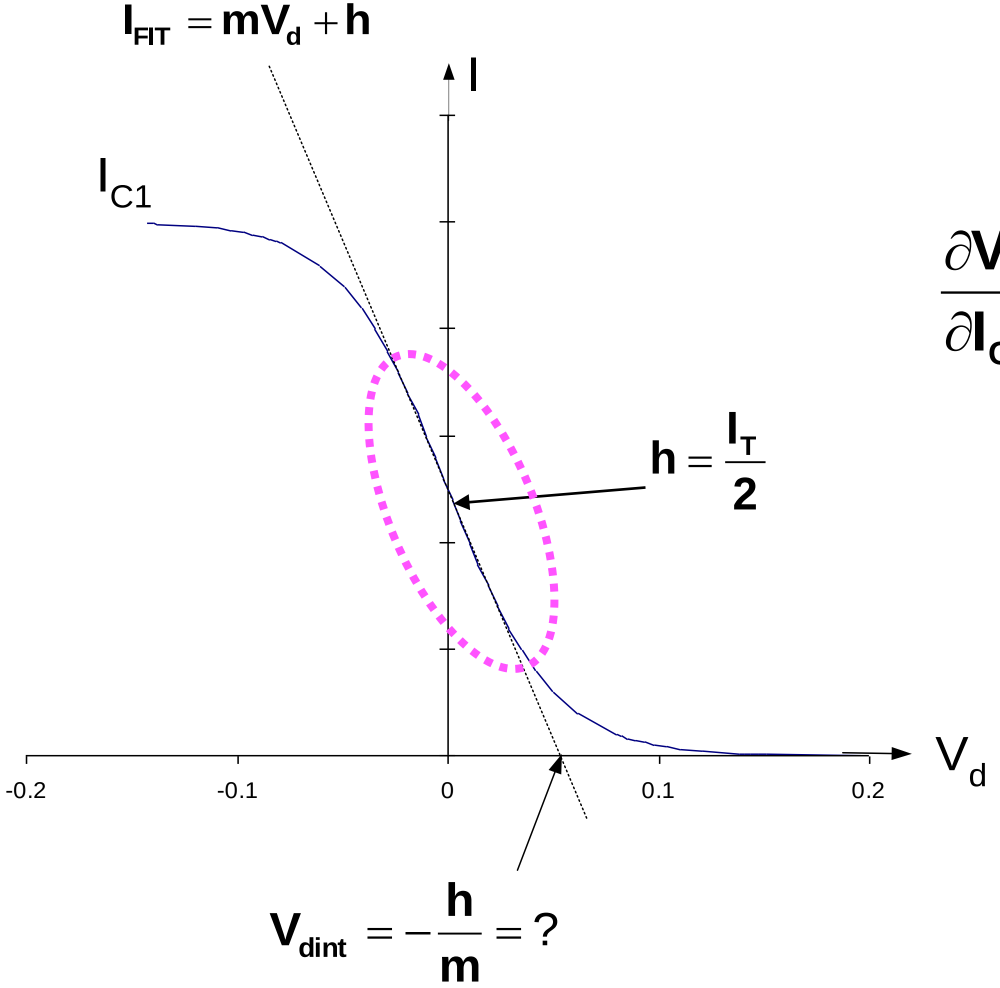
Transition much steeper than for MOS case

Transfer Characteristics of Bipolar Differential Pair



Transition much steeper than for MOS case
Asymptotic Convergence to 0 and I_T

Signal Swing and Linearity of Bipolar Differential Pair



$$m = \left.\frac{\partial I_{C1}}{\partial V_d}\right|_{Q-point}$$

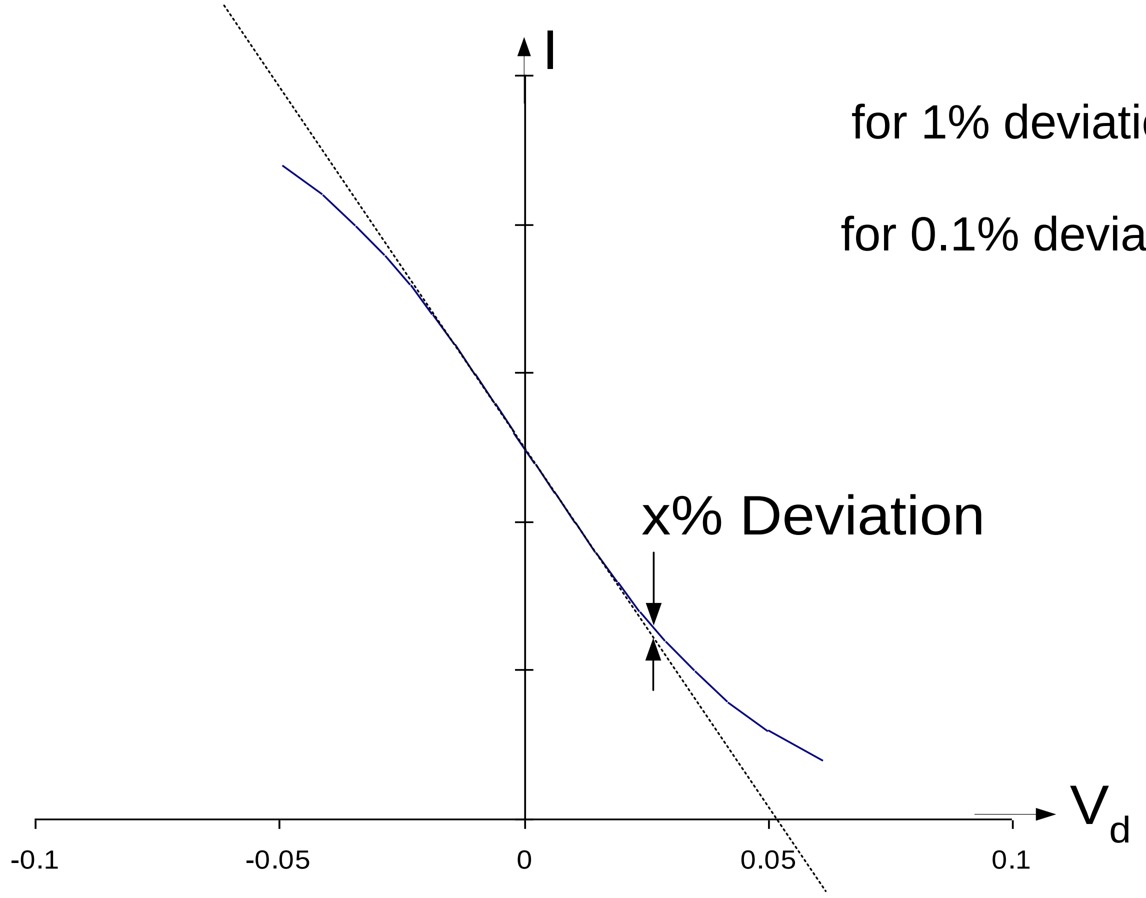
$$\left.\frac{\partial V_d}{\partial I_{C1}}\right|_{Q-point} = -V_t \left.\frac{I_T}{I_{C1}(I_T - I_{C1})}\right|_{I_{C1}=\frac{I_T}{2}}$$

$$\left.\frac{\partial V_d}{\partial I_{C1}}\right|_{Q-point} = -\frac{4V_t}{I_T}$$

$$I_{FIT} = -\frac{I_T}{4V_t}V_d + \frac{I_T}{2}$$

$$V_{dint} = -\frac{h}{m} = 2V_t$$

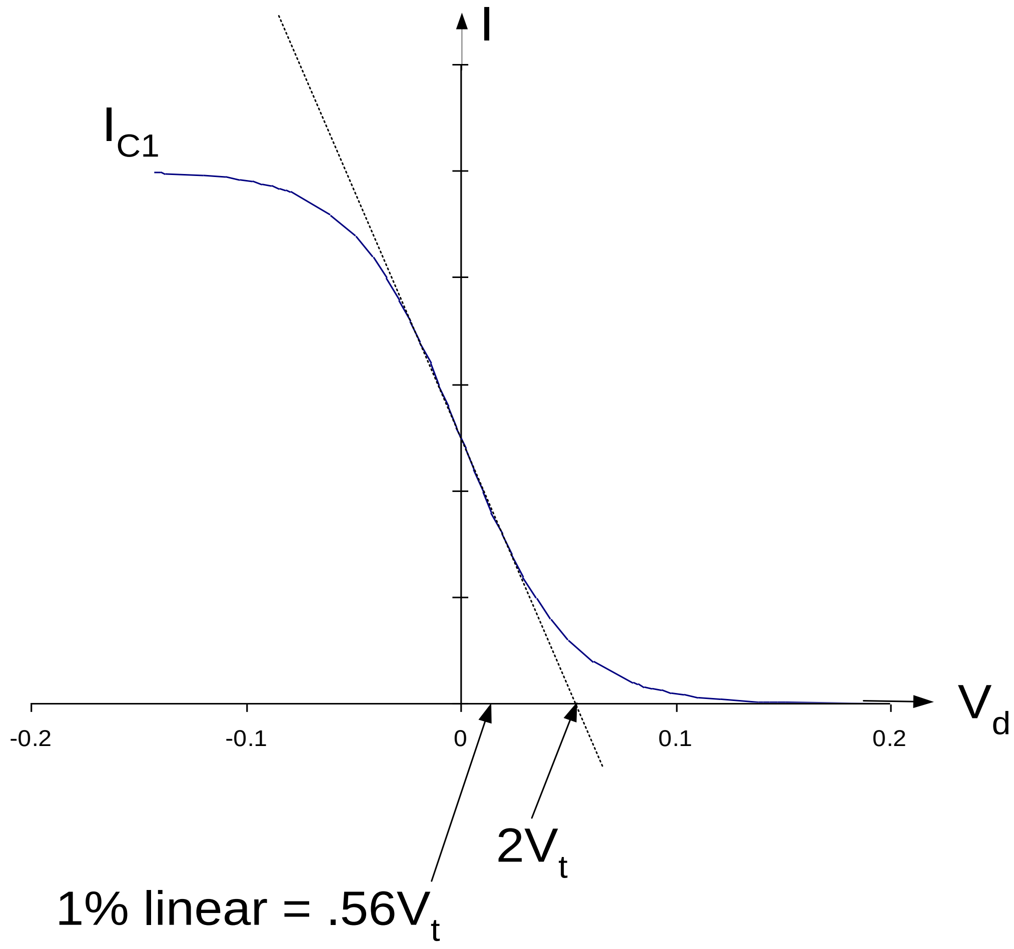
Signal Swing and Linearity of Bipolar Differential Pair



for 1% deviation, V_d=.56V_t

for 0.1% deviation, V_d=.27V_t

Signal Swing and Linearity of Bipolar Differential Pair



Signal Swing and Linearity Summary

- Signal swing of MOSFET can be rather large if V_{EB} is large but this limits gain
- Signal swing of MOSFET degrades significantly if V_{EB} is changed for fixed W/L
- Bipolar swing is very small but independent of g_m
- Multiple-decade adjustment of bipolar g_m is practical
- Even though bipolar input swing is small, since gain is often very large, this small swing does usually not limit performance in feedback applications

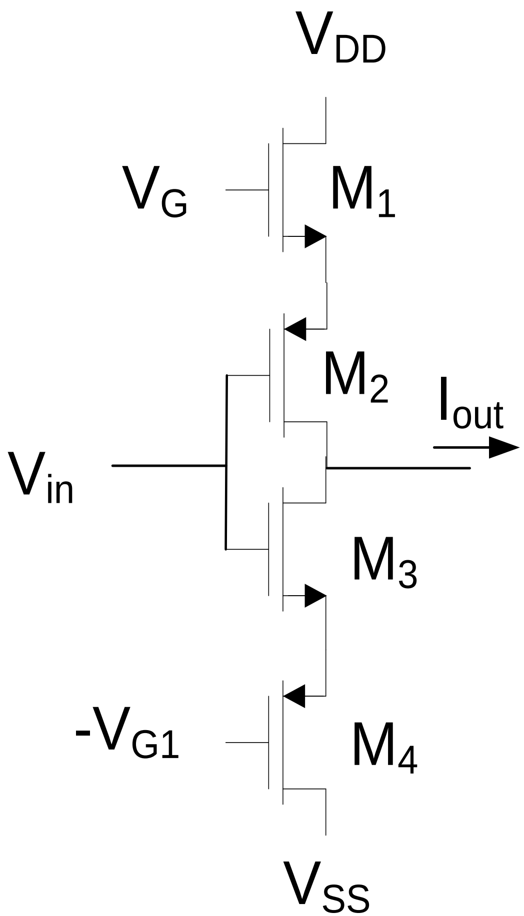
End of Lecture 34

EE 508

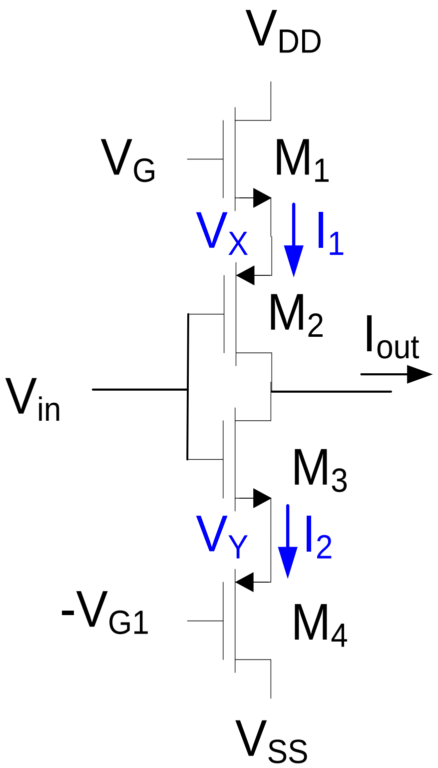
Lecture 35

Transconductor Design and Applications

Simple single-ended OTA



Simple single-ended OTA



$$\left.\begin{aligned}
I_0 &= I_1 - I_2 \\
I_1 &= \beta_1 \left(V_G - V_X - V_{Tn}\right)^2 \\
I_1 &= \beta_2 \left(V_X - V_{in} + V_{Tp}\right)^2 \\
I_2 &= \beta_3 \left(V_{in} - V_Y - V_{Tn}\right)^2 \\
I_2 &= \beta_4 \left(V_Y + V_{G1} + V_{Tp}\right)^2
\end{aligned}\right\}$$

Taking the square root of the two I_1 equations

$$\sqrt{\frac{1}{\beta_1}}\sqrt{I_1} = \left(V_G - V_X - V_{Tn}\right)$$

$$\sqrt{\frac{1}{\beta_2}}\sqrt{I_1} = \left(V_X - V_{in} + V_{Tp}\right)$$

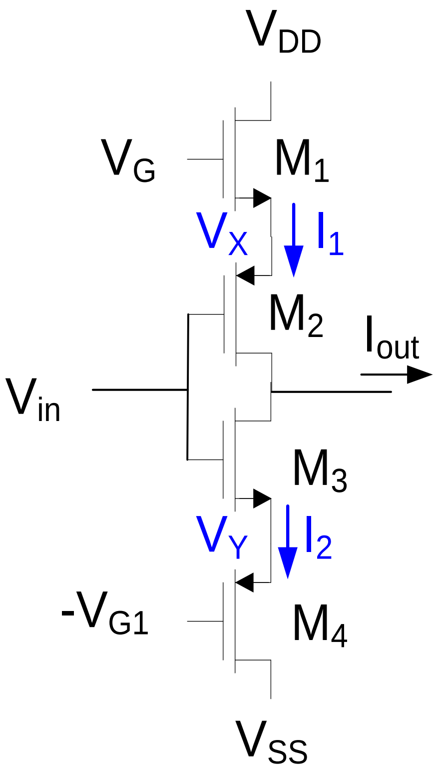
Adding these two equations, we obtain

$$\left(\sqrt{\frac{1}{\beta_2}} + \sqrt{\frac{1}{\beta_1}}\right)\sqrt{I_1} = \left(V_G - V_{in} + V_{Tp} - V_{Tn}\right)$$

Similarly, for the last two equations, obtain

$$\left(\sqrt{\frac{1}{\beta_3}} + \sqrt{\frac{1}{\beta_4}}\right)\sqrt{I_2} = \left(V_{G1} + V_{in} + V_{Tp} - V_{Tn}\right)$$

Simple single-ended OTA



$$I_0 = I_1 - I_2$$

$$\left(\sqrt{\frac{1}{\beta_2}} + \sqrt{\frac{1}{\beta_1}}\right)\sqrt{I_1} = \left(V_G - V_{in} + V_{Tp} - V_{Tn}\right)$$

$$\left(\sqrt{\frac{1}{\beta_3}} + \sqrt{\frac{1}{\beta_4}}\right)\sqrt{I_2} = \left(V_{G1} + V_{in} + V_{Tp} - V_{Tn}\right)$$

Squaring the last two equations we obtain

$$I_1 = \beta_5\left(V_G - V_{in} + V_{Tp} - V_{Tn}\right)^2$$

$$I_2 = \beta_6\left(V_{G1} + V_{in} + V_{Tp} - V_{Tn}\right)^2$$

Define

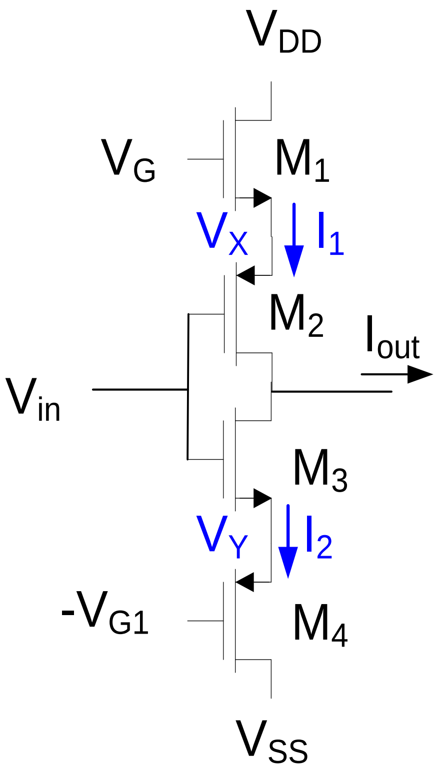
$$\left(\sqrt{\frac{1}{\beta_2}} + \sqrt{\frac{1}{\beta_1}}\right) = \sqrt{\frac{1}{\beta_5}}$$

$$\left(\sqrt{\frac{1}{\beta_3}} + \sqrt{\frac{1}{\beta_4}}\right) = \sqrt{\frac{1}{\beta_6}}$$

Equating the difference to I_0, we obtain

$$\begin{aligned} I_0 &= \left(\beta_5 - \beta_6\right)V_{in}^2 \\ &+ V_{in}\left(2\beta_5\left[V_{Tn} - V_{Tp} - V_G\right] + 2\beta_6\left[V_{Tn} - V_{Tp} + V_{G1}\right]\right) \\ &+ \beta_5\left[V_{Tp} - V_{Tn} + V_G\right]^2 - \beta_6\left[V_{Tp} - V_{Tn} + V_{G1}\right]^2 \end{aligned}$$

Simple single-ended OTA



$$I_0 = (\beta_5 - \beta_6)V_{in}^2 + V_{in}\left(2\beta_5\left[V_{Tn} - V_{Tp} - V_G\right] + 2\beta_6\left[V_{Tn} - V_{Tp} + V_{G1}\right]\right) + \beta_5\left[V_{Tp} - V_{Tn} + V_G\right]^2 - \beta_6\left[V_{Tp} - V_{Tn} + V_{G1}\right]^2$$

If size devices so that $\beta_5=\beta_6$ and $V_G=V_{G1}$, this simplifies to

$$I_0 = V_{in}\left(4\beta_5\left[V_{Tn} - V_{Tp} - V_G\right]\right)$$

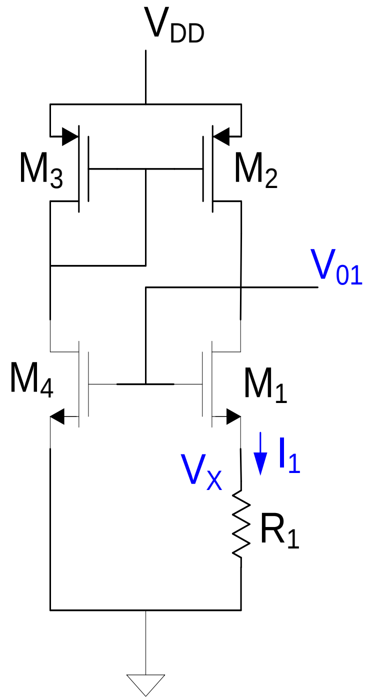
Note this behaves as a linear transconductor !

$$g_m = 4\beta_5\left[V_{Tn} - V_{Tp} - V_G\right]$$

- Since both M_2 and M_3 are driven, this is a power-efficient method for generating a given g_m
- Behavior will degrade with bulk-dependent threshold voltages of n-channel devices
- Would like to generate V_G and V_{G1} independent of V_{DD}

V_{DD} Independent Bias Generators

Two widely-used V_{DD} independent bias generators (start-up ckts not shown)



$$I_{D2} = I_{D1} = MI_{D3}$$

$$I_{D4} = I_{D3} = \frac{\mu C_{OX} W_4}{2L_4}\left(V_{01} - V_{Tn}\right)^2$$

$$I_{D1} = \frac{\mu C_{OX} W_1}{2L_1}\left(V_{01} - V_X - V_{Tn}\right)^2$$

$$V_X = I_{D1} R_1$$

4 equations and 4 unknowns $\{I_{D1}, I_{D3}, V_{01}, V_X\}$

Define:

M is the M_3:M_2 mirror gain

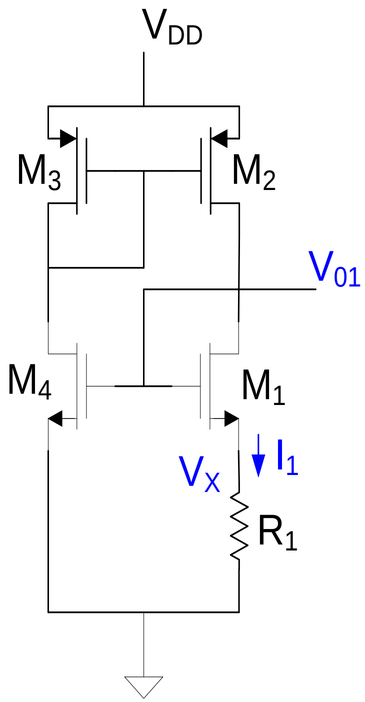
$$\beta_k = \frac{\mu C_{OX} W_k}{2L_k}$$

$$V_X = \frac{1}{R_1}\left(\sqrt{\frac{1}{M\beta_4}} - \sqrt{\frac{1}{\beta_1}}\right)^2$$

$$V_{01} = V_{Tn} + \left(\frac{1}{MR}\right)\left(\frac{1}{\beta_4} - \frac{1}{\sqrt{M\beta_1\beta_4}}\right)$$

V_{DD} Independent Bias Generators

Two widely-used V_{DD} independent bias generators (start-up ckts not shown)



$$V_X = \frac{1}{R_1}\left(\sqrt{\frac{1}{M\beta_4}} - \sqrt{\frac{1}{\beta_1}}\right)^2$$

$$V_{01} = V_{Tn} + \left(\frac{1}{MR}\right)\left(\frac{1}{\beta_4} - \frac{1}{\sqrt{M\beta_1\beta_4}}\right)$$

Define:

M is the M_3:M_2 mirror gain

$$\beta_k = \frac{\mu C_{OX} W_k}{2L_k}$$

Observe V_X is independent of both V_T and V_{DD}

Offers some attractive properties when used as part of a temperature sensor as well

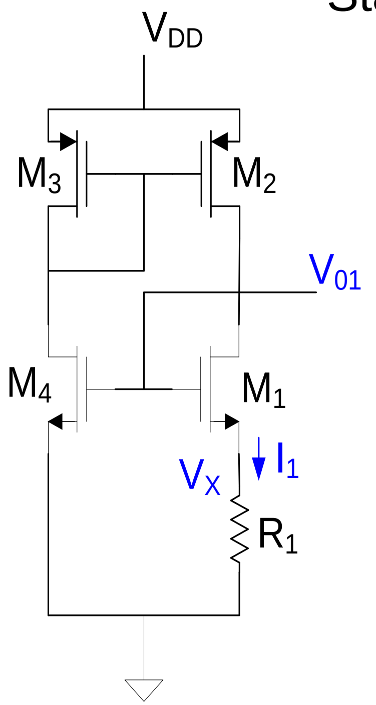
Requires Start-up Circuit

May need compensation for stability

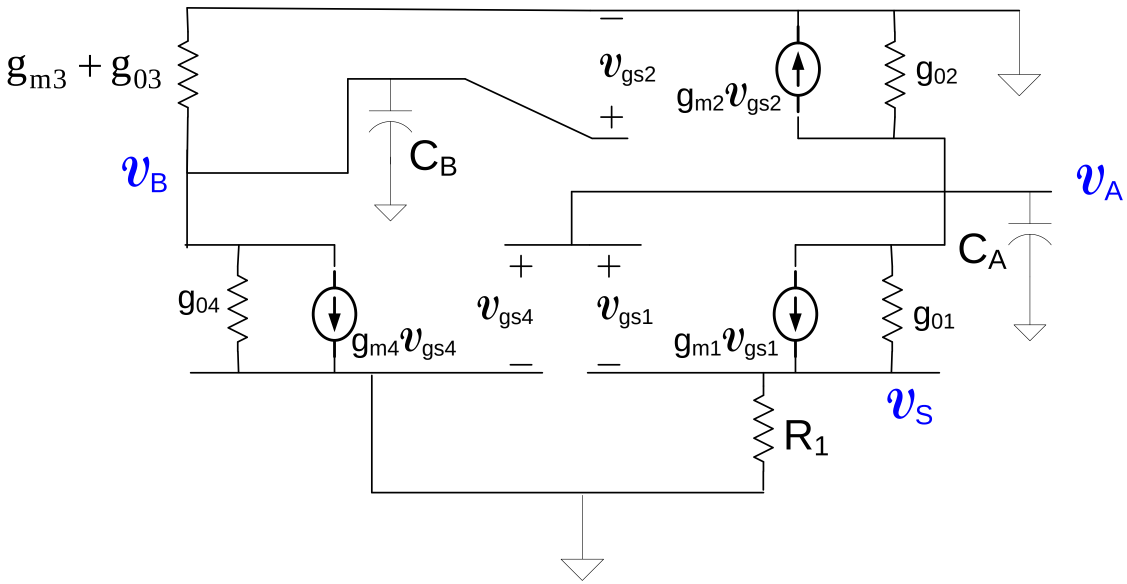
Pseudo-static operation so frequency response of little concern

V_{DD} Independent Bias Generators

Stability Concerns

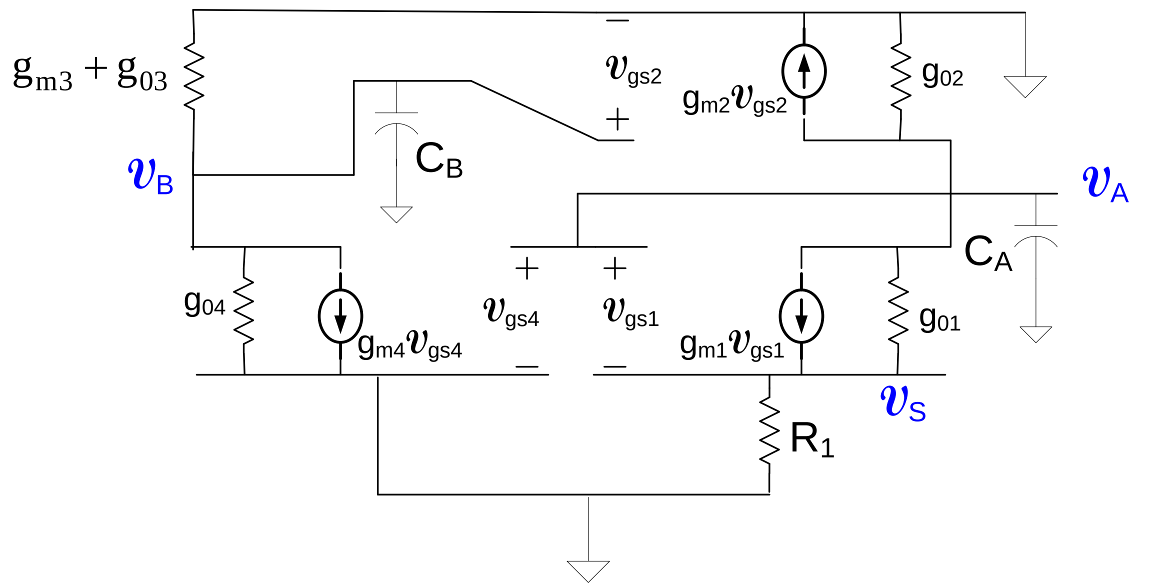


Since there is a local feedback loop, the issue of stability must be addressed. To do this, consider the small-signal equivalent circuit



V_{DD} Independent Bias Generators

Stability Concerns



Summing current at the three nodes, we obtain

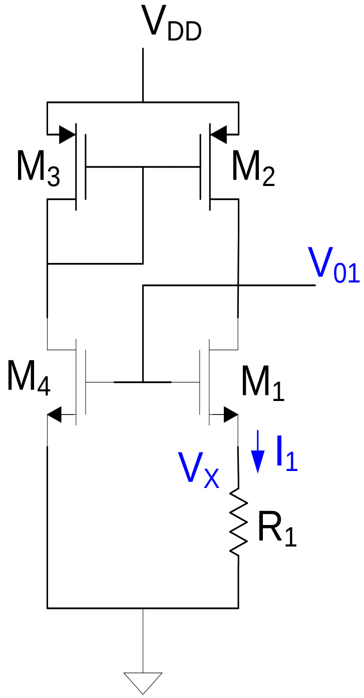
$$
\left.
\begin{aligned}
&v_A\left(g_{01}+g_{02}+sC_A\right)+g_{m2}v_B+g_{m1}\left(v_A-v_S\right)=g_{01}v_S\\
&v_B\left(g_{m3}+g_{01}+g_{03}+sC_B\right)+g_{m1}v_A=0\\
&v_S\left(g_{01}+g_1\right)=g_{01}v_A+g_{m1}\left(v_A-v_S\right)
\end{aligned}
\right\}
$$

Solving and neglecting g_0 terms compared to g_m terms, we obtain the characteristic polynomial

$$
D(s)=s^2C_AC_B+s\left(C_Ag_{m3}-C_B\frac{g_{m1}^2}{g_1+g_{m1}}\right)+g_{m1}\left(g_{m3}-g_{m2}\right)
$$

V_{DD} Independent Bias Generators

Stability Concerns



$$D(s) = s^2 C_A C_B + s\left(C_A g_{m3} - C_B \frac{g_{m1}^2}{g_1 + g_{m1}}\right) + g_{m1}(g_{m3} - g_{m2})$$

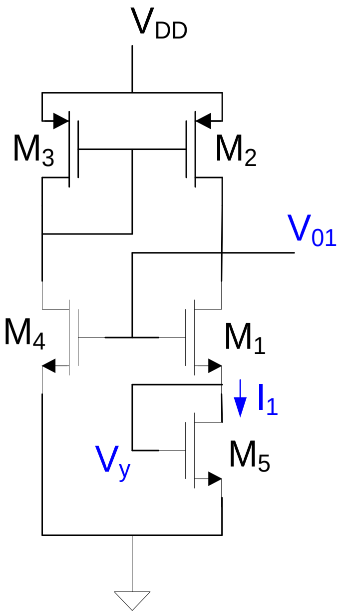
Thus, for stability, must have

$$g_{m3} > g_{m2}$$

$$C_A g_{m3} > C_B \frac{g_{m1}^2}{g_1 + g_{m1}}$$

V_{DD} Independent Bias Generators

Two widely-used V_{DD} independent bias generators (start-up ckts not shown)



$$I_{D2} = I_{D1} = I_{D5} = MI_{D3}$$

$$I_{D4} = I_{D3} = \frac{\mu C_{OX} W_4}{2L_4}\left(V_{01} - V_{Tn}\right)^2$$

$$I_{D1} = \frac{\mu C_{OX} W_1}{2L_1}\left(V_{01} - V_Y - V_{Tn}\right)^2$$

$$I_{D5} = \frac{\mu C_{OX} W_5}{2L_5}\left(V_Y - V_{Tn}\right)^2$$

4 equations and 4 unknowns
$\{I_{D1}, I_{D3}, V_{01}, V_Y\}$

$$V_{01} = V_{Tn}\left(\frac{\sqrt{M}\left(1+\sqrt{\frac{W_1L_5}{W_5L_1}}\right)-2}{\sqrt{M}\left(1+\sqrt{\frac{W_1L_5}{W_5L_1}}\right)-1}\right)$$

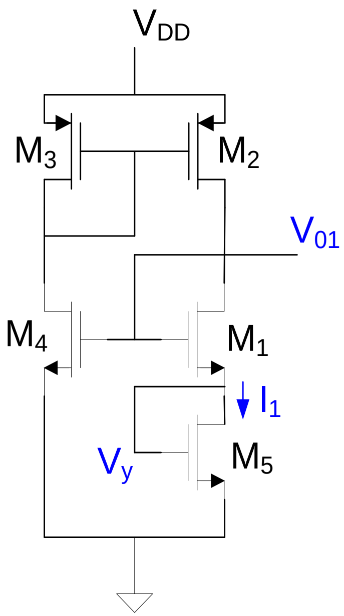
$$V_Y = V_{Tn}\left[\left(\sqrt{\frac{W_5L_1}{W_1L_5}}-1\right)+\left(\frac{1}{1+\sqrt{\frac{W_5L_1}{W_1L_5}}}\right)\left(\frac{\sqrt{M}\left(1+\sqrt{\frac{W_1L_5}{W_5L_1}}\right)-2}{\sqrt{M}\left(1+\sqrt{\frac{W_1L_5}{W_5L_1}}\right)-1}\right)\right]$$

Define:

M to be the M_3:M_2 mirror gain

V_{DD} Independent Bias Generators

Two widely-used V_{DD} independent bias generators (start-up ckts not shown)



$$V_{01} = V_{Tn}\left(\frac{\sqrt{M}\left(1+\sqrt{\frac{W_1L_5}{W_5L_1}}\right)-2}{\sqrt{M}\left(1+\sqrt{\frac{W_1L_5}{W_5L_1}}\right)-1}\right)$$

$$V_Y = V_{Tn}\left[\left(\sqrt{\frac{W_5L_1}{W_1L_5}}-1\right)+\left(\frac{1}{1+\sqrt{\frac{W_5L_1}{W_1L_5}}}\right)\left(\frac{\sqrt{M}\left(1+\sqrt{\frac{W_1L_5}{W_5L_1}}\right)-2}{\sqrt{M}\left(1+\sqrt{\frac{W_1L_5}{W_5L_1}}\right)-1}\right)\right]$$

Define:

M to be the M_3:M_2 mirror gain

Note V_{01} and V_Y are dependent only upon V_T

Applications well beyond this biasing requirement

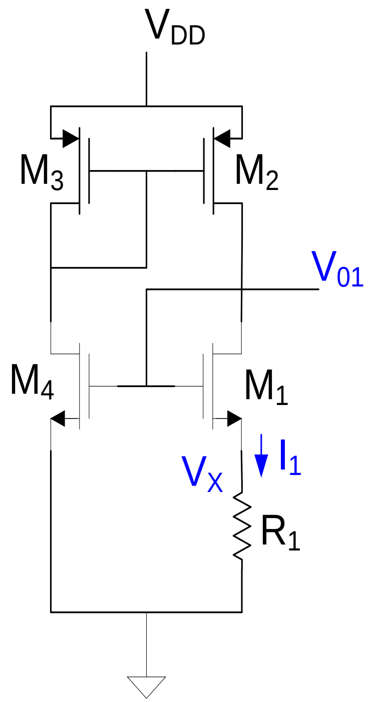
Requires Start-up Circuit

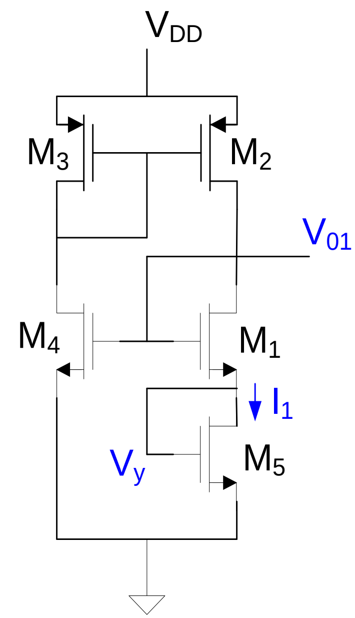
May need compensation for stability

Pseudo-static operation so frequency response of little concern

V_{DD} Independent Bias Generators

Two widely-used V_{DD} independent bias generators (start-up ckts not shown)





$$V_X = \frac{1}{R_1}\left(\sqrt{\frac{1}{M\beta_4}} - \sqrt{\frac{1}{\beta_1}}\right)^2$$

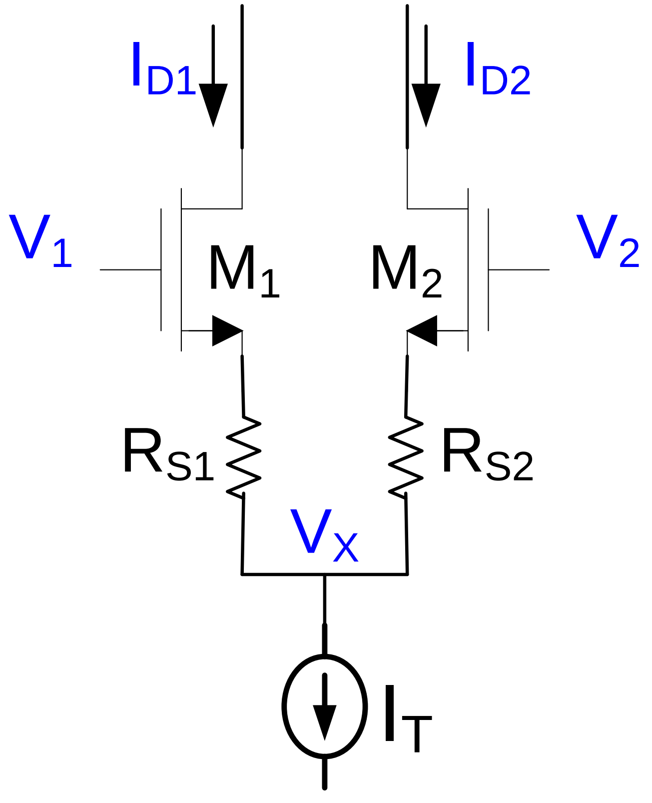
$$V_{01} = V_{Tn} + \left(\frac{1}{MR}\right)\left(\frac{1}{\beta_4} - \frac{1}{\sqrt{M\beta_1\beta_4}}\right)$$

$$V_{01} = V_{Tn}\left(\frac{\sqrt{M}\left(1+\sqrt{\frac{W_1L_5}{W_5L_1}}\right)-2}{\sqrt{M}\left(1+\sqrt{\frac{W_1L_5}{W_5L_1}}\right)-1}\right)$$

$$V_Y = V_{Tn}\left[\left(\sqrt{\frac{W_5L_1}{W_1L_5}}-1\right)+\left(\frac{1}{1+\sqrt{\frac{W_5L_1}{W_1L_5}}}\right)\left(\frac{\sqrt{M}\left(1+\sqrt{\frac{W_1L_5}{W_5L_1}}\right)-2}{\sqrt{M}\left(1+\sqrt{\frac{W_1L_5}{W_5L_1}}\right)-1}\right)\right]$$

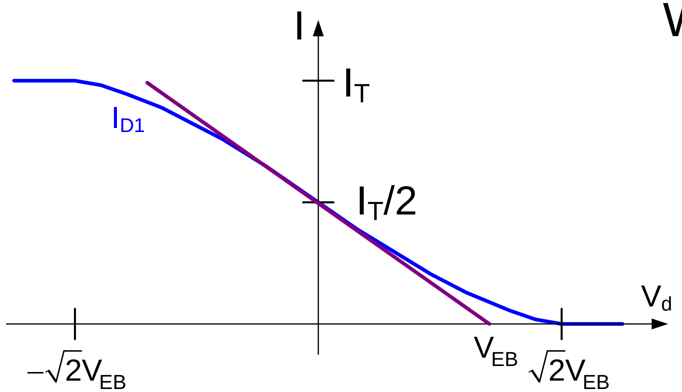
where $\beta_k = \frac{\mu C_{OX} W_k}{2L_k}$ and M is the M_3:M_2 mirror gain

Transconductance Linearization Strategies

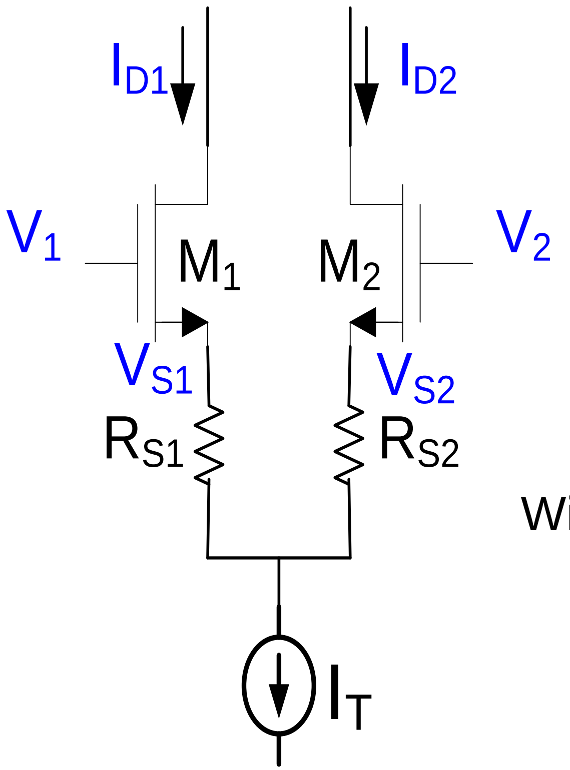


Widely used source degeneration

Recall with R_S=0



Transconductance Linearization Strategies



$$\left.\begin{aligned} I_{D1} &= \beta\left(V_1 - V_{S1} - V_T\right)^2 \\ I_{D2} &= \beta\left(V_2 - V_{S2} - V_T\right)^2 \\ V_{S1} - I_{D1}R_{S1} &= V_{S2} - I_{D2}R_{S2} \\ I_{D1} + I_{D2} &= I_T \end{aligned}\right\}$$

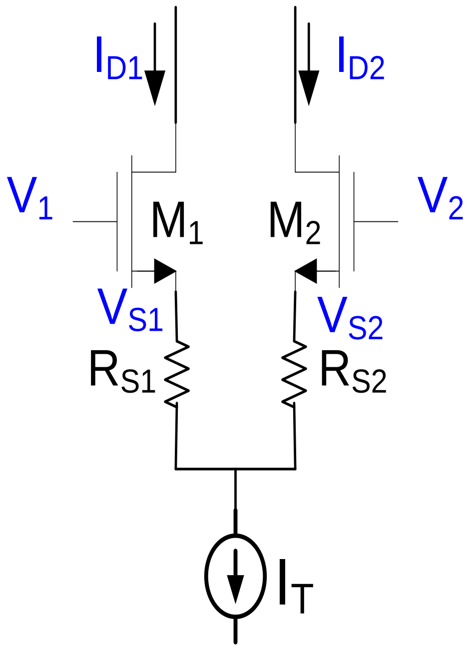
With a straightforward analysis, we obtain the expression

$$\sqrt{\frac{1}{\beta}}\left(\sqrt{I_T - I_{D1}} - \sqrt{I_{D1}}\right) + R_S\left(I_T - 2I_{D1}\right) = V_d$$

The first term on the right is the nonlinear term of the original source coupled pair and the second is linear in I_{D1}

The larger the second term becomes, the more linear the transfer characteristics are

Transconductance Linearization Strategies



$$\sqrt{\frac{1}{\beta}}\left(\sqrt{I_T - I_{D1}} - \sqrt{I_{D1}}\right) + R_S\left(I_T - 2I_{D1}\right) = V_d$$

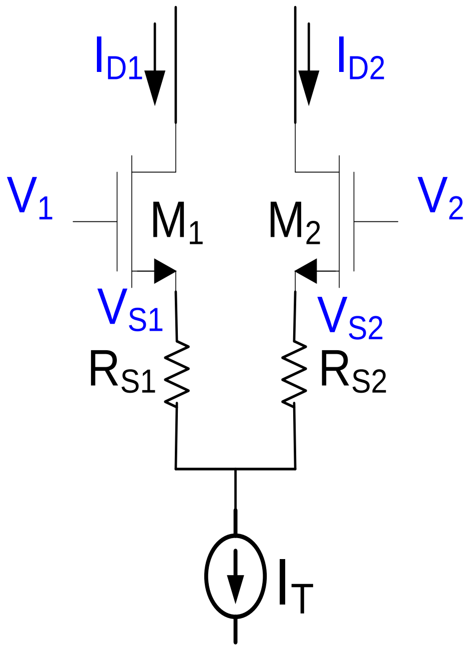
The transconductance of this structure can be readily derived to obtain

$$g_m = \left.\frac{\partial V_d}{\partial I_{D1}}\right|_{Q-pt}^{-1} = \left[\sqrt{\frac{1}{\beta}} \bullet \frac{1}{2}\left(-\left(I_T - I_{D1}\right)^{-1/2} - I_{D1}^{-1/2}\right) - 2R_S\right]\Bigg|_{Q-pt}^{-1}$$

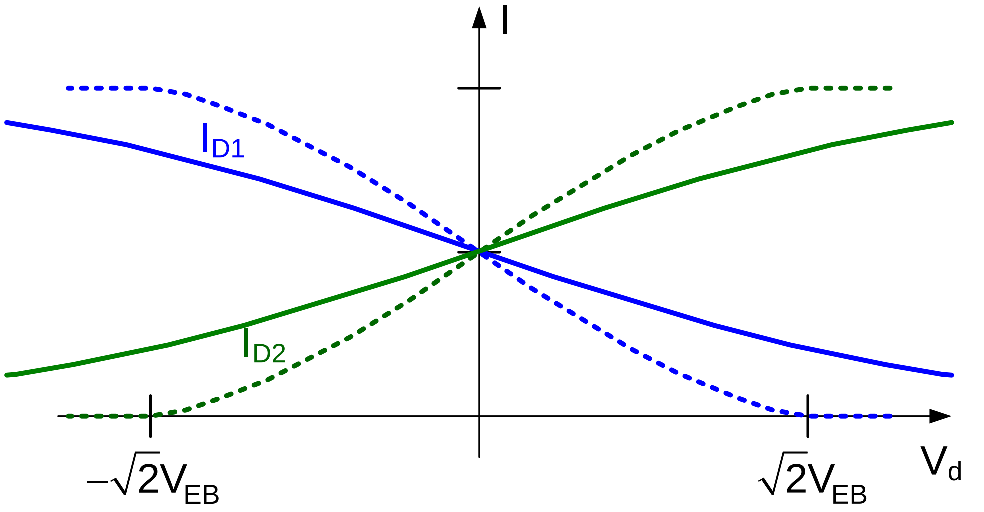
This can be expressed as

$$g_m = \left.\frac{\partial V_d}{\partial I_{D1}}\right|_{Q-pt}^{-1} = -\frac{1}{\left[\sqrt{\frac{2}{\beta I_T}} + 2R_S\right]} = -\frac{\beta V_{EB}}{1 + 2\beta V_{EB} R_S}$$

Transconductance Linearization Strategies



$$\sqrt{\frac{1}{\beta}}\left(\sqrt{I_T - I_{D1}} - \sqrt{I_{D1}}\right) + R_S\left(I_T - 2I_{D1}\right) = V_d$$



Transconductance Linearization Strategies

There are a host of transconductance linearization strategies that have been discussed in the literature

Some are shown below

Many are strongly dependent upon a precise square-law model of the MOS devices and do not provide practical solutions when the devices are not square-law devices

Analysis or simulation with a more realistic model is necessary to validate linearity and practical applications of these structures

Transconductance Linearization Strategies

How good is the square-law model that we have been using for predicting filter performance?

It is reasonably good when analyzing structures whose linearity characteristics are not strongly dependent upon the device model

The circuits considered to date are not particularly linear so the square-law model probably does a pretty good job of predicting their performance

More accurate models are usually unwieldy for hand analysis

Transconductance Linearization Strategies

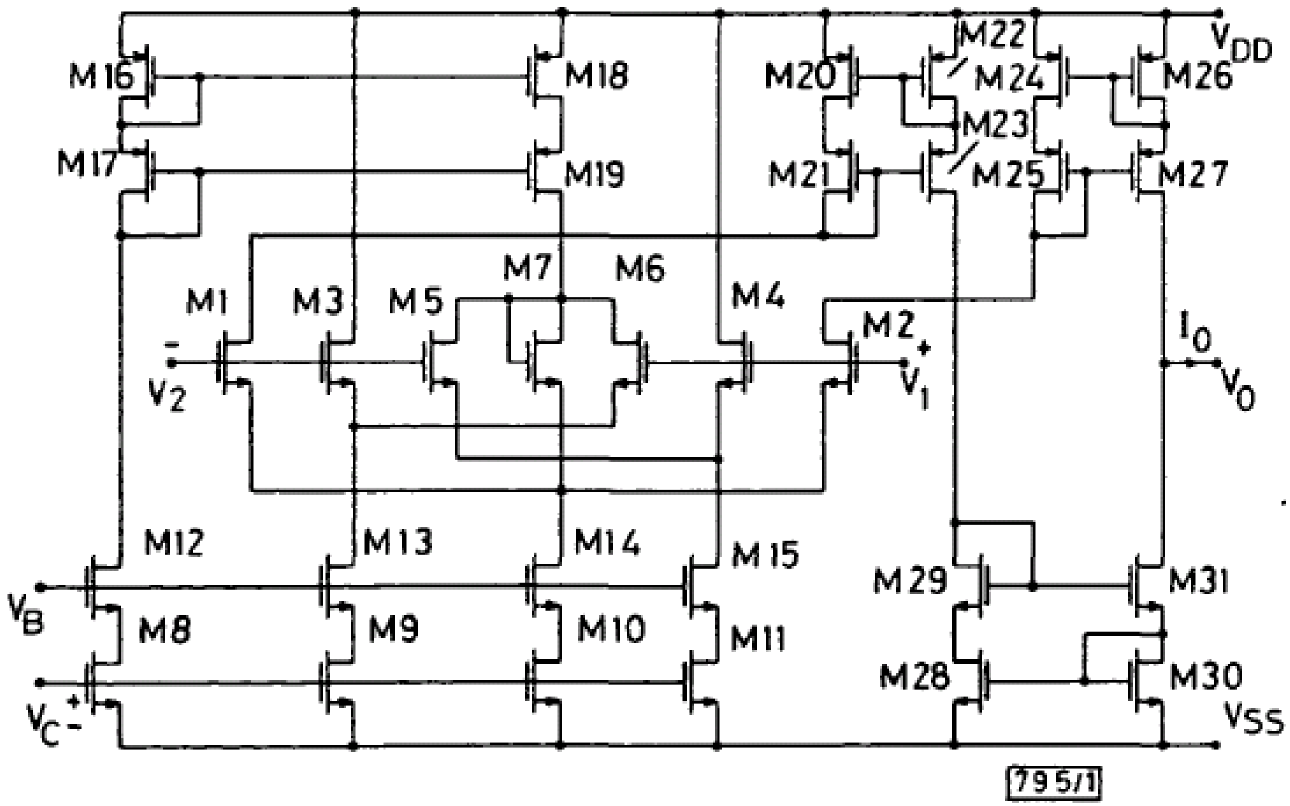
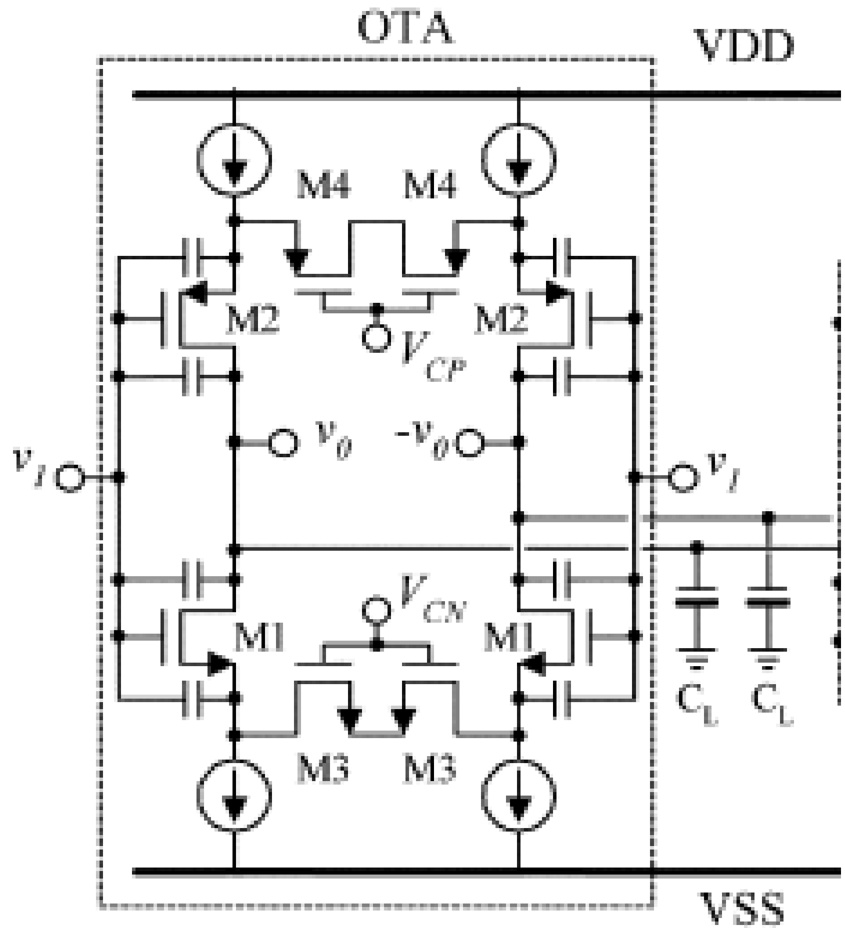


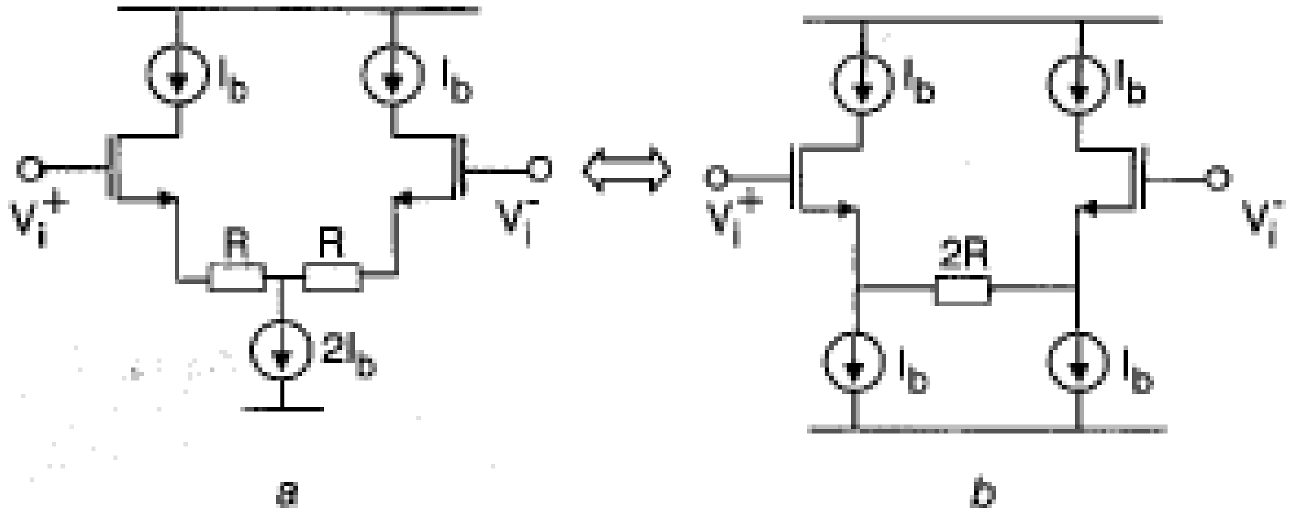
Fig. 1 *Linearised CMOS transconductance circuit*

Transconductance Linearization Strategies



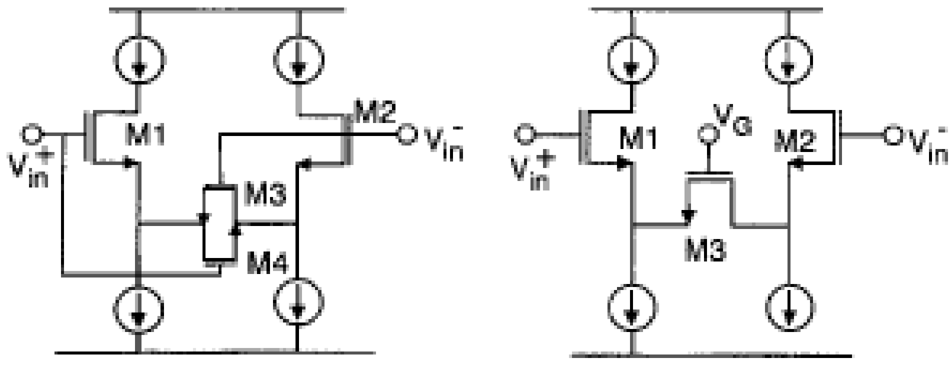
From CAS 2006 P 811 Jose Silva

Transconductance Linearization Strategies

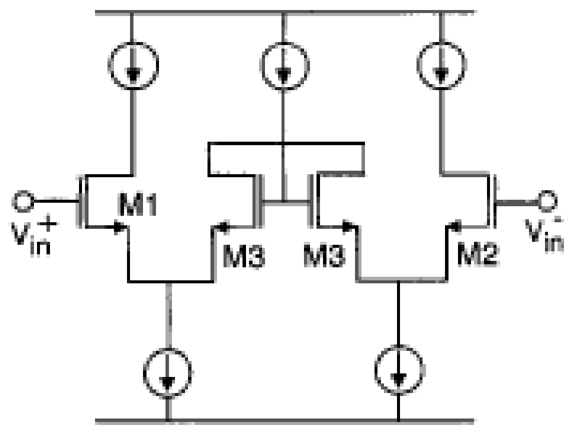


Linearity Enhancement with Source Degeneration

Transconductance Linearization Strategies



a b

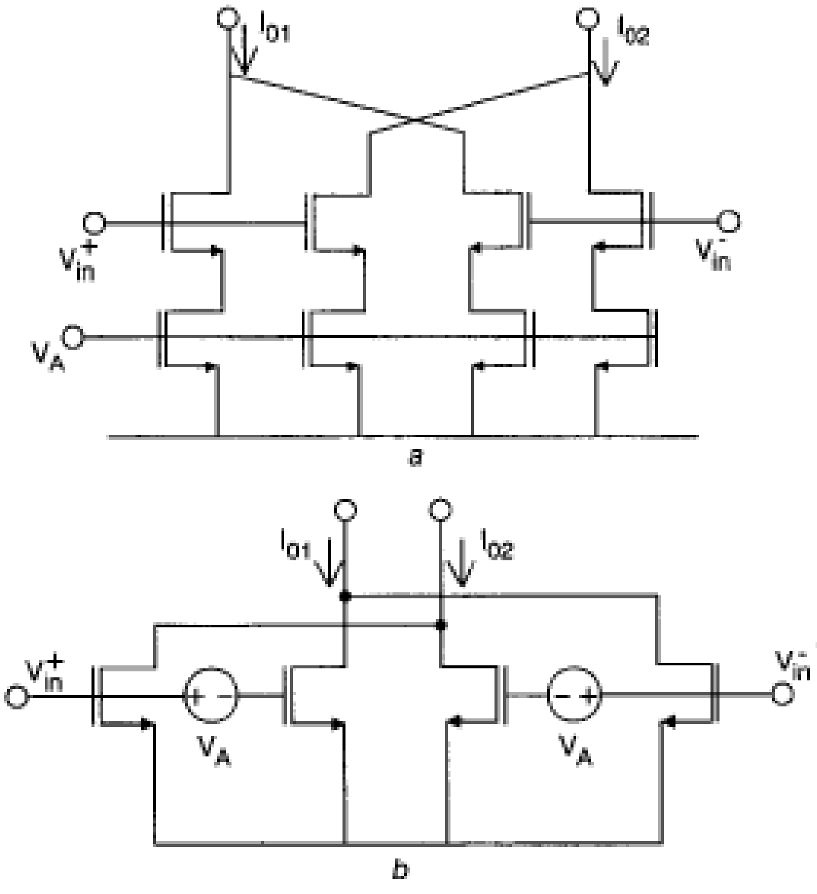


Linearization with active source degeneration

CMOS transconductance amplifiers, architectures and active filters: a tutorial

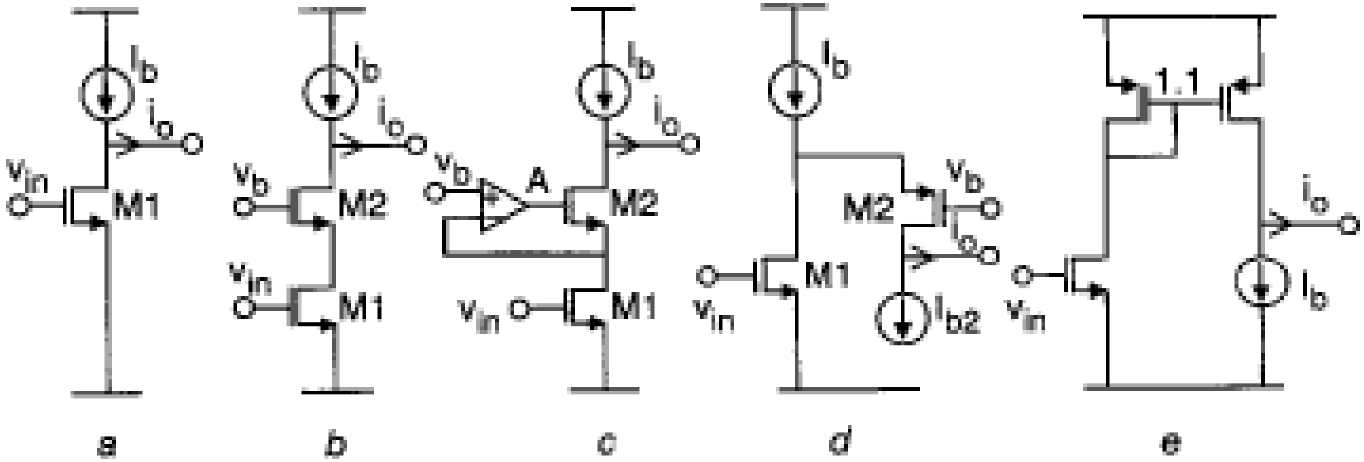
E.Sánchez-Sinencio and J.Silva-Martínez

Abstract: An updated version of a 1985 tutorial paper on active filters using operational transconductance amplifiers (OTAs) is presented. The integrated circuit issues involved in active filters (using CMOS transconductance amplifiers) and the progress in this field in the last 15 years is addressed. CMOS transconductance amplifiers, nonlinearised and linearised, as well as frequency limitations and dynamic range considerations are reviewed. OTA-C filter architectures, current-mode filters, and other potential applications of transconductance amplifiers are discussed.

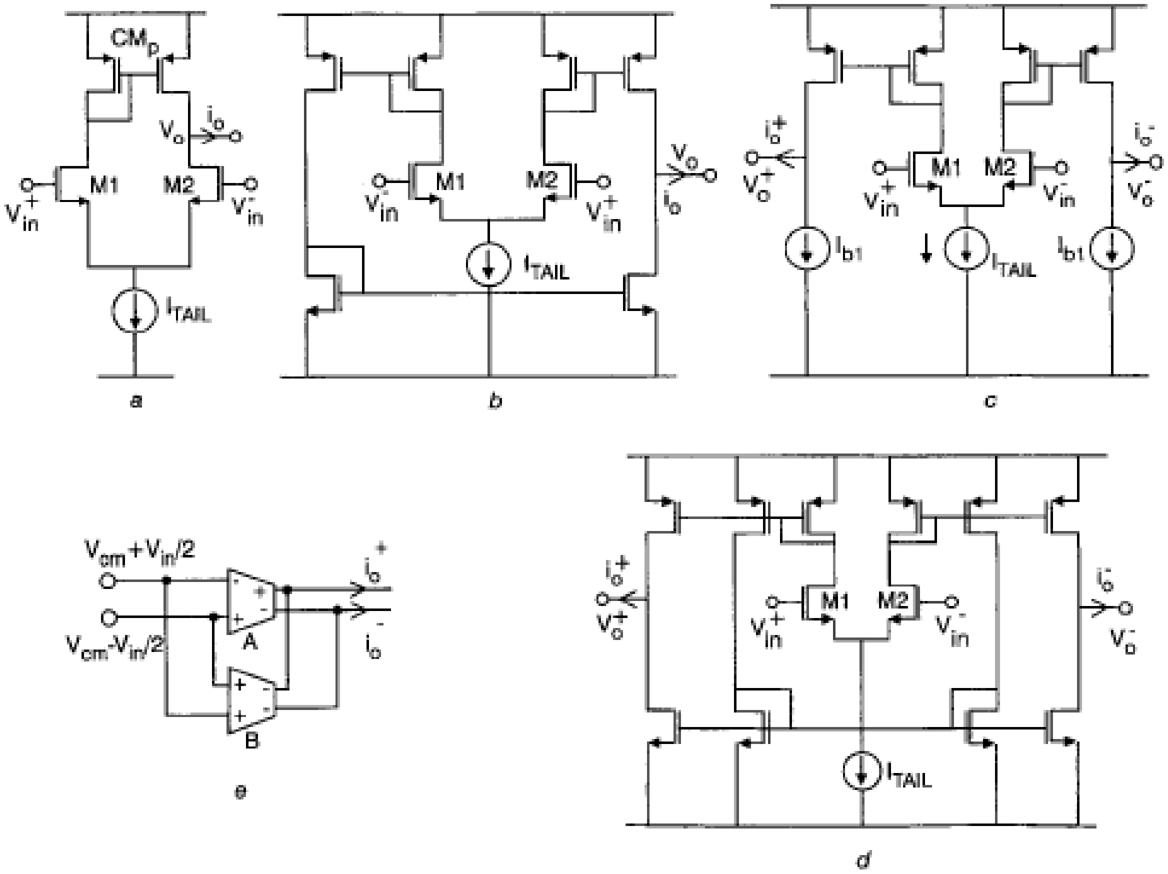


Linearity compensation with cross-coupled feedback

Single-ended input TAs

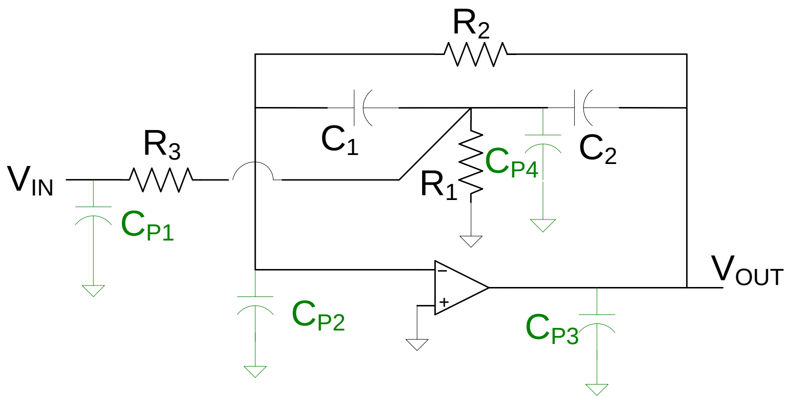


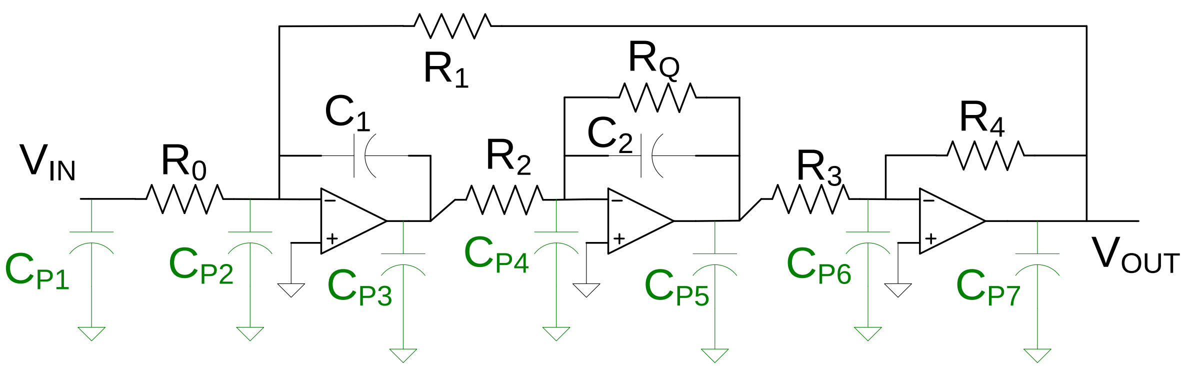
Differential input OTAs

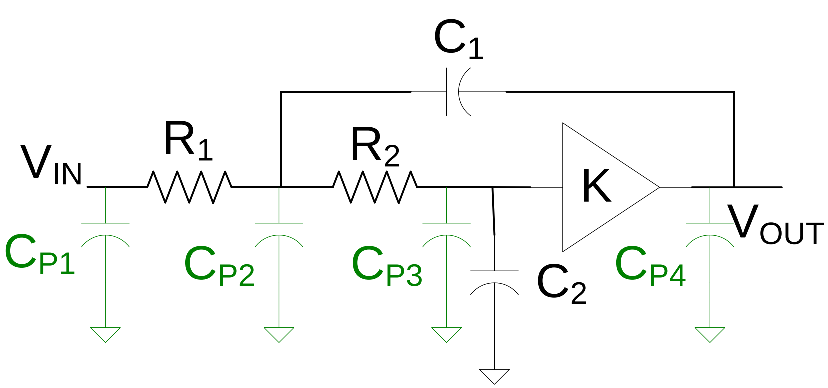


Differential input and output OTAs

Parasitic Capacitances and Floating Nodes





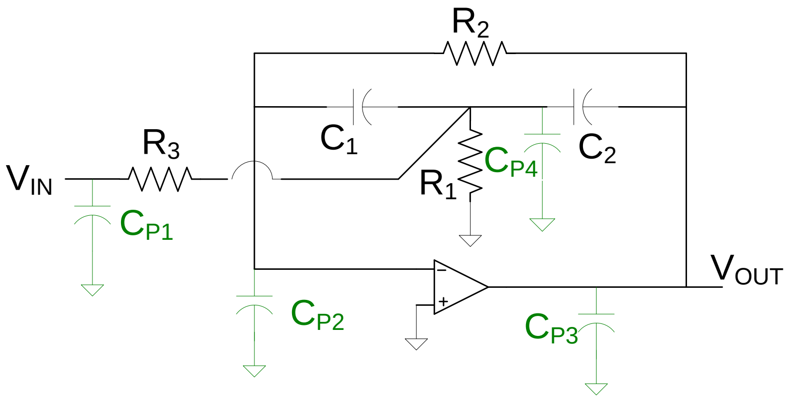


There is invariably a parasitic capacitance associated with every terminal of every element in a filter

These parasitic capacitances can be significant in integrated filters

These can be combined into a single parasitic capacitance on each node

Parasitic Capacitances and Floating Nodes

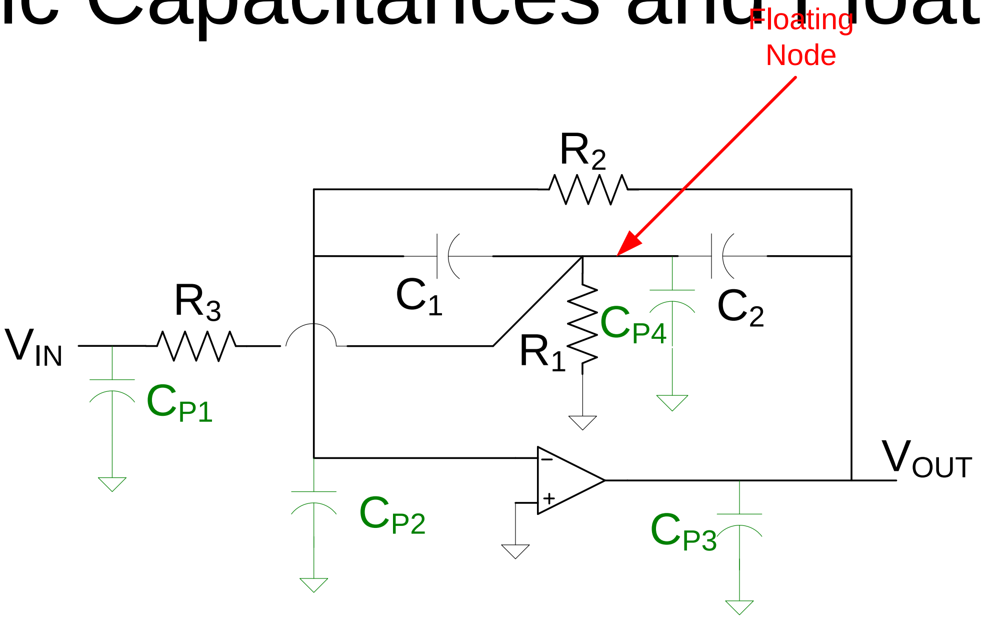


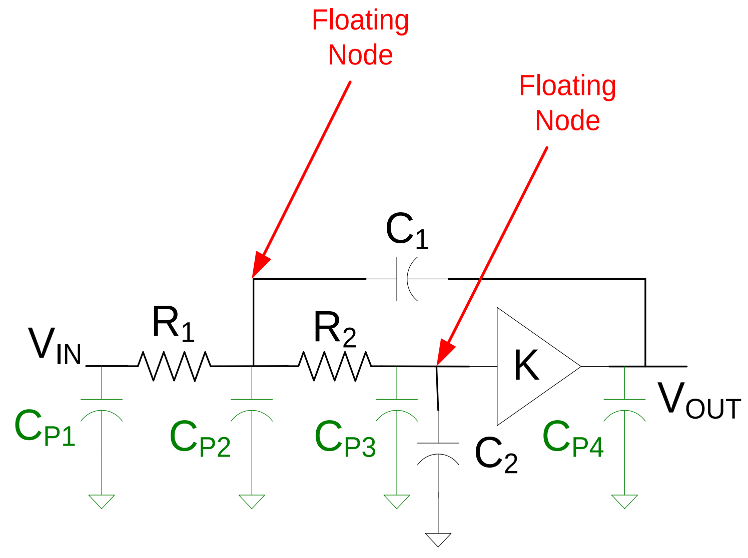
A floating node is a node that is not connected to either a zero-impedance element or across a null-port

Floating nodes are generally avoided in integrated filters because the parasitic capacitances on the floating nodes usually degrades filter performance and often increases the order of the filter

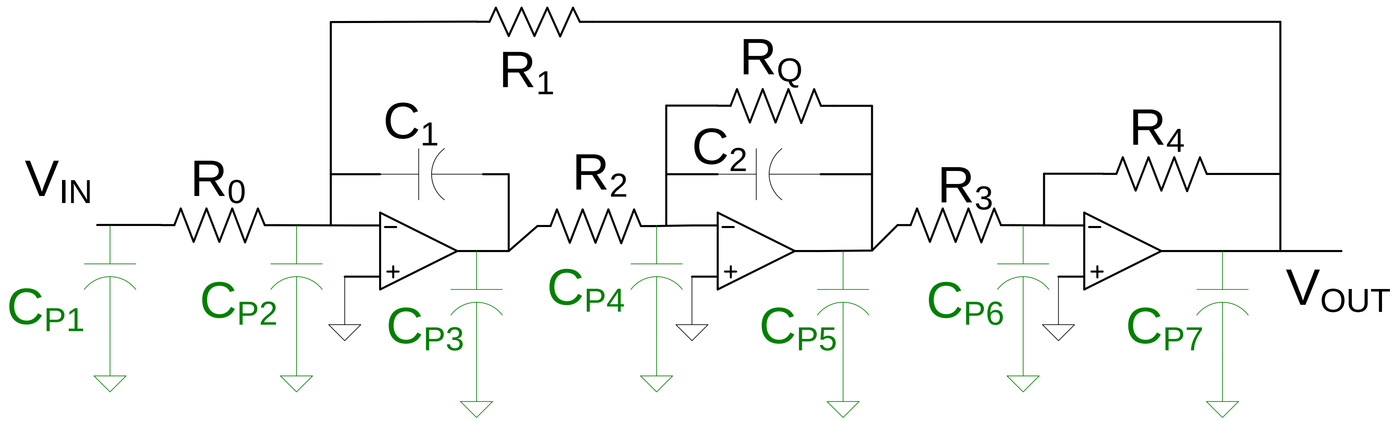
Some filter architectures inherently have no floating nodes, specifically, most of the basic integrator-based filters have no floating nodes

Parasitic Capacitances and Floating Nodes





Parasitic Capacitances and Floating Nodes



No floating nodes !

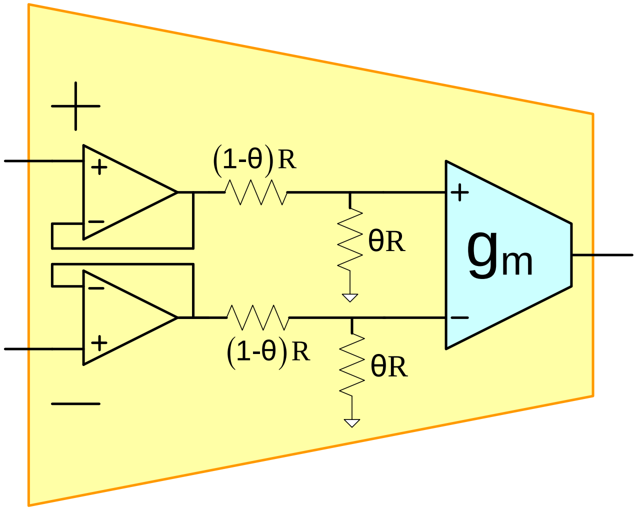
Signal Swing in OTA Circuits

The signal swing for the basic bipolar OTA is limited to a few mV for reasonably linear operation

This limited signal swing limits the use of the OTS

The following circuit (with maybe a 100:1 or more attenuation) can be used to increase the input signal swing to the volt range and although it involves quite a few more components, the functionality can be most significant

Program range is not affected by adding the attenuators



$$g_{meq} = \theta g_m$$

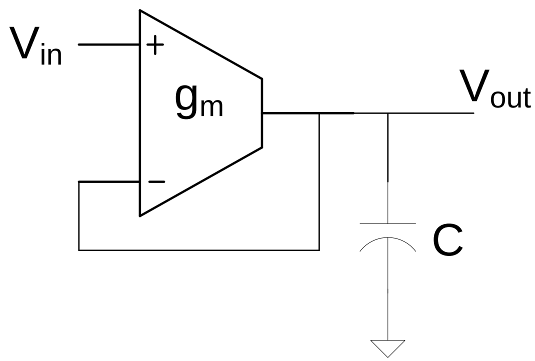
R. L. Geiger and E. Sánchez-Sinencio, "Active Filter Design Using Operational Transconductance Amplifiers: A Tutorial," *IEEE Circuits and Devices Magazine*, Vol. 1, pp.20-32, March 1985.

Active Filter Design Using Operational Transconductance Amplifiers: A Tutorial

Randall L. Geiger and Edgar Sánchez-Sinencio

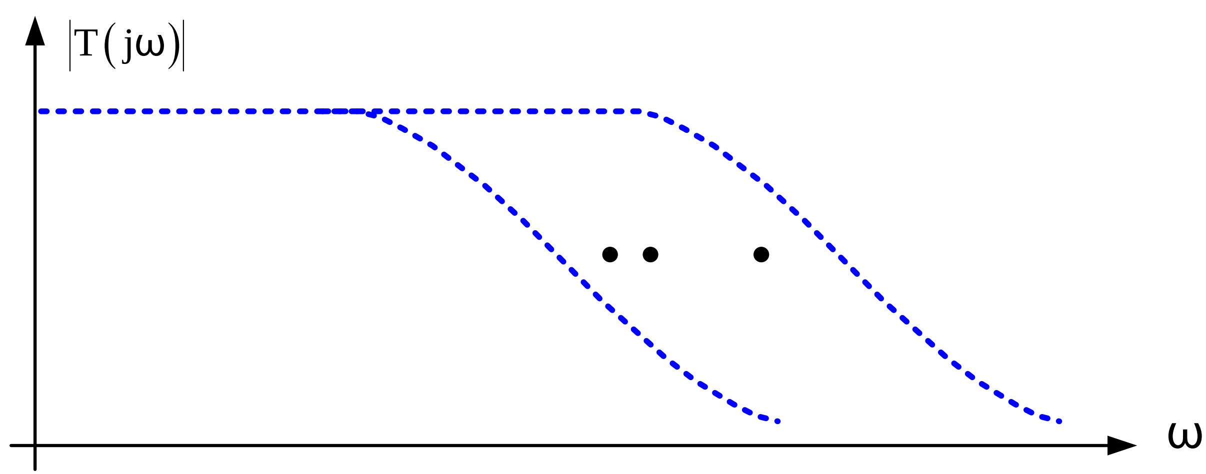
Programmable Filter Structures

It will be assumed that the transconductance gain can be programmed with either a dc current or a dc voltage

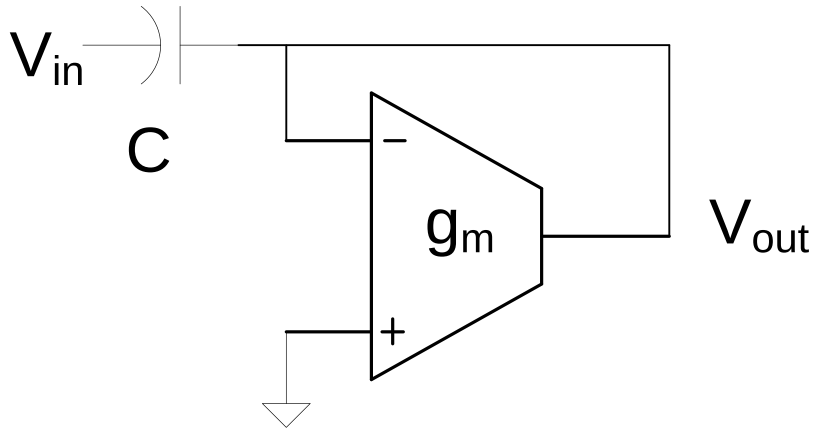


$$T(s) = \frac{g_m}{g_m + sC}$$

Programmable First-Order Low-Pass Filter

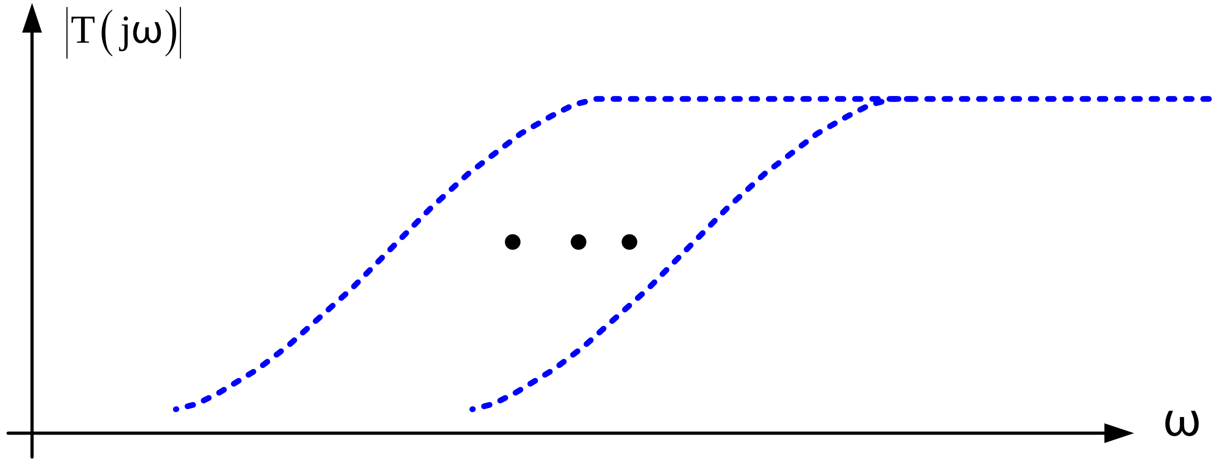


Programmable Filter Structures



$$T(s) = \frac{sC}{g_m + sC}$$

Programmable First-Order High-Pass Filter



End of Lecture 35

EE 508

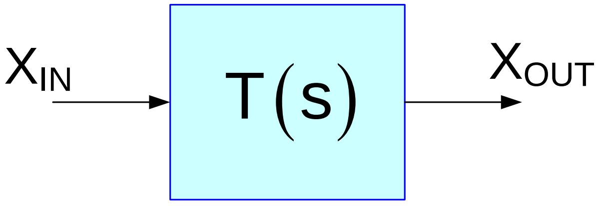
Lecture 36

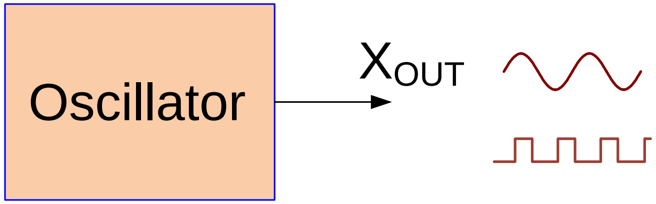
Oscillators, VCOs, and

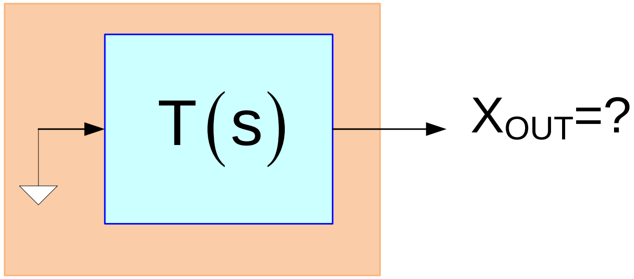
Oscillator/VCO-Derived Filters

Question:

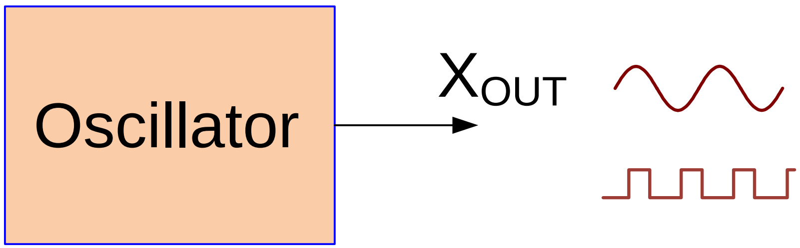
What is the relationship, if any, between a filter and an oscillator or VCO?

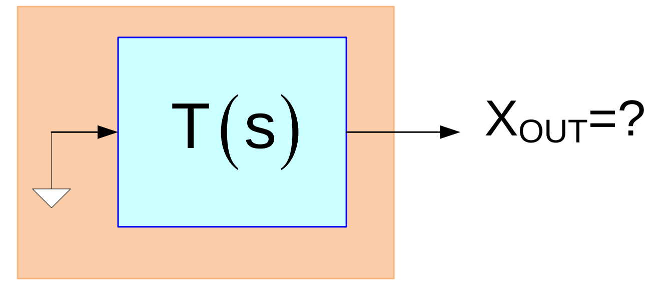






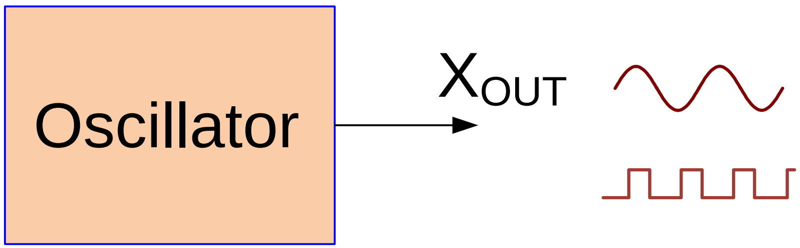
What is the relationship, if any, between a filter and an oscillator or VCO?

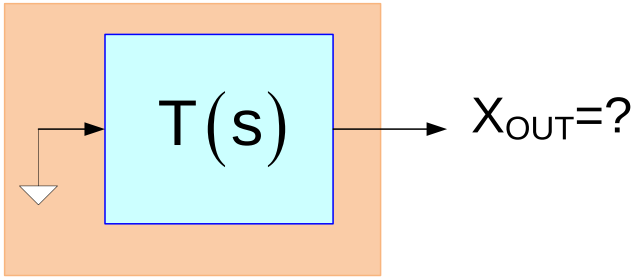




- When power is applied to an oscillator, it initially behaves as a small-signal linear network
- When operating linearly, the oscillator has poles (but no zeros)
- Poles are ideally on the imaginary axis or appear as cc pairs in the RHP
- There is a wealth of literature on the design of oscillators
- Oscillators often are designed to operate at very high frequencies
- If cc poles of a filter are moved to RHP is will become an oscillator
- Can oscillators be modified to become filters?

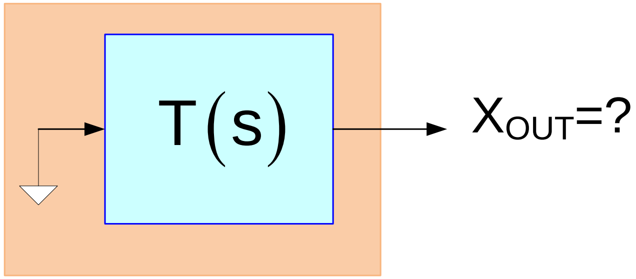
What is the relationship, if any, between a filter and an oscillator or VCO?

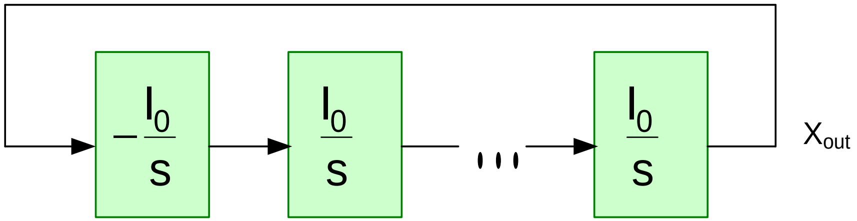




Will focus on modifying oscillator structures to form high frequency narrow-band filters

Consider a cascaded integrator loop comprised of n integrators





$$X_{OUT} = -\left(\frac{I_0}{s}\right)^n X_{OUT}$$

$$X_{OUT}\left(s^n + I_0^n\right) = 0$$

$$D(s) = s^n + I_0^n$$

Consider the poles of $D(s) = s^n + I_0^n$

$$s^n + I_0^n = 0$$

$$s^n = -I_0^n$$

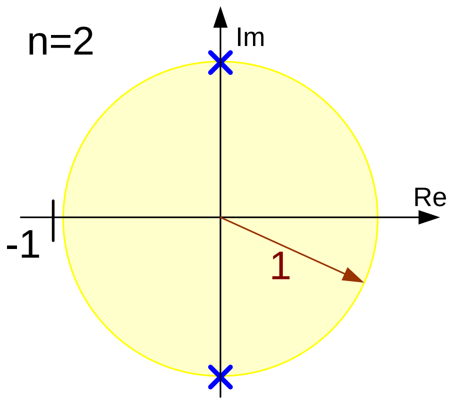
$$s = \left[-I_0^n\right]^{\frac{1}{n}}$$

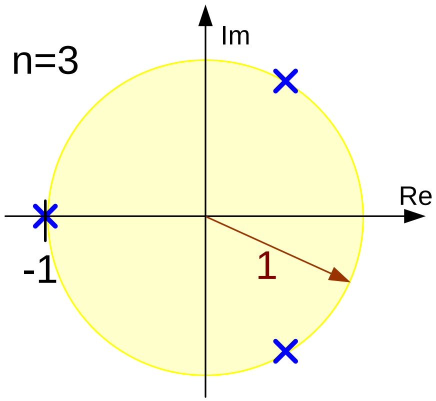
$$s = [-1]^{\frac{1}{n}} \left[I_0^n\right]^{\frac{1}{n}}$$

$$s = I_0 [-1]^{\frac{1}{n}}$$

Poles are the n roots of -1 scaled by I_0

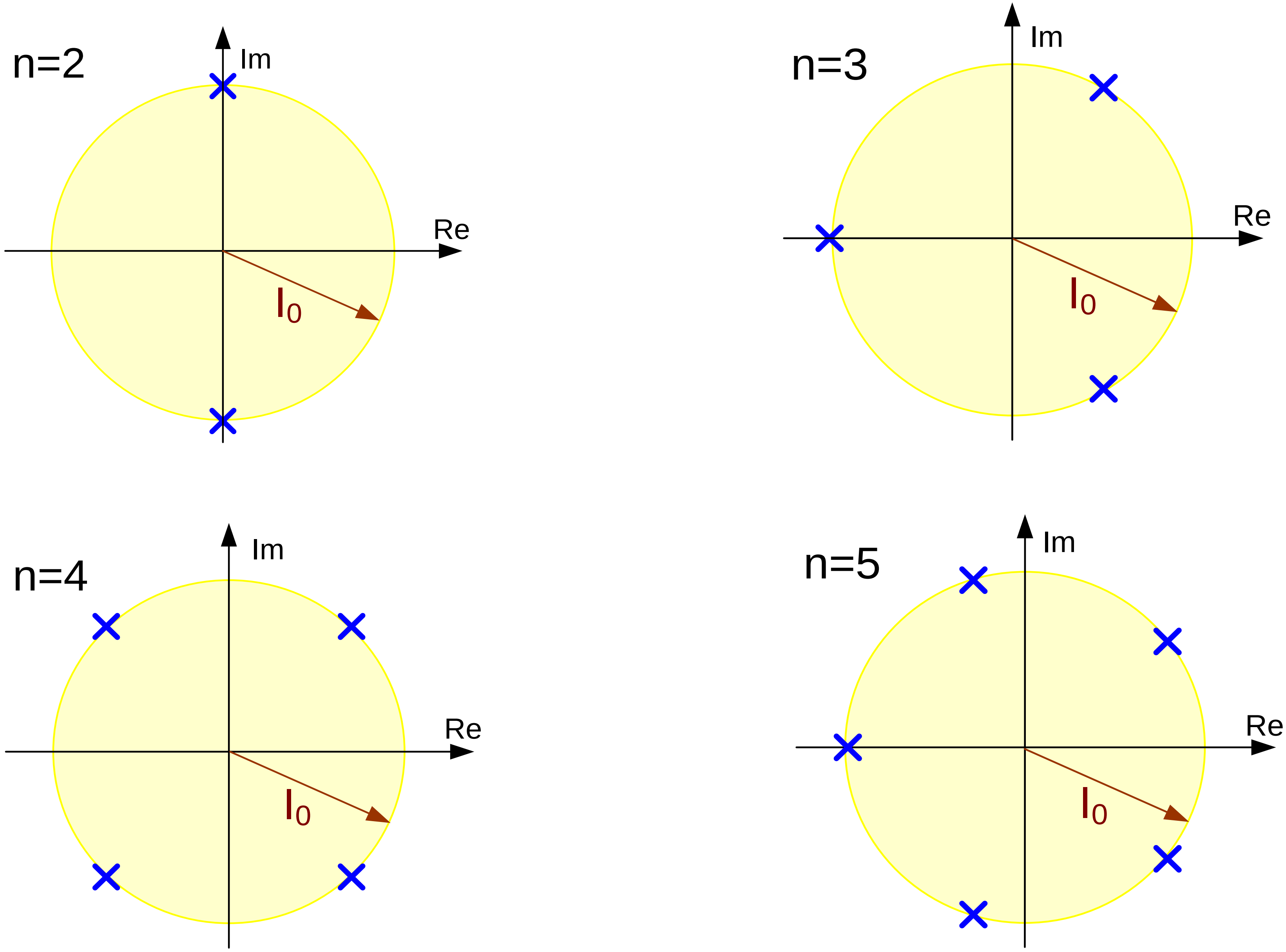
Roots of -1:

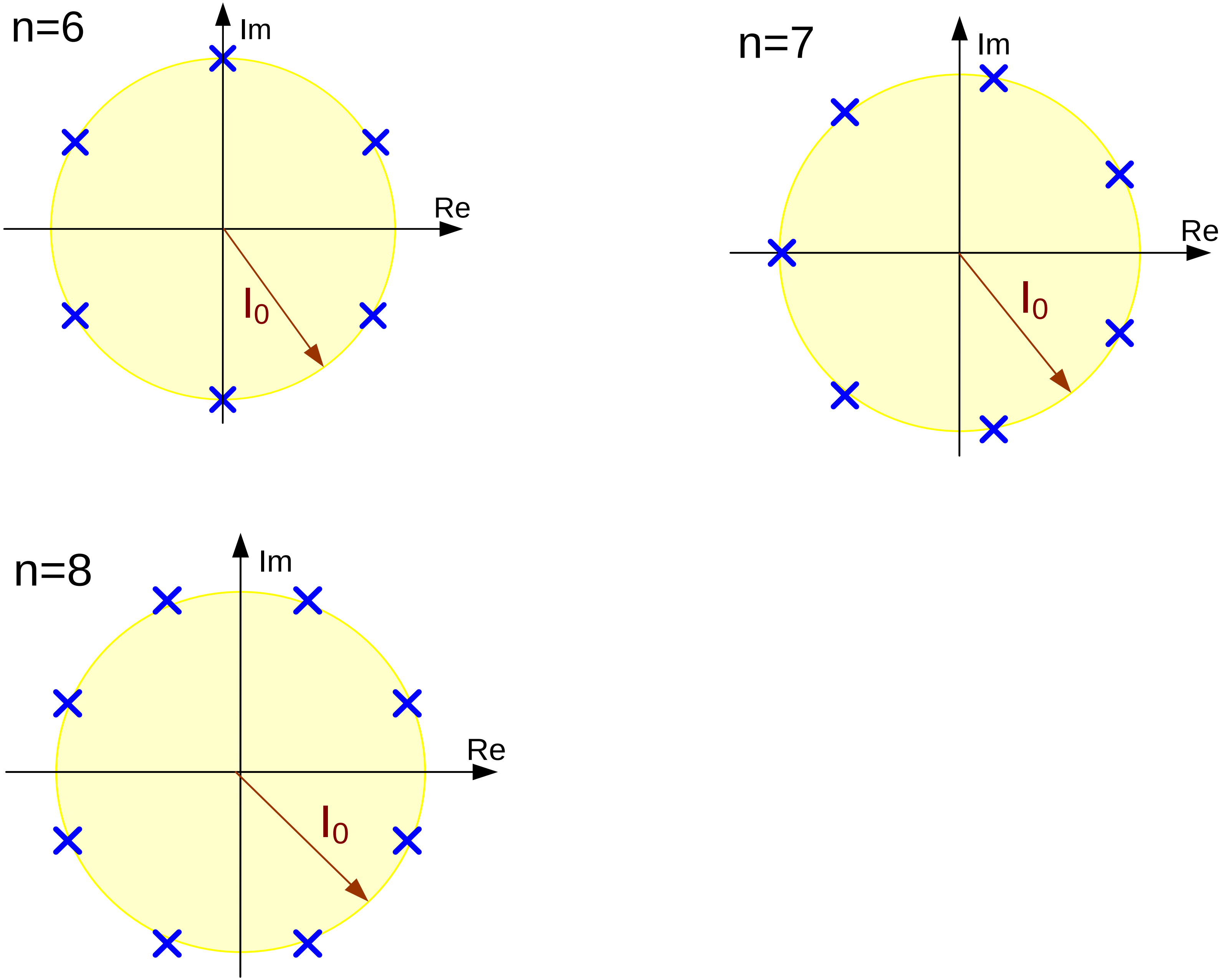




Roots are uniformly spaced on a unit circle

Consider the poles of $D(s) = s^n + I_0^n$





n=6
Im
Re
I_0
n=7
Im
Re
I_0
n=8
Im
Re
I_0

Some useful theorems

Theorem: A rational fraction $T(s) = \frac{N(s)}{\prod_{i=1}^{n}(s-p_i)}$ with simple poles can be expressed

in partial fraction form as $T(s)=\sum_{i=1}^{n}\frac{A_i}{s-p_i}$

where $A_i=(s-p_i)T(s)\big|_{s=p_i}$ for 1 ≤ j ≤ n

Theorem: The impulse response of a rational fraction T(s) with simple poles can be expressed as $T(s)=\sum_{i=1}^{n}A_i e^{p_i t}$ where the numbers A_i are the coefficients in the partial fraction expansion of T(s)

Theorem: If p_i is a simple complex pole of the rational fraction T(s), then the partial fraction expansion terms in the impulse response corresponding to p_i and p_i^* can be expressed as

$$\frac{A_i}{s-p_i} + \frac{A_i^*}{s-p_i^*}$$

Theorem: If $p_i = \alpha_i + j\beta_i$ is a simple pole with non-zero imaginary part of the rational fraction T(s), then the impulse response terms corresponding to the poles p_i and p_i^* in the partial fraction expansion can be expressed as

$$|A_i| e^{\alpha_i t} \cos(\beta_i t + \theta_i)$$

where θ_i is the angle of the complex quantity A_i

Theorem: If all poles of an n-th order rational fraction T(s) are simple and have a non-zero Imaginary part, then the impulse response can be expressed as

$$\sum_{i=1}^{n/2} |A_i| e^{\alpha_i t} \cos(\beta_i t + \theta_i)$$

where θ_i, A_i, α_i, and β_i are as defined before

Theorem: If an odd-order rational fraction has one pole on the negative real axis at α_0 and n simple poles that have a non-zero Imaginary part, then the impulse response can be expressed as

$$A_0 e^{\alpha_0 t} + \sum_{i=1}^{n/2} |A_i| e^{\alpha_i t} \cos(\beta_i t + \theta_i)$$

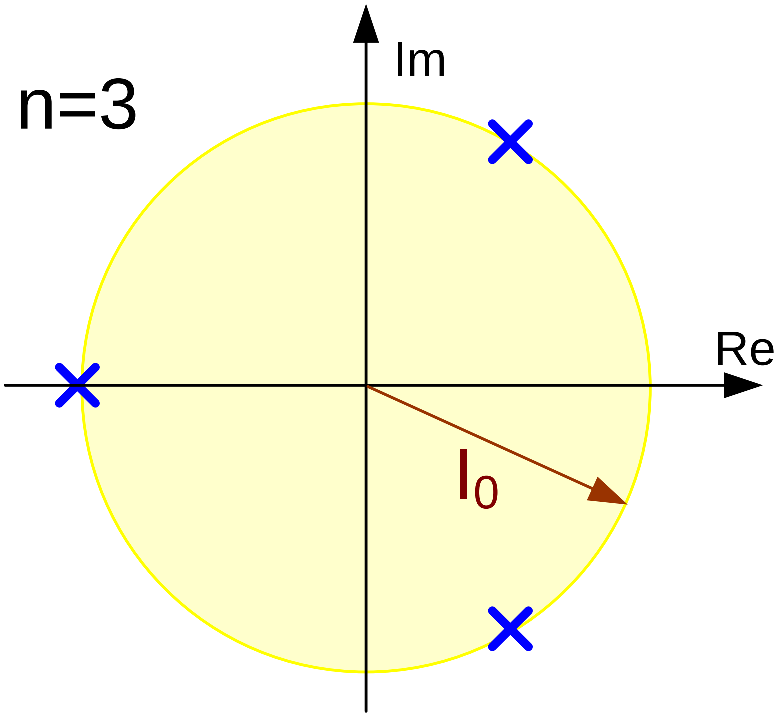
where θ_i, A_i, α_i, and β_i are as defined before

Poles of $D(s) = s^n + I_0^n$

Consider the following

n=3

0.5	-0.866025404
0.5	0.866025404
-1	3.67545E-16



$\alpha = 0.5\ I_0$

$\beta = 0.866\ I_0$

frequency of oscillation: $|A_i| e^{\alpha_i t} \cos(\beta_i t + \theta_i)$

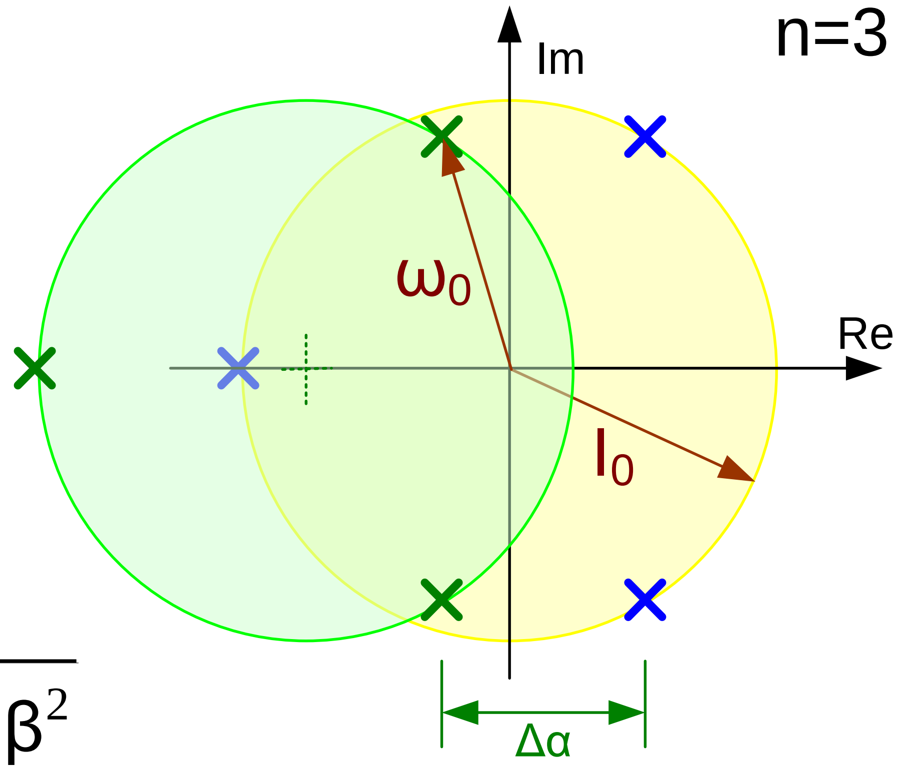
Starts at $\omega = 0.866 I_0$ and will slow down as nonlinearities limit amplitude

Poles of $D(s) = s^n + I_0^n$

Consider the following

$\beta = 0.866\ I_0$

$\alpha = 0.5\ I_0 - \Delta\alpha$



$$\omega_0 = \sqrt{(\alpha - \Delta\alpha)^2 + \beta^2}$$

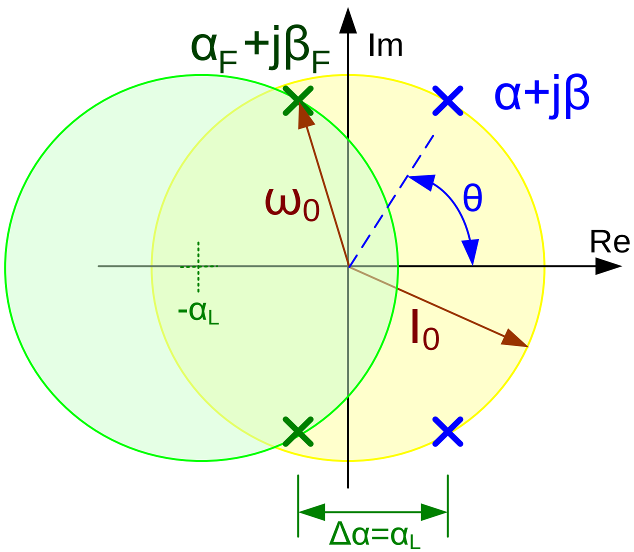
So, to get a high ω_0, want β as large as possible

Consider now the filter by adding a loss of α_L to the integrator

Will now determine α_L and I_0 needed to get a desired pole Q and ω_0

The values of α and β are dependent upon I_0 but the angle θ is only dependent upon the number of integrators in the VCO

$$\alpha + j\beta = I_0\left(\cos\theta + j\sin\theta\right)$$



Define the location of the filter pole to be

$$\alpha_F + j\beta_F$$

It follows that

$$\beta_F = \beta \qquad \alpha_F = \alpha - \alpha_L$$

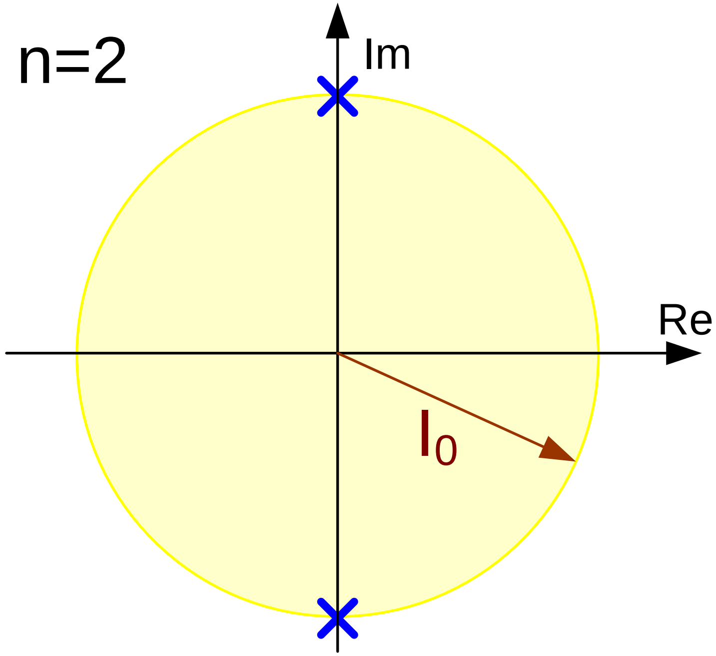
The relationship between the filter parameters is well known

$$\beta_F = \frac{\omega_0}{2Q}\sqrt{4Q^2-1} \qquad \alpha_F = -\frac{\omega_0}{2Q}$$

Thus

$$I_0 = \frac{\omega_0}{(\sin\theta)2Q}\sqrt{4Q^2-1} \qquad \alpha_L = \frac{\omega_0}{2Q} + I_0\cos\theta = \frac{\omega_0}{2Q} + \frac{\omega_0}{2Q(\tan\theta)}\sqrt{4Q^2-1}$$

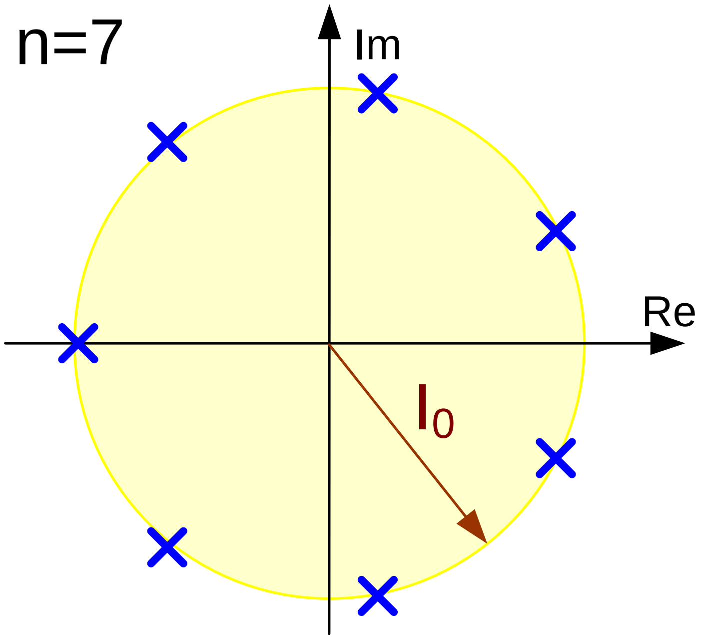
Will a two-stage structure give the highest frequency of operation?



$$\omega_0 = \sqrt{(\alpha - \Delta\alpha)^2 + \beta^2} \longrightarrow \omega_0 = \sqrt{(-\Delta\alpha)^2 + \beta^2}$$

- Even though the two-stage structure may not oscillate, can work as a filter!
- Can add phase lead if necessary

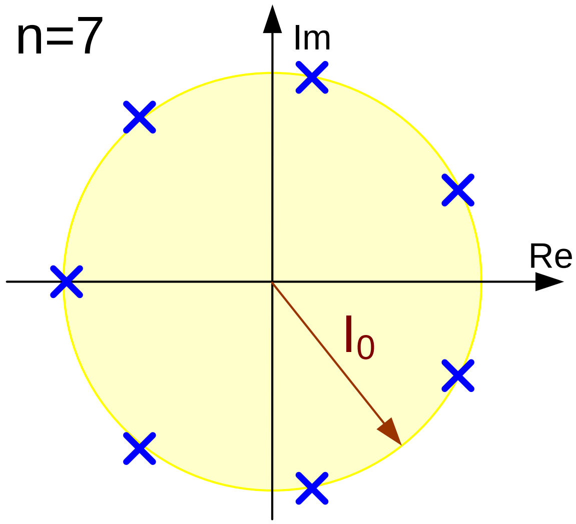
What will happen with a circuit that has two pole-pairs in the RHP?



The impulse response will have three decaying exponential terms and two growing exponential terms

$$A_0 e^{\alpha_0 t} + \sum_{i=1}^{n/2} |A_i| e^{\alpha_i t} \cos(\beta_i t + \theta_i)$$

What will happen with a circuit that has two pole-pairs in the RHP?



-0.62349	-0.781831482
0.222521	-0.974927912
0.900969	-0.433883739
0.900969	0.433883739
0.222521	0.974927912
-0.62349	0.781831482
-1	3.67545E-16

$\alpha_1=0.2225$ $\beta_1=0.974$

$\alpha_2=0.9009$ $\beta_2=0.4338$

Consider the growing exponential terms and normalize to $I_0=1$

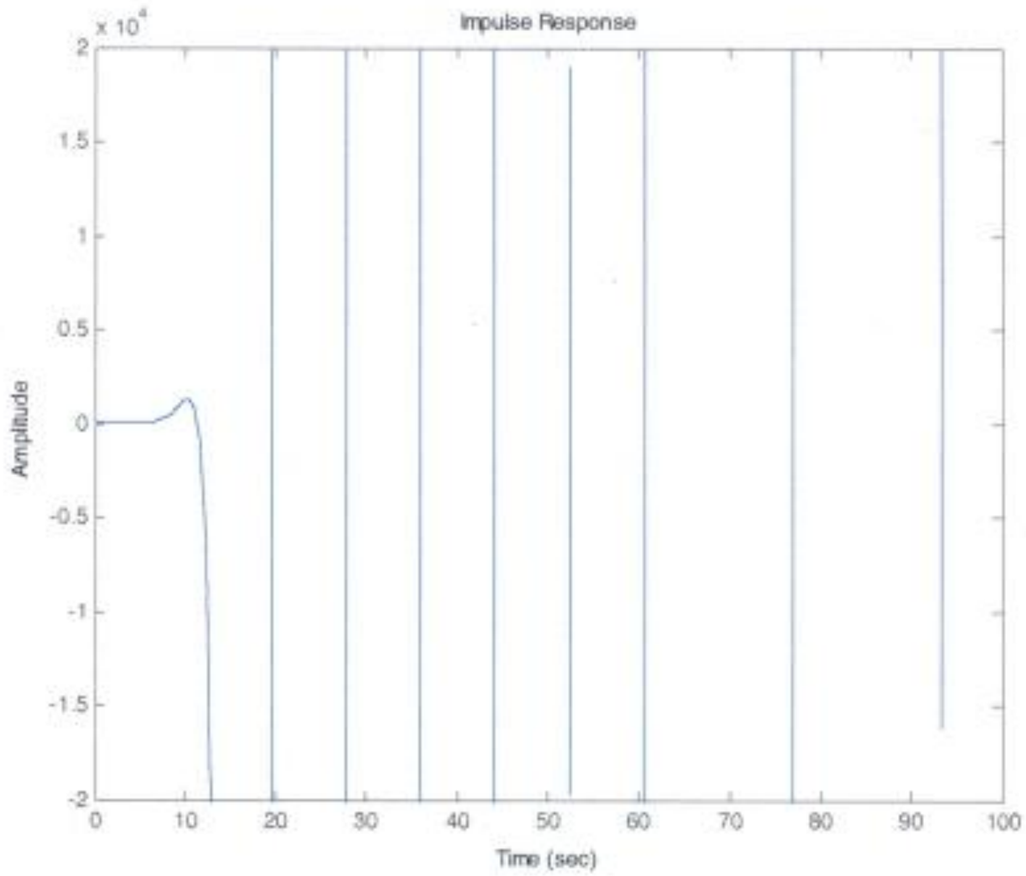
$$|A_1|e^{\alpha_1 t}\cos(\beta_1 t+\theta_1) + |A_2|e^{\alpha_2 t}\cos(\beta_2 t+\theta_2)$$

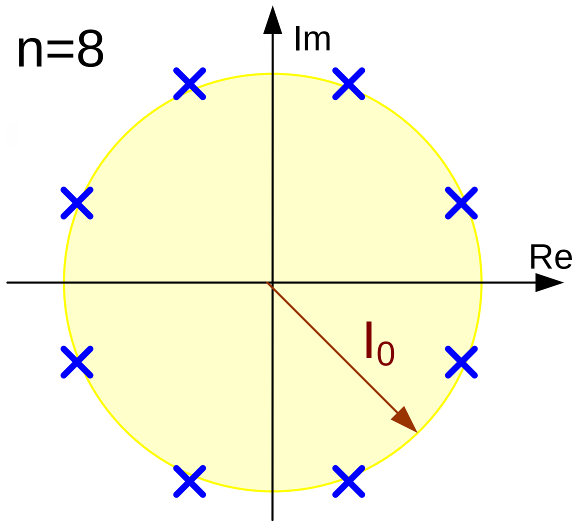
At t=145 (after only 10 periods of the lower frequency signal)

$$r = \left.\frac{e^{\alpha_2 t}}{e^{\alpha_1 t}}\right|_{t=145} = \frac{e^{.9009\bullet 145}}{e^{.2225\bullet 145}} = 5.2\text{x}10^{42}$$

The lower frequency oscillation will completely dominate !

What will happen with a circuit that has two pole-pairs in the RHP?





Thanks to Chen for these plots

Figure 14 N=8 impulse response

Can only see the lower frequency component !

What will happen with a circuit that has two pole-pairs in the RHP?

Thanks to Chen for these plots

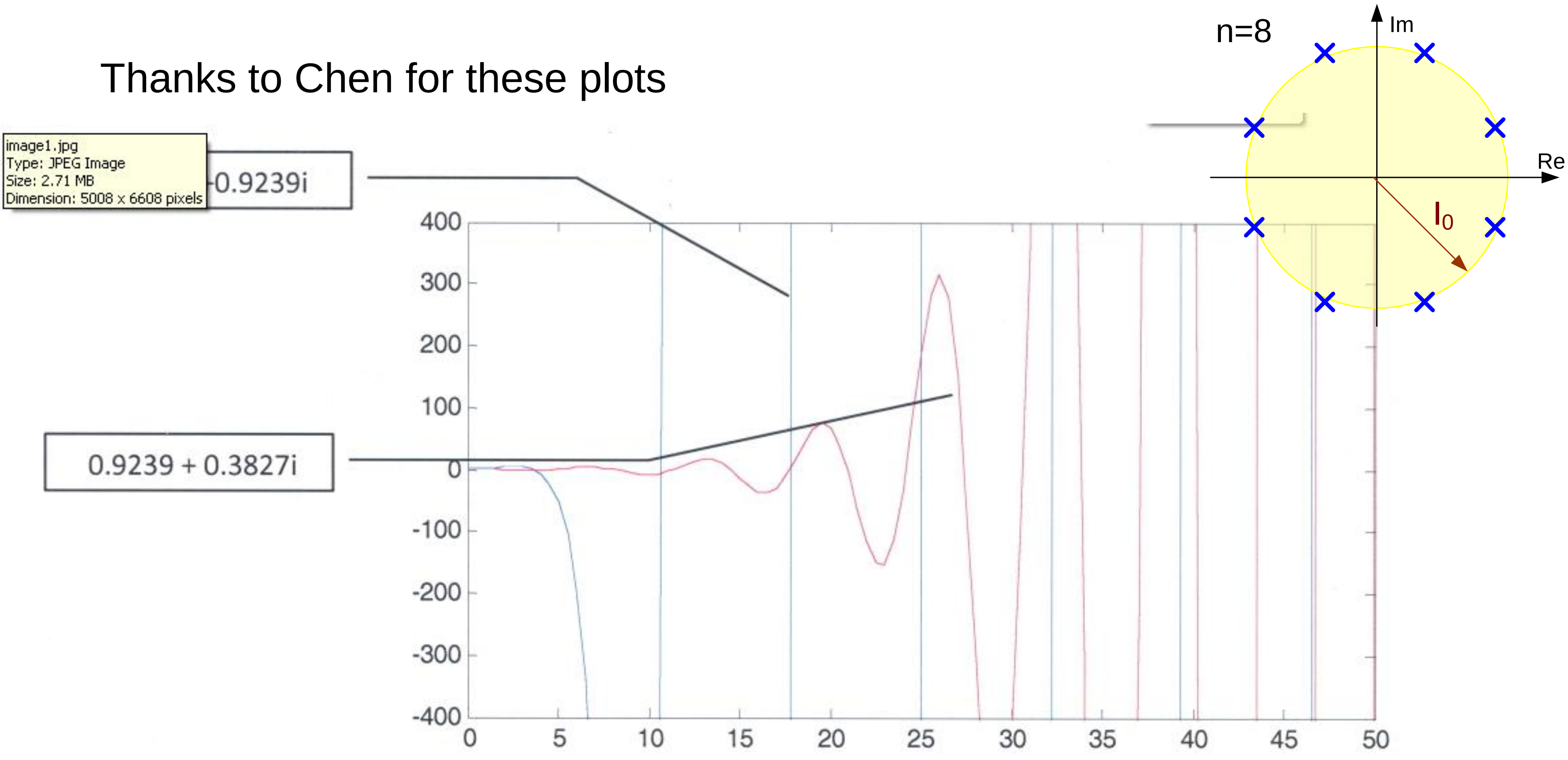
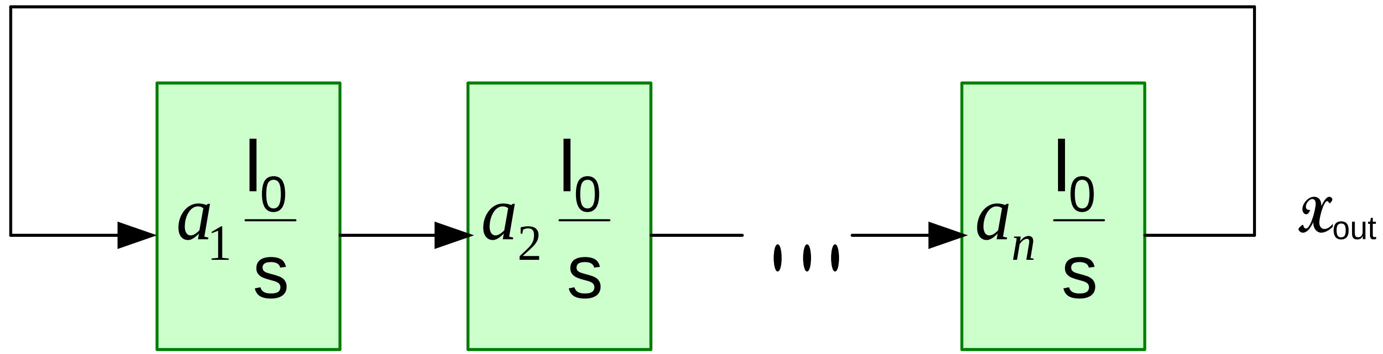


Figure 7 N=8 the impulse response of two poles

After even only two periods of the lower frequency waveform, it completely dominates !

How do we guarantee that we have a net coefficient of +1 in D(s)?

$$D(s) = s^n + I_0^n$$



$$x_{out} = \left(\prod_{i=1}^{n} a_i \left(\frac{I_0}{s} \right) \right) x_{out} \qquad a_i \in \{-1, 1\}$$

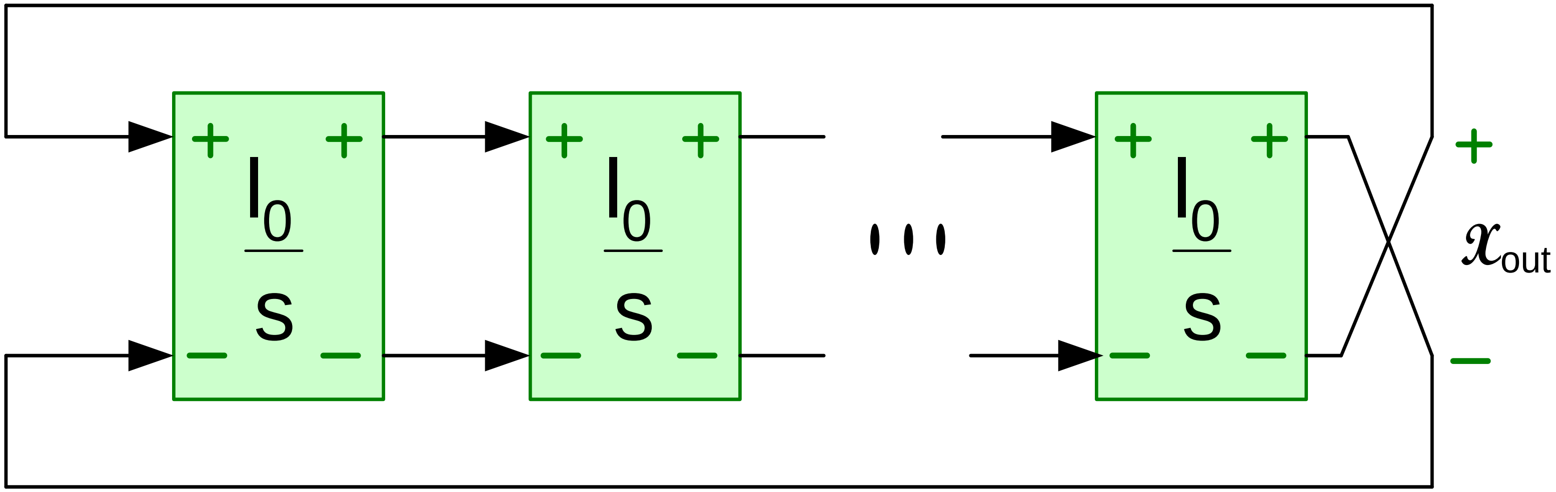
$$D(s) = s^n - \left(\prod_{i=1}^{n} a_i \right) I_0^n \longrightarrow \prod_{i=1}^{n} a_i = -1$$

Must have an odd number of inversions in the loop !

If n is odd, all stages can be inverting and identical !

How do we guarantee that we have a net coefficient of +1 in D(s)?

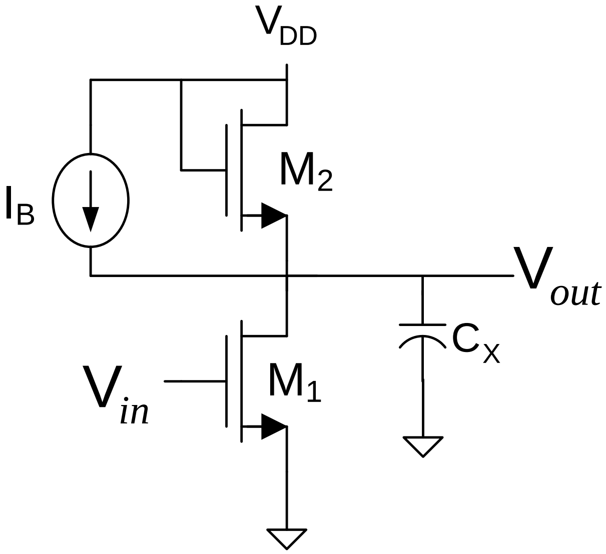
$$D(s) = s^n + I_0^n$$



If fully differential or fully balanced, must have an odd number of crossings of outputs

Applicable for both even and odd order loops

A lossy integrator stage

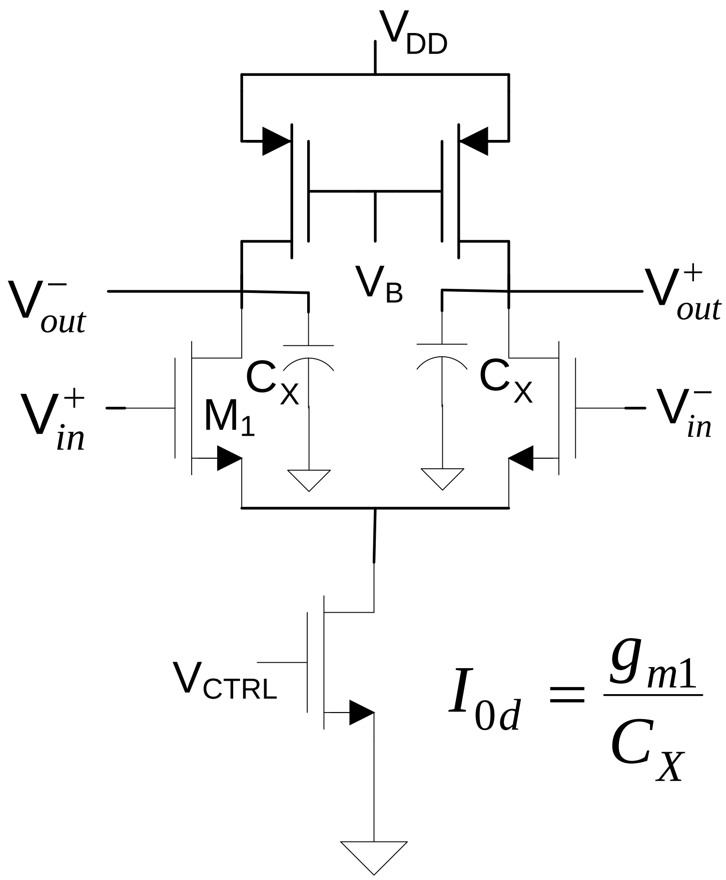


$$I(s) = \frac{-g_{m1}/C_X}{s+g_{m2}/C_X}$$

$$I_0 = g_{m1}/C_X$$

$$\alpha_L = g_{m2}/C_X$$

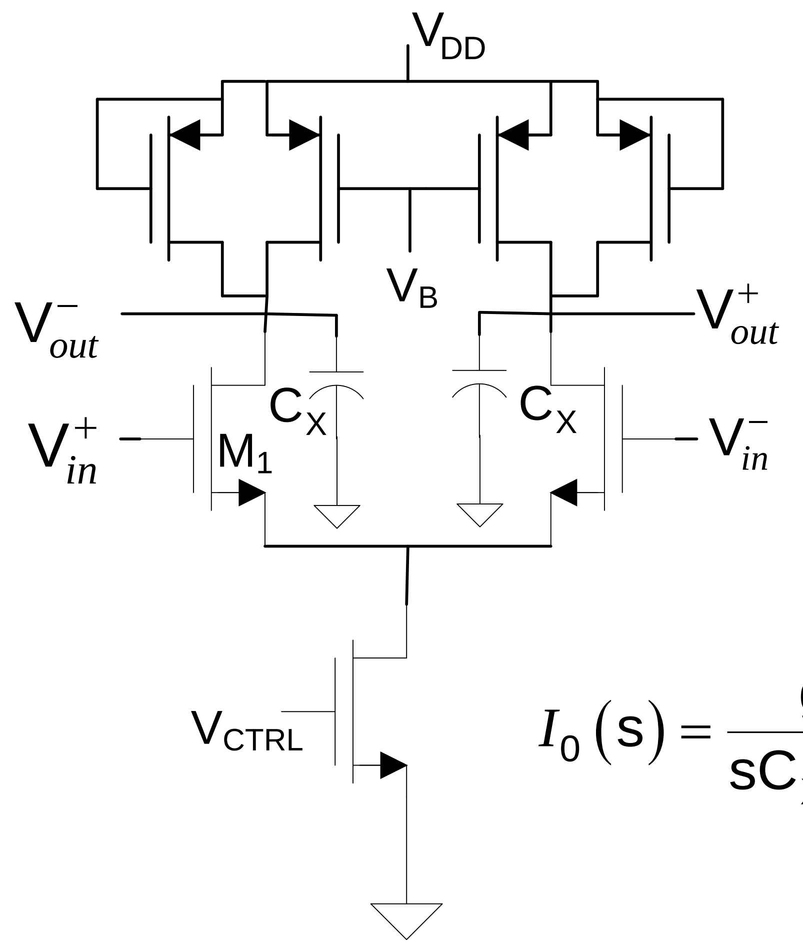
A fully-differential voltage-controlled integrator stage



$$I_{0d} = \frac{g_{m1}}{C_X}$$

Will need CMFB circuit

A fully-differential voltage-controlled integrator stage with loss



$$I_0(s) = \frac{g_{m1}}{sC_X + g_{m3}}$$

Will need CMFB circuit

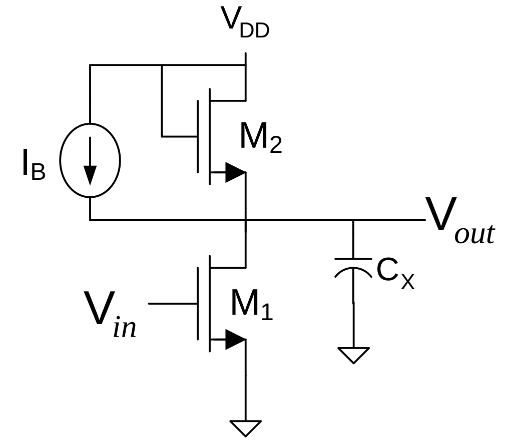
Example:

Using the single-stage lossy integrator, design the integrator to meet a given ω_0 and Q requirement

Recall:

$$I_0 = \frac{\omega_0}{(\sin\theta)2Q}\sqrt{4Q^2-1} \qquad (1)$$

$$\alpha_L = \frac{\omega_0}{2Q} + \frac{\omega_0}{2Q(\tan\theta)}\sqrt{4Q^2-1} \qquad (2)$$



$$I_0 = g_{m1}/C_X$$

$$\alpha_L = g_{m2}/C_X$$

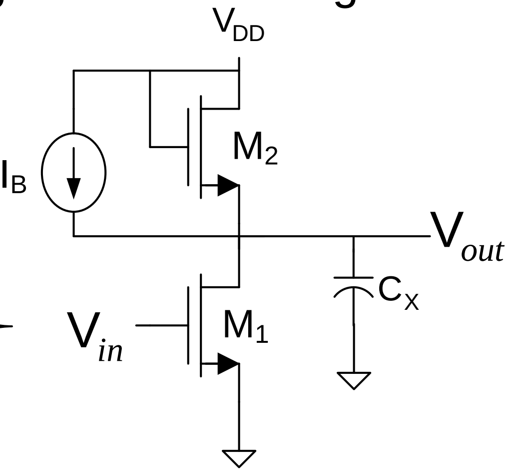
Substituting for I_0 and α_L we obtain:

$$\frac{g_{m1}}{C_X} = \frac{\omega_0}{(\sin\theta)2Q}\sqrt{4Q^2-1} \qquad (3)$$

$$\frac{g_{m2}}{C_X} = \frac{\omega_0}{2Q} + \frac{\omega_0}{2Q(\tan\theta)}\sqrt{4Q^2-1} \qquad (4)$$

Example:

Using the single-stage lossy integrator, design the integrator to meet a given ω_0 and Q requirement



Expressing g_{m1} and g_{m2} in terms of design parameters:

$$\frac{\mu C_{OX} W_1 V_{EB1}}{L_1 C_X} = \frac{\omega_0}{(\sin\theta) 2Q}\sqrt{4Q^2-1} \qquad (5)$$

$$\frac{\mu C_{OX} W_2 V_{EB2}}{L_2 C_X} = \frac{\omega_0}{2Q} + \frac{\omega_0}{2Q(\tan\theta)}\sqrt{4Q^2-1} \qquad (6)$$

$$I_0 = g_{m1}/C_X$$

$$\alpha_L = g_{m2}/C_X$$

If we assume I_B=0, equating drain currents obtain:

$$V_{EB2} = V_{EB1}\sqrt{\frac{W_1 L_2}{W_2 L_1}} \qquad (7)$$

Thus the previous two expressions can be rewritten as :

$$\frac{\mu C_{OX} V_{EB1}}{C_X}\left[\frac{W_1}{L_1}\right] = \frac{\omega_0}{(\sin\theta) 2Q}\sqrt{4Q^2-1} \qquad (8)$$

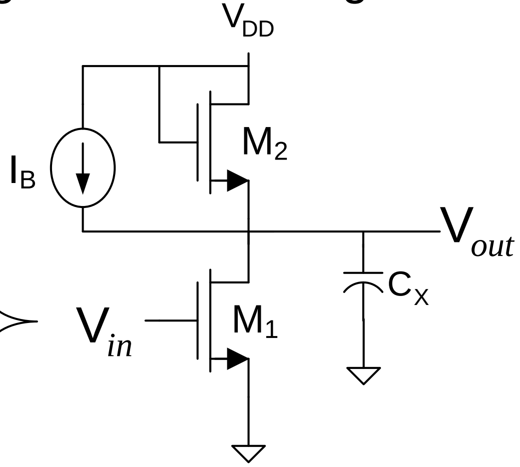
$$\frac{\mu C_{OX} V_{EB1}}{C_X}\left[\frac{W_1 W_2}{L_1 L_2}\right] = \frac{\omega_0}{2Q} + \frac{\omega_0}{2Q(\tan\theta)}\sqrt{4Q^2-1} \qquad (9)$$

Example:

Using the single-stage lossy integrator, design the integrator to meet a given ω_0 and Q requirement

$$\frac{\mu C_{OX} V_{EB1}}{C_X}\left[\frac{W_1}{L_1}\right] = \frac{\omega_0}{(\sin\theta)2Q}\sqrt{4Q^2\text{-}1} \qquad (8)$$

$$\frac{\mu C_{OX} V_{EB1}}{C_X}\left[\frac{W_1 W_2}{L_1 L_2}\right] = \frac{\omega_0}{2Q} + \frac{\omega_0}{2Q(\tan\theta)}\sqrt{4Q^2\text{-}1} \qquad (9)$$



$$I_0 = g_{m1}/C_X$$

$$\alpha_L = g_{m2}/C_X$$

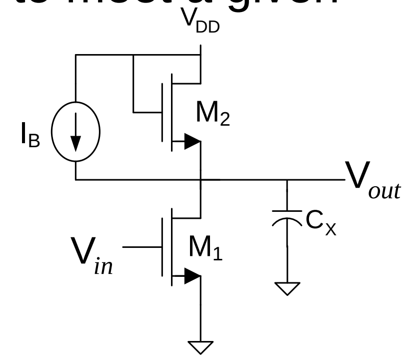
Taking the ratio of these two equations we obtain:

$$\frac{W_2}{L_2} = \frac{\sin\theta + \cos\theta\sqrt{4Q^2\text{-}1}}{\sqrt{4Q^2\text{-}1}} \qquad (10)$$

Observe that the pole Q is determined by the dimensions of the lossy device !

Example:

Using the single-stage lossy integrator, design the integrator to meet a given ω_0 and Q requirement



$$\frac{\mu C_{OX} V_{EB1}}{C_X}\left[\frac{W_1}{L_1}\right]=\frac{\omega_0}{(\sin\theta)2Q}\sqrt{4Q^2-1} \qquad (8)$$

$$\frac{W_2}{L_2}=\frac{\sin\theta+\cos\theta\sqrt{4Q^2-1}}{\sqrt{4Q^2-1}} \qquad (10)$$

Still must obtain W_1/L_1, V_{EB1}, and C_X from either of these equations

Although it appears that there might be 3 degrees of freedom left and only one constraint (one of these equations), if these integrators are connected in a loop, the operating point (Q-point) will be the same for all stages and will be that value where $V_{out}=V_{in}$. So, this adds a second constraint.

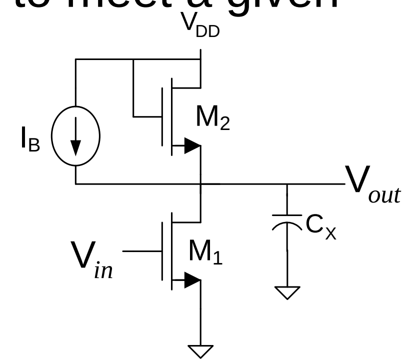
Setting $V_{out}=V_{in}$, and assuming $V_{T1}=V_{T2}$, we obtain from KVL

$$V_{DD}=V_{EB1}+V_{EB2}+2V_T \qquad (11)$$

But V_{EB1} and V_{EB2} are also related in (7)

Example:

Using the single-stage lossy integrator, design the integrator to meet a given ω_0 and Q requirement



$$\frac{\mu C_{OX} V_{EB1}}{C_X}\left[\frac{W_1}{L_1}\right]=\frac{\omega_0}{(\sin\theta)2Q}\sqrt{4Q^2\text{-}1} \quad (8)$$

$$\frac{W_2}{L_2}=\frac{\sin\theta+\cos\theta\sqrt{4Q^2\text{-}1}}{\sqrt{4Q^2\text{-}1}} \quad (10)$$

Still must obtain W_1/L_1, V_{EB1}, and C_X from either of these equations

$$V_{DD}=V_{EB1}+V_{EB2}+2V_T \quad (11)$$

$$V_{EB2}=V_{EB1}\sqrt{\frac{W_1L_2}{W_2L_1}} \quad (7)$$

→

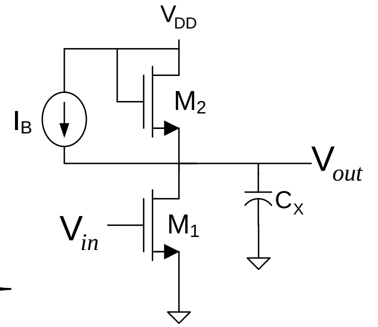
$$V_{EB1}=\frac{V_{DD}\text{-}2V_T}{1+\sqrt{\frac{W_2L_1}{W_1L_2}}} \quad (12)$$

Substituting (10) into (12) and then into (8) we obtain

$$\frac{\mu C_{OX}}{C_X}\left[\frac{W_1}{L_1}\right]\left(\frac{V_{DD}-2V_T}{1+\sqrt{\left(\frac{W_1}{L_1}\right)^{-1}\left(\frac{\sin\theta+\cos\theta\sqrt{4Q^2\text{-}1}}{\sqrt{4Q^2\text{-}1}}\right)}}\right)=\frac{\omega_0}{(\sin\theta)2Q}\sqrt{4Q^2\text{-}1} \quad (13)$$

Example:

Using the single-stage lossy integrator, design the integrator to meet a given ω_0 and Q requirement



$$\frac{W_2}{L_2} = \frac{\sin\theta + \cos\theta\sqrt{4Q^2\text{-}1}}{\sqrt{4Q^2\text{-}1}} \quad (10)$$

$$\frac{\mu C_{OX}}{C_X}\left[\frac{W_1}{L_1}\right]\left(\frac{V_{DD} - 2V_T}{1+\sqrt{\left(\frac{W_1}{L_1}\right)^{-1}\left(\frac{\sin\theta+\cos\theta\sqrt{4Q^2\text{-}1}}{\sqrt{4Q^2\text{-}1}}\right)}}\right) = \frac{\omega_0}{(\sin\theta)2Q}\sqrt{4Q^2\text{-}1} \quad (13)$$

There is still one degree of freedom remaining. Can either pick W_1/L_1 and solve for C_X or pick C_X and solve for W_1/L_1.

Explicit expression for W_1/L_1 not available

Tradeoffs between C_X and W_1/L_1 will often be made

Since $V_{OUTQ}=V_T+V_{EB1}$, it may be preferred to pick V_{EB1}, then solve (12) for W_1/L_1 and then solve (13) for C_X

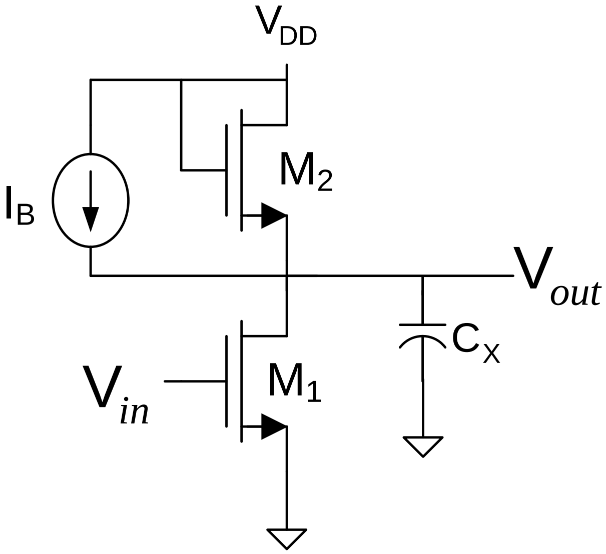
Adding I_B will provide one additional degree of freedom and will relax the relationship between V_{OUTQ} and W_1/L_1 since (7) will be modified

EE 508

Lecture 37

High Frequency Filters

A lossy integrator stage

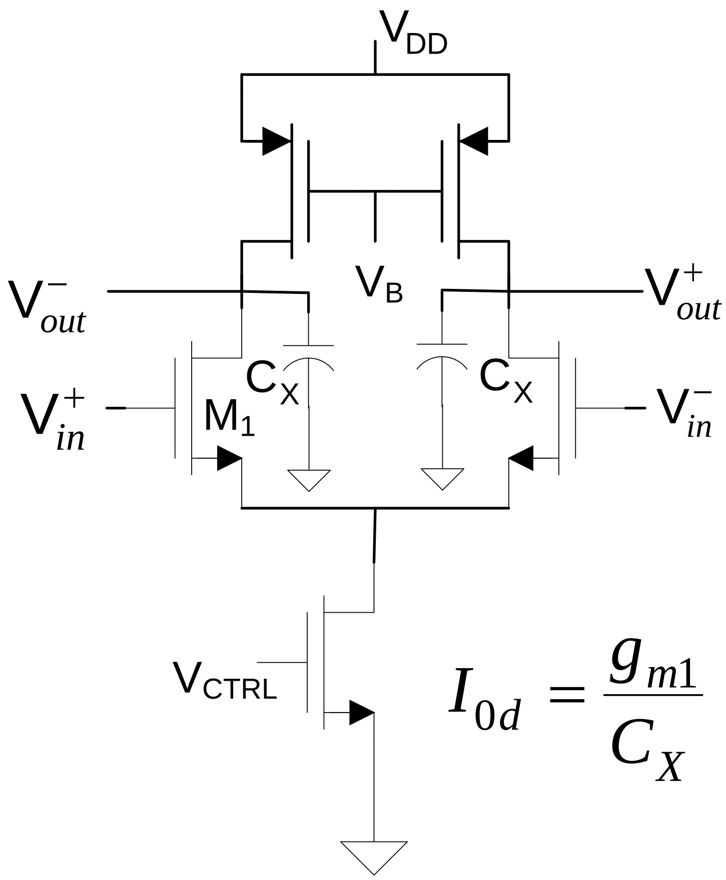


$$I(s) = \frac{-g_{m1}/C_X}{s+g_{m2}/C_X}$$

$$I_0 = g_{m1}/C_X$$

$$\alpha_L = g_{m2}/C_X$$

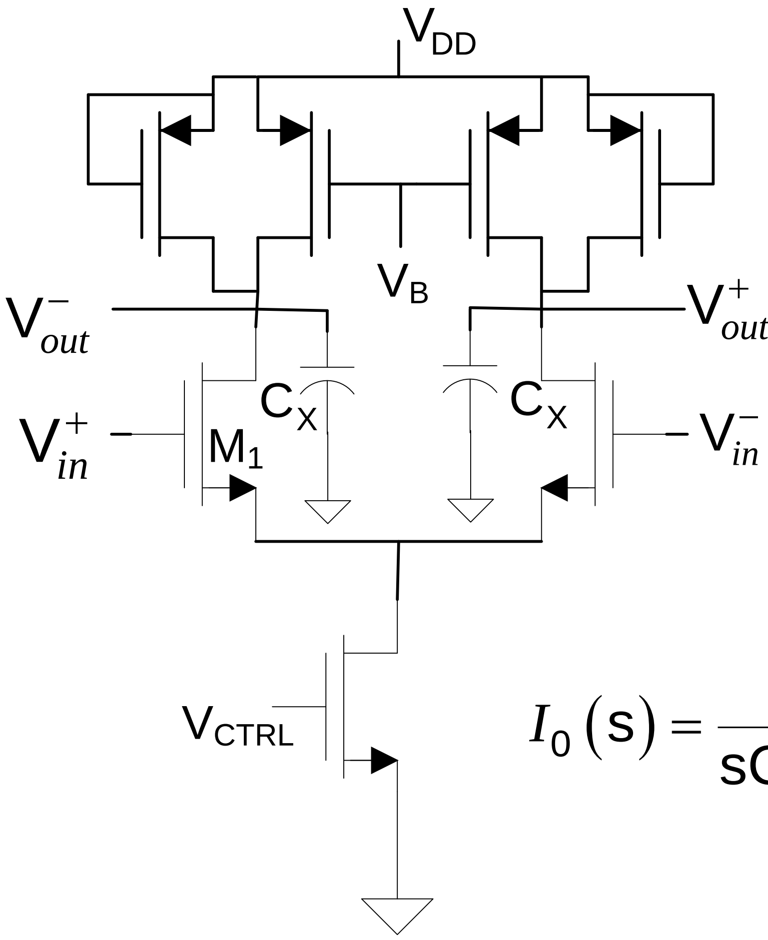
A fully-differential voltage-controlled integrator stage



$$I_{0d} = \frac{g_{m1}}{C_X}$$

Will need CMFB circuit

A fully-differential voltage-controlled integrator stage with loss



$$I_0(s) = \frac{g_{m1}}{sC_X + g_{m3}}$$

Will need CMFB circuit

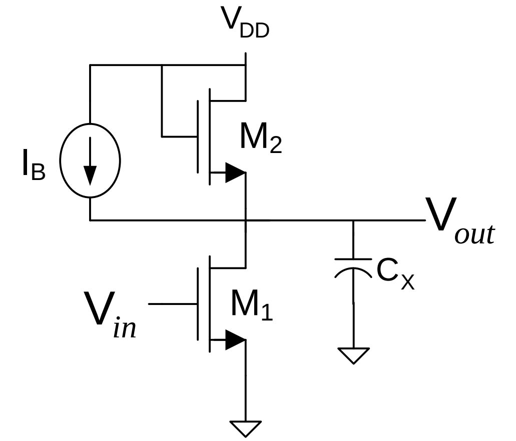
Example:

Using the single-stage lossy integrator, design the integrator to meet a given ω_0 and Q requirement

Recall:

$$I_0 = \frac{\omega_0}{(\sin\theta)2Q}\sqrt{4Q^2-1} \quad (1)$$

$$\alpha_L = \frac{\omega_0}{2Q} + \frac{\omega_0}{2Q(\tan\theta)}\sqrt{4Q^2-1} \quad (2)$$



$$I_0 = g_{m1}/C_X$$

$$\alpha_L = g_{m2}/C_X$$

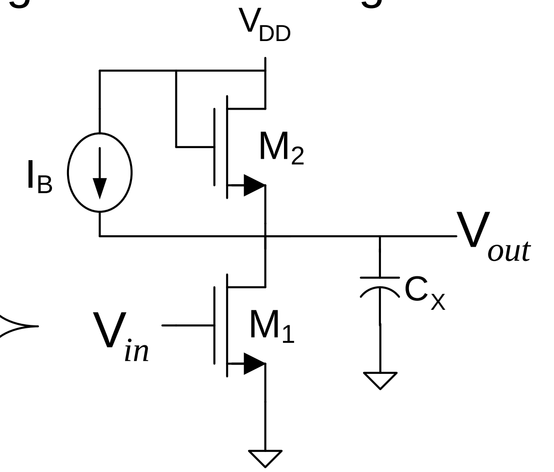
Substituting for I_0 and α_L we obtain:

$$\frac{g_{m1}}{C_X} = \frac{\omega_0}{(\sin\theta)2Q}\sqrt{4Q^2-1} \quad (3)$$

$$\frac{g_{m2}}{C_X} = \frac{\omega_0}{2Q} + \frac{\omega_0}{2Q(\tan\theta)}\sqrt{4Q^2-1} \quad (4)$$

Example:

Using the single-stage lossy integrator, design the integrator to meet a given ω_0 and Q requirement



Expressing g_{m1} and g_{m2} in terms of design parameters:

$$\frac{\mu C_{OX} W_1 V_{EB1}}{L_1 C_X} = \frac{\omega_0}{(\sin\theta)2Q}\sqrt{4Q^2-1} \quad (5)$$

$$\frac{\mu C_{OX} W_2 V_{EB2}}{L_2 C_X} = \frac{\omega_0}{2Q} + \frac{\omega_0}{2Q(\tan\theta)}\sqrt{4Q^2-1} \quad (6)$$

$$I_0 = g_{m1}/C_X$$

$$\alpha_L = g_{m2}/C_X$$

If we assume I_B=0, equating drain currents obtain:

$$V_{EB2} = V_{EB1}\sqrt{\frac{W_1 L_2}{W_2 L_1}} \quad (7)$$

Thus the previous two expressions can be rewritten as :

$$\frac{\mu C_{OX} V_{EB1}}{C_X}\left[\frac{W_1}{L_1}\right] = \frac{\omega_0}{(\sin\theta)2Q}\sqrt{4Q^2-1} \quad (8)$$

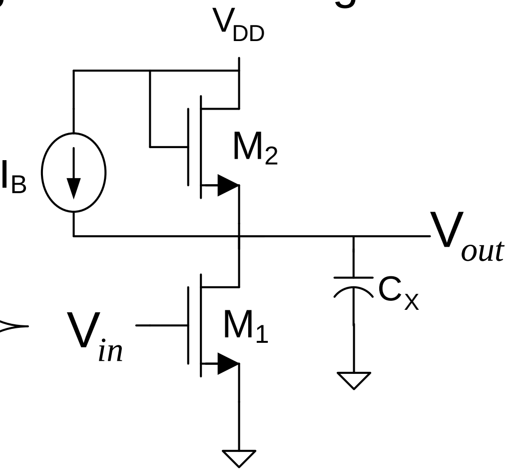
$$\frac{\mu C_{OX} V_{EB1}}{C_X}\left[\frac{W_1 W_2}{L_1 L_2}\right] = \frac{\omega_0}{2Q} + \frac{\omega_0}{2Q(\tan\theta)}\sqrt{4Q^2-1} \quad (9)$$

Example:

Using the single-stage lossy integrator, design the integrator to meet a given ω_0 and Q requirement

$$\frac{\mu C_{OX} V_{EB1}}{C_X}\left[\frac{W_1}{L_1}\right] = \frac{\omega_0}{(\sin\theta) 2Q}\sqrt{4Q^2 - 1} \quad (8)$$

$$\frac{\mu C_{OX} V_{EB1}}{C_X}\left[\frac{W_1 W_2}{L_1 L_2}\right] = \frac{\omega_0}{2Q} + \frac{\omega_0}{2Q(\tan\theta)}\sqrt{4Q^2 - 1} \quad (9)$$



$$I_0 = g_{m1}/C_X$$

$$\alpha_L = g_{m2}/C_X$$

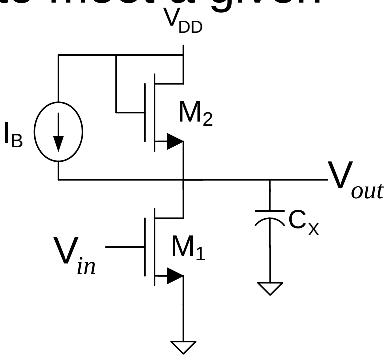
Taking the ratio of these two equations we obtain:

$$\frac{W_2}{L_2} = \frac{\sin\theta + \cos\theta\sqrt{4Q^2 - 1}}{\sqrt{4Q^2 - 1}} \quad (10)$$

Observe that the pole Q is determined by the dimensions of the lossy device !

Example:

Using the single-stage lossy integrator, design the integrator to meet a given ω_0 and Q requirement



$$\frac{\mu C_{OX} V_{EB1}}{C_X}\left[\frac{W_1}{L_1}\right]=\frac{\omega_0}{(\sin\theta)2Q}\sqrt{4Q^2-1} \quad (8)$$

$$\frac{W_2}{L_2}=\frac{\sin\theta+\cos\theta\sqrt{4Q^2-1}}{\sqrt{4Q^2-1}} \quad (10)$$

Still must obtain W_1/L_1, V_{EB1}, and C_X from either of these equations

Although it appears that there might be 3 degrees of freedom left and only one constraint (one of these equations), if these integrators are connected in a loop, the operating point (Q-point) will be the same for all stages and will be that value where $V_{out}=V_{in}$. So, this adds a second constraint.

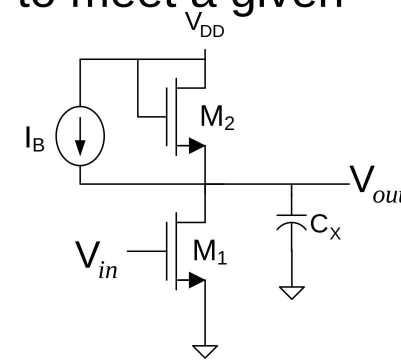
Setting $V_{out}=V_{in}$, and assuming $V_{T1}=V_{T2}$, we obtain from KVL

$$V_{DD}=V_{EB1}+V_{EB2}+2V_T \quad (11)$$

But V_{EB1} and V_{EB2} are also related in (7)

Example:

Using the single-stage lossy integrator, design the integrator to meet a given ω_0 and Q requirement



$$\frac{\mu C_{OX} V_{EB1}}{C_X}\left[\frac{W_1}{L_1}\right]=\frac{\omega_0}{(\sin\theta)2Q}\sqrt{4Q^2\text{-}1} \qquad (8)$$

$$\frac{W_2}{L_2}=\frac{\sin\theta+\cos\theta\sqrt{4Q^2\text{-}1}}{\sqrt{4Q^2\text{-}1}} \qquad (10)$$

Still must obtain W_1/L_1, V_{EB1}, and C_X from either of these equations

$$V_{DD}=V_{EB1}+V_{EB2}+2V_T \qquad (11)$$

$$V_{EB2}=V_{EB1}\sqrt{\frac{W_1L_2}{W_2L_1}} \qquad (7)$$

$$\rightarrow \quad V_{EB1}=\frac{V_{DD}\text{-}2V_T}{1+\sqrt{\frac{W_2L_1}{W_1L_2}}} \qquad (12)$$

Substituting (10) into (12) and then into (8) we obtain

$$\frac{\mu C_{OX}}{C_X}\left[\frac{W_1}{L_1}\right]\left(\frac{V_{DD}-2V_T}{1+\sqrt{\left(\frac{W_1}{L_1}\right)^{-1}\left(\frac{\sin\theta+\cos\theta\sqrt{4Q^2\text{-}1}}{\sqrt{4Q^2\text{-}1}}\right)}}\right)=\frac{\omega_0}{(\sin\theta)2Q}\sqrt{4Q^2\text{-}1} \qquad (13)$$

Example:

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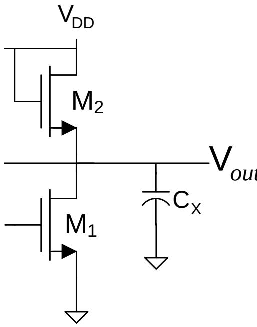
Using the single-stage loss ... iven

ω_0 and Q requirement

Lecture 38

High Frequency Filter Design



$$\frac{W_2}{L_2}=\frac{\sin\theta+\cos\theta\sqrt{4Q^2\text{-}1}}{\sqrt{4Q^2\text{-}1}}$$

$$\frac{\mu C_{OX}}{C_X}\left[\frac{W_1}{L_1}\right]\left(\frac{V_{DD}-2V_T}{1+\sqrt{\left(\frac{W_1}{L_1}\right)^{-1}\left(\frac{\sin\theta+\cos}{\sqrt{4Q^2\text{-}1}}\right)}}\right)$$

There is still one degree of freedom remaining. Can either pick W_1/L_1 and solve for C_X or pick C_X and solve for W_1/L_1.

Explicit expression for W_1/L_1 not available

Tradeoffs between C_X and W_1/L_1 will often be made

Since $V_{OUTQ}=V_T+V_{EB1}$, it may be preferred to pick V_{EB1}, then solve (12) for W_1/L_1 and then solve (13) for C_X

Adding I_B will provide one additional degree of freedom and will relax the relationship between V_{OUTQ} and W_1/L_1 since (7) will be modified

High Frequency Filter Design

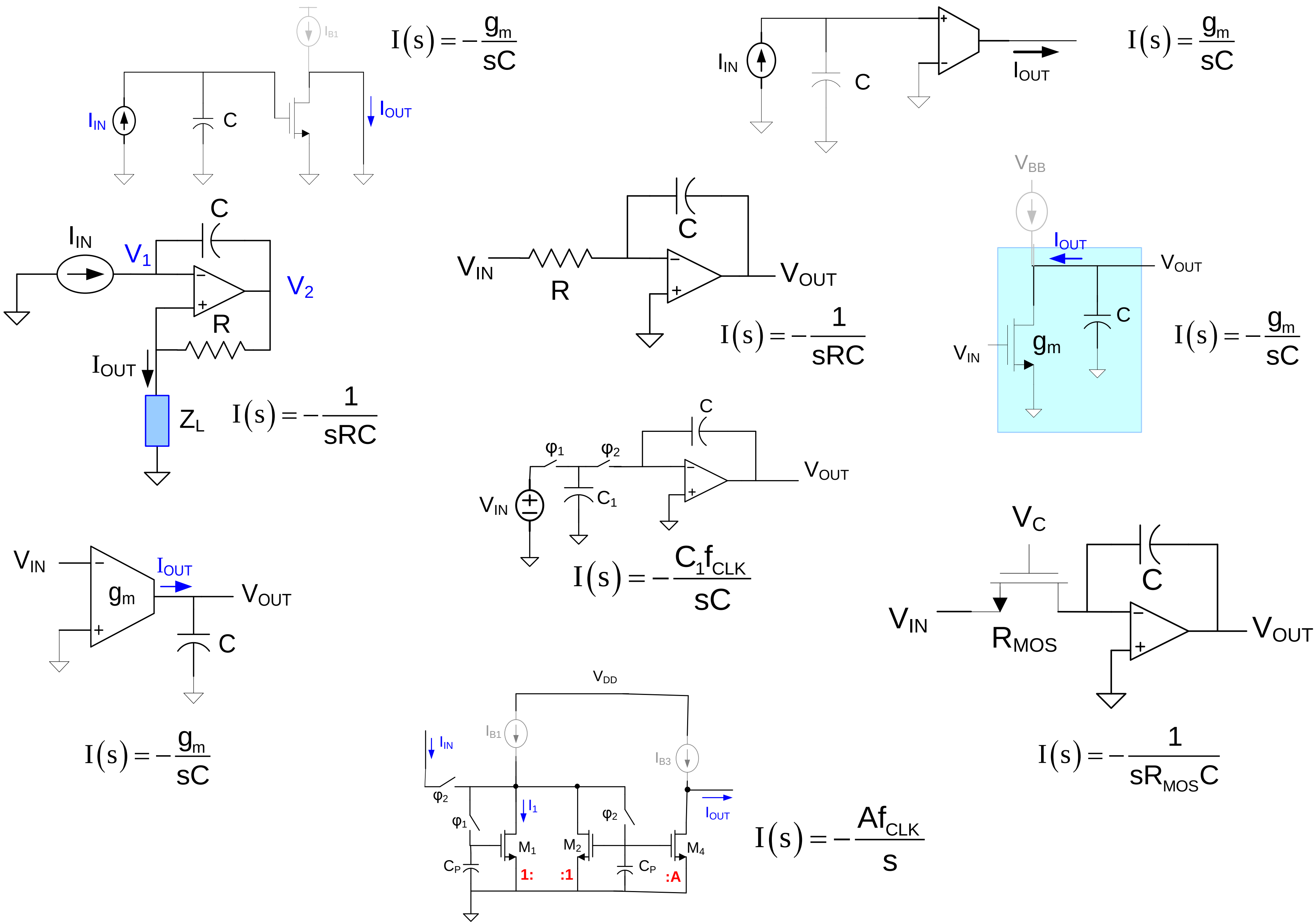
- Architecture selection is critical
- At high frequencies, simplicity of the structures is important
- Parasitic capacitances and their relationship to the time constants that can be achieved provide the ultimate limit on speed
- Will limit discussions to “inductorless” structures

High Frequency Filter Design

Following two methods will provide highest frequency of operation

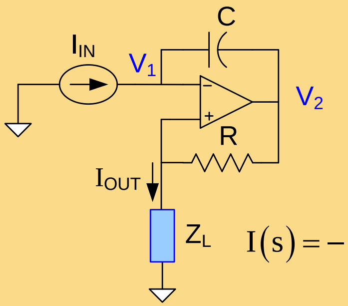
- Degenerate VCOs
- Simple high-frequency integrator-based filters

Integrator Architecture Selection

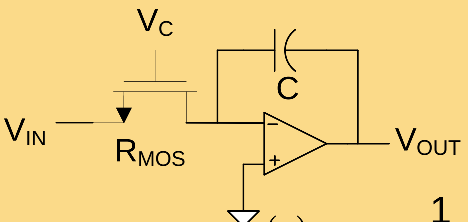


Integrators for High-Speed Operation

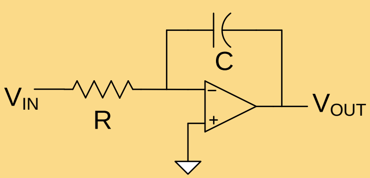
Slow



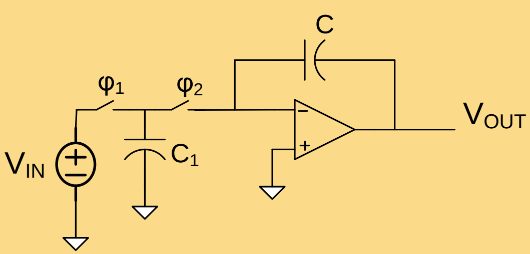
$$I(s) = -\frac{1}{sRC}$$



$$I(s) = -\frac{1}{sR_{MOS}C}$$

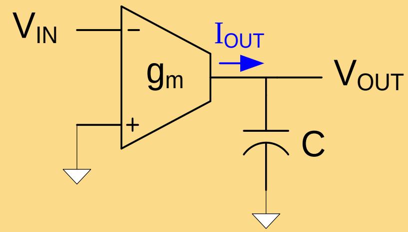


$$I(s) = -\frac{1}{sRC}$$

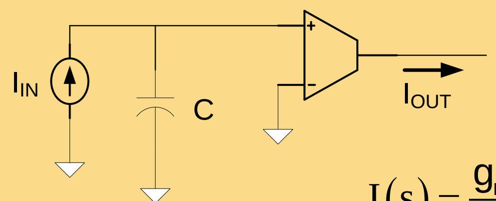


$$I(s) = -\frac{C_1 f_{CLK}}{sC}$$

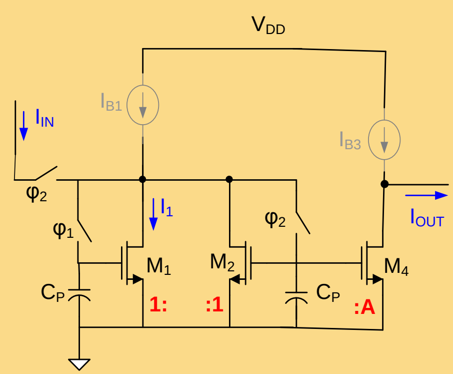
Reasonably Fast



$$I(s) = -\frac{g_m}{sC}$$

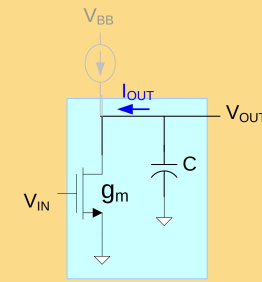


$$I(s) = \frac{g_m}{sC}$$

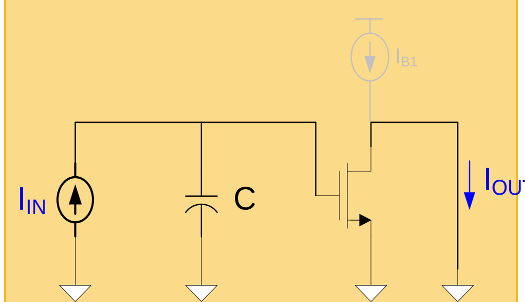


$$I(s) = -\frac{Af_{CLK}}{s}$$

Very Fast



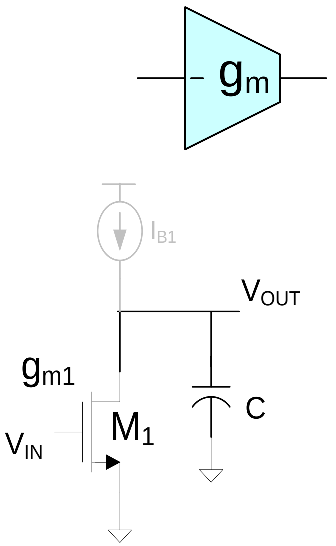
$$I(s) = -\frac{g_m}{sC}$$



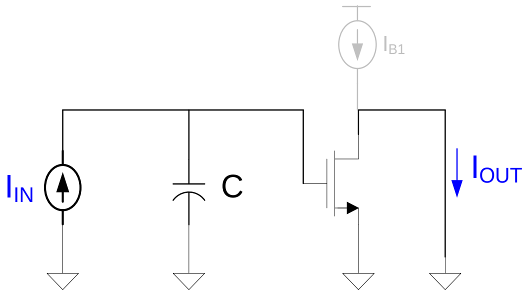
$$I(s) = -\frac{g_m}{sC}$$

Single-ended High-Frequency TA Integrators

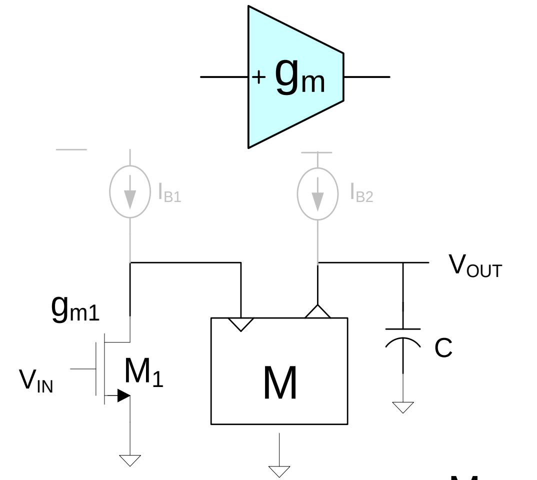
Structures of choice for highest-frequency of operation



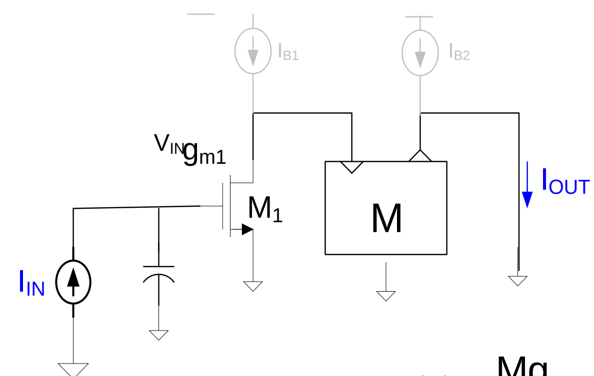
$$I(s) = -\frac{g_m}{sC}$$



$$I(s) = -\frac{g_m}{sC}$$



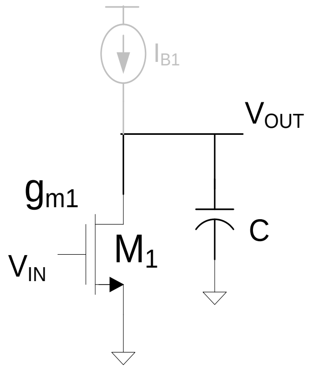
$$I(s) = \frac{Mg_m}{sC}$$



$$I(s) = \frac{Mg_m}{sC}$$

Some authors focus on voltage mode and others on current mode
But overall structures and performance appears to be identical

Single-ended High-Frequency TA Integrators



$$I(s) = -\frac{g_m}{sC}$$

$$I_0 = \frac{g_m}{C}$$

Recall: ω_0 for integrator-based filters generally port ional to I_0

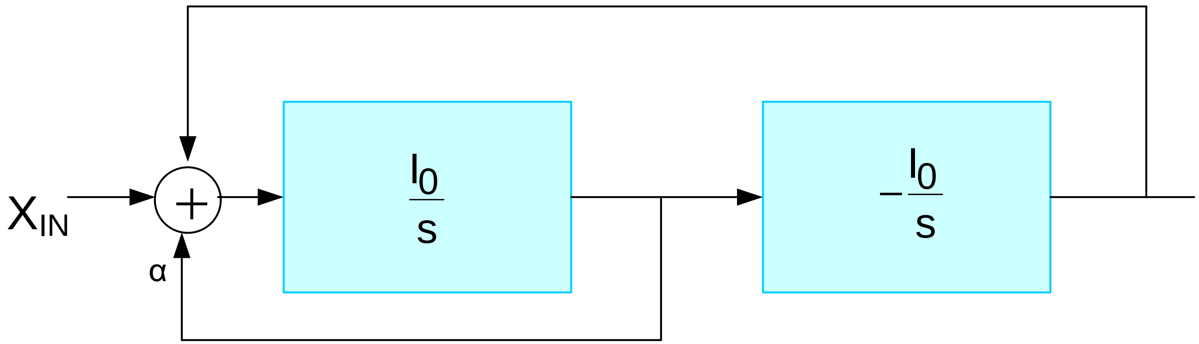
How high can I_0 be?

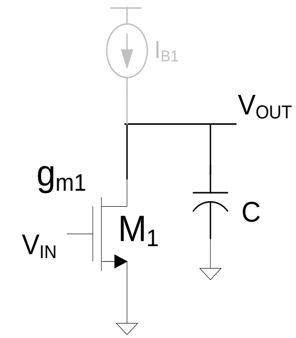
$$I_0 = \frac{\mu C_{OX} W / L V_{EB}}{C}$$

Looks like we can make I_0 as large as we want by making V_{EB} large, C small, L small, and W large

Single-ended High-Frequency TA Integrators

How high can I_0 be?

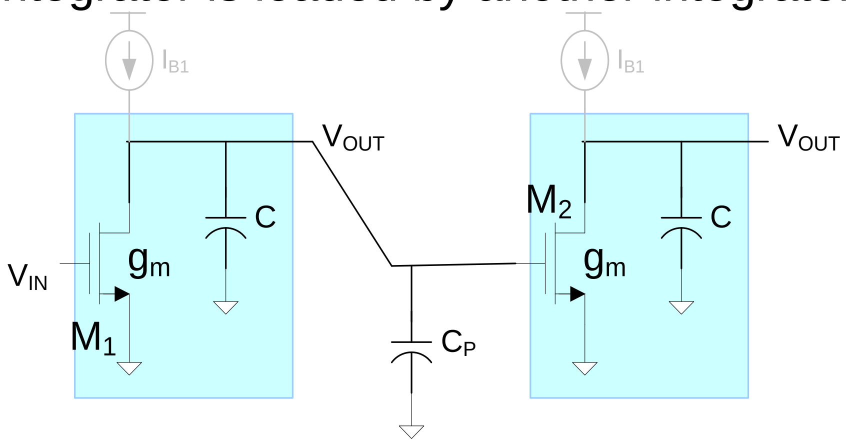




Consider a typical filter – the two integrator loop

$$I_0 = \frac{\mu C_{OX} W / L V_{EB}}{C}$$

Integrator is loaded by another integrator!



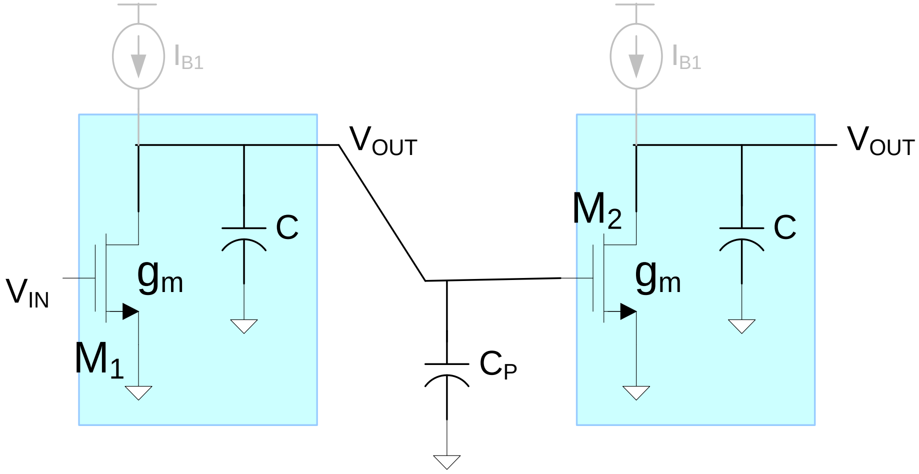
$$I_0 = \frac{\mu C_{OX} W_1 / L_1 V_{EB1}}{C + C_P + C_{OX} W_2 L_2}$$

Even if C goes to 0, I_0 is limited!

C_P is the parasitic capacitances on the output node

Single-ended High-Frequency TA Integrators

How high can I_0 be?



$$I_0 = \frac{\mu C_{OX} W_1 / L_1 V_{EB1}}{C + C_P + C_{OX} W_2 L_2}$$

Setting C to 0 and assuming C_p is small,

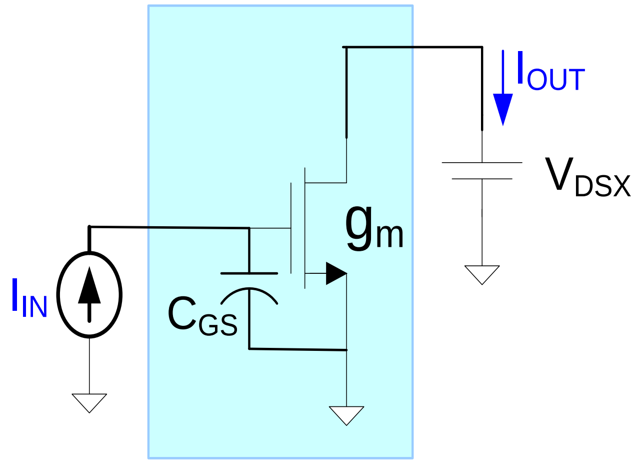
$$I_0 = \frac{\mu C_{OX} W_1 / L_1 V_{EB1}}{C_{OX} W_2 L_2}$$

$$I_0 = \frac{\mu W_1 V_{EB1}}{W_2 L_1 L_2}$$

Assuming the integrator stages are identical, it follows that

$$I_0 = \frac{\mu V_{EB1}}{L_{min}^2}$$

Transition (transit) frequency (f_T) of a process



The transit frequency of a process is the frequency where the short-circuit current gain of the common-source configuration drops to 1.

$$i_{OUT} = g_m v_{gs}$$

$$i_{IN} \bullet \frac{1}{sC_{GS}} = v_{gs}$$

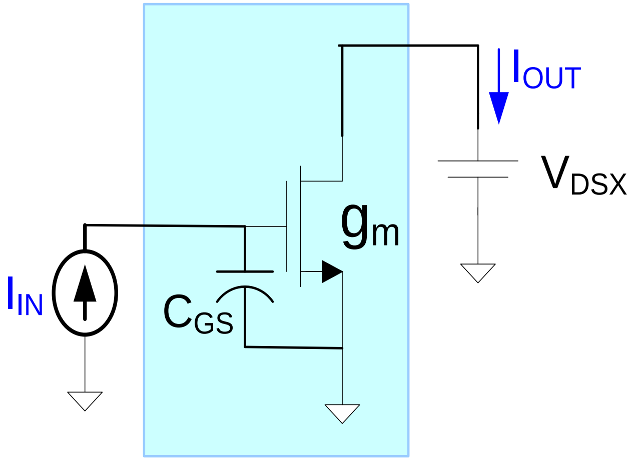
$$\frac{i_{OUT}}{i_{IN}} = \frac{g_m}{sC_{GS}}$$

$$1 = \frac{g_m}{C_{GS}\omega_T}$$

$$\omega_T = \frac{g_m}{C_{GS}} = \frac{\left(\mu C_{OX} \frac{W}{L} V_{EB}\right)}{C_{OX} WL} = \frac{\mu V_{EB}}{L^2}$$

$$\omega_T = \frac{\mu V_{EB}}{L_{min}^2}$$

Transition (transit) frequency (f_T) of a process



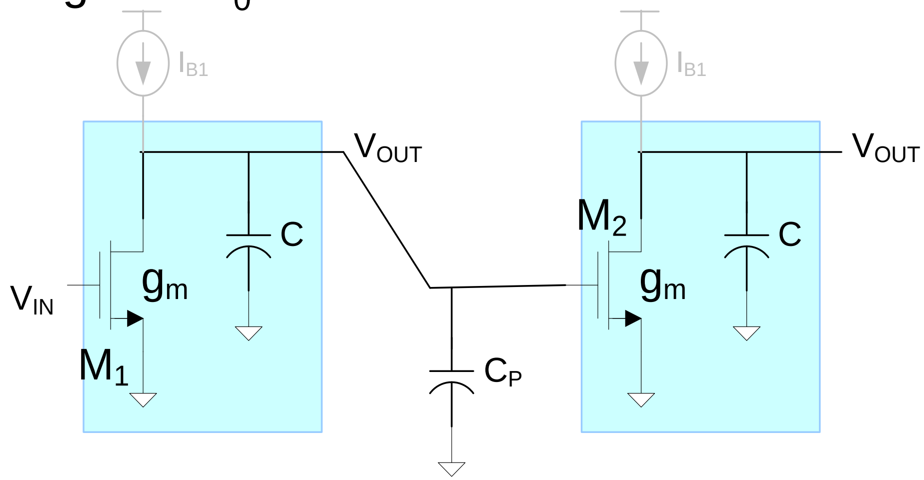
The transit frequency of a process is the frequency where the short-circuit current gain of the common-source configuration drops to 1.

$$\omega_T = \frac{\mu V_{EB}}{L_{min}^2}$$

This is dependent upon V_{EB}
Does not include effects of diffusion capacitances or overlap capacitances
f_{MAX} is another figure that characterizes the speed of a process

Single-ended High-Frequency TA Integrators

How high can I_0 be?



$$I_{0M} = \frac{\mu V_{EB1}}{L_{min}^2}$$

$$I_{0M} = \omega_T$$

Speed of operation increases with V_{EB1}

V_{EB1} is limited by signal swing requirements and V_{DD}

Signal Swing:

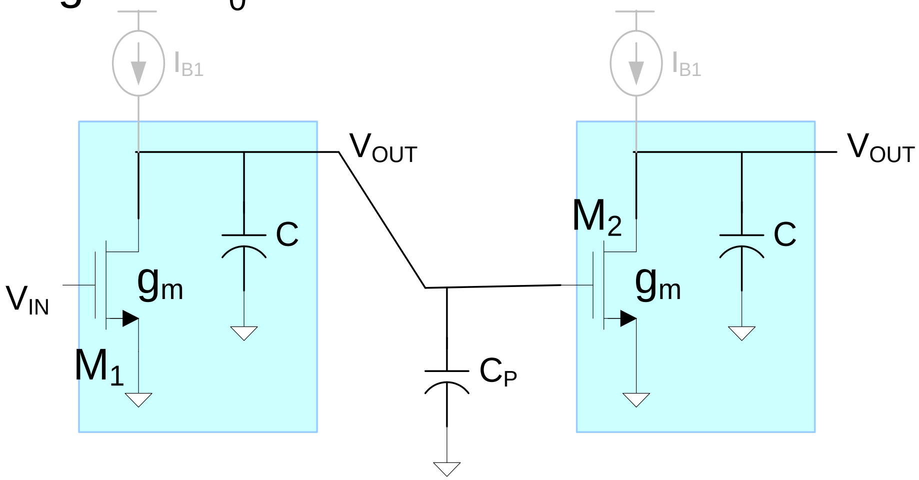
$$V_{SW\text{-}0P} \simeq \min\{V_{DD} - V_{OQ}, V_{OQ} - (V_T + 100mV)\}$$

$$V_{OQ} = V_T + V_{EB}$$

$$V_{SW\text{-}0P} \simeq \min\{V_{DD} - V_T - V_{EB}, V_T + V_{EB} - (V_T + 100mV)\}$$

Single-ended High-Frequency TA Integrators

How high can I_0 be?



$$I_{0M} = \frac{\mu V_{EB1}}{L_{min}^2}$$

$$I_{0M} = \omega_T$$

Speed of operation increases with V_{EB}

V_{EB} is limited by signal swing requirements and V_{DD}

Signal Swing:

$$V_{DD} - V_T - V_{EB} = V_T + V_{EB} - (V_T + 100mV)$$

$$V_{EB} = \frac{V_{DD} + 100mV - V_T}{2}$$

$$I_{OMAX} \simeq \frac{\mu\left(V_{DD} + 100mV - V_T\right)}{2L_{min}^2}$$

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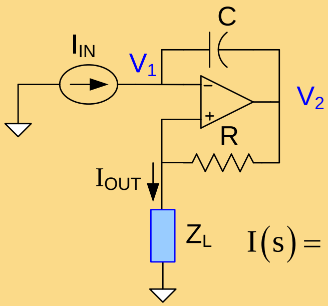
Lecture 38

High Frequency Filters

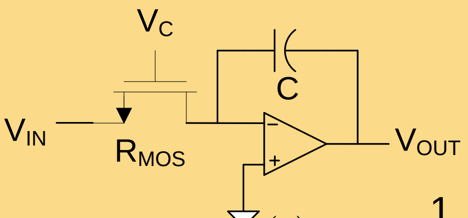
Integrators for High-Speed Operation

Review from last time

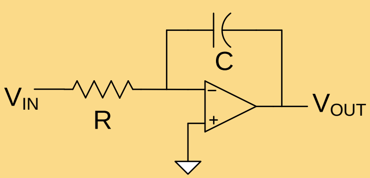
Slow



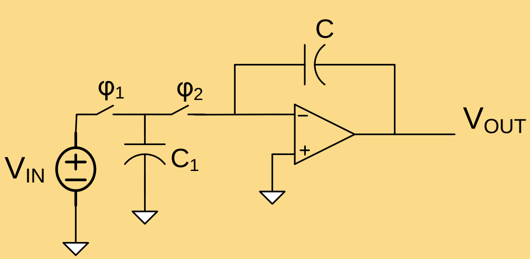
$$I(s) = -\frac{1}{sRC}$$



$$I(s) = -\frac{1}{sR_{MOS}C}$$

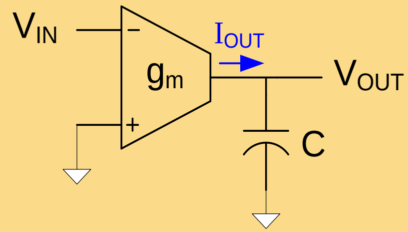


$$I(s) = -\frac{1}{sRC}$$

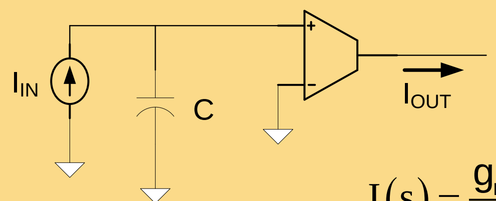


$$I(s) = -\frac{C_1 f_{CLK}}{sC}$$

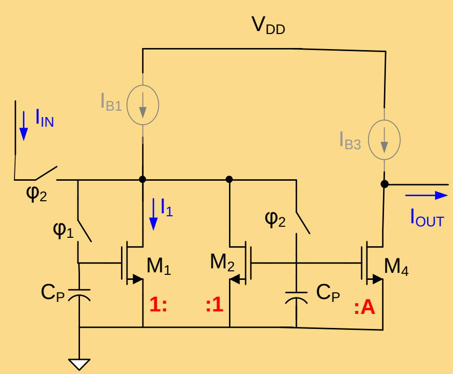
Reasonably Fast



$$I(s) = -\frac{g_m}{sC}$$

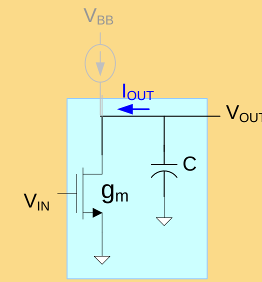


$$I(s) = \frac{g_m}{sC}$$

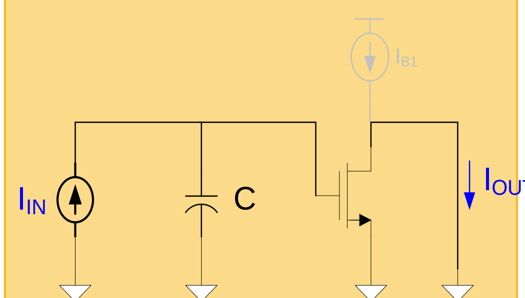


$$I(s) = -\frac{Af_{CLK}}{s}$$

Very Fast



$$I(s) = -\frac{g_m}{sC}$$

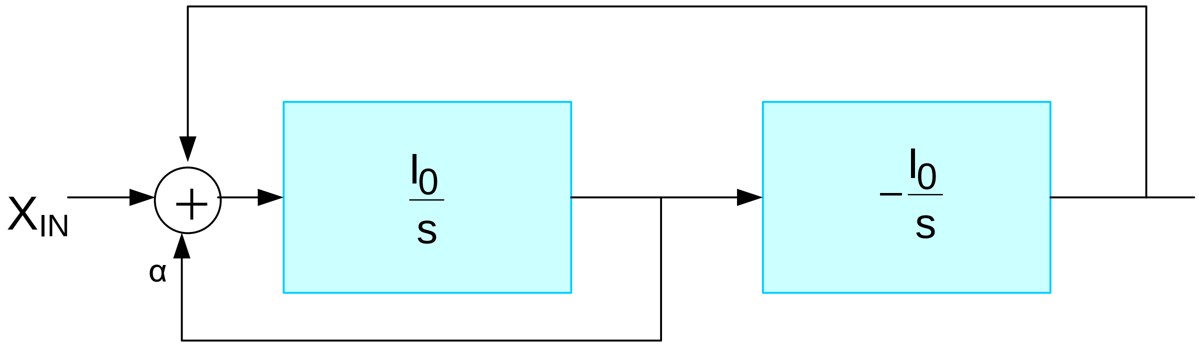


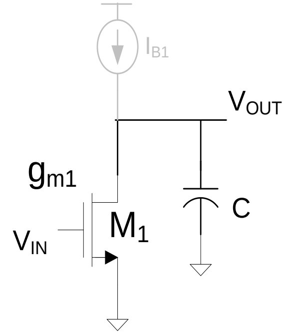
$$I(s) = -\frac{g_m}{sC}$$

Review from last time

Single-ended High-Frequency TA Integrators

How high can I_0 be?

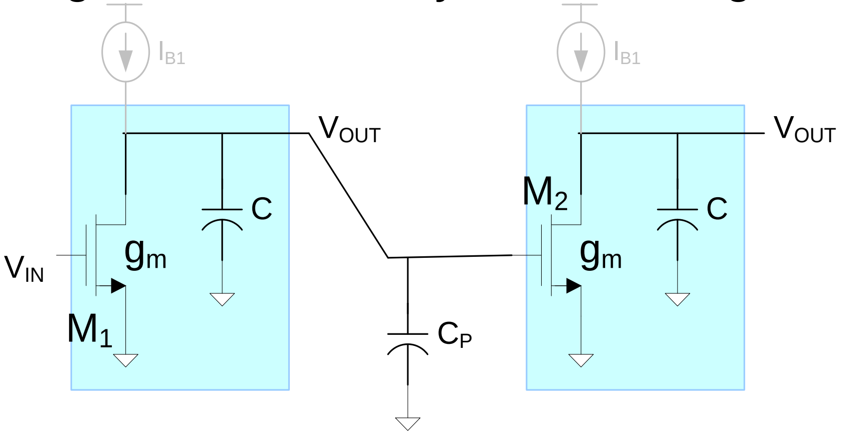




Consider a typical filter – the two integrator loop

$$I_0 = \frac{\mu C_{OX} W / L V_{EB}}{C}$$

Integrator is loaded by another integrator!



$$I_0 = \frac{\mu C_{OX} W_1 / L_1 V_{EB1}}{C + C_P + C_{OX} W_2 L_2}$$

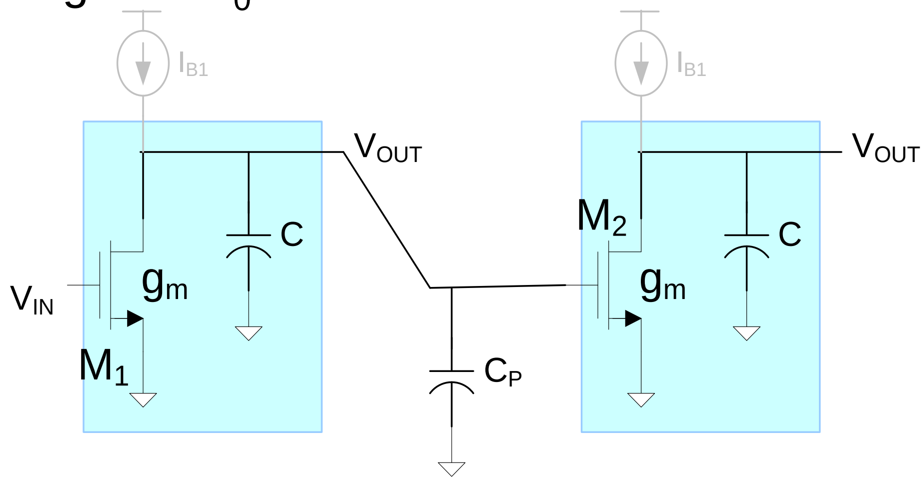
Even if C goes to 0, I_0 is limited!

C_P is the parasitic capacitances on the output node

Review from last time

Single-ended High-Frequency TA Integrators

How high can I_0 be?



$$I_{0M} = \frac{\mu V_{EB1}}{L_{min}^2}$$

$$I_{0M} = \omega_T$$

Speed of operation increases with V_{EB1}

V_{EB1} is limited by signal swing requirements and V_{DD}

Signal Swing:

$$V_{SW\text{-}0P} \simeq \min\{V_{DD} - V_{OQ}, V_{OQ} - (V_T + 100mV)\}$$

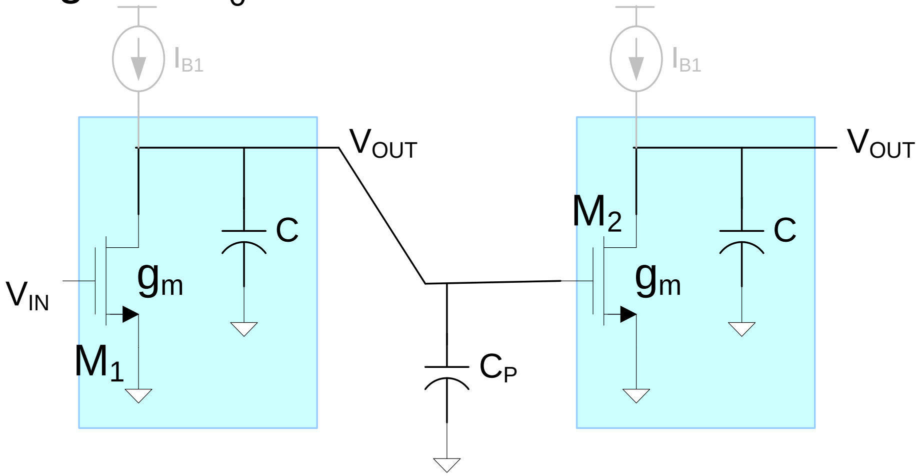
$$V_{OQ} = V_T + V_{EB}$$

$$V_{SW\text{-}0P} \simeq \min\{V_{DD} - V_T - V_{EB}, V_T + V_{EB} - (V_T + 100mV)\}$$

Review from last time

Single-ended High-Frequency TA Integrators

How high can I_0 be?



$$I_{0M}=\frac{\mu V_{EB1}}{L_{min}^2}$$

$$I_{0M}=\omega_T$$

Speed of operation increases with V_{EB}

V_{EB} is limited by signal swing requirements and V_{DD}

Signal Swing:

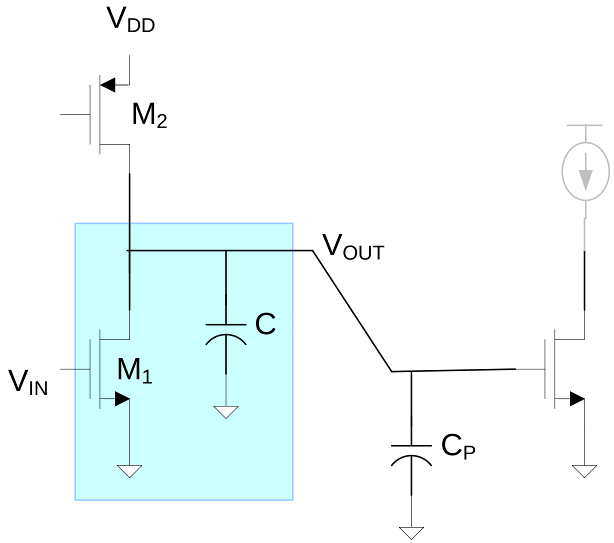
$$V_{DD}-V_T-V_{EB}=V_T+V_{EB}-(V_T+100mV)$$

$$V_{EB}=\frac{V_{DD}+100mV-V_T}{2}$$

$$I_{OMAX}\simeq\frac{\mu\left(V_{DD}+100mV-V_T\right)}{2L_{min}^2}$$

Single-ended High-Frequency TA Integrators

How high can I_0 be?



$$I_0 = \frac{\mu C_{OX} W_1 / L_1 V_{EB1}}{C + C_P + C_{OX} W_1 L_1}$$

Neglecting Cp and C, obtained

$$I_{0M} = \omega_T$$

$$I_{0M} = \frac{\mu V_{EB1}}{L_{min}^2}$$

Note this is independent of W_1

$$I_{OMAX} \simeq \frac{\mu\left(V_{DD} + 100mV - V_T\right)}{2L_{min}^2}$$

How much power is required to realize I_{0MAX}?

$$P_{QPT} = V_{DD} I_D$$

$$P_{QPT} = V_{DD} \frac{\mu C_{OX} W_1 V_{EB1}^2}{2L_{min}}$$

Note this is proportional to W_1

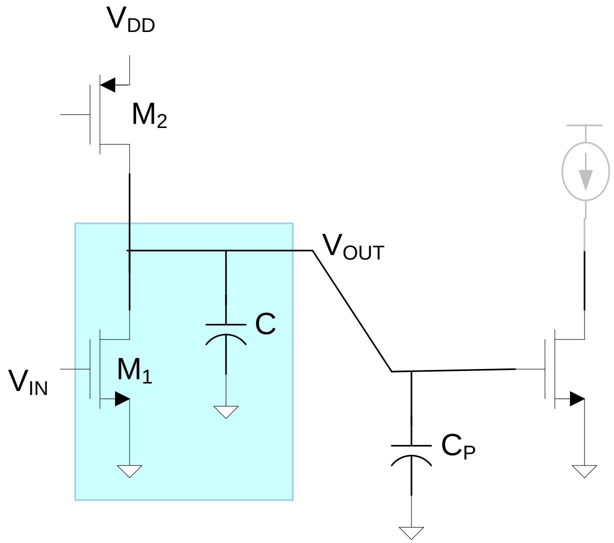
$$P_{QPT} = V_{DD} \frac{\mu C_{OX} W_{min} V_{EB1}^2}{2L_{min}} \overset{W_{min}=L_{min}}{\simeq} V_{DD} \frac{\mu C_{OX} V_{EB1}^2}{2}$$

$$= V_{DD} \frac{\mu C_{OX}}{2}\left(\frac{V_{DD} + 100mV - V_T}{2}\right)^2 \overset{V_T = 0.25V_{DD}}{\simeq} V_{DD} \frac{\mu C_{OX}}{2}\left(\frac{0.75V_{DD}}{2}\right)^2 \simeq .07 \mu C_{OX} V_{DD}^3$$

Single-ended High-Frequency TA Integrators

How high can I_0 be?

$$I_0 = \frac{\mu C_{OX} W_1 / L_1 V_{EB1}}{C + C_P + C_{OX} W_1 L_1}$$



Neglecting Cp and C, obtained

$$I_{0M} = \omega_T$$

$$I_{0M} = \frac{\mu V_{EB1}}{L_{min}^2}$$

$$I_{OMAX} \simeq \frac{\mu(V_{DD} + 100mV - V_T)}{2L_{min}^2}$$

C_P will modestly reduce the speed of the circuit

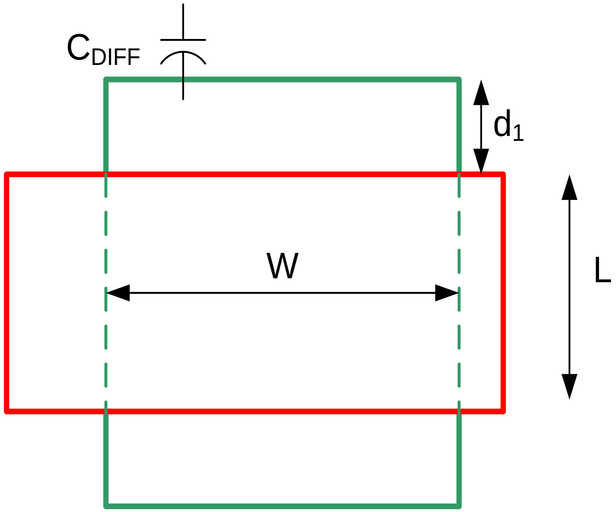
$$I_0 = \frac{\mu C_{OX} W_1 V_{EB1}}{L_{min} C_P + C_{OX} W_1 L_{min}^2}$$

Consider the diffusion capacitances on M_1 and M_2

$$I_0 = \frac{\mu C_{OX} W_1 V_{EB1}}{L_{min}(C_{P1} + C_{P2}) + C_{OX} W_1 L_{min}^2}$$

How high can I_0 be?

The parasitic diffusion capacitances are strongly layout dependent



Consider a basic layout of a transistor

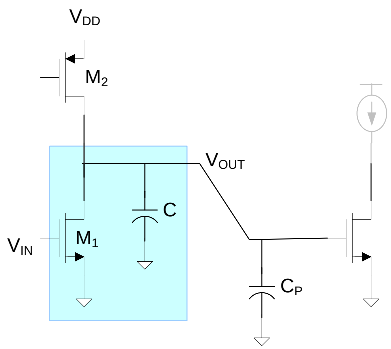
The capacitance density along the sw of the drain is usually somewhat less than that along the outer perimeters but may not easily be modeled separately

Assuming the same, drain diffusion capacitance of a transistor is given by

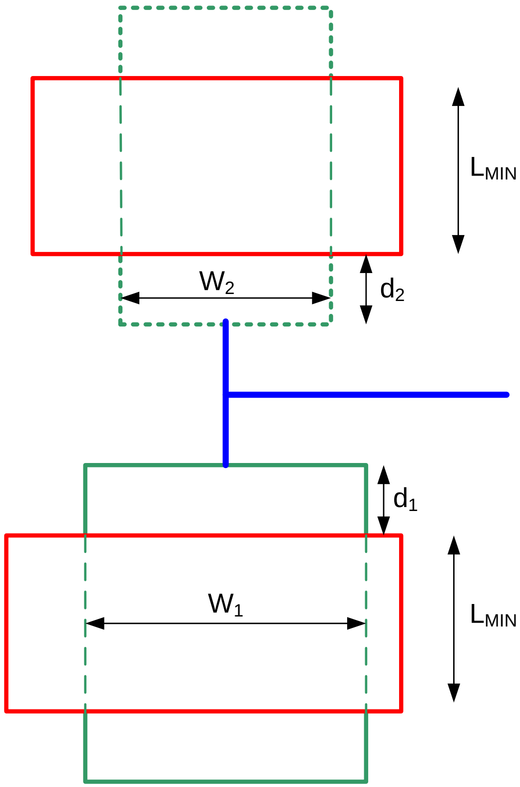
$$C_{DIFF} = C_{BOT}\left[W d_1\right] + C_{SW}\left[2d_1 + 2W\right]$$

C_{BOT} is the bottom diffusion capacitance density
C_{SW} is the sidwall diffusion capacitance density

How high can I_0 be?



Consider a basic layout



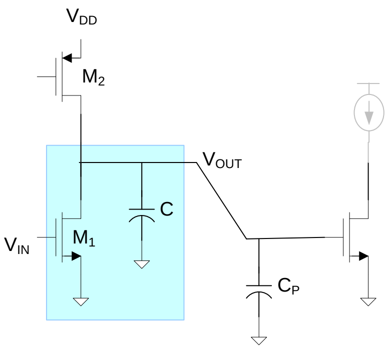
$$I_0 = \frac{\mu C_{OX} W_1 V_{EB1}}{L_{min}\left(C_{P1} + C_{P2}\right) + C_{OX} W_1 L_{min}^2}$$

$$I_0 = \frac{\mu C_{OX} W_1 V_{EB1}}{L_{min}\left(C_{BOTn}\left[W_1 d_1\right] + C_{SWn}\left[2d_1 + 2W_1\right] + C_{BOTp}\left[W_2 d_2\right] + C_{SWp}\left[2d_2 + 2W_2\right]\right) + C_{OX} W_1 L_{min}^2}$$

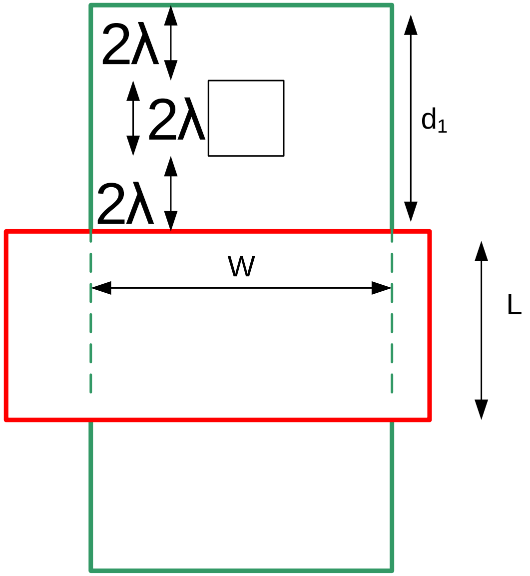
Assume $L_{MIN}=2\lambda$

$$I_0 = \frac{\mu C_{OX} W_1 V_{EB1}}{2\lambda\left(C_{BOTn}\left[W_1 d_1\right] + C_{SWn}\left[2d_1 + 2W_1\right] + C_{BOTnp}\left[W_2 d_2\right] + C_{SWp}\left[2d_2 + 2W_2\right]\right) + C_{OX} W_1 4\lambda^2}$$

How high can I_0 be?



Consider a basic layout of a transistor

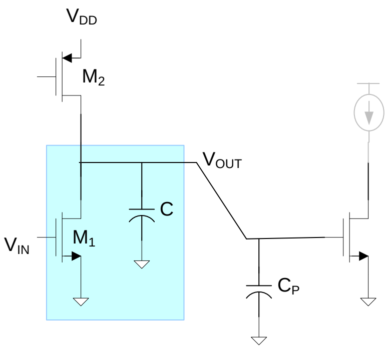


$$d_1 = 6\lambda$$

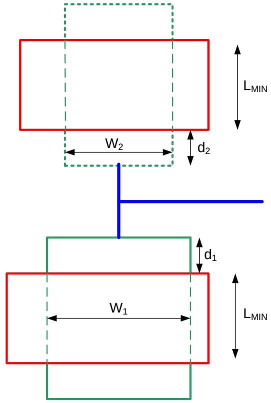
$$I_0 = \frac{\mu C_{OX} W_1 V_{EB1}}{2\lambda\left(C_{BOTn}\left[W_1 d_1\right] + C_{SWn}\left[2d_1 + 2W_1\right] + C_{BOTp}\left[W_2 d_2\right] + C_{SWp}\left[2d_2 + 2W_2\right]\right) + C_{OX} W_1 4\lambda^2}$$

$$I_0 = \frac{\mu C_{OX} W_1 V_{EB1}}{2\lambda\left(C_{BOTn}\left[W_1 6\lambda\right] + C_{SWn}\left[12\lambda + 2W_1\right] + C_{BOTp}\left[W_2 6\lambda\right] + C_{SWp}\left[12\lambda + 2W_2\right]\right) + C_{OX} W_1 4\lambda^2}$$

How high can I_0 be?



Consider a basic layout



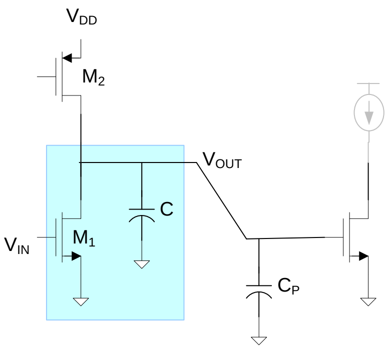
$$I_0 = \frac{\mu_n C_{OX} W_1 V_{EB1}}{2\lambda\left(C_{BOTn}\left[W_1 6\lambda\right] + C_{SWn}\left[12\lambda + 2W_1\right] + C_{BOTp}\left[W_2 6\lambda\right] + C_{SWp}\left[12\lambda + 2W_2\right]\right) + C_{OX} W_1 4\lambda^2}$$

$$I_0 = \frac{\mu_n V_{EB1}}{4\lambda^2 + 2\lambda\left(\frac{C_{BOTn}}{C_{OX}}\left[6\lambda\right] + \frac{C_{SWn}}{C_{OX}}\left[12\frac{\lambda}{W_1} + 2\right] + \frac{C_{BOTp}}{C_{OX}}\left[\frac{W_2}{W_1}\right]\left[6\lambda\right] + \frac{C_{SWp}}{C_{OX}}\left[12\frac{\lambda}{W_1} + 2\frac{W_2}{W_1}\right]\right)}$$

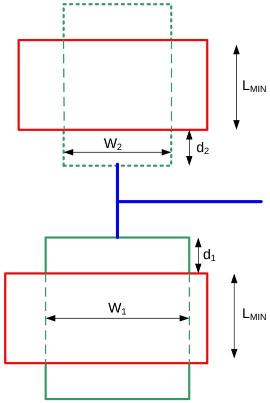
Define and assume $h_{BOT} = \frac{C_{BOTn}}{C_{OX}} = \frac{C_{BOTp}}{C_{OX}}$ $\quad h_{SW} = \frac{C_{SWn}}{\lambda C_{OX}} = \frac{C_{SWp}}{\lambda C_{OX}}$

$$I_0 = \frac{\mu_n V_{EB1}}{4\lambda^2 + 2\lambda\left(h_{BOT}\left[6\lambda\right] + \lambda h_{SW}\left[12\frac{\lambda}{W_1} + 2\right] + h_{BOT}\left[\frac{W_2}{W_1}\right]\left[6\lambda\right] + \lambda h_{SW}\left[12\frac{\lambda}{W_1} + 2\frac{W_2}{W_1}\right]\right)}$$

How high can I_0 be?



Consider a basic layout

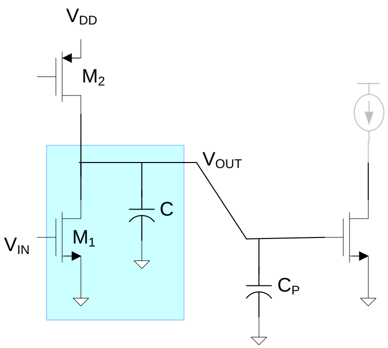


$$I_0 = \frac{\mu_n V_{EB1}}{4\lambda^2 + 2\lambda\left(h_{BOT}[6\lambda] + \lambda h_{SW}\left[12\frac{\lambda}{W_1} + 2\right] + h_{BOT}\left[\frac{W_2}{W_1}\right][6\lambda] + \lambda h_{SW}\left[12\frac{\lambda}{W_1} + 2\frac{W_2}{W_1}\right]\right)}$$

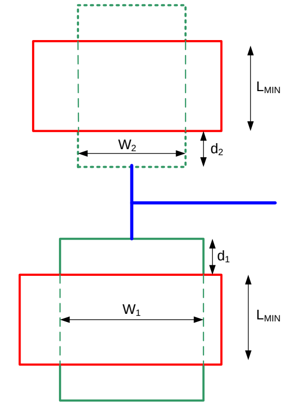
$$I_0 = \frac{\mu_n V_{EB1}}{4\lambda^2 + 4\lambda^2\left(3h_{BOT}\left[1 + \frac{W_2}{W_1}\right] + h_{SW}\left[12\frac{\lambda}{W_1} + 1 + \frac{W_2}{W_1}\right]\right)}$$

$$I_0 = \frac{\dfrac{\mu_n V_{EB1}}{4\lambda^2}}{1 + \left(3h_{BOT}\left[1 + \frac{W_2}{W_1}\right] + h_{SW}\left[12\frac{\lambda}{W_1} + 1 + \frac{W_2}{W_1}\right]\right)}$$

How high can I_0 be?



Consider a basic layout



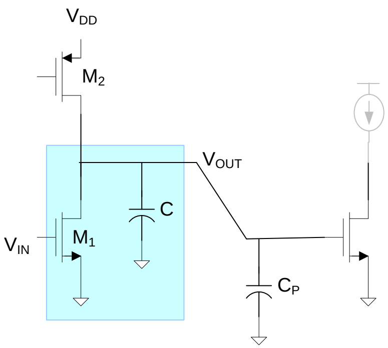
$$I_0 = \frac{\dfrac{\mu_n V_{EB1}}{4\lambda^2}}{1+\left(3h_{BOT}\left[1+\dfrac{W_2}{W_1}\right]+h_{SW}\left[12\dfrac{\lambda}{W_1}+1+\dfrac{W_2}{W_1}\right]\right)}$$

Recall

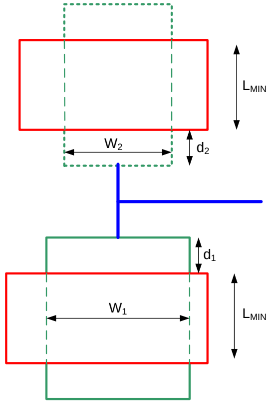
$$\frac{W_2}{W_1} = \frac{\mu_n}{\mu_p}\left(\frac{V_{EB1}}{V_{EB2}}\right)^2 \qquad \omega_T = \frac{\mu_n V_{EB1}}{4\lambda^2}$$

$$I_0 = \frac{\omega_T}{1+\left(3h_{BOT}\left[1+\dfrac{\mu_n}{\mu_p}\left(\dfrac{V_{EB1}}{V_{EB2}}\right)^2\right]+h_{SW}\left[12\dfrac{\lambda}{W_1}+1+\dfrac{\mu_n}{\mu_p}\left(\dfrac{V_{EB1}}{V_{EB2}}\right)^2\right]\right)}$$

How high can I_0 be?



Consider a basic layout



$$I_0 = \frac{\omega_T}{1+\left(3h_{BOT}\left[1+\frac{\mu_n}{\mu_p}\left(\frac{V_{EB1}}{V_{EB2}}\right)^2\right]+h_{SW}\left[12\frac{\lambda}{W_1}+1+\frac{\mu_n}{\mu_p}\left(\frac{V_{EB1}}{V_{EB2}}\right)^2\right]\right)}$$

Example: Consider the 0.25u TSMC CMOS Process

$$C_{OX} = 5.8\text{fF}/\mu^2$$

$$C_{SWn} = .440\text{fF}/\mu$$

$$C_{SWp} = .350\text{fF}/\mu$$

$$C_{BOT} = 1.8\text{fF}/\mu^2$$

$$\frac{\mu_n}{\mu_p} = 4.1$$

$$\mu_n = 3.74\text{E}10$$

$$\lambda = 0.125\mu$$

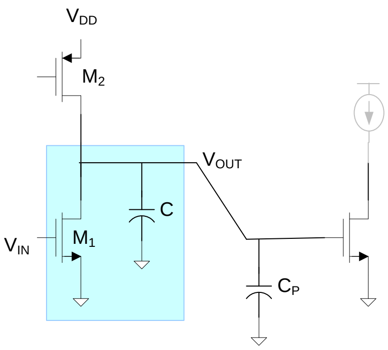
$$h_{BOT} = \frac{C_{BOTn}}{C_{OX}} = \frac{C_{BOTp}}{C_{OX}}$$

$$h_{BOT} = 0.31$$

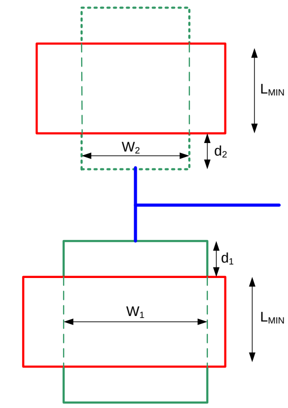
$$h_{SW} = \frac{C_{SWn}}{\lambda C_{OX}} = \frac{C_{SWp}}{\lambda C_{OX}}$$

$$h_{SW} = 0.61$$

How high can I_0 be?



Consider a basic layout



$$I_0 = \frac{\omega_T}{1+\left(3h_{BOT}\left[1+\frac{\mu_n}{\mu_p}\left(\frac{V_{EB1}}{V_{EB2}}\right)^2\right]+h_{SW}\left[12\frac{\lambda}{W_1}+1+\frac{\mu_n}{\mu_p}\left(\frac{V_{EB1}}{V_{EB2}}\right)^2\right]\right)}$$

Example: Consider the 0.25u TSMC CMOS Process

$$I_0 = \frac{\omega_T}{1+\left(3\bullet 0.31\left[1+4.1\left(\frac{V_{EB1}}{V_{EB2}}\right)^2\right]+0.61\left[12\frac{0.125}{W_1}+1+4.01\left(\frac{V_{EB1}}{V_{EB2}}\right)^2\right]\right)}$$

$$I_0 = \frac{\omega_T}{1+\left(0.931\left[1+4.1\left(\frac{V_{EB1}}{V_{EB2}}\right)^2\right]+0.61\left[\frac{1.5}{W_1}+1+4.01\left(\frac{V_{EB1}}{V_{EB2}}\right)^2\right]\right)}$$

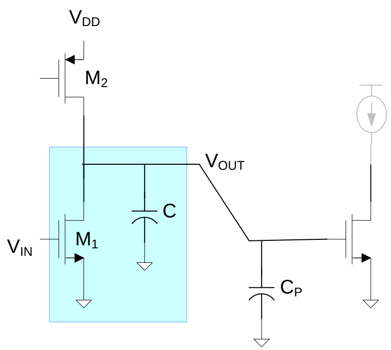
GATE term — BOT term — SW term

$$h_{BOT} = 0.31$$

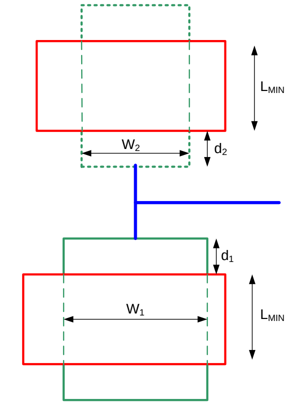
$$h_{SW} = 0.61$$

$$\frac{\mu_n}{\mu_p} = 4.1$$

How high can I_0 be?



Consider a basic layout



Example: Consider the 0.25u TSMC CMOS Process

$$I_0 = \frac{\omega_T}{1+\left(\underbrace{0.93\left[1+4.1\left(\frac{V_{EB1}}{V_{EB2}}\right)^2\right]}_{\text{BOT term}}+\underbrace{0.61\left[\frac{1.5}{W_1}+1+4.01\left(\frac{V_{EB1}}{V_{EB2}}\right)^2\right]}_{\text{SW term}}\right)}$$

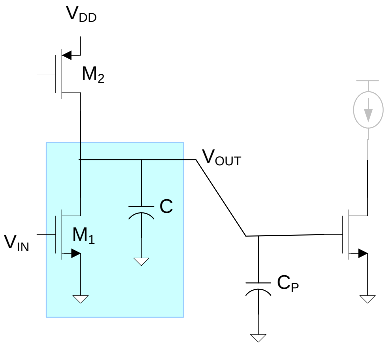
SW term

BOT term

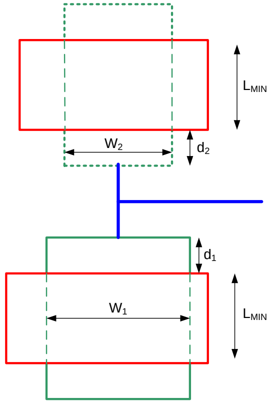
If W_1=1.5u and $V_{EB1}=V_{EB2}$ $$I_0 = \frac{\omega_T}{1+\left(4.73+4.03\right)} = .102\omega_T$$

- The diffusion capacitance term can dominate the C_{GS} term
- The SW capacitance can be the biggest contributor to the speed limitations
- A factor of 10 or even much more reduction in speed is possible due to the diffusion parasitics and layout
- Maximizing W_1 will minimize I_0 but power will get very large for marginal improvement in speed

How high can I_0 be?



Consider a basic layout



Example: Consider the 0.25u TSMC CMOS Process

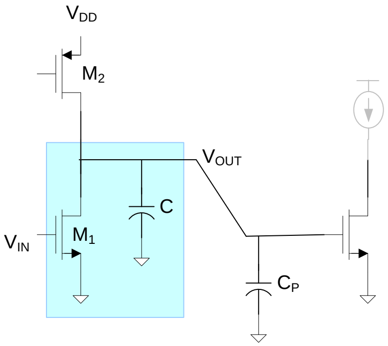
$$I_0 = \frac{\omega_T}{1+\left(0.93\left[1+4.1\left(\frac{V_{EB1}}{V_{EB2}}\right)^2\right]+0.61\left[\frac{1.5}{W_1}+1+4.01\left(\frac{V_{EB1}}{V_{EB2}}\right)^2\right]\right)}$$

BOT term SW term

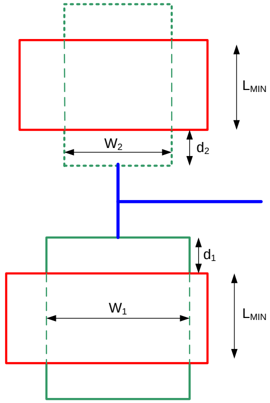
This example shows that layout is really critical when high speed operation is needed

What can be done with layout to improve performance?

How high can I_0 be?



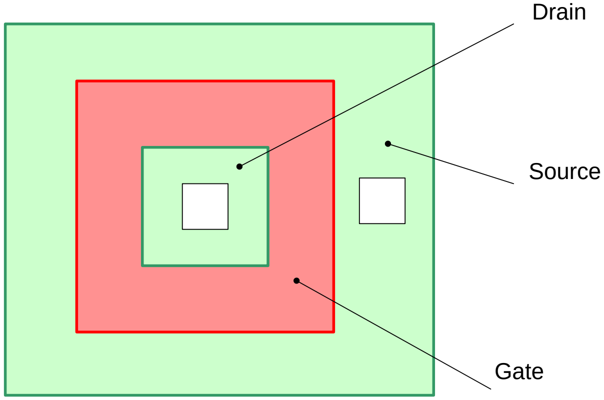
Consider a basic layout

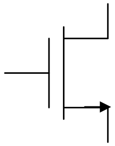


What can be done with layout to improve performance?

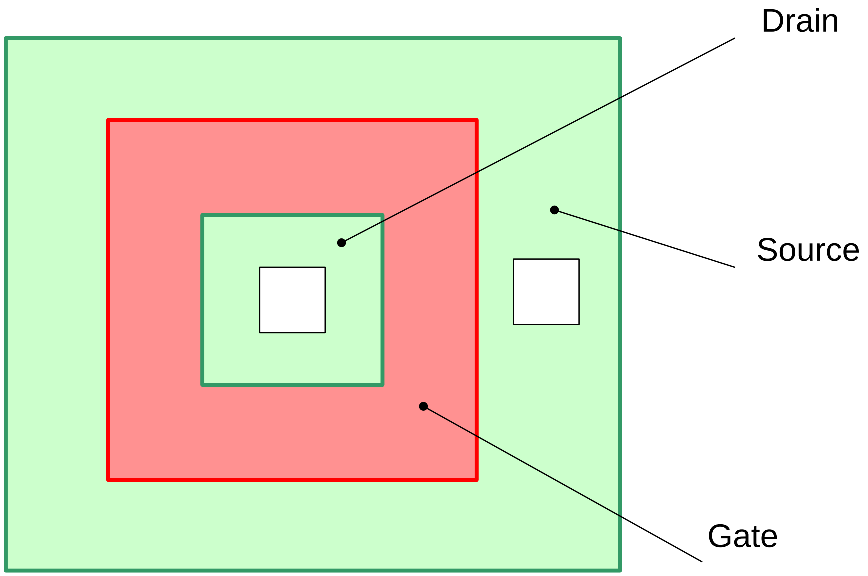
Reducing the diffusion capacitances on the drains will have a major impact on speed!

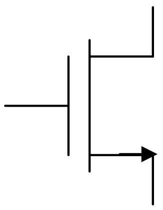
Consider a concentric layout approach:





Concentric Layouts





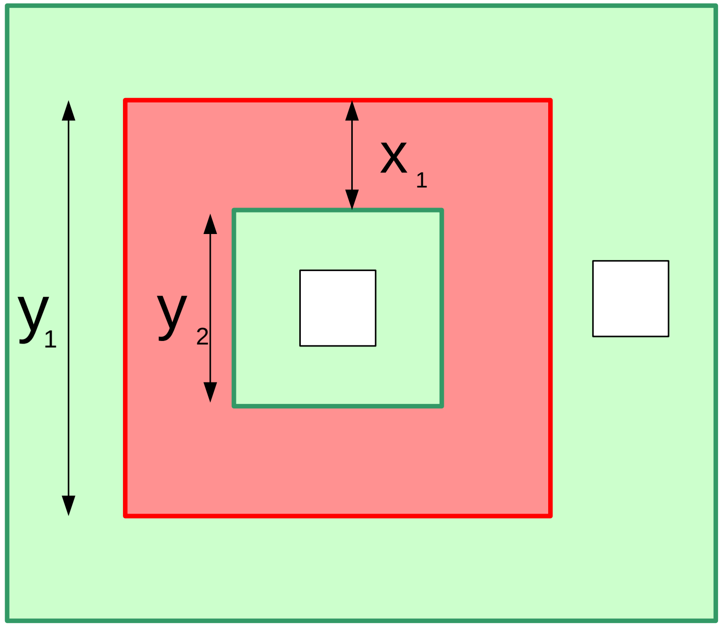
Can be shown this is equivalent to a rectangular transistor (W_{EQ}/L_{EQ})

Drain area and perimeter dramatically reduced

Source area and perimeter dramatically increased (but does not degrade performance)

Only sidewall is adjacent to the gate and C_{SW} is usually considerably lower here though some models do not provide separate characterization

Concentric Layouts



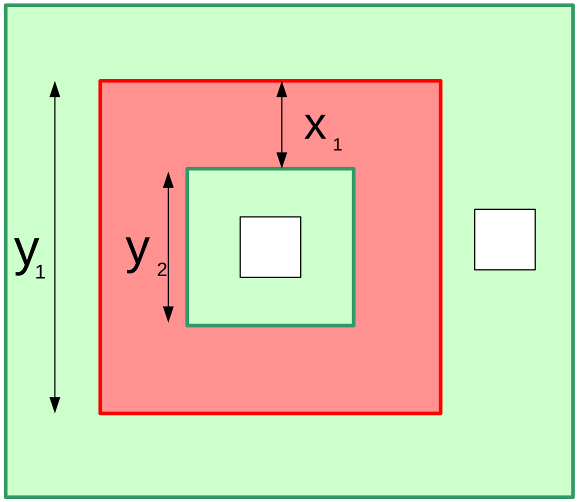
$$W_{EQ} \simeq 4\left(\frac{y_1 + y_2}{2}\right) \quad \text{or} \quad W_{EQ} \simeq 4\left(y_2 + \sqrt{2}\left[\frac{y_1 - y_2}{4}\right]\right)$$

$$L_{EQ} \simeq x_1$$

Exact closed-form expressions exist which are somewhat more complicated

How high can I_0 be?

Consider concentric layouts for M_1 and M_2



Recall $$\frac{W_2}{W_1} = \frac{\mu_n}{\mu_p}\left(\frac{V_{EB1}}{V_{EB2}}\right)^2$$

Assume $W_2 > W_1$

Will minimize the diffusion capacitance by starting with a minimum-sized concentric device

Thus $y_2 = 6\lambda$ $x_2 = 2\lambda$ $y_1 = 10\lambda$ $W_{1min} \simeq 4\lambda\left(6+\sqrt{2}\right)$

Define K_1 to be the scaling factor of W_1 above that of the minimum-sized concentric device

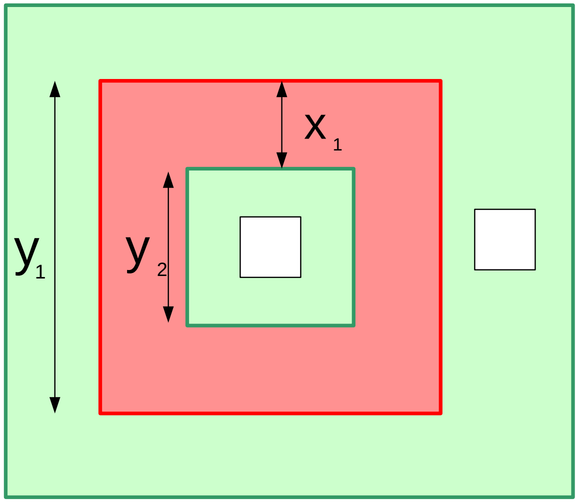
$$K_1 = \frac{W_1}{W_{1min}}$$

Assume, for convenience, that K is an integer

M_1 realized by placing K_1 minimum-sized concentric devices in parallel

How high can I_0 be?

Consider concentric layouts for M_1 and M_2



$y_2=6\lambda$ $x_2=2\lambda$ $y_1=10\lambda$

$$W_{1min} \simeq 4\lambda\left(6+\sqrt{2}\right)$$

$$K_1 = \frac{W_1}{W_{1min}}$$

Consider now the concentric layout for M_1

$P_{D1}=K_1 24\lambda$ $A_{D1}=K_1(6\lambda)^2$ $A_{GATE1}=K_1(48\lambda^2+16\lambda^2)$

Consider now the concentric layout for M_2

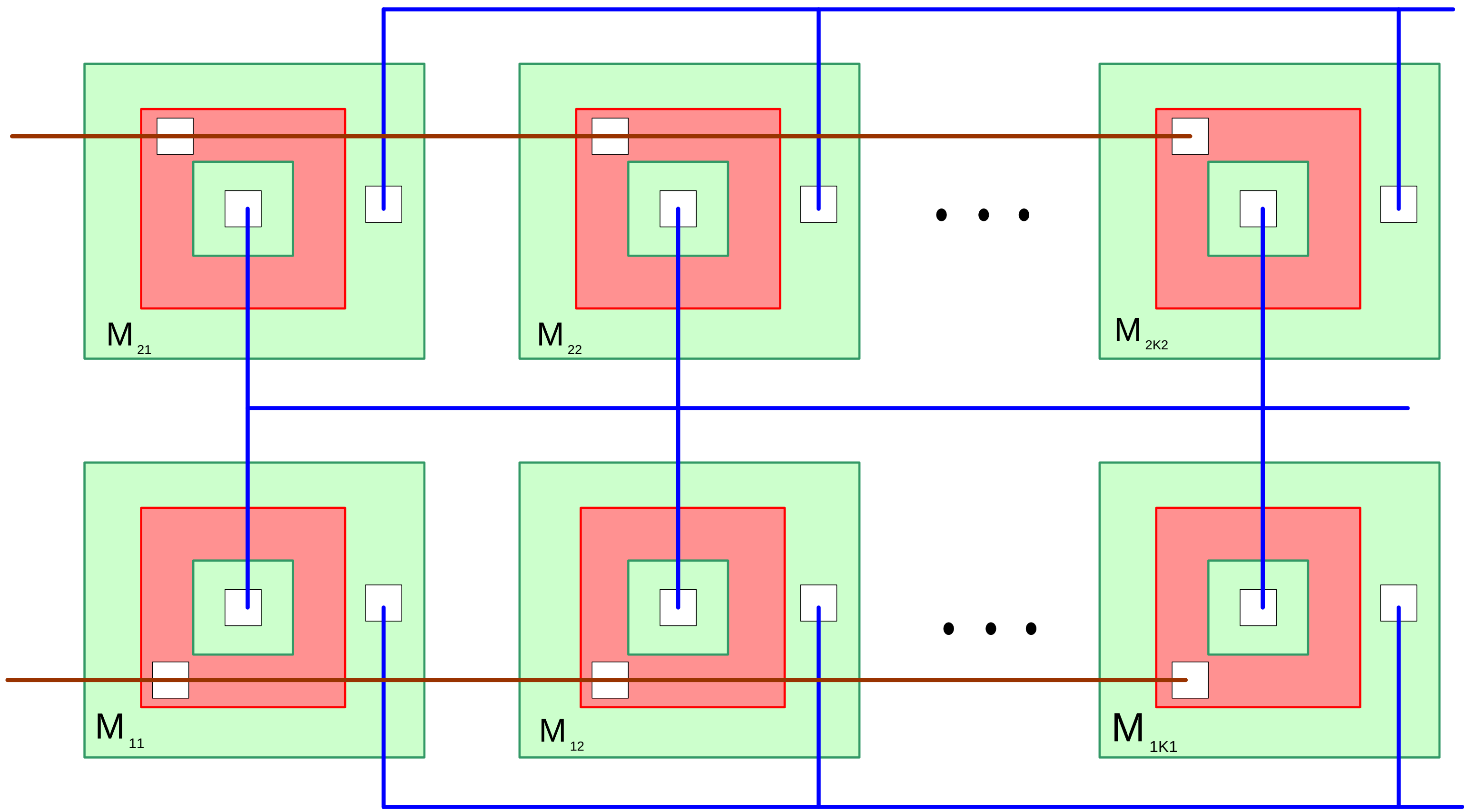
The minimum-sized layout (gate,source, and drain) for the p-channel transistors are identical to those for n-channel transistors

Define K_2 to be the scaling factor for W_2 above that of a minimum-sized concentric device

$P_{D2}=K_2 24\lambda$ $A_{D2}=K_2(6\lambda)^2$

How high can I_0 be?

Consider concentric layouts for M_1 and M_2

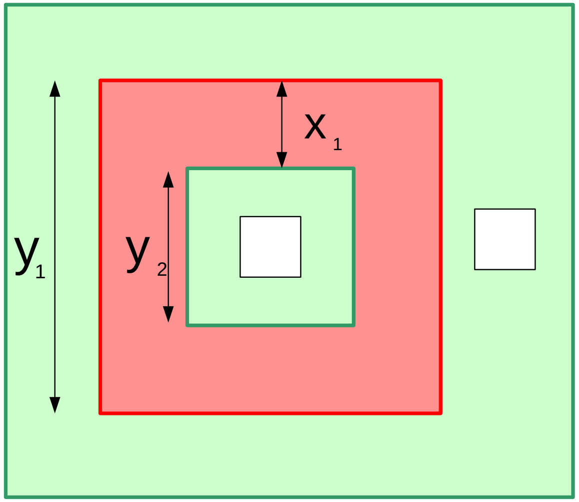


Individual segments can be a little bigger than minimum sized w/o major change in performance

May select $K_1=K_2=1$

How high can I_0 be?

Consider concentric layouts for M_1 and M_2



$$K_2 = \frac{W_2}{W_{1min}} \qquad W_2 = W_1 \frac{\mu_n}{\mu_p}\left(\frac{V_{EB1}}{V_{EB2}}\right)^2$$

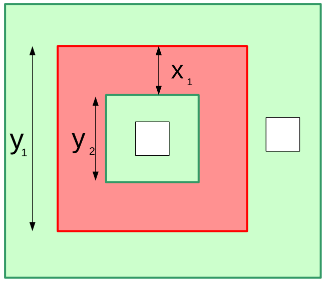
$$K_2 = \frac{W_1}{W_{1min}} \frac{\mu_n}{\mu_p}\left(\frac{V_{EB1}}{V_{EB2}}\right)^2 = K_1 \frac{\mu_n}{\mu_p}\left(\frac{V_{EB1}}{V_{EB2}}\right)$$

$$I_0 = \frac{\mu C_{OX} W_1 V_{EB1}}{L_{min}\left(C_{P1}+C_{P2}\right)+C_{OX} W_1 L_{min}^2} \quad \rightarrow \quad I_0 = \frac{\dfrac{\mu C_{OX} W_1 V_{EB1}}{L_{min}}}{\left(C_{P1}+C_{P2}\right)+C_{GS1}}$$

$$I_0 = \frac{\dfrac{\mu V_{EB1}}{L_{min}^2}}{\dfrac{\left(C_{P1}+C_{P2}\right)+C_{GS1}}{C_{OX} L_{min} W_1}} \qquad I_0 = \frac{\omega_T}{\dfrac{\left(C_{P1}+C_{P2}\right)+C_{GS1}}{2\lambda C_{OX} W_1}}$$

How high can I_0 be?

Consider concentric layouts for M_1 and M_2



$$I_0 = \frac{\omega_T}{\dfrac{\left(C_{P1} + C_{P2}\right) + C_{GS1}}{2\lambda C_{OX} W_1}}$$

$P_{D1}=K_1 24\lambda$ $A_{D1}=K_1(6\lambda)^2$ $A_{GATE1}=K_1(48\lambda^2+16\lambda^2)$

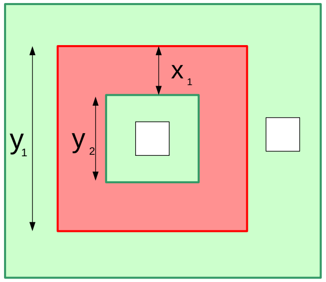
$P_{D2}=K_2 24\lambda$ $A_{D2}=K_2(6\lambda)^2$ $W_1 \simeq 4K_1\lambda\left(6+\sqrt{2}\right)$

$$I_0 = \frac{\omega_T}{\dfrac{C_{OX}K_1\left(48\lambda^2+16\lambda^2\right)+\left(C_{SWn}K_1 24\lambda + C_{BOTn}K_1(6\lambda)^2 + C_{SWp}K_2 24\lambda + C_{BOTp}K_2(6\lambda)^2\right)}{2\lambda C_{OX}4K_1\lambda\left(6+\sqrt{2}\right)}}$$

$$I_0 = \frac{\omega_T}{\dfrac{C_{OX}K_1\left(48\lambda^2+16\lambda^2\right)+C_{BOT}(6\lambda)^2(K_1+K_2)+C_{SW}24\lambda(K_1+K_2)}{2\lambda C_{OX}4K_1\lambda\left(6+\sqrt{2}\right)}}$$

How high can I_0 be?

Consider concentric layouts for M_1 and M_2

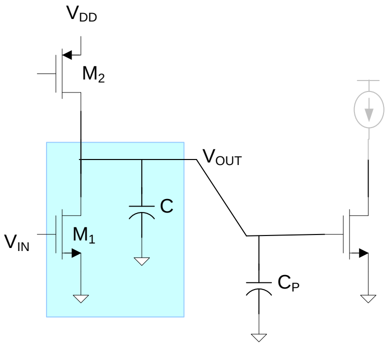


$$I_0 = \frac{\omega_T}{\dfrac{C_{OX}K_1\left(48\lambda^2+16\lambda^2\right)+C_{BOT}(6\lambda)^2(K_1+K_2)+C_{SW}24\lambda(K_1+K_2)}{2\lambda C_{OX}4K_1\lambda\left(6+\sqrt{2}\right)}}$$

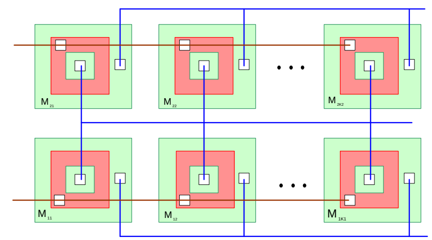
$$I_0 = \frac{\omega_T}{\dfrac{(8)+h_{BOT}4.5(1+K_2/K_1)+h_{SW}3(1+K_2/K_1)}{\left(6+\sqrt{2}\right)}}$$

$$I_0 = \frac{\omega_T}{1.08+h_{BOT}.61(1+K_2/K_1)+h_{SW}0.4(1+K_2/K_1)}$$

How high can I_0 be?



Consider concentric layout



$$I_0 = \frac{\omega_T}{1.08 + h_{BOT}.61(1 + K_2/K_1) + h_{SW}0.4(1 + K_2/K_1)}$$

Example: Consider the 0.25u TSMC CMOS Process with W_1=1.5u and $V_{EB1}=V_{EB2}$

$$\frac{K_2}{K_1} = \frac{\mu_n}{\mu_p}\left(\frac{V_{EB1}}{V_{EB2}}\right) \qquad \frac{K_2}{K_1} = 4.01\left(\frac{V_{EB1}}{V_{EB2}}\right) \qquad \frac{\mu_n}{\mu_p} = 4.1$$

$$I_0 = \frac{\omega_T}{1.08 + .19(5.01) + 0.24(5.01)}$$

BOT term SW term

$$I_0 = \frac{\omega_T}{1.08 + .95 + 1.2}$$

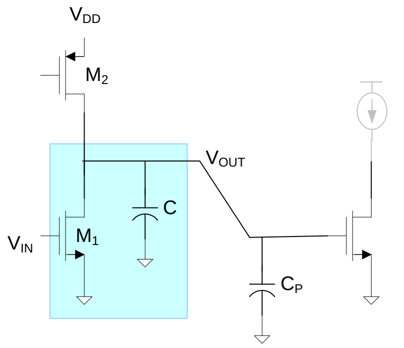
$$I_0 = .31\omega_T$$

Diffusion parasitics still dominate frequency degradation

SW term probably over-estimated since it is an internal SW capacitance

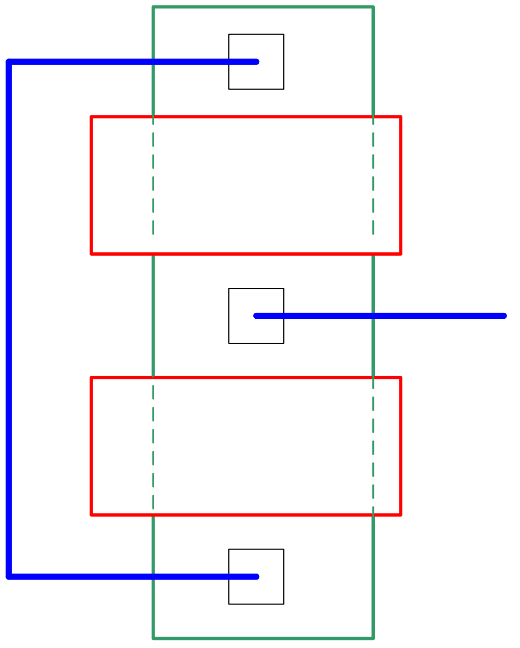
But a factor of 3 faster with the concentric layout compared to standard layout

How high can I_0 be?

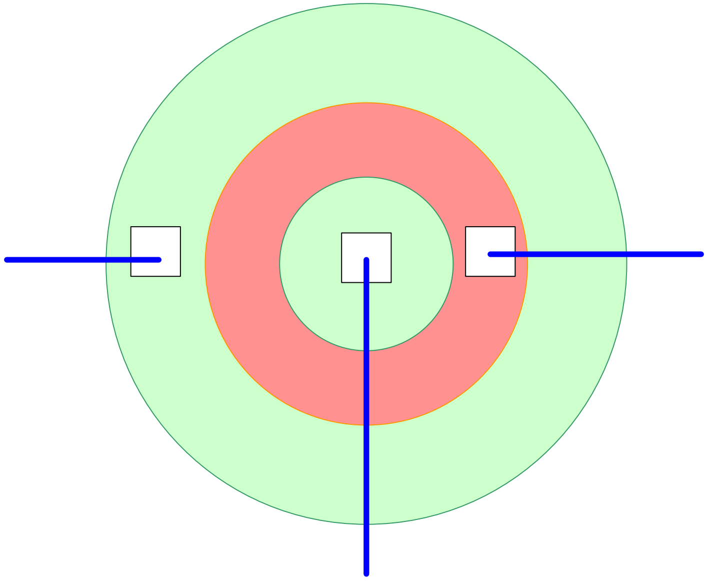


Other layouts for enhancing speed of operation

Goal: reduce area and perimeter on drain



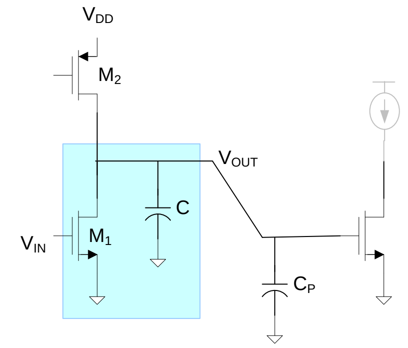
Shared-drain structure



Circular-concentric structure

Though the reduced size drain structures work very well, CAD support may be limited for layout, simulation, and extraction

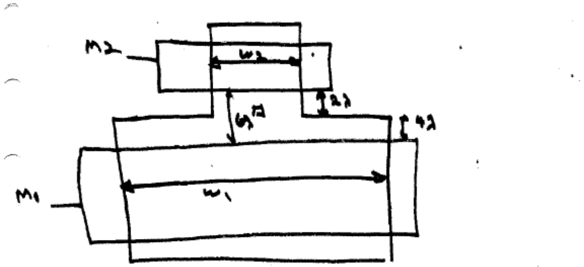
How high can I_0 be?



Other layouts for enhancing speed of operation

Goal: reduce area and perimeter on drain

n-channel load, simple Layout $\quad W_1 = 4W_2$



$$A_D = 4\lambda W_1 + 2\lambda W_2 = 18\lambda W_2$$

$$P_D = 12\lambda + 2W_1 = 12\lambda + 8W_2$$

$$A_{G1} = 2\lambda W_1 = 8\lambda W_2$$

$$A_{G2} = 2\lambda W_2$$

$$\tilde{\omega}_0 = \frac{\sqrt{3}}{2}\frac{g_{m1}}{C_L}$$

$$\hat{\omega}_0 = \frac{\frac{\sqrt{3}}{2}\frac{\mu_n V_{EB1}}{4\lambda^2}}{5/4 + 9/4\, h_{bot} + \left(1 + \frac{1}{4K}\right) h_{sw}} \qquad K = \frac{W_2}{6\lambda}$$

Useful for adding loss or in high-speed gain stages

Parameters from .25u TSMC Process

u	3.74E+10	1/(V*sec)
2*lambda	0.25	u
hsw	0.61	none
ot	0.32	none
/up	4.1	

Integrator Io for Special Layouts

file: integrator-speed-comp

Note: Process parameters may be a little optimistic but relative performance should be as predicted

Conventional Layout

VEB1/ VEB2	K	W1	W2	SWn	SWp	BOTn	BOTp	SW comp Total	Bot comp Total	Load comp	Den	VEB1	Io,no d f GHz	Io GHz
1	1	0.75	3.075	0.92	3.42	0.96	3.94	4.33	4.90	1	10.2	1	95.3	9.3
1	2	1.5	6.15	0.61	3.11	0.96	3.94	3.72	4.90	1	9.6	1	95.3	9.9
1	4	3	12.3	0.46	2.96	0.96	3.94	3.42	4.90	1	9.3	1	95.3	10.2
1	8	6	24.6	0.38	2.88	0.96	3.94	3.26	4.90	1	9.2	1	95.3	10.4
1	16	12	49.2	0.34	2.84	0.96	3.94	3.19	4.90	1	9.1	1	95.3	10.5
0.5	1	0.75	0.769	0.92	1.54	0.96	0.98	2.46	1.94	1	5.4	1	95.3	17.6
0.5	2	1.5	1.538	0.61	1.24	0.96	0.98	1.85	1.94	1	4.8	1	95.3	19.9
0.5	4	3	3.075	0.46	1.08	0.96	0.98	1.54	1.94	1	4.5	1	95.3	21.2
0.5	8	6	6.15	0.38	1.01	0.96	0.98	1.39	1.94	1	4.3	1	95.3	22.0
0.5	16	12	12.3	0.34	0.97	0.96	0.98	1.31	1.94	1	4.3	1	95.3	22.4
2	1	0.75	12.3	0.92	10.92	0.96	15.74	11.83	16.70	1	29.5	1	95.3	3.2
2	2	1.5	24.6	0.61	10.61	0.96	15.74	11.22	16.70	1	28.9	1	95.3	3.3
2	4	3	49.2	0.46	10.46	0.96	15.74	10.92	16.70	1	28.6	1	95.3	3.3
2	8	6	98.4	0.38	10.39	0.96	15.74	10.77	16.70	1	28.5	1	95.3	3.3
2	16	12	196.8	0.34	10.35	0.96	15.74	10.69	16.70	1	28.4	1	95.3	3.4
1	1	0.75	3.075	0.92	3.42	0.96	3.94	4.33	4.90	1	10.2	1.5	142.9	14.0
1	2	1.5	6.15	0.61	3.11	0.96	3.94	3.72	4.90	1	9.6	1.5	142.9	14.9
1	4	3	12.3	0.46	2.96	0.96	3.94	3.42	4.90	1	9.3	1.5	142.9	15.3
1	8	6	24.6	0.38	2.88	0.96	3.94	3.26	4.90	1	9.2	1.5	142.9	15.6
1	16	12	49.2	0.34	2.84	0.96	3.94	3.19	4.90	1	9.1	1.5	142.9	15.7
0.5	1	0.75	0.769	0.92	1.54	0.96	0.98	2.46	1.94	1	5.4	1.5	142.9	26.5
0.5	2	1.5	1.538	0.61	1.24	0.96	0.98	1.85	1.94	1	4.8	1.5	142.9	29.8
0.5	4	3	3.075	0.46	1.08	0.96	0.98	1.54	1.94	1	4.5	1.5	142.9	31.9
0.5	8	6	6.15	0.38	1.01	0.96	0.98	1.39	1.94	1	4.3	1.5	142.9	33.0
0.5	16	12	12.3	0.34	0.97	0.96	0.98	1.31	1.94	1	4.3	1.5	142.9	33.6
2	1	0.75	12.3	0.92	10.92	0.96	15.74	11.83	16.70	1	29.5	1.5	142.9	4.8
2	2	1.5	24.6	0.61	10.61	0.96	15.74	11.22	16.70	1	28.9	1.5	142.9	4.9
2	4	3	49.2	0.46	10.46	0.96	15.74	10.92	16.70	1	28.6	1.5	142.9	5.0
2	8	6	98.4	0.38	10.39	0.96	15.74	10.77	16.70	1	28.5	1.5	142.9	5.0
2	16	12	196.8	0.34	10.35	0.96	15.74	10.69	16.70	1	28.4	1.5	142.9	5.0
1	1	0.75	3.075	0.92	3.42	0.96	3.94	4.33	4.90	1	10.2	2	190.6	18.6
1	2	1.5	6.15	0.61	3.11	0.96	3.94	3.72	4.90	1	9.6	2	190.6	19.8
1	4	3	12.3	0.46	2.96	0.96	3.94	3.42	4.90	1	9.3	2	190.6	20.5
1	8	6	24.6	0.38	2.88	0.96	3.94	3.26	4.90	1	9.2	2	190.6	20.8
1	16	12	49.2	0.34	2.84	0.96	3.94	3.19	4.90	1	9.1	2	190.6	21.0
0.5	1	0.75	0.769	0.92	1.54	0.96	0.98	2.46	1.94	1	5.4	2	190.6	35.3
0.5	2	1.5	1.538	0.61	1.24	0.96	0.98	1.85	1.94	1	4.8	2	190.6	39.8
0.5	4	3	3.075	0.46	1.08	0.96	0.98	1.54	1.94	1	4.5	2	190.6	42.5
0.5	8	6	6.15	0.38	1.01	0.96	0.98	1.39	1.94	1	4.3	2	190.6	44.0
0.5	16	12	12.3	0.34	0.97	0.96	0.98	1.31	1.94	1	4.3	2	190.6	44.8
2	1	0.75	12.3	0.92	10.92	0.96	15.74	11.83	16.70	1	29.5	2	190.6	6.5
2	2	1.5	24.6	0.61	10.61	0.96	15.74	11.22	16.70	1	28.9	2	190.6	6.6
2	4	3	49.2	0.46	10.46	0.96	15.74	10.92	16.70	1	28.6	2	190.6	6.7
2	8	6	98.4	0.38	10.39	0.96	15.74	10.77	16.70	1	28.5	2	190.6	6.7
2	16	12	196.8	0.34	10.35	0.96	15.74	10.69	16.70	1	28.4	2	190.6	6.7

Note: Significant change in speed with optimal choice of design variables

Parameters from .25u TSMC Process

u	3.74E+10	1/(V*sec)
2*lambda	0.25	u
hsw	0.61	none
hbot	0.32	none
µn/up	4.1	

Integrator Io for Special Layouts

file: integrator-speed-comp

Note: Process parameters may be a little optimistic but relative performance should be as predicted

Concentric Layout

VEB1/ VEB2	K	K2	K2^	W1	W2	SWn	SWp	BOTn	BOTp	SW comp Total	Bot comp Total	Load Comp	Den	VEB1	Io,no dif GHz	Io GHz
1	1	4.8		3.7	15.2	0.25	1.19	0.19	4.53	1.44	4.73	1.08	7.24	1	88.3	13.2
1	2	8.9		6.7	27.5	0.27	1.22	0.43	8.56	1.49	8.99	1.04	11.53	1	91.3	8.3
1	4	17.1		12.7	52.1	0.29	1.23	0.91	16.63	1.52	17.53	1.02	20.08	1	93.1	4.7
1	1	4.8		3.7	15.2	0.25	1.19	0.19	4.53	1.44	4.73	1.08	7.24	1.5	132.5	19.7
1	2	8.8		6.7	27.5	0.27	1.22	0.43	8.56	1.49	8.99	1.04	11.53	1.5	136.9	12.4
1	4	17.1		12.7	52.1	0.29	1.23	0.91	16.63	1.52	17.53	1.02	20.08	1.5	139.7	7.1
1	1	4.8		3.7	15.2	0.25	1.19	0.19	4.53	1.44	4.73	1.08	7.24	2	176.6	26.3
1	2	8.9		6.7	27.5	0.27	1.22	0.43	8.56	1.49	8.99	1.04	11.53	2	182.6	16.5
1	4	17.1		12.7	52.1	0.29	1.23	0.91	16.63	1.52	17.53	1.02	20.08	2	186.3	9.5
0.5	1	1.0		3.7	3.8	0.25	0.25	0.19	0.21	0.50	0.40	1.08	1.98	1	88.3	48.1
0.6	2	2.1		6.7	6.9	0.27	0.28	0.43	0.45	0.55	0.88	1.04	2.48	1	91.3	38.4
0.5	4	4.1		12.7	13.0	0.29	0.30	0.91	0.96	0.58	1.86	1.02	3.47	1	93.1	27.5
0.6	1	1.0		3.7	3.8	0.25	0.25	0.19	0.21	0.50	0.40	1.08	1.98	1.5	132.5	72.2
0.5	2	2.1		6.7	6.9	0.27	0.28	0.43	0.45	0.55	0.88	1.04	2.48	1.5	136.9	57.6
0.6	4	4.1		12.7	13.0	0.29	0.30	0.91	0.96	0.58	1.86	1.02	3.47	1.5	139.7	41.2
0.5	1	1.0		3.7	3.8	0.25	0.25	0.19	0.21	0.50	0.40	1.08	1.98	2	176.6	96.2
0.5	2	2.1		6.7	6.9	0.27	0.28	0.43	0.45	0.55	0.88	1.04	2.48	2	182.6	76.8
0.5	4	4.1		12.7	13.0	0.29	0.30	0.91	0.96	0.58	1.86	1.02	3.47	2	186.3	54.9
2	1	20.0		3.7	60.8	0.25	4.94	0.19	77.92	5.19	78.11	1.08	84.38	1	88.3	1.1
2	2	36.4		6.7	110.0	0.27	4.97	0.43	142.47	5.24	142.90	1.04	149.18	1	91.3	0.6
2	4	69.2		12.7	208.4	0.29	4.99	0.91	271.56	5.27	272.47	1.02	278.77	1	93.1	0.3
2	1	20.0		3.7	60.8	0.25	4.94	0.19	77.92	5.19	78.11	1.08	84.38	1.5	132.5	1.7
2	2	36.4		6.7	110.0	0.27	4.97	0.43	142.47	5.24	142.90	1.04	149.18	1.5	136.9	1.0
2	4	69.2		12.7	208.4	0.29	4.99	0.91	271.56	5.27	272.47	1.02	278.77	1.5	139.7	0.5
2	1	20.0		3.7	60.8	0.25	4.94	0.19	77.92	5.19	78.11	1.08	84.38	2	176.6	2.3
2	2	36.4		6.7	110.0	0.27	4.97	0.43	142.47	5.24	142.90	1.04	149.18	2	182.6	1.3
2	4	69.2		12.7	208.4	0.29	4.99	0.91	271.56	5.27	272.47	1.02	278.77	2	186.3	0.7

Segmented Conccentric Layout

VEB1/ VEB2	K	K2	K2^	W1	W2	SWn	SWp	BOTn	BOTp	SW comp Total	Bot comp Total	Load Comp	Den	VEB1	Io,no dif GHz	Io GHz
1	1	4.8	2.3	3.71	16.2	0.25	1.13	0.19	2.05	1.38	2.24	1.08	4.70	1	88.3	20.3
1	2	8.9	4.35	6.71	27.5	0.27	1.19	0.43	4.06	1.46	4.49	1.04	6.99	1	91.3	13.6
1	4	17.1	8.45	12.71	52.1	0.29	1.22	0.91	8.09	1.50	8.99	1.02	11.52	1	93.1	8.3
1	1	4.8	2.3	3.71	15.2	0.25	1.13	0.19	2.05	1.38	2.24	1.08	4.70	1.5	132.5	30.4
1	2	8.9	4.35	6.71	27.5	0.27	1.19	0.43	4.06	1.46	4.49	1.04	6.99	1.5	136.9	20.4
1	4	17.1	8.45	12.71	52.1	0.29	1.22	0.91	8.09	1.50	8.99	1.02	11.52	1.5	139.7	12.4
1	1	4.8	2.3	3.71	15.2	0.25	1.13	0.19	2.05	1.38	2.24	1.08	4.70	2	176.6	40.5
1	2	8.9	4.35	6.71	27.5	0.27	1.19	0.43	4.06	1.46	4.49	1.04	6.99	2	182.6	27.3
1	4	17.1	8.45	12.71	52.1	0.29	1.22	0.91	8.09	1.50	8.99	1.02	11.52	2	188.3	16.5
0.5	1	1.0	0.4	3.71	3.8	0.25	0.20	0.19	0.06	0.44	0.26	1.08	1.78	1	88.3	53.6
0.5	2	2.1	0.91	6.71	6.9	0.27	0.25	0.43	0.18	0.52	0.61	1.04	2.17	1	91.3	43.9
0.6	4	4.1	1.94	12.71	13.0	0.29	0.28	0.91	0.42	0.57	1.33	1.02	2.92	1	93.1	32.6
0.5	1	1.0	0.4	3.71	3.8	0.25	0.20	0.19	0.06	0.44	0.26	1.08	1.78	1.5	132.5	80.4
0.6	2	2.1	0.91	6.71	6.9	0.27	0.25	0.43	0.18	0.52	0.61	1.04	2.17	1.5	136.9	65.8
0.6	4	4.1	1.94	12.71	13.0	0.29	0.28	0.91	0.42	0.57	1.33	1.02	2.92	1.5	139.7	48.9
0.5	1	1.0	0.4	3.71	3.8	0.25	0.20	0.19	0.06	0.44	0.26	1.08	1.78	2	176.6	107.2
0.5	2	2.1	0.91	6.71	6.9	0.27	0.25	0.43	0.18	0.52	0.61	1.04	2.17	2	182.6	87.7
0.5	4	4.1	1.94	12.71	13.0	0.29	0.28	0.91	0.42	0.57	1.33	1.02	2.92	2	186.3	65.2
2	1	20.0	9.9	3.71	60.8	0.25	4.89	0.19	38.05	5.13	38.24	1.08	44.45	1	88.3	2.1
2	2	36.4	18.1	6.71	110.0	0.27	4.94	0.43	70.31	5.21	70.74	1.04	77.00	1	91.3	1.2
2	4	69.2	34.5	12.71	208.4	0.29	4.97	0.91	134.86	5.26	135.77	1.02	142.04	1	93.1	0.7
2	1	20.0	9.9	3.71	60.8	0.25	4.89	0.19	38.05	5.13	38.24	1.08	44.45	1.5	132.5	3.2
2	2	36.4	18.1	6.71	110.0	0.27	4.94	0.43	70.31	5.21	70.74	1.04	77.00	1.5	136.9	1.9
2	4	69.2	34.5	12.71	208.4	0.29	4.97	0.91	134.86	5.26	135.77	1.02	142.04	1.5	139.7	1.0
2	1	20.0	9.9	3.71	60.8	0.25	4.89	0.19	38.05	5.13	38.24	1.08	44.45	2	176.6	4.3
2	2	36.4	18.1	6.71	110.0	0.27	4.94	0.43	70.31	5.21	70.74	1.04	77.00	2	182.6	2.5
2	4	69.2	34.5	12.71	208.4	0.29	4.97	0.91	134.86	5.26	135.77	1.02	142.04	2	186.3	1.3

Parameters from 0.25u TSMC process

u	3.74E+10	1/(V*sec)
2*lambda	0.25	u
hsw	0.61	none
hbot	0.32	none
'up	4.1	

Lossy Integrator

Note: Process parameters may be a little optimistic but relative performance should be as predicted.

File:lossy-integrator-speed-comp

K	W2	W1	SWn	SWp	BOTn	BOTp	SW comp Total	Bot comp Total	Load comp	Den	VEB1	Io,no dif GHz	Io GHz
P-channel Load, Conventional Layout													
1	0.75	0.73	1.24	1.25	0.96	0.98	2.49	1.94	2.03	6.45	1	40.8	12.8
2	1.50	1.46	0.92	0.94	0.96	0.98	1.86	1.94	2.03	5.83	1	40.8	14.2
4	3.00	2.93	0.77	0.78	0.96	0.98	1.55	1.94	2.03	5.52	1	40.8	15.0
1	0.75	0.73	1.24	1.25	0.96	0.98	2.49	1.94	2.03	6.45	1.5	61.1	19.2
2	1.50	1.46	0.92	0.94	0.96	0.98	1.86	1.94	2.03	5.83	1.5	61.1	21.2
4	3.00	2.93	0.77	0.78	0.96	0.98	1.55	1.94	2.03	5.52	1.5	61.1	22.4
1	0.75	0.73	1.24	1.25	0.96	0.98	2.49	1.94	2.03	6.45	2	81.5	25.6
2	1.50	1.46	0.92	0.94	0.96	0.98	1.86	1.94	2.03	5.83	2	81.5	28.3
4	3.00	2.93	0.77	0.78	0.96	0.98	1.55	1.94	2.03	5.52	2	81.5	29.9
P-channel Load, Concentric Layout													
1	3.80	3.71	0.25	0.254	0.194	0.206	0.50	0.40	2.18	3.08	1	37.8	26.7
2	6.87	6.71	0.27	0.28	0.429	0.454	0.55	0.88	2.11	3.55	1	39.1	23.3
4	13.02	12.71	0.29	0.296	0.907	0.955	0.58	1.86	2.07	4.52	1	39.8	18.3
1	3.80	3.71	0.25	0.254	0.194	0.206	0.50	0.40	2.18	3.08	1.5	56.7	40.1
2	6.87	6.71	0.27	0.28	0.429	0.454	0.55	0.88	2.11	3.55	1.5	58.6	34.9
4	13.02	12.71	0.29	0.296	0.907	0.955	0.58	1.86	2.07	4.52	1.5	59.8	27.4
1	3.80	3.71	0.25	0.254	0.194	0.206	0.50	0.40	2.18	3.08	2	75.6	53.5
2	6.87	6.71	0.27	0.28	0.429	0.454	0.55	0.88	2.11	3.55	2	78.1	46.5
4	13.02	12.71	0.29	0.296	0.907	0.955	0.58	1.86	2.07	4.52	2	79.7	36.5
N-Channel Load, Simple Layout													
1	0.75	3.00					0.76	0.72	1.25	2.73	1	66.0	30.2
2	1.50	6.00					0.69	0.72	1.25	2.66	1	66.0	31.1
4	3.00	12.00					0.65	0.72	1.25	2.62	1	66.0	31.5
1	0.75	3.00					0.76	0.72	1.25	2.73	1.5	99.0	45.3
2	1.50	6.00					0.69	0.72	1.25	2.66	1.5	99.0	46.6
4	3.00	12.00					0.65	0.72	1.25	2.62	1.5	99.0	47.3
1	0.75	3.00					0.76	0.72	1.25	2.73	2	132.0	60.4
2	1.50	6.00					0.69	0.72	1.25	2.66	2	132.0	62.1
4	3.00	12.00					0.65	0.72	1.25	2.62	2	132.0	63.0
N-Channel Load, Concentric Layout													
1	3.71	14.83					0.31	0.24	1.35	1.90	1	61.2	43.4
2	6.71	26.83					0.34	0.54	1.30	2.18	1	63.3	37.8
4	12.71	50.83					0.36	1.13	1.28	2.77	1	64.5	29.8
1	3.71	14.83					0.31	0.24	1.35	1.90	1.5	91.8	65.1
2	6.71	26.83					0.34	0.54	1.30	2.18	1.5	94.9	56.7
4	12.71	50.83					0.36	1.13	1.28	2.77	1.5	96.8	44.7
1	3.71	14.83					0.31	0.24	1.35	1.90	2	122.4	86.9
2	6.71	26.83					0.34	0.54	1.30	2.18	2	126.5	75.6
4	12.71	50.83					0.36	1.13	1.28	2.77	2	129.1	59.5

EE 508

Lecture 39

Current Mode Filters

Current-Mode Filters

Current-Mode Filters have become a topic of considerable interest in recent years

Consider first a brief background about filters

Recall:

John Hughes introduced the concept of the switched-current filter in 1989

This was considered a revolutionary concept since it offered potential for operating at very high frequencies with simple amplifiers (current mirrors) but no operational amplifiers. Some properties of Hughes's current-mode filters

1. **Filter parameters dependend only on geometric ratios and clock frequency**
2. **Insensitive to value of parasitic capacitors**
3. **Operates at both low and high frequencies**
4. **Very small**
5. **Can operate at very low voltages (one V_T and one V_{EB} between rails if switches are neglected)**

Others argued that these properties were inherent in the current-mode of operation and that continuous-time structures may perform even better !

A current-conveyor community had been struggling for years to get any adoption and this seemed to propel them to the forefront of the technology field

Literally hundreds of researchers jumped on the current-mode filter bandwagon

Recall:

John Hughes introduced the concept of the switched-current filter in 1989

Hughes has been recognized as a renowned filter design expert for many years and has had the benefits of an industrial research environment to support his work

Update on Hughes Work

Recall the Hughes integrator:

$$H(z) = \frac{B}{1 + Az^{-1}}$$

Hughes found the sensitivity of the parameter A was too large in his original structure to make an acceptable lossless integrator

He made some modifications to this approach to improve the sensitivity problem

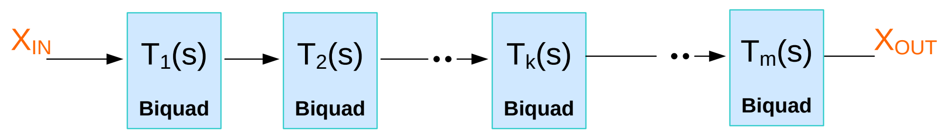
He worked for about another 10 years to develop practical switched-current filters at Phillips but struggled to get good practical experimental results. He retired several years ago

There appears to be little work going on today on the switched-current filter and there appears to be little adoption of the concept

Filter Background

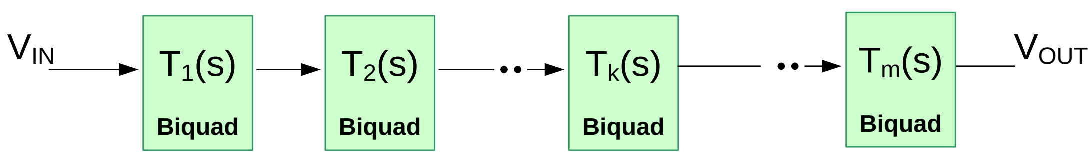
Conventional Wisdom:

- A current-mode filter is a filter where the input and output variables are currents
- A voltage-mode filter is a filter where the input and output variables are voltages



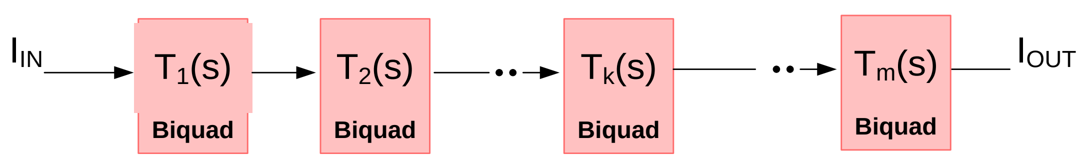
$$T(s) = T_1 T_2 \bullet\bullet T_m$$

Voltage Mode Filter



$$T(s) = \frac{V_{OUT}}{V_{IN}} = T_1 T_2 \bullet\bullet T_m$$

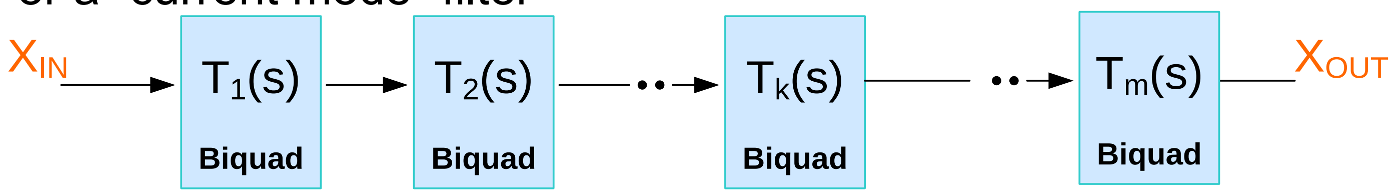
Current Mode Filter



$$T(s) = \frac{I_{OUT}}{I_{IN}} = T_1 T_2 \bullet\bullet T_m$$

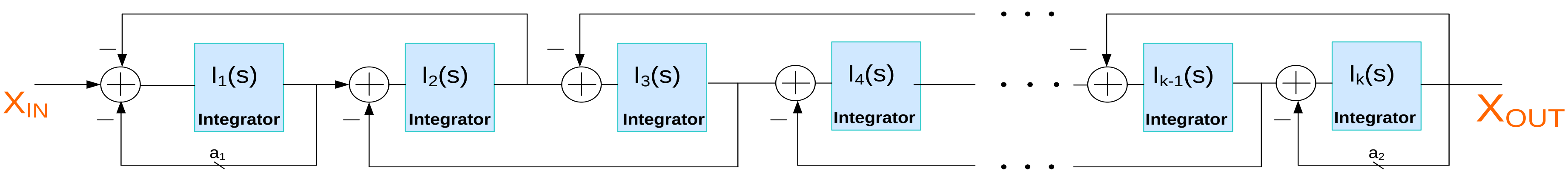
Filter Background

- Most higher-order filters today are built around one of the following architectures
- These basic structures have evolved because of their performance capabilities (e.g. sensitivities, component spread, …)
- These basic structures are used irrespective of whether the filter is a "voltage mode" or a "current mode" filter

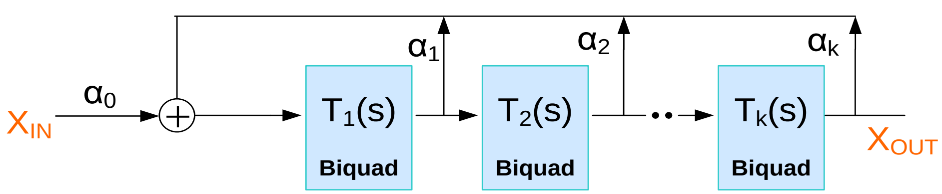


Cascaded Biquad

$$T(s) = T_1 T_2 \bullet\bullet T_m$$



Leapfrog



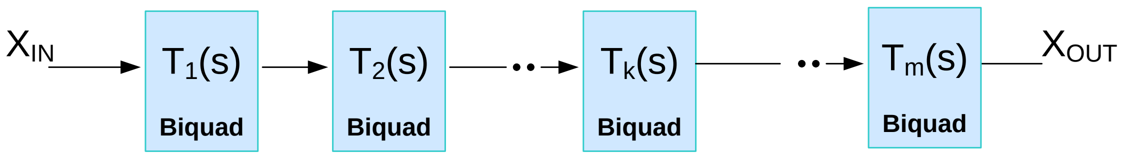
Primary Resonator Block

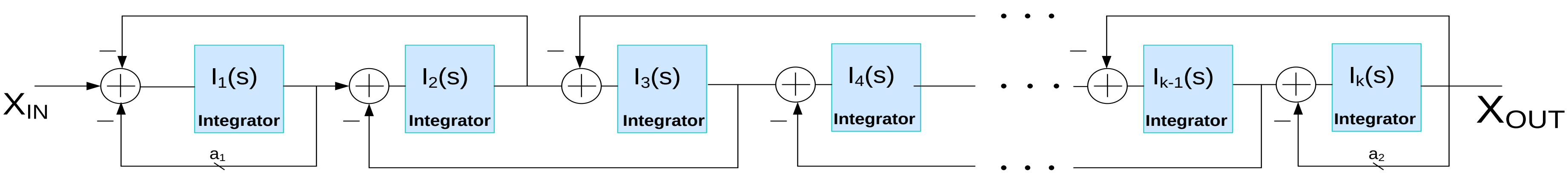
$$T(s) = \frac{-a_0 T_1 T_2 \bullet\bullet T_k}{1 + a_1 T_1 + a_2 T_1 T_2 + \bullet\bullet + a_k T_1 T_2 \bullet\bullet T_k}$$

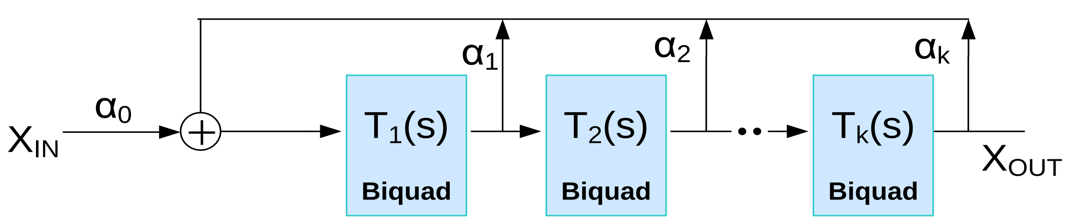
$$a_k = \frac{\alpha_F}{\alpha_k}$$

Filter Background

Most filters today, particularly integrated structures, are built with integrators





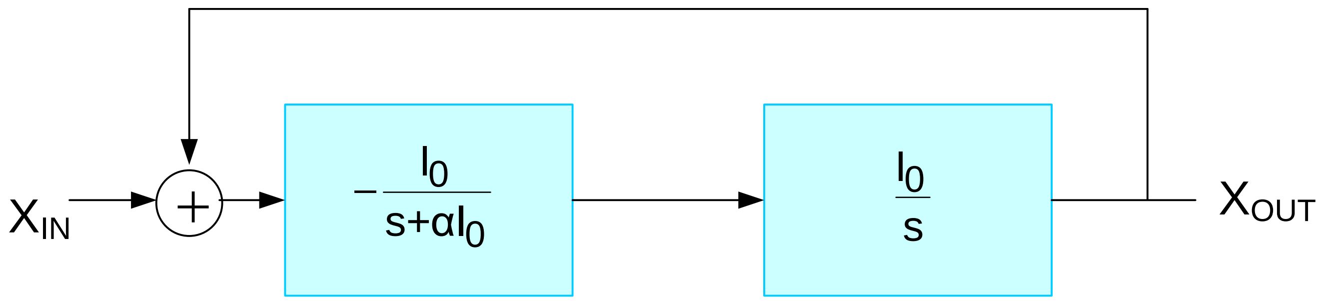


Biquads are usually built with integrators !

Filter Background

Most filters today, particularly integrated structures, are built with integrators

Typical integrator-based biquadratic structure (shown LP only)



$$T(s)=\frac{-I_0^2}{s^2+\alpha I_0 s+I_0^2}$$

Tow-Thomas Biquad

- State Variable Biquad
- Two Integrator Loop

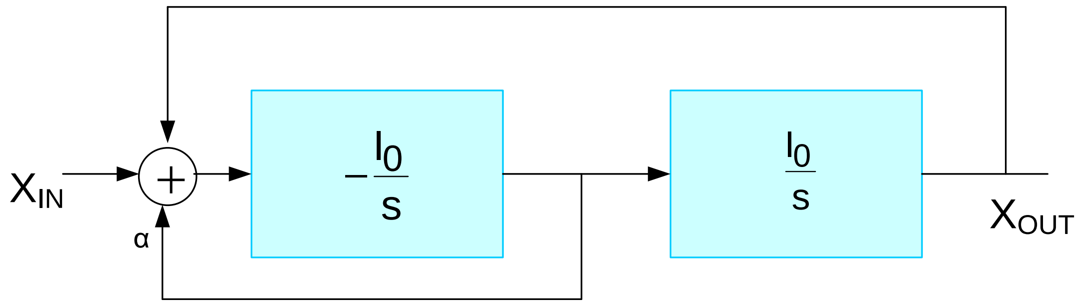
Similar to:

- KHN Biquad
- Lead and Lag in a Loop

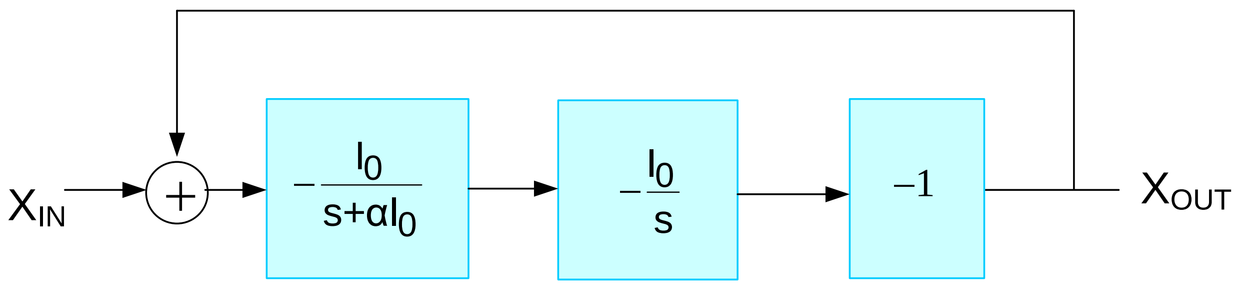
Filter Background

Most filters today, particularly integrated structures, are built with integrators

Variants of two integrator loop



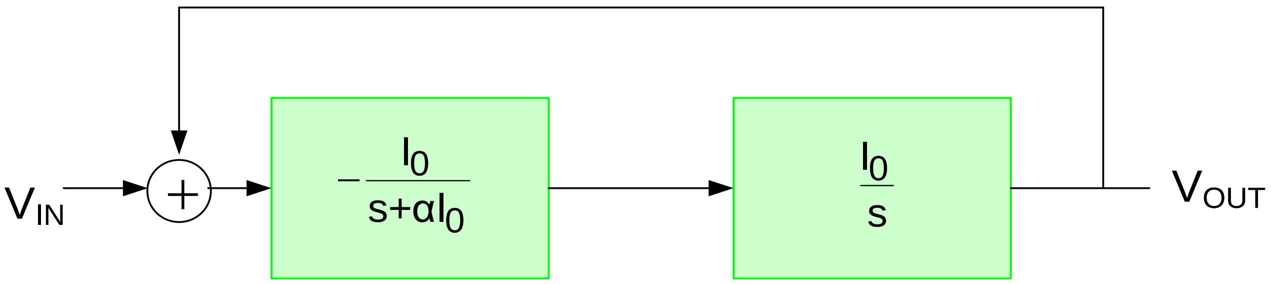
$$T(s)=\frac{-I_0^2}{s^2+\alpha I_0 s+I_0^2}$$



$$T(s)=\frac{-I_0^2}{s^2+\alpha I_0 s+I_0^2}$$

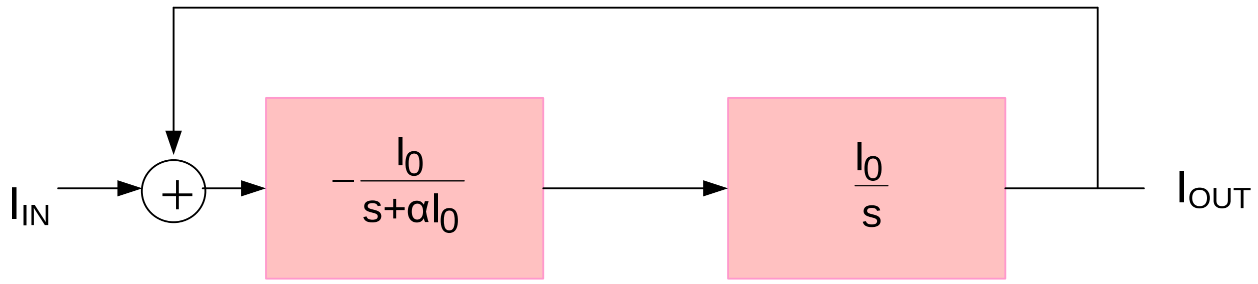
Filter Background

Voltage-Mode Biquad



$$T(s) = \frac{V_{OUT}}{V_{IN}} = \frac{-I_0^2}{s^2 + \alpha I_0 s + I_0^2}$$

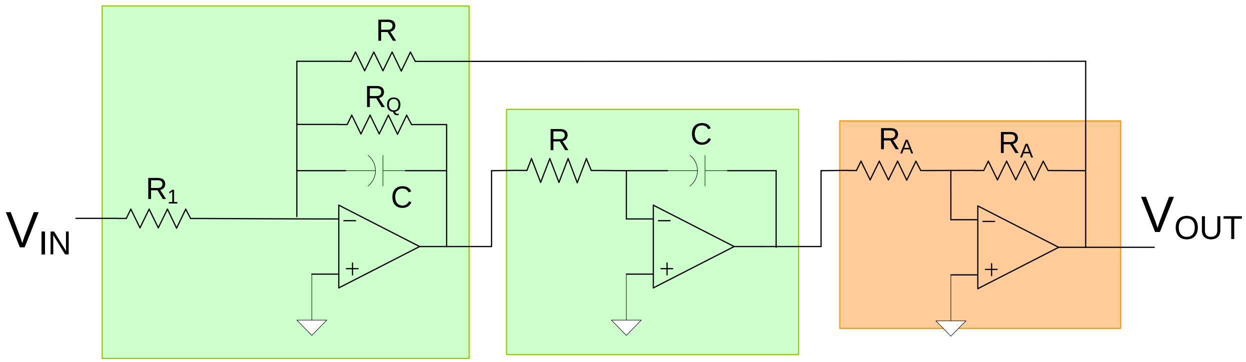
Current-Mode Biquad



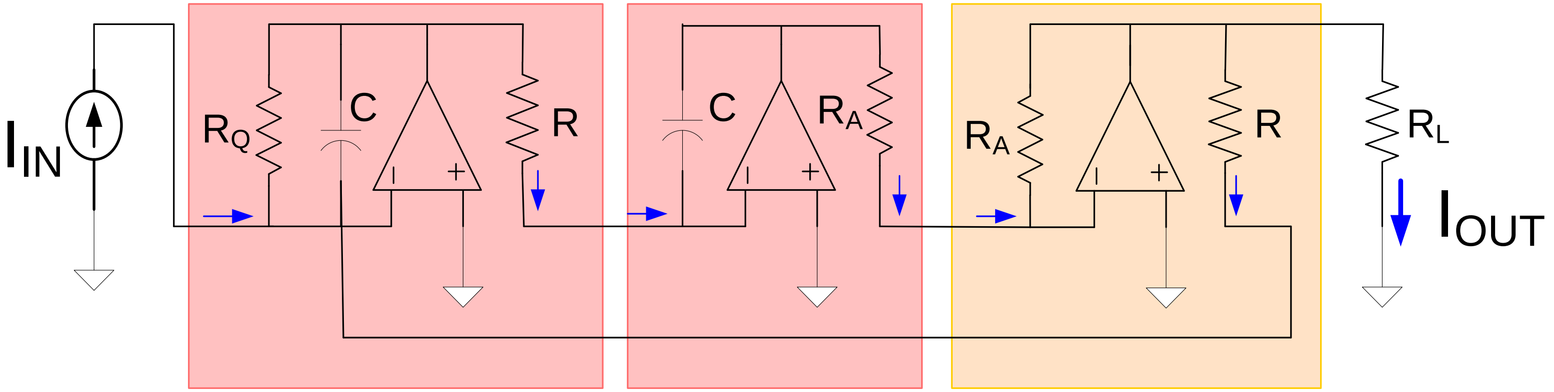
$$T(s) = \frac{I_{OUT}}{I_{IN}} = \frac{-I_0^2}{s^2 + \alpha I_0 s + I_0^2}$$

Filter Background

Voltage-Mode Biquad

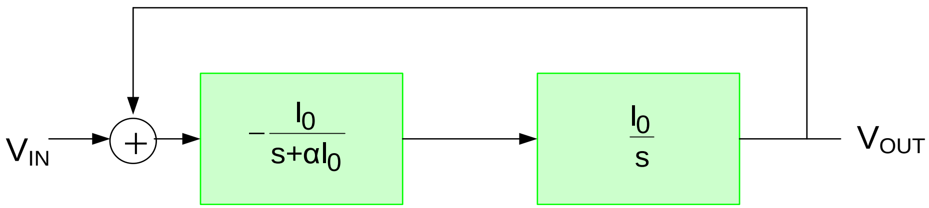


Current-Mode Biquad

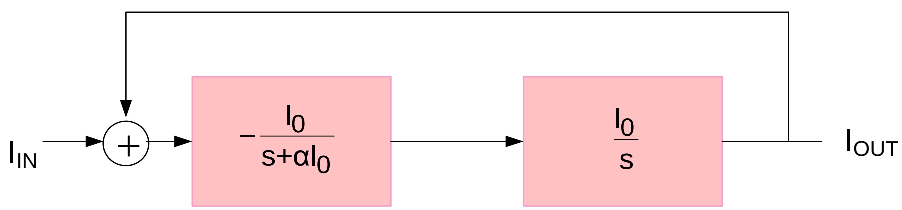


Notice considerable differences in the circuit-level implementations

Filter Background



$$T(s) = \frac{V_{OUT}}{V_{IN}} = \frac{-I_0^2}{s^2+\alpha I_0 s+I_0^2}$$

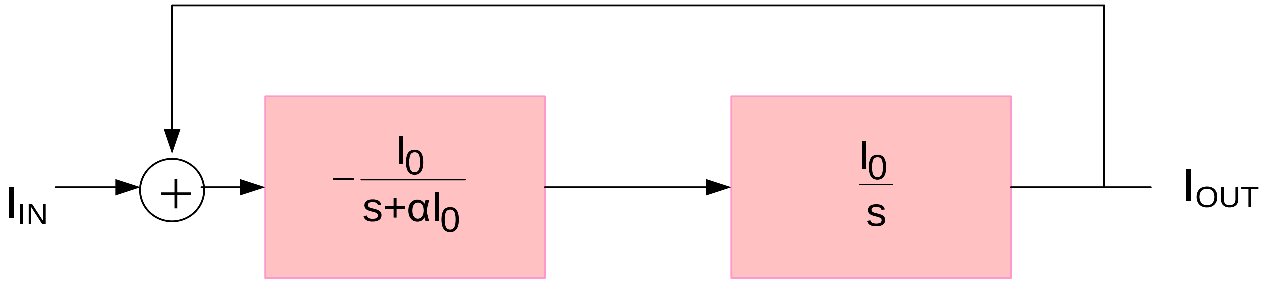


$$T(s) = \frac{I_{OUT}}{I_{IN}} = \frac{-I_0^2}{s^2+\alpha I_0 s+I_0^2}$$

Observations:

- Structures for voltage mode and current mode Integrators are often the same
- Structures for voltage mode and current mode filters are often the same
- Circuit-level implementations appear to be quite different

Current-Mode Filters



$$T(s)=\frac{I_{OUT}}{I_{IN}}=\frac{-I_0^2}{s^2+\alpha I_0 s+I_0^2}$$

Concept of Current-Mode Filters is Somewhat Recent:

Key paper that generated interest in current-mode filters:

Switched currents-a new technique for analog sampled-data signal processing

JB **Hughes**, NC **Bird**, IC **Macbeth** - Circuits and Systems, 1989., …, 1989 - ieeexplore.ieee.org

Abstract A technique calledswitched currents,'for analog sampled-data signal processing in the current domain, is introduced. A family of modules that are capable of various computational and memory functions is described. The modules are well suited to system **...**

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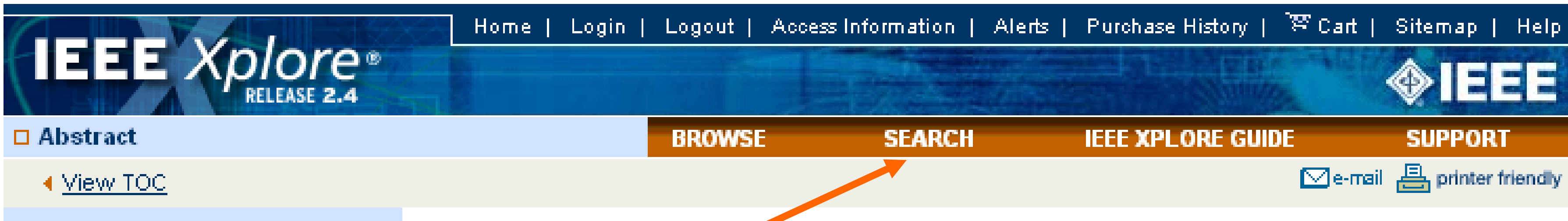
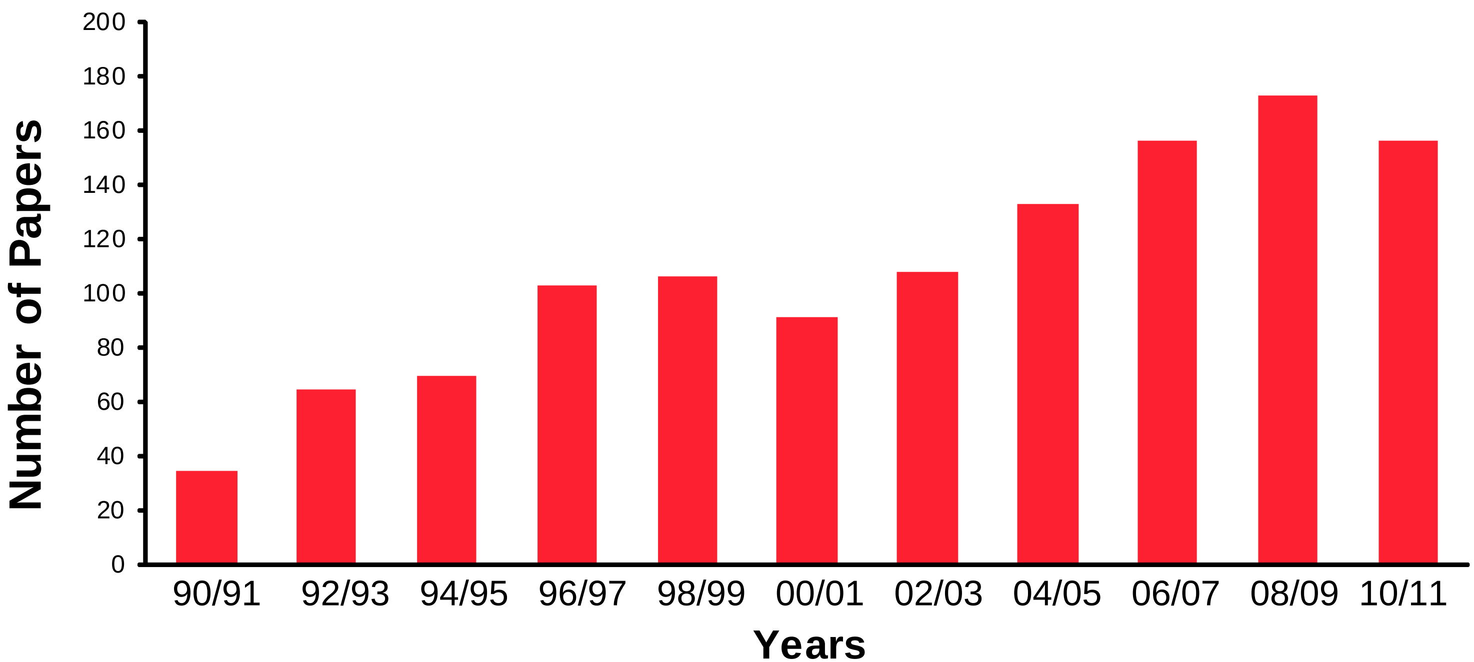
Current-Mode Filters



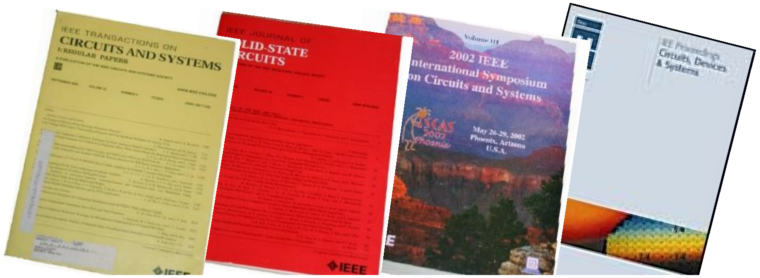
Advanced Search for "current-mode" and "filters" in Metadata



Updated Nov 22, 2012

Current-Mode Filters

IEEE Xplore® RELEASE 2.4



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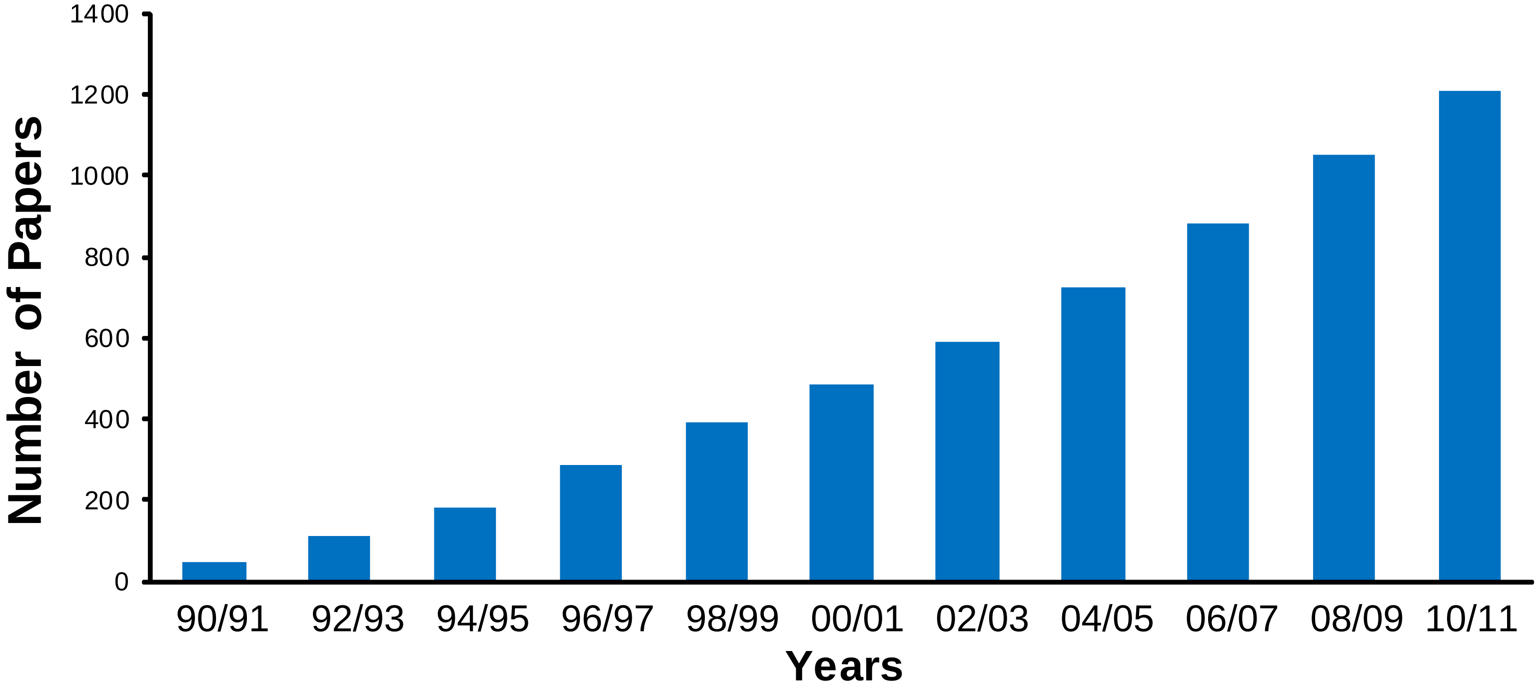


Steady growth in research in the area since 1990 and publication rate is growing with time !!

Updated Nov 22, 2012

Current-Mode Filters

The Conventional Wisdom:

TSP 2012:

RECENTLY, current-mode circuits have been receiving considerable attention due to their potential advantages such as inherently wide bandwidth, higher slew-rate, greater linearity, wider dynamic range, simpler circuitry and lower power consumption [1].

CECNet 2012

In analog circuit design, many researches have been performed on current-mode active filters using different active elements [1–22]. The use of current-mode active devices has many other advantages such as larger dynamic, higher bandwidth, greater linearity, simple circuitry and low power consumption compared to that of voltage-mode counterparts for example operational amplifiers [1–2].

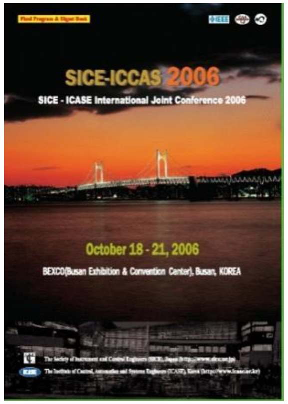
Current-Mode Filters

The Conventional Wisdom:

Proc. IEE Dec 2006:

1 Introduction

Current-mode circuits have been proven to offer advantages over their voltage-mode counterparts [1, 2]. They possess wider bandwidth, greater linearity and wider dynamic range. Since the dynamic range of the analogue circuits using low-voltage power supplies will be low, this problem can be solved by employing current-mode operation.



Proc. SICE-ICASE, Oct. 2006

1. INTRODUCTION

It is well known that current-mode circuits have been receiving significant attention owing to its advantage over the voltage-mode counterpart, particularly for higher frequency of operation and simpler filtering structure [1].

Current-Mode Filters

The Conventional Wisdom:

JSC April 1998:

"… current-mode functions exhibit higher frequency potential, simpler architectures, and lower supply voltage capabilities than their voltage-mode counterparts."

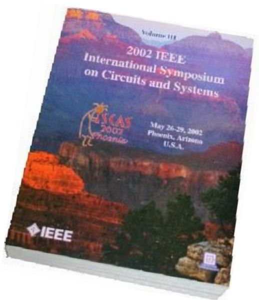
CAS June 1992

"Current-mode signal processing is a very attractive approach due to the simplicity in implementing operations such as … and the potential to operate at higher signal bandwidths than their voltage mode analogues" … "Some voltage-mode filter architectures using transconductance amplifiers and capacitors (TAC) have the drawback that …"

Current-Mode Filters

The Conventional Wisdom:

ISCAS 1993:

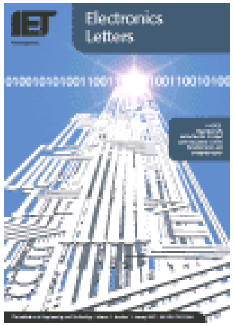


"In this paper we propose a fully balanced high frequency current-mode integrator for low voltage high frequency filters. Our use of the term current mode comes from the use of current amplifiers as the basic building block for signal processing circuits. This fully differential integrator offers significant improvement even over recently introduced circuit with respect to accuracy, high frequency response, linearity and power supply requirements. Furthermore, it is well suited to standard digital based CMOS processes."

Current-Mode Filters

The Conventional Wisdom:

Two key publications where benefits of the current-mode circuits are often referenced:



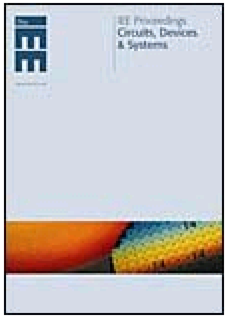
All current-mode frequency selective circuits **GW Roberts**, AS **Sedra** - Electronics Letters, June 1989 - pp. 759-761 Cited by 161

"To make greatest use of the available transistor bandwidth $\boldsymbol{f_T}$ **,** and operate at low voltage supply levels, it has become apparent that analogue signal processing can greatly benefit from processing current signals rather than voltage signals. Besides this, it is well known by electronic circuit designers that the mathematical operations of adding, subtracting or multiplying signals represented by currents are simpler to perform than when they are represented by voltages. This also means that the resulting circuits are simpler and require less silicon area."

Current-Mode Filters

The Conventional Wisdom:

Two key publications where benefits of the current-mode circuits are often referenced:



Recent developments in **current** conveyors and **current**-mode circuits **B Wilson** - Circuits, Devices and Systems, IEE Proceedings G, pp. 63-77, Apr. 1990 Cited by 203

“The **use** of current rather than voltage as the active parameter can result in higher usable gain, accuracy and bandwidth due to reduced voltage excursion at sensitive nodes. A current-mode approach is not just restricted to current processing, but also offers certain important advantages when interfaced to voltage-mode circuits.”

Current-Mode Filters

Conventional Wisdom:

- Current-Mode filters operate at higher-frequencies than voltage-mode counterparts
- Current-Mode filters operate at lower supply voltages and lower power levels than voltage-mode counterparts
- Current-Mode filters are simpler than voltage-mode counterparts
- Current-Mode filters offer better linearity than voltage-mode counterparts
- Integrated Current-Mode filters require less area

Observation

- Many papers have appeared that tout the performance advantages of current-mode circuits
- In all of the current-mode papers that this author has seen, no attempt is made to provide a quantitative comparison of the key performance features of current-mode circuits with voltage-mode counterparts
- All justifications of the advantages of the current-mode circuits this author has seen are based upon qualitative statements

Observations (cont.)

- In selected comparisons this author has made, performance characteristics of current-mode filters do not appear to be substantially better than those reported for “other” approaches
- It appears easy to get papers published that have the term “current-mode” in the title
- Over 1200 papers have been published in IEEE forums alone !

Research Opportunity (for academia)

- Provide a formal justification of the high frequency, low voltage and low power benefits of current-mode circuits
- Gain insight into how these benefits can be further exploited
- Sounds like a simple problem

Questions about the Conventional Wisdom

- Why does a current-mode circuit work better at high frequencies?
- Why is a current-mode circuit better suited for low frequencies?
- Why do some “voltage”-mode circuits have specs that are as good as the current-mode circuits?

Questions about the Conventional Wisdom

- Why are most of the papers on current-mode circuits coming from academia?
- Why haven’t current-mode circuits replaced “voltage”-mode circuits in industrial applications?

Questions about the Conventional Wisdom

- Are current-mode circuits really better than their "voltage-mode" counterparts?

- What is a current-mode circuit?
  - Must have a simple answer since so many authors use the term

- Do all agree on the definition of a current-mode circuit?

Questions about the Conventional Wisdom

What is a current-mode circuit?

- Everybody seems to know what it is
- Few have tried to define it
- Is a current-mode circuit not a voltage-mode circuit?

Questions about the Conventional Wisdom

What is a current-mode circuit?

"Several analog CMOS continuous-time filters for high frequency applications have been reported in the literature… Most of these filters were designed to process voltage signals. It results in high voltage power supply and large power dissipation. To overcome these drawbacks of the voltage-mode filters, the current-mode filters circuits , which process current signals have been developed"

A **3V-50MHz Analog CMOS Current-Mode High Frequency Filter with a Negative Resistance Load**, pp. 260…,,IEEE Great Lakes Symposium March 1996.

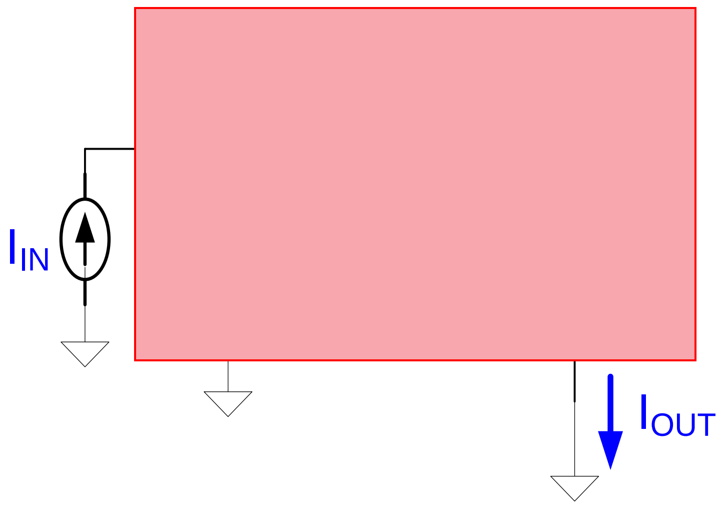
Questions about the Conventional Wisdom

Conventional Wisdom Definition:

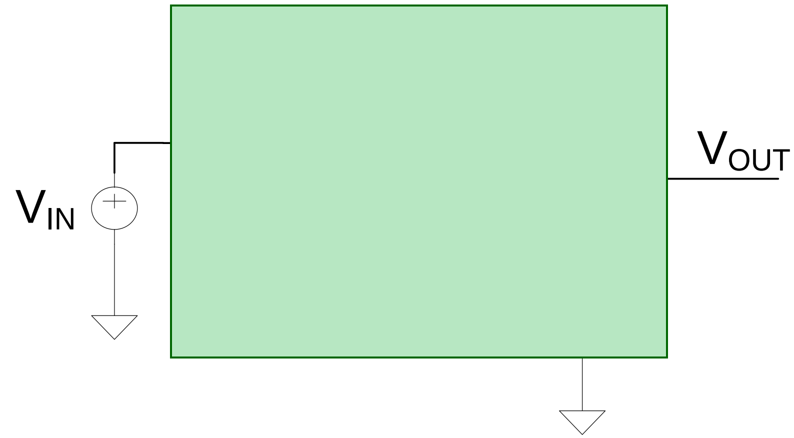
- A current-mode circuit is a circuit that processes current signals
- A current-mode circuit is one in which the defined state variables are currents

Example:

Is this a current-mode circuit?



Is this a voltage-mode circuit?



Conventional Wisdom Definition:

A current-mode circuit is a circuit that processes current signals

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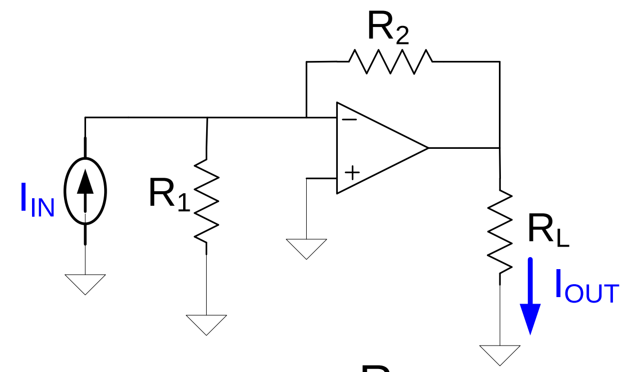
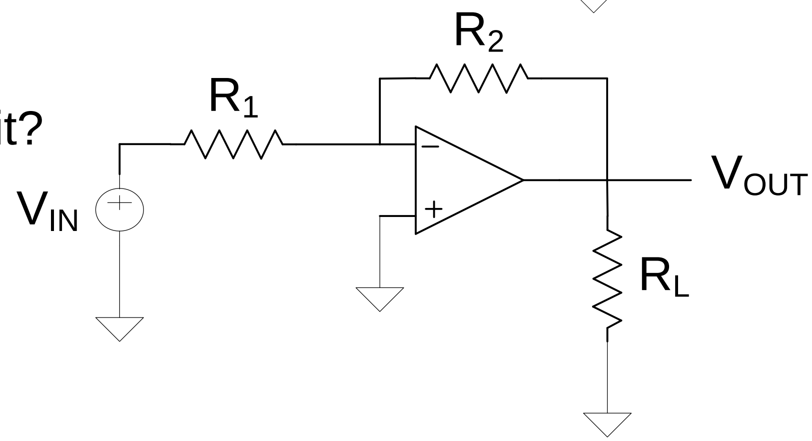
Is this a current-mode circuit?



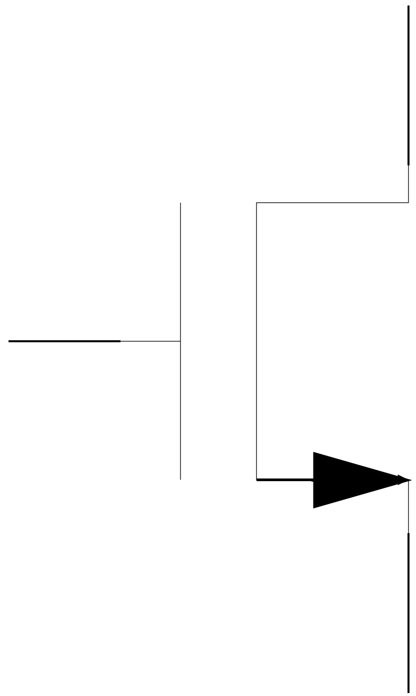
Is this a voltage-mode circuit?



- One is obtained from the other by a Norton to Thevenin Transformation
- **The poles and the BW of the two circuits are identical !**

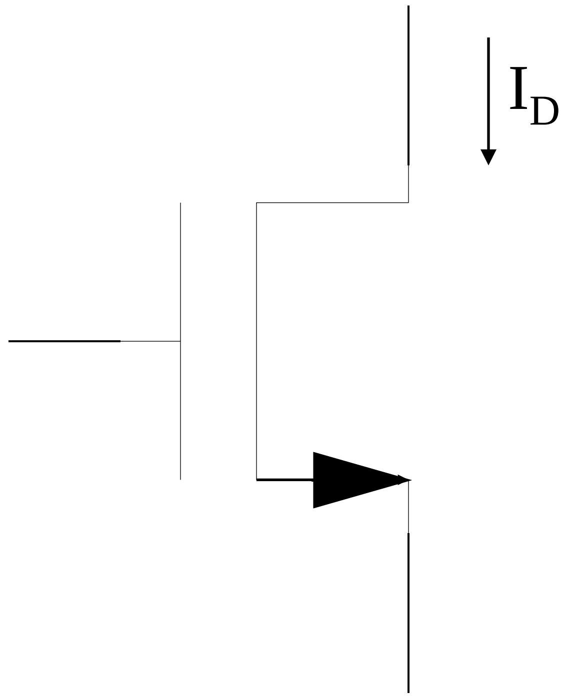
Question?

Is the following circuit a voltage mode-circuit or a current-mode circuit?



Question?

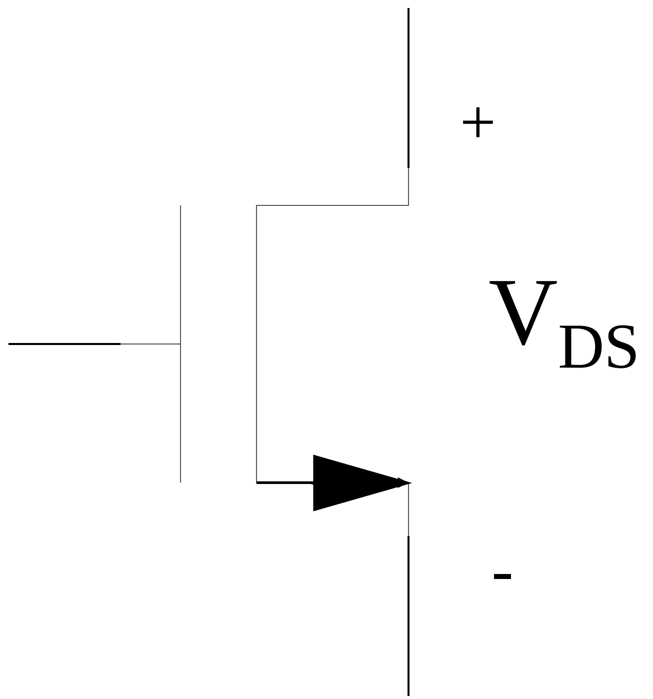
Is the following circuit a voltage mode-circuit or a current-mode circuit?



Current Mode !

Question?

Is the following circuit a voltage mode-circuit or a current-mode circuit?



Voltage Mode !

Observations:

- Voltage-Mode or Current-Mode Operation of a Given Circuit is not Obvious
- All electronic devices have a voltage-current relationship between the port variables that characterizes the device
- The “solution” of all circuits is identical independent of whether voltages or currents are used as the state variables
- The poles of any circuit are independent of whether the variables identified for analysis are currents or voltages or a mixture of the two

Questions about the Conventional Wisdom

Is it possible that there are really no benefits from frequency response, supply voltage and power dissipation viewpoints for "current-mode" circuits?

JSC April 1998:

"… current-mode functions exhibit higher frequency potential, simpler architectures, and lower supply voltage capabilities than their voltage-mode counterparts."

Questions about the Conventional Wisdom

Is it possible that there are really no benefits from a frequency response, supply voltage and power dissipation viewpoints for "current-mode" circuits?

Observation: If any so-called current-mode circuit is analyzed using voltages as the port variables, the poles, sensitivities, frequency response, power dissipation, supply voltage requirements and the power dissipation will all be the same!

Questions about the Conventional Wisdom

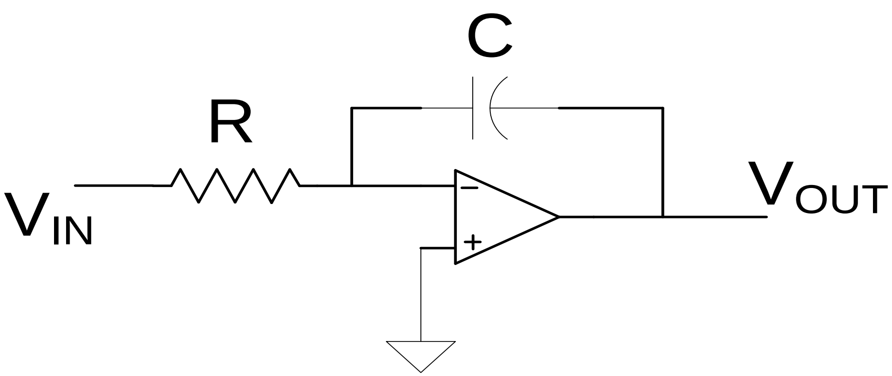
Since a given structure can be implemented with either current-mode or voltage-mode circuits, is there a fundamental relationship between the performance of so-called current-mode circuits and their "voltage-mode" counterparts?

Comparison of Continuous-Time Current-Mode and Voltage-Mode Filters

- Current-Mode and Voltage-Mode Integrators
  - Op-amp based current-mode and voltage-mode integrators
  - g_mC current-mode and voltage-mode integrators
  - High frequency current-mode and voltage-mode integrators
- Structure Comparisons
  - Two integrator loop filters
  - Leapfrog filters

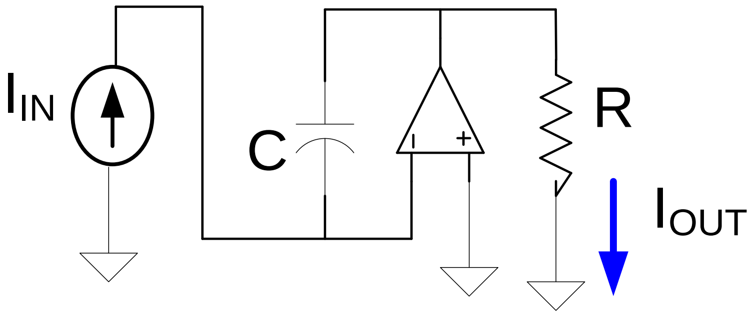
Basic Feedback Inverting Integrators

Voltage-Mode



$$\frac{V_{OUT}}{V_{IN}} = -\frac{1}{sRC}$$

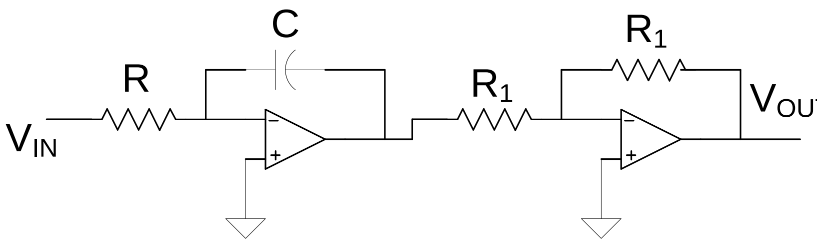
Current-Mode



$$\frac{I_{OUT}}{I_{IN}} = -\frac{1}{sRC}$$

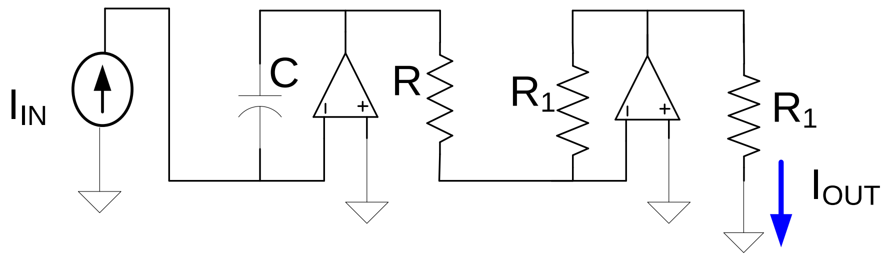
Basic Feedback Non-Inverting Integrators

Voltage-Mode



$$\frac{V_{OUT}}{V_{IN}} = \frac{1}{sRC}$$

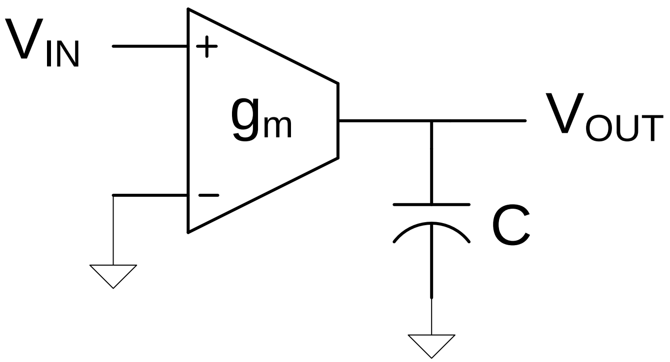
Current-Mode



$$\frac{I_{OUT}}{I_{IN}} = \frac{1}{sRC}$$

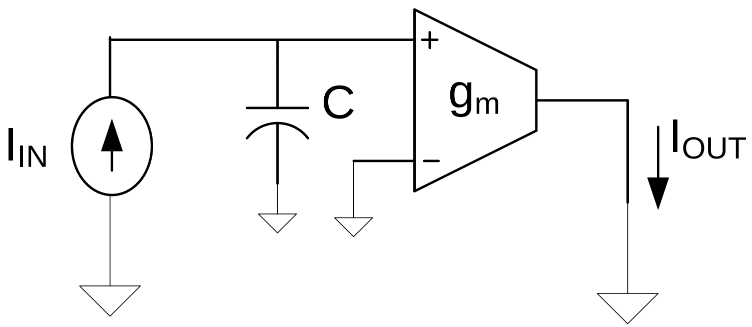
Basic OL Non-Inverting Integrators

Voltage-Mode



$$\frac{V_{OUT}}{V_{IN}} = \frac{g_m}{sC}$$

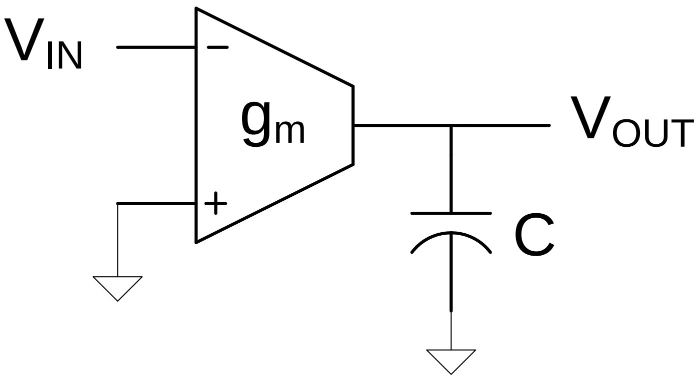
Current-Mode



$$\frac{I_{OUT}}{I_{IN}} = \frac{g_m}{sC}$$

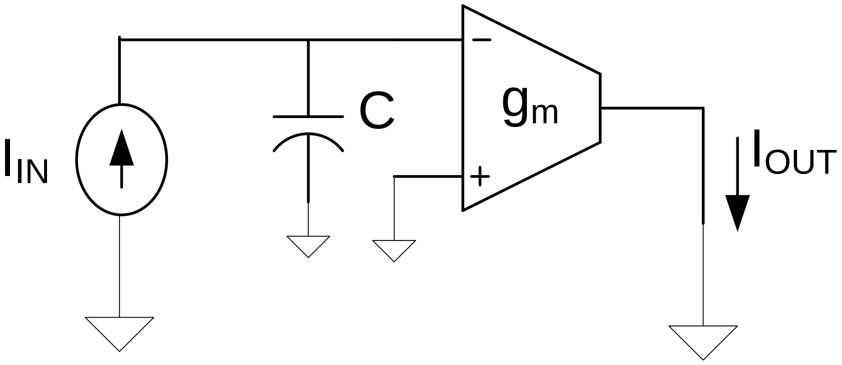
Basic OL Inverting Integrators

Voltage-Mode



$$\frac{V_{OUT}}{V_{IN}} = \frac{-g_m}{sC}$$

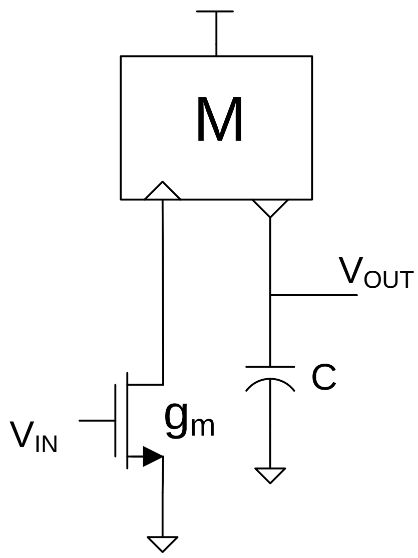
Current-Mode



$$\frac{I_{OUT}}{I_{IN}} = \frac{-g_m}{sC}$$

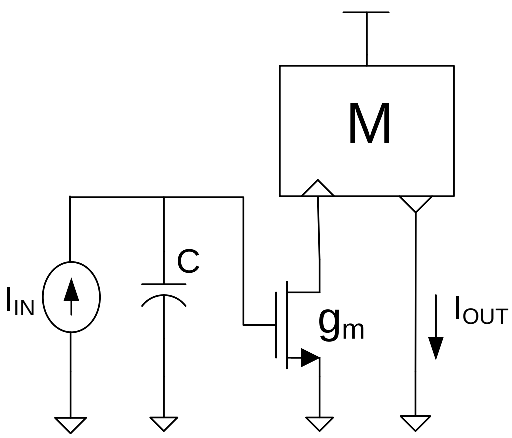
High-Frequency Non-Inverting Integrators

Voltage-Mode



$$\frac{V_{OUT}}{V_{IN}} = \frac{M \bullet g_m}{sC}$$

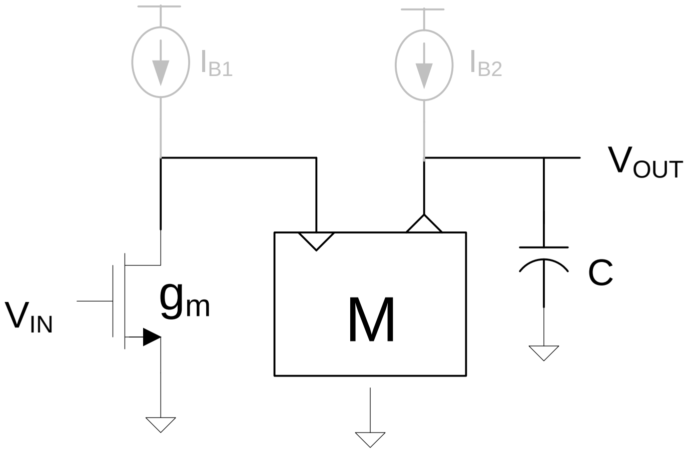
Current-Mode



$$\frac{I_{OUT}}{I_{IN}} = \frac{M \bullet g_m}{sC}$$

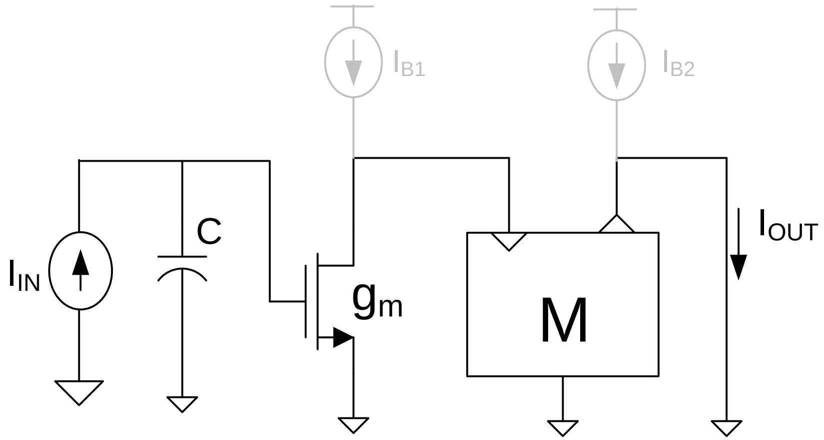
High-Frequency Non-Inverting Integrators

Voltage-Mode



$$\frac{V_{OUT}}{V_{IN}} = \frac{M \bullet g_m}{sC}$$

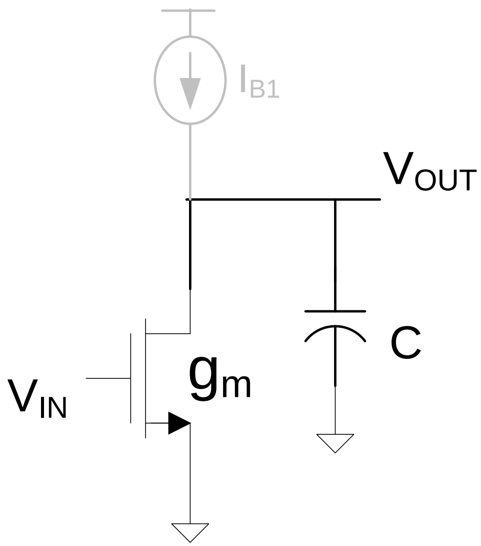
Current-Mode



$$\frac{I_{OUT}}{I_{IN}} = \frac{M \bullet g_m}{sC}$$

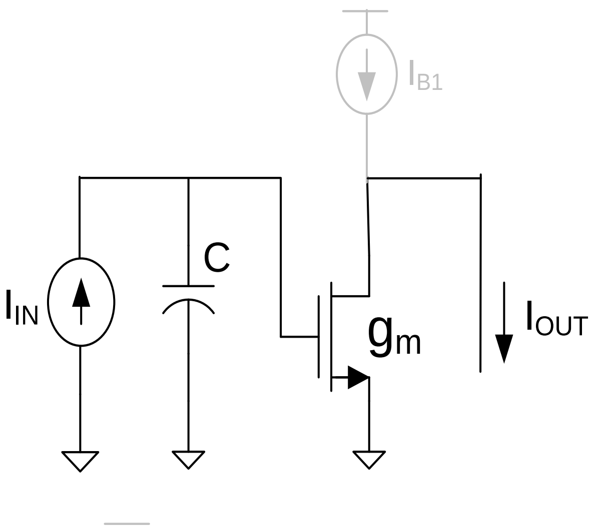
High-Frequency Inverting Integrators

Voltage-Mode



$$\frac{V_{OUT}}{V_{IN}} = \frac{-g_m}{sC}$$

Current-Mode

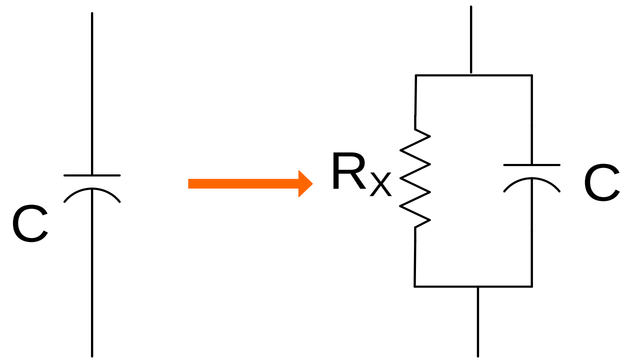


$$\frac{I_{OUT}}{I_{IN}} = \frac{-g_m}{sC}$$

Lossy Integrators

Well-known Property:

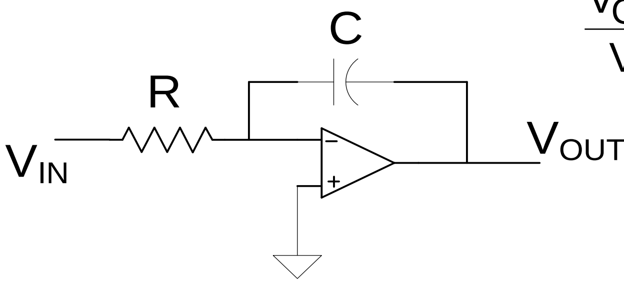
All voltage-mode and current-mode integrators can be made lossy by placing a resistor in shunt with the capacitor

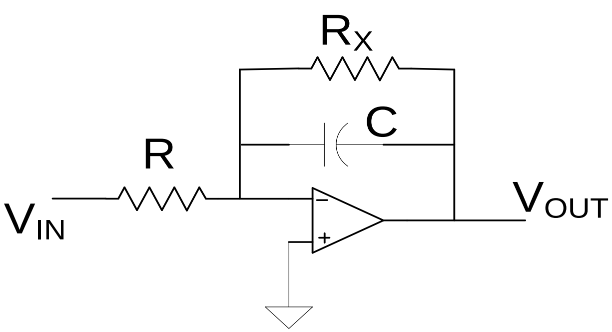


Basic Feedback Lossy Inverting Integrators

Voltage-Mode

$$\frac{V_{OUT}}{V_{IN}} = -\frac{1}{sRC}$$

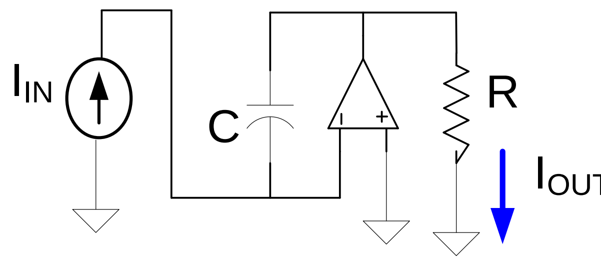


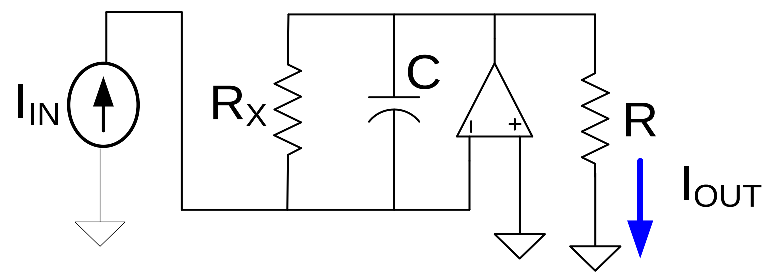


$$\frac{V_{OUT}}{V_{IN}} = -\frac{1}{sRC + R/R_X}$$

Current-Mode

$$\frac{I_{OUT}}{I_{IN}} = -\frac{1}{sRC}$$





$$\frac{I_{OUT}}{I_{IN}} = -\frac{1}{sRC + R/R_X}$$

Question:

How does the performance of filters that use the current-mode and voltage-mode integrators compare?

A fair comparison should be within a common structure and with a common integrator type

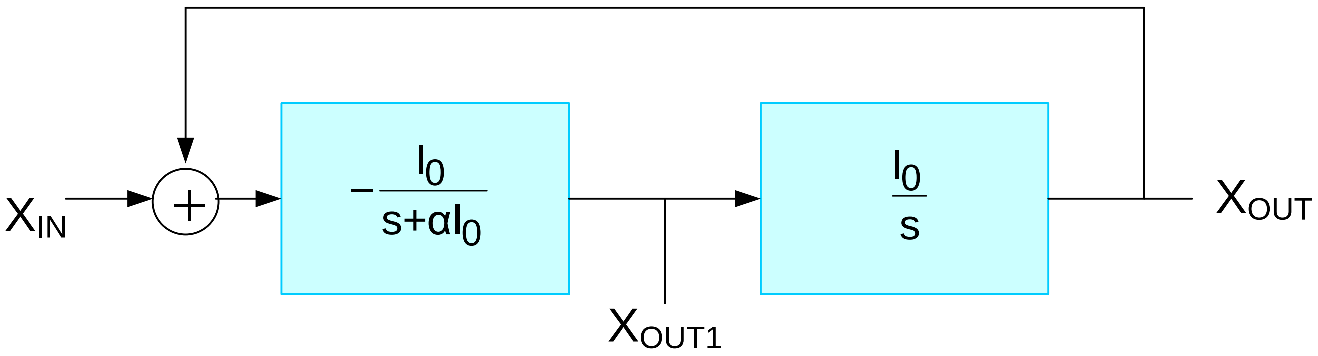
Question:

How does the performance of filters that use the current-mode and voltage-mode integrators compare?

Will compare the filter performance of

- a two-integrator loop biquad
- a leapfrog filter

Two-Integrator-Loop Biquad



"Integrator and Lossy Integrator in a Loop"

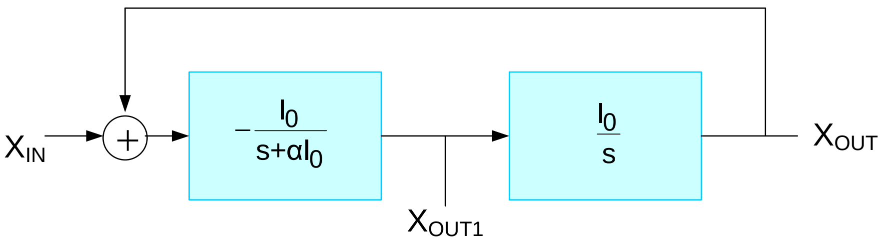
Lowpass output to X_{OUT}

$$T(s) = \frac{-I_0^2}{s^2 + \alpha I_0 s + I_0^2}$$

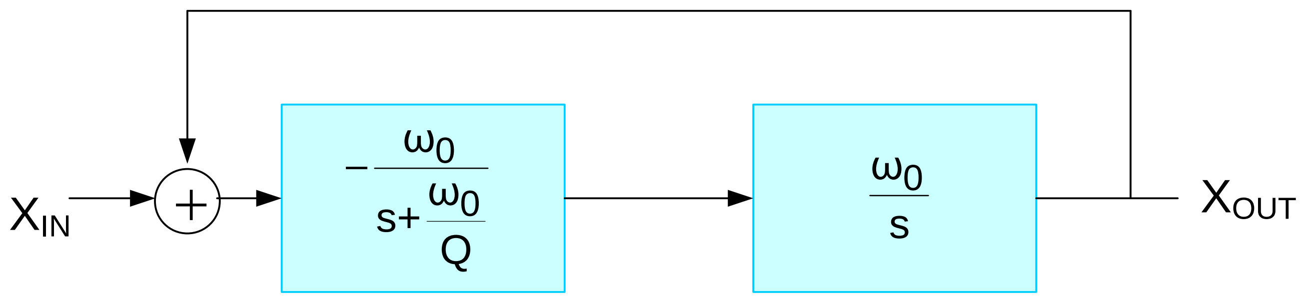
Bandass output to X_{OUT1}

$$T_1(s) = \frac{-sI_0}{s^2 + \alpha I_0 s + I_0^2}$$

Two-Integrator-Loop Biquad



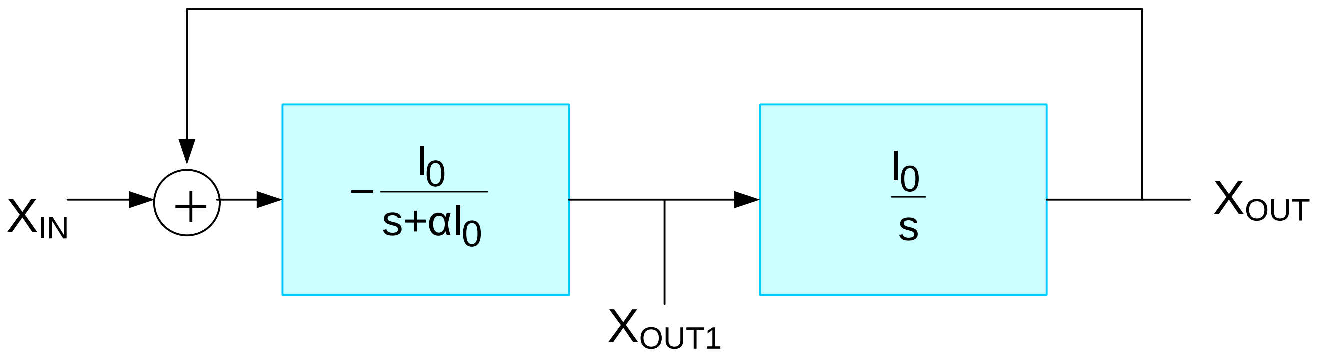
Alternate Equivalent Representation: $I_0 \leftrightarrow \omega_0 \qquad \alpha_0 \leftrightarrow \omega_0$



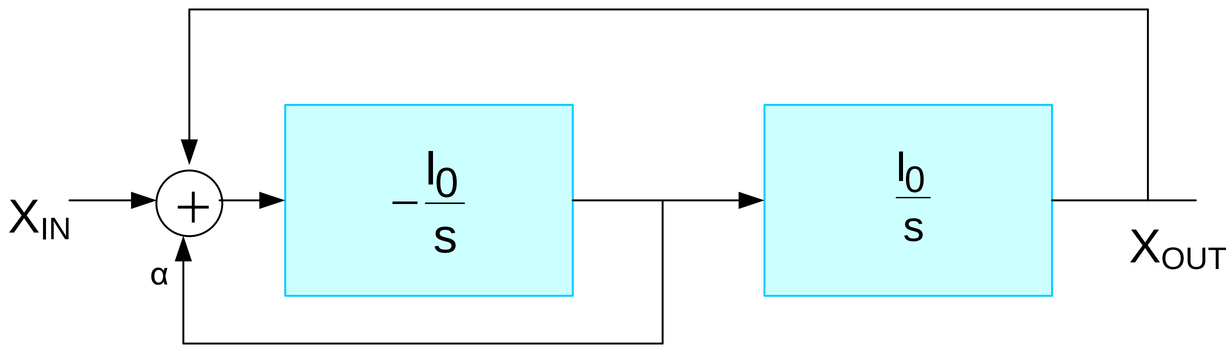
Lowpass output to X_{OUT} $$T(s)=\frac{-\omega_0^2}{s^2+\frac{\omega_0}{Q}s+\omega_0^2}$$

Bandass output to X_{OUT1} $$T(s)=\frac{-s\omega_0}{s^2+\frac{\omega_0}{Q}s+\omega_0^2}$$

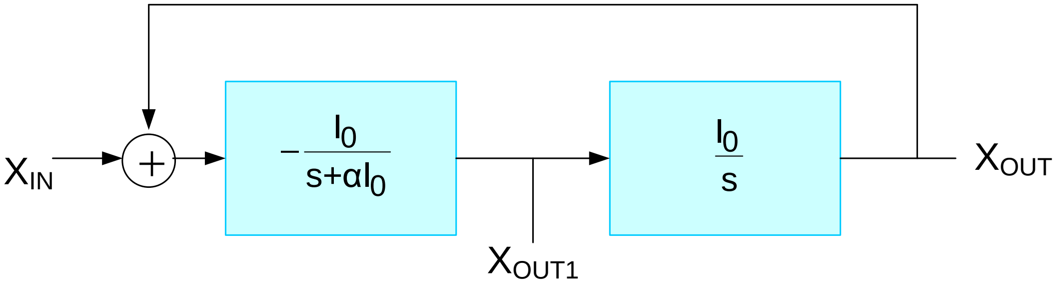
Two-Integrator-Loop Biquad



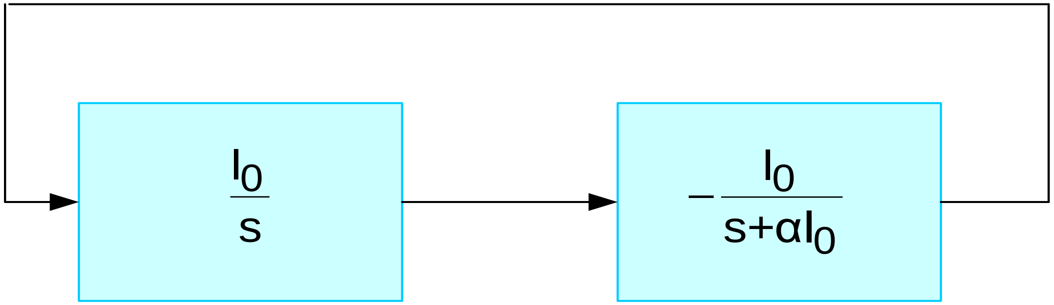
Alternate implementation of Lossy Integrator



Two-Integrator-Loop Biquad

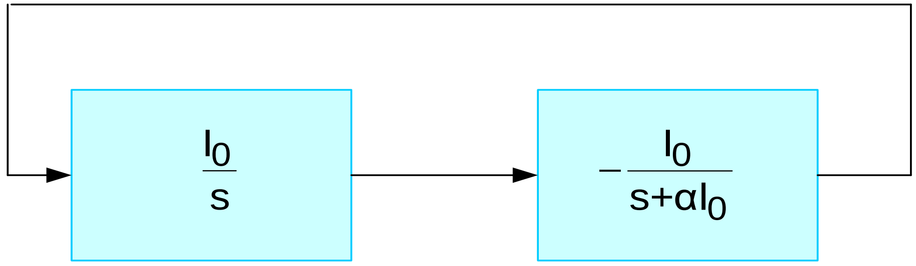


- For notational convenience, the input signal can be suppressed and output will not be designated
- This forms the “dead network”
- Poles for dead network are identical to original network as are key sensitivities

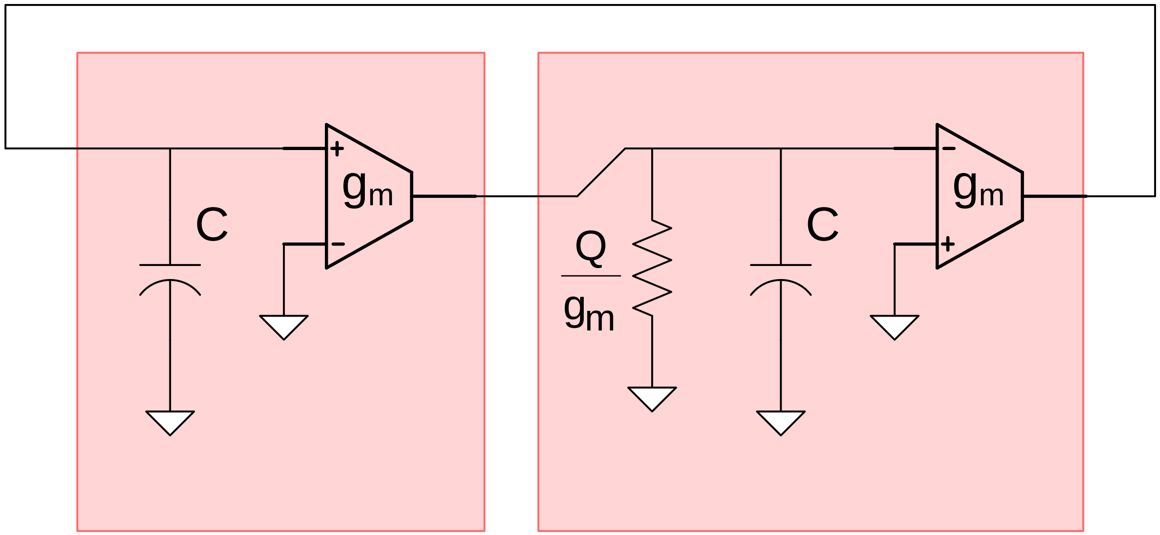


Two Integrator Loop Biquad

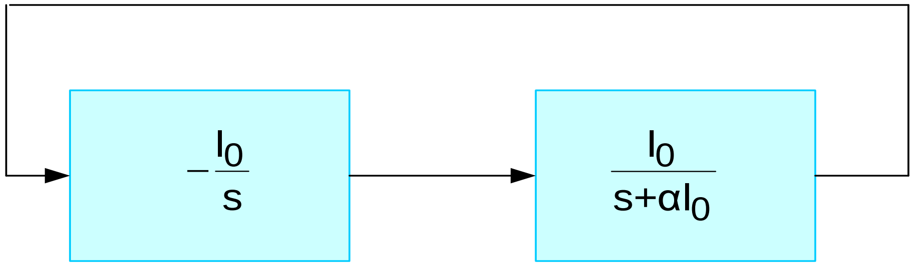
Two-Integrator-Loop Biquad



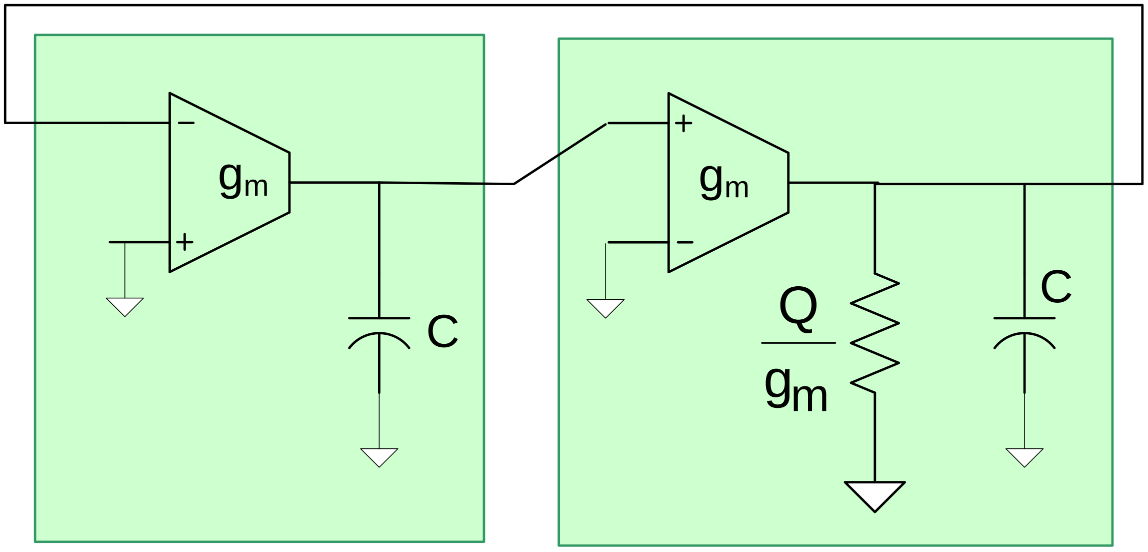
Consider a current-mode implementation:



Two-Integrator-Loop Biquad

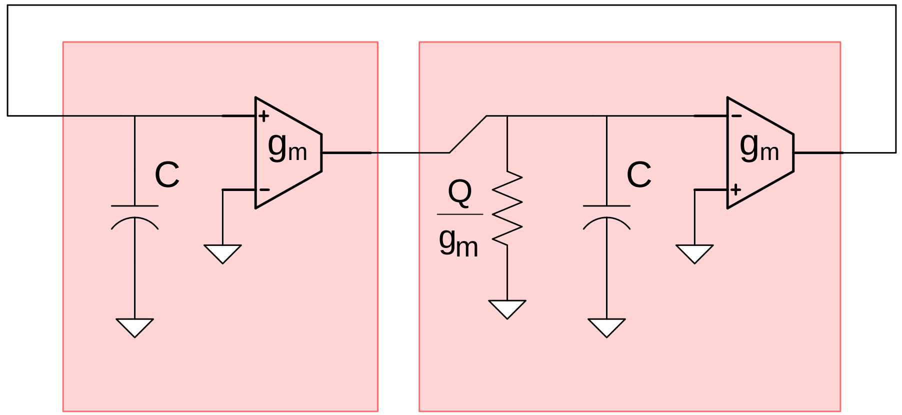


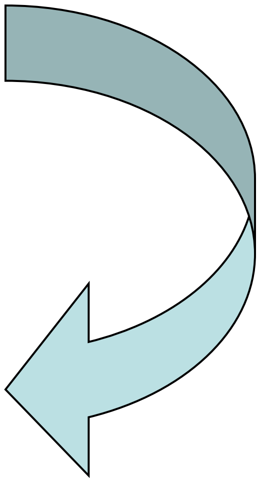
Consider the corresponding voltage-mode implementation:

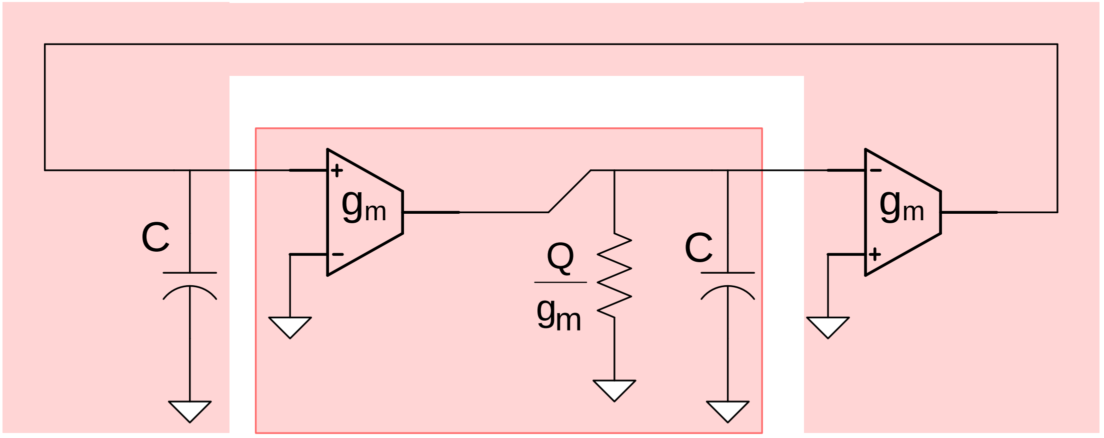


Two-Integrator-Loop Biquad

Current-mode

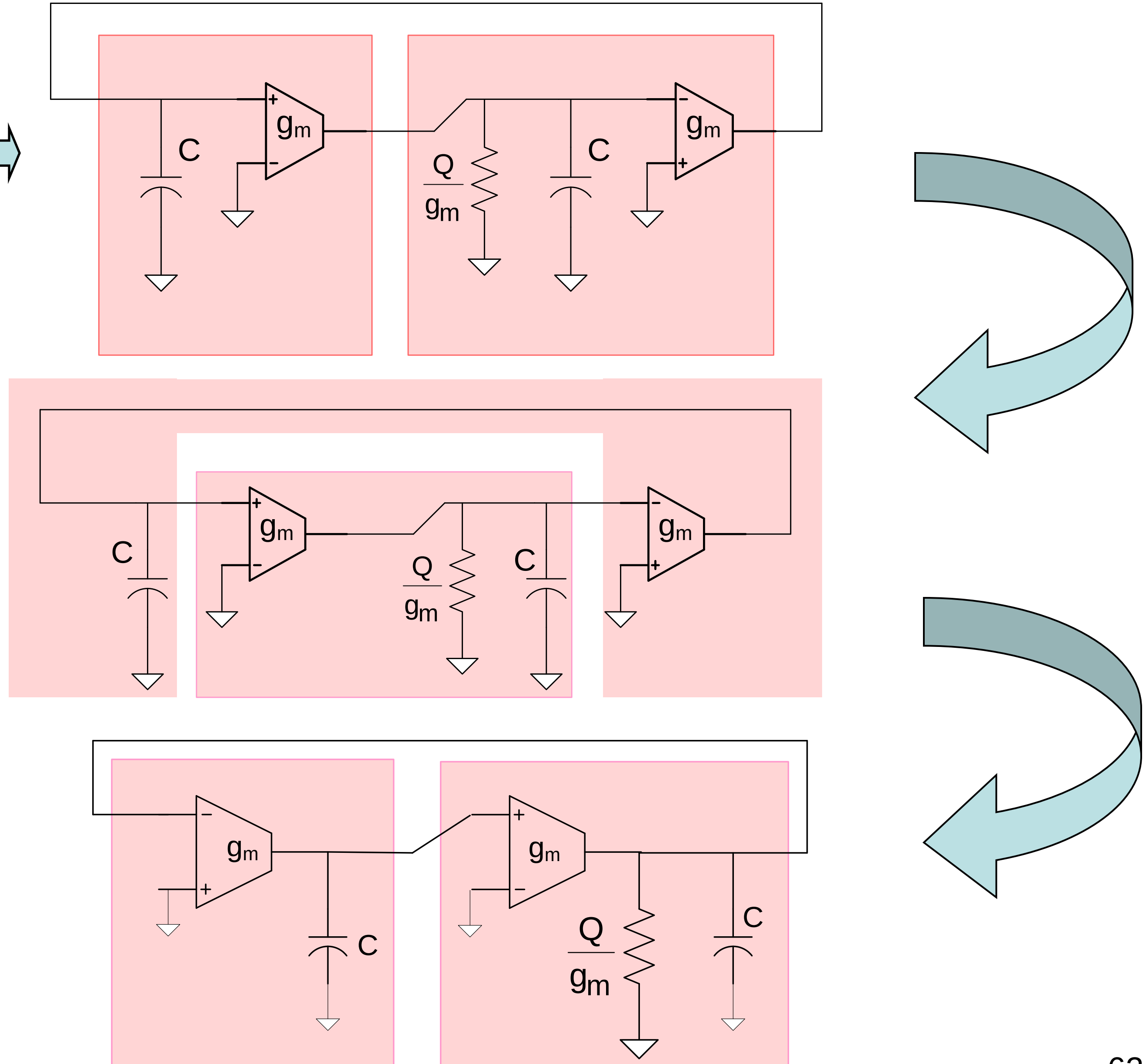




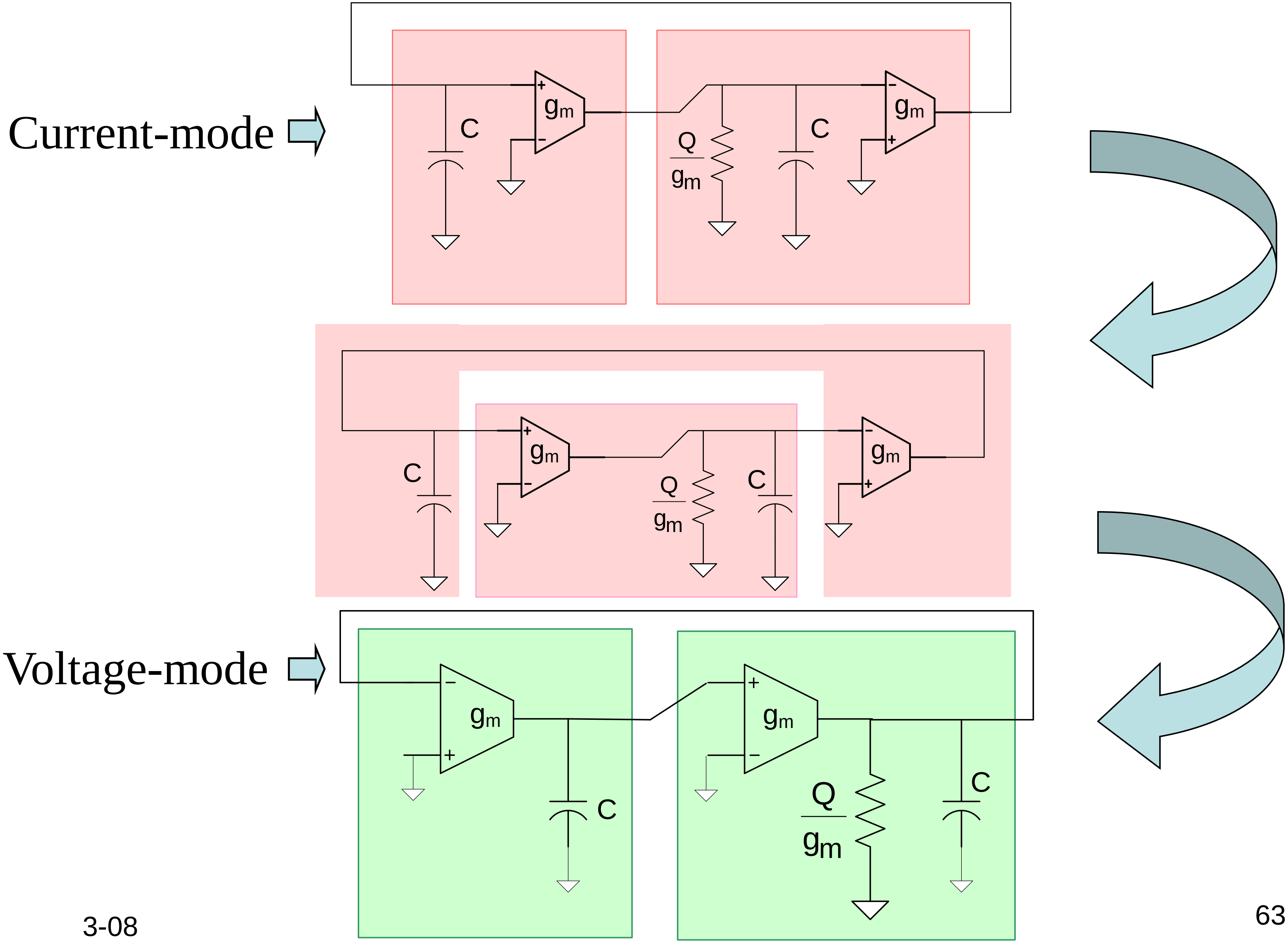


Two-Integrator-Loop Biquad

Current-mode



Two-Integrator-Loop Biquad



Question:

How does the performance of filters that use the current-mode and voltage-mode integrators compare?

Question:

How does the performance of filters that use the current-mode and voltage-mode integrators compare?

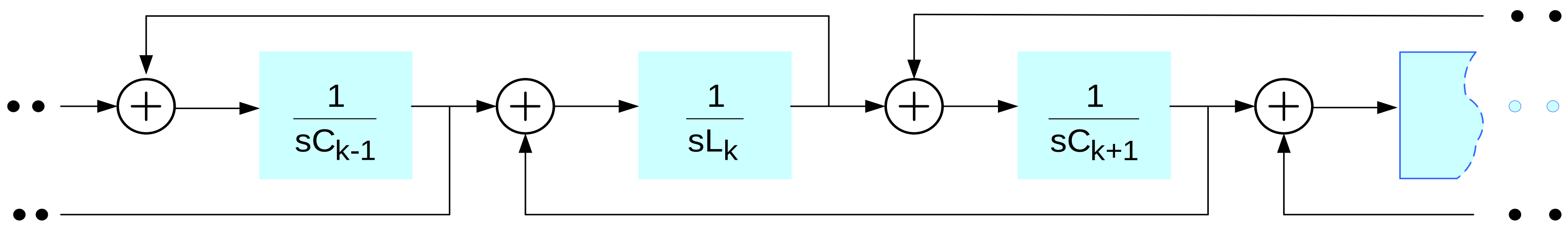
The corresponding current-mode and the voltage-mode two integrator loop biquad filters are identical!

Question:

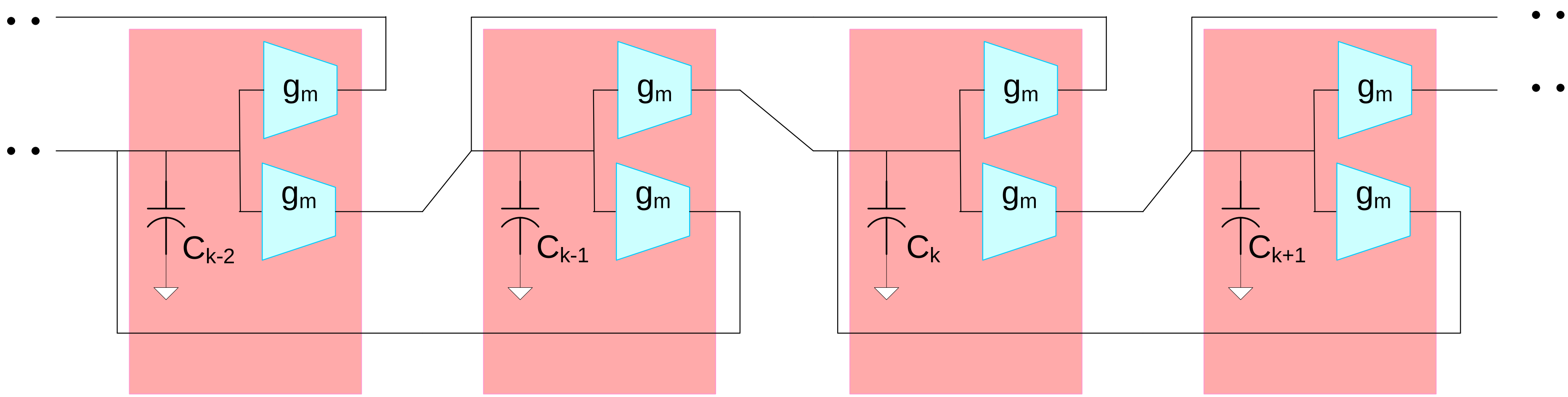
How does the performance of filters that use the current-mode and voltage-mode integrators compare?

The performance (speed, signal swing, sensitivity, linearity,power dissipation, etc.) of these circuits is identical !

Leap-Frog Filter

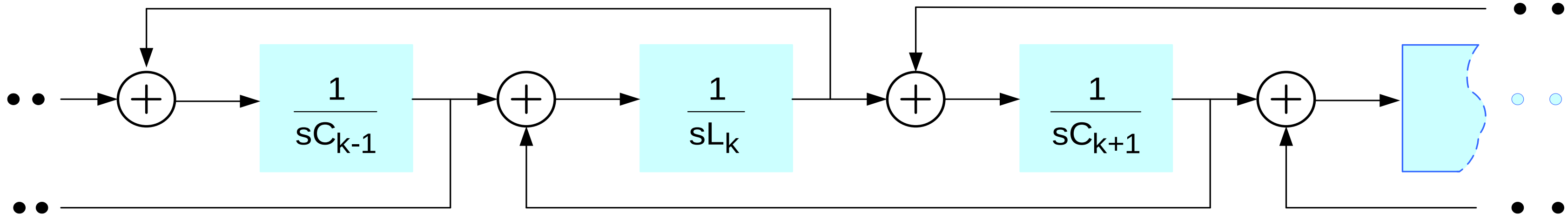


Current-mode:

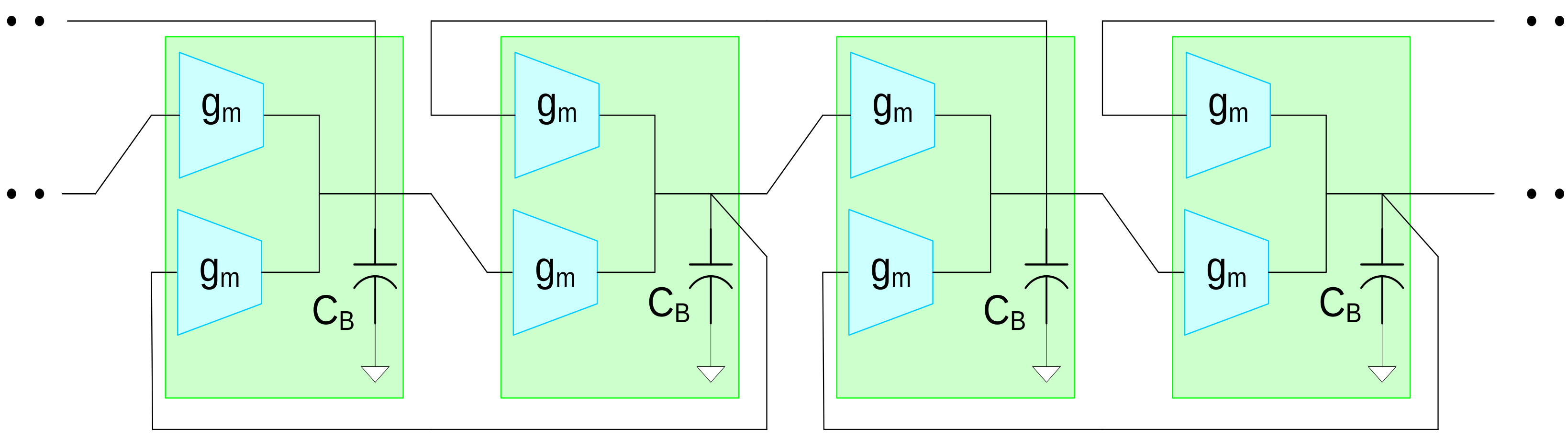


Standard OTA

Leap-Frog Filter



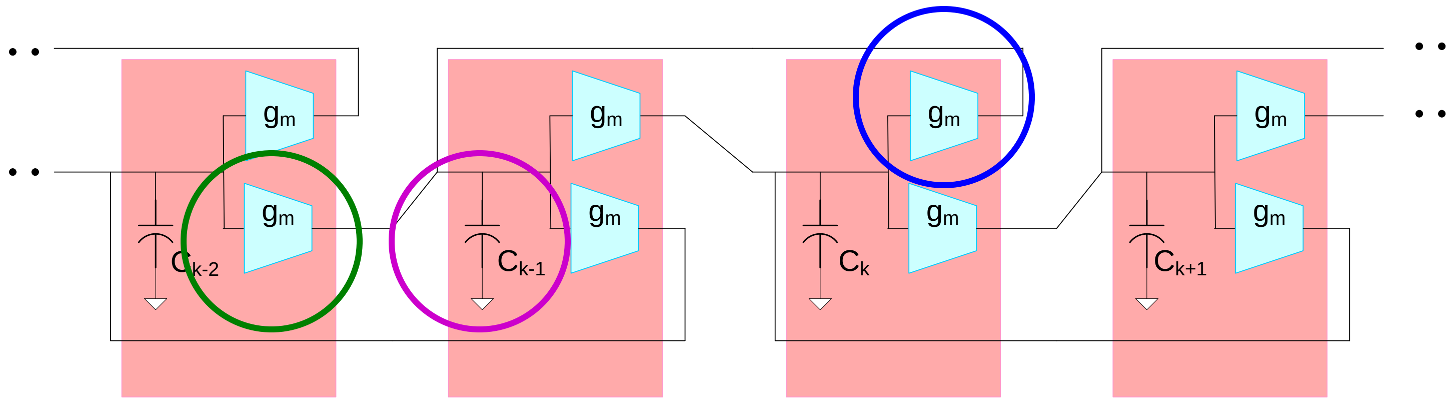
Voltage-mode:



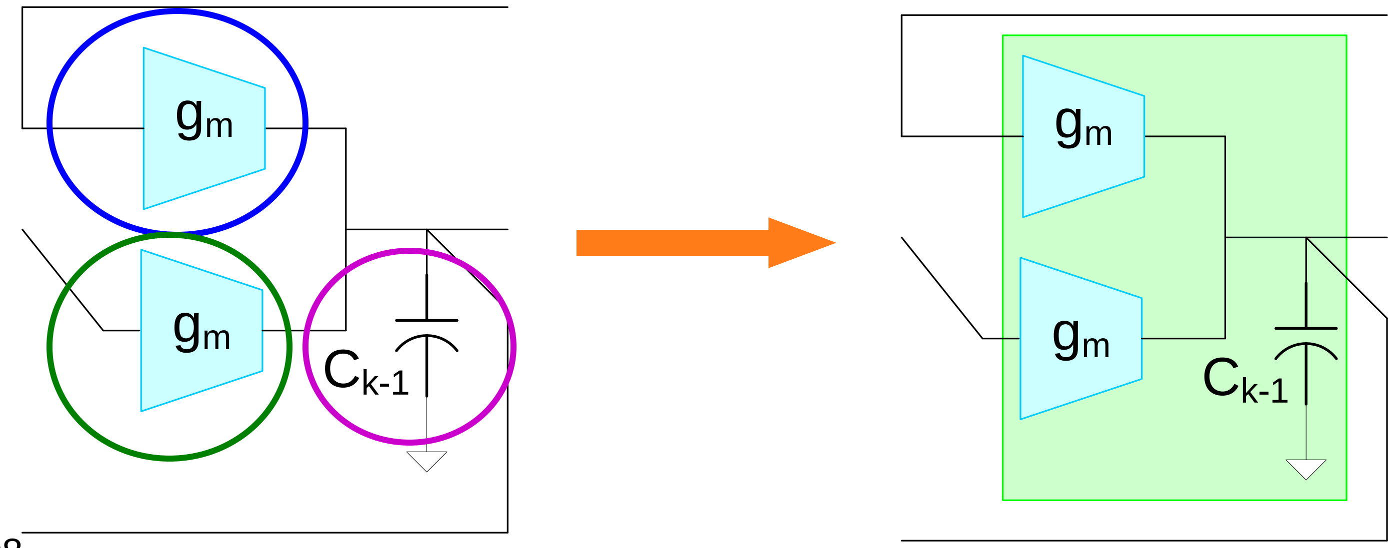
Standard OTA

Leap-Frog Filter

Current-mode

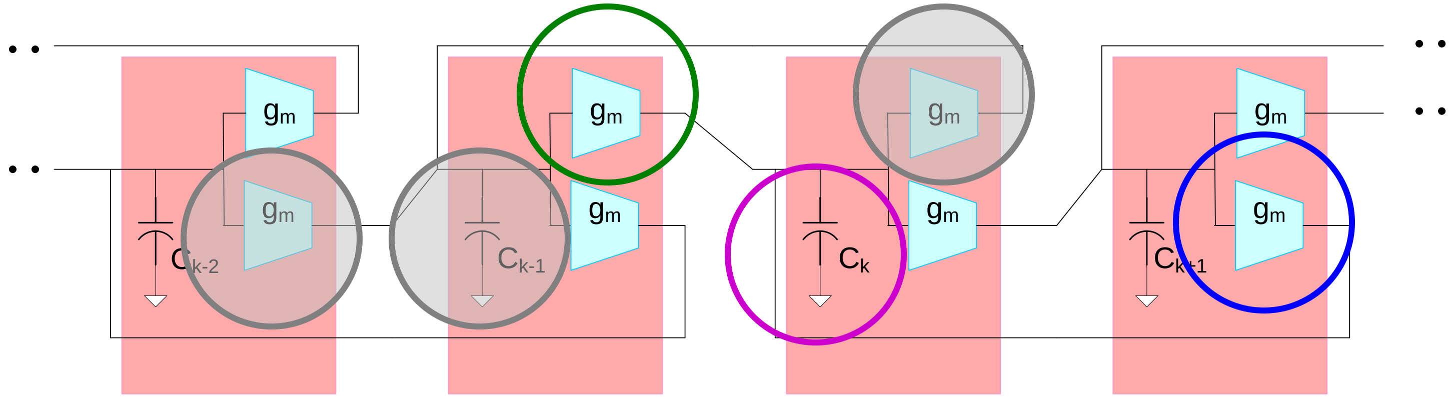


Consider lower OTA in stage k-2, capacitor in stage k-1 and upper OTA in stage k

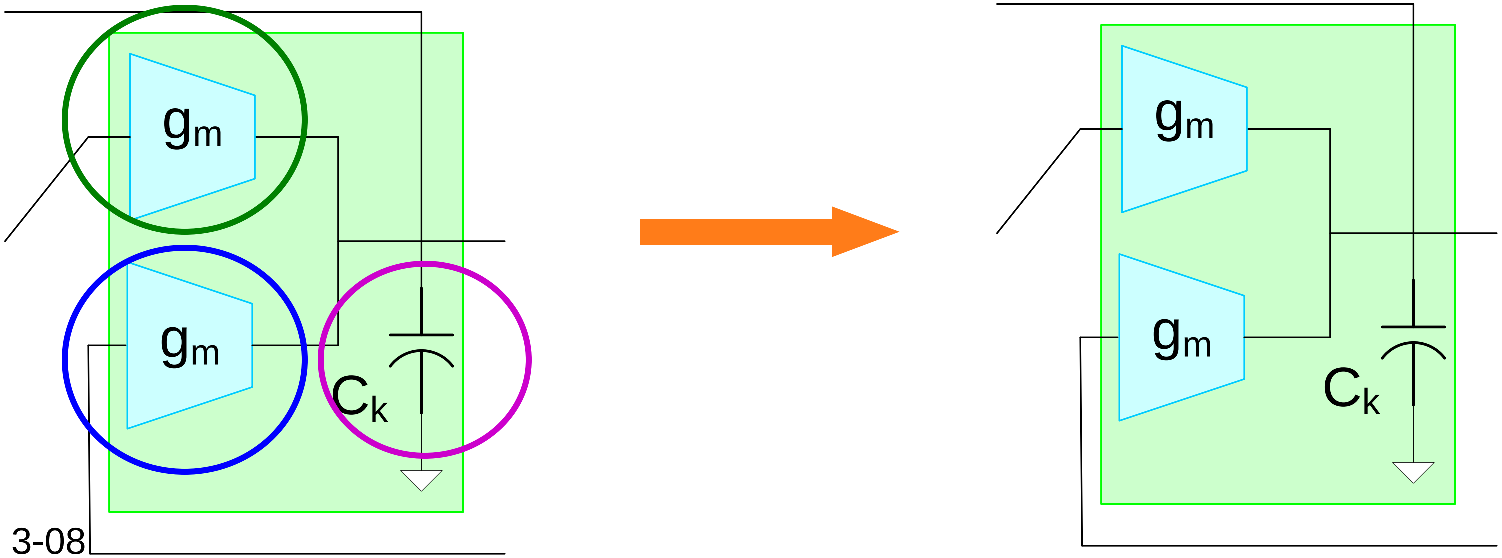


Leap-Frog Filter

Current-mode

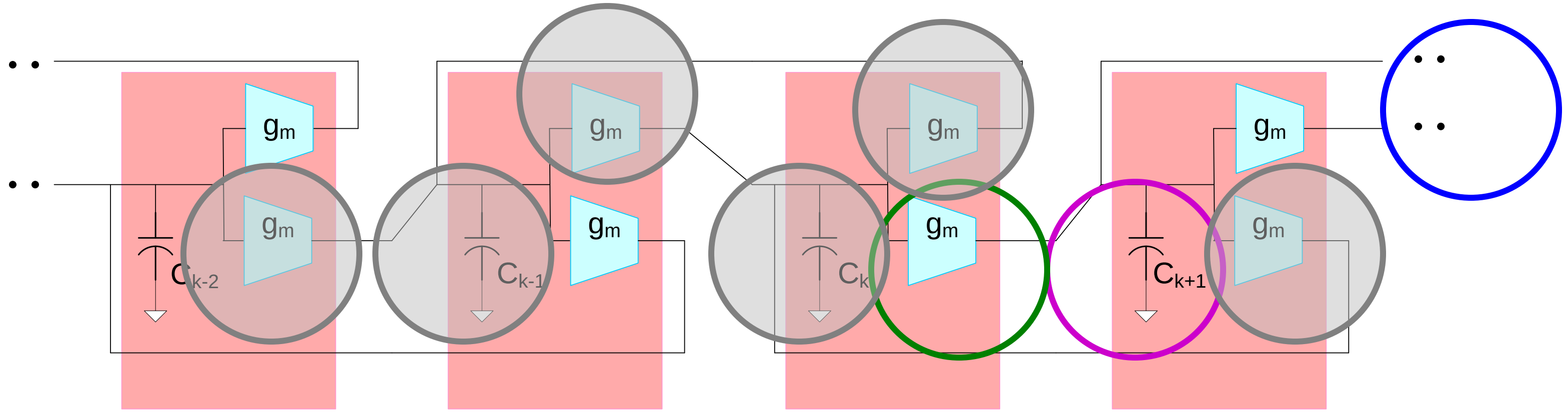


Consider upper OTA in stage k-1, capacitor in stage k and lower OTA in stage k+1

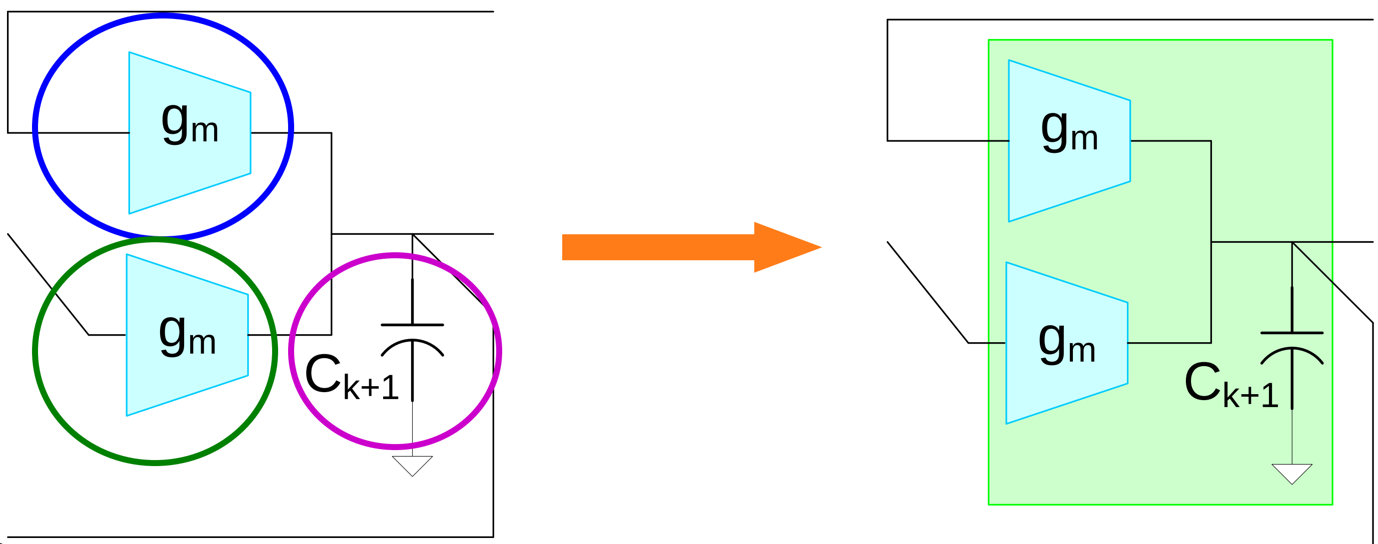


Leap-Frog Filter

Current-mode

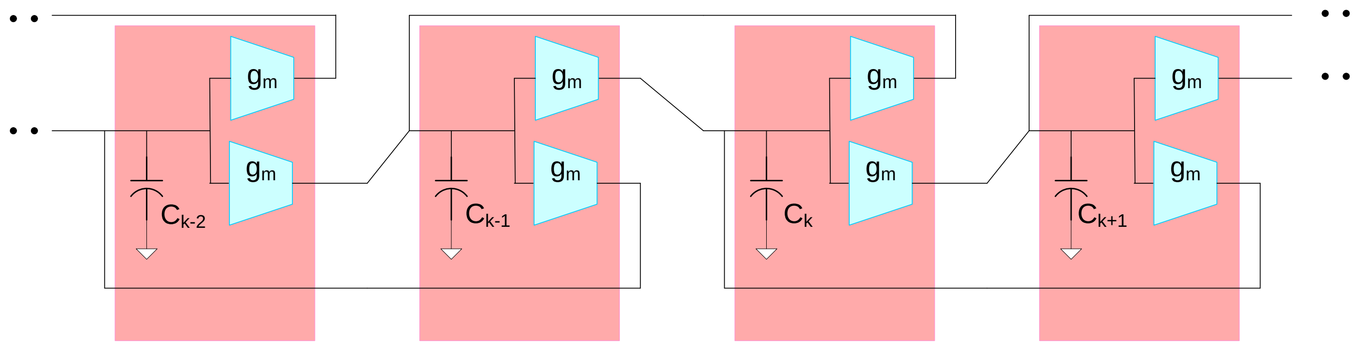


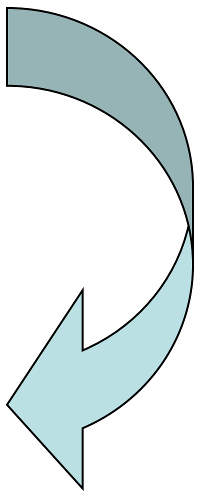
Consider lower OTA in stage k, capacitor in stage k+1 and upper OTA in stage k+2

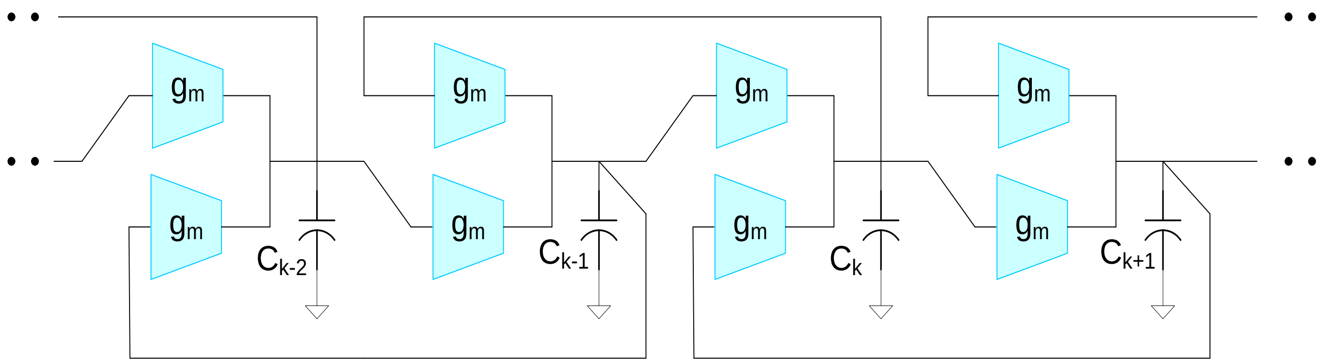


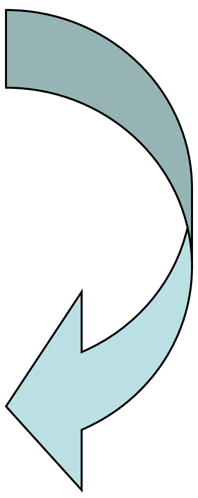
Leap-Frog Filter

Current-mode

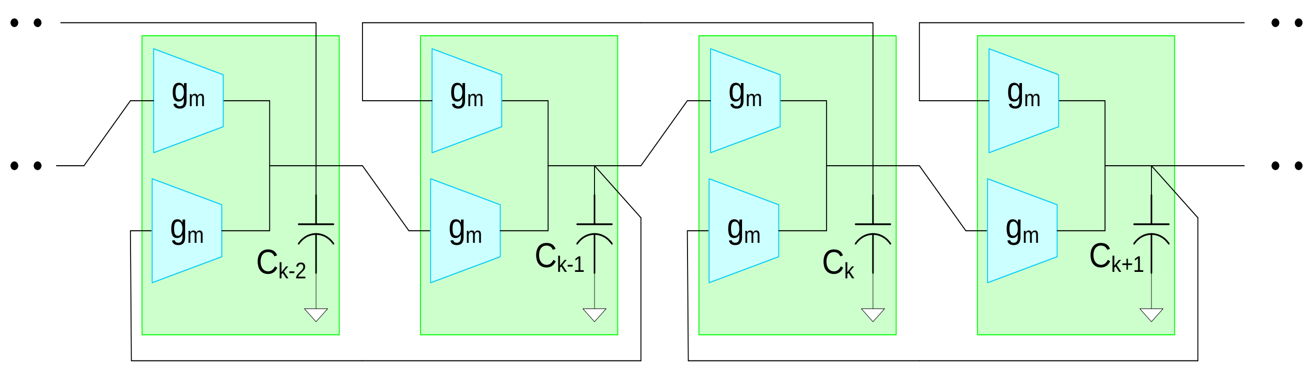






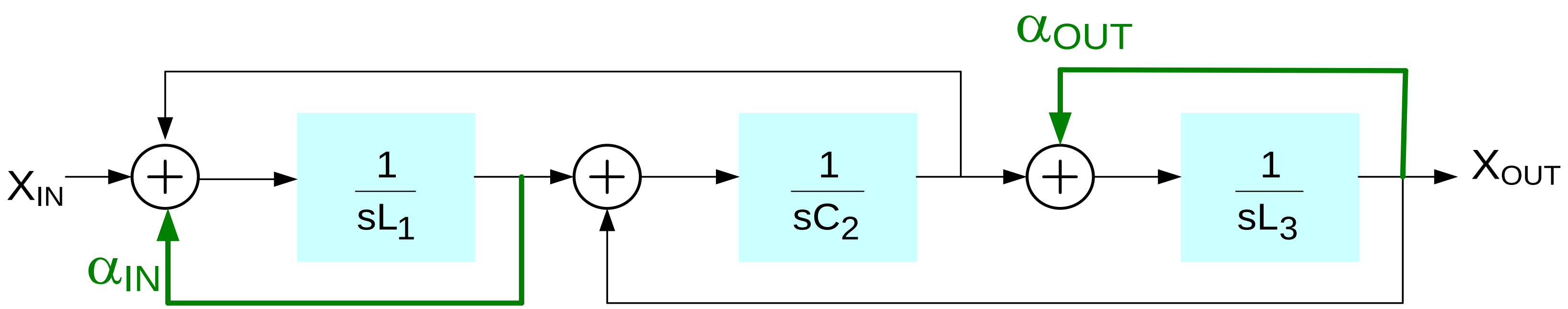


Voltage-mode

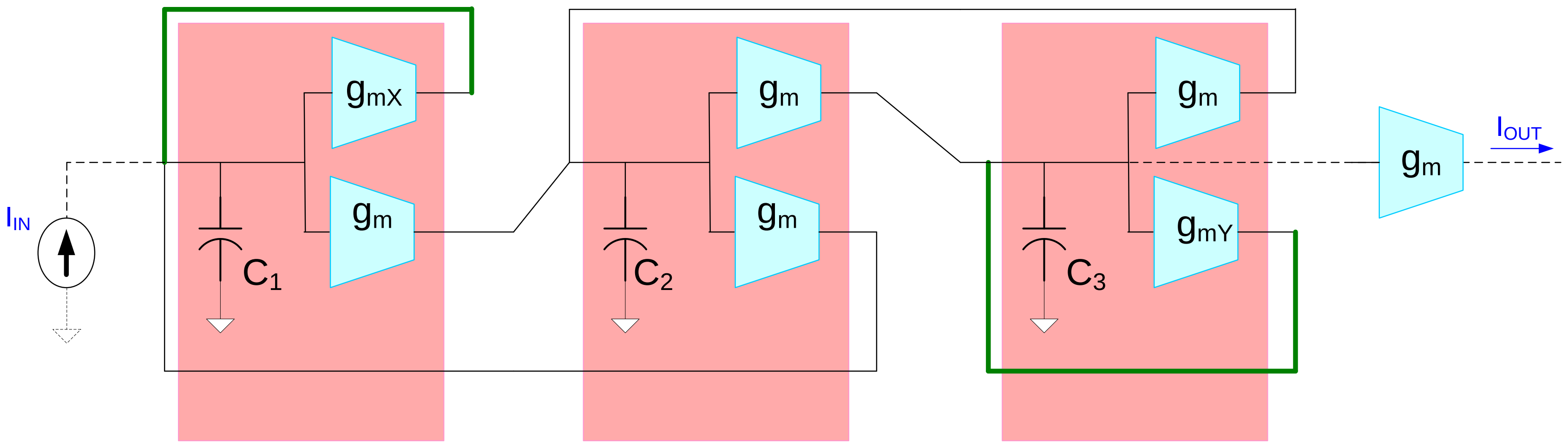


Leap-Frog Filter

Terminated Leap-Frog Filter (3-rd order lowpass)

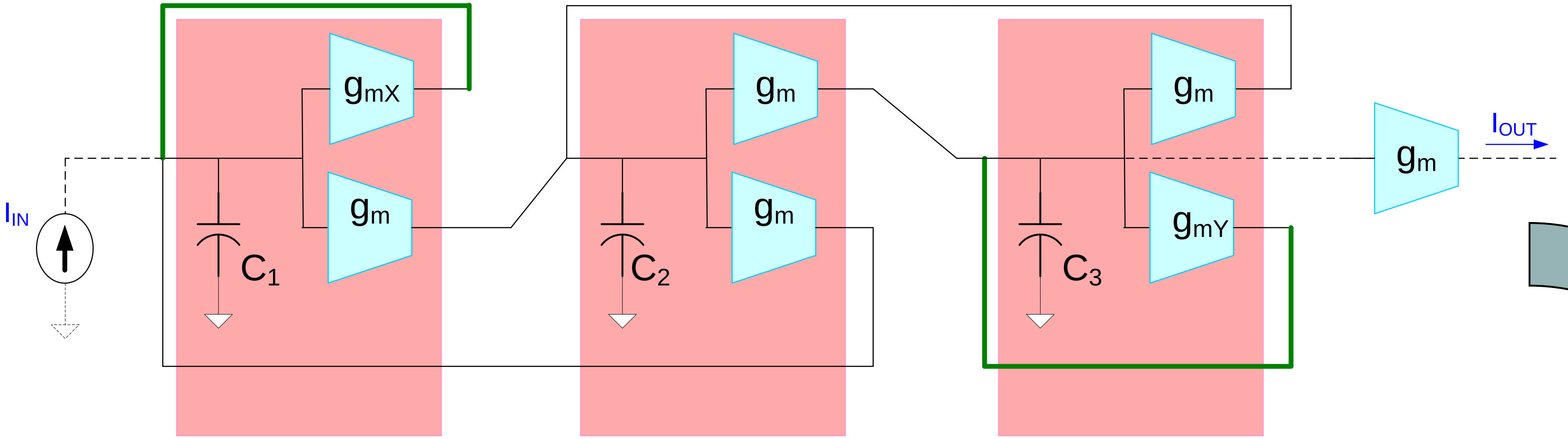


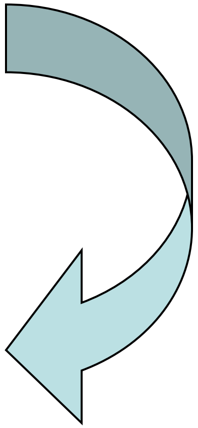
Current-mode implementation



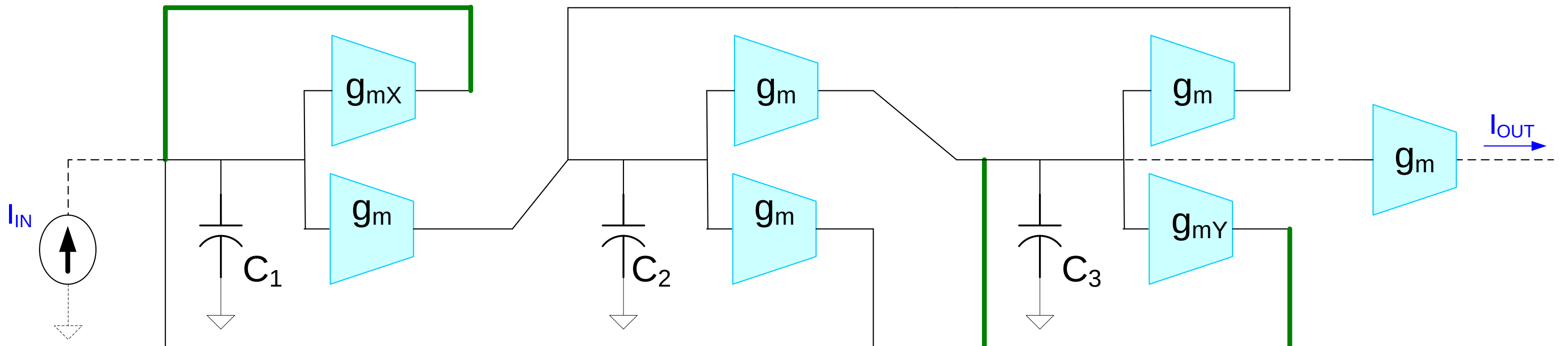
Leap-Frog Filter

Current-mode implementation



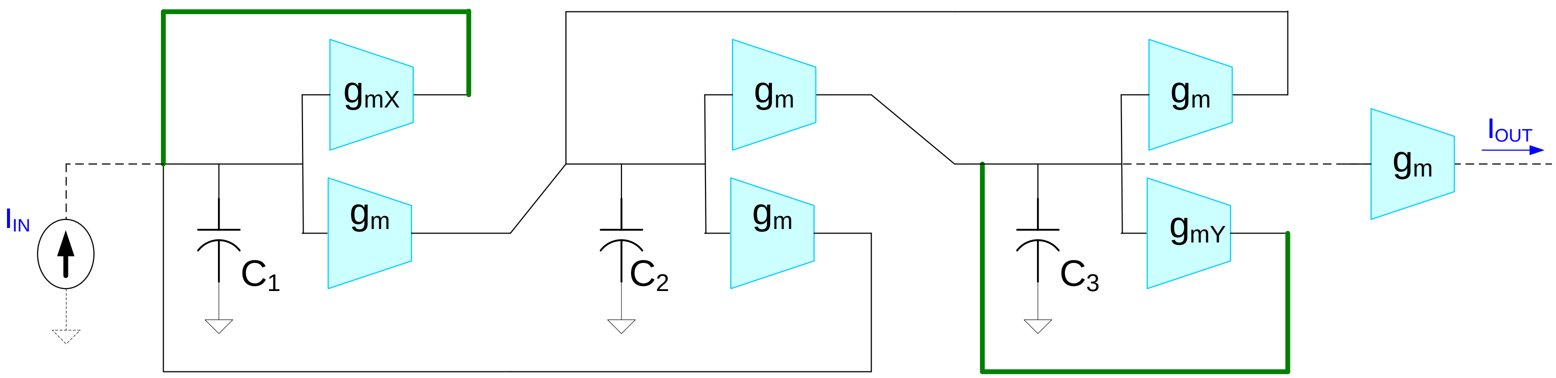


Consider schematic view:

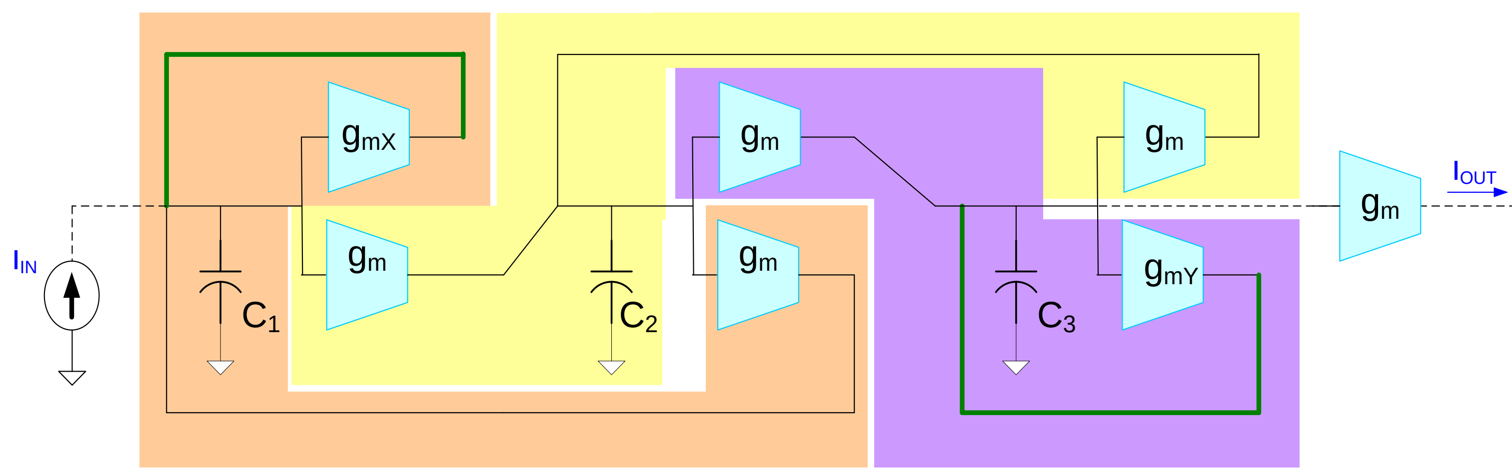


Leap-Frog Filter

Current-mode implementation

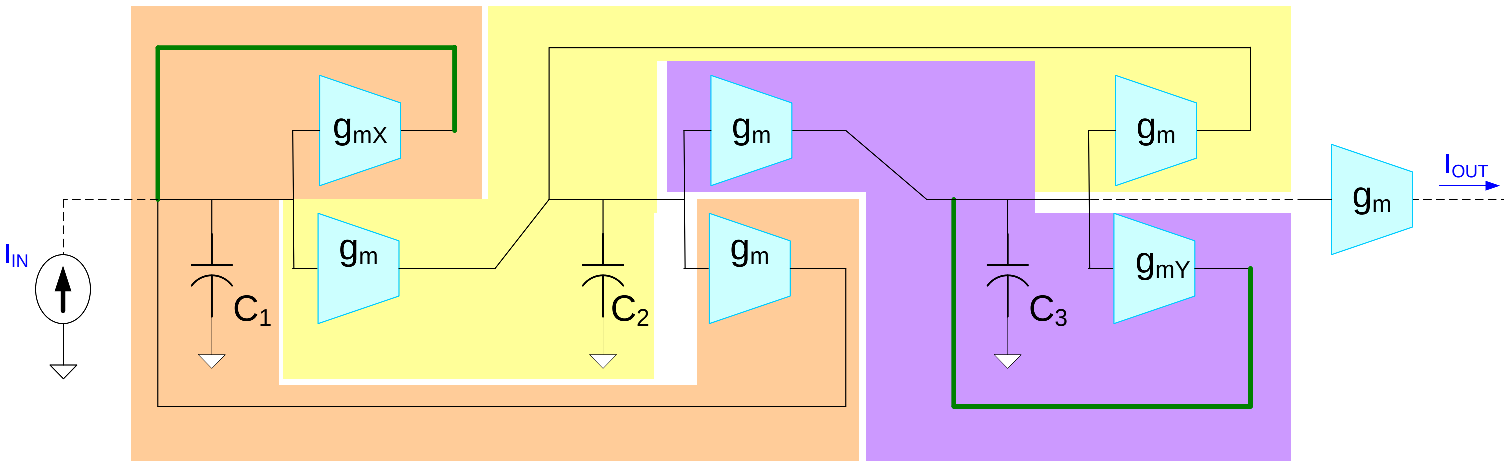


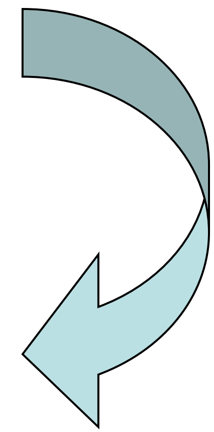
Re-group elements:



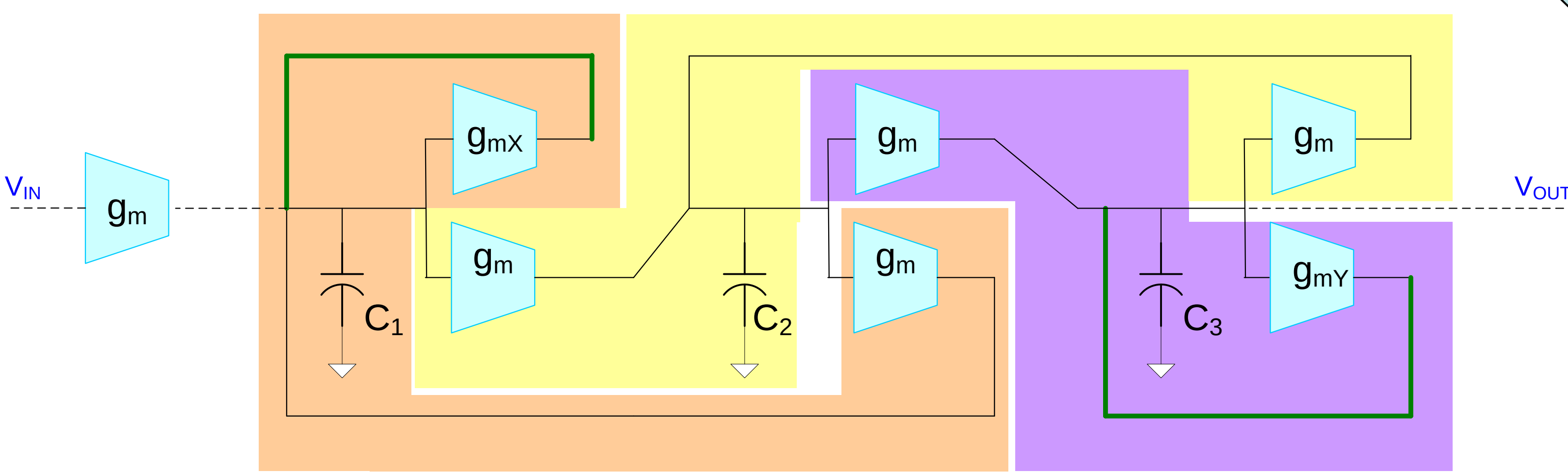
Leap-Frog Filter

Current-mode implementation



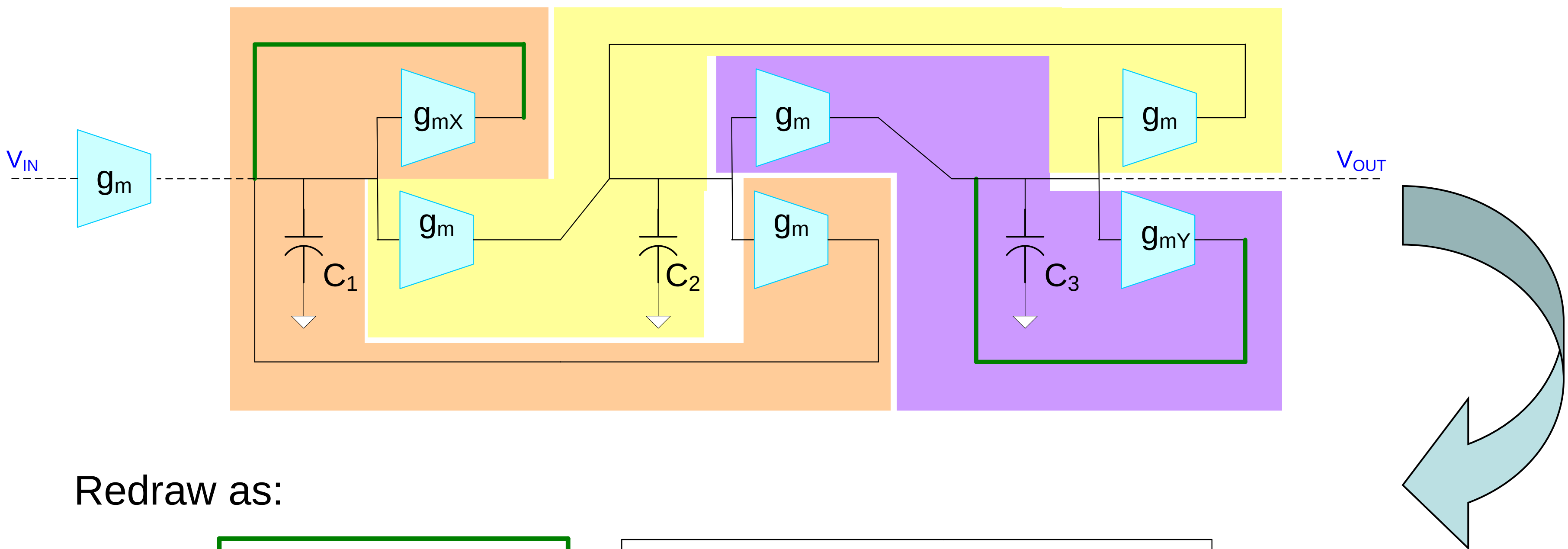


I/O Source Transformation

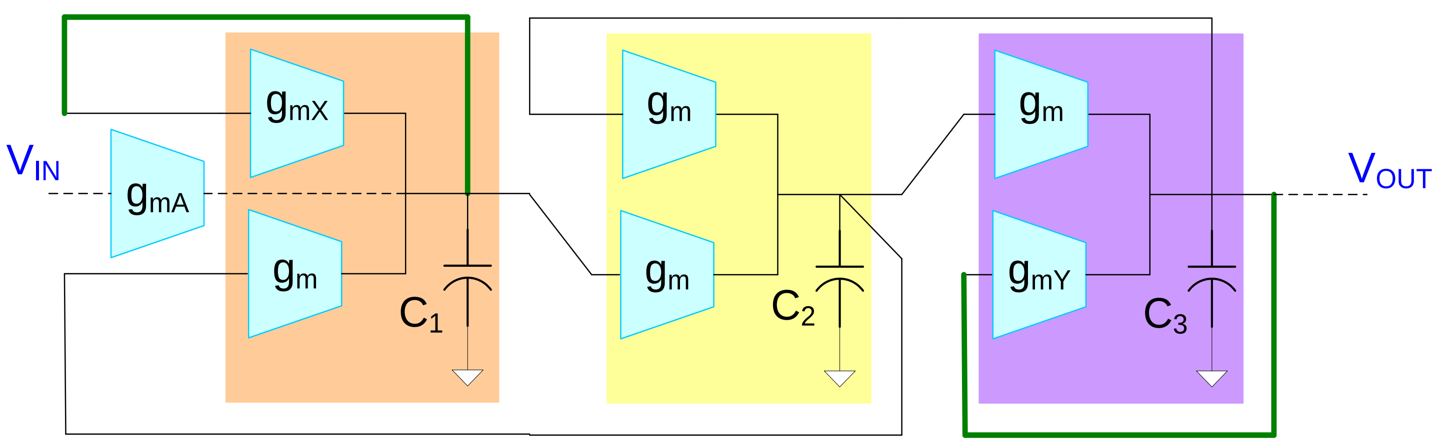


Leap-Frog Filter

Current-mode implementation

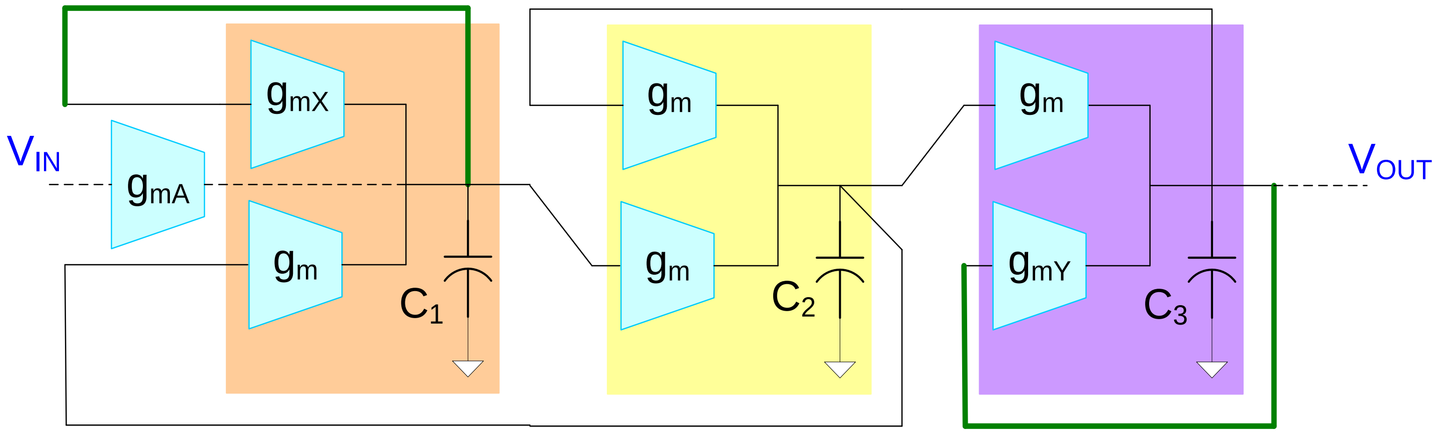


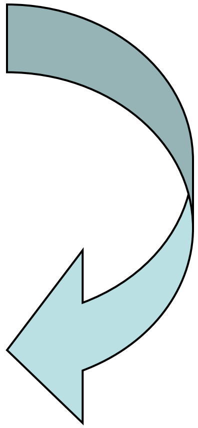
Redraw as:



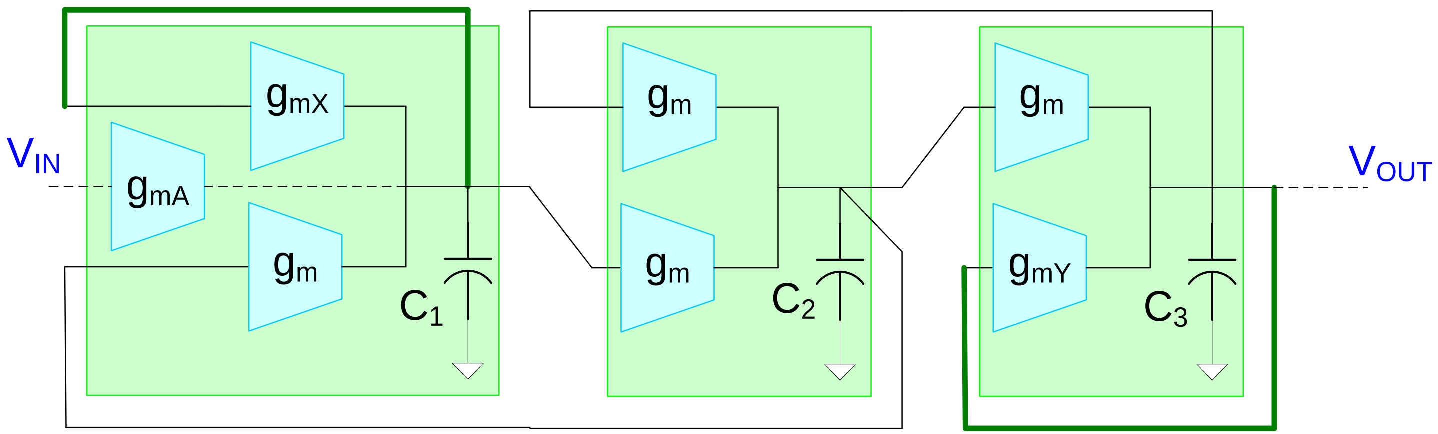
Leap-Frog Filter

Current-mode implementation





Change notation:

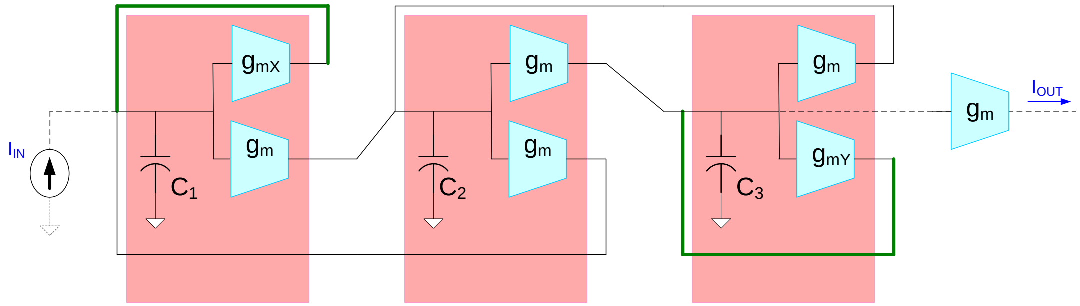


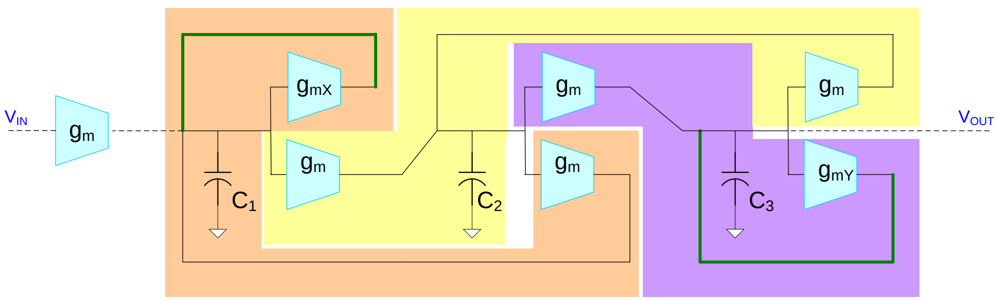
This is a voltage-mode implementation of the Leap-Frog Circuit !

Leap-Frog Filter

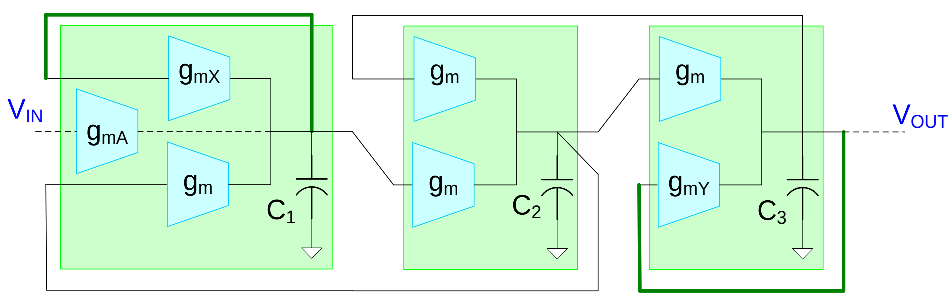
SUMMARY

Current-mode implementation





Voltage-mode implementation

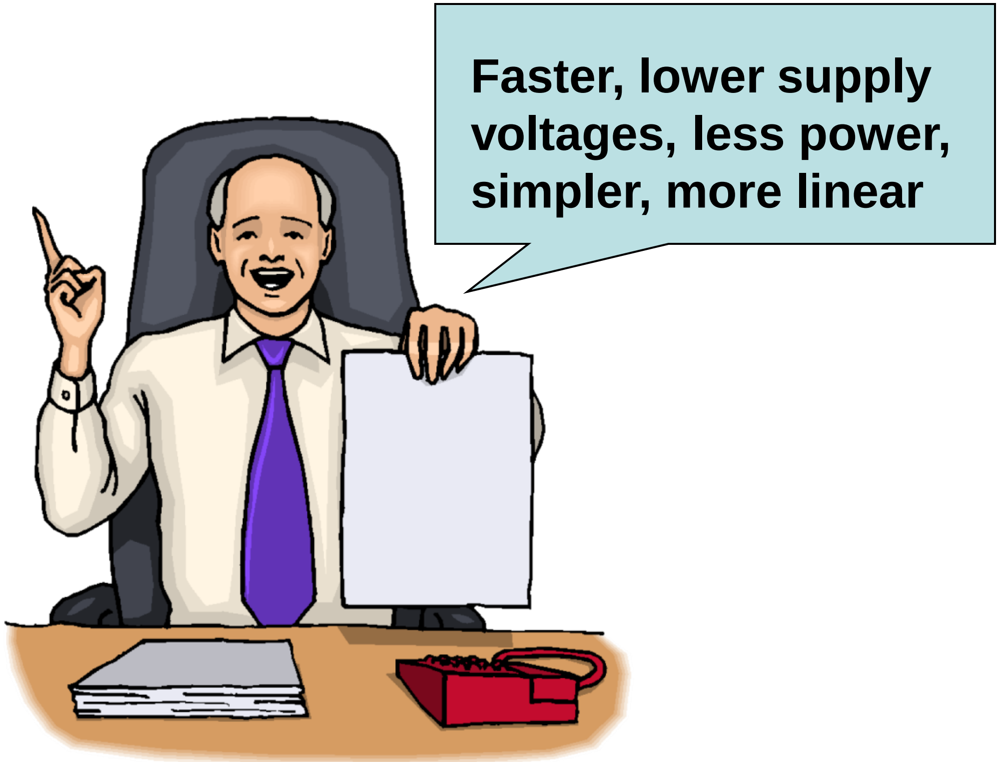


Comment

The current-mode and voltage-mode equivalence also exists for the high-frequency single transistor two-integrator loop filters and leapfrog filter structures

Question:

How does the performance of filters that use the current-mode and voltage-mode integrators compare?



Question:

How does the performance of filters that use the current-mode and voltage-mode integrators compare?

The current-mode and the voltage-mode leapfrog filters are identical!

Question:

How does the performance of filters that use the current-mode and voltage-mode integrators compare?

The performance (speed, signal swing, sensitivity, linearity, etc.) of these circuits is identical !

Current-Mode Filters

Conventional Wisdom

- Current-Mode circuits operate at higher-frequencies than voltage-mode counterparts
- Current-Mode circuits operate at lower supply voltages and lower power levels than voltage-mode counterparts
- Current-Mode circuits are simpler than voltage-mode counterparts
- Current-Mode circuits offer better linearity than voltage-mode counterparts

Reconciliation of Conventional Wisdom and Fundamental Concepts

- The choice of state (or stated) port variables plays no role on the fundamental performance characteristics of a filter
- Many current-mode and voltage-mode filters that have appeared in the literature are identical
- The issue of whether there are any performance advantages from the viewpoint of supply voltage, speed of operation and linearity of continuous-time current-mode filters over volt-mode counterparts is in question

EE 508
Lecture 40

What filter architectures are really being used today?

Today will consider the first 50 responses

A CMOS/Thin-Film Fluorescence Contact Imaging Microsystem for DNA Analysis

Ritu Raj Singh, *Student Member, IEEE*, Derek Ho, *Student Member, IEEE*, Alireza Nilchi, *Student Member, IEEE*, Glenn Gulak, *Member, IEEE*, Patrick Yau, and Roman Genov, *Member, IEEE*

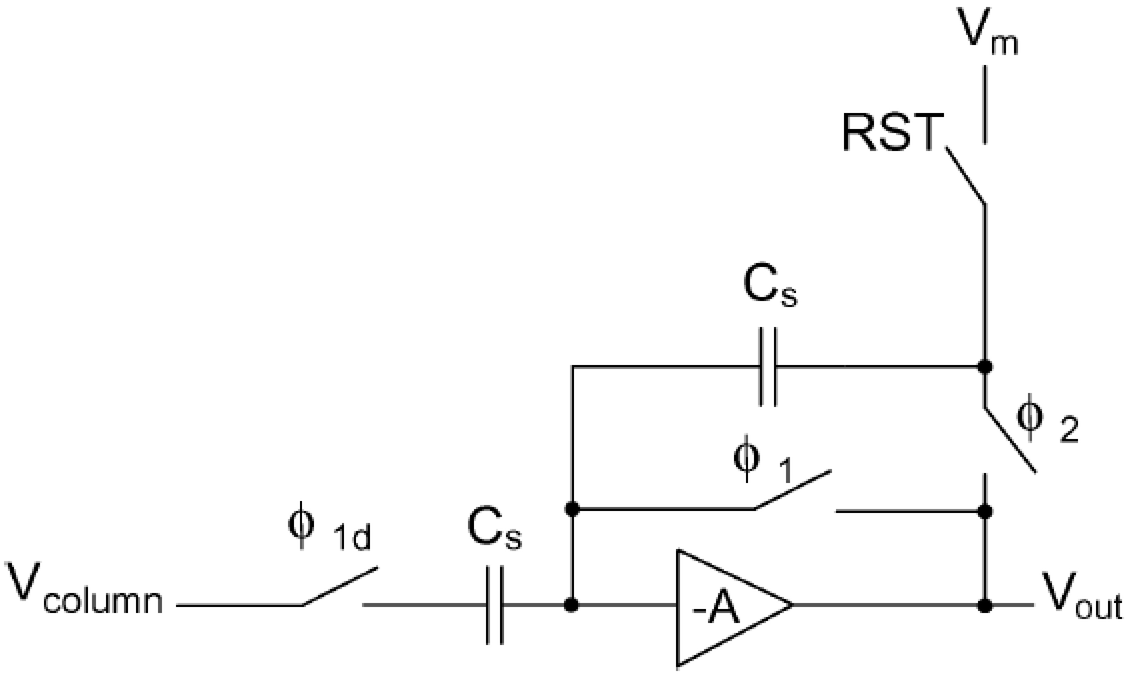


Fig. 5. Column-parallel switched-capacitor difference circuit.

1

An Ultra-Low Voltage, Low-Noise, High Linearity 900-MHz Receiver With Digitally Calibrated In-Band Feed-Forward Interferer Cancellation in 65-nm CMOS

Ajay Balankutty, *Student Member, IEEE*, and Peter R. Kinget, *Fellow, IEEE*

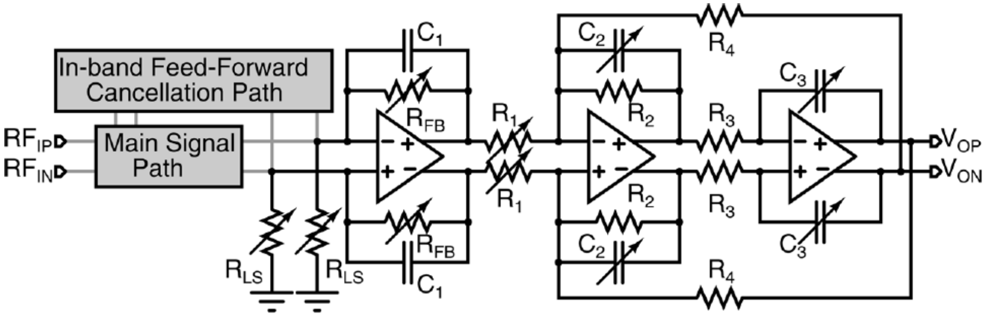


Fig. 5. TIA and channel select filter; the variable resistors and capacitors are implemented as discretely switched banks of resistors and capacitors.

2

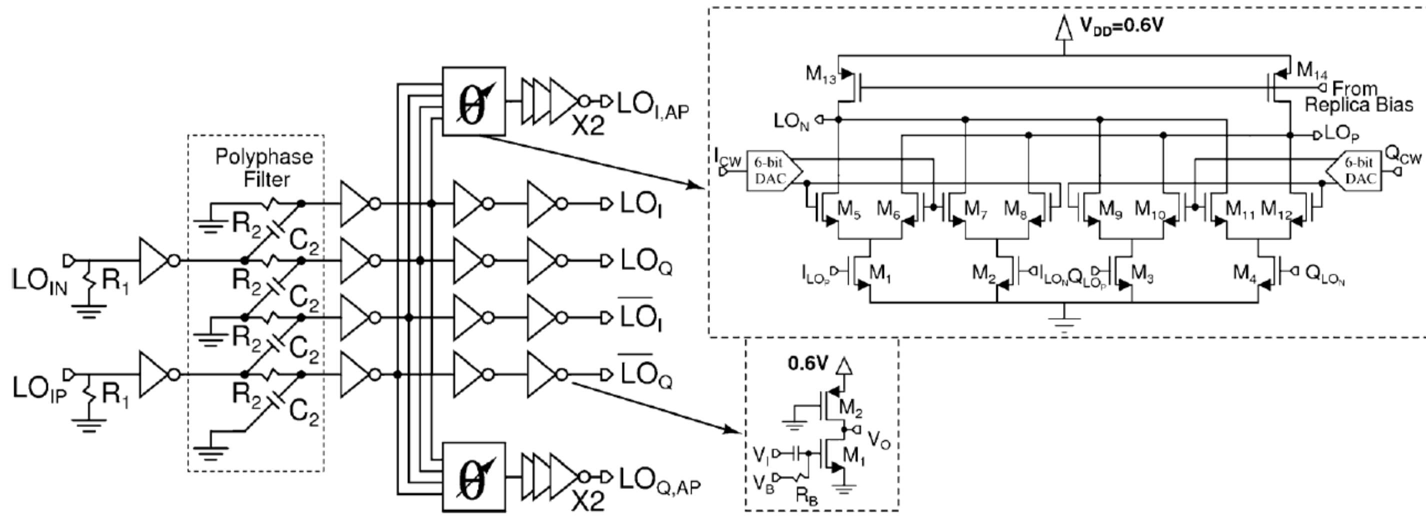


Fig. 6. Schematics of the passive polyphase filter, LO phase shifter, and LO buffers.

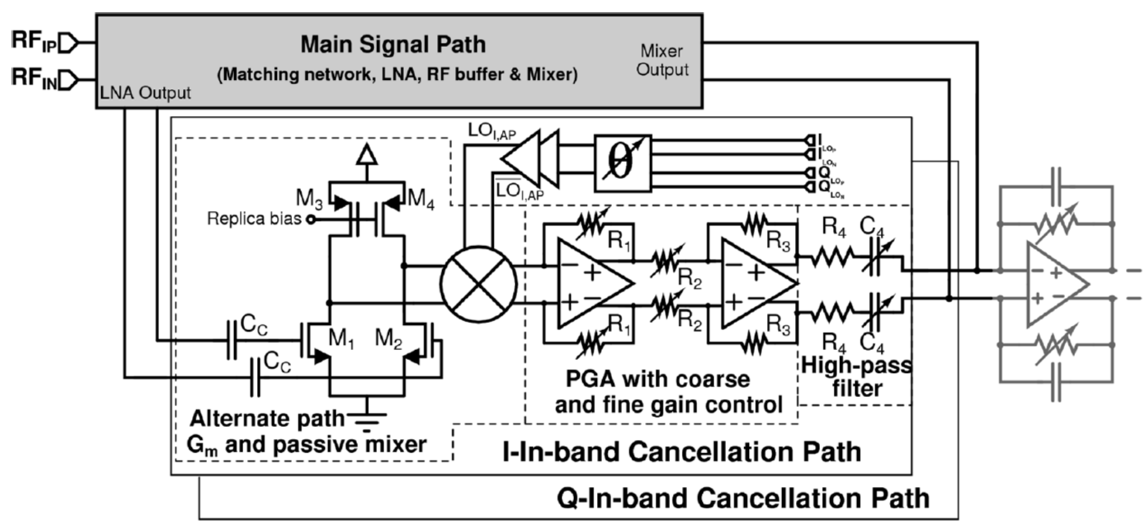
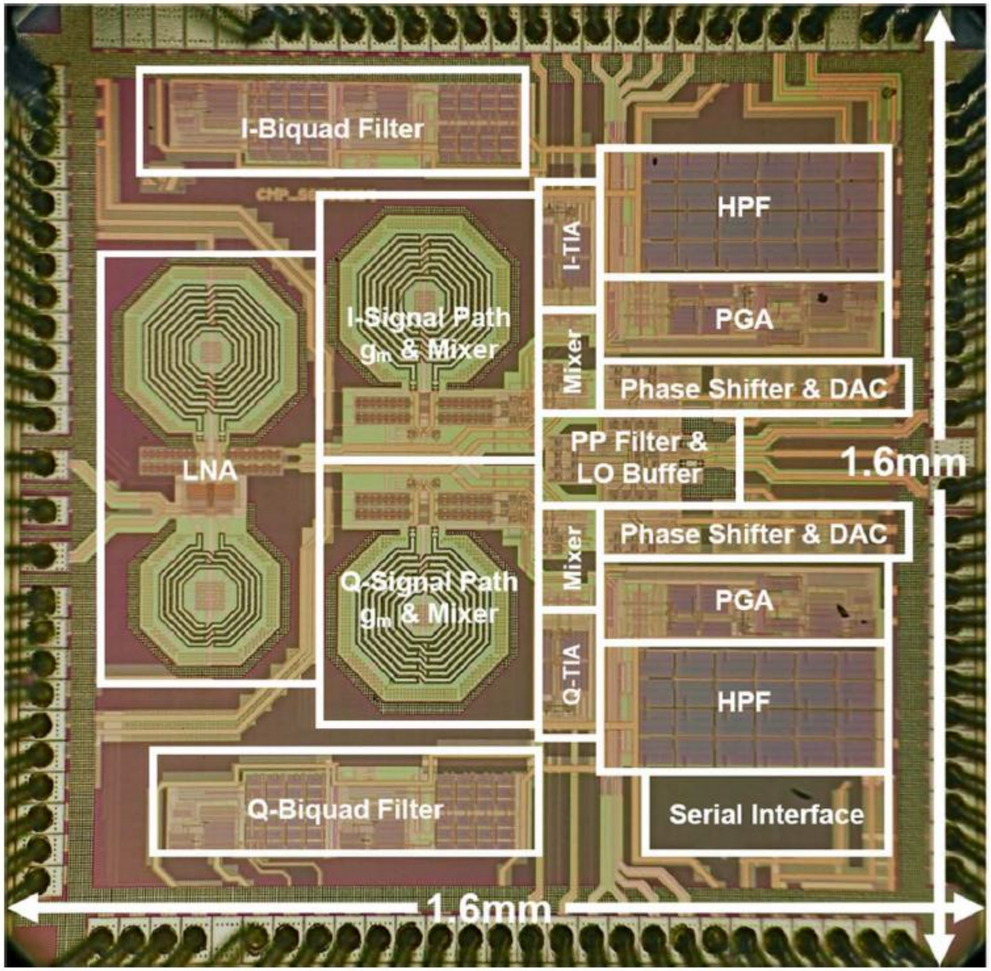


Fig. 7. Alternate path transconductor, downconversion mixer, PGA, and high-pass filter implementation.


I-Biquad Filter
HPF
I-TIA
I-Signal Path
g_m & Mixer
PGA
Mixer
Phase Shifter & DAC
LNA
PP Filter &
LO Buffer
1.6mm
Mixer
Phase Shifter & DAC
Q-Signal Path
g_m & Mixer
PGA
Q-TIA
HPF
Q-Biquad Filter
Serial Interface
1.6mm

A Low Area, Switched-Resistor Based Fractional-N Synthesizer Applied to a MEMS-Based Programmable Oscillator

Michael H. Perrott, *Senior Member, IEEE*, Sudhakar Pamarti, *Member, IEEE*, Eric G. Hoffman, *Member, IEEE*, Fred S. Lee, *Member, IEEE*, Shouvik Mukherjee, Cathy Lee, Vadim Tsinker, Sathi Perumal, Benjamin T. Soto, Niveditha Arumugam, and Bruno W. Garlepp, *Member, IEEE*

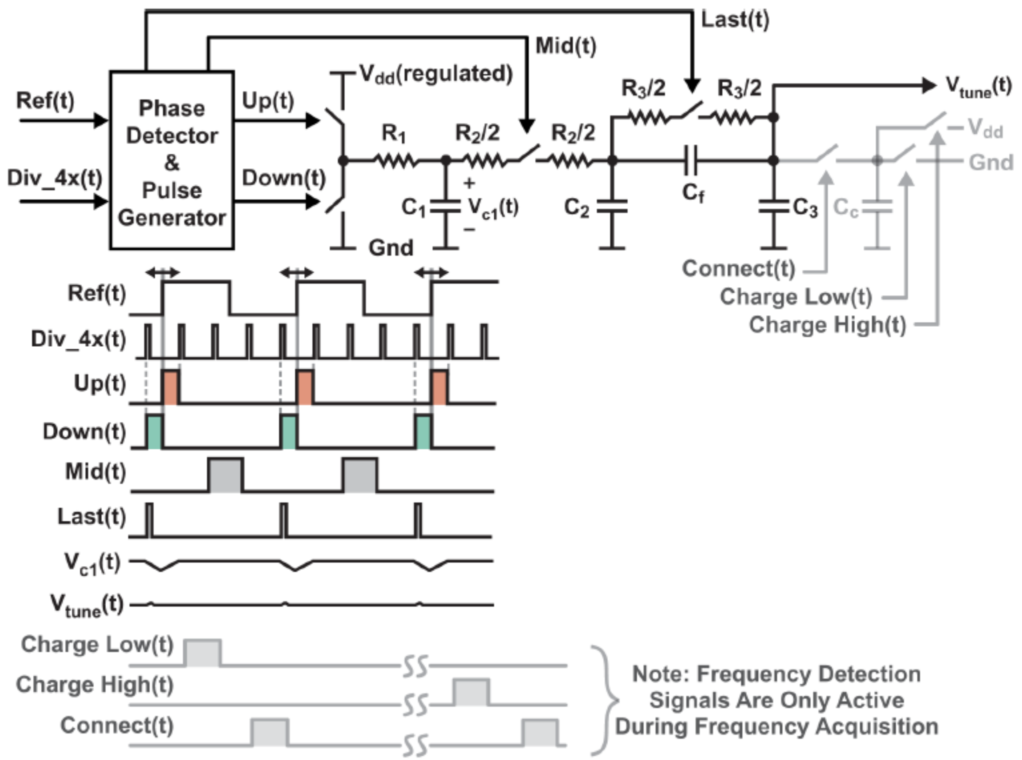


Fig. 3. Proposed switched resistor loop filter. Note that the R_3 pulsing frequency can also be set lower than the reference frequency, and is implemented at 1/4 the reference frequency in the prototype.

3

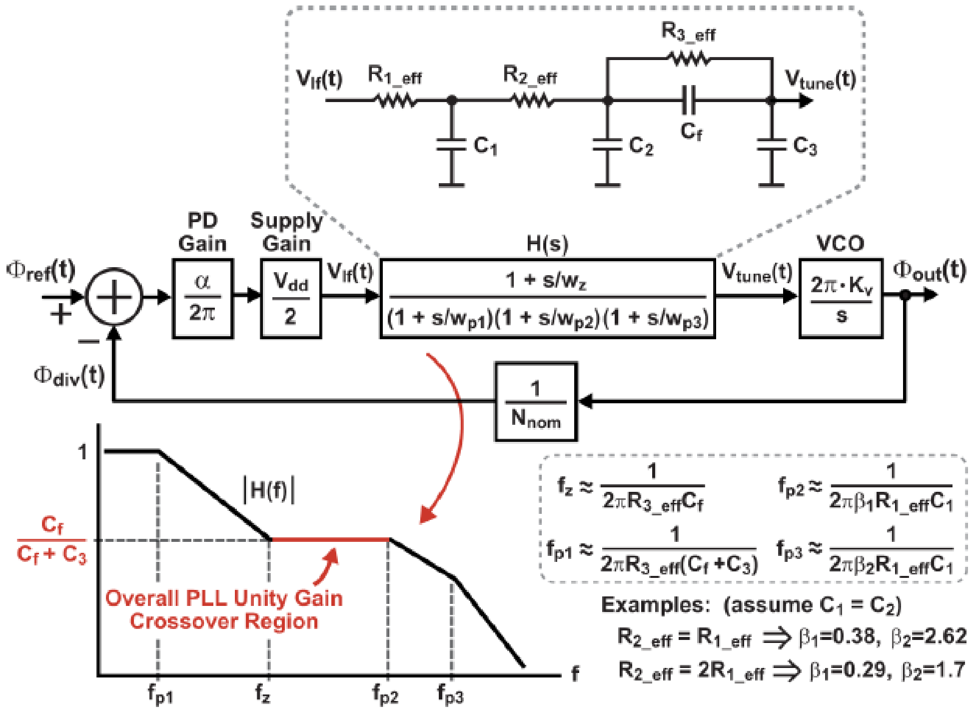


Fig. 5. Transfer function analysis of a switched resistor PLL in which the Bode plot of the loop filter, $H(f)$, is considered within the context of the overall PLL block diagram. Note that the values of β_1 and β_2 are calculated using the Quadratic formula by ignoring the influence of R_{3_eff}, C_f, and C_3 on poles f_{p2} and f_{p3}.

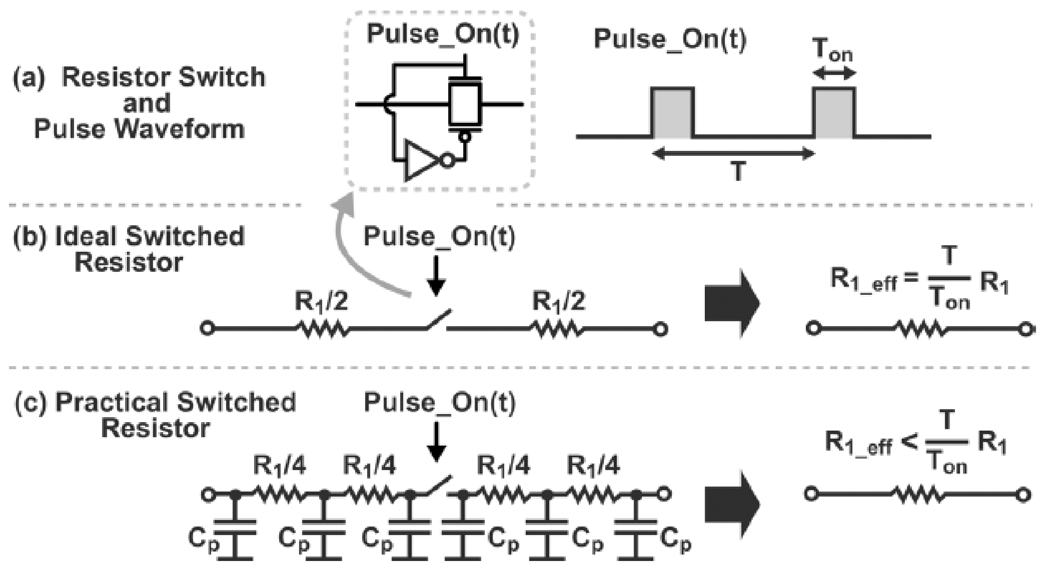


Fig. 4. Implementation of switched resistor using CMOS devices and poly resistors, along with impact of pulsed switching and parasitic capacitance on the effective resistance.

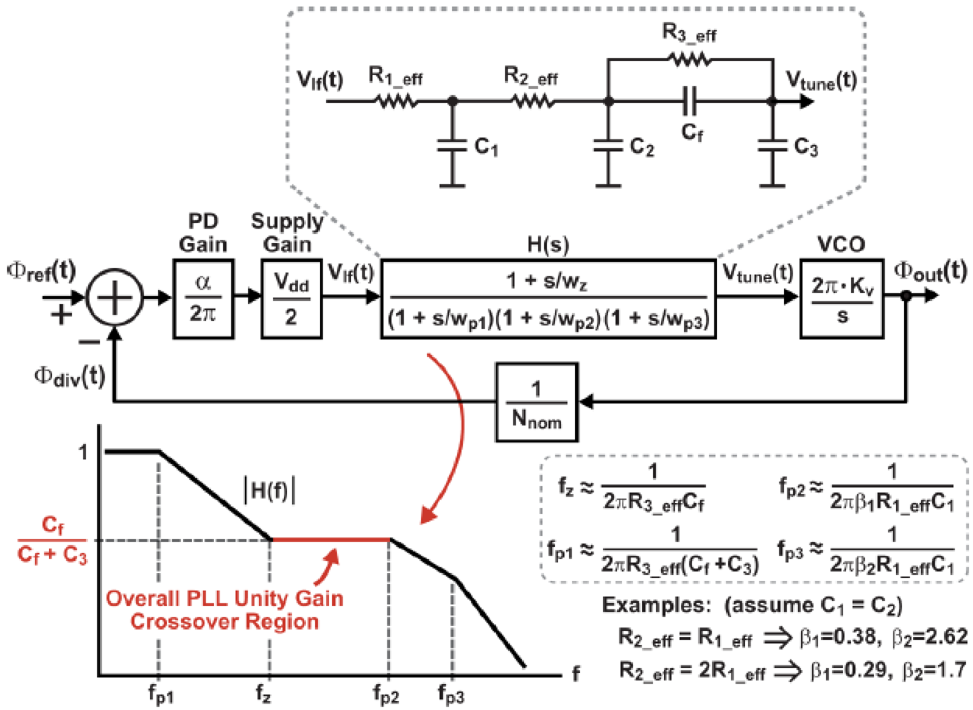


Fig. 5. Transfer function analysis of a switched resistor PLL in which the Bode plot of the loop filter, $H(f)$, is considered within the context of the overall PLL block diagram. Note that the values of β_1 and β_2 are calculated using the Quadratic formula by ignoring the influence of R_{3_eff}, C_f, and C_3 on poles f_{p2} and f_{p3}.

IEEE JOURNAL OF SOLID-STATE CIRCUITS, VOL. 46, NO. 3, MARCH 2011

A Wideband Receiver for Multi-Gbit/s Communications in 65 nm CMOS

Federico Vecchi, *Member, IEEE*, Stefano Bozzola, Enrico Temporiti, Davide Guermandi, Massimo Pozzoni, Matteo Repossi, Marco Cusmai, Ugo Decanis, *Student Member, IEEE*, Andrea Mazzanti, *Member, IEEE*, and Francesco Svelto, *Member, IEEE*

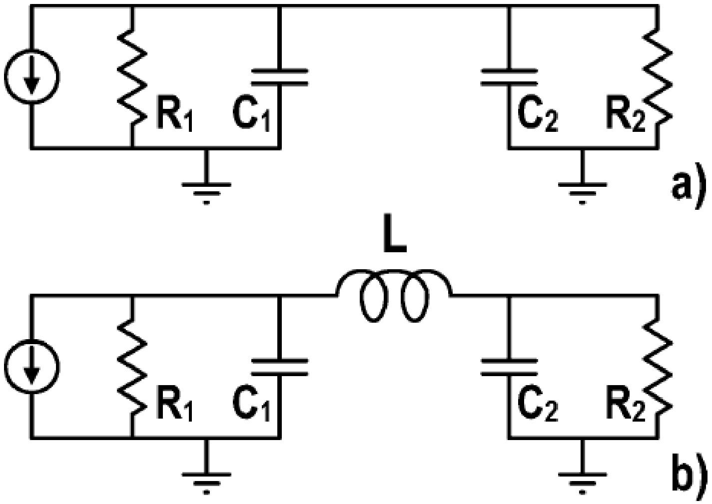


Fig. 4. (a) Equivalent circuit of a baseband amplifier with low-pass transfer function: a transconductor drives two separate *RC* groups (R_1, C_1 and R_2, C_2). (b) The two capacitors are separated by means of a series inductor improving gain–bandwidth.

4

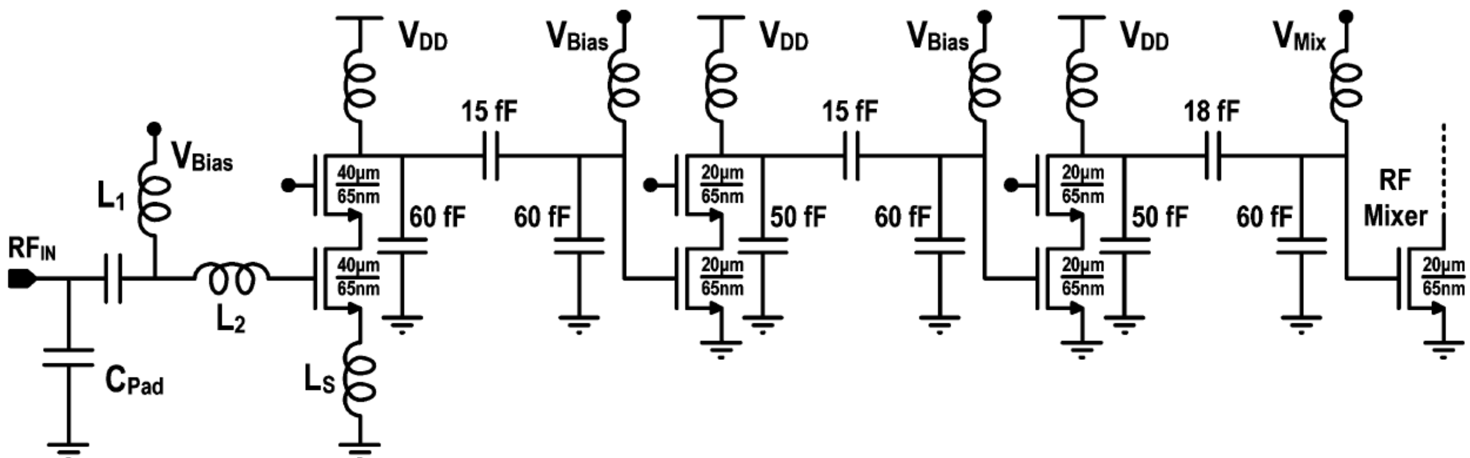


Fig. 7. Circuit diagram of the three stages LNA.

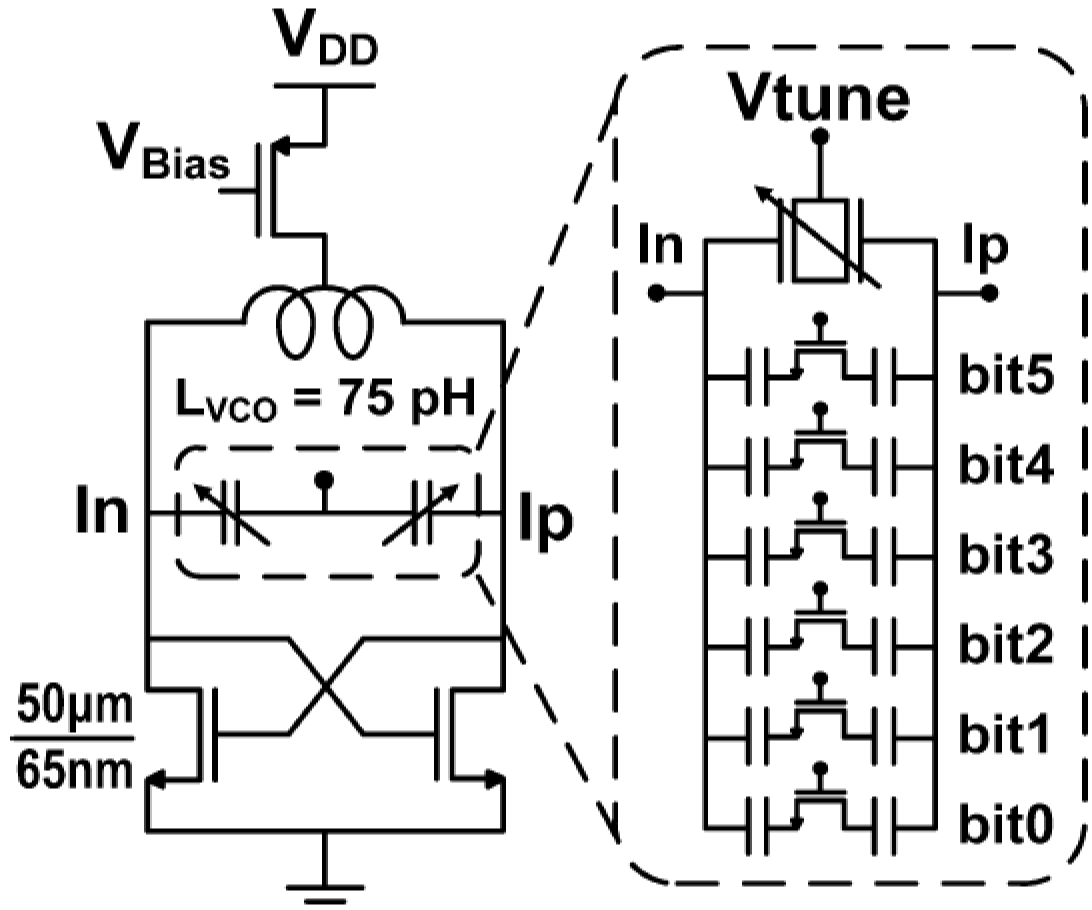


Fig. 11. LC VCO. A 5 bits capacitor tank is adopted for coarse tuning with an nMOS varactor for fine tuning.

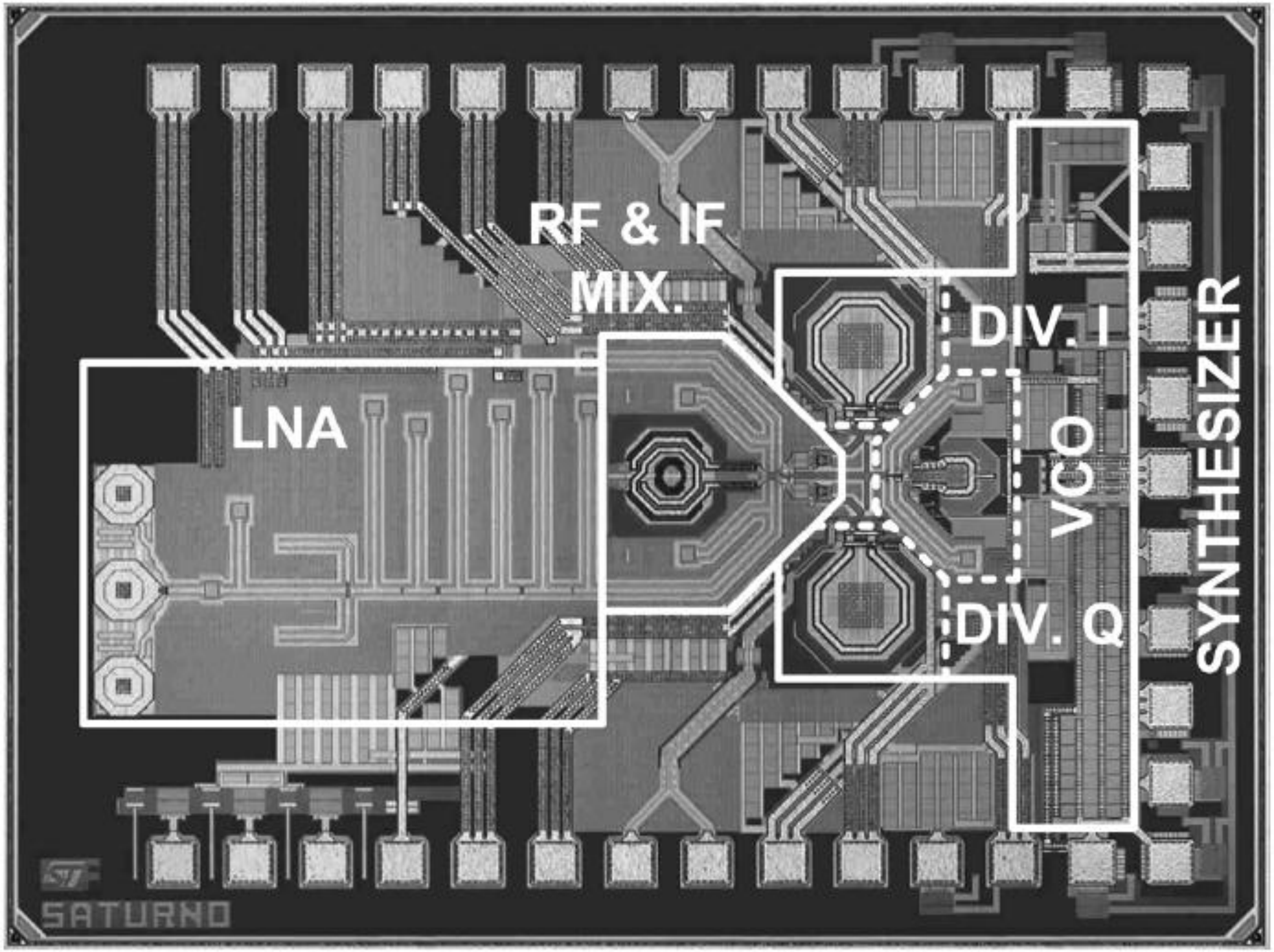


Fig. 13. Chip microphotograph.

A Broadband CMOS RF Front-End for Universal Tuners Supporting Multi-Standard Terrestrial and Cable Broadcasts

Donggu Im, *Student Member, IEEE*, Hongteuk Kim, *Member, IEEE*, and Kwyro Lee, *Senior Member, IEEE*

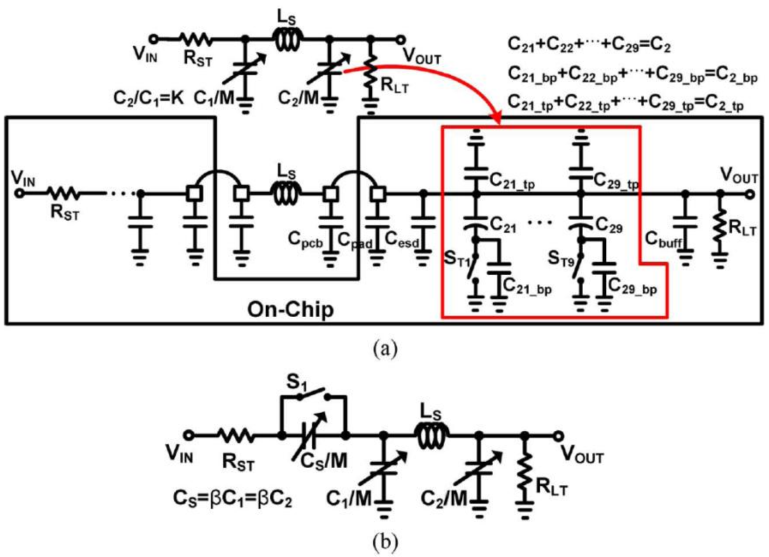


Fig. 7. (a) Third-order pi-section LC filter and its detailed schematic for calculating parasitic capacitance. (b) Modified 3rd-order pi-section LC filter with the variable capacitor C_S to increase the tuning range.

$$\frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{M^2 w_{\text{cL}}^3}{s^3 + [2M]w_{\text{cL}}s^2 + \left[\left(\frac{1+K}{2}\right)M\right]w_{\text{cL}}^2 s + M^2 w_{\text{cL}}^3} \quad (8)$$

5

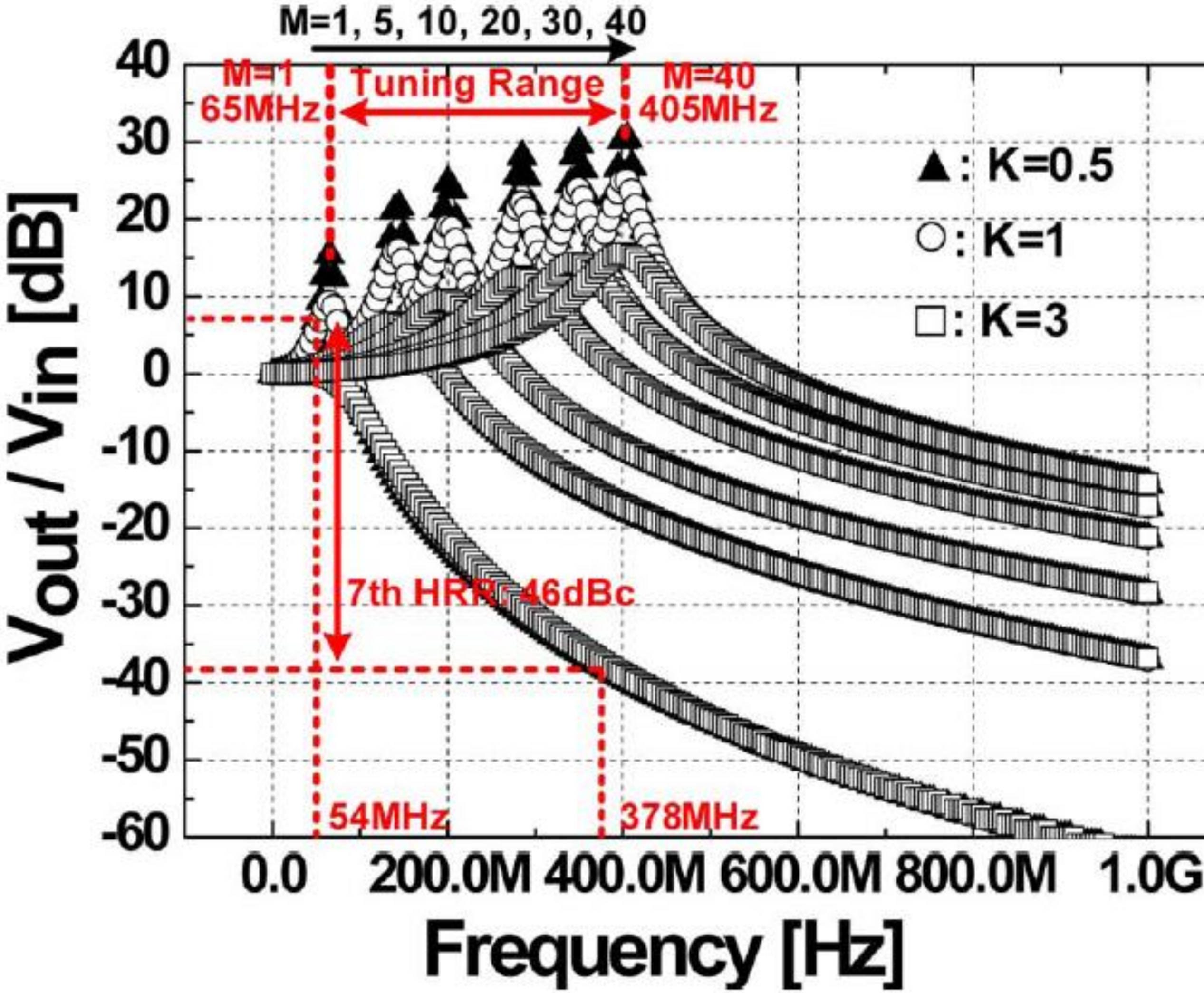


Fig. 8. Frequency response of (8) according to the variation of the M for $K = 0.5$, 1, and 3.

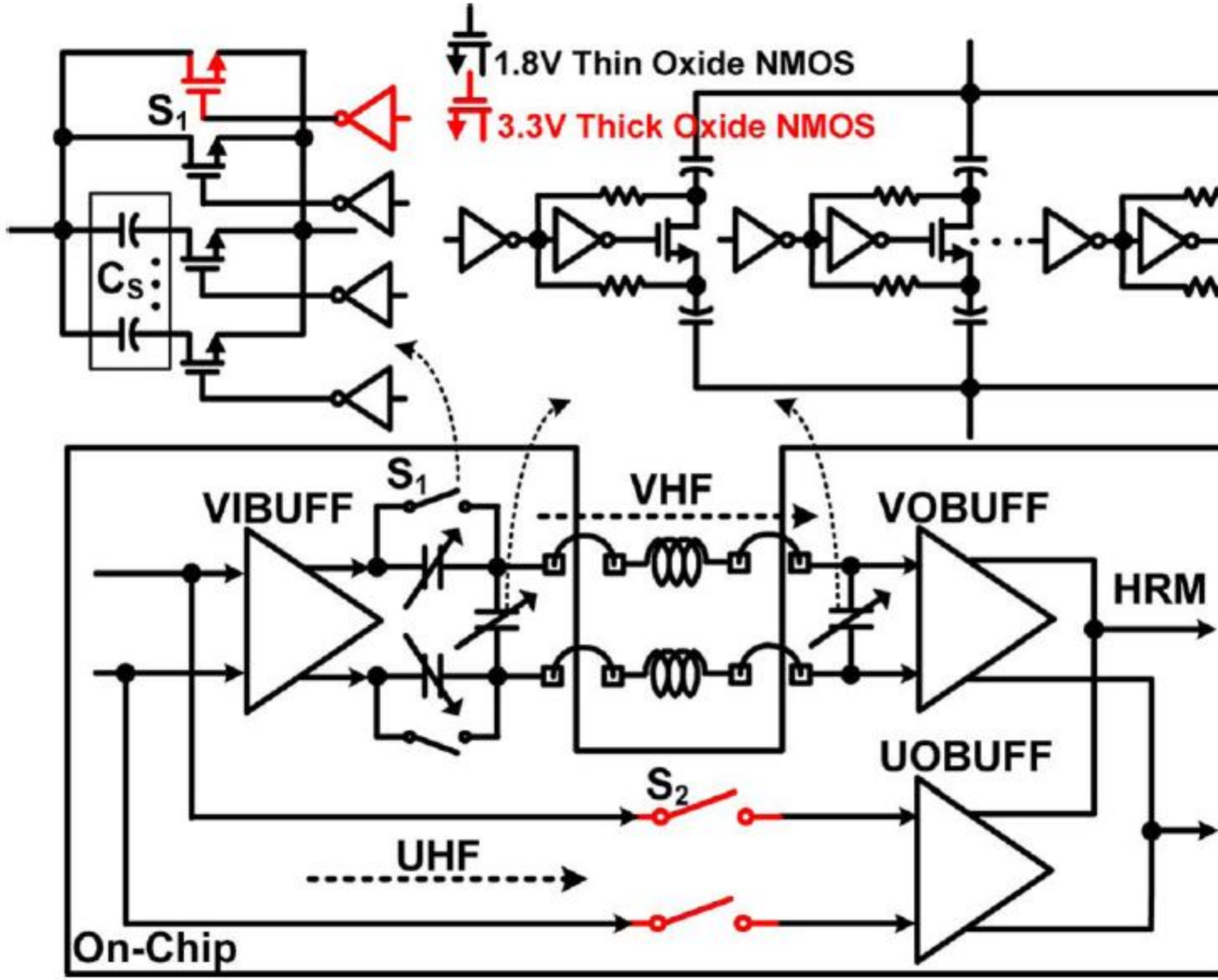


Fig. 11. Complete passive tunable filter.

Motion Image Sensor with On-chip Adaptation and Programmable Filtering

Peng Xu*, Pamela Abshire*, and J. Sean Humbert†
*Dept. of Electrical and Computer Engineering, *Institute for Systems Research, †Dept. of Aerospace Engineering
University of Maryland, College Park, MD 20742

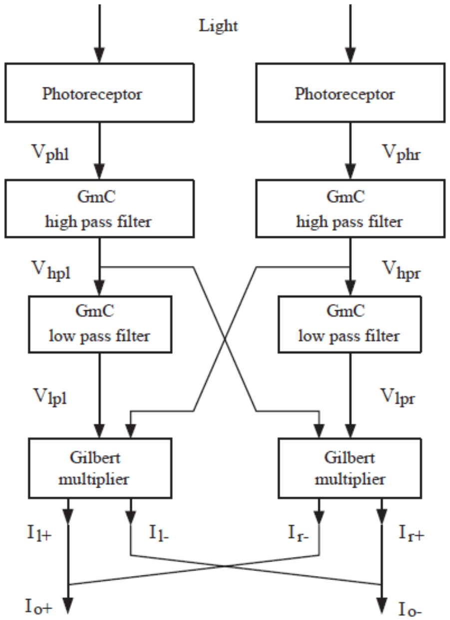


Fig. 1. Block diagram of EMD implementation.

6

The optic flow generated by the EMDs pass on to spatial filters. Each filter coefficient is programmed using nonvolatile storage in floating gate MOS. Bandyopadhyay et al. used a programmable floating-gate array to implement matrix transformations or images [8]. Here programming is accomplished

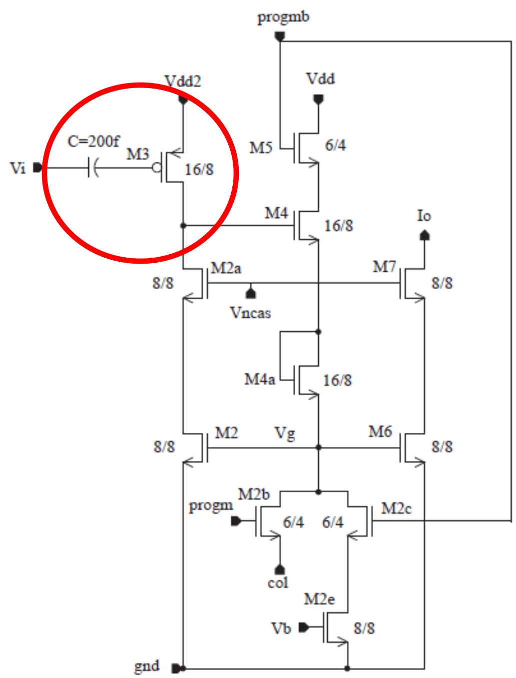


Fig. 2. Schematic of the PCE.

Programmable Current Element

current. $M3$ performs the multiplication operation, and the charge stored on its floating gate functions as the coefficient of the filter.

A Fully Integrated and Reconfigurable Architecture for Coherent Self-Testing of High Speed Analog-to-Digital Converters

Edinei Santin, *Student Member, IEEE*, Luís B. Oliveira, *Member, IEEE*, Blazej Nowacki, and João Goes, *Senior Member, IEEE*

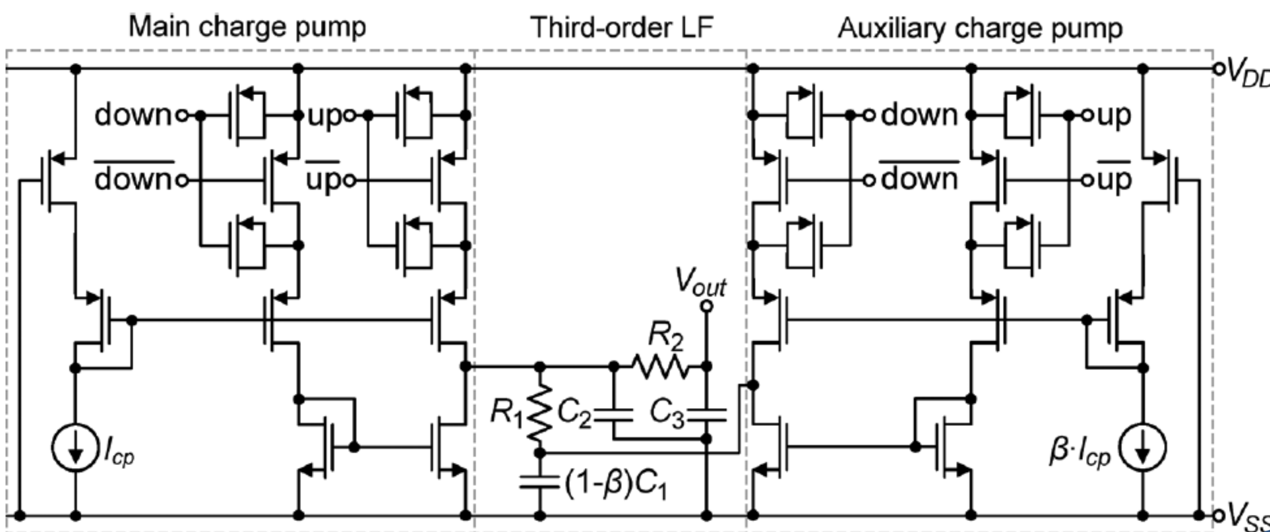


Fig. 5. Schematic of the charge pump and third-order loop filter.

7

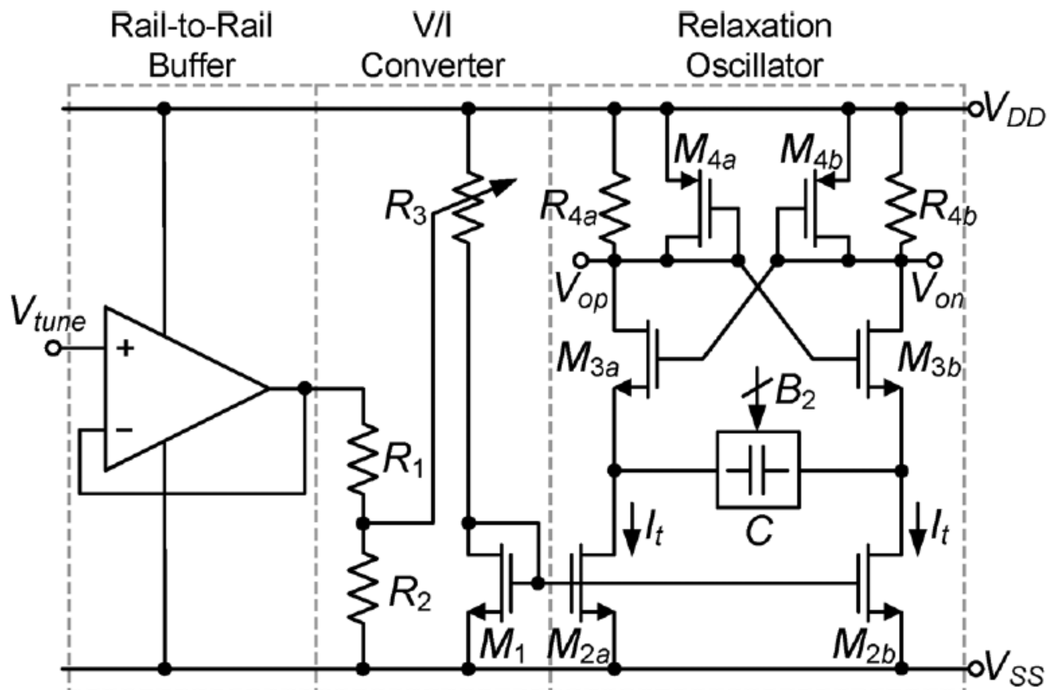


Fig. 3. Simplified schematic of the relaxation oscillator (VCO_2).

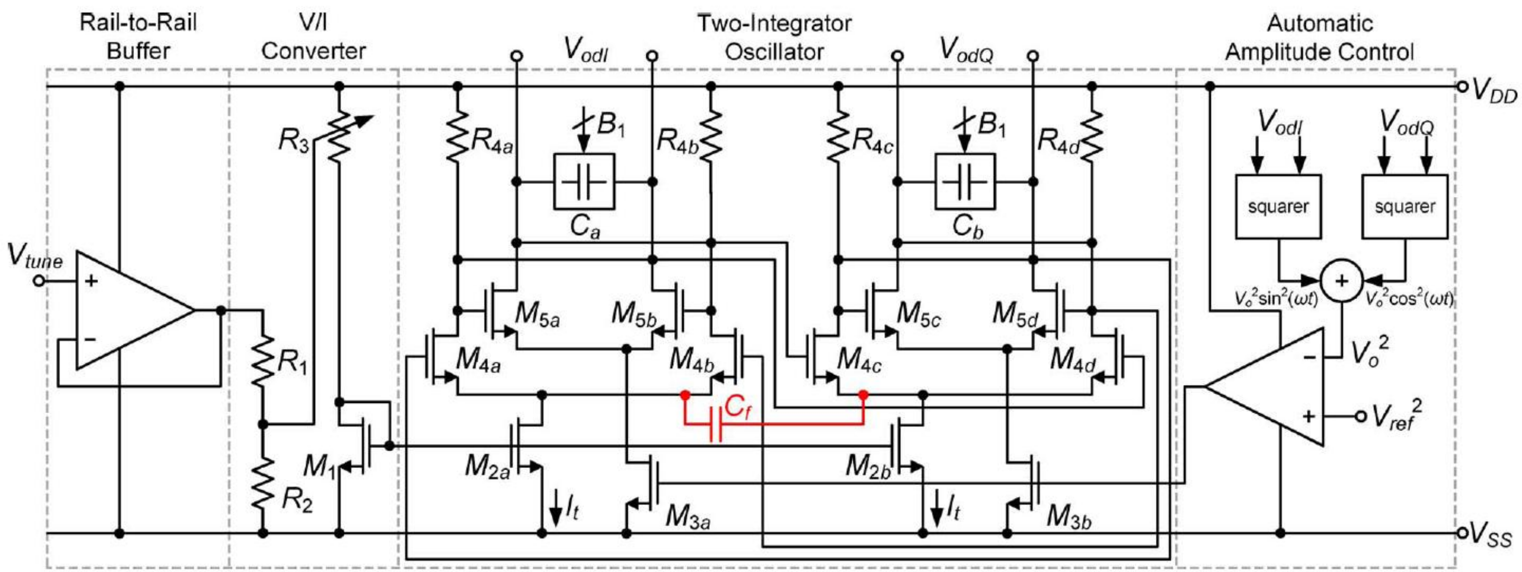


Fig. 4. Simplified schematic of the two-integrator oscillator (VCO_1).

Current-Mode, WCDMA Channel Filter With In-Band Noise Shaping

Alberto Pirola, *Student Member, IEEE*, Antonio Liscidini, *Member, IEEE*, and Rinaldo Castello, *Fellow, IEEE*

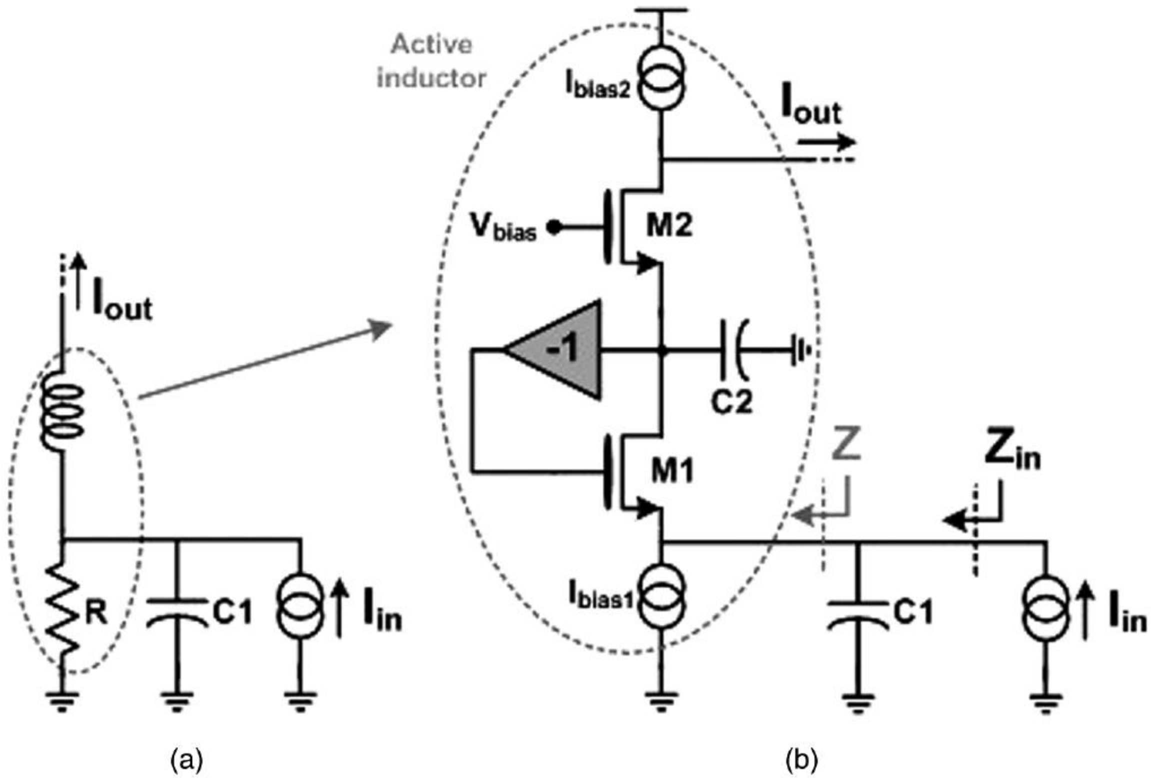


Fig. 4. Current biquad cell.

8

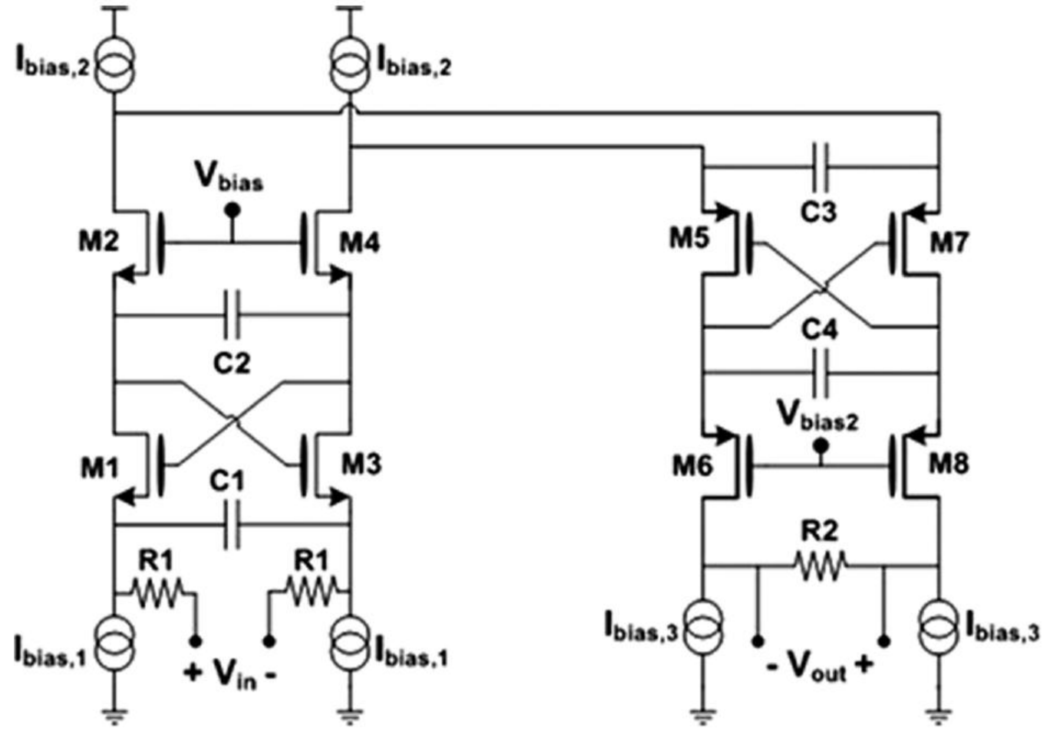
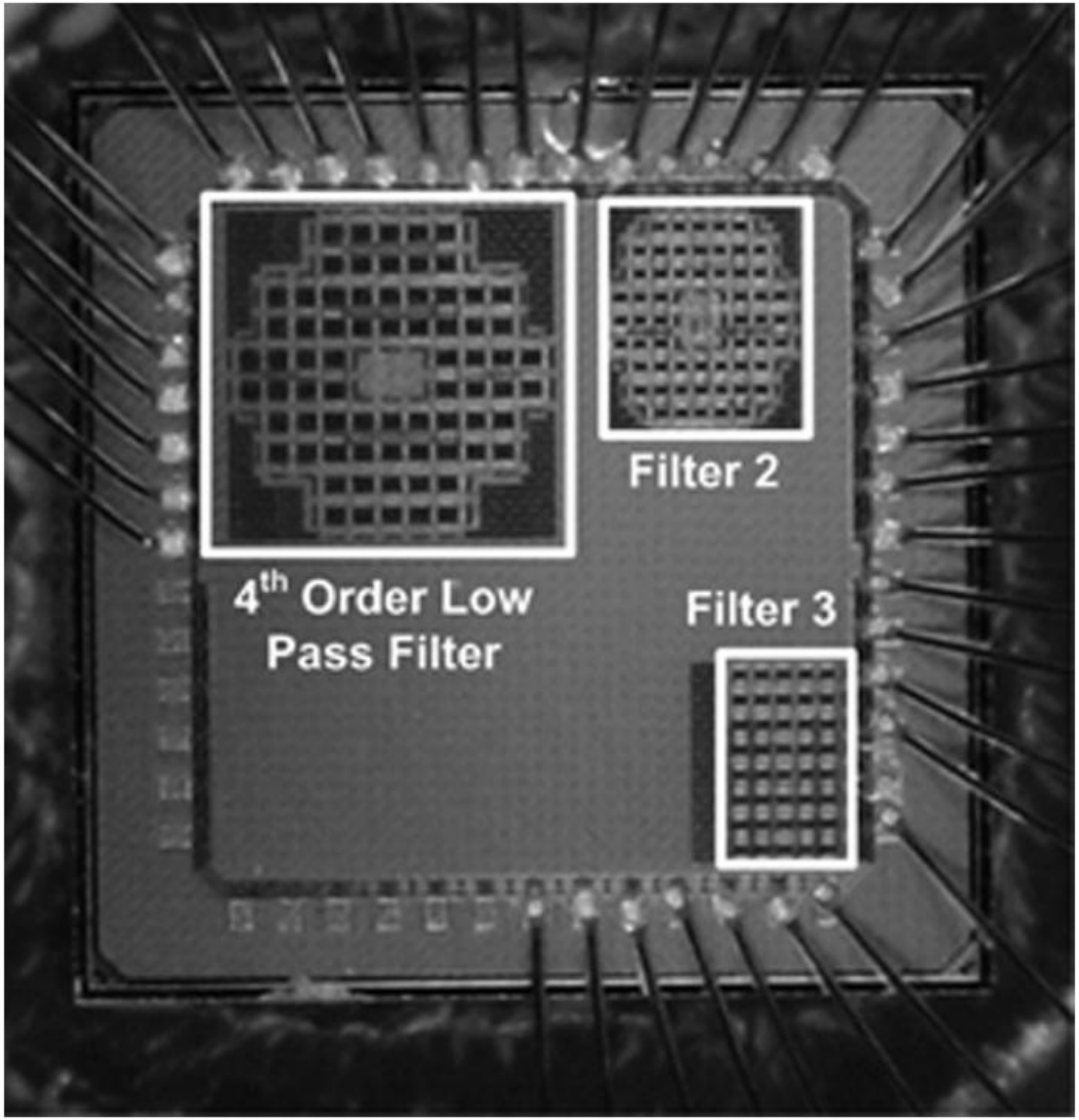


Fig. 8. Fourth-order filter schematic.

The chip micrograph of the filter prototype, fabricated in a 90 nm CMOS process, is shown in Fig. 10. All pads are ESD protected and the active die area is 0.5 mm^2. This area is dominated by low-density MiM capacitors (210 pF), whose placement could be further improved, resulting in a lower area occupation. Moreover, large on-chip MoM bypass capacitors are



Filter 2
4th Order Low Pass Filter
Filter 3

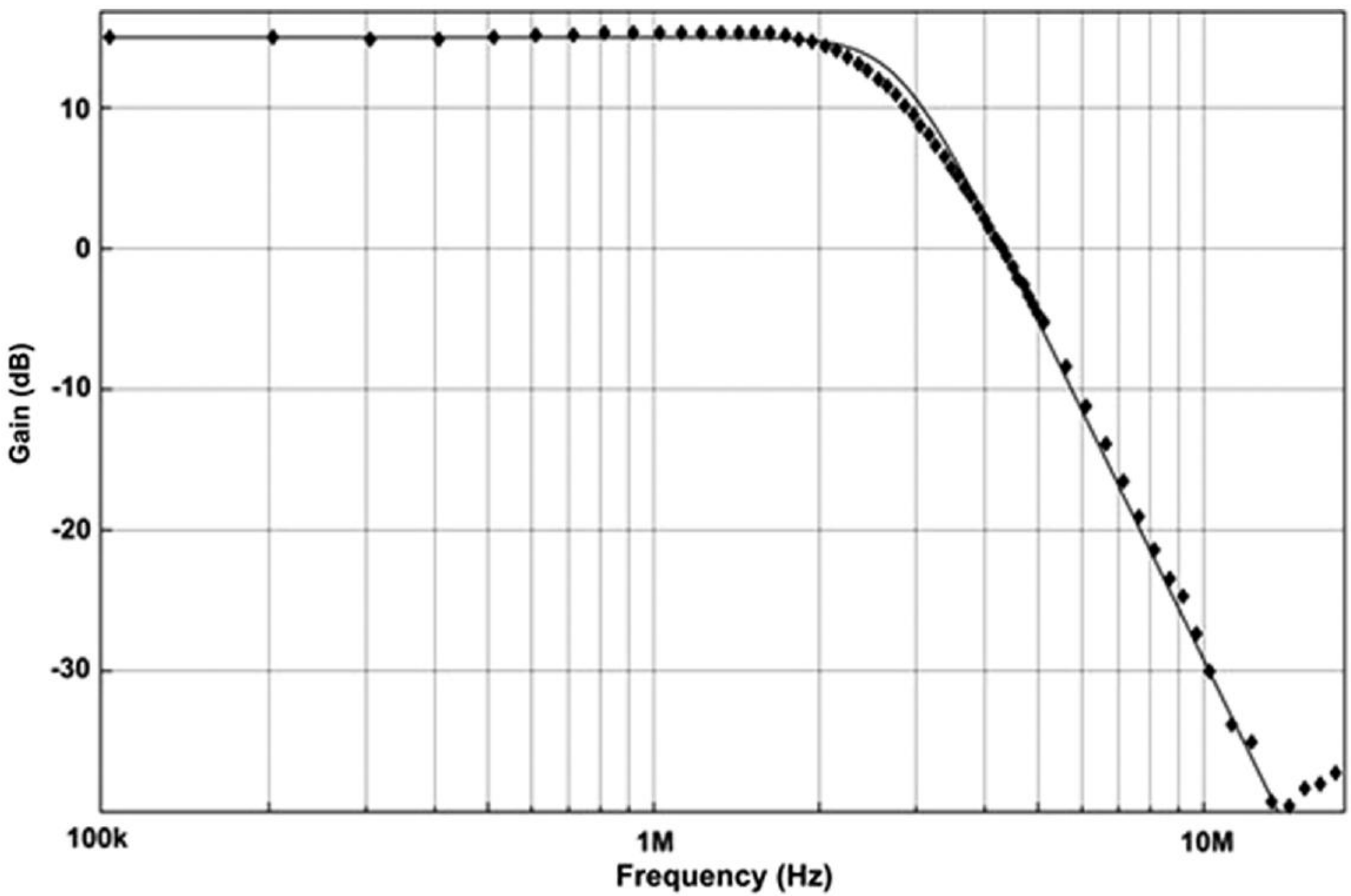


Fig. 11. Measured output transfer function.

Transformed lumped element integrated passive bandpass filters for gigahertz range spectrum monitor receivers

S. Liang and W. Redman-White

ELECTRONICS LETTERS 24th November 2011 Vol. 47 No. 24

Two integrated bandpass filters have been designed and fabricated on standard 130 nm CMOS technology. The lumped element, third-order Butterworth bandpass filters at 9.45 and 1.75 GHz are designed for sub-band filtering in a spectrum monitoring receiver function in a future cognitive radio. A series coupled resonator topology is selected and a novel delta–star transformation technique is applied to obtain element values suitable for reliable fabrication on an integrated circuit. Occupying die areas of 780 × 200 μm and 1750 × 500 μm each, the filters achieve insertion losses of 15.6 and 8.6 dB, and bandwidths of 1 GHz and 210 MHz, respectively, which are suitable for the chosen spectrum monitor receiver architecture.

9

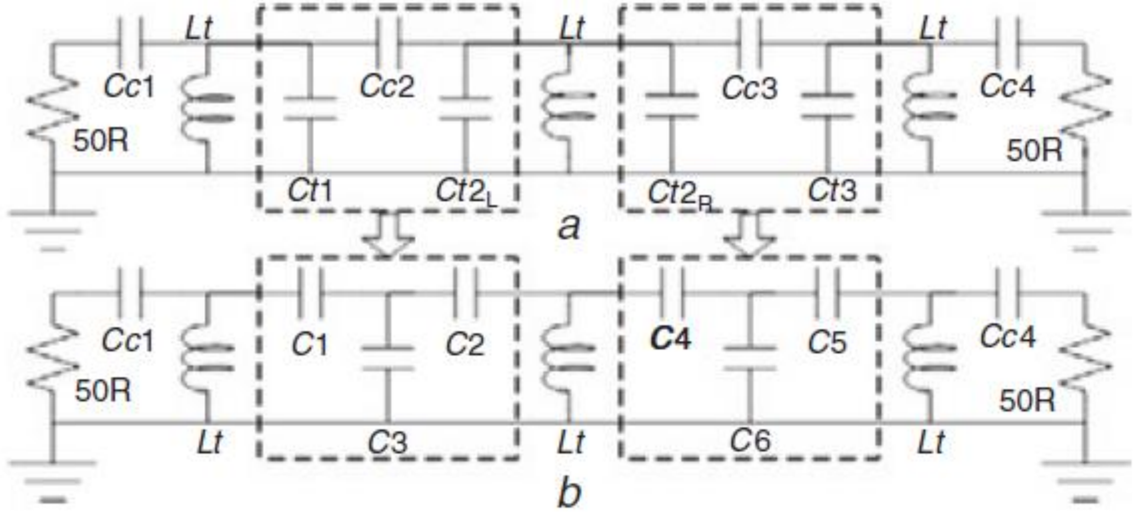


Fig. 2 *Series coupled resonator filter topology and delta–star transformation*

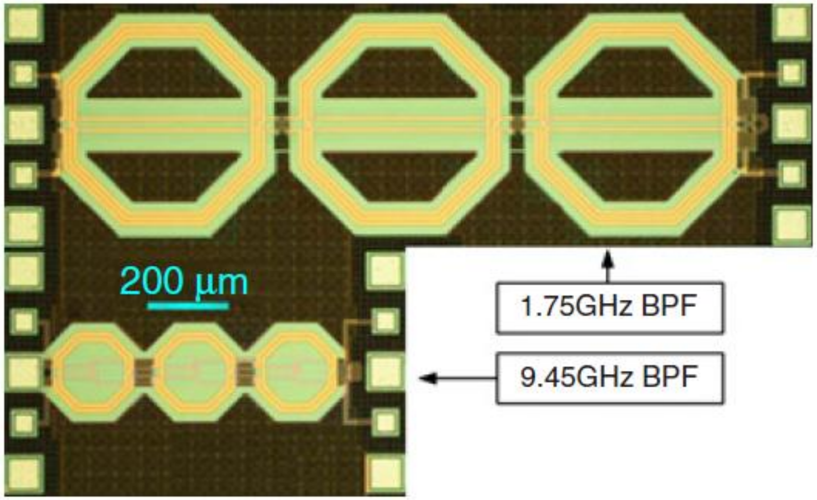


Fig. 3 *Filter die photos including GSGSG pads*

EE 508
Lecture 41

What filter architectures are really being used today?

Tunable Class AB CMOS Gm-C Filter Based on Quasi-Floating Gate Techniques

Coro Garcia-Alberdi, Antonio J. Lopez-Martin, *Senior Member, IEEE*, Lucia Acosta, Ramon G. Carvajal, *Senior Member, IEEE*, and Jaime Ramirez-Angulo, *Fellow, IEEE*

Manuscript received November 06, 2011; revised March 29, 2012, May 25, 2012; accepted July 24, 2012. This work was supported in part by the Spanish

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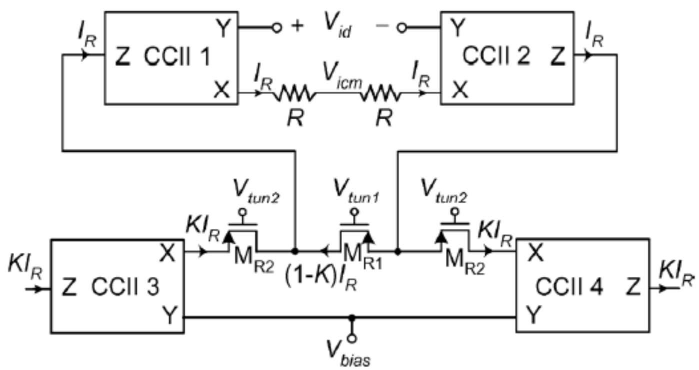


ig. 2. Transconductor proposed.

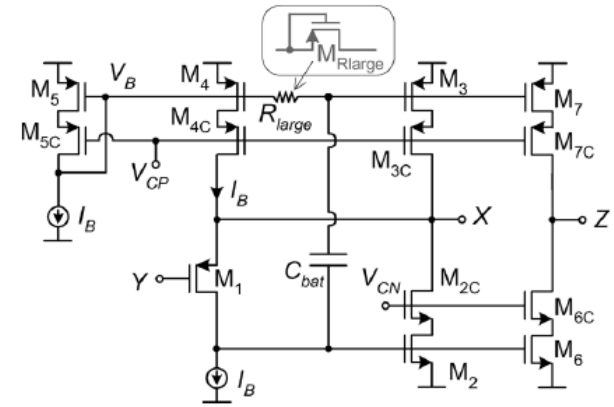


Fig. 3. Class AB CCII.

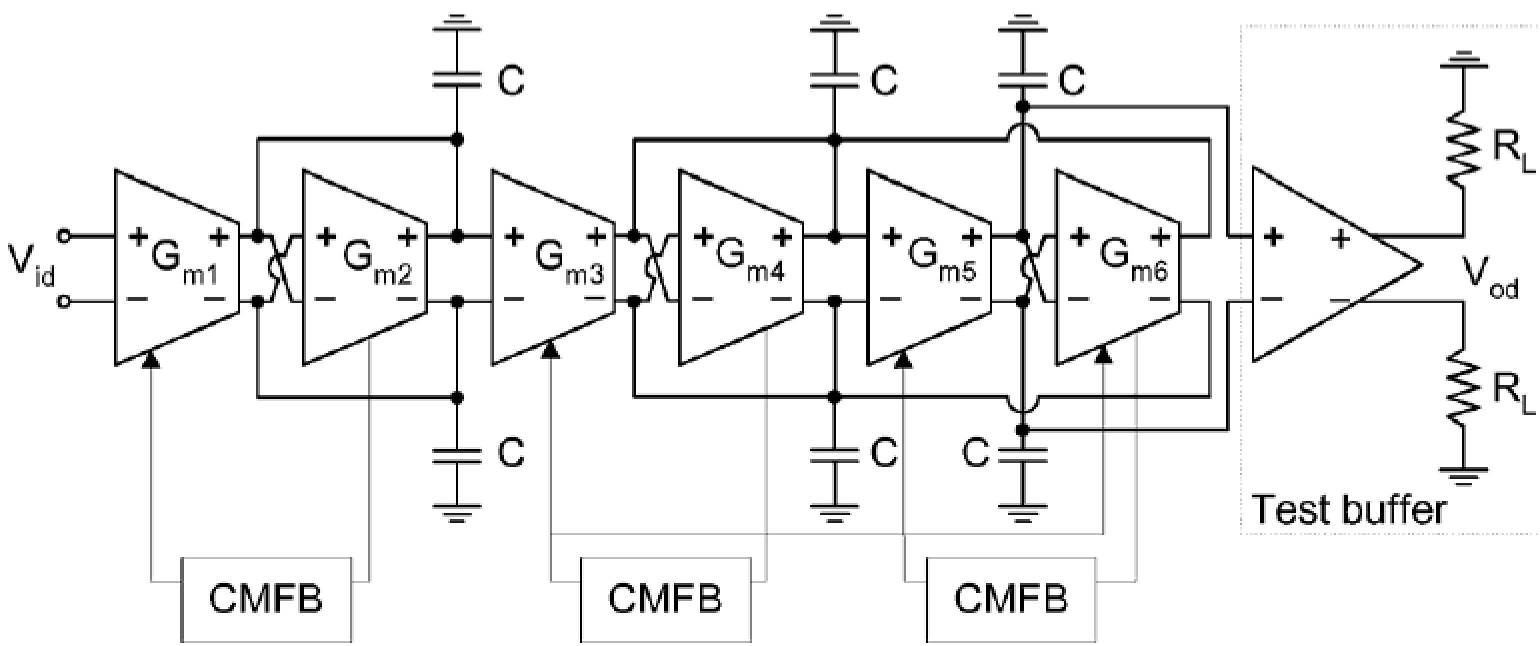


Fig. 5. Third-order Butterworth Gm-C channel filter.

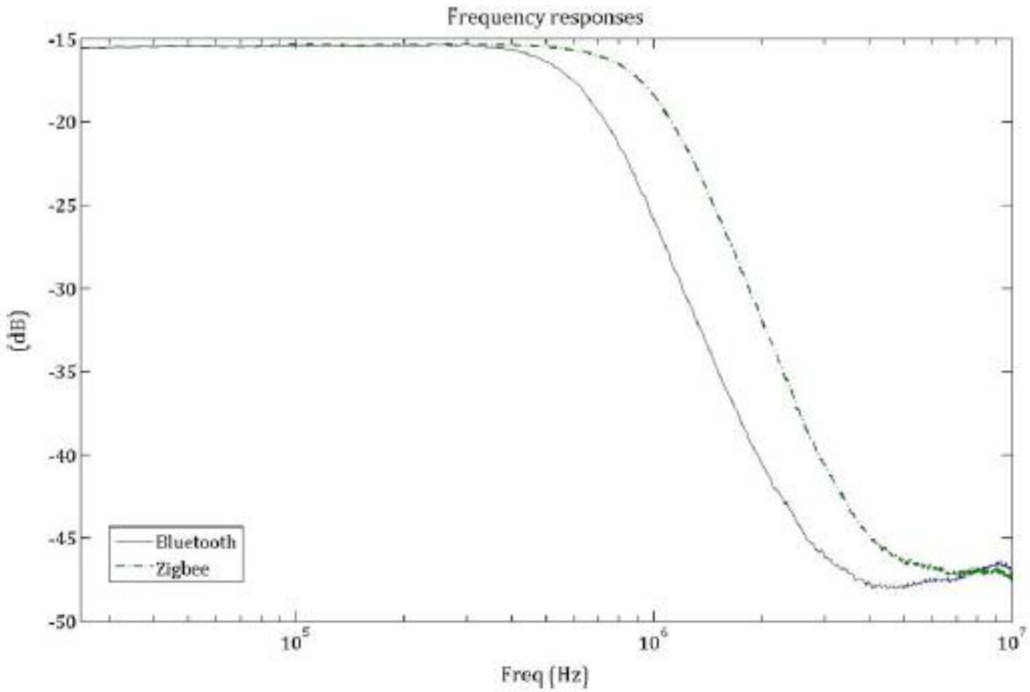
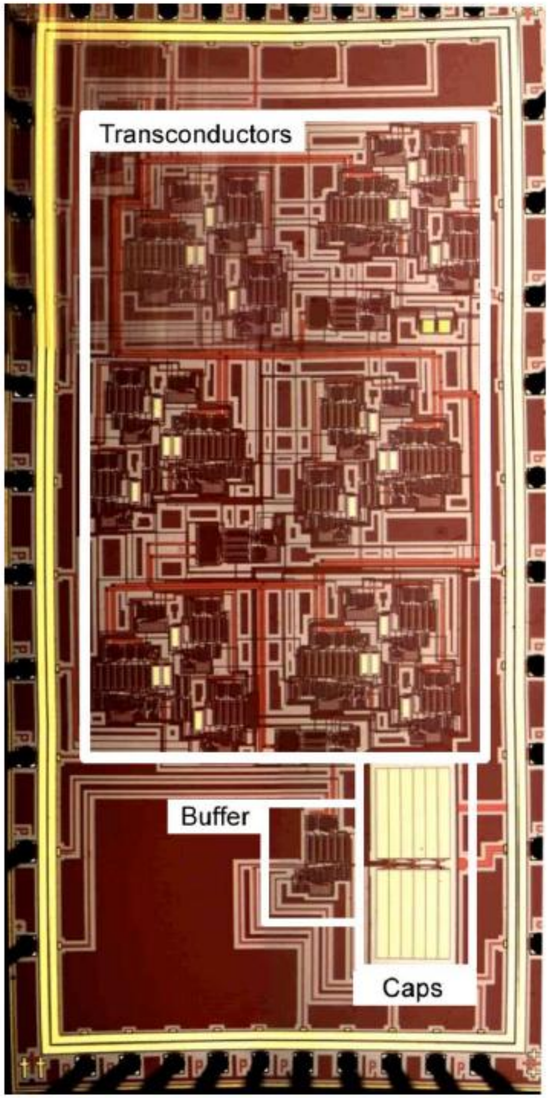


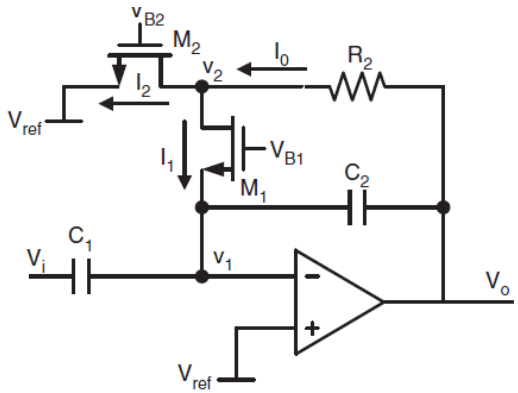
Fig. 10. Measured frequency response of the filter.


Transconductors
Buffer
Caps

5 mHz highpass filter with −80 dB total harmonic distortions

Haixi Li, Jinyong Zhang and Lei Wang

ELECTRONICS LETTERS 7th June 2012 Vol. 48 No. 12



Parameters	Values
C_1	1.5 pF
C_2	1.4 pF
R_2	297 kΩ
M_1	0.3 µm/9 µm
M_2	98 µm/0.3 µm
V_{B1} (at fc = 0.05 Hz)	1.25 V
V_{B2}	1.8 V
V_{ref}	0.9 V

b

Fig. 1 *Architecture and configuration of proposed first-order highpass filter*
a Architecture
b Configuration

11

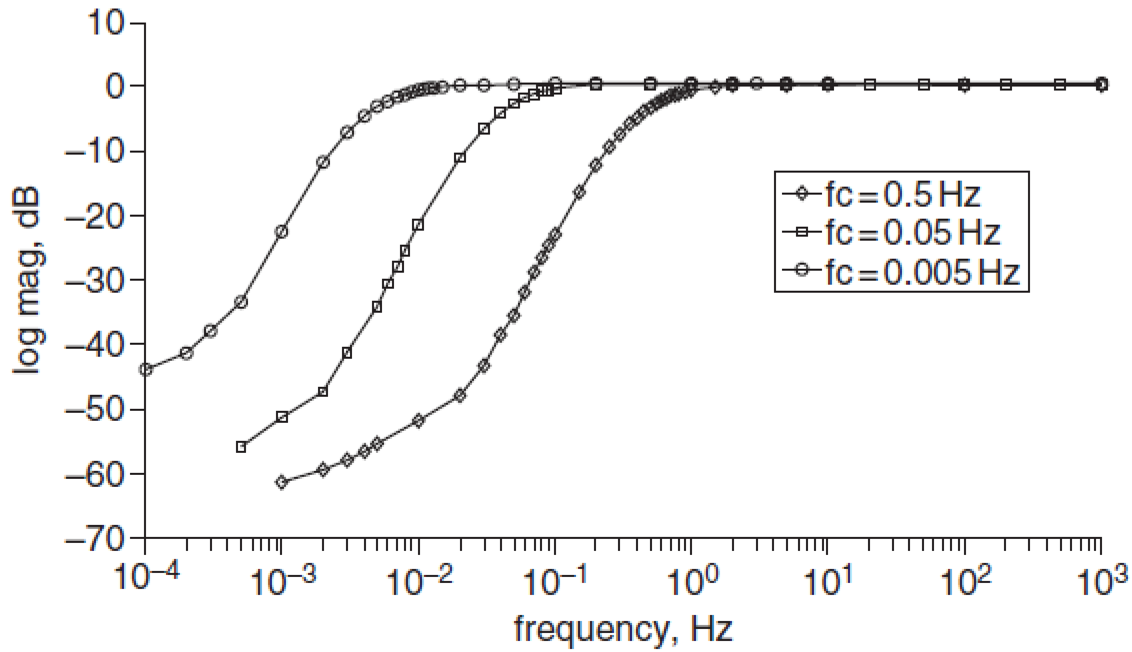


Fig. 2 *Frequency responses to three different −3 dB frequencies (fc) which are tuned by* V_{B1}

A 0.5-V R-MOSFET-C Filter Design Using Subthreshold R-MOSFET Resistors and OTAs With Cross-Forward Common-Mode Cancellation Technique

Phanumas Khumsat, *Member, IEEE*, and Apisak Worapishet, *Senior Member, IEEE*

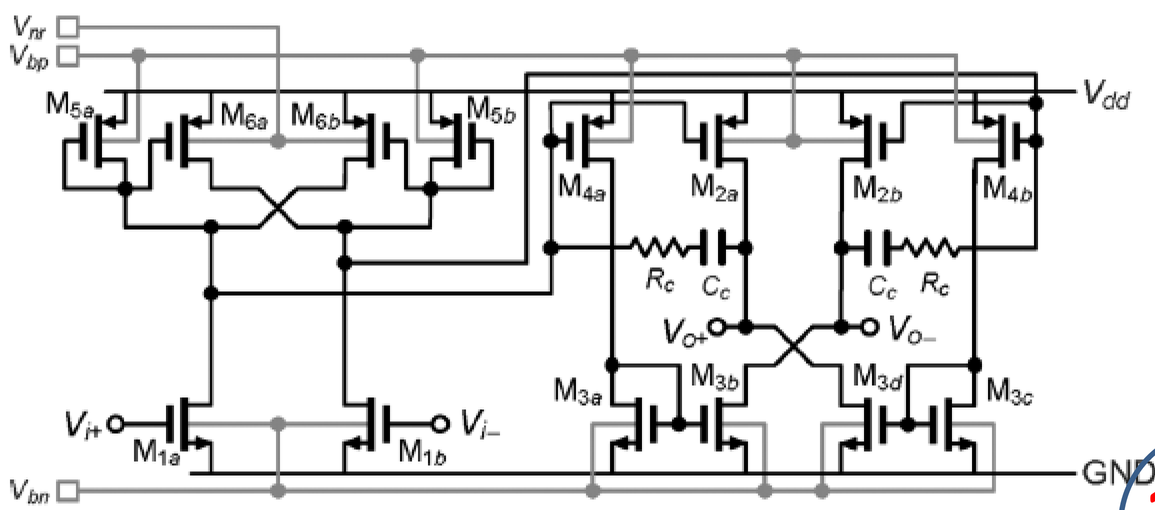
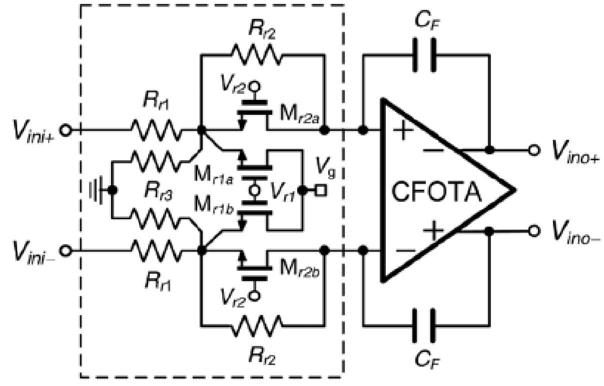


Fig. 1. Fully differential CFOTA with frequency compensation.

12



(a)

Mosfet-R integtator

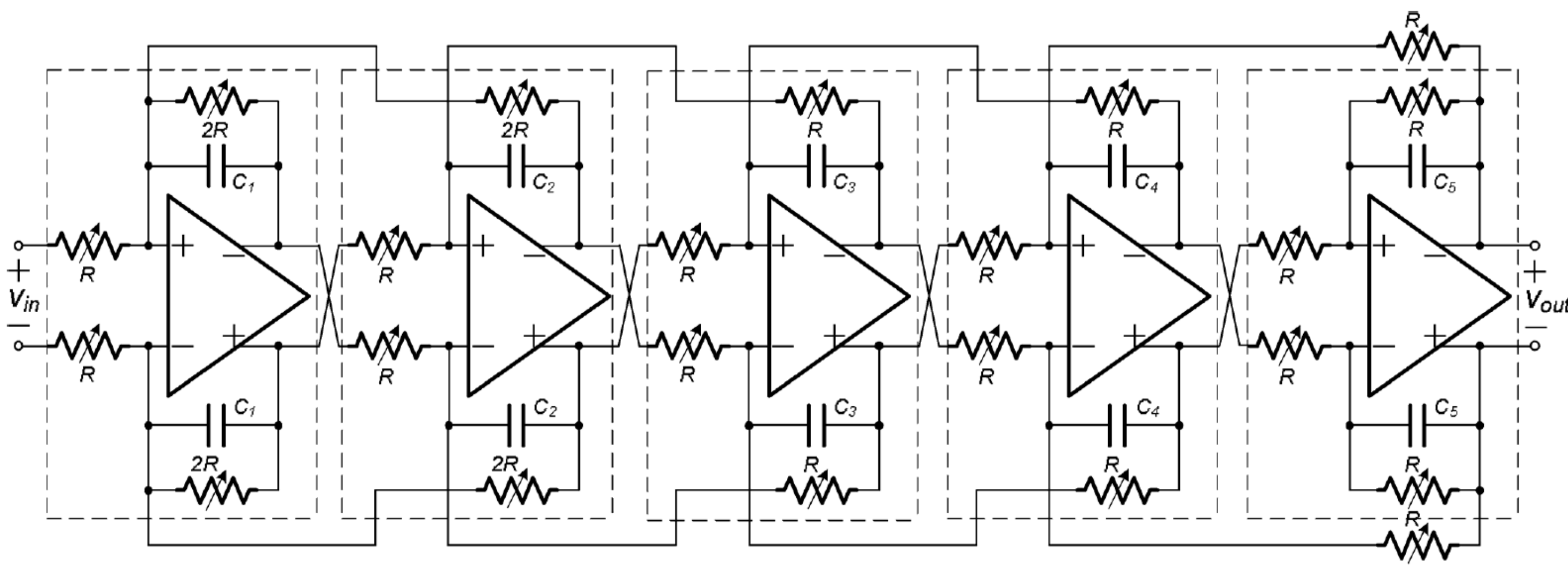


Fig. 9. Fifth-order Chebyshev R-MOSFET-C filter prototype. Each circuit in the dashed boxes is similar to Fig. 6.

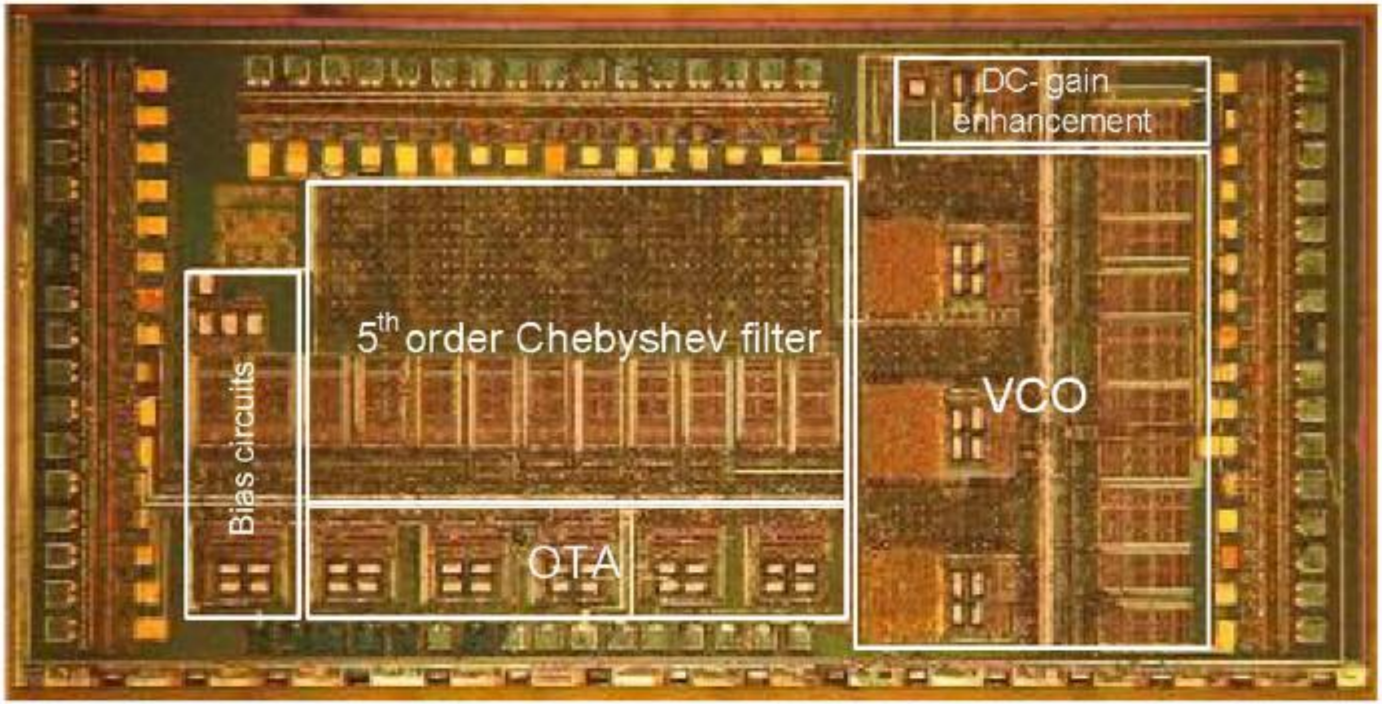
TABLE II
FILTER DESIGN PARAMETERS

Capacitor	Value	Capacitor	Value
C_1	11.31pF	C_5	22.62pF
C_2	16.31pF	Resistor	Value
C_3	33.70pF	R	75kΩ
C_4	16.31pF		

TABLE III
COMPONENTS PARAMETERS OF UNIT SUB-RMOS RESISTOR AND BIAS CIRCUITS [SEE FIG. 6] FOR A NOMINAL RESISTANCE VALUE OF 150 kΩ

Transistors	Sizes	Resistors	Values
M_{r1a}, M_{r1b}, M_{r2a}, M_{r2b}	100μm/1.8μm	R_{r1}, R_{r2}, R_{r3}	75kΩ
M_{c1}, M_{c2}, M_{c3}	192μm/0.36μm	R_{c4}	10kΩ
M_{c4}	30μm/0.48μm	R_g	75kΩ
M_g	48μm/0.48μm		

(a)



(b)

Fig. 10. Experimental 0.5-V filter. (a) Block diagram. (b) Chip microphotograph.

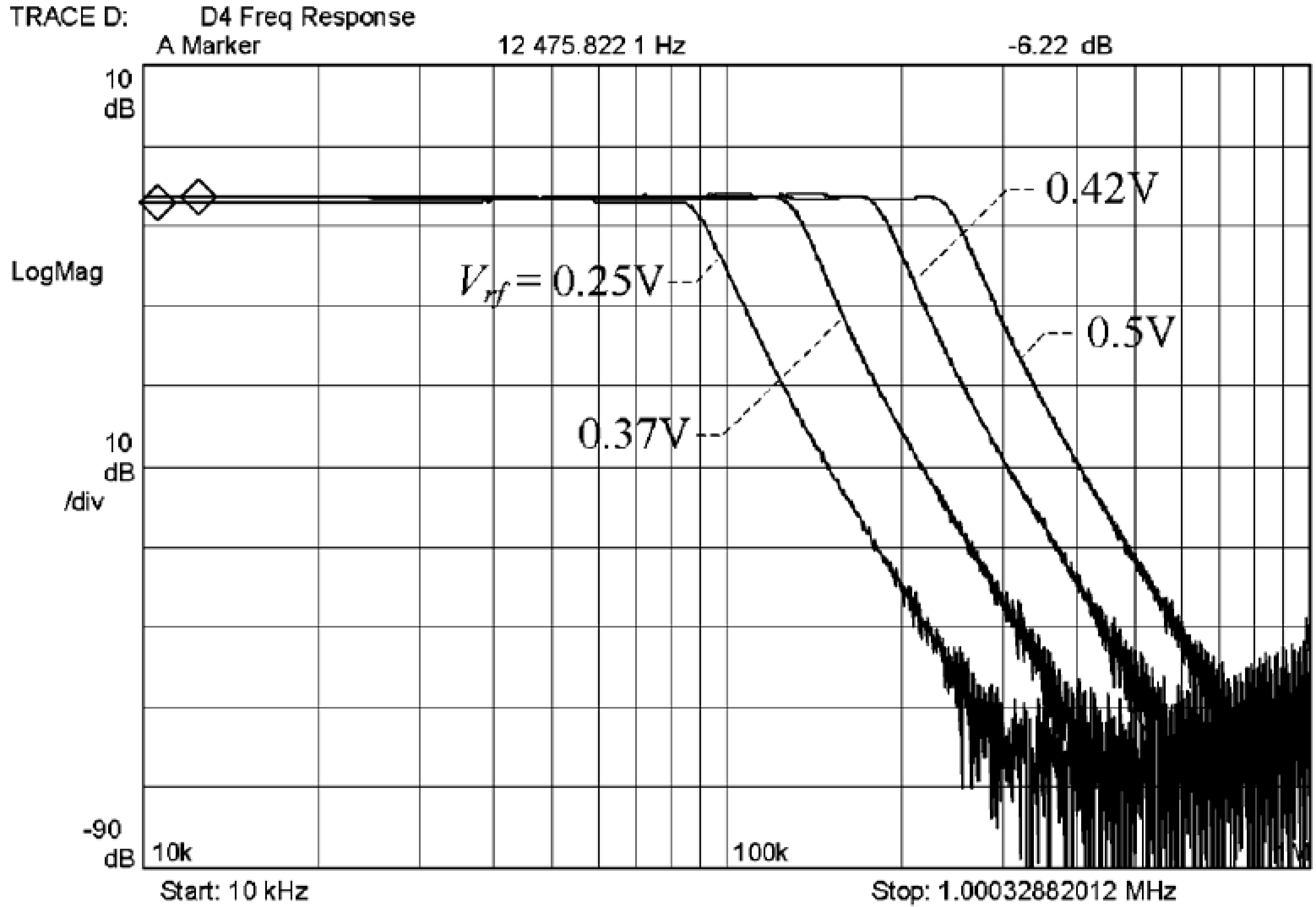


Fig. 11. Differential-to-single-ended frequency responses at different frequency tuning voltages V_{rf}.

TABLE IV

PERFORMANCE SUMMARY AND COMPARISON WITH OTHER VERY LOW-VOLTAGE FILTERS

	This work	[2]	[3]	[5]	[4]
Technology	0.18-μm	0.18-μm	0.18-μm	0.13-μm	0.18-μm
Threshold voltage, V_{th} with $V_b = V_s$	0.5 V	0.5 V	0.5 V	0.3 V	0.5 V
V_{dd}	0.5 V	0.5 V	0.6 V	0.55 V	0.6 V
V_{dd}/V_{th}	1.0	1.0	1.2	1.83	1.2
Filter type	5^{th}-order Chebyshev	5^{th}-order Elliptic	2^{nd}-order Butterworth	4^{th}-order Butterworth	5^{th}-order Chebyshev
Filter structure	Leapfrog, active-*RC*	Leapfrog, active-*RC*	Leapfrog, active-*RC*	Cascaded biquadratic, active-G_m-*RC*	Leapfrog, companding
Nominal –3dB bandwidth	135 kHz	135 kHz	135 kHz	11.3 MHz	100 kHz
DC gain[†]	–0.3 dB	–0.35 dB	0 dB	–1.4dB	–2 dB
Input[†] (1%-THD)	219.2 mV_{rms} (@100 kHz)	50 mV_{rms} (@100 kHz)	212.2 mV_{rms} (@ 2kHz)	70.7 mV_{rms} (@ 1MHz)	-
Input-referred noise[‡]	195.4 μV_{rms}	74 μV_{rms}	-	110 μV_{rms}	-
Dynamic range (1%-THD)	61.0 dB	56.6 dB	64 dB (SNR)	60 dB	89 dB
In-band IIP3	+4 dBV_{rms}	–3 dBV_{rms}	+17 dBV_{rms}	–3 dBV_{rms}	–76 dBA_{rms}
Out-of-band IIP3	+14 dBV_{rms}	+5 dBV_{rms}	-	0 dBV_{rms}	–86 dBA_{rms}
In-band spurious-free dynamic range (SFDR)	53.7 dB (f_{in} = 50, 55kHz) 54.0 dB (f_{in} = 90, 95kHz)	-	-	-	-
Total current consumption	1.2 mA	2.2 mA	-	5.8 mA[**]	-
Total power consumption	0.6 mW 0.4 mW[**]	1.1 mW	1 mW	3.5 mW[**]	0.443 mW
Bandwidth tuning range	91 – 268 kHz	88 – 154 kHz	67 – 203 kHz	-	-
PLL lock range	190 – 400 kHz	-	-	-	-
PLL tone feed-through[*]	11.4 μV_{rms} @ 263 kHz 54.1 μV_{rms} @ 526 kHz 23.7 μV_{rms} @ 789 kHz	85 μV_{rms} @ 280kHz	-	-	-
Chip area (size breakdown) - OTAs - Filter resistors and capacitors - Bias circuits - VCO + DC Enhancement	0.25×1.17 mm^2 (0.29 mm^2) 0.7×1.17 mm^2 (0.82 mm^2) 0.75×0.25 mm^2 (0.19 mm^2) 1.07×0.77 mm^2 + 0.2×0.67 mm^2 (0.96 mm^2)	0.33×0.7 mm^2 (0.23 mm^2) 0.55×0.7 mm^2 (0.38 mm^2) 0.13×1.0 mm^2 (0.13 mm^2) 0.87×0.3 mm^2 (0.26 mm^2)	0.5×1.4 mm^2 (0.70[δ] mm^2)	0.45[δ] mm^2	2.128[δ] mm^2
FOM	0.79 pJ 0.52 pJ[**]	2.41 pJ	2.34 pJ	0.0774 pJ[**]	0.0314 pJ

[†]: Differential [*]: Measured at *single-ended* output with no input signal applied

[‡]: Integrated over 1 kHz to 1 MHz [**]: No automatic-frequency tuning [δ]: Total chip area

A 0.47mW 6th-Order 20MHz Active Filter Using Highly Power-Efficient Opamp

Le Ye, Congyin Shi, Huailin Liao*, and Ru Huang
Institute of Microelectronics, Peking University

978-1-4244-9474-3/11/$26.00 ©2011 IEEE 1640

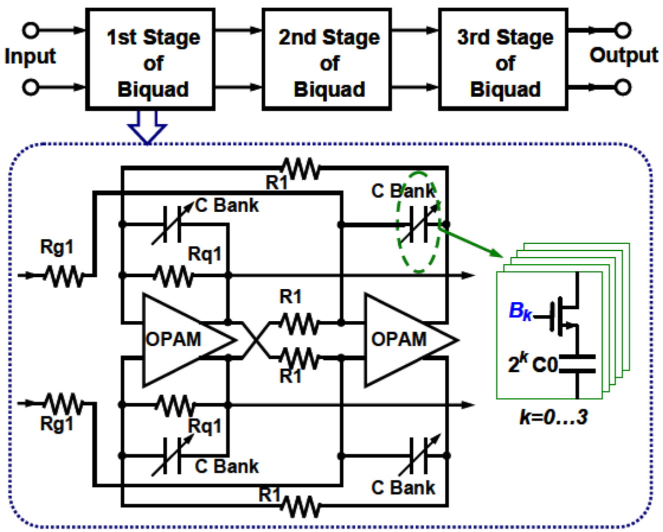


Figure. 5 The architecture of the proposed ultra-low power 6th-order active low-pass filte

Opamps. The tunable frequency bands are realized by switching the capacitor bank.

13

Authors claim Op Amp is highly power efficient

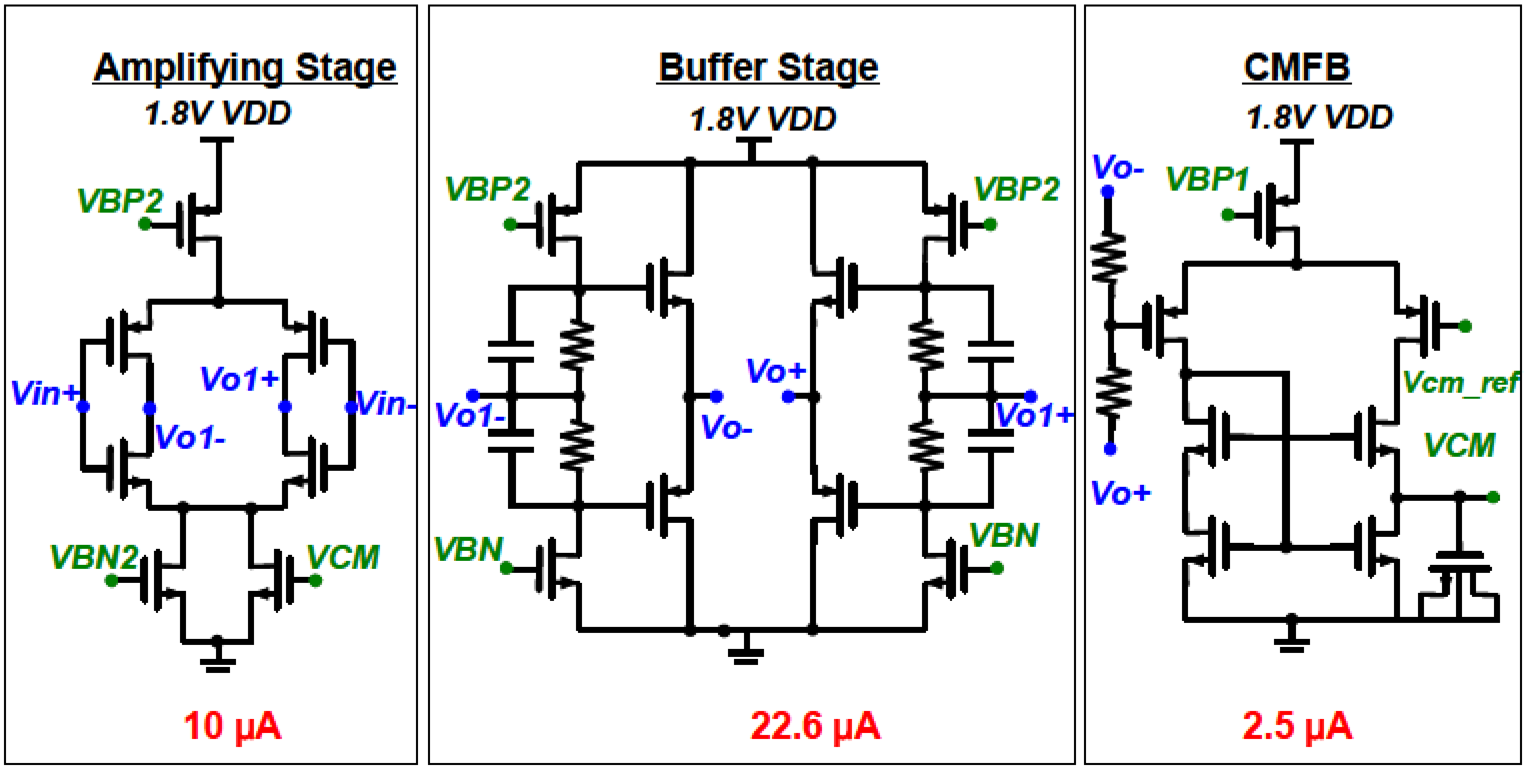


Figure. 4 The schematic of the proposed highly power-efficient Opamp

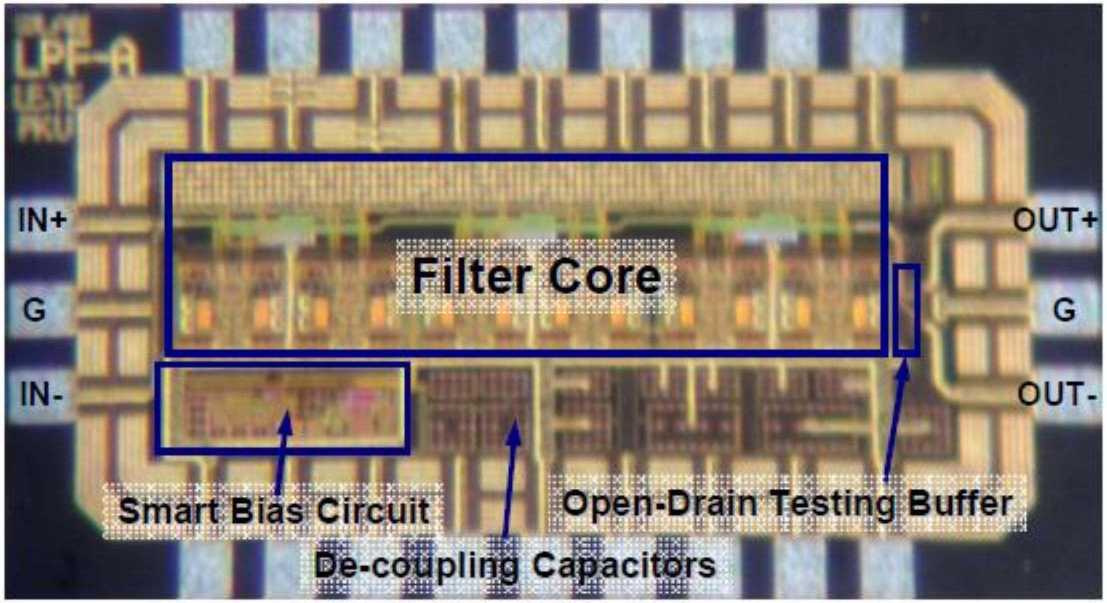


Figure. 6 The chip microphotograph.

Figure. 6 The chip microphotograph.

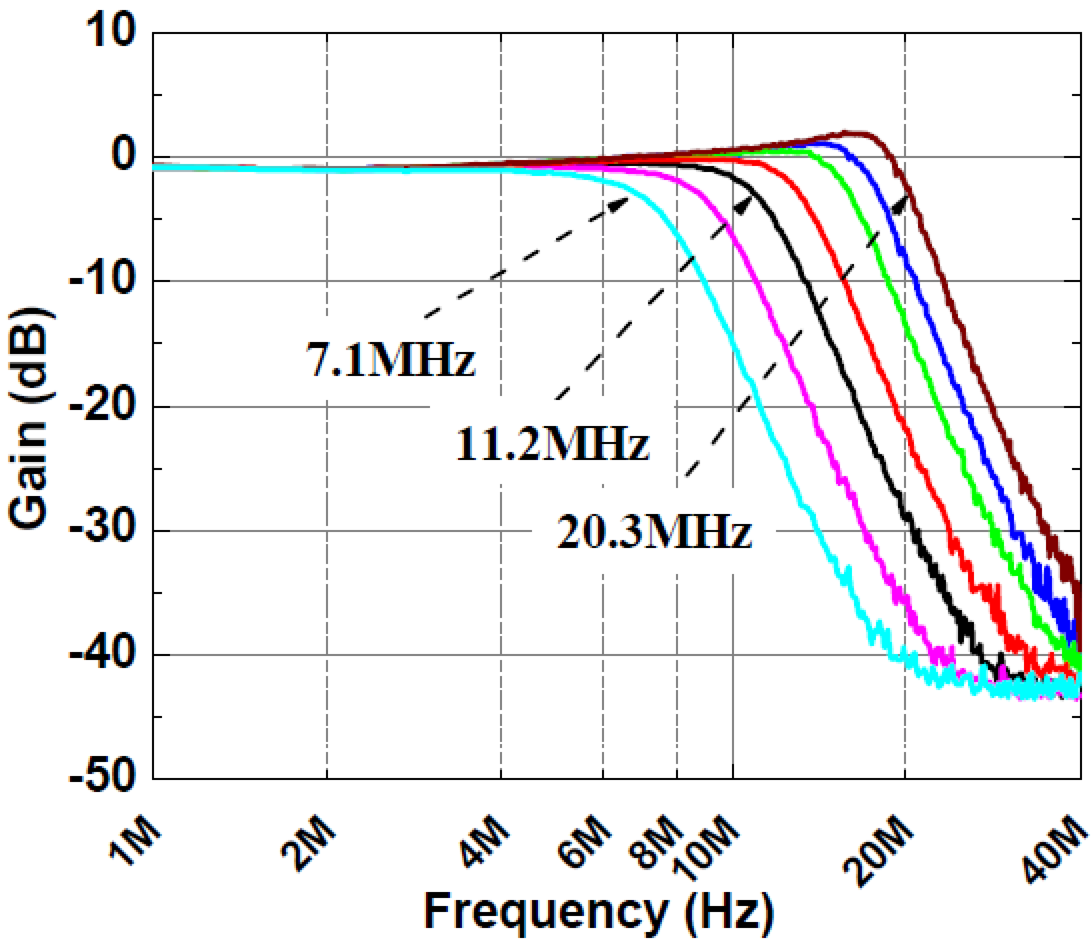


Figure. 7 The measured magnitude response of the filter with selected frequency tuning bands.

TABLE II. MEASURED PERFORMANCE SUMMARY

Technology	0.18 µm CMOS
Supply Voltage	1.8V ±10%
Filter Order	6
-3dB Cutoff Frequency	7.1 MHz ~ 20.3 MHz
DC voltage gain	-0.8 dB
Noise (100 kHz to 20 MHz)	298 µVrms
Noise Density	66.2 nV/√Hz
IIP3 @ 3 MHz, 5 MHz	20.9 dBm
Power Consumption	260 µA (0.47 mW)
Normalized Power	3.86 pW/Hz/pole
Silicon area without Pads	0.21 mm^2

TABLE I. PERFORMANCE COMPARISON OF THE PUBLISHED STATE-OF-THE-ART FILTERS

Ref	[3] RFIC 2005	[4] JSSC 2007	[5] ISCAS 2009	[6] JSSC 2006	[7] ISSCC 2006	This work
Technology	90nm CMOS	0.13 μm CMOS	65 nm CMOS	0.13μm CMOS	0.18 μm CMOS	**0.18 μm CMOS**
VDD	1.4 V	1.5 V	1.3 V	1.2 V	1.8 V	**1.8 V**
Topology	Gm-C	Active-RC	Active-RC	Active-Gm-RC	Source-follow-based	**Active-RC**
Filter Order	6	5	5	4	4	**6**
BW	10/100 MHz	19.7 MHz	9 MHz	11 MHz	10 MHz	**7.1 to 20.3MHz**
Noise Density	19 nV/√Hz	30 nV/√Hz	6.5 pA/√Hz	10.9 nV/√Hz (1)	7.5 nV/√Hz	**66.2 nV/√Hz**
In-band OIP3 (2)	15.5 dBm (3)	17.3 dBm (4)	--	21 dBm	13.5 dBm (5)	**20.1 dBm**
Filter Area	0.55 mm^2	0.20 mm^2	0.46 mm^2	0.9 mm^2	0.26 mm^2	**0.21 mm^2**
Power	13.5 mW	11.3 mW	12.4 mW	14.2 mW	4.1 mW	**0.47 mW**
Normalized Power	**22.5** pW/Hz/pole (6)	**115** pW/Hz/pole	**276** pW/Hz/pole	**323** pW/Hz/pole	**103** pW/Hz/pole	**3.86 pW/Hz/pole**
FOM	**440.6** (7)	**171.7**	--	**304.1**	**615.5**	2779

(1) Calculated from the reported 36 μVrms integrated input-referred noise; (2) For fear comparison, take the voltage gain into account; (3) calculated from the

Wide-Loading-Range Fully Integrated LDR With a Power-Supply Ripple Injection Filter

Edward N. Y. Ho and Philip K. T. Mok, *Senior Member, IEEE*

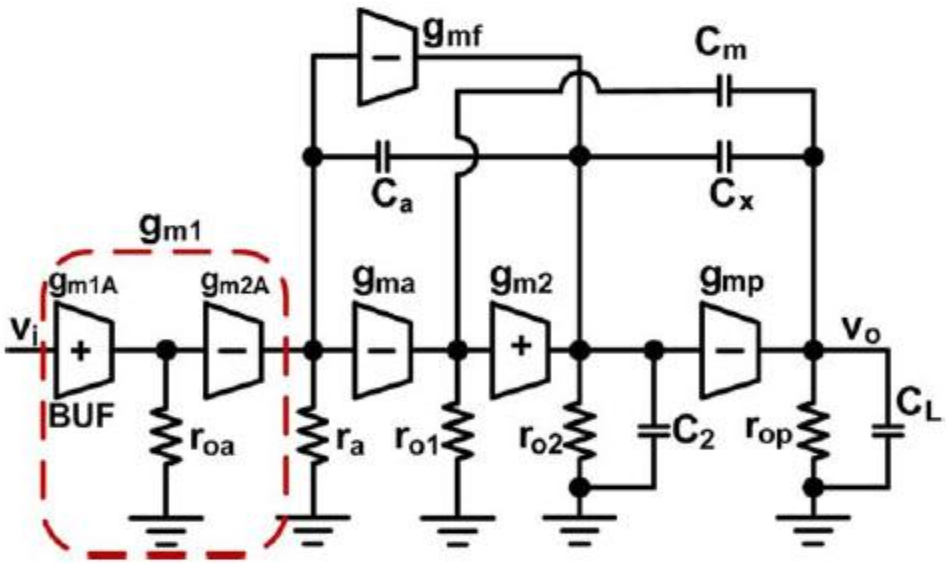


Fig. 3. Small-signal model of the proposed design.

A 30-MHz–2.4-GHz CMOS Receiver With Integrated RF Filter and Dynamic-Range-Scalable Energy Detector for Cognitive Radio Systems

Masaki Kitsunezuka, Hiroshi Kodama, Naoki Oshima, Kazuaki Kunihiro, Tadashi Maeda, *Member, IEEE*, and Muneo Fukaishi, *Member, IEEE*

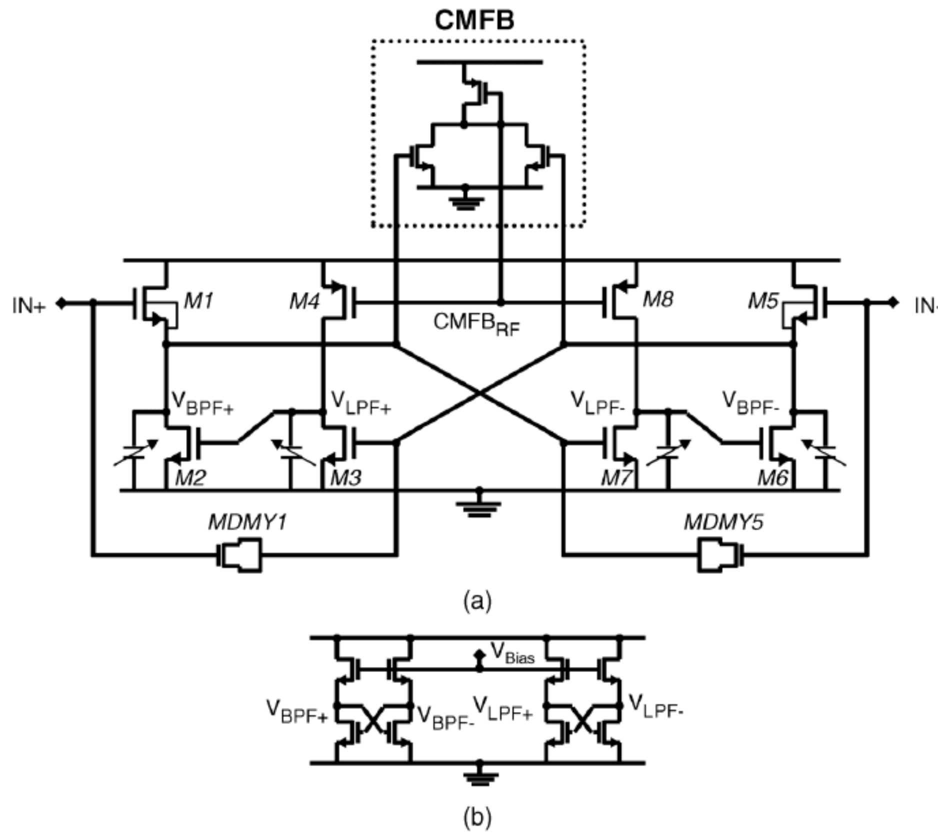


Fig. 4. (a) Circuit diagram of differential RF filter and (b) common-mode rejection/stabilization circuit.

To enable a wide tuning range (30 to 800 MHz) to be covered, 4-bit binary-weighted load-capacitor arrays are used to vary the filter characteristics. In addition, transconductance gain g_m is controlled by tuning the gate bias voltage of M1. Fig. 5

The transfer functions of the BPF and LPF are expressed as

$$H_{\mathrm{BPF}}(s) = \frac{V_{\mathrm{BPF}}}{V_{\mathrm{in}}} = \frac{s\frac{g_{m1}}{C_1}}{s^2 + s\frac{g_{m1}}{C_1} + \frac{g_{m2}g_{m3}}{C_1C_2}} \tag{1}$$

and

$$H_{\mathrm{LPF}}(s) = \frac{V_{\mathrm{LPF}}}{V_{\mathrm{in}}} = \frac{\frac{g_{m1}g_{m3}}{C_1C_2}}{s^2 + s\frac{g_{m1}}{C_1} + \frac{g_{m2}g_{m3}}{C_1C_2}}, \tag{2}$$

respectively, where g_{mi} is the transconductance gain of transistor M_i. The cut-off (or center) frequency and quality factor are respectively written as

$$\omega_c = \sqrt{\frac{g_{m2}g_{m3}}{C_1C_2}} \tag{3}$$

and

$$Q = \frac{\sqrt{g_{m2}g_{m3}}}{g_{m1}}\sqrt{\frac{C_1}{C_2}}. \tag{4}$$

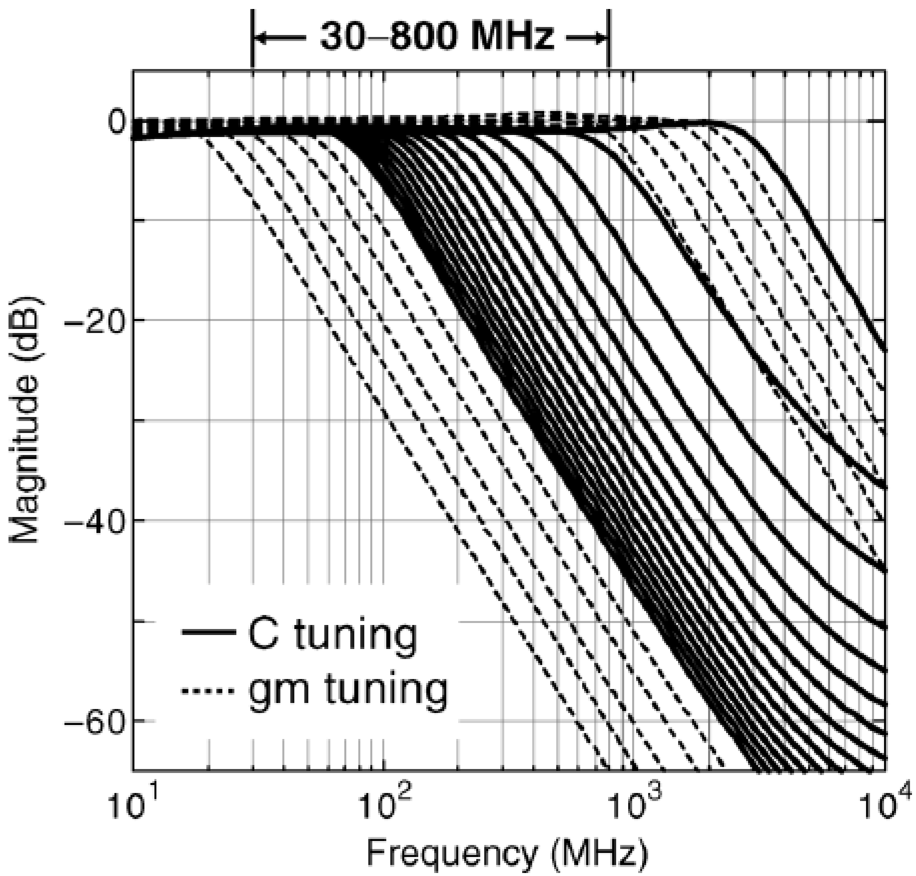
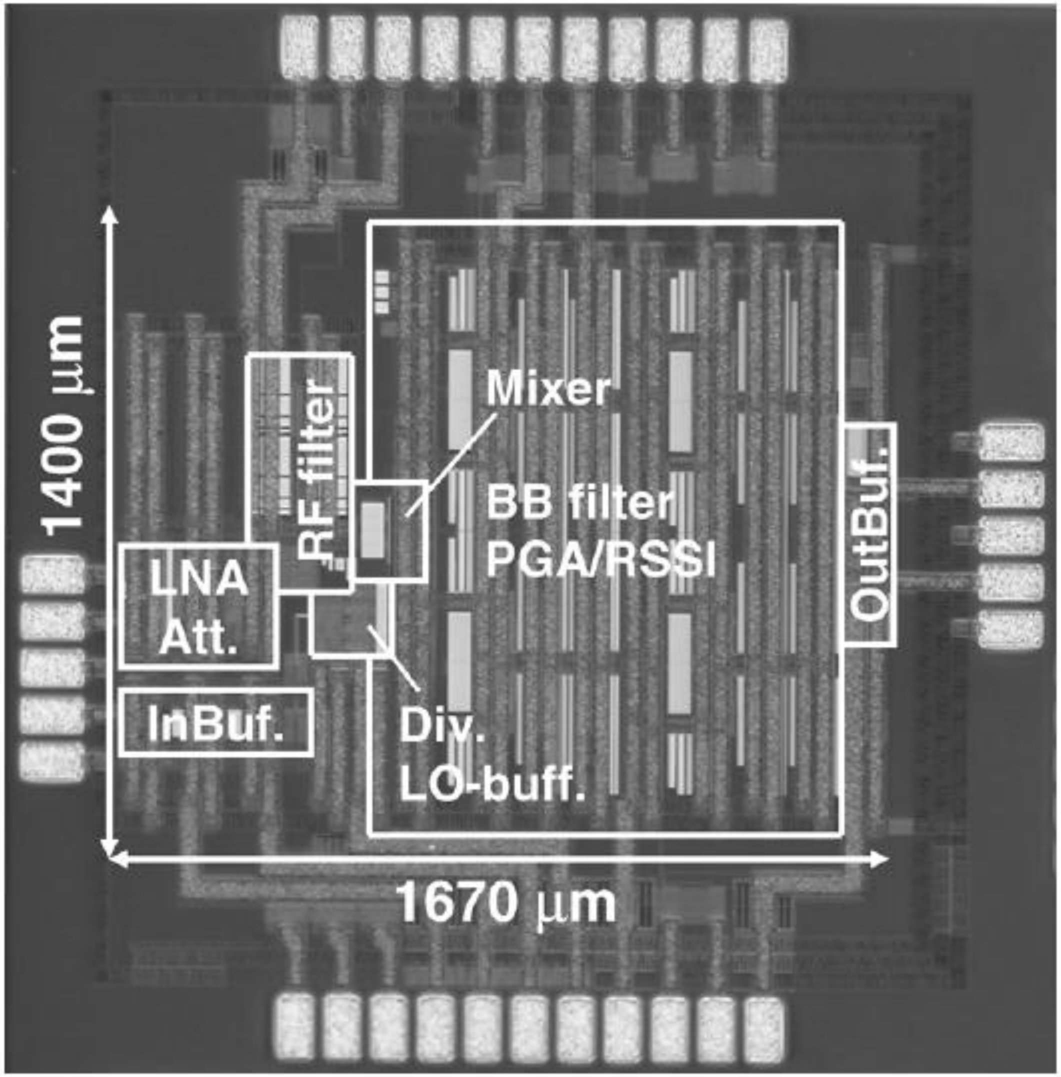


Fig. 5. Simulated frequency characteristics of RF filter.



1400 μm
RF filter
Mixer
BB filter
PGA/RSSI
OutBuf.
LNA
Att.
InBuf.
Div.
LO-buff.
1670 μm

A 4th-Order 84dB-DR CMOS-90nm Low-Pass Filter for WLAN Receivers

M. De Matteis[1], A. Pezzotta[2], A. Baschirotto[2]
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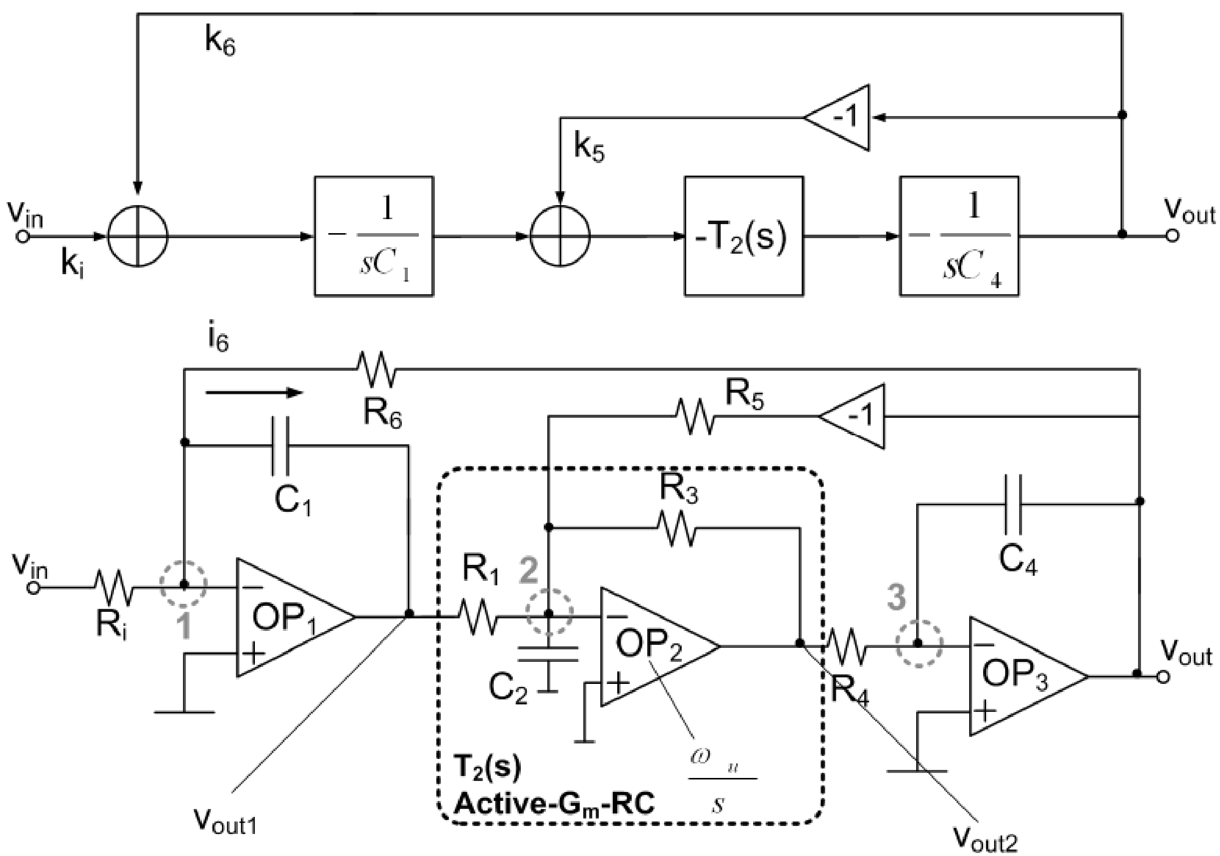

Fig. 1 - WLAN 4th Order Filter Block/Circuital Scheme

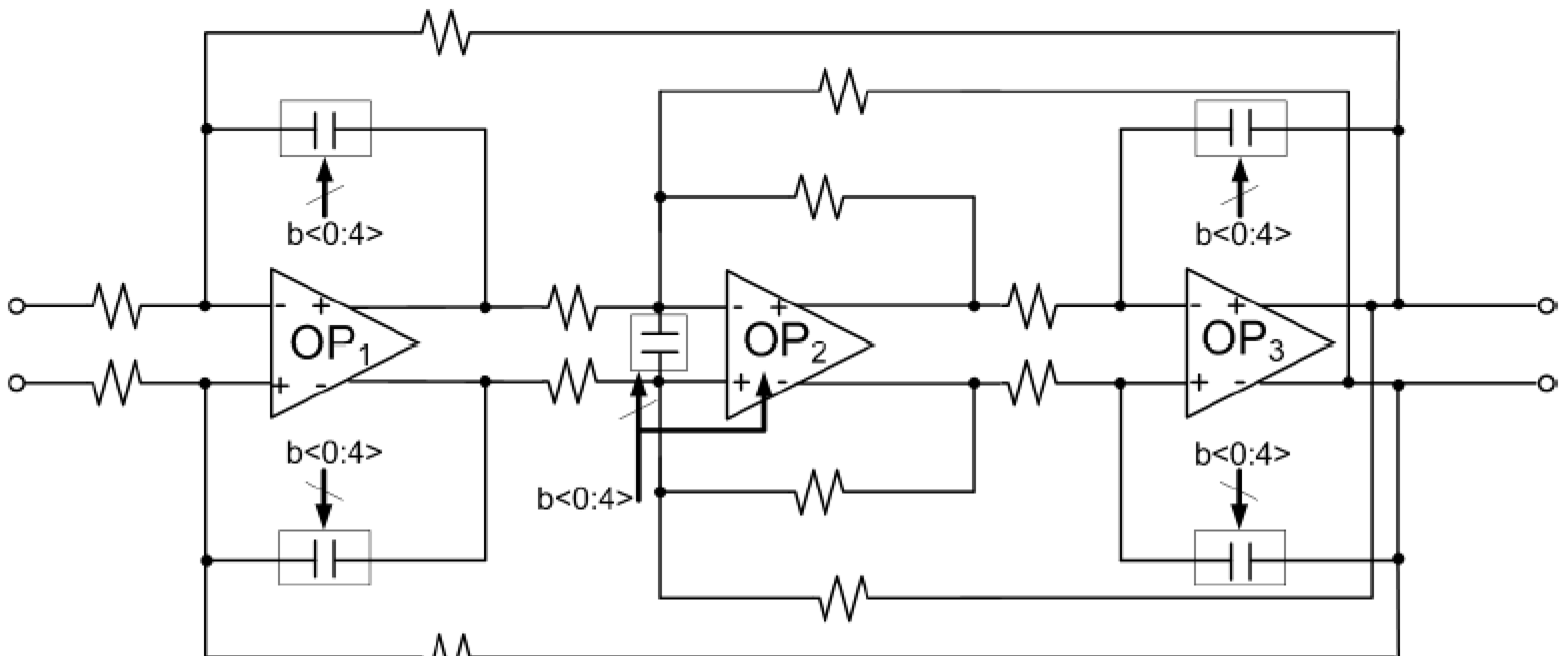

Fig. 5 – 4th Order WLAN Filter – Top-View Schematic

The uncertainty on the passive components nominal value due to the fabrication process and temperature variations can reach the 45%, affecting the filter frequency response. Any process variation and temperature dependencies can be compensated by tuning either the R's or the C's values. The

in the Fig. 6. The array value is, then, set by the external using a proper digital code.

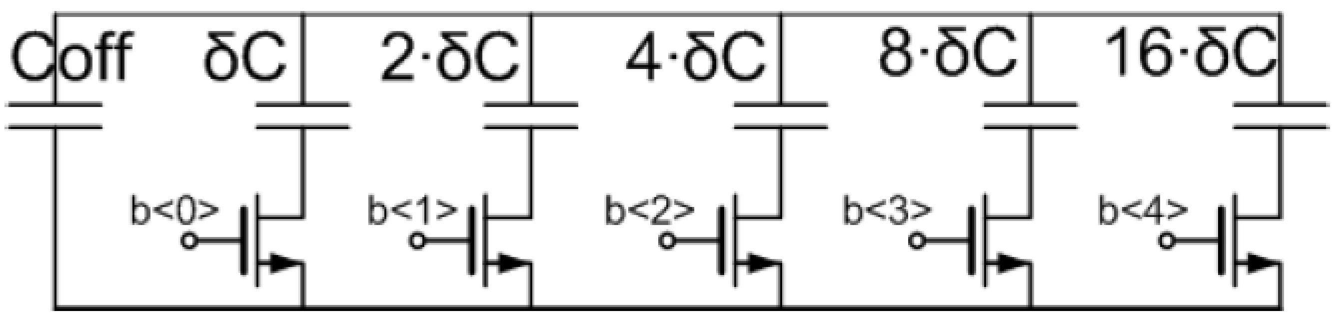

Fig. 6 – Capacitor Array

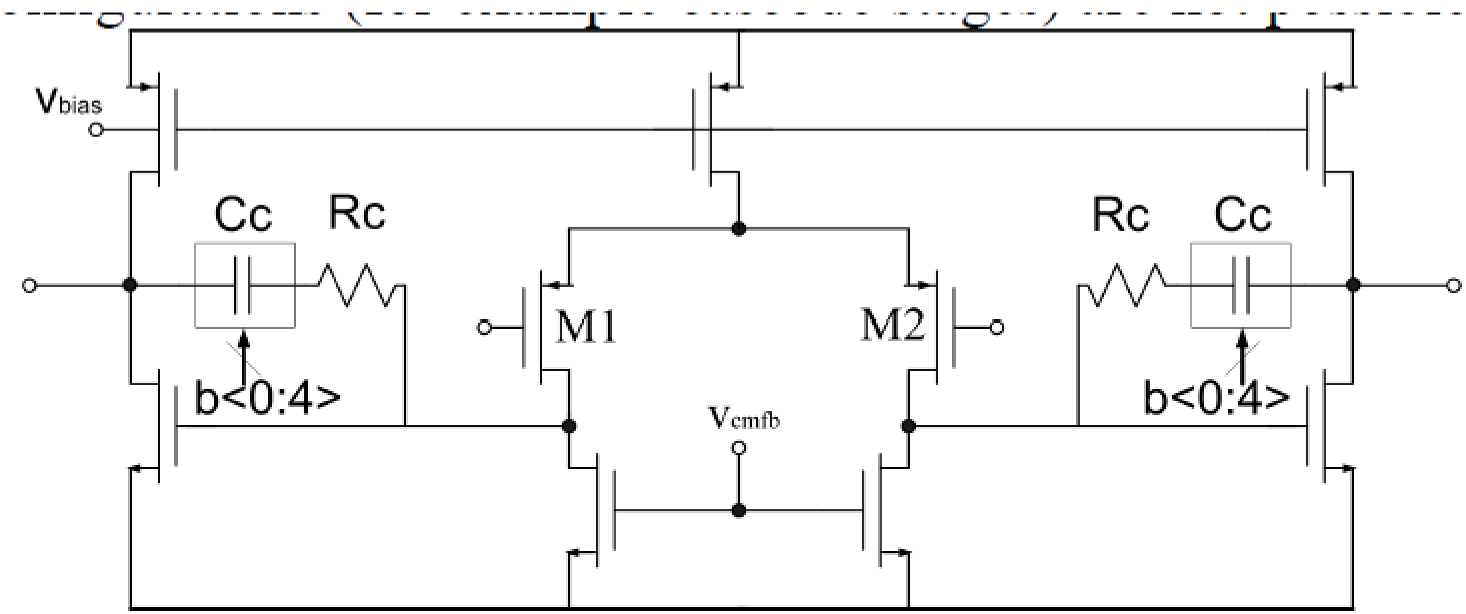

Fig. 7 – Opamp Schematic

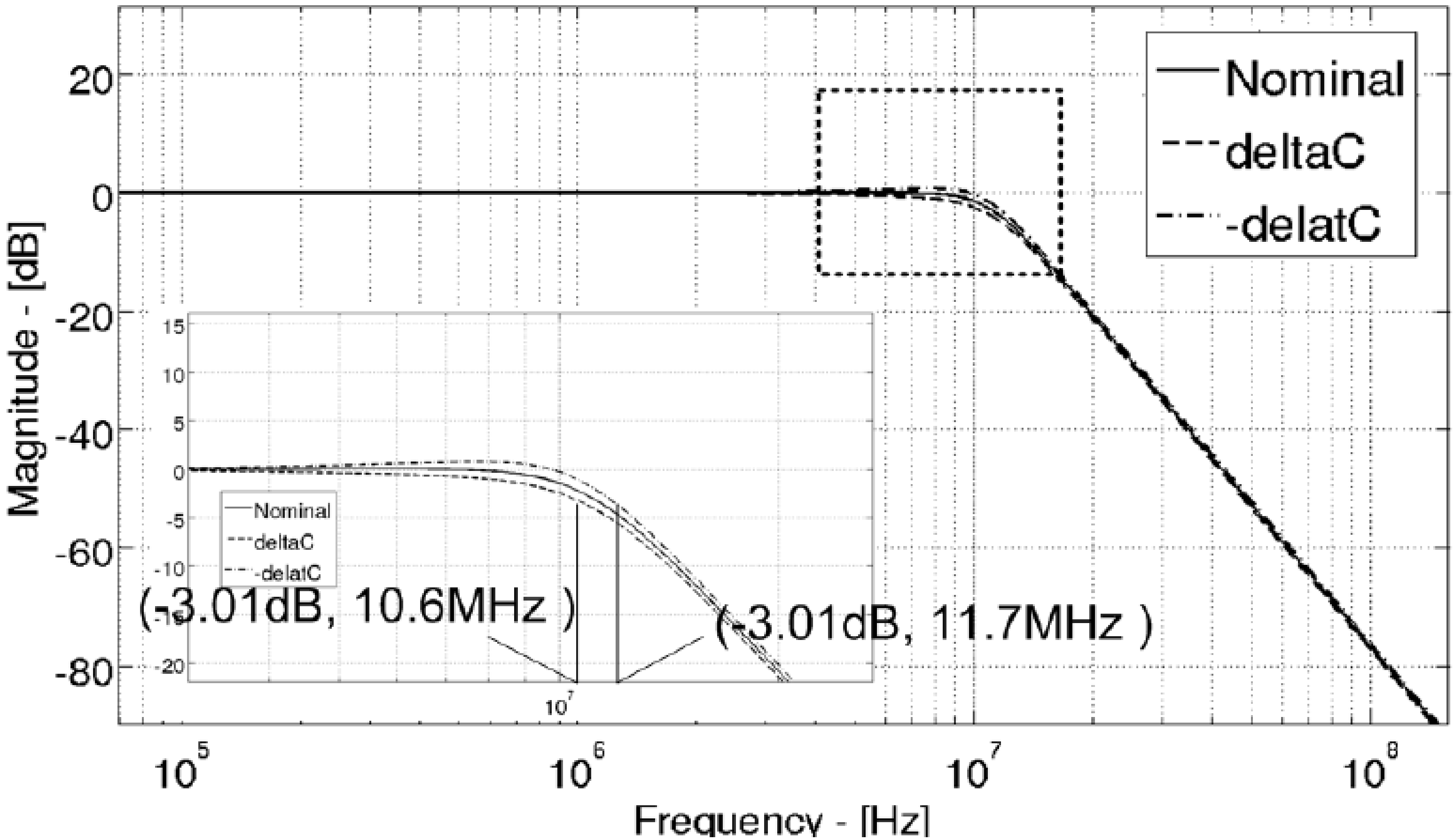

Fig. 8 – Filter Frequency Response

Parameter	**Value**
G[dB]	0
$f_{@-3dB}$[MHz]	11
avdd[V]	1.2
CMOS Technology	90nm
Power Consumption[mW]	14
Output Integrated Noise[μV_{rms}] - (100kHz÷20MHz)	48
IRN Spectral Density@2MHz [$nV/\sqrt{Hz}$]	5
THD[dBc] – v_{out}=1.05$V_{zero-peak}$@4MHz	40
DR@THD=40dBc - [dB]	84
IIP3 [dBm] - $v_{in}=v_{in1}+v_{in2}$ - v_{in1}@4MHz, v_{in2}@5MHz	10

Tab. IV – Filter Performance Resume

A 23.4 mW 68 dB Dynamic Range Low Band CMOS Hybrid Tracking Filter for ATSC Digital TV Tuner Adopting RC and Gm-C Topology

Kuduck Kwon, *Member, IEEE*, and Kwyro Lee, *Senior Member, IEEE*

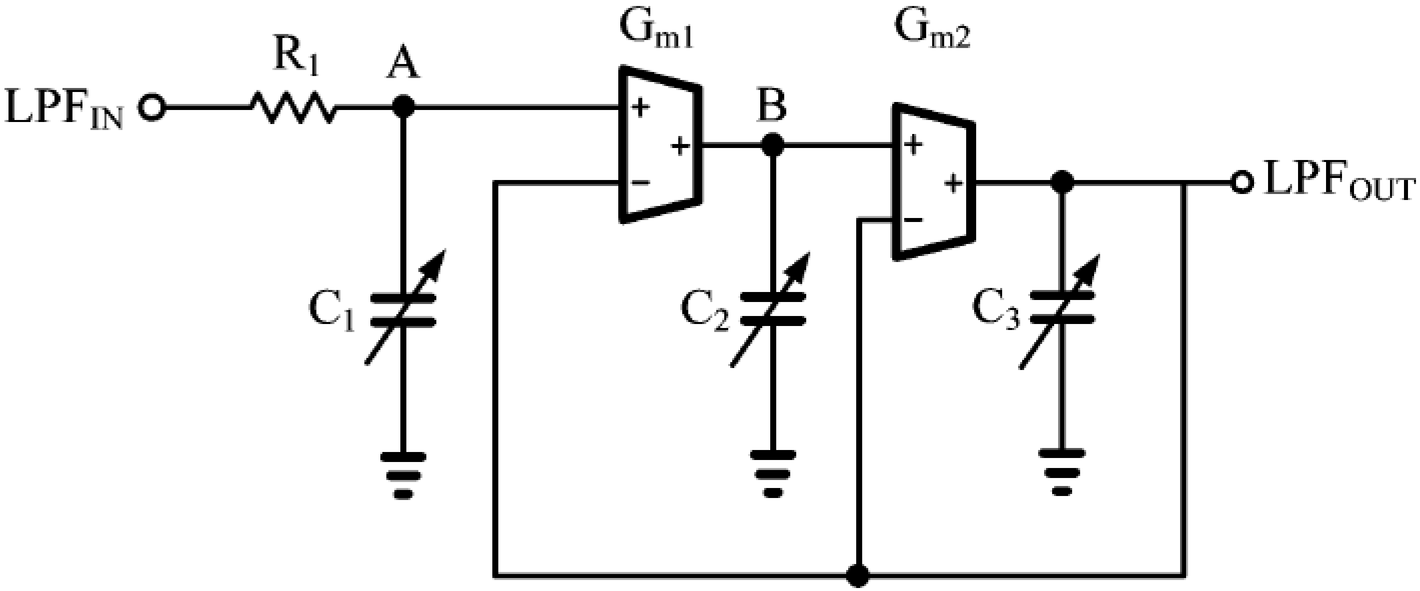

Fig. 3. Proposed third-order Chebyshev hybrid tracking low-pass filter.

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A 23.4 mW 68 dB Dynamic Range Low Band CMOS Hybrid Tracking Filter for ATSC Digital TV Tuner Adopting RC and Gm-C Topology

Kuduck Kwon, *Member, IEEE*, and Kwyro Lee, *Senior Member, IEEE*

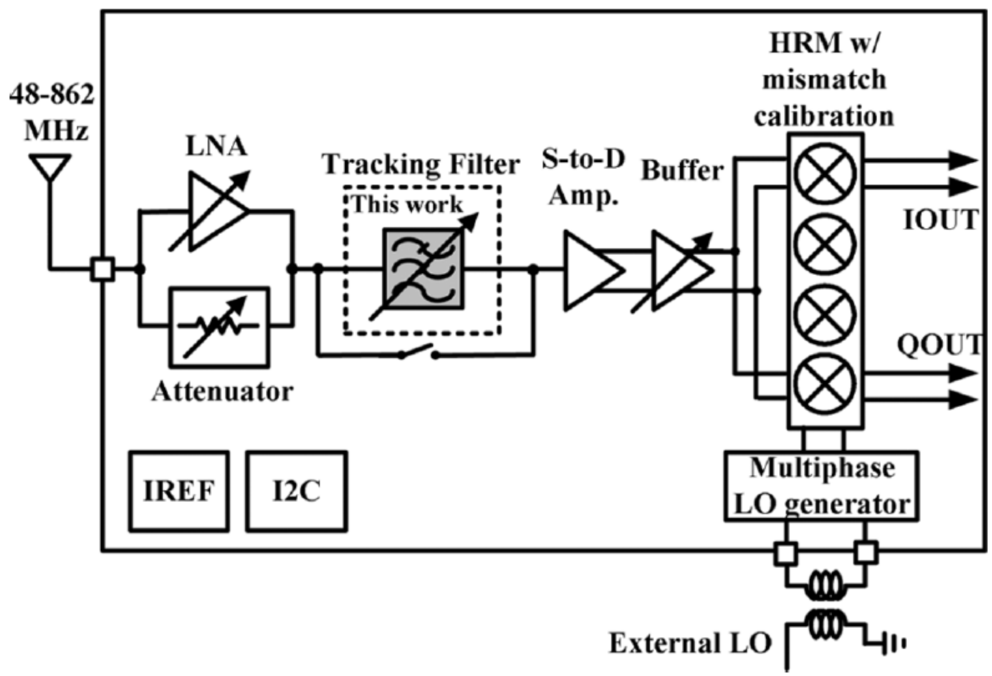

Fig. 1. Block diagram of RF front-end for ATSC terrestrial digital TV tuner.

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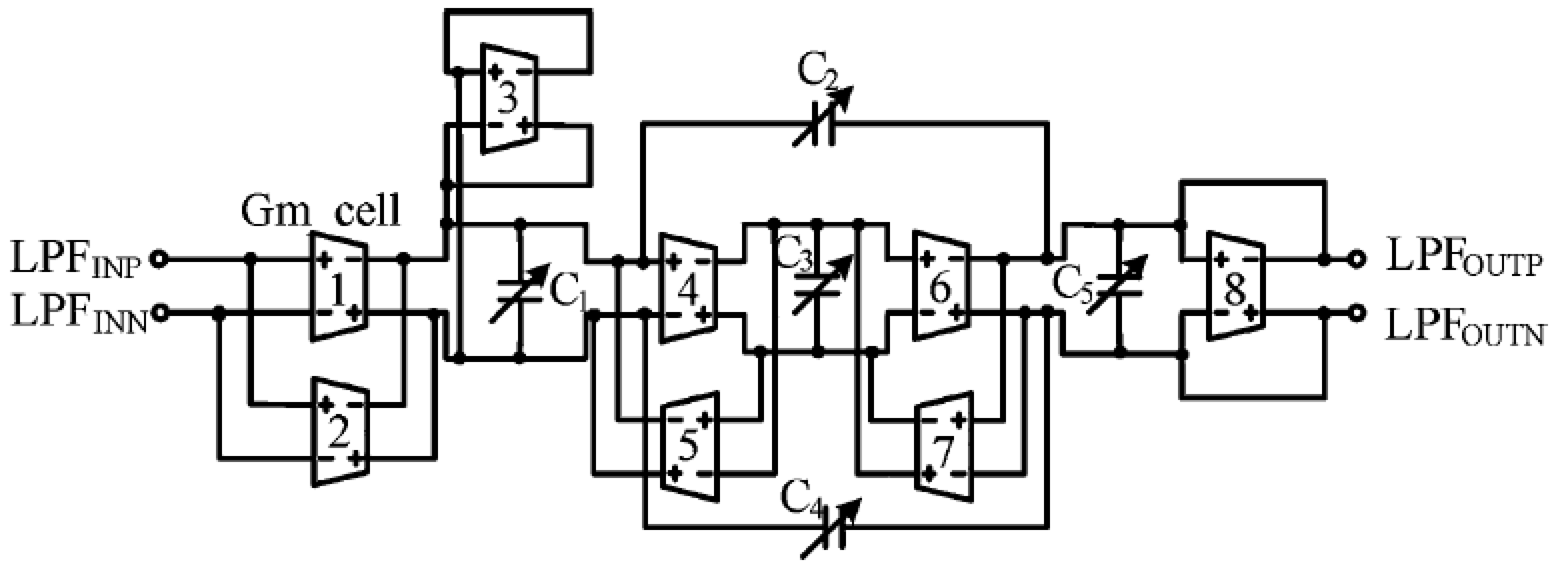


Fig. 2. Third-order elliptic transconductor-C low-pass filter based on ladder filter structure.

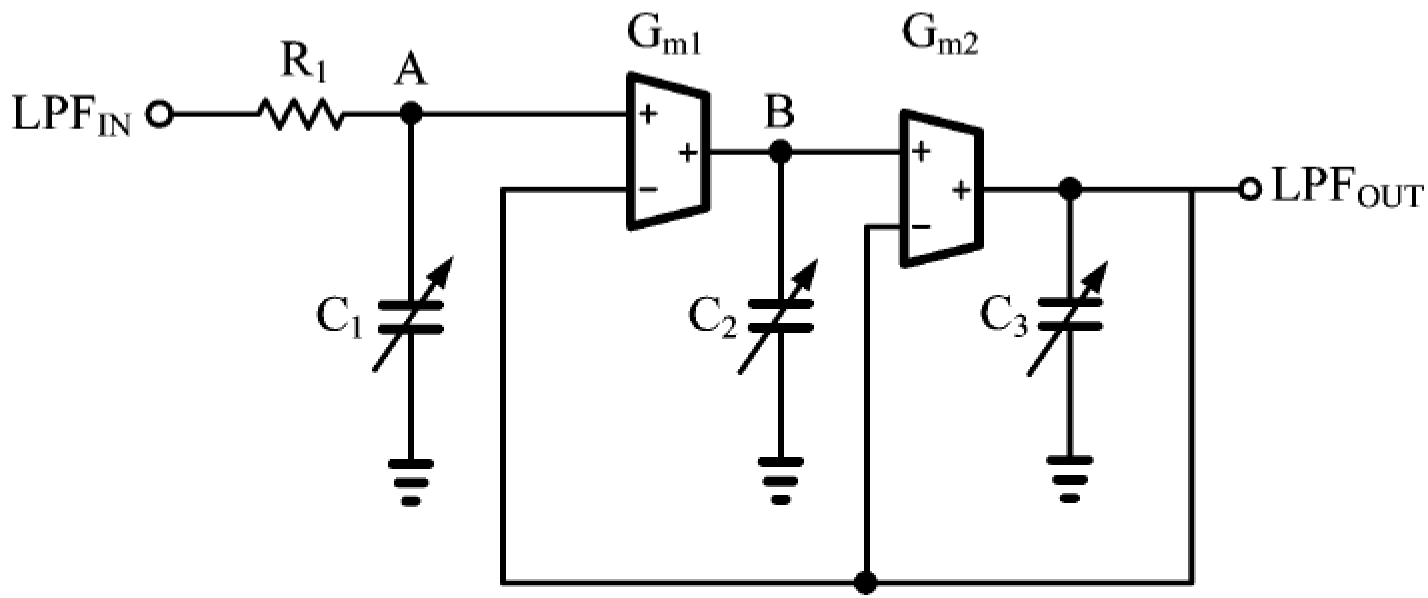


Fig. 3. Proposed third-order Chebyshev hybrid tracking low-pass filter.

COMPONENTS VALUES AND TRANSISTOR SIZES

Components	R_1	C_1*	C_2*	C_3*	G_{m1}	G_{m2}
Values	200 [Ω]	5 [pF]	19 [pF]	36 [pF]	16 [mS]	16 [mS]
Transistor	MT		ST		LT	
Sizes [W/L]	108µm /0.18µm		22µm /0.18µm		186µm /0.35µm	

*C1, C2, and C3 in Table II are values for 100MHz of -1dB cut-off frequency.

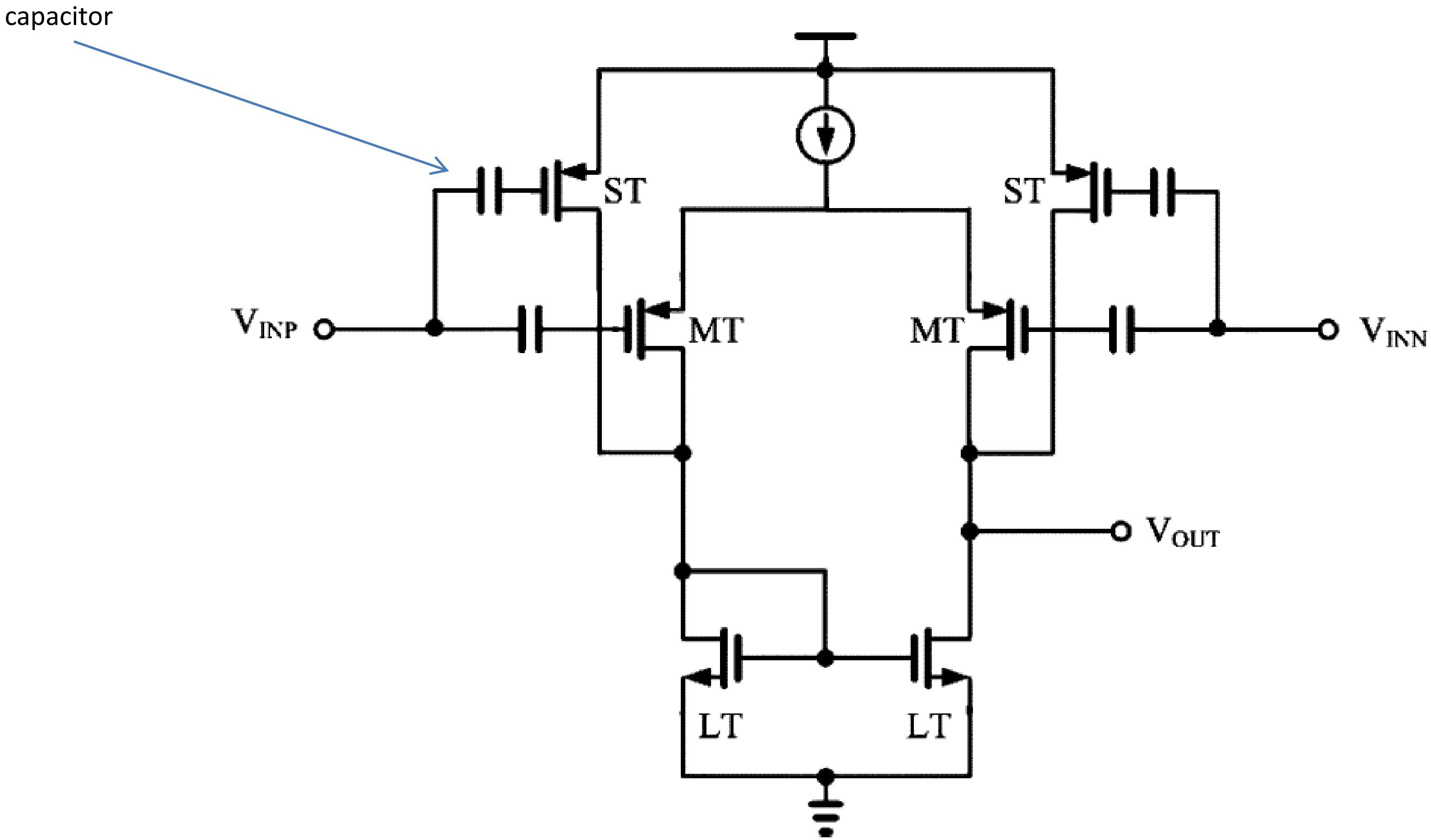
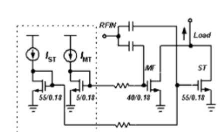
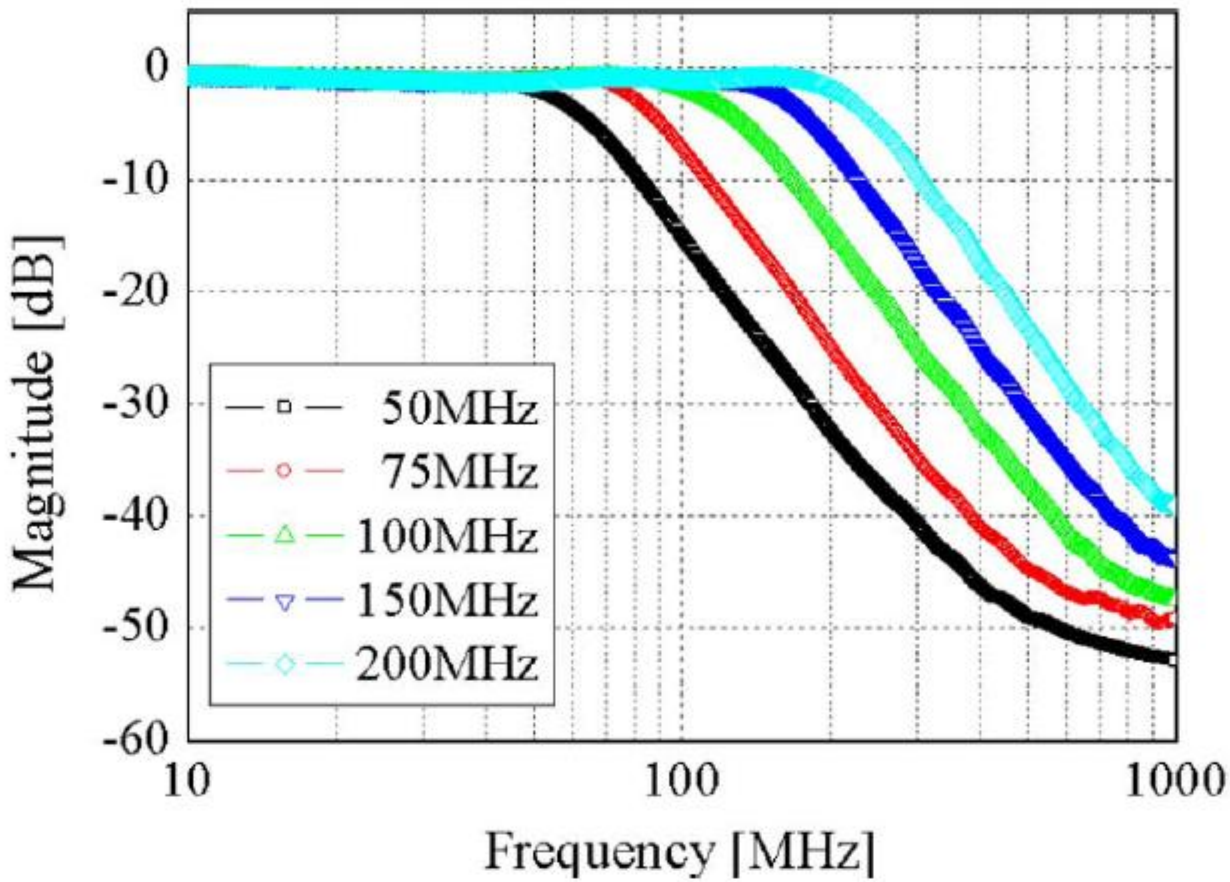


Fig. 4. Transconductor.

ST transistors operating in weak inversion provide linearization of basic transconductance stage. Transistor biasing not shown but described in [11]

For the robust characteristics to variations of process, supply voltage, and temperature, the MT and ST are biased with current-mirror bias circuitry [11].





0
-10
-20
-30
-40
-50
-60
10
100
1000
Magnitude [dB]
Frequency [MHz]
50MHz
75MHz
100MHz
150MHz
200MHz

TABLE IV
COMPARISON WITH OTHER REPORTED FILTERS

	[1]	[18]	[19]*	[25]	[26]	[27]	[28]	This work
Technology	0.18μm CMOS	0.13μm CMOS	0.18μm CMOS	0.25μm CMOS	0.25μm CMOS	0.35μm CMOS	0.13μm CMOS	0.18μm CMOS
Cut-off freq.(*f*) [MHz]	50-300	50-300	50-300	80-200	30-120	200	200	50-200
OIP3 [dBm]	14	11	16.9	18.8	10.1	17	14	17.3
NF [dB]	17	20	14	-	-	-	32	15
Filter order (N)	4	4	3	7	8	7	2	3
Single (S) or Differential (D)	D	D	D	D	D	D	D	S
Pdc [mW]	72	7.6	72	210	120	60	20.8	23.4
FOM (FOM2**) (FOM3***)	51 (2511) (51)	120 (11926) (120)	152 (3673) (152)	- (1264) -	- (327) -	- (1170) -	0.3 (483) (0.3)	193 (5901) (96.5)

*[19] is our previous work of the tracking filter utilized for both ATSC terrestrial and Cable digital TV standards
**FOM2 is the equation excluding the (F-1) term in the FOM of equation (14) because [25]-[27] do not indicate the noise performance

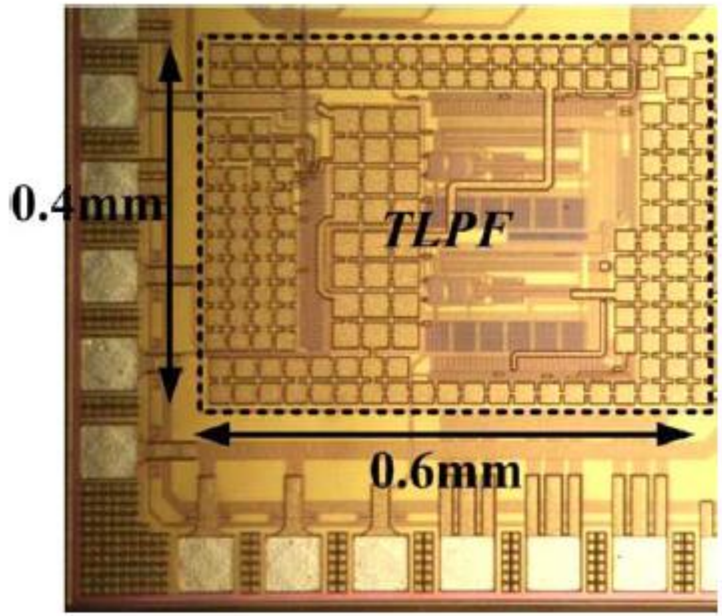
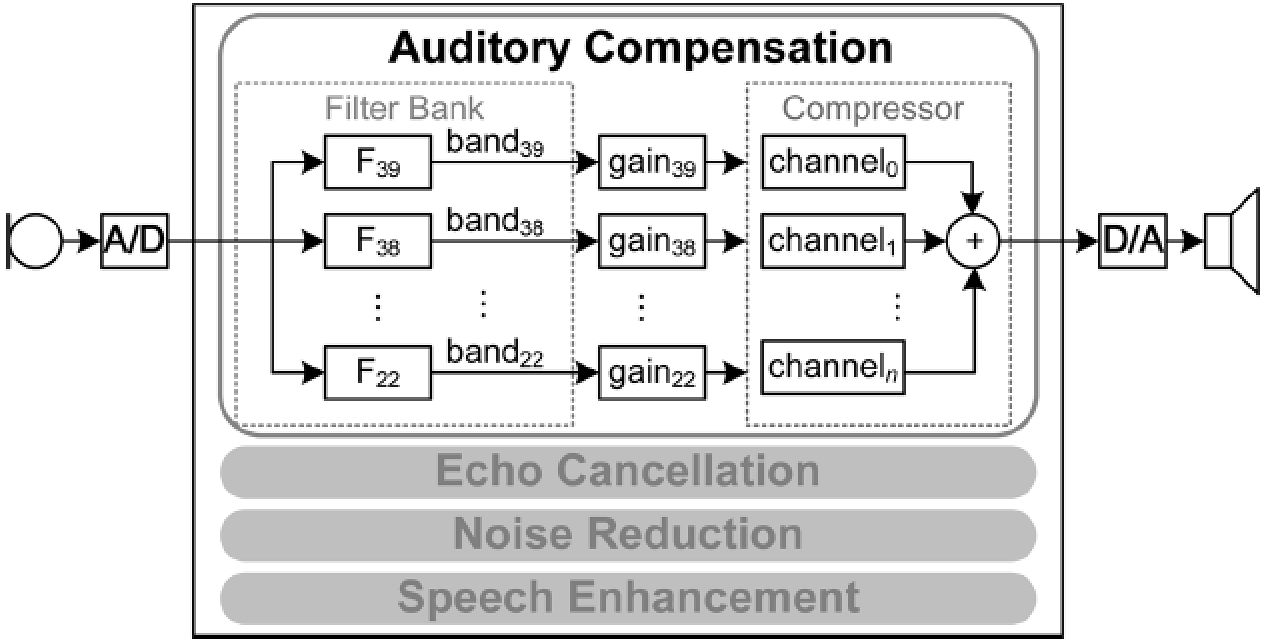


Fig. 7. Microphotograph of the hybrid tracking low-pass filter.

Design and Implementation of Low-Power ANSI S1.11 Filter Bank for Digital Hearing Aids

Yu-Ting Kuo, Tay-Jyi Lin, *Member, IEEE*, Yueh-Tai Li, and Chih-Wei Liu



ck diagram of the advanced hearing aid.

18

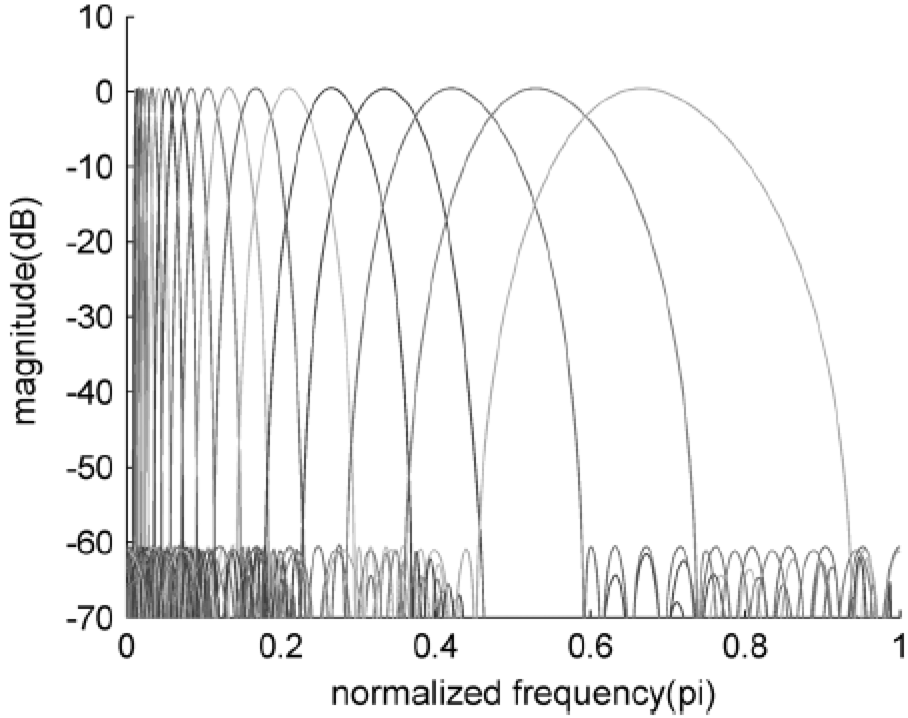
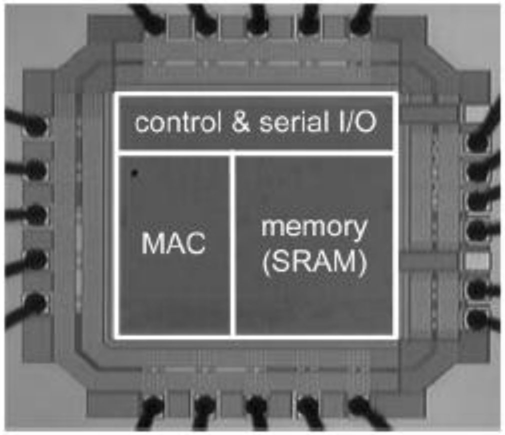


Fig. 14. Magnitude response of the proposed 18-band 1/3-octave filter bank.



Sub-modules	Gate count
MAC	2,847
memory	5,594
system controller	1,010
memory controller	301
serial I/O	1,103

This paper addresses the low-power filter bank design for advanced digital hearing aids. In the literature, the standard ANSI S1.11 1/3-octave filter bank is rarely adopted in hearing aids due to high computation complexity even though it has the advantage of well matching the human hearing characteristics. We develop an efficient multirate filter bank algorithm to implement an 18-band ANSI S1.11 1/3-octave FIR filter bank. The proposed architecture needs only 4% of multiplications and additions of a straightforward parallel FIR filter bank design. We also investigate and apply several lower-power

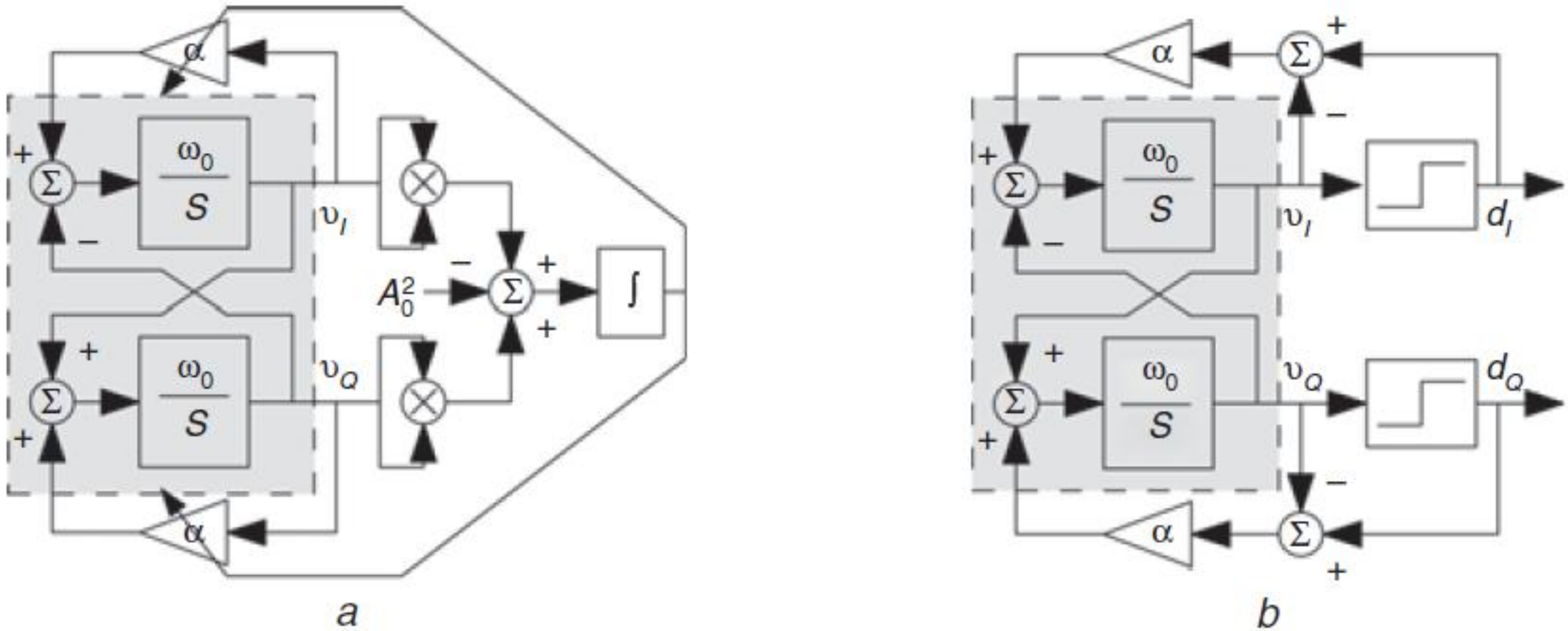
The test chip consumes only 87 μW,

Most existing ANSI S1.11 filter banks are implemented by infinite-impulse response (IIR) filters [8], [9]. Indeed, the researches in psychoacoustics had shown that human ear is not sensitive to phase-distortion. The filter bank with IIR filters may be a good design with low computation complexity; however, FIR filters are still preferred and adopted in [3]–[7], not only for their linear phase but also for the stability and regular structure. The round-off error of FIR filters is easier to analyze and

Simple quadrature oscillator for BIST

J. Raman, P. Rombouts and L. Weyten

ELECTRONICS LETTERS 18th February 2010 Vol. 46 No. 4



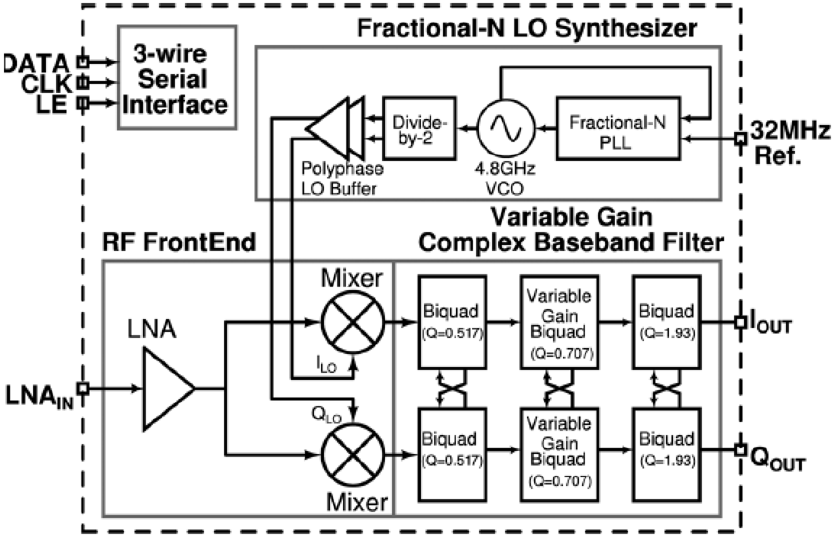
Two integrator loop oscillators – different methods for controlling the loss

EE 508
Lecture 42

What filter architectures are really being used today?

A 0.6-V Zero-IF/Low-IF Receiver With Integrated Fractional-N Synthesizer for 2.4-GHz ISM-Band Applications

Ajay Balankutty, *Student Member, IEEE*, Shih-An Yu, *Student Member, IEEE*, Yiping Feng, *Student Member, IEEE*, and Peter R. Kinget, *Senior Member, IEEE*



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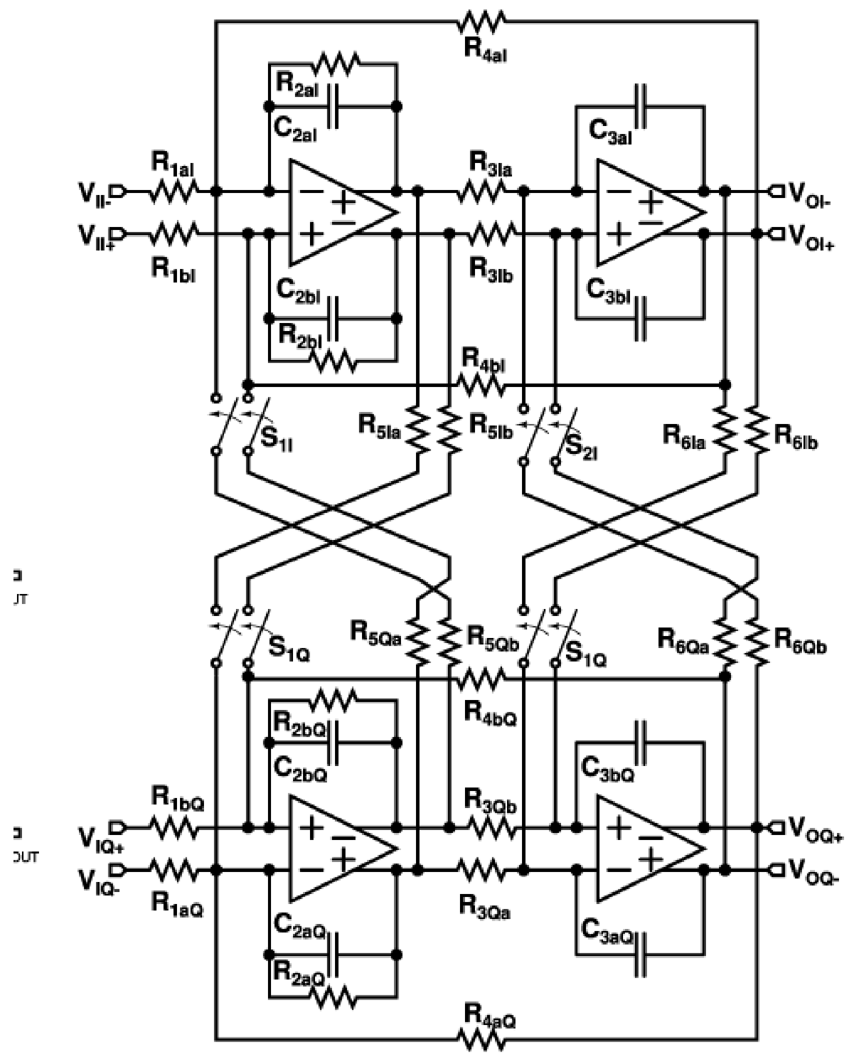
filter is only 2. To reduce the number of OTAs required for implementing the filter, a lower-order filter transfer function would be preferred. However, biquad-based filters have the drawback that the filter characteristics are more easily affected by parasitics and OTA non-idealities and this sensitivity of the filter characteristics typically is a function of the Q of the filter poles [24]. To make the biquad more tolerant to parasitics and OTA non-idealities, a Tow–Thomas implementation for the biquad is used [25]. To further reduce the sensitivity to parasitics, low Q biquads are preferred. The 6th-order Butterworth filter is chosen for the channel select filter and is implemented as a cascade of three biquads with pole Qs of 0.515, 0.707 and 1.93. The ordering of the biquads has a significant impact on the baseband performance and is discussed in the next section. Even though

System requirements appear to not have played a role in defining the filter type


R4aI
R2aI
C2aI
C3aI
R1aI
R3Ia
VII-
VOI-
VII+
VOI+
R1bI
R3Ib
C2bI
C3bI
R2bI
R4bI
S1I
R5Ia
R5Ib
S2I
R6Ia
R6Ib
S1Q
R5Qa
R5Qb
S1Q
R6Qa
R6Qb
R4bQ
R2bQ
C2bQ
C3bQ
R1bQ
R3Qb
VIQ+
VOQ+
VIQ-
VOQ-
R1aQ
R3Qa
C2aQ
C3aQ
R2aQ
R4aQ

(b)

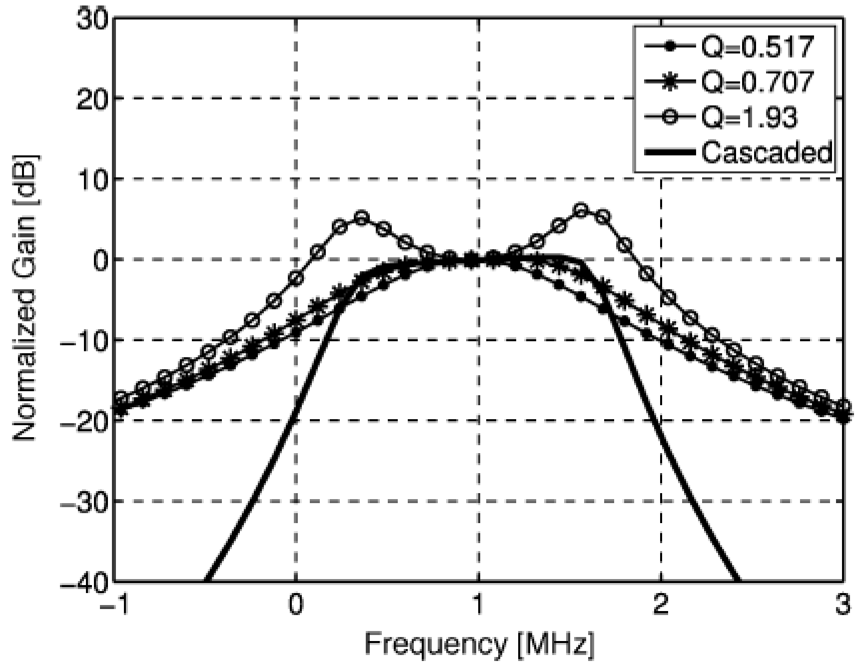
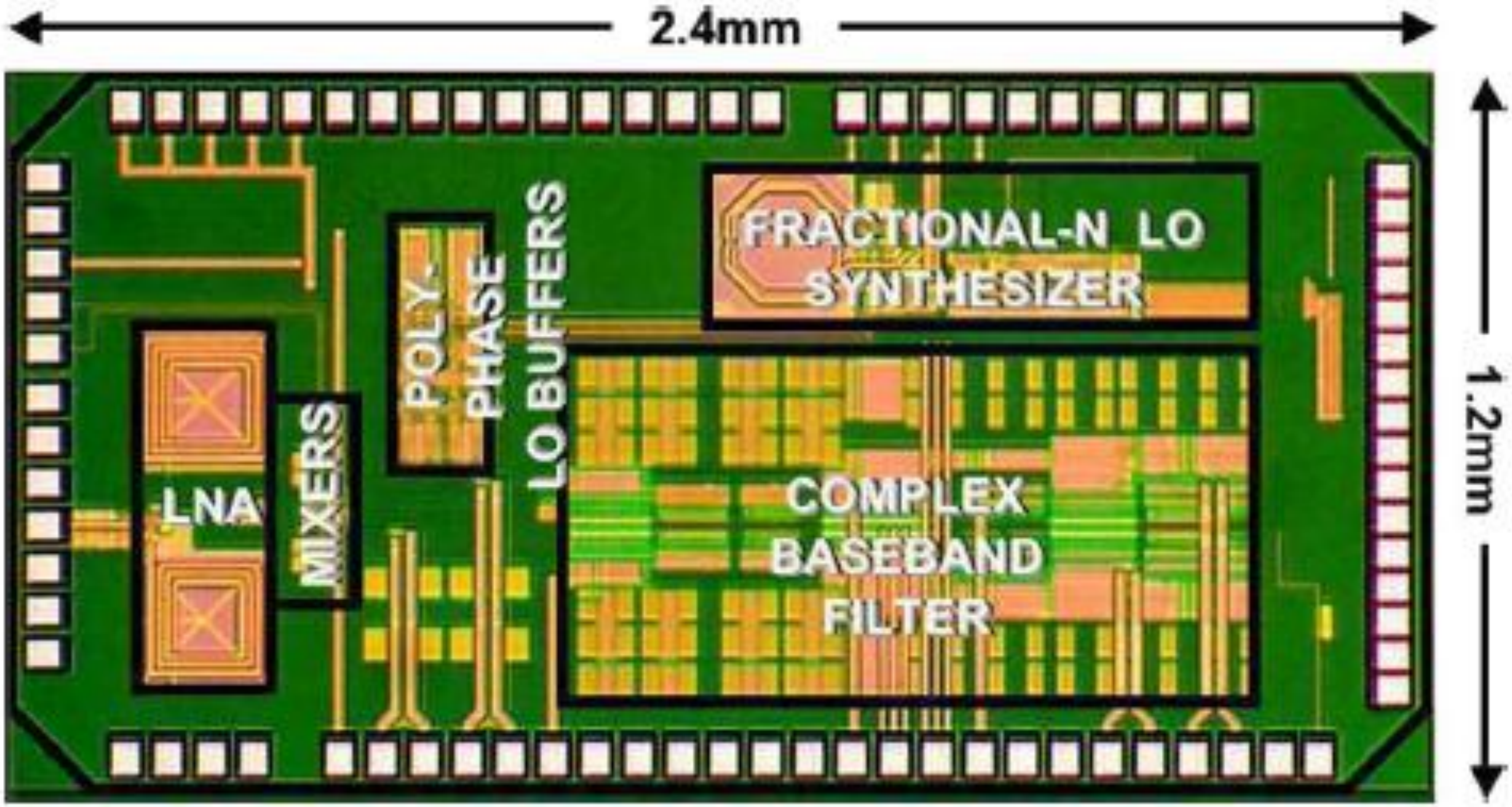


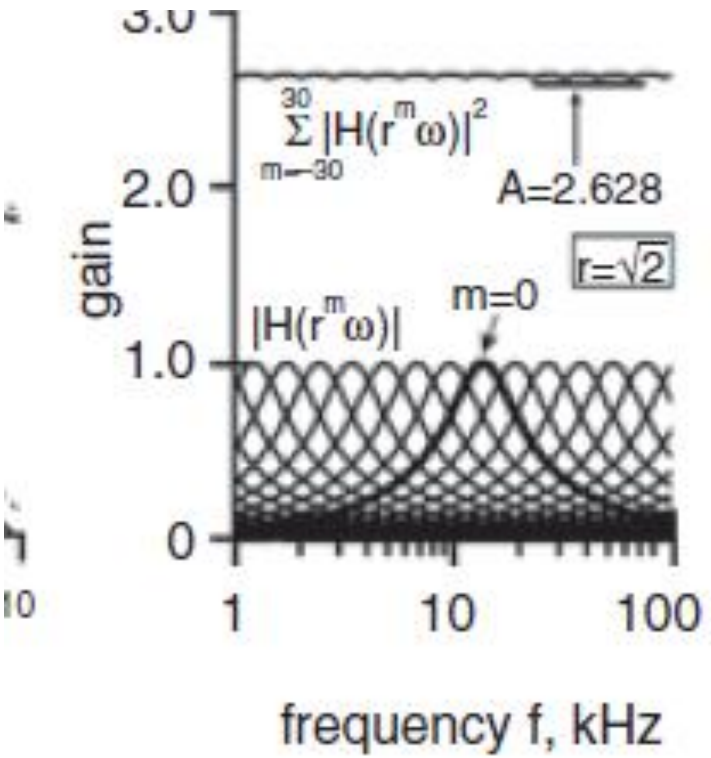
Fig. 5. Simulated frequency response for the different complex biquad stages. The ordering of the biquads is determined by their blocker attenuation. For maximal out-of-channel attenuation as early as possible, the biquads are ordered such that the biquad with Q = 0.517 is followed by the biquad with Q = 0.707 and biquad with Q = 1.93.


2.4mm
1.2mm
FRACTIONAL-N LO
SYNTHESIZER
POLY-
PHASE
LO BUFFERS
LNA
MIXERS
COMPLEX
BASEBAND
FILTER

Analogue wavelet transform with single biquad stage per scale

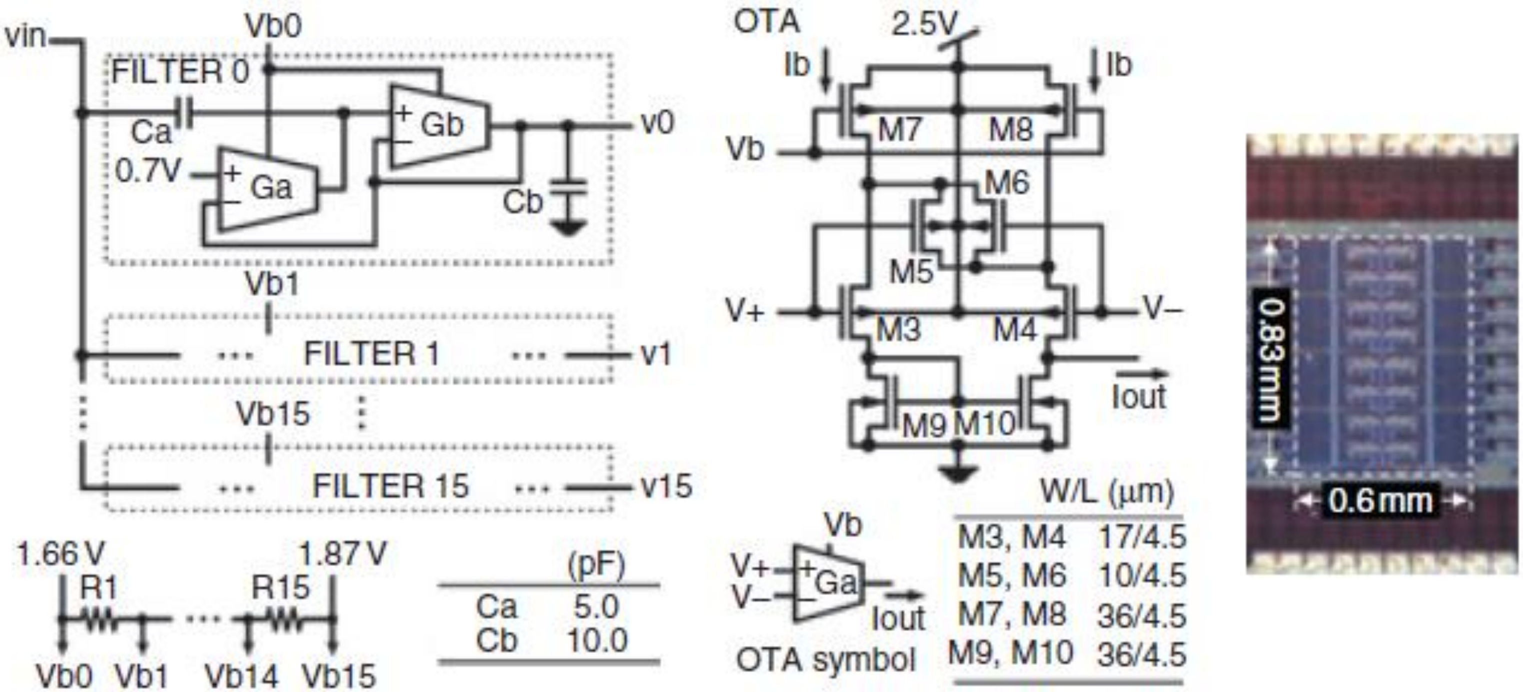
M.A. Gurrola-Navarro and G. Espinosa-Flores-Verdad

ELECTRONICS LETTERS 29th April 2010 Vol. 46 No. 9



c

22



a *b* *c*

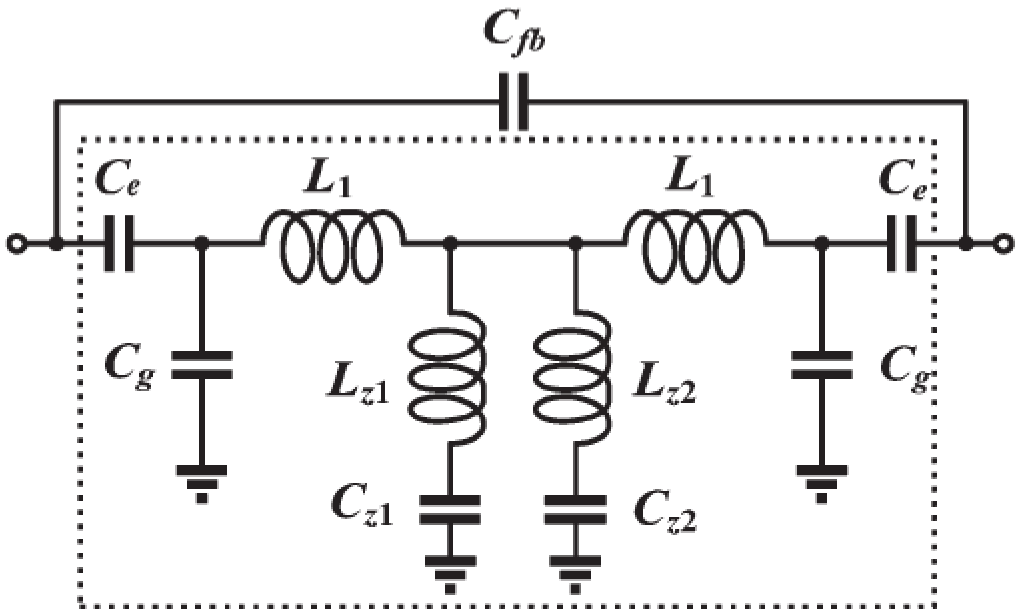
The OTAs are biased in the subthreshold region.

Design of a K-Band Chip Filter With Three Tunable Transmission Zeros Using a Standard 0.13-μm CMOS Technology

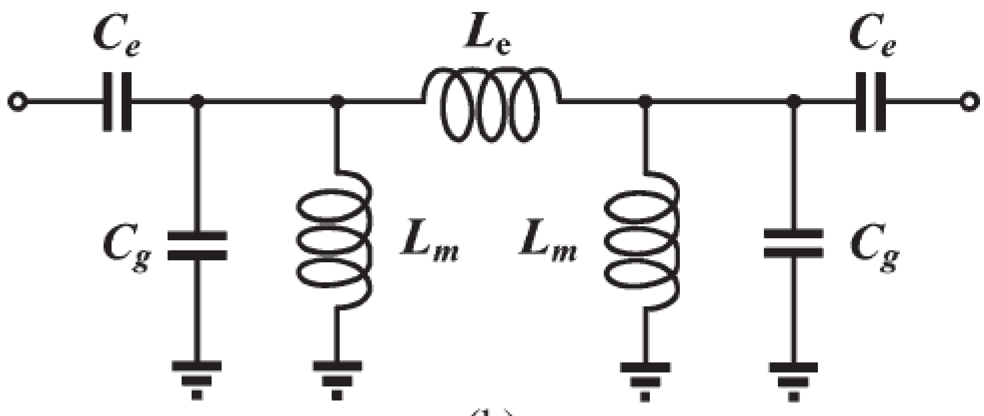
Chin-Lung Yang, Shin-Yi Shu, and Yi-Chyun Chiang, *Member, IEEE*

is presented for 24-GHz automotive ultrawideband (UWB) radar systems. The filter combines a second-order asymmetrically compact resonator filter with a source–load coupling mechanism to realize three transmission zeros; two zeros are arranged in the lower stopband, and one zero is located in the upper stopband. To achieve a compact layout size and a low insertion loss, a semi-lumped approach, which is accomplished with mixed utilization of high-impedance coplanar waveguide lines and lumped capacitors, is used to construct the chip filter. A K-band experimental proto-

23



(a)



(b)

Power-Efficient and Cost-Effective 2-D Symmetry Filter Architectures

Pei-Yu Chen, *Student Member, IEEE*, Lan-Da Van, *Member, IEEE*, I-Hung Khoo, *Member, IEEE*,
Hari C. Reddy, *Fellow, IEEE*, and Chin-Teng Lin, *Fellow, IEEE*

***Abstract*—This paper presents two-dimensional (2-D) VLSI digital filter structures possessing various symmetries in the filter magnitude response. For this purpose, four Type-1 and four Type-2 power-efficient and cost-effective 2-D magnitude sym-**

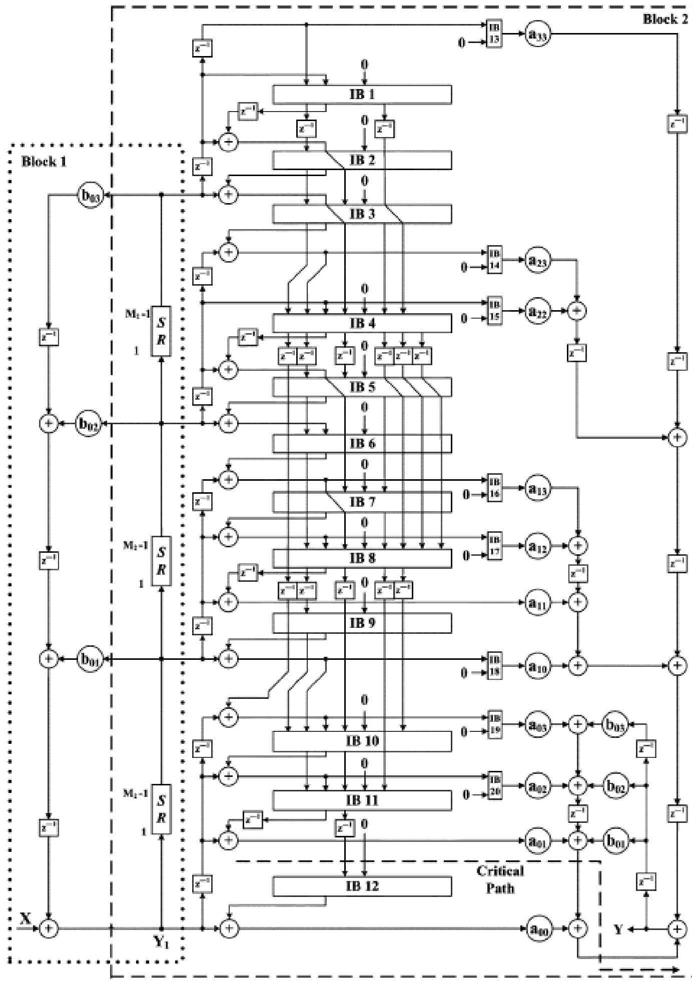


Fig. 11. Proposed multimode 2-D filter architecture with four symmetries for $N = 3$.

area size of 718.95 μm $\times$ 711.05 μm. The corresponding power consumption of the proposed multimode 2-D filter is 29.34 mW on average. In terms of power comparison, the DSM, FRSM,

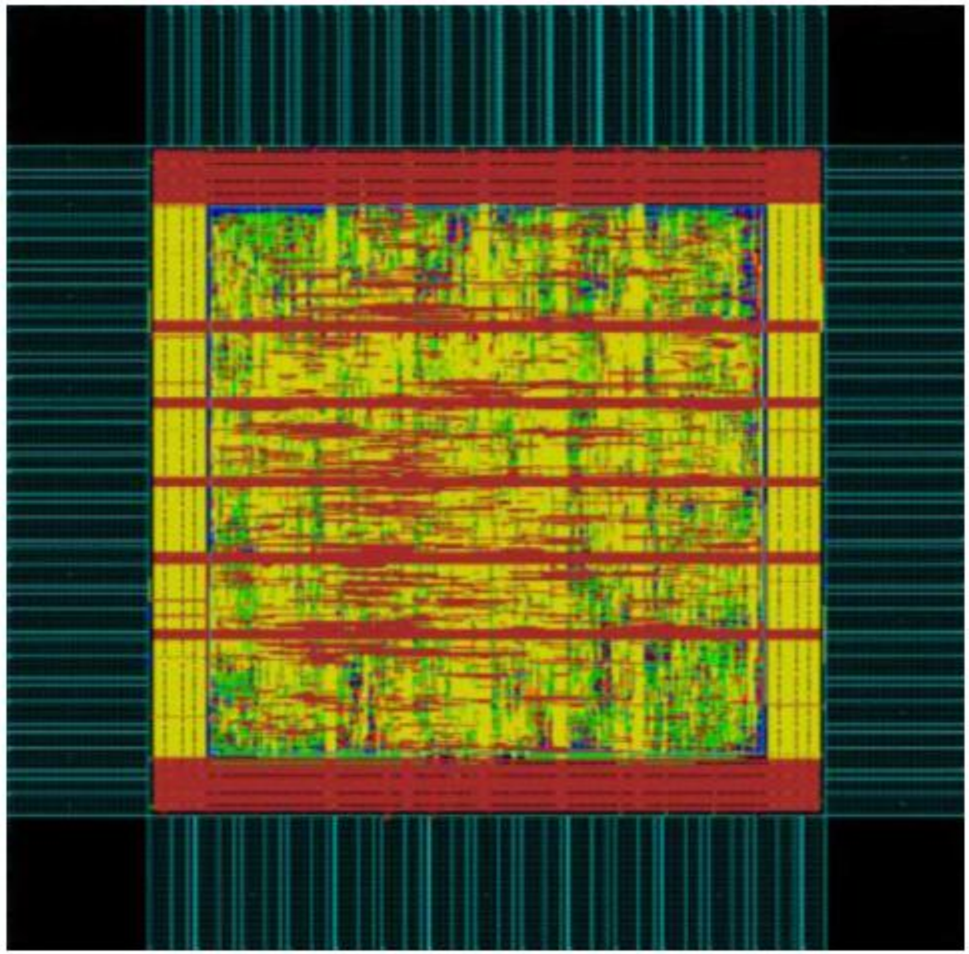


Fig. 18. Layout of the proposed multimode symmetry filter for $N = 3$.

A Subharmonic Receiver in SiGe Technology for 122 GHz Sensor Applications

Klaus Schmalz, Wolfgang Winkler, Johannes Borngräber, Wojciech Debski, Bernd Heinemann, and J. Christoph Scheytt, *Member, IEEE*

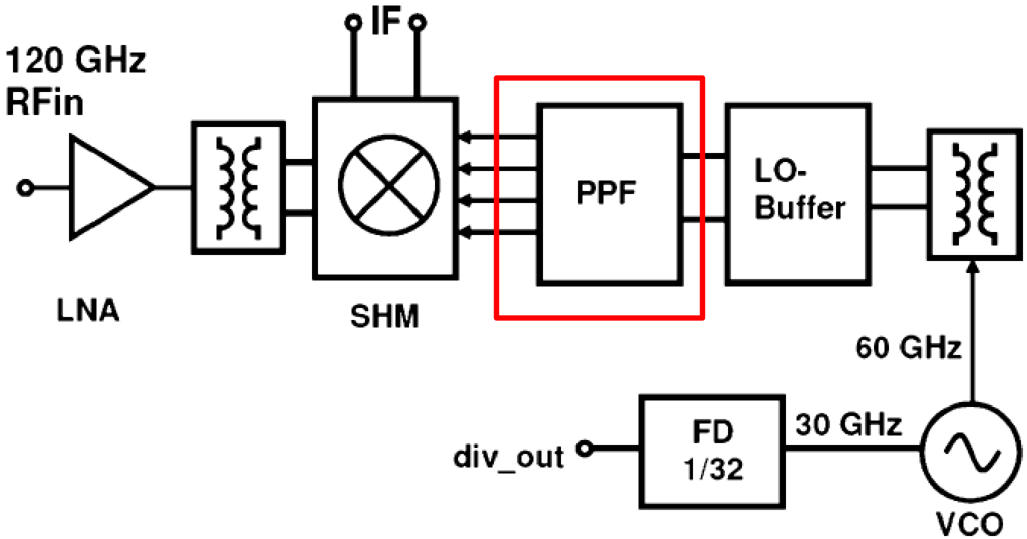


Fig. 1. Topology of the subharmonic receiver.

is fabricated in SiGe:C BiCMOS technology with $f_T/f_{\max}$ of 255 GHz/315 GHz. The receiver was optimized by an

25

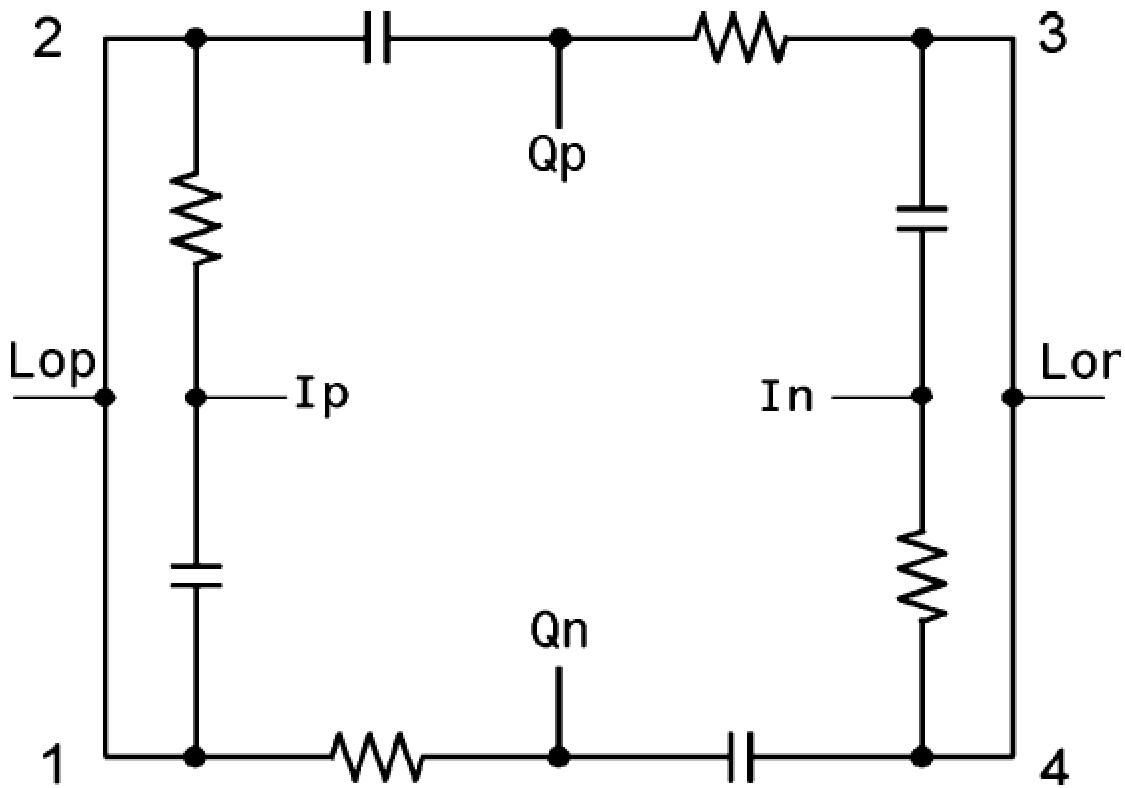


Fig. 4. Schematic of 5 GHz polyphase filter

$R = 67\ \Omega$ and $C = 40$ fF

unsalicided, p-doped gate polysilicon as resistor

MIM capacitors with 1 fF/μm^2.

Low-Power and Widely Tunable Linearized Biquadratic Low-Pass Transconductor-C Filter

Armin Tajalli, *Member, IEEE*, and Yusuf Leblebici, *Fellow, IEEE*

***Abstract*—A sixth-order low-pass transconductor-C filter with a very wide tuning range ($f_c = 100$ Hz to 10 MHz) is presented. The wide tuning range has been achieved without using switchable components or programmable building blocks.**

ı 0.18-μm CMOS technology

the input devices are biased in the subthreshold regime

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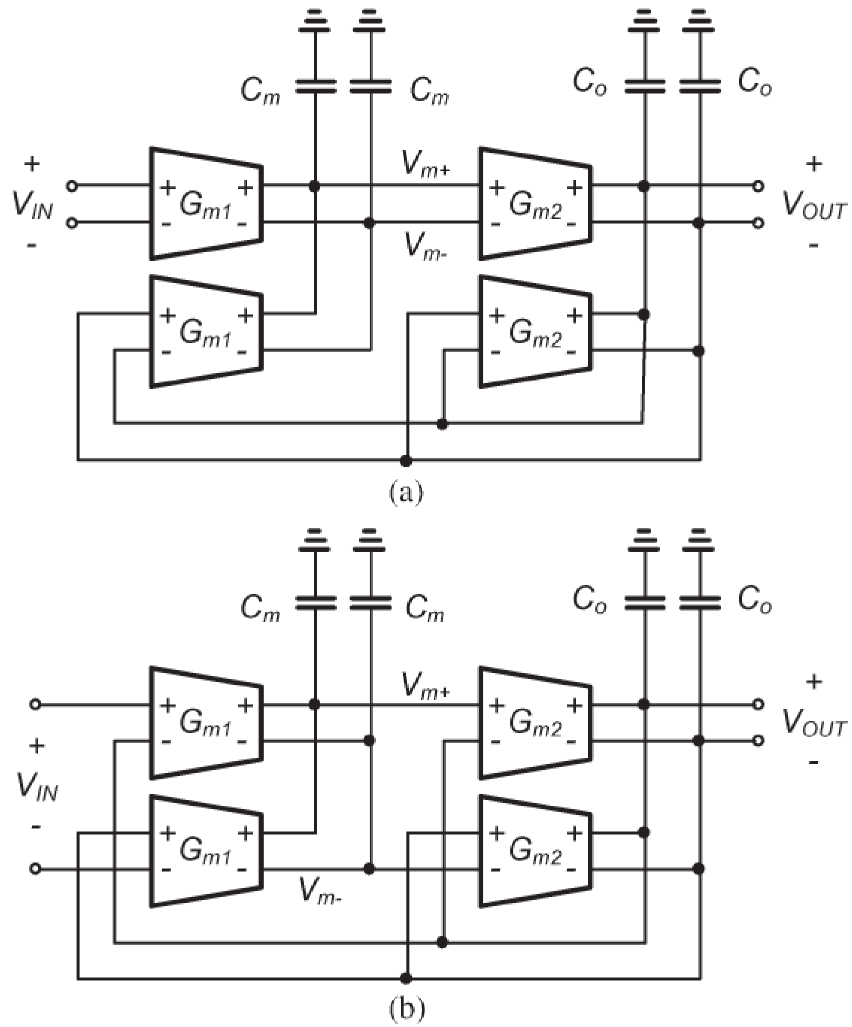


Fig. 2. Biquadratic g_m-C filter. (a) Conventional topology. (b) Modified topology with improved linearity performance.

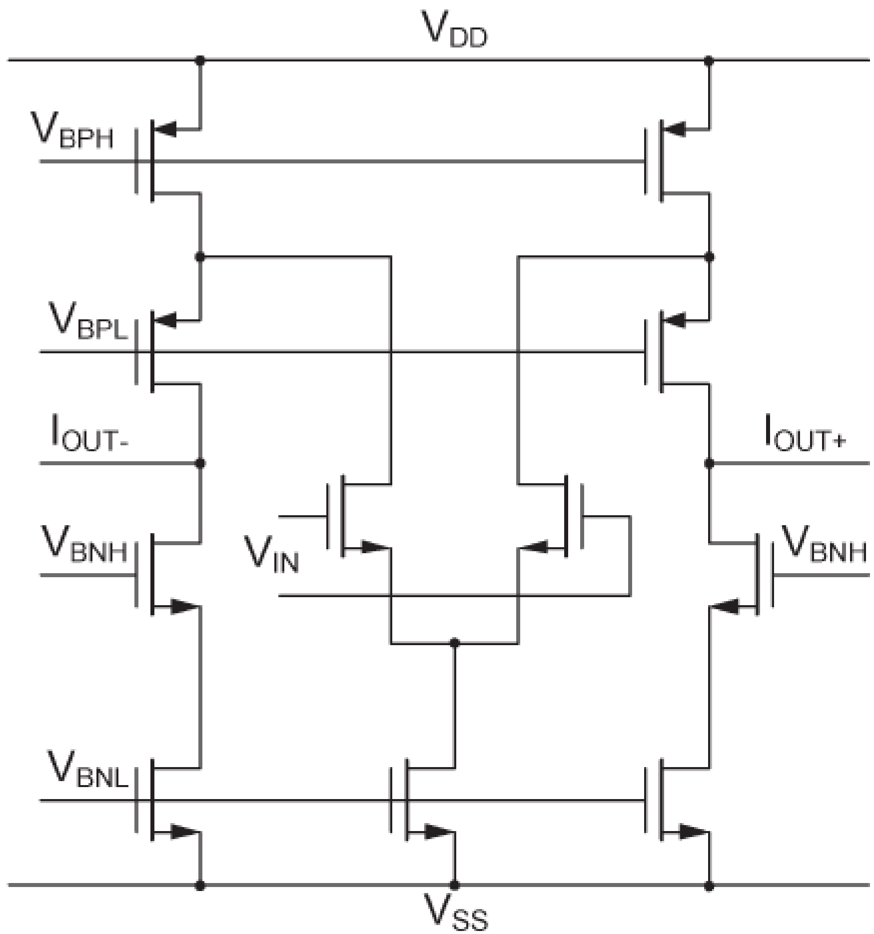
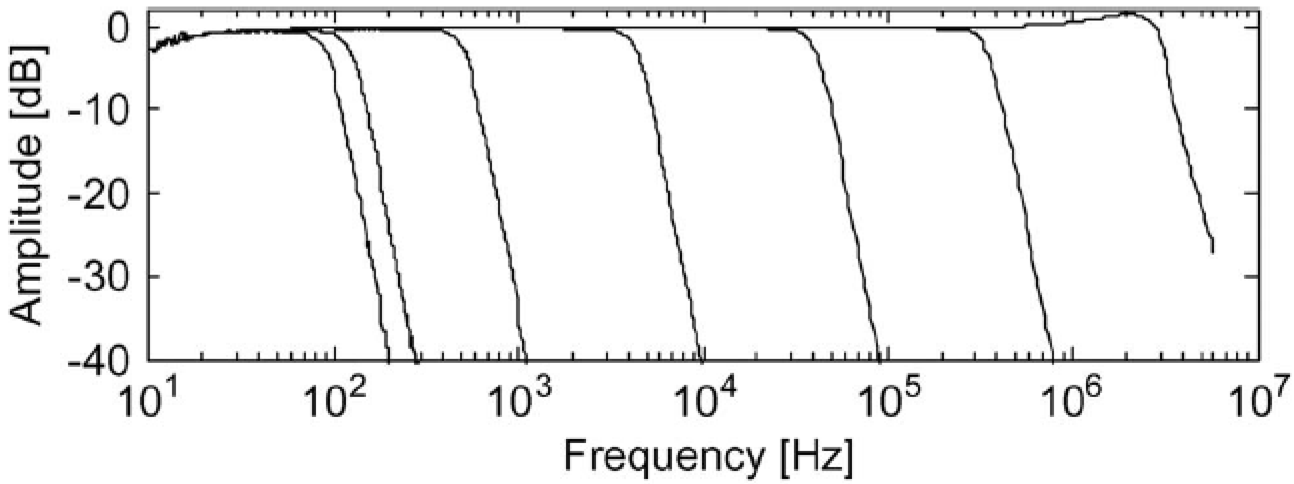
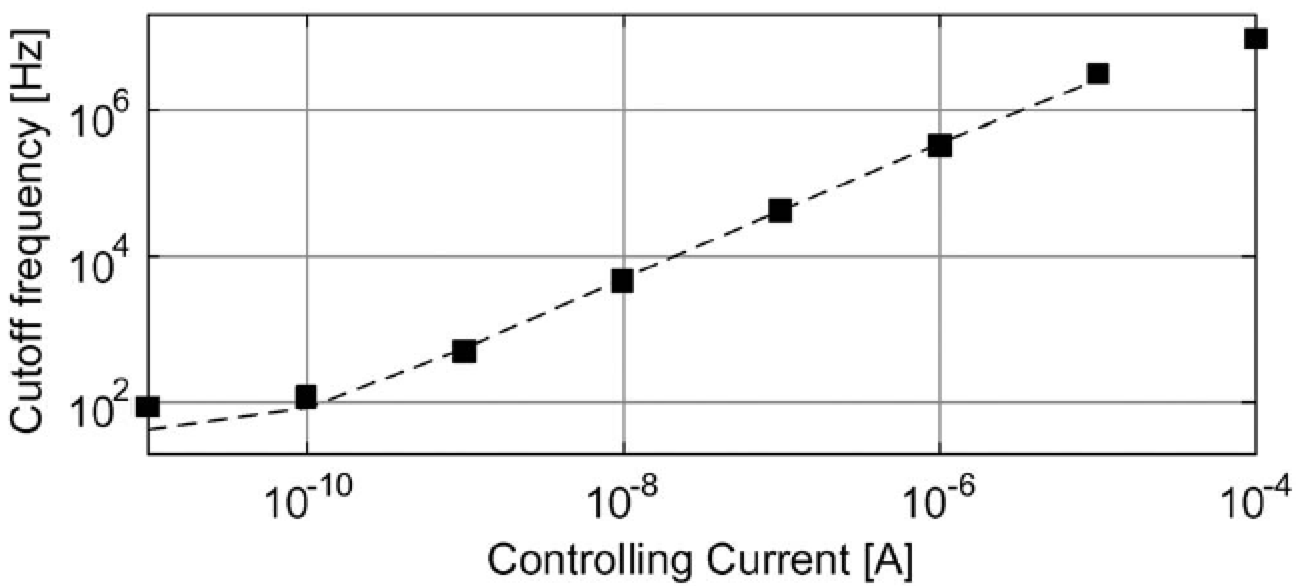


Fig. 4. Schematic of the folded cascode transconductor used to implement the widely tunable filter.



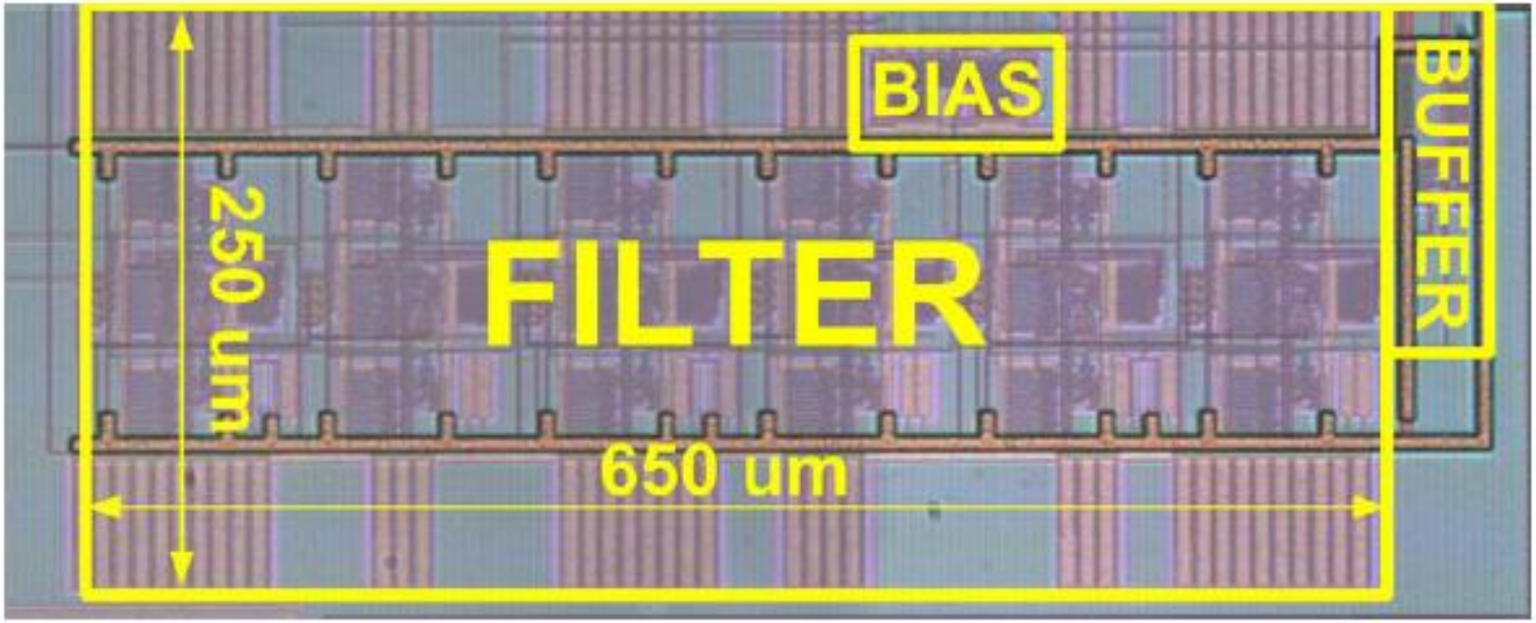
Amplitude [dB]
0
-10
-20
-30
-40
10¹
10²
10³
10⁴
10⁵
10⁶
10⁷
Frequency [Hz]

(a)



Cutoff frequency [Hz]
10²
10⁴
10⁶
10⁻¹⁰
10⁻⁸
10⁻⁶
10⁻⁴
Controlling Current [A]

(b)


BIAS
BUFFER
250 um
FILTER
650 um

CMOS on-chip active RF tracking filter for digital TV tuner ICs

Y. Sun, C.J. Jeong, S.K. Han and S.G. Lee

ELECTRONICS LETTERS 17th March 2011 Vol. 47 No. 6

The fabricated tracking filter based on a 0.13 µm CMOS process shows 48–780 MHz tracking range with 15–60 MHz bandwidth,

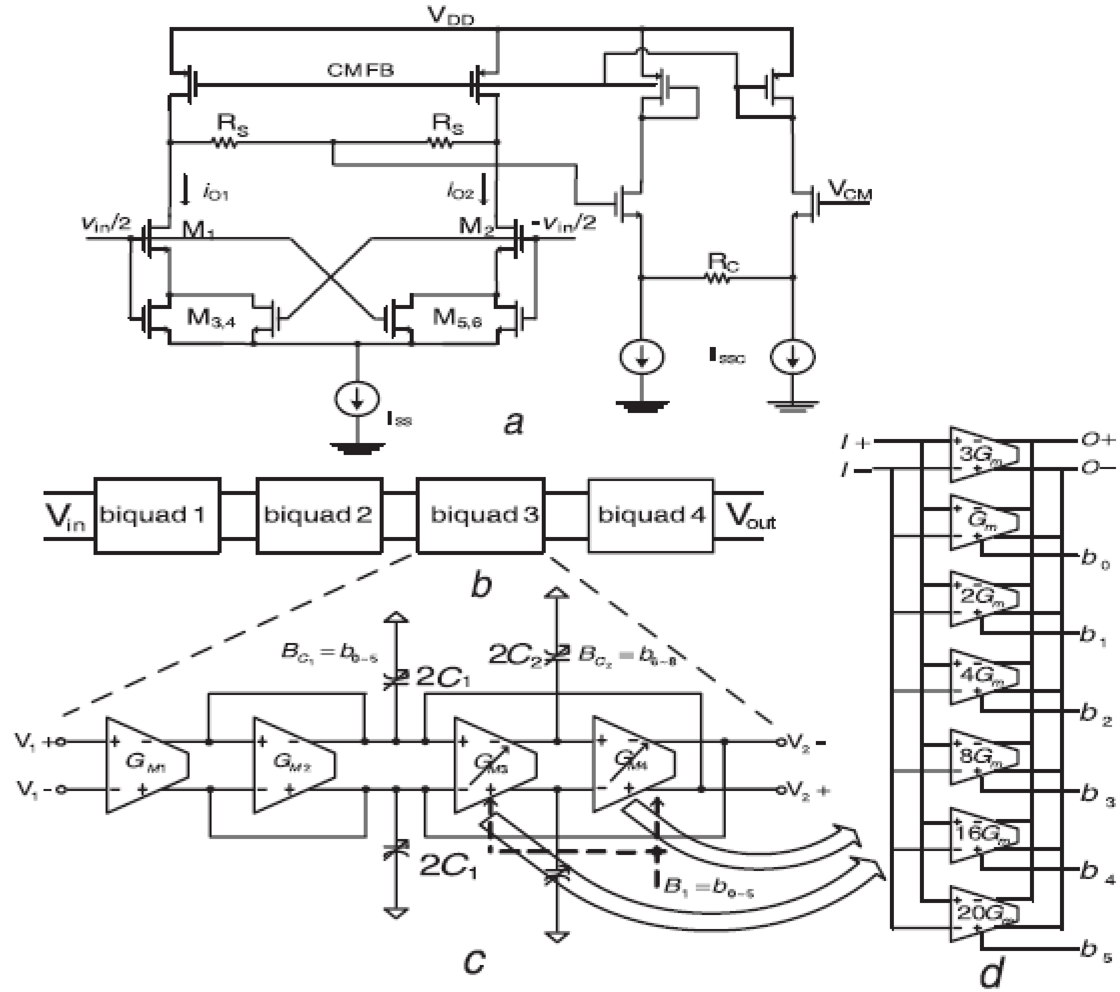


Fig. 1 *Proposed RF tracking filter design*

a Schematic of unit G_m-cell used in proposed RF tracking filter
b RF tracking filter architecture
c Biquad architecture
d G_M-cell in biquad

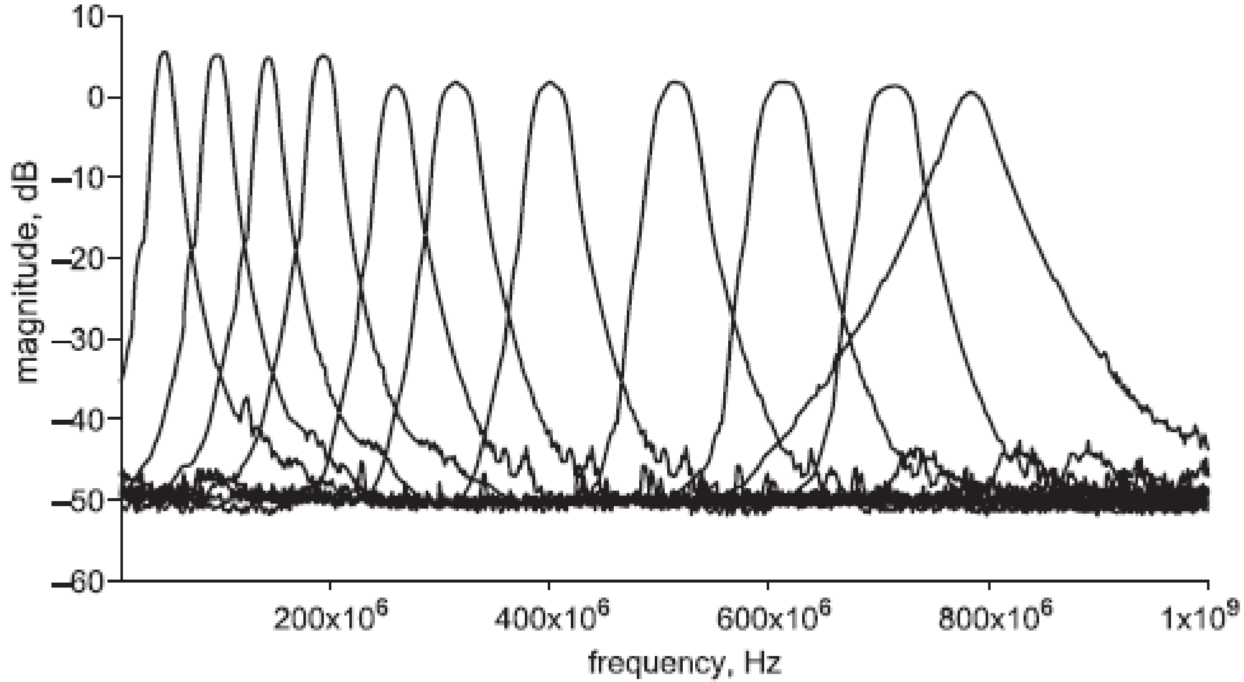


Fig. 2 *Measured frequency tuning and rejection performance*

IEEE JOURNAL OF SOLID-STATE CIRCUITS, VOL. 46, NO. 5, MAY 2011

Tunable High-Q N-Path Band-Pass Filters: Modeling and Verification

Amir Ghaffari, *Student Member, IEEE*, Eric A. M. Klumperink, *Senior Member, IEEE*, Michiel C. M. Soer, *Student Member, IEEE*, and Bram Nauta, *Fellow, IEEE*

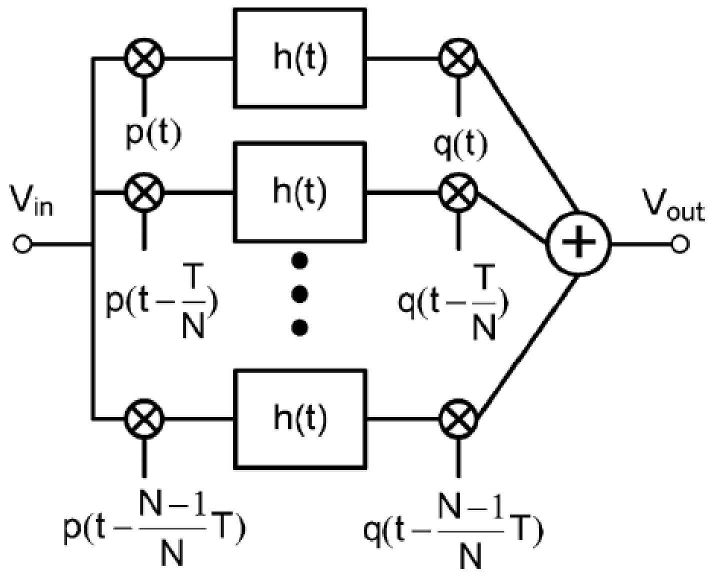


Fig. 1. Architecture of an N-path filter [5] (p and q are the mixing functions and T is the period of the mixing frequency).

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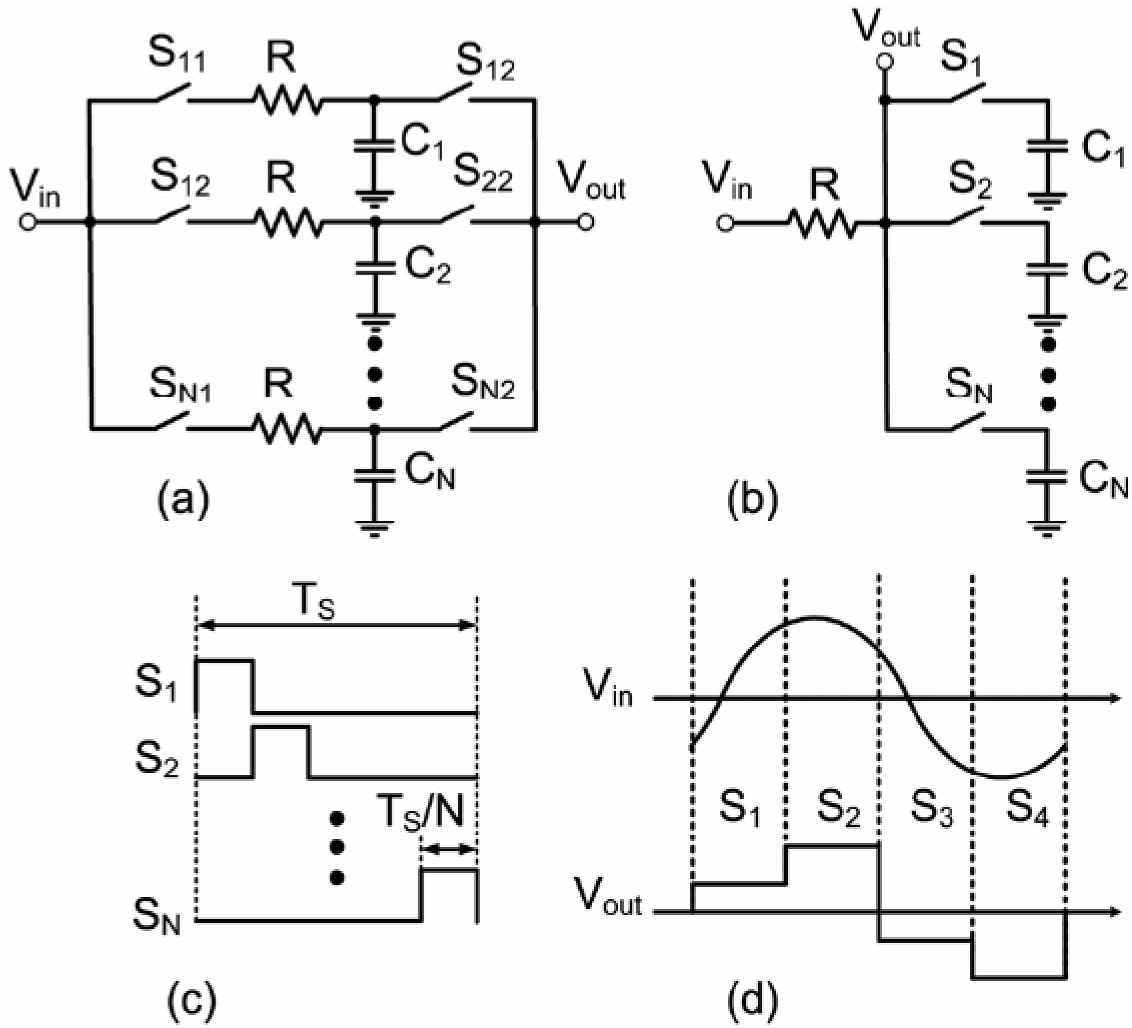
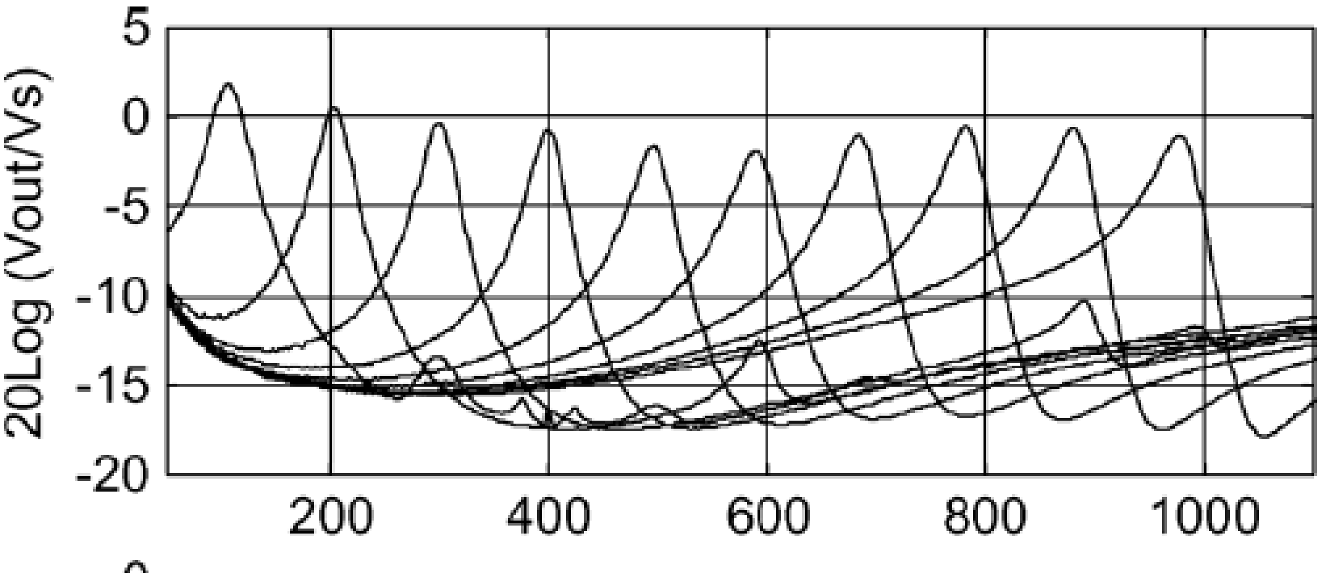


Fig. 2. (a) Switched-RC N-path filter. (b) Single port, single ended N-path filter. (c) Multiphase clocking. (d) Typical (in-band) input and output signal.



20Log (Vout/Vs)
5
0
-5
-10
-15
-20
200
400
600
800
1000

Performance	This Work	[9]	[4]
Process	65nm CMOS	0.35um CMOS	0.18um CMOS
Active Area	$0.07mm^2$	$1.9mm^2$	$0.81mm^2$
Power Consumption	2 to 16mW	63mW	17mW
Frequency Tuning Range	0.1 to 1GHz	240 to 530MHz	2 to 2.06GHz
-3dB Band Width	35MHz	1.75 to 4.6MHz	130MHz
Voltage Gain	-2dB	-2dB	0dB
Quality Factor (Q)	3 to 29	301 to 114	15.4 to 15.8
P_{1dB}	2dBm	-5dBm	-6.6dBm
IIP3	14dBm	NA	2.5dBm
Noise Figure	3-5dB	9dB	15dB

A 400 μW Hz-Range Lock-In A/D Frontend Channel for Infrared Spectroscopic Gas Recognition

Stepan Sutula, *Student Member, IEEE*, Carles Ferrer, and Francisco Serra-Graells, *Member, IEEE*

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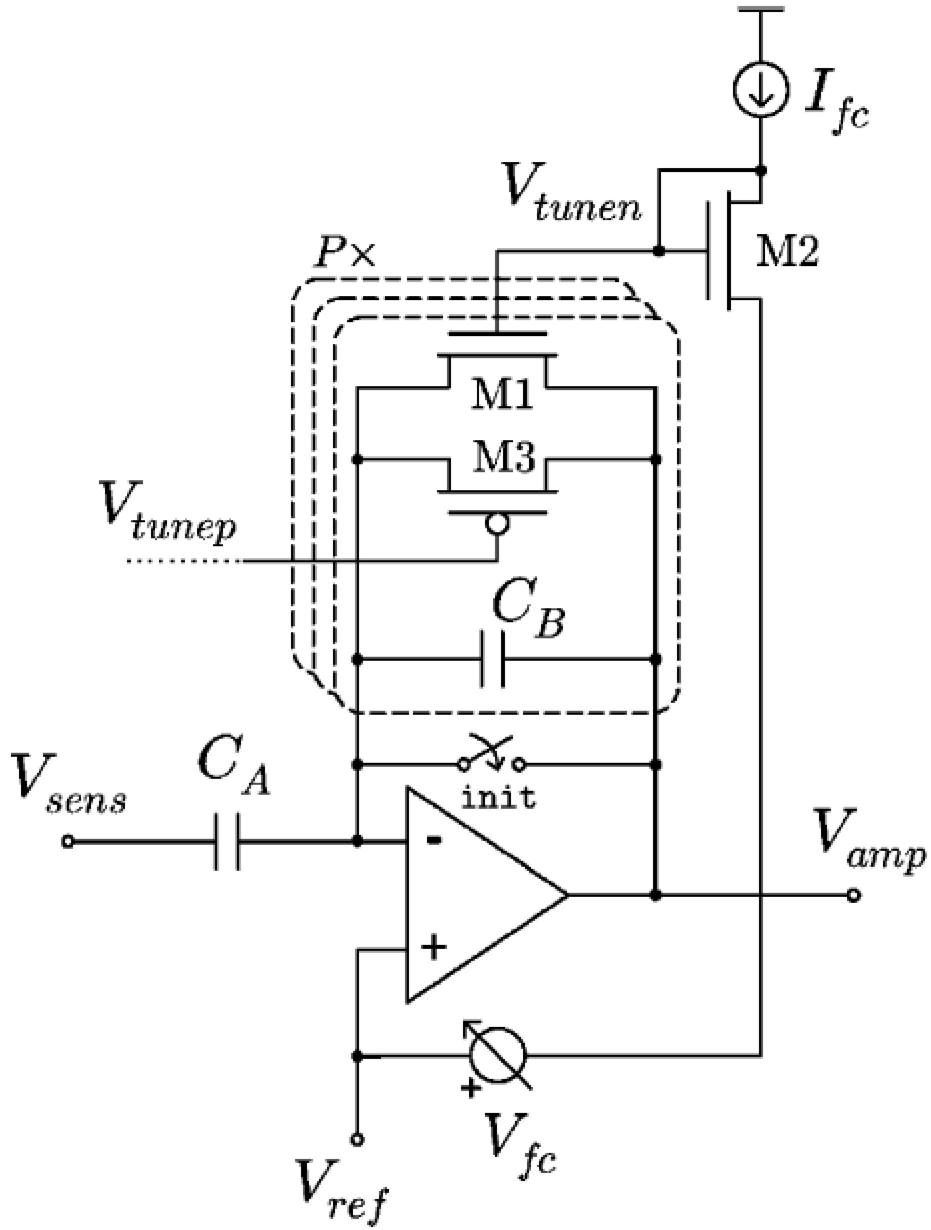


Fig. 3. Proposed sub-Hz programmable MOS-C high-pass preamplifier.

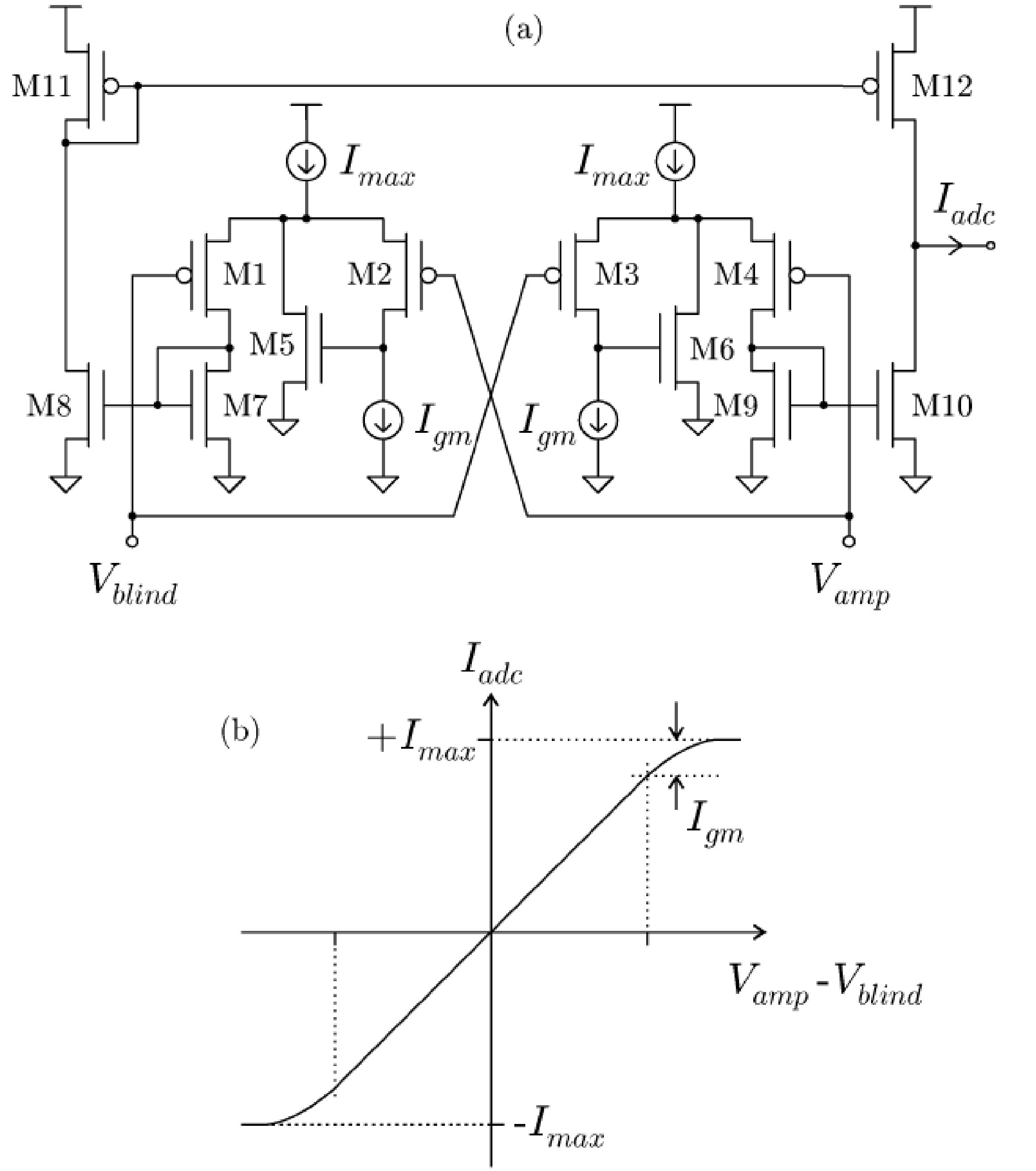
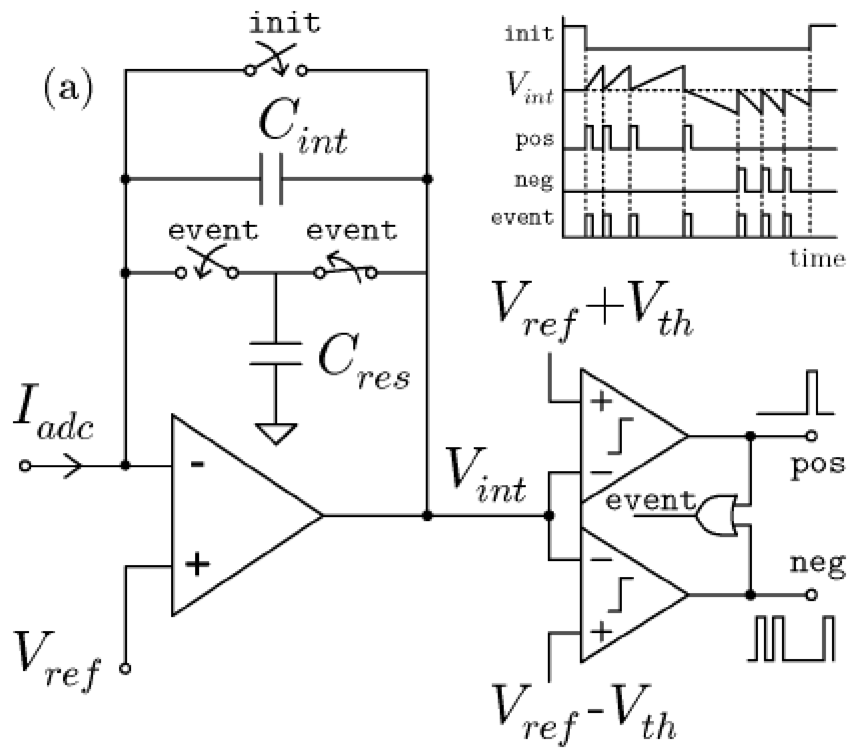
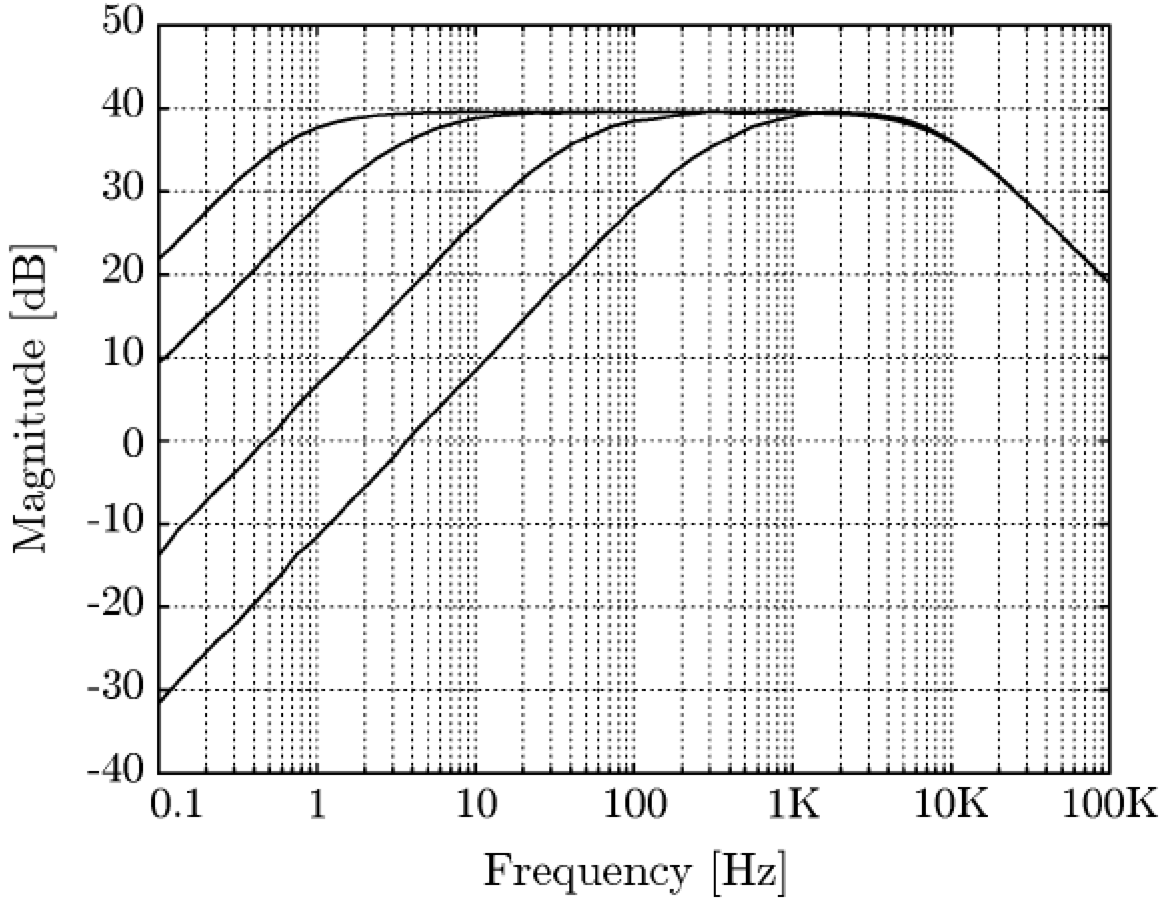


Fig. 6. (a) Proposed linear transconductor and (b) equivalent built-in limiter



(a)
init
C_{int}
event
event
C_{res}
I_{adc}
-
+
V_{ref}
V_{int}
$V_{ref}+V_{th}$
$V_{ref}-V_{th}$
+
−
−
+
event
pos
neg
init
V_{int}
pos
neg
event
time



9. Experimental transfer function of the high-pass preamplifier stage for
pendent gain (top) and corner frequency (bottom) digital programming.

An Electronically Fine-Tunable Multi-Input–Single-Output Universal Filter

Indrit Myderrizi, *Member, IEEE*, Shahram Minaei, *Senior Member, IEEE*, and Erkan Yuce, *Member, IEEE*

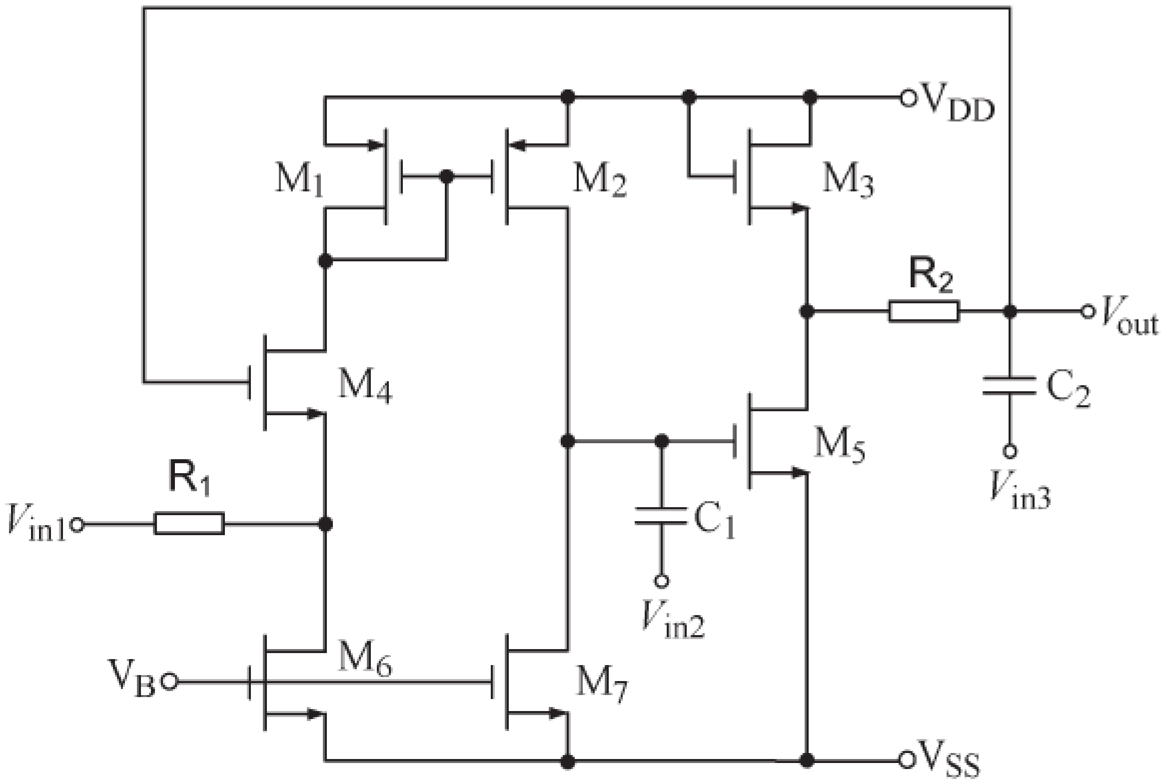


Fig. 1. CMOS realization of the proposed VM MISO universal filter.

30

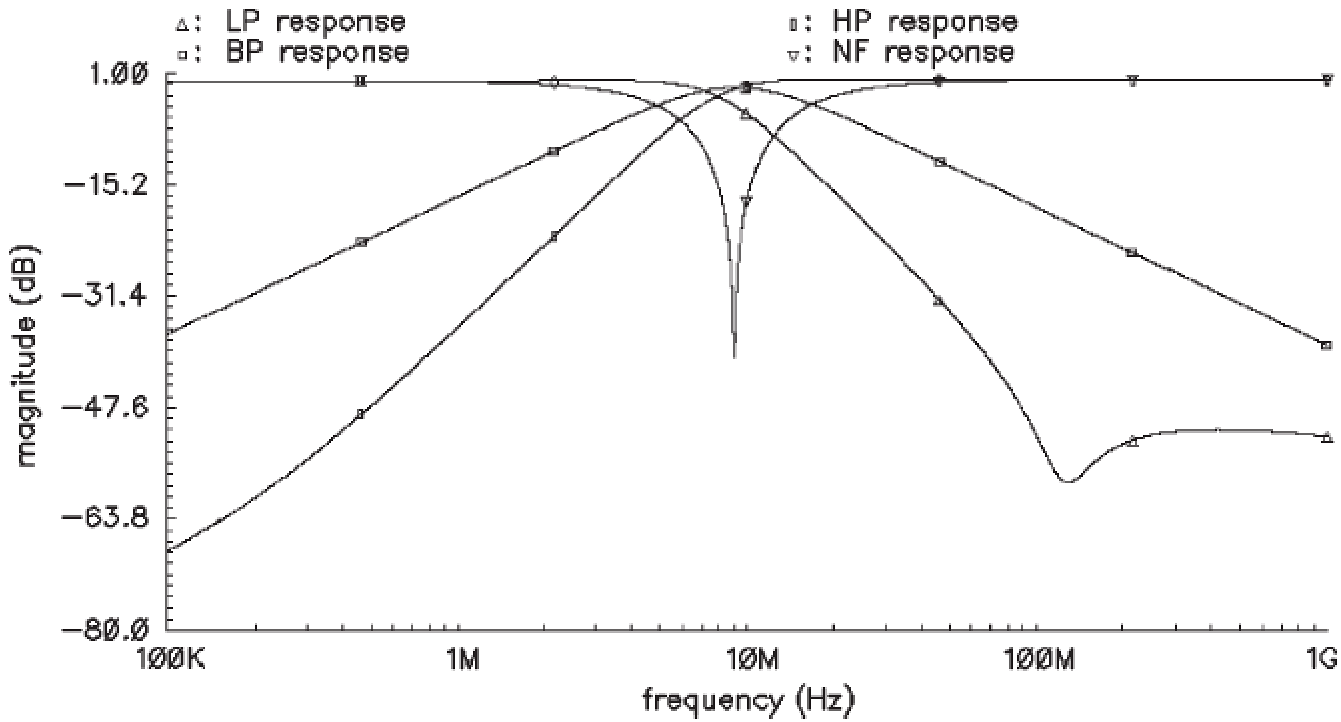


Fig. 3. Magnitude frequency responses of the MISO multifunctional filter.

for a high-Q BP response of the filter. In this case, $Q = 5$, and the component values are selected as $C_1 = 0.457$ pF, $C_2 = 2$ pF, $R_1 = 6.5$ kΩ, and $R_2 = 37.2$ kΩ.

A 16-Channel Low-Noise Programmable System for the Recording of Neural Signals

Carolina Mora López, Dries Braeken, Carmen Bartic, Robert Puers*, Georges Gielen* and Wolfgang Eberle
Imec, Leuven, Belgium

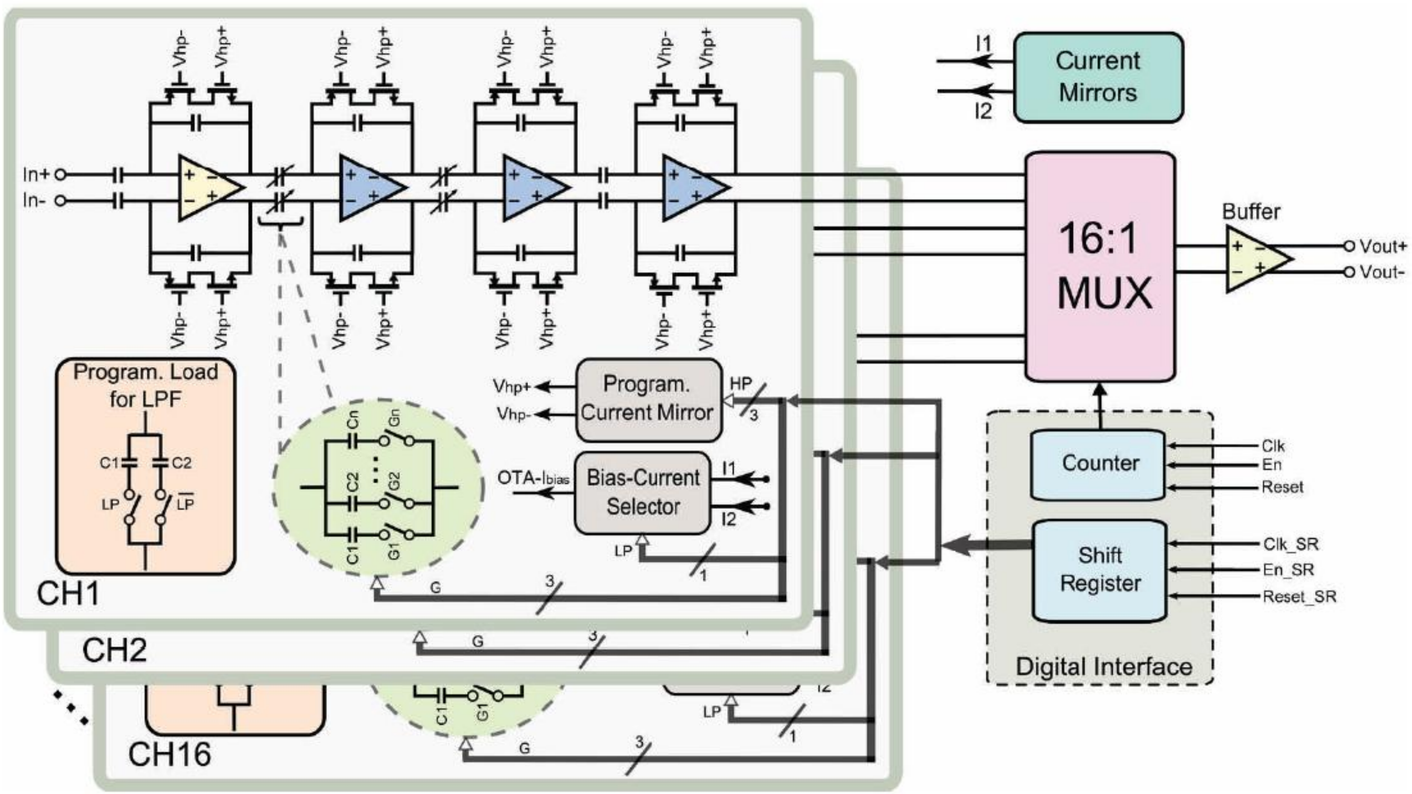


Figure 1. Architecture of the 16-channel neural recording system.

The high-pass filtering characteristic in each channel is achieved by the parallel combination of the feedback capacitors and the feedback MOS-bipolar pseudoresistors (formed by a NMOS and a PMOS transistors). These voltage-controlled pseudoresistor elements [7] can achieve very high resistance values in the order of 10^{12} Ω, providing an area-efficient way to implement very low frequency filters. The cutoff frequency can be tuned via a current-mode digital-to-analog converter (Fig. 3) that changes the gate voltages of the transistors (i.e. the resistance), in order to accept or reject the LFP signal frequencies. This circuit consists of a wide-swing cascode current mirror that copies the selected current to the transistors *Mn* and *Mp*, which are diode-connected and sized with large *W/L*. A similar method was previously described by Yin *et al* [8].

The fourth-order low-pass filter characteristic is implemented by the cascade of first-order voltage integrators, consisting of an OTA and a load capacitor. The load capacitor is implemented as a capacitor array that allows the selection of two different frequency ranges: one for the LFP signals and another for the AP signals. Also, for low-frequency LFP recordings, the supply current of the amplifiers is lowered to save power.

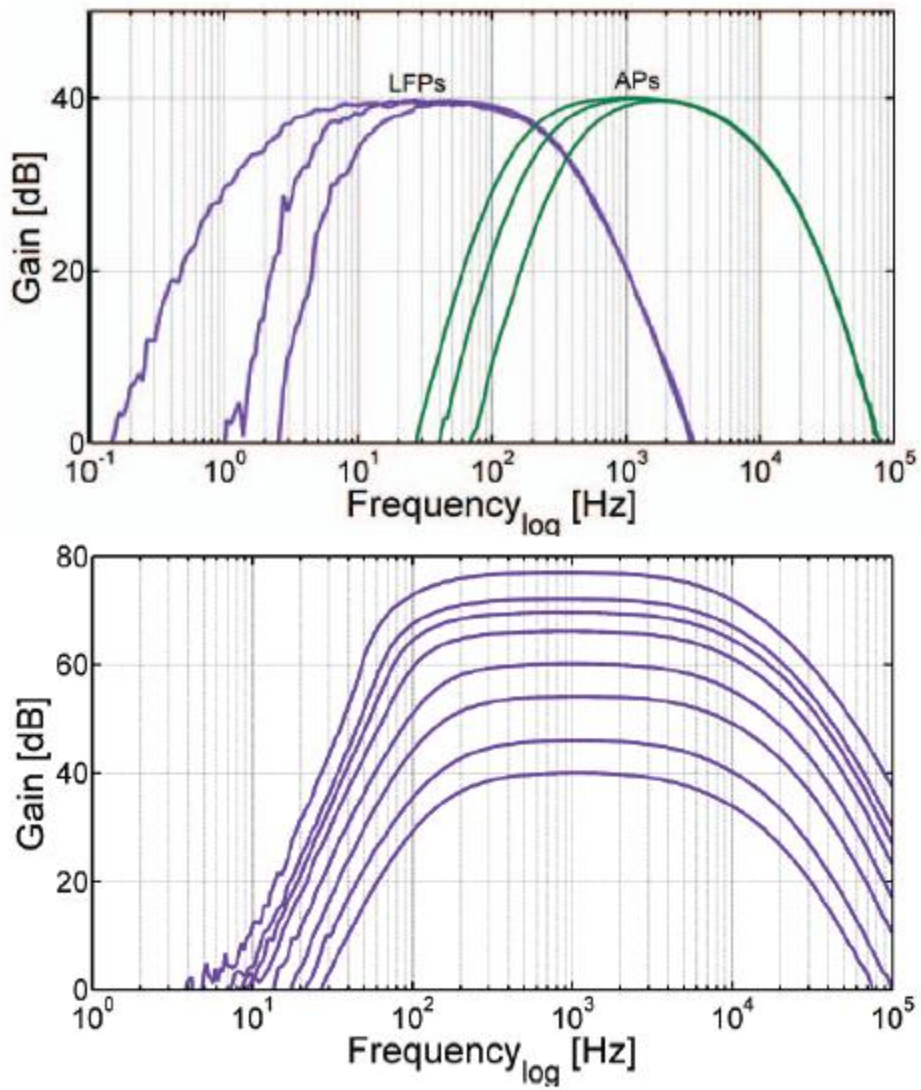


Figure 5. Transfer function of one channel, measued for different bandwiths and gains.

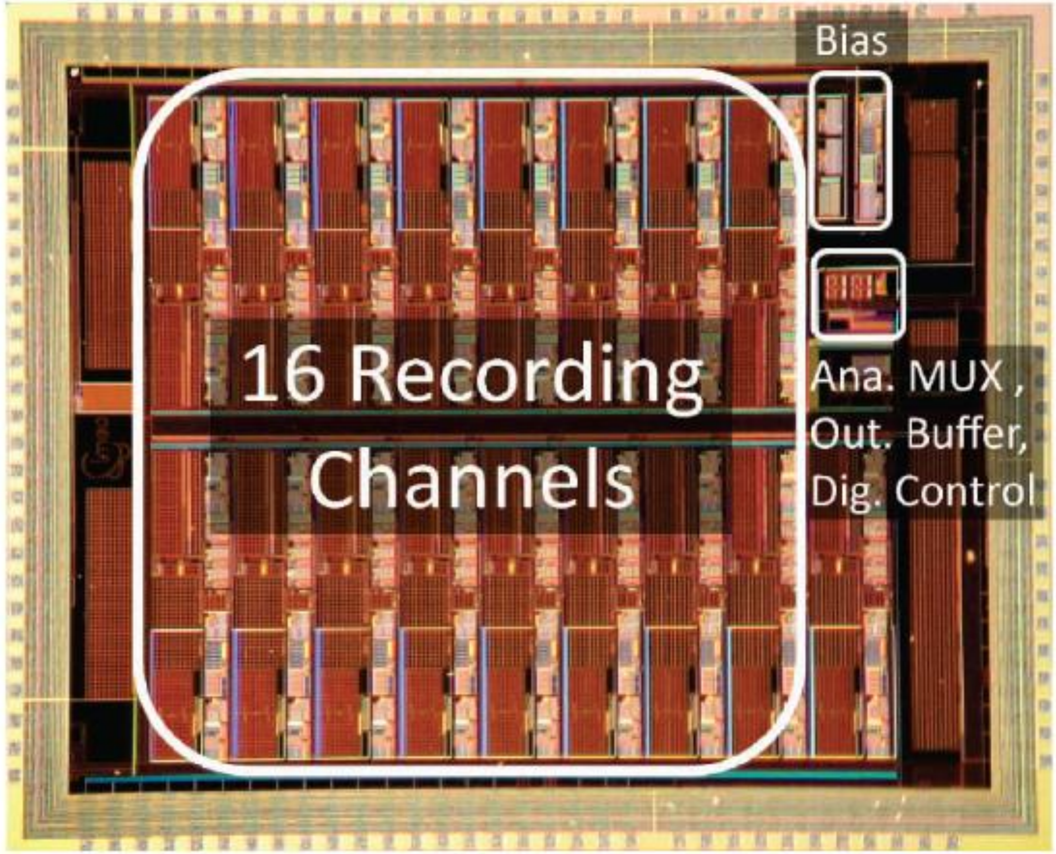


Figure 4. Die photo of the 16-channel neural recording system. The total area is 5.6 mm × 4.5 mm and the core area is 4.1mm × 3.8 mm.

The 16-channel neural recording system in Fig.1 has been implemented and fabricated in a 0.35 μm On Semiconductor CMOS technology. The capacitors are implemented as metal-insulator-metal capacitors and the resistors as polysilicon resistors. The die (Fig. 4) occupies a core area of 4.1 × 3.8 mm^2 and a total area of 5.6 × 4.5 mm^2. The area of one channel is 0.76 mm^2.

A High-Performance PLL With a Low-Power Active Switched-Capacitor Loop Filter

Yu Song, *Student Member, IEEE*, and Zeljko Ignjatovic, *Member, IEEE*

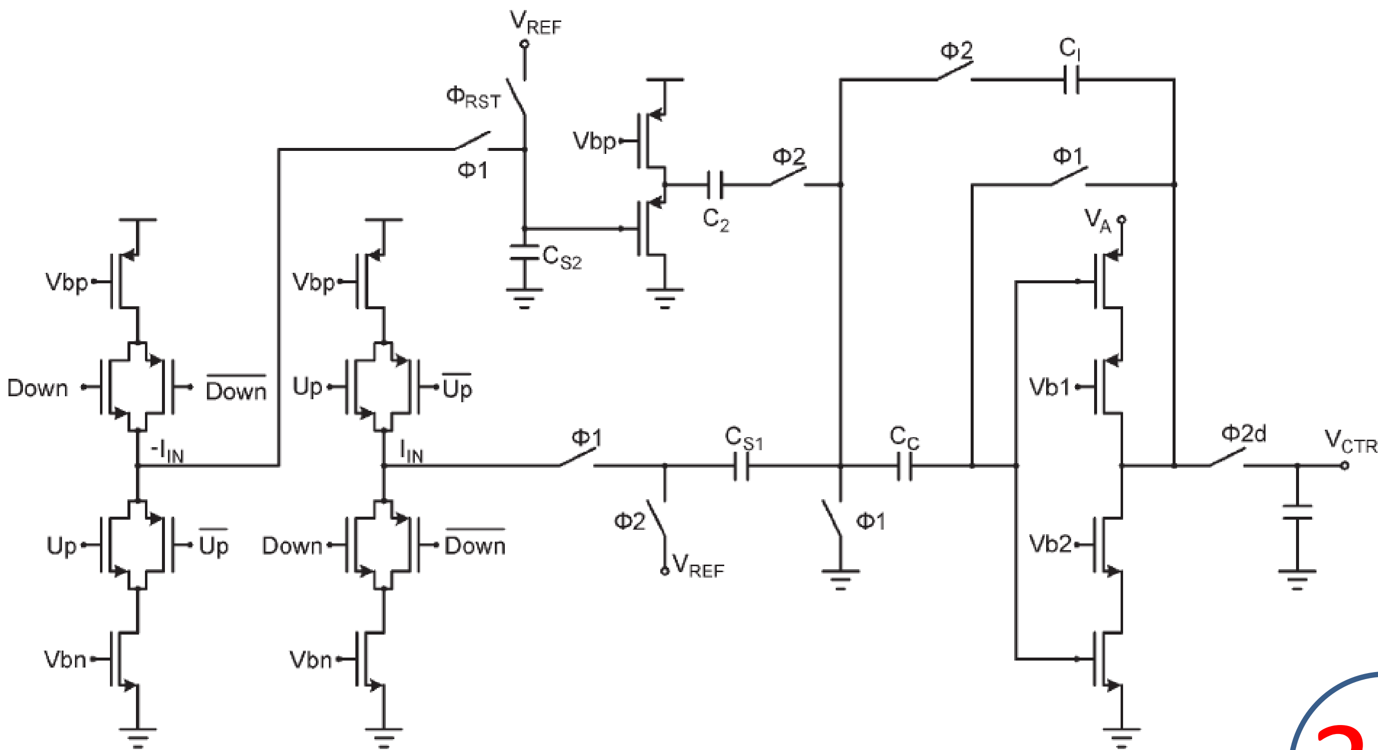


Fig. 1. Proposed SC PLL loop filter with complimentary charge pumps, single-ended shown.

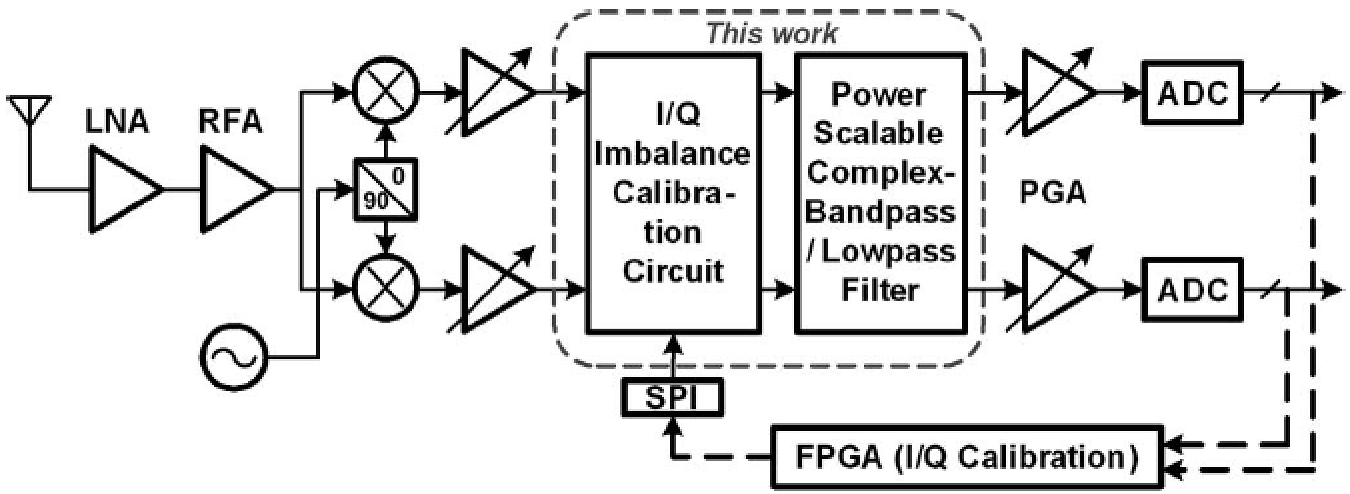
A low-power low-noise differential SC loop filter is designed based on the above analysis, with another filter, as shown in Fig. 1, sharing the complimentary charge pumps. The input reference frequency is selected to be 10 MHz, and the loop bandwidth is around 350 kHz. The discrepancy between the

TABLE I
PLL PERFORMANCE COMPARISON

	[1]	[3]	[4]	This Work
Freq. (GHz)	2.4	2.4	3.6	2.5
Technology	0.25µm CMOS	0.18µm CMOS	0.18µm CMOS	0.18µm CMOS
Loop Filter Structure	Passive SC	Passive SC	Hybrid	Active SC
Reference Frequency	1MHz	12MHz	50MHz	10MHz
Power (mW)	?	48.8 (core)	110 (core)	16
Area (mm^2)	?	4.8	2.7 (acitve)	0.36
Reference Spur	-62dBc	-70dBc	-45dBc	-64dBc
Phase Noise	-126dBc/Hz @2MHz	-125dBc/Hz @3MHz	-155dBc/Hz @20MHz	-124dBc/Hz @3MHz

Power-Scalable, Complex Bandpass/Low-Pass Filter With I/Q Imbalance Calibration for a Multimode GNSS Receiver

Yang Xu, Baoyong Chi, *Member, IEEE*, Xiaobao Yu, Nan Qi, Patrick Chiang, *Member, IEEE*, and Zhihua Wang, *Senior Member, IEEE*



Mode	GPS			Compass			GLONASS		Galileo		
Channel	L1	L2	L5	B1	B2	B3	L1	L2	E1	E5a	E5b
BW (MHz)	2.2/18	18	18	4.2	4.2/18	18	10/18	8/18	2.2	18	18

Fig. 1. Low-IF/zero-IF multimode GNSS receiver.

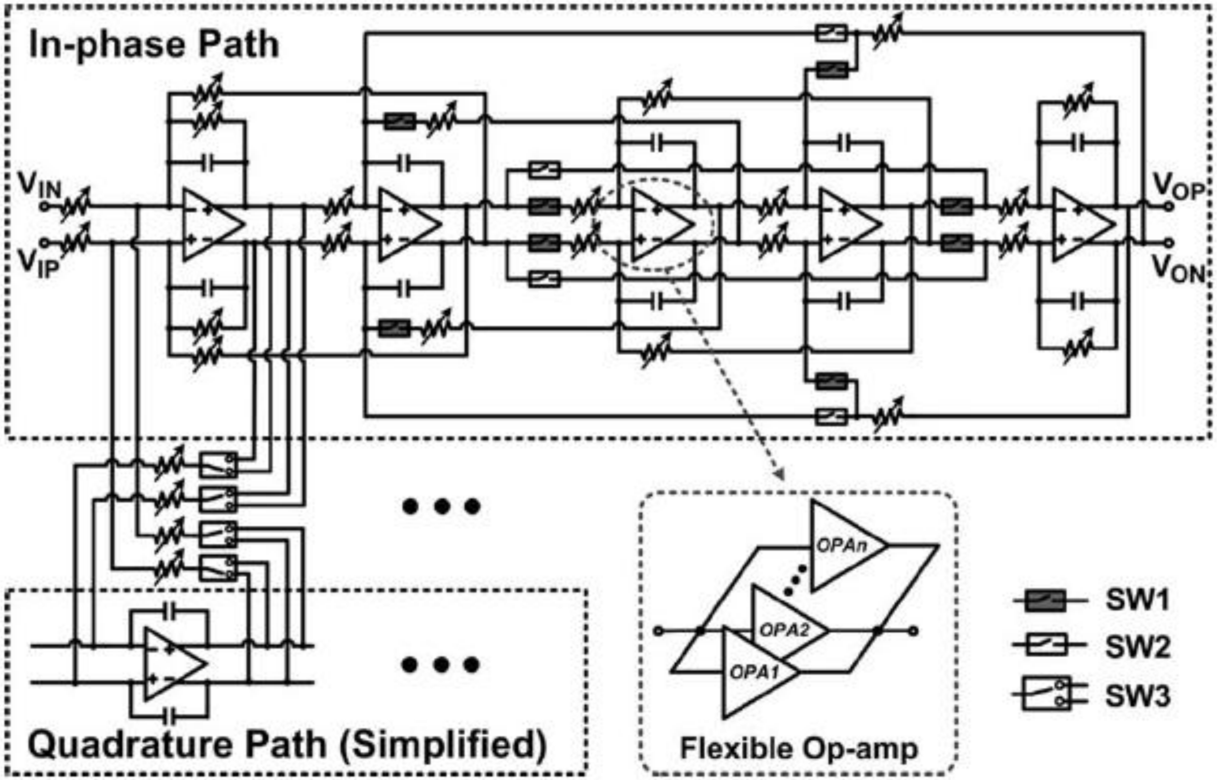


Fig. 2. Architecture of the reconfigurable fifth-order CBPF/third-order LPF.

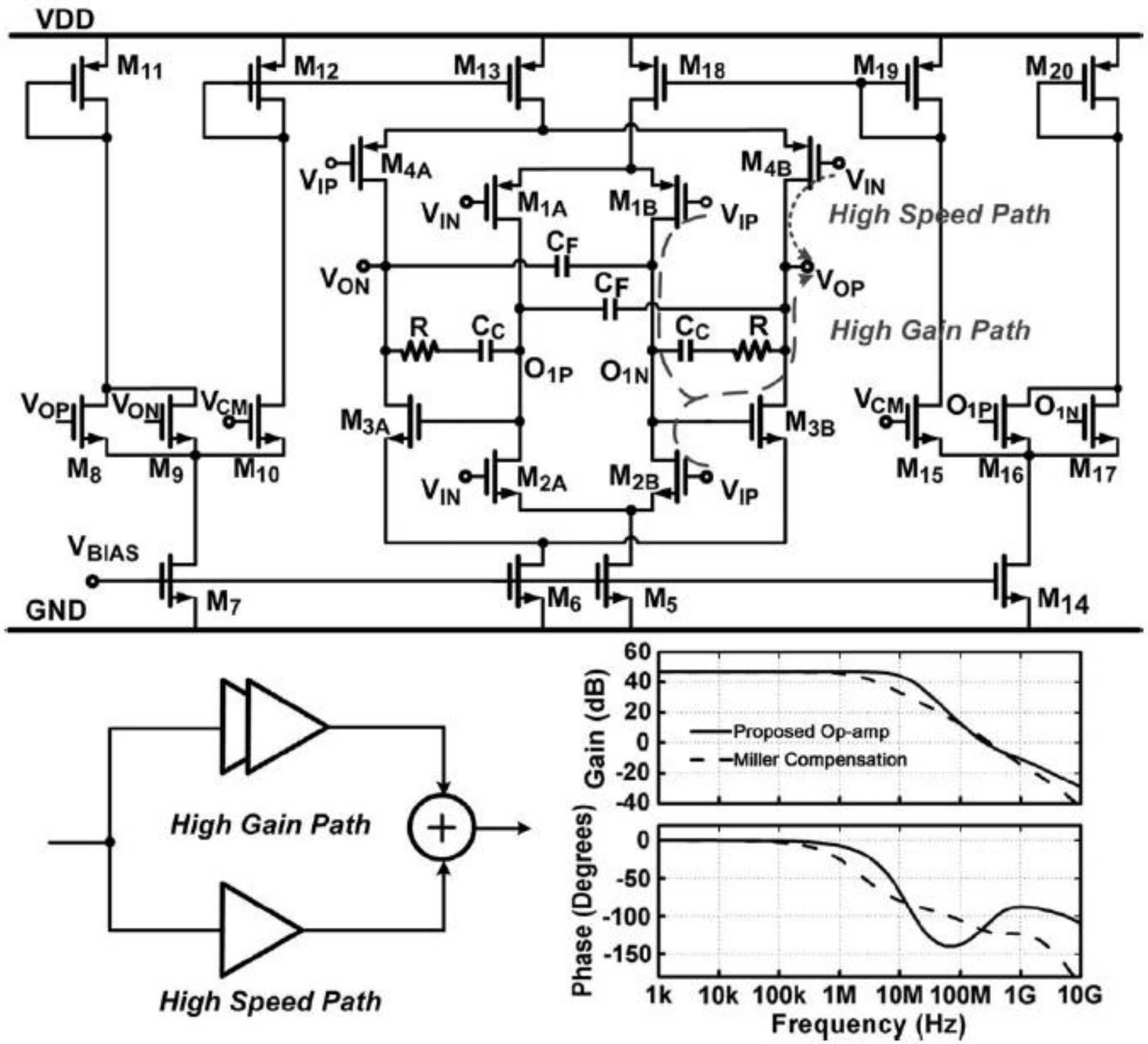


Fig. 3. Simplified op-amp cell of the FLOA.

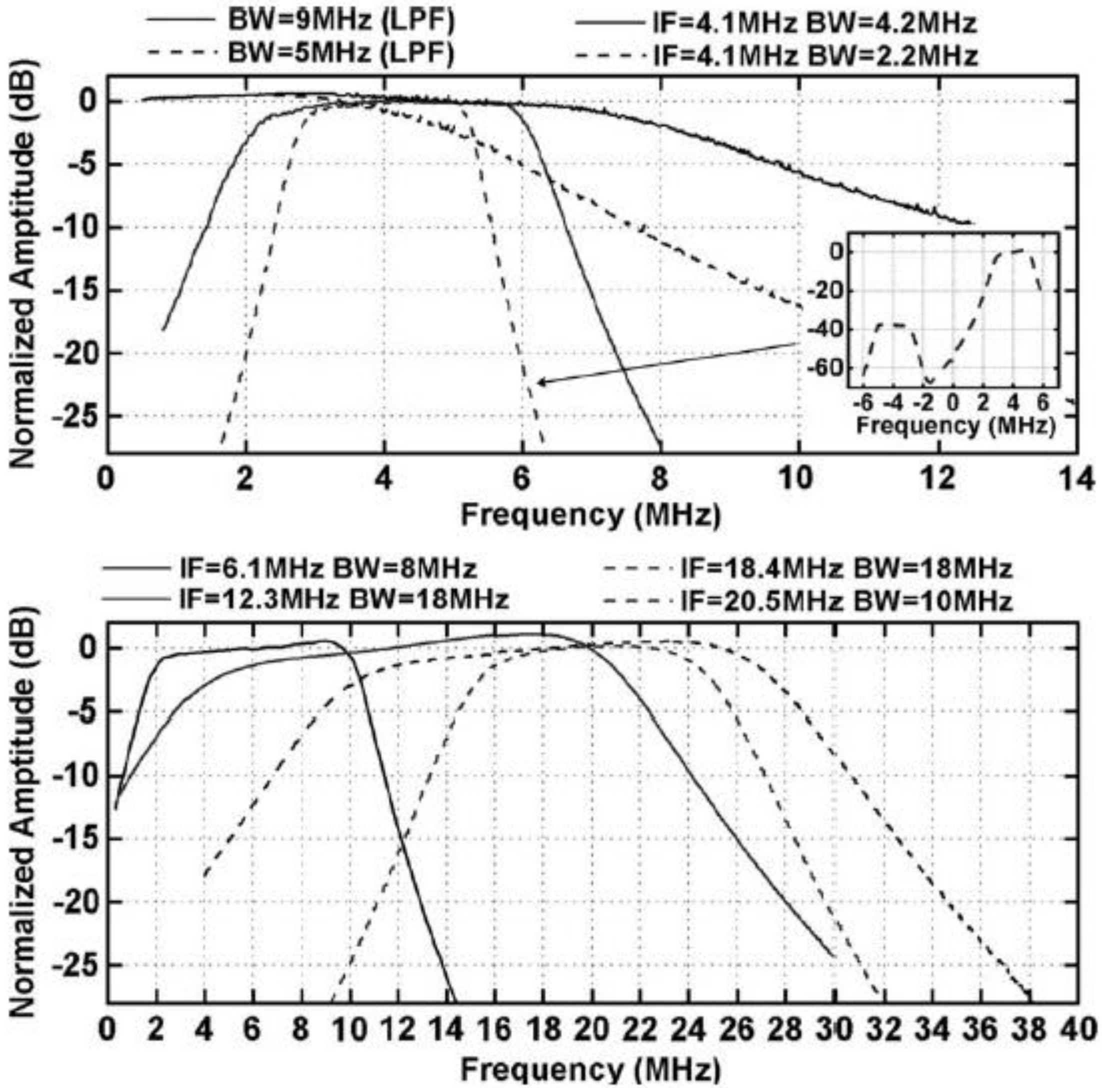


Fig. 7. Measured ac response of the LPF and the CBPF.

EE 508

Lecture 43

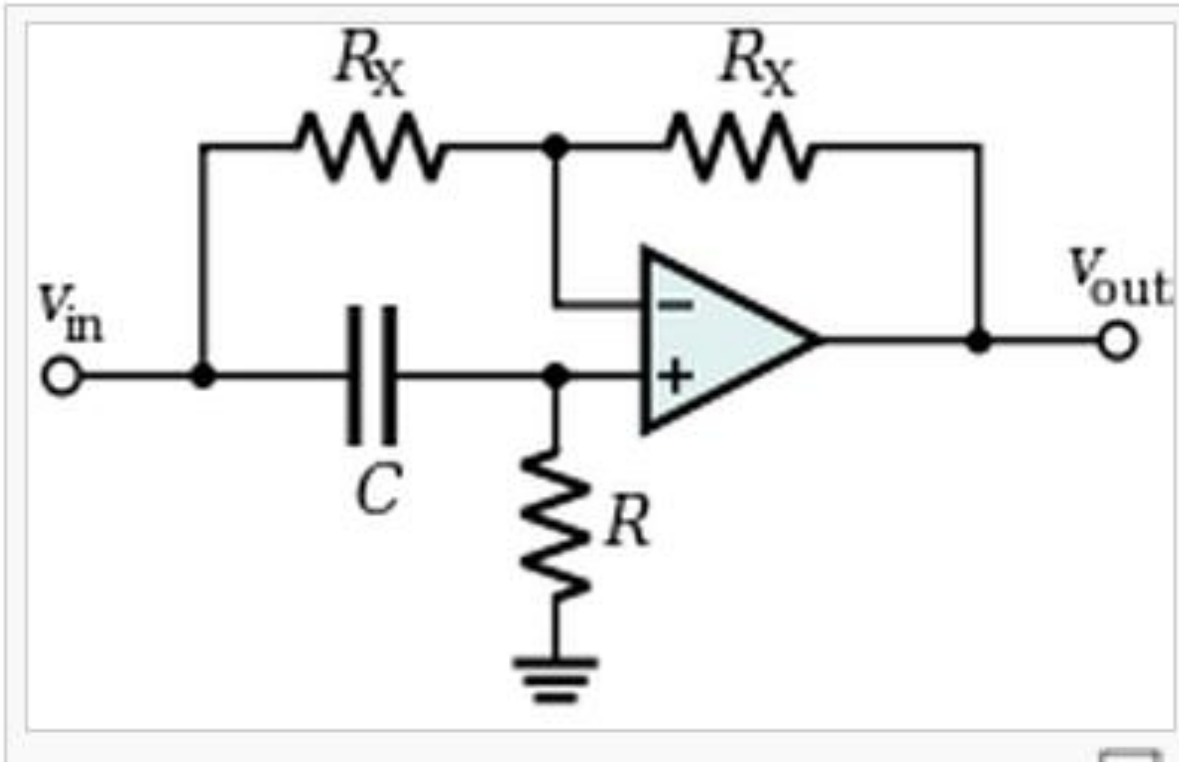
Basic Filter Components

- All Pass Networks
- Arbitrary Transfer Function Synthesis
- Impedance Transformation Circuits
- Equalizers

All-Pass Circuits

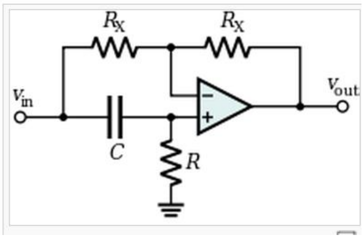
- Magnitude of Gain is Constant
- Phase Changes with Frequency
- Used to correct undesired phase characteristics of a filter

First-Order All Pass

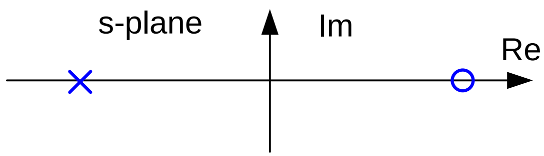


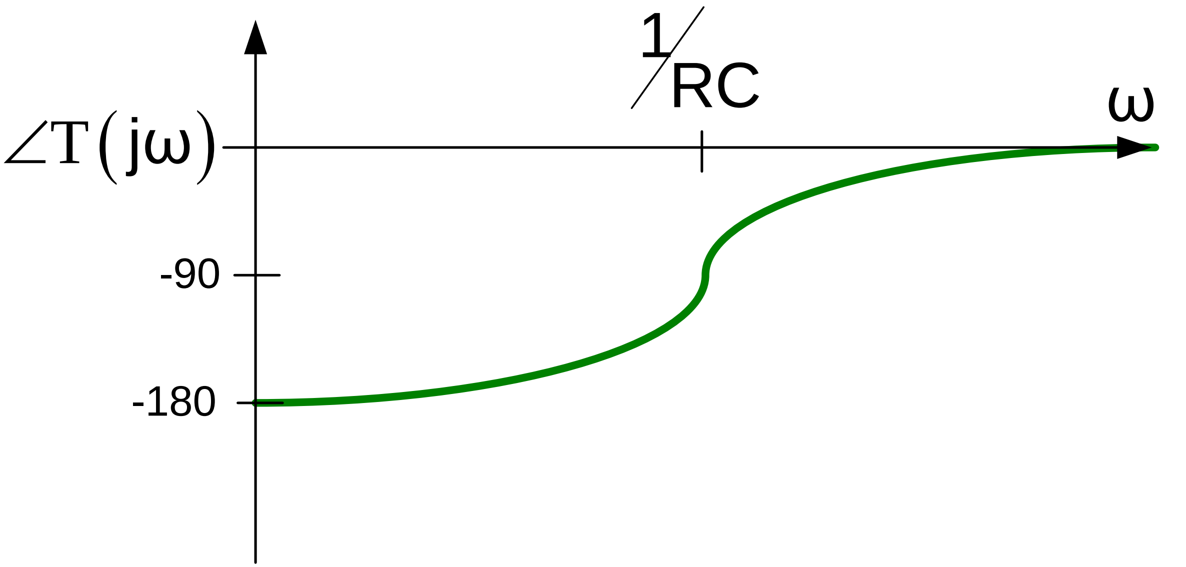
$$T(s) = \frac{s - \frac{1}{RC}}{s + \frac{1}{RC}}$$

First-Order All Pass

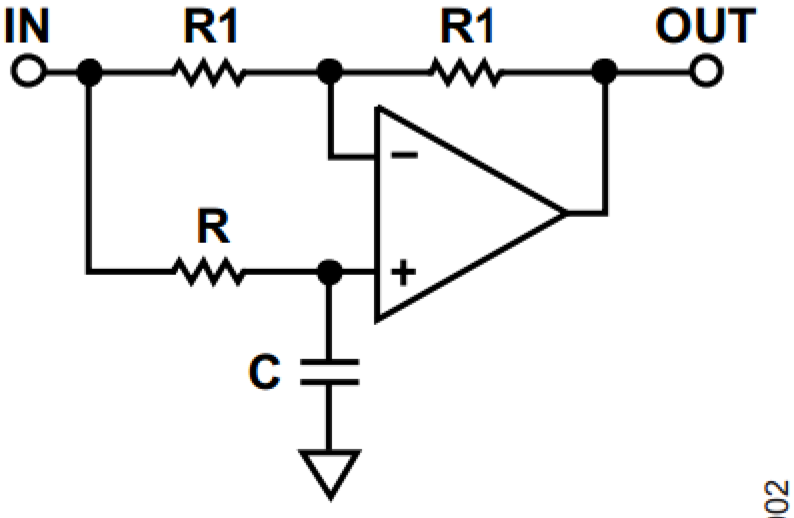


$$T(s) = \frac{s - \frac{1}{RC}}{s + \frac{1}{RC}}$$



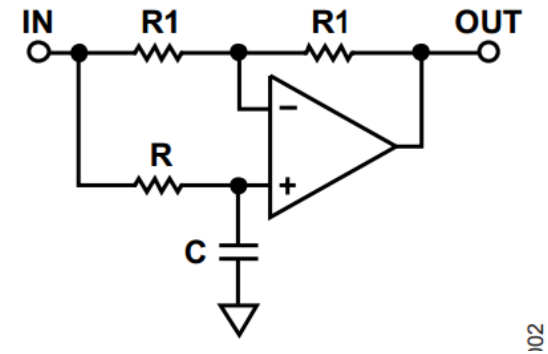


First-Order All Pass

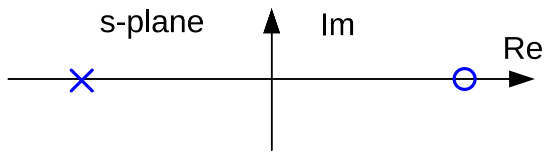


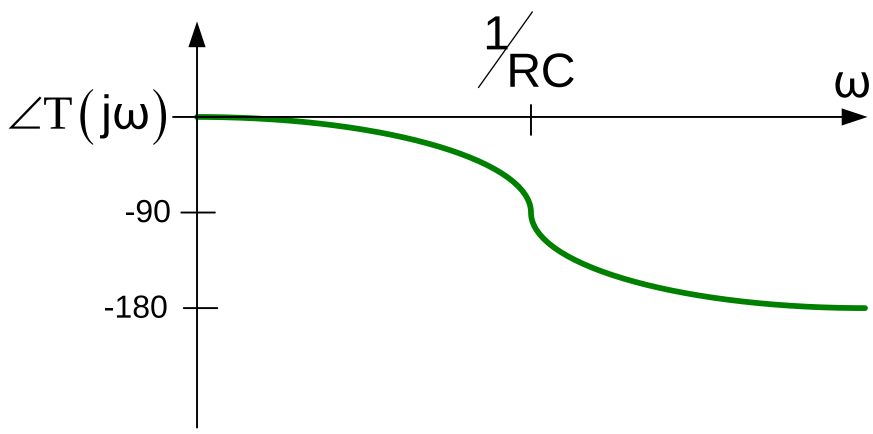
$$T(s) = -\frac{s - \frac{1}{RC}}{s + \frac{1}{RC}}$$

First-Order All Pass

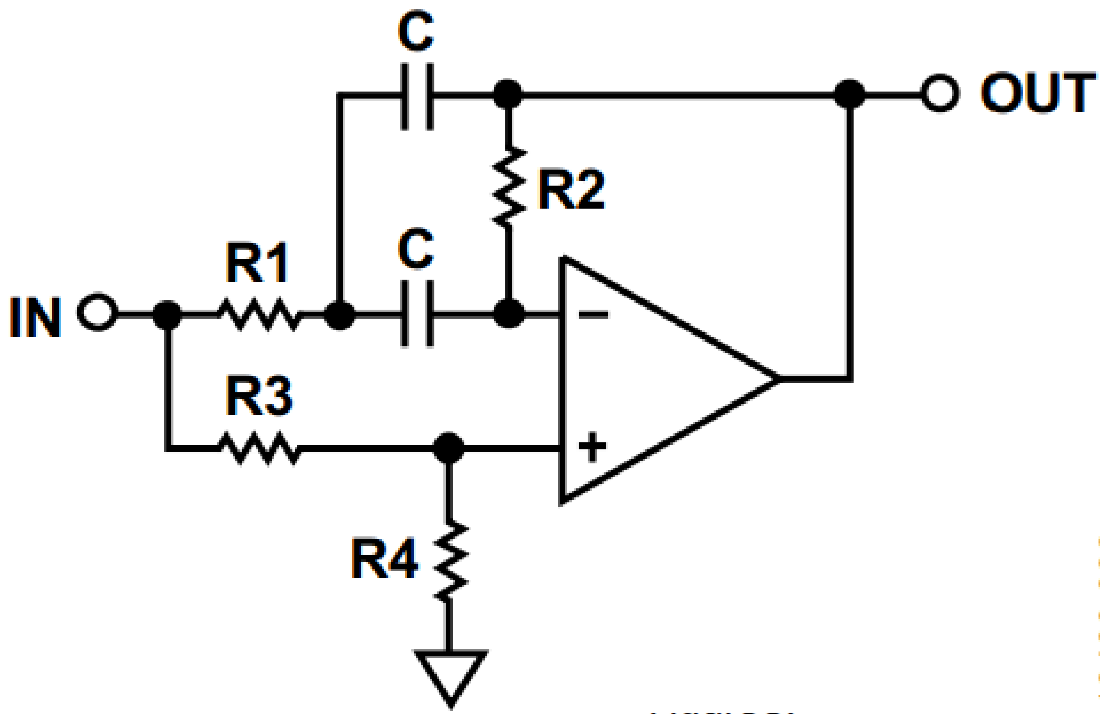


$$T(s) = -\frac{s - \frac{1}{RC}}{s + \frac{1}{RC}}$$





Second-Order All Pass

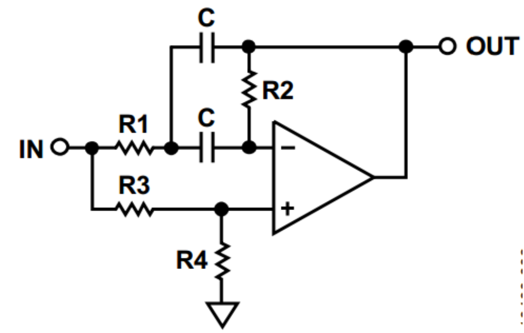


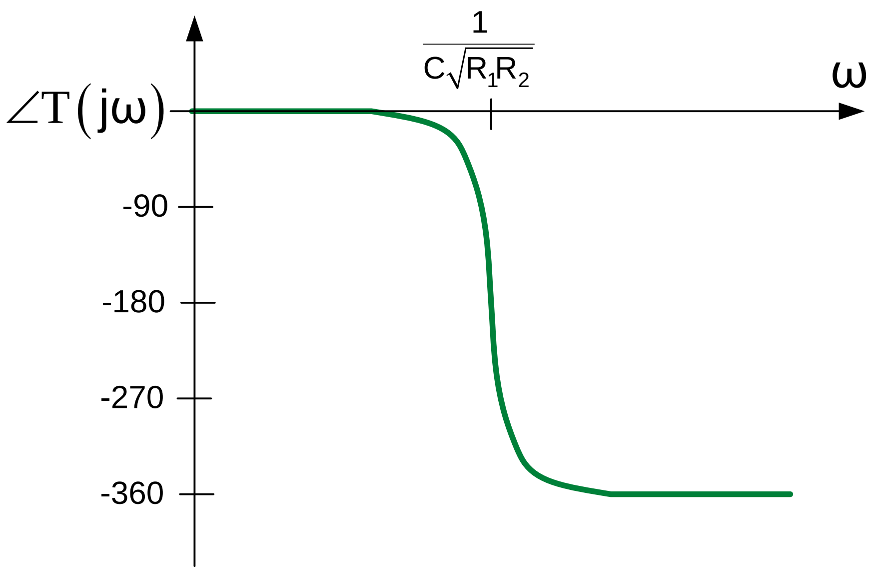
$$\frac{V_O}{V_{IN}} = \frac{s^2 - s\left(\frac{2}{R2\,C}\right) + \frac{1}{R1\ R2\ C^2}}{s^2 + s\left(\frac{2}{R2\,C}\right) + \frac{1}{R1\ R2\ C^2}}$$

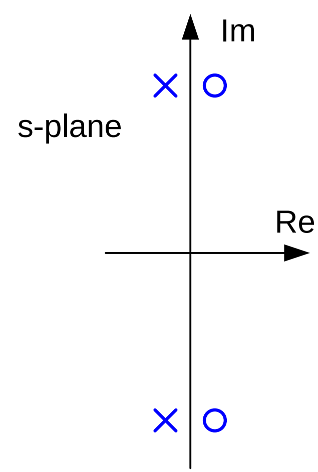
Based upon Bridged-T Feedback Structure

Second-Order All Pass

$$\frac{V_O}{V_{IN}} = \frac{s^2 - s\left(\frac{2}{R2\,C}\right) + \frac{1}{R1\;R2\;C^2}}{s^2 + s\left(\frac{2}{R2\,C}\right) + \frac{1}{R1\;R2\;C^2}}$$



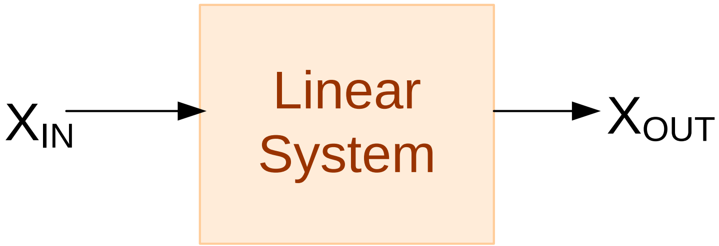




Arbitrary Transfer Function Synthesis

- Based upon coefficient derivation
- Can be used to implement/solve an arbitrary differential equation
- Versatile
- Basic concept of Analog Computer

Applications of integrators to solving differential equations



Standard Integral form of a differential equation

$$X_{OUT} = b_1 \int X_{OUT} + b_2 \iint X_{OUT} + b_3 \iiint X_{OUT} + \ldots + a_0 X_{IN} + \int X_{IN} + \iint X_{IN} + \ldots$$

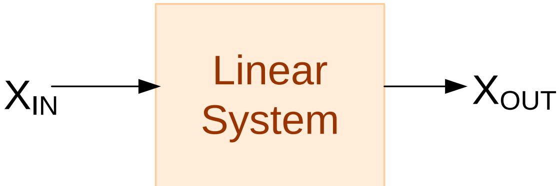
Standard differential form of a differential equation

$$X_{OUT} = \alpha_1 X_{OUT}^{'} + \alpha_2 X_{OUT}^{''} + \alpha_3 X_{OUT}^{'''} + \ldots + \beta_1 X_{IN} + \beta_2 X_{IN}^{'} + \beta_3 X_{IN}^{''} + \ldots$$

Initial conditions not shown

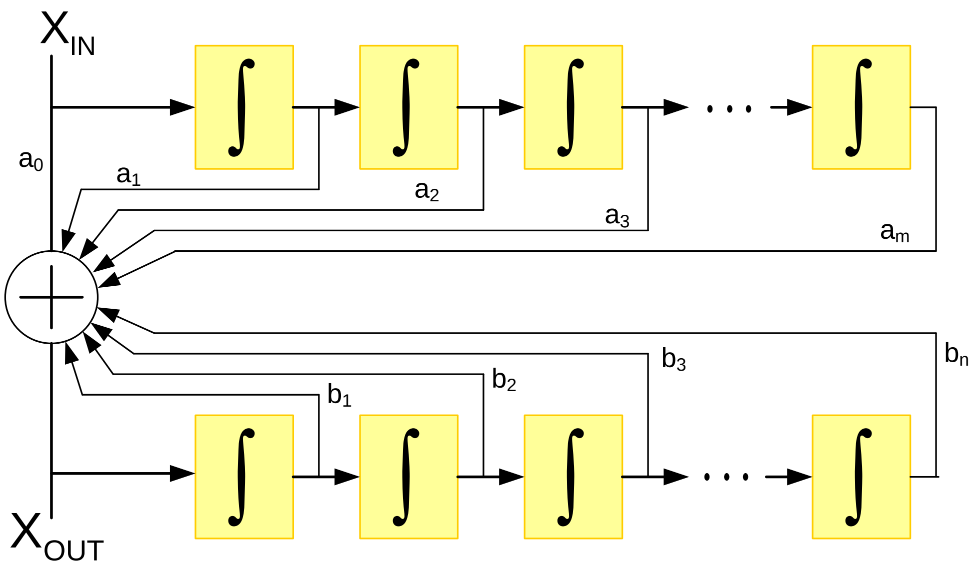
Can express any system in either differential or integral form

Applications of integrators to solving differential equations



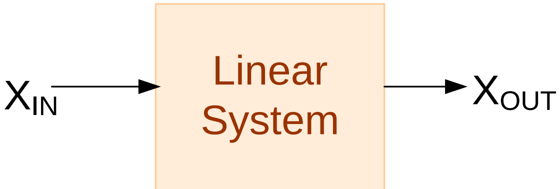
Consider the standard integral form

$$X_{OUT} = b_1 \int X_{OUT} + b_2 \iint X_{OUT} + b_3 \iiint X_{OUT} + \ldots + a_0 X_{IN} + \int X_{IN} + \iint X_{IN} + \ldots$$



This circuit is comprised of summers and integrators
Can solve an arbitrary linear differential equation
This concept was used in Analog Computers in the past

Applications of integrators to solving differential equations



Consider the standard integral form

$$X_{OUT} = b_1 \int X_{OUT} + b_2 \iint X_{OUT} + b_3 \iiint X_{OUT} + \ldots + a_0 X_{IN} + \int X_{IN} + \iint X_{IN} + \ldots$$

Take the Laplace transform of this equation

$$\mathcal{X}_{OUT} = b_1 \frac{1}{s}\mathcal{X}_{OUT} + b_2 \frac{1}{s^2}\mathcal{X}_{OUT} + b_3 \frac{1}{s^3}\mathcal{X}_{OUT} + \ldots + b_n \frac{1}{s^n} + a_0 \mathcal{X}_{IN} + a_1 \frac{1}{s}\mathcal{X}_{IN} + a_2 \frac{1}{s^2}\mathcal{X}_{IN} + a_3 \frac{1}{s^3}\mathcal{X}_{IN} + \ldots + a_m \frac{1}{s^m}$$

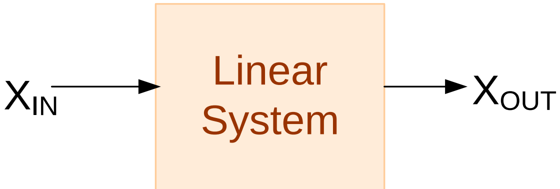
Multiply by s^n and assume m=n (some of the coefficients can be 0)

$$s^n \mathcal{X}_{OUT} = b_1 s^{n-1}\mathcal{X}_{OUT} + b_2 s^{n-2}\mathcal{X}_{OUT} + b_3 s^{n-3}\mathcal{X}_{OUT} + \ldots + b_n + a_0 s^n \mathcal{X}_{IN} + a_1 s^{n-1}\mathcal{X}_{IN} + a_2 s^{n-2}\mathcal{X}_{IN} + a_3 s^{n-3}\mathcal{X}_{IN} + \ldots + a_n$$

$$\mathcal{X}_{OUT}\left(s^n - b_1 s^{n-1} - b_2 s^{n-2} - b_3 s^{n-3} - \ldots - b_n\right) = \mathcal{X}_{IN}\left(a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3} + \ldots + a_n\right)$$

$$T(s) = \frac{\mathcal{X}_{OUT}}{\mathcal{X}_{IN}} = \frac{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3} + \ldots + a_n}{s^n - b_1 s^{n-1} - b_2 s^{n-2} - b_3 s^{n-3} - \ldots - b_n}$$

Applications of integrators to solving differential equations



Consider the standard integral form

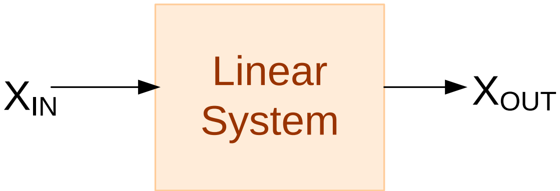
$$X_{OUT} = b_1 \int X_{OUT} + b_2 \iint X_{OUT} + b_3 \iiint X_{OUT} + \ldots + a_0 X_{IN} + \int X_{IN} + \iint X_{IN} + \ldots$$

$$T(s) = \frac{\mathcal{X}_{OUT}}{\mathcal{X}_{IN}} = \frac{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3} + \ldots + a_n}{s^n - b_1 s^{n-1} - b_2 s^{n-2} - b_3 s^{n-3} - \ldots - b_n}$$

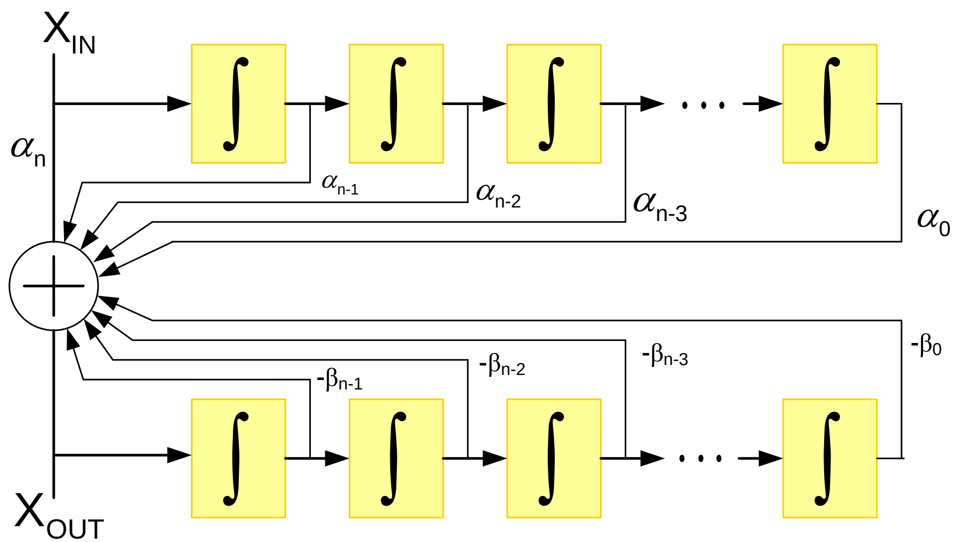
This can be written in more standard form

$$T(s) = \frac{\alpha_n s^n + \alpha_{n-1} s^{n-1} + \ldots \alpha_1 s + \alpha_0}{s^n + \beta_{n-1} s^{n-1} + \ldots + \beta_1 s + \beta_0}$$

Applications of integrators to filter design



$$T(s) = \frac{\alpha_n s^n + \alpha_{m-1} s^{n-1} + \ldots \alpha_1 s + \alpha_0}{s^n + \beta_{n-1} s^{n-1} + \ldots + \beta_1 s + \beta_0}$$



Can design (synthesize) any T(s) with just integrators and summers !

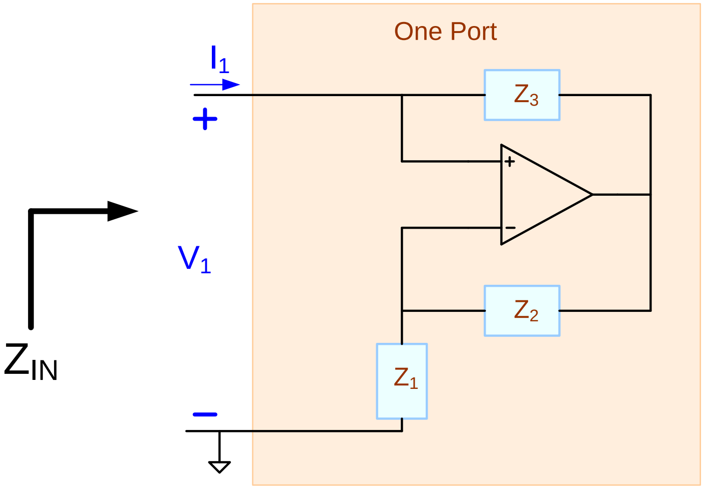
Integrators are not used "open loop" so loss is not added

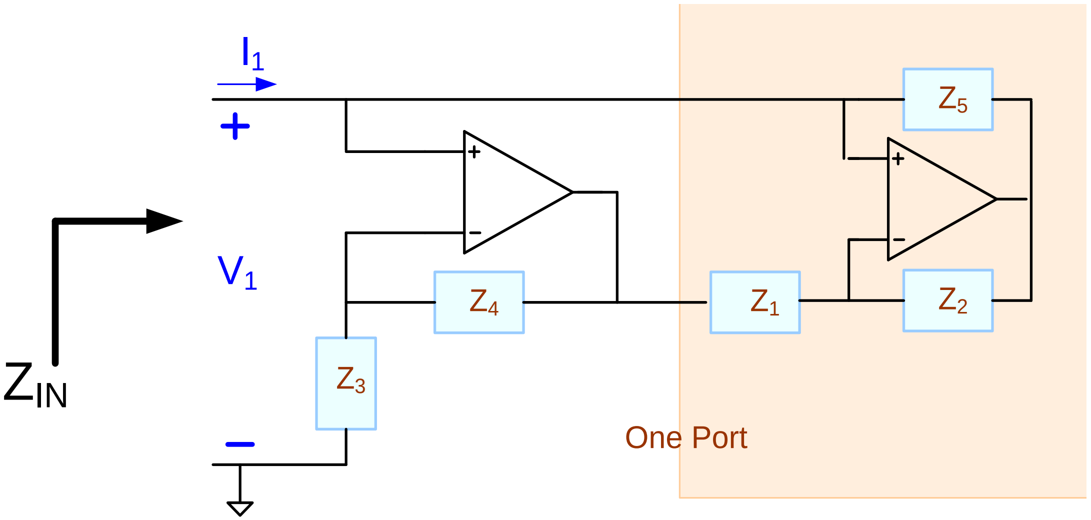
Although this approach to filter design works, often more practical methods are used

Impedance Synthesis

- Focus on synthesizing impedance rather than transfer function
- Gyrators will provide inductance simulation
- Capacitance Multiplication
- Synthesis of super components

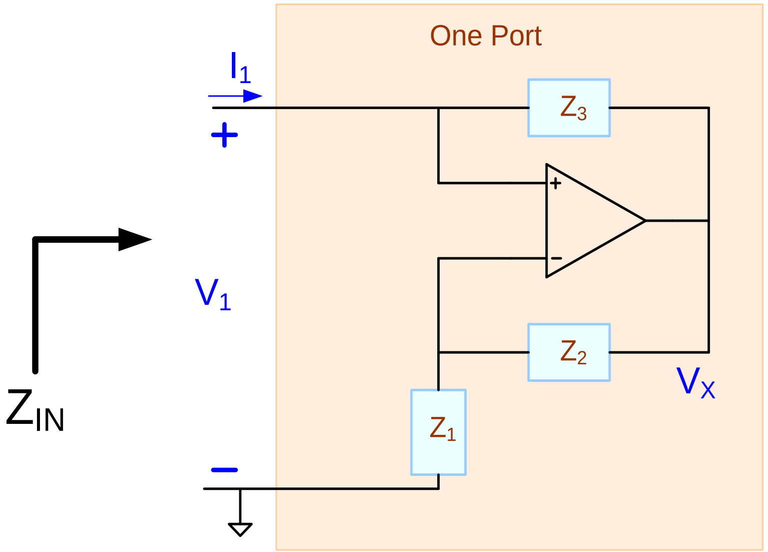
Impedance Converters





Note these circuits are strictly one-ports and have no output node

Impedance Converters



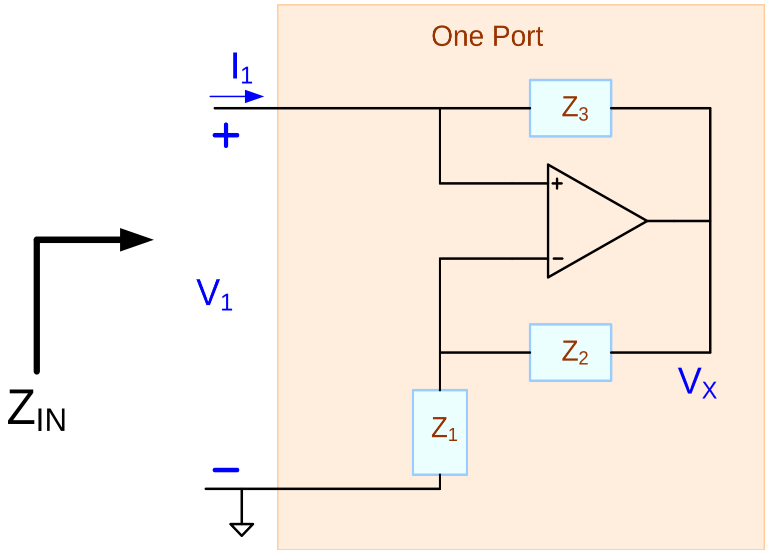
$$V_1(G_1+G_2) = V_X G_2$$

$$I_1=(V_1-V_X)G_3$$

$$Z_{IN} = -\frac{Z_1 Z_3}{Z_2}$$

Observe this input impedance is negative!

Impedance Converters



$$Z_{IN} = -\frac{Z_1 Z_3}{Z_2}$$

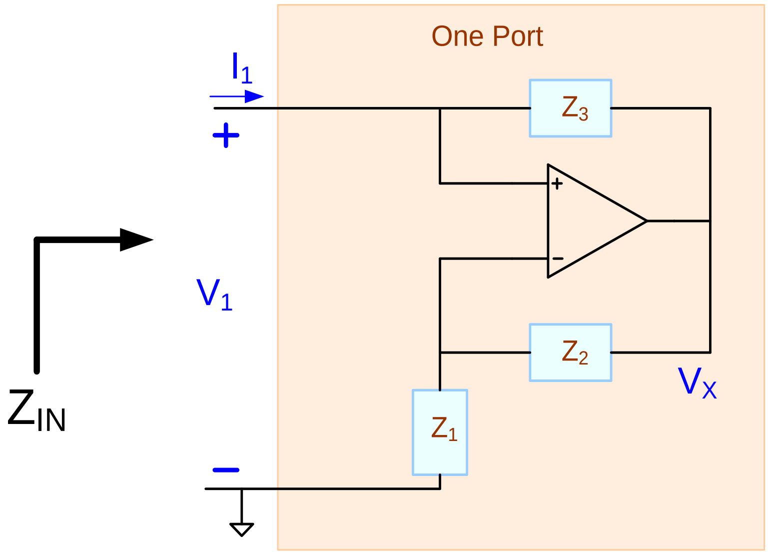
If $Z_1=R_1$, $Z_2=R_2$ and $Z_3=R_3$, $\quad Z_{IN} = -\frac{R_1 R_3}{R_2}$ This is a negative resistor !

If $Z_2=1/sC$, $Z_1=R_1$ and $Z_3=R_3$, $\quad Z_{IN} = -sCR_1R_3$ This is a negative inductor !

If $Z_2=R_2$, $Z_1=1/sC$ and $Z_3=R_3$, $\quad Z_{IN} = -\frac{R_3}{sCR_2}$ This is a negative capacitor !

This is termed a Negative Impedance Converter

Impedance Converters



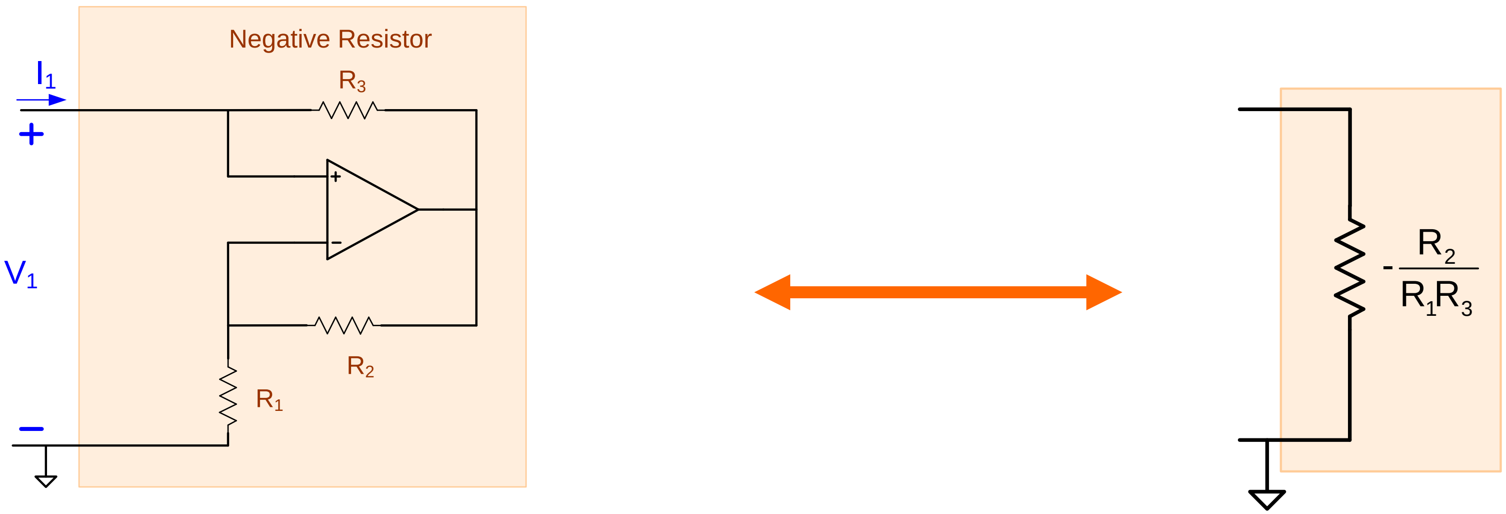
$$Z_{IN} = -\frac{Z_1 Z_3}{Z_2}$$

If $Z_2 = 1/sC$, $Z_1 = R_1$ and $Z_3 = R_3$, $\quad Z_{IN} = -sCR_1R_3$

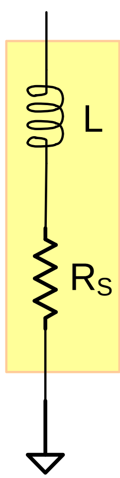
Modification of NIC to provide a positive inductance:

Replace Z_1 itself with a second NIC that has a negative input impedance

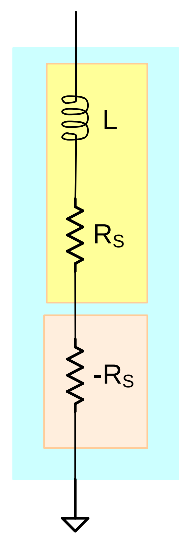
Negative Impedance Converter



One application of NIC



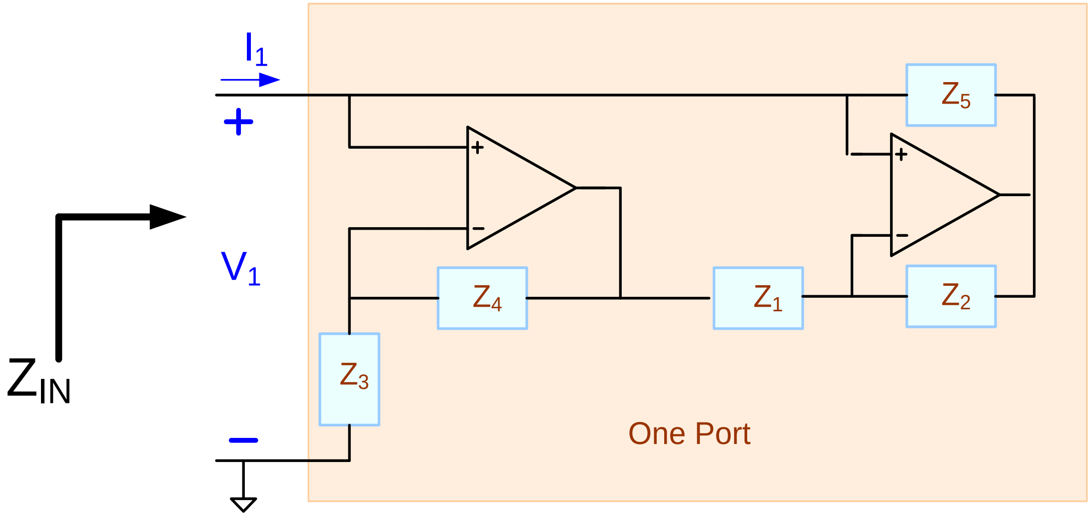
Lossy Inductor



If select components so that $R_S = \frac{R_2}{R_1 R_3}$

Lossless Inductor

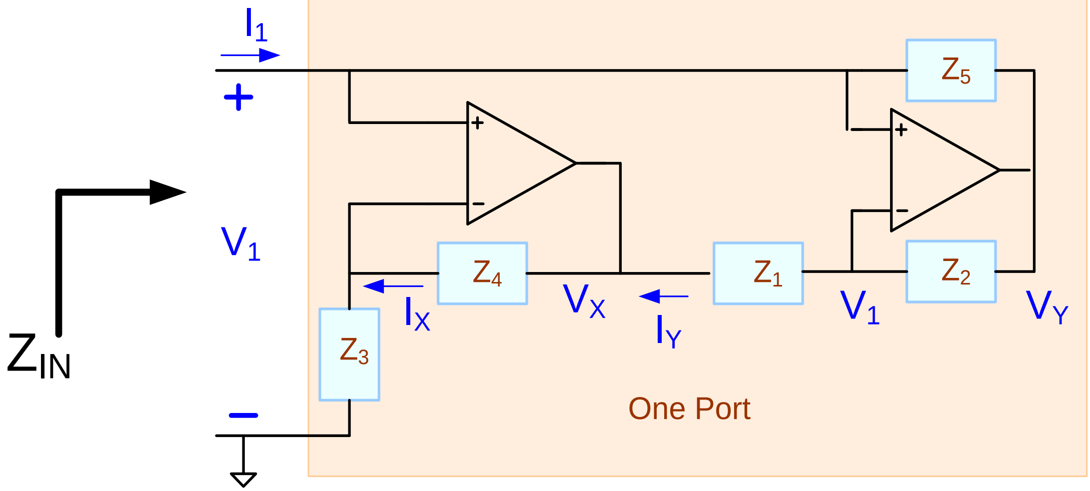
Impedance Converters



$$Z_{IN} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$

This circuit is often called a Gyrator

Gyrator Analysis



$$I_X = V_1 G_3$$

$$V_X = V_1 + V_1 G_3 / G_4 \;= V_1\left(1 + \frac{G_3}{G_4}\right)$$

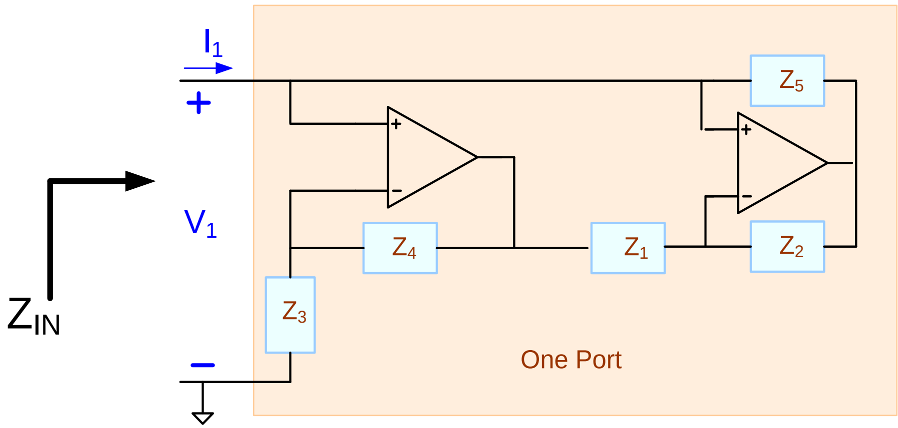
$$I_Y = (V_1 - V_X) G_1 \;= V_1\left(-\frac{G_3}{G_4}\right) G_1$$

$$V_Y = V_1 + I_Y / G_2 \;= V_1\left(1 - \frac{G_3}{G_4}\left(\frac{G_1}{G_2}\right)\right)$$

$$I_1 = (V_1 - V_Y) G_5 = V_1\left(\frac{G_3}{G_4}\left(\frac{G_1}{G_2}\right)\right) G_5$$

$$Z_{IN} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$

Gyrator Applications



$$Z_{IN} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$

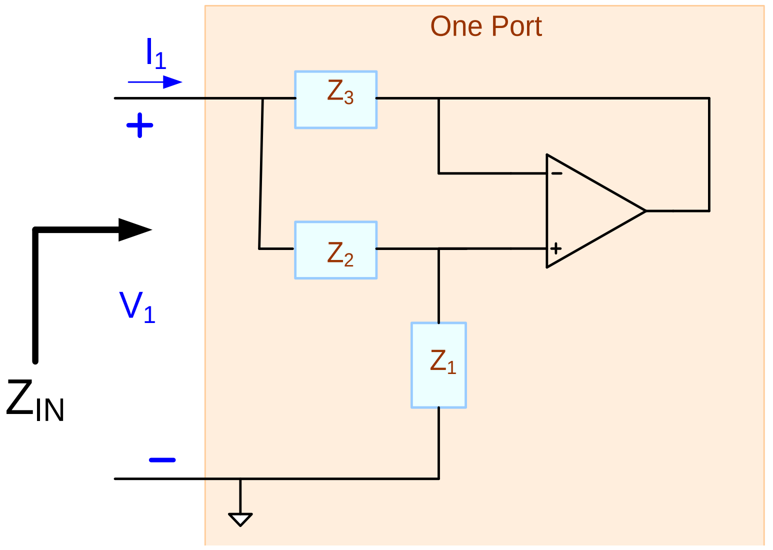
If $Z_1=Z_3=Z_4=Z_5=R$ and $Z_2=1/sC$ $\quad Z_{IN} = \left(R^2C\right)s$ $\quad$ This is an inductor of value $L=R^2C$

If $Z_2=R_2$, $Z_3=R_3$, $Z_4=R_4$, $Z_5=R_5$ and $Z_1=1/sC$ $\quad Z_{IN} = \frac{R_3R_5}{sCR_2R_4}$

This is a capacitor of value $\quad C_{EQ} = C\frac{R_2R_4}{R_3R_5}$ $\quad$ (can scale capacitance up or down)

If $Z_2=Z_4=Z_5=R$ and $Z_1=Z_3=1/sC$ $\quad Z_{IN} = \left(R^3C^2\right)s^2$ $\quad$ This is a “super” capacitor of value R^3C^2

Impedance Converters

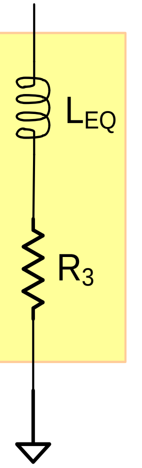


$$I_1 = \left(V_1 - \left(\frac{Z_1}{Z_1 + Z_2} \right) V_1 \right) G_3$$

$$Z_{IN} = Z_3 \left(1 + \frac{Z_2}{Z_1} \right)$$

If $Z_3 = R_3$, $Z_2 = R_2$ and $Z_1 = 1/sC$

$$Z_{IN} = R_3 + s\left(CR_2R_3\right)$$

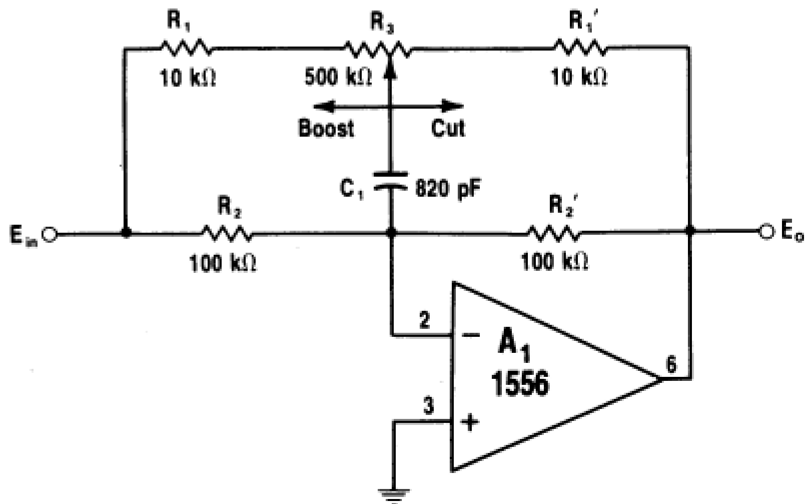


$$L_{EQ} = CR_2R_3$$

Shelving Equalizers

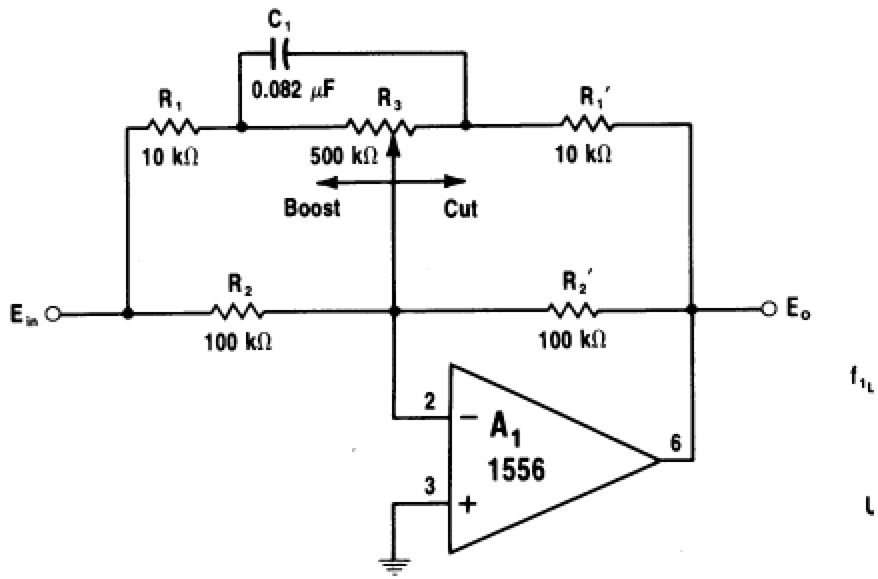
- Widely used in audio applications
- User-programmable filter response

Shelving Equalizers



(A) High frequency.

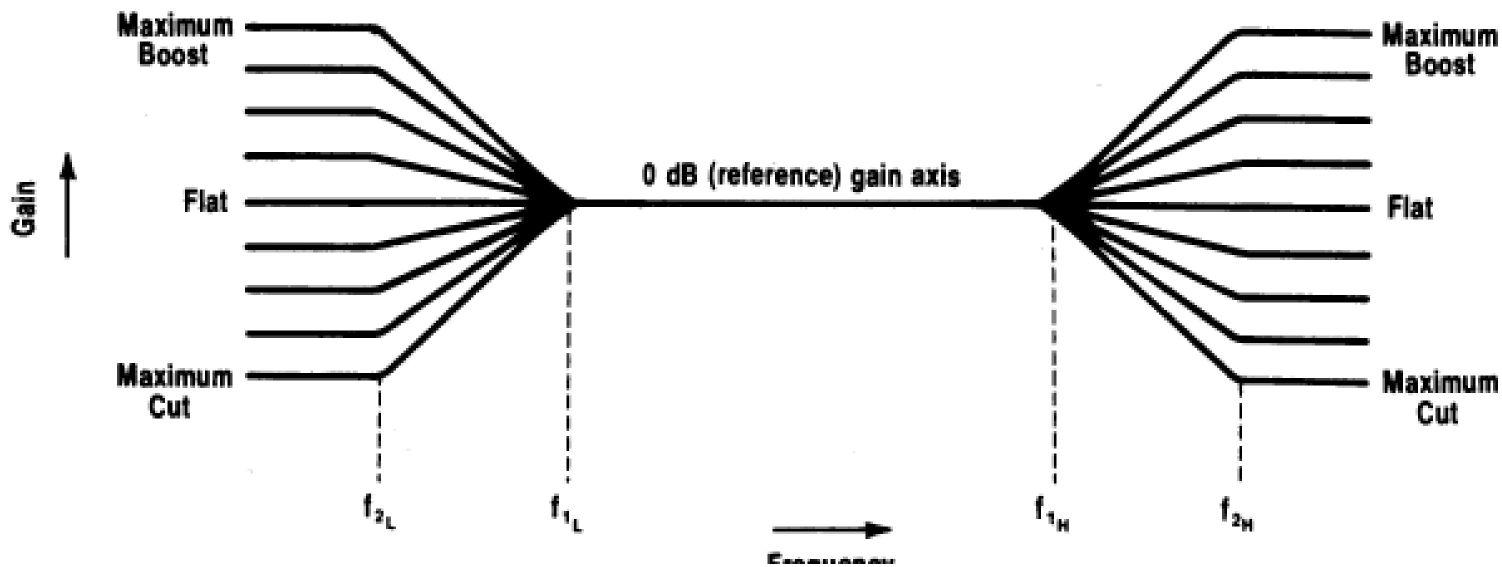
Shelving Equalizers



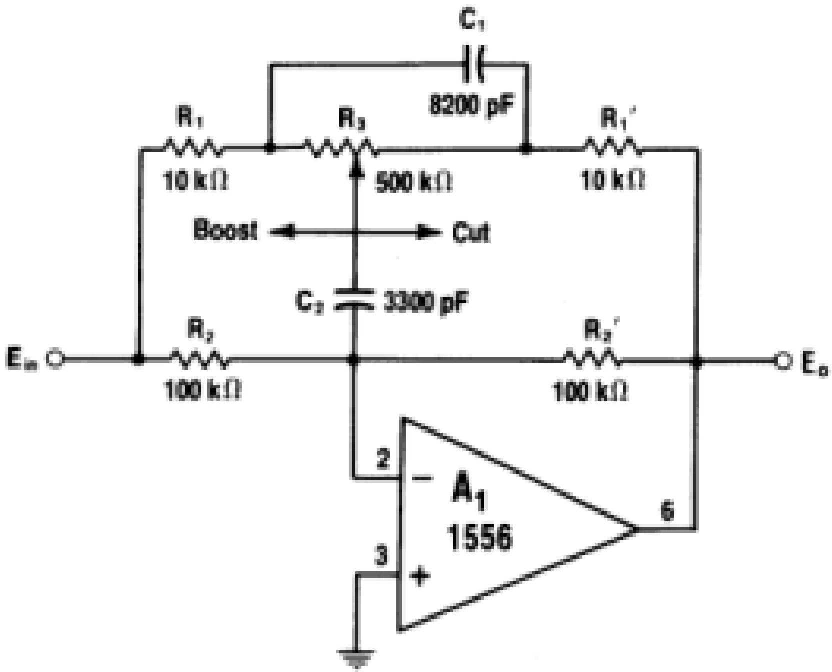
(B) Low frequency.

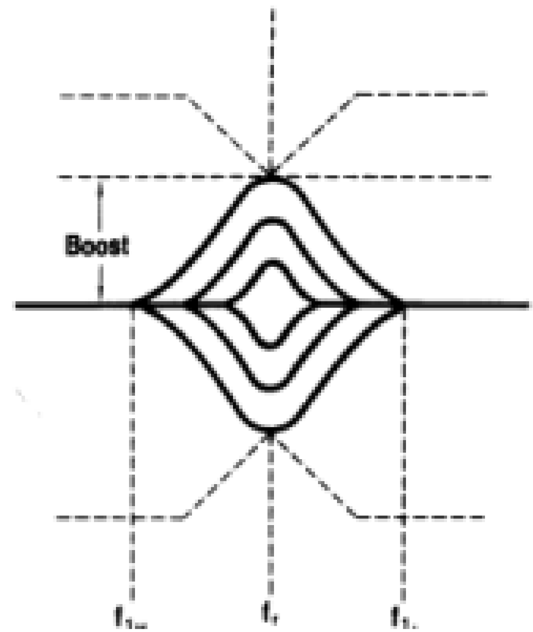
Fig. 6-37. Shelving equalizers.

Shelving Equalizers



- The expressions for f_L and f_H for the previous two circuits show a small movement with the potentiometer position in contrast to the fixed point location depicted in this figure
- The OTA-C filters discussed earlier in the course can be designed to have fixed values for f_L and f_H when cut or boost is used.


C1
8200 pF
R1
10 kΩ
R3
500 kΩ
R1'
10 kΩ
Boost
Cut
C2
3300 pF
Ein
R2
100 kΩ
R2'
100 kΩ
Eo
2
3
6
A1
1556


Boost

End of Lecture 43

EE 508

Lecture 44

Conventional Wisdom – Benefits and Consequences of Annealing Understanding of Engineering Principles

by Randy Geiger
Iowa State University

Summary of Recent Published Filter Architectures thanks to Yongjie Jiang

Summary by Application

I. Channel Selection Filter in communication system

Architecture	Applicati0on	Interests	Reference
active RC	GSM channel selection filter, signal bandwidth: 100kHz	Low voltage design	2011, JSSC, pp.2268-
active RC (active L) RC	WCDMA channel selection filter, Signal bandwidth 2MHz	New (Active L) RC current biquads cell	2010,JSSC, pp.1770-
active RC	Channel selection and anti-aliasing filter in receiver Cut-off frequency 11MHz	Using dominant poles of opamps to create 4-th order filter in a single cell	2011,ISCAS,pp.1644
active RC	Low IF Channel selection band-pass filter IF Central frequency 1MHz		2010,JSSC,pp.538
gm-C	Channel selection filter for blue tooth and zigbee cut off frequency 200kHz to 2Mhz	Linearization skills	Garcia-Alberdi, 2012, TCAS1
active RC	Channel selection filter for zero IF and low IF receiver. IF frequency: 4.1MHz to 20.6Mhz, Bandwidth: 2.2 to 18MHz		2012,TCASII,pp.30

II. RF Filters in communication system

High frequency filter in receiver used as LO harmonic rejection, Sub-band filtering

Architecture	Application	Interests	Reference
Passive RLC	RF filter, reject LO harmonics LO frequency: 300Mhz to 800Mhz		2012,JSSC,pp.392
gm-C	RF Filter, reject LO harmonics LO frequency: 30Mhz to 900Mhz	The author claims that linearity problem can be relaxed due to harmonic Rejection Mixer	2012,JSSC,pp.1084
Passive RLC	RF filter, reject LO harmonics filter sub-band noise in spectrum monitor receiver Signal bandwidth 1G and 200MHz LO frequency: 9.45G and 1.75G	New method to select passives.	2011 Electronics Letter November, No.24
Passive RLC	reject LO harmonics LO frequency: 24GHz		2010,TCASII,pp.522
Passive RC	Poly phase filter operate in 60GHz for sub-harmonic Mixers		2010,JSSC,pp.1644

gm-C	Digital TV tuner LO frequency: 48MHz to 780MHz bandwidth: 15MHz	RF tracking technique	2011,Electronics Letter, pp.407
Hybrid RC-gm-C filter	Digital TV tuner, LO frequency: 48Mhz to 200Mhz	Linearization skills	2011,TCASI,pp.2346
Switched resistor C	Digital TV tuner, LO frequency:100MHz to 1GHz bandwidth 35MHz		2011,JSSC,pp.998

III. Sub-audio frequency application

Architecture	Application	Interests	Reference
Active MOSFET-C	5mHz cut off frequency for electrophysiological signal acquisitions		2012, Electronics Letter, pp. 698
MOSFET-C	sub-Hz high-pass filter for sensors		2011,TCASI,pp.1561
MOSFET-C	Neural signal recording HPF: 2.3Hz to 572Hz LPF: 200Hz and 6.2kHz		2011,ISCAS,pp.1451

IV. Loop Filter In PLL

Architecture	Application	Interests	Reference
Switched resistor filter	Loop Filter in PLL PLL bandwidth 30KHz PLL reference frequency 5MHz	Using Switched resistor filter to eliminate charge pump and reduce reference spur	2011, JSSC, pp.2566-
Switched capacitor filter	Used as loop filter in PLL with reference frequency 10MHz for reducing reference spur and capacitor size		2011,TCASII,pp.555

V. Others

Architecture	Application	Interests	Reference
R-MOSFET-C	91 to 268khz tuning range	Cross couple linear region transistor to improve the linearity	2012,JSSC,pp.2751
Active RC	7Mhz to 20Mhz cut-off frequency	Power efficient op-amp	2011,ISCAS,pp.2751
gm-C	100Hz to 10Mhz multiple purpose filter	Modified biquads structure to improve the linearity	2011,TCASII.pp.159
gm-C	Motion Sensor	Floating gate array to program cutoff frequency	2011,ISCAS,pp.2425
MISO universal filter, 7 transistors biquad cell	10MHz Cut off frequency	Simple 7 transistors new bi-quads cell for HP,LP and BP application	2011,TCASII, pp.356
Digital Filter	Digital Hearing aids		2010,TCASI,pp.584

Conventional Wisdom:

Conventional wisdom is the collective understanding of fundamental engineering concepts and principles that evolves over time through interactions of practicing engineers around the world

Conventional Wisdom:

- Guides engineers in daily practice of the Profession
- Widely use to enhance productivity
- Heavily emphasized in universities around the world when educating next-generation engineers
- Often viewed as a fundamental concept or principle
- Validity of conventional wisdom seldom questioned

Are Conventional Wisdom and Fundamental Concepts and Principles Always Aligned?

Much of Society till 1200AD to 1600AD and later

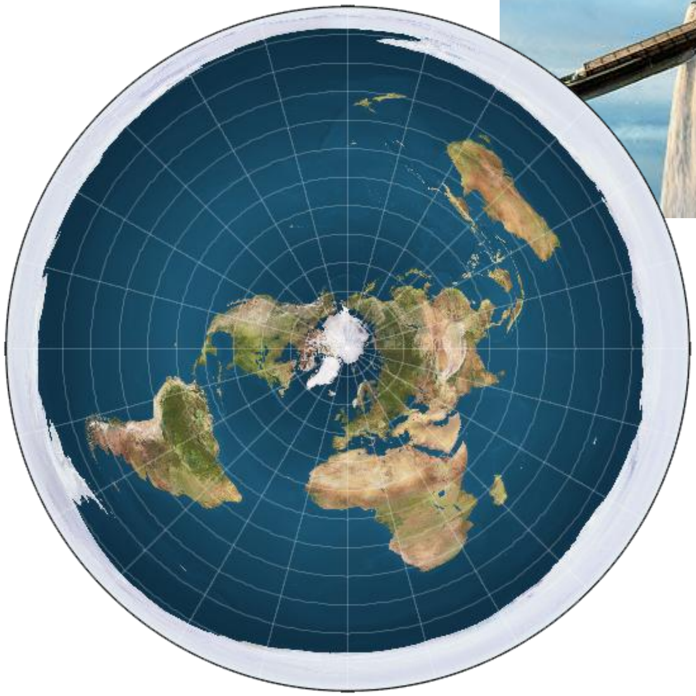
http://greenfunkdan.blogspot.com/2008/11/csiro-warns-of-climate-change-doomsday.html

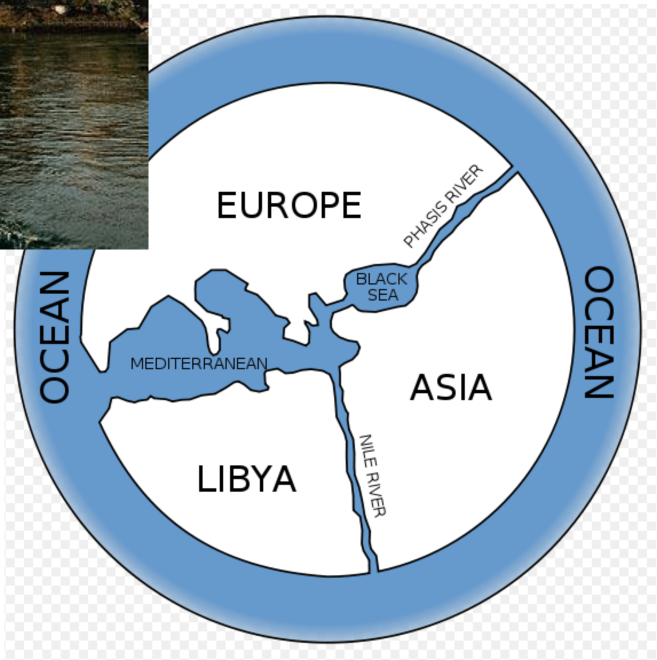
Pythagoras 520BC Aristotle 300BC

http://www.christiananswers.net/q-aig/aig-c034.html

Sometimes the differences can be rather significant !

Conventional wisdom, when not correctly representing fundamental principles, can provide conflicting perceptions or irresolvable paradoxes





Are Conventional Wisdom and Fundamental Concepts always aligned in the Microelectronics Field ?

Introduction: This is "CW" who reflects the Conventional Wisdom that has evolved.

CW will share his views with us, on occasion, throughout this presentation

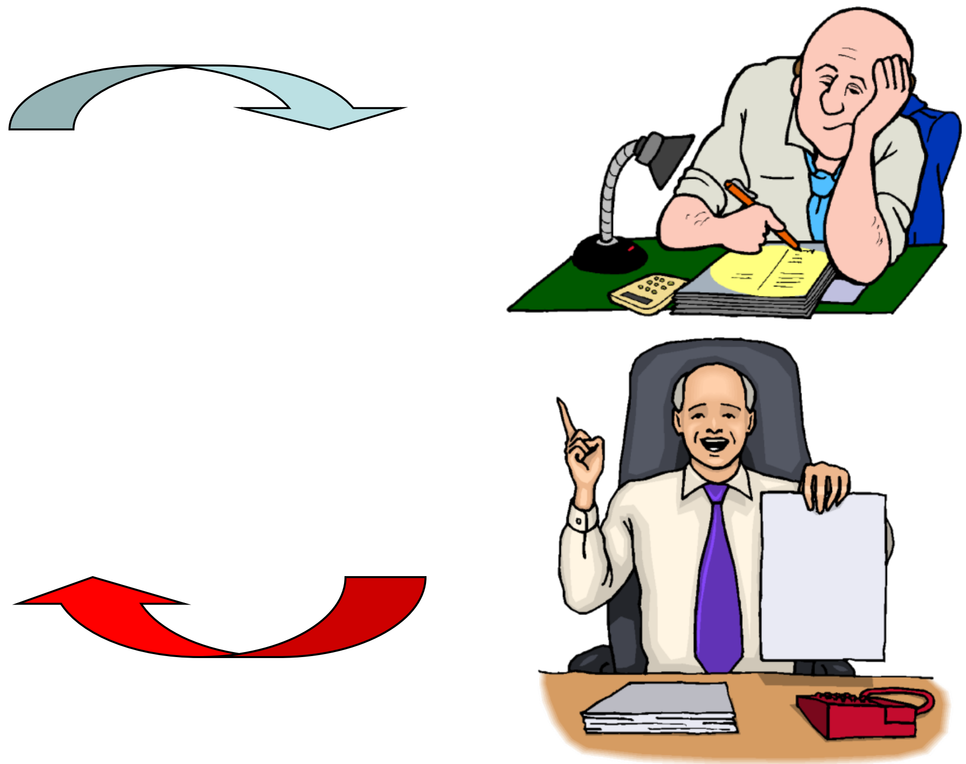
Are Conventional Wisdom and Fundamental Concepts always aligned in the Microelectronics Field ?

Records of

- **Conventional Wisdom**
- **Fundamental Concepts**
- **Occasional Oversight of Error**
- **Key information embedded in tremendous volume of materials (noise)**

Conventional Wisdom

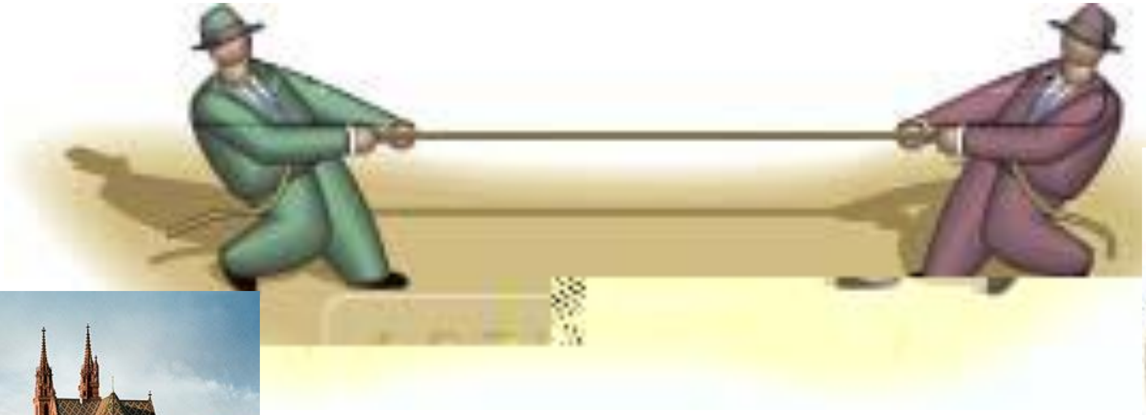
Do Conventional Wisdom and Fundamental Concepts Differ In the Microelectronics Field ?



Reliability ?

The process is good but not perfect !

What Happens When Fundamental Concepts and Conventional Wisdom Differ?

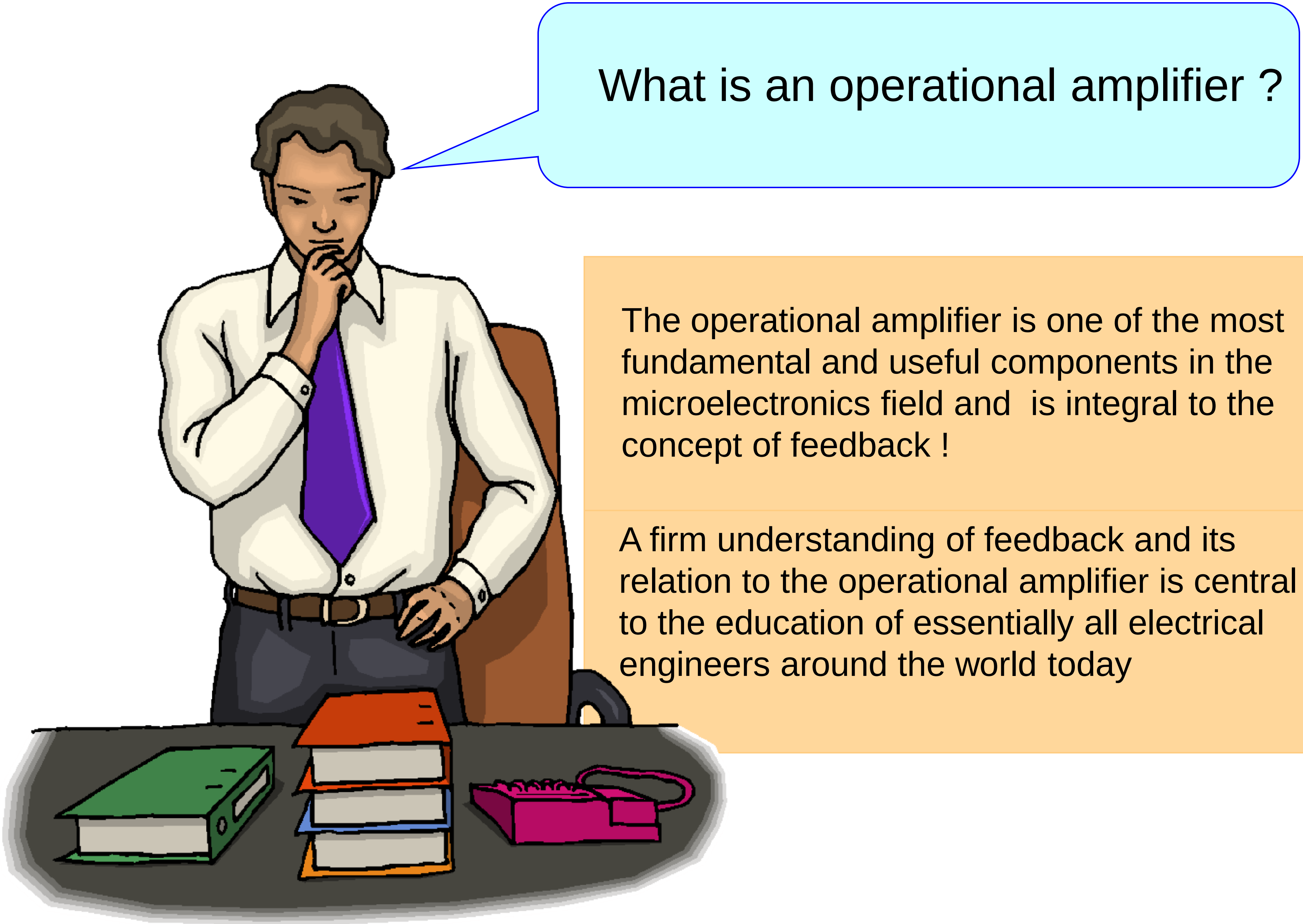


- Confusion Arises
- Progress is Slowed
- Principles are not correctly understood
- Errors Occur
- Time is Wasted

Are Conventional Wisdom and Fundamental Concepts always aligned in the Microelectronics Field ?

Will consider 4 basic examples in this discussion

- Op Amp
- Positive Feedback Compensation
- Current Mode Filters
- Current Dividers



The operational amplifier is one of the most fundamental and useful components in the microelectronics field and is integral to the concept of feedback !

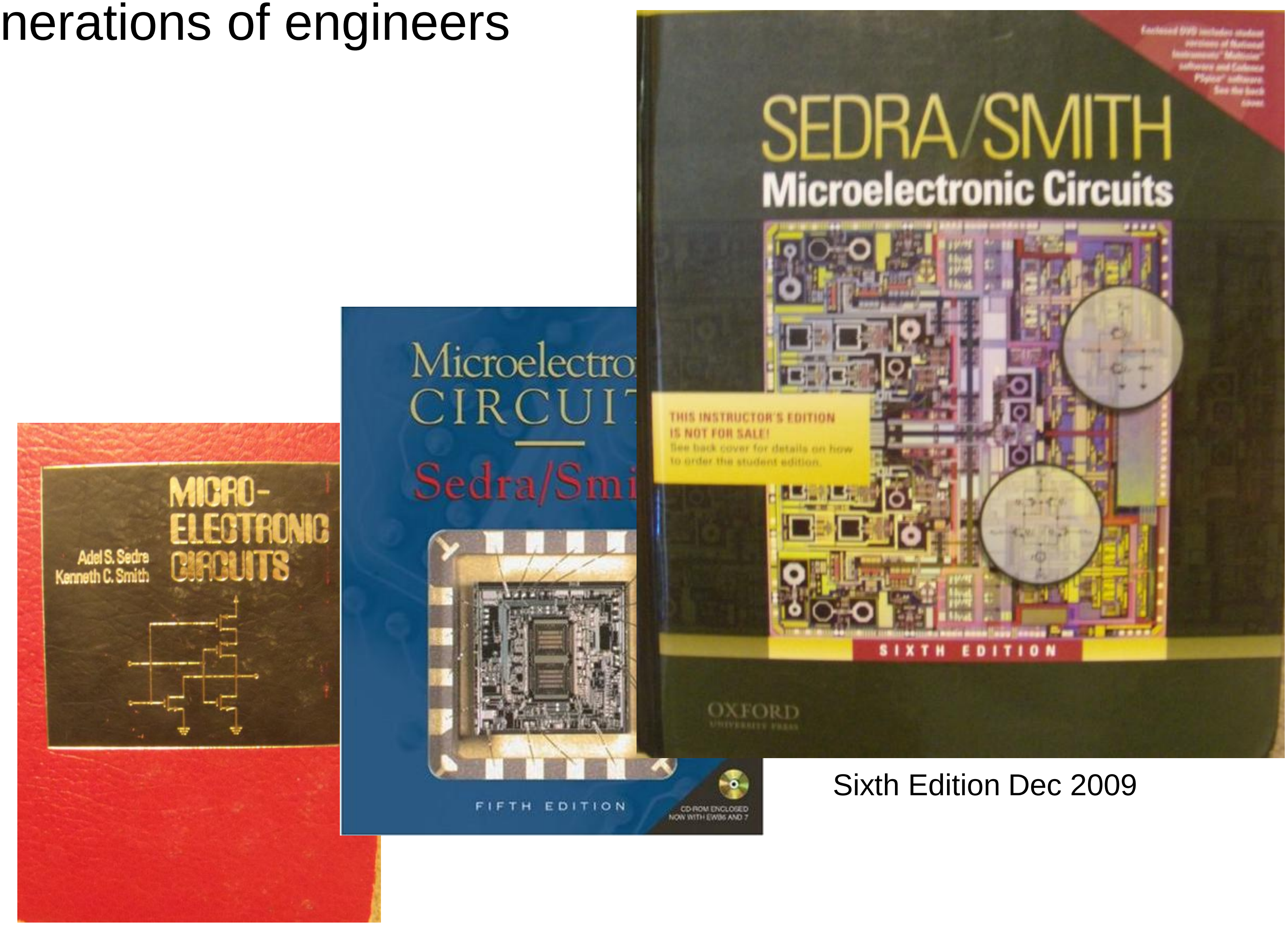
A firm understanding of feedback and its relation to the operational amplifier is central to the education of essentially all electrical engineers around the world today

What is an Operational Amplifier?

Lets see what the experts say !

Consider one of the most popular textbooks on the subject used in the world today

A classic textbook that has helped educate two generations of engineers



Sixth Edition Dec 2009

First Edition 1982

In all editions, concept of the op amp has remained unchanged

2.1.2 Function and Characteristics of the Ideal Op Amp

We now consider the circuit function of the op amp. The op amp is designed to sense the difference between the voltage signals applied at its two input terminals (i.e., the quantity $v_2 - v_1$), multiply this by a number A, and cause the resulting voltage $A(v_2 - v_1)$ to appear at output terminal 3. Here it should be emphasized that when we talk about the voltage at a terminal we mean the voltage between that terminal and ground; thus v_1 means the voltage applied between terminal 1 and ground.

The ideal op amp is not supposed to draw any input current; that is, the signal current into terminal 1 and the signal current into terminal 2 are both zero. In other words, *the input impedance of an ideal op amp is supposed to be infinite.*

How about the output terminal 3? This terminal is supposed to act as the output terminal of an ideal voltage source. That is, the voltage between terminal 3 and ground will always be equal to $A(v_2 - v_1)$, independent of the current that may be drawn from terminal 3 into a load impedance. In other words, *the output impedance of an ideal op amp is supposed to be zero.*

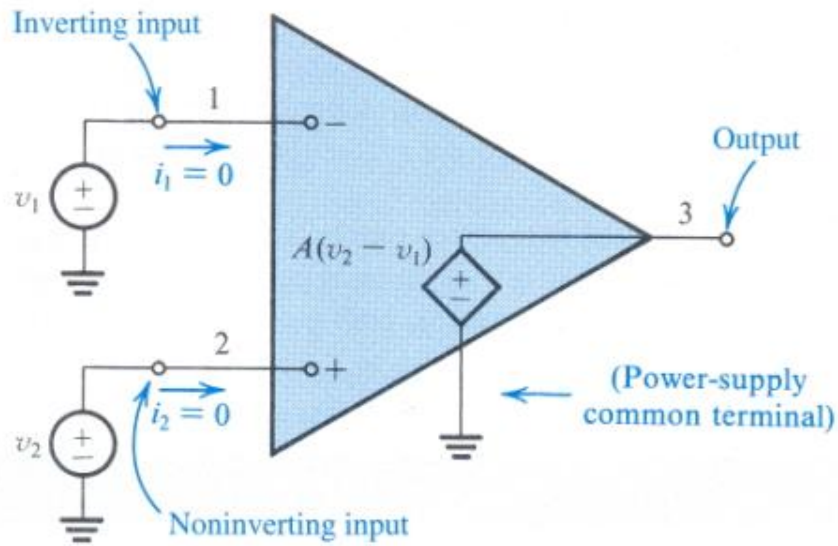
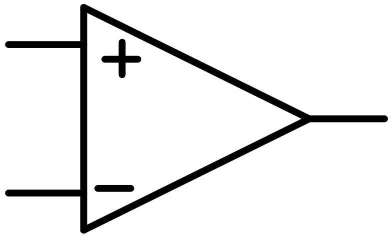


FIGURE 2.3 Equivalent circuit of the ideal op amp.

TABLE 2.1 Characteristics of the Ideal Op Amp

1. Infinite input impedance
2. Zero output impedance
3. Zero common-mode gain or, equivalently, infinite common-mode rejection
4. Infinite open-loop gain A
5. Infinite bandwidth

What is an Operational Amplifier?



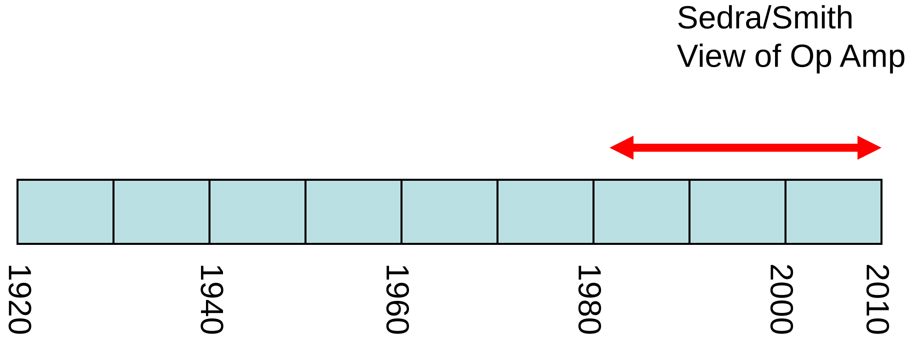
Textbook Definition:

- Voltage Amplifier with Very Large Gain
  - Very High Input Impedance
  - Very Low Output Impedance
- Differential Input and Single-Ended Output

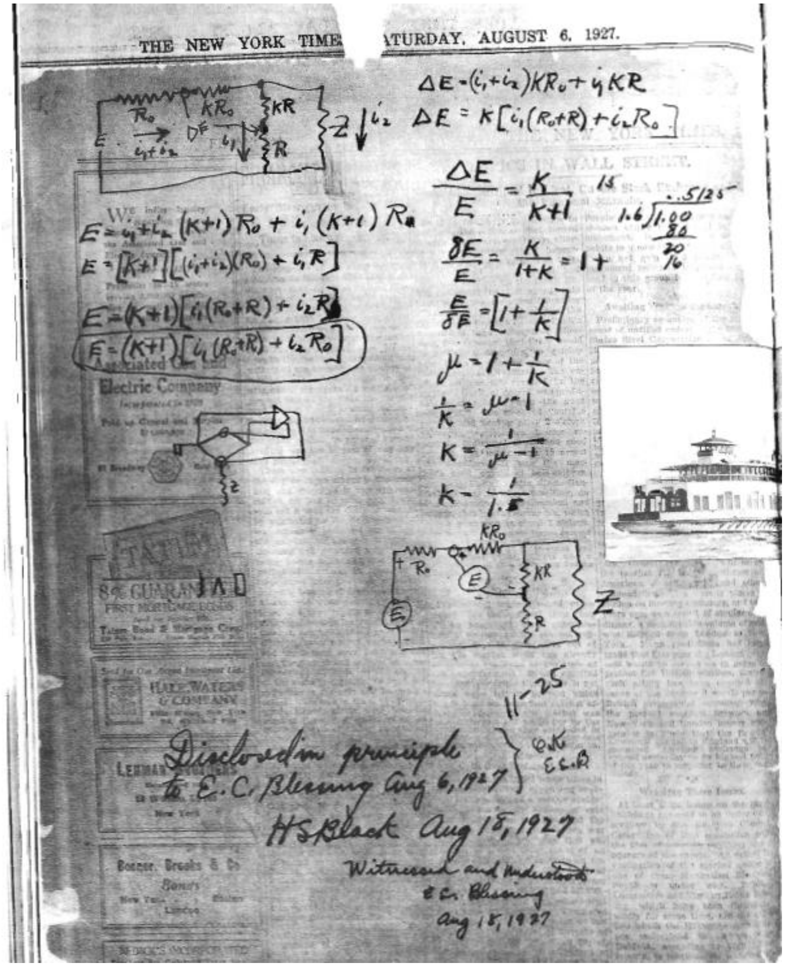
This represents the Conventional Wisdom !

Does this correctly reflect what an operational amplifier really is?

Operational Amplifier Evolution in Time Perspective



Consider some history leading up to the present concept of the operational amplifier



H.S. Black sketch of basic concept of feedback on Aug 6, 1927

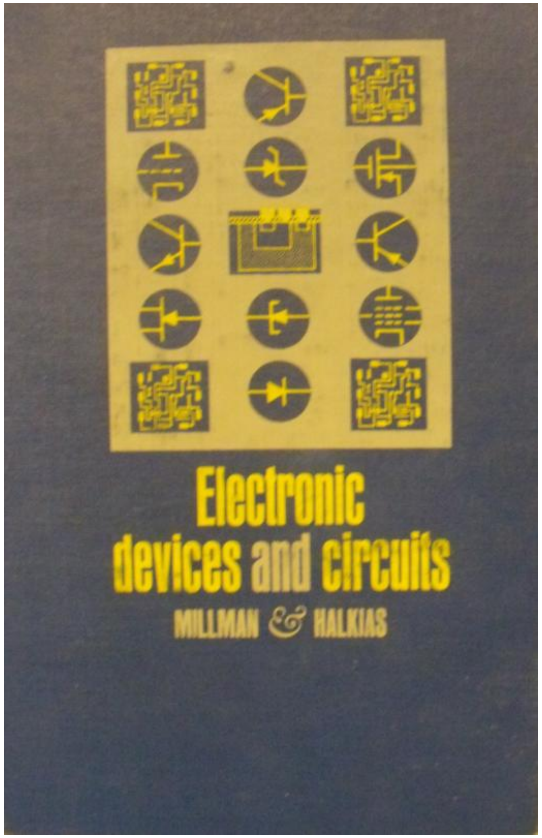
Black did not use the term operational amplifier but rather focused on basic concepts of feedback involving the use of high-gain amplifiers

A classic textbook sequence that has helped educate the previous two generations of engineers

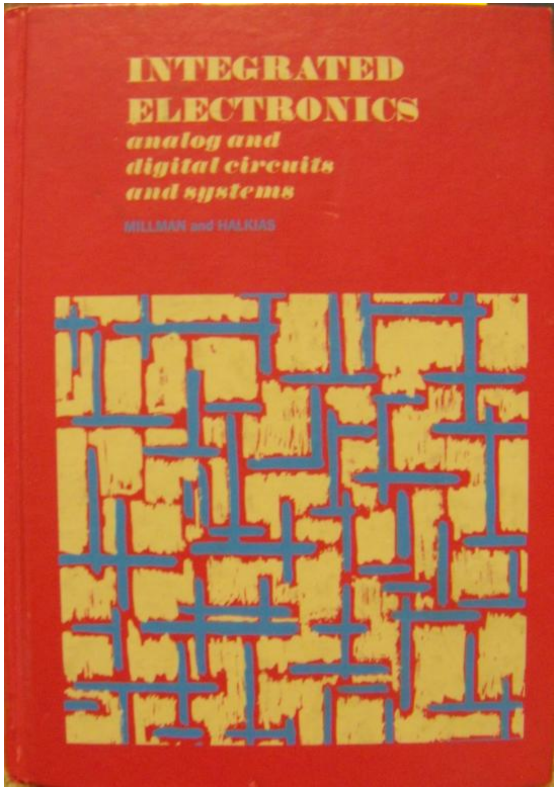
Vacuum Tube and Semiconductor Electronics

By Millman

First Edition 1958



First Edition 1967



First Edition 1972

Millman view of an operational amplifier in 1967

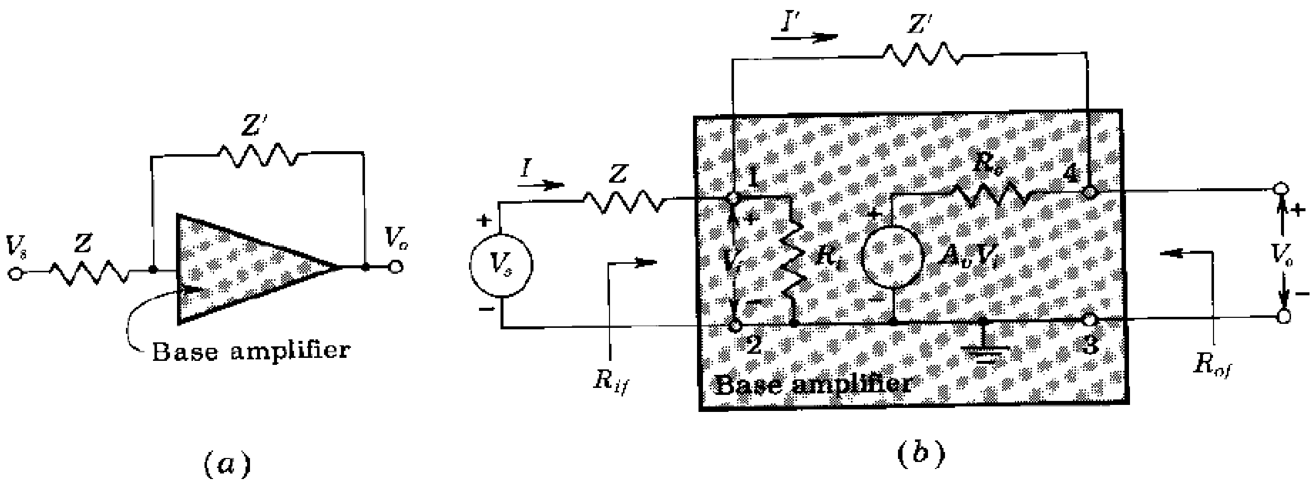
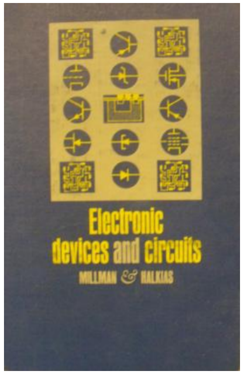


Fig. 17-26 (a) Schematic diagram and (b) equivalent circuit of an operational amplifier. The open-circuit voltage gain A_v is negative.

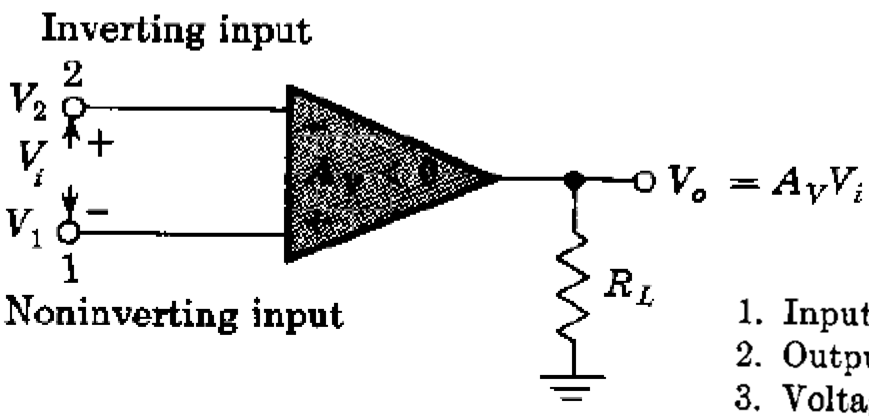


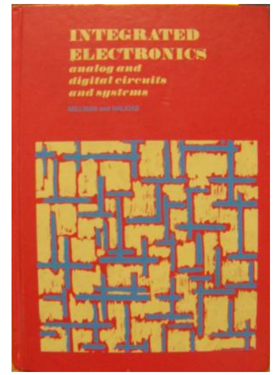
Operational Amplifier refers to the entire feedback circuit

Concept of a “Base Amplifier” as the high-gain amplifier block

Note Base Amplifier is modeled as a voltage amplifier with single-ended input and output

Millman view of an operational amplifier in 1972





1. Input resistance $R_i = \infty$
2. Output resistance $R_o = 0$
3. Voltage gain $A_v = -\infty$
4. Bandwidth $= \infty$
5. $V_o = 0$ when $V_1 = V_2$ independent of the magnitude of V_1
6. Characteristics do not drift with temperature.

This book was published several years after the first integrated op amps were introduced by industry

This fundamentally agrees with that in use today by most authors

Major change in the concept from his own earlier works

Seminal source for “Operational Amplifier” notation:

Analysis of Problems in Dynamics by Electronic Circuits*

JOHN R. RAGAZZINI†, MEMBER, I.R.E., ROBERT H. RANDALL‡, AND FREDERICK A. RUSSELL§, MEMBER, I.R.E.

The term “operational amplifier” is a generic term applied to amplifiers whose gain functions are such as to enable them to perform certain useful operations such as summation, integration, differentiation, or a combination of such operations.

Seminal source introduced a fundamentally different definition than what is used today

Consistent with the earlier use of the term by Millman

Seminal Publication of Feedback Concepts:

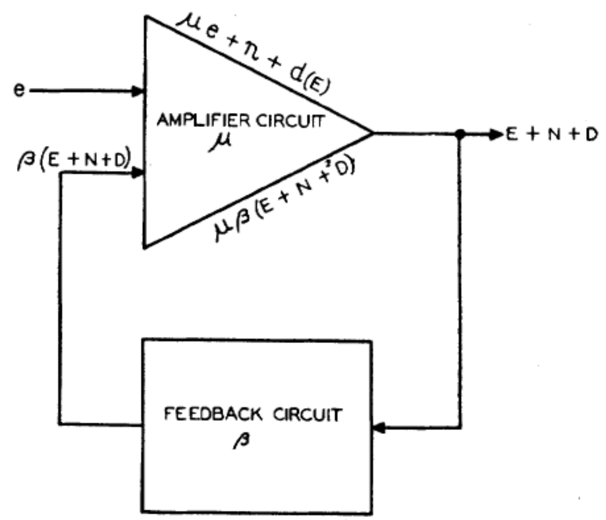
Stabilized Feed-Back Amplifiers

By
H. S. BLACK
MEMBER A.I.E.E.

Bell Telephone Laboratories, Inc.,
New York, N. Y.

Transactions of the American Institute of Electrical Engineers, Jan. 1934

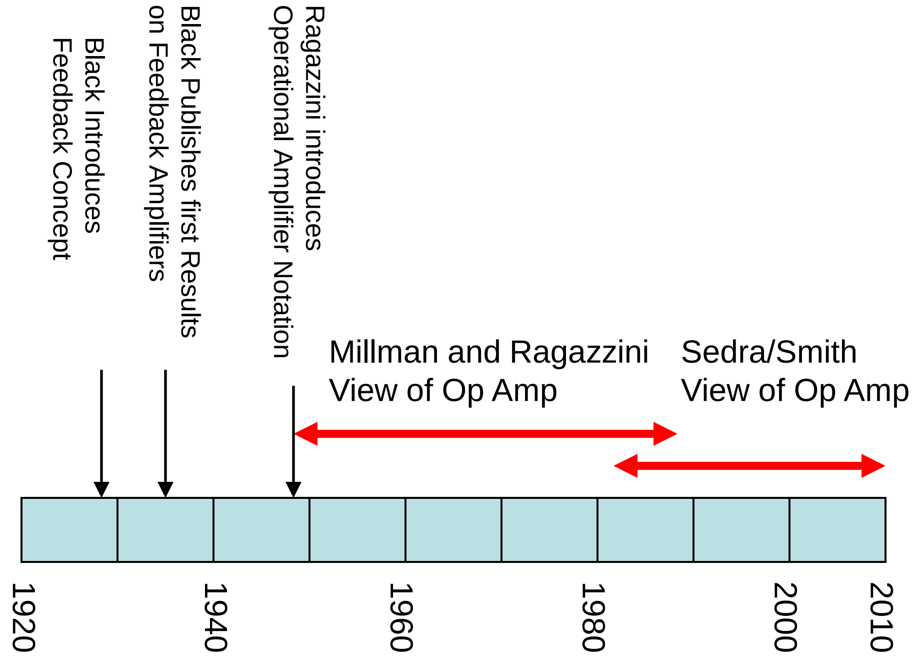
Fig. 1. Amplifier system with feed-back



Uses a differential input high-gain voltage amplifier (voltage series feedback)

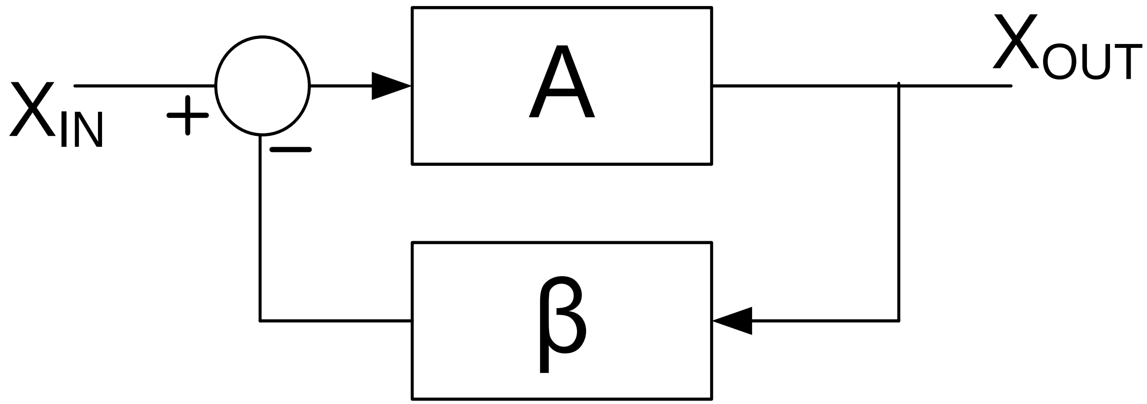
Subsequent examples of feedback by Black relaxed the differential input requirement

Operational Amplifier Evolution in Time Perspective



Do we have it right now?

Why are Operational Amplifiers Used?

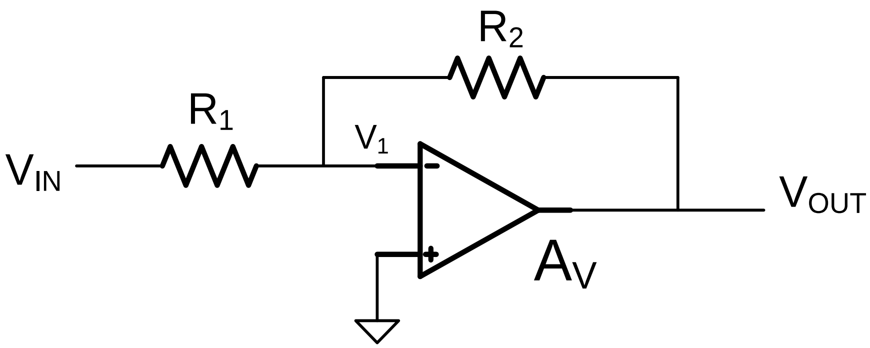


Input and Output Variables intentionally designated as "X" instead of "V"

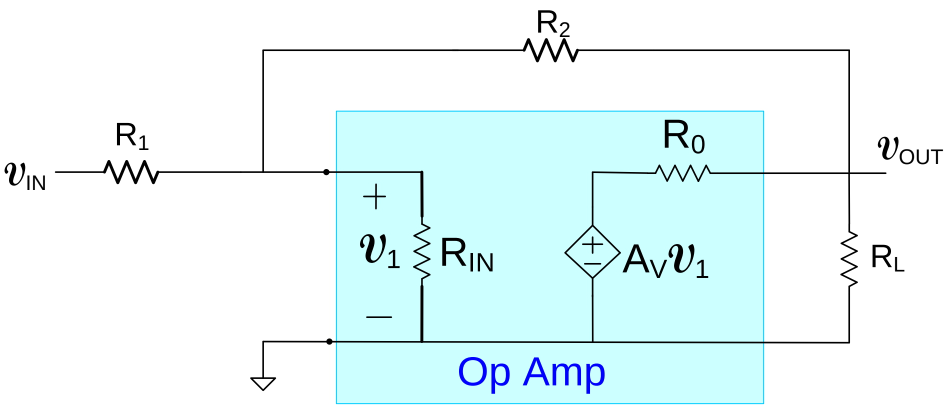
$$\frac{Xout}{Xin} = A_F = \frac{A}{1+A\beta} = \overset{A\to\infty}{\approx} \frac{1}{\beta}$$

Op Amp is Enabling Element Used to Build Feedback Networks !

One of the Most Basic Op Amp Applications



Model of Op Amp/Amplifier including A_V, R_{IN}, R_O and R_L

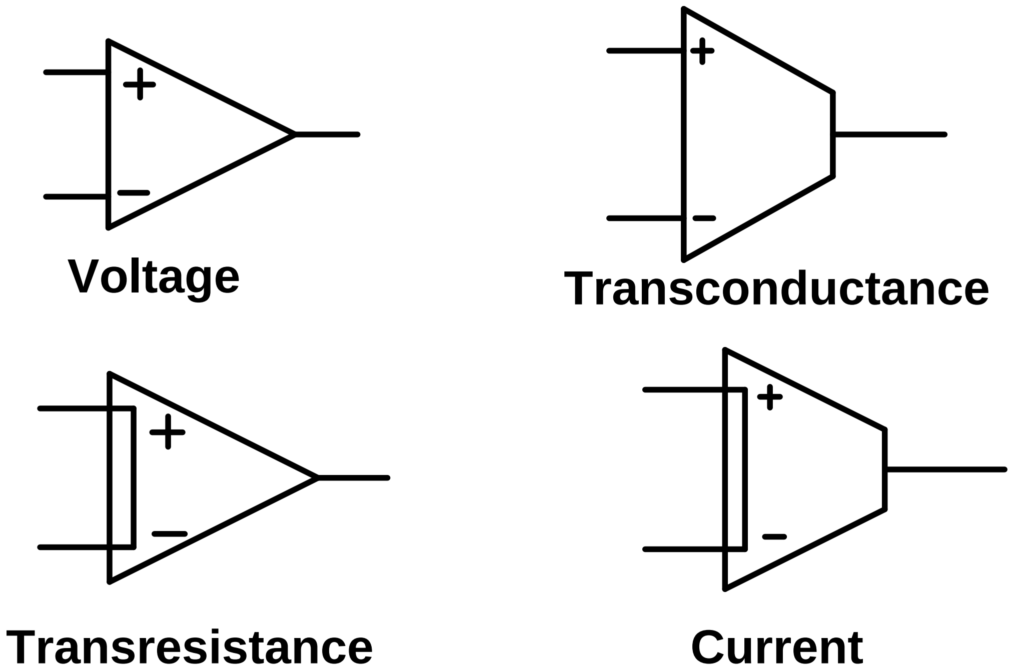


If it is assumed that A_V is large,

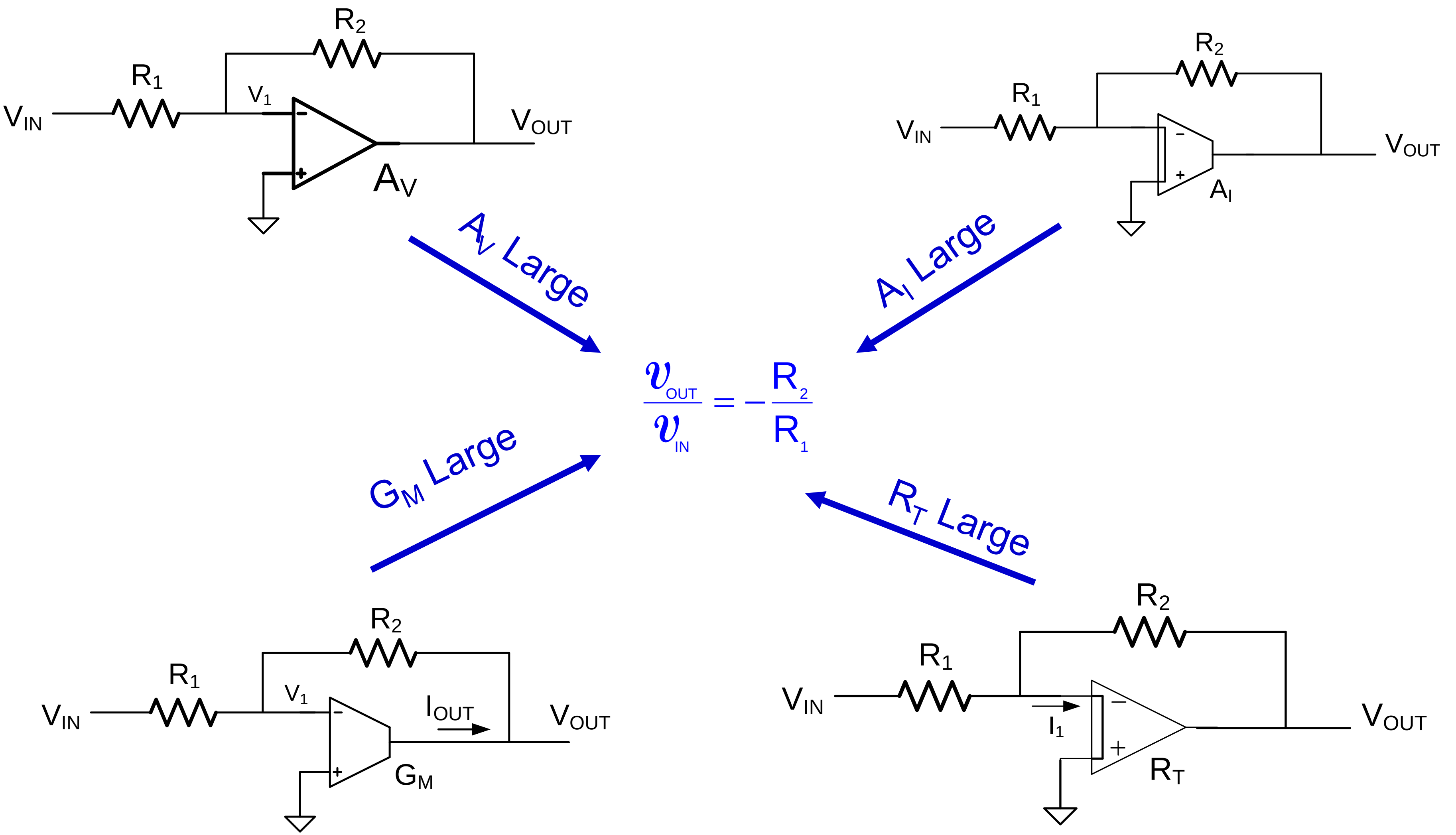
$$A_{VF} = \frac{v_{OUT}}{v_{IN}} \simeq -\frac{R_2}{R_1}$$

This result is not dependent upon R_{IN}, R_0 or R_L

The Four Basic Types of Amplifiers:



Four Feedback Circuits with Same β Network



All have same closed-loop gain and all are independent of R_{IN}, R_{OUT} and R_L if gain is large

Concept well known

AN-88 CMOS LINEAR APPLICATIONS

Gene Taatjes
JULY 1973

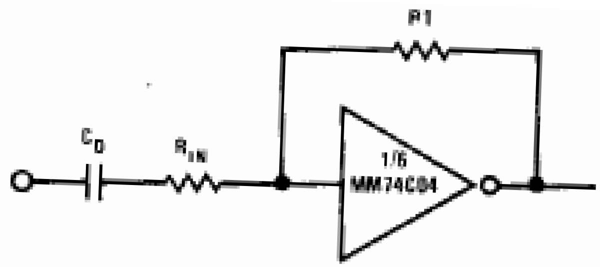
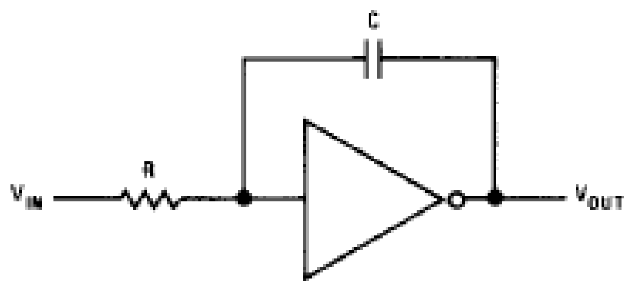


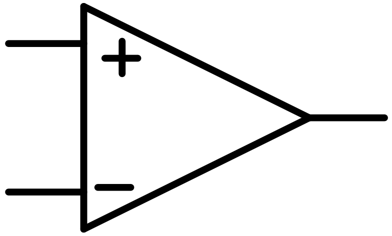
FIGURE 2. A 74CMOS Invertor Biased for Linear Mode Operation.



Integrator Using
Any Inverting CMOS Gate

Hex Inverters in 74C04 much less costly than 6 op amps at the time!

What is an Operational Amplifier?



Textbook Definition:

- Voltage Amplifier with Very Large Gain
  - Very High Input Impedance
  - Very Low Output Impedance

This represents the Conventional Wisdom !

Do we have it right now?

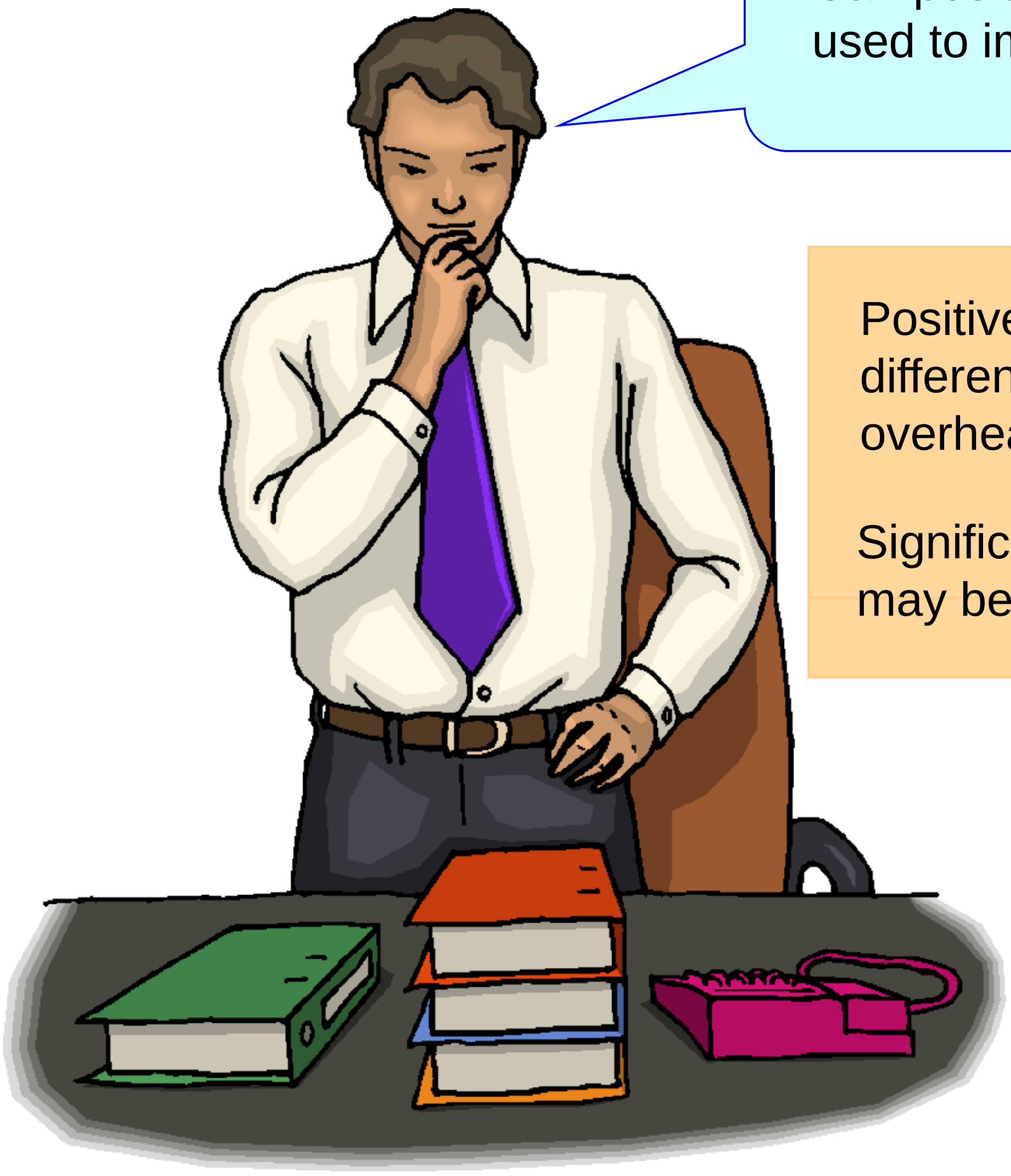
~~Voltage Amplifier?~~ ~~Low Output Impedance?~~ ~~Single-Ended Output?~~

~~High Input Impedance?~~ ~~Differential Input?~~ Large Gain !!!

Are Conventional Wisdom and Fundamental Concepts always aligned in the Microelectronics Field ?

Will consider 4 basic examples in this discussion

- Op Amp
- Positive Feedback Compensation
- Current Mode Filters
- Current Dividers

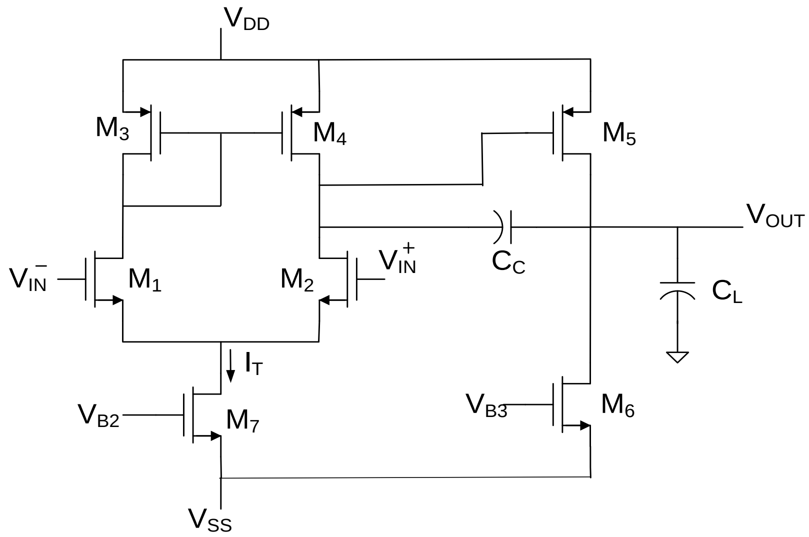


Positive feedback can be easily applied in differential structures with little circuit overhead

Significant gain enhancement in the op amp may be possible if positive feedback is used

Compensation of two-stage amplifiers

To illustrate concept consider basic two-stage op amp with internal compensation



$$A_V = \left(A_0 \frac{p_1 p_2}{z} \right) \frac{-s+z}{(s+p_1)(s+p_2)}$$

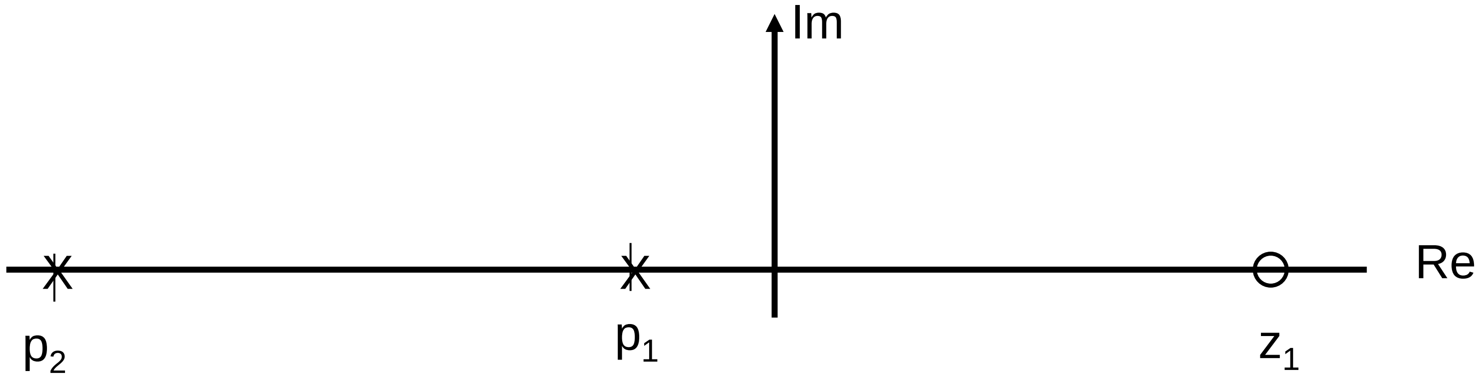
Miller Effect on C_C provides dominant pole on first stage

Compensation requires a large ratio of p_2/p_1 be established

Two-stage amplifier with LHP Zero Compensation

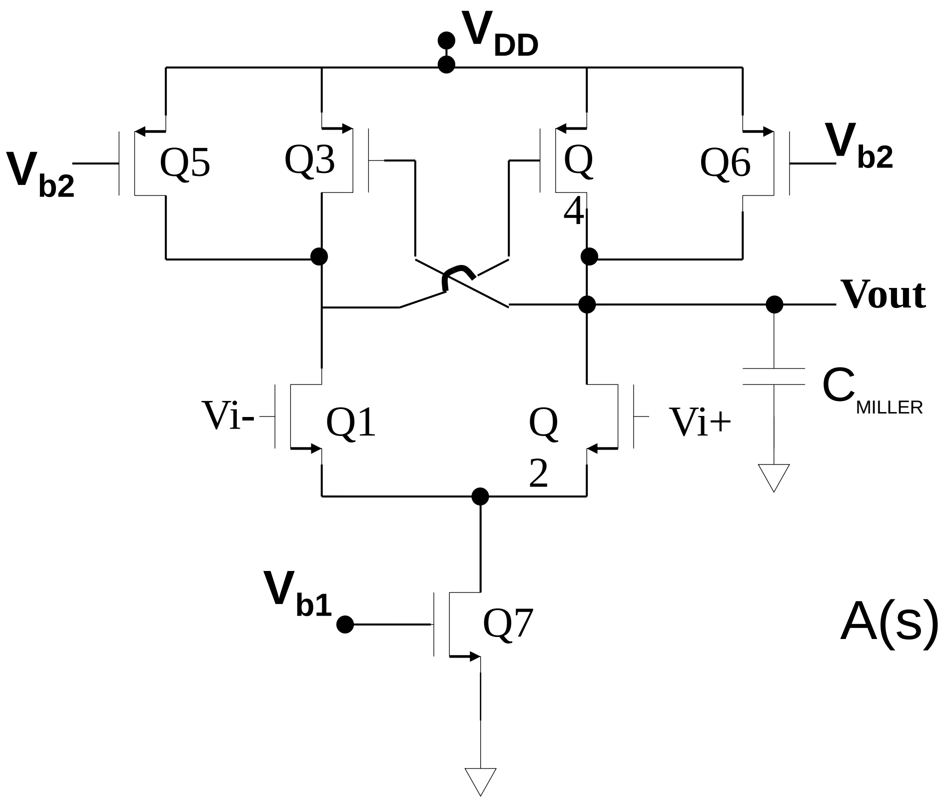
$$p_1 = -\frac{g_{o1} + g_{o5}}{C_C\left(\frac{g_{m5}}{g_{05} + g_{o6}}\right)}$$

$$p_2 = -\frac{g_{m5}}{C_L}$$



To make p_1 sufficiently dominant requires a large value for C_C

Positive Feedback on First-Stage for gain enhancement and pole control



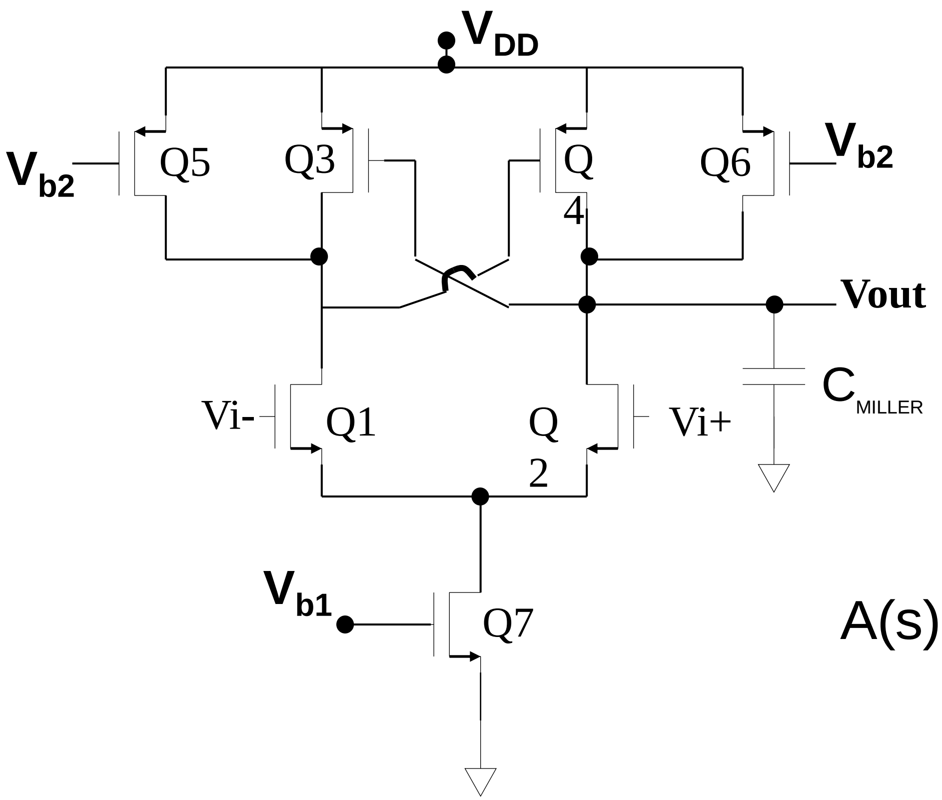
$$A(s) = \frac{(1/2)g_{m1}}{sC_{MILLER} + [g_{o2} + g_{o4} + g_{o6} - g_{m4}]}$$

$$p_1 \simeq -\frac{g_{o1} + g_{o5} + g_{o6} - g_{m4}}{C_{MILLER}}$$

Can reduce size of C_{MILLER} and enhance dc gain by appropriate choice of g_{m4}

Can actually move p_1 into RHP if g_{m4} is too big

Positive Feedback on First-Stage for gain enhancement and pole control



$$A(s) = \frac{(1/2)g_{m1}}{sC_{MILLER} + [g_{o2} + g_{o4} + g_{o6} - g_{m4}]}$$

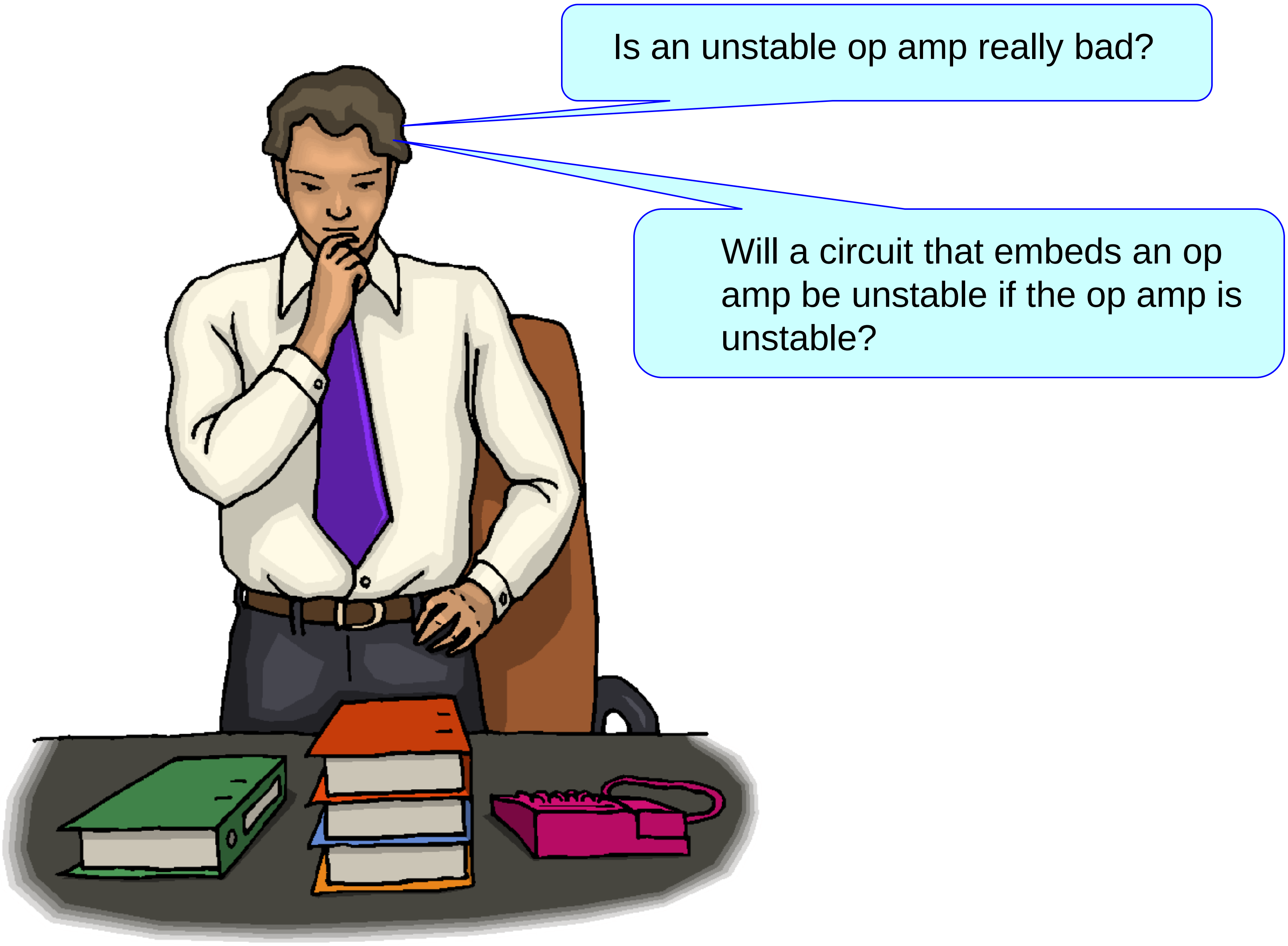
$$A_{DC} \simeq -\frac{(1/2)g_{m1}}{g_{o1}+g_{o5}+g_{o6}-g_{m4}}$$

Dc gain actually goes to ∞ when $g_{m1} = g_{02} + g_{04} + g_{06}$!

This technique is not practical since Op Amp pole can move into RHP making it unstable!

$$p_1 \simeq -\frac{g_{o1}+g_{o5}+g_{o6}-g_{m4}}{C_{MILLER}}$$

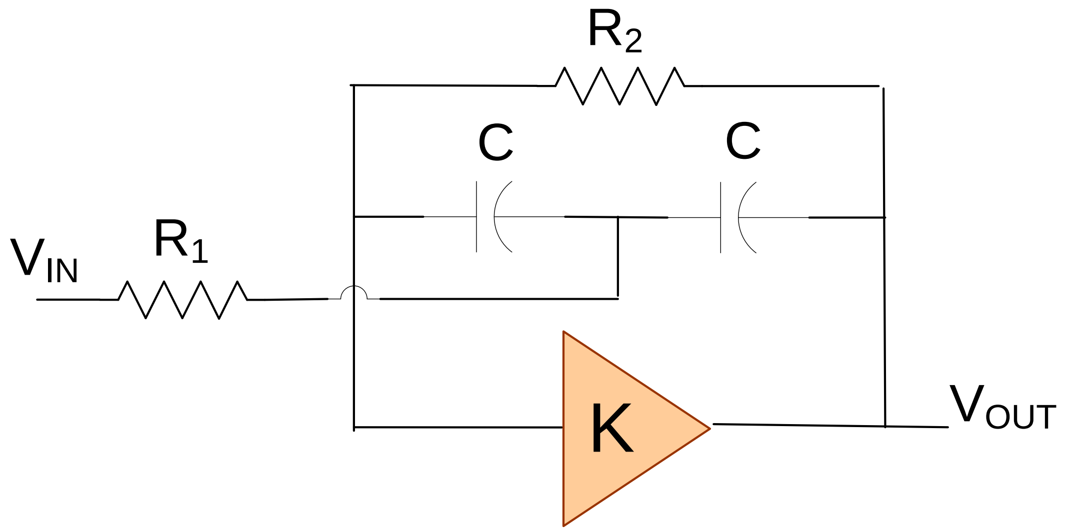
Several authors have discussed this approach in the literature but place a major emphasis on limiting the amount of positive feedback used so that under PVT variations, op amp remains stable


Is an unstable op amp really bad?
Will a circuit that embeds an op amp be unstable if the op amp is unstable?

Example: Filter Structure with Feedback Amplifier

Bridged-T Feedback

(Termed SAB, STAR, Friend/Delyannis Biquad)

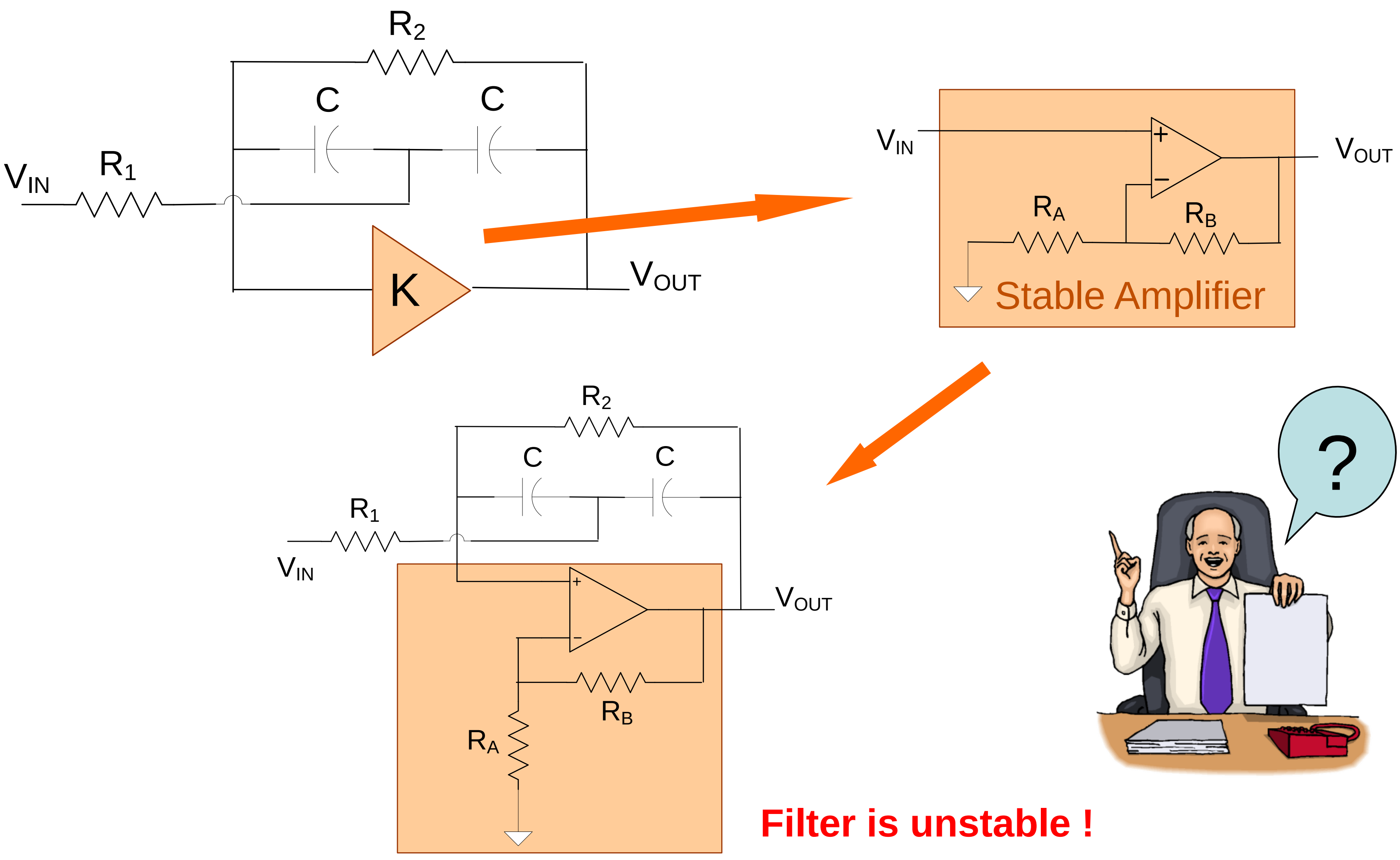


K is a small positive gain

want high input impedance on "K" amplifier

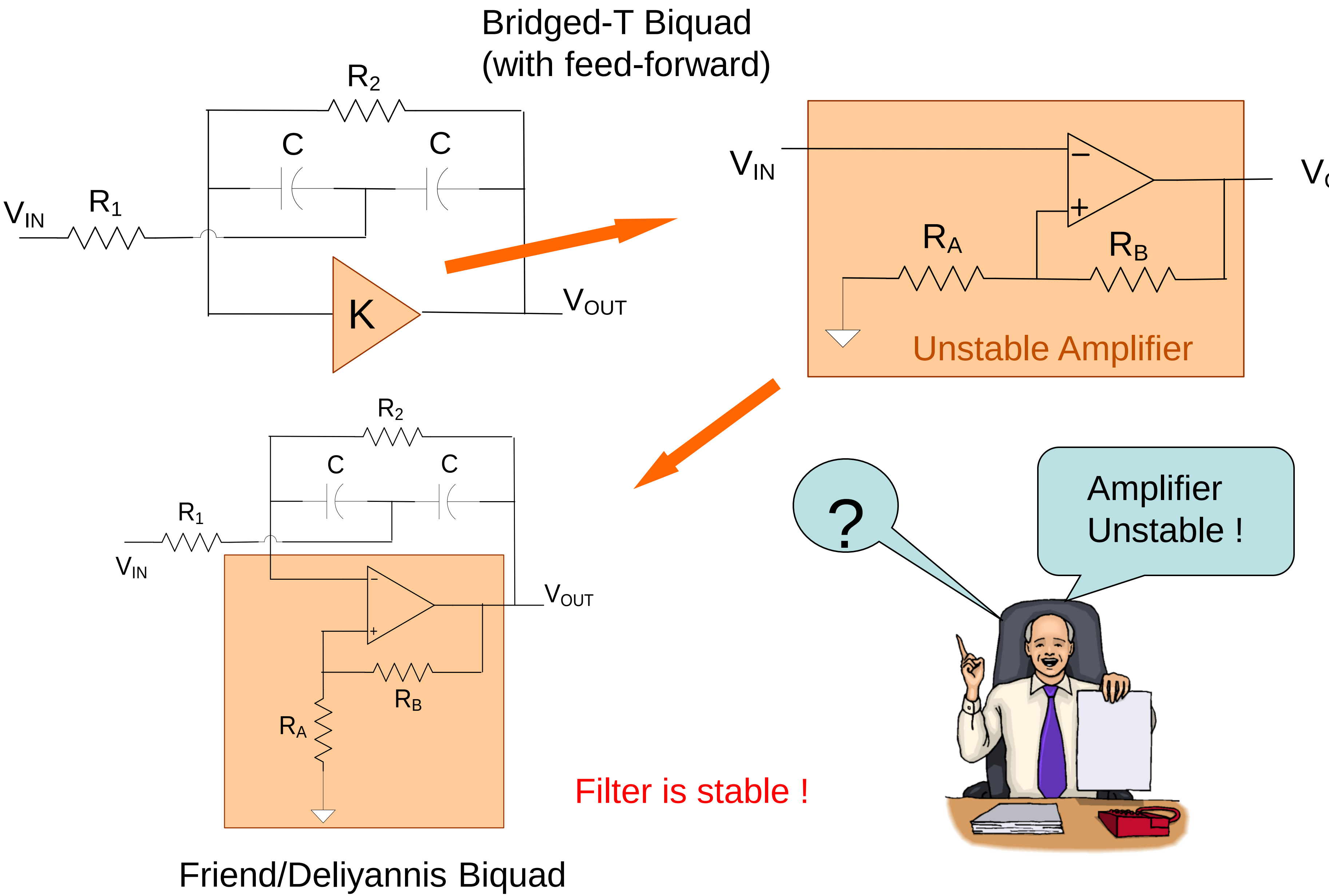
- Very popular filter structure
- One of the best 2nd-order BP filters
- Widely used by Bell System in 70's

Example: Filter Structure with Feedback Amplifier



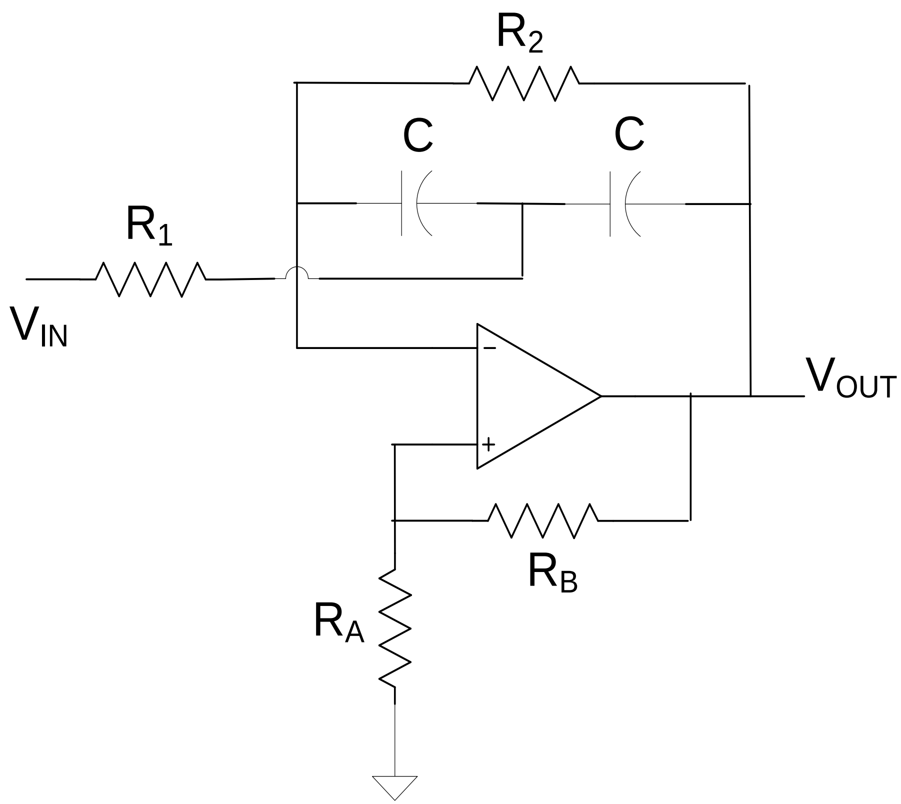
Filter is unstable !

Example: Filter Structure with Feedback Amplifier



Very Popular Bandpass Filter

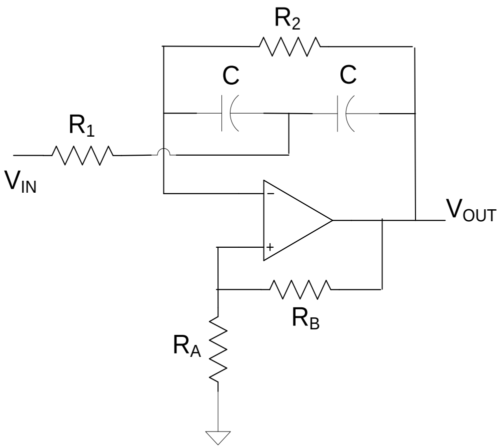
Friend-Deliyannis Biquad



One of the best bandpass filters !!

Embedded finite gain amplifier is unstable!!

Stability of embedded amplifier is not necessary (or even desired)

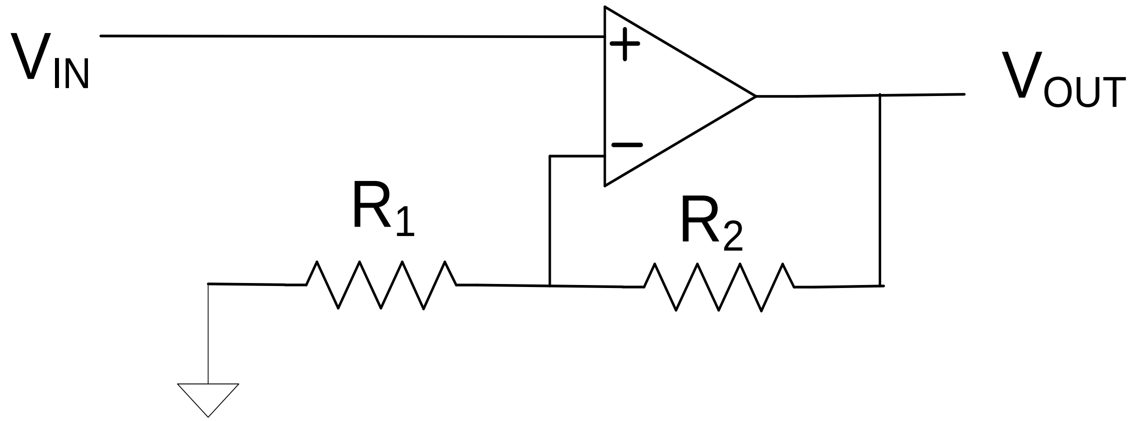


- Filter structure unstable with stable finite gain amplifier
- Filter structure stable with unstable finite gain amplifier
- Stability of feedback network not determined by stability of amplifier!

Is an unstable op amp really bad?

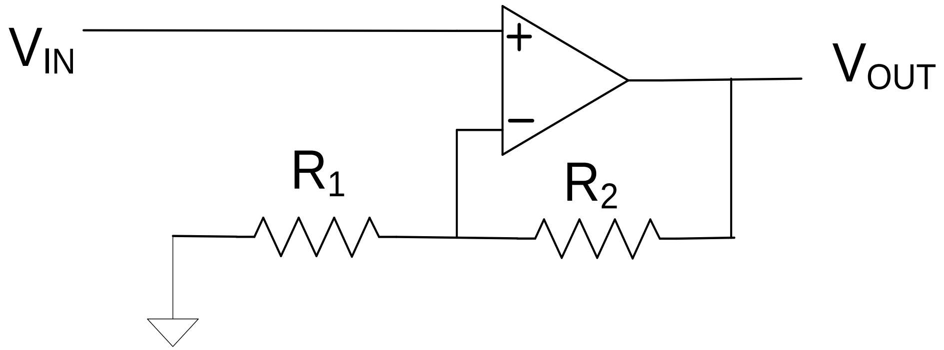
Will a circuit that embeds an op amp be unstable if the op amp is unstable? Not necessarily !

Example: Voltage Amplifier with Unstable Op Amp



$$A(s) = \frac{-A_o}{\frac{s}{-p} + 1} \qquad p > 0$$

Example: Voltage Amplifier with Unstable Op Amp

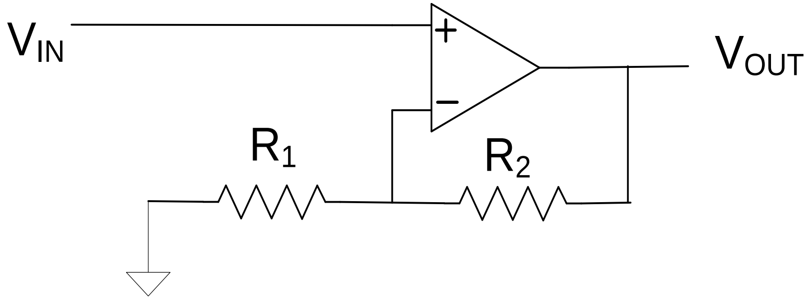


$$\beta = \frac{R_1}{R_2 + R_1}$$

$$A(s) = \frac{-A_o}{\frac{s}{-p} + 1} \qquad p > 0$$

$$A_{FB}(s) = \frac{A(s)}{1 + \beta A(s)}$$

Example: Voltage Amplifier with Unstable Op Amp

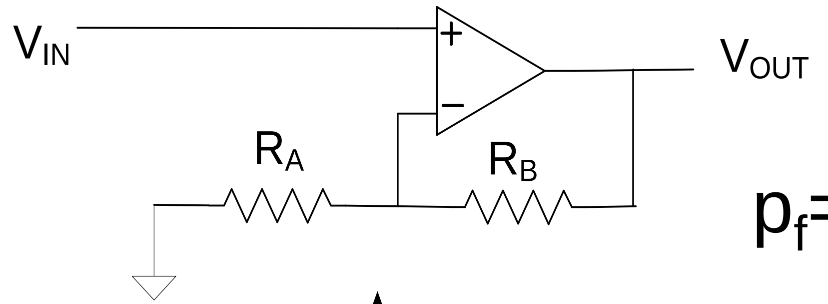


$$A_{FB}(s) = \frac{A(s)}{1+\beta A(s)} = \frac{A_o p}{s+p\left(\beta A_o - 1\right)} \qquad p > 0$$

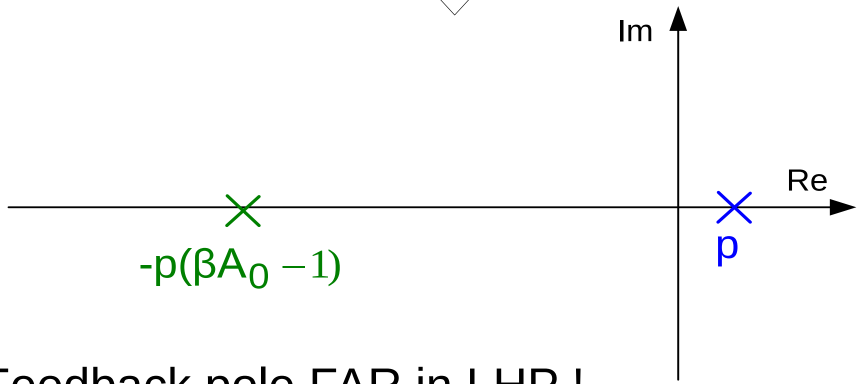
$$p_f = p(1-\beta A_0)$$

For $\beta A_o > 1$, Feedback Amplifier is Stable !!!

Example: Voltage Amplifier with Unstable Op Amp



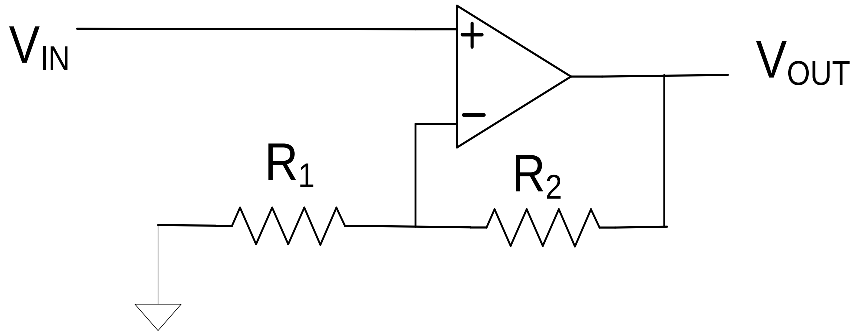
$$p_f = p(1-\beta A_0)$$



Feedback pole FAR in LHP !

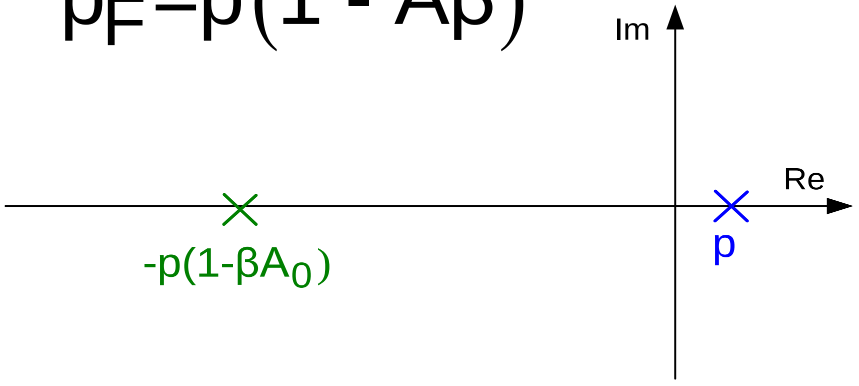
How does this compare to the feedback pole of a stable op amp with a pole In the LHP at –p?

Example: Voltage Amplifier with Unstable Op Amp



$p > 0$

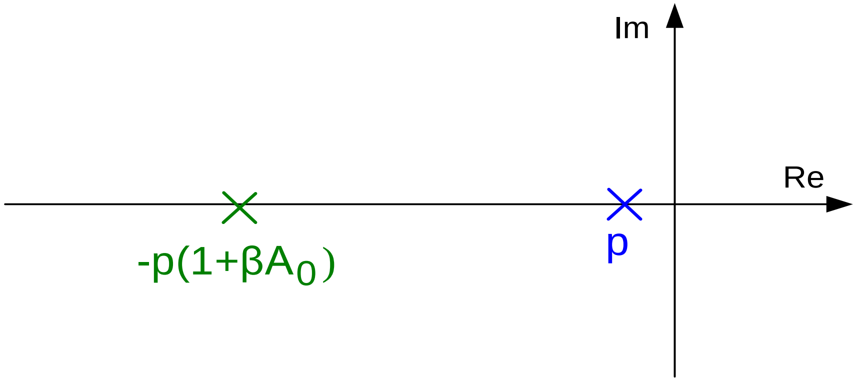
$$p_F = p(1 - A\beta)$$



Feedback pole FAR in LHP !

$p < 0$

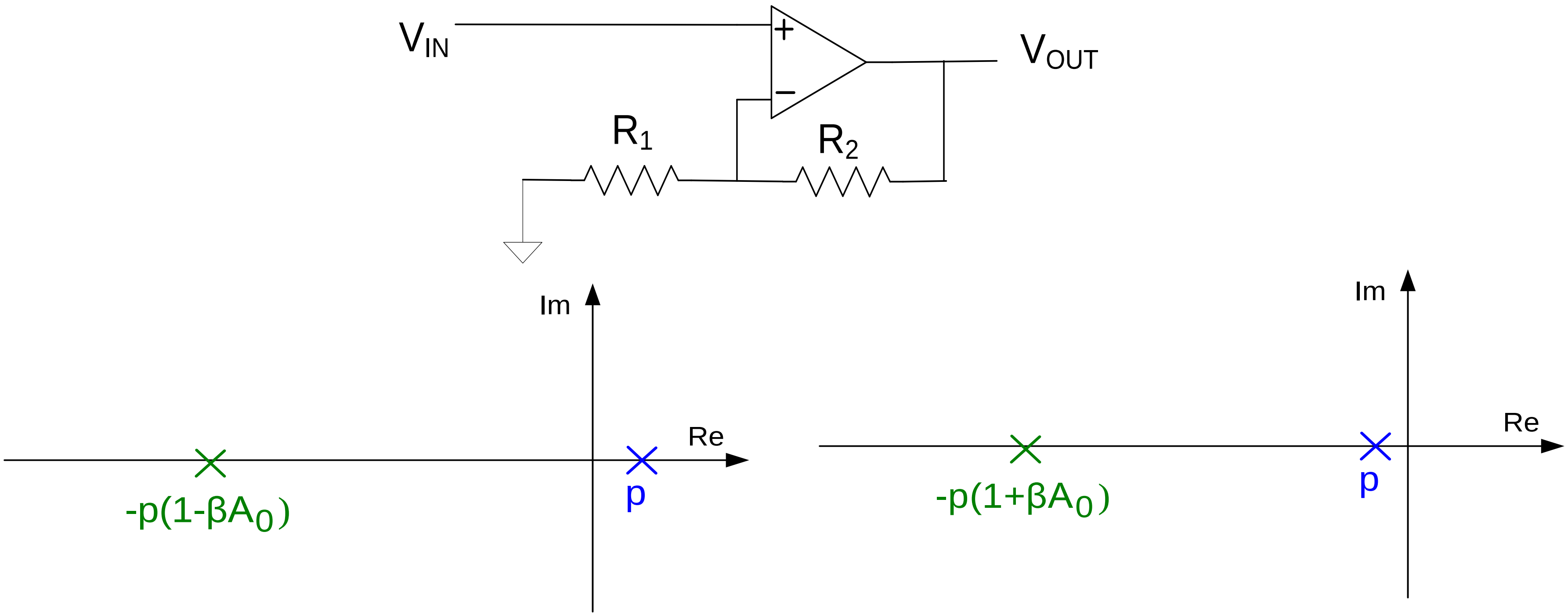
$$p_F = p(1 + A\beta)$$



Feedback pole FAR in LHP !

Can show that some improvements in feedback performance can be realized if the open-loop pole is at the orgin or modestly in the RHP!

Example: Voltage Amplifier with Unstable Op Amp



Stability of open-loop amplifier is not a factor in determining the stability of the feedback structure in practical structures when |p| is small!

It can actually be shown that the performance of the feedback amplifier can be improved if the open-loop pole is moved modestly into the RHP

This is contrary to the Conventional Wisdom !

Is an unstable op amp really bad?

No, and it can actually improve performance of FB circuit!

Will a circuit that embeds an op amp be unstable if the op amp is unstable?   Not necessarily !

Are Conventional Wisdom and Fundamental Concepts always aligned in the Microelectronics Field ?

Will consider 4 basic examples in this discussion

- Op Amp
- Positive Feedback Compensation
- Current Mode Filters
- Current Dividers


What are the advantages of current-mode signal processing ?

EVERYBODY knows that Current-Mode circuits operate at lower supply voltages, are faster, are smaller, consume less power, and take less area than their voltage-mode counterparts !

And I've heard there are even some more benefits but with all of these, who really cares?

Have considered Current Mode Filters in Lecture 31 and 32

Showed by example that an Active RC Current-Mode Filter was identical to a Voltage-Mode Counterpart

Will now look at more general Current-Mode Architectures

Questions about the Conventional Wisdom

- Why does a current-mode circuit work better at high frequencies?
- Why is a current-mode circuit better suited for low frequencies?
- Why do some “voltage”-mode circuits have specs that are as good as the current-mode circuits?

Questions about the Conventional Wisdom

- Why are most of the papers on current-mode circuits coming from academia?
- Why haven't current-mode circuits replaced "voltage"-mode circuits in industrial applications?

Questions about the Conventional Wisdom

What is a current-mode circuit?

- Everybody seems to know what it is
- Few have tried to define it
- Is a current-mode circuit not a voltage-mode circuit?

Questions about the Conventional Wisdom

What is a current-mode circuit?

"Several analog CMOS continuous-time filters for high frequency applications have been reported in the literature… Most of these filters were designed to process voltage signals. It results in high voltage power supply and large power dissipation. To overcome these drawbacks of the voltage-mode filters, the current-mode filters circuits , which process current signals have been developed"

A **3V-50MHz Analog CMOS Current-Mode High Frequency Filter with a Negative Resistance Load**, pp. 260…,,IEEE Great Lakes Symposium March 1996.

Questions about the Conventional Wisdom

- Are current-mode circuits really better than their "voltage-mode" counterparts?
- What is a current-mode circuit?
  - Must have a simple answer since so many authors use the term
- Do all agree on the definition of a current-mode circuit?

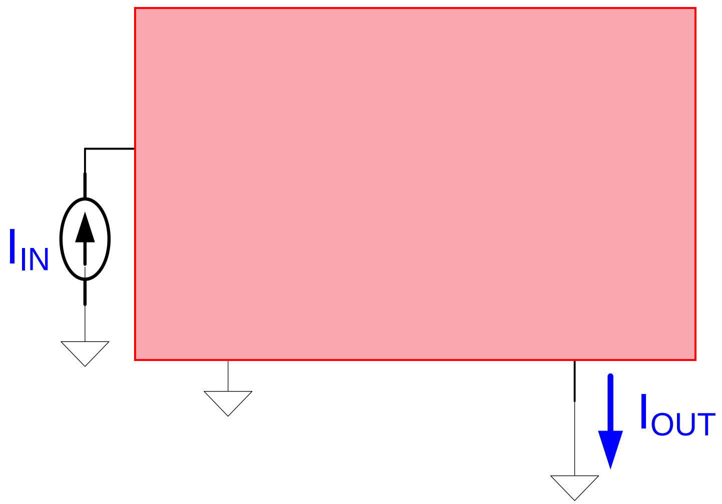
Questions about the Conventional Wisdom

Conventional Wisdom Definition:

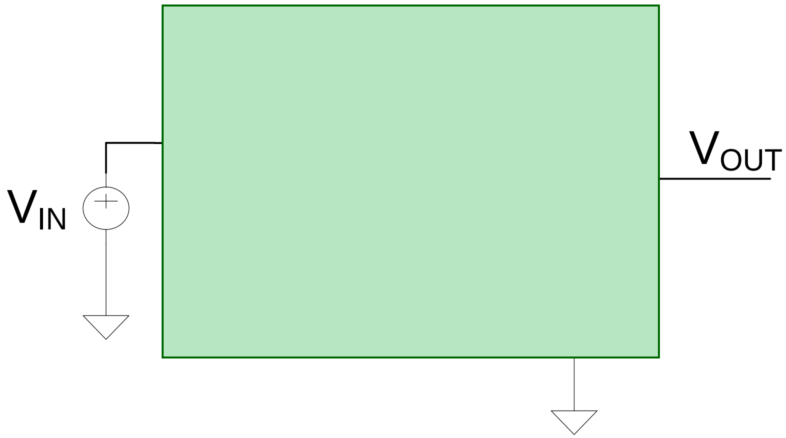
- A current-mode circuit is a circuit that processes current signals
- A current-mode circuit is one in which the defined state variables are currents

Example:

Is this a current-mode circuit?



Is this a voltage-mode circuit?

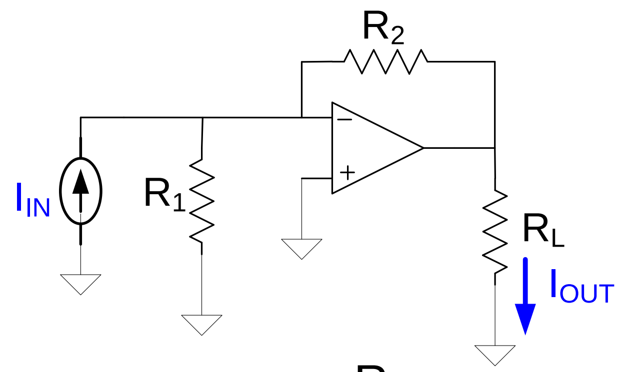


Conventional Wisdom Definition:

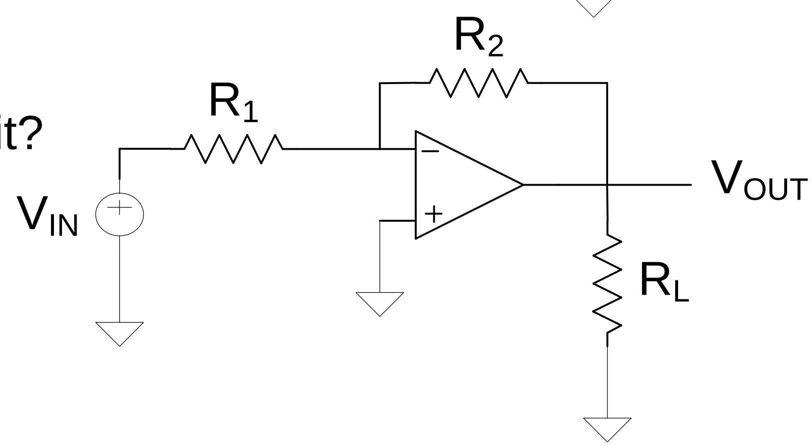
A current-mode circuit is a circuit that processes current signals

Example:

Is this a current-mode circuit?



Is this a voltage-mode circuit?



- One is obtained from the other by a Norton to Thevenin Transformation
- **The poles and the BW of the two circuits are identical !**

Current-Mode Filters

Concept of Current-Mode Filters is Somewhat Recent:

Key paper that generated interest in current-mode filters (ISCAS 1989):

Switched currents-a new technique for analog sampled-data signal processing

JB Hughes, NC Bird... - Circuits and Systems, 1989 ..., 2002 - ieeexplore.ieee.org

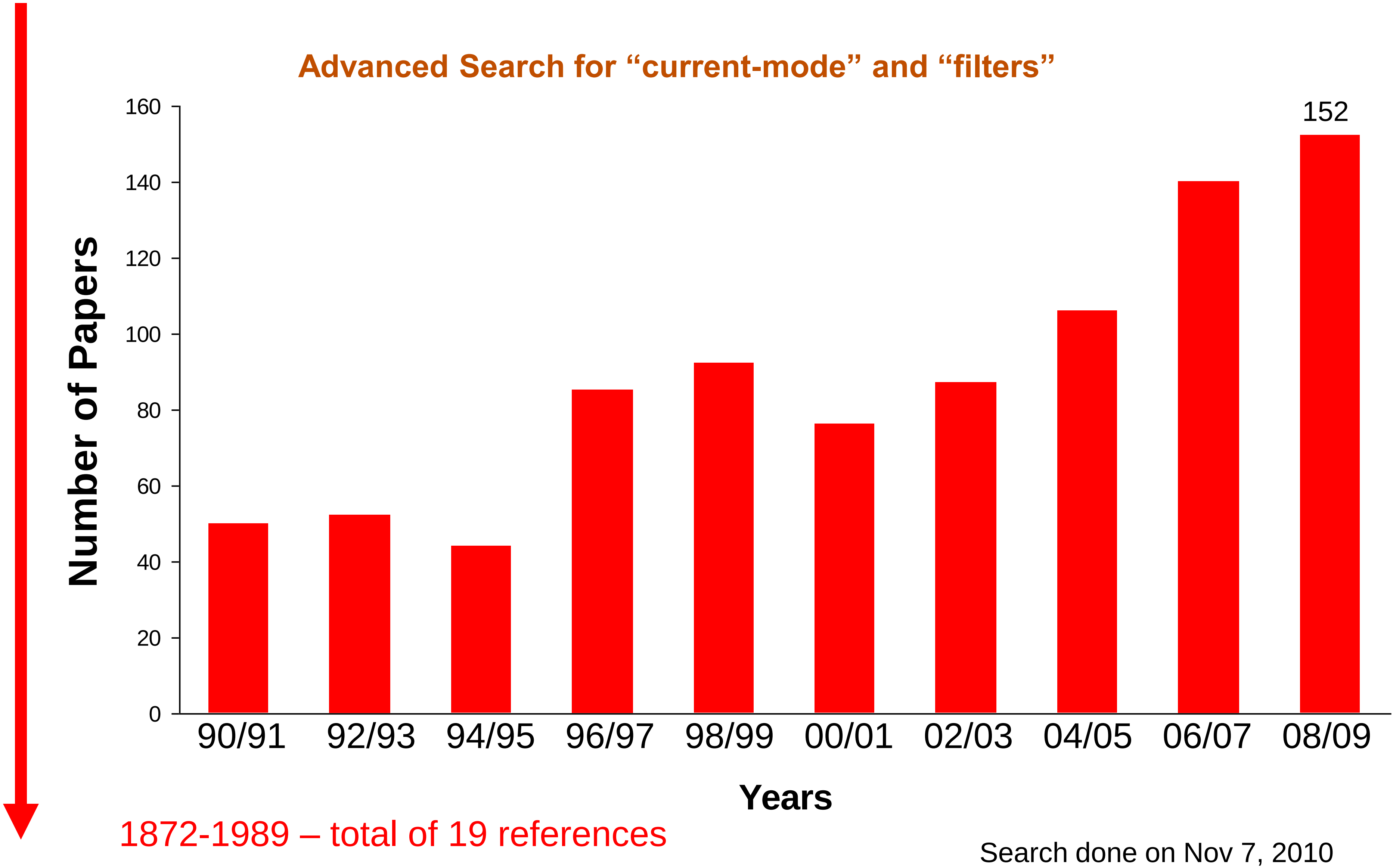
NTRODUCTION The enormous complexity available in state-of-the-art CMOS processing has made possible the integration of complete systems, including both digital and analog signal processing functions, within the same chip Through the last decade, the **switched** capacitor technique **...**

Cited by 151 - Related articles

This paper is one of the most significant contributions that has ever come from ISCAS

Current-Mode Filters

Current-Mode Filters

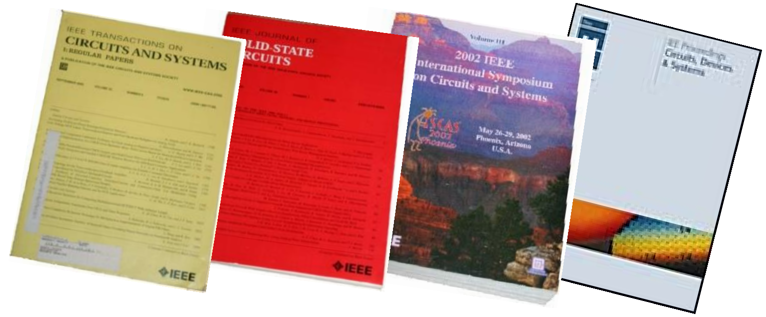


Search done on Nov 7, 2010

Review from Earlier Lecture

Current-Mode Filters

IEEE Xplore® RELEASE 2.4



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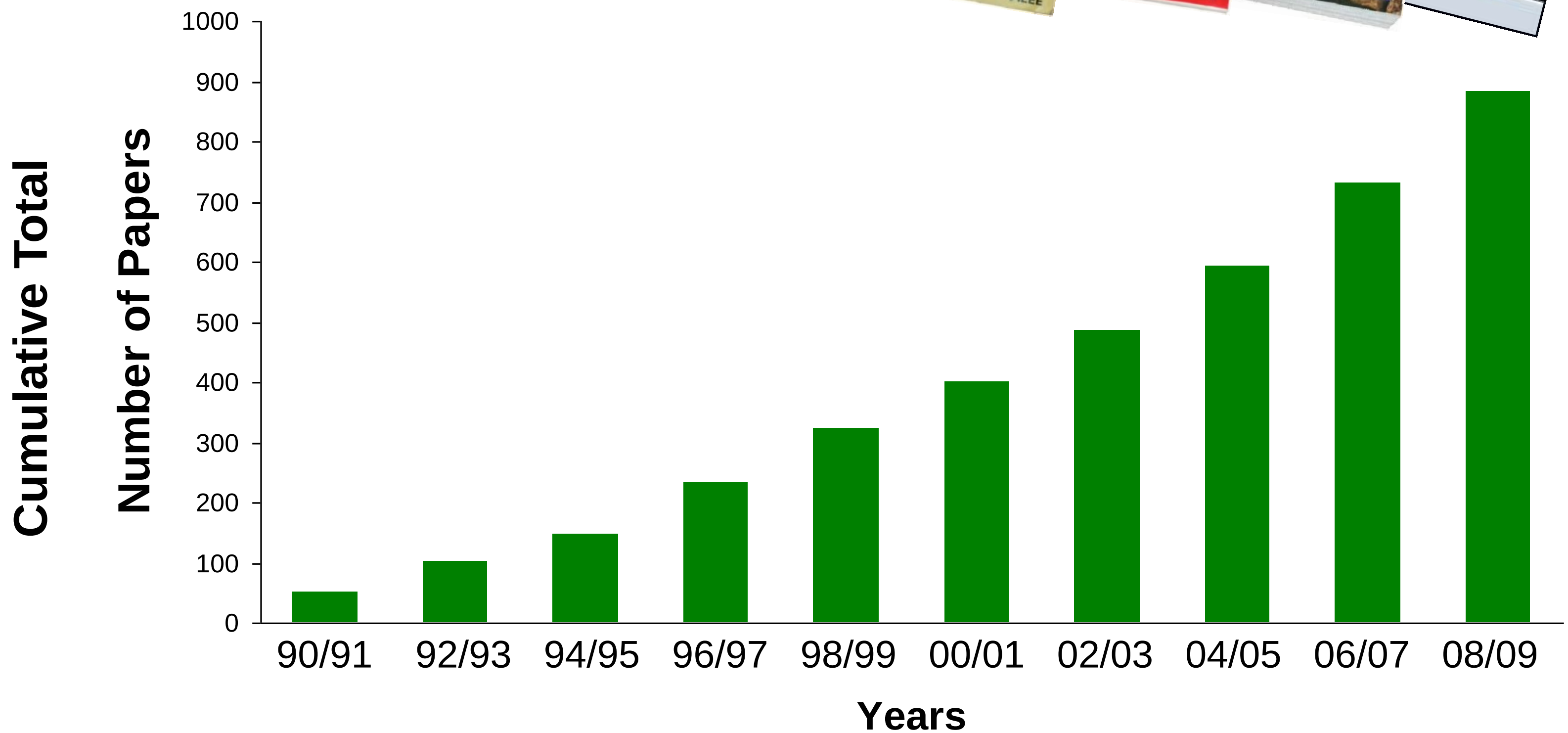


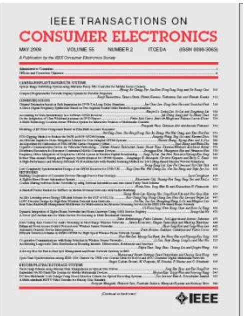
Steady growth in research in the area since 1990 and publication rate is growing with time !!

Current-Mode Filters

The Conventional Wisdom:

Proc. ICASP May 2010:

It is well known that current-mode circuits can offer many advantages, such as simplicity of circuit structure, high-frequency operation, wide dynamic range, and so on, compared with their voltage-mode counterparts.



IEEE Trans. On Consumer Electronics, Feb 2009

Current mode signal processing is a better solution than conventional voltage mode processing for high speed, low power and low voltage analog circuit design.

Current-Mode Filters

The Conventional Wisdom:

Proc. IEE Dec 2006:

Current-mode circuits have been proven to offer advantages over their voltage-mode counterparts [1, 2]. They possess wider bandwidth, greater linearity and wider dynamic range. Since the dynamic range of the analogue circuits using low-voltage power supplies will be low, this problem can be solved by employing current-mode operation.

Proc. SICE-ICASE, Oct. 2006

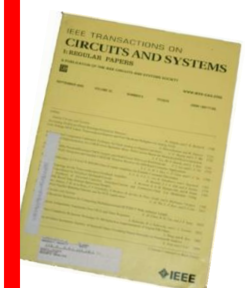
It is well known that current-mode circuits have been receiving significant attention owing to its advantage over the voltage-mode counterpart, particularly for higher frequency of operation and simpler filtering structure [1].

Current-Mode Filters

The Conventional Wisdom:

JSC April 1998:

"… current-mode functions exhibit higher frequency potential, simpler architectures, and lower supply voltage capabilities than their voltage-mode counterparts."



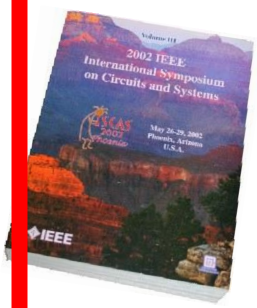
CAS June 1992

"Current-mode signal processing is a very attractive approach due to the simplicity in implementing operations such as … and the potential to operate at higher signal bandwidths than their voltage mode analogues" … "Some voltage-mode filter architectures using transconductance amplifiers and capacitors (TAC) have the drawback that …"

Current-Mode Filters

The Conventional Wisdom:

ISCAS 1993:

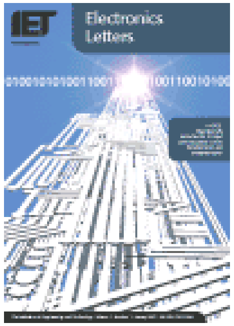


"In this paper we propose a fully balanced high frequency current-mode integrator for low voltage high frequency filters. Our use of the term current mode comes from the use of current amplifiers as the basic building block for signal processing circuits. This fully differential integrator offers significant improvement even over recently introduced circuit with respect to accuracy, high frequency response, linearity and power supply requirements. Furthermore, it is well suited to standard digital based CMOS processes."

Current-Mode Filters

The Conventional Wisdom:

Two key publications where benefits of the current-mode circuits are often referenced:



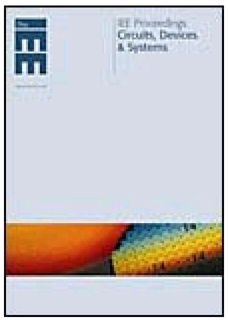
All current-mode frequency selective circuits **GW Roberts**, AS **Sedra** - Electronics Letters, June 1989 - pp. 759-761 Cited by 228

“To make greatest use of the available transistor bandwidth f_T, and operate at low voltage supply levels, it has become apparent that analogue signal processing can greatly benefit from processing current signals rather than voltage signals. Besides this, it is well known by electronic circuit designers that the mathematical operations of adding, subtracting or multiplying signals represented by currents are simpler to perform than when they are represented by voltages. This also means that the resulting circuits are simpler and require less silicon area.”

Current-Mode Filters

The Conventional Wisdom:

Two key publications where benefits of the current-mode circuits are often referenced:



Recent developments in **current** conveyors and **current**-mode circuits **B Wilson** - Circuits, Devices and Systems, IEE Proceedings G, pp. 63-77, Apr. 1990 Cited by 288

“The **use** of current rather than voltage as the active parameter can result in higher usable gain, accuracy and bandwidth due to reduced voltage excursion at sensitive nodes. A current-mode approach is not just restricted to current processing, but also offers certain important advantages when interfaced to voltage-mode circuits.”

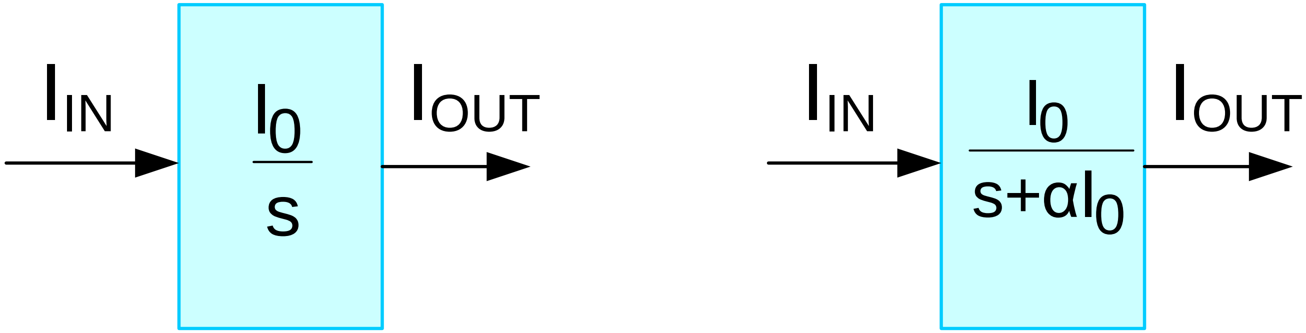
Current-Mode Filters

The Conventional Wisdom:

- Current-Mode circuits operate at higher-frequencies than voltage-mode counterparts
- Current-Mode circuits operate at lower supply voltages and lower power levels than voltage-mode counterparts
- Current-Mode circuits are simpler than voltage-mode counterparts
- Current-Mode circuits offer better linearity than voltage-mode counterparts

This represents four really significant benefits of current-mode circuits!

Current-Mode Filters



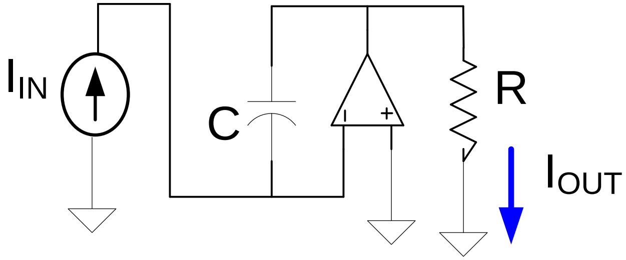
Integrator

Lossy Integrator

As with voltage-mode filters, most integrated current-mode filters are built with integrators and lossy integrators

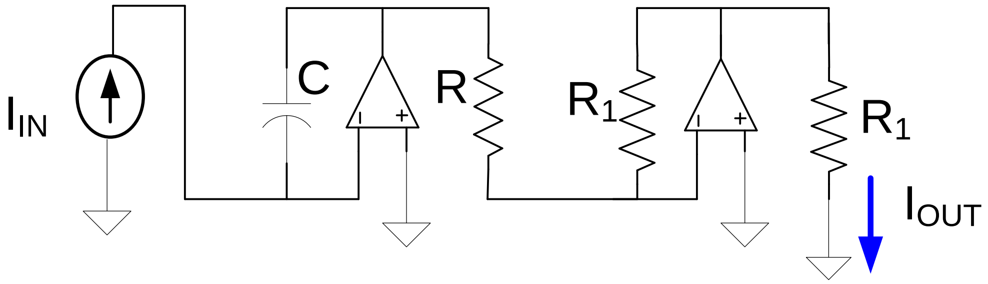
Some Current-Mode Integrators

Active RC



$$I_{OUT} = \left(\frac{-1}{RCs}\right) I_{IN}$$

Inverting



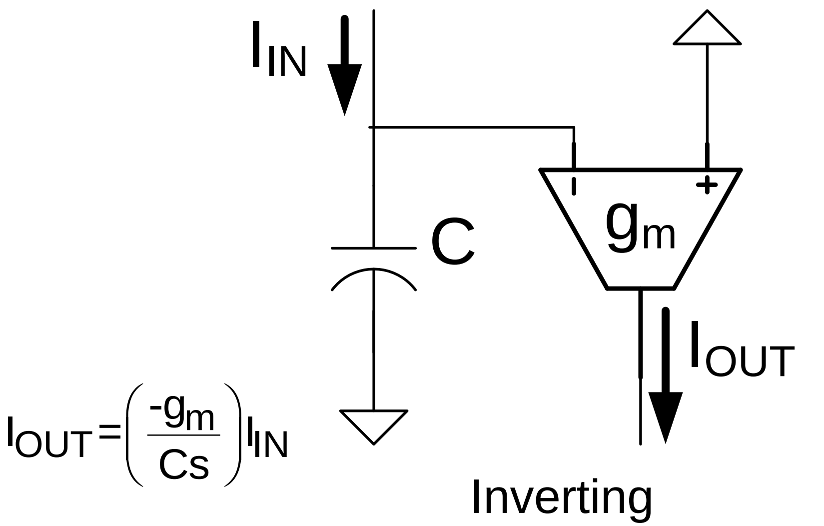
$$I_{OUT} = \left(\frac{1}{RCs}\right) I_{IN}$$

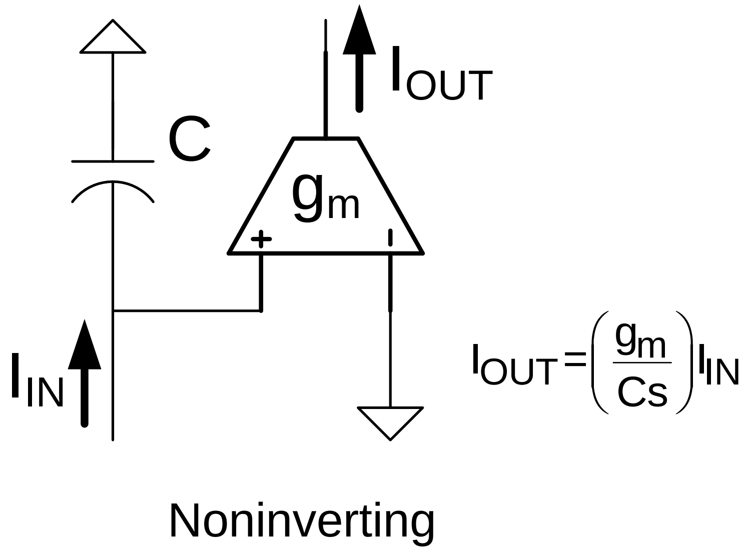
Noninverting

- Summing inputs really easy to obtain
- Loss is easy to add
- Some argue that since only interested in currents, can operate at lower voltages

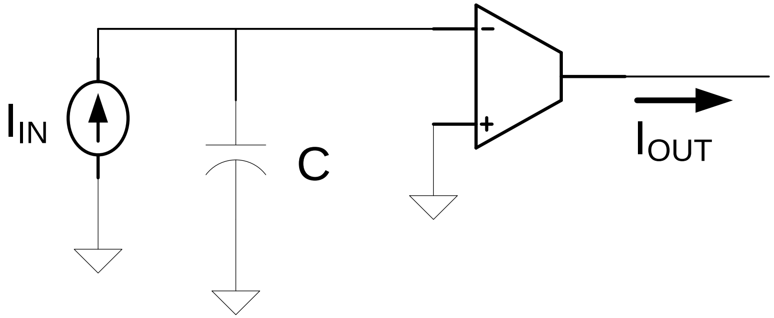
Some Current-Mode Integrators

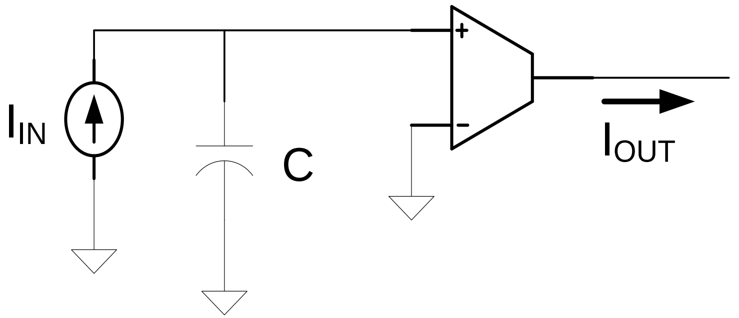
OTA-C





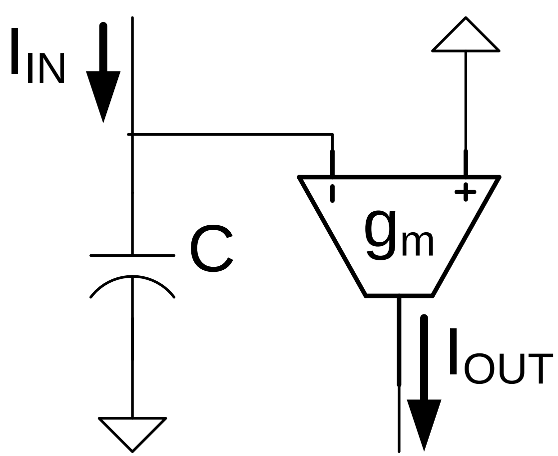
Alternate representation



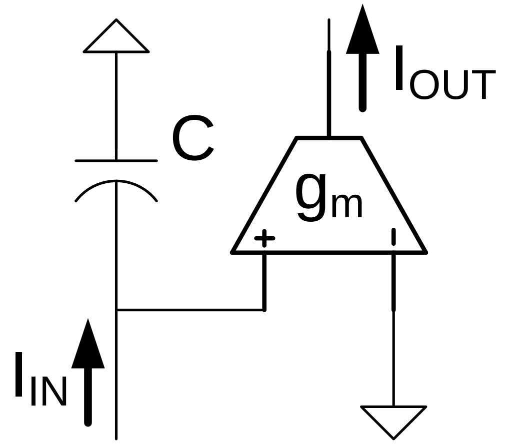


Some Current-Mode Integrators

OTA-C



Inverting

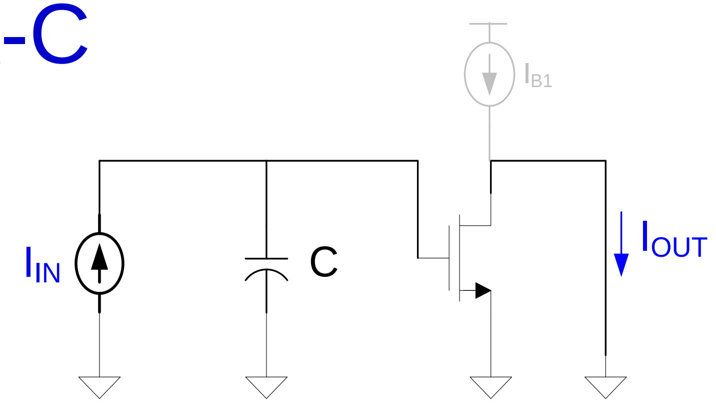


Noninverting

- Summing inputs really easy to obtain
- Loss is easy to add
- Many argue that since only interested in currents, can operate at lower voltages and higher frequencies

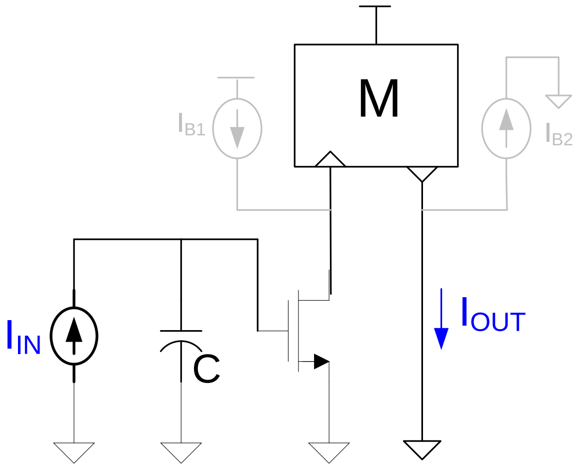
Some Current-Mode Integrators

TA-C



$$I_{OUT} = \left(\frac{-g_m}{Cs}\right) I_{IN}$$

Inverting

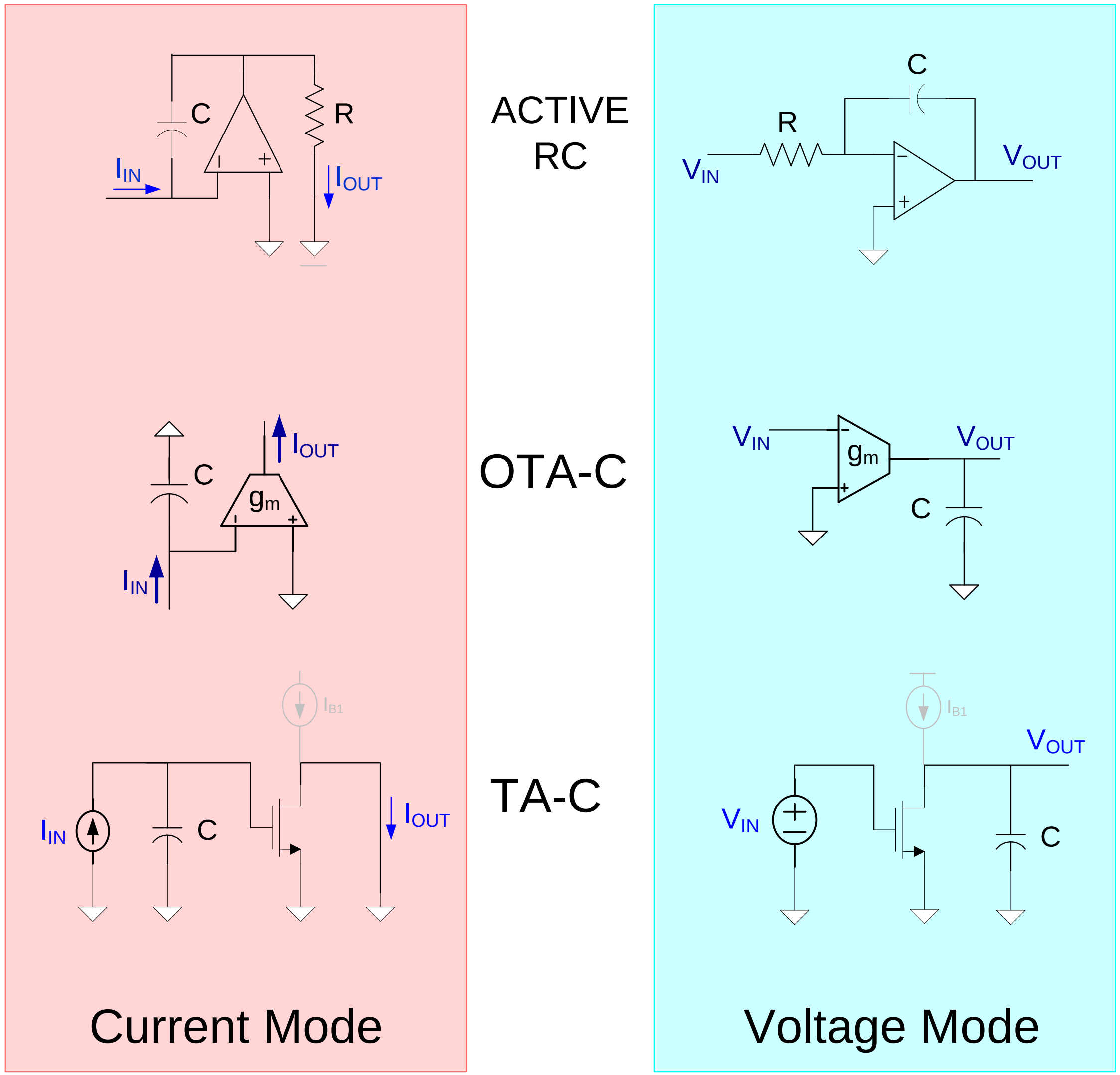


$$I_{OUT} = \left(\frac{g_m}{Cs}\right) I_{IN}$$

Noninverting

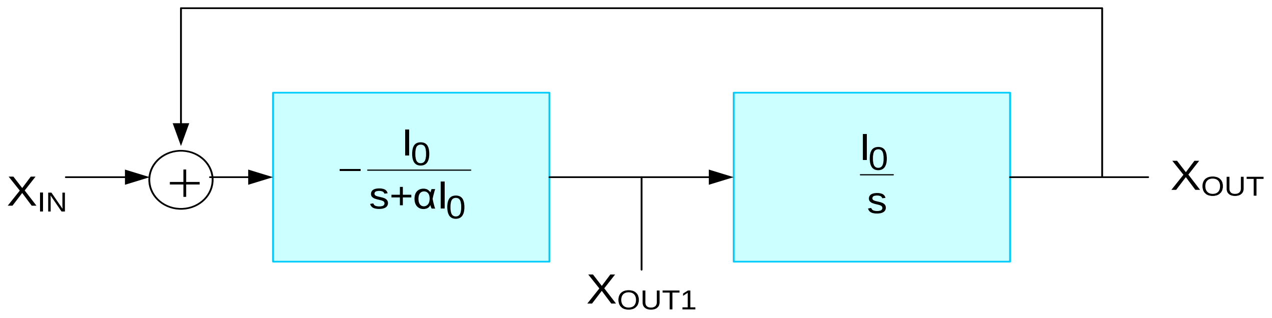
- Summing inputs really easy to obtain
- Loss is easy to add
- Many argue that since only interested in currents, can operate at lower voltages and higher frequencies

Comparison of Current Mode and Voltage Mode Integrators



- Current Mode and Voltage Mode Inverting integrators have same device counts
- Same is true of noninverting and lossy structures

Two-Integrator-Loop Biquad

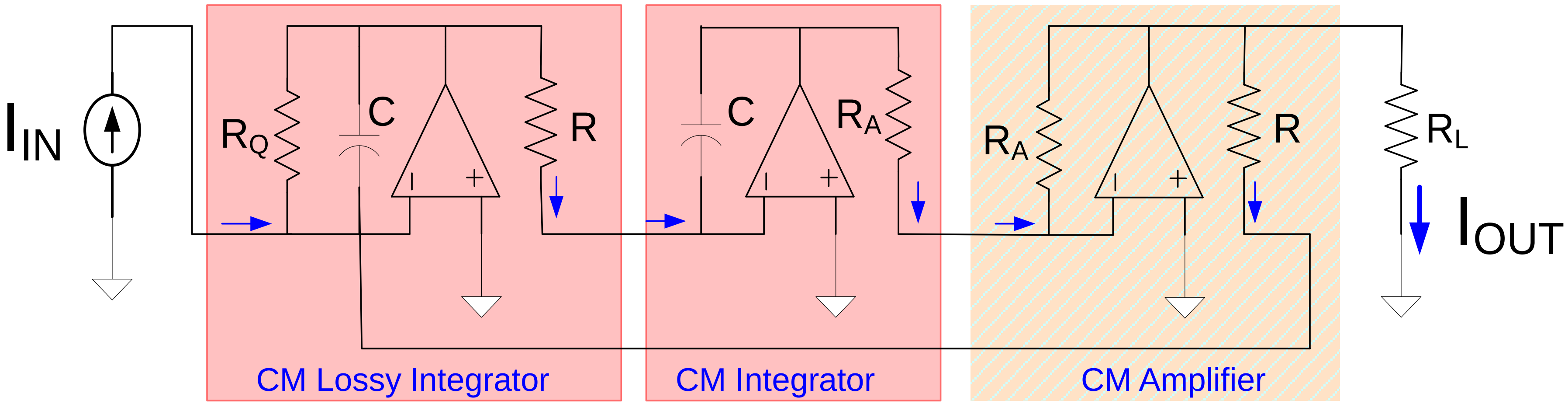


One of the most widely used architectures for implementing integrated filters

Review from Earlier Lecture

Current-Mode Two Integrator Loop

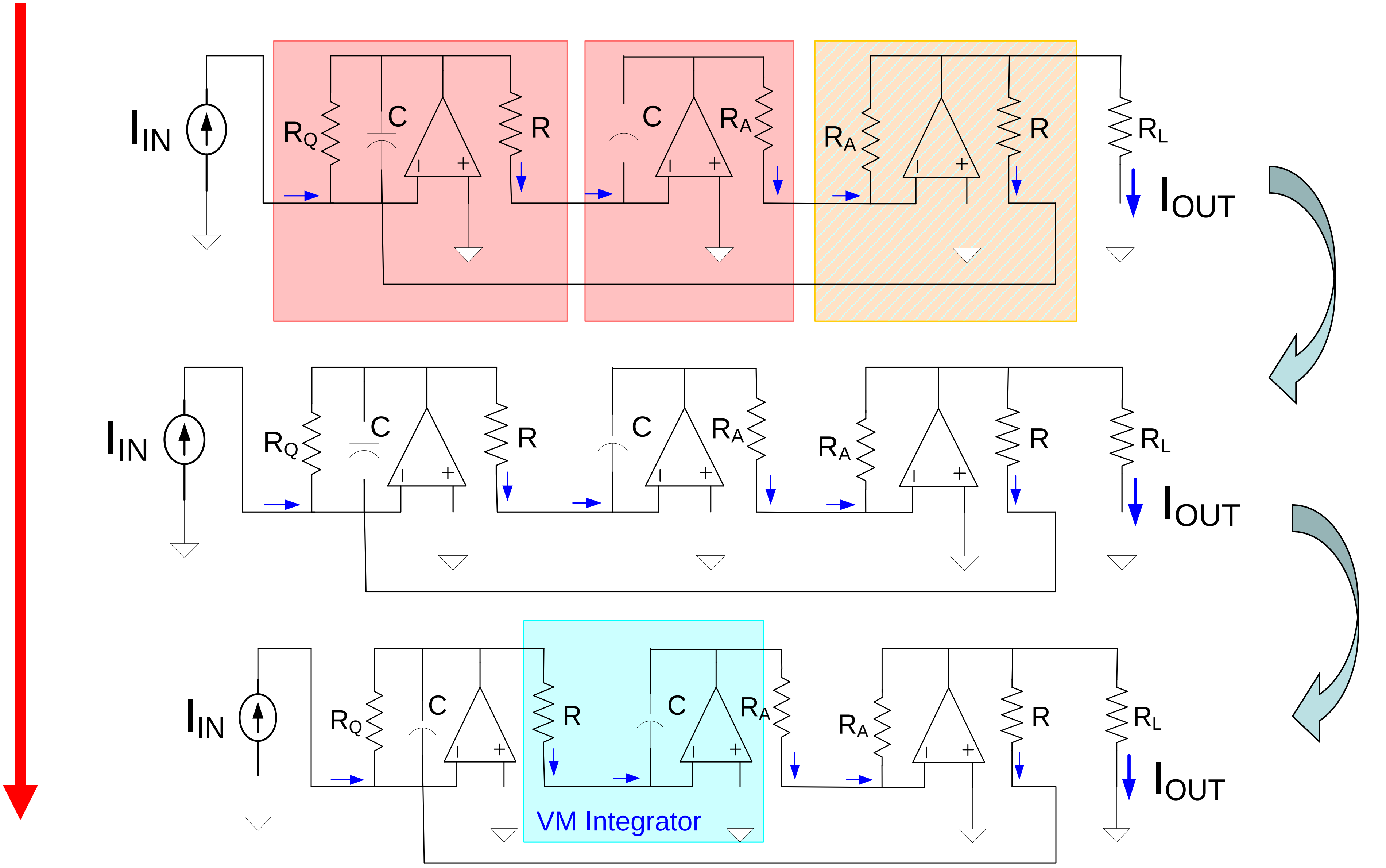
Active RC Current-Mode implementation



- Straightforward implementation of the two-integrator loop
- Simple structure

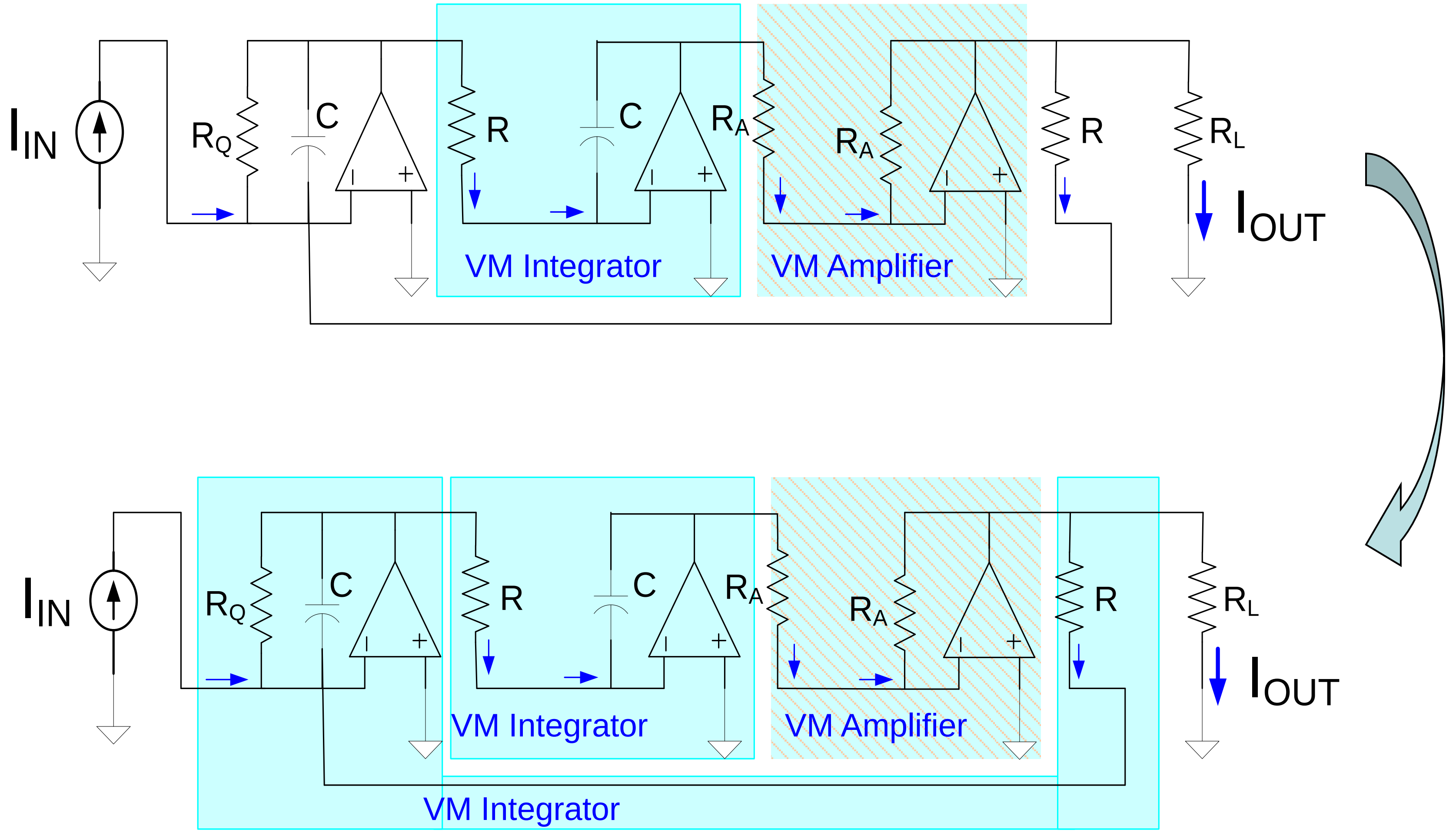
Current-Mode Two Integrator Loop

An Observation:



Current-Mode Two Integrator Loop

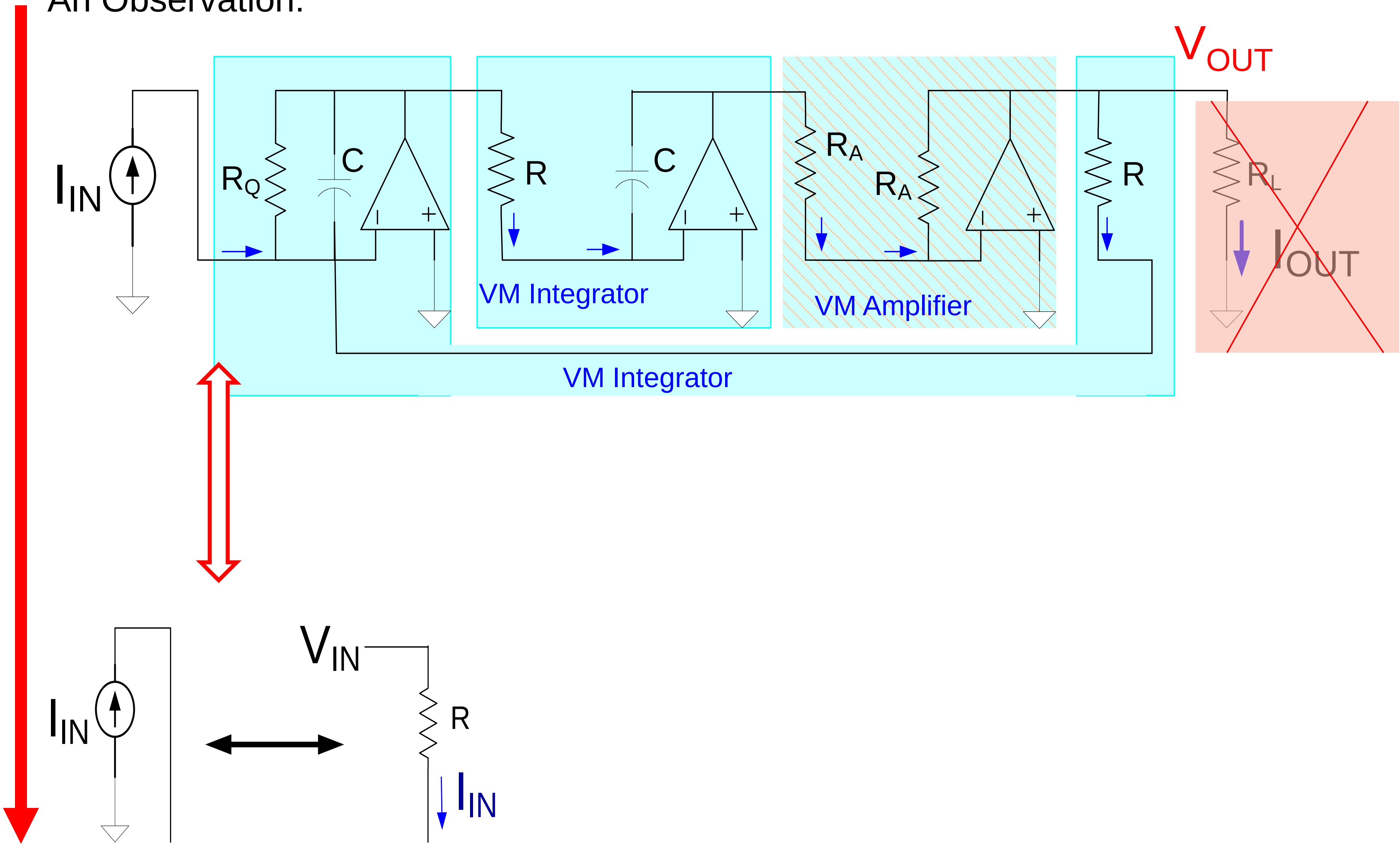
An Observation:



This circuit is identical to another one with two voltage-mode integrators and a voltage-mode amplifier !

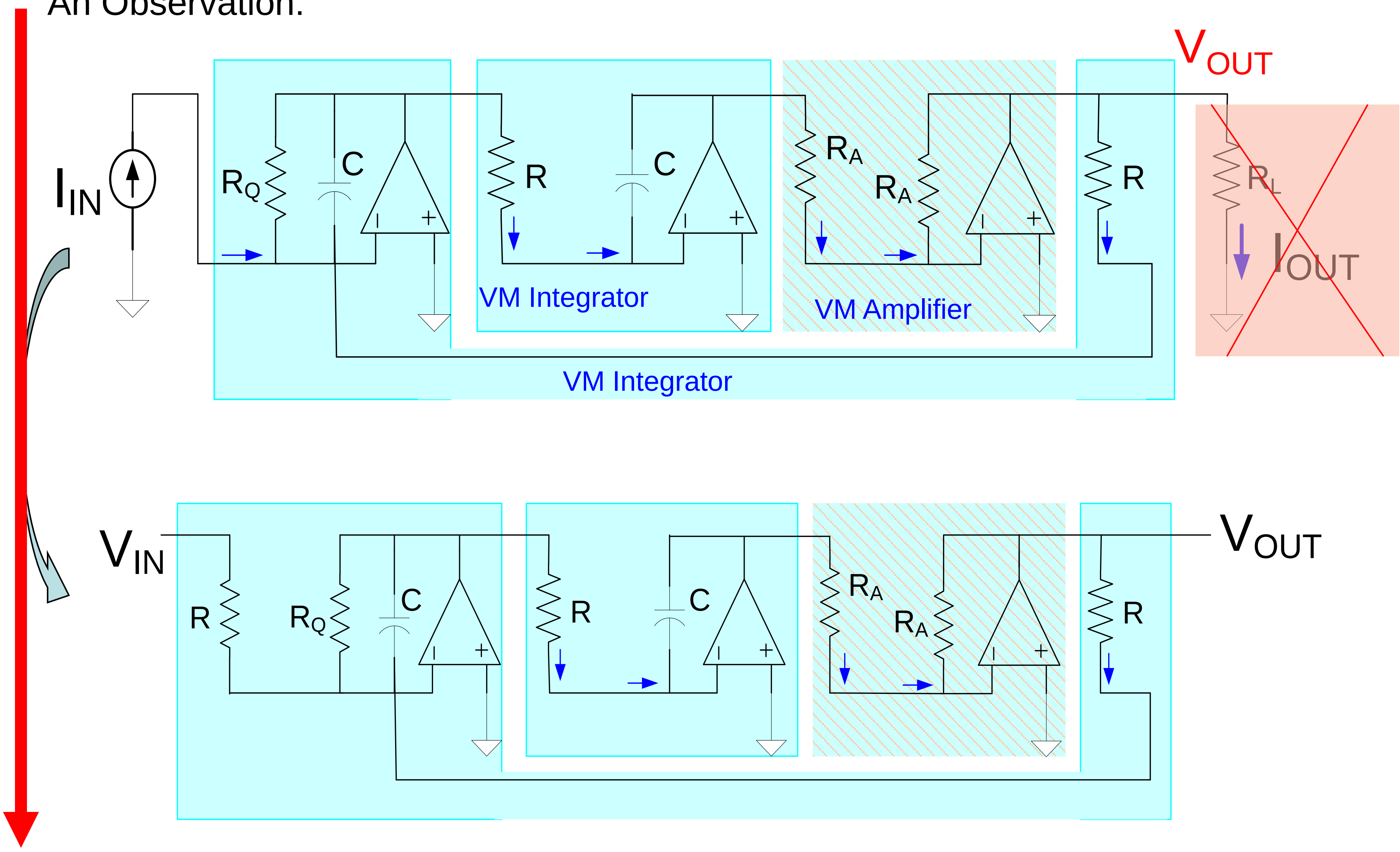
Current-Mode Two Integrator Loop

An Observation:



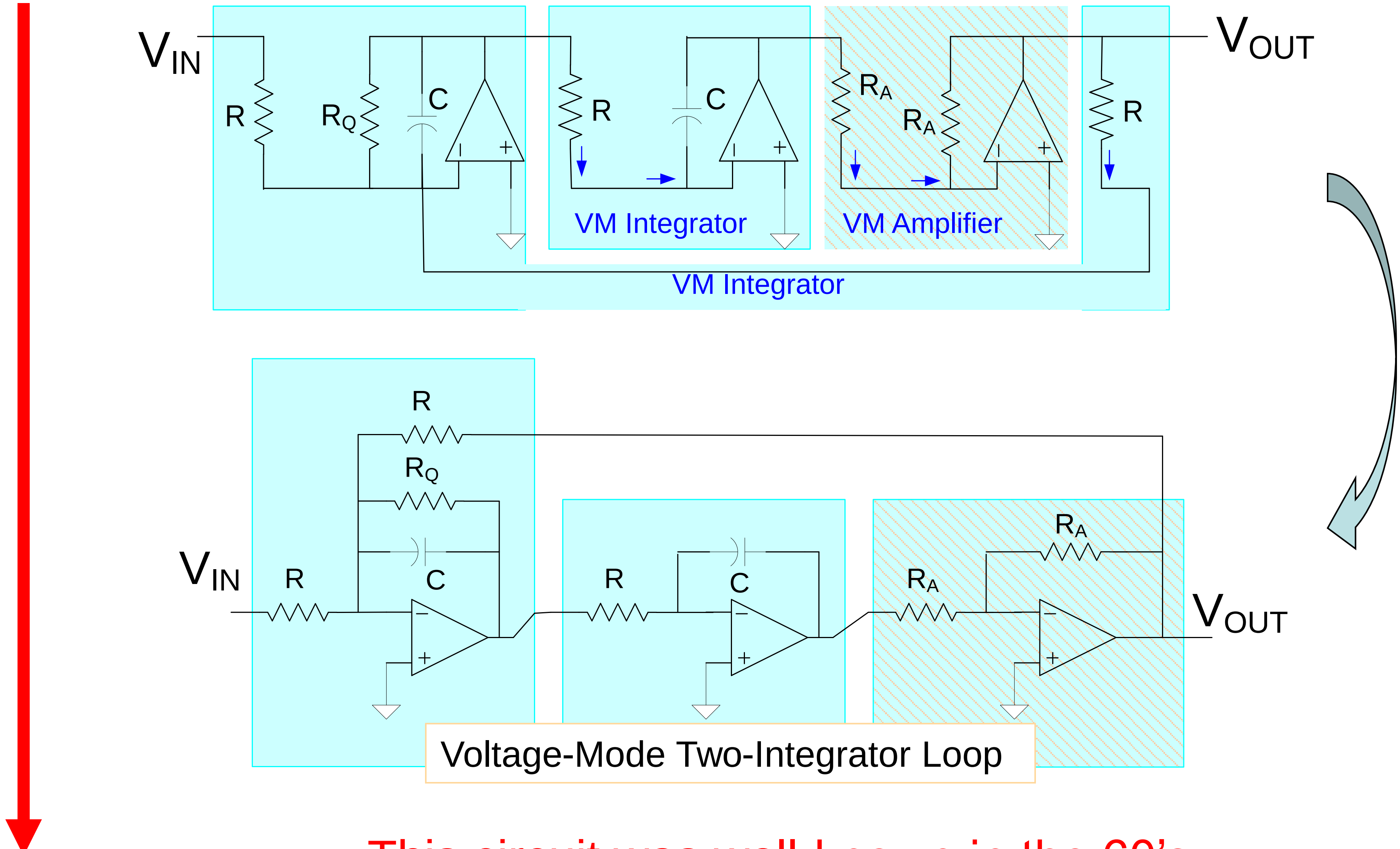
Current-Mode Two Integrator Loop

An Observation:



Current-Mode Two Integrator Loop

An Observation:

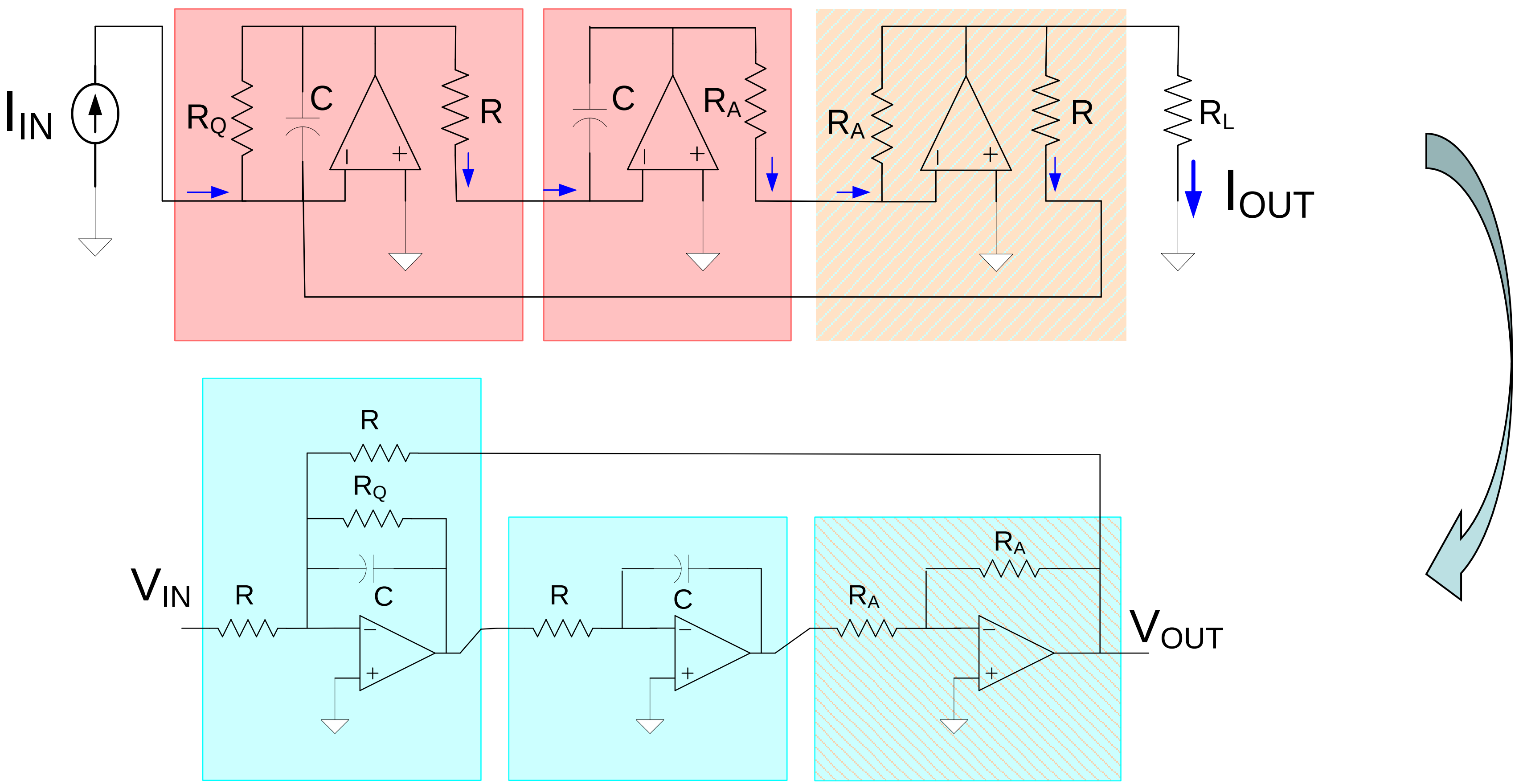


This circuit was well-known in the 60's

Review from Earlier Lecture

Current-Mode Two Integrator Loop

Active RC Current-Mode implementation



Current-mode and voltage-mode circuits have same component count
Current-mode and voltage-mode circuits are identical !
Current-mode and voltage-mode properties are identical !

Current-mode circuit offers NO benefits over voltage-mode counterpart

Observation

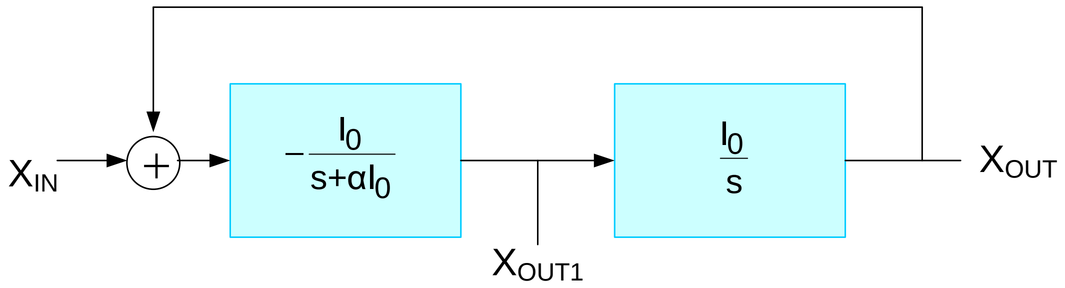
- Many papers have appeared that tout the performance advantages of current-mode circuits
- In all of the current-mode papers that this instructor has seen, no attempt is made to provide a quantitative comparison of the key performance features of current-mode circuits with voltage-mode counterparts
- All justifications of the advantages of the current-mode circuits this instructor has seen are based upon qualitative statements

Observations (cont.)

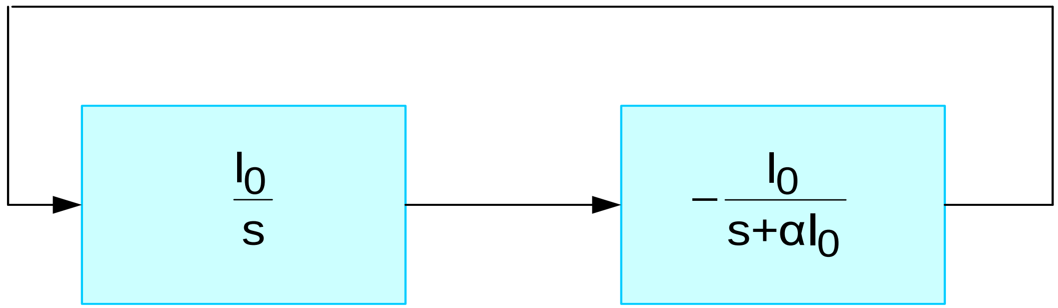
- It appears easy to get papers published that have the term “current-mode” in the title
- Over 900 papers have been published in IEEE forums alone !
- Some of the “current-mode” filters published perform better than other “voltage-mode” filters that have been published
- We are still waiting for even one author to quantitatively show that current-mode filters offer even one of the claimed four advantages over their voltage-mode counterparts

Will return to a discussion of Current-Mode filters later

Two-Integrator-Loop Biquad



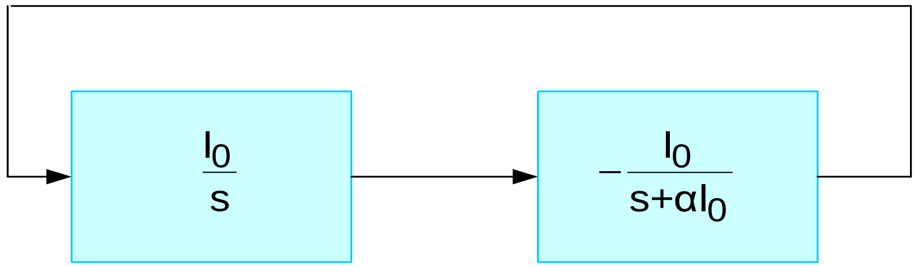
- For notational convenience, the input signal can be suppressed and output will not be designated
- This forms the “dead network”
- Poles for dead network are identical to original network as are key sensitivities



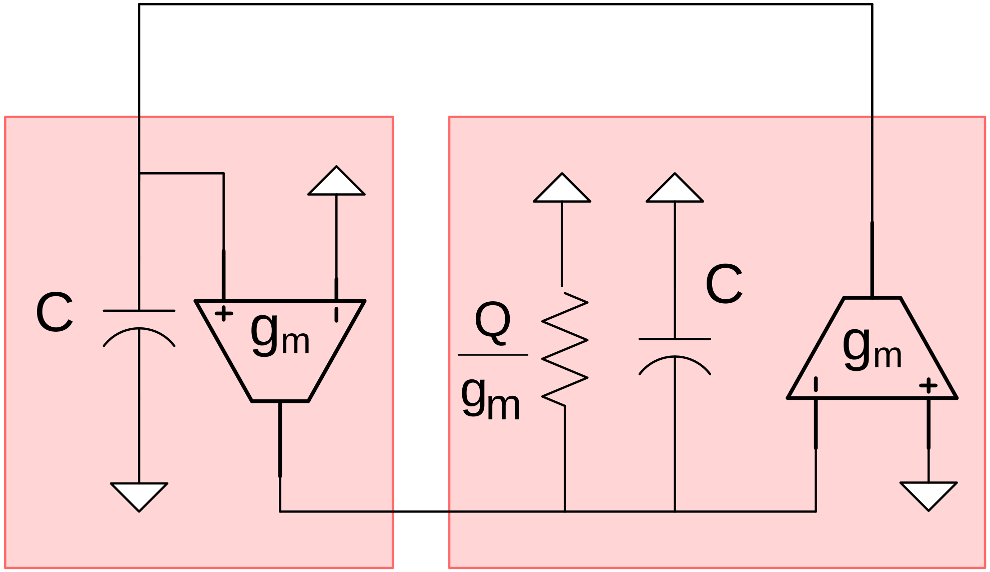
Two Integrator Loop Biquad

Two-Integrator-Loop Biquad

OTA-C implementation

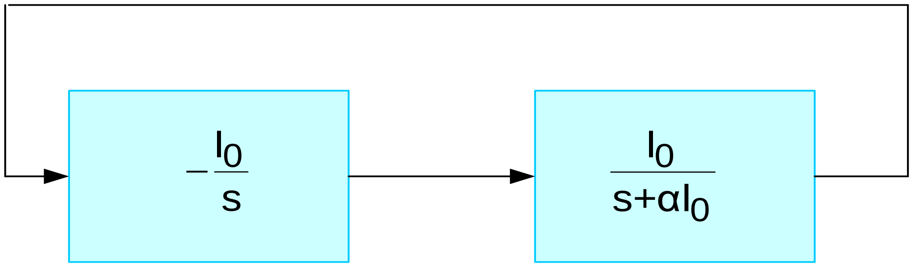


Consider a current-mode implementation:

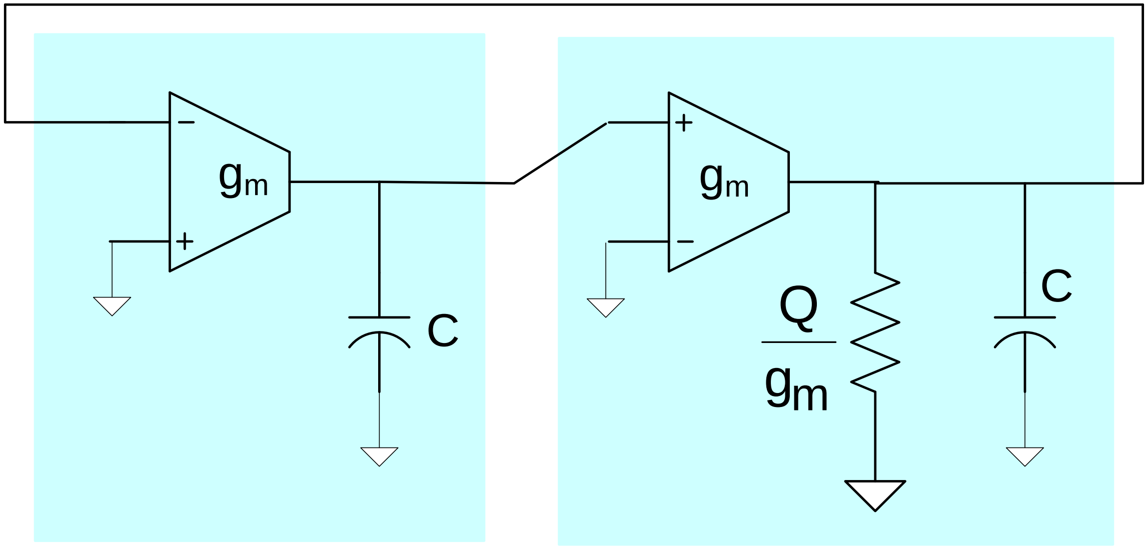


Numerous current-mode filter papers use this basic structure

Two-Integrator-Loop Biquad



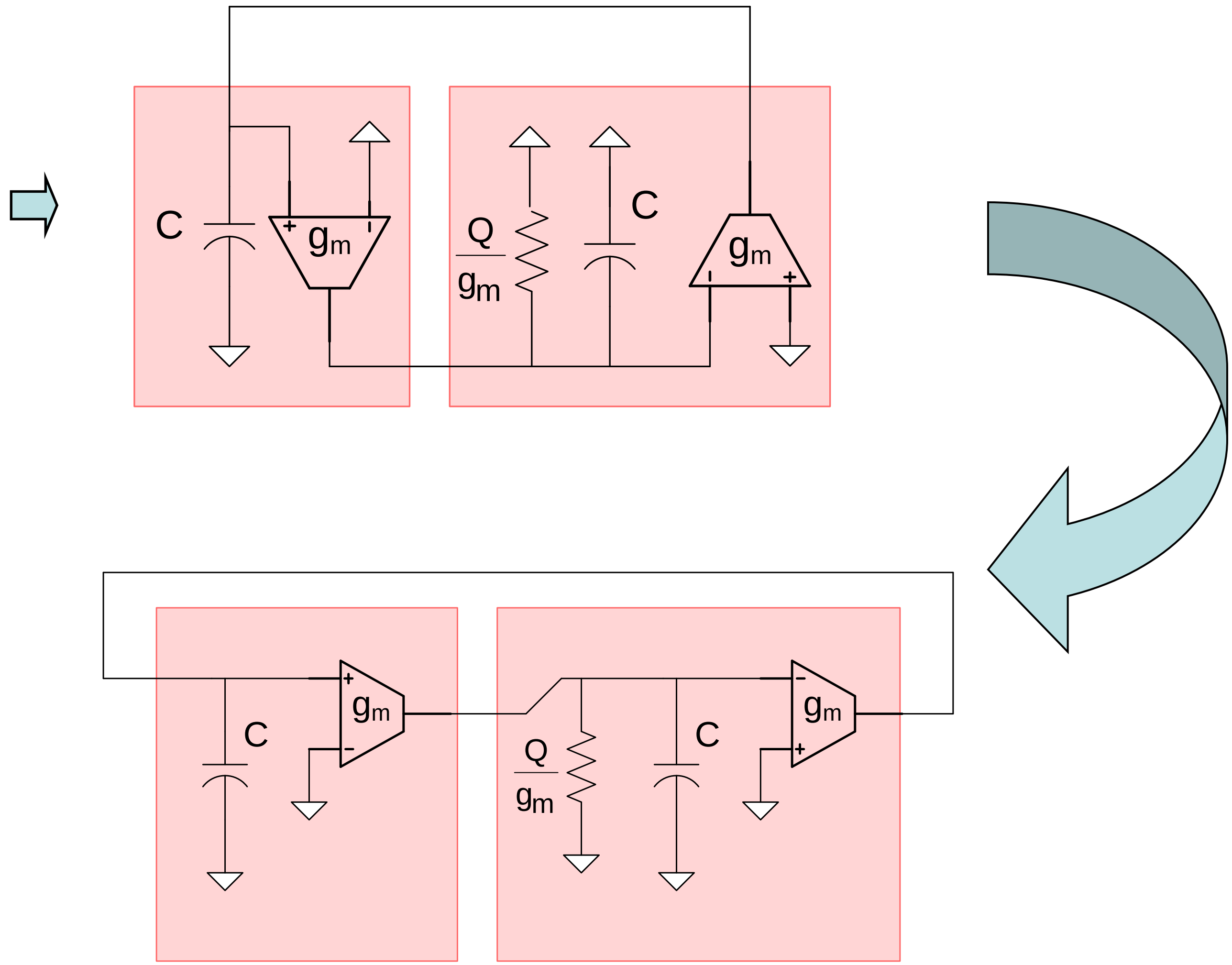
Consider the corresponding voltage-mode implementation:



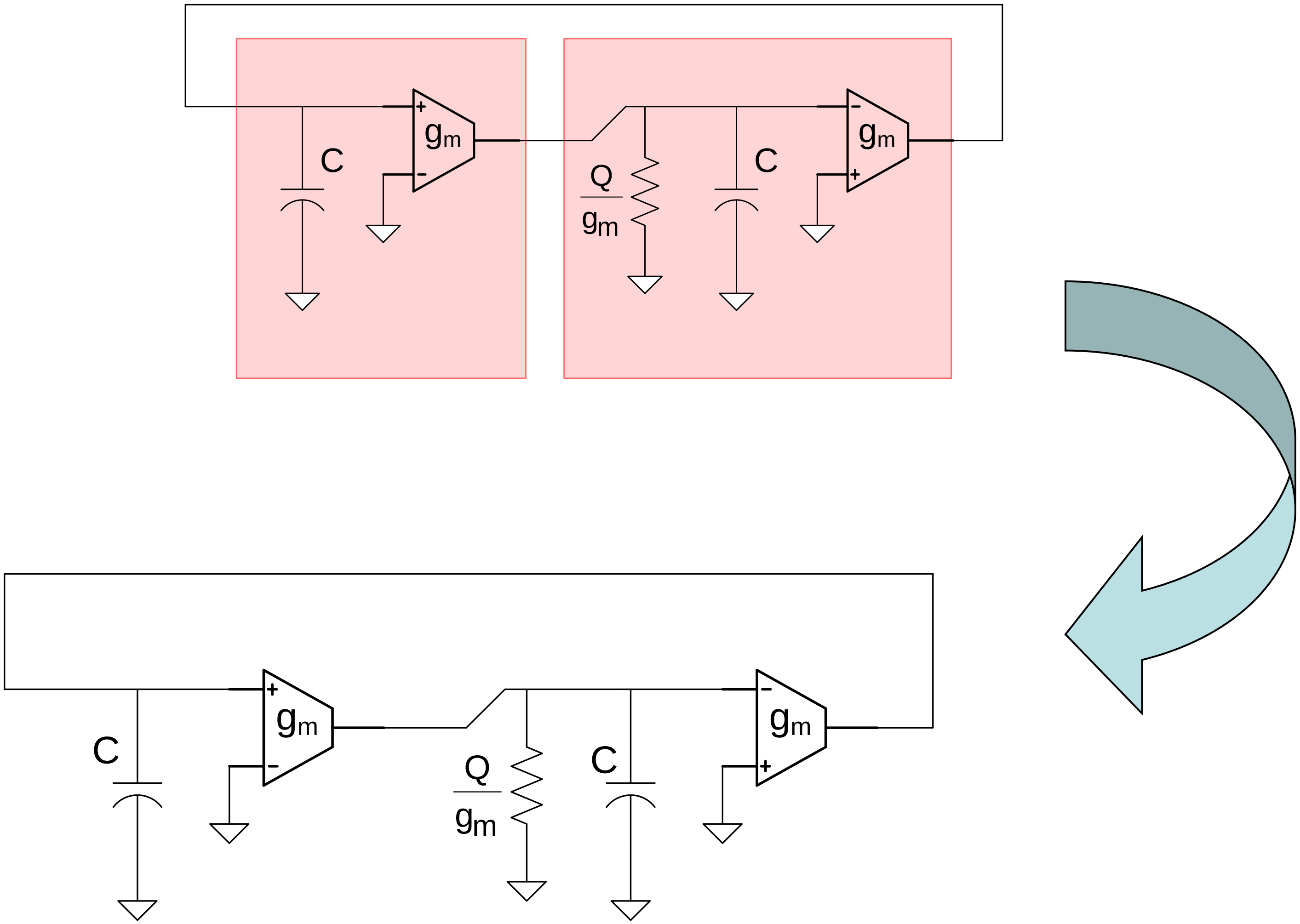
Two-Integrator-Loop Biquad

An Observation:

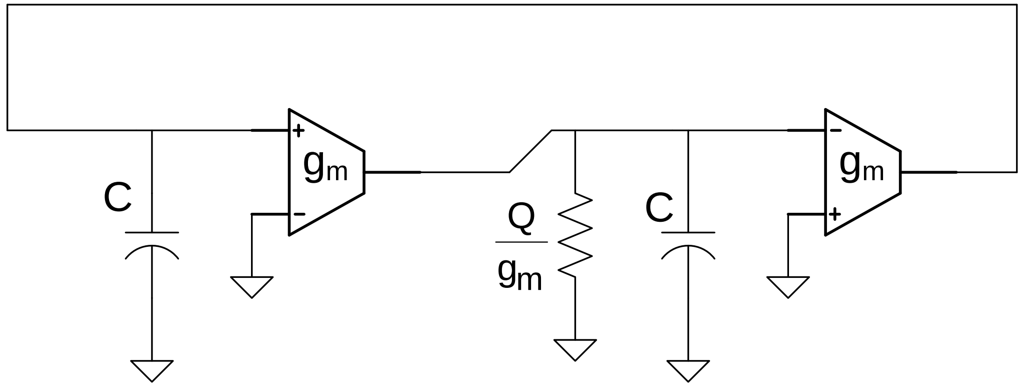
Current-mode

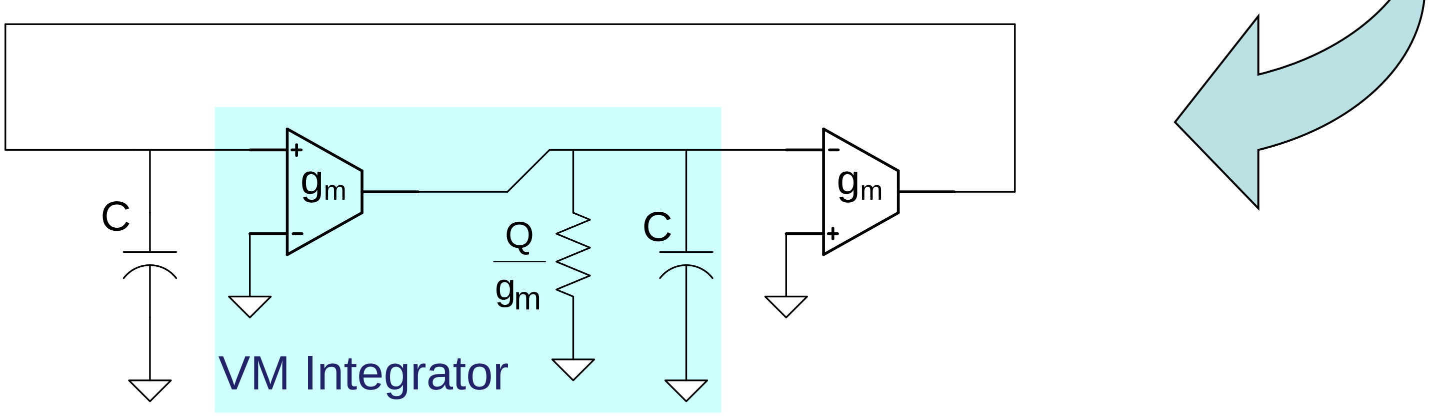


Two-Integrator-Loop Biquad

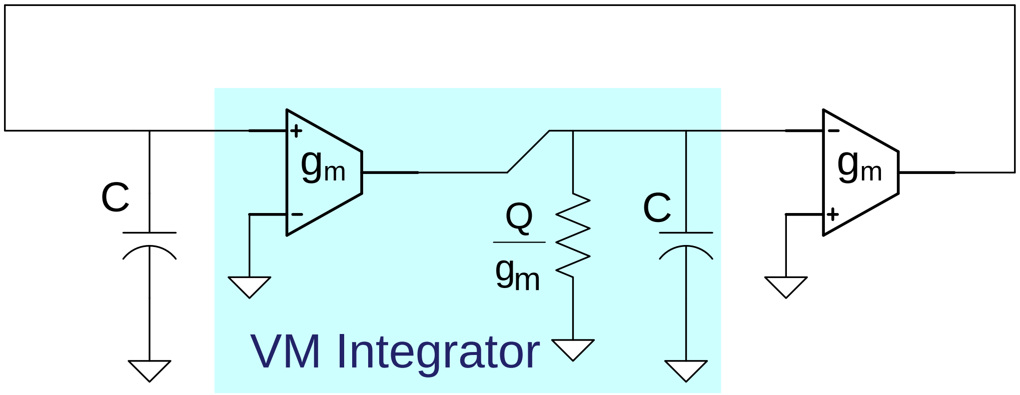


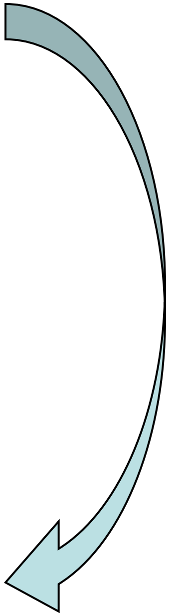
Two-Integrator-Loop Biquad

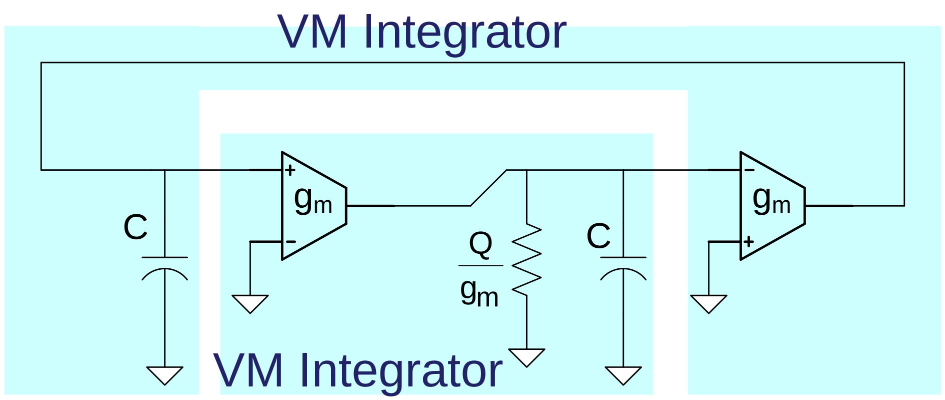




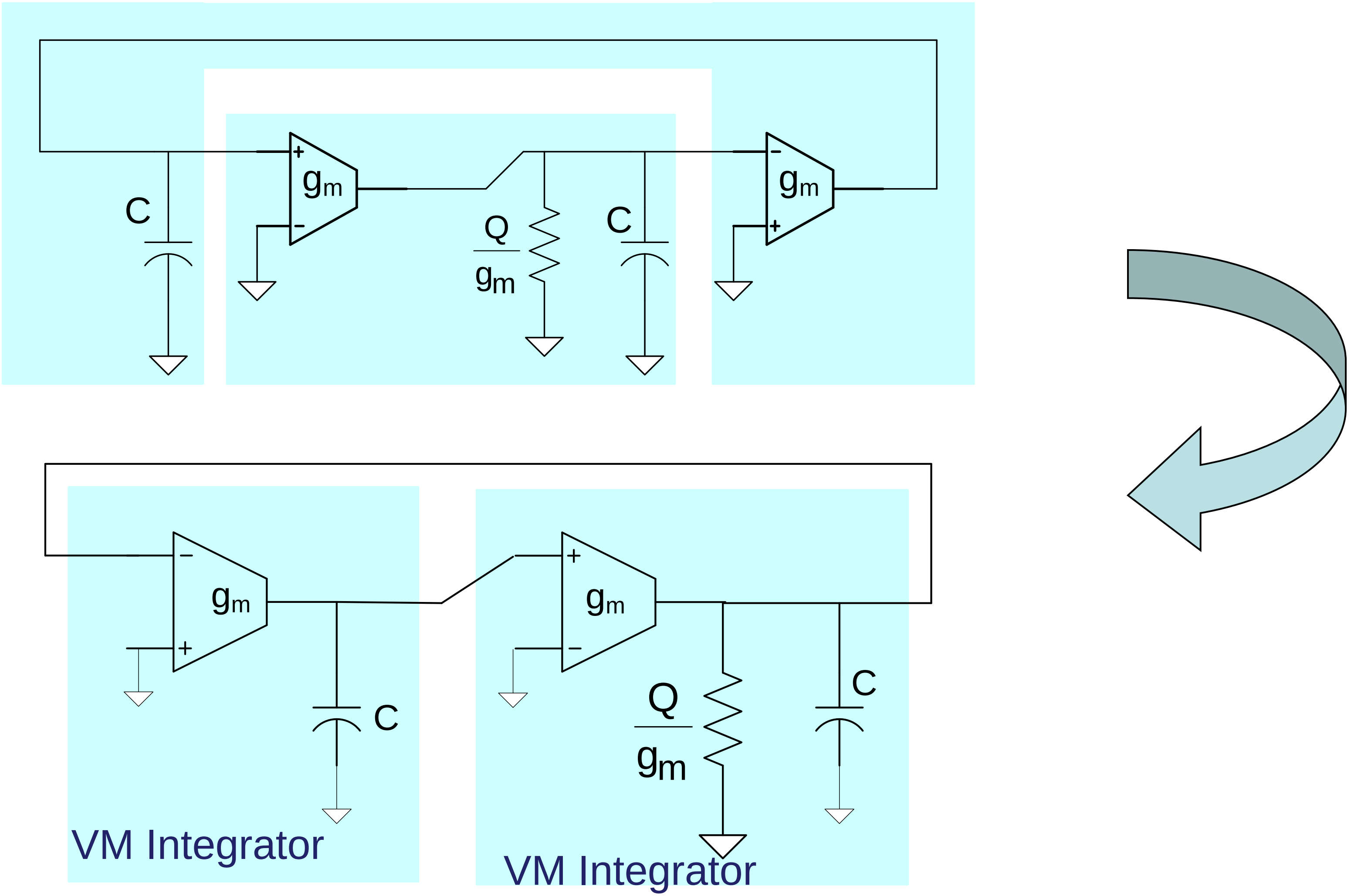
Two-Integrator-Loop Biquad







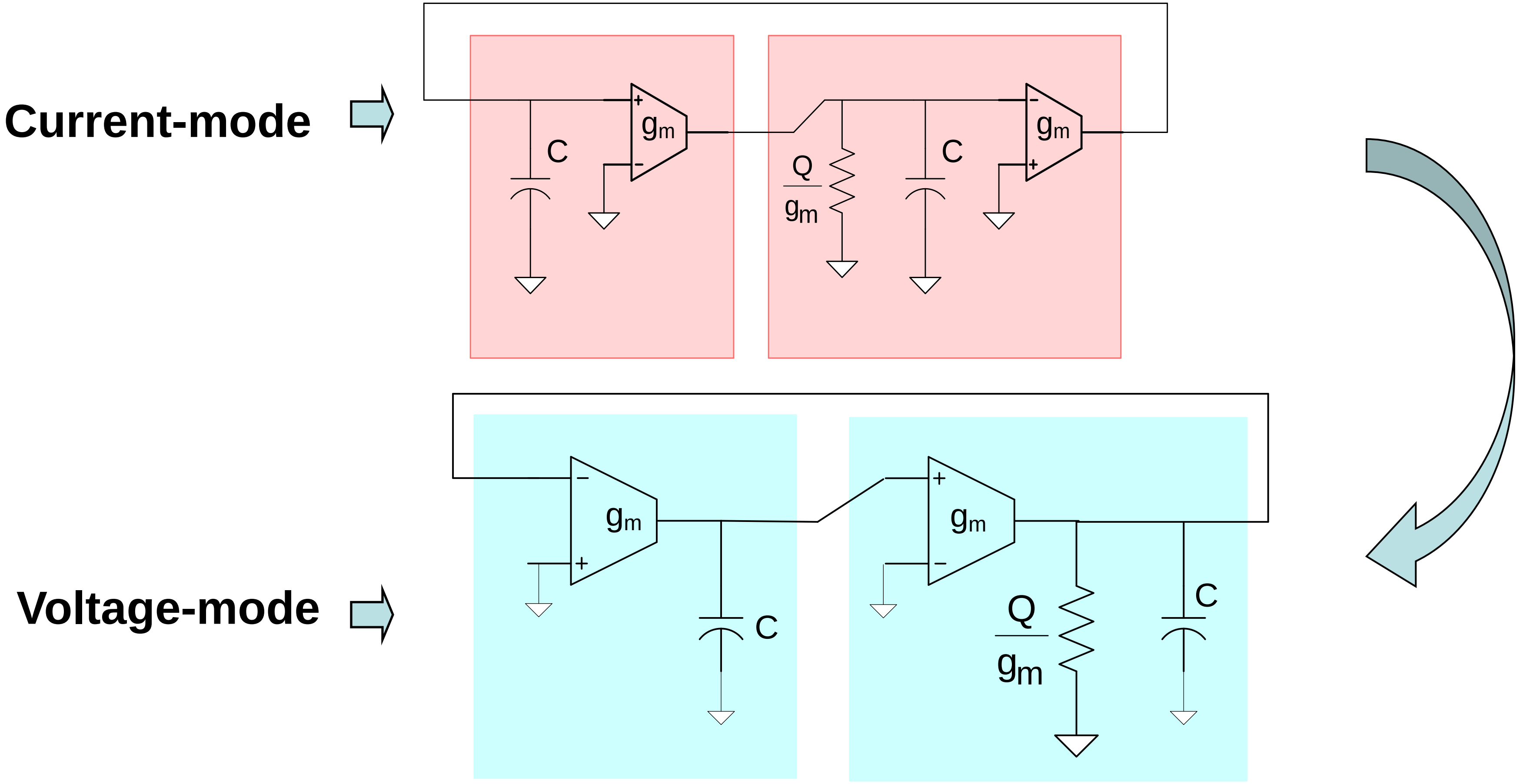
Two-Integrator-Loop Biquad



This circuit was well-known in the 80's

Two-Integrator-Loop Biquad

OTA-C implementation



Current-mode and voltage-mode circuits have same component count
Current-mode and voltage-mode circuits are identical !
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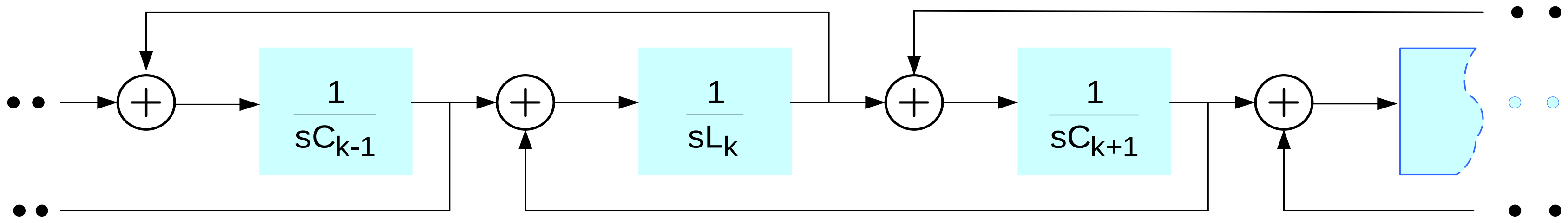
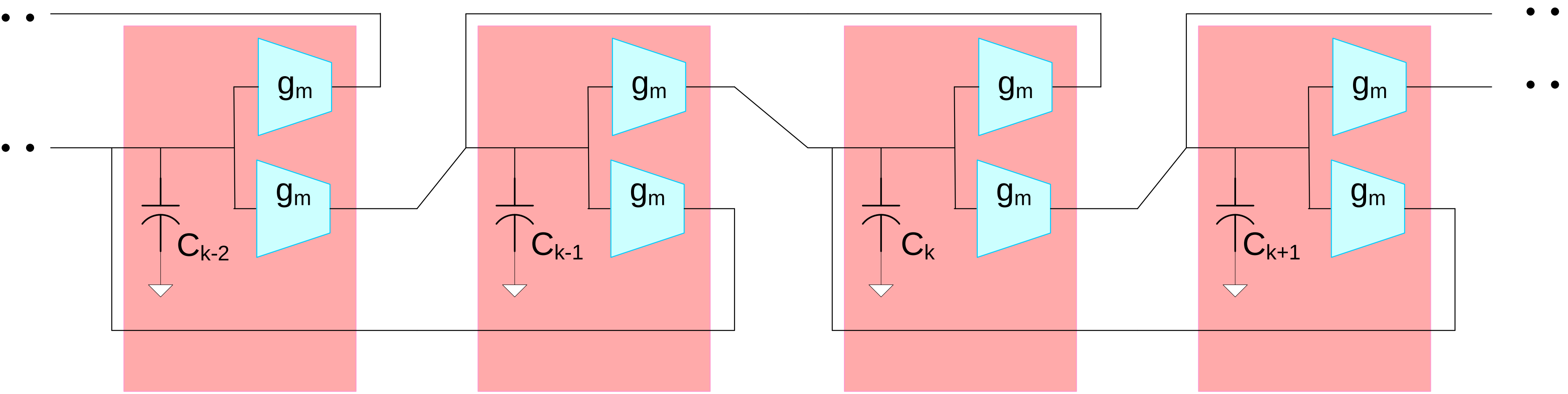
Leap-Frog Filter

OTA-C implementation

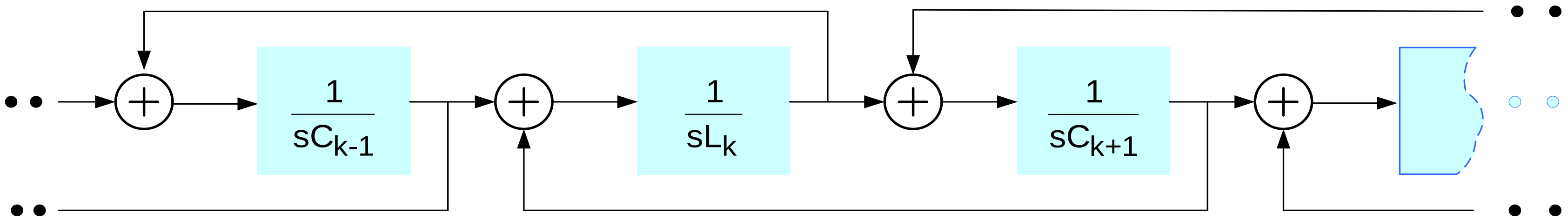


Consider a current-mode implementation:

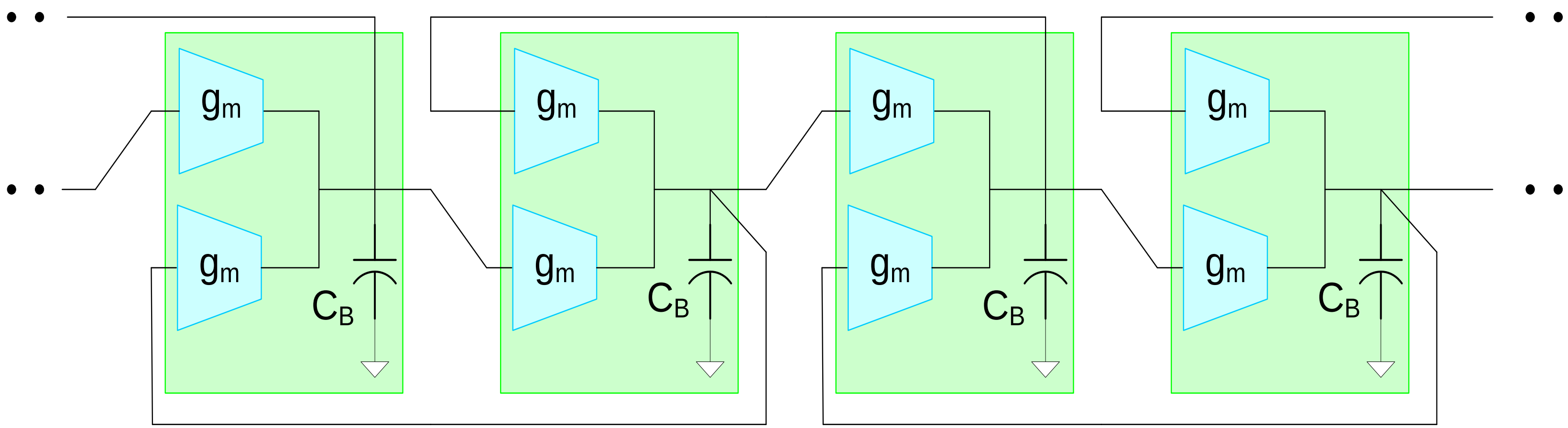


Numerous current-mode filter papers use this basic structure

Leap-Frog Filter

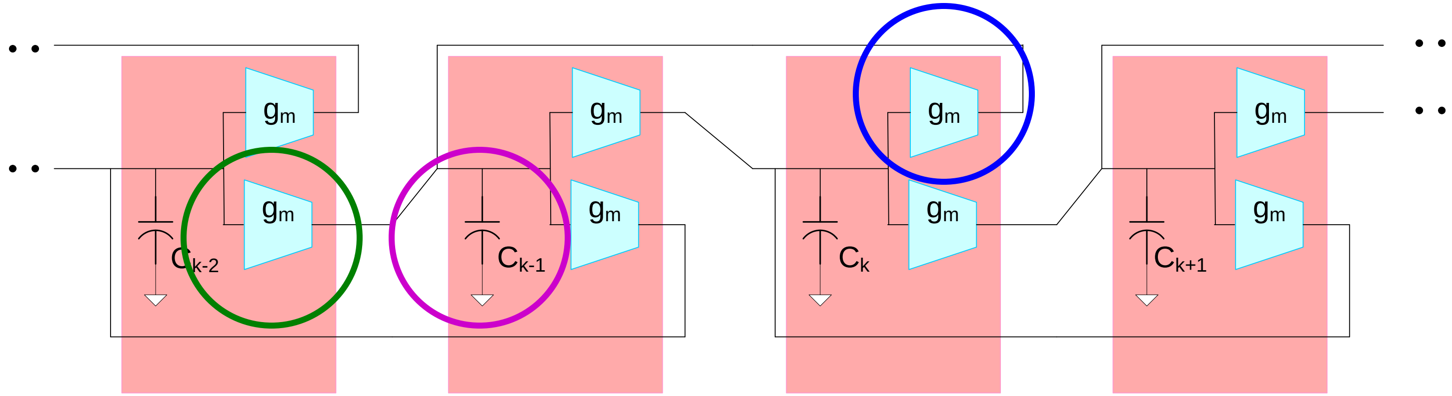


Consider a voltage-mode implementation:

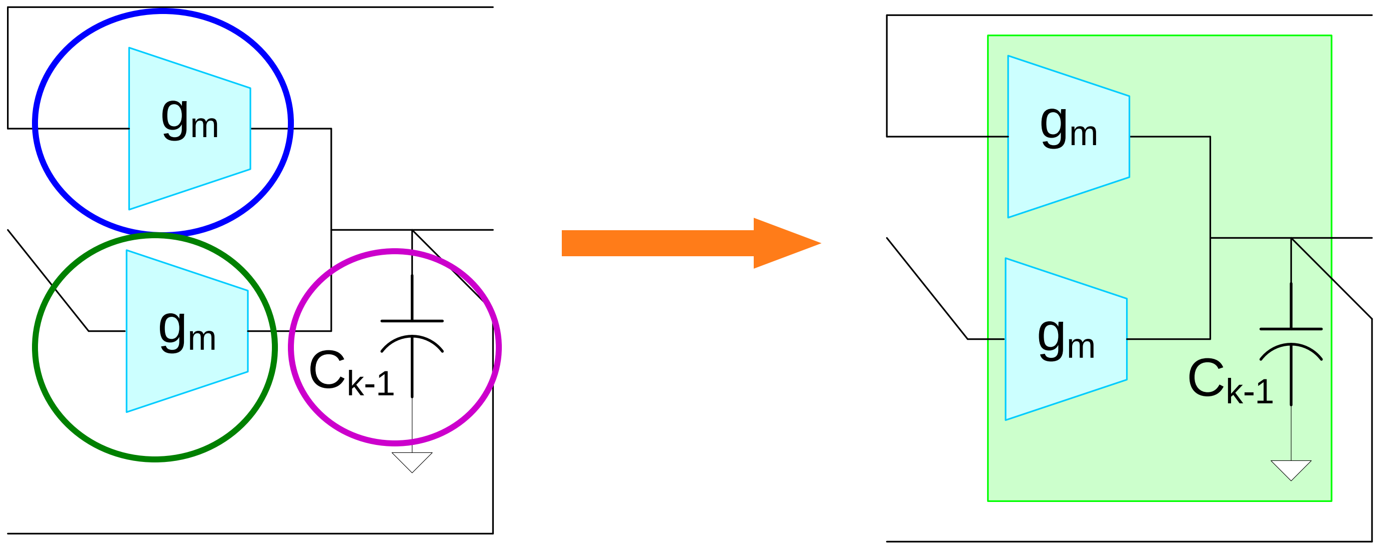


Leap-Frog Filter

An Observation:

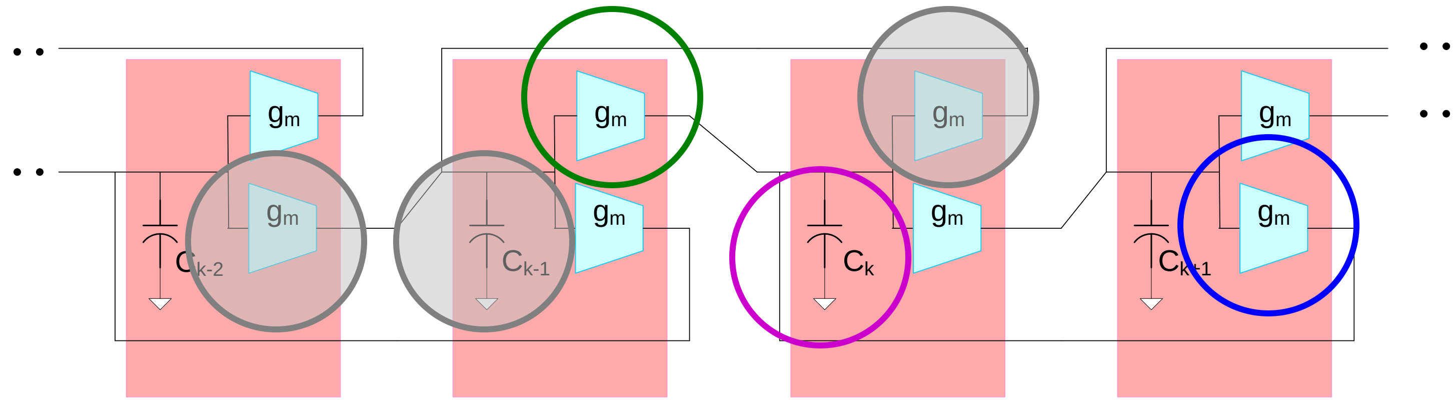


Consider lower OTA in stage k-2, capacitor in stage k-1 and upper OTA in stage k

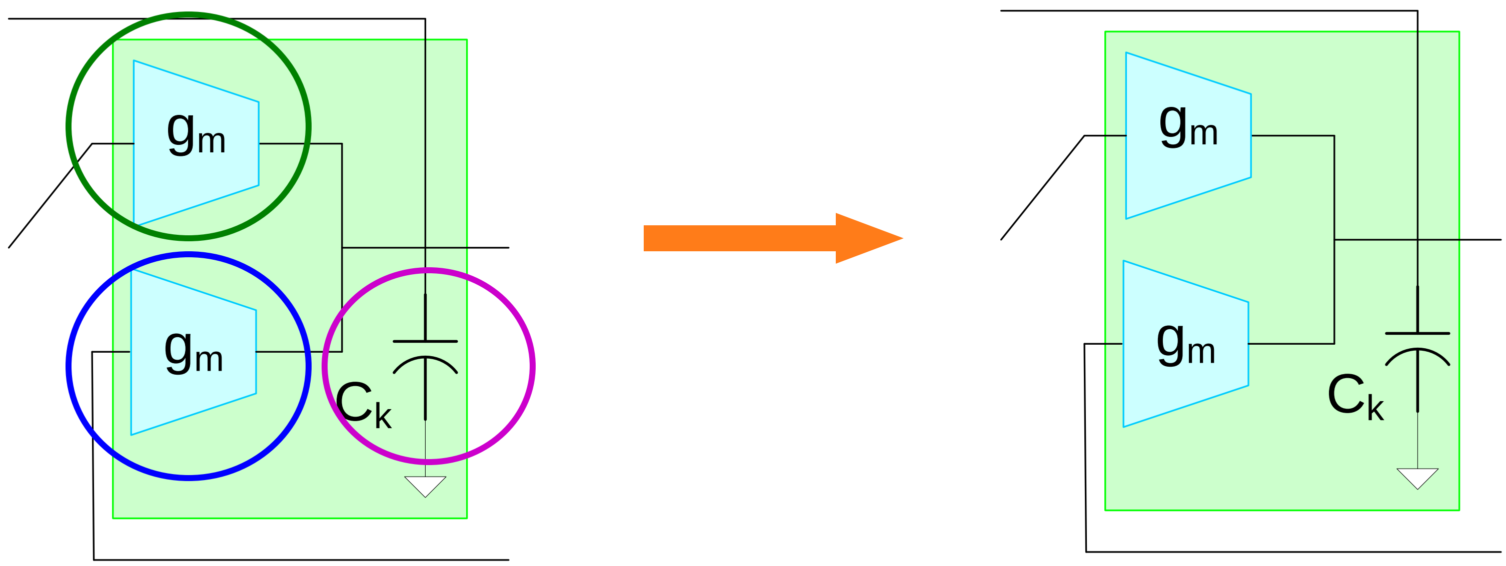


Leap-Frog Filter

Current-mode

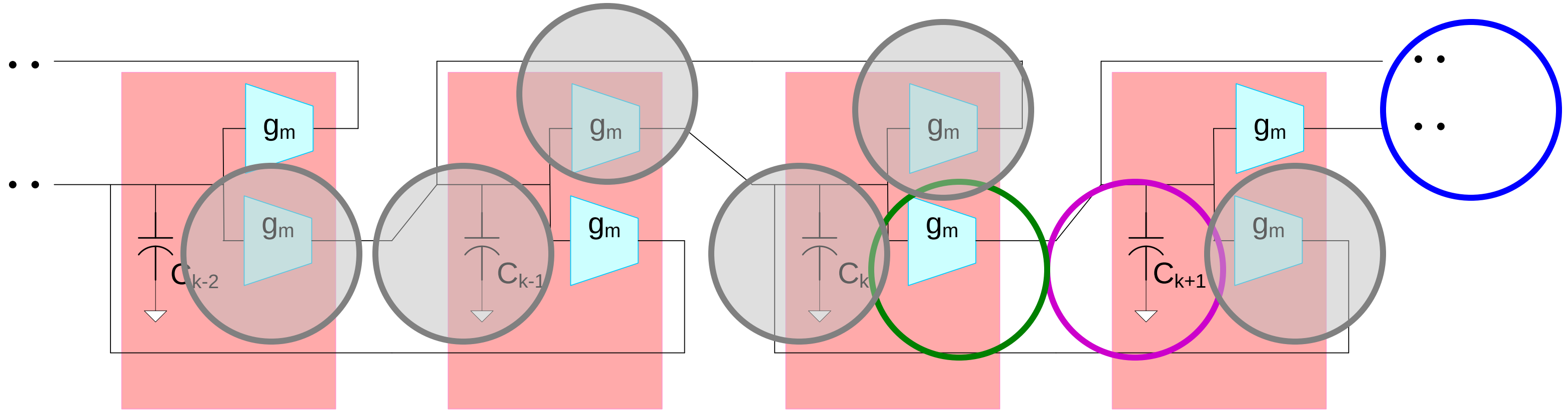


Consider upper OTA in stage k-1, capacitor in stage k and lower OTA in stage k+1

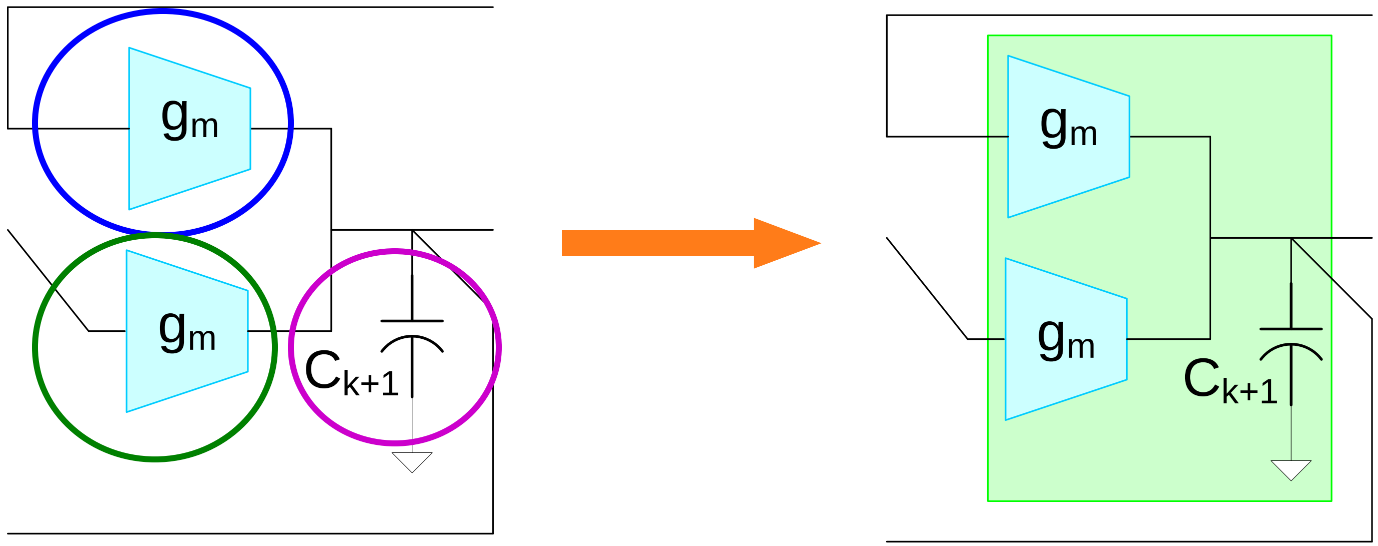


Leap-Frog Filter

Current-mode

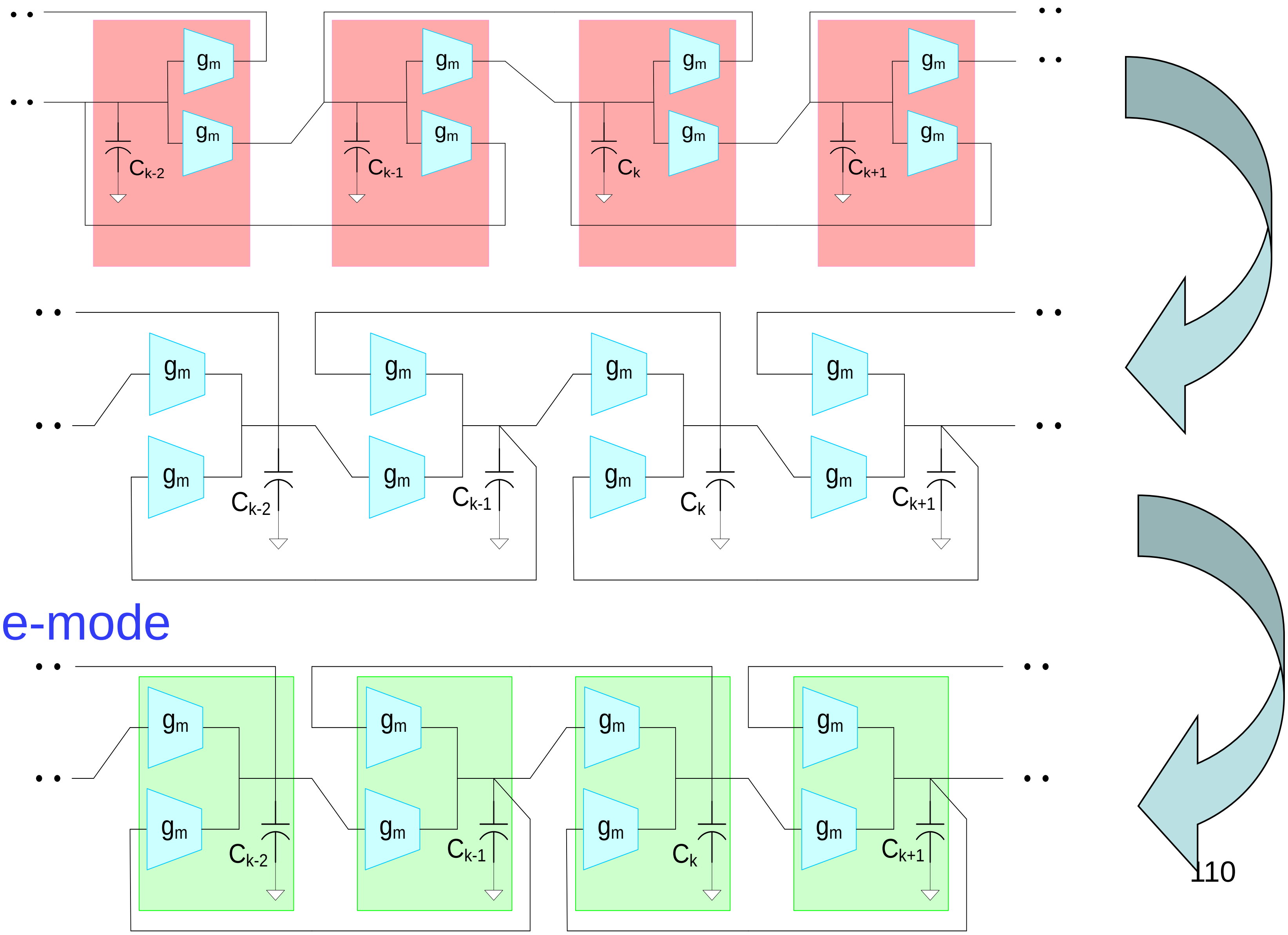


Consider lower OTA in stage k, capacitor in stage k+1 and upper OTA in stage k+2



Leap-Frog Filter

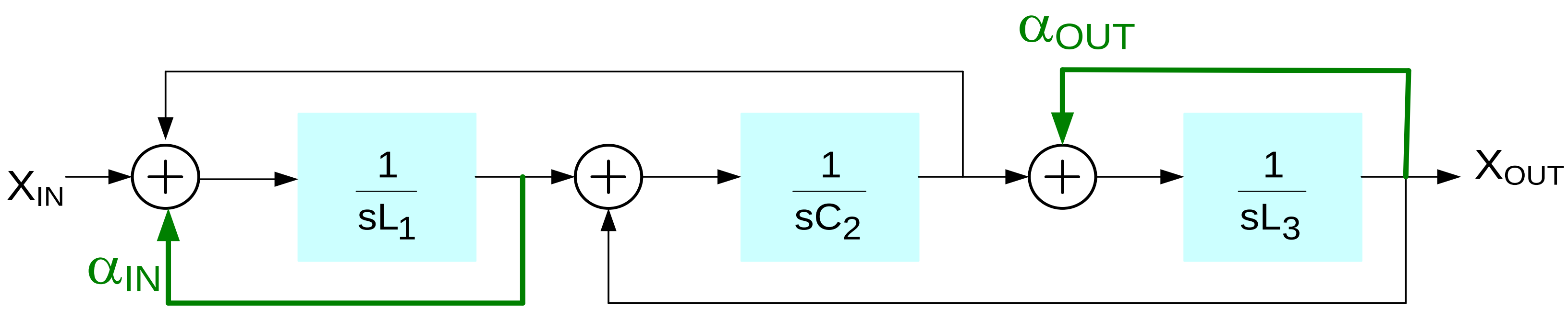
Current-mode



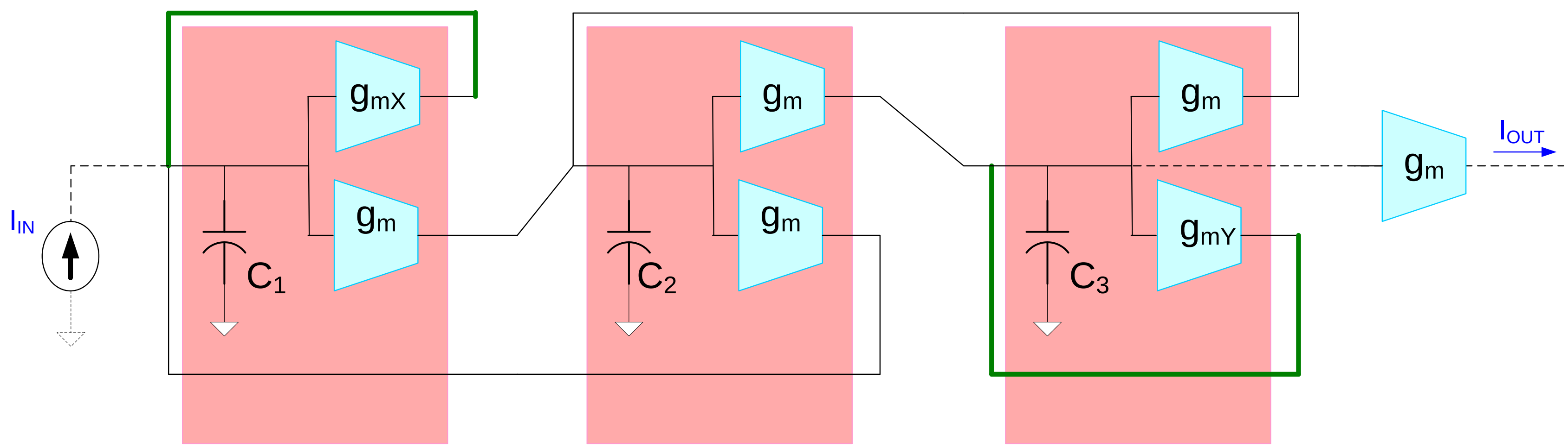
Voltage-mode

Leap-Frog Filter

Terminated Leap-Frog Filter (3-rd order lowpass)

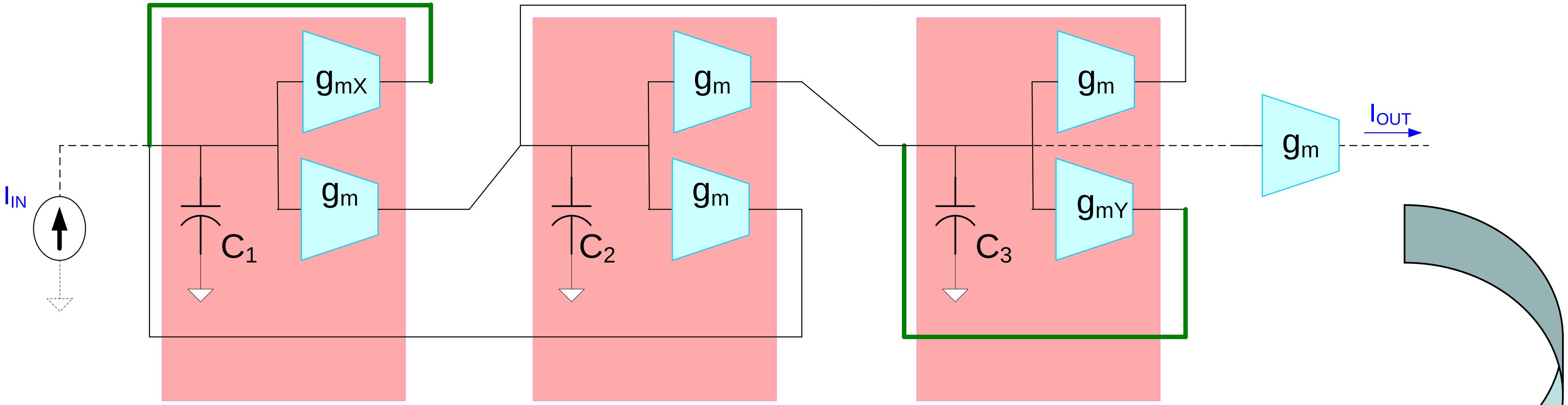


Current-mode implementation

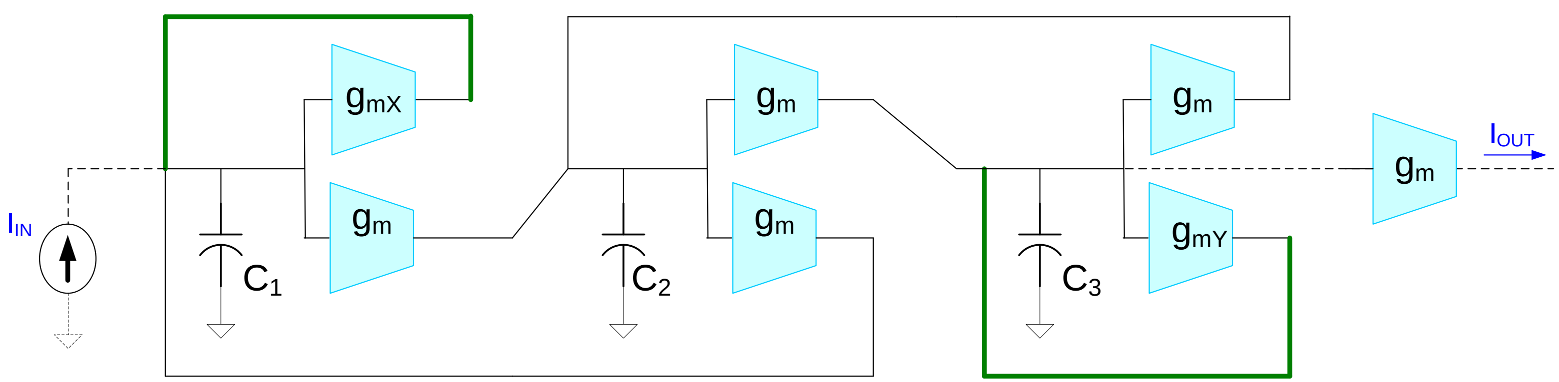


Leap-Frog Filter

Current-mode implementation

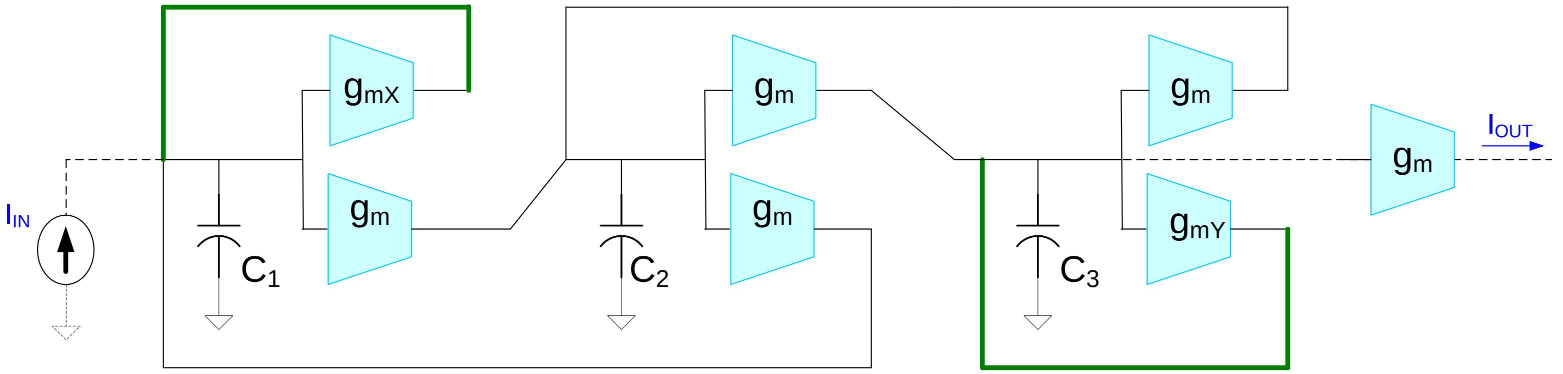


Consider schematic view:

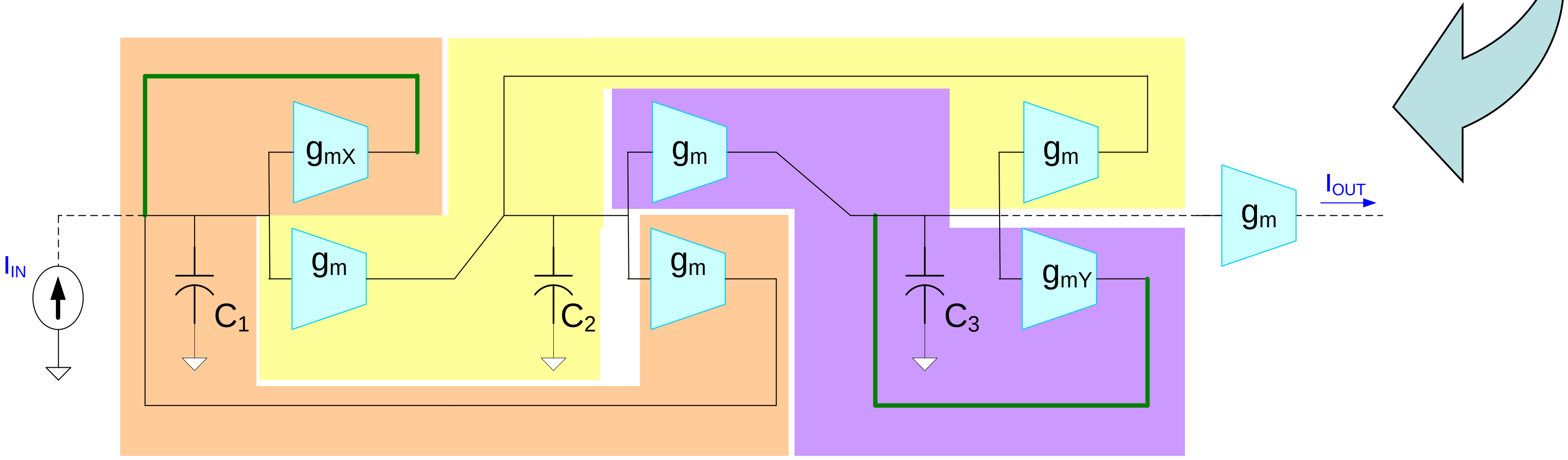


Leap-Frog Filter

Current-mode implementation

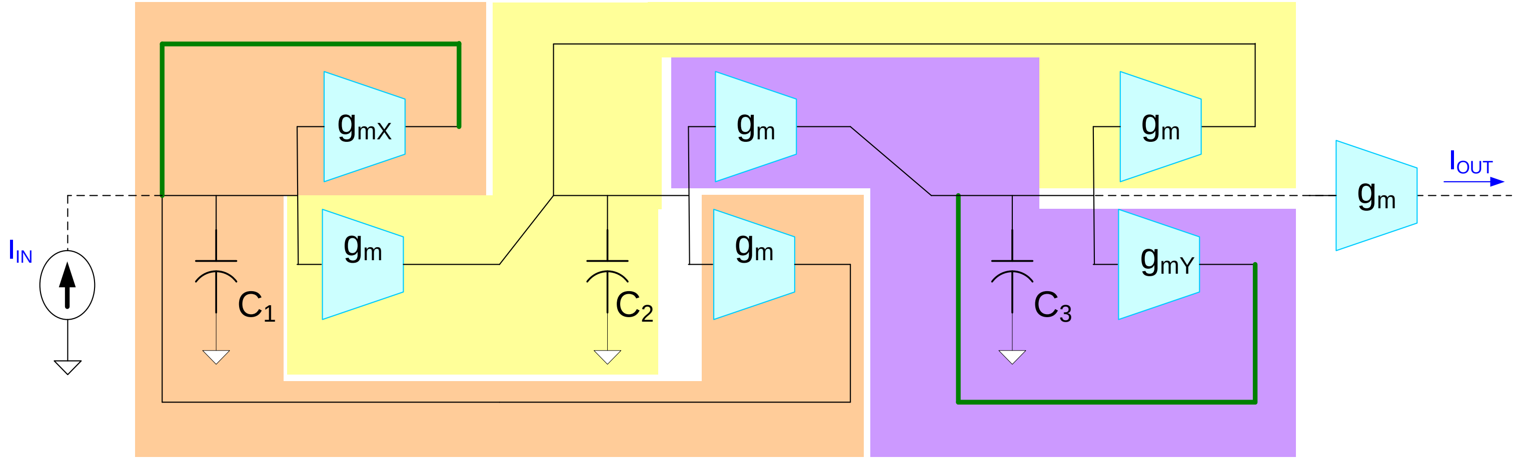


Re-group elements:

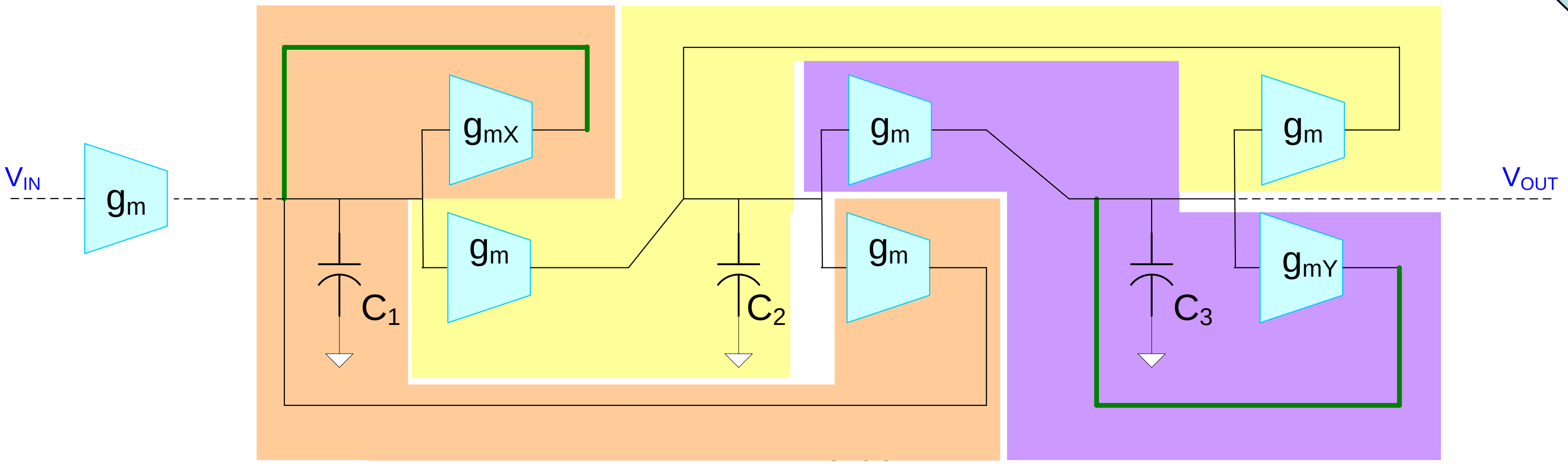


Leap-Frog Filter

Current-mode implementation

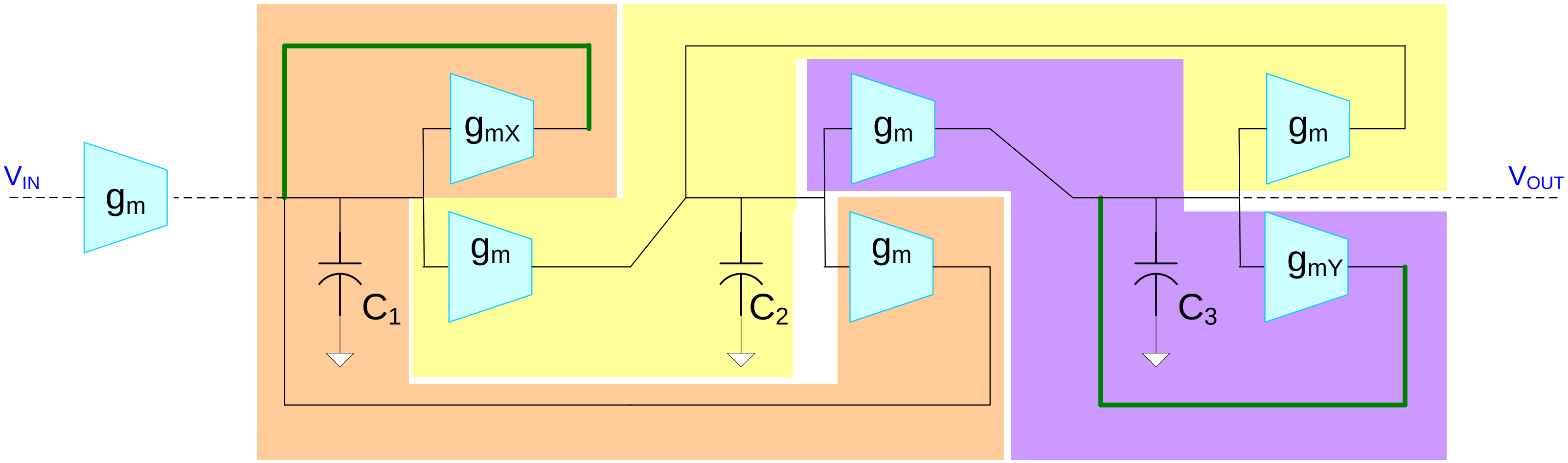


I/O Source Transformation

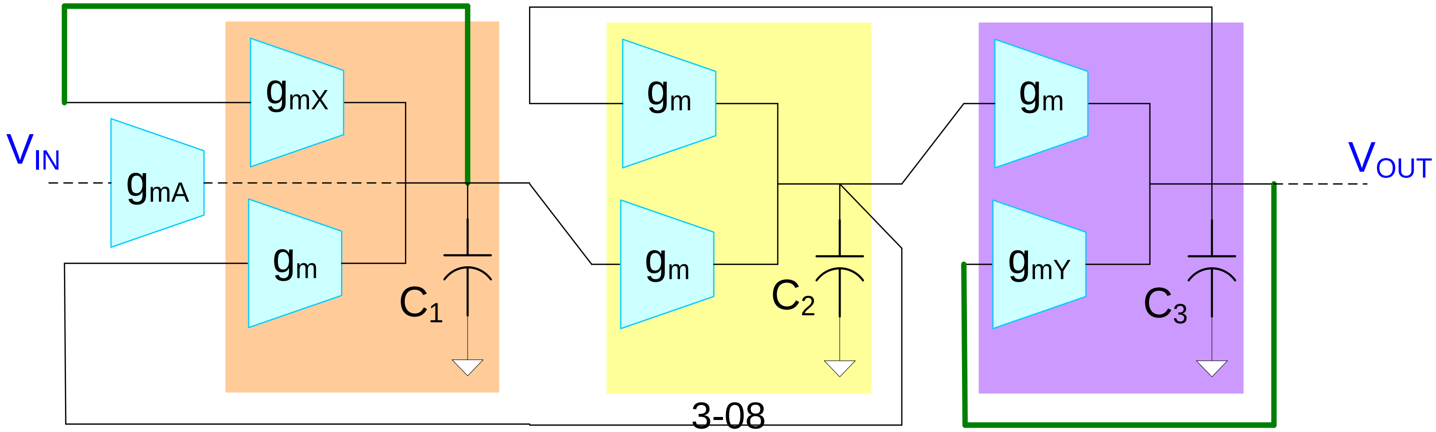


Leap-Frog Filter

Current-mode implementation

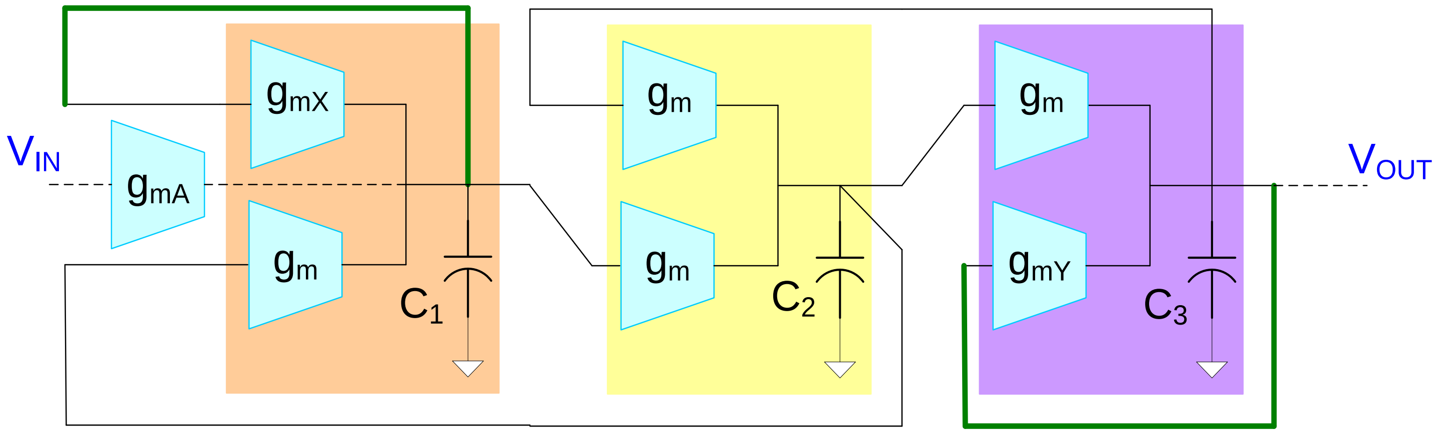


Redraw as:

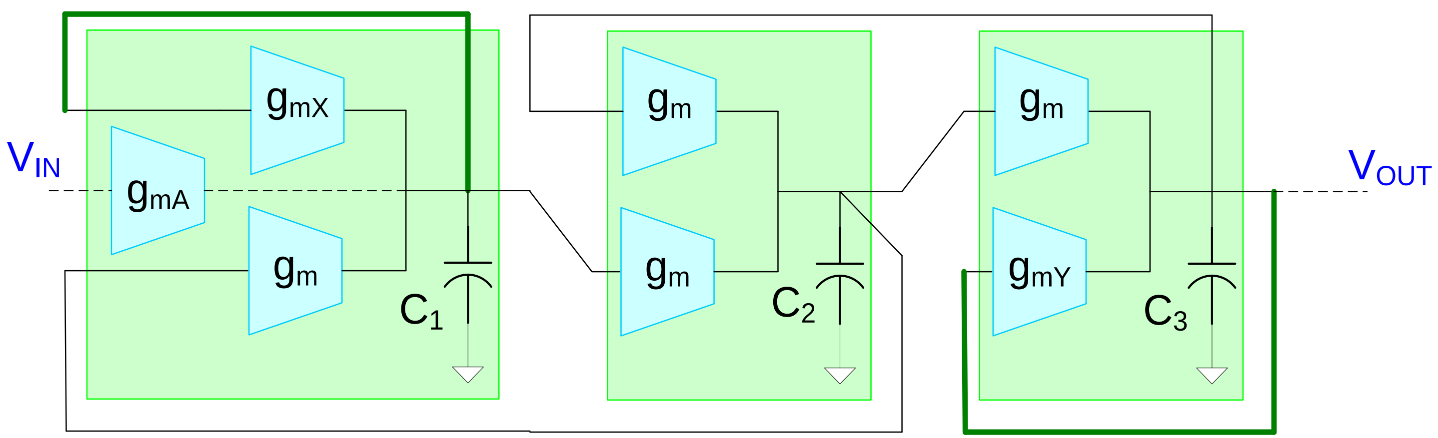


Leap-Frog Filter

Current-mode implementation



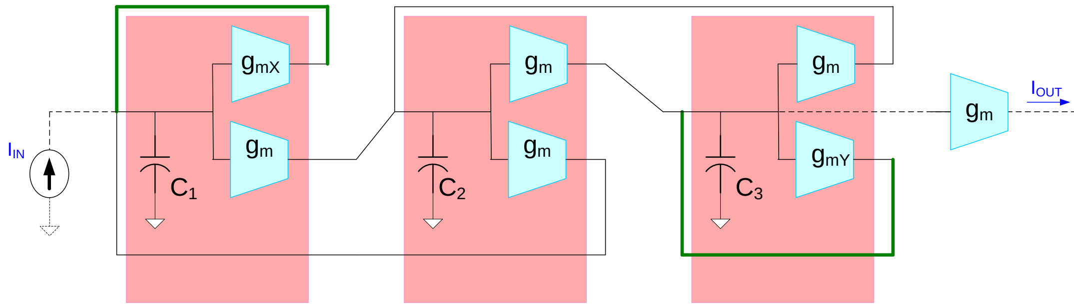
Change notation:



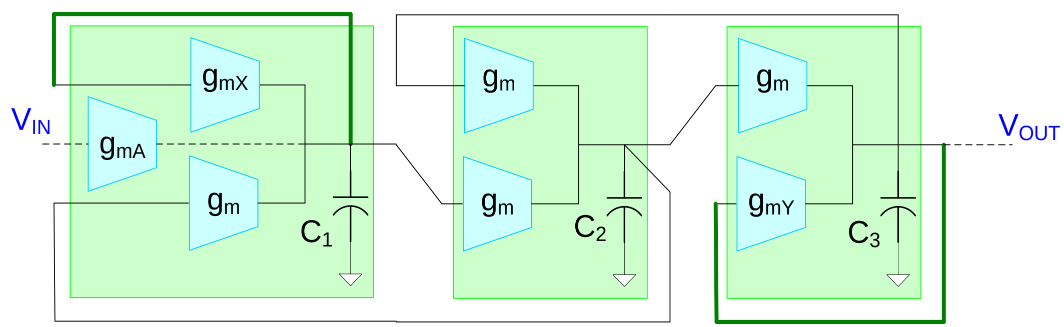
This is a voltage-mode implementation of the Leap-Frog Circuit !

Leap-Frog Filter

Current-mode



Voltage-mode



Current-mode and voltage-mode circuits have same component count

Current-mode and voltage-mode circuits are identical !

Current-mode and voltage-mode properties are identical !

Current-mode circuit offers NO benefits over voltage-mode counterpart

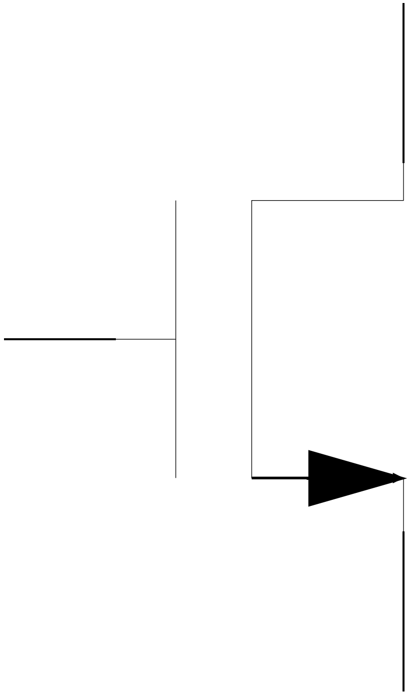
Questions about the Conventional Wisdom

What is a current-mode circuit?

- Everybody seems to know what it is
- Few have tried to define it
- Is a current-mode circuit not a voltage-mode circuit?

Question?

Is the following circuit a voltage mode-circuit or a current-mode circuit?



Question?

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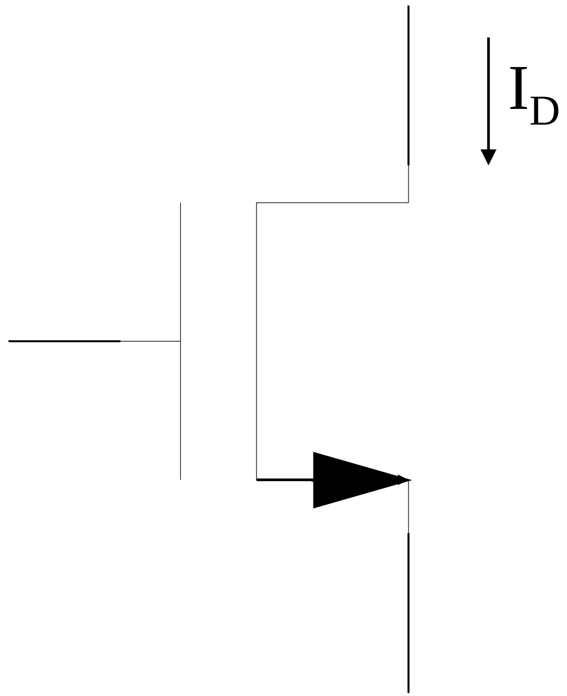
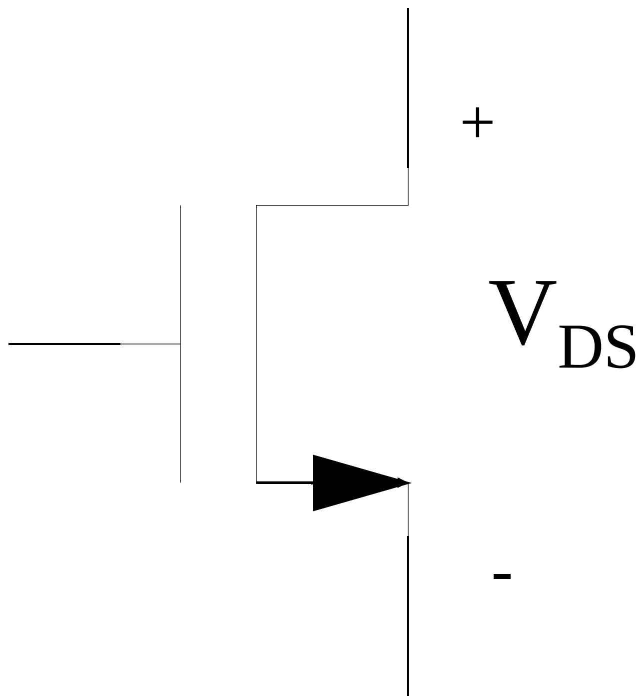


Current Mode !

Question?

Is the following circuit a voltage mode-circuit or a current-mode circuit?



Voltage Mode !

Observations:

- Voltage-Mode or Current-Mode Operation of a Given Circuit is not Obvious
- All electronic devices have a voltage-current relationship between the port variables that characterizes the device
- The “solution” of all circuits is identical independent of whether voltages or currents are used as the state variables
- The poles of any circuit are independent of whether the variables identified for analysis are currents or voltages or a mixture of the two

Observation

- Conventional wisdom suggests numerous performance advantages of current-mode circuits
- Some of the “current-mode” filters published perform better than other “voltage-mode” filters that have been published
- Few, if any, papers provide a quantitative comparison of the key performance features of current-mode circuits with voltage-mode counterparts
- It appears easy to get papers published that have the term “current-mode” in the title

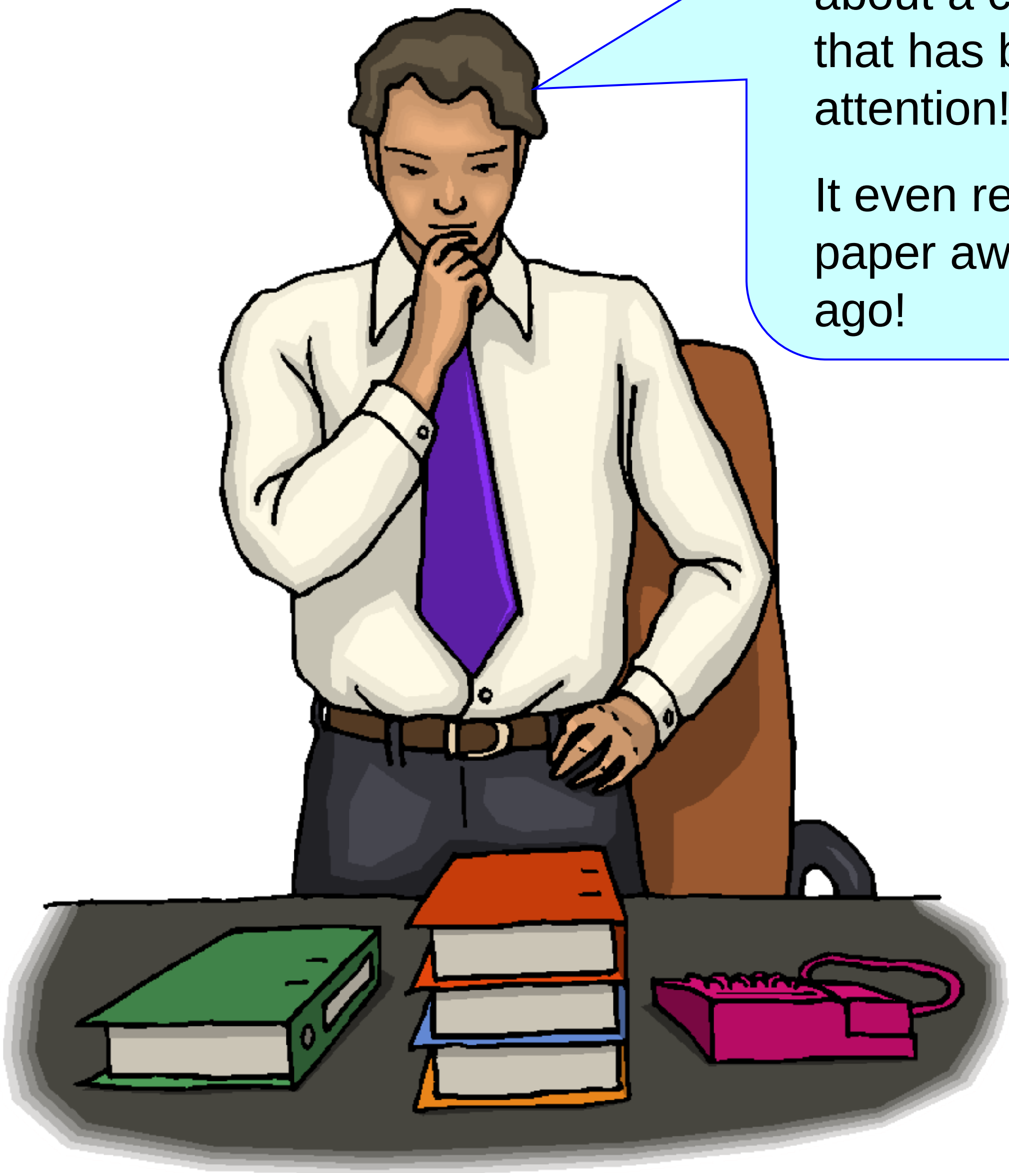
Observations (cont.)

- Over 900 current-mode papers have been published in IEEE forums alone !
- Most, if not all, current-mode circuits are IDENTICAL to a voltage-mode counterpart
- We are still waiting for even one author to quantitatively show that current-mode filters offer even one of the claimed four advantages over their voltage-mode counterparts

Are Conventional Wisdom and Fundamental Concepts always aligned in the Microelectronics Field ?

Will consider 4 basic examples in this discussion

- Op Amp
- Positive Feedback Compensation
- Current Mode Filters
- Current Dividers



I’ve heard of some amazing claims about a clever current divider circuit that has been receiving lots of attention!

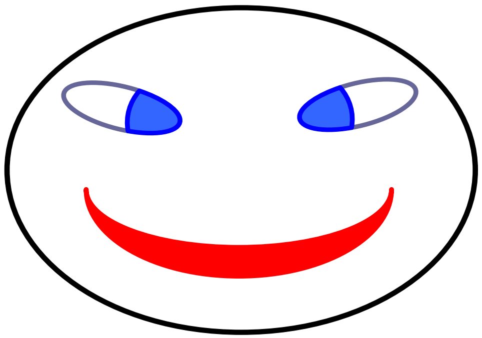
It even received the outstanding paper award at ISSCC a few years ago!

Current Dividers

- ➡ Background
- Objective
- Concept of Current Divider
- Characterization of Inherently Linear Current Divider
- Inherent Current Division in Symmetric Circuits
- Conclusionhs

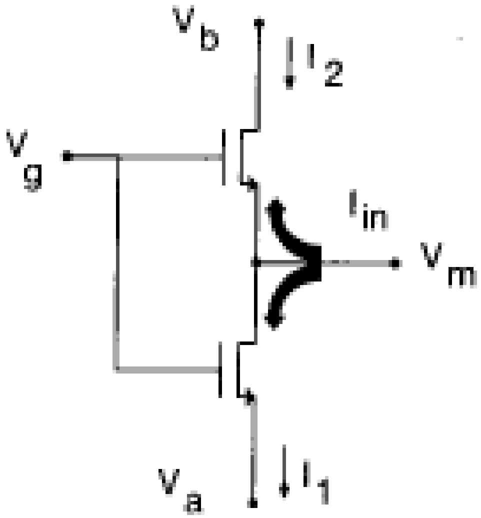
Current Dividers

Motivation: Circuits that do accurate current division in the presence of varying loading conditions could be among the most useful building blocks that are available



Background Introduction

Current divider with “Inherent Linearity”



. The basic principle of current division.

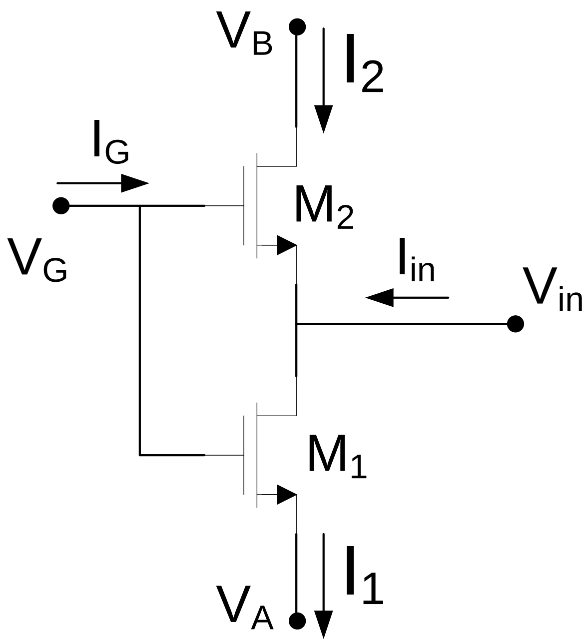
- constant and independent of I_{in} (implying low distortion),
- independent of the values of V_a and V_b,
- independent of whether one or both devices are saturated or nonsaturated,
- and also independent of whether one or both devices operate in strong or in weak inversion.

above we have assumed that V_a and V_b are ideal voltage sources, i.e., having zero output impedance.

- Examples that were given did not have zero impedance on V_A and V_B nodes
- Experimentally reported THD from -80dB to -85dB
- Experimentally measured Dynamic Range in excess of 100dB
- All digital standard CMOS process

Bult and Geelen, ISSCC Feb1992, JSC Dec 1992 “An Inherently Linear and Compact MOST-only Current Division Technique”

Background Introduction



Current divider with “Inherent Linearity”

Current Division Factor

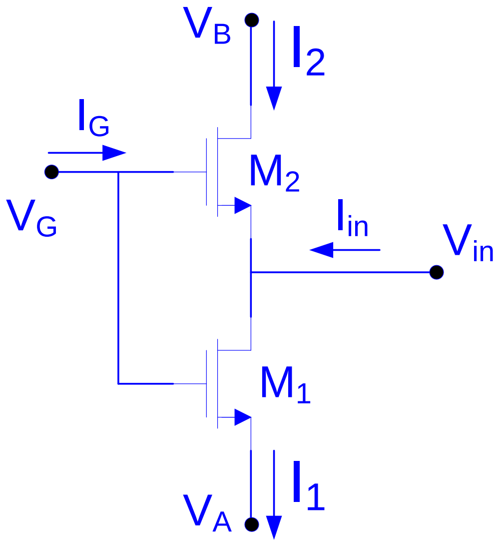
$$\theta = \frac{(W/L)_1}{(W/L)_2}$$

Very Simple and Compact

Elegant !

Bult and Geelen, ISSCC Feb1992, JSC Dec 1992 “An Inherently Linear and Compact MOST-only Current Division Technique”

Background Introduction

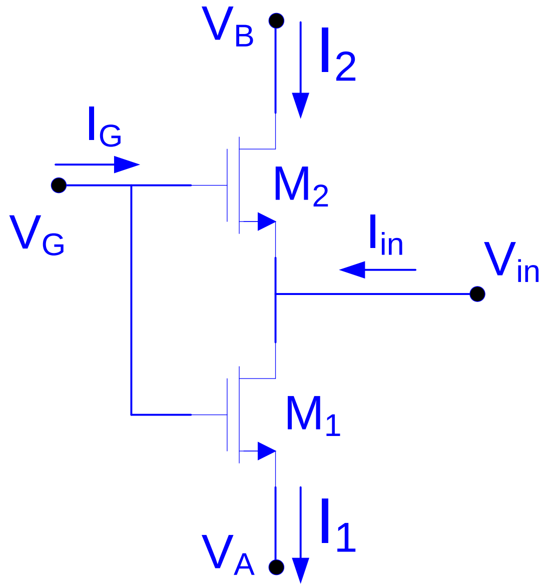


Inherently Linear Current Divider

Conventional Wisdom: current division factor independent of

- I_{IN}
- V_A and V_B
- Device operation region (weak, moderate, or strong inversion; triode or saturation region)
- body effect, mobility degradation

Background Introduction



Inherently Linear Current Divider

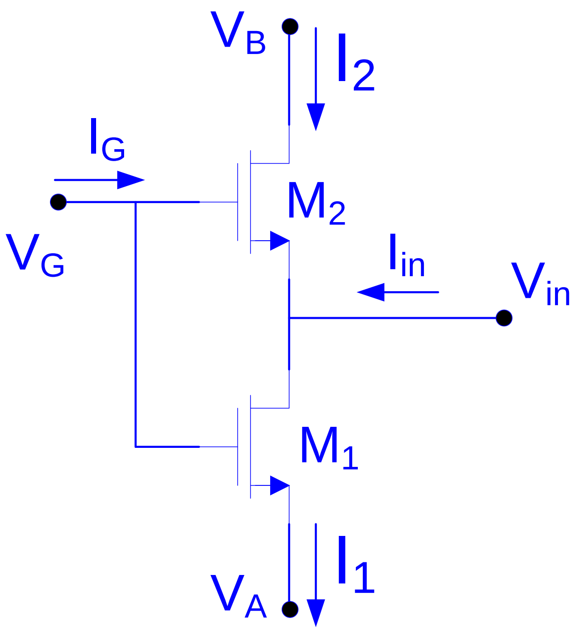
only weakly dependent upon second-order effects

THD better than -85dB in audio range
Dynamic Range better than 100dB
Experimentally verified

Very impressive linearity properties !

Influential Concept

- Outstanding paper of ISSCC 1992
- Cited 180 times Google Scholar
- Reported applications include
  - Volume controller
  - Data converter
  - Tunable filters
  - Variable gain amplifier
  - Accurate current generator
  - Sensors
  - Other circuits
- Numerous reported works experimentally verify the high linearity



Inherently Linear Current Divider

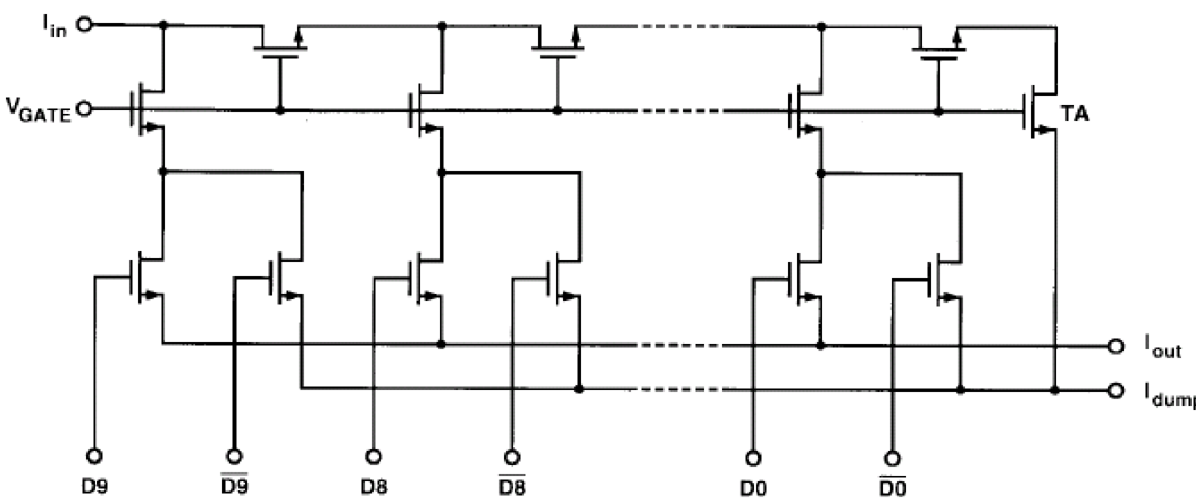
An example application of the concept and the circuit

IEEE JOURNAL OF SOLID-STATE CIRCUITS, VOL. 33, NO. 8, AUGUST 1998

Design and Implementation of an Untrimmed MOSFET-Only 10-Bit A/D Converter with −79-dB THD

Clemens M. Hammerschmied, *Student Member, IEEE*, and Qiuting Huang, *Senior Member, IEEE*

The MOSFET ladder is based on a linear current division principle instead, the basic circuit of which is depicted in Fig. 4 [14]. An input current I_{in} is divided into two currents



40 Google Scholar Citations (Dec. 15, 2010)

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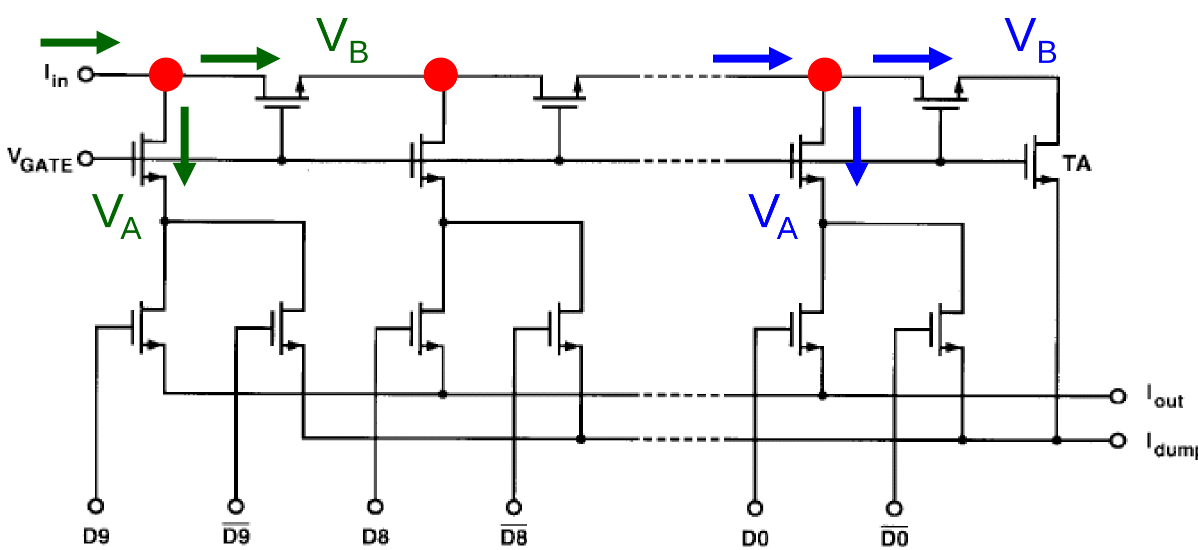

V_A and V_B not even at zero impedance nodes !

An example application of the concept and the circuit

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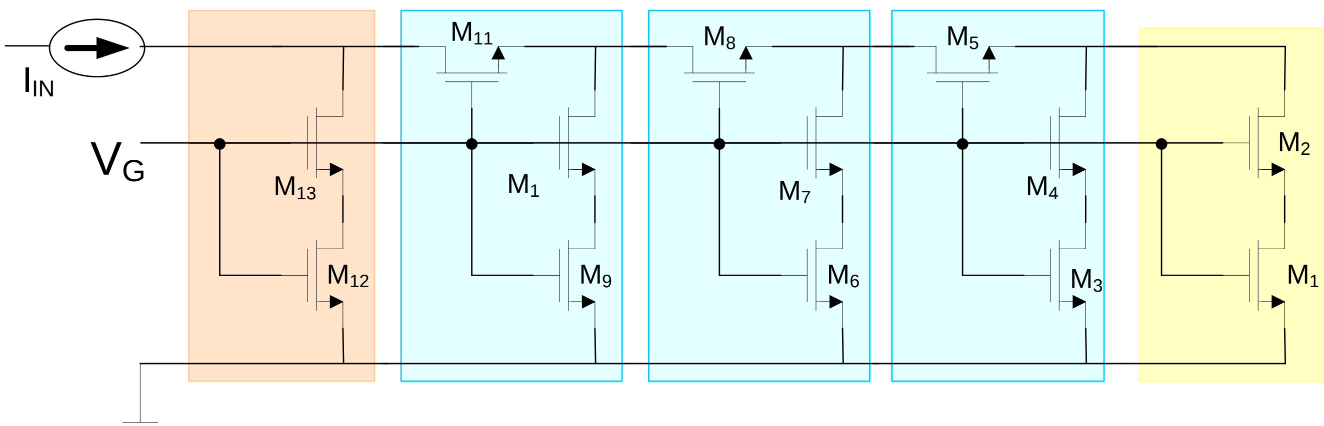
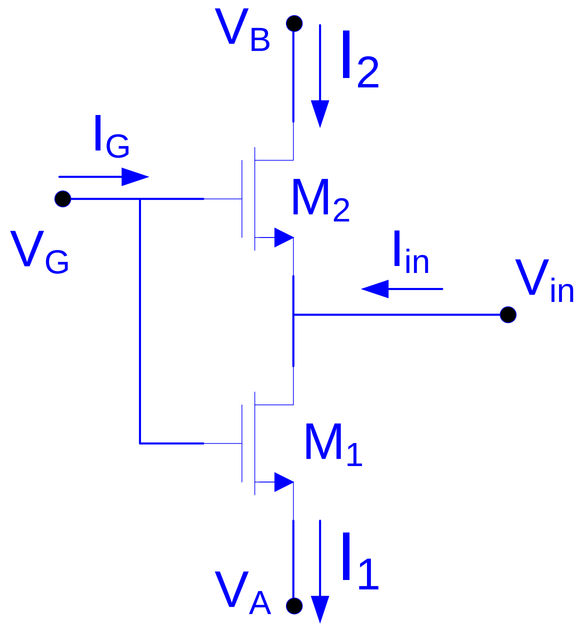


But



Inherently Linear Current Divider

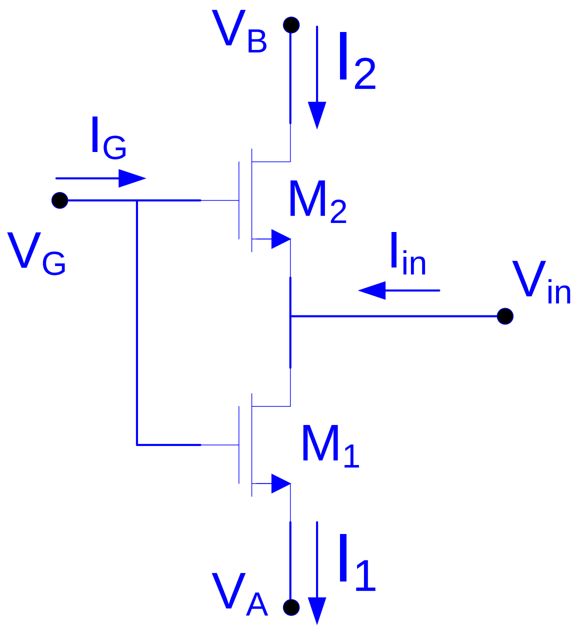
We have been unable to achieve linearity that is even close to that reported in different but closely related applications of this circuit

(e.g. -40dB or less linearity in contrast to -85dB or better performance)

Outline

- Background
- Objective
- Concept of Current Divider
- Characterization of Inherently Linear Current Divider
- Inherent Current Division in Symmetric Circuits
- Conclusionhs

Purpose of this work



Clarify and quantify the potential and limitations of the “inherently linear current divider”

(Do not question the reported experimental results attributed to this circuit)

Current Dividers

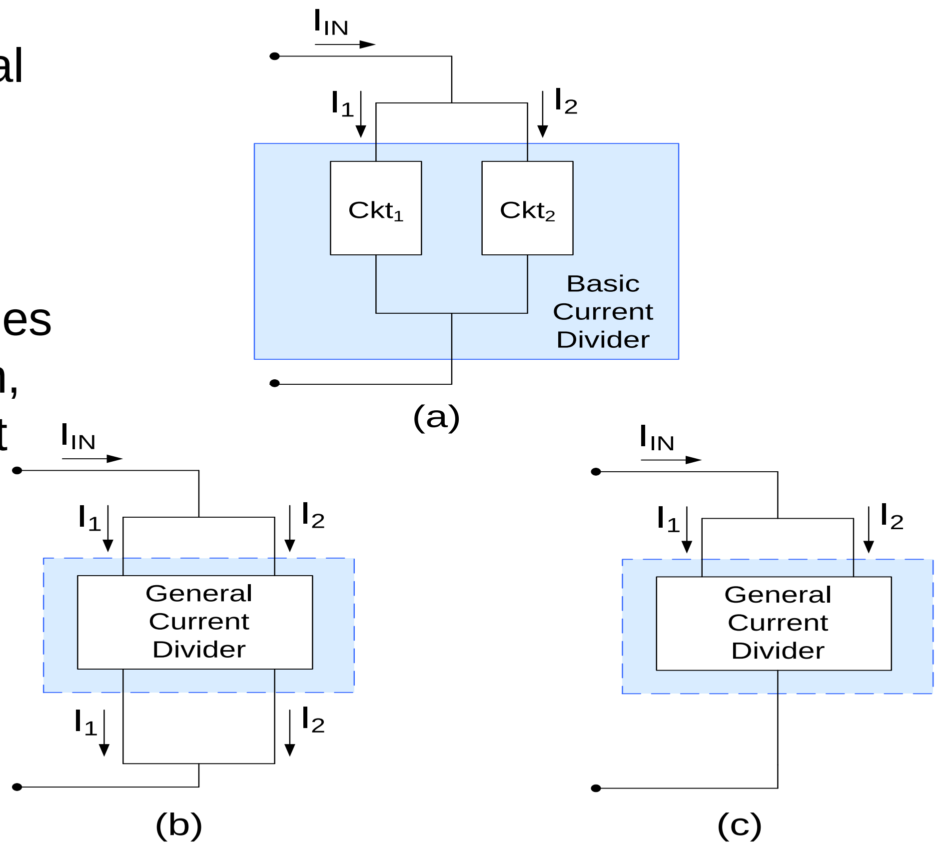
- Background
- Objective
- ➡ Concept of Current Divider
- Characterization of Inherently Linear Current Divider
- Inherent Current Division in Symmetric Circuits
- Conclusionhs

Concept of Current Divider

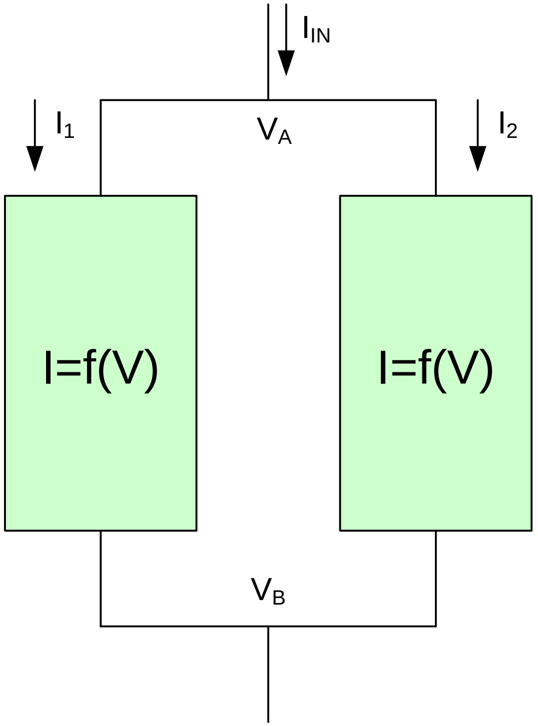
What is a current divider ?

- Although the term is widely used, formal definitions seldom if ever given
- Consider a node with three incident branches in a circuit
- If the current in one of the three branches is proportional to that in another branch, we will define this circuit to be a current divider

$$I_1 = \theta I_{IN}$$



Observations That Will Become Relevant



$$I_1 = \frac{1}{2} I_{IN}$$

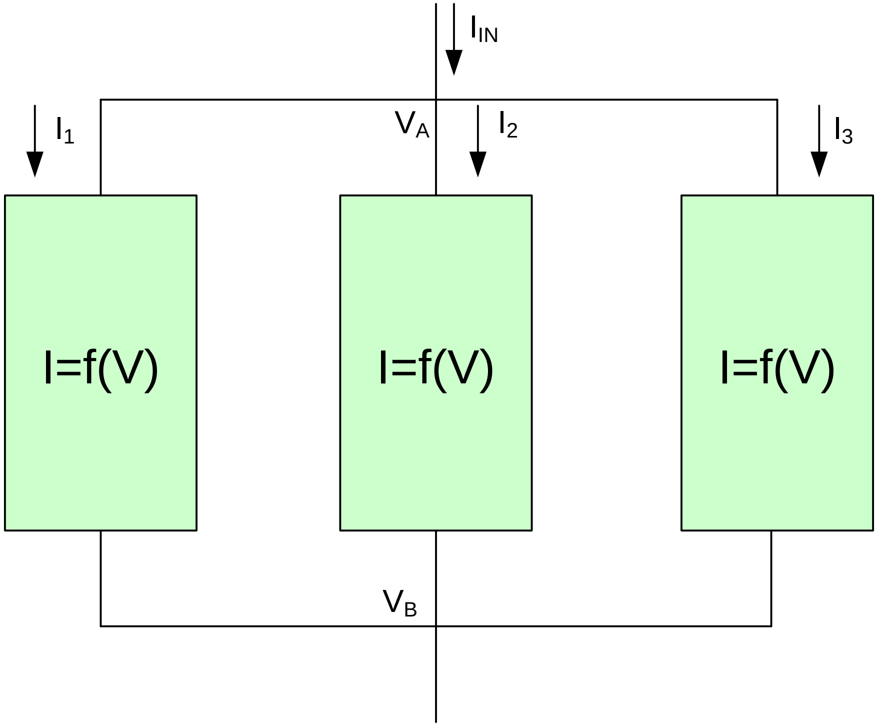
Independent of V_A, V_B, $I_{IN,}$, f

Inherent property of symmetric network

Current Divider !

Concept that has probably been known for well over 100 years

Observations that Will Become Relevant

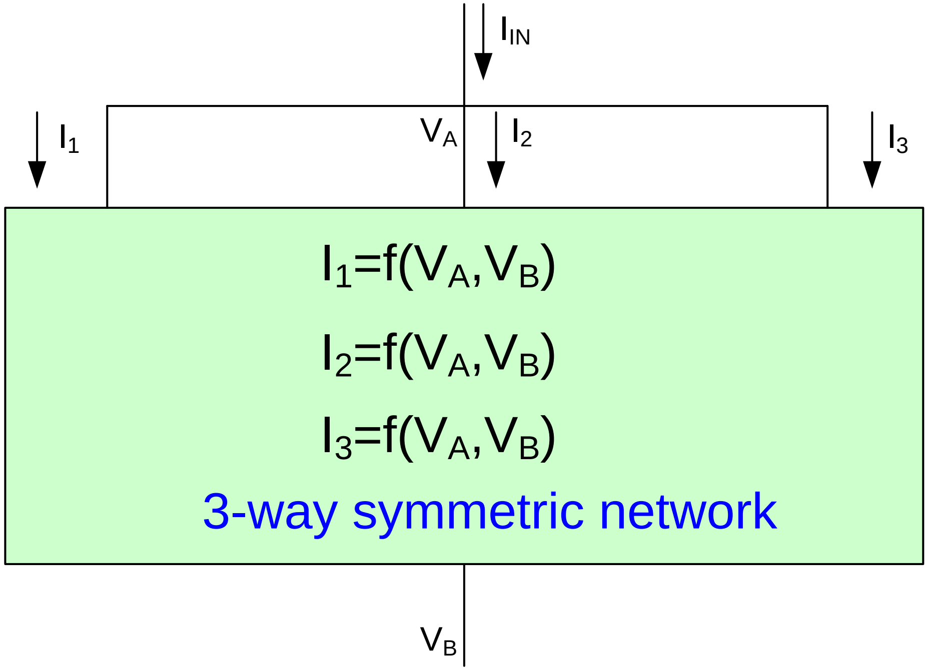


$$\mathbf{I_1 = \frac{1}{3} I_{IN}}$$

Independent of V_A, V_B, $I_{IN,}$, f

Inherent property of symmetric network

Observations that Will Become Relevant



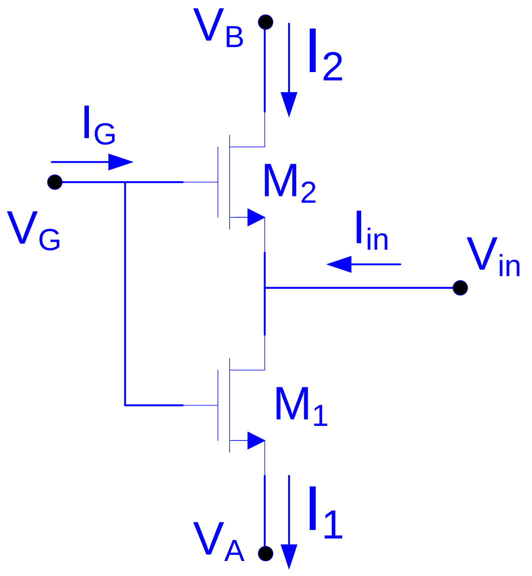
$$\mathbf{I_1 = \frac{1}{3} I_{IN}}$$

Independent of V_A, V_B, $I_{IN,}$, f

Inherent property of symmetric network

Concept that has probably been known for well over 100 years

Consider the Inherently Linear Current Divider with Linearity Challenges



Conventional Wisdom: current division factor independent of

- I_{IN}
- V_A and V_B
- Device operation region (weak, intermediate, or strong inversion; triode or saturation region of operation)
- body effect, mobility degradation

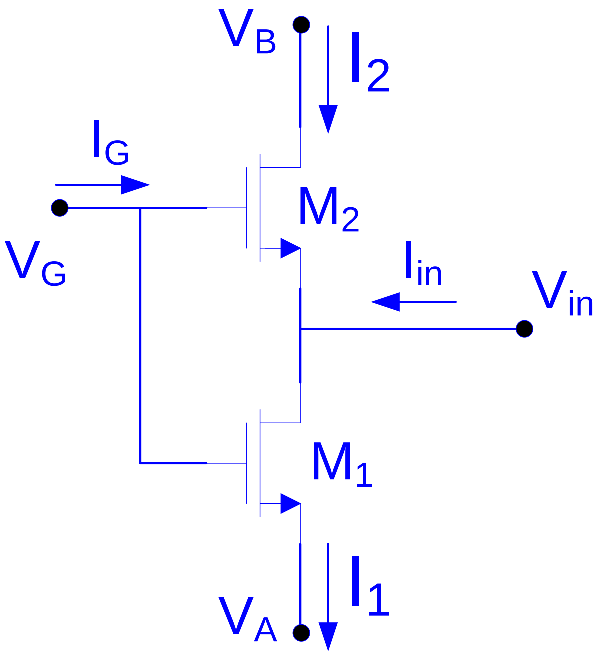
Current Dividers

- Background
- Objective
- Concept of Current Divider
- ➡ Characterization of Inherently Linear Current Divider
- Inherent Current Division in Symmetric Circuits
- Conclusionhs

Assumptions

- Square-law model
- Identical V_{th}
- No Body or Output Conductance Effects

- $\{I_{in}, V_{GA}, V_{BA}\}$ independent variables

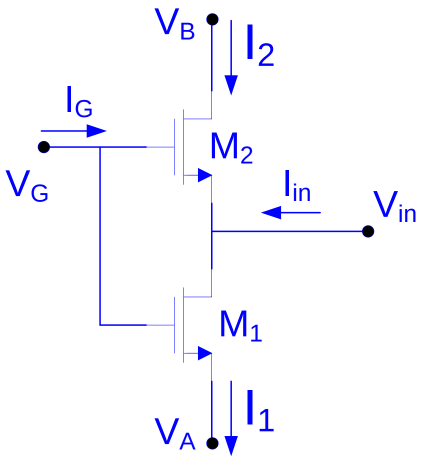


$\eta_1 = \mu C_{OX}(W_1/L_1)$

$\eta_2 = \mu C_{OX}(W_2/L_2)$

From a straightforward but tedious analysis

If M_1 in the triode region and M_2 in the triode region



$$I_1 = \left[\frac{\eta_1}{\eta_1+\eta_2}\right] I_{in} + \frac{\eta_1\eta_2}{\eta_1+\eta_2} V_{BA}\left(V_{GA}-V_T-\frac{V_{BA}}{2}\right)$$

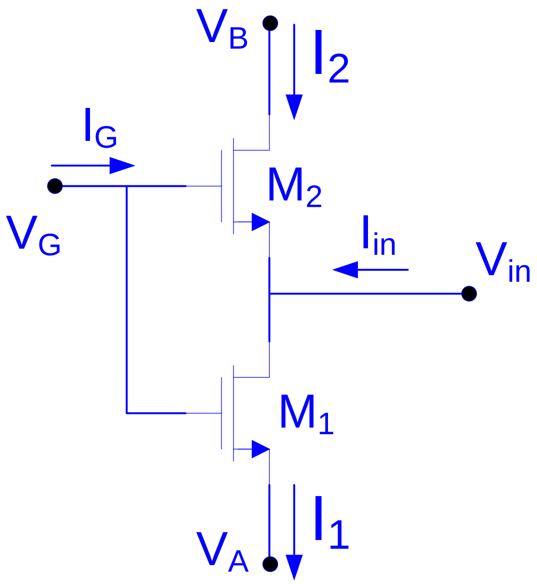
$$V_{inA} = V_{GA} - V_T - \sqrt{\left(V_{GA}-V_T\right)^2 - 2\left(\left[\frac{1}{\eta_1+\eta_2}\right] I_{in} + \frac{\eta_2}{\eta_1+\eta_2} V_{BA}\left(V_{GA}-V_T-\frac{V_{BA}}{2}\right)\right)}$$

Oddly, the driving point voltage is dependent upon the driving point current !

From a straightforward but tedious analysis

If M_1 in the triode region and M_2 in the saturation region

$$\mathbf{I_1} = \left[\frac{\boldsymbol{\eta_1}}{\boldsymbol{\eta_1}+\boldsymbol{\eta_2}}\right]\mathbf{I_{in}} + \frac{\boldsymbol{\eta_1\eta_2}}{\mathbf{2}(\boldsymbol{\eta_1}+\boldsymbol{\eta_2})}\left(\mathbf{V_{GA}}-\mathbf{V_T}\right)^2$$

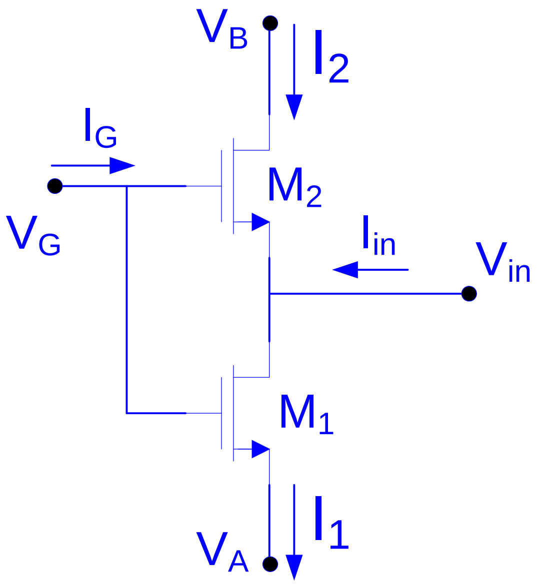


$$V_{inA} = \left(V_{GA}-V_T\right)\left(1-\sqrt{\frac{\eta_1-\dfrac{2I_{in}}{\left(V_{GA}-V_T\right)^2}}{\eta_1-\eta_2}}\right)$$

Oddly, the driving point voltage is dependent upon the driving point current !

From a straightforward but tedious analysis using the basic square-law model

If V_{GA} and V_{GB} do not depend upon I_{IN}, then
- the circuit performs as a linear current divider with an offset
- the current divider ratio does not change as M_1 and M_2 change from the triode region to the saturation region



But, if these conditions are not satisfied, will the circuit still perform as a linear current divider ?

Some things ignored in previous analysis

- Device model errors (not exactly square-law)
- Threshold voltages mismatches
- Finite output impedance of transistors
- Body effect
- Finite output impedance of the current source

More Accurate Analysis

- Analytical study is unwieldy with highly complicated model
- Computer simulation helpful for predicting linearity

Linearity Metrics

- Static linearity defined as deviation from fit line

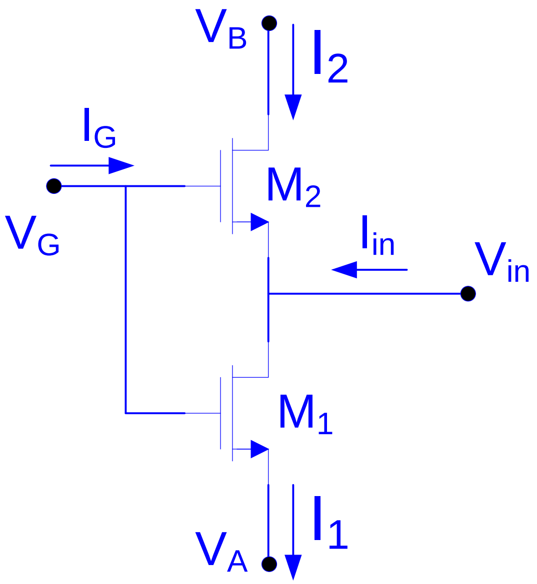
$$I_{1FIT}(I_{in}) = I_{1Q} + \left.\frac{\partial I_1}{\partial I_{in}}\right|_{\{I_{inQ}, V_{GAQ}, V_{inAQ}\}} \cdot (I_{in} - I_{inQ})$$

$$\Delta = \left[\frac{I_1(I_{in}) - I_{1FIT}(I_{in})}{I_{1FIT}(I_{in})}\right] \times 100\%$$

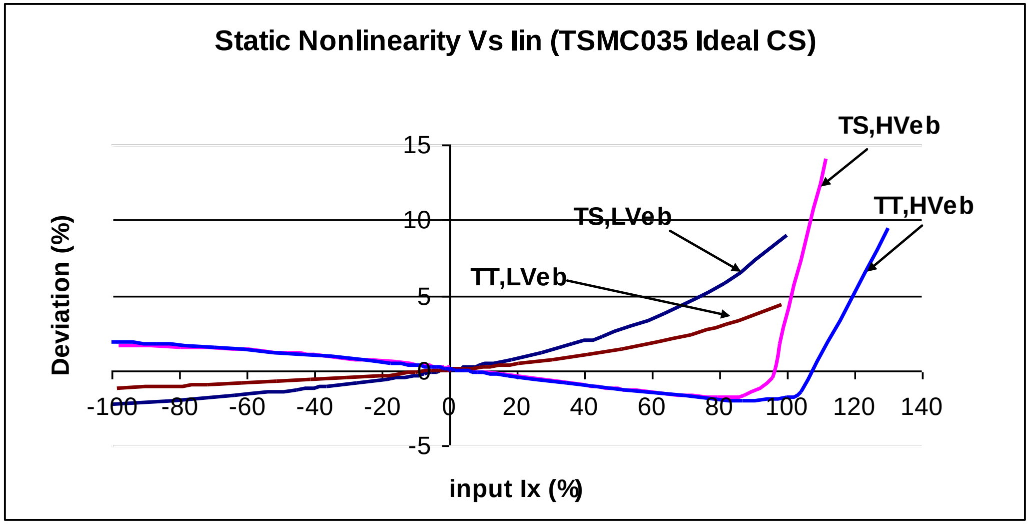
- Dynamic linearity defined as the THD performance with continuous sinusoid excitation

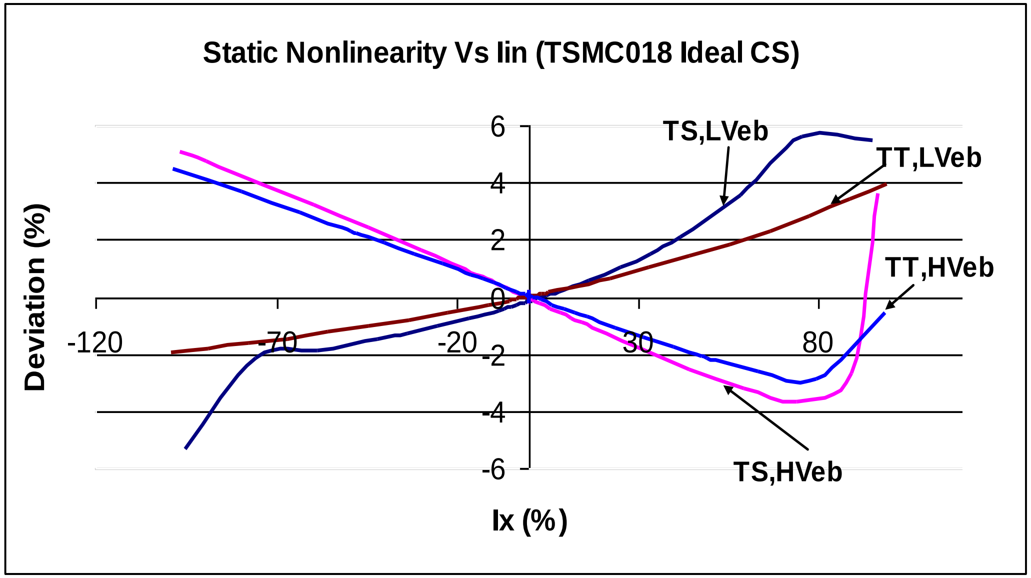
Simulation Environments

- Different operation regions (M_1, M_2)
  - Triode, Triode ("TT")
  - Triode, Saturation ("TS")
- Different bias level
  - Large V_{EB}
  - Small V_{EB}
- Different size devices (width, length)
- Different process
  - TSMC 0.18um
  - TSMC 0.35um
- V_{AS}, V_{BS}, V_{GS} fixed
- Ideal current source excitation

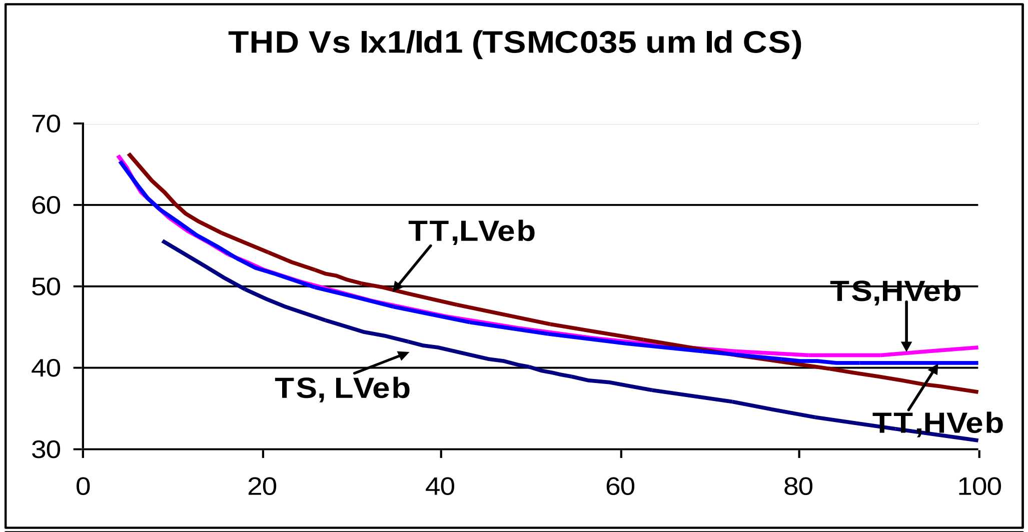


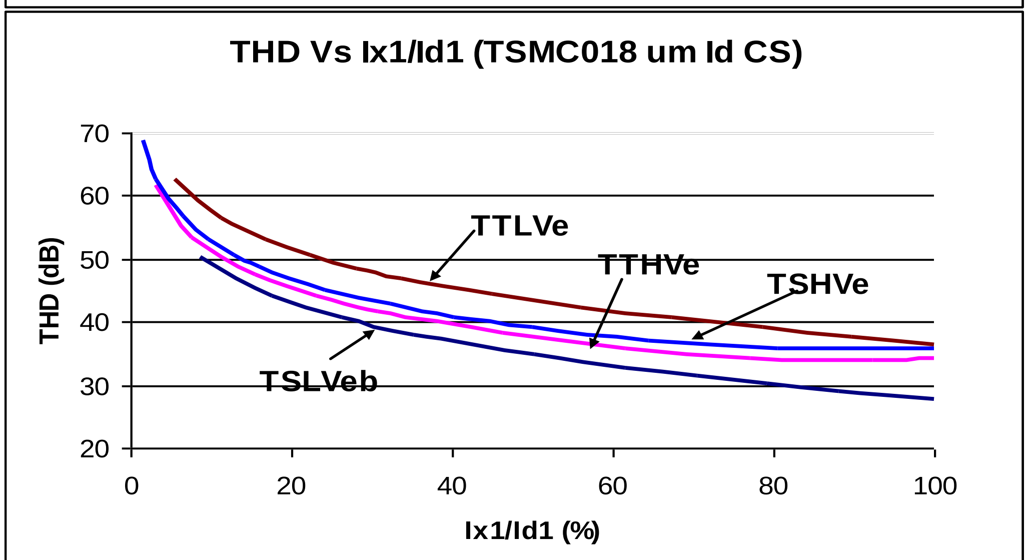
Static Linearity Simulation





Dynamic Linearity Simulation

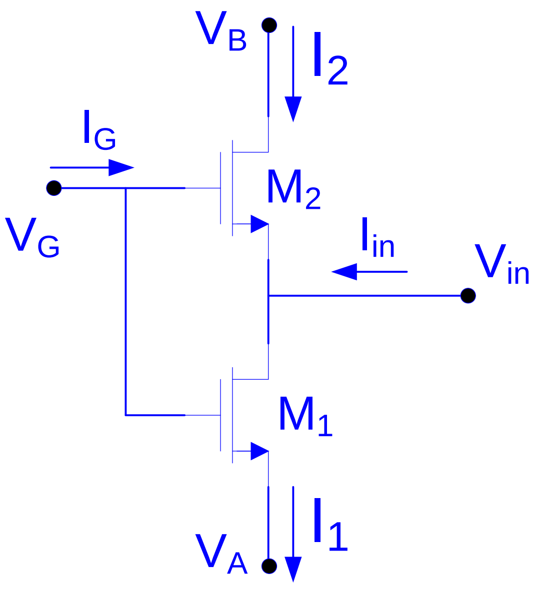




Observations about Linearity

- Static nonlinearity in the few percent range
- Dynamic linearity is quite limited with even moderate input current levels
  - limited to about 30~40 dB level if reasonable input current swings occur
- Including effects of output impedance of current source and circuit dependence of V_{AS} and V_{BS} will further degrade performance

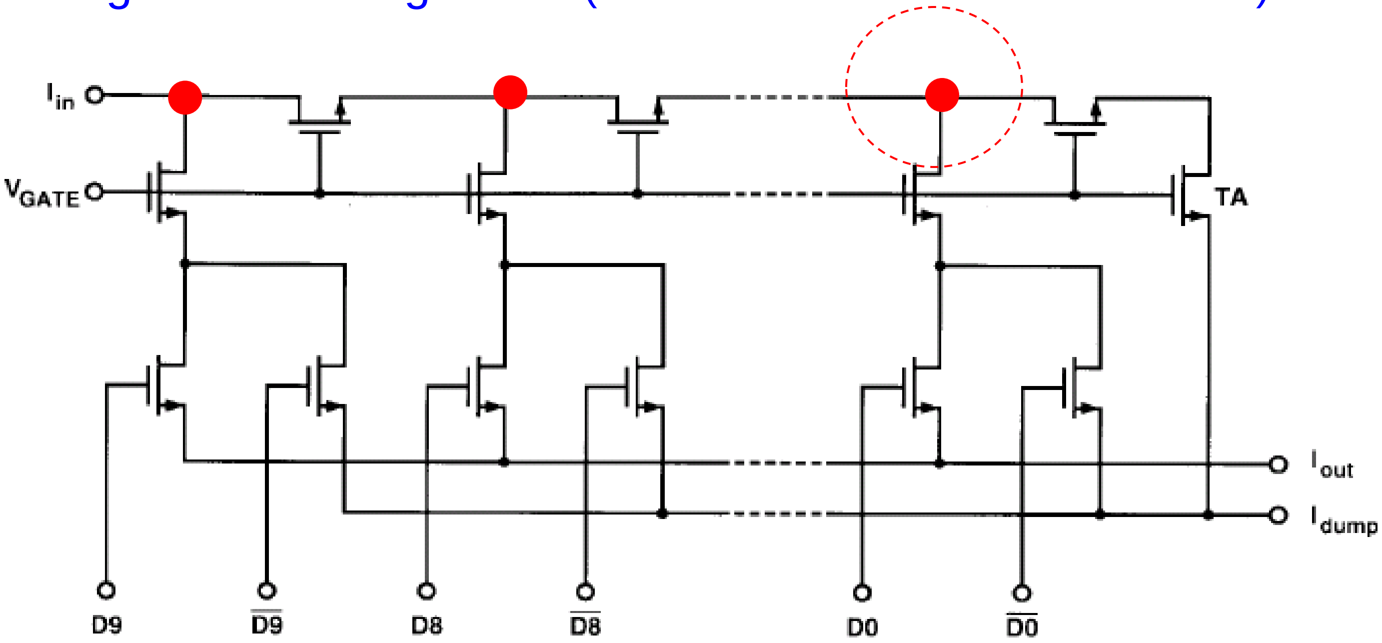
Observations about inherently linear current divider



- Performance as a current divider is somewhat questionable
- Not inherently linear (appears to be strongly dependent upon model)

Question: How was the excellent linearity obtained in the author's own work and that reported in the literature if it is difficult to verify the linearity?

Consider again the Huang circuit (in which all transistors are identical)

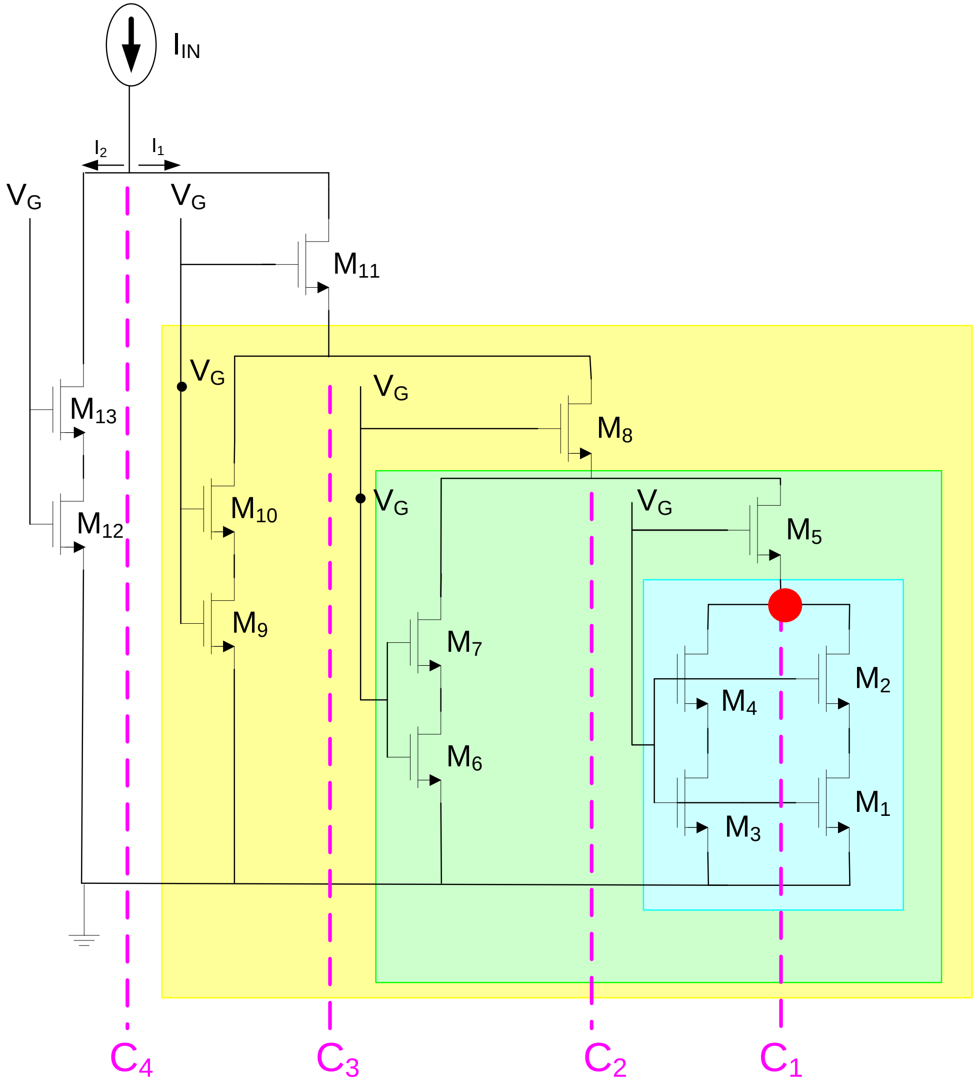


For proper operation, it is critical that currents divide equally at each of The current division nodes !

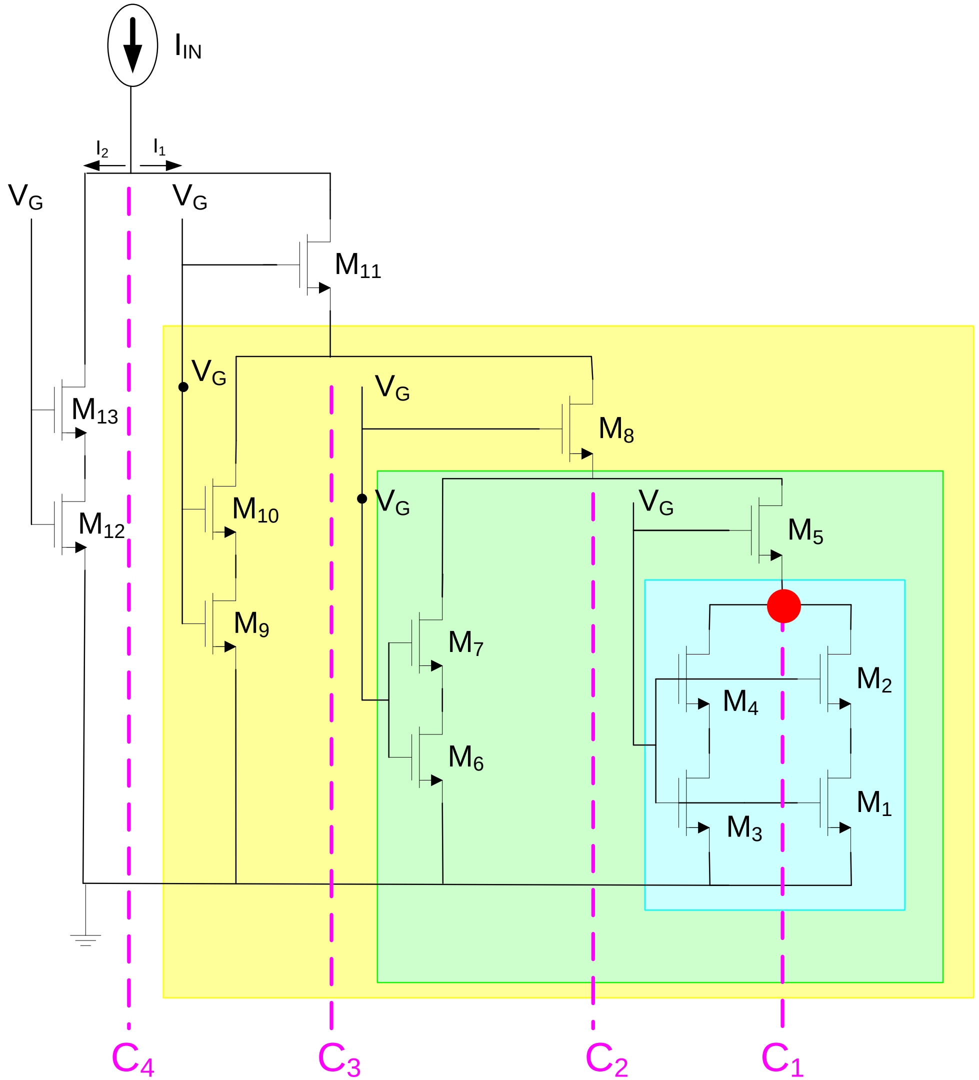
Even the assumption that the voltages V_A and V_B must be zero-impedance sources was not required to obtain the good performance (79 dB range) !

Question: How was the excellent linearity obtained in the author's own work and that reported in the literature if it is difficult to verify the linearity?

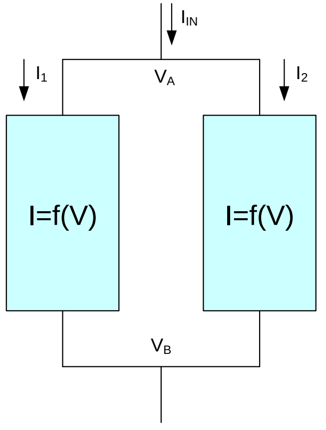
Redraw the Huang Circuit and Consider the right-most Current Divider node



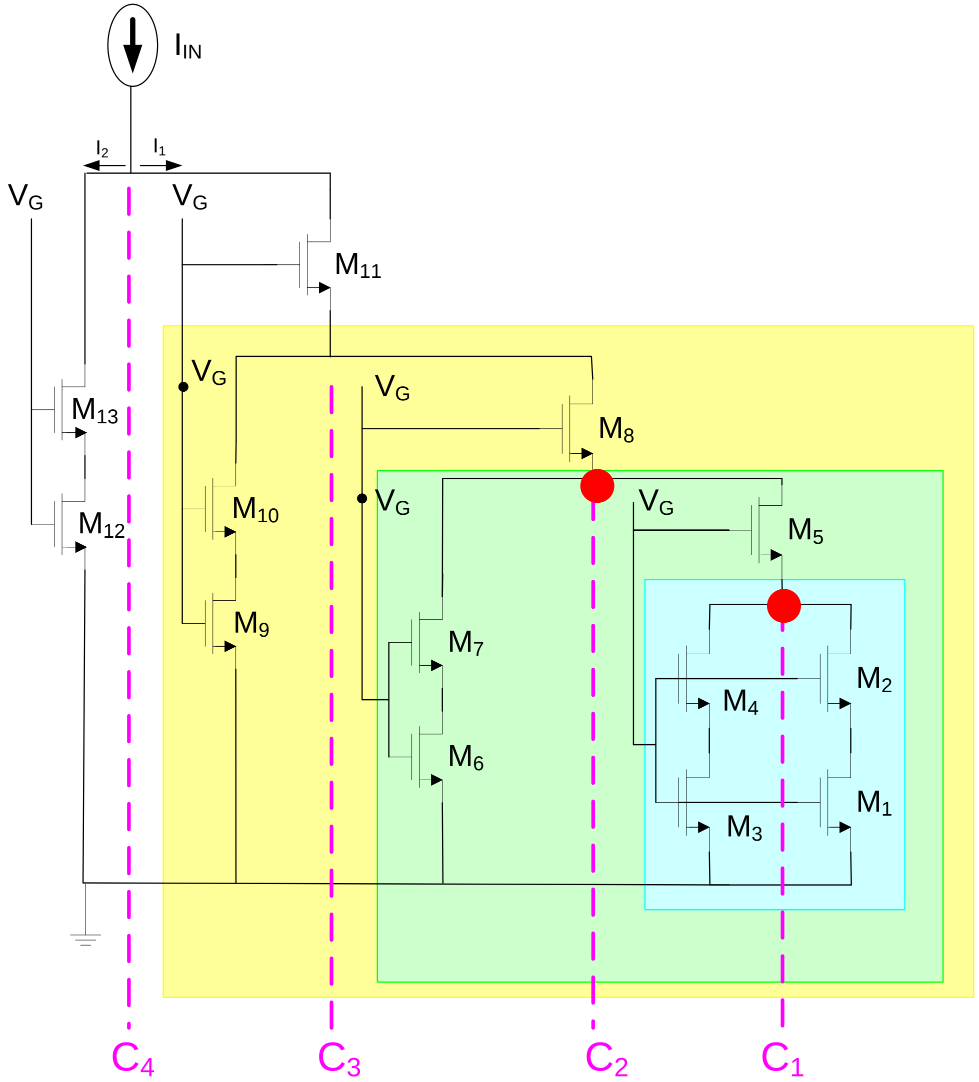
Question: How was the excellent linearity obtained in the author's own work and that reported in the literature if it is difficult to verify the linearity?



- Circuit in blue is completely symmetric on C_1 and is the well-known current divider
- it is not dependent upon any specific properties of the transistors !
- This was the right-most node where the "inherently linear" current divider was used !

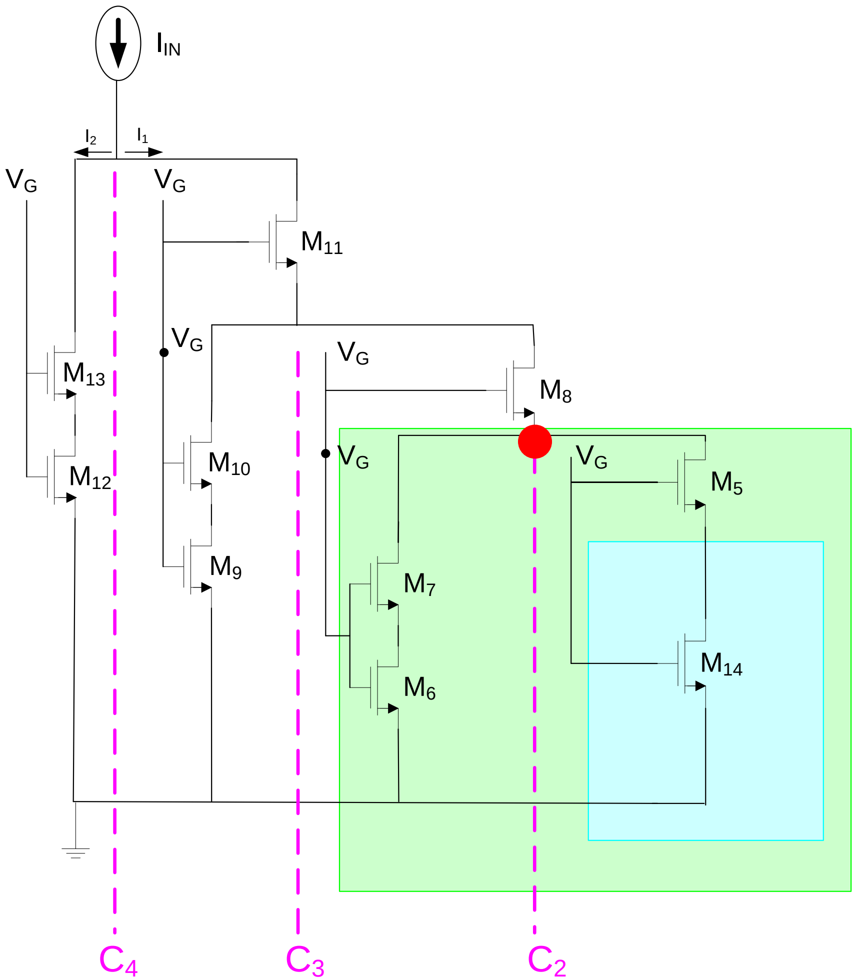


Question: How was the excellent linearity obtained in the author's own work and that reported in the literature if it is difficult to verify the linearity?

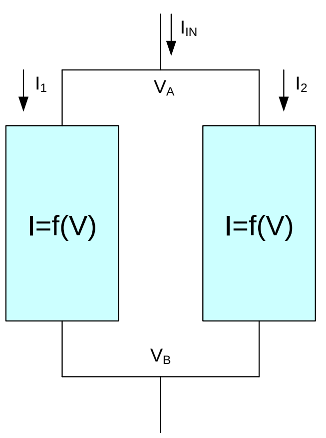


- Observe that M_1,M_2,M_3,M_4 can be modeled as a single transistor
  that is of the same size as M_1
- Call this M_{14}
- Consider now the next closest current-divider node

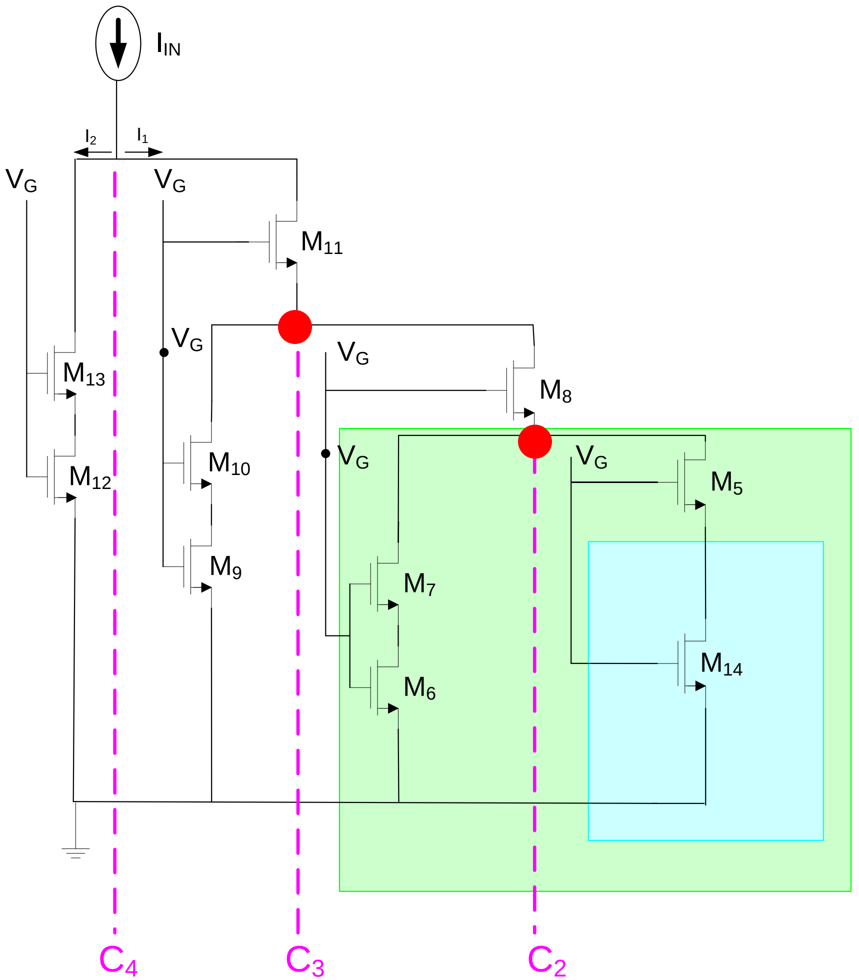
Question: How was the excellent linearity obtained in the author's own work and that reported in the literature if it is difficult to verify the linearity?



- Circuit in green is completely symmetric about C_2 and is the well-known current divider
- it is not dependent upon any specific properties of the transistors !

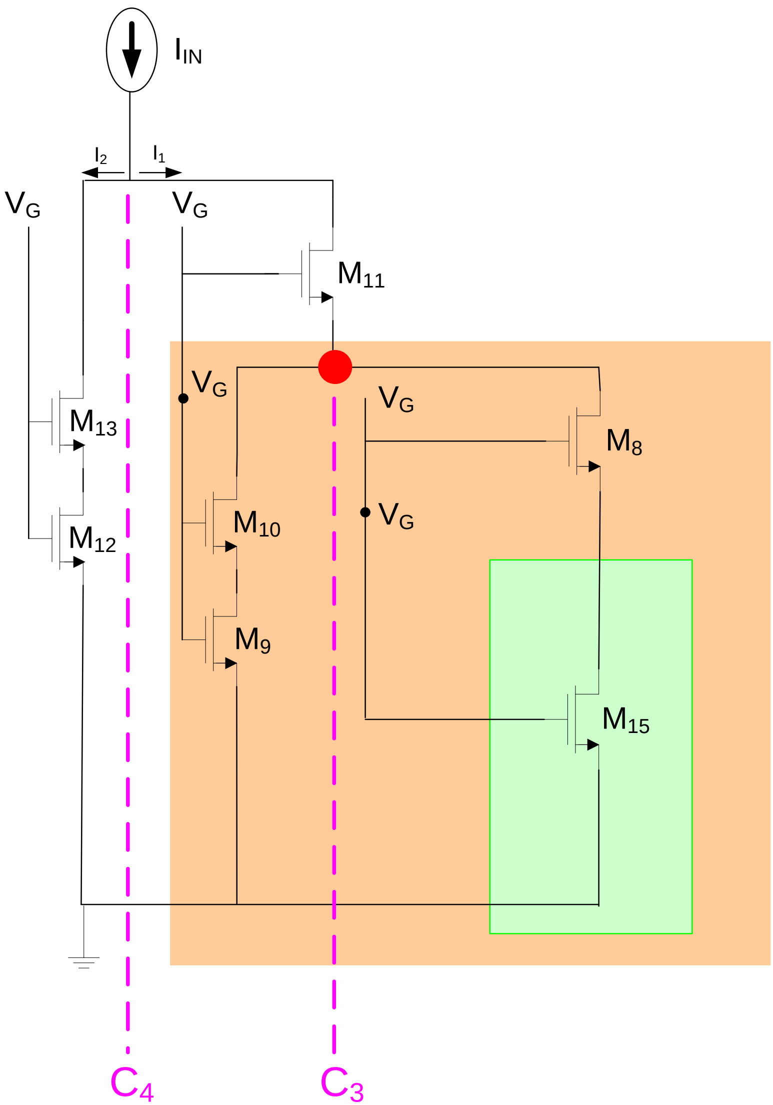


Question: How was the excellent linearity obtained in the author's own work and that reported in the literature if it is difficult to verify the linearity?

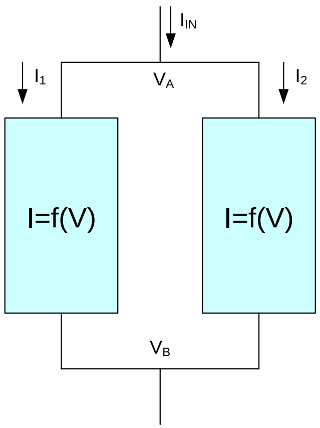


- Observe that M_6,M_7,M_5,M_{14} can be modeled as a single transistor that is of the same size as M_1
- Call this M_{15}
- Consider now the next closest current-divider node

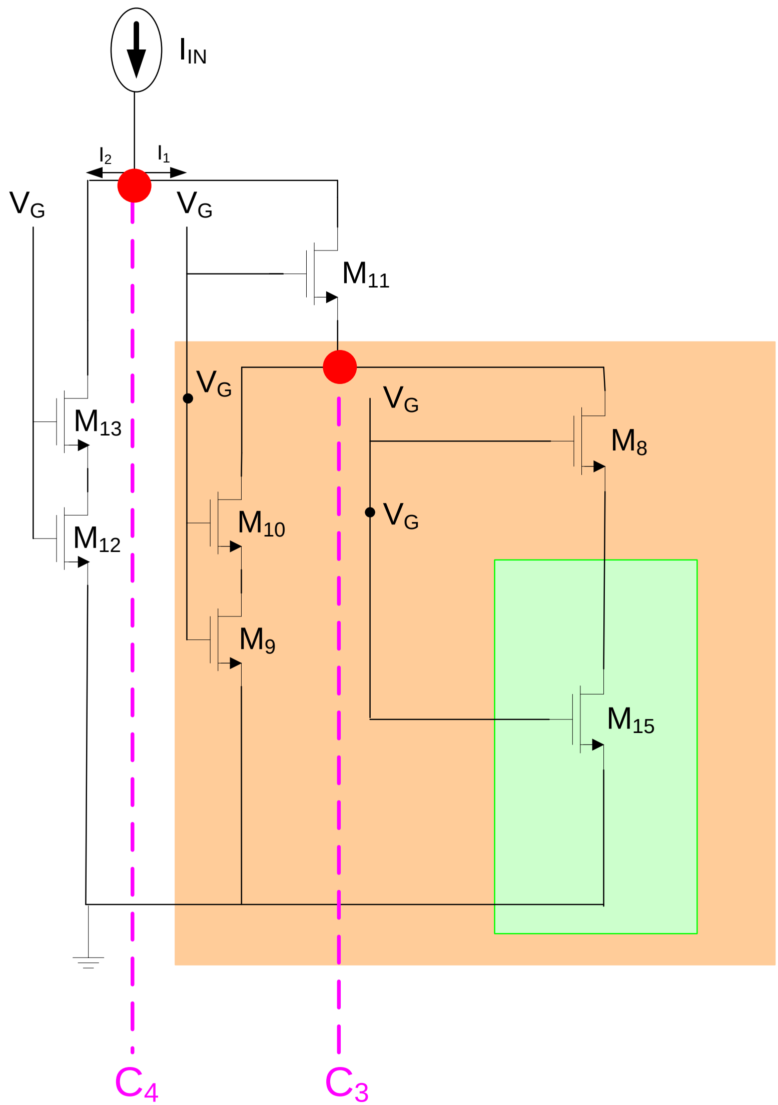
Question: How was the excellent linearity obtained in the author's own work and that reported in the literature if it is difficult to verify the linearity?



- Circuit in brown is completely symmetric on C_3 and is the well-known current divider
- it is not dependent upon any specific properties of the transistors !

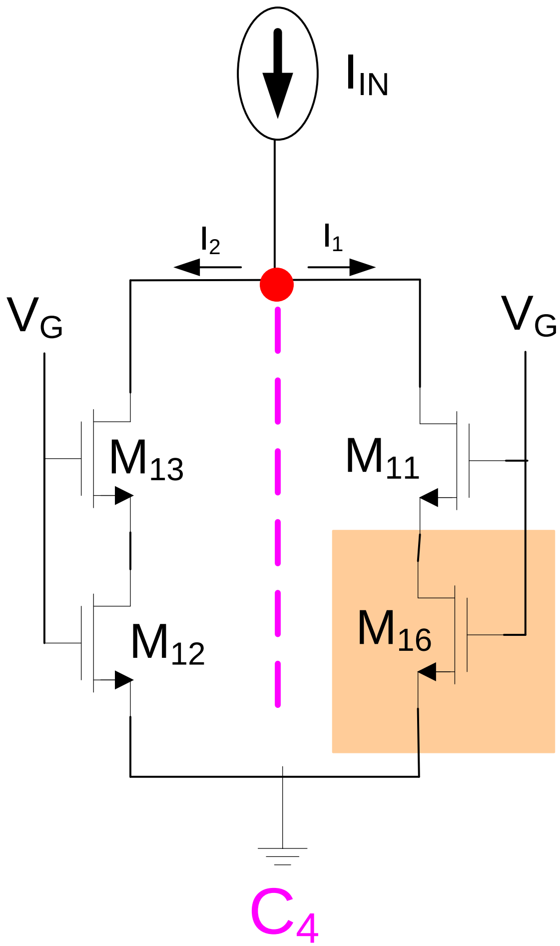


Question: How was the excellent linearity obtained in the author's own work and that reported in the literature if it is difficult to verify the linearity?

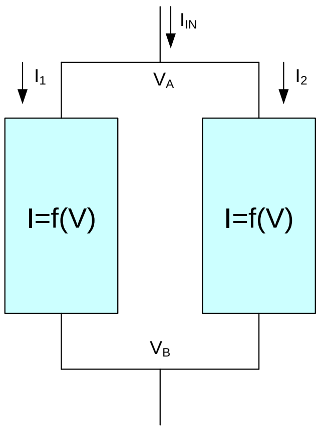


- Observe that M_9,M_{10},M_8,M_{15} can be modeled as a single transistor that is of the same size as M_1
- Call this M_{16}
- Consider now the next closest current-divider node

Question: How was the excellent linearity obtained in the author's own work and that reported in the literature if it is difficult to verify the linearity?

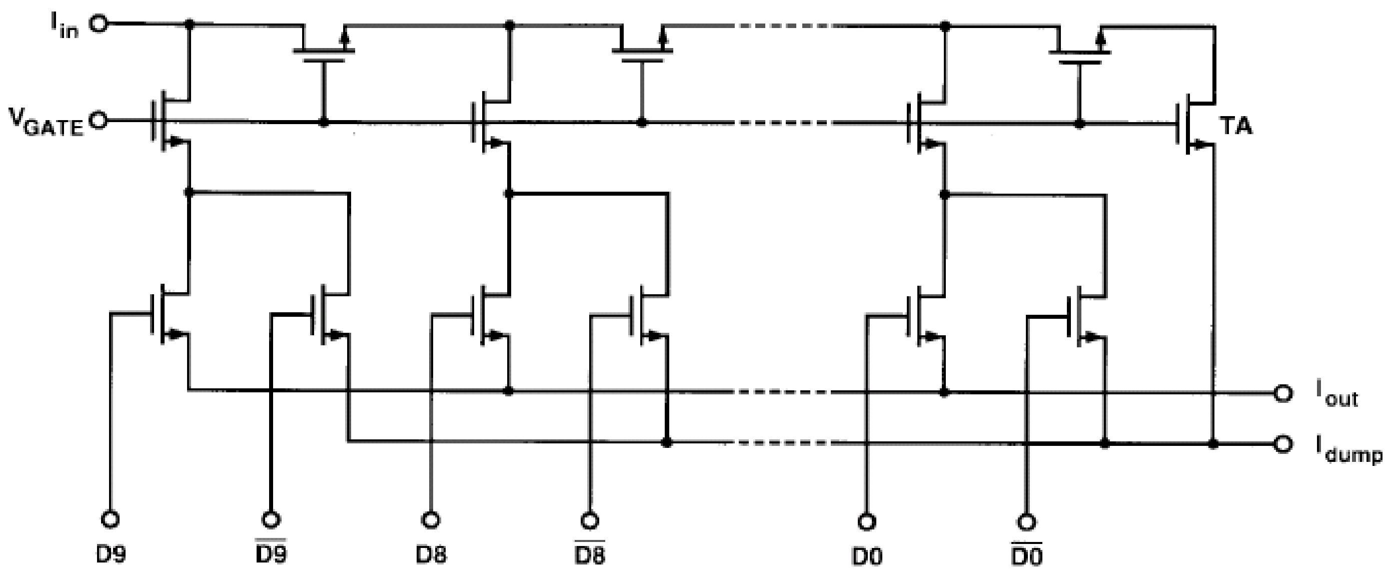


- Circuit shown is completely symmetric on C_3 and is the well-known current divider
- it is not dependent upon any specific properties of the transistors !



Question: How was the excellent linearity obtained in the author's own work and that reported in the literature if it is difficult to verify the linearity?

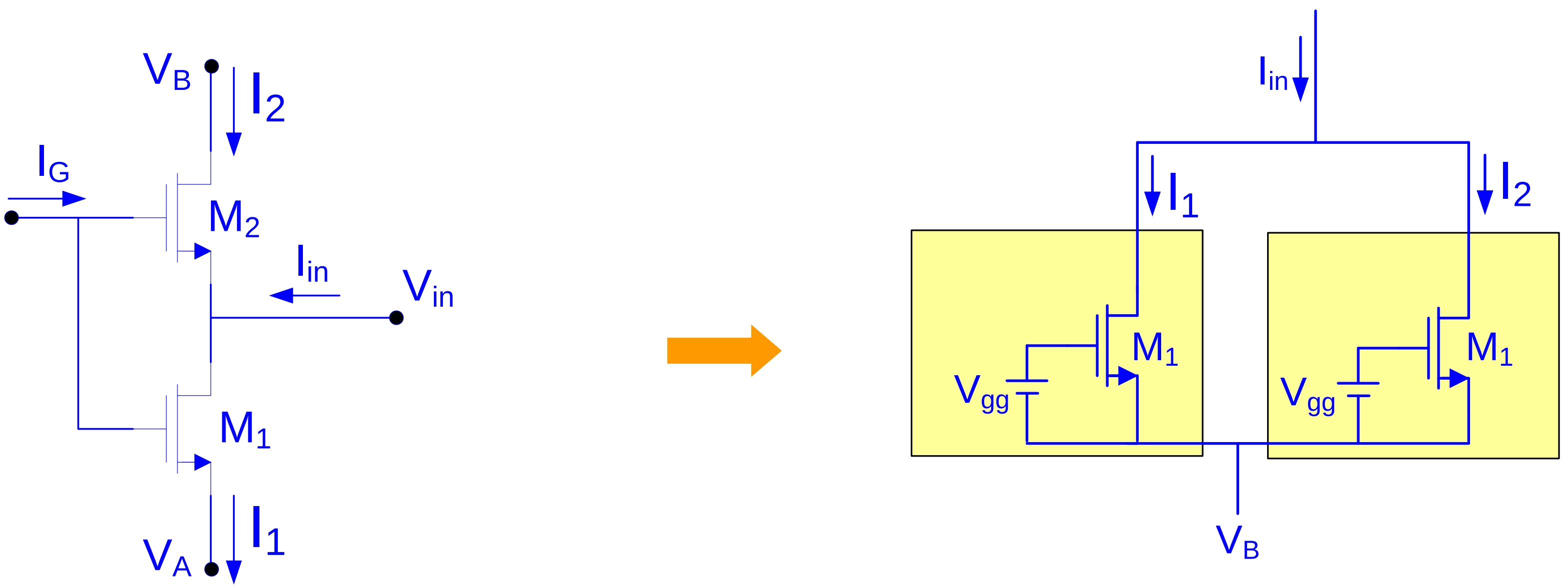
Current divider properties of the Huang DAC (ADC) were all dependent upon the general current division property of symmetric networks and had nothing to do with the current division in two transistors !



Current divider properties of the experimentally reported work of the original author were all dependent upon the general current division property of symmetric networks and had nothing to do with the current division in two transistors !

How was the very good performance of the "inherently linear" current divider obtained?

About 12 months ago one of our Ph.D. students looked at all SCI citations that referenced the "inherently linear" current divider and the performance in all cases was a special case of the general symmetric circuit



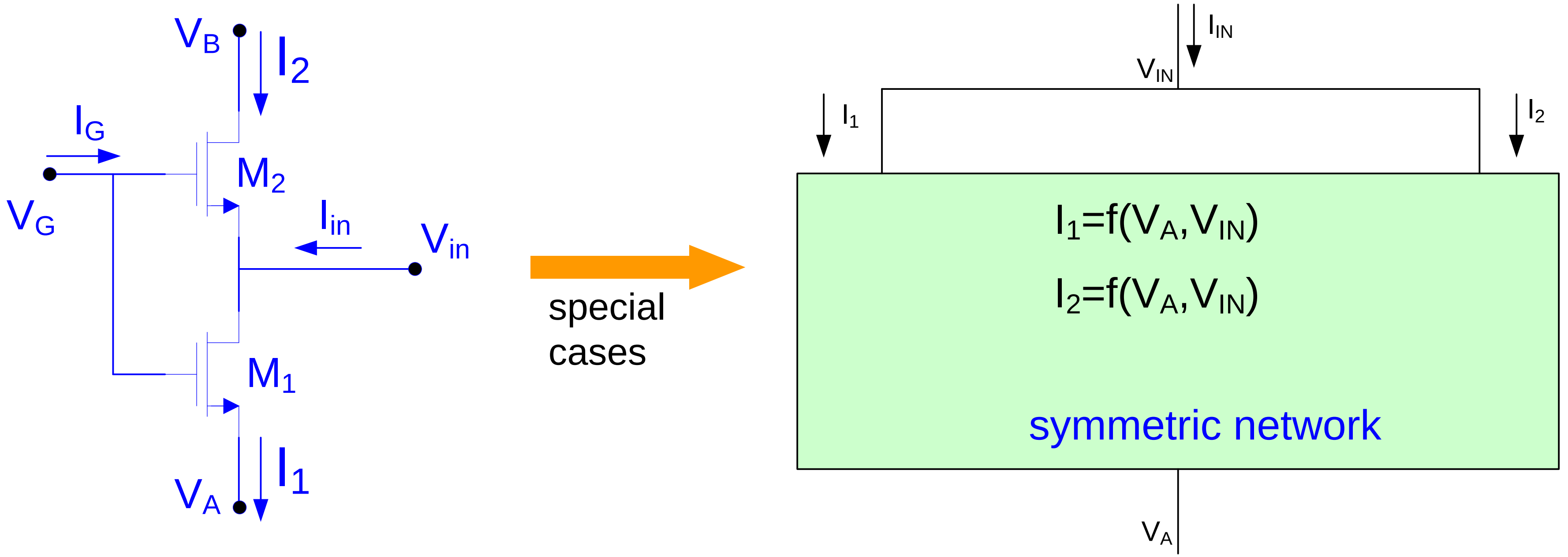
Symmetric Circuit

$I_1 = I_2$

Current Dividers

- Background
- Objective
- Concept of Current Divider
- Characterization of Inherently Linear Current Divider
- Inherent Current Division in Symmetric Circuits
- Conclusionhs

Good linearity properties of “inherently linear” current divider for those we found in the literature are due to well-known symmetry properties of circuits, not due to unique properties of the two-transistor current-divider structure



Conclusion

- The linearity properties are not apparent with existing device models
- Based upon existing models, operation as a current divider in question and linearity can be orders of magnitude worse than previously reported
- Good linearity properties of all applications found in literature survey for this circuit are due to well-known symmetry properties, not inherent characteristics of the two-transistor structure
- Experimental evidence appears to be lacking to support the inherently linearity properties of the current divider
- Is it possible that the circuit performs as an inherently linear current divider that has not yet been experimentally verified?
- Is it possible that there are major errors in existing device models used in circuit simulators that cause dramatic linearity errors in the simple 2-transistor current divider ?

Are Conventional Wisdom and Fundamental Concepts always aligned in the Microelectronics Field ?

Just considered conventional wisdom in 4 basic examples

- Op Amp
- Positive Feedback Compensation
- Current Mode Filters
- Current Dividers

Are Conventional Wisdom and Fundamental Concepts always aligned in the Microelectronics Field ?

Four examples involving some of the most basic concepts in the microelectronics field were identified where the alignment of conventional wisdom and fundamental concepts are weak

Many more examples exist where alignment is weak

Are Conventional Wisdom and Fundamental Concepts always aligned in the Microelectronics Field ?

Conventional Wisdom is VERY USEFUL for enhancing productivity and identifying practical approaches to engineering design and problem solving

Conventional Wisdom, however, should not be viewed as a basic principle or fundamental concept

Keep an OPEN MIND when using Conventional Wisdom to recognize both the benefits and limitations and recognize that even some of the most reputable sources and reputable engineers/scholars do not always distinguish between conventional wisdom and fundamental concepts

Thank you
for your attention !

End of Lecture 44