EE 508
Lecture 19

Basic Biquadratic Active Filters

Sensitivity Functions

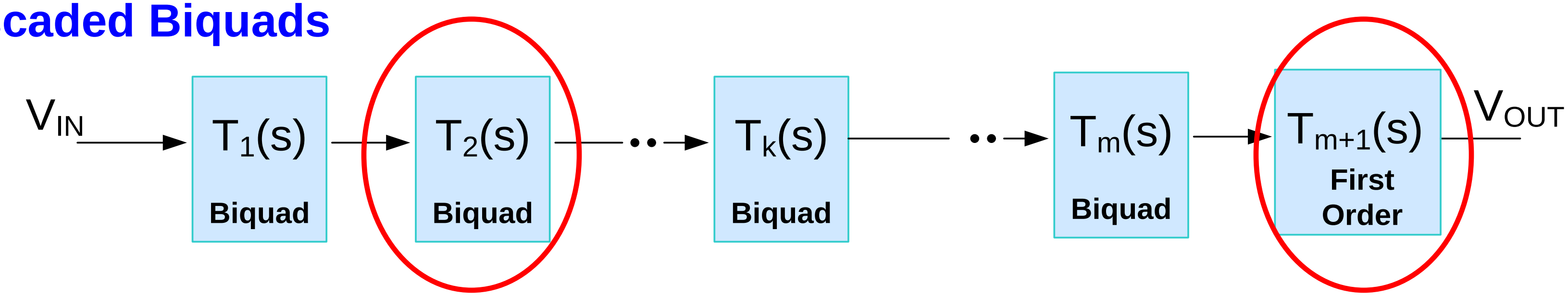
- Comparison of Filter Structures
- Performance Prediction
- Design Characterization

Review from last time

Filter Design/Synthesis Considerations

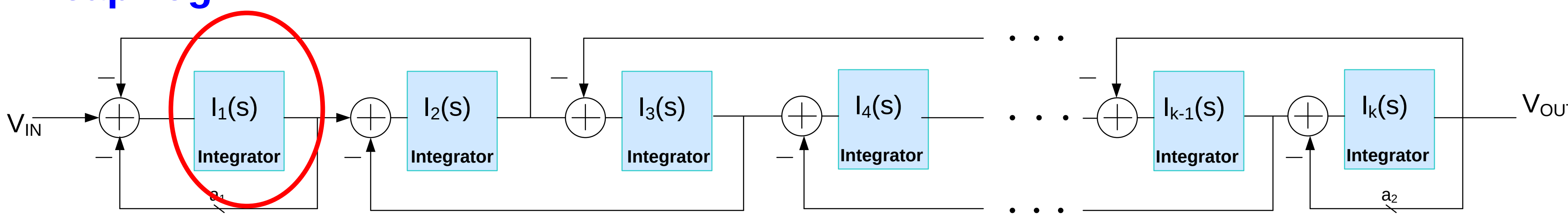
Most odd-ordered designs today use one of the following three basic architectures

Cascaded Biquads

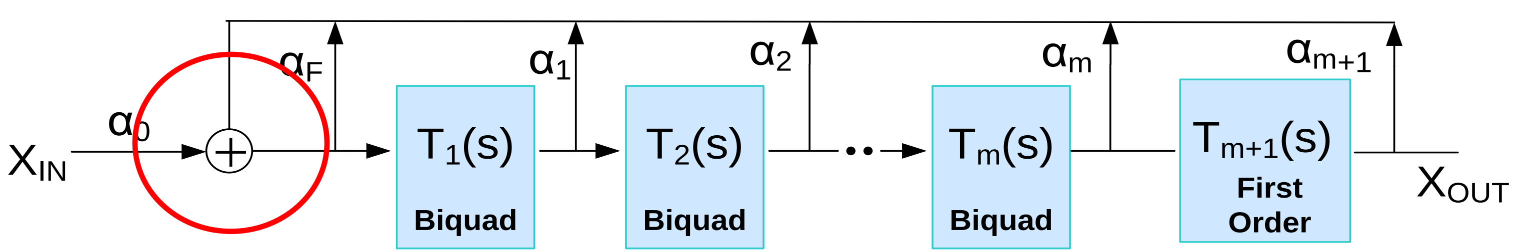


$$T(s) = T_1 T_2 \bullet\bullet T_m$$

Leapfrog



Multiple-loop Feedback (less popular)

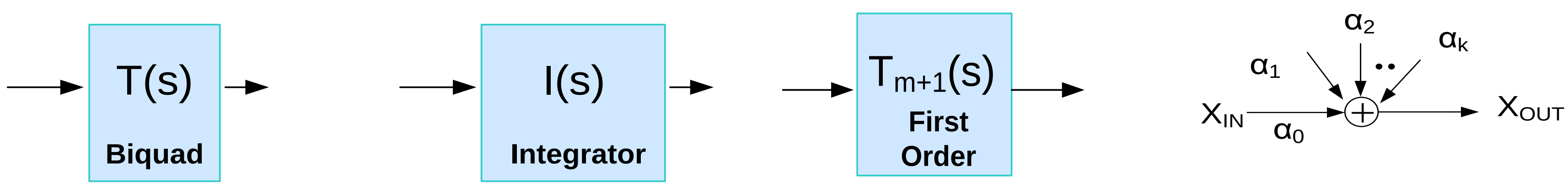


What's unique in all of these approaches?

Review from last time

Filter Design/Synthesis Considerations

What's unique in all of these approaches?



$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0}$$

$$I(s) = \frac{I_0}{s}$$

$$T(s) = \frac{a_1 s + a_0}{s + b_0}$$

$$X_{OUT} = \sum_{i=0}^{k} \alpha_i$$

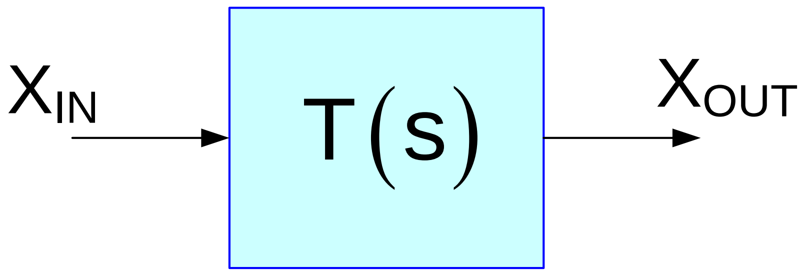
- Most effort on synthesis can focus on synthesizing these four blocks
  (the summing function is often incorporated into the Biquad or Integrator)
  (the first-order block is much less challenging to design than the biquad)
- Some issues associated with their interconnections
- And, in many integrated structures, the biquads are made with integrators
  (thus, much filter design work simply focuses on the design of integrators)

Review from last time

Filter Design/Synthesis Considerations

There are many different filter architectures that can realize a given transfer function

Will first consider second-order Bandpass filter structures



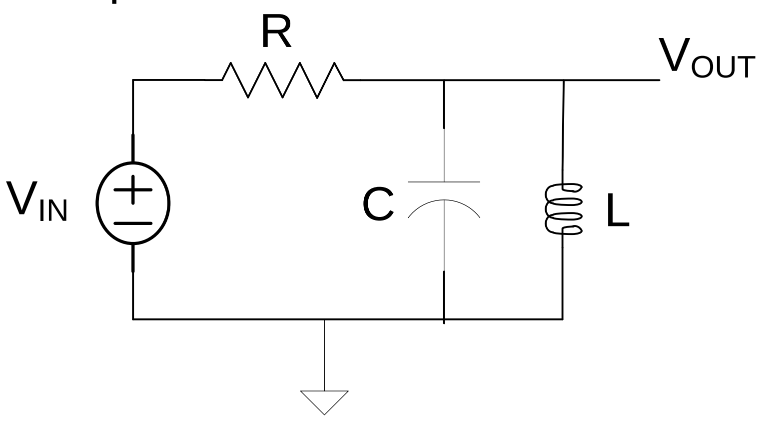
$$\left|T(s)\right| = H\frac{s\left(\frac{\omega_0}{Q}\right)}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

$$BW = \omega_B - \omega_A = \frac{\omega_0}{Q}$$

$$\omega_{PEAK} = \omega_0$$

Review from last time

Example 1:



$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{1}{RC} \frac{s}{s^2 + s\left(\frac{1}{RC}\right) + \frac{1}{LC}}$$

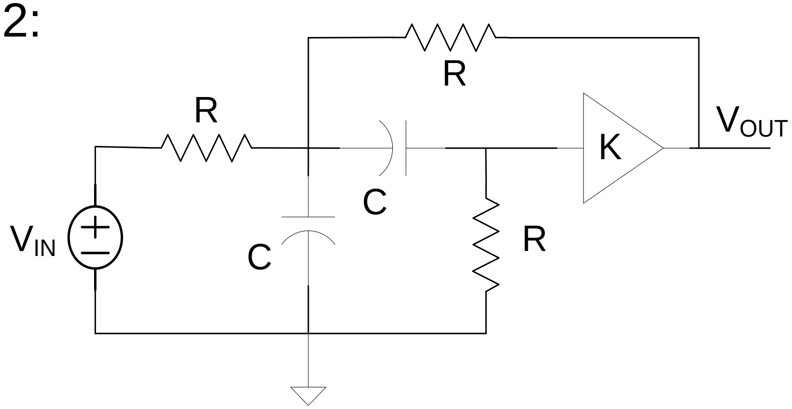
$$\omega_0 = \sqrt{\frac{1}{LC}} \qquad Q = R\sqrt{\frac{C}{L}} \qquad BW = \frac{1}{RC}$$

Can realize an arbitrary 2nd order bandpass function within a gain factor

Simple design process (sequential but not independent control of ω_0 and Q)

Review from last time

Example 2:



Equal R, Equal C Realization

$$\frac{V_{OUT}}{V_{IN}} = T(s) = \frac{K}{RC} \frac{s}{s^2 + s\left(\frac{4-K}{RC}\right) + \frac{2}{(RC)^2}}$$

$$\omega_0 = \frac{\sqrt{2}}{RC} \qquad Q = \frac{\sqrt{2}}{4-K} \qquad BW = \frac{4-K}{RC}$$

3 degrees of freedom (effectively 2 since dimensionless)

Can satisfy arbitrary 2nd=order BP constraints within a gain factor with this circuit

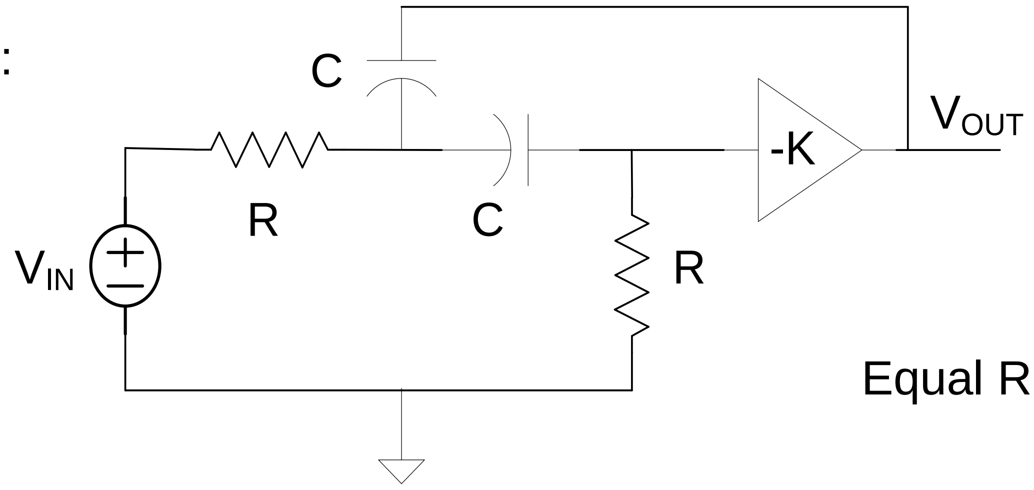
Very simple circuit structure

Independent control of ω_0 and Q but requires tuning more than one component

Can actually move poles in RHP by making K >4

Review from last time

Example 3:



Equal R, Equal C Realization

$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{K}{(1+K)RC}\frac{s}{s^2+s\left(\left[\frac{3}{RC}\right]\frac{1}{(1+K)}\right)+\frac{1}{(1+K)(RC)^2}}$$

$$\omega_0 = \frac{1}{RC\sqrt{1+K}} \qquad Q = \frac{\sqrt{1+K}}{3} \qquad BW = \frac{3}{RC(1+K)}$$

3 degrees of freedom (2 effective since dimensionless)

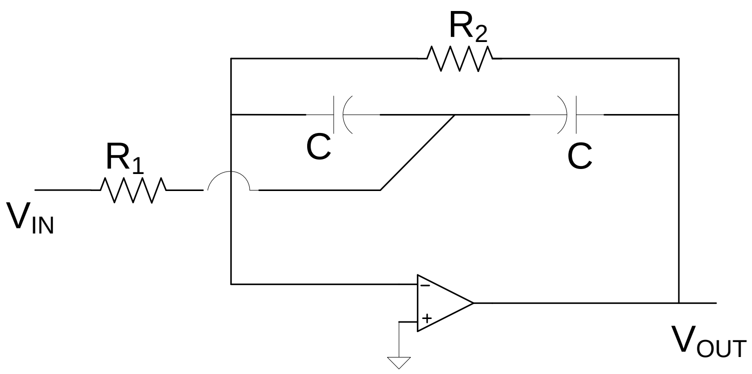
Can satisfy arbitrary 2nd=order BP constraints within a gain factor with this circuit

Very simple circuit structure

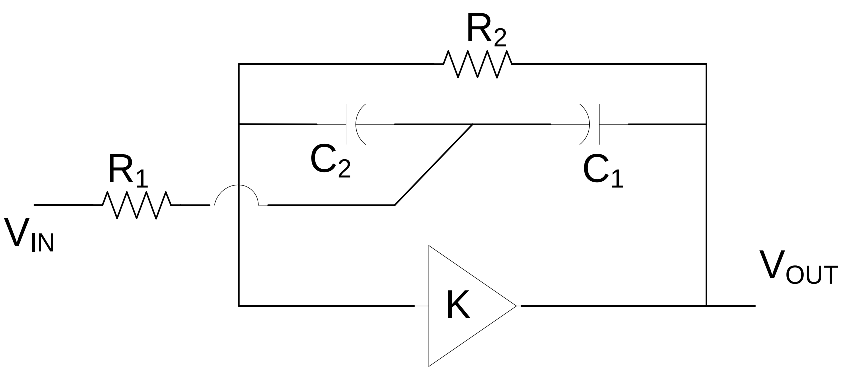
Simple design process (sequential but not independent control of ω_0 and Q, requires tuning of more than 1 component if Rs used)

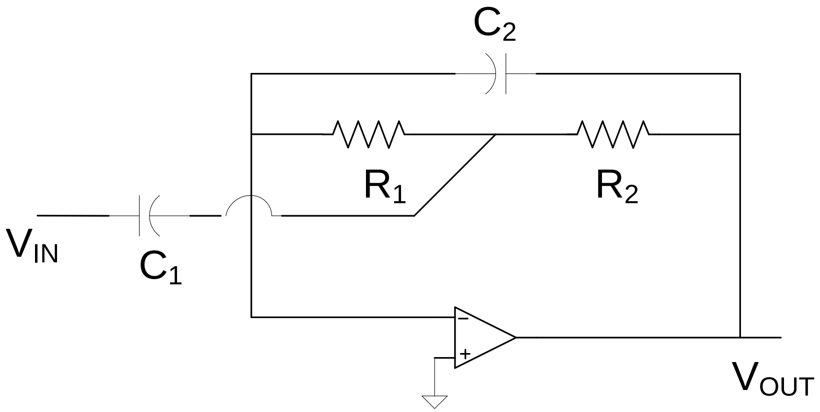
Review from last time

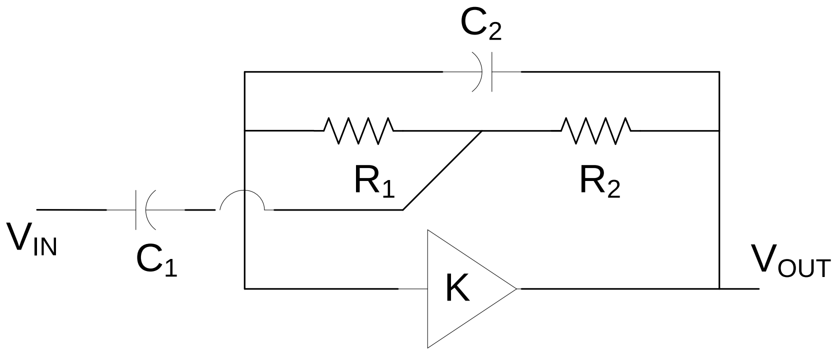
Example 4:



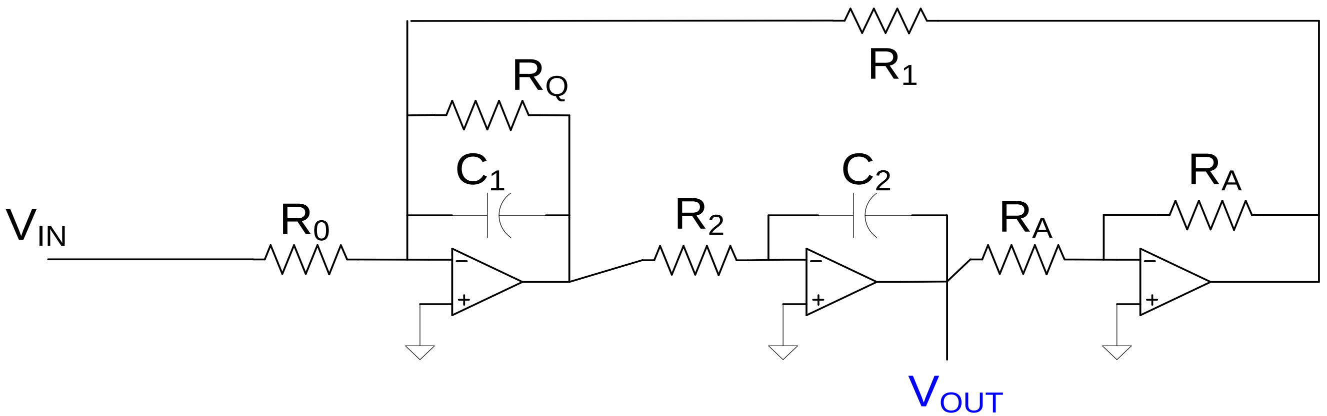
Some variants of the bridged-T feedback structure







Example 5:



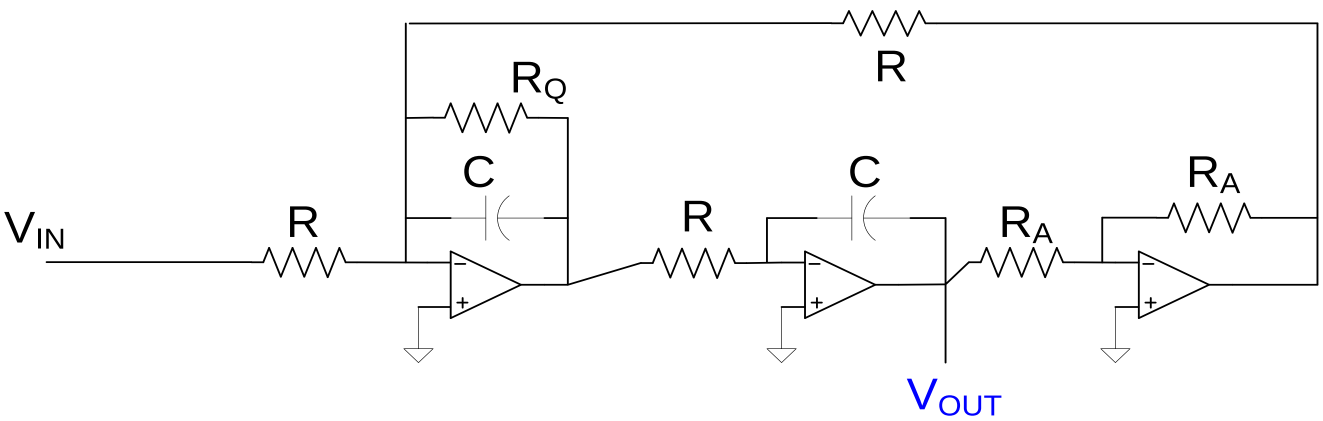
$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{1}{R_0 C_2}\frac{s}{s^2 + s\left(\frac{1}{R_Q C_2}\right) + \frac{1}{R_1 R_2 C_1 C_2}}$$

Second-order Bandpass Filter

8 degrees of freedom (effectively 7 since dimensionless)

Denote as a two-integrator-loop structure

Example 5:



Equal R Equal C (except R_Q)

$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{1}{RC}\frac{s}{s^2+s\left(\left[\frac{R}{R_Q}\right]\frac{1}{RC}\right)+\frac{1}{(RC)^2}}$$

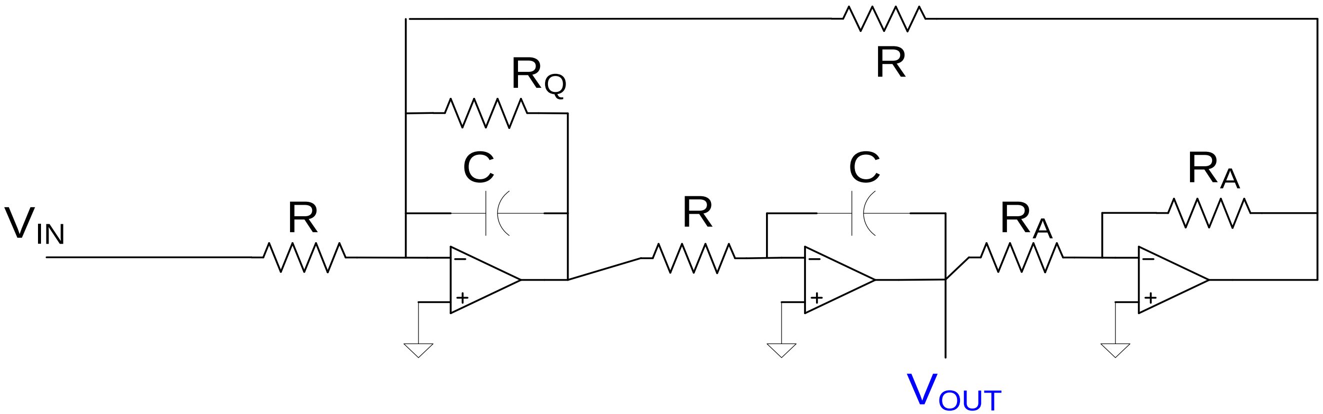
3 degrees of freedom (effectively 2 since dimensionless)

$\omega_0 = ?$

Q = ?

BW = ?

Example 5:



Equal R Equal C (except R_Q)

$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{1}{RC}\frac{s}{s^2+s\left(\left[\frac{R}{R_Q}\right]\frac{1}{RC}\right)+\frac{1}{(RC)^2}}$$

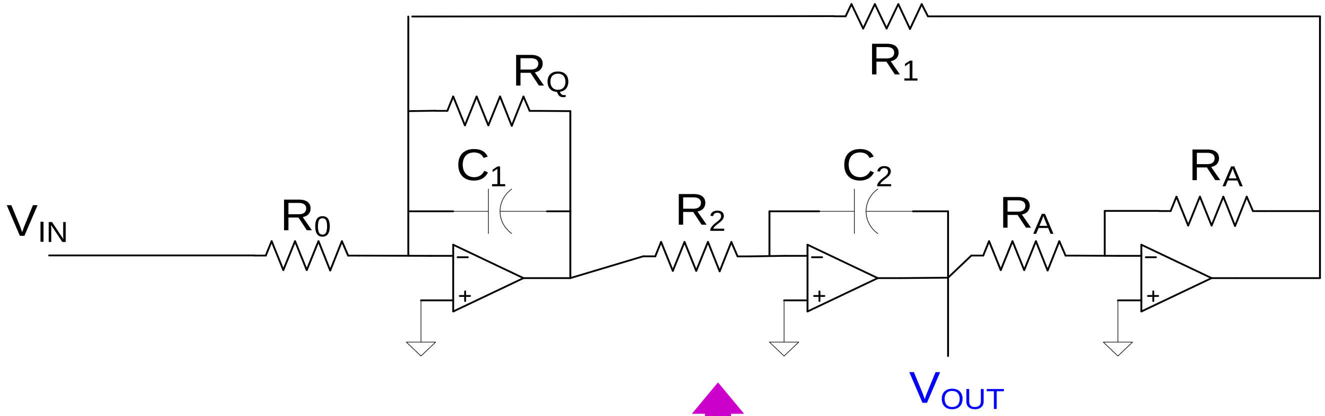
3 degrees of freedom (effectively 2 since dimensionless)

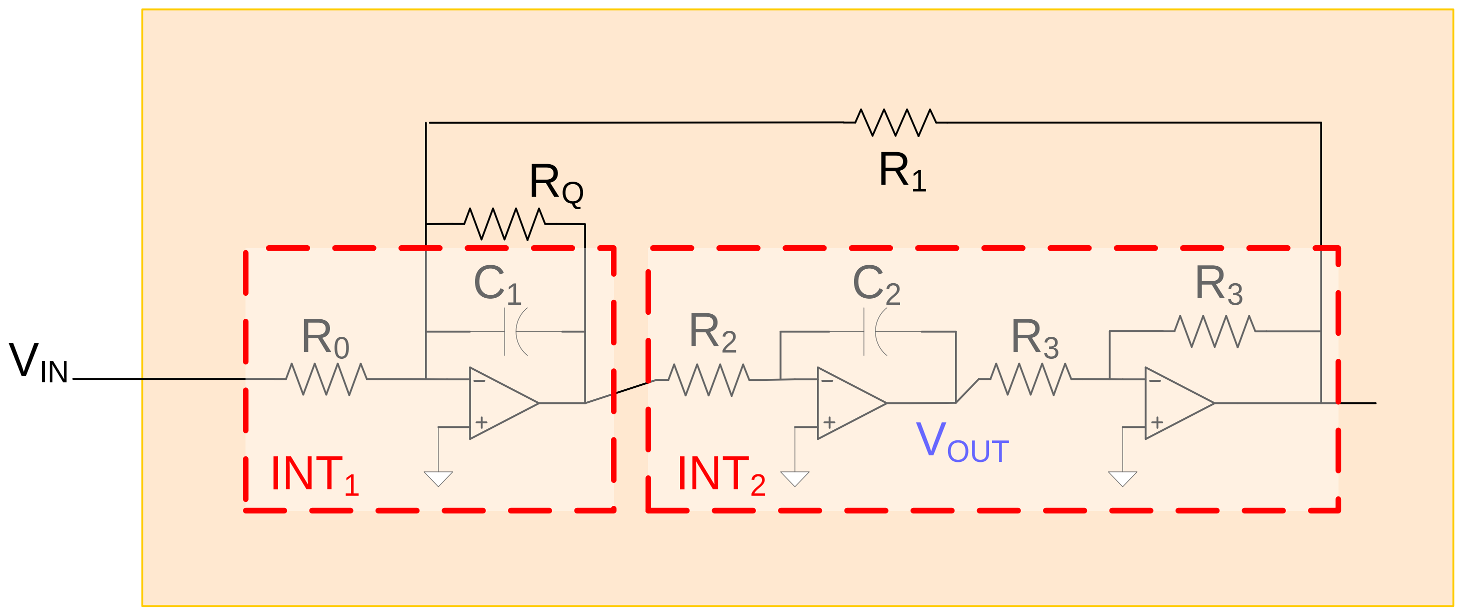
$$\omega_0 = \frac{1}{RC} \qquad Q = \frac{R_Q}{R} \qquad BW = \left[\frac{R}{R_Q}\right]\frac{1}{RC}$$

Simple design process (sequential but not independent control of ω_0 and Q with Rs, requires more tuning more than one R if Cs fixed)

Modest component spread even for large Q

Example 5:

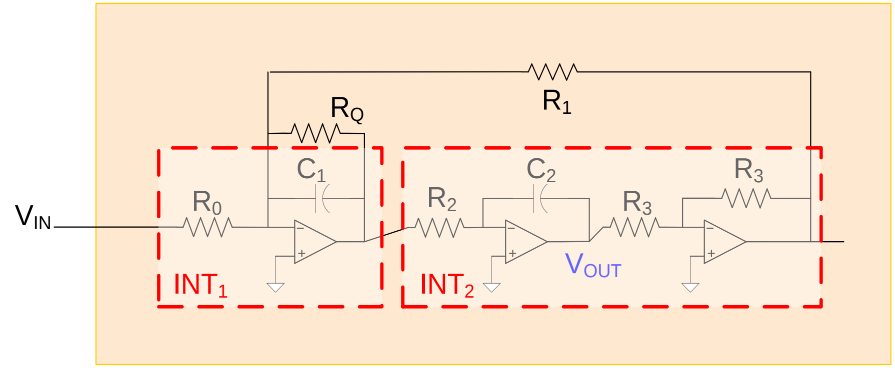


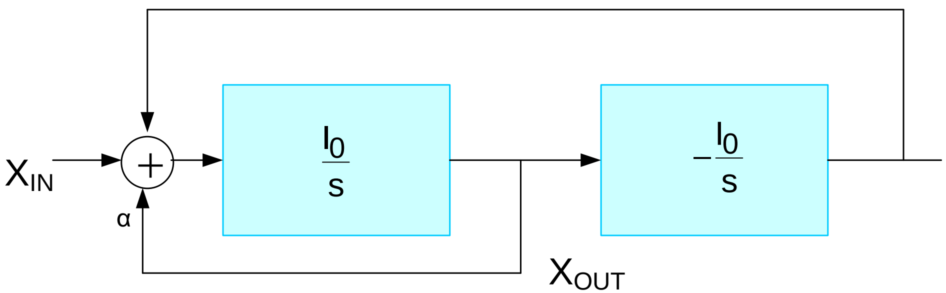


Two Integrator Loop Representation

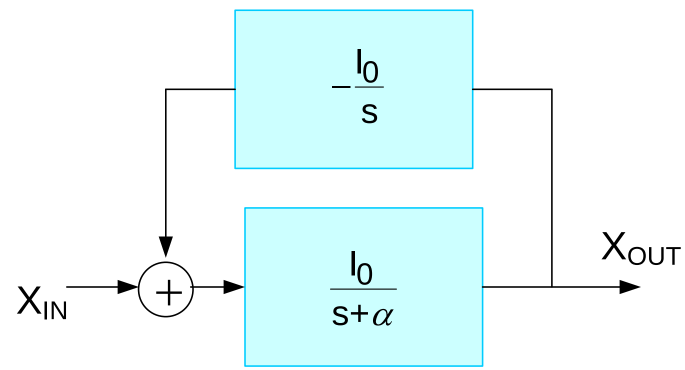
Example 5:

Two Integrator Loop Representation

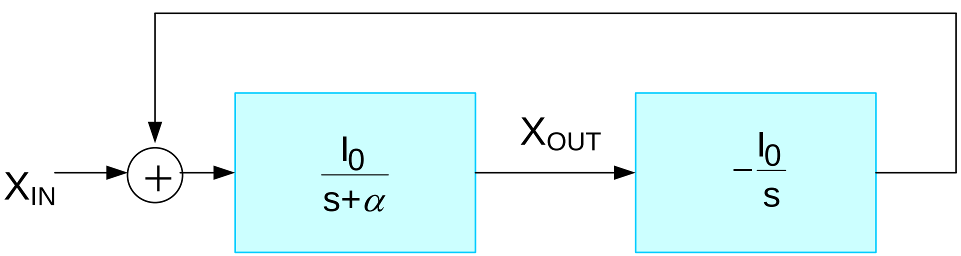




Inverting and Noninverting Integrator Loop

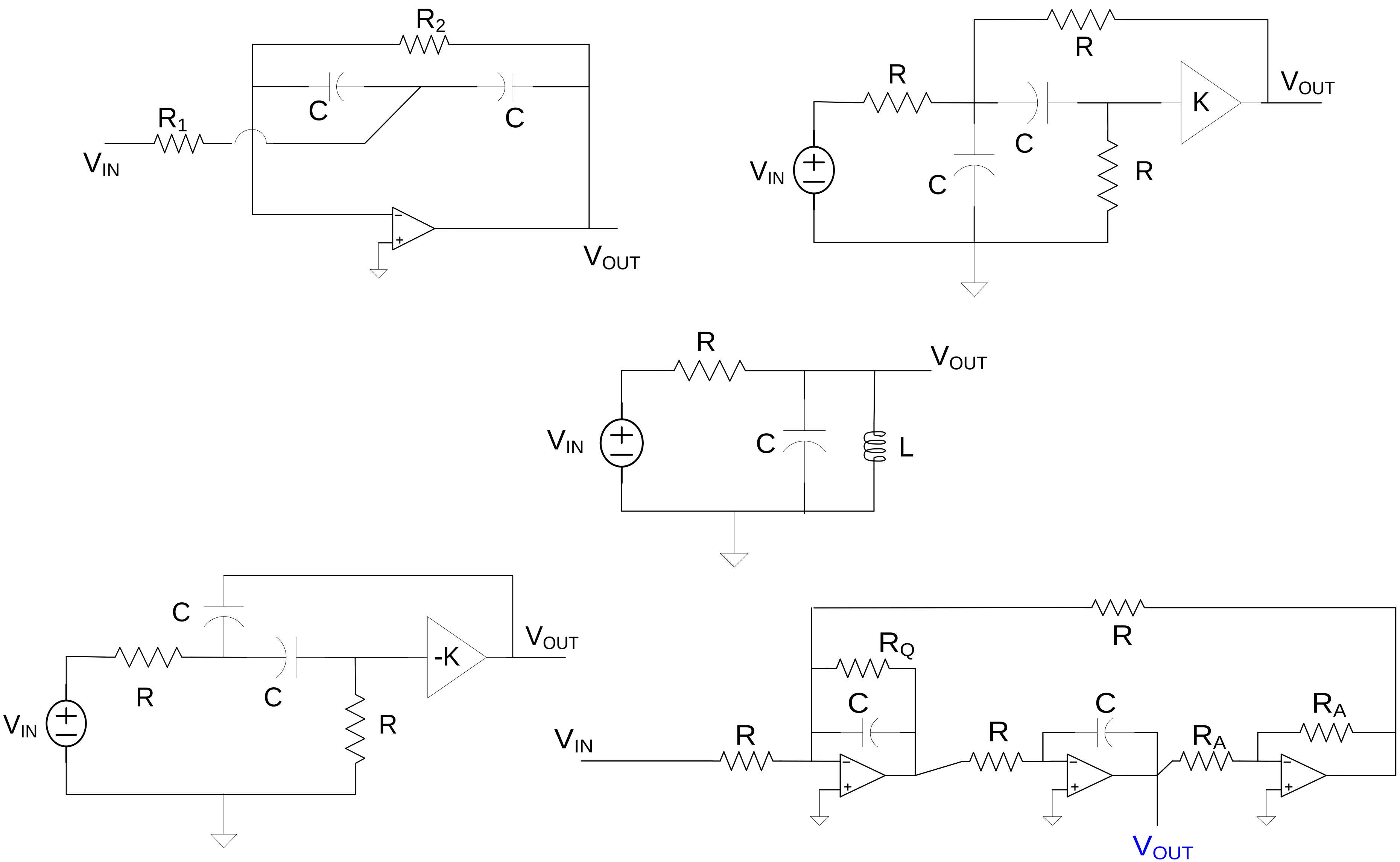


Integrator and Lossy Integrator Loop



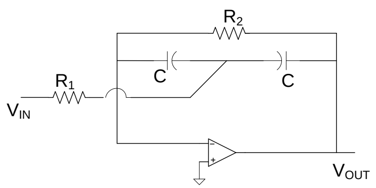
Integrator and Lossy Integrator Loop

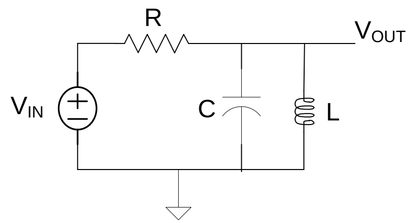
How does the performance of these bandpass filters compare?

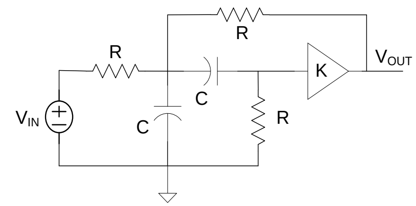


Ideally, all give same performance (within a gain factor)

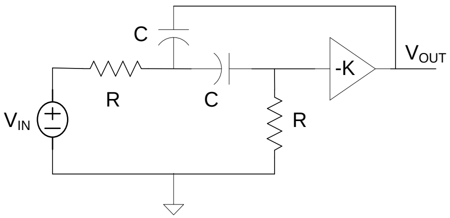
How does the performance of these bandpass filters compare?

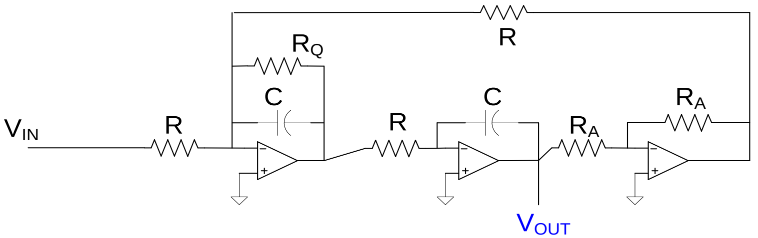






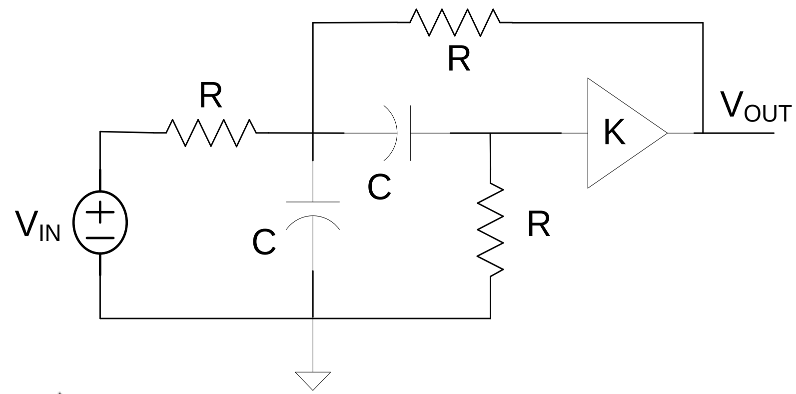
- Component Spread
- Number of Op Amps
- Is the performance strongly dependent upon how DOF are used?
- Ease of tunability/calibration (but practical structures often are not calibrated)
- Total capacitance or total resistance
- Power Dissipation
- Sensitivity
- Effects of Op Amps



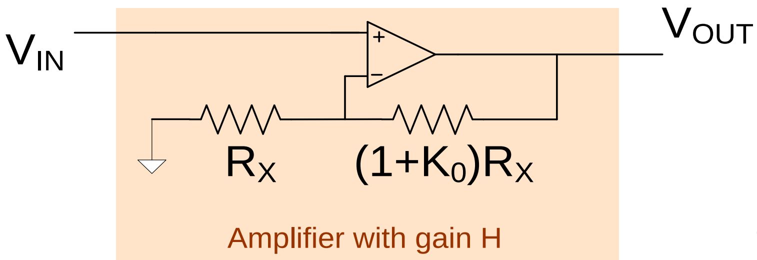


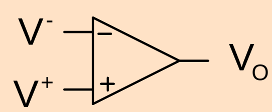
Consider effects of Op Amp on +KRC Bandpass with Equal R, Equal C

$$\omega_0 = \frac{\sqrt{2}}{RC} \qquad Q = \frac{\sqrt{2}}{4-K}$$



Assume K realized with standard Op Amp Circuit





$$A(s) = \frac{V_0}{V^+ - V^-} = \frac{GB}{s}$$

$$K(s) = \frac{K_0}{1 + \frac{K_0}{GB}s}$$

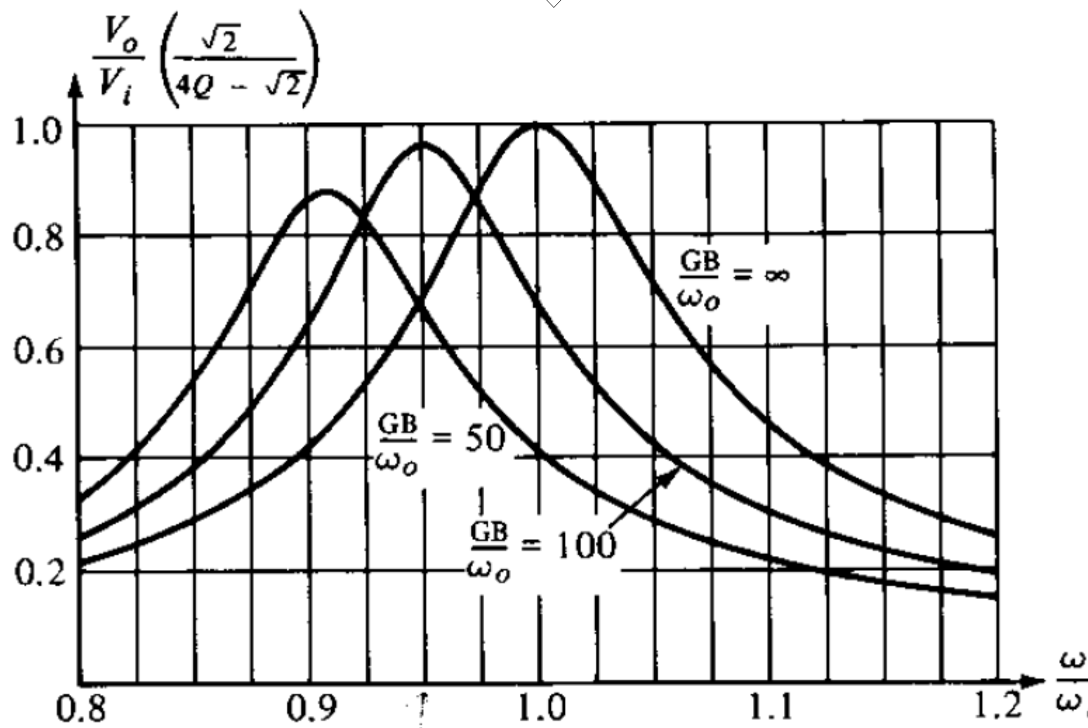
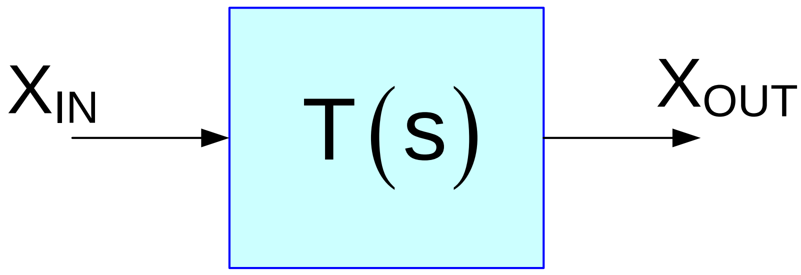


Fig. 11-4 Effect of GB on the magnitude curve for $Q = 10$

- Significant shift in peak frequency
- BW does not change very much
- Some drop in gain at peak frequency

Practically, GB/ω_0 must be must less than 100

Consider 2nd Order Lowpass Biquads

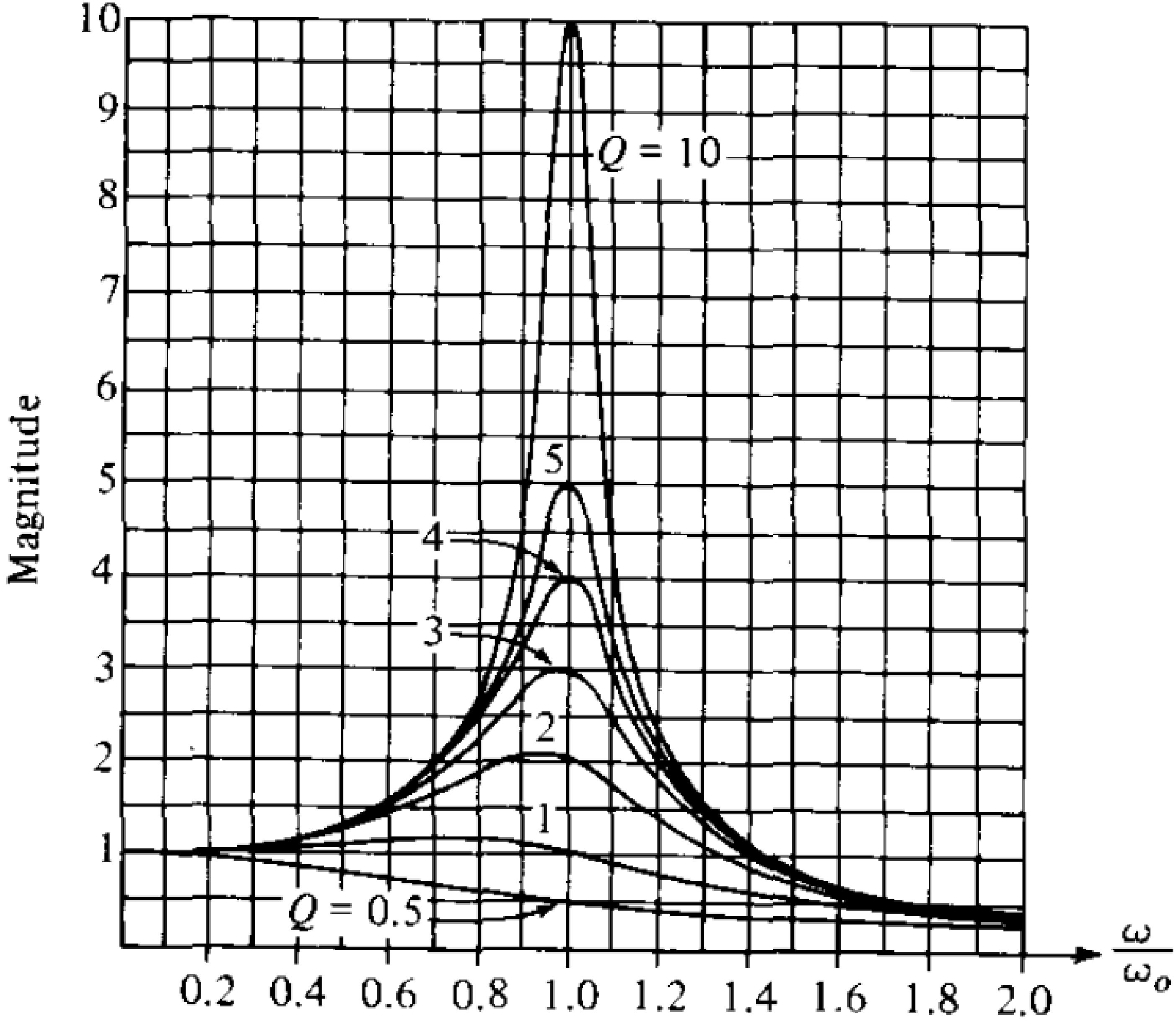


$$\left|T(s)\right| = H\frac{\omega_0^2}{s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2}$$

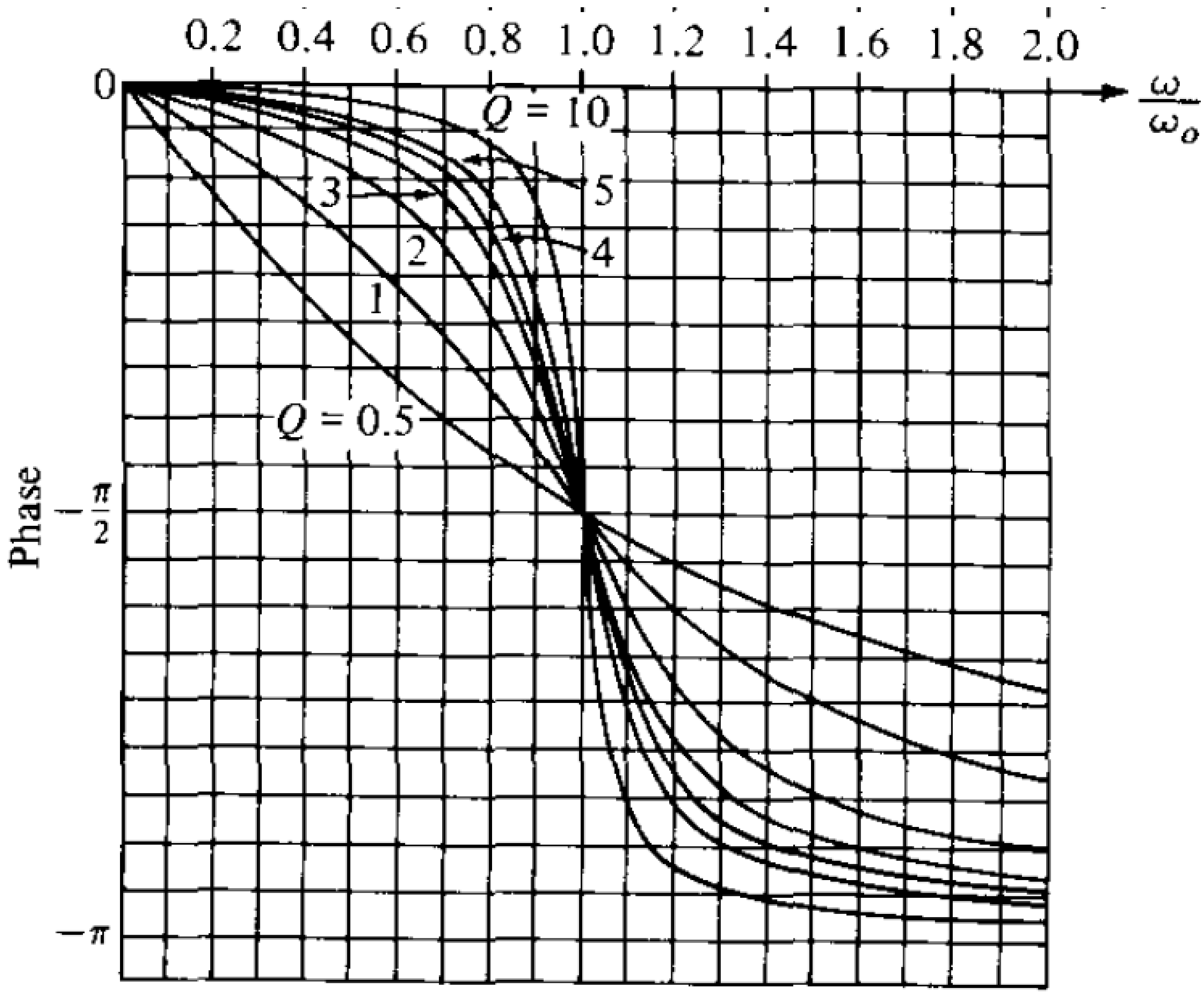
$$BW = \omega_B - \omega_A \neq \frac{\omega_0}{Q}$$

$$\omega_{PEAK} \neq \omega_0$$

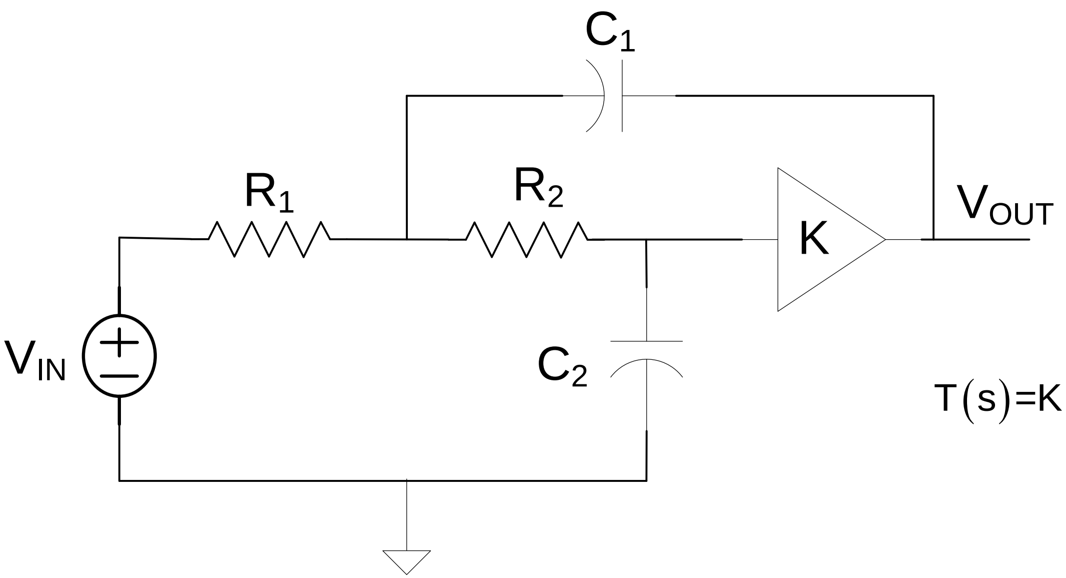
Consider 2nd Order Lowpass Biquads



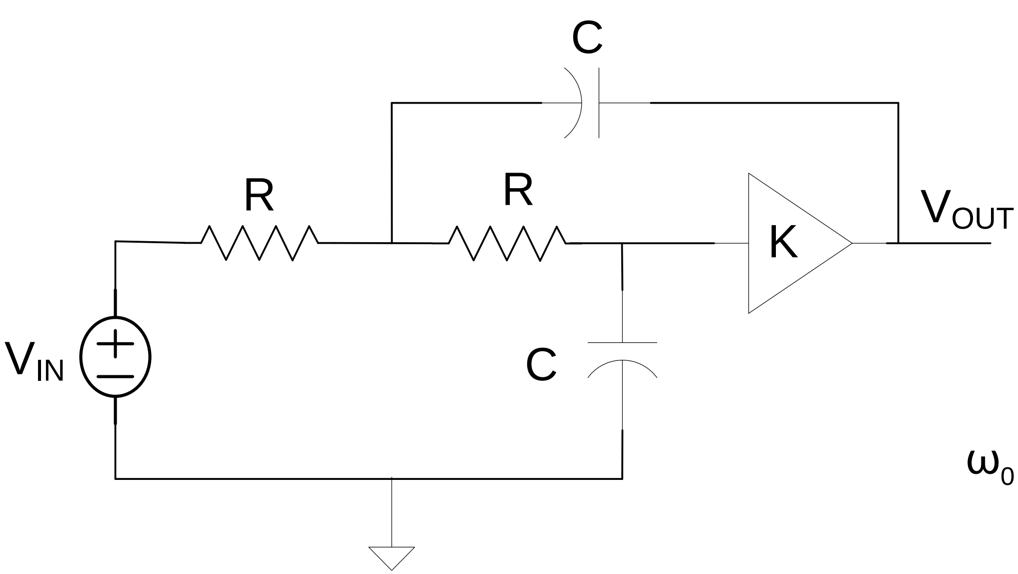
Consider 2nd Order Lowpass Biquads



Example: 2nd Order +KRC Lowpass



$$T(s)=K\frac{\frac{1}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1-K}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}}$$

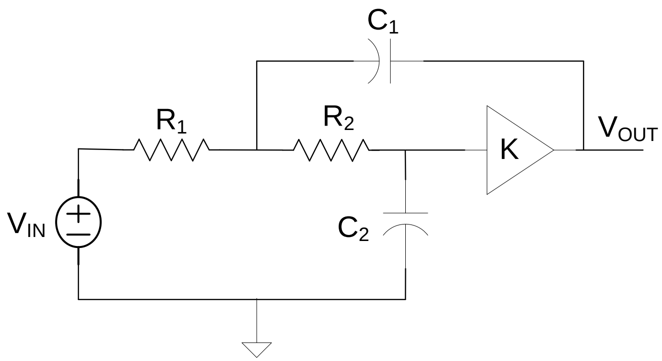


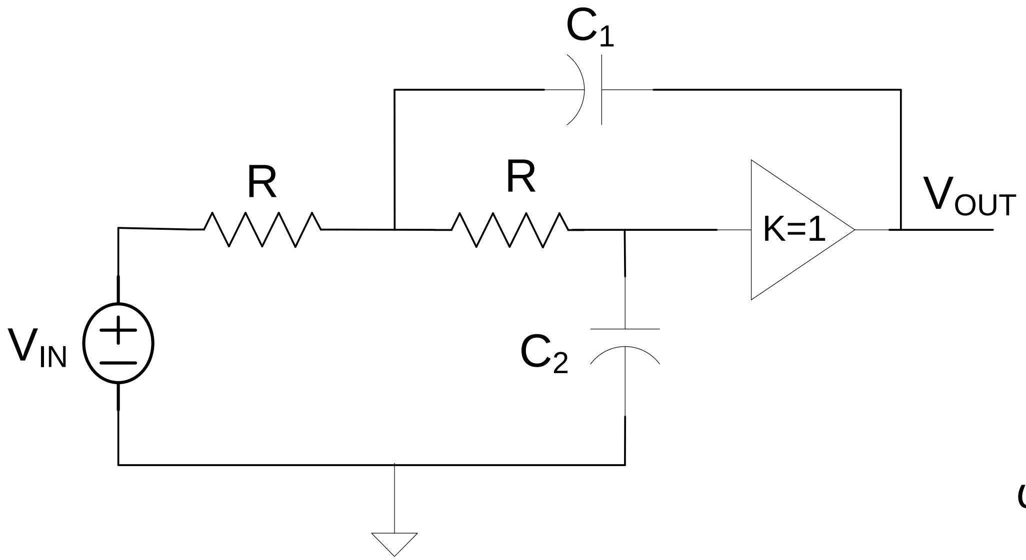
Equal R, Equal C

$$T(s)=K\frac{\frac{1}{R^2C^2}}{s^2+s\left[\frac{(3-K)}{RC}\right]+\frac{1}{R^2C^2}}$$

$$\omega_0=\frac{1}{RC} \qquad Q=\frac{1}{3-K}$$

Example: 2nd Order +KRC Lowpass

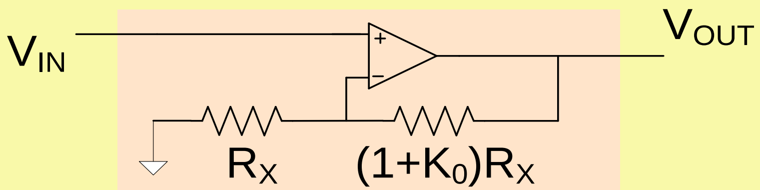




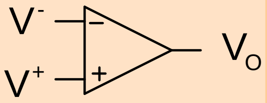
Equal R, K=1

$$T(s)=K\frac{\frac{1}{R^2C_1C_2}}{s^2+s\left[\frac{2}{RC_1}\right]+\frac{1}{R^2C_1C_2}}$$

$$\omega_0=\frac{1}{R\sqrt{C_1C_2}} \qquad Q=\frac{1}{2}\sqrt{\frac{C_1}{C_2}}$$



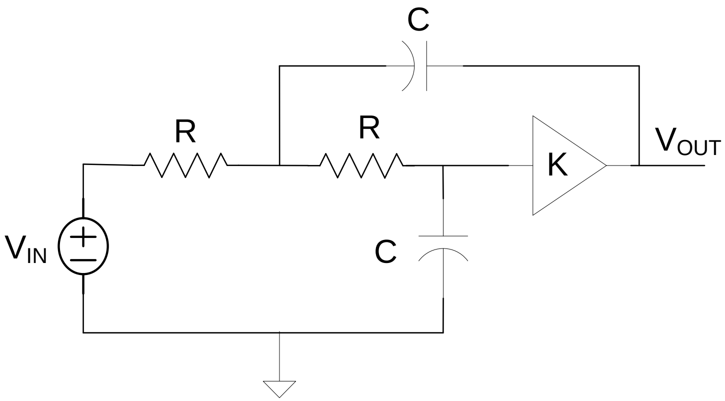
Amplifier with gain H

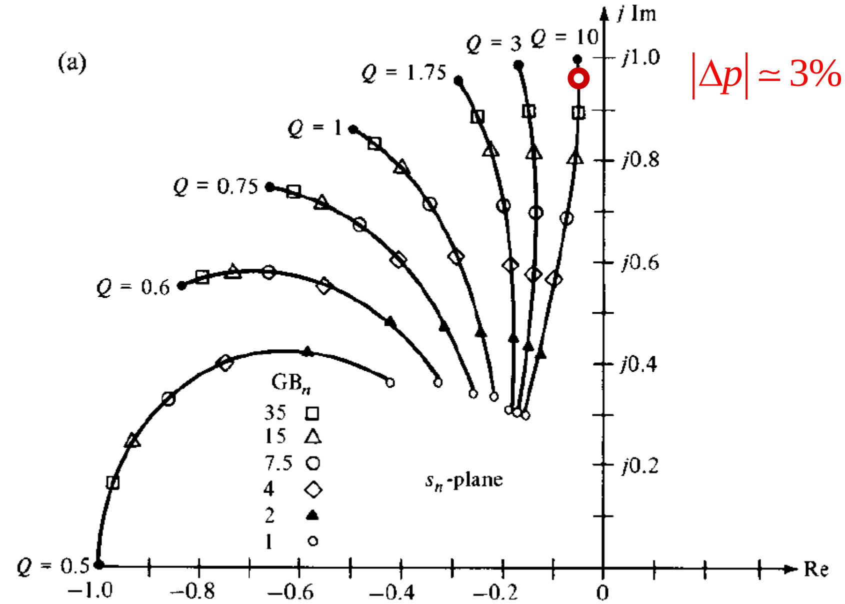


$$A(s)=\frac{V_0}{V^+-V^-}=\frac{GB}{s}$$

$$K(s)=\frac{K_0}{1+\frac{K_0}{GB}s}$$

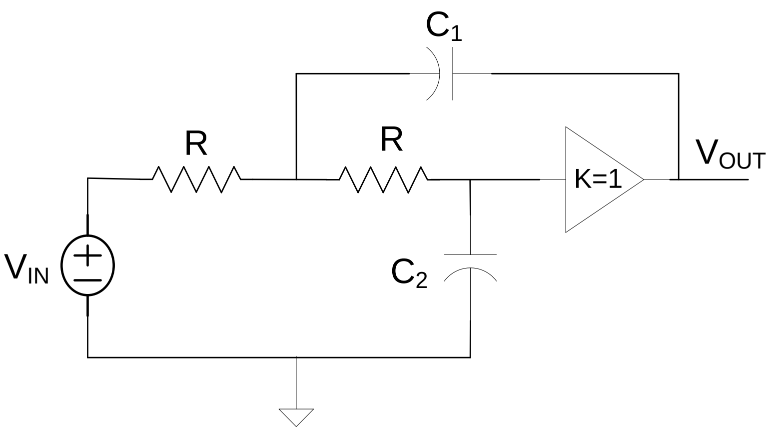
Example: 2nd Order +KRC Lowpass

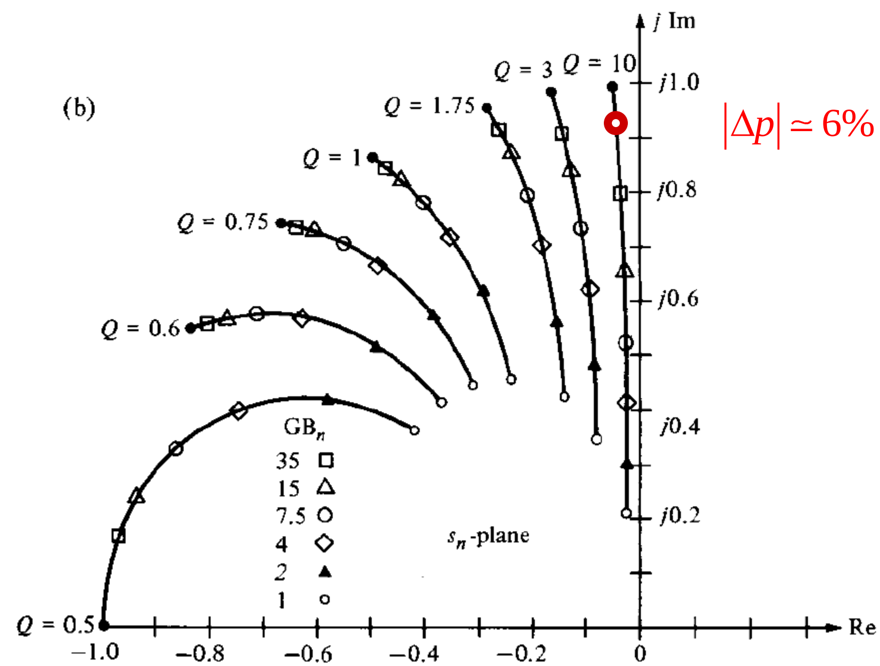




Equal R, Equal C

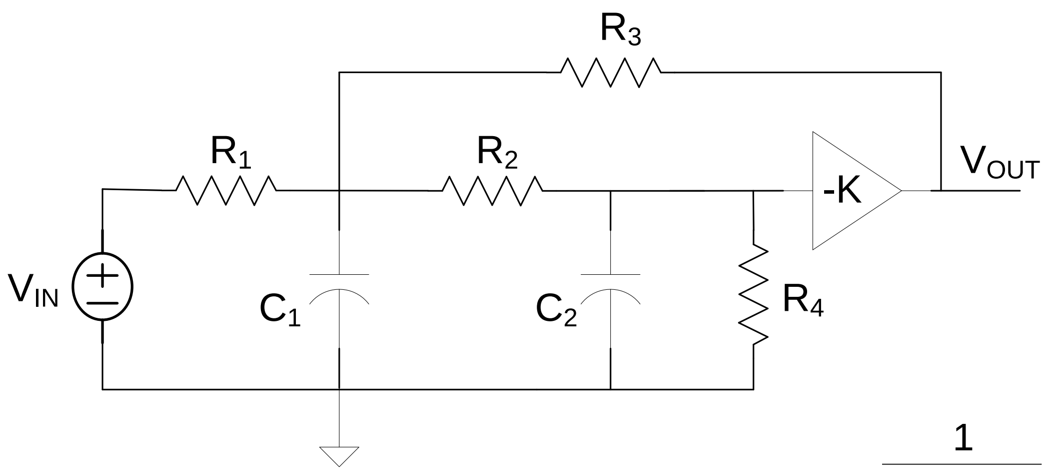
consider ⊙ ↔ $GB_n = \frac{GB}{\omega_0} = .01$



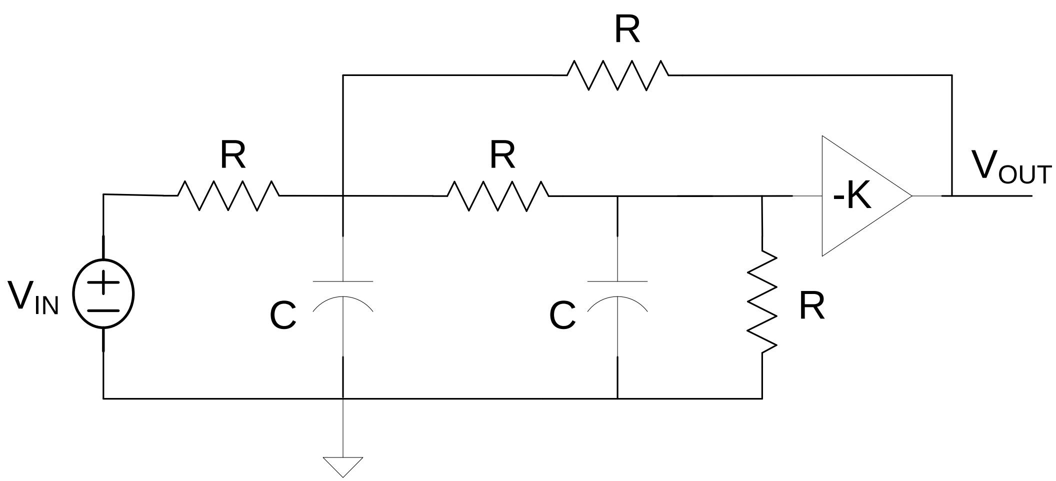


Equal R, K=1

Example: 2nd Order -KRC Lowpass



$$T(s) = -K \frac{\dfrac{1}{R_1R_2C_1C_2}}{s^2 + s\left[\dfrac{1}{R_1C_1}\left(1+\dfrac{R_1}{R_3}\right) + \dfrac{1}{R_4C_2} + \dfrac{1}{R_2C_2}\left(1+\dfrac{C_2}{C_1}\right)\right] + \left[\dfrac{1+(R_1/R_3)(1+K)+(R_1/R_4)\left(1+(R_2/R_3)+(R_2/R_1)\right)}{R_1R_2C_1C_2}\right]}$$

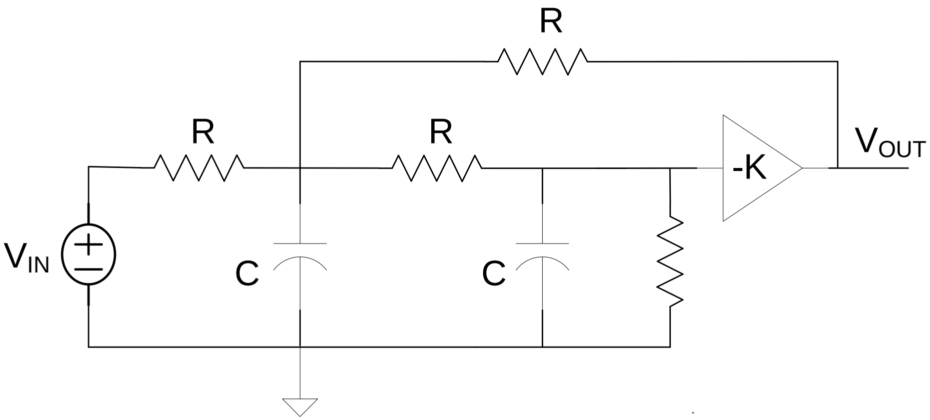


Equal R, Equal C

$$T(s) = -K \frac{\dfrac{1}{R^2C^2}}{s^2 + s\left[\dfrac{5}{RC}\right] + \left[\dfrac{5+K}{R^2C^2}\right]}$$

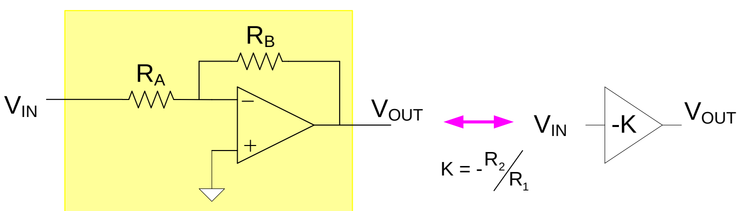
$$\omega_0 = \frac{\sqrt{5+K}}{RC} \qquad Q = \frac{\sqrt{5+K}}{5}$$

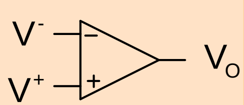
Example: 2nd Order -KRC Lowpass



consider ⊙ ⟷ $GB_n = \frac{GB}{\omega_0} = .01$

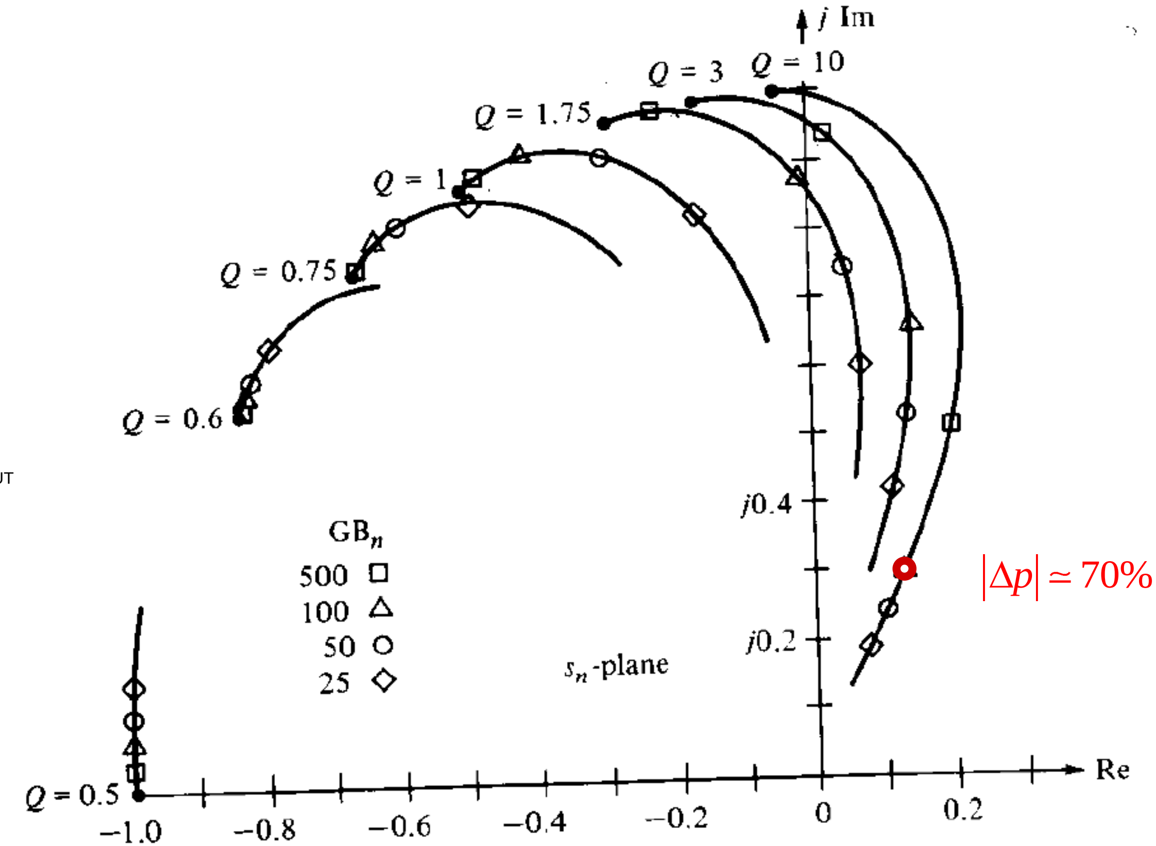
$$\omega_0 = \frac{\sqrt{5+K}}{RC} \qquad Q = \frac{\sqrt{5+K}}{5}$$





$$A(s) = \frac{V_0}{V^+ - V^-} = \frac{GB}{s}$$

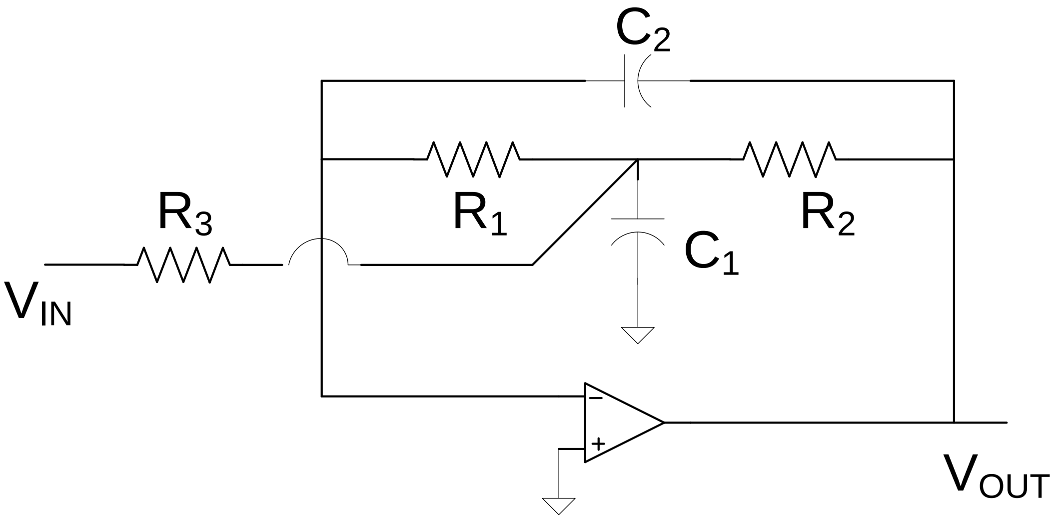
$$K(s) = -\frac{K_0}{1+\frac{(1+K_0)s}{GB}}$$



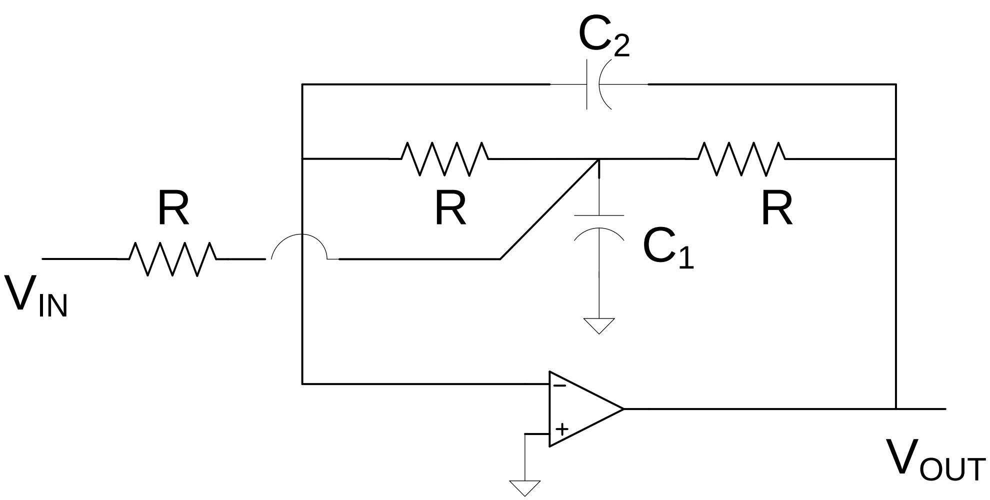
Poles "move" towards RHP as GB degrades

Even very large values of GB will cause instability

Example: 2nd Bridged-T FB Lowpass



$$T(s) = -\frac{\frac{1}{R_2R_3C_1C_2}}{s^2 + s\frac{1}{C_1}\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]+\frac{1}{R_1R_2C_1C_2}}$$

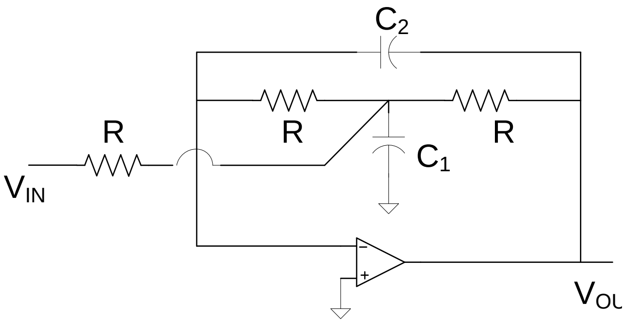


Equal R

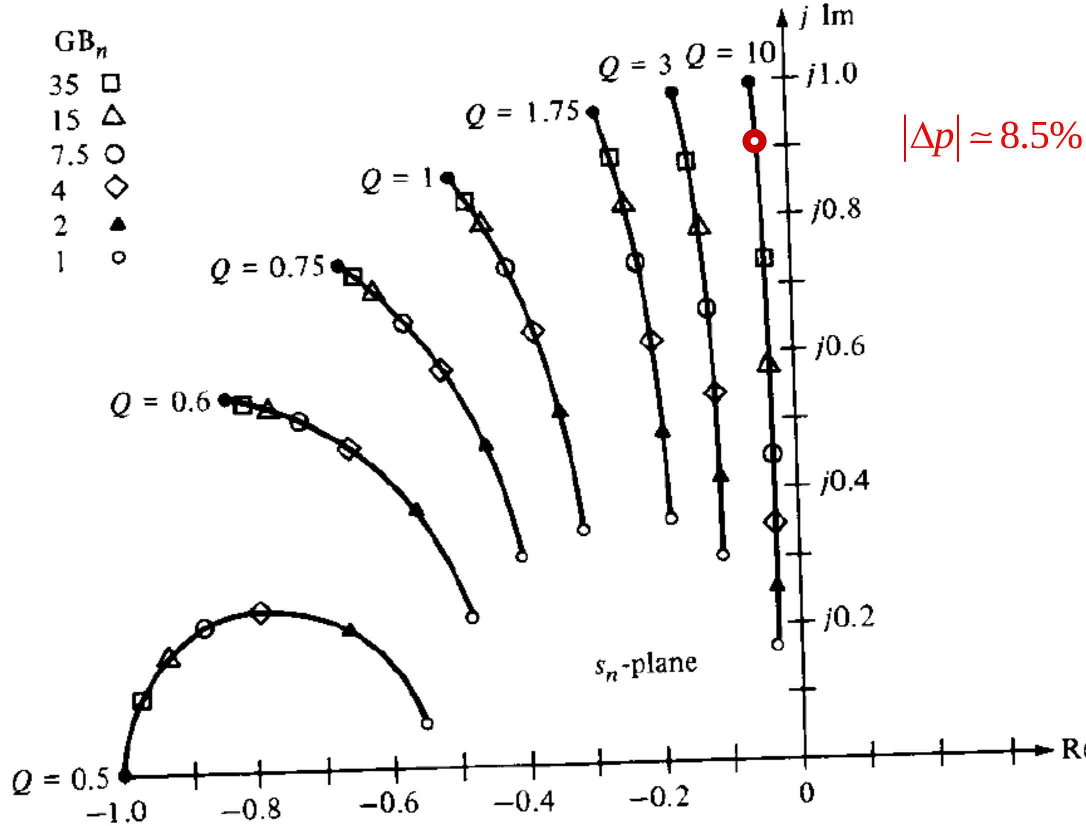
$$T(s) = -\frac{\frac{1}{R^2C_1C_2}}{s^2 + s\left(\frac{3}{RC_1}\right)+\frac{1}{R^2C_1C_2}}$$

$$\omega_0 = \frac{1}{R\sqrt{C_1C_2}} \qquad Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}}$$

Example: 2nd Bridged-T FB Lowpass

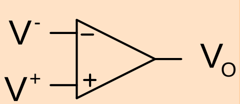


$$\omega_0 = \frac{1}{R\sqrt{C_1C_2}} \qquad Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}}$$



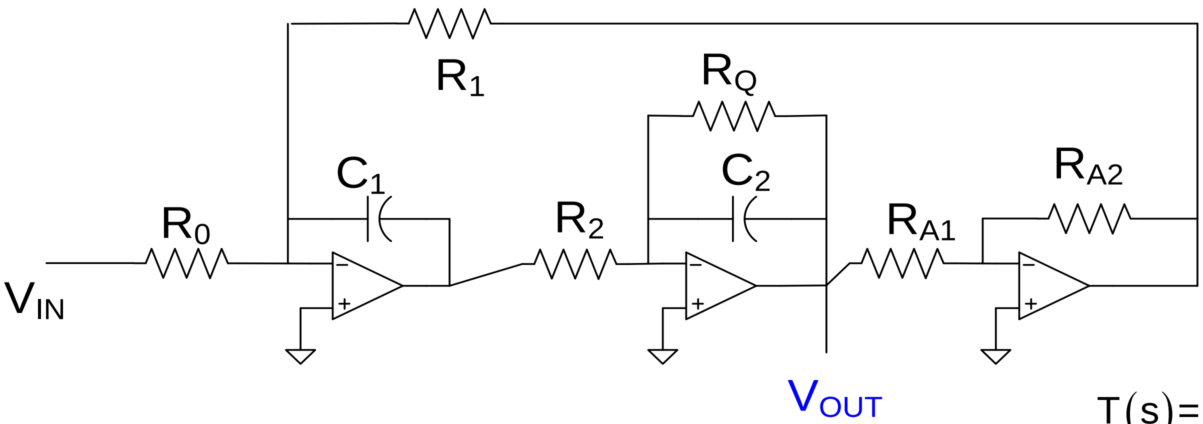
consider

⊙ ⟷ $GB_n = \frac{GB}{\omega_0} = .01$

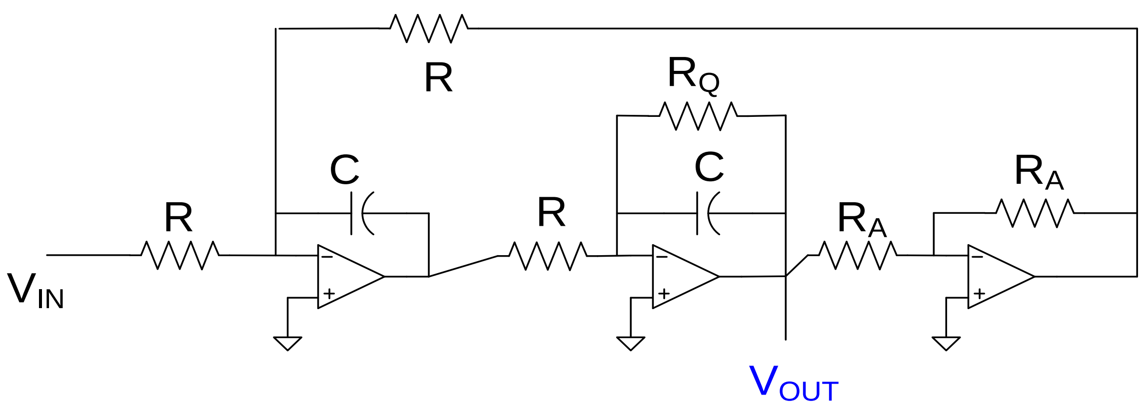


$$A(s) = \frac{V_0}{V^+ - V^-} = \frac{GB}{s}$$

Example: 2nd Two-Integrator-Loop Lowpass



$$T(s) = -\frac{\frac{1}{R_0R_2C_1C_2}}{s^2+s\left(\frac{1}{C_2R_Q}\right)+\frac{R_{A2}/R_{A1}}{R_1R_2C_1C_2}}$$

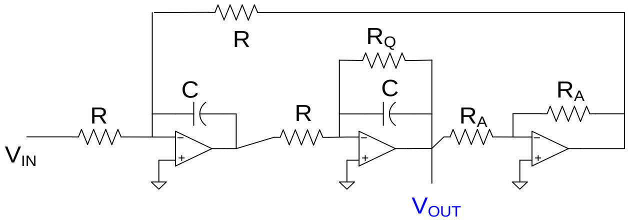


Equal R, Equal C
(except R_Q)

$$T(s) = -\frac{\frac{1}{R^2C^2}}{s^2+s\left(\frac{1}{CR_Q}\right)+\frac{1}{R^2C^2}}$$

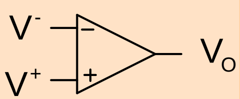
$$\omega_0 = \frac{1}{RC} \qquad Q = \frac{R_Q}{R}$$

Example: 2nd Two-Integrator-Loop Lowpass

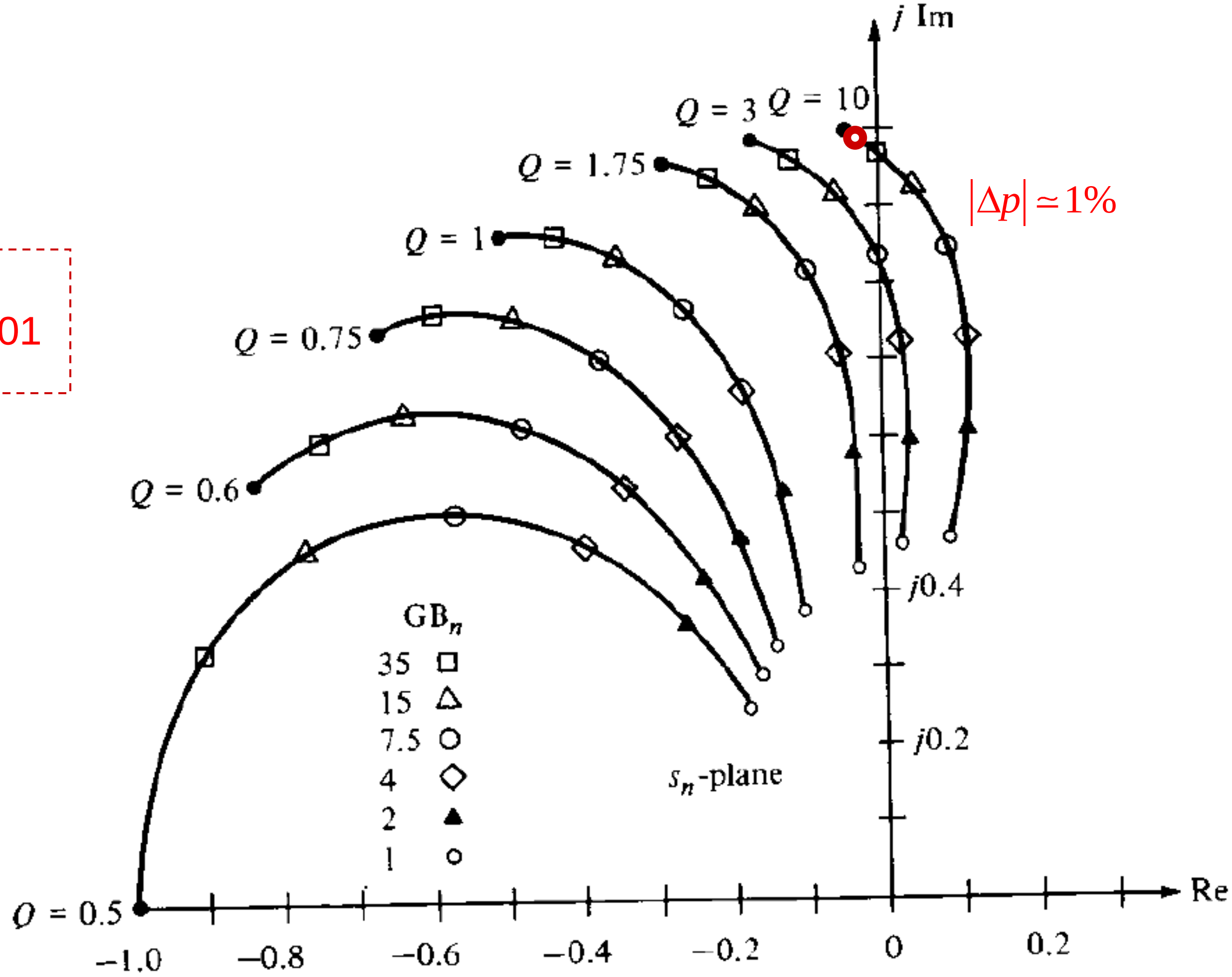


$$\omega_0 = \frac{1}{RC} \qquad Q = \frac{R_Q}{R}$$

consider ⊙ ⟷ $GB_n = \frac{GB}{\omega_0} = .01$

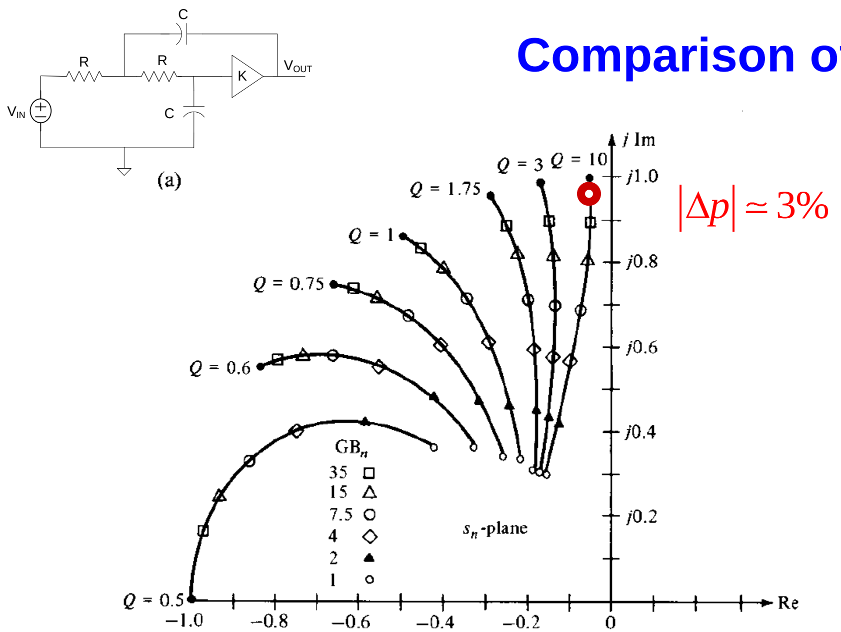


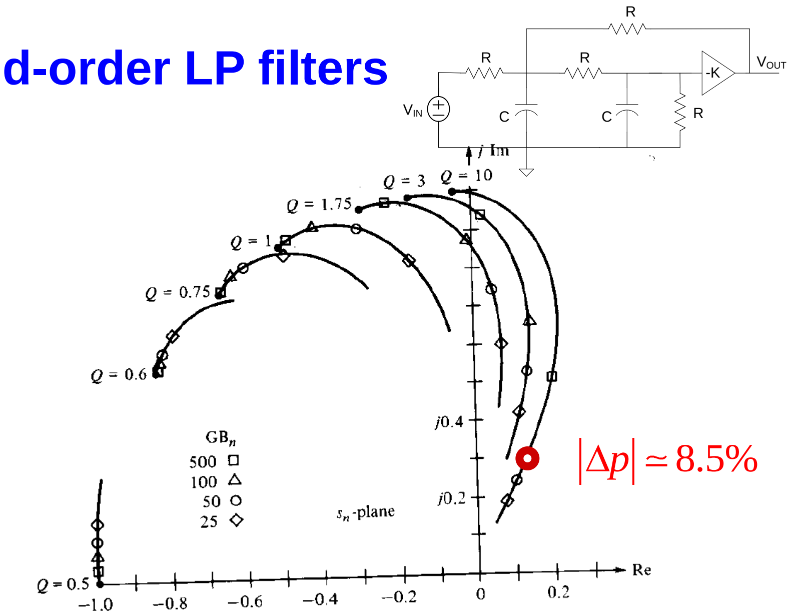
$$A(s) = \frac{V_0}{V^+ - V^-} = \frac{GB}{s}$$

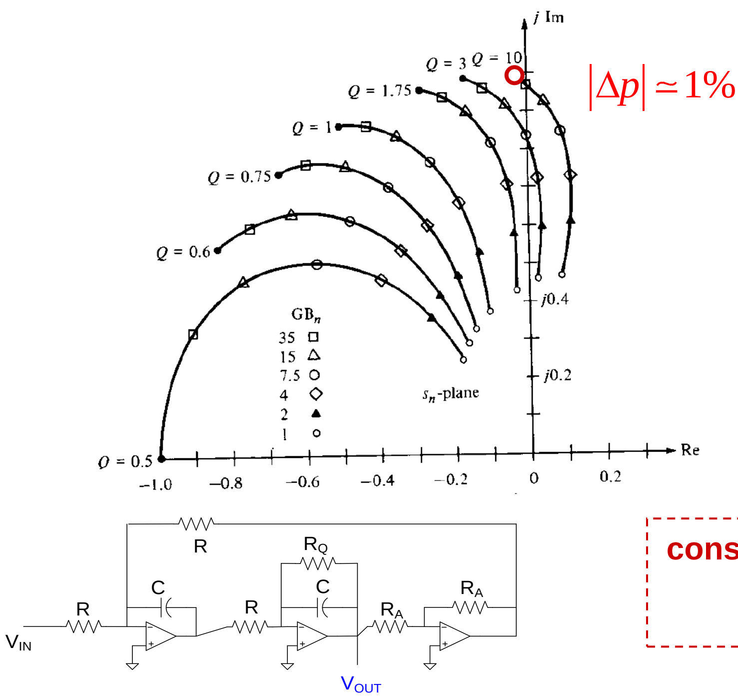


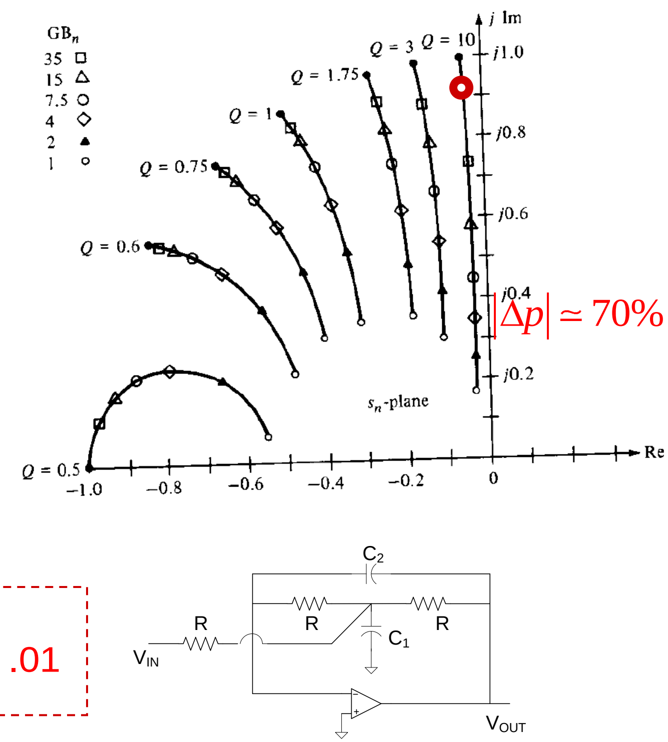
Poles "move" towards RHP as GB degrades

Comparison of 4 second-order LP filters









consider ● ⟷ $GB_n = \dfrac{GB}{\omega_0} = .01$

Some Observations

- Seemingly similar structures have dramatically different sensitivity to frequency response of the Op Amp
- Critical to have enough GB if filter is to perform as desired
- Performance strongly affected by both magnitude and direction of pole movement
- Some structures appear to be totally impractical – at least for larger Q
- Different use of the Degrees of Freedom produces significantly different results

Sensitivity analysis is useful for analytical characterization of the performance of a filter