EE 508
Lecture 24

Integrator Design

Switched Capacitor Integrators

Voltage Mode Integrators

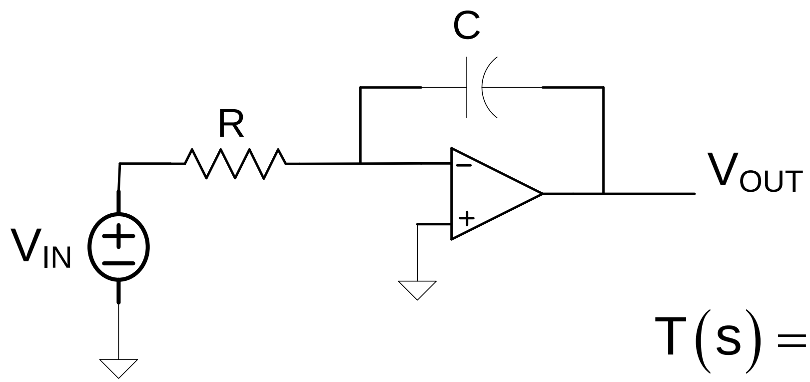
- Active RC (Feedback-based)
- MOSFET-C (Feedback-based)
- OTA-C
- TA-C

Sometimes termed "current mode"

- Other Continuous-time Structures
- Switched Capacitor
- Switched Resistor

Discrete Time

Consider the Basic Integrator



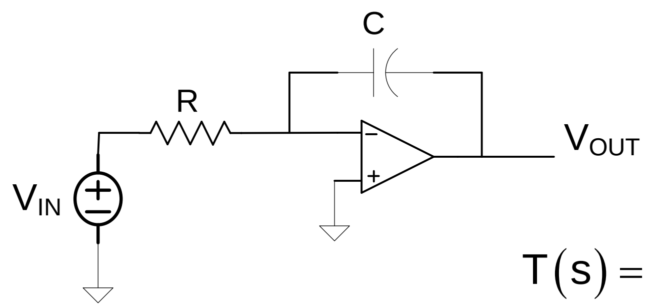
$$T(s) = -\frac{1}{RCs}$$

$$I_0 = \frac{1}{RC}$$

Key performance of integrator (and integrator-based filter) is determined by the integrator time constant I_0

Precision of time constants of a filter invariably determined by precision of I_0

Consider the Basic Integrator



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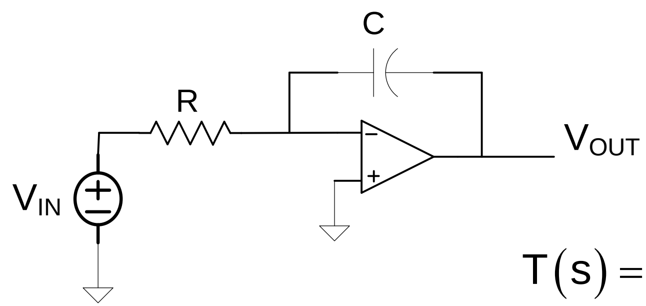
Precision of time constants of a filter invariably determined by precision of I_0

How much area required if $\omega_0 = I_O = 2\pi$ 1KHz , $R_\square = 100\Omega/\square$ and C=1pF?

Consider the Basic Integrator



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Precision of time constants of a filter invariably determined by precision of I_0

How much area required if $\omega_0 = I_O = 2\pi$ 1KHz , R□=100Ω/□ and C=1pF?

$$R = \frac{1}{2\pi \bullet 10^3 C} = 160M\Omega$$

Number of squares: n_s

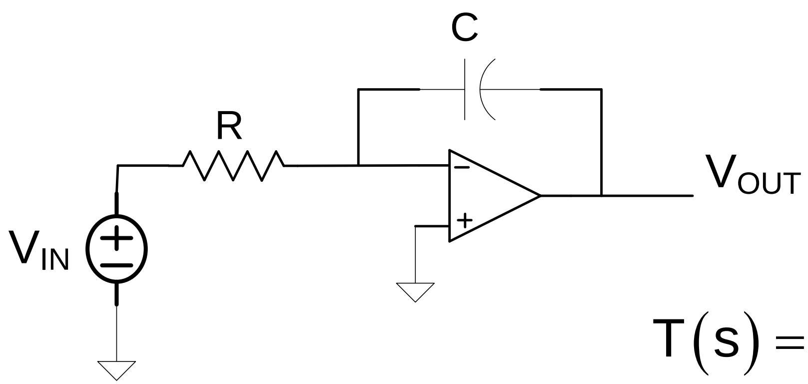
$$n_s = \frac{R}{R_\square} = \frac{160M\Omega}{100\Omega/\square} = 1.6x10^6$$

Define:

A_{SQ}=area of resistor square
C_d=capacitance density

$$H(z) = -\frac{z^{C_1/C}}{z-1}$$

Consider the Basic Integrator



$$T(s) = -\frac{1}{RCs}$$

$$I_0 = \frac{1}{RC}$$

1. Accuracy of R and C difficult to accurately control – particularly in integrated applications (often 2 or 3 orders of magnitude to variable)

2. Size of R and C unacceptably large if I_0 is in audio frequency range (2 or 3 orders of magnitude too large)

3. Amplifier GB limits performance

Incredible Challenge to Building Filters on Silicon!

Challenges for Integration of Active Filters

- Passive Component Variability
- Passive Component Size
- Op Amp Limitations

Historical Perspective

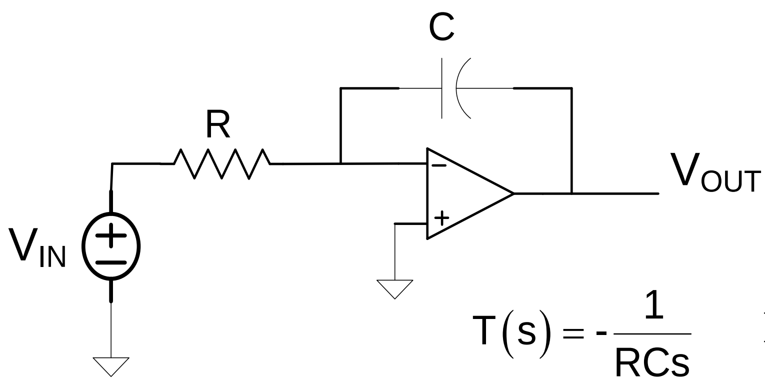
Filters were widely viewed as one of the most fundamental applications of integrated circuit technology

Considerable effort was expended on developing methods to build integrated filters but these three issues were viewed for years as a fundamental roadblock

Practical solution required finding SIMULTANEOUS solutions to three problems which were each 2 or 3 orders of magnitude problematic

This problem was not solved from the invention of the integrated circuit in 1959 up until the late 1970s

Switched-Capacitor Circuits



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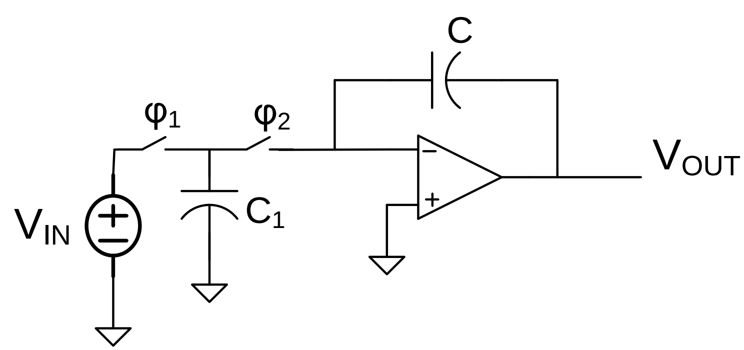
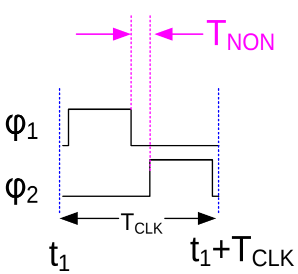
Consider:



$$V_{IN} = V_M \sin\left(2\pi f_{SIG} t + \theta\right)$$



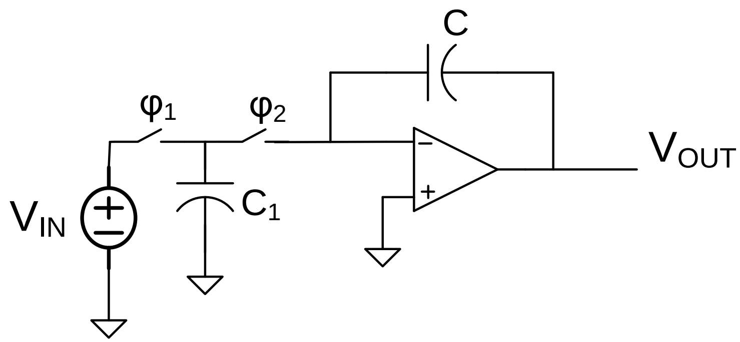
Assume $T_{CLK} << T_{SIG}$

Φ_1 and Φ_2 are complimentary non-overlapping clocks

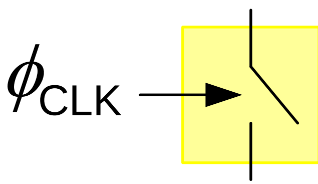
Termed a Switched-Capacitor circuit

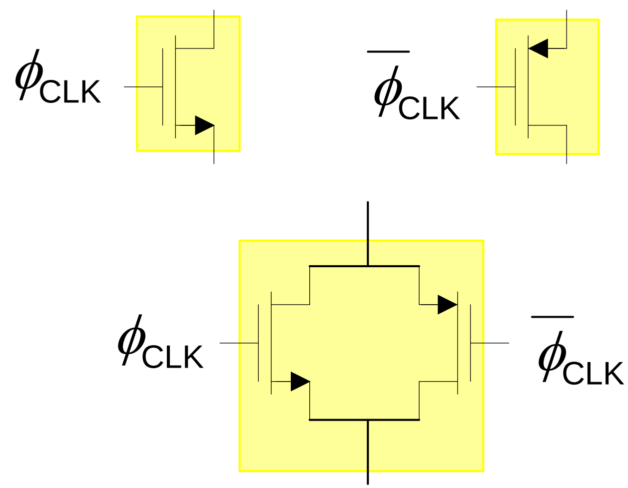
Φ_1 and Φ_2 are periodic signals
"clocks" shown for one period

Switched-Capacitor Circuits



How are the switches made?

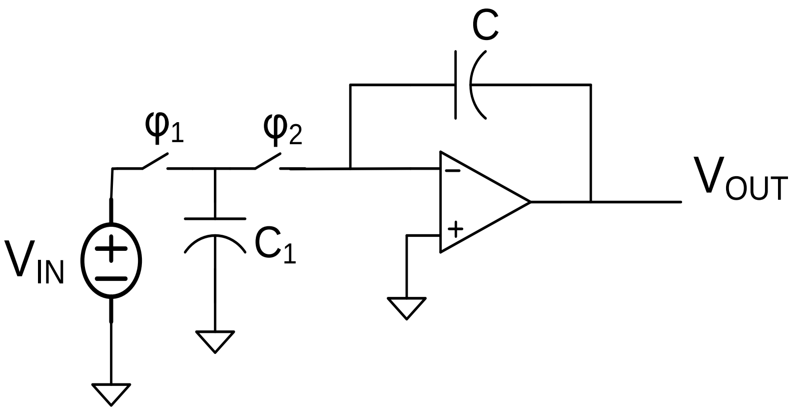




- Often single transistor
- Occasionally complimentary transistors
- On rare occasion more complicated
- Area overhead for switches small, clock routing a little more of concern
- Sizing of devices is important
- Clocking of switches may be important

Although originating in SC filters, switched charge redistribution circuits widely used in other non-filtering applications

Consider the Switched-Capacitor Circuit



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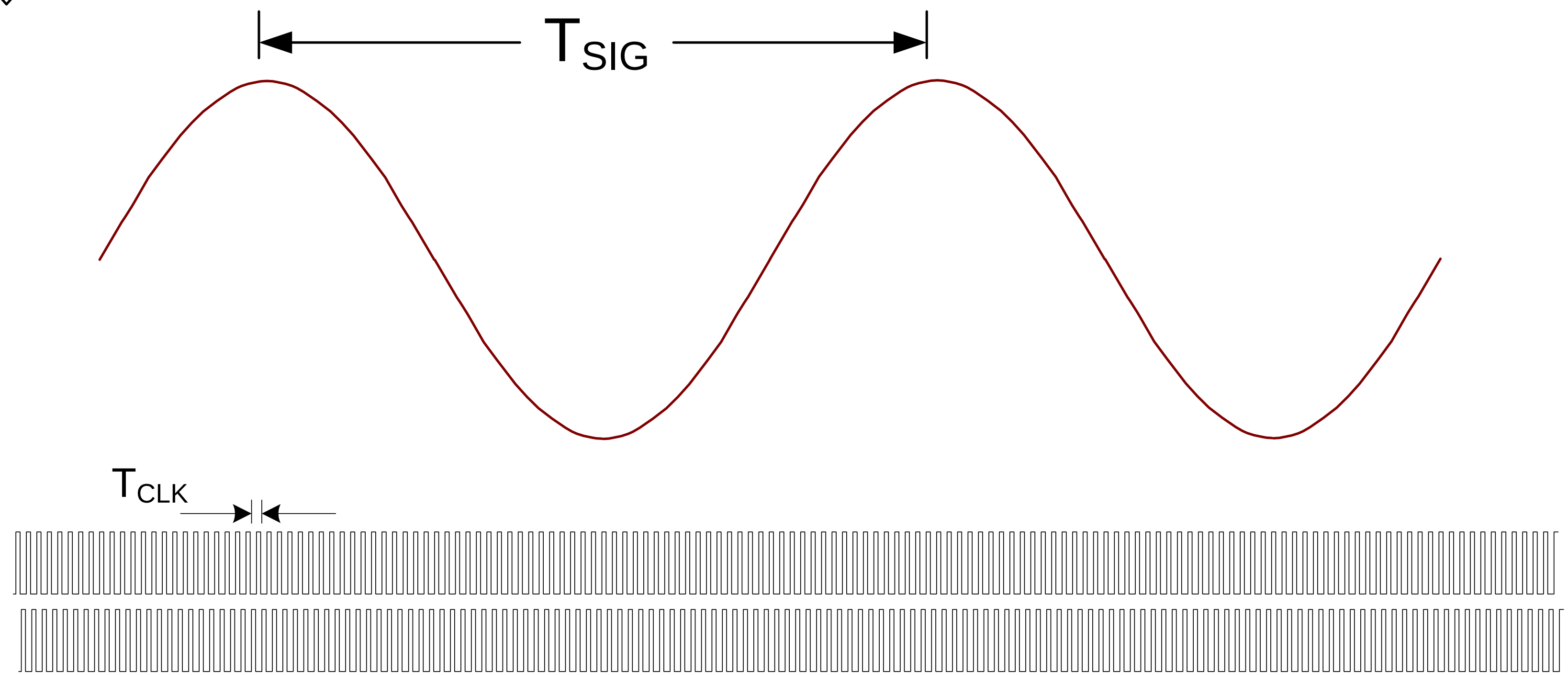
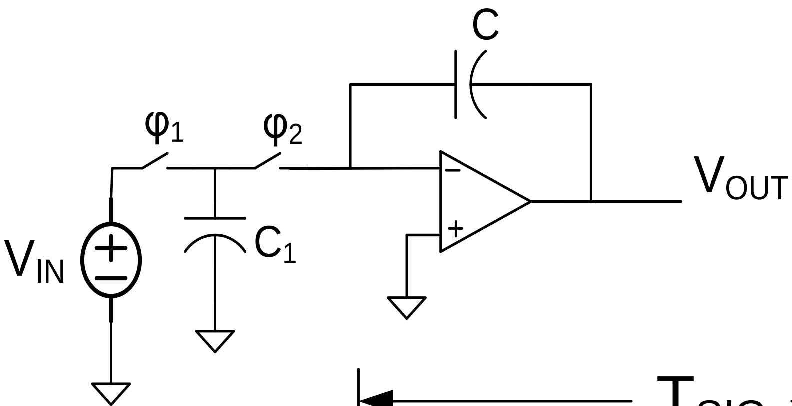
Φ_1 and Φ_2 are complimenary nonoverlapping clocks



Lets now zoom in on the clock period

Consider the Switched-Capacitor Circuit



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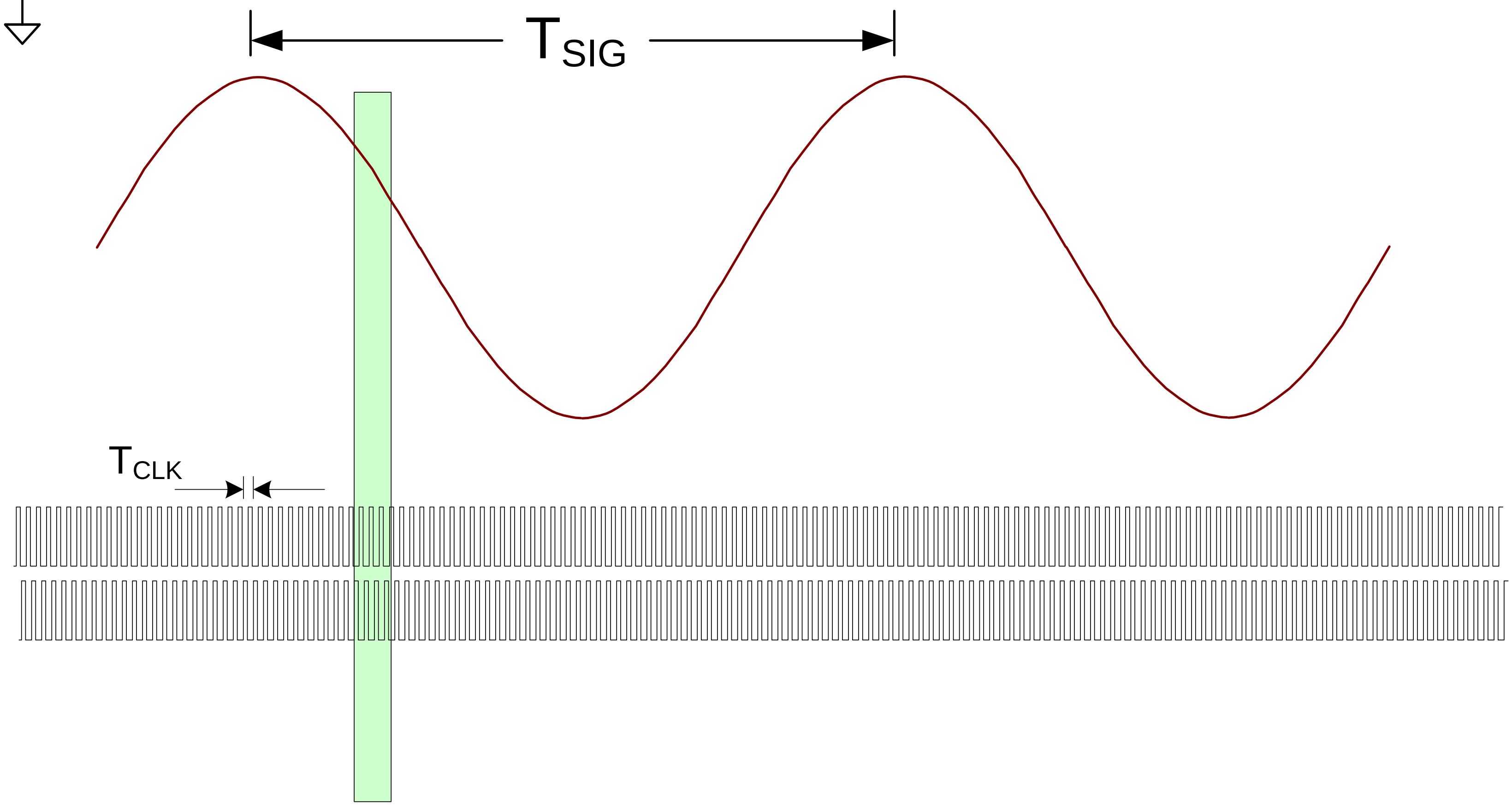
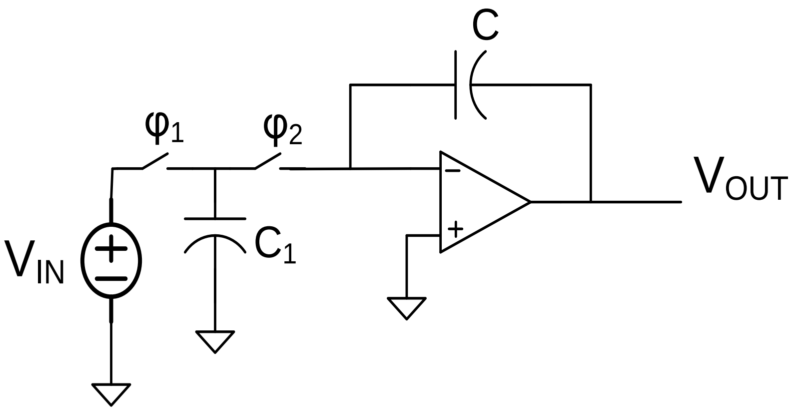


Consider the Switched-Capacitor Circuit



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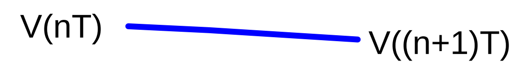
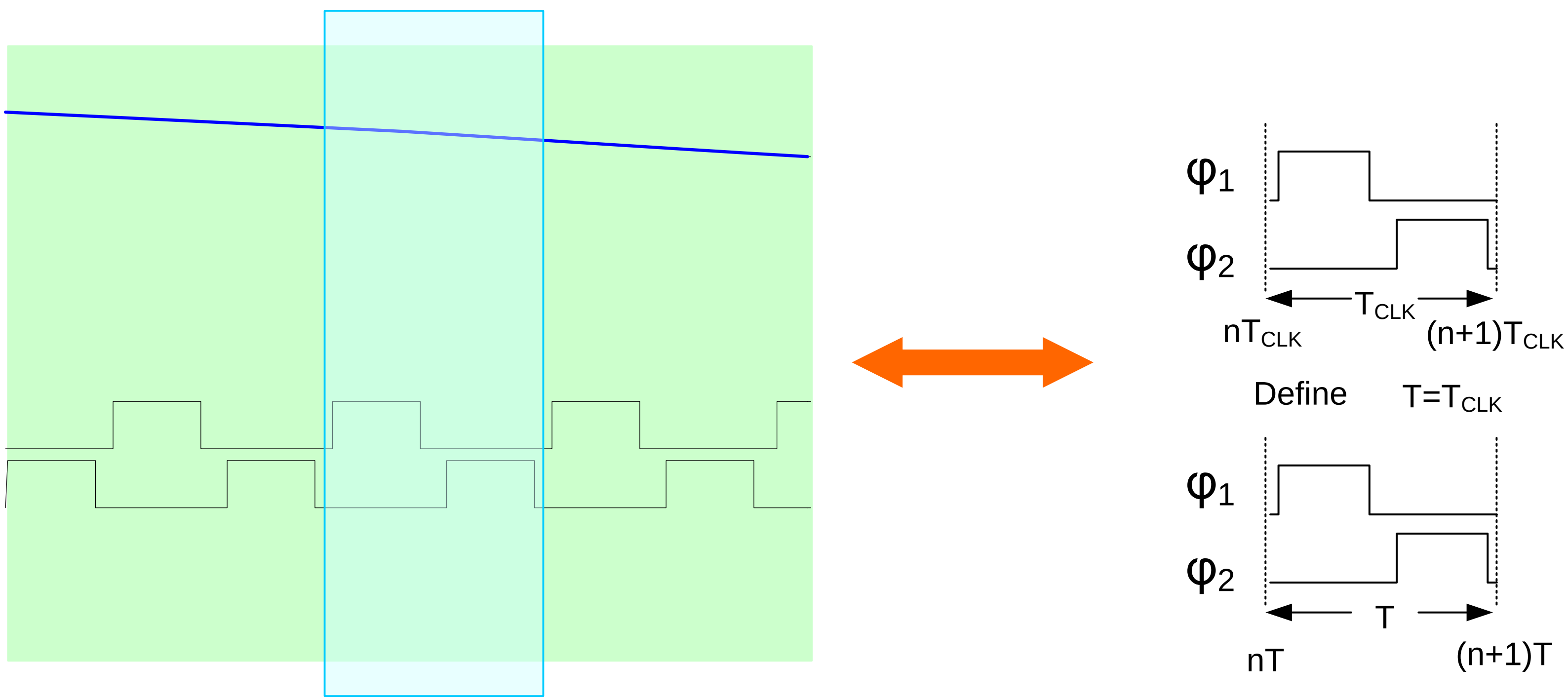
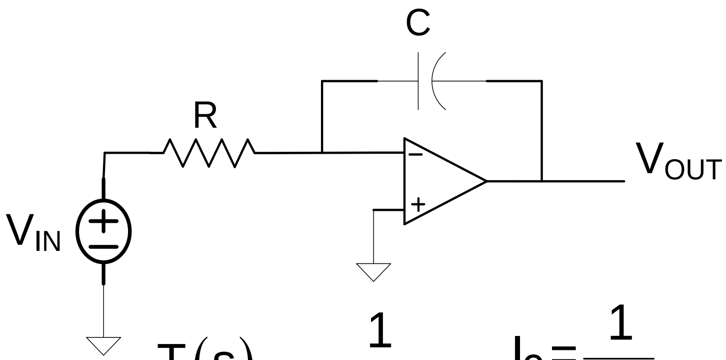
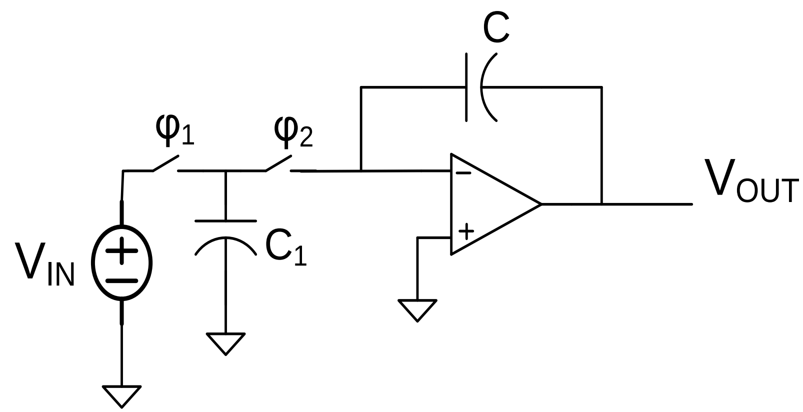




Compare the performance of the following two circuits



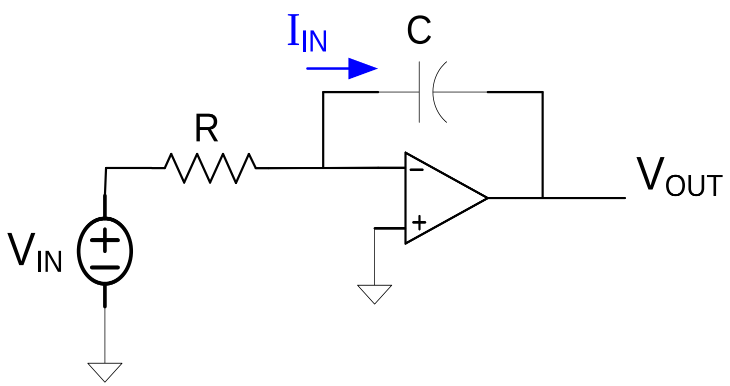
$$T(s) = -\frac{1}{RCs} \qquad I_0 = \frac{1}{RC}$$



Will now consider the charge transferred to the feedback capacitor for both circuits in an interval of length T_{CLK} at time t_1

Consider the charge transferred to the feedback capacitor for both circuits in an interval of length T_{CLK} at time t_1

For the RC circuit:



$$Q_{RC} = \int_{t_1}^{t_1+T_{CLK}} I_{in}(t)\,dt$$

$$Q_{RC} = \int_{t_1}^{t_1+T_{CLK}} \frac{V_{in}(t)}{R}\,dt$$

Since V_{in} changes slowly

$$Q_{RC} \cong \int_{t_1}^{t_1+T_{CLK}} \frac{V_{in}(t_1)}{R}\,dt$$

$$Q_{RC} \cong \left[\frac{V_{in}(t_1)}{R}\right] \int_{t_1}^{t_1+T_{CLK}} 1\,dt$$

$$Q_{RC} \cong \left[\frac{V_{in}(t_1)}{R}\right] T_{CLK}$$

Consider the charge transferred to the feedback capacitor for both circuits in an interval of length T_{CLK} at time t_1

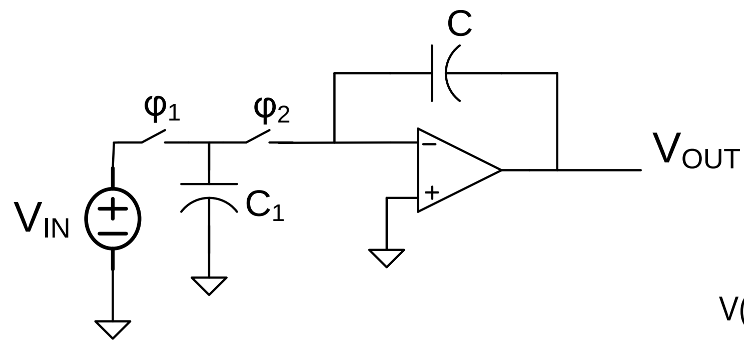
For the RC circuit:

$$Q_{RC} \cong \left[\frac{V_{in}(t_1)}{R}\right] T_{CLK}$$

Observe that a resistor

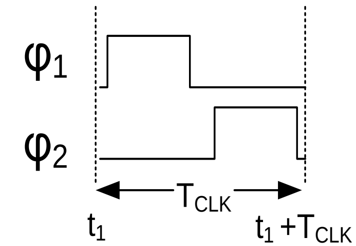
a) "transfers" charge proportional to V_{in} in a short interval of T_{CLK}
b) Transfers about the same amount of charge during closely spaced intervals

For the SC circuit



$$Q_{C1} = C_1 V_{in}\left(t_1 + \frac{T_{CLK}}{2} - \varepsilon\right)$$

Since $V_{in}(t)$ is slowly varying



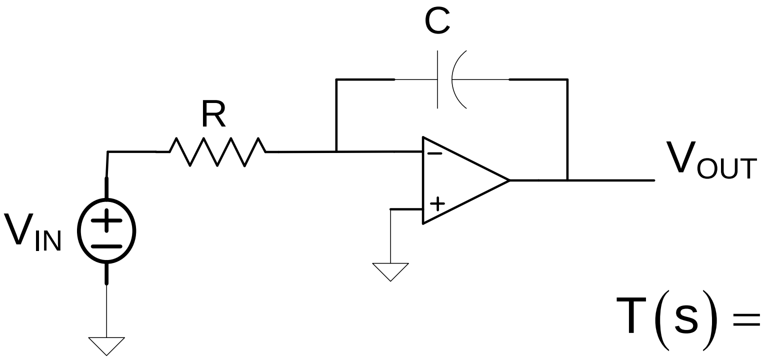
$$Q_{C1} \cong C_1 V_{in}\left(t_1\right)$$

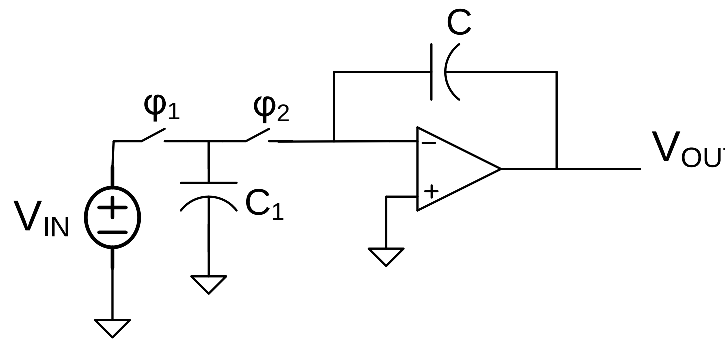
But this is the charge that will be transferred to C during phase Φ_2

$$Q_{SC} \cong C_1 V_{in}\left(t_1\right)$$

Observe that the SC circuit also transfers charge proportional to V_{in} in short intervals of length T_{CLK}

This is precisely what a resistor does so the switched capacitor behaves as a resistor





$$T(s) = -\frac{1}{RCs}$$

$$I_0 = \frac{1}{RC}$$

Comparing the two circuits

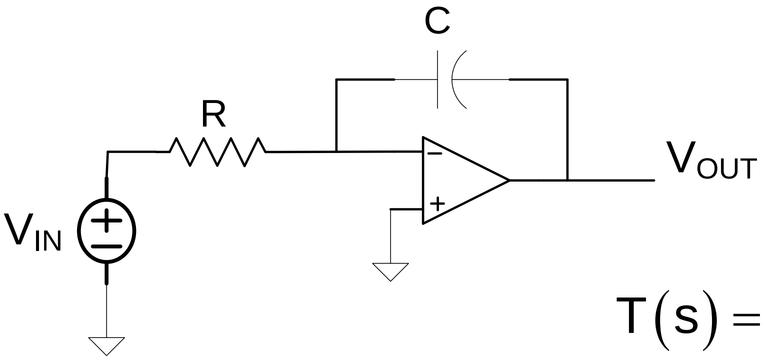
$$Q_{RC} \cong \left[\frac{V_{in}(t_1)}{R}\right] T_{CLK}$$

$$Q_{SC} \cong C_1 V_{in}(t_1)$$

Equating charges since both proportional to $V_{in}(t_1)$

$$C_1 \cong \left[\frac{1}{R}\right] T_{CLK}$$

$$R_{EQ} \cong \frac{1}{f_{CLK} C_1}$$



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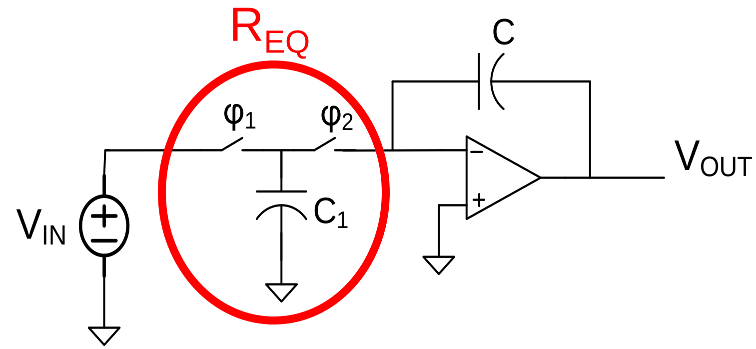


$$R_{EQ} \cong \frac{1}{f_{CLK} C_1}$$

Observe that a switched-capacitor behaves as a resistor!

This is an interesting observation that was made by Maxwell over 100 years ago but in and of itself was of almost no consequence

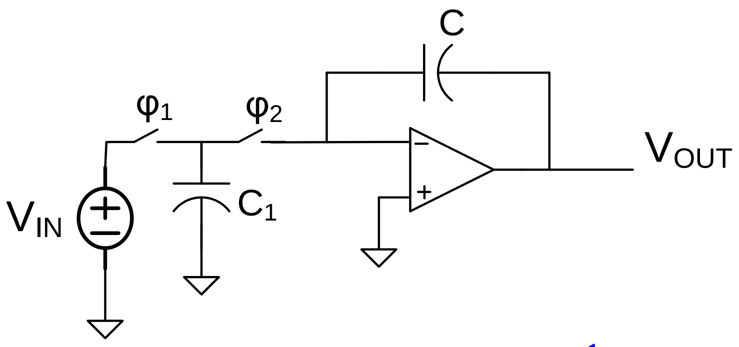
Observation by Maxwell was forgotton and rediscovered several times over the years but remained of no conseqeunce

Note that large resistors require small capacitors !

This offers potential for overcoming <u>one</u> of the critical challenges for Implementing integrators on silicon at audio frequencies!

- Passive Component Variability
- Passive Component Size
- Op Amp Limitations

Consider again the SC integrator



$$R_{EQ} \cong \frac{1}{f_{CLK} C_1}$$

$$T_{SC}(s) \cong \frac{-1}{R_{EQ} C s}$$

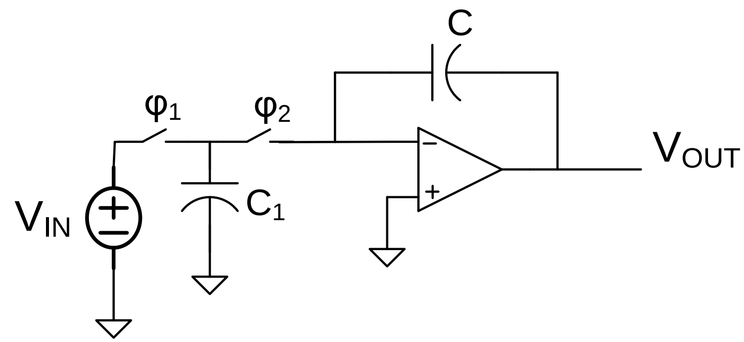
$$I_{0eq} = \frac{1}{R_{EQ} C}$$

$$I_{0eq} = \frac{1}{R_{EQ} C} = \frac{C_1 f_{CLK}}{C}$$

$$I_{0eq} = \left[\frac{C_1}{C}\right] f_{CLK}$$

This is a frequency referenced filter!

The SC integrator



$$T_{SC}(s) \cong \frac{-1}{R_{EQ}Cs}$$

$$R_{EQ} \cong \frac{1}{f_{CLK}C_1}$$

$$I_{0eq} = \left[\frac{C_1}{C}\right] f_{CLK}$$

The expressions $S_{C}^{I_0}$ and $S_{C_1}^{I_0}$ have the same magnitude as for the RC integrator

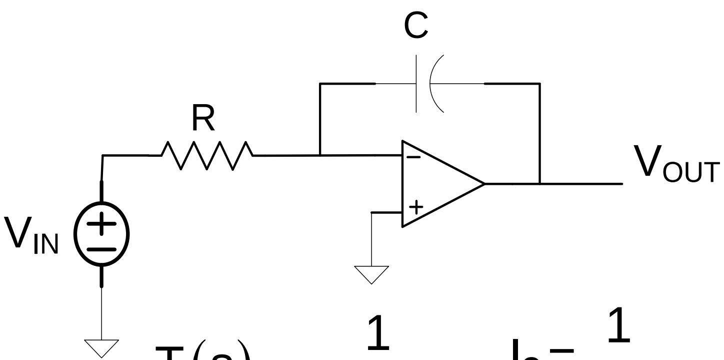
On-chip capacitor values CAN be highly correlated with proper selection and layout

- The ratio of capacitors CAN be accurately controlled in IC processes
  (1% to .01% is achievable with careful layout)
- f_{CLK} CAN be VERY accurately controlled with a low cost crystal (1 part in 10^6 or better)
- Variability of I_{0eq} is very small

The SC integrator CAN dramatically reduce the first main concern for building integrated integrators

- ✓ Passive Component Variability
- Passive Component Size
- Op Amp Limitations

The SC integrator



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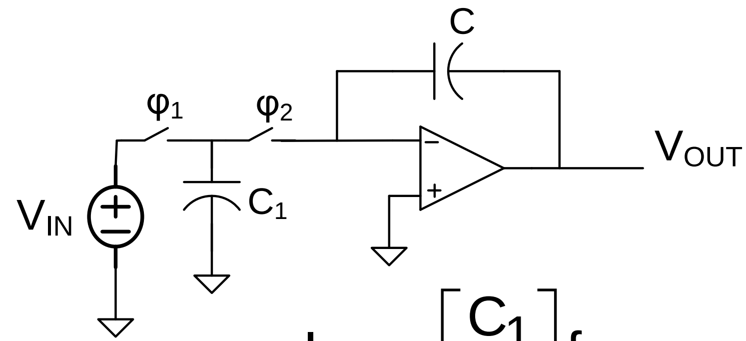


$$I_{0eq} = \left[\frac{C_1}{C}\right] f_{CLK}$$

1. **Accuracy of R and C difficult to accurately control (often 2 or 3 orders of magnitude to variable)**
2. **Area of R and C too large in audio frequency range (2 or 3 orders of magnitude too large)**
3. **Amplifier GB limits performance**

1. **Accuracy of cap ratio and f_{CLK} very good**
2. **Area of C1 and C not too large**
3. **Amplifier GB limits performance less**

Two of these properties were discovered independently by Gray, Broderson and Hosticka at Berkeley and by Copeland of Carelton

[1] J. T. Caves, M. A. Copeland, C. F. Rahim, and S. D. Rosenbaum, "Sampled analog filtering using switched capacitors as resistor equivalents," *IEEE J. Solid-State Circuits*, vol. SC-12, pp. 592–599, Dec. 1977.

[2] B. J. Hosticka, R. W. Brodersen, and P. R. Gray, "MOS sampled data recursive filters using switched capacitor integrators," *IEEE J. Solid-State Circuits*, vol. SC-12, pp. 600–608, Dec. 1977.

207 citations

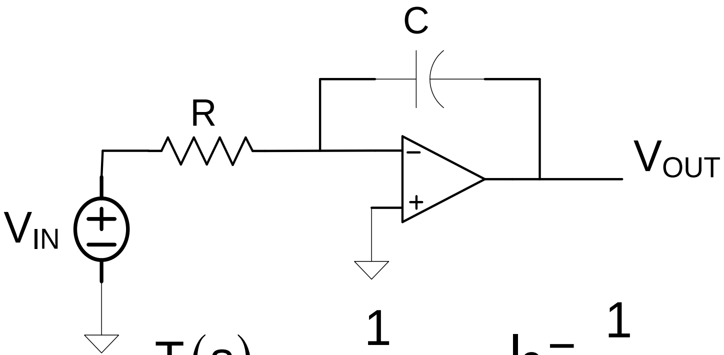
303 citations

Updated Oct 9, 2020

Seminal source of SC concept received few citations!

But cited as a key contribution when Brodersen and Gray elected to NAE

The SC integrator



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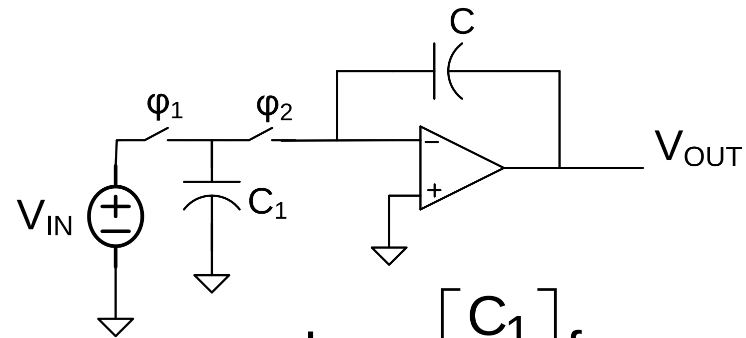

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Updated Oct 26, 2018

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EE Times BETA News & Analysis

Switched-Capacitor Filters Beat Active Filters at Their Own Game

Charles Yager and Carlos Laber

6/29/2000 12:00 AM EDT

Switched capacitor filters are growing increasingly popular because they have many advantages over active filters. Switched capacitor filters don't require external precision capacitors like active filters do. Their cutoff frequencies have a typical accuracy of ±0.3% and they are less sensitive to temperature changes. These characteristics allow consistent, repeatable filter designs.

Another distinct advantage of switched capacitor filters is that their cutoff frequency can be adjusted by changing the clock frequency. Switched capacitor filters offer higher integration at a lower system cost. Center frequencies of up to 150-kHz with Q values up to 20 are achievable.

Switched Capacitor Filters

The realization that a small switched capacitor was equivalent to a resistor was of little consequence

The realization that a switched capacitor was dependent upon frequency was of little consequence

The realization that RC time constants could be accurately controlled with a small amount of area in silicon was of considerable consequence

The experimental validation and the efforts to convince industry that the SC techniques offered practical solutions was the MAJOR contribution !!

Basic Building Blocks in Both Cascaded Biquads and Multiple Feedback Structures

- **Developed from observations from feedback implementations**

1. Integrators
2. Summers
3. Op Amps (inc OTAs)
4. Switches

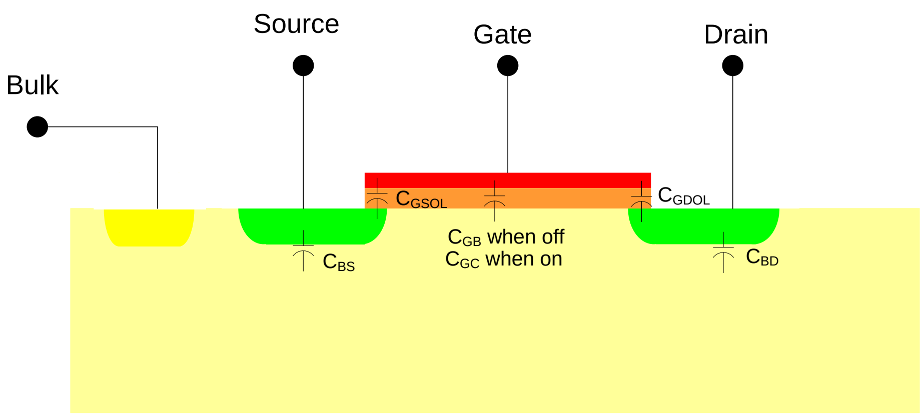
- First-order filter blocks
- Biquads

- **Same building blocks used in open-loop applications as well**

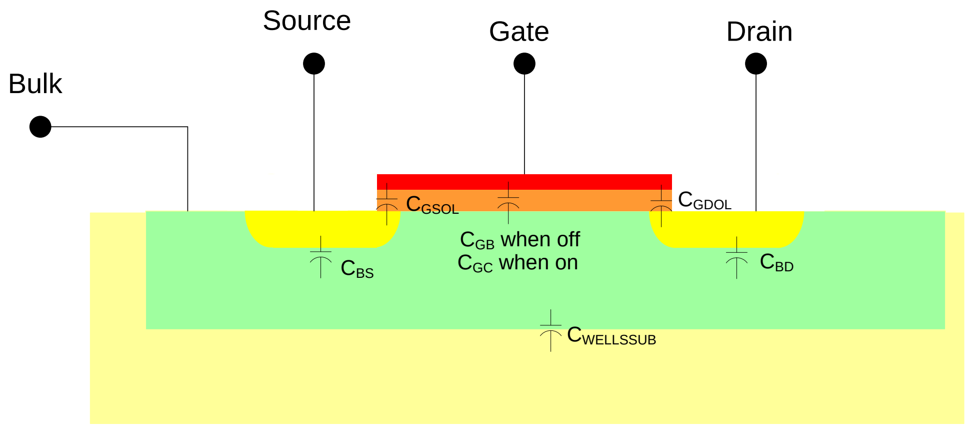
Nonideal Effects in Switched Capacitor Circuits

- Parasitic Capacitances
- Charge Injection
- Aliasing
- Redundant Switch Removal
- Matching
- Noise
- Op Amp Bandwidth

Parasitic Capacitors in MOS Transistors

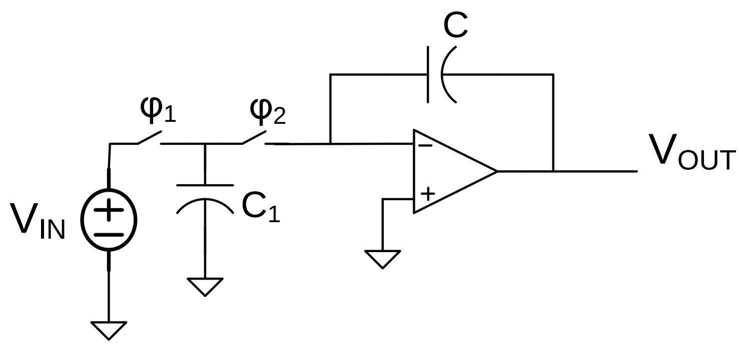


n-channel MOSFET



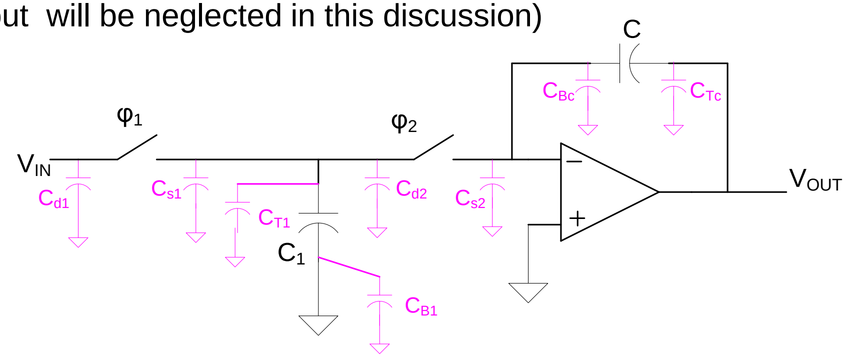
p-channel MOSFET

The SC integrator



$$I_{0eq}=\left[\frac{C_1}{C}\right]f_{CLK}$$

Observe this circuit has considerable parasitics (gate parasitics cause offset in this circuit and some signal-dependent distortion but will be neglected in this discussion)



$$C_{1EQ}=C_1+C_{s1}+C_{d2}+C_{T1}$$

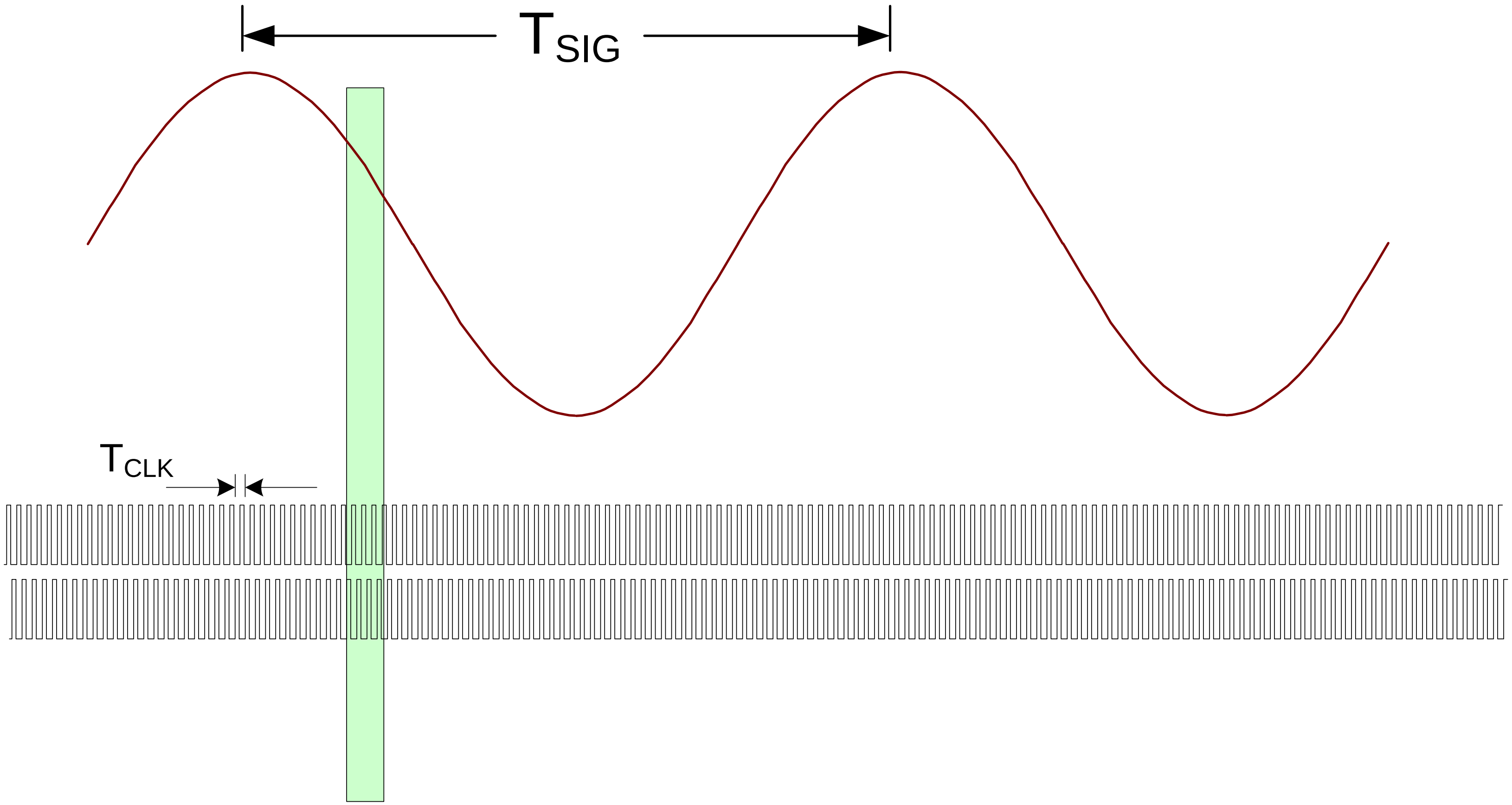
Parasitic capacitors $C_{s1}+C_{d2}+C_{T1}$ difficult to accurately match

- Parasitic capacitors of THIS SC integrator limit performance
- Other SC integrators (discussed later) offer same benefits but are not affected by parasitic capacitors

Switched-Capacitor Filter Issues

What if T_{CLK} is not much-much smaller than T_{SIG}?

For $T_{CLK} << T_{SIG}$



Switched-Capacitor Filter Issues

What if T_{CLK} is not much-much smaller than T_{SIG}?

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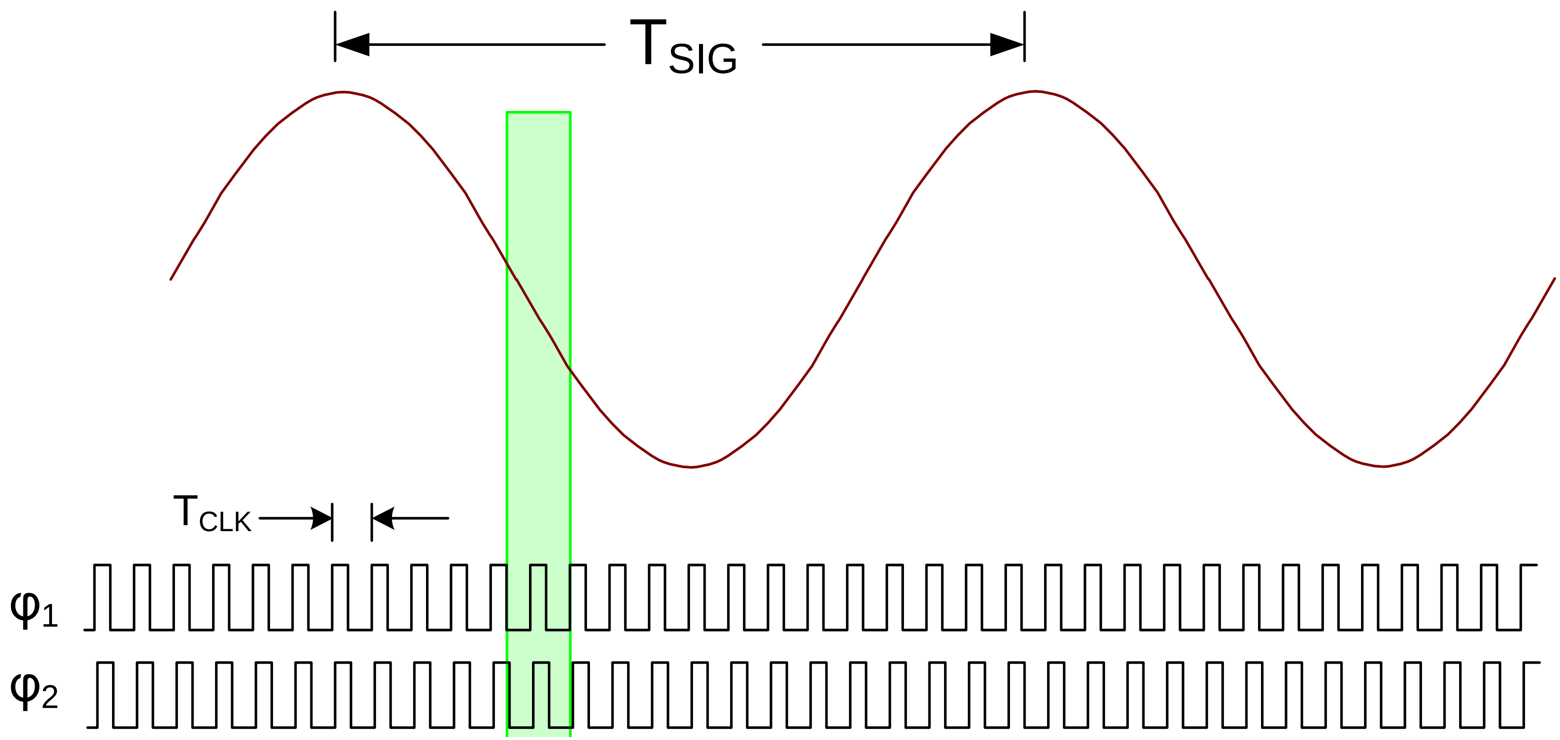

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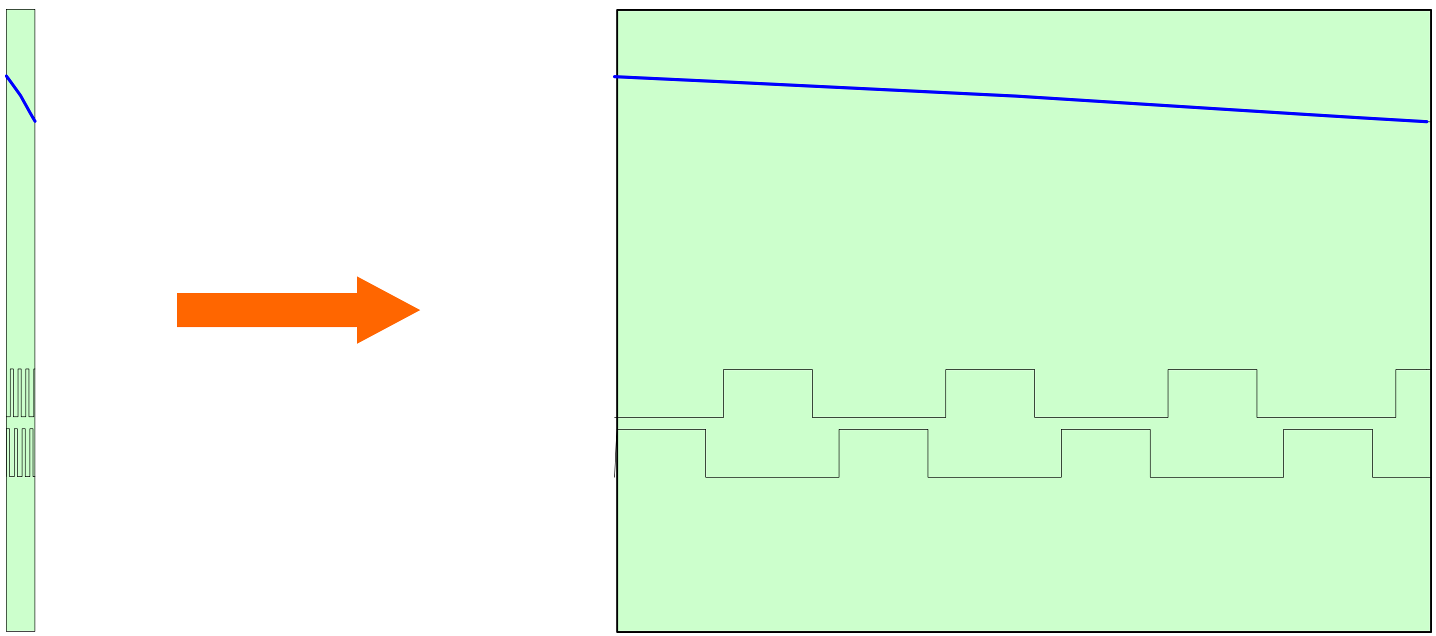

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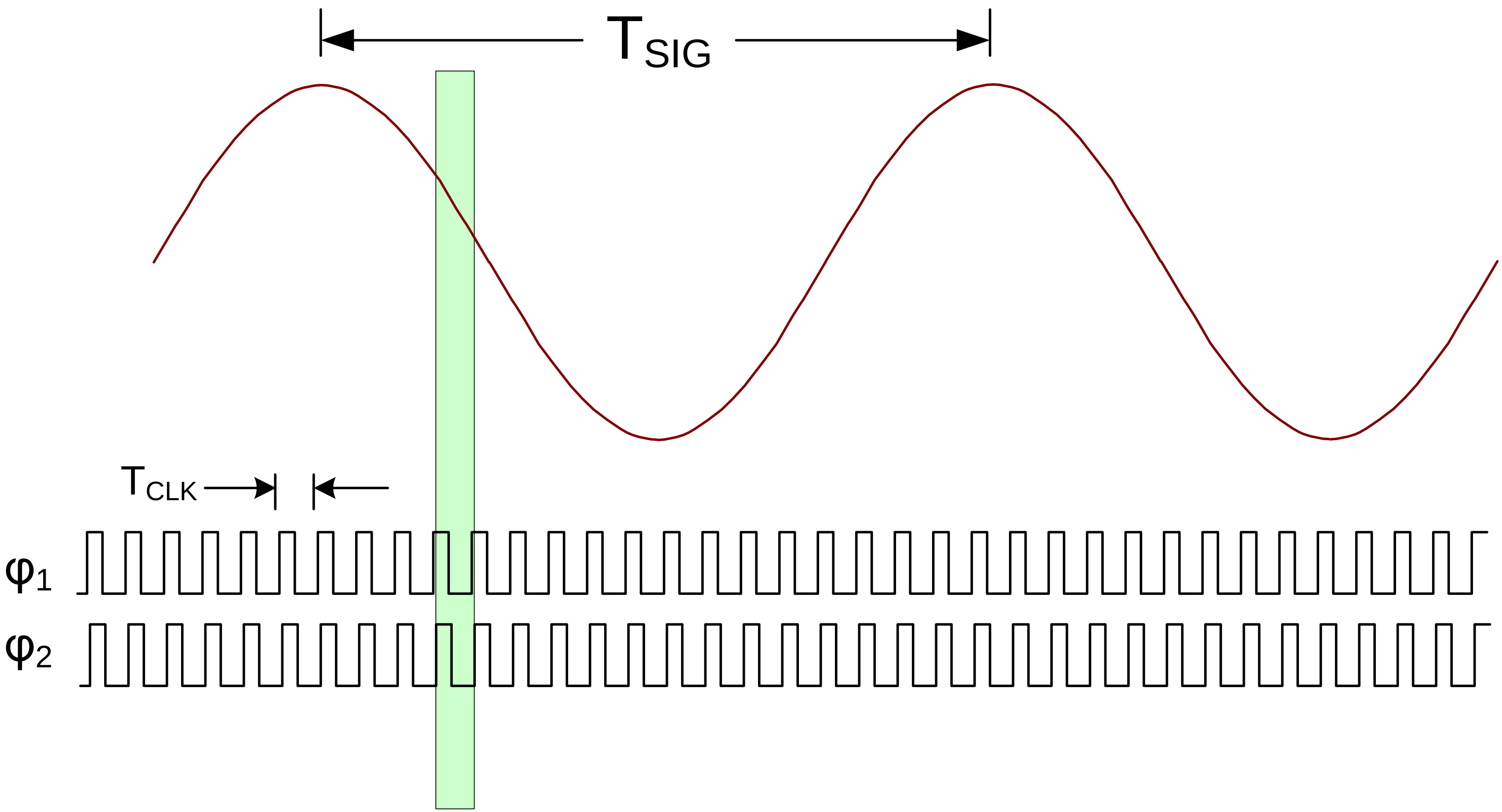

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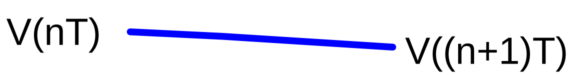
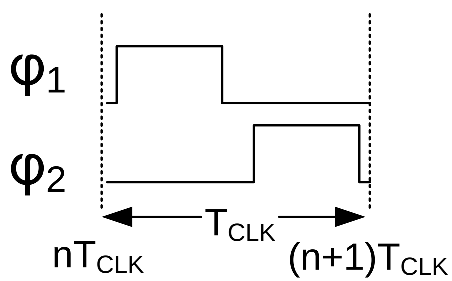
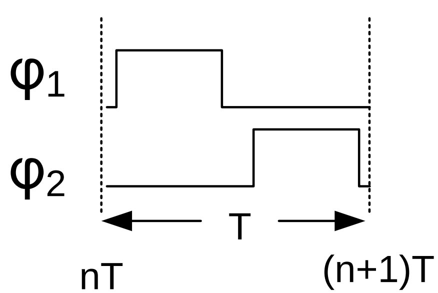
What if T_{CLK} is not much-much smaller than T_{SIG}?

For $T_{CLK} << T_{SIG}$

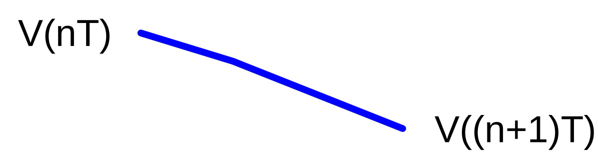


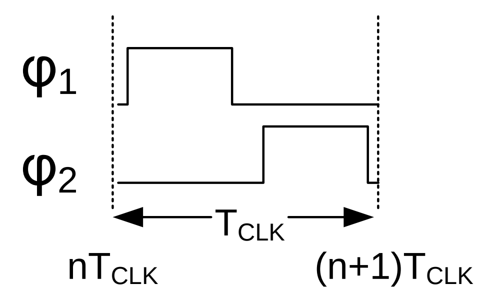


Define $T = T_{CLK}$

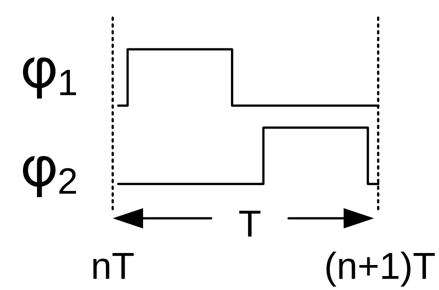


For $T_{CLK} < T_{SIG}$





Define $T = T_{CLK}$

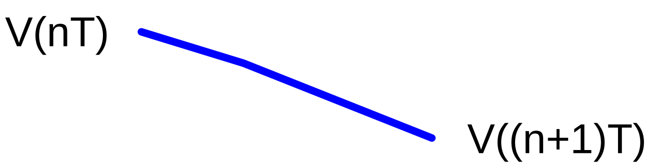


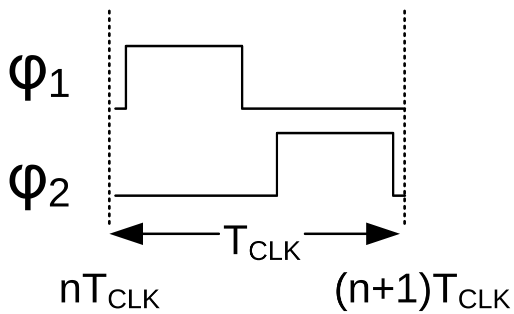
Considerable change in V(t) in clock period

Switched-Capacitor Filter Issues

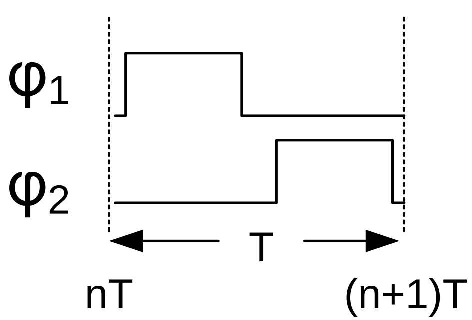
What if T_{CLK} is not much-much smaller than T_{SIG}?

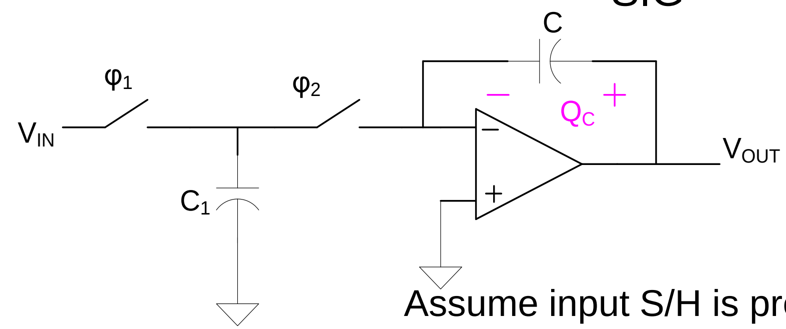
For $T_{CLK} < T_{SIG}$





Define $T = T_{CLK}$





Assume input S/H is present

$$V_0(nT+T) = V_0(nT) + \frac{\Delta Q_C}{C}$$

but $-\Delta Q_c$ is the charge on C_1 at the time φ_1 opens

$$-\Delta Q_c \simeq C_1 V_{IN}(nT+T/2)$$

$$\therefore V_{OUT}(nT+T) = V_{OUT}(nT) - (C_1/C)V_{IN}(nT+T/2)$$

Due to input S/H, V_{IN} constant over periods of length T thus, assume $V_{IN}(nT+T/2) \simeq V_{IN}(nT)$

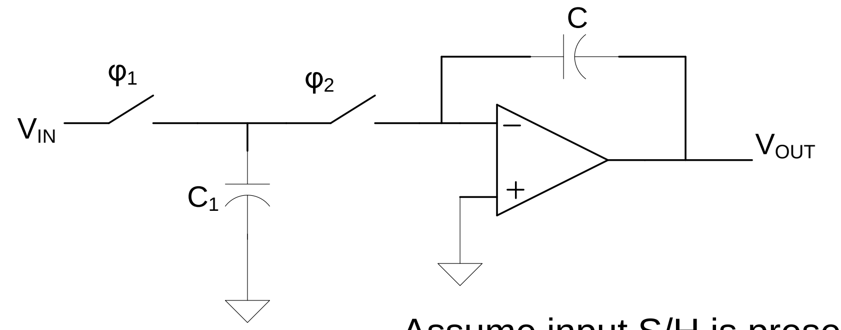
So obtain

$$V_{OUT}(nT+T) = V_{OUT}(nT) - (C_1/C)V_{IN}(nT)$$

How does this analysis differ from what we did earlier?

Switched-Capacitor Filter Issues

What if T_{CLK} is not much-much smaller than T_{SIG}?



Assume input S/H is present

$$V_{OUT}(nT+T)=V_{OUT}(nT)-(C_1/C)V_{IN}(nT)$$

for any T_{CLK}, characterized in time domain by difference equation

This can be characterized in the discrete-time frequency domain by transfer function obtained by taking z-transform of the difference equation

$$zV_{OUT}(z)=V_{OUT}(z)-(C_1/C)V_{IN}(z)$$

$$H(z)=-\frac{C_1/C}{z-1}$$

This is a standard integrator transfer function in the z-domain (but not unique)

Note pole at z=1

Switched-Capacitor Filter Issues

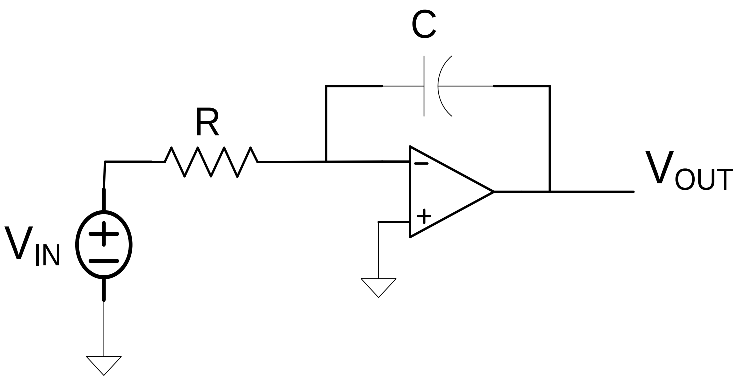
What if T_{CLK} is not much-much smaller than T_{SIG}?

Claim: The transfer function of any Switched-Capacitor Filter is a rational fraction in z with all coefficients in both the numerator and denominator determined totally by capacitor ratios

$$H(z) = \frac{\sum_{i=0}^{m} a_i z^i}{\sum_{i=0}^{n} b_i z^i}$$

What is really required for building a filter that has high-performance features?

Consider an integrator:



Frequency domain:

Transfer function

$$T(s) = -\frac{1}{RCs}$$

Time domain:

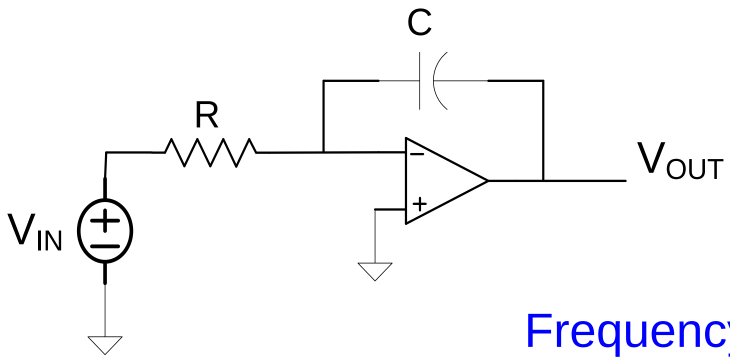
Differential Equation

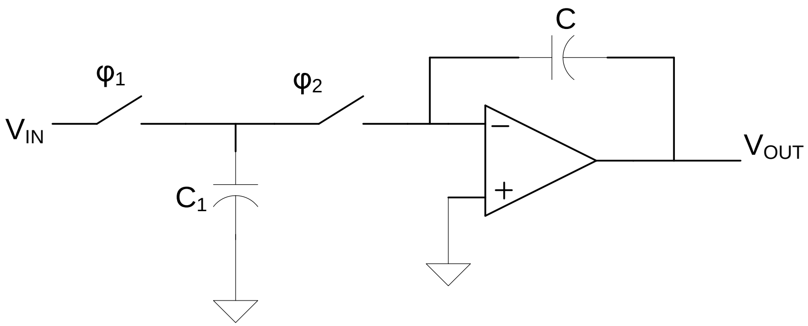
$$V_{OUT}(t) = V_{OUT}(t_0) - \frac{1}{RC}\int_{t_0}^{t} V_{IN}(\tau)\,d\tau$$

- Accurate control of polynomial coefficients in transfer function or accurate control of coefficients in the differential equation
- Absence of over-ordering terms due to parasitics

What is really required for building a filter that has high-performance features?

Consider continuous-time and discrete-time integrators:





Frequency domain:

Transfer function

$$T(s) = -\frac{1}{RCs}$$

$$H(z) = -\frac{C_1/C}{z-1}$$

Time domain:

Differential Equation

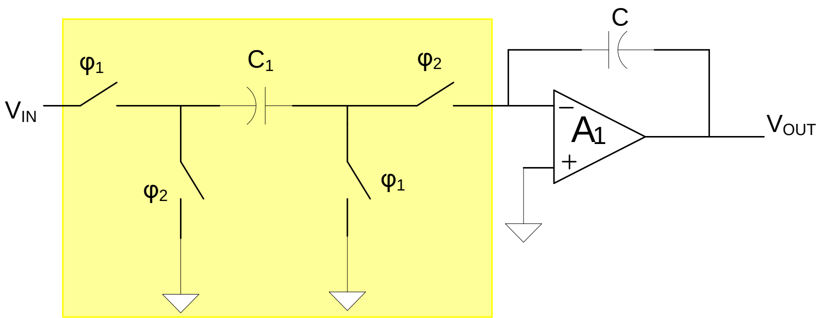
$$V_{OUT}(t) = V_{OUT}(t_0) - \frac{1}{RC}\int_{t_0}^{t} V_{IN}(\tau)\,d\tau$$

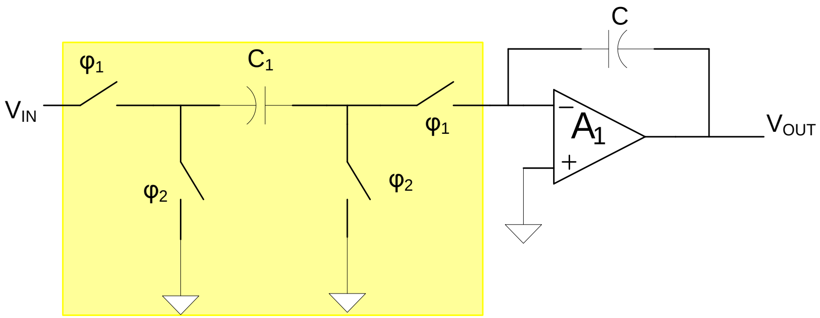
Difference Equation

$$V_{OUT}(nT+T) = V_{OUT}(nT) - (C_1/C)V_{IN}(nT)$$

- Accurate control of polynomial coefficients in transfer function or accurate control of coefficients in the differential/difference equation needed for good filter performance
- Absence of over-ordering terms due to parasitics

Switched-Capacitor Integrators





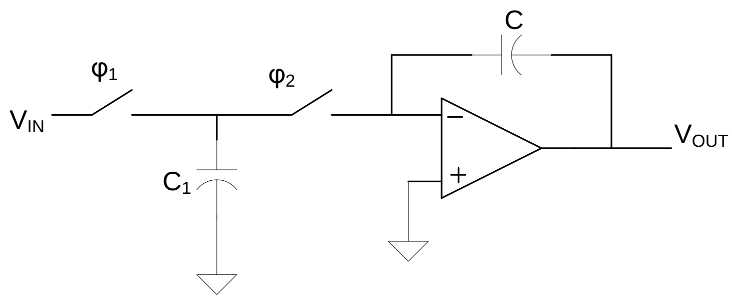
Switched-capacitor filters are characterized in the z-domain

SC filters have continuous-amplitude inputs but outputs valid only at discrete times

Digital filters implemented with ADC/DAC approach have discrete amplitude and discrete time

What effects does the discrete-time property of a SC filter have on the filter performance?

Switched-Capacitor Filter Issues



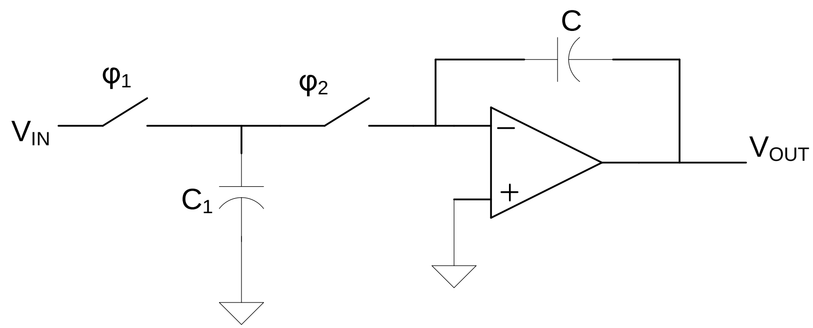
Transfer function of any SC filter of form:

$$H(z) = \frac{\sum_{i=0}^{m} a_i z^i}{\sum_{i=0}^{n} b_i z^i}$$

Switched-capacitor circuits have potential for good accuracy and attractive area irrespective of how T_{CLK} relates to T_{SIG}

But good layout techniques and appropriate area need to be allocated to realize this potential !

Switched-Capacitor Integrators

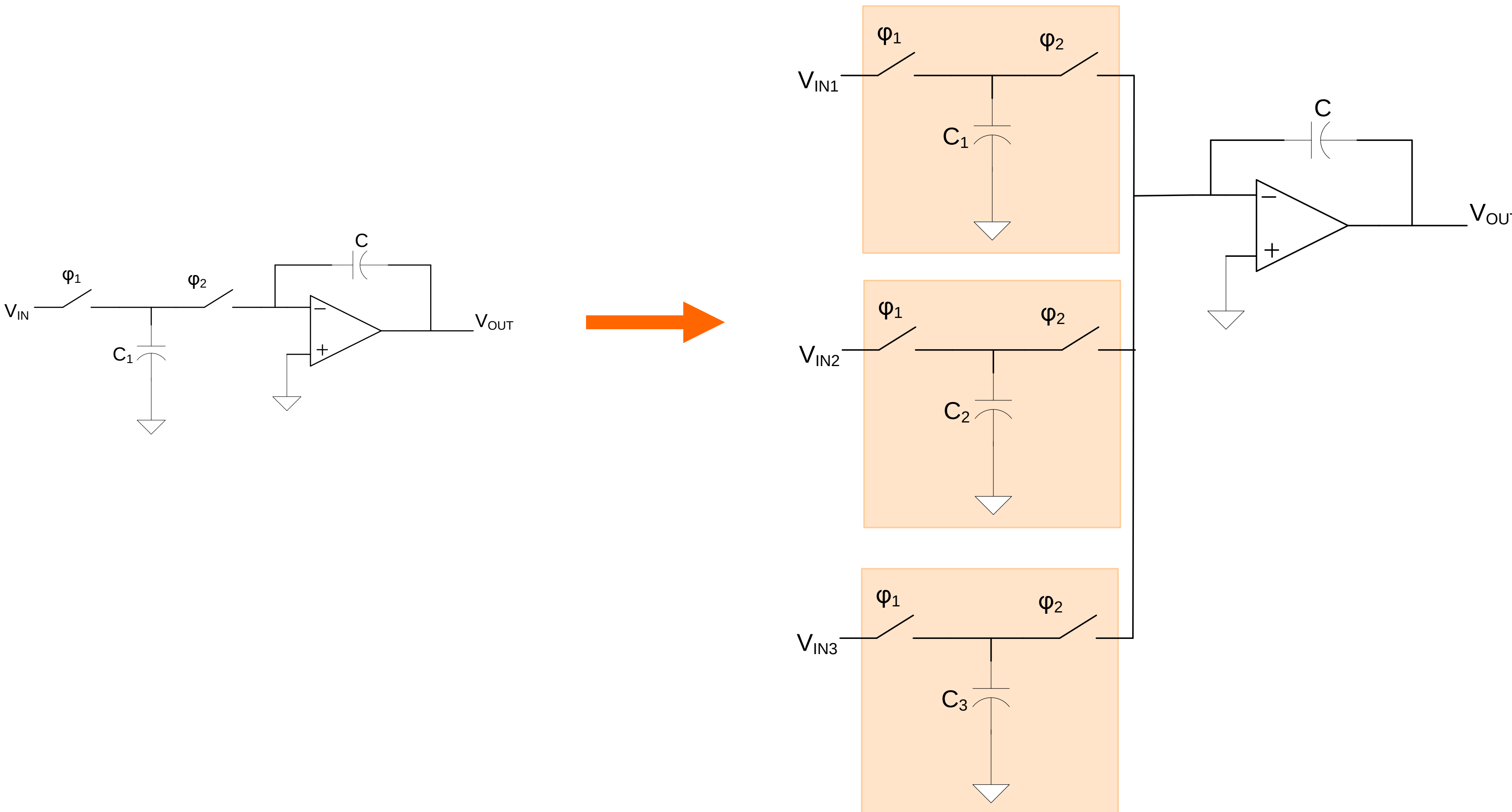


$$V_{OUT}(nT+T)=V_{OUT}(nT)-(C_1/C)V_{IN}(nT)$$

$$H(z)=-\frac{C_1/C}{z-1}$$

Sensitive to parasitic capacitances

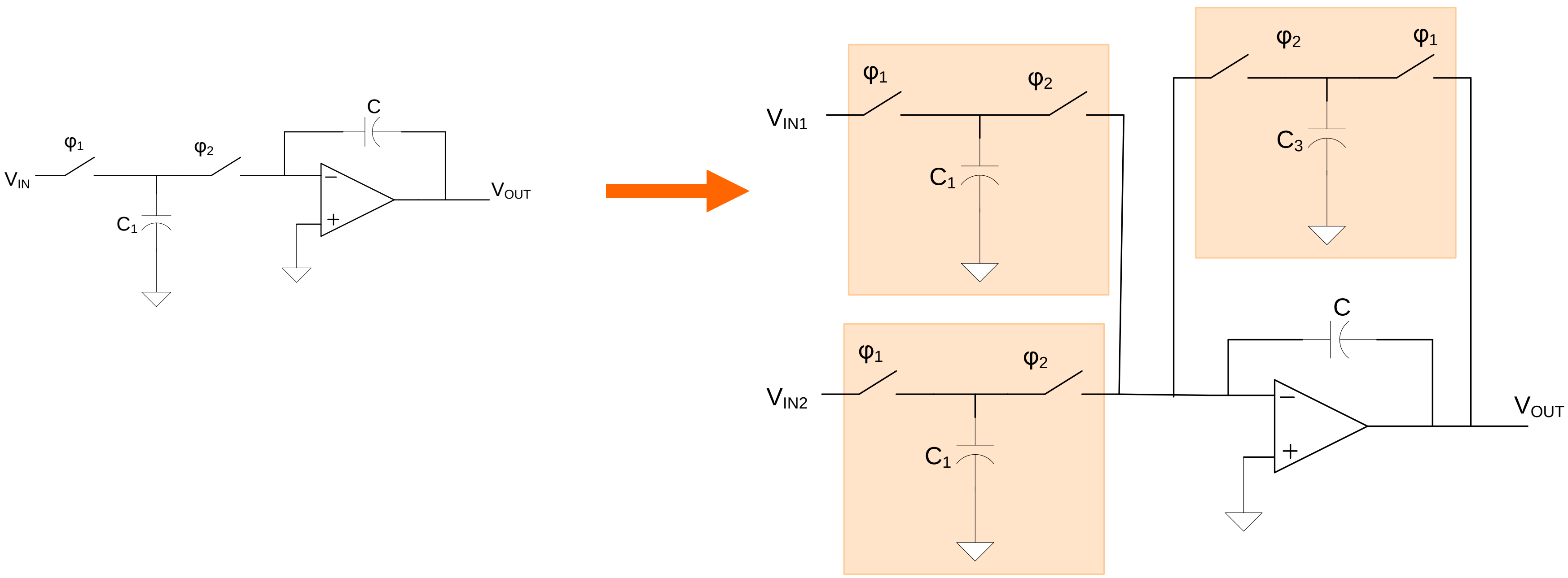
Switched-Capacitor Integrators



Summing Inputs

Sensitive to parasitic capacitances

Switched-Capacitor Integrators

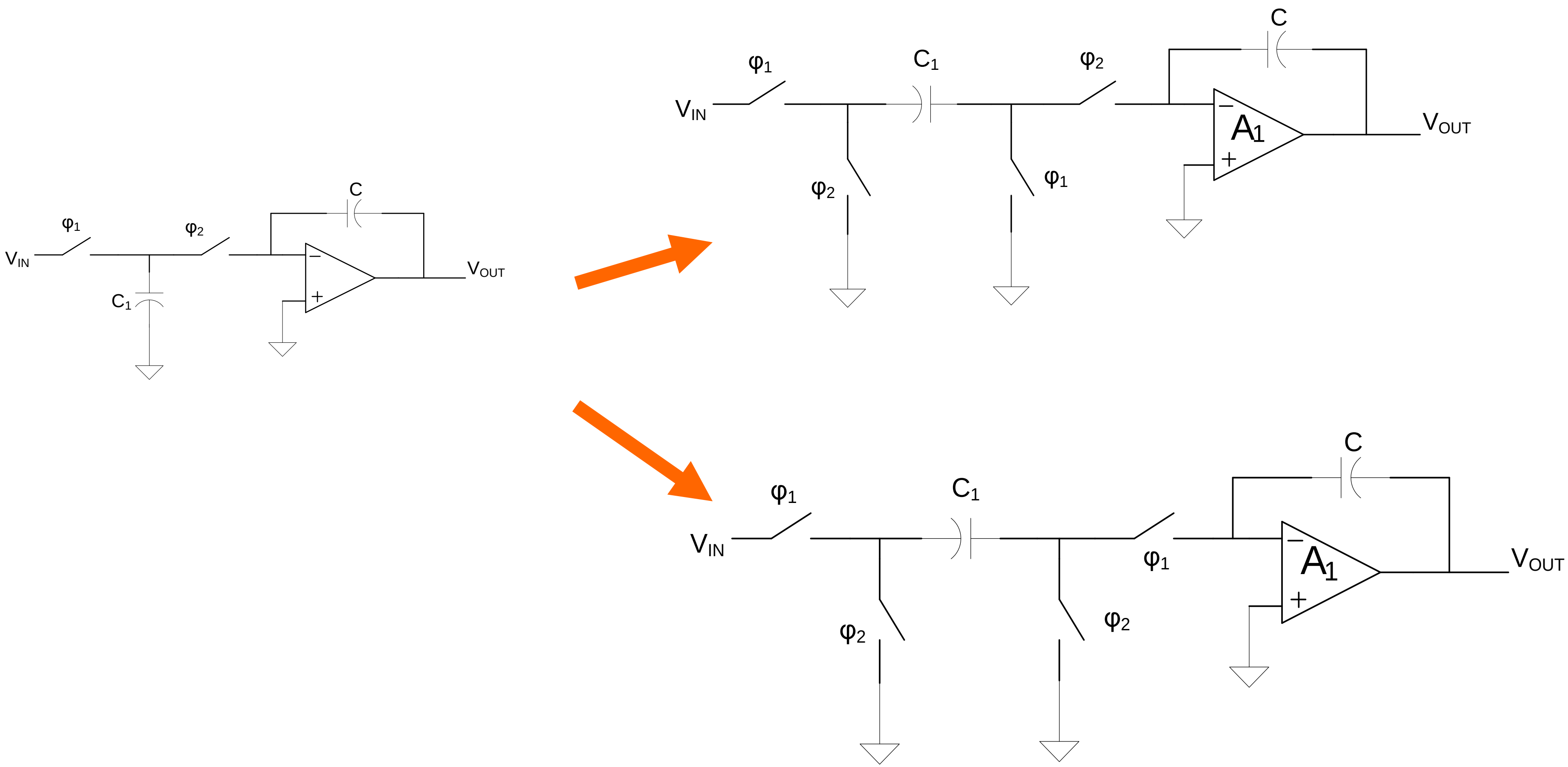


Summing Inputs and Lossy

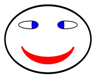
Sensitive to parasitic capacitances

Switched-Capacitor Integrators

Consider the following two SC circuits

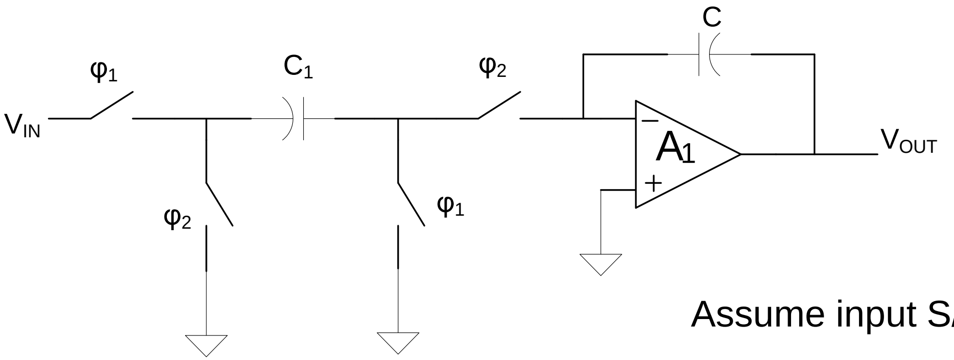


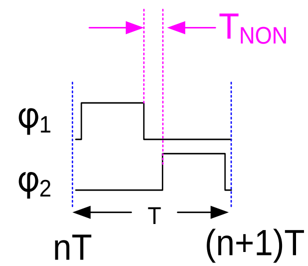
Still have two capacitors but twice as many switches ! 

But switches can be pretty small ! 

Switched-Capacitor Integrators

Consider the first SC circuit





Assume input S/H

During phase Φ_1, capacitor V_{IN} is charged up to $V_{IN}(nT)$

During phase Φ_2, this charge is transferred to C and increasing V_{OUT}

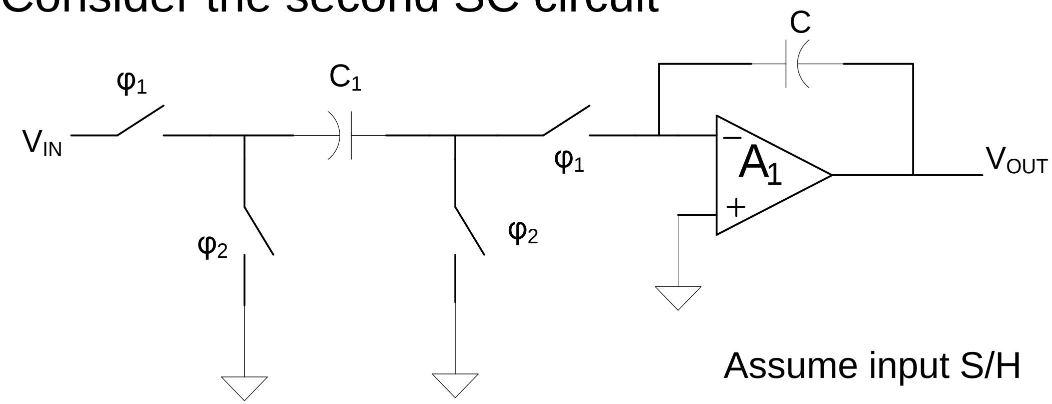
$$V_{OUT}(nT+T) = V_{OUT}(nT) + \frac{C_1}{C} V_{IN}(nT)$$

$$H(z) = \frac{C_1/C}{z-1}$$

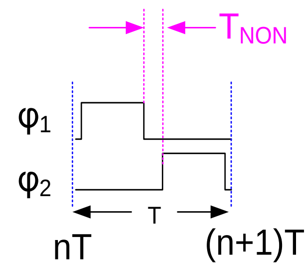
Serves as a non-inverting integrator

Switched-Capacitor Integrators

Consider the second SC circuit



Assume input S/H



Prior to the start of phase Φ_1, the capacitor C_1 was discharged by Φ_2

During phase Φ_1, capacitor V_{IN} charges up to $V_{IN}(nT)$

While charge is flowing into C_1, it is also flowing into C thus decreasing V_{OUT}

$$V_{OUT}(nT+T) = V_{OUT}(nT) - \frac{C_1}{C} V_{IN}(nT+T)$$

$$H(z) = -\frac{z\,{}^{C_1}\!/_{C}}{z-1}$$

Since $|z|_{z=e^{j\omega T}} = 1$

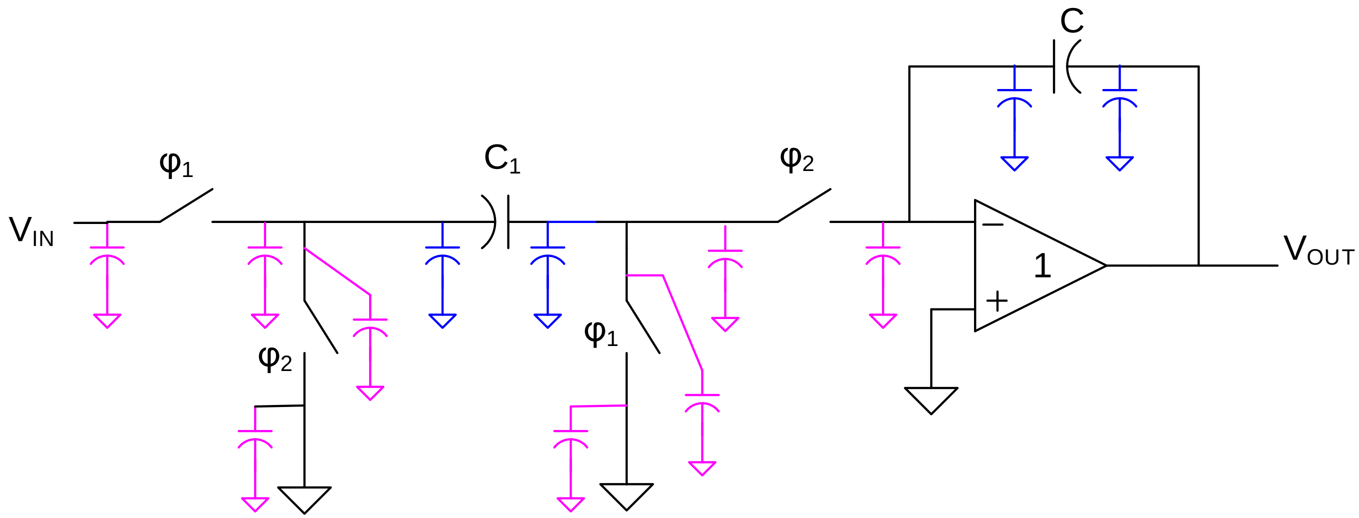
Serves as an inverting integrator

Nonideal Effects in Switched Capacitor Circuits

- Parasitic Capacitances
- Op Amp Affects
- Charge Injection
- Aliasing
- Redundant Switch Removal
- Matching
- Noise

Switched-Capacitor Integrators

Drain and Source Parasitic Capacitors shown in purple
Capacitor Parasitics shown in blue

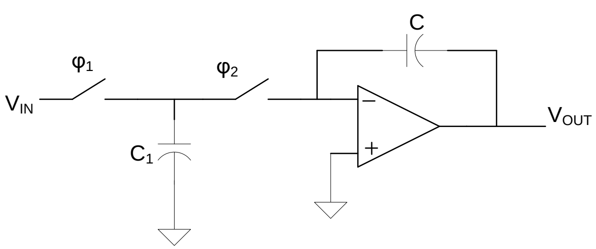


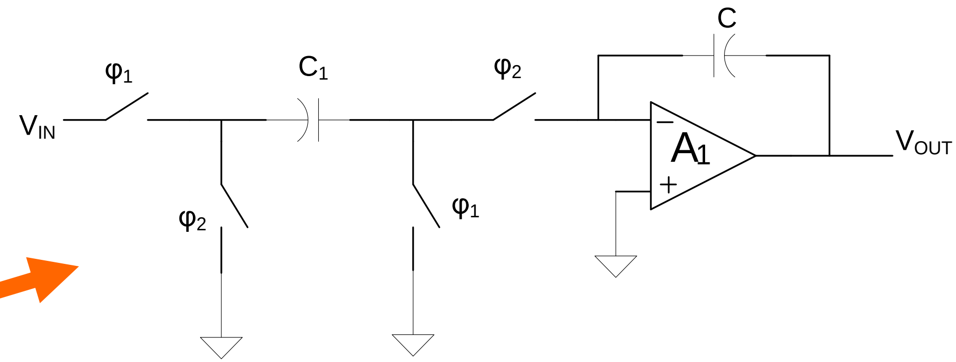
C_{GD} and C_{GS} do not affect gain of integrator
$C_{GCHANNEL}$ does not affect gain of integrator if switch is not too fast

Stray-Insensitive Properties

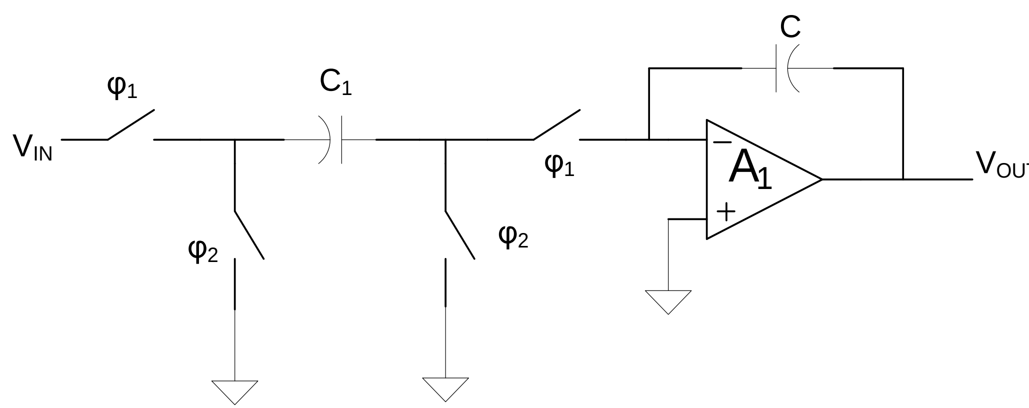
- This structure has more switches and parasitic capacitances than previous SC integrator
- But none of the parasitic capacitances affect the charge transfer thus none affect the gain of the integrator

Switched-Capacitor Integrators





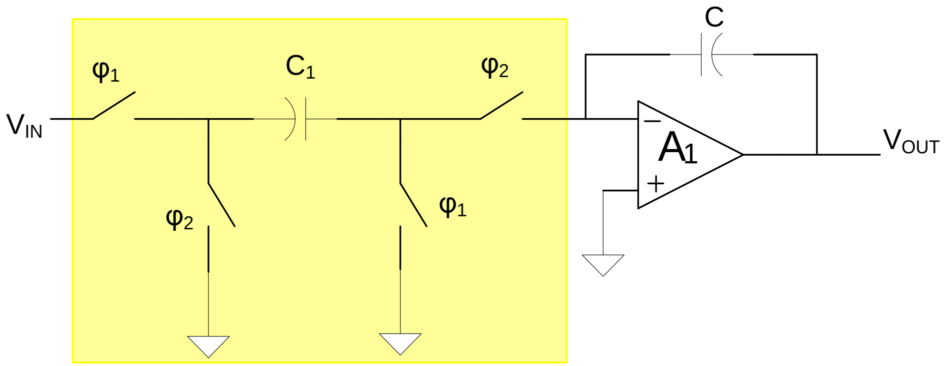
Stray-Insensitive Noninverting

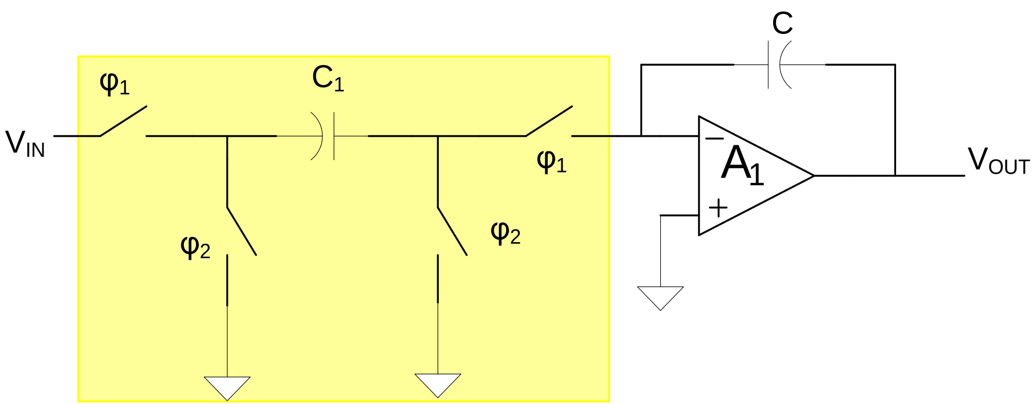


Stray-Insensitive Inverting

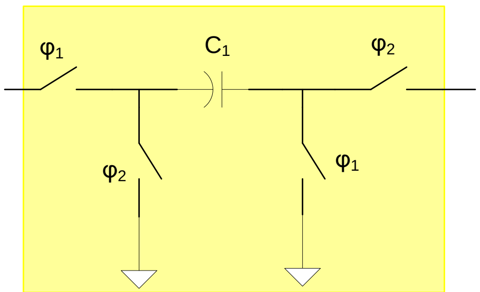
Switched-Capacitor Integrators

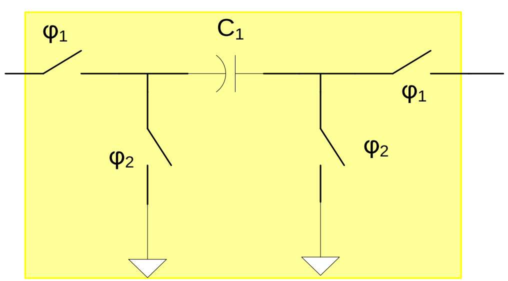
Stray-Insensitive SC Integrators





“Resistor Blocks”

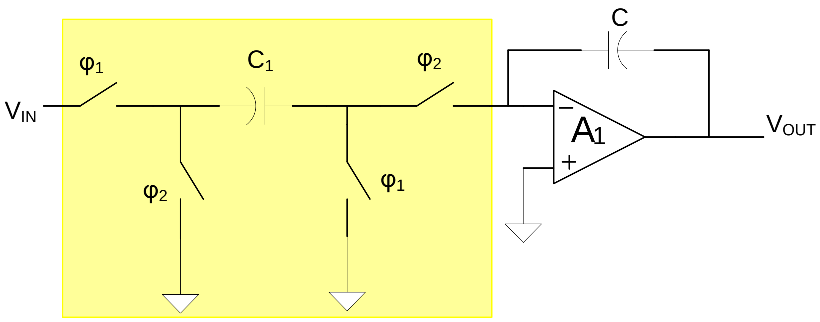


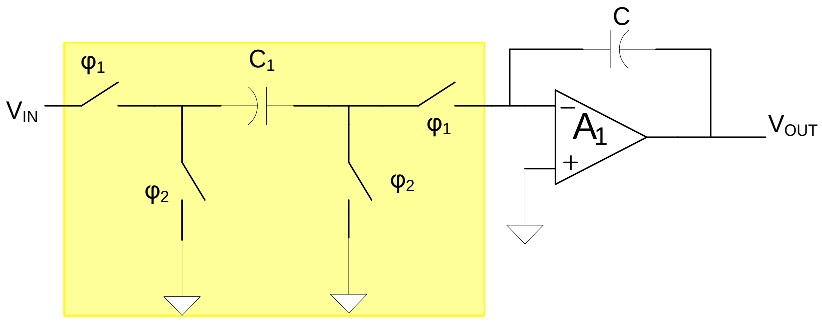


- Resistor blocks can be repeated and combined to provide summing inverting or noninverting inputs
- Resistor block can be placed in FB path to form lossy SC integrator

Switched-Capacitor Integrators

Stray-Insensitive SC Integrators





Many different SC filter structures have been proposed

But most that are actually used are based upon these two circuits with the summing inputs or loss added as needed

Stay Safe and Stay Healthy !

End of Lecture 24