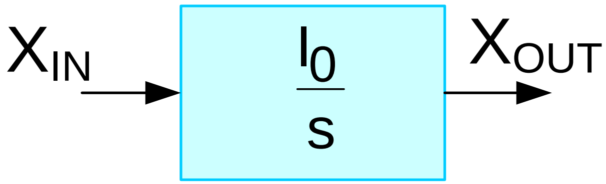
EE 508
Lecture 26

Integrator Design

TA-C Integrators
Other Integrator Structures

Review from last time

Integrator Characteristics of Interest



$$I(s) = \frac{I_0}{s}$$

Properties of an ideal integrator:

$$\left|I(j\omega)\right| = \frac{I_0}{\omega}$$

Gain decreases with 1/ω

$$\angle I(j\omega) = -90^0$$

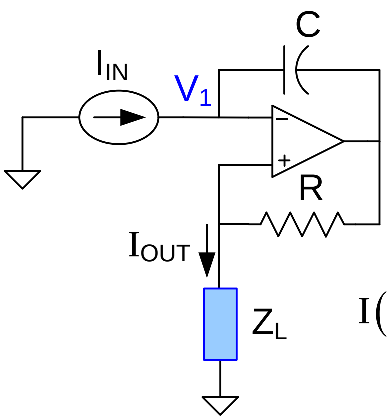
Phase is a constant -90°

$$\left|I(jI_0)\right| = 1$$

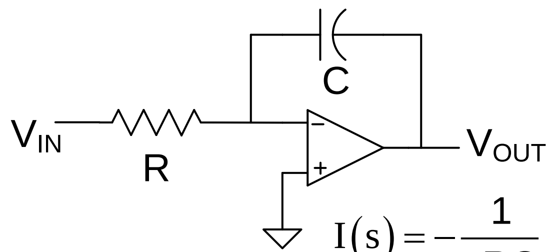
Unity Gain Frequency = 1

How important is it that an integrator have all 3 of these properties?

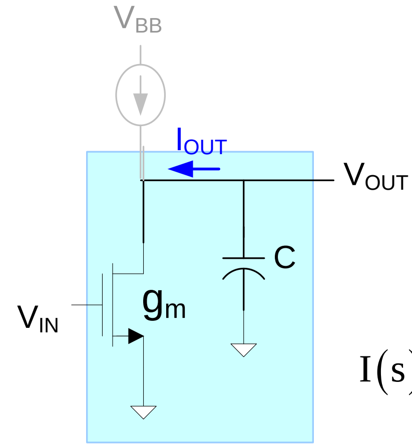
Some integrator structures



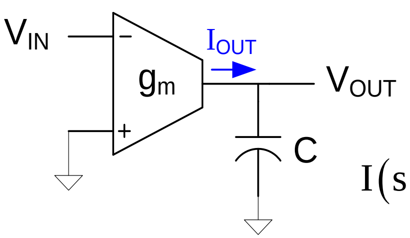
$$I(s) = -\frac{1}{sRC}$$



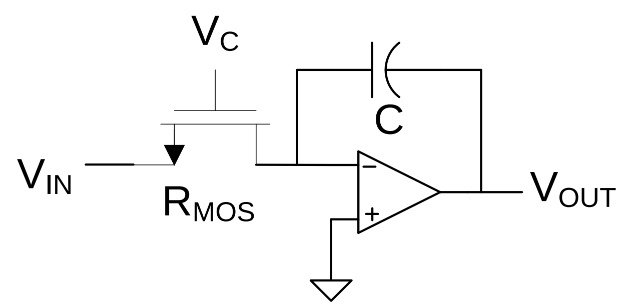
$$I(s) = -\frac{1}{sRC}$$



$$I(s) = -\frac{g_m}{sC}$$



$$I(s) = -\frac{g_m}{sC}$$



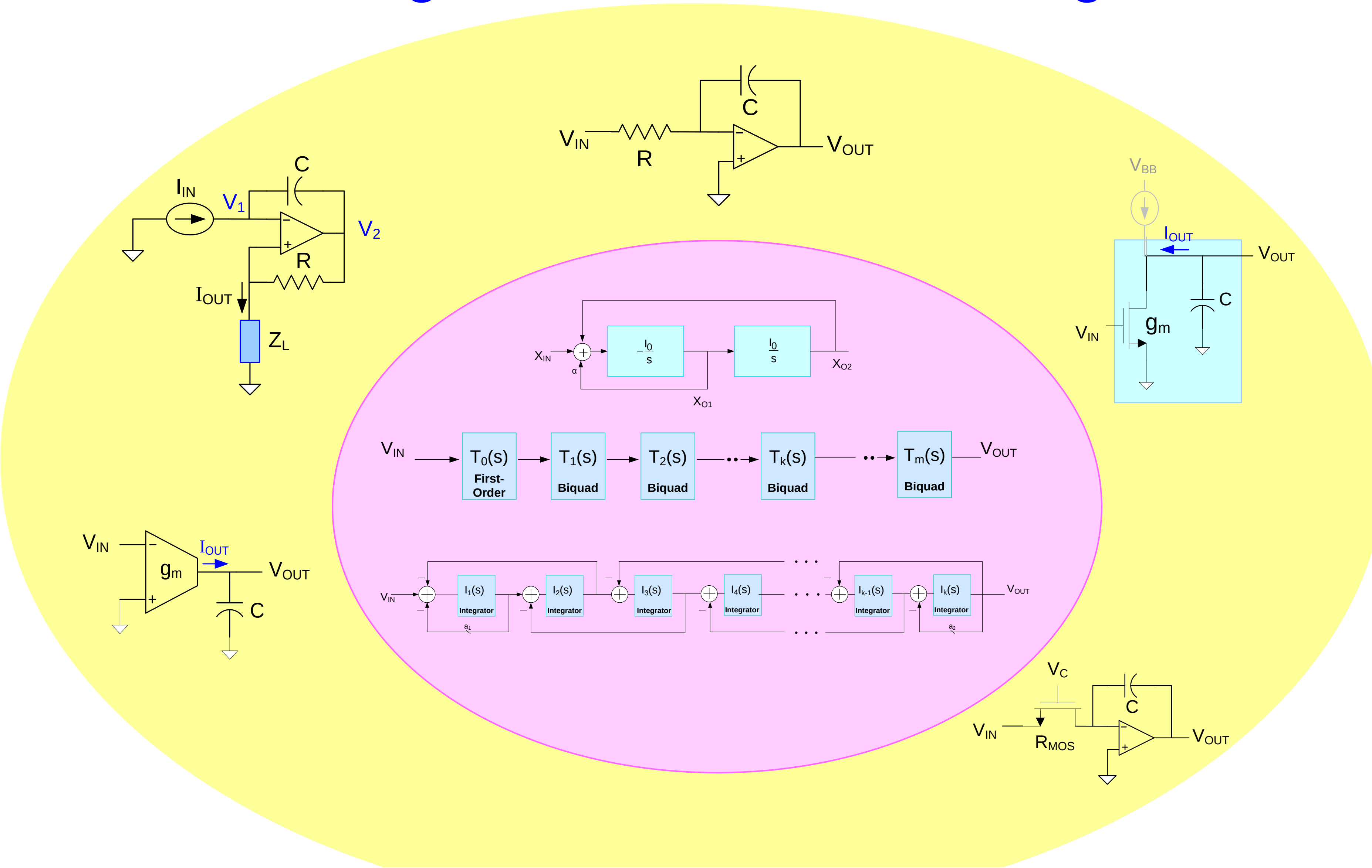
$$I(s) = -\frac{1}{sR_{MOS}C}$$

There are other useful integrator structures (some will be introduced later)

There are many different ways to build an inverting integrator

Review from last time

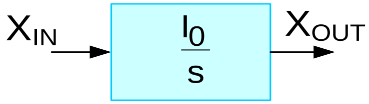
Integrator-Based Filter Design



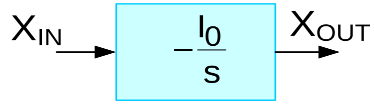
Any of these different types of integrators can be used to build integrator-based filters

Review from last time

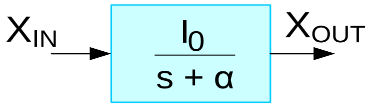
Basic Integrator Functionality



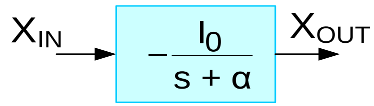
Noninverting



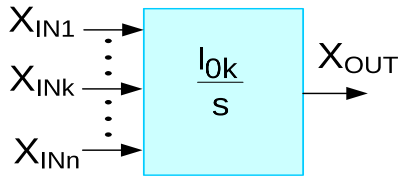
Inverting



Lossy Noninverting

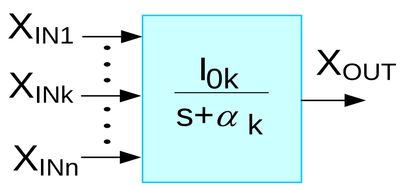


Lossy Inverting



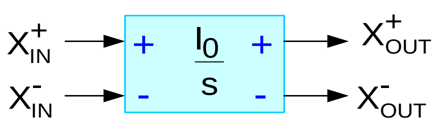
$$X_{OUT} = \sum_{k=1}^{n} \frac{\pm I_{0k}}{s}$$

Summing (Multiple-Input) Inverting/Noninverting



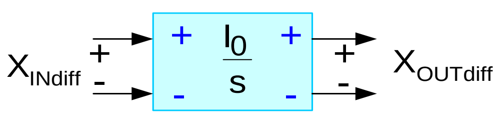
$$X_{OUT} = \sum_{k=1}^{n} \frac{\pm I_{0k}}{s+\alpha_k}$$

Summing (Multiple-Input) Lossy Inverting/Noninverting



$$X_{OUT}^{+} - X_{OUT}^{+} = \frac{I_0}{s}\left(X_{IN}^{+} - X_{IN}^{+}\right)$$

Balanced Differential

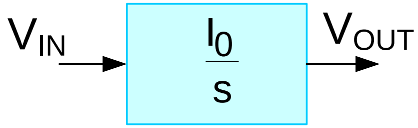


$$X_{OUTdiff} = \frac{I_0}{s} X_{INdiff}$$

Fully Differential

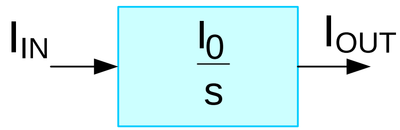
Review from last time

Integrator Types



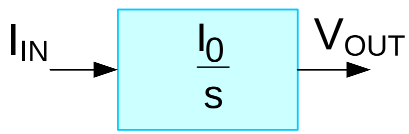
Voltage Mode

$$V_{OUT} = \frac{I_o}{s} V_{IN}$$



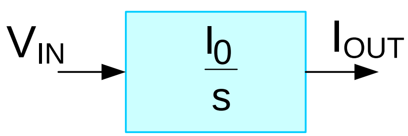
Current Mode

$$I_{OUT} = \frac{I_o}{s} I_{IN}$$



Transresistance Mode

$$V_{OUT} = \frac{I_o}{s} I_{IN}$$



Transconductance Mode

$$I_{OUT} = \frac{I_o}{s} V_{IN}$$

Will consider first the Voltage Mode type of integrators

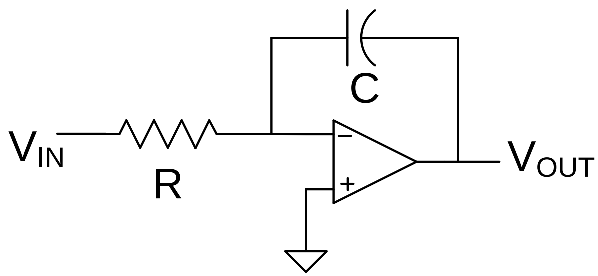
Voltage Mode Integrators

- Active RC (Feedback-based)
- MOSFET-C (Feedback-based)
- OTA-C
- TA-C

  } Sometimes termed "current mode"
- Switched Capacitor
- Switched Resistor

  } Will discuss later
- Other Structures

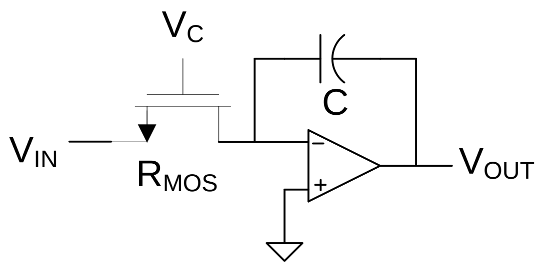
Active RC Voltage Mode Integrator



$$V_{OUT} = -\frac{1}{CRs} V_{IN}$$

- Limited to low frequencies because of Op Amp limitations
- No good resistors for monolithic implementations
  - Area for passive resistors is too large at low frequencies
    - Some recent work by Haibo Fei shows promise for some audio frequency applications
- Capacitor area too large at low frequencies for monolithic implementatins
- Active devices are highly temperature dependent, proc. dependent, and nonlinear
- No practical tuning or trimming scheme for integrated applications with passive resistors

MOSFET-C Voltage Mode Integrator

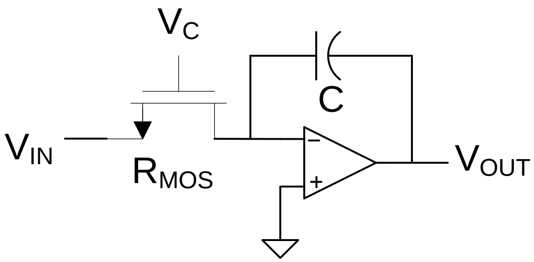


$$V_{OUT} = -\frac{1}{CR_{MOS}s} V_{IN}$$

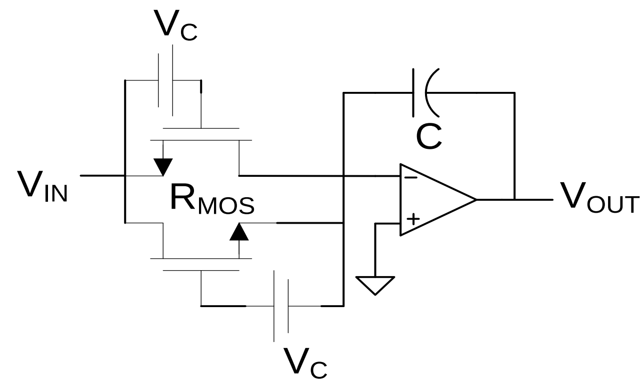
- Limited to low frequencies because of Op Amp limitations
- Area for R_{MOS} is manageable !
- Active devices are highly temperature dependent, process dependent
- Potential for tuning with V_C
- Highly Nonlinear (can be partially compensated with cross-coupled input

A Solution without a Problem

MOSFET-C Voltage Mode Integrator



$$V_{OUT} = -\frac{1}{C\,R_{MOS}s}V_{IN}$$

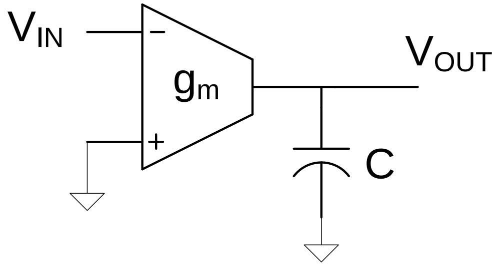


$$V_{OUT} = -\frac{1}{C\,R_{MOS}s}V_{IN}$$

- Improved Linearity
- Some challenges for implementing V_C

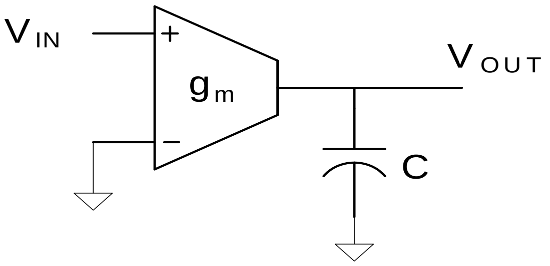
Still A Solution without a Problem

OTA-C Voltage Mode Integrator



$$V_{OUT} = -\frac{g_m}{sC} V_{IN}$$

Inverting



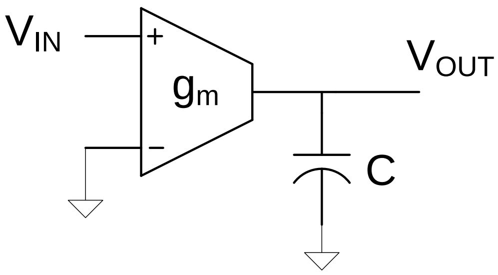
$$V_{OUT} = \frac{g_m}{sC} V_{IN}$$

Noninverting

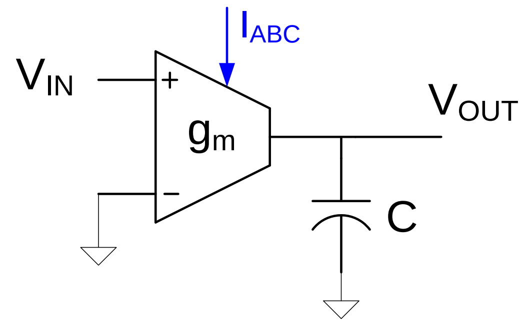
- Requires only two components
- Inverting and Noninverting structures of same complexity
- Good high-frequency performance
- Small area
- Linearity is limited (no feedback in integrator)
- Susceptible to process and temperature variations
- Tuning control can be readily added

Widely used in high frequency applications

OTA-C Voltage Mode Integrator



$$V_{OUT} = \frac{g_m}{sC} V_{IN}$$

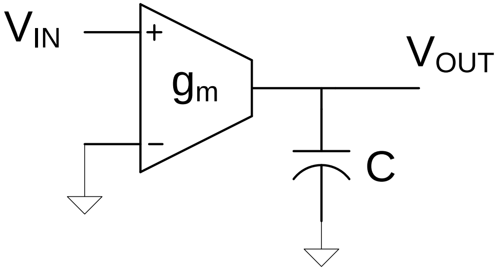


$$V_{OUT} = \frac{g_m}{sC} V_{IN}$$

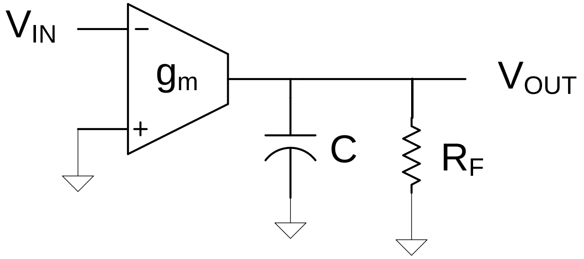
$$g_m = f\left(I_{ABC}\right)$$

Programmable Integrator

OTA-C Voltage Mode Integrator



$$V_{OUT} = \frac{g_m}{sC} V_{IN}$$

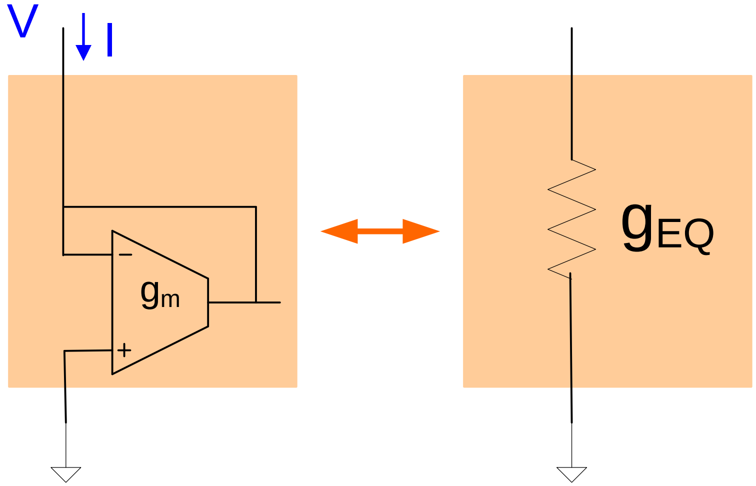


$$\frac{V_{OUT}(s)}{V_{IN}(s)} = \frac{g_m R_F}{1+s(R_F C)}$$

Lossy Integrator

But R_F is typically too large for integrated applications

OTA-C Voltage Mode Integrator



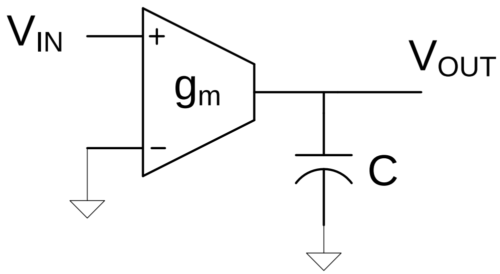
$$I = -\,-g_m V$$

$$g_{EQ} = \frac{I}{V}$$

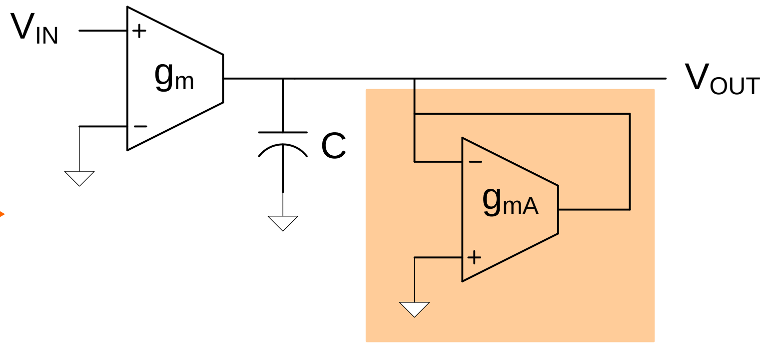
$$g_{EQ} = g_m$$

OTA is generally much smaller than a resistor

OTA-C Voltage Mode Integrator



$$V_{OUT} = \frac{g_m}{sC} V_{IN}$$

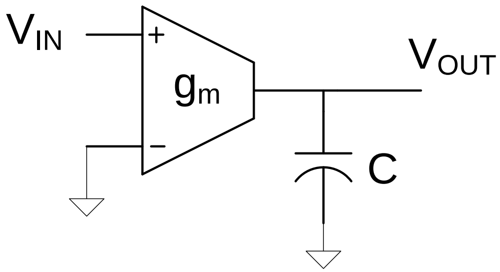


$$\frac{V_{OUT}(s)}{V_{IN}(s)} = \frac{g_m/g_{mA}}{1+s(C/g_{mA})}$$

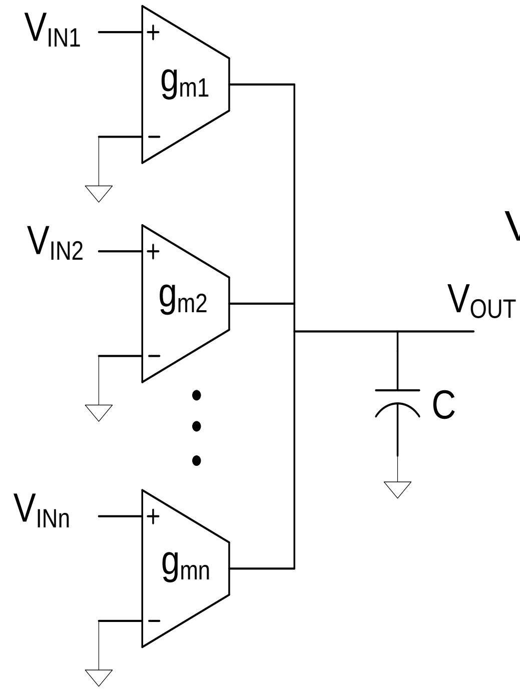
Lossy Integrator

- Practical implementation
- Both OTAs can be readily programmable

OTA-C Voltage Mode Integrator



$$V_{OUT} = \frac{g_m}{sC} V_{IN}$$

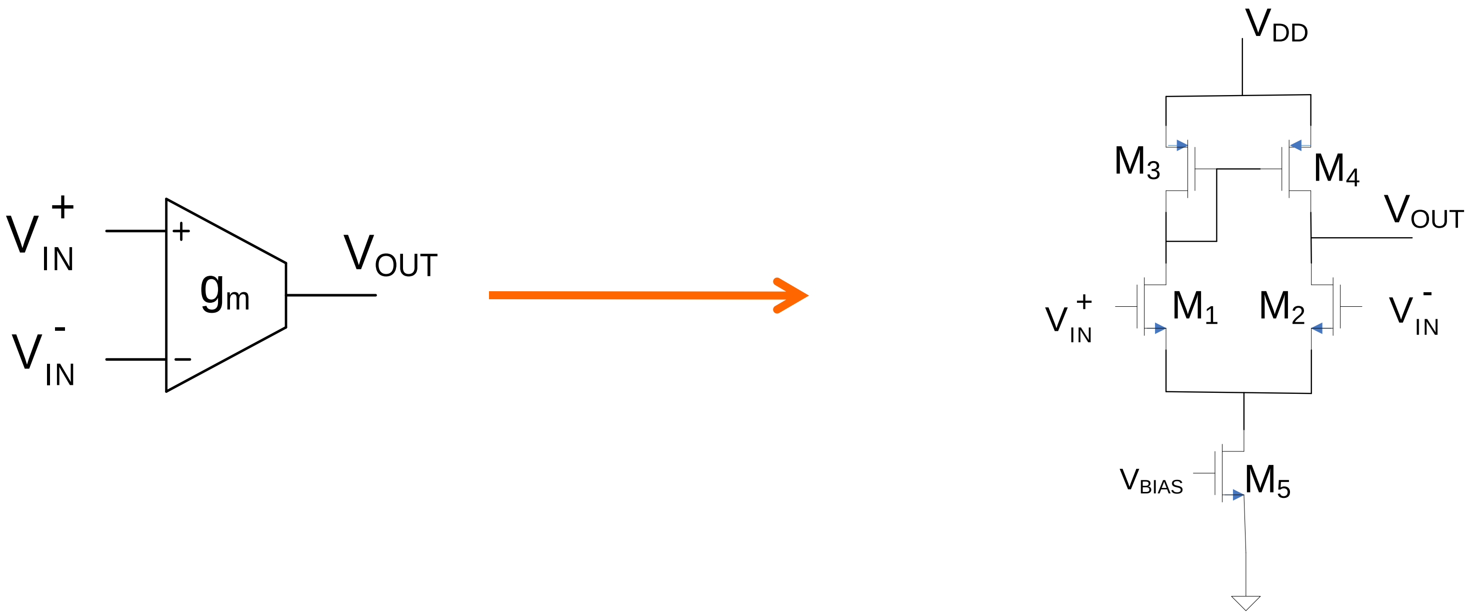


$$V_{OUT} = \frac{\sum_{k=1}^{n} g_{mk} V_{INk}}{sC}$$

Summing Integrator

- Inverting and noninverting functions can be combined in single summer
- All transconductance gains can be programmable

OTA Architecture



Mid-complexity OTA

- M_1 and M_2 matched
- M_2 and M_4 matched
- Define M to be the gain of the current mirror formed with M_2 and M_4
- g_m programmable with V_{BIAS}

$$g_m = \frac{g_{m1}}{2}(1+M)$$

Often M=1

$$g_m = g_{m1}$$

Other OTAs exist, considerable effort expended over past two decades on OTA design

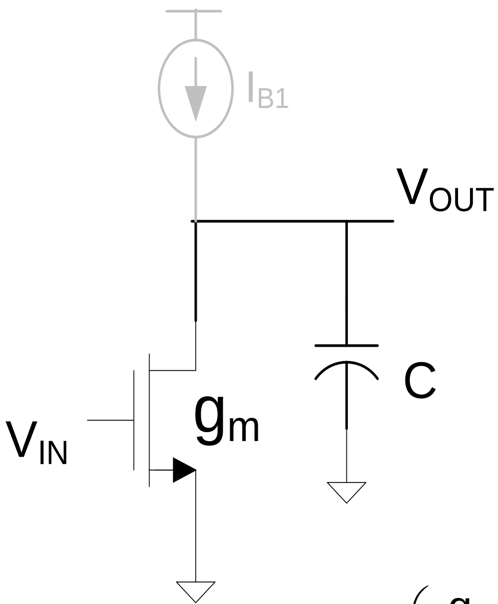
Voltage Mode Integrators

- Active RC (Feedback-based)
- MOSFET-C (Feedback-based)
- OTA-C
- TA-C

  Sometimes termed "current mode" (OTA-C, TA-C)
- Switched Capacitor
- Switched Resistor

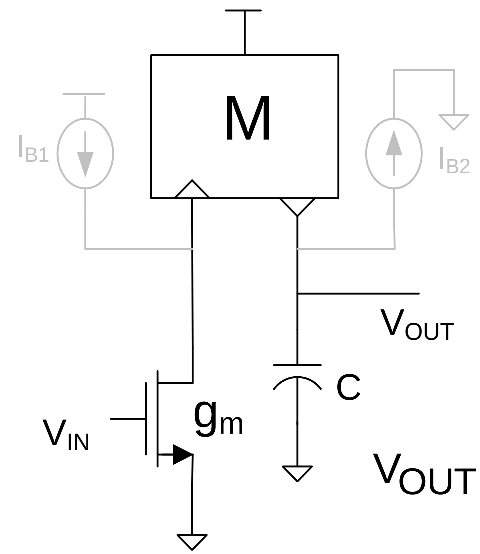
  Will discuss later (Switched Capacitor, Switched Resistor)
- Other Structures

TA-C Voltage Mode Integrator



$$V_{OUT}=\left(\frac{-g_m}{sC}\right)V_{IN}$$

Inverting Integrator



$$V_{OUT}=\left(\frac{g_m M}{sC}\right)V_{IN}$$

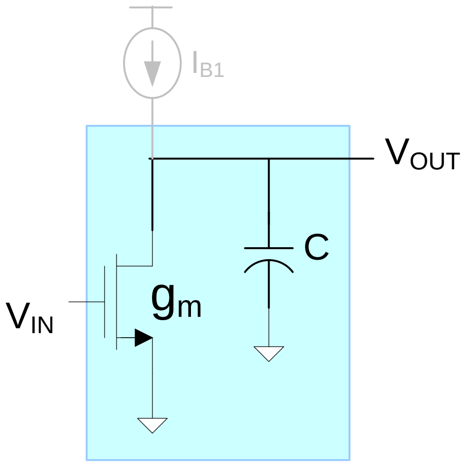
Typically M=1

Noninverting Integrator

- Can operate at very high frequencies
- Low device count circuit
- Simplicity is important for operating at very high frequencies
- I_0 is process and temperature dependent
- Linearity is limited

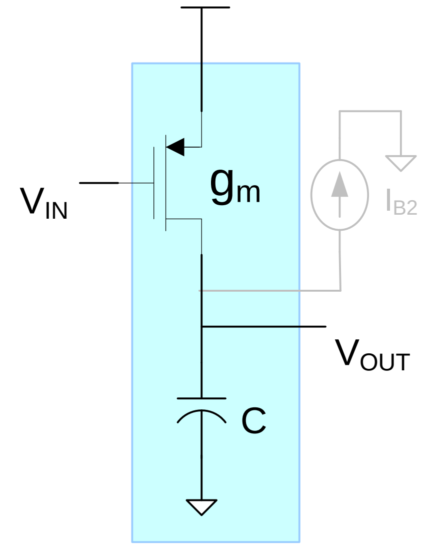
TA-C Voltage Mode Integrator

Some other perspectives



n-channel input

$$V_{OUT} = \left(\frac{-g_m}{sC}\right) V_{IN}$$

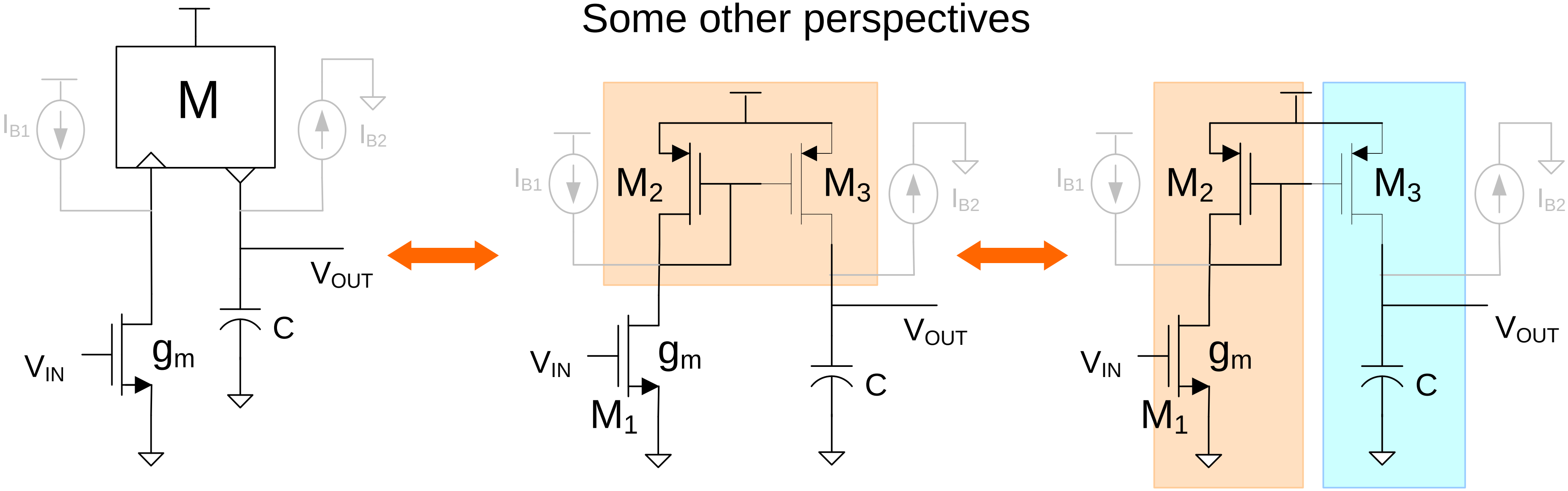


p-channel input

Inverting Integrators

TA-C Voltage Mode Integrator

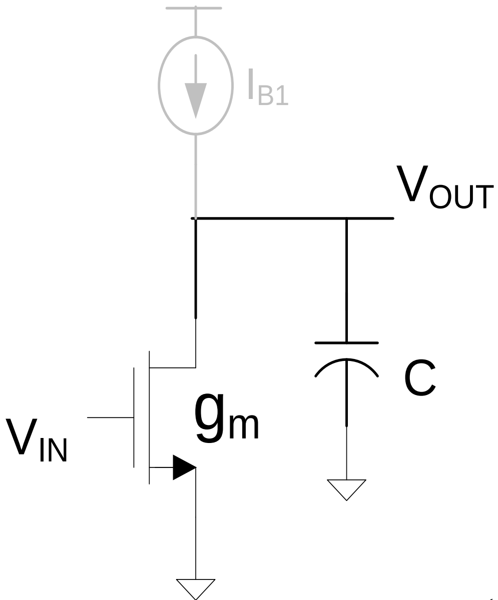
Some other perspectives



Noninverting Integrator

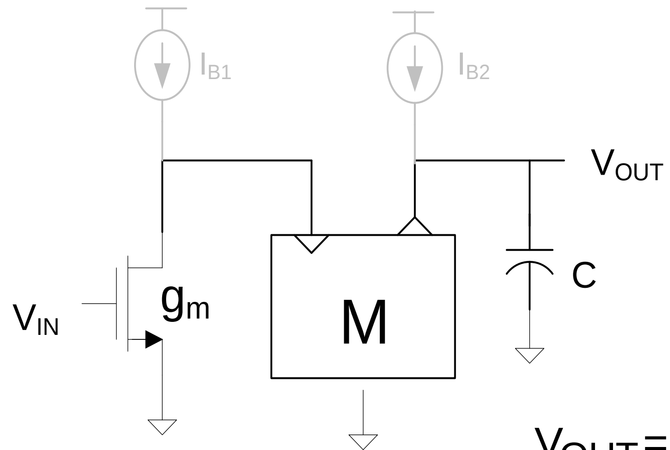
Can be viewed either as n-channel input with current mirror or as low-gain inverter driving a p-channel input inverting integrator

TA-C Voltage Mode Integrator



$$V_{OUT} = \left(\frac{-g_m}{sC}\right) V_{IN}$$

Inverting Integrator

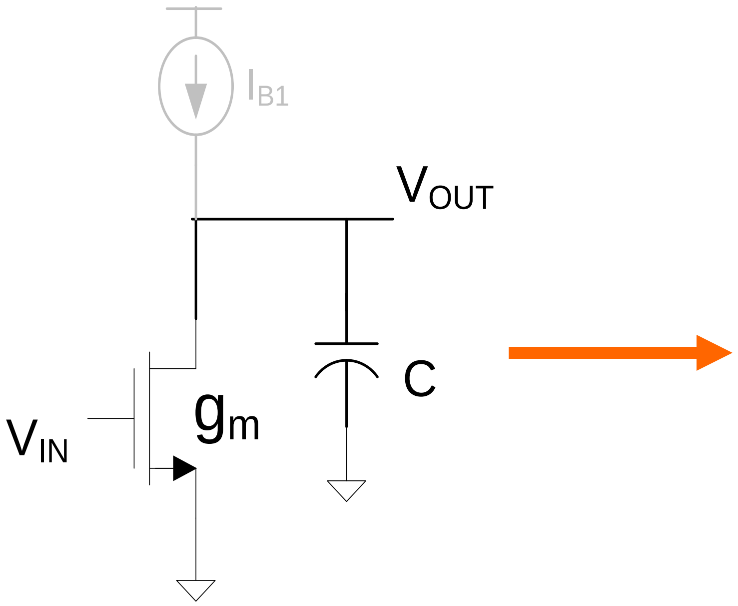


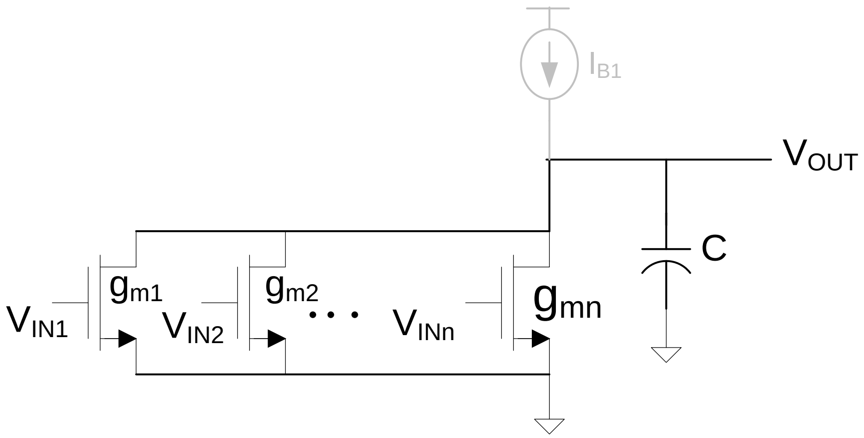
$$V_{OUT} = \left(\frac{g_m M}{sC}\right) V_{IN}$$

Typically M=1

Alternate noninverting Integrator

TA-C Voltage Mode Integrator





Summing Inverting Integrator

Voltage Mode Integrators

- Active RC (Feedback-based)
- MOSFET-C (Feedback-based)
- OTA-C
- TA-C

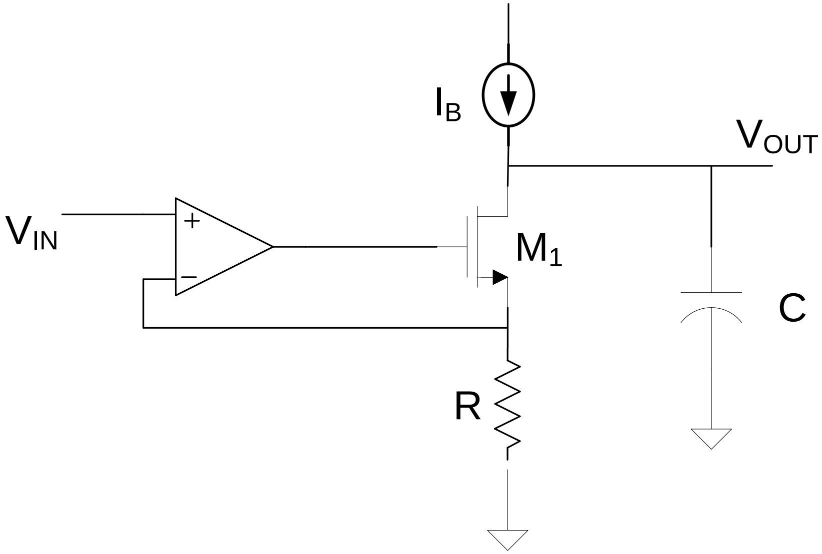
Sometimes termed "current mode" (OTA-C, TA-C)

- Switched Capacitor
- Switched Resistor

Will discuss later (Switched Capacitor, Switched Resistor)

- Other Structures

Another Voltage Mode Integrator

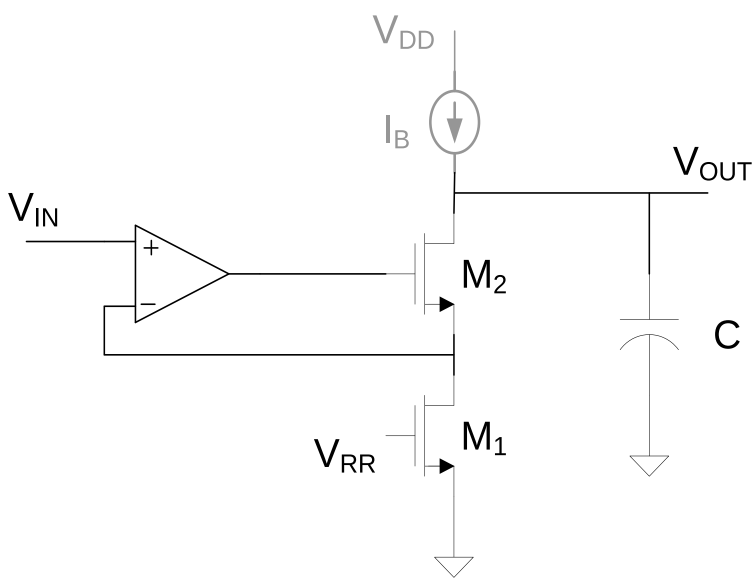


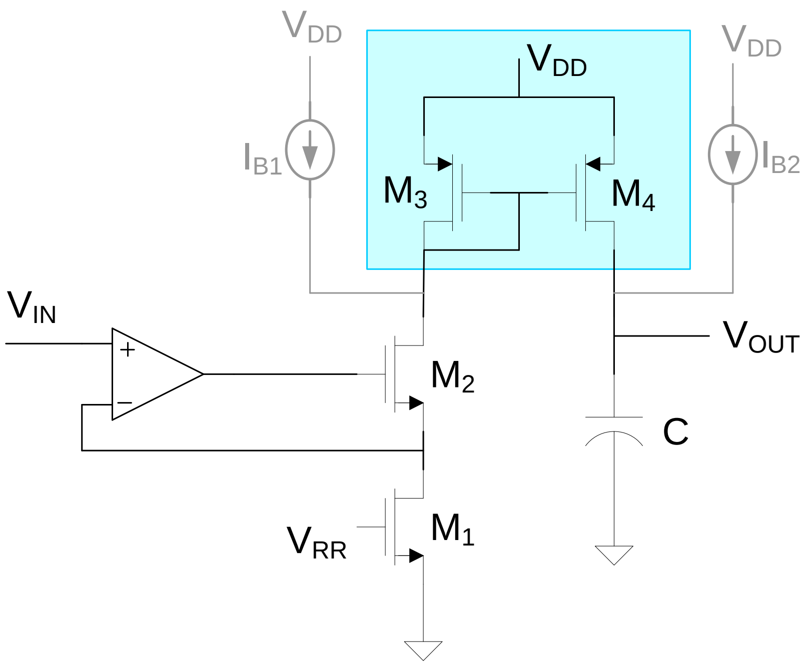
Inverting Integrator

$$V_{OUT} = \left(\frac{-1}{sRC}\right) V_{IN}$$

- **Infinite input impedance (in contrast to basic Active RC Integrator)**
- **Both R and C have one terminal grounded**
- **Requires integrated process**
- **Accuracy limited by process and temperature**
- **Size limitations same as basic Active RC Integrator**
- **Limited to lower frequencies because of Op Amp**
- **Good linearity**

Another Voltage Mode Integrator





Inverting Integrator

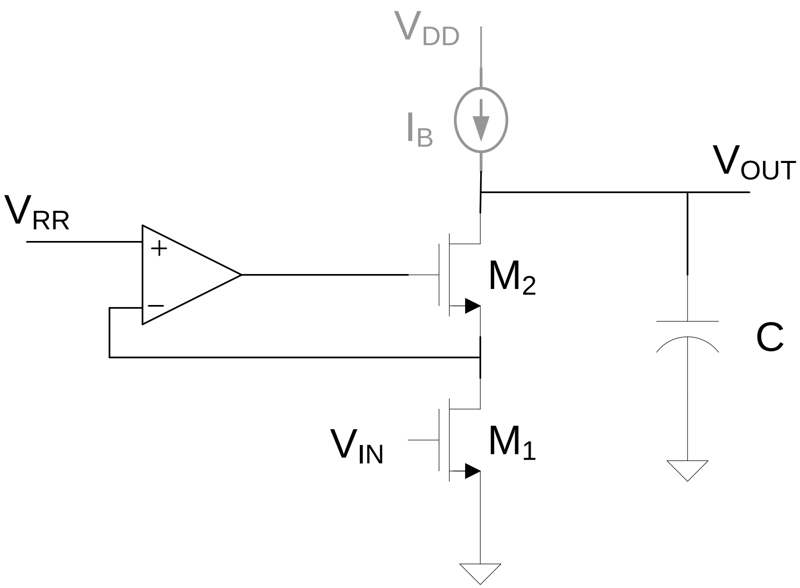
$$V_{OUT} = \left(\frac{-1}{sR_{FET}C} \right) V_{IN}$$

Noninverting Integrator

$$V_{OUT} = \left(\frac{1}{sR_{FET}C} \right) V_{IN}$$

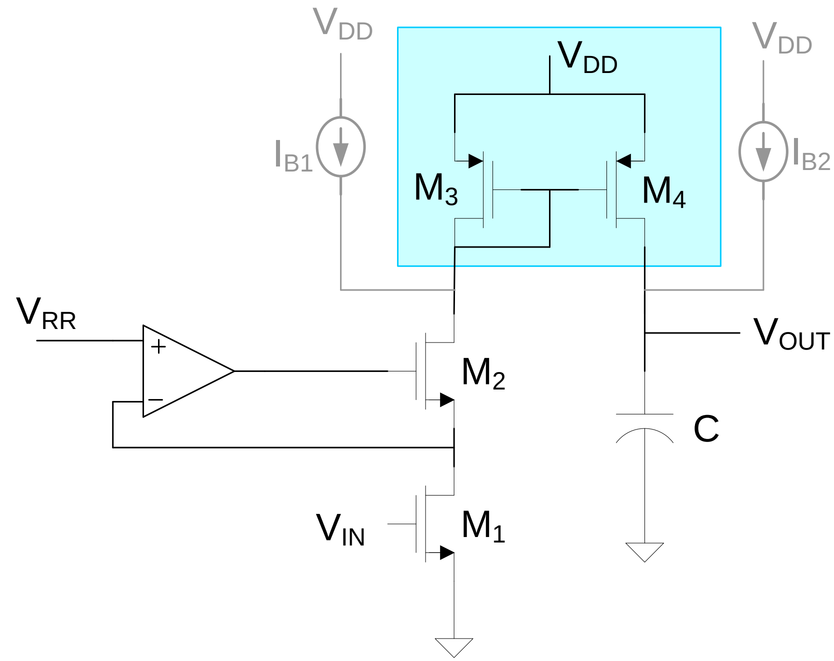
- M_1 in triode region
- Reduces Area Concerns but Loss of Linearity
- I_0 is programmable with V_{RR}
- Accurate control of I_B critical

Regulated Cascode Voltage Mode Integrator



Inverting Integrator

$$V_{OUT} = \left(\frac{-g_{mT}}{sC}\right) V_{IN}$$



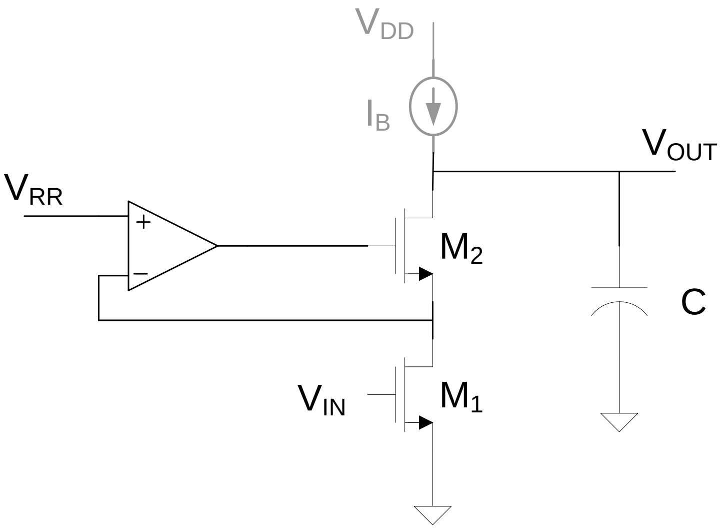
Noninverting Integrator

$$V_{OUT} = \left(\frac{g_{mT}}{sC}\right) V_{IN}$$

g_{MT} is triode region transconductance of M_1

- **M_1 operating in triode region**
- **R_{FET} programmable with V_{RR}**
- **Very good linearity properties**
- **Input impedance still infinite**

Regulated Cascode Voltage Mode Integrator



$$V_{OUT} = \left(\frac{-g_{mT}}{sC}\right) V_{IN}$$

Linearity Properties:

Assuming square-law triode model

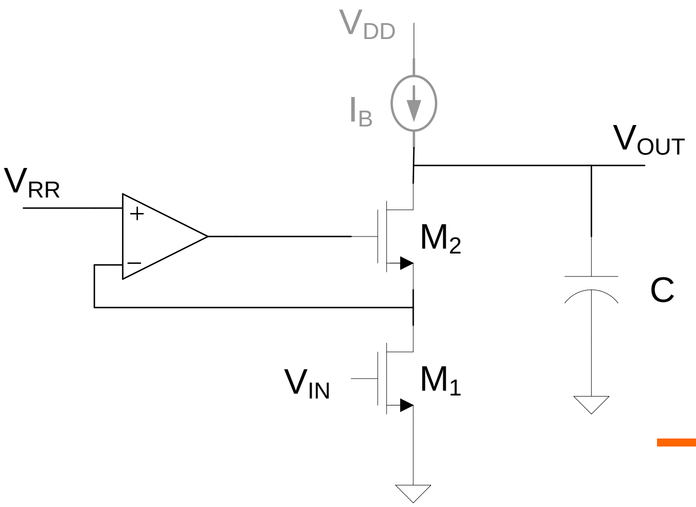
$$I_{D1} = \frac{\mu C_{OX} W}{L}\left(V_{GS} - V_T - \frac{V_{RR}}{2}\right) V_{RR}$$

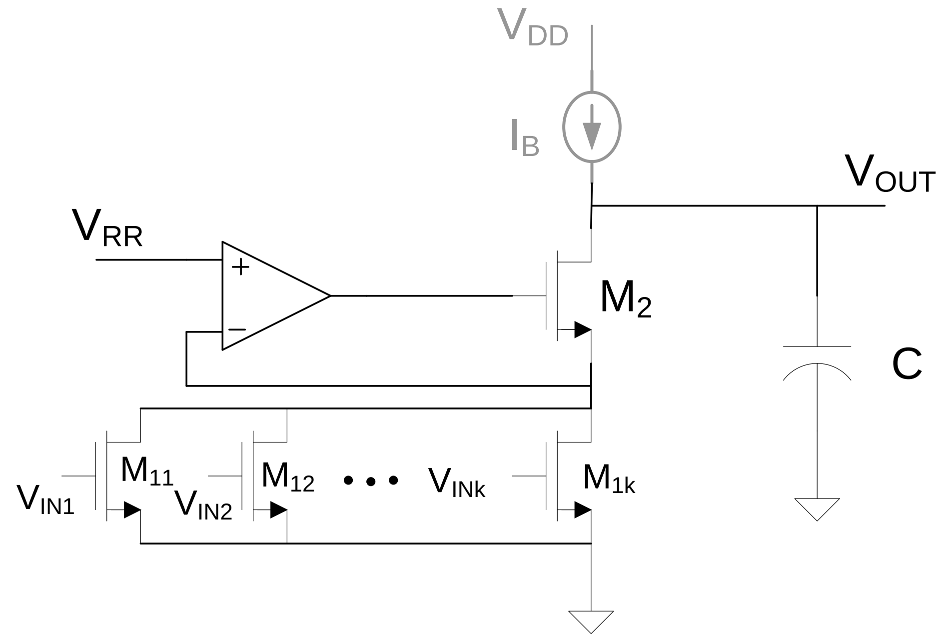
$$I_{D1} = \left[\frac{\mu C_{OX} W}{L} V_{RR}\right] V_{IN} + \left[\frac{\mu C_{OX} W}{L}\left(V_T + \frac{V_{RR}}{2}\right) V_{RR}\right]$$

Note linear dependence on V_{IN}

$$g_{mT} = \left[\frac{L}{\mu C_{OX} W \ V_{RR}}\right]$$

Regulated Cascode Voltage Mode Integrator



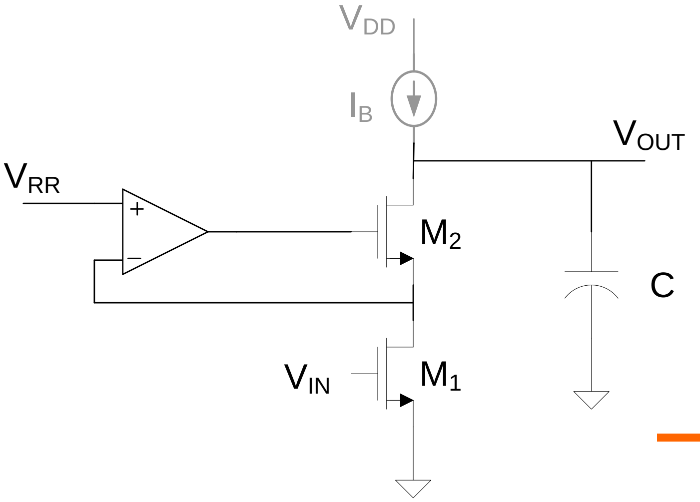


Inverting Integrator

$$V_{OUT} = \left(\frac{-1}{sR_{FET}C} \right) V_{IN}$$

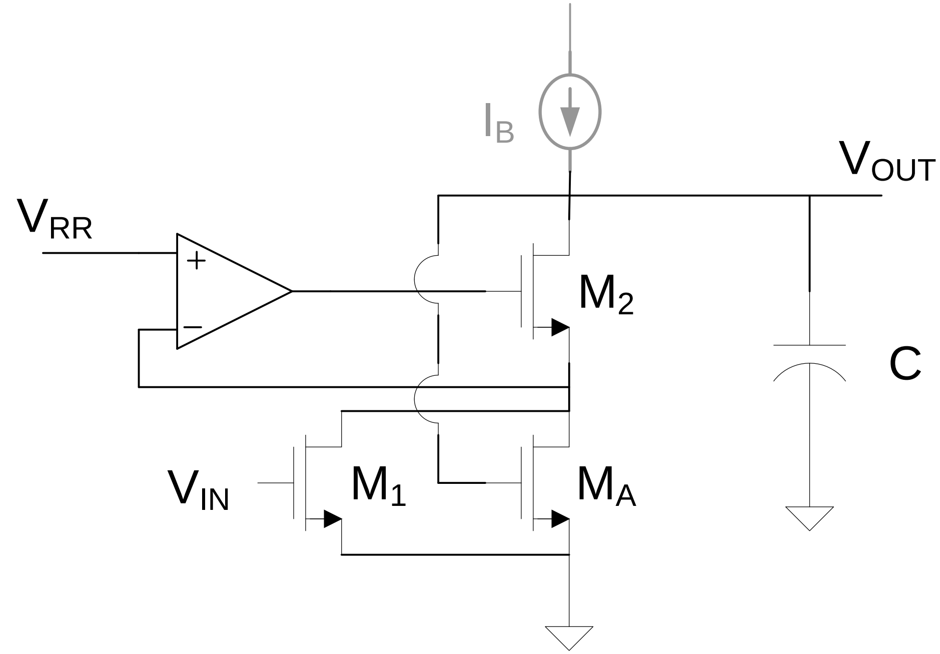
- **Multiple inputs require single additional transistor**
- **Accurate ratioing of gains practical**
- **Can also sum currents on C**

Regulated Cascode Voltage Mode Integrator



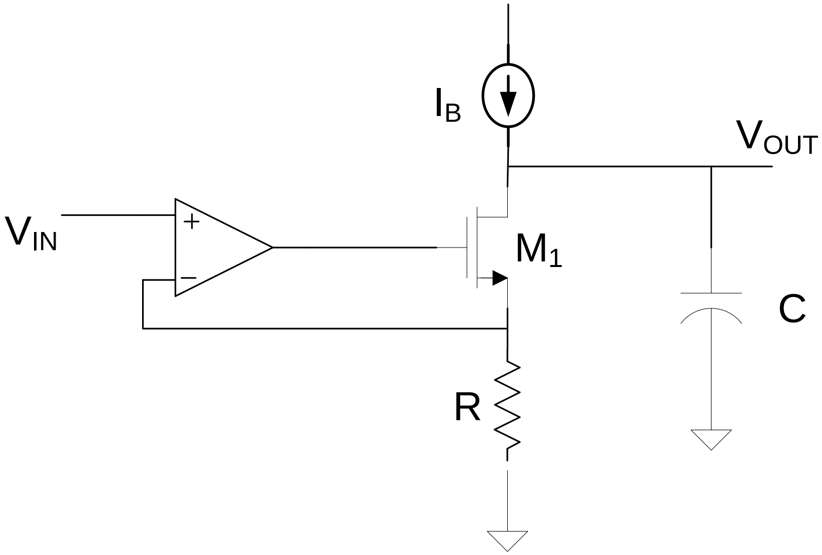
Inverting Integrator

$$V_{OUT} = \left(\frac{-1}{sR_{FET}C}\right)V_{IN}$$



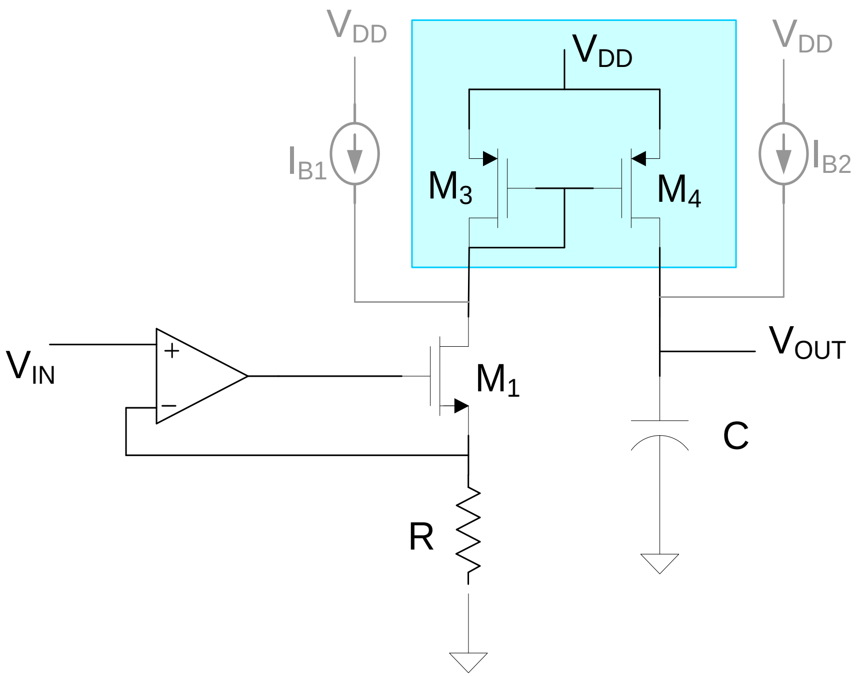
Inverting Lossy Integrator

Another Voltage Mode Integrator



Inverting Integrator

$$V_{OUT} = \left(\frac{-1}{sRC}\right) V_{IN}$$



Noninverting Integrator

$$V_{OUT} = \left(\frac{1}{sRC}\right) V_{IN}$$

Voltage Mode Integrators

- Active RC (Feedback-based)
- MOSFET-C (Feedback-based)
- OTA-C
- TA-C

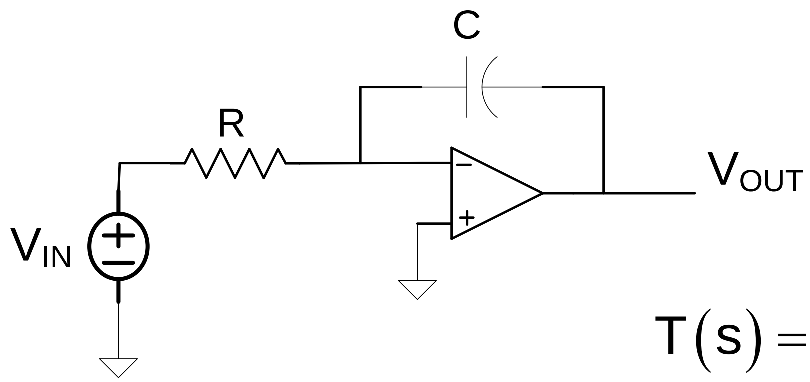
Sometimes termed "current mode"

- Switched Capacitor
- Switched Resistor

Will discuss later

- Other Structures

Consider the Basic Integrator



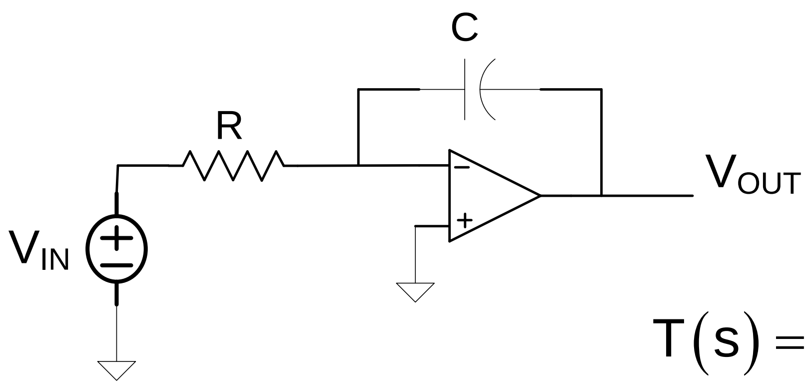
$$T(s) = -\frac{1}{RCs}$$

$$I_0 = \frac{1}{RC}$$

Key performance of integrator (and integrator-based filter) is determined by the integrator time constant I_0

Precision of time constants of a filter invariably determined by precision of I_0

Consider the Basic Integrator



$$T(s) = -\frac{1}{RCs}$$

$$I_0 = \frac{1}{RC}$$

1. Accuracy of R and C difficult to accurately control – particularly in integrated applications (often 2 or 3 orders of magnitude to variable)

2. Size of R and C unacceptably large if I_0 is in audio frequency range (2 or 3 orders of magnitude too large)

3. Amplifier GB limits performance

Incredible Challenge to Building Filters on Silicon!

Challenges for Integration of Active Filters

- Passive Component Variability
- Passive Component Size
- Op Amp Limitations

Historical Perspective

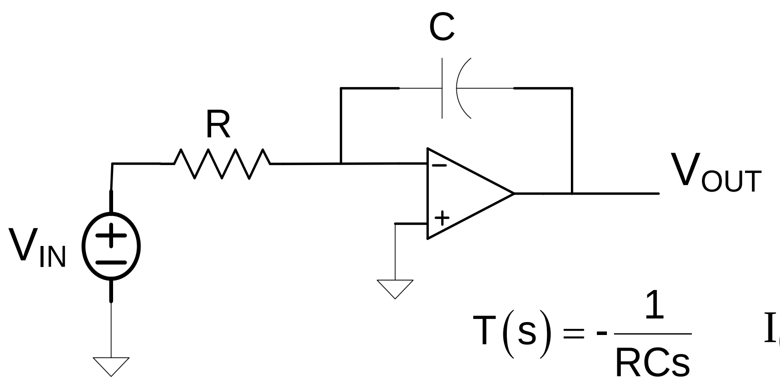
Filters were widely viewed as one of the most fundamental applications of integrated circuit technology

Considerable effort was expended on developing methods to build integrated filters but these three issues were viewed for years as a fundamental roadblock

Practical solution required finding SIMULTANEOUS solutions to three problems which were each 2 or 3 orders of magnitude problematic

This problem was not solved from the invention of the integrated circuit in 1959 up until the late 1970s

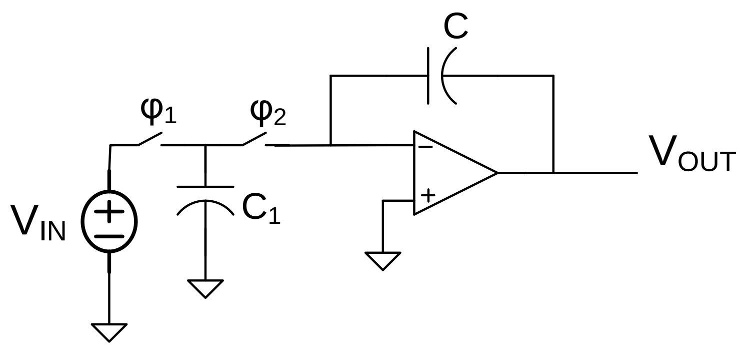
Switched-Capacitor Circuits

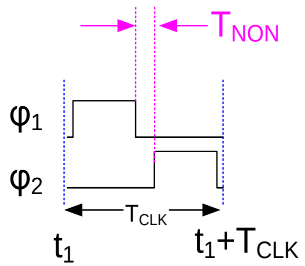


$$T(s) = -\frac{1}{RCs} \qquad I_0 = \frac{1}{RC}$$

Consider:

$$V_{IN} = V_M \sin(2\pi f_{SIG} t + \theta)$$





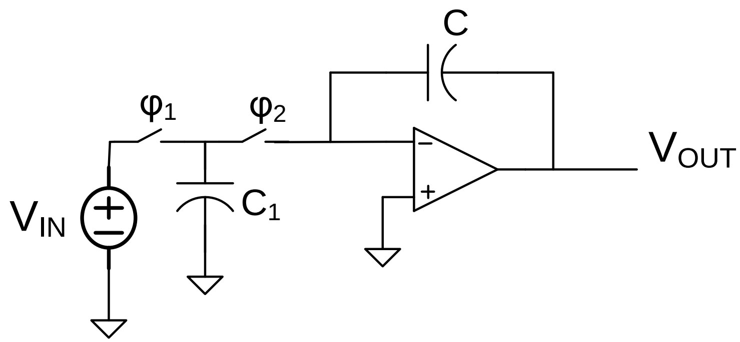
Assume $T_{CLK} << T_{SIG}$

Φ_1 and Φ_2 are complimentary non-overlapping clocks

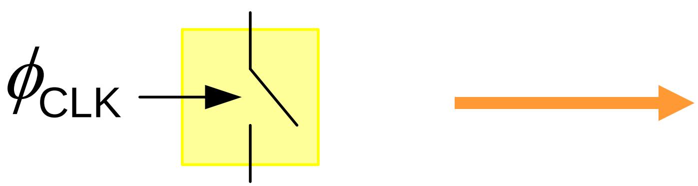
Φ_1 and Φ_2 are periodic signals "clocks" shown for one period

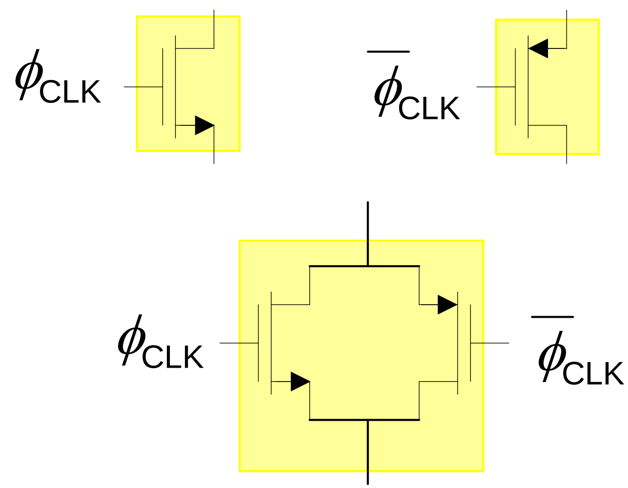
Termed a Switched-Capacitor circuit

Switched-Capacitor Circuits



How are the switches made?

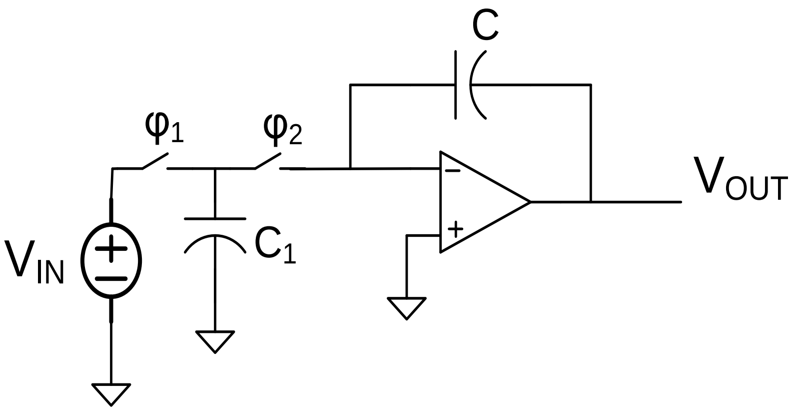




- Often single transistor
- Occasionally complimentary transistors
- On rare occasion more complicated
- Area overhead for switches small, clock routing a little more of concern
- Sizing of devices is important
- Clocking of switches may be important

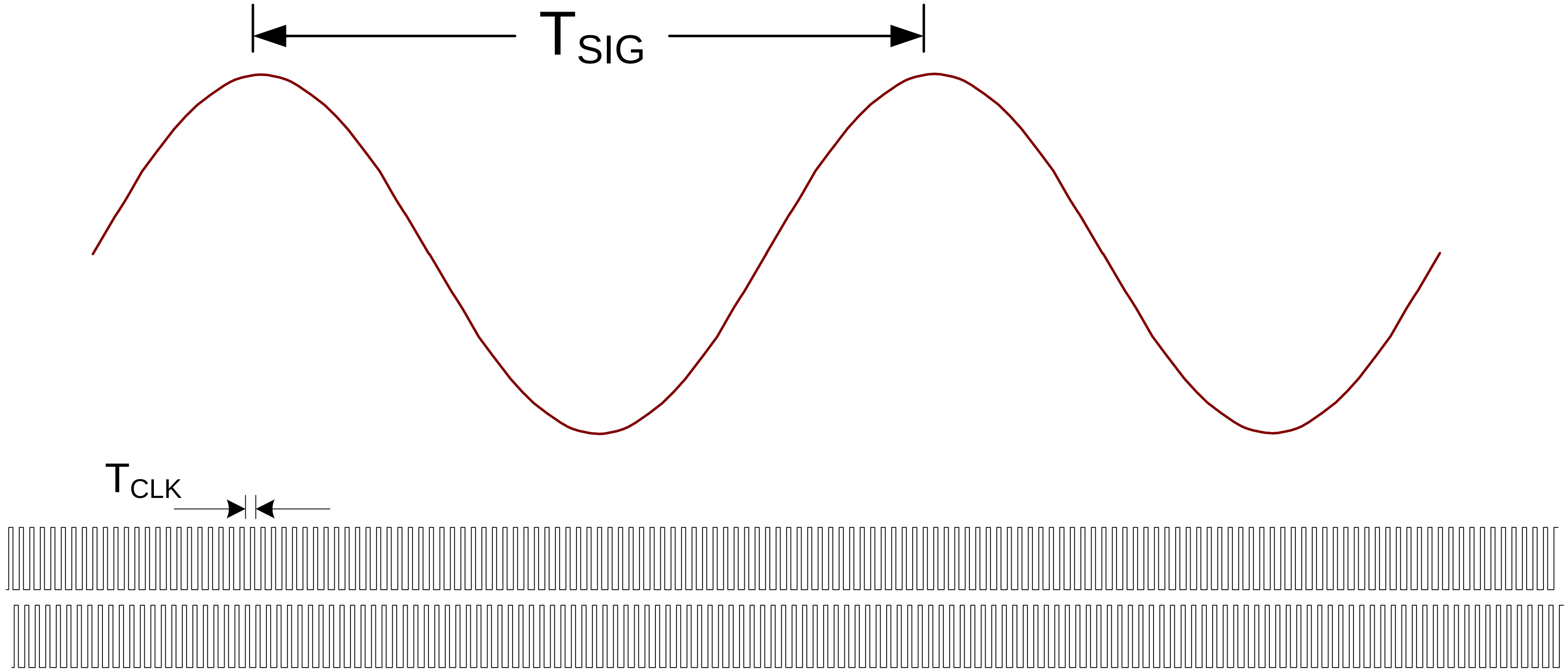
Although originating in SC filters, switched charge redistribution circuits widely used in other non-filtering applications

Consider the Switched-Capacitor Circuit



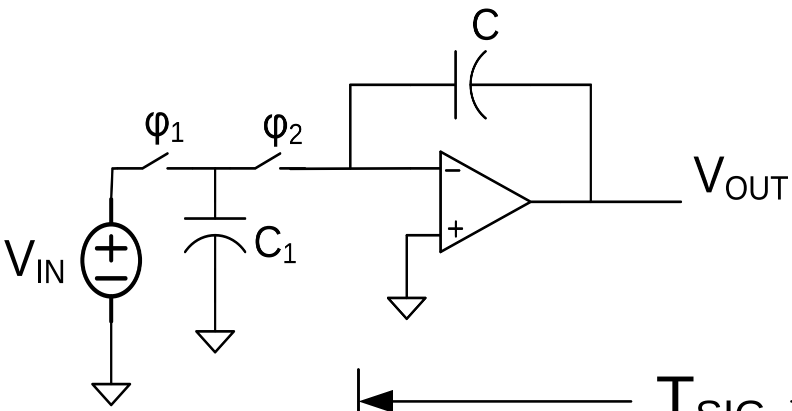
Assume $T_{CLK} << T_{SIG}$

Φ_1 and Φ_2 are complimenary nonoverlapping clocks



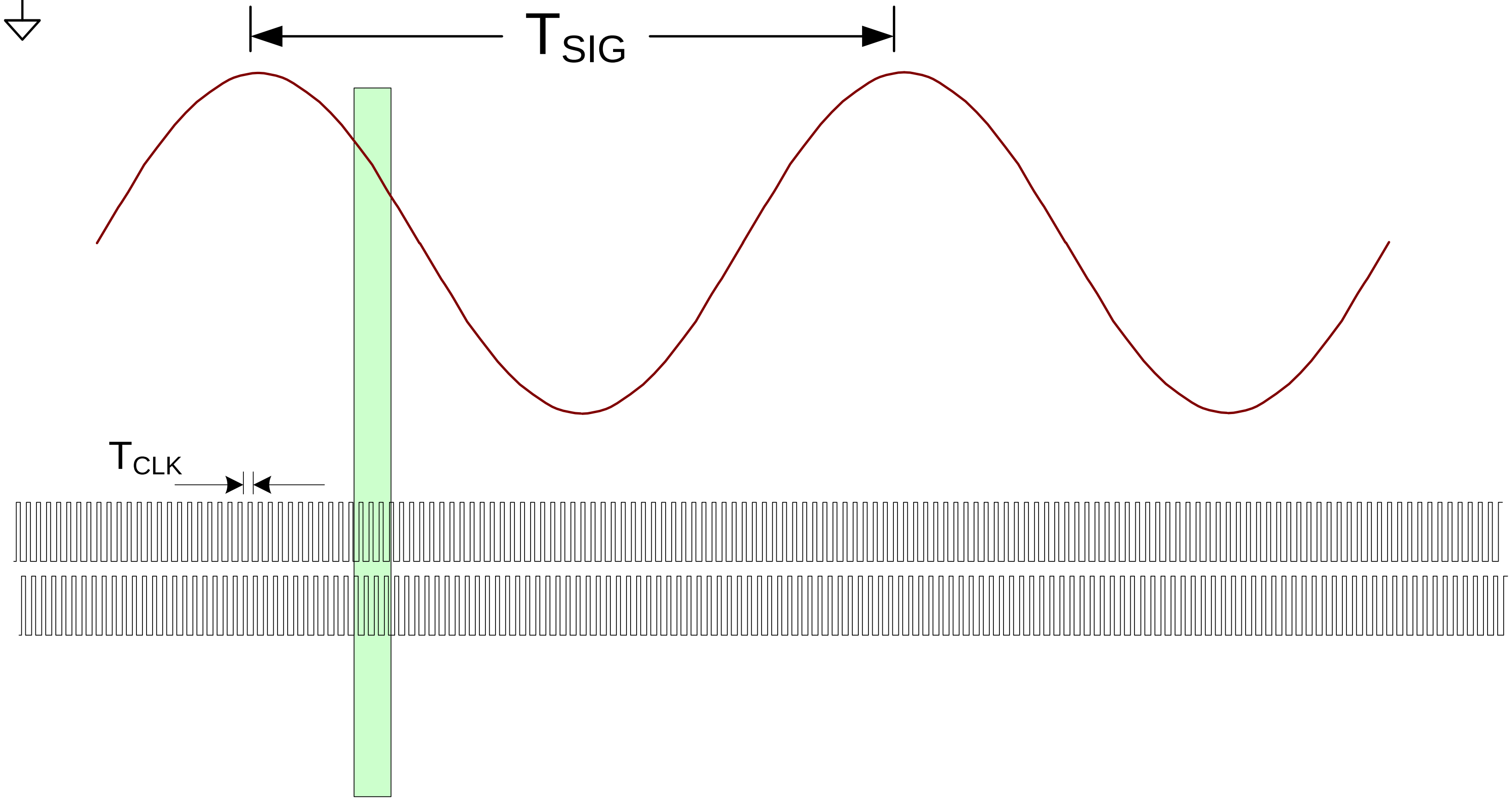
Lets now zoom in on the clock period

Consider the Switched-Capacitor Circuit

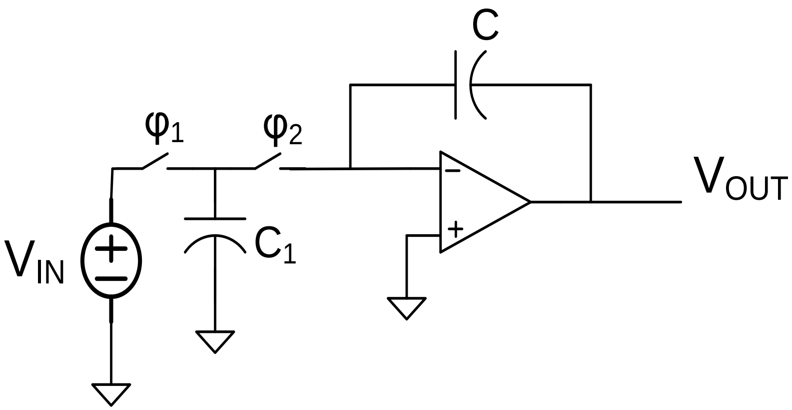


Assume $T_{CLK} << T_{SIG}$

Φ_1 and Φ_2 are complimenary nonoverlapping clocks

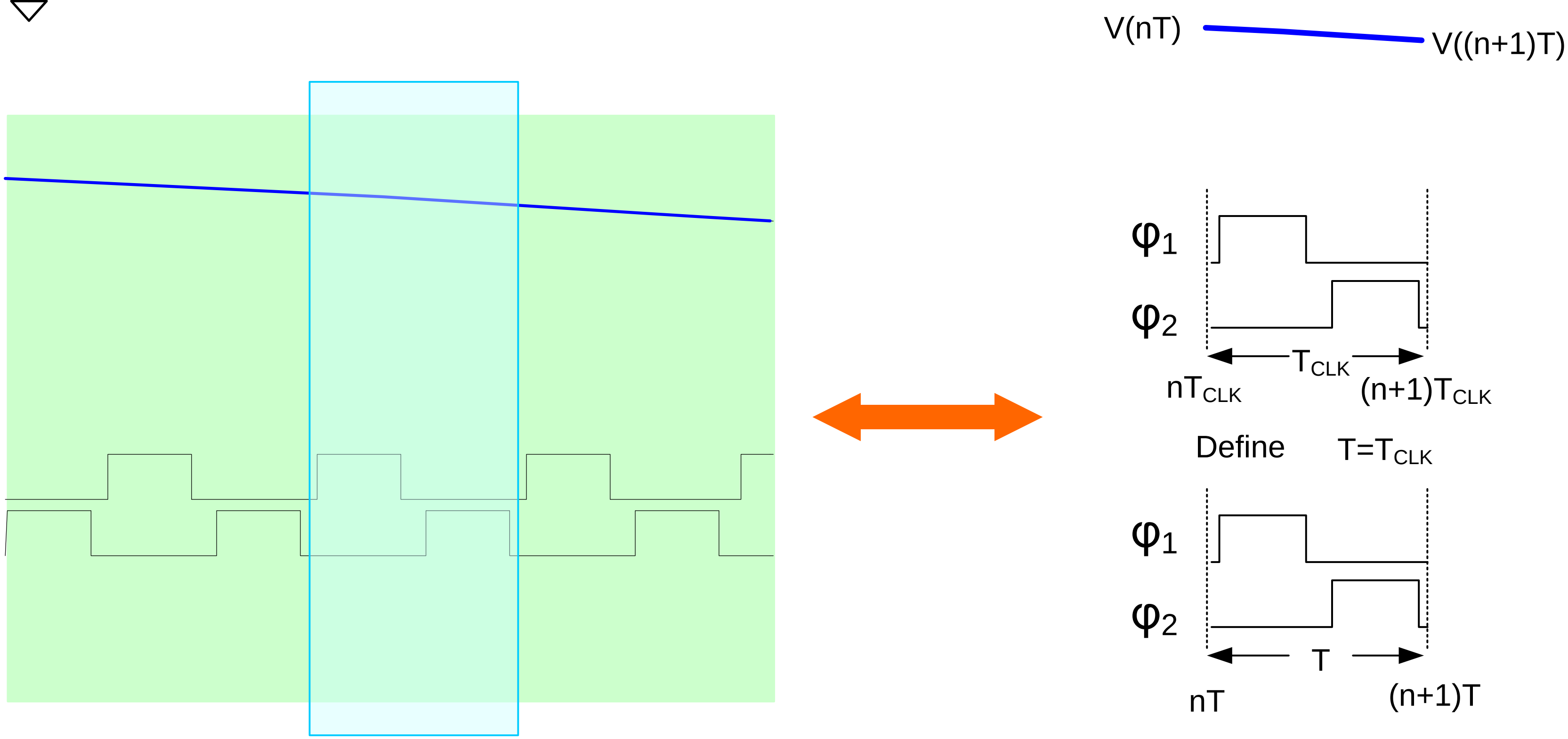


Consider the Switched-Capacitor Circuit

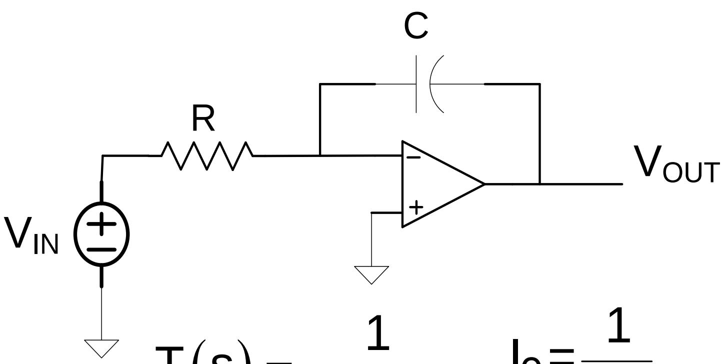


Assume $T_{CLK} << T_{SIG}$

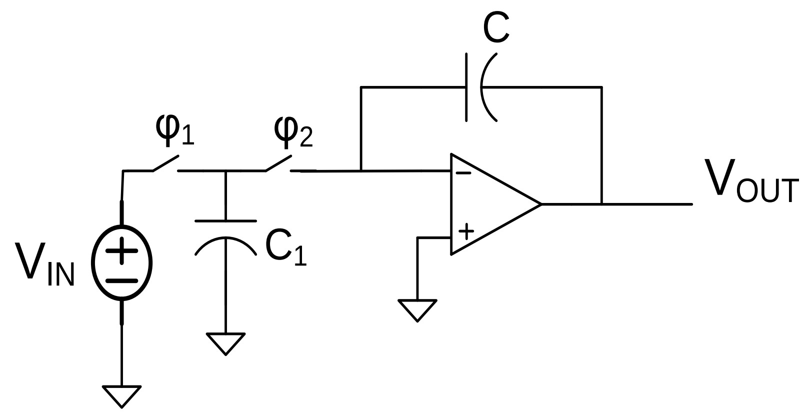
Φ_1 and Φ_2 are complimenary nonoverlapping clocks



Compare the performance of the following two circuits

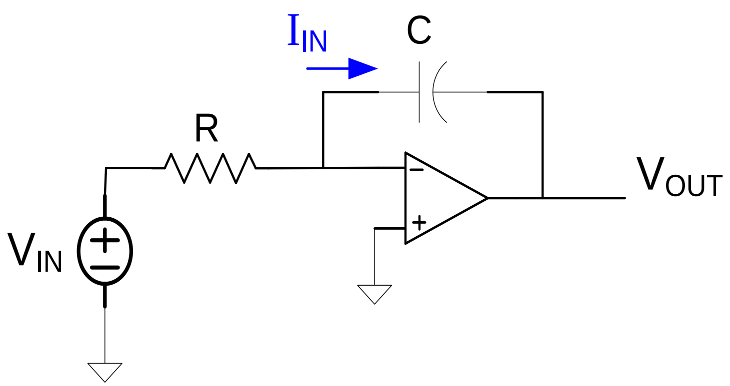


$$T(s) = -\frac{1}{RCs} \qquad I_0 = \frac{1}{RC}$$



Consider the charge transferred to the feedback capacitor for both circuits in an interval of length T_{CLK} at time t_1

For the RC circuit:



$$Q_{RC} = \int_{t_1}^{t_1+T_{CLK}} I_{in}(t)\,dt$$

$$Q_{RC} = \int_{t_1}^{t_1+T_{CLK}} \frac{V_{in}(t)}{R}\,dt$$

Since V_{in} changes slowly

$$Q_{RC} \simeq \int_{t_1}^{t_1+T_{CLK}} \frac{V_{in}(t_1)}{R}\,dt$$

$$Q_{RC} \simeq \left[\frac{V_{in}(t_1)}{R}\right] \int_{t_1}^{t_1+T_{CLK}} 1\,dt$$

$$Q_{RC} \simeq \left[\frac{V_{in}(t_1)}{R}\right] T_{CLK}$$

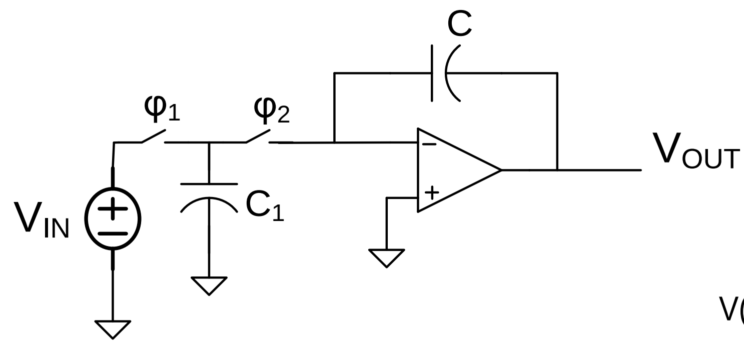
Consider the charge transferred to the feedback capacitor for both circuits in an interval of length T_{CLK} at time t_1

For the RC circuit:

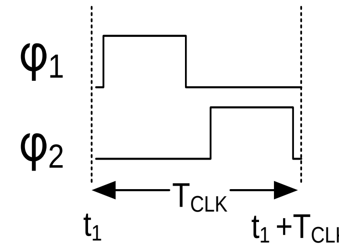
$$Q_{RC} \simeq \left[\frac{V_{in}(t_1)}{R}\right] T_{CLK}$$

Observe that a resistor "transfers" charge proportional to V_{in} in a short interval of T_{CLK}

For the SC circuit



$$Q_{C1} = C_1 V_{in}\left(t_1 + \frac{T_{CLK}}{2} - \varepsilon\right)$$



Since $V_{in}(t)$ is slowly varying

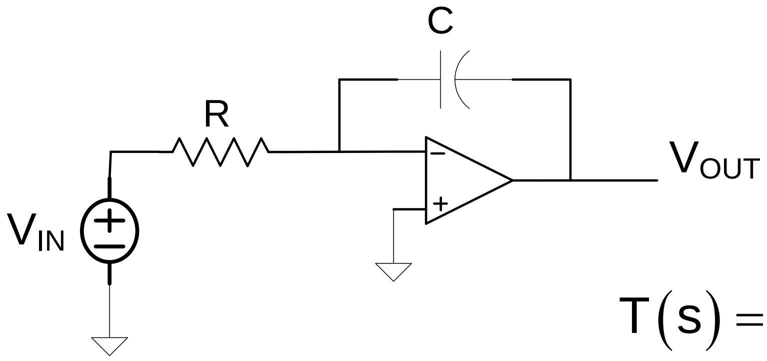
$$Q_{C1} \simeq C_1 V_{in}(t_1)$$

But this is the charge that will be transferred to C during phase Φ_2

$$Q_{SC} \simeq C_1 V_{in}(t_1)$$

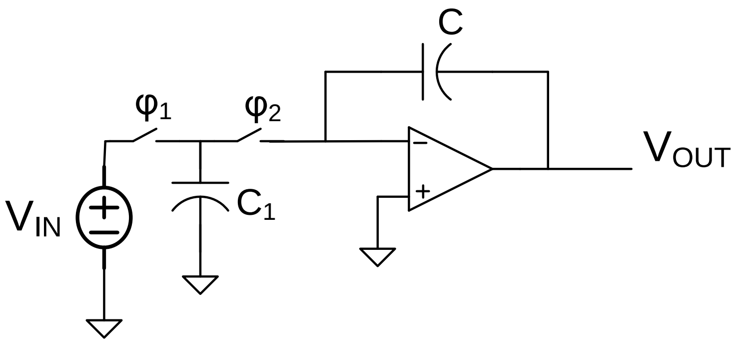
Observe that the SC circuit also transfers charge proportional to V_{in} in short intervals of length T_{CLK}

This is precisely what a resistor does so the switched capacitor behaves as a resistor



$$T(s) = -\frac{1}{RCs}$$

$$I_0 = \frac{1}{RC}$$



Comparing the two circuits

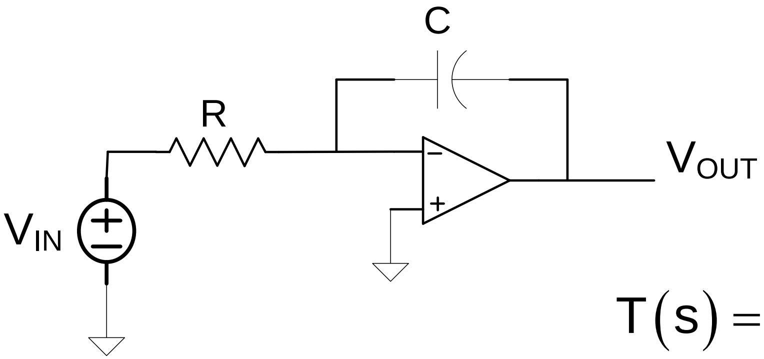
$$Q_{RC} \simeq \left[\frac{V_{in}(t_1)}{R}\right] T_{CLK}$$

$$Q_{SC} \simeq C_1 V_{in}(t_1)$$

Equating charges since both proportional to $V_{in}(t_1)$

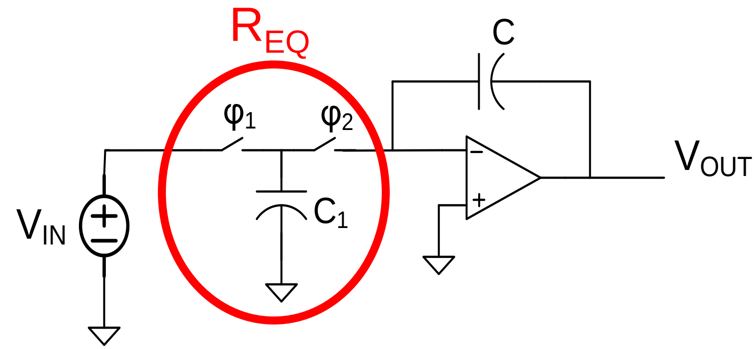
$$C_1 \simeq \left[\frac{1}{R}\right] T_{CLK}$$

$$R_{EQ} \simeq \frac{1}{f_{CLK} C_1}$$



$$T(s) = -\frac{1}{RCs}$$

$$I_0 = \frac{1}{RC}$$



$$R_{EQ} \simeq \frac{1}{f_{CLK} C_1}$$

Observe that a switched-capacitor behaves as a resistor!

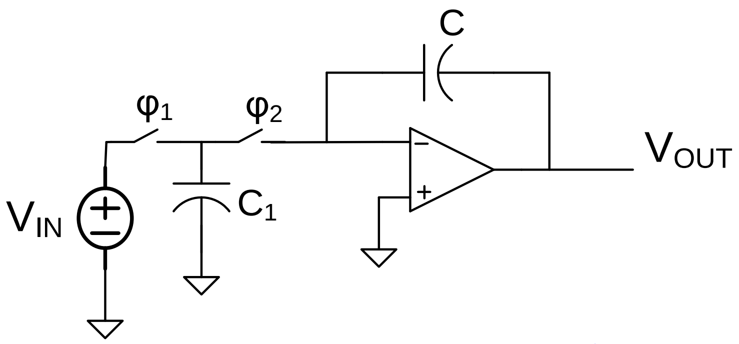
This is an interesting observation that was made by Maxwell over 100 years ago but in and of itself was of almost no consequence

Observation by Maxwell was forgotton and rediscovered several times over the years but remained of no conseqeunce

Note that large resistors require small capacitors !

This offers potential for overcoming <u>one</u> of the critical challenges for Implementing integrators on silicon at audio frequencies!

Consider again the SC integrator



$$T_{SC}(s) \simeq \frac{-1}{R_{EQ}Cs}$$

$$R_{EQ} \simeq \frac{1}{f_{CLK}C_1}$$

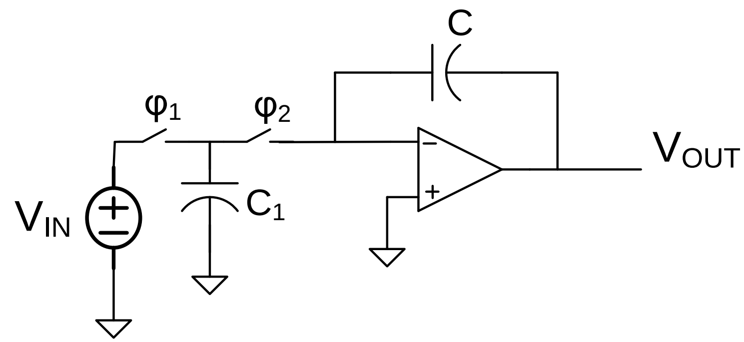
$$I_{0eq} = \frac{1}{R_{EQ}C}$$

$$I_{0eq} = \frac{1}{R_{EQ}C} = \frac{C_1 f_{CLK}}{C}$$

$$I_{0eq} = \left[\frac{C_1}{C}\right] f_{CLK}$$

This is a frequency referenced filter!

The SC integrator



$$T_{SC}(s) \simeq \frac{-1}{R_{EQ}Cs}$$

$$R_{EQ} \simeq \frac{1}{f_{CLK}C_1}$$

$$I_{0eq} = \left[\frac{C_1}{C}\right] f_{CLK}$$

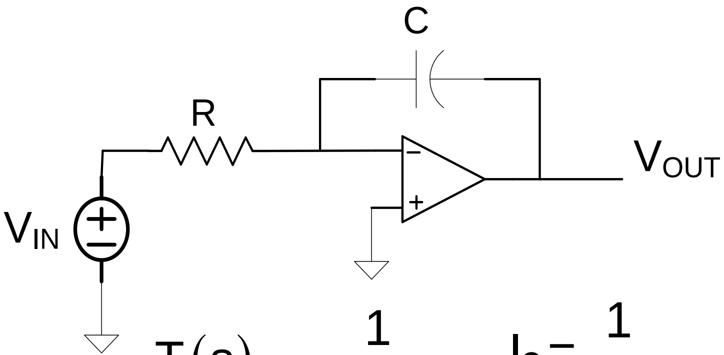
The expressions $S_{C}^{I_0}$ and $S_{C_1}^{I_0}$ have the same magnitude as for the RC integrator

On-chip capacitor values CAN be highly correlated with proper selection and layout

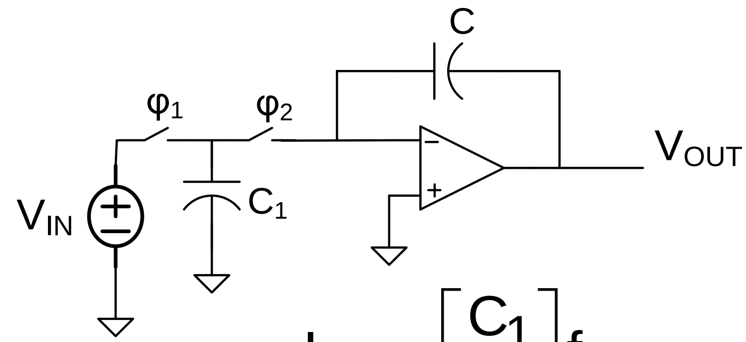
- The ratio of capacitors CAN be accurately controlled in IC processes (1% to .01% is achievable with careful layout)
- f_{CLK} CAN be VERY accurately controlled with a c low cost crystal (1 part in 10^6 or better)
- Variability of I_{0eq} is very small

The SC integrator CAN dramatically reduce the second main concern for building integrated integrators

The SC integrator



$$T(s) = -\frac{1}{RCs} \qquad I_0 = \frac{1}{RC}$$



$$I_{0eq} = \left[\frac{C_1}{C}\right] f_{CLK}$$

1. **Accuracy of R and C difficult to accurately control (often 2 or 3 orders of magnitude to variable)**
2. **Area of R and C too large in audio frequency range (2 or 3 orders of magnitude too large)**
3. **Amplifier GB limits performance**

1. **Accuracy of cap ratio and f_{CLK} very good**
2. **Area of C1 and C not too large**
3. **Amplifier GB limits performance less**

Two of these properties were discovered independently by Gray, Broderson and Hosticka at Berkeley and by Copeland of Carelton

[1] J. T. Caves, M. A. Copeland, C. F. Rahim, and S. D. Rosenbaum, "Sampled analog filtering using switched capacitors as resistor equivalents," *IEEE J. Solid-State Circuits*, vol. SC-12, pp. 592–599, Dec. 1977.
[2] B. J. Hosticka, R. W. Brodersen, and P. R. Gray, "MOS sampled data recursive filters using switched capacitor integrators," *IEEE J. Solid-State Circuits*, vol. SC-12, pp. 600–608, Dec. 1977.

77 citations

108 citations

Seminal source of SC concept received few citations!

But cited as a key contribution when Brodersen and Gray elected to NAE

EE Times BETA News & Analysis

Switched-Capacitor Filters Beat Active Filters at Their Own Game

Charles Yager and Carlos Laber

6/29/2000 12:00 AM EDT

Switched capacitor filters are growing increasingly popular because they have many advantages over active filters. Switched capacitor filters don't require external precision capacitors like active filters do. Their cutoff frequencies have a typical accuracy of ±0.3% and they are less sensitive to temperature changes. These characteristics allow consistent, repeatable filter designs.

Another distinct advantage of switched capacitor filters is that their cutoff frequency can be adjusted by changing the clock frequency. Switched capacitor filters offer higher integration at a lower system cost. Center frequencies of up to 150-kHz with Q values up to 20 are achievable.

Switched Capacitor Filters

The realization that a switched-capacitor was equivalent to a resistor was of little consequence

The realization that a small switched capacitor was equivalent to a resistor was of little consequence

The realization that a switched capacitor was dependent upon frequency was of little consequence

The realization that RC time constants could be accurately controlled with a small amount of area in silicon was of considerable consequence

The experimental validation and the efforts to convince industry that the SC techniques offered practical solutions was the MAJOR contribution !!

Basic Building Blocks in Both Cascaded Biquads and Multiple Feedback Structures

- **Developed from observations from feedback implementations**

1. Integrators
2. Summers
3. Op Amps (inc OTAs)
4. Switches

- First-order filter blocks
- Biquads

- **Same building blocks used in open-loop applications as well**

End of Lecture 26