EE 508
Lecture 16

Filter Transformations

Lowpass to Bandpass
Lowpass to Highpass
Lowpass to Band-reject

Review from Last Time

Theorem: If the perimeter variations and contact resistance are neglected, the standard deviation of the local random variations of a resistor of area A is given by the expression

$$\sigma_{\frac{R}{R_N}} = \frac{A_\rho}{\sqrt{A}}$$

Theorem: If the perimeter variations are neglected, the standard deviation of the local random variations of a capacitor of area A is given by the expression

$$\sigma_{\frac{C}{C_N}} = \frac{A_C}{\sqrt{A}}$$

Theorem: If the perimeter variations are neglected, the variance of the local random variations of the normalized threshold voltage of a rectangular MOS transistor of dimensions W and L is given by the expression

$$\sigma^2_{\frac{V_T}{V_{T_N}}} = \frac{A^2_{VTO}}{V^2_{T_N} WL} \quad \text{or as} \quad \sigma^2_{\frac{V_T}{V_{T_N}}} = \frac{A^2_{VT}}{WL}$$

Review from Last Time

Theorem: If the perimeter variations are neglected, the variance of the local random variations of the normalized C_{OX} of a rectangular MOS transistor of dimensions W and L is given by the expression

$$\sigma^2_{\frac{C_{OX}}{C_{OXN}}} = \frac{A^2_{COX}}{WL}$$

Theorem: If the perimeter variations are neglected, the variance of the local random variations of the normalized mobility of a rectangular MOS transistor of dimensions W and L is given by the expression

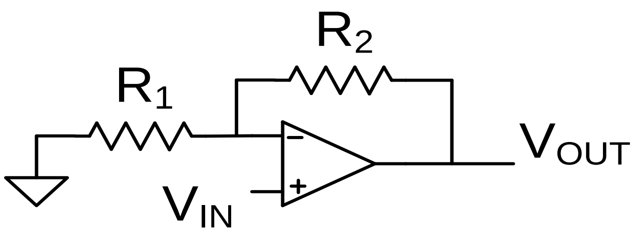
$$\sigma^2_{\frac{\mu_R}{\mu_N}} = \frac{A_\mu^2}{WL}$$

where the parameters A_X are all constants characteristic of the process (i.e. model parameters)

- The effects of edge roughness on the variance of resistors, capacitors, and transistors can readily be included but for most layouts is dominated by the area dependent variations
- There is some correlation between the model parameters of MOS transistors but they are often ignored to simplify calculations

Statistical Modeling of dimensionless parameters - example

$$K = 1 + \frac{R_2}{R_1}$$



Assume common centroid layout

area of R_1 is 100u^2 A_ρ=.01µm

Determine the yield if the nominal gain is 10 $\pm 1\%$

$$\frac{K}{K_N} \sim N(1,\ 0.00095)$$

$$\frac{\frac{K}{K_N} - 1}{0.00095} \sim N(0,1)$$

$$9.9 < K < 10.1$$

$$.99 < \frac{K}{K_N} < 1.01$$

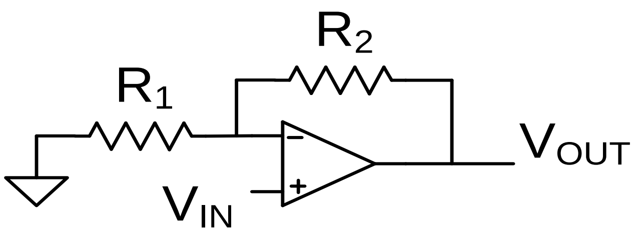
$$-.01 < \frac{K}{K_N} - 1 < .01$$

$$-10 < \frac{\frac{K}{K_N} - 1}{.00095} < 10$$

The gain yield is essentially 100%

Could substantially decrease area or increase gain accuracy if desired

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the yield if the nominal gain is 10 $\pm 1\%$

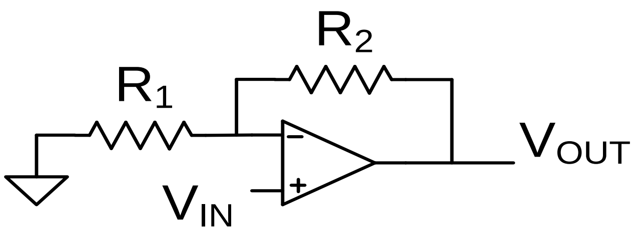
Assume a common centroid layout of R_1 and R_2 has been used and the area of R_1 is $10u^2$ and both resistors have the same resistance density and R_2 is comprised of K-1 copies of R_1 . Neglect variable edge effects in the layout

A_ρ=.025µm

$$\sigma_{\frac{R_{PROC}}{R_{NOM}}} = 0.2$$

Review from Last Time

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the standard deviation of the voltage gain K

$$\sigma_K \simeq \frac{A_\rho}{\sqrt{A_{R1}}}\sqrt{K_N(K_N - 1)}$$

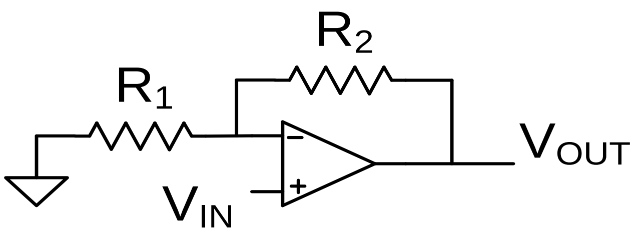
A_ρ=.025um A_{R1}=10um^2

$$\sigma_{\frac{R_{PROC}}{R_{NOM}}} = 0.2$$

$$\sigma_K \simeq \frac{.025}{\sqrt{10}}\sqrt{K_N(K_N - 1)} = .0079\sqrt{K_N(K_N - 1)}$$

$$\sigma_{\frac{K}{K_N}} \simeq .0079\sqrt{1 - \frac{1}{K_N}}$$

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the standard deviation of the voltage gain K

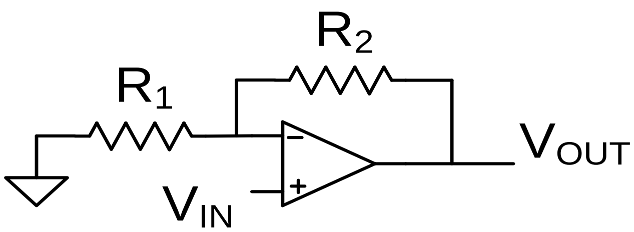
$$\sigma_{\frac{K}{K_N}} \simeq .0079\sqrt{1 - \frac{1}{K_N}}$$

Determine the yield if the nominal gain is $10 \pm 1\%$

$$\sigma_{\frac{K}{K_N}} \simeq .0079\sqrt{1 - \frac{1}{10}} = .0075$$

$$\frac{K}{K_N} \sim N(1,\ 0.0075)$$

Statistical Modeling of dimensionless parameters - example



$$K = 1 + \frac{R_2}{R_1}$$

Determine the yield if the nominal gain is 10 ±1%

$$\frac{K}{K_N} \sim N(1,\ 0.0075)$$

$$9.9 < K < 10.1$$

$$.99 < \frac{K}{K_N} < 1.01$$

$$-.01 < \frac{K}{K_N} - 1 < .01$$

$$\frac{\frac{K}{K_N} - 1}{0.0075} \sim N(0,1)$$

$$-1.33 < \frac{\frac{K}{K_N} - 1}{.0075} < 1.33$$

$$Y = 2F_{N(0,1)}(1.33) - 1 = 2{*}.9082 - 1 = 0.8164$$

Dramatic drop from 100% yield to about 82% yield!

Filter Transformations

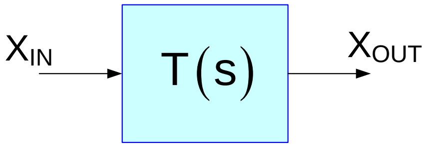
Lowpass to Bandpass **(LP to BP)**
Lowpass to Highpass **(LP to HP)**
Lowpass to Band-reject **(LP to BR)**

Approach will be to take advantage of the results obtained for the standard LP approximations

Will focus on flat passband and zero-gain stop-band transformations

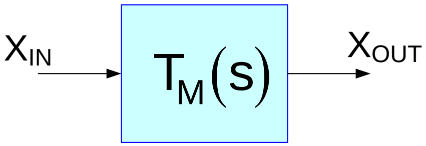
Will focus on transformations that map passband to passband and stopband to stopband

Filter Transformations



$$s \rightarrow f(s)$$

$$T_M(s) = T(f(s))$$

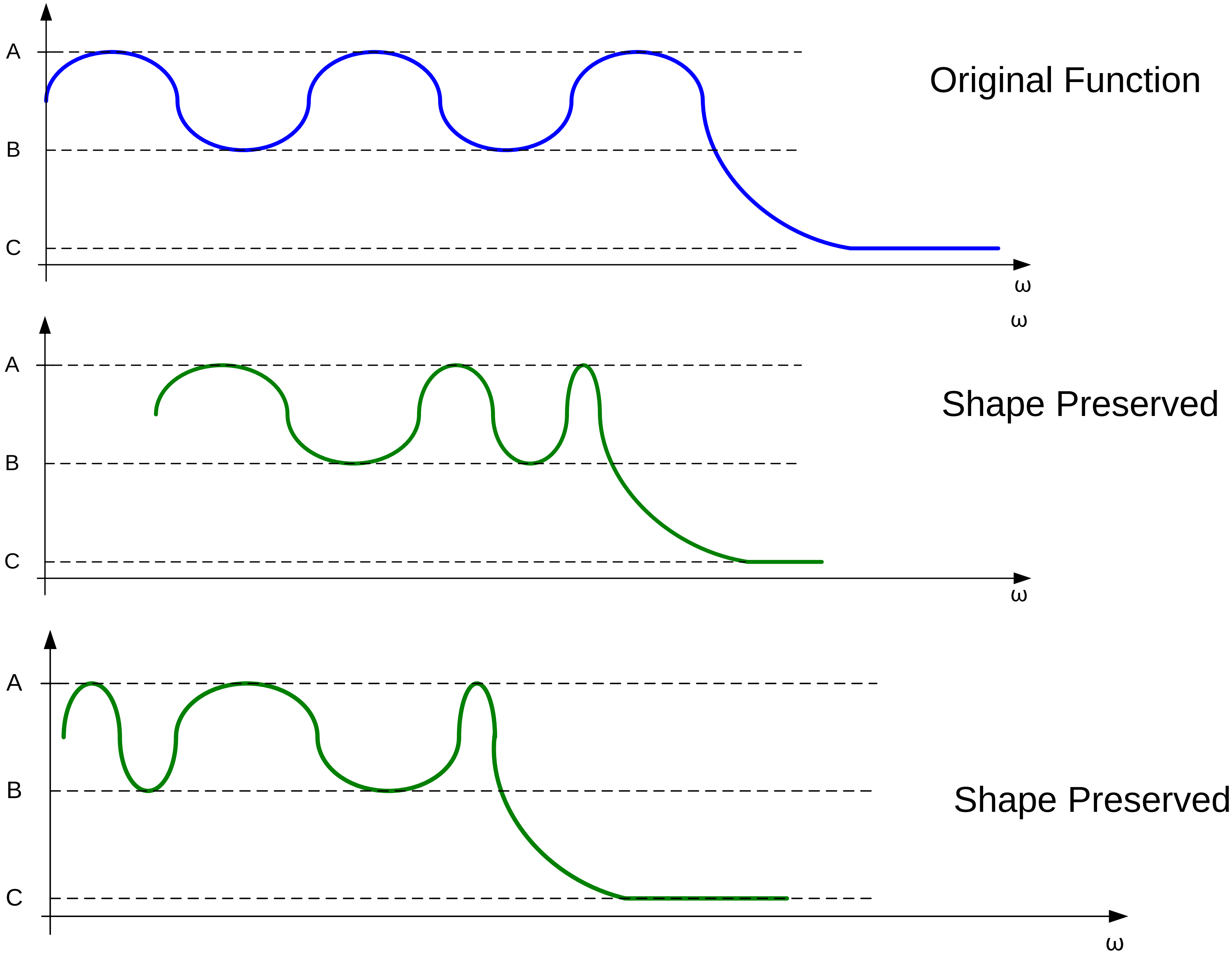


Claim:

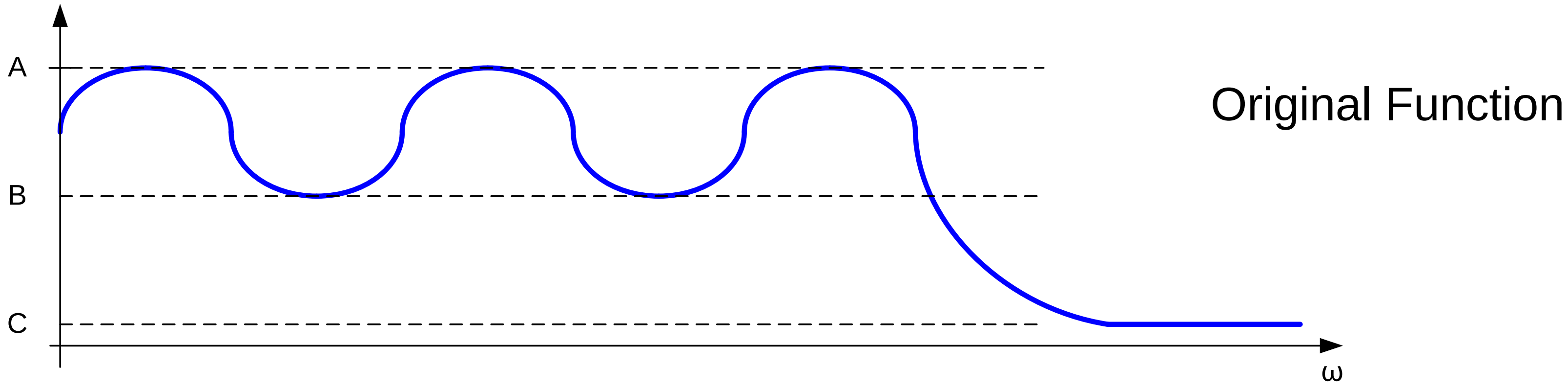
If the imaginary axis in the s-plane is mapped to the imaginary axis in the s-plane with a variable mapping function, the basic shape of the function T(s) will be preserved in the function F(T(s)) but the frequency axis may be warped and/or folded in the magnitude domain

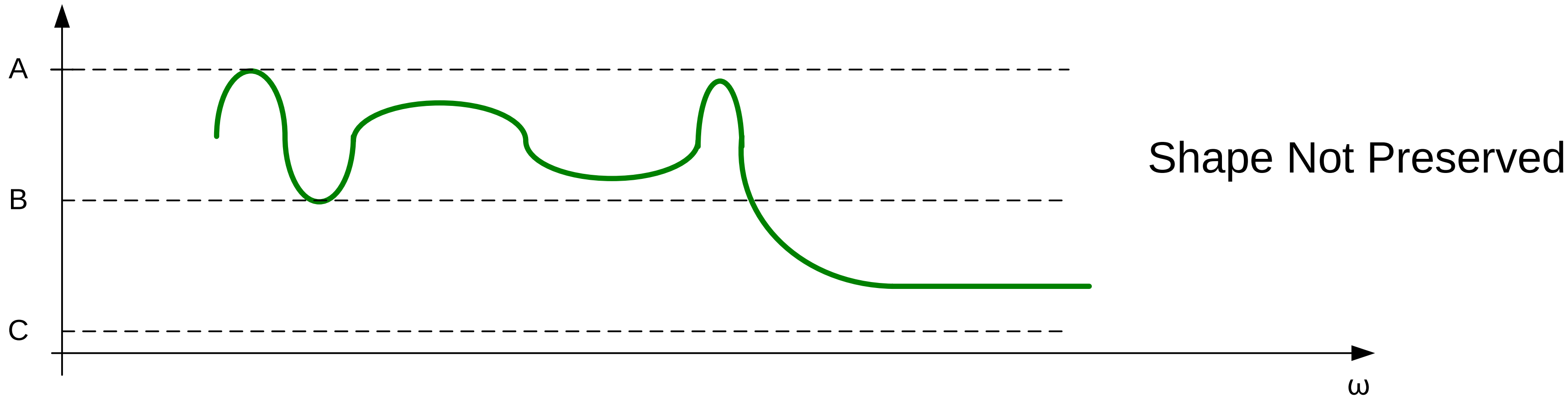
Preserving basic shape, in this context, constitutes maintaining features in the magnitude response of F(T(s)) that are in T(s) including, but not limited to, the peak amplitude, number of ripples, peaks of ripples,

Example: Shape Preservation

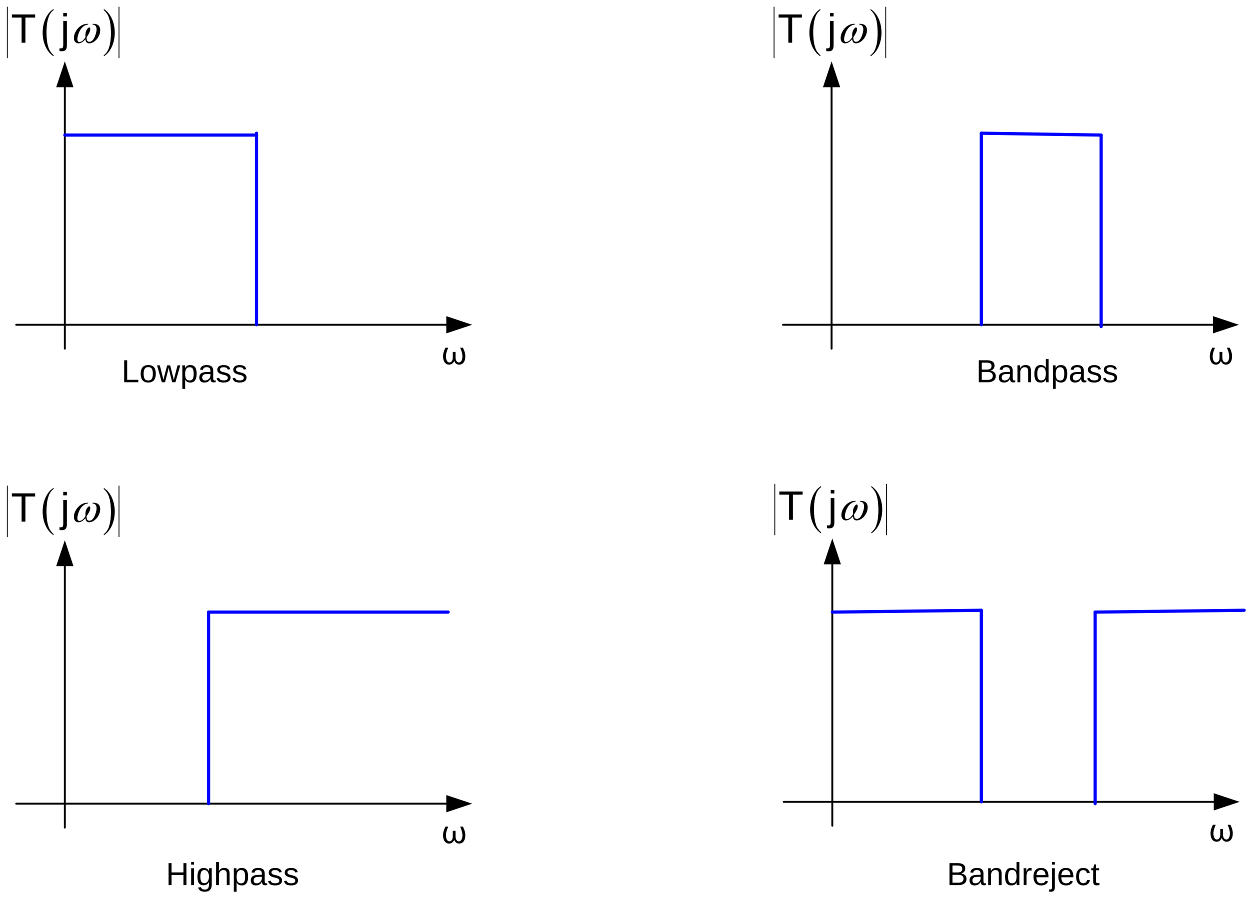


Example: Shape Preservation





Flat Passband/Stopband Filters



Filter Transformations

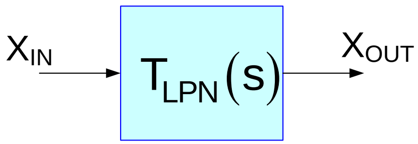
Lowpass to Bandpass **(LP to BP)**
Lowpass to Highpass **(LP to HP)**
Lowpass to Band-reject **(LP to BR)**

- Approach will be to take advantage of the results obtained for the standard LP approximations

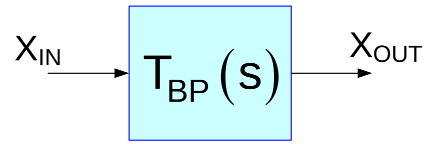
- Will focus on flat passband and zero-gain stop-band transformations

- Will focus on transformations that map passband to passband and stopband to stopband

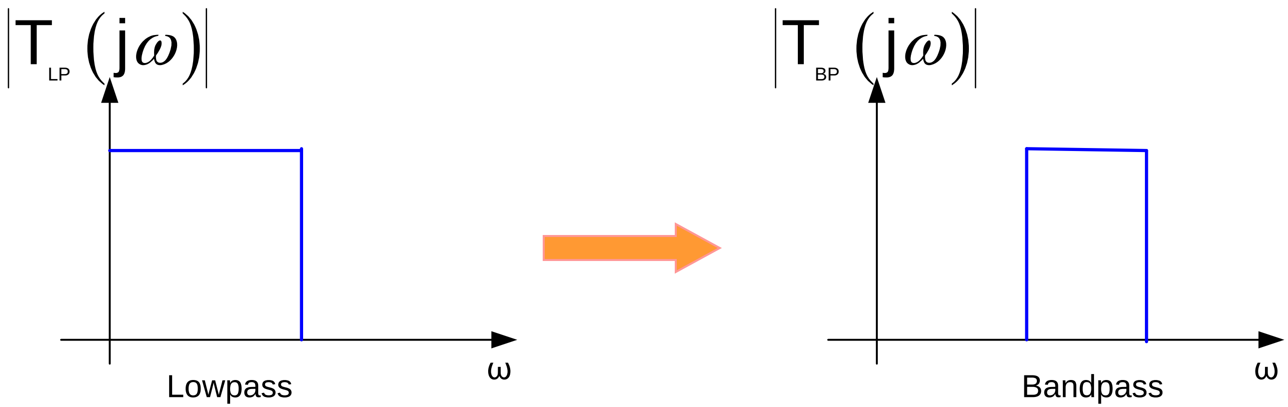
LP to BP Filter Transformations



$$s \rightarrow f(s)$$



$$T_{BP}(s) = T_{LPN}(f(s))$$



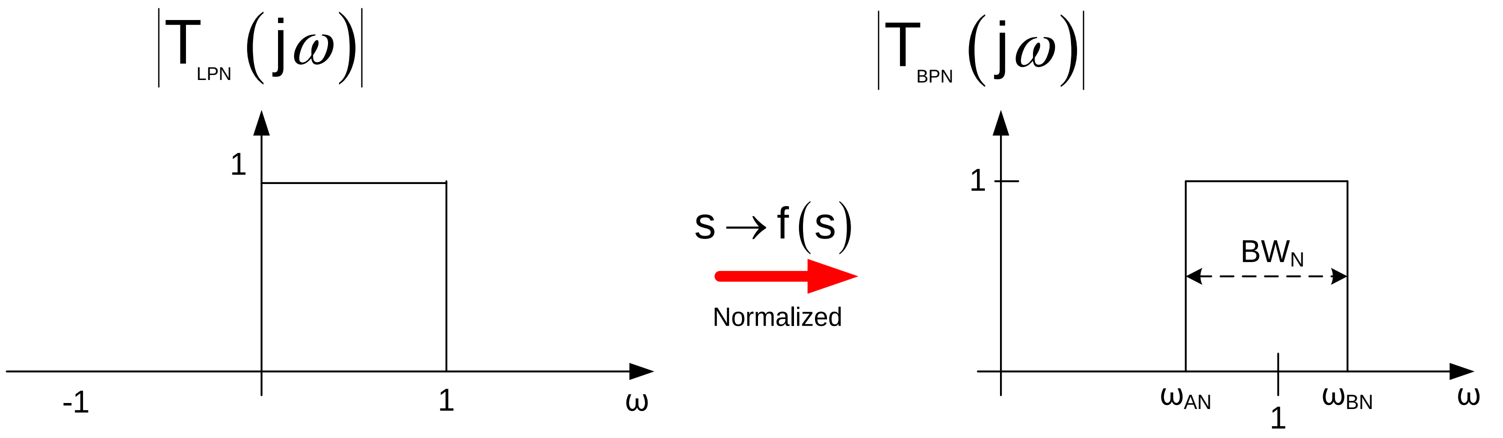
Will consider rational fraction mappings

$$f(s) = \frac{\sum_{i=0}^{m_T} a_{Ti} s^i}{\sum_{i=0}^{n_T} b_{Ti} s^i}$$

- Not all rational fraction mappings will map Im axis to the Im axis
- Not all rational fraction mappings will map passband to passband and stopband to stopband
- Consider only that subset of those mappings with these properties

LP to BP Transformation

Mapping Strategy: Consider first a mapping to a normalized BP approximation



$$BW_N = \omega_{BN} - \omega_{AN}$$

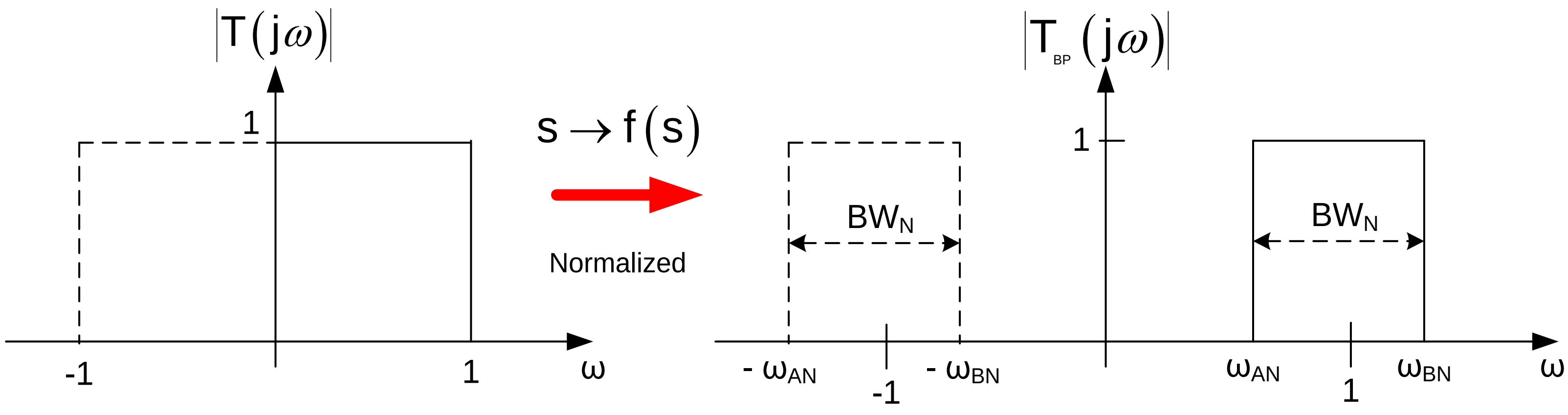
$$\sqrt{\omega_{AN}\omega_{BN}} = 1$$

LP to BP Transformation

Mapping Strategy: Consider first a mapping to a normalized BP approximation

A mapping from s → f(s) will map the entire imaginary axis in the frequency domain

Thus, must consider both positive and negative frequencies. Since $|T(j\omega)|$ is a function of ω^2, the magnitude response on the negative ω axis will be a mirror image of that on the positive ω axis

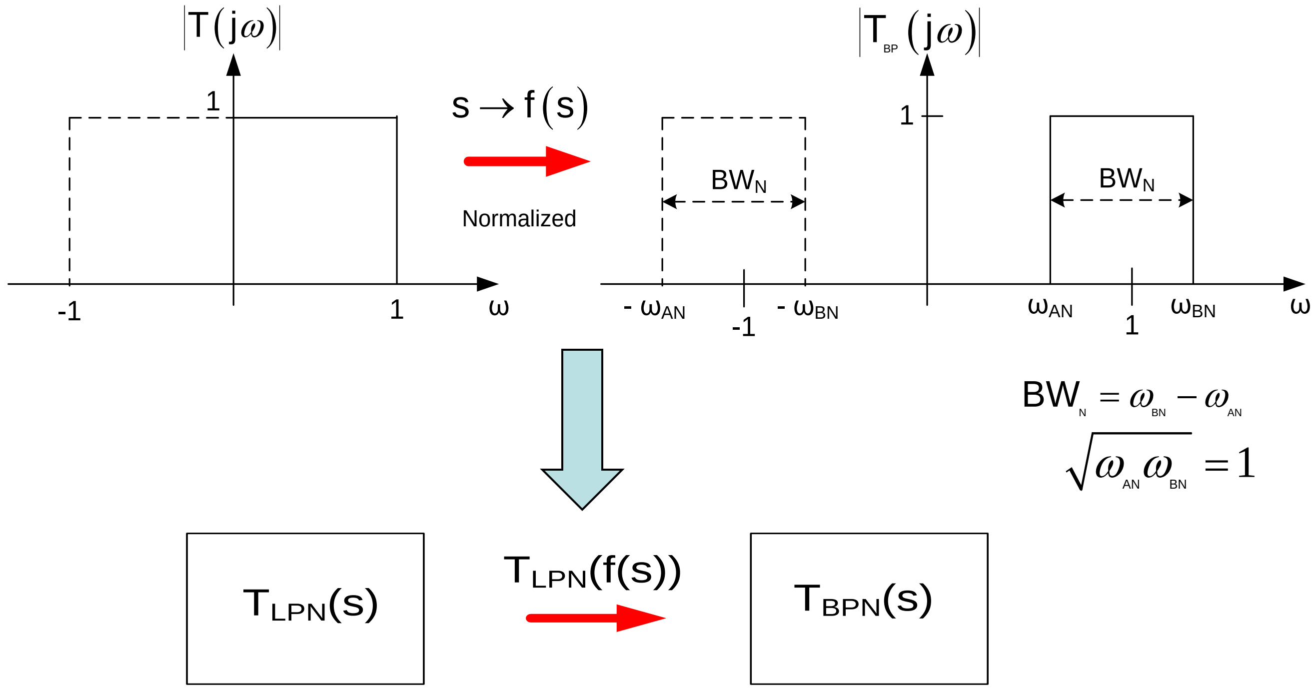


$$BW_N = \omega_{BN} - \omega_{AN}$$

$$\sqrt{\omega_{AN}\omega_{BN}} = 1$$

Standard LP to BP Transformation

Normalized LP to Normalized BP mapping Strategy:



$$BW_N = \omega_{BN} - \omega_{AN}$$

$$\sqrt{\omega_{AN}\omega_{BN}} = 1$$

Variable Mapping Strategy to Preserve Shape of LP function:

consider:

s-domain

map $s=j0$ to $s=j1$
map $s=j1$ to $s=j\omega_{BN}$
map $s=-j1$ to $s=j\omega_{AN}$

$T_{LPN}(f(s))$

ω-domain

map $\omega=0$ to $\omega=1$
map $\omega=1$ to $\omega=\omega_{BN}$
map $\omega=-1$ to $\omega=\omega_{AN}$

This mapping will introduce 3 constraints

Standard LP to BP Transformation

Mapping Strategy: $T_{LPN}(s)$ $\xrightarrow{T_{LPN}(f(s))}$ $T_{BPN}(s)$

s-domain

map s=0 to s= j1
map s=j1 to s=$j\omega_{BN}$
map s= –j1 to s= $j\omega_{AN}$

$T_{LPN}(f(s))$

ω-domain

map ω=0 to ω=1
map ω=1 to ω=ω_{BN}
map ω= –1 to ω=ω_{AN}

Consider variable mapping

$$f(s) = \frac{a_{T2}s^2 + a_{T1}s + a_{T0}}{b_{T1}s + b_{T0}}$$

With this mapping, there are 5 D.O.F and 3 mathematical constraints and the additional constraints that the Im axis maps to the Im axis and maps PB to PB and SB to SB

Will now show that the following mapping will meet these constraints

$$f(s) = \frac{s^2+1}{s \bullet BW_N} \qquad \text{or equivalently} \qquad s \rightarrow \frac{s^2+1}{s \bullet BW_N}$$

This is the lowest-order mapping that will meet these constraints and it doubles the order of the approximation

Standard LP to BP Transformation

s-domain

map s=0 to s= j1
map s=j1 to s=$j\omega_{BN}$
map s= –j1 to s= $j\omega_{AN}$

$T_{LPN}(f(s))$ ⟷

ω-domain

map ω=0 to ω=1
map ω=1 to ω=ω_{BN}
map ω= –1 to ω=ω_{AN}

Verification of mapping Strategy:

$$s \rightarrow \frac{s^2+1}{s \bullet BW_N}$$

$$\left.\frac{s^2+1}{s \bullet BW_N}\right|_{j1} = 0 \qquad \Rightarrow \qquad 0 \rightarrow j1$$

$$\left.\frac{s^2+1}{s \bullet BW_N}\right|_{j\omega_{BN}} = \frac{1-\omega_{BN}^2}{j\omega_{BN}\left(\omega_{BN}-\omega_{AN}\right)} = j\frac{\omega_{BN}^2-1}{\omega_{BN}^2-\omega_{AN}\omega_{BN}} = j\frac{\omega_{BN}^2-1}{\omega_{BN}^2-1} = j \qquad \Rightarrow \qquad j1 \rightarrow j\omega_{BN}$$

$$\left.\frac{s^2+1}{s \bullet BW_N}\right|_{j\omega_{AN}} = \frac{1-\omega_{AN}^2}{j\omega_{AN}\left(\omega_{BN}-\omega_{AN}\right)} = j\frac{\omega_{AN}^2-1}{\omega_{AN}\omega_{BN}-\omega_{AN}^2} = j\frac{\omega_{AN}^2-1}{1-\omega_{AN}^2} = -j \qquad \Rightarrow \qquad -j1 \rightarrow j\omega_{AN}$$

Must still show that the Im axis maps to the Im axis and maps PB to PB and SB to SB

Standard LP to BP Transformation

s-domain

map $s=0$ to $s=j1$
map $s=j1$ to $s=j\omega_{BN}$
map $s=-j1$ to $s=j\omega_{AN}$

$T_{LPN}(f(s))$

ω-domain

map $\omega=0$ to $\omega=1$
map $\omega=1$ to $\omega=\omega_{BN}$
map $\omega=-1$ to $\omega=\omega_{AN}$

Verification of mapping Strategy:

$$s \rightarrow \frac{s^2+1}{s \bullet BW_N}$$

Image of Im axis:

$$j\omega = \frac{s^2+1}{s \bullet BW_N}$$

solving for s, obtain

$$s = \frac{j\omega \bullet BW_N \pm \sqrt{(BW_N \bullet j\omega)^2 - 4}}{2} = j\left(\frac{\omega \bullet BW_N \pm \sqrt{(BW_N \bullet \omega)^2 + 4}}{2}\right)$$

this has no real part so the imaginary axis maps to the imaginary axis

Can readily show this mapping maps PB to PB and SB to SB

The mapping $s \rightarrow \dfrac{s^2+1}{s \bullet BW_N}$ is termed the standard LP to BP transformation

Standard LP to BP Transformation

The standard LP to BP transformation $$s \rightarrow \frac{s^2+1}{s \bullet BW_N}$$

If we add a subscript to the LP variable for notational convenience, can express this mapping as

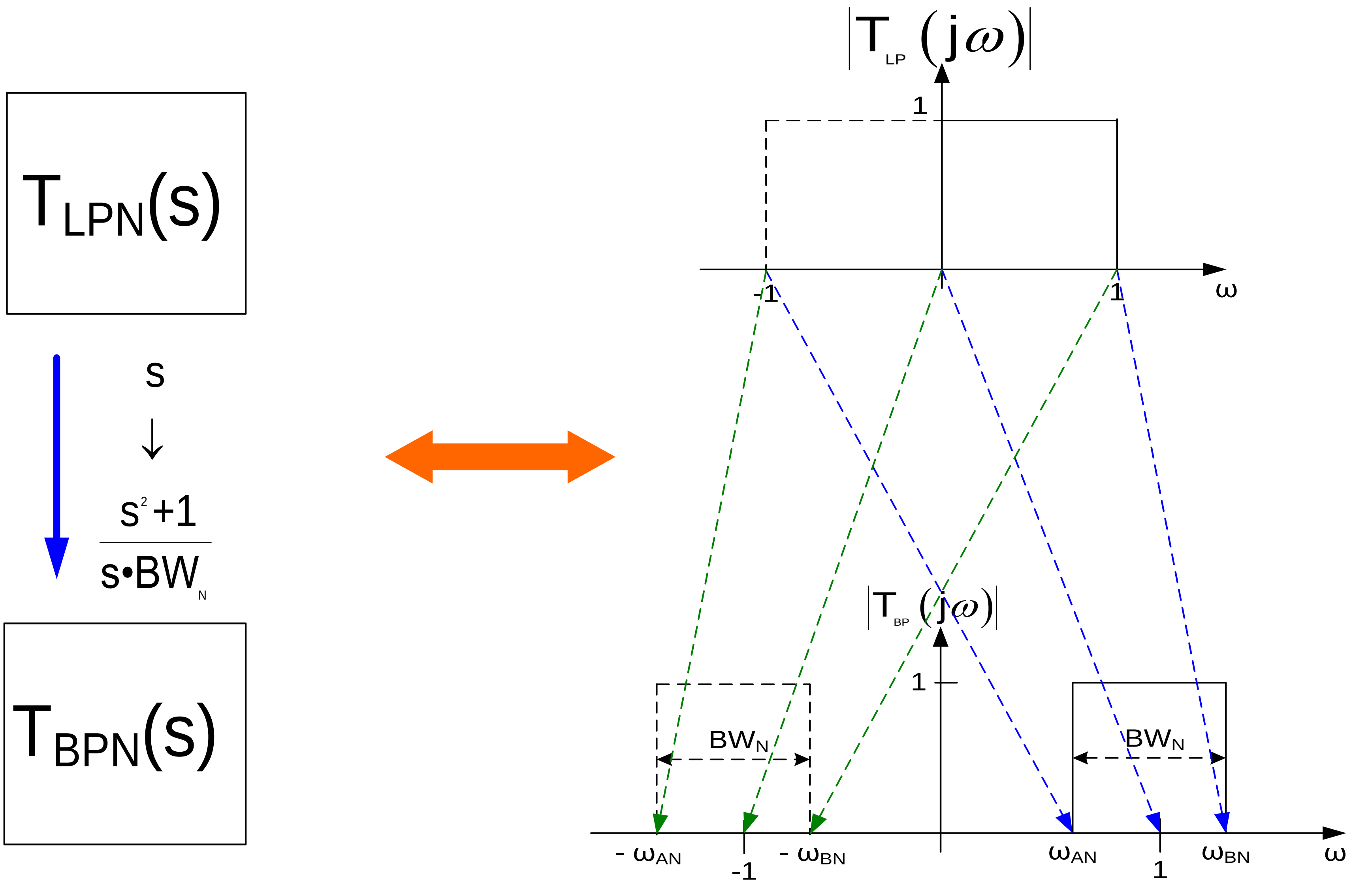
$$s_x = \frac{s^2+1}{s \bullet BW_N}$$

Question: Is this mapping dimensionally consistent ?

- The dimensions of the constant “1” in the numerator must be set so that this is dimensionally consistent

- The dimensions of BW_N must be set so that this is dimensionally consistent

Standard LP to BP Transformation



Standard LP to BP Transformation

Frequency and s-domain Mappings

(subscript variable in LP approximation for notational convenience)

$T_{LPN}(s_X)$

$$s_X \downarrow \frac{s^2+1}{s \bullet BW_N}$$

$T_{BPN}(s)$

$$s_X \rightarrow \frac{s^2+1}{s \bullet BW_N}$$

$$\omega_X \rightarrow \frac{\omega^2-1}{\omega \bullet BW_N}$$

solving for s or ω

$$s \leftarrow \frac{s_X \bullet BW_N \pm \sqrt{\left(BW_N \bullet s_X\right)^2 - 4}}{2}$$

$$\omega \leftarrow \frac{\omega_X \bullet BW_N \pm \sqrt{\left(BW_N \bullet \omega_X\right)^2 + 4}}{2}$$

Exercise: Resolve the dimensional consistency in the last equation

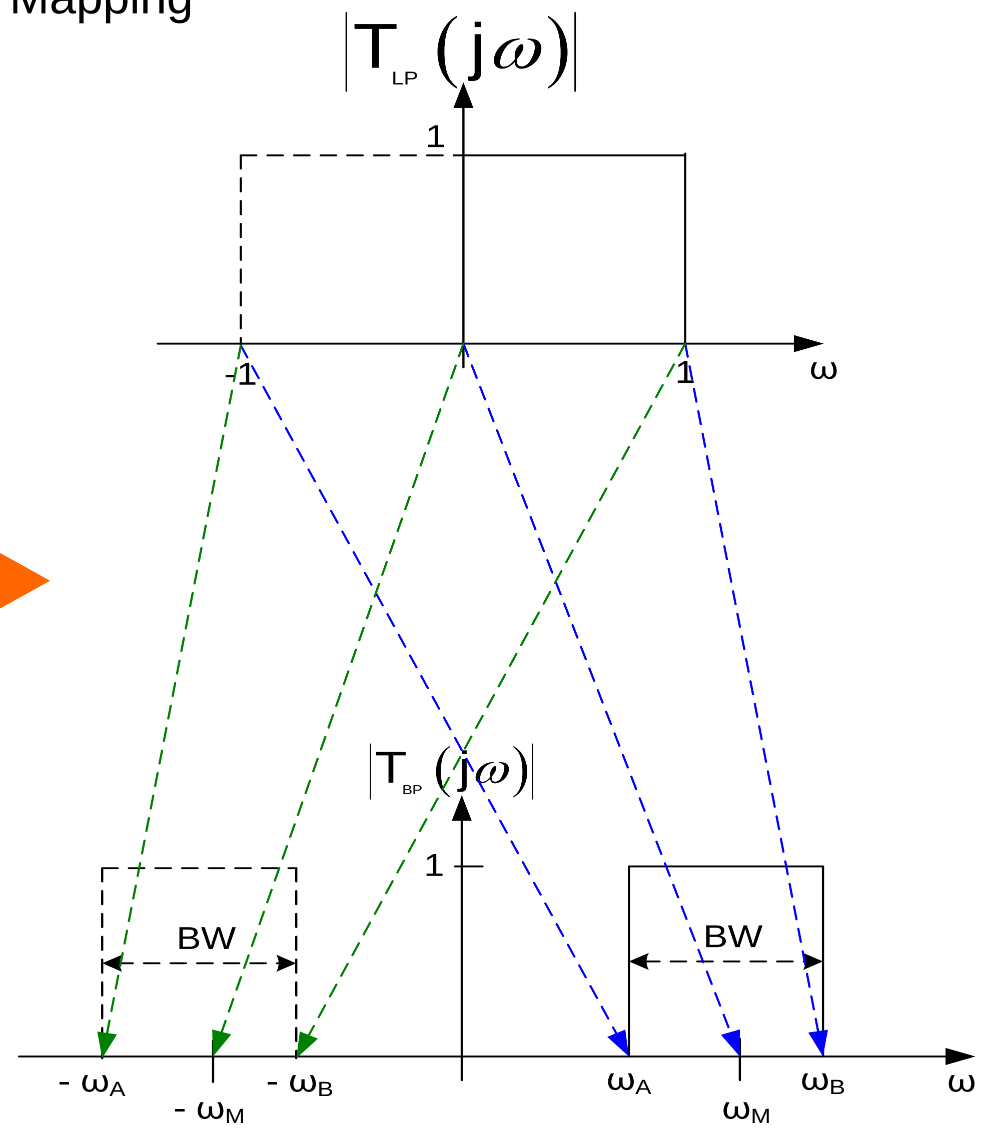
Standard LP to BP Transformation

Denormalized Mapping

$T_{LPN}(s)$

$$s \rightarrow \frac{s^2+\omega_M^2}{s \cdot BW}$$

$T_{BP}(s)$



Standard LP to BP Transformation

Frequency and s-domain Mappings - Denormalized

(subscript variable in LP approximation for notational convenience)

$$T_{LPN}(s_X)$$

$$s_X \downarrow \frac{s^2+\omega_M^2}{s \bullet BW}$$

$$T_{BP}(s)$$

$$s_X \rightarrow \frac{s^2+\omega_M^2}{s \bullet BW}$$

$$\omega_X \rightarrow \frac{\omega^2-\omega_M^2}{\omega \bullet BW}$$

$$s \leftarrow \frac{s_X \bullet BW \pm \sqrt{(BW \bullet s_X)^2 - 4\omega_M^2}}{2}$$

$$\omega \leftarrow \frac{\omega_X \bullet BW \pm \sqrt{(BW \bullet \omega_X)^2 + 4\omega_M^2}}{2}$$

Exercise: Resolve the dimensional consistency in the last equation

Standard LP to BP Transformation

Frequency and s-domain Mappings - Denormalized

(subscript variable in LP approximation for notational convenience)

Approach 1:

$$T_{LPN}(s_X) \xrightarrow{s_N \rightarrow \frac{s}{W_N}} T_{LP}(s) \xrightarrow{s \rightarrow \frac{s^2+1}{s \bullet BW_N}} T_{BP}(s)$$

Approach 2:

$$T_{LPN}(s_X) \xrightarrow{s_X \rightarrow \frac{s_N^2+1}{s_N \bullet BW_N}} T_{BPN}(s_N) \xrightarrow{s_N \rightarrow \frac{s}{W_N}} T_{BP}(s)$$

Approach 3:

$$T_{LPN}(s_X) \xrightarrow{s_X \rightarrow \frac{s^2+\omega_M^2}{s \bullet BW}} T_{BP}(s)$$

All three approaches give same approximation

Which is most practical to use? Often none of them !

Standard LP to BP Transformation

Frequency and s-domain Mappings - Denormalized

(subscript variable in LP approximation for notational convenience)

$$T_{LPN}(s_X)$$

$$s_X \rightarrow \frac{s_N^2+1}{s_N \bullet BW_N}$$

$$T_{BPN}(s_N)$$

Often most practical to synthesize directly from the T_{BPN} and then do the frequency scaling of components at the circuit level rather than at the approximation level

Standard LP to BP Transformation

Frequency and s-domain Mappings

(subscript variable in LP approximation for notational convenience)

Poles and Zeros of the BP approximations

$$s_X \xrightarrow{f} \frac{s^2+1}{s \bullet BW_N} \quad \xrightarrow{\text{solving for s}} \quad s \xleftarrow{f^{-1}} \frac{s_X \bullet BW_N \pm \sqrt{\left(BW_N \bullet s_X\right)^2 - 4}}{2}$$

$$T_{BP}(s) = T_{LPN}(f(s))$$

$$T_{LPN}(p_X) = 0$$

$$T_{LPN}(f(p)) = 0$$

$$T_{BP}(p) = T_{LPN}(f(p)) = 0$$

Since this relationship maps the complex plane to the complex plane, it also maps the poles and zeros of the LP approximation to the poles and zeros of the BP approximation

Standard LP to BP Transformation

Pole Mappings

Claim: With a variable mapping transform, the variable mapping naturally defines the mapping of the poles of the transformed function

$$T_{LPN}(s_X)$$

$$s_X \downarrow \frac{s^2+1}{s \bullet BW_N}$$

$$T_{BPN}(s)$$

$$p_X \rightarrow \frac{p^2+1}{p \bullet BW_N}$$

$$p \leftarrow \frac{p_X \bullet BW_N \pm \sqrt{\left(BW_N \bullet p_X\right)^2 - 4}}{2}$$

Exercise: Resolve the dimensional consistency in the last equation

Standard LP to BP Transformation

Pole Mappings

$$p \leftarrow \frac{p_X \bullet BW_N \pm \sqrt{\left(BW_N \bullet p_X\right)^2 - 4}}{2}$$

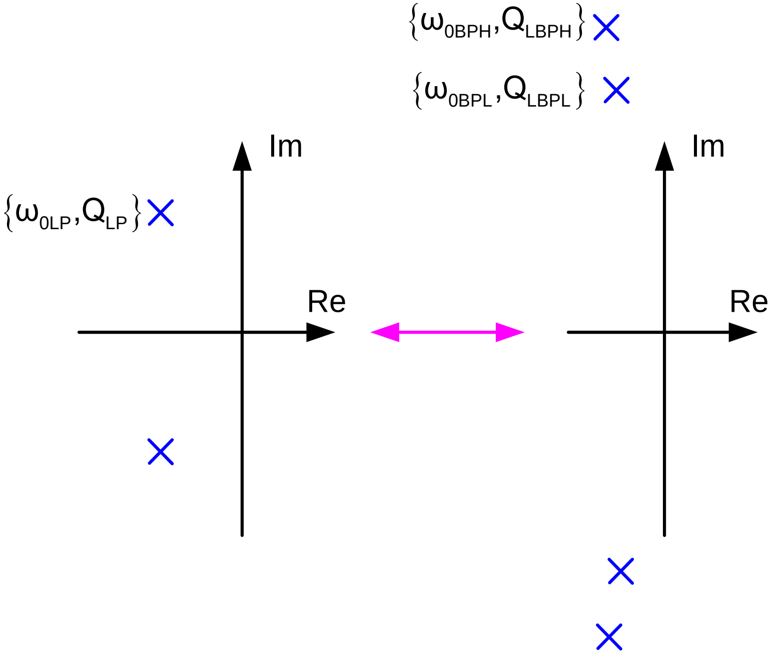
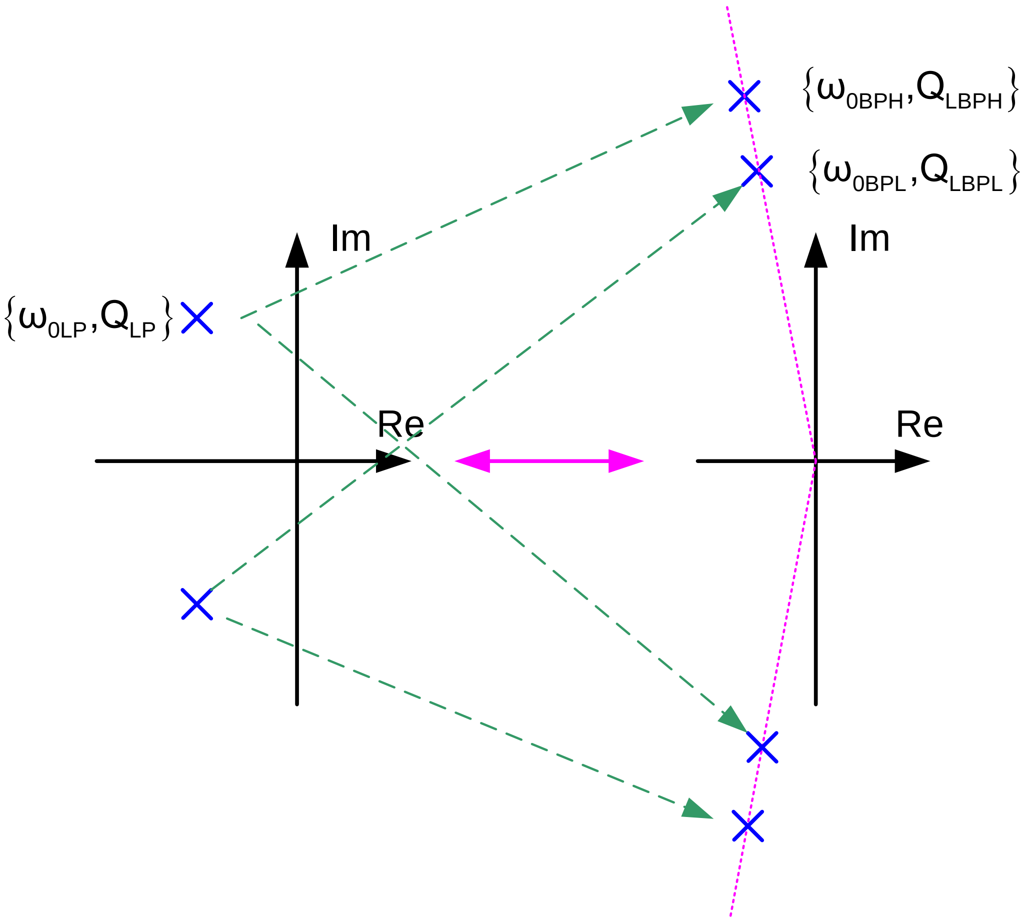


Image of the cc pole pair is the two pairs of poles

Standard LP to BP Transformation

Pole Mappings

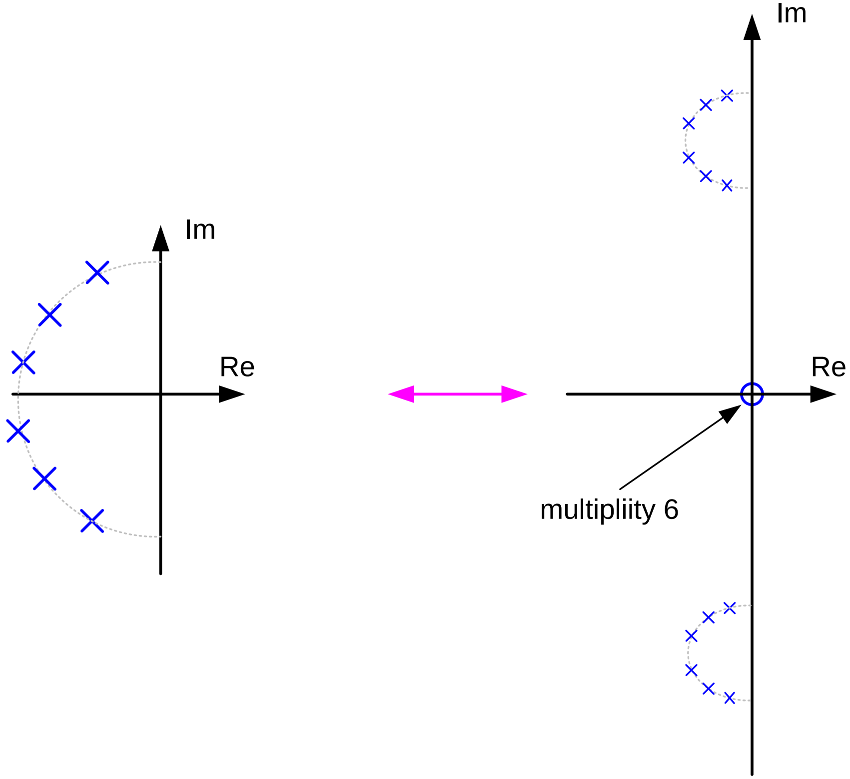


Can show that the upper hp pole maps to one upper hp pole and one lower hp pole as shown. Corresponding mapping of the lower hp pole is also shown

Standard LP to BP Transformation

Pole Mappings

$$p \leftarrow \frac{p_X \bullet BW_N \pm \sqrt{\left(BW_N \bullet p_X\right)^2 - 4}}{2}$$



Note doubling of poles, addition of zeros, and likely Q enhancement

End of Lecture 16