Integrator Design

TA-C Integrators
Other Integrator Structures
Integrator Characteristics of Interest

Properties of an ideal integrator:

\[ |I(j\omega)| = \frac{I_0}{\omega} \quad \text{Gain decreases with } 1/\omega \]

\[ \angle I(j\omega) = -90^\circ \quad \text{Phase is a constant } -90^\circ \]

\[ |I(jI_0)| = 1 \quad \text{Unity Gain Frequency} = 1 \]

How important is it that an integrator have all 3 of these properties?
There are many different ways to build an inverting integrator
Integrator-Based Filter Design

Any of these different types of integrators can be used to build integrator-based filters.
Basic Integrator Functionality

1. **Noninverting**
   - \( X_{IN} \) → \( \frac{l_0}{s} \) → \( X_{OUT} \)

2. **Inverting**
   - \( X_{IN} \) → \( -\frac{l_0}{s} \) → \( X_{OUT} \)

3. **Lossy Noninverting**
   - \( X_{IN} \) → \( \frac{l_0}{s + \alpha} \) → \( X_{OUT} \)

4. **Lossy Inverting**
   - \( X_{IN} \) → \( -\frac{l_0}{s + \alpha} \) → \( X_{OUT} \)

5. **Summing (Multiple-Input) Inverting/Noninverting**
   - \( X_{IN1}, X_{IN2}, \ldots, X_{INn} \) → \( \frac{l_0k}{s} \) → \( X_{OUT} \)
   - \( X_{OUT} = \sum_{k=1}^{n} \frac{\pm l_{Ok}}{s} \)

6. **Summing (Multiple-Input) Lossy Inverting/Noninverting**
   - \( X_{IN1}, X_{IN2}, \ldots, X_{INn} \) → \( \frac{l_0k}{s + \alpha_k} \) → \( X_{OUT} \)
   - \( X_{OUT} = \sum_{k=1}^{n} \frac{\pm l_{Ok}}{s + \alpha_k} \)

7. **Balanced Differential**
   - \( X^+_{IN}, X^-_{IN} \) → \( +\frac{l_0}{s} + \) → \( X^+_{OUT} \)
   - \( X^-_{IN} \) → \( -\frac{l_0}{s} - \) → \( X^-_{OUT} \)
   - \( X^+_{OUT} - X^-_{OUT} = \frac{l_0}{s} (X^+_{IN} - X^-_{IN}) \)

8. **Fully Differential**
   - \( X^{\text{Indiff}}_{IN} \) → \( +\frac{l_0}{s} + \) → \( X^{\text{OUTdiff}} \)
   - \( X^{\text{OUTdiff}} = \frac{l_0}{s} X^{\text{Indiff}} \)
An inverting/noninverting integrator pair define a family of integrators
All integrator functional types can usually be obtained from the inverting/noninverting integrator pair
Suffices to focus primarily on the design of the inverting/noninverting integrator pair since properties of class primarily determined by properties of integrator pair
Integrator Types

Will consider first the Voltage Mode type of integrators.
Voltage Mode Integrators

- Active RC (Feedback-based)
- MOSFET-C (Feedback-based)
- OTA-C
- TA-C
- Sometimes termed “current mode”
- Other Continuous-time Structures
- Switched Capacitor
- Switched Resistor
- Discrete Time
Active RC Voltage Mode Integrator

\[ V_{OUT} = -\frac{1}{CR_s} V_{IN} \]

- Limited to low frequencies because of Op Amp limitations
- No good resistors for monolithic implementations
  - Area for passive resistors is too large at low frequencies
    Some recent work by Haibo Fei shows promise for some audio frequency applications
- Capacitor area too large at low frequencies for monolithic implementatins
- Active devices are highly temperature dependent, proc. dependent, and nonlinear
- No practical tuning or trimming scheme for integrated applications with passive resistors
Voltage Mode Integrators

- Active RC (Feedback-based)
- MOSFET-C (Feedback-based)
- OTA-C
- TA-C

\[ \text{Sometimes termed "current mode"} \]

- Other Continuous-time Structures
- Switched Capacitor
- Switched Resistor

\[ \text{Discrete Time} \]
MOSFET-C Voltage Mode Integrator

- Limited to low frequencies because of Op Amp limitations
- Area for $R_{MOS}$ is manageable!
- Active devices are highly temperature dependent, process dependent
- Potential for tuning with $V_C$
- Highly Nonlinear (can be partially compensated with cross-coupled input)

\[ V_{OUT} = -\frac{1}{CR_{MOS}s} V_{IN} \]

A Solution without a Problem
MOSFET-C Voltage Mode Integrator

\[ V_{\text{OUT}} = -\frac{1}{C R_{\text{MOS}} s} V_{\text{IN}} \]

Still A Solution without a Problem

- Improved Linearity
- Some challenges for implementing \( V_C \)
Voltage Mode Integrators

- Active RC (Feedback-based)
- MOSFET-C (Feedback-based)
- OTA-C
- TA-C
  - Sometimes termed “current mode”
  - Other Continuous-time Structures
  - Switched Capacitor
  - Switched Resistor
    - Discrete Time
OTA-C Voltage Mode Integrator

- Requires only two components
- Inverting and Noninverting structures of same complexity
- Good high-frequency performance
- Small area
- Linearity is limited (no feedback in integrator)
- Susceptible to process and temperature variations
- Tuning control can be readily added

Widely used in high frequency applications

\[ V_{OUT} = -\frac{g_m}{sC} V_{IN} \]

Inverting

\[ V_{OUT} = \frac{g_m}{sC} V_{IN} \]

Noninverting
OTA-C Voltage Mode Integrator

\[
V_{OUT} = \frac{g_m}{sC} V_{IN}
\]

Programmable Integrator

\[
V_{OUT} = \frac{g_m}{sC} V_{IN}
\]

\[
g_m = f(I_{ABC})
\]
OTA-C Voltage Mode Integrator

\[ V_{OUT} = \frac{g_m}{sC} V_{IN} \]

\[ \frac{V_{OUT}(s)}{V_{IN}(s)} = \frac{g_m R_F}{1 + sR_FC} \]

Lossy Integrator

But \( R_F \) is typically too large for integrated applications
OTA-C Voltage Mode Integrator

\[ I = -g_m V \]

\[ g_{EQ} = \frac{I}{V} \]

\[ g_{EQ} = g_m \]

OTA is generally much smaller than a resistor
**OTA-C Voltage Mode Integrator**

\[ V_{OUT} = \frac{g_m}{sC} V_{IN} \]

### Practical implementation
- Both OTAs can be readily programmable

### Lossy Integrator

\[ \frac{V_{OUT}(s)}{V_{IN}(s)} = \frac{g_m/g_{mA}}{1+s(C/g_{mA})} \]
OTA-C Voltage Mode Integrator

- Inverting and noninverting functions can be combined in single summer
- All transconductance gains can be programmable

\[ V_{OUT} = \frac{g_m}{sC} V_{IN} \]

\[ V_{OUT} = \sum_{k=1}^{n} g_{mk} V_{INk} \]

\[ V_{O U T} = \frac{\sum_{k=1}^{n} g_{mk} V_{INk}}{sC} \]
OTA Architecture

- $M_1$ and $M_2$ matched
- $M_2$ and $M_4$ matched
- Define $M$ to be the gain of the current mirror formed with $M_2$ and $M_4$
- $g_m$ programmable with $V_{BIAS}$

$$g_m = \frac{g_{m1}}{2} (1+M)$$

Often $M=1$

$$g_m = g_{m1}$$

Other OTAs exist, considerable effort expended over past two decades on OTA design
Voltage Mode Integrators

- Active RC  (Feedback-based)
- MOSFET-C  (Feedback-based)
- OTA-C
- TA-C
- Other Continuous-time Structures
  - Switched Capacitor
  - Switched Resistor  (Discrete Time)

Sometimes termed “current mode”
TA-C Voltage Mode Integrator

- Can operate at very high frequencies
- Low device count circuit
- Simplicity is important for operating at very high frequencies
- $I_0$ is process and temperature dependent
- Linearity is limited

**Inverting Integrator**

$$V_{OUT} = \left( -\frac{g_m}{sC} \right) V_{IN}$$

**Noninverting Integrator**

$$V_{OUT} = \left( \frac{g_m M}{sC} \right) V_{IN}$$

Typically $M=1$
TA-C Voltage Mode Integrator

Some other perspectives

n-channel input

\[ V_{\text{OUT}} = \left( -\frac{g_m}{sC} \right) V_{\text{IN}} \]

Inverting Integrators

p-channel input
TA-C Voltage Mode Integrator

Can be viewed either as n-channel input with current mirror or as low-gain inverter driving a p-channel input inverting integrator.
**TA-C Voltage Mode Integrator**

\[ V_{OUT} = \left( \frac{-g_m}{sC} \right) V_{IN} \]

Inverting Integrator

\[ V_{OUT} = \left( \frac{g_m M}{sC} \right) V_{IN} \]

Typically \( M = 1 \)

Alternate noninverting Integrator
TA-C Voltage Mode Integrator

Summing Inverting Integrator
Voltage Mode Integrators

- Active RC (Feedback-based)
- MOSFET-C (Feedback-based)
- OTA-C
- TA-C

Sometimes termed “current mode”

- Other Continuous-time Structures
- Switched Capacitor
- Switched Resistor

Discrete Time
Another Voltage Mode Integrator

\[
V_{OUT} = \left( \frac{-1}{sRC} \right) V_{IN}
\]

Inverting Integrator

- Infinite input impedance (in contrast to basic Active RC Integrator)
- Both R and C have one terminal grounded
- Requires integrated process
- Accuracy limited by process and temperature
- Size limitations same as basic Active RC Integrator
- Limited to lower frequencies because of Op Amp
- Good linearity
Another Voltage Mode Integrator

Inverting Integrator

\[ V_{OUT} = \left( \frac{-1}{sR_{FET}C} \right) V_{IN} \]

- \( M_1 \) in triode region
- Reduces Area Concerns but Loss of Linearity
- \( I_0 \) is programmable with \( V_{RR} \)
- Accurate control of \( I_B \) critical

Noninverting Integrator

\[ V_{OUT} = \left( \frac{1}{sR_{FET}C} \right) V_{IN} \]
Regulated Cascode Voltage Mode Integrator

Inverting Integrator

\[ V_{\text{OUT}} = \left( -\frac{g_{mT}}{sC} \right) V_{\text{IN}} \]

Noninverting Integrator

\[ V_{\text{OUT}} = \left( \frac{g_{mT}}{sC} \right) V_{\text{IN}} \]

\( g_{mT} \) is triode region transconductance of \( M_1 \)

- \( M_1 \) operating in triode region
- \( R_{\text{FET}} \) programmable with \( V_{RR} \)
- Very good linearity properties
- Input impedance still infinite
Regulated Cascode Voltage Mode Integrator

Linearity Properties:
Assuming square-law triode model

\[ I_{D1} = \frac{\mu C_{Ox} W}{L} \left( V_{GS} - V_T - \frac{V_{RR}}{2} \right) V_{RR} \]

\[ I_{D1} = \left[ \frac{\mu C_{Ox} W}{L} V_{RR} \right] V_{IN} + \left[ \frac{\mu C_{Ox} W}{L} \left( V_T + \frac{V_{RR}}{2} \right) V_{RR} \right] \]

Note linear dependence on \( V_{IN} \)

\[ g_{mT} = \left[ \frac{L}{\mu C_{Ox} W V_{RR}} \right] \]
**Regulated Cascode Voltage Mode Integrator**

**Inverting Integrator**

\[
V_{\text{OUT}} = \left(-\frac{1}{sR_{\text{FET}}C}\right)V_{\text{IN}}
\]

- Multiple inputs require single additional transistor
- Accurate ratioing of gains practical
- Can also sum currents on C
Regulated Cascode Voltage Mode Integrator

Inverting Integrator

\[ V_{OUT} = \left( \frac{-1}{sR_{FET}C} \right) V_{IN} \]

Inverting Lossy Integrator
Another Voltage Mode Integrator

Inverting Integrator

\[ V_{\text{OUT}} = \left( \frac{-1}{sRC} \right) V_{\text{IN}} \]

Noninverting Integrator

\[ V_{\text{OUT}} = \left( \frac{1}{sRC} \right) V_{\text{IN}} \]
End of Lecture 25