Integrator Design Issues

The Switched Capacitor Integrator
Review from Last Time

Filter Synthesis Methods (with op amp feedback)

- Cascaded Biquads

- Multiple Feedback Structures
  - Leapfrog
  - Direct Synthesis
  - Primary Resonator Block
Basic Building Blocks in Both Cascaded Biquads and Multiple Feedback Structures

- Developed from observations from feedback implementations
  1. Integrators
  2. Summers
  3. First-order filter blocks
  4. Biquads

- Same building blocks used in open-loop applications as well
Consider the Basic Integrator

Key performance of integrator (and integrator-based filter) is determined by the integrator time constant $I_0$

Precision of time constants of a filter invariably determined by precision of $I_0$
Consider the Basic Integrator

\[ T(s) = -\frac{1}{RCs} \]

\[ I_0 = \frac{1}{RC} \]

1. Accuracy of R and C difficult to accurately control – particularly in integrated applications (often 2 or 3 orders of magnitude to variable)

2. Size of R and C unacceptably large if \( I_0 \) is in audio frequency range (2 or 3 orders of magnitude too large)

3. Amplifier GB limits performance

Incredible Challenge to Building Filters on Silicon!
Integrator Design Issues

Consider:

Assume $T_{CLK} \ll T_{SIG}$

$\Phi_1$ and $\Phi_2$ are complimentary nonoverlapping clocks

Termed a switched-capacitor circuit
Consider the Switched-Capacitor Circuit

Assume $T_{CLK} \ll T_{SIG}$

$\Phi_1$ and $\Phi_2$ are complimentary nonoverlapping clocks
Consider the Switched-Capacitor Circuit

Assume $T_{CLK} \ll T_{SIG}$

$\Phi_1$ and $\Phi_2$ are complimentary nonoverlapping clocks
Compare the performance of the following two circuits

\[ T(s) = -\frac{1}{RCs} \quad I_0 = \frac{1}{RC} \]
Consider the charge transferred to the feedback capacitor for both circuits in an interval of length $T_{CLK}$ at time $t_1$

For the RC circuit:

$$Q_{RC} = \int_{t_1}^{t_1+T_{CLK}} i_{in}(t) dt$$

$$Q_{RC} = \int_{t_1}^{t_1+T_{CLK}} \frac{V_{in}(t)}{R} dt$$

Since $V_{in}$ changes slowly

$$Q_{RC} \approx \int_{t_1}^{t_1+T_{CLK}} \frac{V_{in}(t_1)}{R} dt$$

$$Q_{RC} \approx \left[ \frac{V_{in}(t_1)}{R} \right] \int_{t_1}^{t_1+T_{CLK}} 1 dt$$

$$Q_{RC} \approx \left[ \frac{V_{in}(t_1)}{R} \right] T_{CLK}$$
Consider the charge transferred to the feedback capacitor for both circuits in an interval of length $T_{CLK}$ at time $t_1$.

For the RC circuit:

$$Q_{RC} \approx \left[ \frac{V_{in}(t_1)}{R} \right] T_{CLK}$$

Observe that a resistor “transfers” charge proportional to $V_{in}$ in a short interval of $T_{CLK}$. 
For the SC circuit

\[ Q_{C1} = C_1 V_{in} \left( t_1 + \frac{T_{CLK}}{2} - \varepsilon \right) \]

Since \( V_{in}(t) \) is slowly varying

\[ Q_{C1} \approx C_1 V_{in} \left( t_1 \right) \]

But this is the charge that will be transferred to \( C \) during phase \( \Phi_2 \)

\[ Q_{SC} \approx C_1 V_{in} \left( t_1 \right) \]

Observe that the SC circuit also transfers charge proportional to \( V_{in} \) in short intervals of length \( T_{CLK} \)
Comparing the two circuits

\[ T(s) = -\frac{1}{RCs} \]

\[ I_0 = \frac{1}{RC} \]

Equating charges since both proportional to \( V_{in}(t_1) \)

\[ Q_{RC} \approx \left[ \frac{V_{in}(t_1)}{R} \right] T_{CLK} \]

\[ Q_{SC} \approx C_1 V_{in}(t_1) \]

\[ C_1 \approx \left[ \frac{1}{R} \right] T_{CLK} \]

\[ R_{EQ} \approx \frac{1}{f_{CLK}C_1} \]
\[ T(s) = -\frac{1}{RC_s} \]

\[ I_0 = \frac{1}{RC} \]

\[ R_{EQ} \approx \frac{1}{f_{CLK}C_1} \]

Observe that a switched-capacitor behaves as a resistor!

This is an interesting observation that was made by Maxwell over 100 years ago but in and of itself was of almost no consequence.

Note that large resistors require small capacitors!

This offers potential for overcoming one of the critical challenges for implementing integrators on silicon at audio frequencies!
Consider again the SC integrator

\[ T_{SC}(s) \approx -\frac{1}{R_{EQ}C_s} \]

\[ I_{0eq} = \frac{1}{R_{EQ}C} \]

\[ I_{0eq} = \frac{1}{R_{EQ}C} = \frac{C_1f_{CLK}}{C} \]

\[ I_{0eq} = \left[ \frac{C_1}{C} \right] f_{CLK} \]

\[ R_{EQ} \approx \frac{1}{f_{CLK}C_1} \]

This is a frequency referenced filter!
Consider again the SC integrator

\[ T_{SC}(s) = \frac{-1}{R_{EQ}C_s} \]

The expressions \( S'_C \) and \( S'_C \) have the same magnitude as for the RC integrator

- But the ratio of capacitors can be accurately controlled in IC processes
  
  (1% to .01% is achievable with careful layout)

- \( f_{CLK} \) can be VERY accurately controlled with a crystal  
  
  (1 part in \( 10^6 \) or better)

- Variability of \( I_{0eq} \) is very small

The SC integrator can dramatically reduce the second main concern for building integrated integrators
Consider again the SC integrator

\[ T(s) = -\frac{1}{RC_s}, \quad I_0 = \frac{1}{RC} \]

1. Accuracy of R and C difficult to accurately control (often 2 or 3 orders of magnitude to variable)
2. Area of R and C too large in audio frequency range (2 or 3 orders of magnitude too large)
3. Amplifier GB limits performance

\[ I_{0eq} = \left[ \frac{C_1}{C} \right] f_{CLK} \]

1. Accuracy of cap ratio and \( f_{CLK} \) very good
2. Area of \( C_1 \) and C not too large
3. Amplifier GB limits performance less
Consider again the SC integrator

\[
I_{0eq} = \left[ \frac{C_1}{C} \right] f_{CLK}
\]

Parasitic capacitors \(C_{s1}+C_{d2}+C_{T1}\) difficult to accurately match

Parasitic capacitors of THIS SC integrator limit performance

Other SC integrators (discussed later) offer same benefits but are not affected by parasitic capacitors

\[
C_{1EQ} = C_1 + C_{s1} + C_{d2} + C_{T1}
\]
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  3. First-order filter blocks
  4. Biquads
  5. Switches

- Same building blocks used in open-loop applications as well
Switched-Capacitor Filter Issues

What if $T_{\text{CLK}}$ is not much much smaller than $T_{\text{SIG}}$?

For $T_{\text{CLK}} \ll T_{\text{SIG}}$
Switched-Capacitor Filter Issues

What if $T_{\text{CLK}}$ is not much-much smaller than $T_{\text{SIG}}$?

For $T_{\text{CLK}} < T_{\text{SIG}}$
Switched-Capacitor Filter Issues

What if $T_{CLK}$ is not much-much smaller than $T_{SIG}$?

For $T_{CLK}<<T_{SIG}$
Switched-Capacitor Filter Issues

What if $T_{CLK}$ is not much-much smaller than $T_{SIG}$?

For $T_{CLK} < T_{SIG}$
Switched-Capacitor Filter Issues

What if $T_{CLK}$ is not much-much smaller than $T_{SIG}$?

For $T_{CLK} \ll T_{SIG}$

\[ V(nT) \rightarrow V((n+1)T) \]

Considerable change in $V(t)$ in clock period

For $T_{CLK} < T_{SIG}$

\[ V(nT) \rightarrow V((n+1)T) \]
Switched-Capacitor Filter Issues

What if $T_{CLK}$ is not much much smaller than $T_{SIG}$?

For $T_{CLK} < T_{SIG}$

$V(nT)$

$V((n+1)T)$

Define $T = T_{CLK}$

$V_0(nT+T) = V_0(nT) + \frac{\Delta Q}{C}$

but $-\Delta Q$ is the charge on $C_1$ and the time $\phi_1$ opens

$-\Delta Q \approx C_1 V_{IN}(nT+T/2)$

$\therefore V_{OUT}(nT+T) = V_{OUT}(nT) - (C_1/C)V_{IN}(nT+T/2)$

If an input S/H, $V_{IN}$ constant over periods of length $T$

thus, assume $V_{IN}(nT+T/2) \approx V_{IN}(nT)$

So obtain

$V_{OUT}(nT+T) = V_{OUT}(nT) - (C_1/C)V_{IN}(nT)$
Switched-Capacitor Filter Issues

What if $T_{CLK}$ is not much smaller than $T_{SIG}$?

$$V_{OUT}(nT+T)=V_{OUT}(nT)-(C_1/C)V_{IN}(nT)$$

for any $T_{CLK}$, characterized in time domain by difference equation

or in frequency domain characterized by transfer function obtained by taking $z$-transform of the difference equation

$$H(z)=-\frac{C_1}{C}\frac{1}{z-1}$$
What is really required for building a filter that has high-performance features?

**Frequency domain:**

Transfer function

\[ T(s) = \frac{1}{RCs} \]

**Time domain:**

Differential Equation

\[ V_{OUT}(t) = V_{OUT}(t_0) + \frac{1}{RC} \int_{t_0}^{t} V_{IN}(\tau) d\tau \]

Accurate control of polynomial coefficients in transfer function or accurate control of coefficients in the differential equation
What is really required for building a filter that has high-performance features?

**Frequency domain:**

**Transfer function**

\[ T(s) = \frac{1}{RCs} \]

\[ H(z) = -\frac{c_1}{z-1} \]

**Time domain:**

**Differential Equation**

\[ V_{OUT}(t) = V_{OUT}(t_0) + \frac{1}{RC} \int_{t_0}^{t} V_{IN}(\tau) d\tau \]

**Difference Equation**

\[ V_{OUT}(nT+T) = V_{OUT}(nT) - \left(\frac{C_1}{C}\right)V_{IN}(nT) \]

Accurate control of polynomial coefficients in transfer function or accurate control of coefficients in the differential/difference equation.
Switched-Capacitor Filter Issues

What if $T_{CLK}$ is not much-much smaller than $T_{SIG}$?

\[
V_{OUT}(nT+T)=V_{OUT}(nT)-(\frac{C_1}{C})V_{IN}(nT)
\]

\[
H(z)=-\frac{\frac{C_1}{C}}{z-1}
\]

Switched-capacitor circuits have potential for good accuracy and attractive area irrespective of how $T_{CLK}$ relates to $T_{SIG}$

But good layout techniques and appropriate area need to be allocated to realize this potential!
Practical Background Issues

• Symmetric Circuits

• Parasitic Capacitances with Floating Nodes
How can integrator-based performance be improved?

- Better opamps
- Better Integrator Architectures