Basic Biquadratic Active Filters

Second-order Bandpass
Second-order Lowpass
Effects of Op Amp on Filter Performance
Review from Last Time

Standard LP to BS Transformation

map \( \omega = 0 \) to \( \omega = \pm \infty \)
map \( \omega = 0 \) to \( \omega = 0 \)
map \( \omega = 1 \) to \( \omega = \omega_A \)
map \( \omega = 1 \) to \( \omega = -\omega_B \)
map \( \omega = -1 \) to \( \omega = \omega_B \)
map \( \omega = -1 \) to \( \omega = -\omega_A \)
LP to BS Transformation

Pole Q of BS Approximations

\[
\begin{align*}
M_{0LP} & = \frac{\omega}{\gamma} \\
B_W & = \omega_B - \omega_A \\
M_{AN} & = \frac{\omega}{\gamma} \\
B_N & = \omega_B - \omega_A
\end{align*}
\]

It can be shown that

\[
Q_{BSL} = Q_{BSH} = \frac{Q_{LP}}{\sqrt{2}} \sqrt{1 + \frac{4}{\gamma^2} + \sqrt{1 + \frac{4}{\gamma^2}}^2} - \frac{4}{\gamma^2 Q_{LP}^2}
\]

For \( \gamma \) small,

\[
Q_{BS} = \frac{2Q_{LP}}{\gamma}
\]

It can be shown that

\[
\omega_{0BS} = \frac{\omega_M}{2} \left[ \gamma \frac{Q_{BS}}{Q_{LP}} \pm \sqrt{\left( \gamma \frac{Q_{BS}}{Q_{LP}} \right)^2 - 4} \right]
\]

Note for \( \gamma \) small, \( Q_{BS} \) can get very large
Standard LP to BS Transformation

\[ S_x \rightarrow \frac{s \cdot BW}{s^2 + \omega_M^2} \]

- Standard LP to BS transformation is a variable mapping transform
- Maps \( j\omega \) axis to \( j\omega \) axis in the s-plane
- Preserves basic shape of an approximation but warps frequency axis
- Order of BS approximation is double that of the LP Approximation
- Pole Q and \( \omega_0 \) expressions are identical to those of the LP to BP transformation
- Pole Q of BS approximation can get very large for narrow BW
- Other variable transforms exist but the standard is by far the most popular
LP to HP Transformation
(Normalized Transform)

\[ |T(j\omega)| \]

\[ T_{\text{HP}}(j\omega) \]

\( \omega = -\infty \)
\( \omega = \infty \)
Review from Last Time

Comparison of Transforms

LP to BP
\[ s \rightarrow \frac{s^2 + 1}{s \cdot BW_N} \]

LP to BS
\[ s \rightarrow \frac{s \cdot BW_N}{s^2 + 1} \]

LP to HP
\[ s \rightarrow \frac{1}{s} \]
Filter Design
Process

Establish Specifications
- possibly $T_D(s)$ or $H_D(z)$
- magnitude and phase characteristics or restrictions
- time domain requirements

Approximation
- obtain acceptable transfer functions $T_A(s)$ or $H_A(z)$
- possibly acceptable realizable time-domain responses

Synthesis
- build circuit or implement algorithm that has response close to $T_A(s)$ or $H_A(z)$
- actually realize $T_R(s)$ or $H_R(z)$

Filter
Filter Design/Synthesis Considerations

There are many different filter architectures that can realize a given transfer function.

Considerable effort has been focused over the years on “inventing” these architectures and on determining which is best suited for a given application.
Filter Design/Synthesis Considerations

Most even-ordered designs today use one of the following three basic architectures:

**Cascaded Biquads**

\[ T(s) = T_1 T_2 \cdots T_m \]

**Leapfrog**

**Multiple-loop Feedback** (less popular)

What’s unique in all of these approaches?
Filter Design/Synthesis Considerations

Most odd-ordered designs today use one of the following three basic architectures:

**Cascaded Biquads**

\[ T(s) = T_1 T_2 \cdots T_m \]

**Leapfrog**

**Multiple-loop Feedback** (less popular)

What’s unique in all of these approaches?
Filter Design/Synthesis Considerations

What’s unique in all of these approaches?

- Most effort on synthesis can focus on synthesizing these four blocks
  (the summing function is often incorporated into the Biquad or Integrator)
  (the first-order block is much less challenging to design than the biquad)
- Some issues associated with their interconnections

- And, in many integrated structures, the biquads are made with integrators
  (thus, much filter design work simply focuses on the design of integrators)
Biquads

How many biquad filter functions are there?

\[ T(s) = \frac{a_2s^2 + a_1s + a_0}{s^2 + b_1s + b_0} \]

\[ T(s) = \frac{a_2s^2 + a_1s + a_0}{s^2 + b_1s + b_0} \quad a_0 \neq 0, a_1 \neq 0, a_2 \neq 0 \]

\[ T(s) = \frac{a_0}{s^2 + b_1s + b_0} \quad a_0 \neq 0 \]

\[ T(s) = \frac{a_1s}{s^2 + b_1s + b_0} \quad a_1 \neq 0 \]

\[ T(s) = \frac{a_2s^2}{s^2 + b_1s + b_0} \quad a_2 \neq 0 \]
Filter Design/Synthesis Considerations

Review: Second-order bandpass transfer function

\[ |T_{2BP}(j\omega)| \]

\[ H_{\sqrt{2}} \]

\[ T_{2BP}(s) = \frac{s\left(\frac{\omega_0}{Q}\right)}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2} \]

\[ BW = \omega_B - \omega_A = \frac{\omega_0}{Q} \]

\[ \omega_{PEAK} = \omega_0 \]
Filter Design/Synthesis Considerations

There are many different filter architectures that can realize a given transfer function.

Will first consider second-order Bandpass filter structures.

\[
|T(s)| = H \frac{s\left(\frac{\omega_0}{Q}\right)}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}
\]

\[
BW = \omega_B - \omega_A = \frac{\omega_0}{Q}
\]

\[
\omega_{\text{PEAK}} = \omega_0
\]
Filter Design/Synthesis Considerations

There are many different filter architectures that can realize a given transfer function.

Will first consider second-order Bandpass filter structures.

Example 1:

\[ \frac{V_{\text{OUT}}}{V_{\text{IN}}} = T(s) = \frac{1}{RC} \frac{s}{s^2 + \frac{1}{RC} + \frac{1}{LC}} \]

Second-order Bandpass Filter

3 degrees of freedom

2 degrees of freedom for determining dimensionless transfer function

(impedance values scale)

\[ \omega_0 = ? \quad Q = ? \quad \text{BW} = ? \]
Example 1:

Can realize an arbitrary 2nd order bandpass function within a gain factor

Simple design process (sequential but not independent control of $\omega_0$ and $Q$)
Example 2:

\[
\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{K}{R_1C_1} \frac{s}{s^2 + \frac{1}{R_1C_1} + \frac{1}{R_3C_1} + \frac{1}{R_3C_2} + \frac{1-K}{R_2C_1}} + \frac{R_1+R_2}{R_1R_2R_3C_1C_2}
\]

Second-order Bandpass Filter

6 degrees of freedom  (effectively 5 because dimensionless)

Denote as a +KRC filter

\[\omega_0 = ?\]
\[Q = ?\]
\[BW = ?\]
Example 2:

Equal R, Equal C Realization

$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{K}{RC} \frac{s}{s^2 + \left(\frac{4-K}{RC}\right) + \frac{2}{(RC)^2}}$$

$$\omega_0 = ?$$

$$Q = ?$$

$$BW = ?$$
Example 2:

\[ T(s) = \frac{\frac{s}{s^2 + s\left(\frac{4-K}{RC}\right) + \frac{2}{(RC)^2}}}{s^2 + \frac{2}{(RC)^2}} \]

\[ \omega_0 = \frac{\sqrt{2}}{RC} \quad Q = \frac{\sqrt{2}}{4-K} \quad BW = \frac{4-K}{RC} \]

3 degrees of freedom (effectively 2 since dimensionless)

- Can satisfy arbitrary 2\textsuperscript{nd}-order BP constraints within a gain factor with this circuit
- Very simple circuit structure
- Independent control of \( \omega_0 \) and Q but requires tuning more than one component
- Can actually move poles in RHP by making \( K > 4 \)
Example 2:

Unity Gain, Equal R

\[
\frac{V_{\text{OUT}}}{V_{\text{IN}}} = T(s) = \frac{1}{R C_1} \frac{s}{s^2 + s \left( \frac{1}{R C_1} + \frac{2}{C_1 + C_2} \right) + \frac{2}{R^2 C_1 C_2}}
\]

\( \omega_0 = ? \)

\( Q = ? \)

\( BW = ? \)
Example 2:

Unity Gain, Equal R

\[ T(s) = \frac{1}{R C_1} \frac{s}{s^2 + s \left( \frac{1}{R} \left( \frac{2}{C_1} + \frac{1}{C_2} \right) \right) + \frac{2}{R^2 C_1 C_2}} \]

\[ \frac{V_{\text{OUT}}}{V_{\text{IN}}} = T(s) \]

\[ \omega_0 = \frac{\sqrt{2}}{R \sqrt{C_1 C_2}} \]

\[ Q = \sqrt{2} \sqrt{\frac{C_2}{C_1} + \frac{1}{\sqrt{2}} \sqrt{\frac{C_1}{C_2}}} \]

\[ \text{BW} = \left[ \frac{1}{R} \left( \frac{2}{C_1} + \frac{1}{C_2} \right) \right] \]
Example 3:

\[
\frac{V_{\text{OUT}}}{V_{\text{IN}}} = T(s) = -\frac{K}{(1+K)R_1C_1} \frac{s}{s^2 + s\left[\frac{1}{R_1C_1} + \frac{1}{R_2C_2} + \frac{1}{R_2C_1}\right] + \frac{1}{1+K} + \frac{1}{(1+K)R_1R_2C_1C_2}}
\]

Second-order Bandpass Filter

5 degrees of freedom  \(4\) effective since dimensionless

Denote as a -KRC filter

\[\omega_0 = ?\]  \[Q = ?\]  \[BW = ?\]
Example 3:

\[ \frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{K}{(1+K)RC} \frac{s}{s^2 + \frac{3}{RC} \left( \frac{1}{1+K} \right)} + \frac{1}{(1+K)(RC)^2} \]

3 degrees of freedom

\[ \omega_0 = ? \quad Q = ? \quad BW = ? \]
Example 3:

\[ \frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{K}{(1+K)RC} \frac{s}{s^2 + s \left[ \frac{3}{RC}(1+K) \right] + \frac{1}{(1+K)(RC)^2}} \]

\[ \omega_0 = \frac{1}{RC\sqrt{1+K}} \quad Q = \frac{\sqrt{1+K}}{3} \quad BW = \frac{3}{RC(1+K)} \]

3 degrees of freedom (2 effective since dimensionless)

- Can satisfy arbitrary 2\textsuperscript{nd}-order BP constraints within a gain factor with this circuit
- Very simple circuit structure
- Simple design process (sequential but not independent control of \( \omega_0 \) and \( Q \), requires tuning of more than 1 component if Rs used)
Observation:

These are often termed Sallen and Key filters

Sallen and Key introduced a host of filter structures

Sallen and Key structures comprised of summers, RC network, and finite gain amplifiers

These filters were really ahead of their time and appeared long before practical implementations were available

*IRE TRANSACTIONS—CIRCUIT THEORY  March 1955*

**A Practical Method of Designing RC Active Filters**

R. P. SALLEN† AND E. L. KEY†
Example 4:

\[
\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{1}{R_1C_1} \frac{s}{s^2 + s \left( \frac{1}{R_2 \left( \frac{1}{C_1} + \frac{1}{C_2} \right)} \right) + \frac{1}{R_1R_2C_1C_2}}
\]

Second-order Bandpass Filter

4 degrees of freedom  (3 effective since dimensionless)

Denote as a bridged T feedback structure
Example 4:

\[
\frac{V_{\text{OUT}}}{V_{\text{IN}}} = T(s) = -\frac{1}{R_1C} \frac{s}{s^2 + s\left(\frac{2}{CR_2}\right) + \frac{1}{R_1R_2C^2}}
\]

3 degrees of freedom (2 effective since dimensionless)

\[\omega_0 = ? \quad Q = ? \quad BW = ?\]
Example 4:

Equal C implementation

\[
\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{1}{R_1C} \frac{s}{s^2 + s\left(\frac{2}{CR_2}\right) + \frac{1}{R_1R_2C^2}}
\]

\[
\omega_0 = \frac{1}{C\sqrt{R_1R_2}} \quad Q = \frac{1}{2\sqrt{\frac{R_2}{R_1}}} \quad BW = \frac{2}{R_2C}
\]

Simple circuit structure

More tedious design/calibration process for \(\omega_0\) and \(Q\) (iterative if \(C\) is fixed)

Resistor ratio is \(4Q^2\)
Example 4:

Some variants of the bridged-T feedback structure
Are there more 2nd order bandpass filter structures?

Yes, many other 2nd-order bandpass filter structures exist

But, if we ask the question differently

Are there more 2nd-order bandpass filter structures comprised of one amplifier and four passive components?

Yes, but not too many more

Are there more 2nd-order bandpass filter structures comprised of one amplifier, two capacitors, and three resistors?

Yes, but not too many more

Similar comments can be made about 2nd-order LP, BP, and BR

\[
T(s) = \frac{H\omega_0^2}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}
\]

\[
T(s) = \frac{Hs^2}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}
\]

\[
T(s) = \frac{H(s^2 + \omega_0^2)}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}
\]

Similar comments can be made about full 2nd-order biquadratic function

\[
T(s) = H\frac{s^2 + s\left(\frac{\omega_{0N}}{Q_N}\right) + \omega_{0N}^2}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}
\]
End of Lecture 18