Sensitivity Functions

- Comparison of Circuits
- Predistortion and Calibration
Sensitivity Comparisons

Consider 5 second-order lowpass filters

(all can realize same $T(s)$ within a gain factor)

Review from last time
Sensitivity Comparisons

Consider 5 second-order lowpass filters
(all can realize same $T(s)$ within a gain factor)

Review from last time

-KRC Lowpass
How do these five circuits compare?

a) From a passive sensitivity viewpoint?
   - If $Q$ is small
   - If $Q$ is large

b) From an active sensitivity viewpoint?
   - If $Q$ is small
   - If $Q$ is large
   - If $\tau \omega_0$ is large
Comparison: Calculate all $\omega_0$ and $Q$ sensitivities

Consider passive sensitivities first

a) – Passive RLC

\[
\begin{align*}
S^{\omega_0}_R &= 0 \\
S^{\omega_0}_L &= -\frac{1}{2} \\
S^{\omega_0}_C &= -\frac{1}{2} \\
S^Q_R &= -1 \\
S^Q_C &= -\frac{1}{2} \\
S^Q_L &= \frac{1}{2}
\end{align*}
\]

\[Q = \frac{1}{R} \sqrt{\frac{L}{C}}\]

\[\omega_0 = \sqrt{\frac{1}{LC}}\]
Case b1: \(+\text{KRC}\) Equal R, Equal C

\[
\omega_0 = \sqrt{\frac{1}{R_1R_2C_1C_2}}
\]

\[
Q = \frac{1}{\left(\sqrt{\frac{R_1C_1}{R_2C_2}} + \sqrt{\frac{R_2C_2}{R_1C_1}} + \sqrt{\frac{R_1C_1}{R_2C_2}} - K\sqrt{\frac{R_1C_1}{R_2C_2}}\right)}
\]

\[
S^\omega_{R_1} = S^\omega_{R_2} = S^\omega_{C_1} = S^\omega_{C_2} = -\frac{1}{2}
\]

\[
S^Q_{R_1} = Q - \frac{1}{2}
\]

\[
S^Q_{R_2} = -Q + \frac{1}{2}
\]

\[
S^Q_{C_1} = 2Q - \frac{1}{2}
\]

\[
S^Q_{C_2} = -2Q + \frac{1}{2}
\]

\[
S^Q_K = 3Q - 1
\]
If \( Q_w = 10 \), what happens when \( R_1 \) increases by 10%?

\[
\frac{\Delta Q}{Q} = S_{R_1} \cdot \frac{\Delta R_1}{R_1} = (\theta - 1/2)(0.01) = 0.095
\]

\[\therefore Q\text{ changes by 9.5\%} \]

\[
\frac{\Delta R_1}{R_1} = 0.1
\]

\[
\frac{\Delta Q}{Q} = S_{R_1} \cdot \frac{\Delta R_1}{R_1} = (9.5)(0.1) = 0.95
\]

\[\therefore Q\text{ changes by 95\%} \]

---

**Actual**: 10 → 11.04 for \( \frac{\Delta R}{R} = 0.01 \)
10 → 105 for \( \frac{\Delta R}{R} = 0.1 \)
Case b2 : +KRC Equal R, K=1

\[ \omega_0 = \sqrt{\frac{1}{R_1 R_2 C_1 C_2}} \quad Q = \frac{1}{\left(\sqrt{\frac{R_1 C_1}{R_2 C_2}} + \sqrt{\frac{R_2 C_2}{R_1 C_1}} + \sqrt{\frac{R_1 C_1}{R_2 C_2}} - K\sqrt{\frac{R_1 C_1}{R_2 C_2}}\right)} \]

\[ S_{R_1}^{\omega_0} = S_{R_2}^{\omega_0} = S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -\frac{1}{2} \quad S_{K}^{\omega_0} = 0 \]

\[ S_{R_1}^{Q} = 0 \]
\[ S_{R_2}^{Q} = 0 \]
\[ S_{C_1}^{Q} = \frac{1}{2} \]
\[ S_{C_2}^{Q} = -\frac{1}{2} \]
\[ S_{K}^{Q} = 2Q^2 \]

\[ \omega_0 = \frac{1}{RC} \]
\[ Q = \frac{1}{2} \sqrt{\frac{C_1}{C_2}} \]
c) Bridged T Feedback

\[ \omega_0 = \sqrt{\frac{1}{R_1 R_2 C_1 C_2}} \]

\[ Q = \frac{1}{\left( \sqrt{\frac{C_2}{C_1}} + \sqrt{\frac{R_1}{R_3}} + \sqrt{\frac{R_2}{R_1}} + \sqrt{\frac{R_1 R_2}{R_3}} \right)} \]

For \( R_1 = R_2 = R_3 = R \)

\[ S_{R_1}^{\omega_0} = S_{R_2}^{\omega_0} = S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -\frac{1}{2} \]

\[ S_{R_3}^{\omega_0} = 0 \]

\[ S_{R_1}^{Q} = -\frac{1}{6} \]

\[ S_{R_2}^{Q} = -\frac{1}{6} \]

\[ S_{R_3}^{Q} = \frac{1}{3} \]

\[ S_{C_1}^{Q} = -\frac{1}{2} \]

\[ S_{C_2}^{Q} = \frac{1}{2} \]

\[ \omega_0 = \frac{3Q}{RC_1} \]

\[ Q = \frac{1}{3} \sqrt{\frac{C_1}{C_2}} \]
d) 2 integrator loop

\[ \omega_0 = \sqrt{\frac{R_4}{R_3 R_0 R_2 C_1 C_2}} \]

\[ Q = \frac{R_Q}{\sqrt{R_0 R_2}} \sqrt{\frac{C_2}{C_1}} \]

For: \( R_0 = R_1 = R_2 = R \) \hspace{1cm} \( C_1 = C_2 = C \) \hspace{1cm} \( R_3 = R_4 \)

\[ S_{R_1}^{\omega_0} = S_{R_2}^{\omega_0} = S_{R_3}^{\omega_0} = S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} - \frac{1}{2} \]
\[ S_{R_4}^{\omega_0} = \frac{1}{2} \]

\[ S_{R_1}^{Q} = S_{R_2}^{Q} = S_{R_3}^{Q} = S_{C_1}^{Q} = -\frac{1}{2} \]
\[ S_{R_4}^{Q} = S_{C_2}^{Q} = \frac{1}{2} \]

\[ S_{R_2}^{Q} = 1 \]
\[ S_{R_0}^{Q} = 0 \]

\[ \omega_0 = \frac{1}{RC} \]

\[ Q = \frac{R_Q}{R} \]
d) -KRC passive sensitivities

\[ \omega_0 = \sqrt{\frac{1+(R_1/R_3)(1+K)+(R_1/R_4)(1+R_2/R_3+R_2/R_1)}{R_1R_2C_1C_2}} \]

\[ Q = \frac{\sqrt{5+K_0}}{5} \]

\[ \omega_0 = \frac{\sqrt{5+K}}{RC} \]

For \( R_1=R_2=R_3=R_4=R, \ C_1=C_2=C \)

\[ S_{R_1}^{\omega_0} = -\frac{1}{25Q^2} \quad S_{R_2}^{\omega_0} = -\frac{1}{2} + \frac{1}{25Q^2} \quad S_{R_3}^{\omega_0} = -\frac{1}{2} + \frac{3}{50Q^2} \]

\[ S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -\frac{1}{2} \quad S_{R_4}^{\omega_0} = -\frac{3}{50Q^2} \quad S_K^{\omega_0} = \frac{1}{2} + \frac{1}{10Q^2} \]

\[ S_{R_1}^Q = \frac{1}{5} - \frac{1}{25Q^2} \quad S_{R_2}^Q = -\frac{1}{10} + \frac{1}{25Q^2} \quad S_{R_3}^Q = -\frac{3}{10} + \frac{3}{50Q^2} \]

\[ S_{R_4}^Q = \frac{1}{5} - \frac{3}{50Q^2} \quad S_{C_2}^Q = -\frac{1}{10} \quad S_{C_1}^Q = \frac{1}{10} \quad S_K^Q = \frac{1}{2} - \frac{1}{10Q^2} \]
### Passive Sensitivity Comparisons

<table>
<thead>
<tr>
<th>Architecture</th>
<th>$S_{x}^{\omega_0}$</th>
<th>$S_{x}^{Q}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passive RLC</td>
<td>$\leq \frac{1}{2}$</td>
<td>1,1/2</td>
</tr>
<tr>
<td>+KRC</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equal R, Equal C (K=3-1/Q)</td>
<td>0,1/2</td>
<td>Q, 2Q, 3Q</td>
</tr>
<tr>
<td>Equal R, K=1 (C_1=4Q^2C_2)</td>
<td>0,1/2</td>
<td>0,1/2, 2Q^2</td>
</tr>
<tr>
<td>Bridged-T Feedback</td>
<td>0,1/2</td>
<td>1/3,1/2, 1/6</td>
</tr>
<tr>
<td>Two-Integrator Loop</td>
<td>0,1/2</td>
<td>1,1/2, 0</td>
</tr>
</tbody>
</table>

- KRC

Substantial Differences Between (or in) Architectures
How do active sensitivities compare?

\[ S^\omega_\pm = ? \quad S^\Omega_\pm = ? \]

Recall \[ S^f_x = \frac{\partial f}{\partial x} \frac{x}{f} \]

so \[ \frac{\partial f}{f} \approx \frac{x}{x} S^f_x \]

but if \( x \) is ideally \( 0 \), not useful

\[ S^f_x = \frac{\partial f}{\partial x} \]

\[ \frac{\partial f}{f} \approx S^f_x \frac{\Delta x}{f} \]
Where we are at with sensitivity analysis:

Considered a group of five second-order filters

Passive Sensitivity Analysis

- Closed form expressions were obtained for $\omega_0$ and $Q$
- Tedious but straightforward calculations provided passive sensitivities directly from the closed form expressions

Active Sensitivity Analysis

- Closed form expressions for $\omega_0$ and $Q$ are very difficult or impossible to obtain

If we consider higher-order filters

Passive Sensitivity Analysis

- Closed form expressions for $\omega_0$ and $Q$ are very difficult or impossible to obtain for many useful structures

Active Sensitivity Analysis

- Closed form expressions for $\omega_0$ and $Q$ are very difficult or impossible to obtain

Need some better method for obtaining sensitivities when closed-form expressions are difficult or impractical to obtain or manipulate!!
Relationship between pole sensitivities and $\omega_0$ and Q sensitivities

$p = -\alpha + j\beta$

$D_2(s) = (s-p)(s-p^*)$

$D_2(s) = (s+\alpha-j\beta)(s+\alpha+j\beta)$

$D_2(s) = s^2 + s(2\alpha) + (\alpha^2 + \beta^2)$
Relationship between active pole sensitivities and \( \omega_0 \) and Q sensitivities

Define \( D(s) = D_0(s) + t D_1(s) \) (from bilinear form of T(s))

Recall: \( s^p_{\tau} = \frac{-D_1(p)}{\frac{\partial D(s)}{\partial s}} \bigg|_{s=p, \tau=0} \)

Theorem: \( \Delta p \cong \tau s^p_{\tau} \)

Theorem: \( \Delta \alpha \cong \tau \text{Re}(s^p_{\tau}) \)
\( \Delta \beta \cong \tau \text{Im}(s^p_{\tau}) \)

Theorem:
\[
\frac{\Delta \omega_0}{\omega_0} \approx \frac{1}{2Q \omega_0} + \sqrt{1 - \frac{1}{4Q^2}} \frac{\Delta \beta}{\omega_0} \\
\frac{\Delta Q}{Q} \approx -2Q \left(1 - \frac{1}{4Q^2}\right) \frac{\Delta \alpha}{\omega_0} + \sqrt{1 - \frac{1}{4Q^2}} \frac{\Delta \beta}{\omega_0}
\]

Claim: These theorems, with straightforward modification, also apply to other parameters (R, C, L, K, \ldots) where, \( D_0(s) \) and \( D_1(s) \) will change since the parameter is different
Table 10-1  KRC Realization
(see Fig. 10-3b)

<table>
<thead>
<tr>
<th>Equal-R, Equal-C</th>
<th>Unity-gain, Equal-R</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_c = \frac{1}{RC}$, $Q = \frac{1}{3 - K}$</td>
<td>$\omega_c = \frac{1}{R \sqrt{C_1 C_2}}$, $Q = \frac{1}{2} \sqrt{\frac{C_1}{C_2}}$</td>
</tr>
<tr>
<td>$V_c = \frac{\omega_c^2}{s^2 + \frac{\omega_c}{Q} + \omega_c^2 + \frac{s}{GB} [s^2 + s \omega_c (2Q + \frac{1}{Q}) + \omega_c^2]}$</td>
<td>$V_c = \frac{\omega_c^2}{s^2 + \frac{s}{\omega_c} \omega_c + \frac{s}{GB} \omega_c [s^2 + s \omega_c (2Q + \frac{1}{Q}) + \omega_c^2]}$</td>
</tr>
<tr>
<td>$-\frac{\Delta \omega_c}{\omega_c} \approx \frac{1}{2Q} \left( 3 - \frac{1}{Q} \right)^2 \frac{\omega_c}{GB}$, $\frac{\Delta \beta}{\omega_c} \approx -\frac{1}{2} \left( 3 - \frac{1}{Q} \right)^2 \frac{1 - \frac{1}{4Q^2}}{\sqrt{1 - \frac{1}{4Q^2}}} \frac{\omega_c}{GB}$</td>
<td>$-\frac{\Delta \omega_c}{\omega_c} \approx -Q \frac{\omega_c}{GB}$, $\frac{\Delta \beta}{\omega_c} \approx -Q \frac{1 - \frac{1}{2Q^2}}{\sqrt{1 - \frac{1}{4Q^2}}} \frac{\omega_c}{GB}$</td>
</tr>
<tr>
<td>$\frac{\Delta \omega_c}{\omega_c} \approx -\frac{1}{2} \left( 3 - \frac{1}{Q} \right)^2 \frac{\omega_c}{GB}$, $\frac{\Delta Q}{Q} \approx \frac{1}{2} \left( 3 - \frac{1}{Q} \right)^2 \frac{\omega_c}{GB}$</td>
<td>$\frac{\Delta \omega_c}{\omega_c} \approx -Q \frac{\omega_c}{GB}$, $\frac{\Delta Q}{Q} \approx \frac{\omega_c}{GB}$</td>
</tr>
</tbody>
</table>

where

$s_n = s_{\omega_c} = \frac{s}{\omega_c}$, $GB_n = \frac{GB}{\omega_c}$. 
Fig. 10-5a  Plot of upper half-plane root of

\[ s^2 + 2Q \left( \frac{Q G B_a}{3Q - 1} \right) + \frac{Q G B_a}{3Q - 1} = 0 \]  
(Equal-R, equal-C)

Fig. 10-5b  Plot of upper half plane root of

\[ s^2 + 2Q \left( \frac{1 + GB_a}{Q} \right) + \frac{1 + GB_a}{Q} + GB_a = 0 \]  
(Unity-gain, equal-R)
c) Bridged-T structure

Table 10-3 Infinite-gain Realization
(see Fig. 10-10b)

**Equal-R**

\[
\omega_e = \frac{1}{R \sqrt{C_1 C_2}}; \quad Q = \frac{1}{3} \sqrt{\frac{C_1}{C_1}}
\]

\[
\frac{V_s}{V_i} = \frac{\omega_e^2}{s^2 + s \frac{\omega_e}{Q} + \omega_e^2 + \frac{s}{\mathrm{GB}} \left[ s^2 + 3s \omega_e \left( 3Q + \frac{1}{Q} \right) + 2 \omega_e^2 \right]}
\]

\[
-\frac{\Delta \alpha}{\omega_e} \approx \frac{\omega_e}{\mathrm{GB}} \quad -\frac{\Delta \beta}{\omega_e} \approx \frac{3Q - \frac{1}{Q}}{2} \frac{\omega_e}{\mathrm{GB}} \quad -\sqrt{1 - \frac{1}{4Q^2}} \frac{\omega_e}{\mathrm{GB}}
\]

\[
-\frac{\Delta \omega_s}{\omega_s} \approx -\frac{3Q}{2} \frac{\omega_e}{\mathrm{GB}} \quad -\frac{\Delta Q}{Q} \approx \frac{Q}{2} \frac{\omega_e}{\mathrm{GB}}
\]
Fig. 10-12 Plot of upper half-plane root of

\[ s^4 + s^2 \left( 3Q + \frac{1}{Q} + GB_s \right) + s \left( 2 + \frac{GB_s}{Q} \right) + GB_s = 0 \]
d) Two integrator loop architecture

Table 10-4  Three-Amplifier Realization
(see Fig. 10-16)

*Equal-*R (except *R*<sub>2</sub>) and *Equal-*C

\[ \omega_c = \frac{1}{RC}, \quad Q = \frac{R_0}{R} \]

\[ \frac{V_o}{V_i} \approx \frac{\omega_c^3 \left( \frac{2}{GB} s + 1 \right)}{s^2 + s \frac{\omega_c}{Q} + \omega_c^2 + \frac{1}{GB} \left( 4s \left[ s^2 + s \omega_c \left( \frac{1}{2} + \frac{1}{Q} \right) + \frac{\omega_c}{4Q} \right] \right)} \]

\[ \left( \omega_c \approx \frac{\omega_c}{2Q} \right) \]

\[ -\frac{\Delta \alpha}{\omega_c} \approx 2 \left( 1 + \frac{1}{4Q} \right) \frac{\omega_c}{GB}, \quad \frac{\Delta \beta}{\omega_c} \approx -\frac{\left( 1 - \frac{1}{Q} - \frac{1}{4Q^2} \right) \omega_c}{\sqrt{1 - \frac{1}{4Q^2}}} \frac{\omega_c}{GB} \]

\[ \frac{\Delta \omega_c}{\omega_c} = -\frac{\omega_c}{GB}, \quad \frac{\Delta Q}{Q} \approx 4Q \frac{\omega_c}{GB} \]
d) Two integrator loop architecture
Equal-R, Equal-C

\[ \omega_o = \frac{\sqrt{5 + K_0}}{RC}, \quad Q = \frac{\sqrt{5 + K_0}}{5} \]

\[ \frac{V_o}{V_i} = -\frac{\omega_o^2 \left( 1 - \frac{1}{5Q^2} \right)}{s^2 + s \frac{\omega_o}{Q} + \omega_o^2 + \frac{s}{GB} \left[ s^2(25Q^2 - 4) + s\omega_o \left( 20Q - \frac{3}{Q} \right) + \left( 2 - \frac{1}{5Q^2} \right) \omega_o^2 \right]} \]

\[ \left( \omega_o \ll \frac{\omega_o}{2Q} \right) \]

\[ -\frac{\Delta \alpha}{\omega_o} \approx \frac{25Q^2}{2} \left( 1 - \frac{1}{5Q^2} \right) \left( 1 - \frac{6}{25Q^2} \right) \frac{\omega_o}{GB}, \quad \frac{\Delta \beta}{\omega_o} \approx \frac{35Q}{4} \left( 1 - \frac{1}{5Q^2} \right) \left( 1 - \frac{6}{35Q^2} \right) \frac{\omega_o}{GB} \]

\[ \frac{\Delta \omega_o}{\omega_o} \approx \frac{5Q}{2} \left( 1 - \frac{1}{5Q^2} \right) \frac{\omega_o}{GB}, \quad \frac{\Delta Q}{Q} \approx 25Q^3 \left( 1 - \frac{1}{5Q^2} \right) \left( 1 - \frac{7}{5Q^2} \right) \frac{\omega_o}{GB} \]
Active Sensitivity Comparisons

Passive RLC

+KRC

Equal R, Equal C \( (K=3-1/Q) \)

\[
\frac{\Delta \omega}{\omega_0} = -\frac{1}{2} \left( 3 - \frac{1}{Q} \right)^2 \tau \omega_0 \\
\frac{\Delta Q}{Q} = -\frac{1}{2} \left( 3 - \frac{1}{Q} \right)^2 \tau \omega_0
\]

Equal R, \( K=1 \) \( (C_1=4Q^2C_2) \)

\[
\frac{\Delta \omega}{\omega_0} = -Q \tau \omega_0 \\
\frac{\Delta Q}{Q} = Q \tau \omega_0
\]

Bridged-T Feedback

\[
\frac{\Delta \omega}{\omega_0} = -\frac{3}{2} Q \tau \omega_0 \\
\frac{\Delta Q}{Q} = \frac{1}{2} Q \tau \omega_0
\]

Two-Integrator Loop

-KRC

\[
\frac{\Delta \omega}{\omega_0} = -\tau \omega_0 \\
\frac{\Delta Q}{Q} = 4Q \tau \omega_0
\]

Substantial Differences Between Architectures
Are these passive sensitivities acceptable?

\[
\begin{align*}
\left| S_{x}^{\omega_0} \right| & \leq \frac{1}{2} \\
\left| S_{x}^{Q} \right| & = 1,1/2 \\
\end{align*}
\]

Passive RLC

- Equal R, Equal C \quad (K=3-1/Q) \quad 0,1/2
- Equal R, K=1 \quad (C_1=4Q^2C_2) \quad 0,1/2

+KRC

- Equal R, Equal C \quad (K=3-1/Q) \quad 0,1/2
- Equal R, K=1 \quad (C_1=4Q^2C_2) \quad 0,1/2, 2Q^2

Bridged-T Feedback

- \quad 0,1/2
- \quad 1/3,1/2, 1/6

Two-Integrator Loop

- \quad 0,1/2
- \quad 1,1/2, 0

-KRC

less than or equal to 1/2

less than or equal to 1/2
Are these active sensitivities acceptable?

**Active Sensitivity Comparisons**

\[
\frac{\Delta \omega_0}{\omega_0} \quad \frac{\Delta Q}{Q}
\]

**Passive RLC**

+KRC

Equal R, Equal C \((K=3-1/Q)\)
\[-\frac{1}{2\left(3-\frac{1}{Q}\right)^2} \tau \omega_0\]

Equal R, \(K=1\) \((C_1=4Q^2C_2)\)
\[-Q \tau \omega_0\]

**Bridged-T Feedback**

Two-Integrator Loop
\[-\tau \omega_0\]

-KRC
\[\frac{5}{2} Q \tau \omega_0\]

\[\frac{1}{2} Q \tau \omega_0\]

\[\frac{1}{2} Q \tau \omega_0\]

\[4Q \tau \omega_0\]

\[25Q^3 \tau \omega_0\]
Are these sensitivities acceptable?

**Passive Sensitivities:**

\[
\frac{\Delta \omega_0}{\omega_0} \approx S_{\omega_0} \frac{\Delta x}{x}
\]

In integrated circuits, \( \Delta \frac{R}{R} \) and \( \Delta \frac{C}{C} \) due to process variations can be \( K \) 30% or larger due to process variations

Many applications require \( \frac{\Delta \omega_0}{\omega_0} < 0.001 \) or smaller and similar requirements on \( \Delta \frac{Q}{Q} \)

Even if sensitivity is around \( \frac{1}{2} \) or 1, variability is often orders of magnitude too large

**Active Sensitivities:**

All are proportional to \( \tau \omega_0 \)

Some architectures much more sensitive than others

Can reduce \( \tau \omega_0 \) by making GB large but this is at the expense of increased power and even if power is not of concern, process presents fundamental limits on how large GB can be made
What can be done to address these problems?

1. Predistortion

Design circuit so that after component shift, correct pole locations are obtained.

Predistortion is generally used in integrated circuits to remove the bias associated with inadequate amplifier bandwidth.

Tedious process after fabrication since depends on individual components.

Temperature dependence may not track.

Difficult to maintain over time and temperature.

Over-ordering will adversely affect performance.

Seldom will predistortion alone be adequate to obtain acceptable performance. Bell Labs did to this in high-volume production (STAR Biquad).
What can be done to address these problems?

1. Predistortion

Design circuit so that after component shift, correct pole locations are obtained.

Pole shift due to parametric variations (e.g. inadequate GB)
What can be done to address these problems?

1. Predistortion

Design circuit so that after component shift, correct pole locations are obtained

Pre-distorted Pole Location → Desired Pole Location → Actual Pole Location due to parameter variations

Pre-distortion concept
What can be done to address these problems?

1. **Predistortion**

   Design circuit so that after component shift, correct pole locations are obtained.

   - Over-ordering Limitations with Pre-distortion
   - Parasitic Pole Affects Response
   - Predistortion almost always done even if benefits only modest
   - Not effective if significant deviations exist before predistortion.
What can be done to address these problems?

2. **Trimming**

   a) **Functional Trimming**

   • trim parameters of actual filter based upon measurements
   • difficult to implement in many structures
   • manageable for cascaded biquads

   b) **Deterministic Trimming (much preferred)**

   • Trim component values to their ideal value
     - Continuous-trims of resistors possible in some special processes
     - Continuous-trim of capacitors is more challenging
     - Link trimming of Rs or Cs is possible with either metal or switches
   • If all components are ideal, the filter should also be ideal
     - R-trimming algorithms easy to implement
     - Limited to unidirectional trim
     - Trim generally done at wafer level for laser trimming, package for link trims
   • Filter shifts occur due to stress in packaging and heat cycling

   c) **Master-slave reference control (depends upon matching in a process)**

   • Can be implemented in discrete or integrated structures
   • Master typically frequency or period referenced
   • Most effective in integrated form since good matching possible
   • Widely used in integrated form
Master-slave Control (depends upon matching in a process)

- Automatically adjust R in the Master Circuit to match RC to T
- Rely on matching to match RC products in Slave Circuit to T
- Matching can be very good (1% or 0.1% or better)
Master-slave Example:

- Key parameter of integrator is unity gain frequency $T_0 = 1/RC$
- Adjust $R$ in Master Circuit so that $T_0 = 1$ at the input frequency $f$
- With matching, unity gain frequency of all integrators in Slave Circuit will also be 1

$$T(s) = \frac{V_{OUT}}{V_{TEST}} = -\frac{1}{RCs}$$
Master-slave Example:

- Over-ordering will limit accuracy of master-slave approach even if unity gain frequency of master circuit is precisely obtained
- Technique is often used to maintain good control of effective RC products

\[
T(s) = \frac{V_{OUT}}{V_{TEST}} = -\frac{1}{RCs}
\]

\[
T_{ACT}(s) = \frac{V_{OUT}}{V_{TEST}} = -\frac{1}{RCs+\tau} \left( \frac{1}{s+RCs^2} \right)
\]
What can be done to address these problems?

3. Select Appropriate Architecture

   Helps a lot

   Best architectures are not known

   Performance of good architectures often not good enough
What can be done to address these problems?

4. Different Approach for Filter Implementation

- Frequency Referenced Filters
- Switched-Capacitor Filters
- DSP- Based Filter Implementation
- Other Niche Methods
Summary of Sensitivity Observations

• Sensitivity varies substantially from one implementation to another

• Variability too high, even with low sensitivity, for more demanding applications

• Methods of managing high variability
  ➢ Select good structures
  ➢ Trimming
    Functional
    Deterministic
  ➢ Predistortion
    In particular, for active sensitivities
    Useful but not a total solution
  ➢ Frequency Referenced Techniques
    Master-Slave Control
    Depends upon matching
    Can self-trim or self-compensate
    Switched-Capacitor Filters
    AD/digital filter/D/A
  ➢ Alternate Design Approach
    Other methods
Filter Design
Process

Establish Specifications
- possibly $T_D(s)$ or $H_D(z)$
- magnitude and phase characteristics or restrictions
- time domain requirements

Approximation
- obtain acceptable transfer functions $T_A(s)$ or $H_A(z)$
- possibly acceptable realizable time-domain responses

Synthesis
- build circuit or implement algorithm that has response close to $T_A(s)$ or $H_A(z)$
- actually realize $T_R(s)$ or $H_R(z)$

Filter

Where we are at
Filter Design/Synthesis Considerations

Most designs today use one of the following three basic architectures:

**Cascaded Biquads**

\[ T(s) = T_1 T_2 \cdots T_m \]

**Leapfrog**

**Multiple-loop Feedback – One type shown (less popular)**
Filter Design/Synthesis Considerations

Multiple-loop Feedback – Another type

\[
X = V_{IN} - X \cdot \sum_{k=1}^{n} b_{n-k} \left( \frac{I_0}{s} \right)^k
\]

\[
V_{OUT} = X \cdot \sum_{k=0}^{n} a_{n-k} \left( \frac{I_0}{s} \right)^k
\]

\[
T(s) = \frac{\sum_{k=0}^{n} a_{n-k} l_0^k s^{n-k}}{s^n + \sum_{k=1}^{n} b_{n-k} l_0^k s^{n-k}}
\]

- Termed the direct synthesis method
- Directly implements the coefficients in the numerator and denominator
- Approach followed in the Analog Computers
- Not particularly attractive from an overall performance viewpoint
Filter Design/Synthesis Considerations

Cascaded Biquads

\[ T(s) = T_1 T_2 \cdots T_m \]

Leapfrog

Multiple-loop Feedback – One type shown

Will study details of all three types of architectures later

Observation: All filters are comprised of summers, biquads and integrators

Consider now the biquads
Biquad Filters Design Considerations

Several different Biquads were considered and other implementations exist:

Sallen-Key Type (Dependent Sources)

Infinite Gain Amplifiers

Integrator Based Structures

Which type is really used?
Floating Nodes

A node in a circuit is termed a **floating node** if it is not an output node of a ground-referenced voltage-output amplifier (dependent or independent), not connected to a ground-referenced voltage source, or not connected to a ground-referenced null-port.
Parasitic Capacitances on Floating Nodes

Parasitic capacitances ideally have no affect on filter when on a non-floating node but directly affect transfer function when they appear on a floating node.

Parasitic capacitances are invariably large, nonlinear, and highly process dependent in integrated filters. Thus, it is difficult to build accurate integrated filters if floating nodes are present.

Generally avoid floating nodes, if possible, in integrated filters.
Which type of Biquad is really used?

- Not Floating Node
- Floating Node

Sallen-Key Type (Dependent Sources)

Infinite Gain Amplifiers

Integrator Based Structures
Which type of Biquad is really used?

- Not Floating Node
- Floating Node

Integrator-based structures with no floating nodes dominantly used in integrated filters with floating nodes are used.

Some high-frequency or programmable integrated filters with floating nodes are used.

Integrator Based Structures

Infinite Gain Amplifiers

Integrator Based Structures (Dependent Sources)
Integrator-based Biquads

\[ \begin{align*}
V_{\text{IN}} & \rightarrow \frac{l_0}{s} \rightarrow \frac{l_0}{s} \rightarrow -1 \\
X_{\text{IN}} & \rightarrow \frac{l_0}{s} \rightarrow \frac{l_0}{s} \rightarrow -1 \\
X_{\text{IN}} & \rightarrow \frac{l_0}{s} \rightarrow \frac{l_0}{s} \rightarrow X_{O2}
\end{align*} \]
Integrator-based Biquads

State Variable Biquad (Alt KHN Biquad)

Integrator and lossy integrator in a loop
Integrator-based Biquads

With arbitrary zero locations

Tow-Thomas Biquad
Integrator-based Biquads

Two-Integrator Loop

Tow Thomas Biquad

\[ \frac{V_{\text{IN}}}{s} \Rightarrow \frac{V_{\text{OUT}}}{s^2} \]
Integrator-based Biquads

• Integrator-based biquads all involve two integrators in a loop

• All integrator-based biquads discussed have no floating nodes

• Most biquads in integrated filters are based upon two integrator loop structures
  • The summers are usually included as summing inputs on the integrators

• The loss can be combined with the integrator to form a lossy integrator

• Performance of the minor variants of the two integrator loop structures are comparable
Filter Design/Synthesis Considerations

Cascaded Biquads

\[ T(s) = T_1 T_2 \cdots T_m \]

Leapfrog

Multiple-loop Feedback – One type shown

Observation: All filters are comprised of summers, biquads and integrators

And biquads usually made with summers and integrators

Integrated filter design generally focused on design of integrators, summers, and amplifiers (Op Amps)

Will now focus on the design of integrators, summers, and op amps
End of Lecture 23