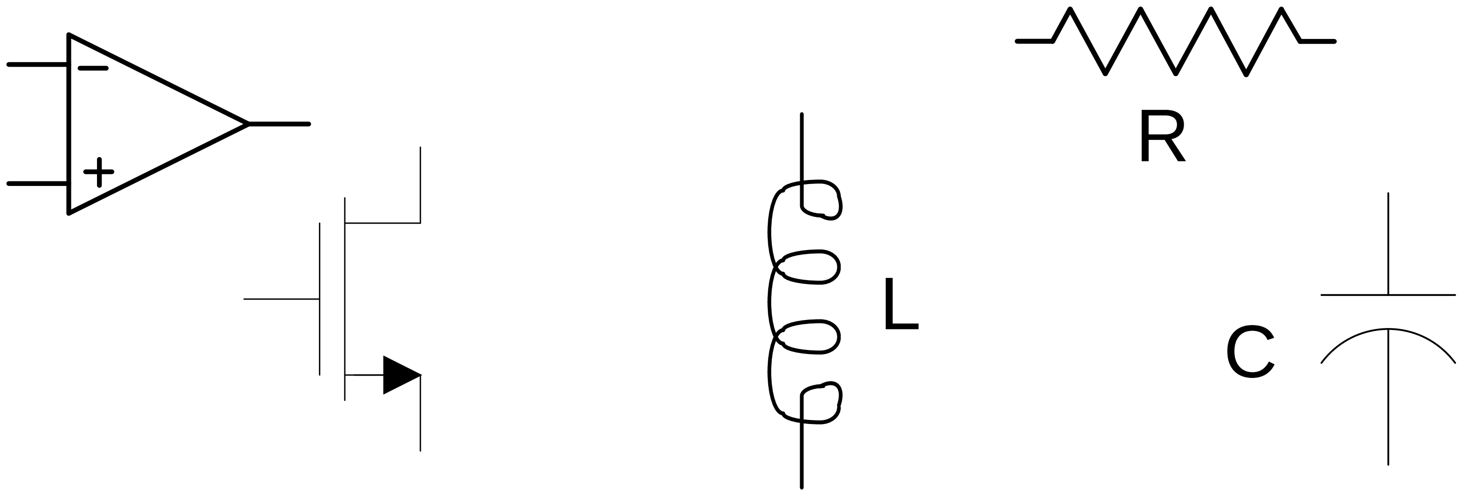
EE 508
Lecture 15

Statistical Characterization of Filter Characteristics

Review from last lecture

Effects of manufacturing variations on components

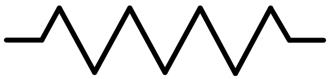


- A rigorous statistical analysis can be used to analytically predict how components vary and how component variations impact circuit performance

- Montecarlo simulations are often used to simulate effects of component variations
  - Requires minimal statistical knowledge to use MC simulations
  - Simulation times may be prohibitively long to get useful results
  - Gives little insight into specific source of problems
  - Must be sure to correctly include correlations in setup
- Often key statistical information is not readily available from the foundry

Review from last lecture

Modeling process variations in semiconductor processes



R

$$X = X_{NOM} + x_{RPROC} + x_{RWAFER} + x_{RDIE} + x_{RLGRAD} + x_{RLVAR}$$

x_{RPROC}, x_{RWAFER}, x_{RDIE}, x_{RLVAR} often assumed to be Gausian with zero mean

Magnitude of x_{RLGRAD} is usually assumed Gaussian with zero mean, direction is uniform from 0^o to 360^o

$$\sigma_{PROC} >> \sigma_{WAFER} >> \sigma_{DIE}$$

$$\sigma_{DIE} >> \sigma_{LVAR}$$

$$\sigma_{DIE} >> \sigma_{|GRAD|}$$

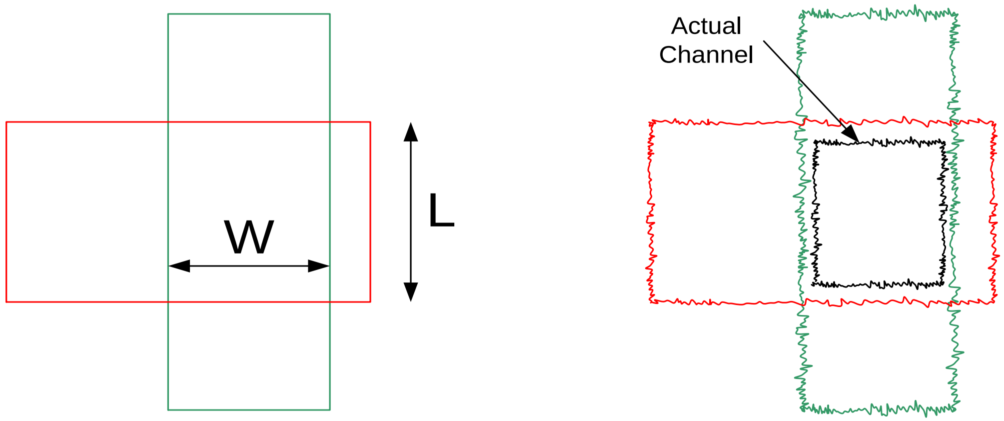
σ_{LVAR} Strongly dependent upon area and layout

$$\sigma_{LVAR} \sim \frac{1}{\sqrt{\text{Area}}}$$

$$\sigma_{LVAR} \sim \text{Perimeter}$$

Relative size between σ_{LVAR} and $\sigma_{|GRAD|}$ dependent upon A, P, and process

Effects of layout on local random variations



Drawn and Actual Features for MOS Transistor

Variations also occur vertically in both oxide thickness and doping levels/profiles and often these will dominate the lateral effects

Review from last lecture

Modeling process variations in semiconductor processes

R

- Statistics associated with value of dimensioned parameters (poles, GB, SR,R,C,transresistance gains, transconductance gains, … dominated by x_{RPROC})

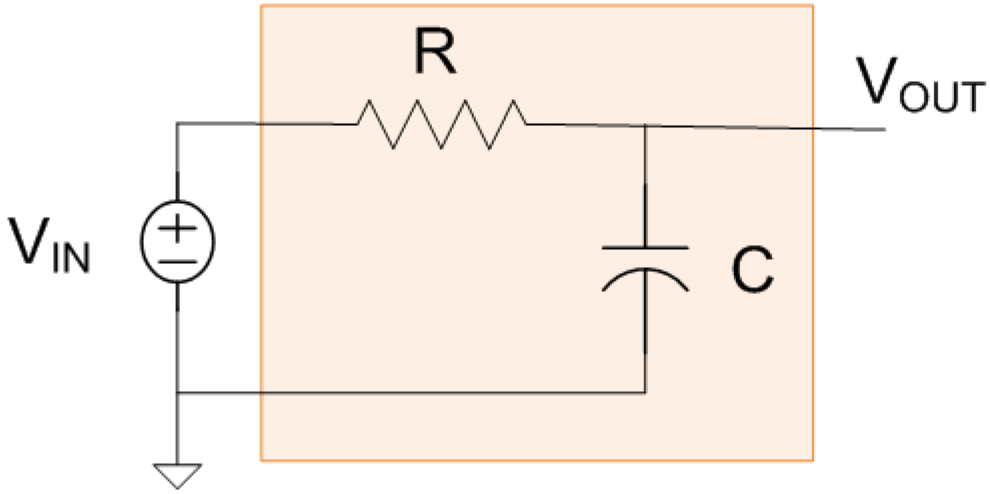
- Statistics associated with matching/sensitive dimensionless parameters such as voltage or current gains, component ratios, pole Q, … (almost always closely placed) dominated by x_{RLGRAD} and x_{RLVAR} (because locally x_{RPROC}, x_{RWAFER}, x_{RDIE} are all correlated and equal)

- Gradients are dominantly linear if spacing is not too large

- Special layout techniques using common centroid approaches can be used to eliminate (or dramatically reduce) linear gradient effects so, if employed, matching/sensitive parameters dominated by x_{RLVAR} but occasionally common centroid layouts become impractical or areas become too large so that gradients become nonlinear and in these cases gradient effects will still limit performance

- Higher-order gradient effects can be eliminated with layout approaches that cancel higher “moments” but area and effort may not be attractive

Statistical Modeling of dimensioned parameters - example

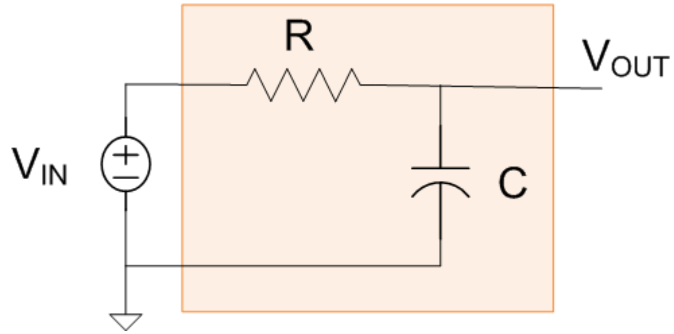


Determine the standard deviation of the pole frequency (or band edge) of the first-order passive filter.

Assume the process variables are zero mean with standard deviations given by

$$\sigma_{\frac{R_{PROC}}{R_{NOM}}} = 0.2 \qquad \sigma_{\frac{C_{PROC}}{C_{NOM}}} = 0.1$$

Review from last lecture



$$p = \frac{1}{RC}$$

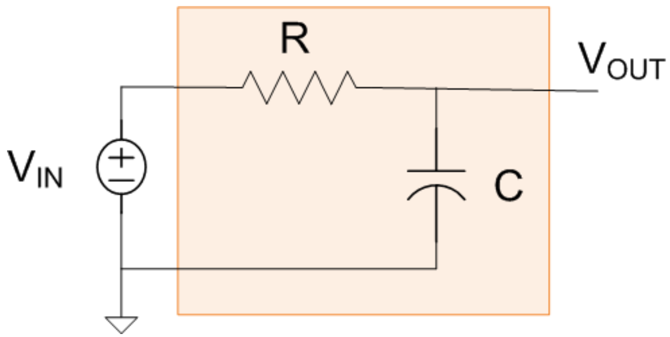
Since R and C are random variables, the pole p is also a random variable

Theorem: The sum of uncorrelated Gaussian random variables is a multivariate Gaussian random variable

Theorem: If $X_1 \dots X_m$ are uncorrelated random variables with standard deviations $\sigma_1, \sigma_2, \dots \sigma_m$, and $a_1, a_2, \dots a_m$ are constants, then the standard deviation of the random variable $y = \sum_{i=1}^{m} a_i X_i$ is given by the expression

$$\sigma_y = \sqrt{\sum_{i=1}^{m} a_i^2 \sigma_i^2}$$

Review from last lecture



$$p = \frac{1}{RC}$$

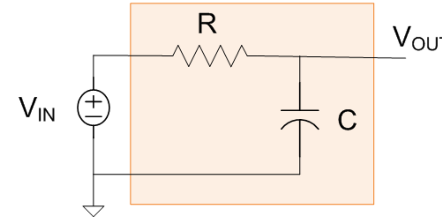
Since R and C are random variables, the pole p is also a random variable

$$p = \frac{1}{(R_{NOM} + R_{RAN})(C_{NOM} + C_{RAN})}$$

Unfortunately the pdf p which is the reciprocal of the product of Gaussian variables is very difficult to obtain

Observe can express p as

$$p = \frac{1}{(R_{NOM} + R_{RAN})(C_{NOM} + C_{RAN})} = \left(\frac{1}{R_{NOM}C_{NOM}}\right)\left(\frac{1}{\left[1+\frac{R_{RAN}}{R_{NOM}}\right]\left[1+\frac{C_{RAN}}{C_{NOM}}\right]}\right)$$



$$p = \frac{1}{RC}$$

$$p = \frac{1}{(R_{NOM} + R_{RAN})(C_{NOM} + C_{RAN})} = \left(\frac{1}{R_{NOM}C_{NOM}}\right)\left(\frac{1}{\left[1+\frac{R_{RAN}}{R_{NOM}}\right]\left[1+\frac{C_{RAN}}{C_{NOM}}\right]}\right)$$

But $R_{RAN} << R_{NOM}$ and $C_{RAN} << C_{NOM}$

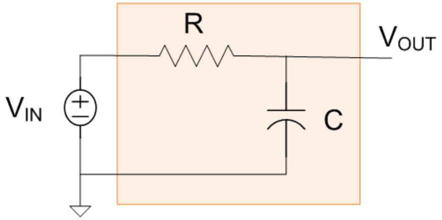
It thus follows from a truncated power series expansion of the two-variable fraction that

$$p \simeq \left(\frac{1}{R_{NOM}C_{NOM}}\right)\left(\left[1-\frac{R_{RAN}}{R_{NOM}}\right]\left[1-\frac{C_{RAN}}{C_{NOM}}\right]\right)$$

$$p \simeq \left(\frac{1}{R_{NOM}C_{NOM}}\right)\left(1-\frac{R_{RAN}}{R_{NOM}}-\frac{C_{RAN}}{C_{NOM}}\right)$$

These operations were used to linearize p in terms of the random variables !

Note that p is the sum of two Gaussian random variables that are assumed to be uncorrelated so p is also Gaussian



$$p = \frac{1}{RC}$$

$$p \simeq \left(\frac{1}{R_{NOM}C_{NOM}}\right)\left(1 - \frac{R_{RAN}}{R_{NOM}} - \frac{C_{RAN}}{C_{NOM}}\right)$$

It thus follows from the theorem that

$$\sigma_p \simeq \left(\frac{1}{R_{NOM}C_{NOM}}\right)\sqrt{\sigma^2_{\frac{R_{RAN}}{R_{NOM}}} + \sigma^2_{\frac{C_{RAN}}{C_{NOM}}}}$$

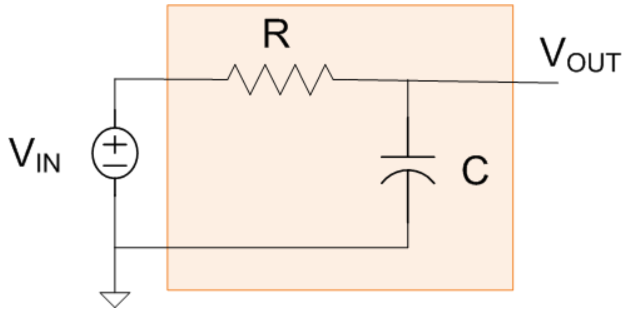
But the nominal value of the pole is $p_{NOM} \simeq \dfrac{1}{R_{NOM}C_{NOM}}$

It thus follows that

$$\sigma_{\frac{p}{p_{NOM}}} \simeq \sqrt{\sigma^2_{\frac{R_{RAN}}{R_{NOM}}} + \sigma^2_{\frac{C_{RAN}}{C_{NOM}}}}$$

Observe:

$$\frac{p}{p_{NOM}} \sim N\left(1, \sigma_{\frac{p}{p_{NOM}}}\right)$$



$$p = \frac{1}{RC}$$

$$\sigma_{\frac{p}{p_{NOM}}} \simeq \sqrt{\sigma^2_{\frac{R_{RAN}}{R_{NOM}}} + \sigma^2_{\frac{C_{RAN}}{C_{NOM}}}}$$

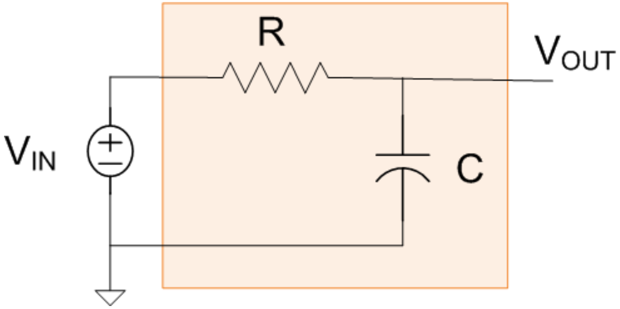
But R_{RAN} and C_{RAN} are approximately R_{PROC} and C_{PROC}

$$\sigma_{\frac{p}{p_{NOM}}} \simeq \sqrt{\sigma^2_{\frac{R_{PROC}}{R_{NOM}}} + \sigma^2_{\frac{C_{PROC}}{C_{NOM}}}}$$

recall

$$\sigma_{\frac{R_{PROC}}{R_{NOM}}} = 0.2 \qquad \sigma_{\frac{C_{PROC}}{C_{NOM}}} = 0.1$$

$$\sigma_{\frac{p}{p_{NOM}}} \simeq \sqrt{0.2^2 + 0.1^2} = 0.22$$



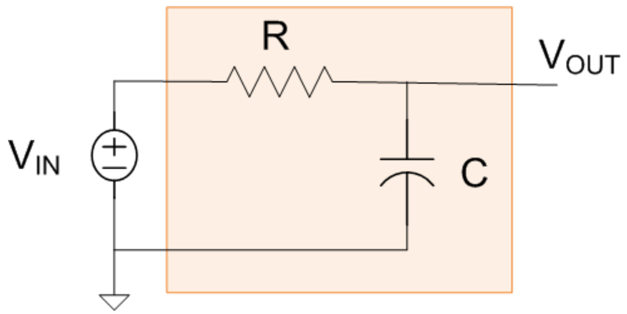
$$p = \frac{1}{RC}$$

$$\sigma_{\frac{p}{p_{NOM}}} \simeq \sqrt{0.2^2 + 0.1^2} = 0.22$$

1. Determine the 3σ range in the pole location

2. Determine the percent of the process lots that will have a pole with mean that is within 10% of the nominal value

3. What can the designer do to tighten the band edge of this filter?



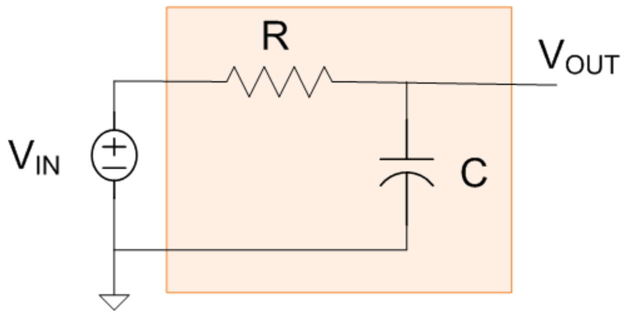
$$p = \frac{1}{RC}$$

$$\sigma_{\frac{p}{p_{NOM}}} \simeq \sqrt{0.2^2 + 0.1^2} = 0.22$$

1. Determine the 3σ range in the pole location

The 3σ range is simply $$0.34 \le \frac{p}{p_{NOM}} \le 1.66$$

So, if the nominal pole location is 10KHz, the average value of the pole location from lot to lot will vary between 3.4KHz and 16.6KHz



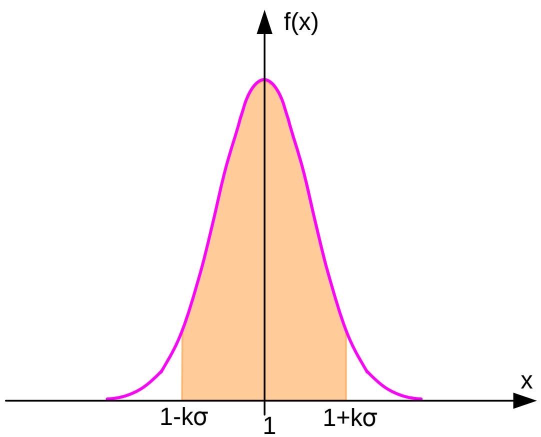
$$p = \frac{1}{RC}$$

$$\sigma_{\frac{p}{p_{NOM}}} \simeq \sqrt{0.2^2 + 0.1^2} = 0.22$$

2. Determine the percent of the process lots that will have a pole with mean that is within 10% of the nominal value

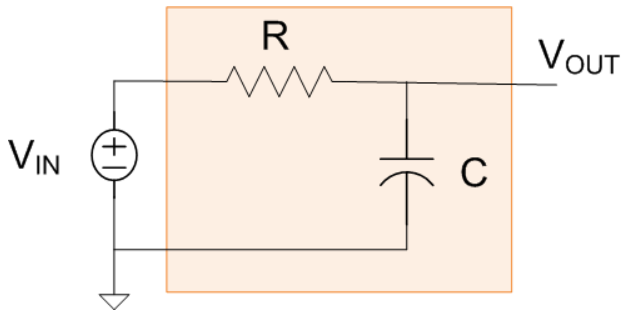
Observe a 10% window is $\left(\frac{.1}{.22}\right)\sigma_{\frac{p}{p_{NOM}}} = 0.45\sigma_{\frac{p}{p_{NOM}}}$

Recall $\frac{p}{p_{NOM}} \sim N\left(1, \sigma_{\frac{p}{p_{NOM}}}\right)$ For a kσ window the probability of being inside that window is the area under the pdf curve between 1– kσ and 1+kσ



Observe

$$\tilde{p} = \frac{\frac{p}{p_{NOM}} - 1}{\sigma_{\frac{p}{p_{NOM}}}} \sim N(0,1)$$



$$p = \frac{1}{RC}$$

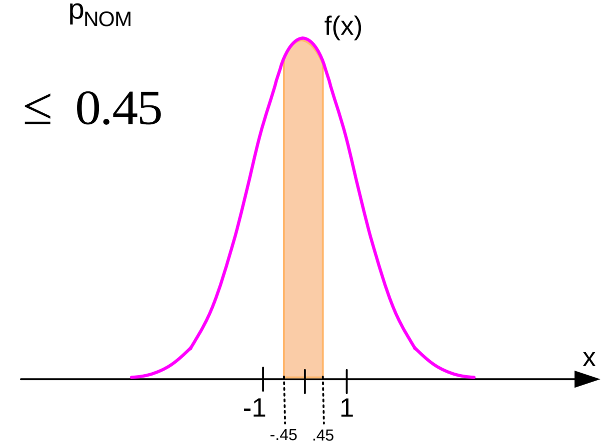
$$\sigma_{\frac{p}{p_{NOM}}} \simeq \sqrt{0.2^2 + 0.1^2} = 0.22$$

2. Determine the percent of the process lots that will have a pole with mean that is within 10% of the nominal value

Observe a 10% window is $\left(\frac{.1}{.22}\right)\sigma_{\frac{p}{p_{NOM}}} = 0.45\sigma_{\frac{p}{p_{NOM}}}$

$$1 - 0.45\sigma_{\frac{p}{p_{NOM}}} \le p \le 1 - 0.45\sigma_{\frac{p}{p_{NOM}}} \quad \Rightarrow \quad -0.45 \le \tilde{p} \le 0.45$$

$$\tilde{p} = \frac{\frac{p}{p_{NOM}} - 1}{\sigma_{\frac{p}{p_{NOM}}}}$$

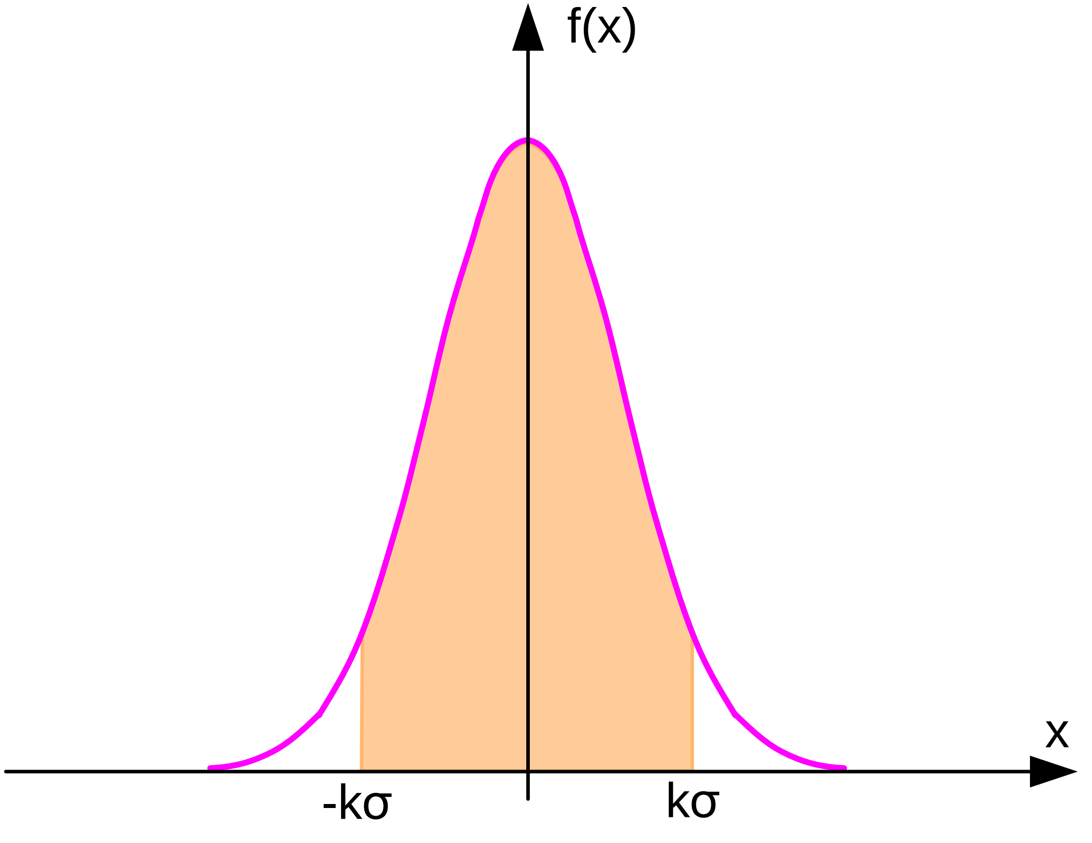


For a Gaussian variable, this area is given by

$$\theta_{prob} = 2F_{N(0,1)}(k) - 1 = 2F_{N(0,1)}(0.45) - 1$$

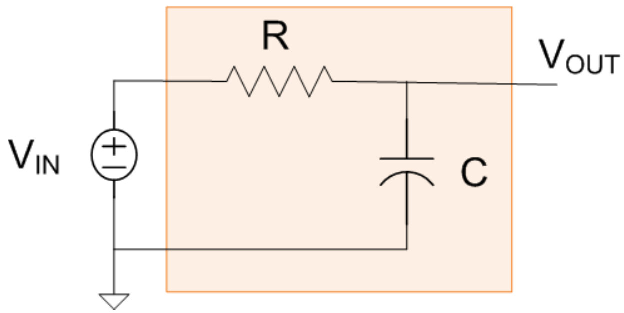
Offset Voltage Distribution

Pdf of zero-mean Gaussian distribution



Percent between:	±σ	68.3%
	±2σ	95.5%
	±3σ	99.73%

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91308	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997
4.0	0.99997	0.99997	0.99997	0.99997	0.99997	0.99997	0.99998	0.99998	0.99998	0.99998



$$p = \frac{1}{RC}$$

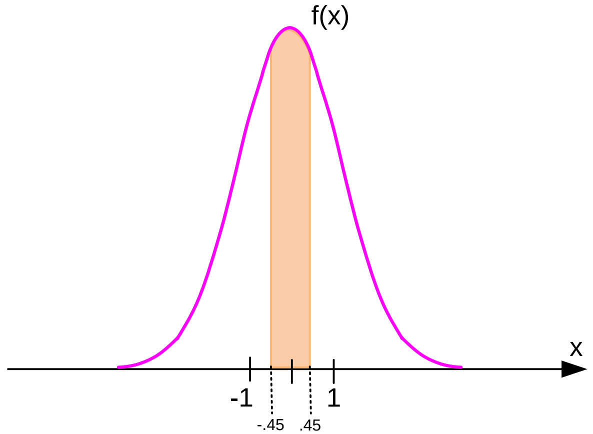
$$\sigma_{\frac{p}{p_{NOM}}} \simeq \sqrt{0.2^2 + 0.1^2} = 0.22$$

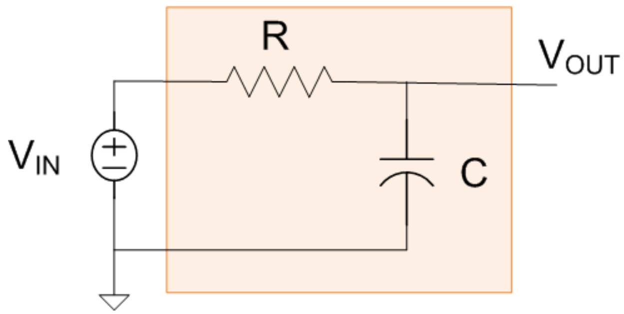
2. Determine the percent of the process lots that will have a pole with mean that is within 10% of the nominal value

$$\theta_{prob} = 2F_{N(0,1)}(0.45) - 1$$

$$\theta_{prob} = 2 \bullet .6736 - 1 = 0.347$$

Thus, approximately 35% of the wafer lots will have a pole within 10% of the nominal value



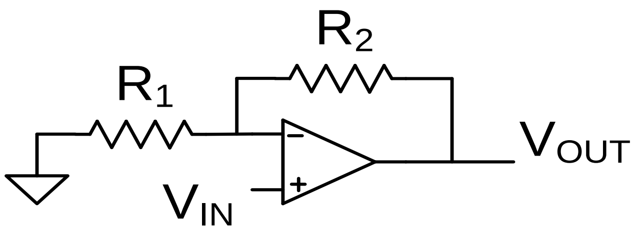


$$p = \frac{1}{RC}$$

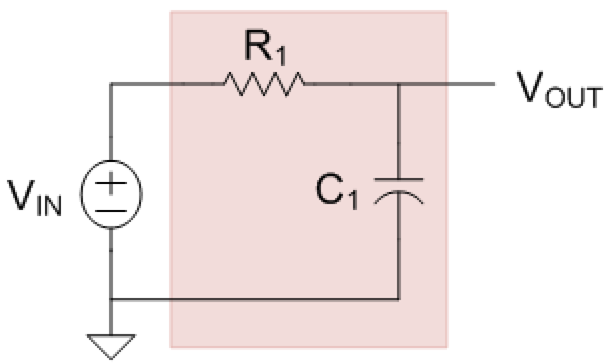
$$\sigma_{\frac{p}{p_{NOM}}} \simeq \sqrt{0.2^2 + 0.1^2} = 0.22$$

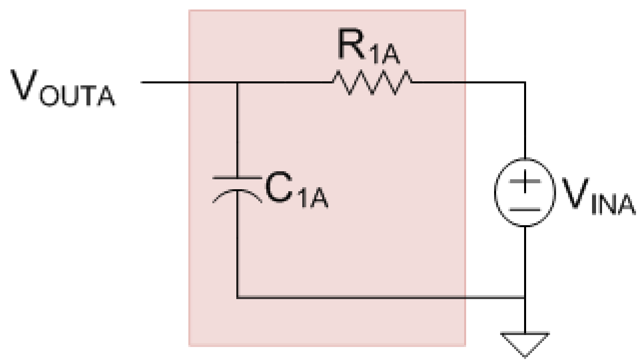
3. What can the designer do to tighten the band edge of this filter?

Statistical Modeling of Dimensionless Parameters



$$K = 1 + \frac{R_2}{R_1}$$



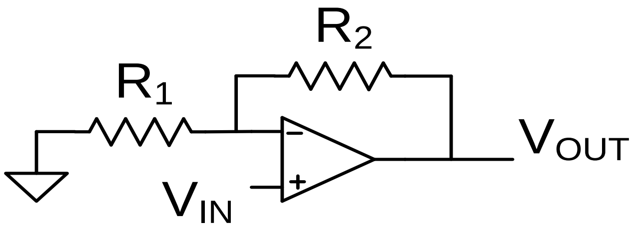


$$p_1 = \frac{1}{RC}$$

$$p_A = \frac{1}{R_{1A}C_{1A}}$$

$$\theta = \frac{p_A - p_1}{p_1}$$

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the standard deviation of the voltage gain K

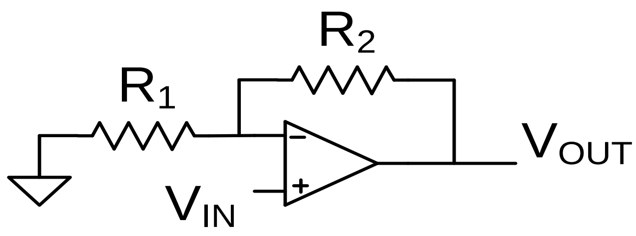
Determine the yield if the nominal gain is 10 $\pm 1\%$

Assume a common centroid layout of R_1 and R_2 has been used and the area of R_1 is 100u^2 and both resistors have the same resistance density and R_2 is comprised of K-1 copies of R_1 . Neglect variable edge effects in the layout

A_ρ=.01µm

$$\sigma_{\frac{R_{PROC}}{R_{NOM}}} = 0.2$$

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the standard deviation of the voltage gain K

$$K = 1 + \frac{R_{2N} + R_{2R}}{R_{1N} + R_{1N}}$$

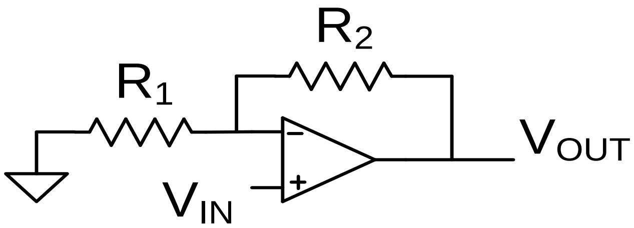
$$K \simeq 1 + \frac{R_{2N}}{R_{1N}}\left(1 + \frac{R_{2R}}{R_{2N}} - \frac{R_{1R}}{R_{1N}}\right)$$

$$K = 1 + \frac{R_{2N}\left(1 + \frac{R_{2R}}{R_{2N}}\right)}{R_{1N}\left(1 + \frac{R_{1R}}{R_{1N}}\right)}$$

$$K \simeq \left(1 + \frac{R_{2N}}{R_{1N}}\right) + \frac{R_{2N}}{R_{1N}}\left(\frac{R_{2R}}{R_{2N}} - \frac{R_{1R}}{R_{1N}}\right)$$

$$K \simeq 1 + \frac{R_{2N}}{R_{1N}}\left(1 + \frac{R_{2R}}{R_{2N}}\right)\left(1 - \frac{R_{1R}}{R_{1N}}\right)$$

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the standard deviation of the voltage gain K

$$K \simeq \left(1 + \frac{R_{2N}}{R_{1N}}\right) + \frac{R_{2N}}{R_{1N}}\left(\frac{R_{2R}}{R_{2N}} - \frac{R_{1R}}{R_{1N}}\right)$$

$$K \simeq K_N + \left(K_N - 1\right)\left(\frac{R_{2R}}{R_{2N}} - \frac{R_{1R}}{R_{1N}}\right)$$

$$R_{2R} \simeq R_{2RPROC} + R_{2RFRAD} + R_{2RLVAR}$$

$$R_{1R} \simeq R_{1RPROC} + R_{1RFRAD} + R_{1RLVAR}$$

$$K \simeq K_N + \left(K_N - 1\right)\left(\frac{R_{2RPROC} + R_{2RFRAD} + R_{2RLVAR}}{R_{2N}} - \frac{R_{1RPROC} + R_{1RFRAD} + R_{1RLVAR}}{R_{1N}}\right)$$

But R_{2RPROC} and R_{1RPROC} are correlated

$$R_{2RPROC} = \left(K_N - 1\right) R_{1RPROC}$$

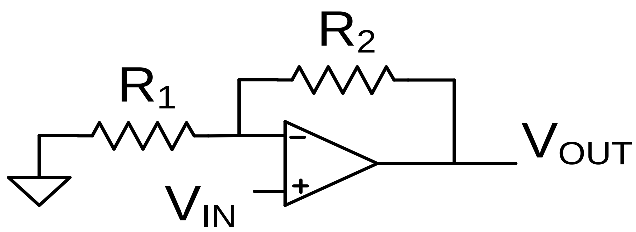
And, since a common centroid layout is used,

R_{2RGRAD} and R_{1RGRAD} are correlated

$$R_{2RGRAD} = \left(K_N - 1\right) R_{1RGRAD}$$

R_{2RLVAR} and R_{1RLVAR} are uncorrelated

Statistical Modeling of dimensionless parameters - example



$$K = 1+\frac{R_2}{R_1}$$

Determine the standard deviation of the voltage gain K

$$K \simeq K_N+(K_N-1)\left(\frac{R_{2RPROC}+R_{2RFRAD}+R_{2RLVAR}}{R_{2N}}-\frac{R_{2RPROC}+R_{2RFRAD}+R_{2RLVAR}}{R_{1N}}\right)$$

$$K \simeq K_N+(K_N-1)\left(\frac{(K_N-1)R_{1RPROC}+(K_N-1)R_{1RPROC}+R_{2RLVAR}}{R_{2N}}-\frac{R_{1RPROC}+R_{1RFRAD}+R_{1RLVAR}}{R_{1N}}\right)$$

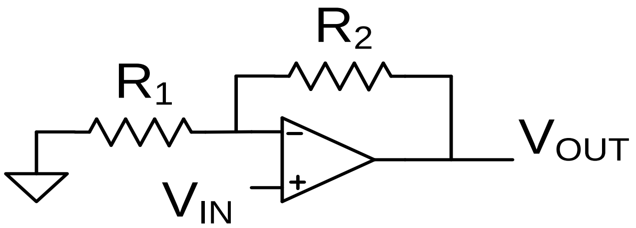
Since $R_{2N}=(K_N-1)R_{1N}$

$$K \simeq K_N+(K_N-1)\left(\frac{(K_N-1)R_{1RPROC}+(K_N-1)R_{1RPROC}}{(K_N-1)R_{1N}}+\frac{R_{2RLVAR}}{R_{2N}}-\frac{R_{1RPROC}+R_{1RFRAD}+R_{1RLVAR}}{R_{1N}}\right)$$

$$K \simeq K_N+(K_N-1)\left(\left[\frac{(K_N-1)R_{1RPROC}+(K_N-1)R_{1RPROC}}{(K_N-1)R_{1N}}-\frac{R_{1RPROC}+R_{1RFRAD}}{R_{1N}}\right]+\frac{R_{2RLVAR}}{R_{2N}}-\frac{R_{1RLVAR}}{R_{1N}}\right)$$

$$K \simeq K_N+(K_N-1)\left(\frac{R_{2RLVAR}}{R_{2N}}-\frac{R_{1RLVAR}}{R_{1N}}\right)$$

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the standard deviation of the voltage gain K

$$K \simeq K_N + (K_N - 1)\left(\frac{R_{2RLVAR}}{R_{2N}} - \frac{R_{1RLVAR}}{R_{1N}}\right)$$

$$\sigma_K \simeq (K_N - 1)\sqrt{\sigma^2_{\frac{R_{2R}}{R_{2N}}} + \sigma^2_{\frac{R_{1R}}{R_{1N}}}}$$

$$\sigma_{\frac{K}{K_N}} \simeq \left(1 - \frac{1}{K_N}\right)\sqrt{\sigma^2_{\frac{R_{2R}}{R_{2N}}} + \sigma^2_{\frac{R_{1R}}{R_{1N}}}}$$

Theorem: If the perimeter variations and contact resistance are neglected, the standard deviation of the local random variations of a resistor of area A is given by the expression

$$\sigma_{\frac{R}{R_N}} = \frac{A_\rho}{\sqrt{A}}$$

Theorem: If the perimeter variations are neglected, the standard deviation of the local random variations of a capacitor of area A is given by the expression

$$\sigma_{\frac{C}{C_N}} = \frac{A_C}{\sqrt{A}}$$

Theorem: If the perimeter variations are neglected, the variance of the local random variations of the normalized threshold voltage of a rectangular MOS transistor of dimensions W and L is given by the expression

$$\sigma^2_{\frac{V_T}{V_{T_N}}} = \frac{A^2_{VTO}}{V^2_{T_N} WL}$$

or as

$$\sigma^2_{\frac{V_T}{V_{T_N}}} = \frac{A^2_{VT}}{WL}$$

Theorem: If the perimeter variations are neglected, the variance of the local random variations of the normalized C_{OX} of a rectangular MOS transistor of dimensions W and L is given by the expression

$$\sigma^2_{\frac{C_{OX}}{C_{OXN}}} = \frac{A^2_{COX}}{WL}$$

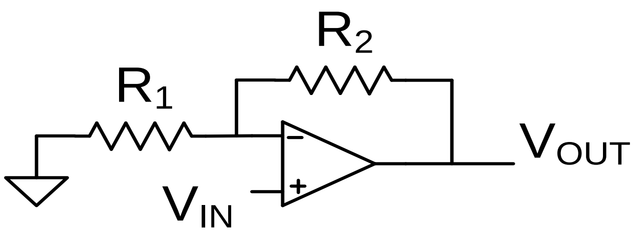
Theorem: If the perimeter variations are neglected, the variance of the local random variations of the normalized mobility of a rectangular MOS transistor of dimensions W and L is given by the expression

$$\sigma^2_{\frac{\mu_R}{\mu_N}} = \frac{A^2_{\mu}}{WL}$$

where the parameters A_X are all constants characteristic of the process (i.e. model parameters)

- The effects of edge roughness on the variance of resistors, capacitors, and transistors can readily be included but for most layouts is dominated by the area dependent variations
- There is some correlation between the model parameters of MOS transistors but they are often ignored to simplify calculations

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the standard deviation of the voltage gain K

$$K \simeq K_N + (K_N - 1)\left(\frac{R_{2RLVAR}}{R_{2N}} - \frac{R_{1RLVAR}}{R_{1N}}\right)$$

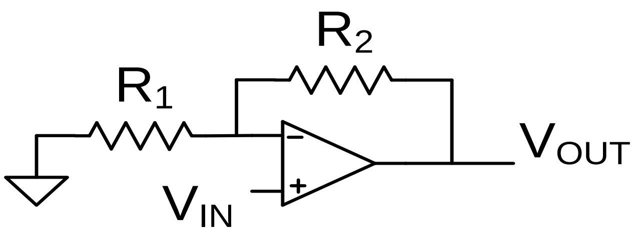
$$\sigma_K \simeq (K_N - 1)\sqrt{\sigma^2_{\frac{R_{2R}}{R_{2N}}} + \sigma^2_{\frac{R_{1R}}{R_{1N}}}}$$

$$\sigma_{\frac{R}{R_N}} = \frac{A_\rho}{\sqrt{A}}$$

$$\sigma_K \simeq (K_N - 1)A_\rho\sqrt{\frac{1}{A_{R2}} + \frac{1}{A_{R1}}}$$

$$\sigma_K \simeq (K_N - 1)A_\rho\sqrt{\frac{1}{(K_N - 1)A_{R1}} + \frac{1}{A_{R1}}}$$

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the standard deviation of the voltage gain K

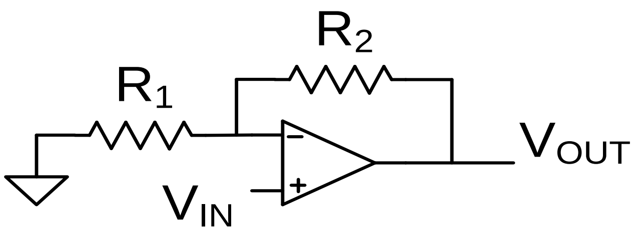
$$\sigma_K \simeq (K_N - 1) A_\rho \sqrt{\frac{1}{(K_N-1) A_{R1}} + \frac{1}{A_{R1}}}$$

$$\sigma_K \simeq (K_N - 1)\frac{A_\rho}{\sqrt{A_{R1}}} \sqrt{\frac{1}{(K_N-1)} + 1} = \frac{A_\rho}{\sqrt{A_{R1}}}(K_N-1)\sqrt{\frac{K_N}{(K_N-1)}}$$

$$\sigma_K \simeq \frac{A_\rho}{\sqrt{A_{R1}}}\sqrt{K_N (K_N - 1)}$$

$$\sigma_{\frac{K}{K_N}} \simeq \frac{A_\rho}{\sqrt{A_{R1}}}\sqrt{1 - \frac{1}{K_N}}$$

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the standard deviation of the voltage gain K

$$\sigma_K \simeq \frac{A_\rho}{\sqrt{A_{R1}}}\sqrt{K_N\left(K_N - 1\right)}$$

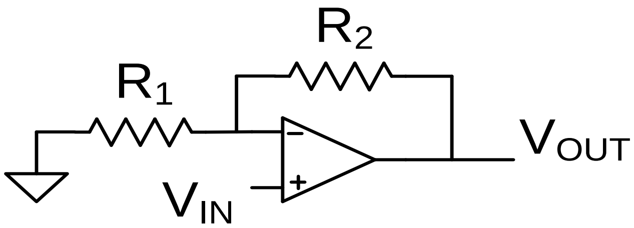
A_ρ=.01u $\quad$ A_{R1}=100u^2 $\quad$ $\sigma_{\frac{R_{PROC}}{R_{NOM}}} = 0.2$

$$\sigma_K \simeq \frac{.01}{10}\sqrt{K_N\left(K_N - 1\right)} = .001\sqrt{K_N\left(K_N - 1\right)}$$

$$\sigma_{\frac{K}{K_N}} \simeq .001\sqrt{1 - \frac{1}{K_N}}$$

- The standard deviation can be improved by increasing area but a 4X increase in area is needed for a 2X reduction in sigma
- Note the standard deviation of the normalized gain is much smaller than the standard deviation of the process variations

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the standard deviation of the voltage gain K

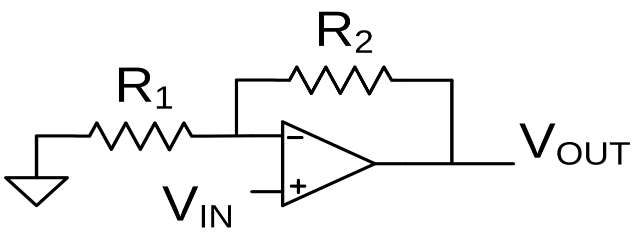
$$\sigma_{\frac{K}{K_N}} \simeq .001\sqrt{1 - \frac{1}{K_N}}$$

Determine the yield if the nominal gain is 10 $\pm 1\%$

$$\sigma_{\frac{K}{K_N}} \simeq .001\sqrt{1 - \frac{1}{10}} = .00095$$

$$\frac{K}{K_N} \sim N(1,\ 0.00095)$$

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the yield if the nominal gain is 10 $\pm 1\%$

$$\frac{K}{K_N} \sim N(1,\ 0.00095)$$

$$\frac{\frac{K}{K_N} - 1}{0.00095} \sim N(0,1)$$

$$9.9 < K < 10.1$$

$$.99 < \frac{K}{K_N} < 1.01$$

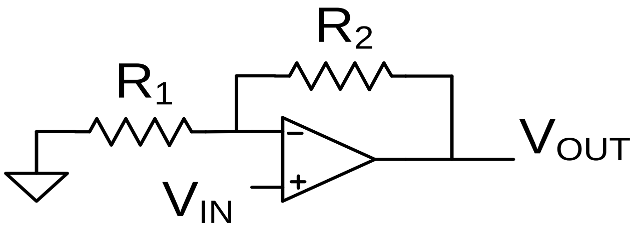
$$-.01 < \frac{K}{K_N} - 1 < .01$$

$$-10 < \frac{\frac{K}{K_N} - 1}{.00095} < 10$$

The gain yield is essentially 100%

Could substantially decrease area or increase gain accuracy if desired

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

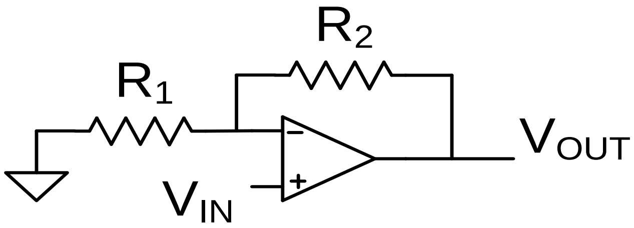
Determine the yield if the nominal gain is 10 $\pm 1\%$

Assume a common centroid layout of R_1 and R_2 has been used and the area of R_1 is $10u^2$ and both resistors have the same resistance density and R_2 is comprised of K-1 copies of R_1 . Neglect variable edge effects in the layout

A_ρ=.025µm

$$\sigma_{\frac{R_{PROC}}{R_{NOM}}} = 0.2$$

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the standard deviation of the voltage gain K

$$\sigma_K \simeq \frac{A_\rho}{\sqrt{A_{R1}}} \sqrt{K_N (K_N - 1)}$$

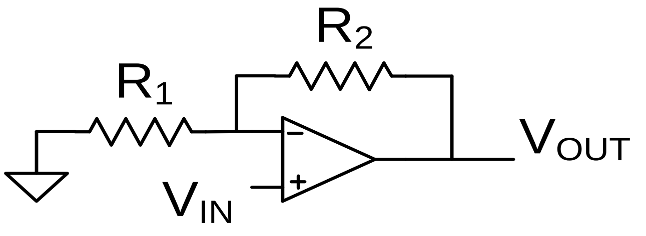
A_ρ=.025u $\quad A_{R1}$=10u^2

$$\sigma_{\frac{R_{PROC}}{R_{NOM}}} = 0.2$$

$$\sigma_K \simeq \frac{.025}{\sqrt{10}} \sqrt{K_N (K_N - 1)} = .0079 \sqrt{K_N (K_N - 1)}$$

$$\sigma_{\frac{K}{K_N}} \simeq .0079 \sqrt{1 - \frac{1}{K_N}}$$

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the standard deviation of the voltage gain K

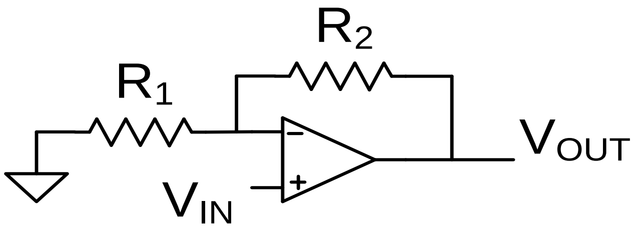
$$\sigma_{\frac{K}{K_N}} \simeq .0079\sqrt{1 - \frac{1}{K_N}}$$

Determine the yield if the nominal gain is 10 $\pm 1\%$

$$\sigma_{\frac{K}{K_N}} \simeq .0079\sqrt{1 - \frac{1}{10}} = .0075$$

$$\frac{K}{K_N} \sim N(1,\ 0.0075)$$

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the yield if the nominal gain is 10 $\pm 1\%$

$$\frac{K}{K_N} \sim N(1,\ 0.0075)$$

$$9.9 < K < 10.1$$

$$.99 < \frac{K}{K_N} < 1.01$$

$$-.01 < \frac{K}{K_N} - 1 < .01$$

$$\frac{\frac{K}{K_N} - 1}{0.0075} \sim N(0,1)$$

$$-1.33 < \frac{\frac{K}{K_N} - 1}{.00095} < 1.33$$

$$Y = 2F_{N(0,1)}(1.33) - 1 = 2*.9082 - 1 = 0.8164$$

Dramatic drop from 100% yield to about 82% yield!

End of Lecture 15