EE 508
Lecture 18

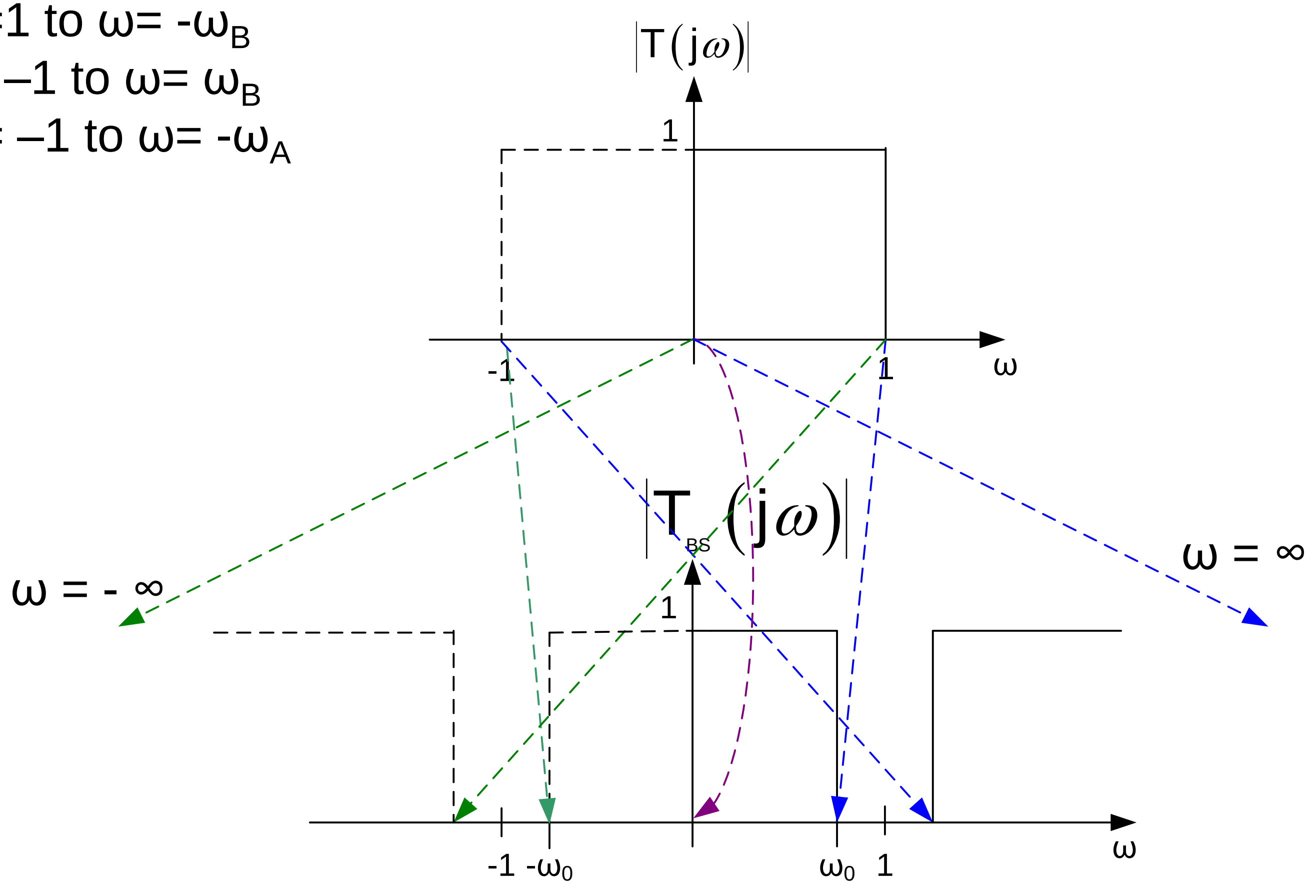
Basic Biquadratic Active Filters

Second-order Bandpass
Second-order Lowpass
Effects of Op Amp on Filter Performance

Review from Last Time

Standard LP to BS Transformation

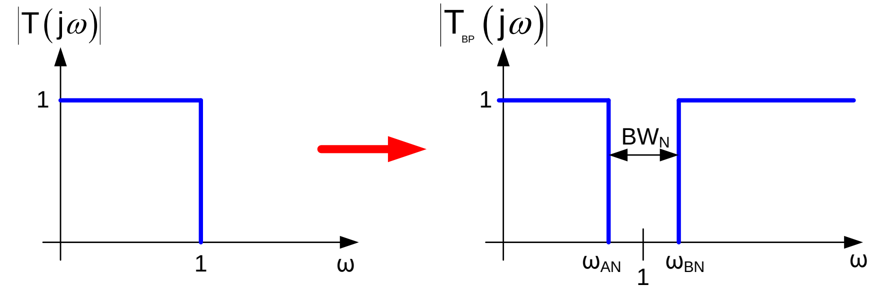
map ω=0 to $\omega = \pm\infty$
map ω=0 to ω = 0
map ω=1 to $\omega = \omega_A$
map ω=1 to $\omega = -\omega_B$
map ω= –1 to $\omega = \omega_B$
map ω= –1 to $\omega = -\omega_A$

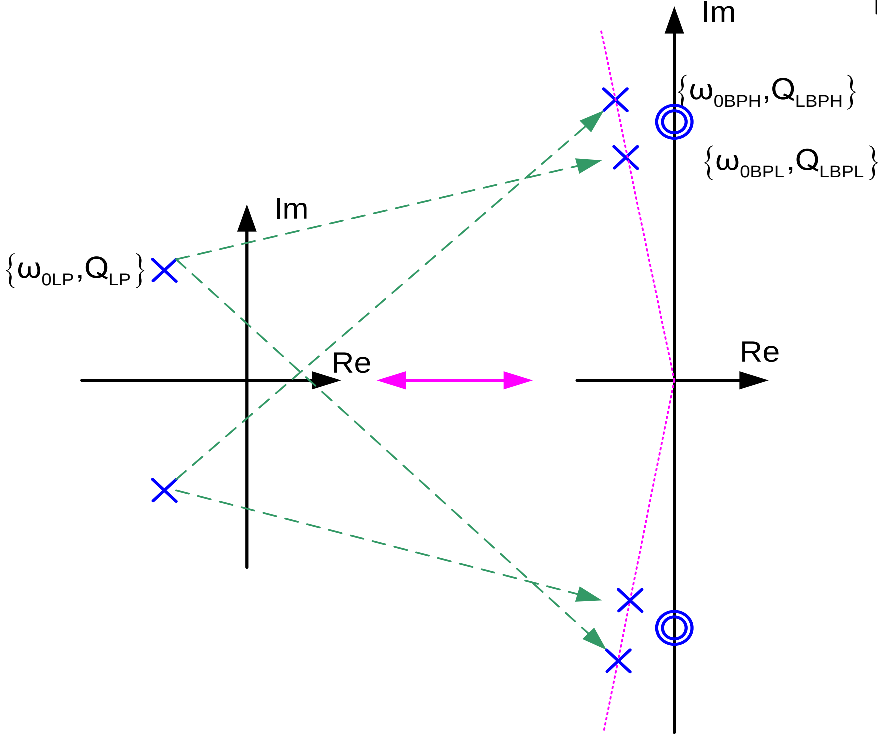


Review from Last Time

LP to BS Transformation

Pole Q of BS Approximations





$$BW = \omega_{BN} - \omega_{AN}$$

$$\omega_M = \sqrt{\omega_{AN}\omega_{BN}}$$

Define: $$\gamma = \left(\frac{BW}{\omega_M \omega_{0LP}}\right)$$

It can be shown that

$$Q_{BSL} = Q_{BSH} = \frac{Q_{LP}}{\sqrt{2}}\sqrt{1+\frac{4}{\gamma^2}+\sqrt{\left(1+\frac{4}{\gamma^2}\right)^2-\frac{4}{\gamma^2 Q_{LP}^2}}}$$

For γ small, $$Q_{BS} \simeq \frac{2Q_{LP}}{\gamma}$$

It can be shown that

$$\omega_{0BS} = \frac{\omega_M}{2}\left[\gamma\frac{Q_{BS}}{Q_{LP}} \pm \sqrt{\left(\gamma\frac{Q_{BS}}{Q_{LP}}\right)^2-4}\right]$$

Note for γ **small, Q_{BS} can get very large**

Standard LP to BS Transformation

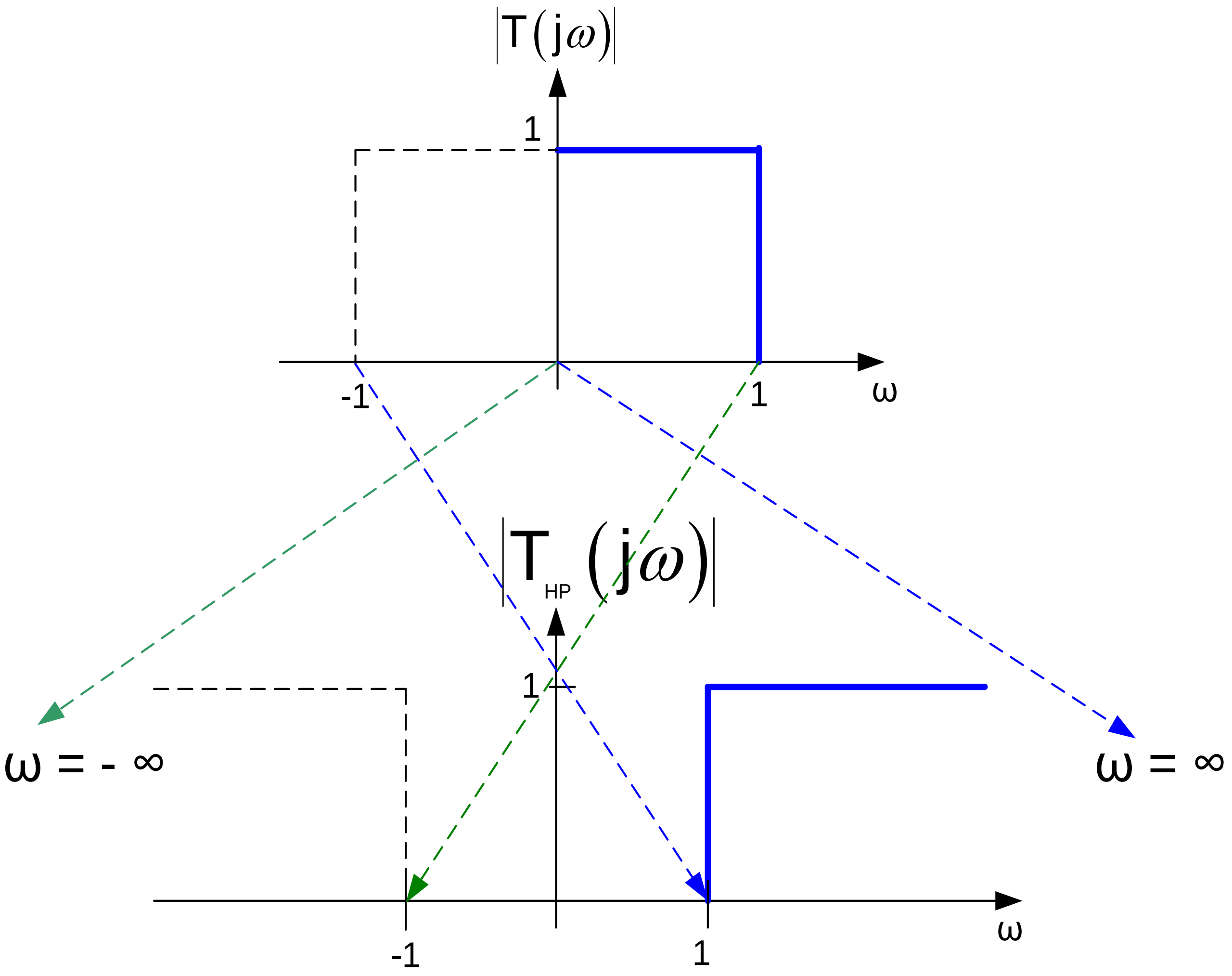
$$s_X \rightarrow \frac{s \bullet BW}{s^2 + \omega_M^2}$$

- Standard LP to BS transformation is a variable mapping transform
- Maps jω axis to jω axis in the s-plane
- Preserves basic shape of an approximation but warps frequency axis
- Order of BS approximation is double that of the LP Approximation
- Pole Q and ω_0 expressions are identical to those of the LP to BP transformation
- Pole Q of BS approximation can get very large for narrow BW
- Other variable transforms exist but the standard is by far the most popular

Review from Last Time

LP to HP Transformation

(Normalized Transform)

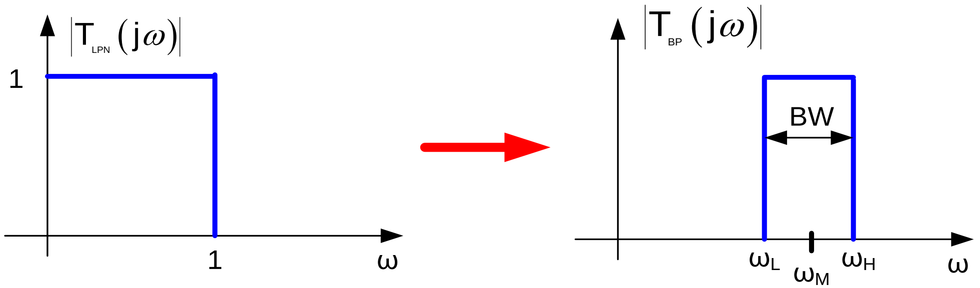


Review from Last Time

Comparison of Transforms

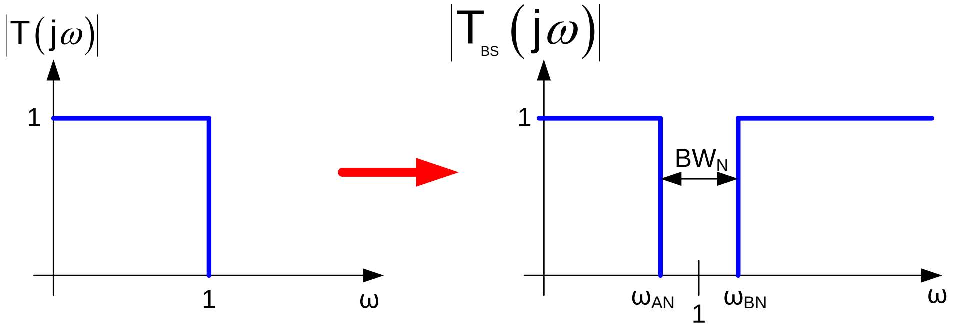
LP to BP

$$s \rightarrow \frac{s^2+1}{s \bullet BW_N}$$



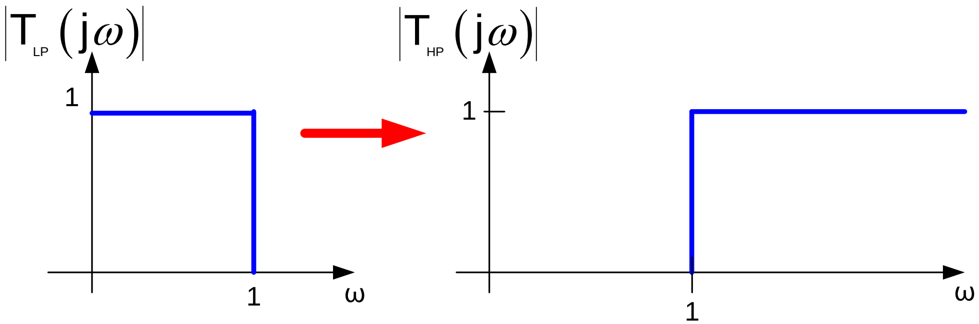
LP to BS

$$s \rightarrow \frac{s \bullet BW_N}{s^2+1}$$

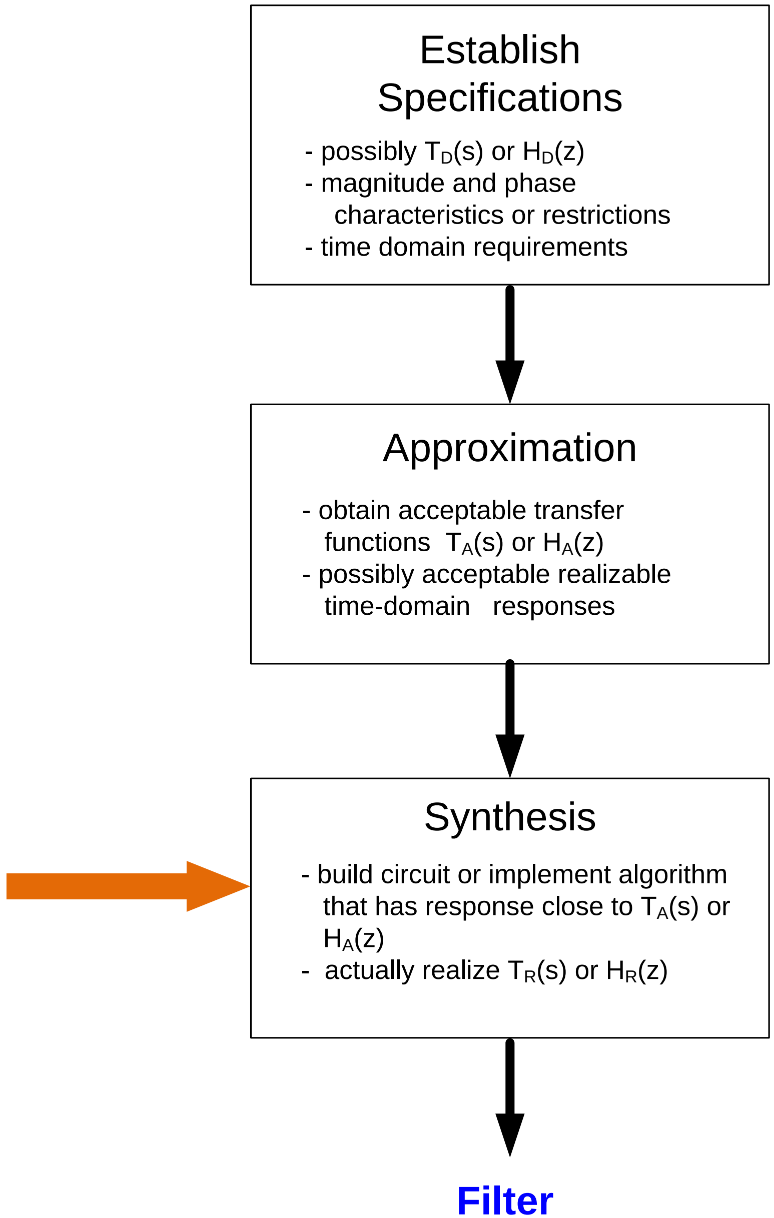


LP to HP

$$s \rightarrow \frac{1}{s}$$



Filter Design Process



Filter Design/Synthesis Considerations

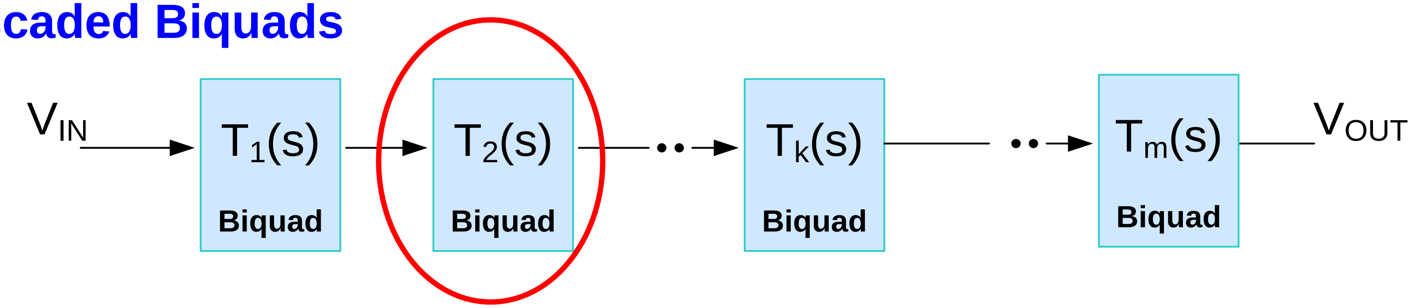
There are many different filter architectures that can realize a given transfer function

Considerable effort has been focused over the years on “inventing” these architectures and on determining which is best suited for a given application

Filter Design/Synthesis Considerations

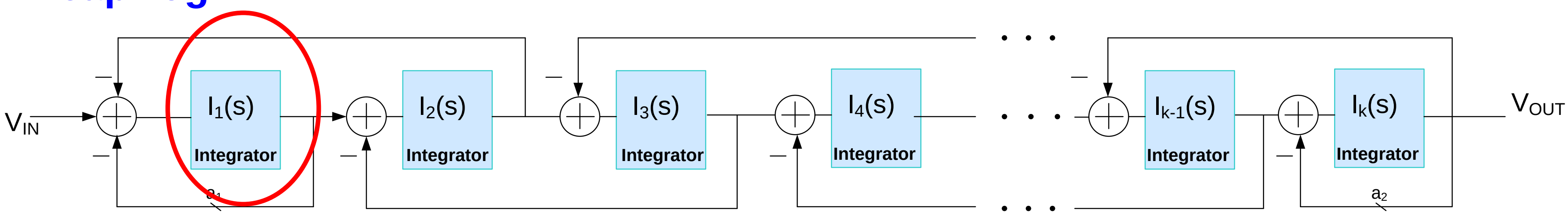
Most even-ordered designs today use one of the following three basic architectures

Cascaded Biquads

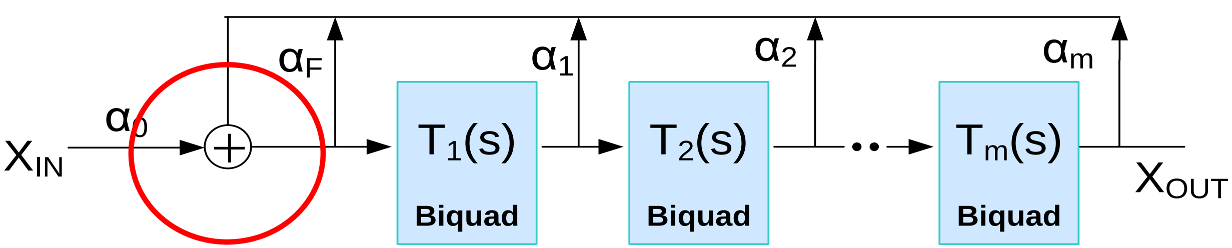


$$T(s) = T_1 T_2 \bullet\bullet T_m$$

Leapfrog



Multiple-loop Feedback (less popular)

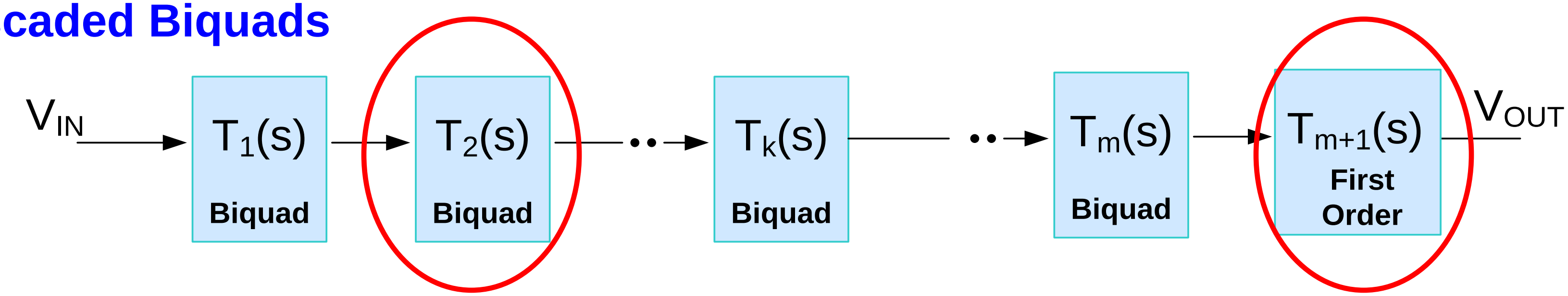


What's unique in all of these approaches?

Filter Design/Synthesis Considerations

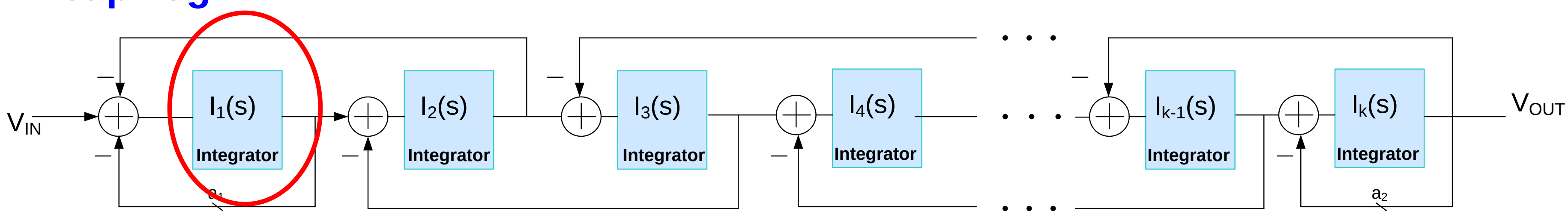
Most odd-ordered designs today use one of the following three basic architectures

Cascaded Biquads

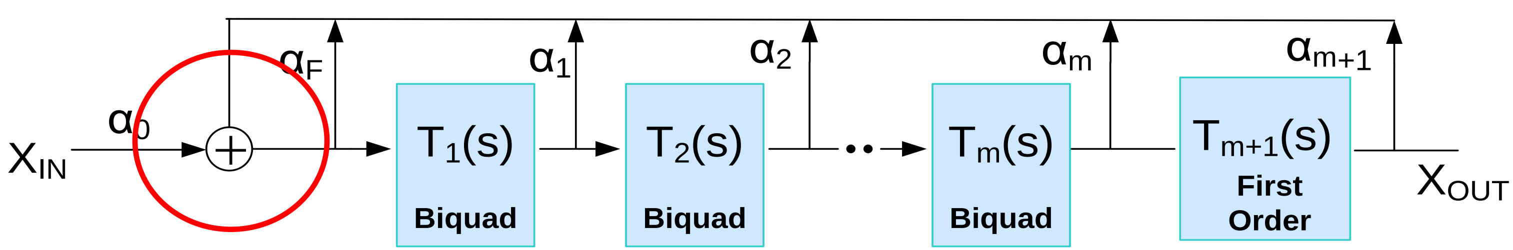


$$T(s) = T_1 T_2 \bullet\bullet T_m$$

Leapfrog



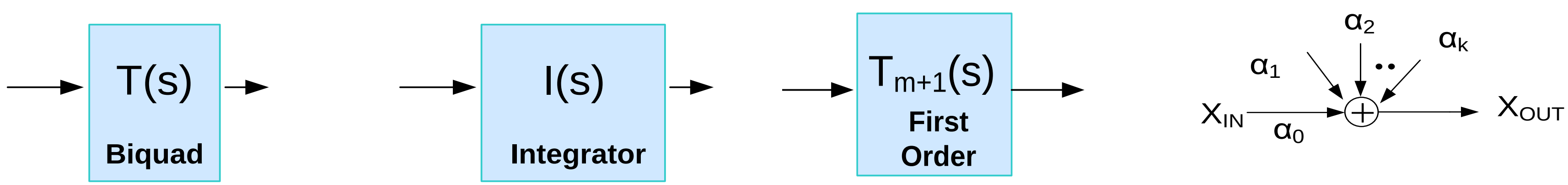
Multiple-loop Feedback (less popular)



What's unique in all of these approaches?

Filter Design/Synthesis Considerations

What's unique in all of these approaches?



$$T(s) = \frac{a_2s^2+a_1s+a_0}{s^2+b_1s+b_0}$$

$$I(s) = \frac{I_0}{s}$$

$$T(s) = \frac{a_1s+a_0}{s+b_0}$$

$$X_{OUT} = \sum_{i=0}^{k} \alpha_i$$

- Most effort on synthesis can focus on synthesizing these four blocks

  (the summing function is often incorporated into the Biquad or Integrator)

  (the first-order block is much less challenging to design than the biquad)

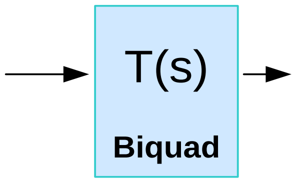
- Some issues associated with their interconnections

- And, in many integrated structures, the biquads are made with integrators

  (thus, much filter design work simply focuses on the design of integrators)

Biquads

How many biquad filter functions are there?



$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0}$$

$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0} \qquad a_0 \neq 0,\ a_1 \neq 0,\ a_2 \neq 0$$

$$T(s) = \frac{a_0}{s^2 + b_1 s + b_0} \qquad a_0 \neq 0$$

$$T(s) = \frac{a_2 s^2 + a_0}{s^2 + b_1 s + b_0} \qquad a_0 \neq 0,\ a_2 \neq 0$$

$$T(s) = \frac{a_1 s}{s^2 + b_1 s + b_0} \qquad a_1 \neq 0$$

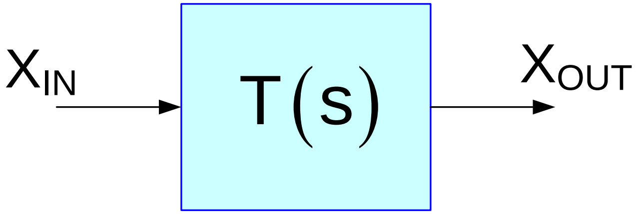
$$T(s) = \frac{a_1 s + a_0}{s^2 + b_1 s + b_0} \qquad a_0 \neq 0,\ a_1 \neq 0$$

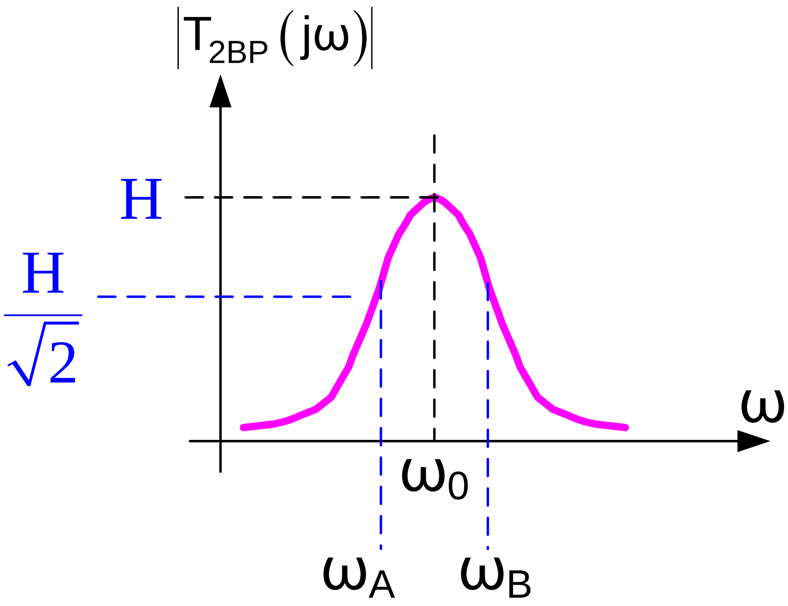
$$T(s) = \frac{a_2 s^2}{s^2 + b_1 s + b_0} \qquad a_2 \neq 0$$

$$T(s) = \frac{a_2 s^2 + a_1 s}{s^2 + b_1 s + b_0} \qquad a_2 \neq 0,\ a_1 \neq 0$$

Filter Design/Synthesis Considerations

Review: Second-order bandpass transfer function





$$|T_{2BP}(s)| = H\frac{s\left(\frac{\omega_0}{Q}\right)}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

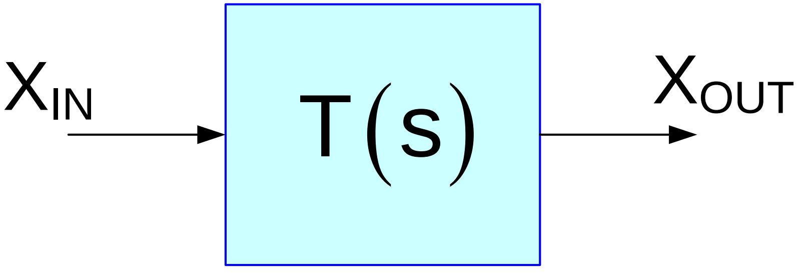
$$BW = \omega_B - \omega_A = \frac{\omega_0}{Q}$$

$$\omega_{PEAK} = \omega_0$$

Filter Design/Synthesis Considerations

There are many different filter architectures that can realize a given transfer function

Will first consider second-order Bandpass filter structures



$$\left|T(s)\right| = H\frac{s\left(\frac{\omega_0}{Q}\right)}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

$$BW = \omega_B - \omega_A = \frac{\omega_0}{Q}$$

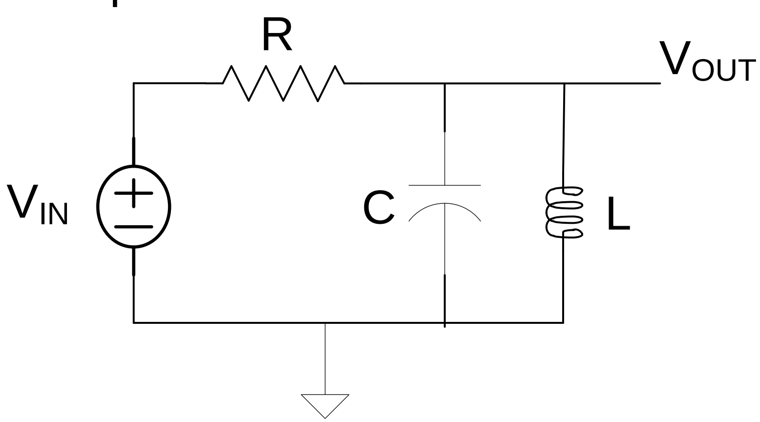
$$\omega_{PEAK} = \omega_0$$

Filter Design/Synthesis Considerations

There are many different filter architectures that can realize a given transfer function

Will first consider second-order Bandpass filter structures

Example 1:



$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{1}{RC}\frac{s}{s^2 + s\left(\frac{1}{RC}\right) + \frac{1}{LC}}$$

Second-order Bandpass Filter

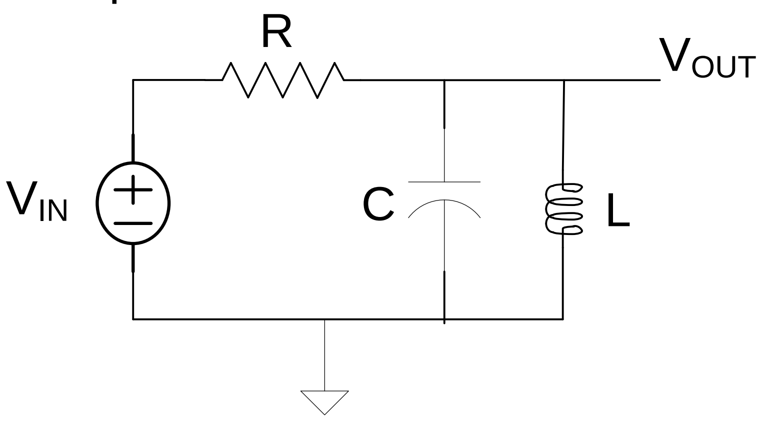
3 degrees of freedom

2 degrees of freedom for determining dimensionless transfer function

(impedance values scale)

$\omega_0 = ?$ Q = ? BW = ?

Example 1:



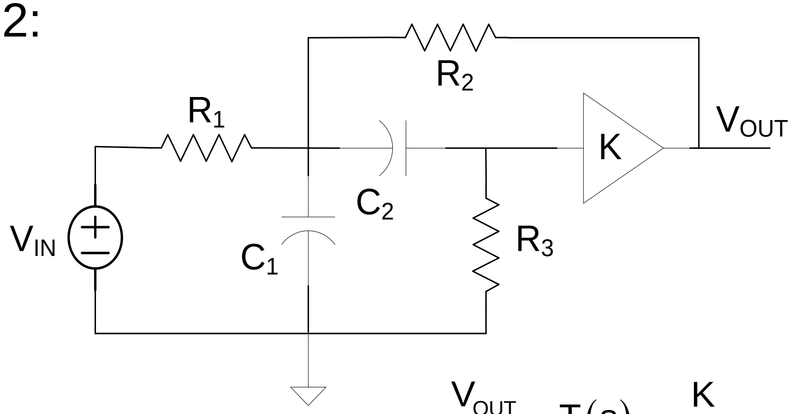
$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{1}{RC}\frac{s}{s^2 + s\left(\frac{1}{RC}\right) + \frac{1}{LC}}$$

$$\omega_0 = \sqrt{\frac{1}{LC}} \qquad Q = R\sqrt{\frac{C}{L}} \qquad BW = \frac{1}{RC}$$

Can realize an arbitrary 2nd order bandpass function within a gain factor

Simple design process (sequential but not independent control of ω_0 and Q)

Example 2:



$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{K}{R_1C_1}\frac{s}{s^2+s\left(\frac{1}{R_1C_1}+\frac{1}{R_3C_1}+\frac{1}{R_3C_2}+\frac{1\text{-}K}{R_2C_1}\right)+\frac{R_1+R_2}{R_1R_2R_3C_1C_2}}$$

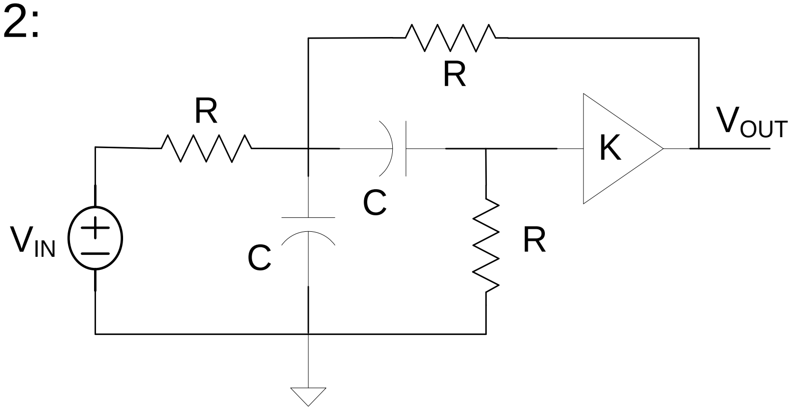
Second-order Bandpass Filter

6 degrees of freedom (effectively 5 because dimensionless)

Denote as a +KRC filter

$\omega_0 = ?$ Q = ? BW = ?

Example 2:



Equal R, Equal C Realization

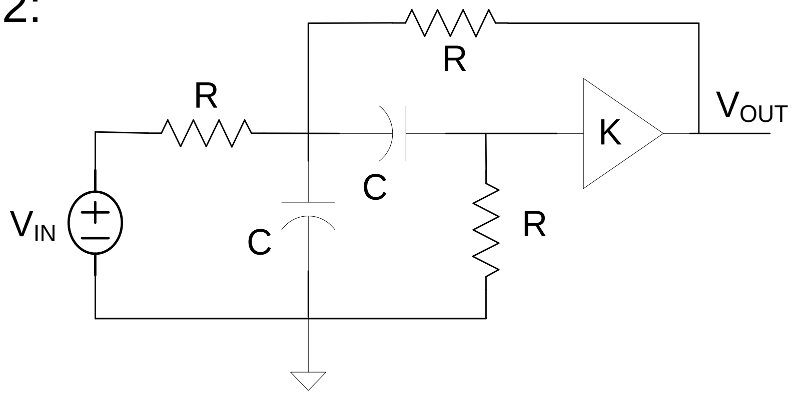
$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{K}{RC}\frac{s}{s^2+s\left(\frac{4-K}{RC}\right)+\frac{2}{(RC)^2}}$$

$\omega_0 = ?$

Q = ?

BW = ?

Example 2:



Equal R, Equal C Realization

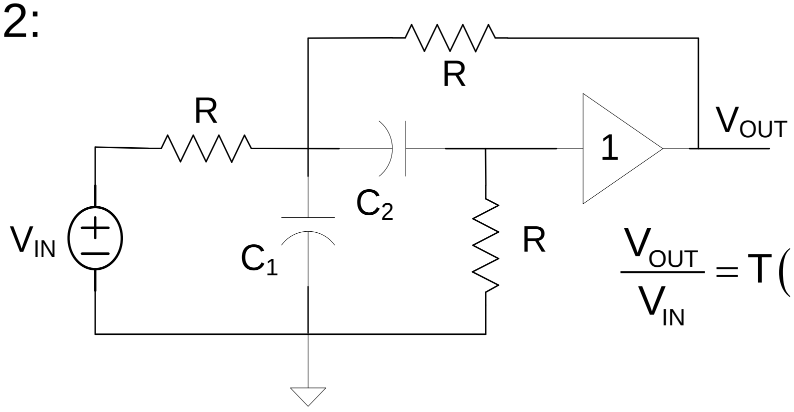
$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{K}{RC} \frac{s}{s^2 + s\left(\frac{4-K}{RC}\right) + \frac{2}{(RC)^2}}$$

$$\omega_0 = \frac{\sqrt{2}}{RC} \qquad Q = \frac{\sqrt{2}}{4-K} \qquad BW = \frac{4-K}{RC}$$

3 degrees of freedom (effectively 2 since dimensionless)

- Can satisfy arbitrary 2nd=order BP constraints within a gain factor with this circuit
- Very simple circuit structure
- Independent control of ω_0 and Q but requires tuning more than one component
- Can actually move poles in RHP by making K >4

Example 2:



Unity Gain, Equal R

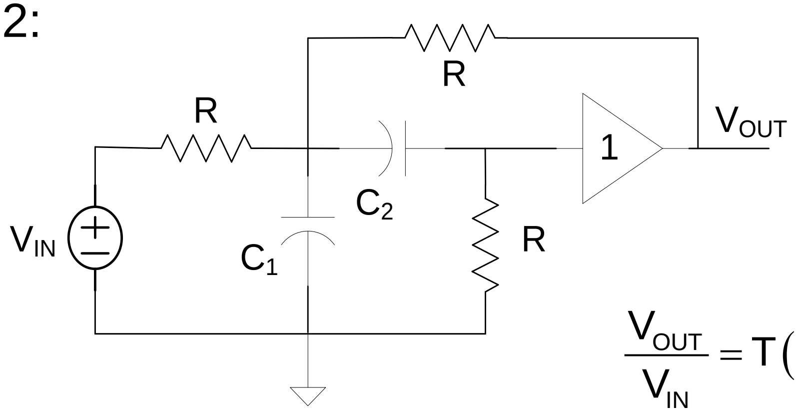
$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{1}{R\,C_1}\,\frac{s}{s^2 + s\left(\left[\frac{1}{R}\right]\left(\frac{2}{C_1} + \frac{1}{C_2}\right)\right) + \frac{2}{R^2 C_1 C_2}}$$

$\omega_0 = ?$

Q = ?

BW = ?

Example 2:



Unity Gain, Equal R

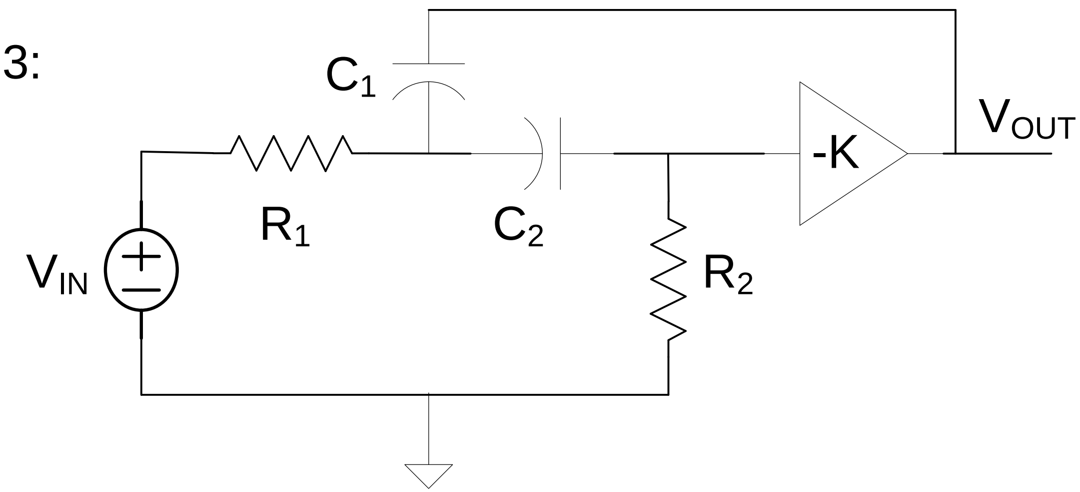
$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{1}{R\,C_1}\frac{s}{s^2 + s\left(\left[\frac{1}{R}\right]\left(\frac{2}{C_1} + \frac{1}{C_2}\right)\right) + \frac{2}{R^2 C_1 C_2}}$$

$$\omega_0 = \frac{\sqrt{2}}{R\sqrt{C_1 C_2}}$$

$$Q = \sqrt{2}\sqrt{\frac{C_2}{C_1}} + \frac{1}{\sqrt{2}}\sqrt{\frac{C_1}{C_2}}$$

$$BW = \left[\frac{1}{R}\right]\left(\frac{2}{C_1} + \frac{1}{C_2}\right)$$

Example 3:



$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{K}{(1+K)R_1C_1}\frac{s}{s^2+s\left(\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1}\right]\frac{1}{(1+K)}\right) + \frac{1}{(1+K)R_1R_2C_1C_2}}$$

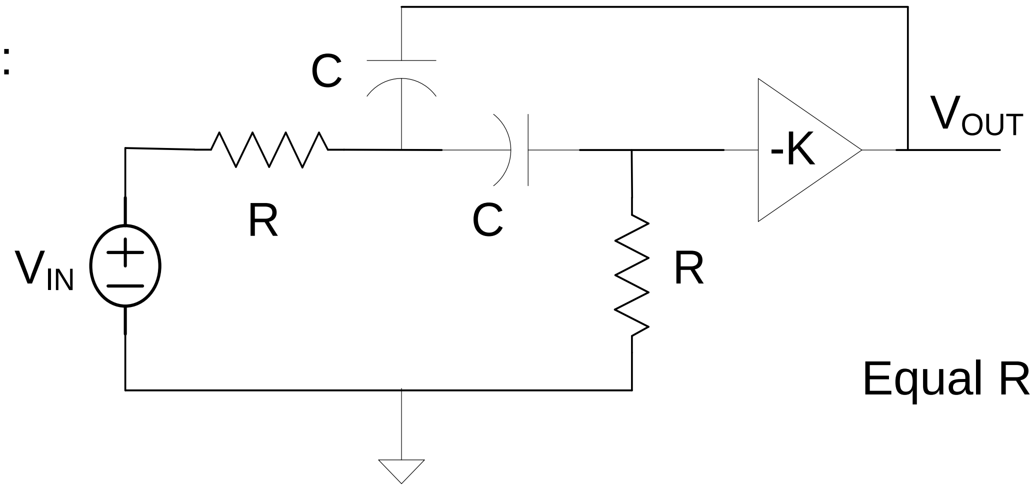
Second-order Bandpass Filter

5 degrees of freedom (4 effective since dimensionless)

Denote as a -KRC filter

$\omega_0 = ?$ $Q = ?$ $BW = ?$

Example 3:



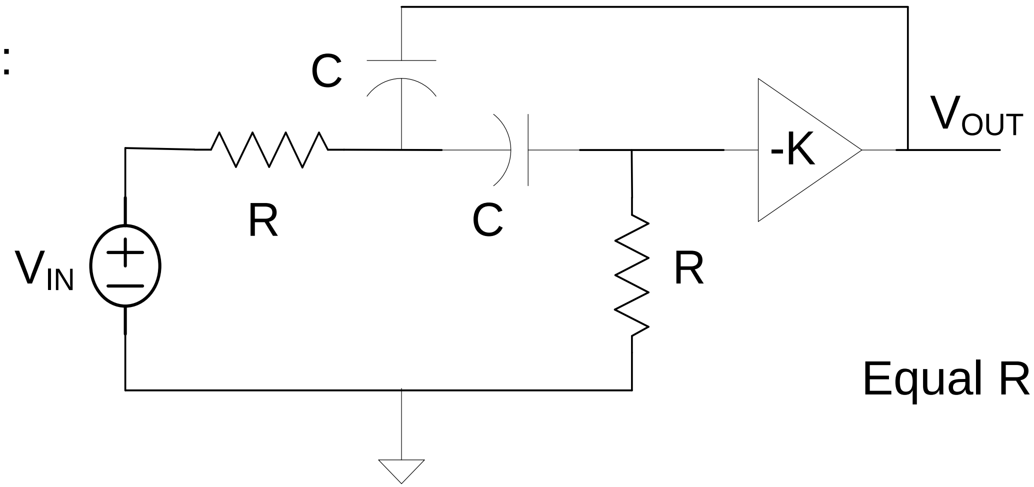
Equal R, Equal C Realization

$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{K}{(1+K)RC}\frac{s}{s^2+s\left(\left[\frac{3}{RC}\right]\frac{1}{(1+K)}\right)+\frac{1}{(1+K)(RC)^2}}$$

3 degrees of freedom

$\omega_0 = ?$ Q = ? BW = ?

Example 3:



Equal R, Equal C Realization

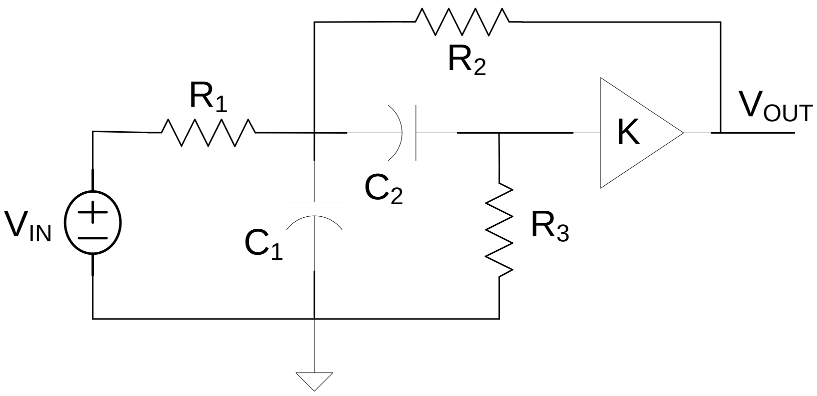
$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{K}{(1+K)RC}\frac{s}{s^2+s\left(\left[\frac{3}{RC}\right]\frac{1}{(1+K)}\right)+\frac{1}{(1+K)(RC)^2}}$$

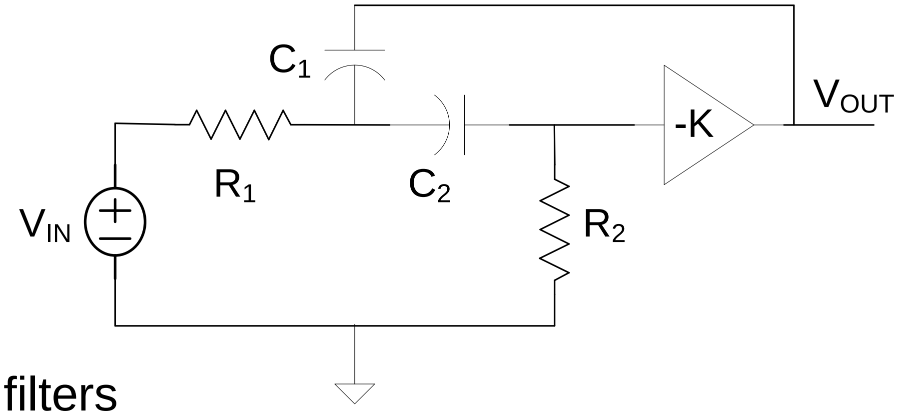
$$\omega_0 = \frac{1}{RC\sqrt{1+K}} \qquad Q = \frac{\sqrt{1+K}}{3} \qquad BW = \frac{3}{RC(1+K)}$$

3 degrees of freedom (2 effective since dimensionless)

- Can satisfy arbitrary 2nd=order BP constraints within a gain factor with this circuit
- Very simple circuit structure
- Simple design process (sequential but not independent control of ω_0 and Q, requires tuning of more than 1 component if Rs used)

Observation:





These are often termed Sallen and Key filters

Sallen and Key introduced a host of filter structures

Sallen and Key structures comprised of summers,
RC network, and finite gain amplifiers

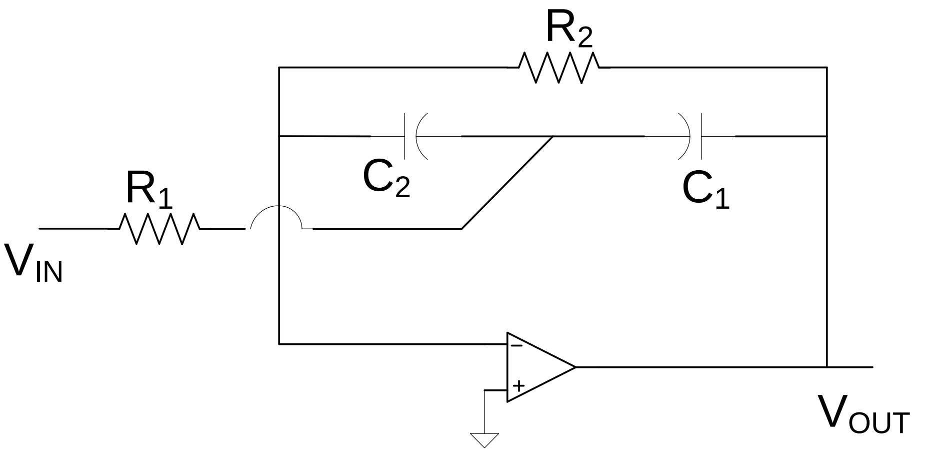
These filters were really ahead of their time and appeared long before practical implementations were available

IRE TRANSACTIONS—CIRCUIT THEORY *March* 1955

A Practical Method of Designing RC Active Filters*

R. P. SALLEN† AND E. L. KEY†

Example 4:



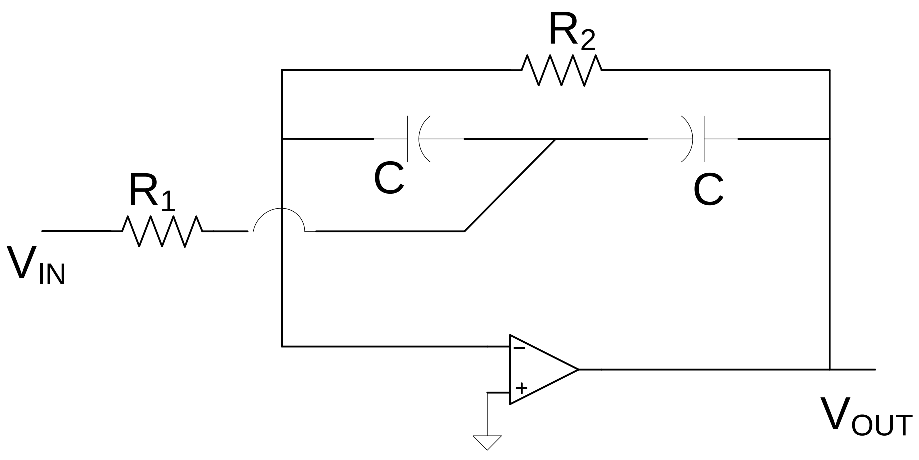
$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{1}{R_1C_1}\frac{s}{s^2+s\left(\frac{1}{R_2}\left[\frac{1}{C_1}+\frac{1}{C_2}\right]\right) + \frac{1}{R_1R_2C_1C_2}}$$

Second-order Bandpass Filter

4 degrees of freedom (3 effective since dimensionless)

Denote as a bridged T feedback structure

Example 4:



Equal C implementation

$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{1}{R_1 C} \frac{s}{s^2 + s\left(\frac{2}{CR_2}\right) + \frac{1}{R_1 R_2 C^2}}$$

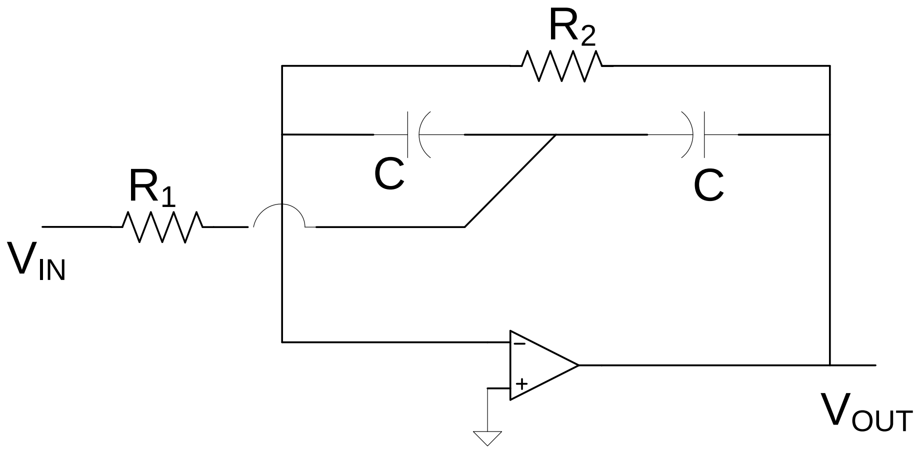
3 degrees of freedom (2 effective since dimensionless)

$\omega_0 = ?$

Q = ?

BW = ?

Example 4:



Equal C implementation

$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{1}{R_1 C}\frac{s}{s^2 + s\left(\frac{2}{CR_2}\right) + \frac{1}{R_1 R_2 C^2}}$$

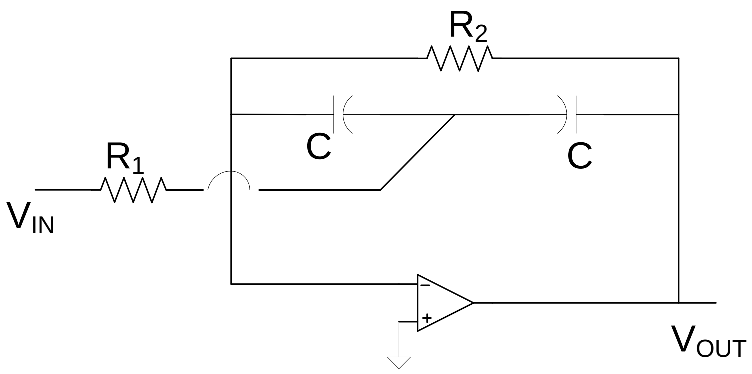
$$\omega_0 = \frac{1}{C\sqrt{R_1 R_2}} \qquad Q = \frac{1}{2}\sqrt{\frac{R_2}{R_1}} \qquad BW = \frac{2}{R_2 C}$$

Simple circuit structure

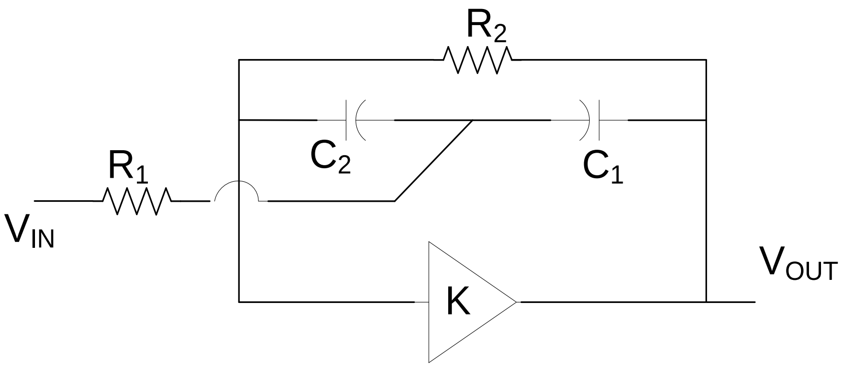
More tedious design/calibration process for ω_0 and Q (iterative if C is fixed)

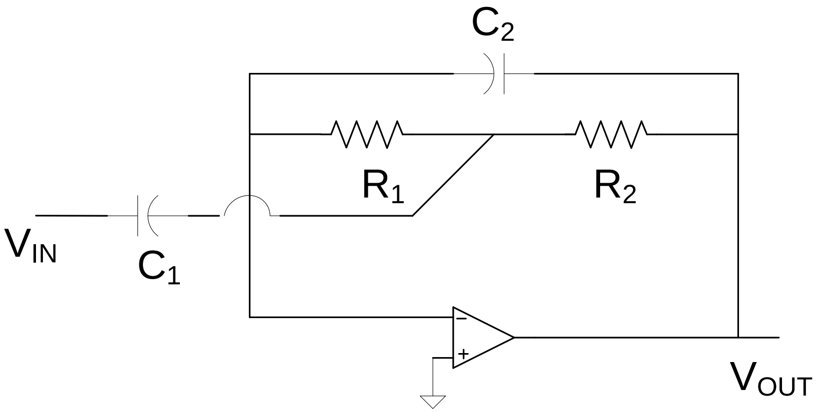
Resistor ratio is $4Q^2$

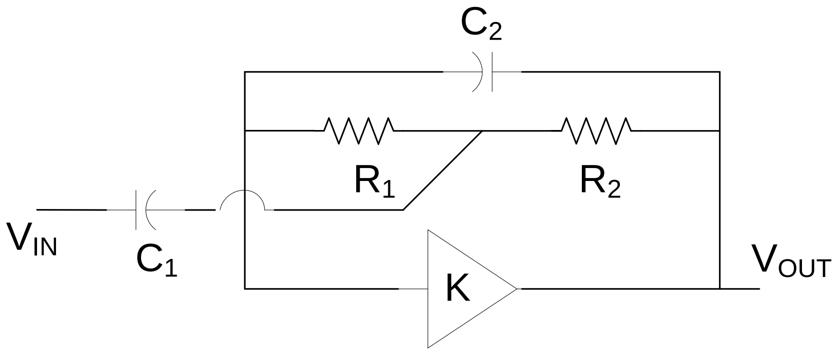
Example 4:



Some variants of the bridged-T feedback structure







Are there more 2^{nd} order bandpass filter structures?

Yes, many other 2^{nd}-order bandpass filter structures exist

But, if we ask the question differently

Are there more 2^{nd}-order bandpass filter structures comprised of one amplifier and four passive components?

Yes, but not too many more

Are there more 2^{nd}-order bandpass filter structures comprised of one amplifier, two capacitors, and three resistors?

Yes, but not too many more

Similar comments can be made about 2^{nd}-order LP, BP, and BR

$$T(s)=\frac{H\omega_0^2}{s^2+s\left(\frac{\omega_0}{Q}\right)+\omega_0^2} \qquad T(s)=\frac{Hs^2}{s^2+s\left(\frac{\omega_0}{Q}\right)+\omega_0^2} \qquad T(s)=\frac{H\left(s^2+\omega_0^2\right)}{s^2+s\left(\frac{\omega_0}{Q}\right)+\omega_0^2}$$

Similar comments can be made about full 2^{nd}-order biquadratic function

$$T(s)=H\frac{s^2+s\left(\frac{\omega_{0N}}{Q_N}\right)+\omega_{0N}^2}{s^2+s\left(\frac{\omega_0}{Q}\right)+\omega_0^2}$$

End of Lecture 18