

EE 508

Lecture 29

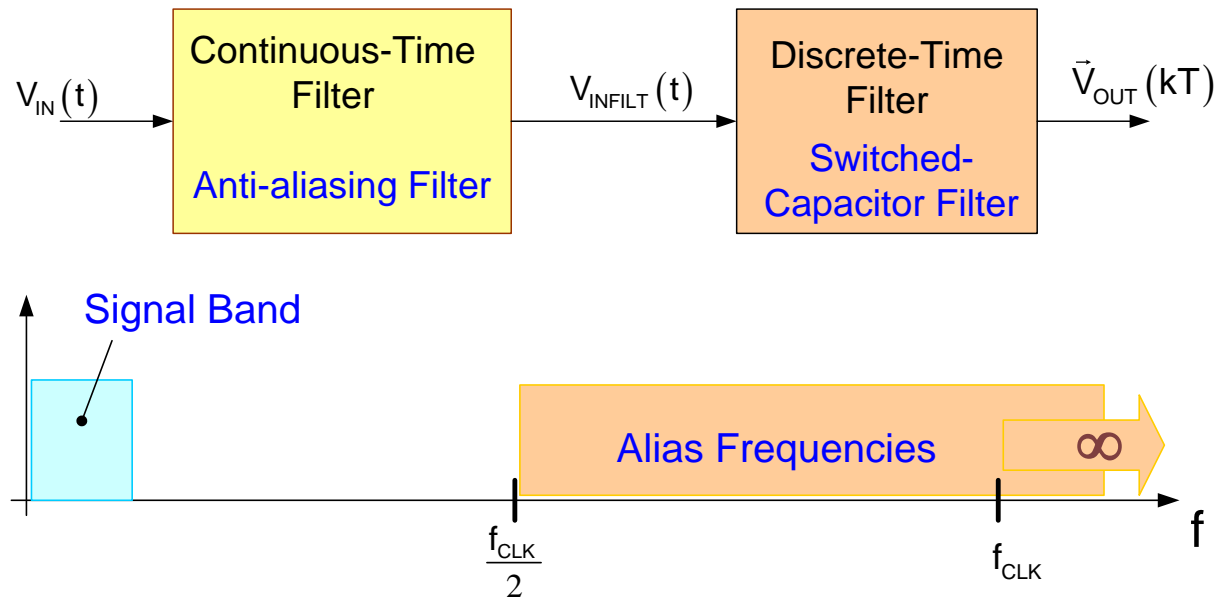
Integrator Design

Metrics for comparing integrators

Current-Mode Integrators

Review from last time

Anti-aliasing filter often required to limit frequency content at input to SC filters



Why not just make the clock frequency \gg signal band edge ?

Recall in the continuous-time RC-SC counterparts

$$f_{POLES} \sim \frac{1}{RC} \approx f_{CLK} \frac{C_1}{C}$$

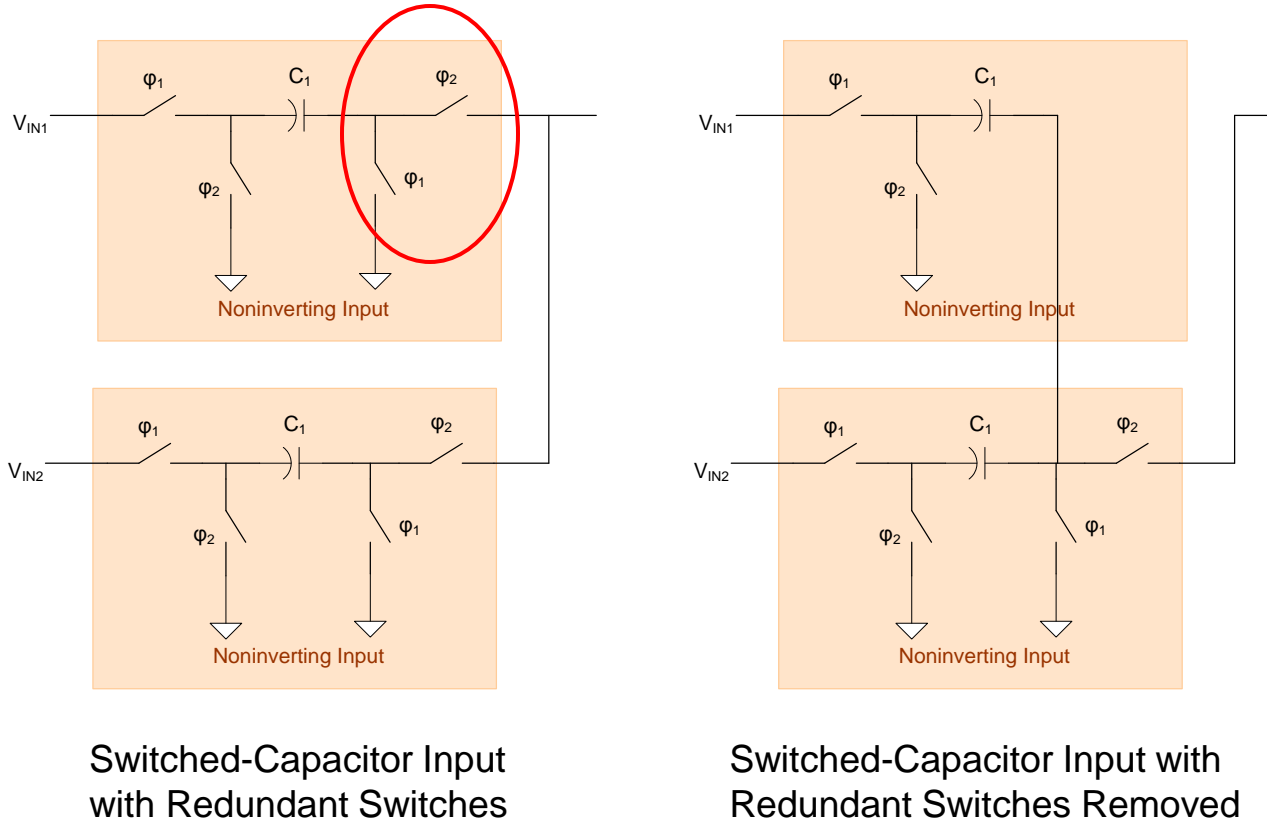
Since f_{POLES} will be in the signal band (that is why we are building a filter) large f_{CLK} will require large capacitor ratios if $f_{CLK} \gg f_{POLES}$

- Large capacitor ratios not attractive on silicon (area and matching issues)
- High f_{CLK} creates need for high GB in the op amps (area, power, and noise increase)

Often f_{CLK}/f_{POLES} in the 10:1 range proves useful (20:1 to 5:1 typical)

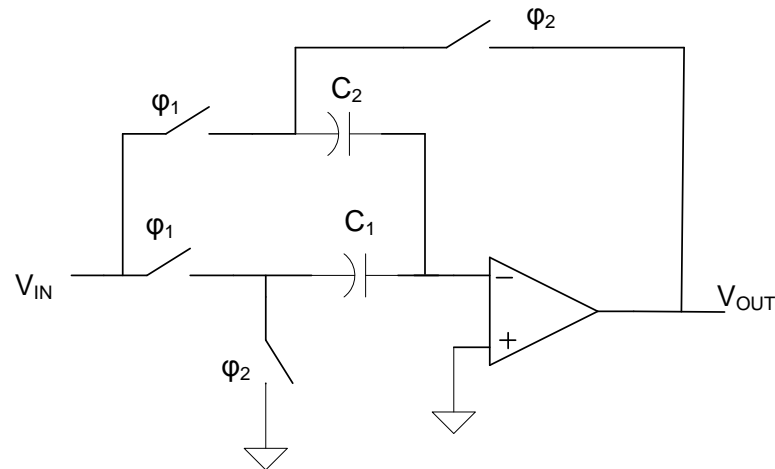
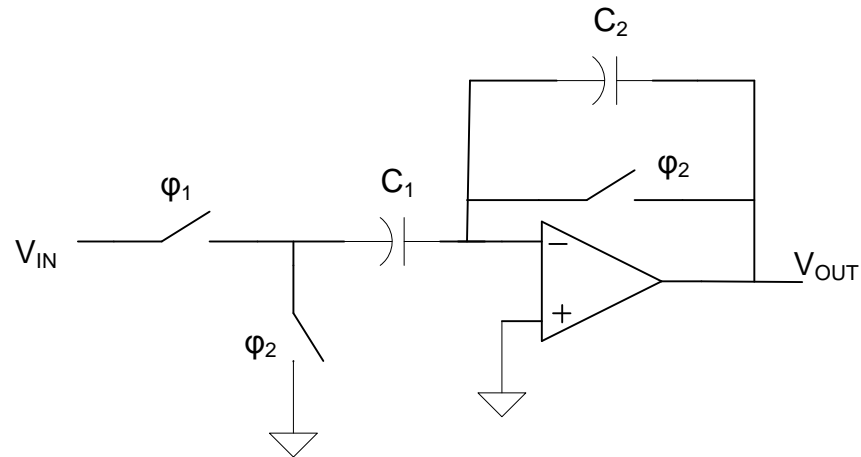
Elimination of Redundant Switches

Redundant Switches



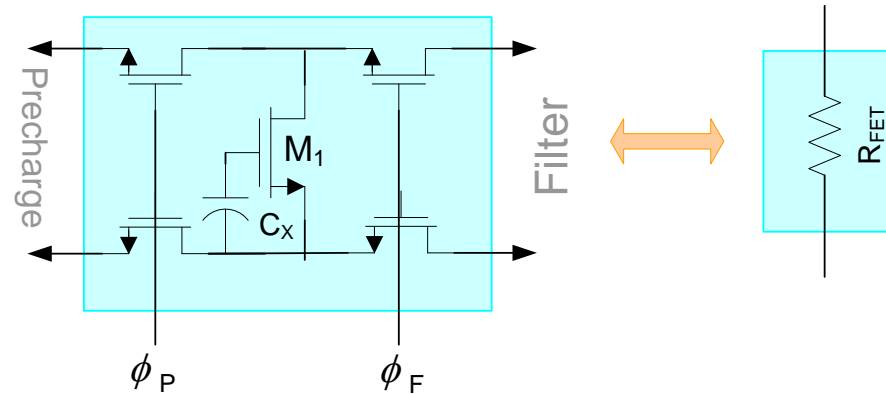
Although developed from the concept of SC-resistor equivalence, SC circuits often have no Resistor-Capacitor equivalents

Switched Capacitor Amplifiers

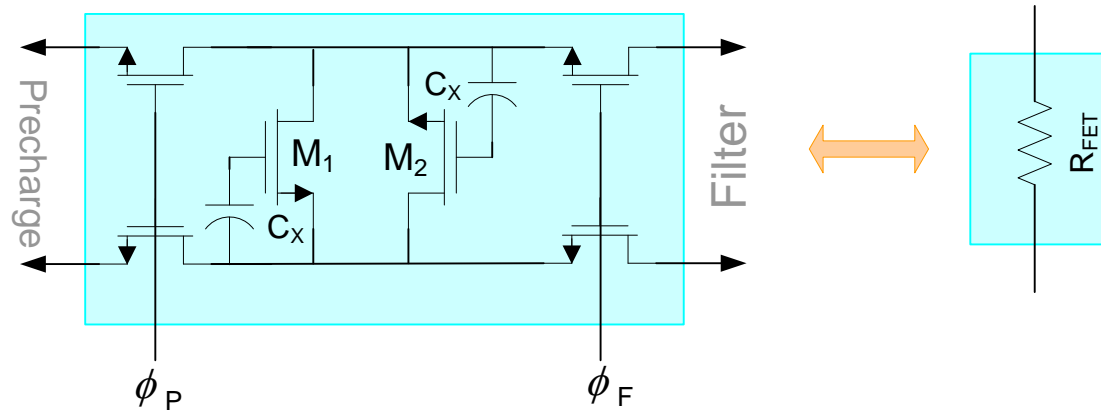


- Summing, Differencing, Inverting, and Noninverting SC Amplifiers Widely Used
- Significant reduction in switches from what we started with by eliminating C in SC integrator
- Must be stray insensitive in most applications

Switched-Resistor Voltage Mode Integrators

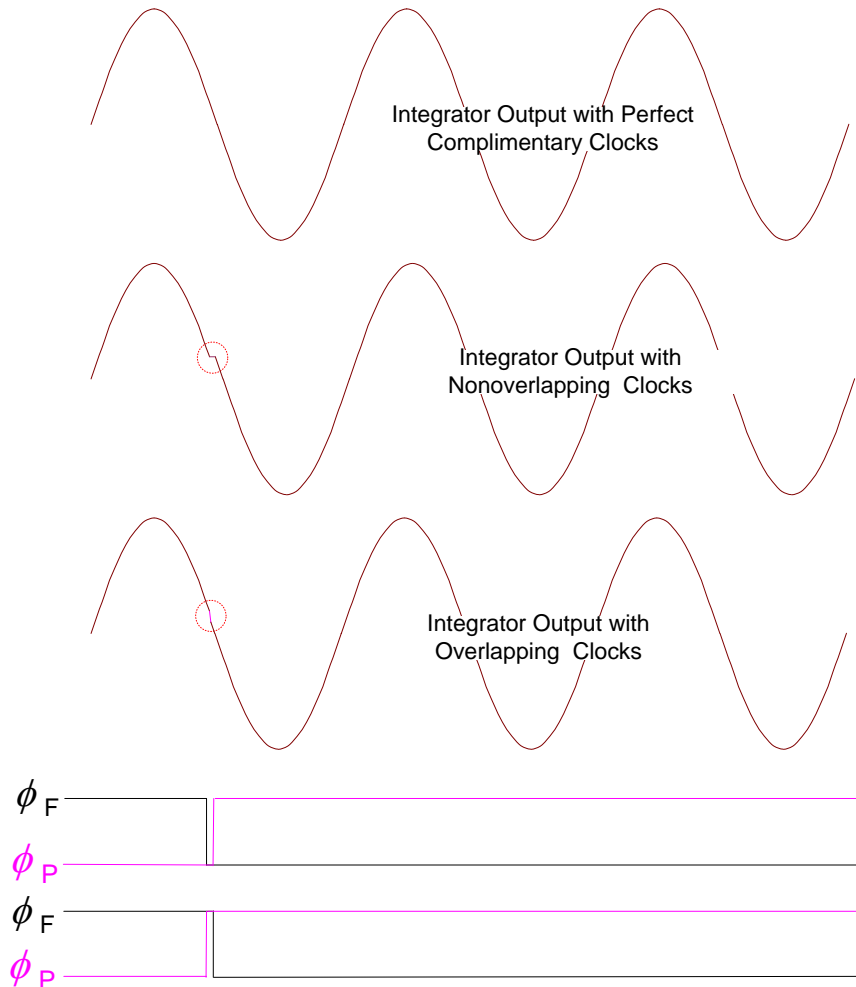


There are some modest nonlinearities in this MOSFET when operating in the triode region



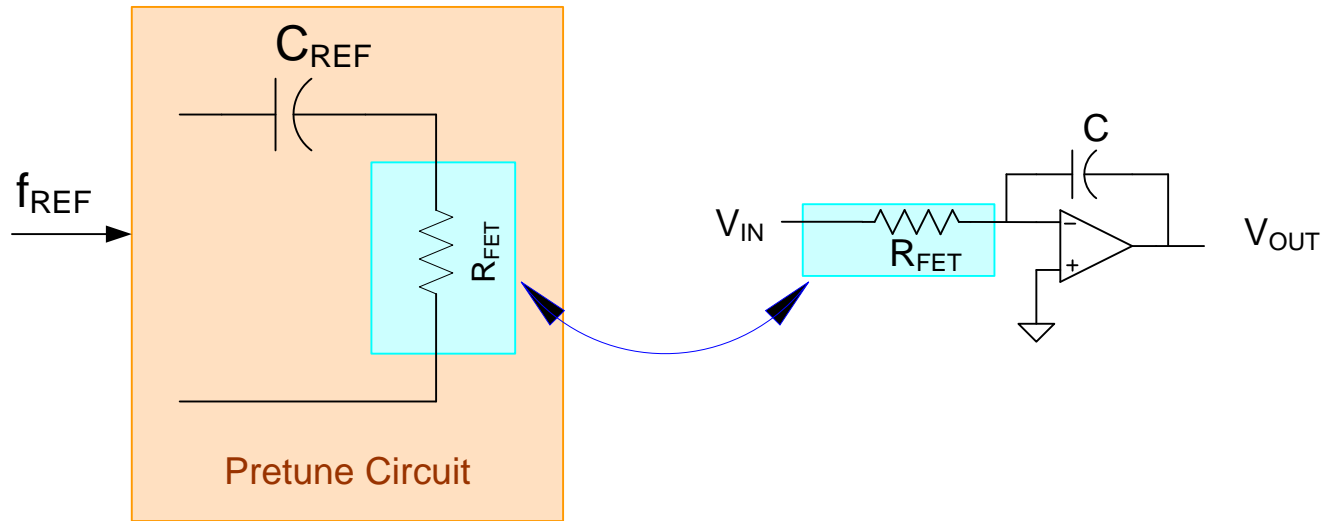
- Significant improvement in linearity by cross-coupling a pair of triode region resistors
- Perfectly cancels nonlinearities if square law model is valid for M_1 and M_2
- Only modest additional complexity in the Precharge circuit

Switched-Resistor Voltage Mode Integrators



- Aberrations are very small, occur very infrequently, and are further filtered
- Play almost no role on performance of integrator or filter

Switched-Resistor Voltage Mode Integrators



Switched-resistor integrator

- Accurate CR_{FET} products is possible
- Area reduced compared to Active RC structure because R_{FET} small
- Single pretune circuit can be used to “calibrate” large number of resistors
- Clock frequency not fast and not critical (but accuracy of f_{REF} is important)
- Since resistors are memoryless elements, no transients associated with switching
- Since filter is a feedback structure, speed limited by BW of op amp

Voltage Mode Integrators

- Active RC (Feedback-based)
 - MOSFET-C (Feedback-based)
 - OTA-C
 - TA-C
 - Switched Capacitor
 - Switched Resistor
 - Other Structures
- Sometimes termed “current mode”
- Will discuss later

Have introduced a basic voltage-mode integrators in each of these approaches

All of these structures have applications where they are useful

Performance of all are limited by variability or Op Amp BW

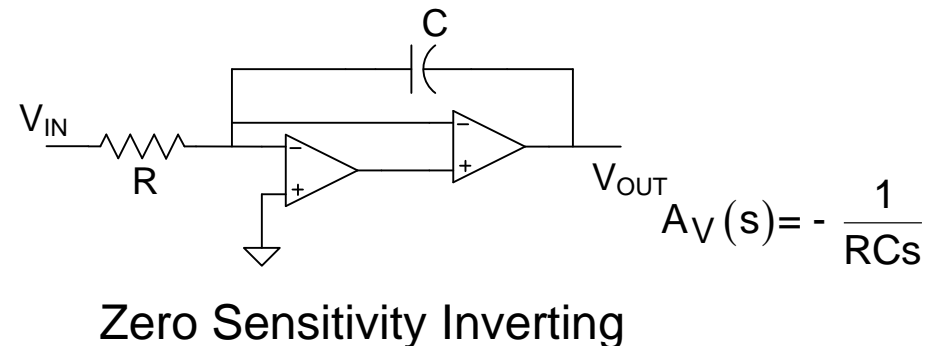
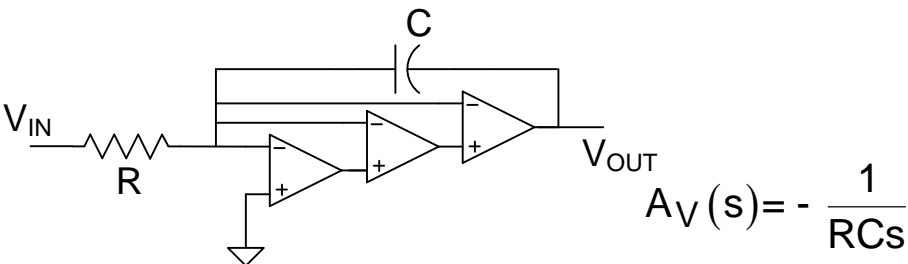
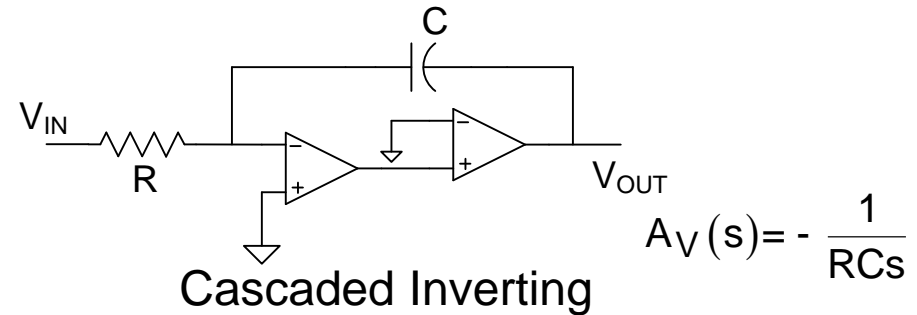
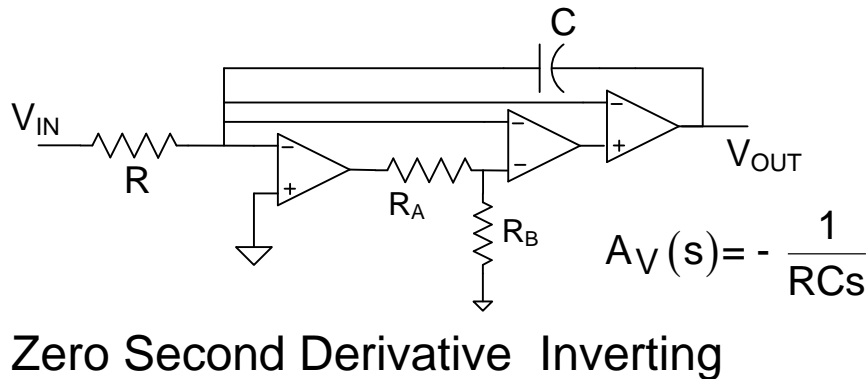
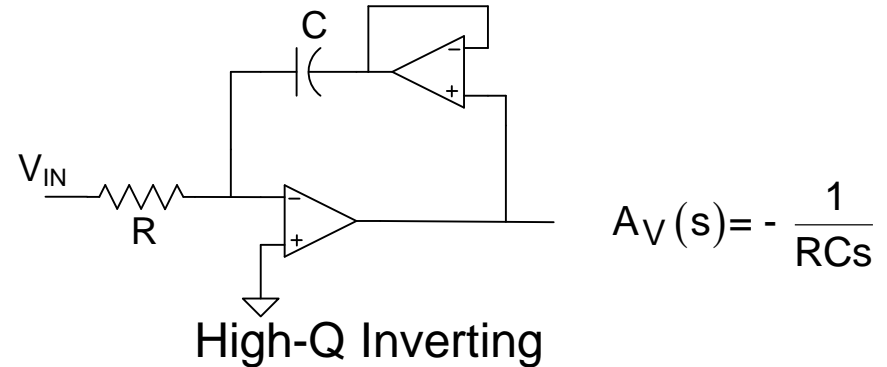
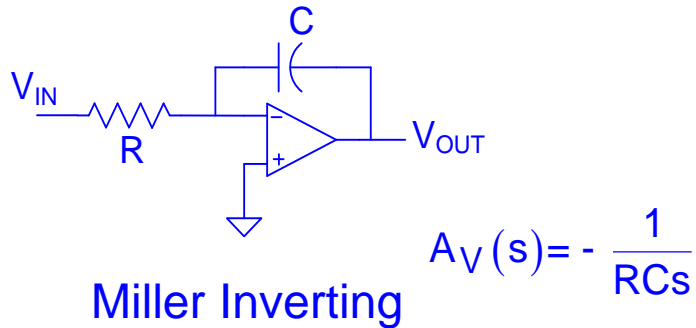
How can integrator performance be improved?

- Better op amps
- Better Integrator Architectures

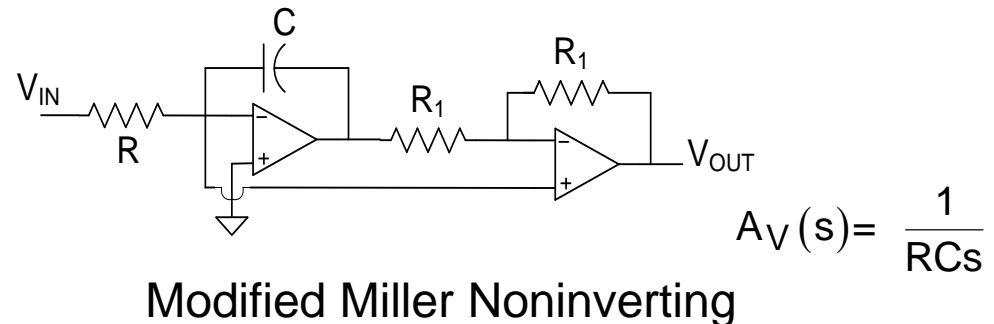
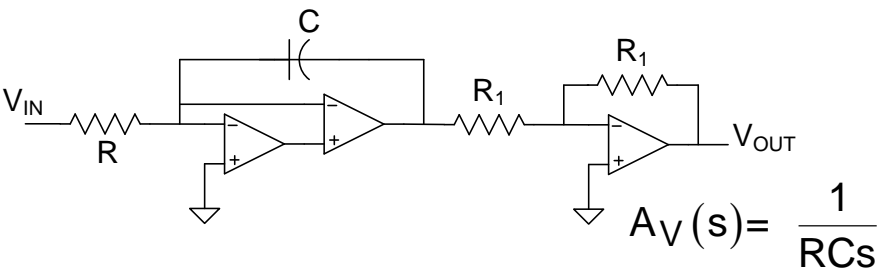
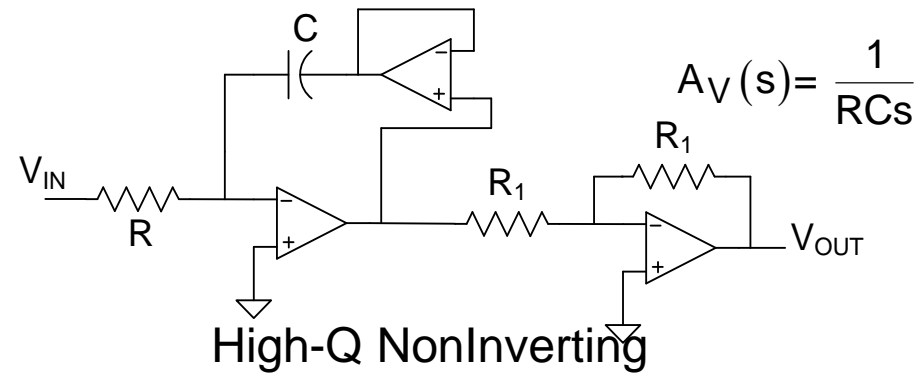
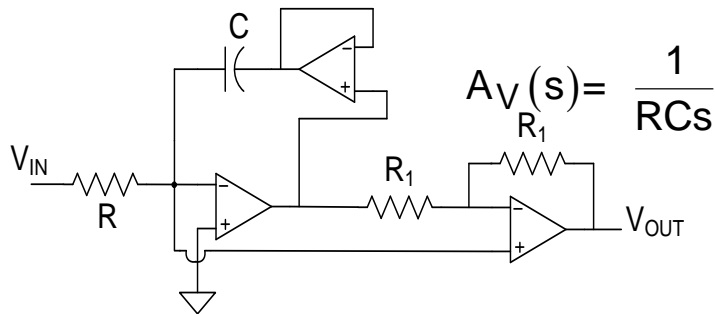
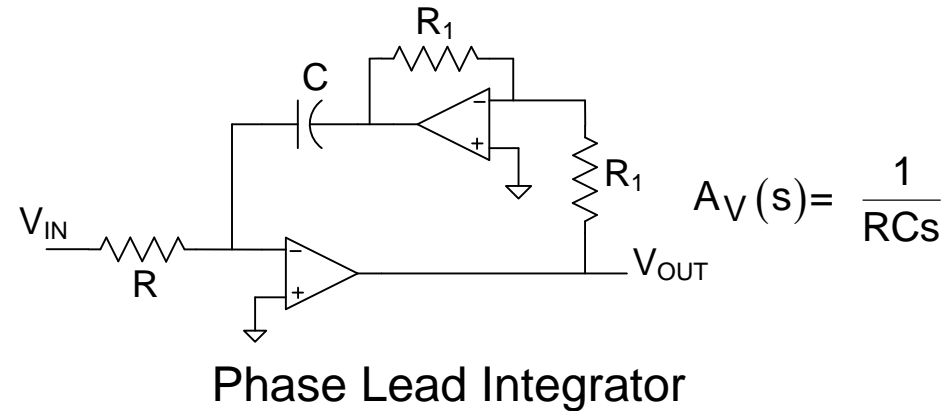
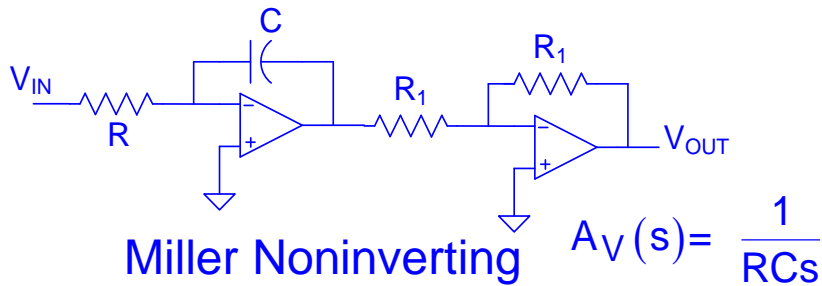
How can the performance of integrator structures be compared?

Need metric for comparing integrator performance

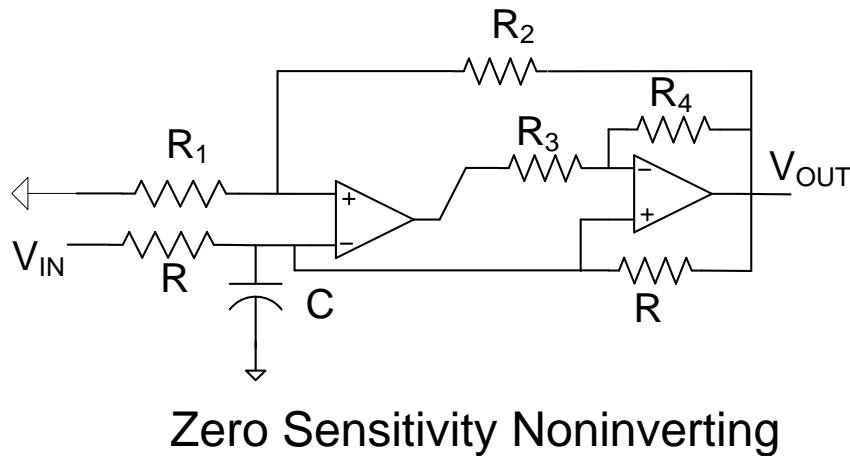
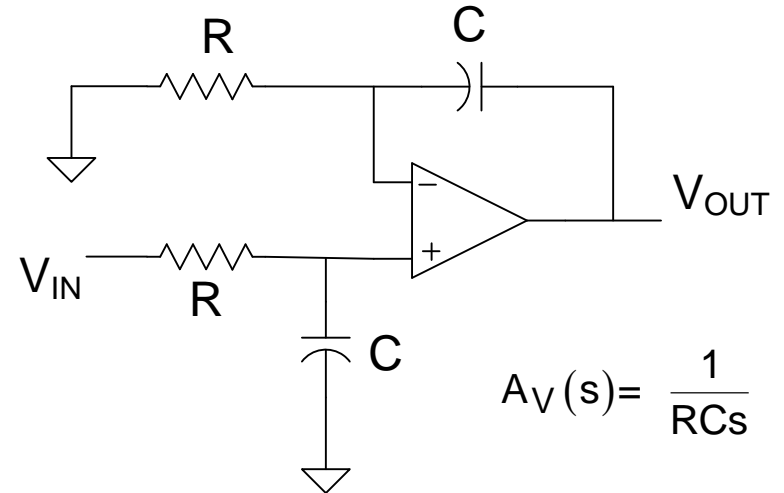
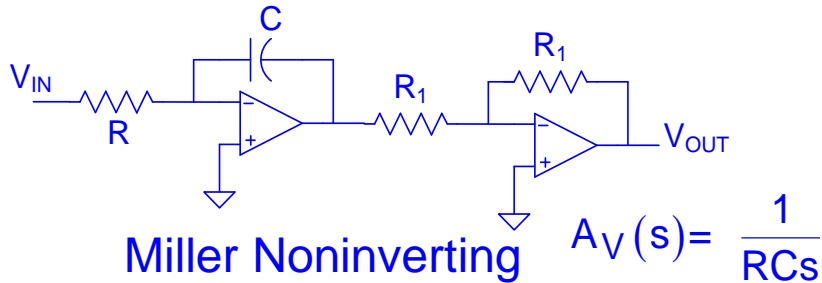
Are there other integrators in the basic classes that have been considered?



Are there other integrators in the basic classes that have been considered?



Are there other integrators in the basic classes that have been considered?

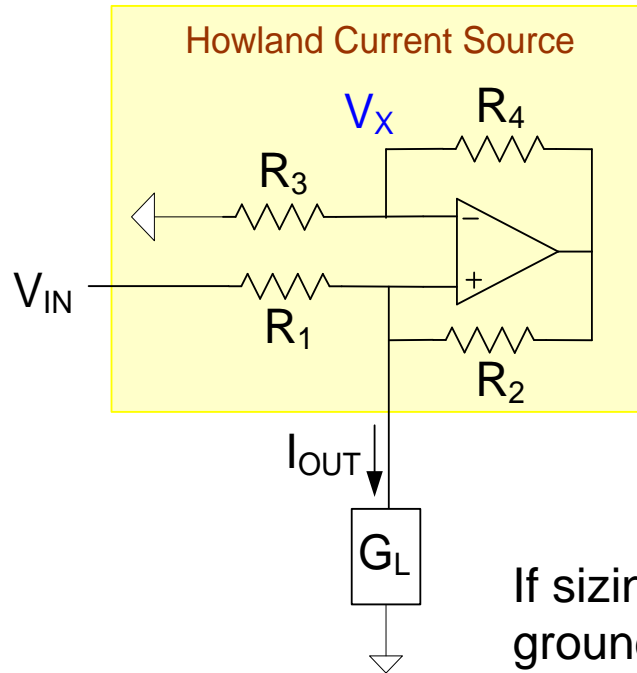


$$A_V(s) = \frac{2}{RCs}$$

If $R_1=R_2$ and $R_3=R_4$

De Boo Integrator

Consider the Howland Current Source



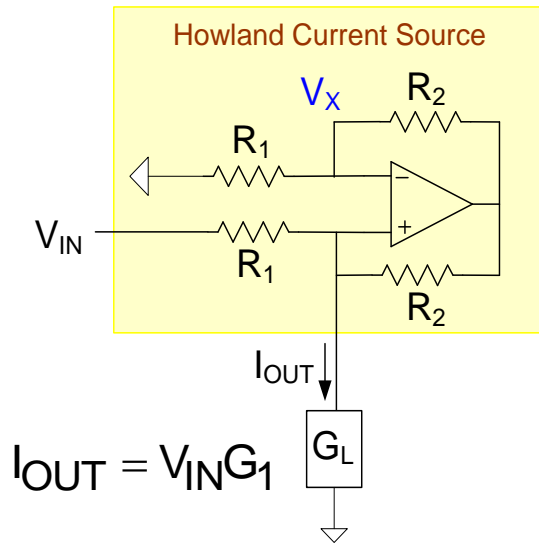
$$I_{OUT} = V_{IN}G_1 + \left[V_X \left(\frac{G_2G_3}{G_4} - G_1 \right) \right]$$

If resistors sized so that $G_1 = \frac{G_2G_3}{G_4}$

$$I_{OUT} = V_{IN}G_1$$

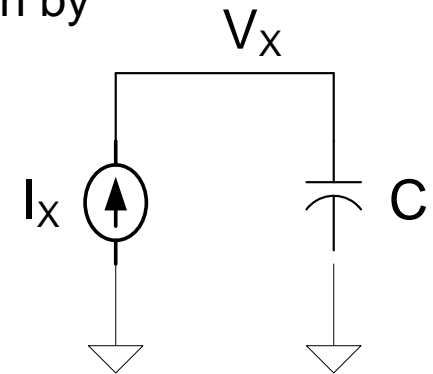
If sizing constraints are satisfied, behaves as a grounded constant-current source

DeBoo Integrator



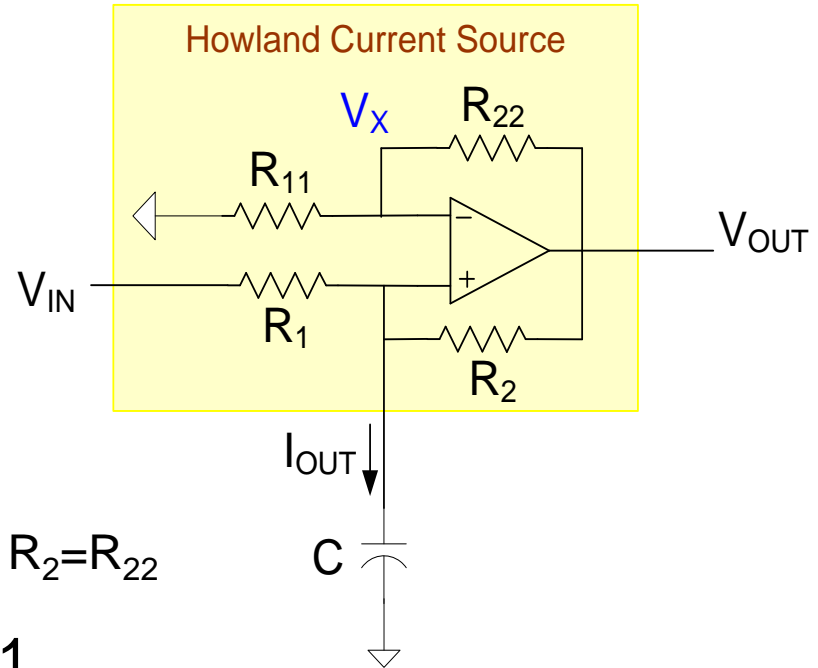
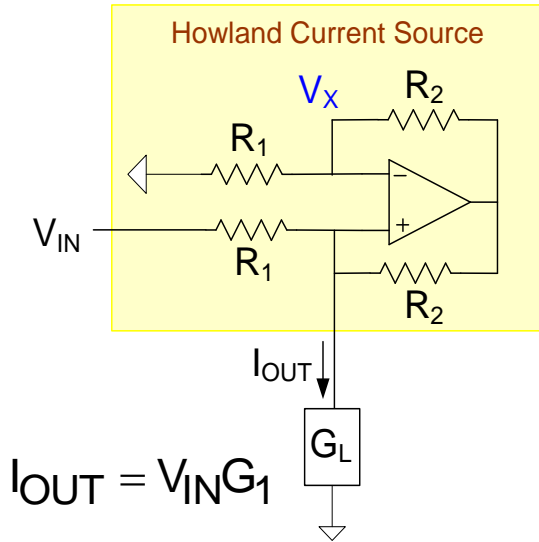
Observe that if a current source drives a grounded capacitor, then the nodal voltage on the capacitor is given by

$$V_X = I_X \frac{1}{sC}$$



Thus, if we could make I_X proportional to V_{IN} , the voltage on the capacitor would be a weighted Integral of V_{IN}

De Boo Integrator



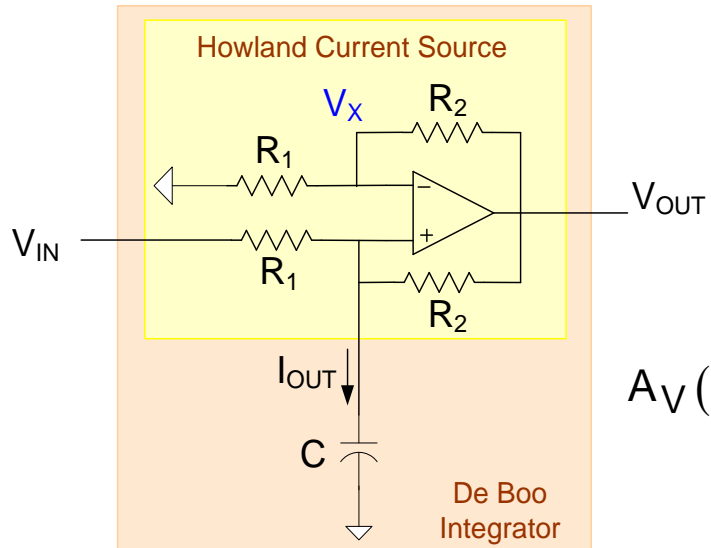
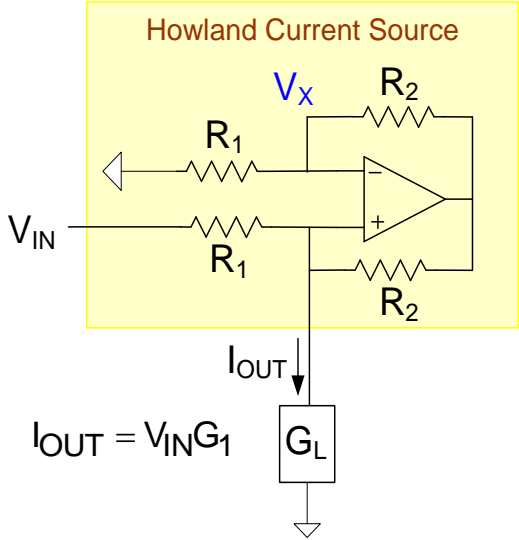
If $R_1=R_{11}$ and $R_2=R_{22}$

$$V_X = \frac{V_{IN}}{R_1} \frac{1}{sC}$$

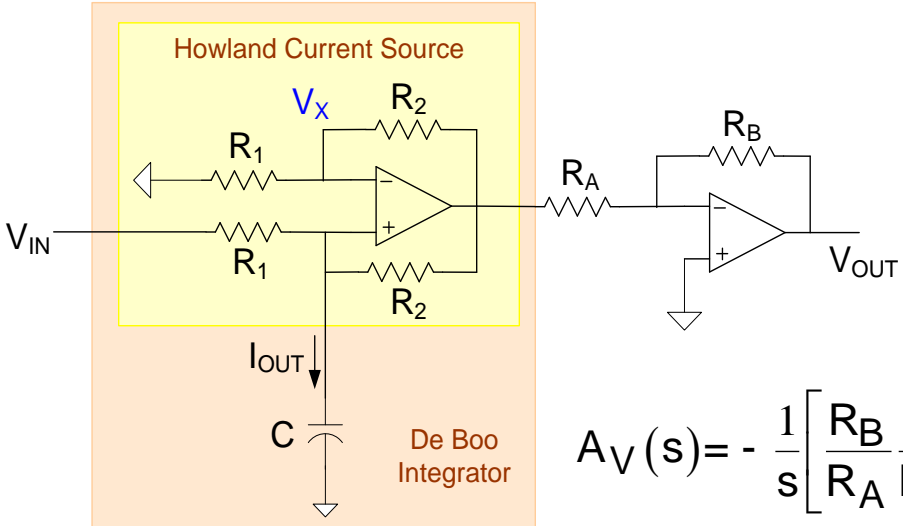
$$V_{OUT} = V_X \left(1 + \frac{R_{22}}{R_{11}} \right) = \frac{V_{IN}}{R_1} \frac{1}{sC} \left(1 + \frac{R_{22}}{R_{11}} \right)$$

$$A_V(s) = \frac{1}{s} \left[\frac{1}{R_1 C} \left(1 + \frac{R_{22}}{R_{11}} \right) \right]$$

De Boo Integrator



$$A_V(s) = \frac{1}{s} \left[\frac{1}{R_1 C} \left(1 + \frac{R_{22}}{R_{11}} \right) \right]$$



$$A_V(s) = - \frac{1}{s} \left[\frac{R_B}{R_A} \frac{1}{R_1 C} \left(1 + \frac{R_{22}}{R_{11}} \right) \right]$$

Many different integrator architectures that ideally provide the same gain

Similar observations can be made for other classes of integrators

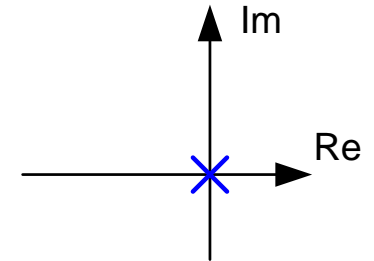
How can the performance of an integrator be characterized and how can integrators be compared?

How can the performance of an integrator be characterized and how can integrators be compared?

Consider Ideal Integrator Gain Function

$$A_V(s) = \frac{I_0}{s} \quad A_V(j\omega) = \frac{I_0}{j\omega}$$

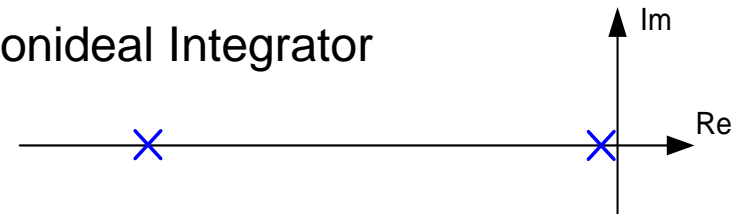
Ideal Integrator



Consider a nonideal integrator Gain Function

$$A_V(s) = \frac{\alpha I_0}{s + \alpha} A_{OO}(s)$$

Nonideal Integrator



Key characteristics of an ideal integrator:

- Magnitude of the gain at $I_0=1$
- Phase of integrator always 90°
- Gain decreases with $1/\omega$

Are any of these properties more critical than others?

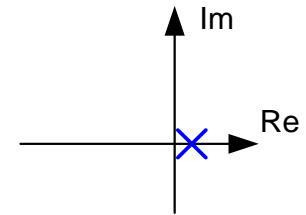
In many applications:

Key property of ideal integrator is a phase shift of 90° at frequencies around I_0 !

How can the performance of an integrator be characterized and how can integrators be compared?

Is stability of an integrator of concern?

Ideal Integrator



- Ideal integrator is not stable
- Integrator function is inherently ill-conditioned
- Integrator is almost never used open-loop
- Stability of integrator not of concern, stability of filter using integrator is of concern
- Some integrators may cause unstable filters, others may result in stable filters

How can the performance of an integrator be characterized and how can integrators be compared?

Express $A_V(j\omega)$ as

$$A_V(j\omega) = \frac{\pm 1}{R(\omega) + jX(\omega)}$$

where $R(\omega)$ and $X(\omega)$ are real and represent the real and imaginary parts of the denominator respectively

$$\text{Phase} = -\tan^{-1} \left(\frac{X(\omega)}{R(\omega)} \right)$$

Ideally $R(\omega) = 0$

Definition: The Integrator Q factor is the ratio of the imaginary part of the denominator to the real part of the denominator

$$Q_{\text{INT}} = \left(\frac{X(\omega)}{R(\omega)} \right)$$

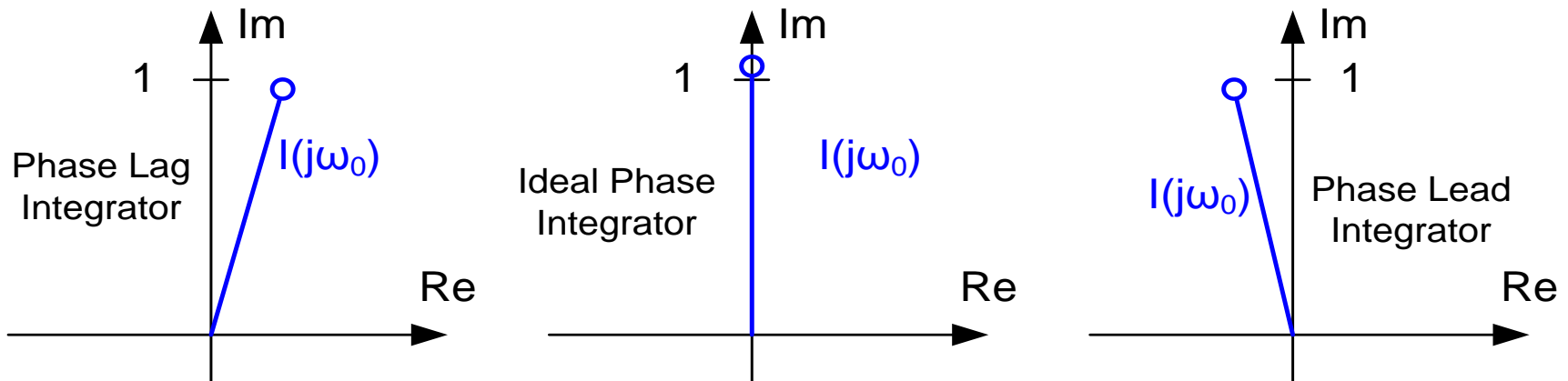
Typically most interested in Q_{INT} at the nominal unity gain frequency of the integrator

How can the performance of an integrator be characterized and how can integrators be compared?

Express $A_V(j\omega)$ as

$$I_V(j\omega) = \frac{\pm 1}{R(\omega) + jX(\omega)}$$

Lead/Lag Characteristics for Inverting Integrators

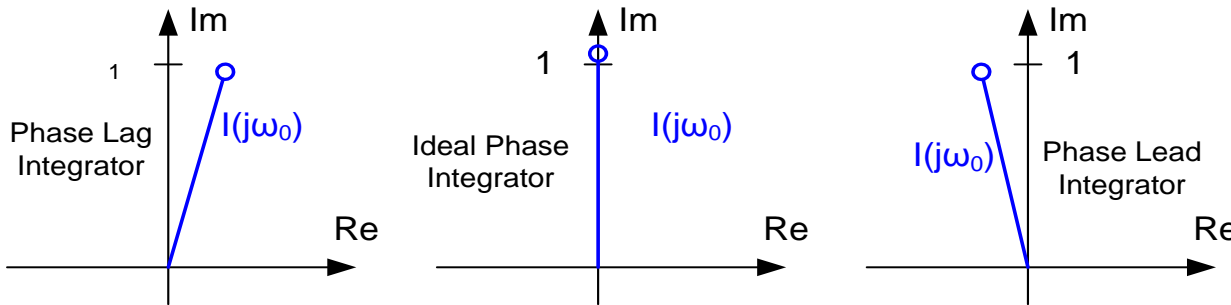


For Phase Lag Integrators, $R(\omega)$ is negative
For Phase Lead integrators, $R(\omega)$ is positive

How can the performance of an integrator be characterized and how can integrators be compared?

Lead/Lag Characteristics for Inverting Integrators

$$I_V(j\omega) = \frac{-1}{R(\omega) + jX(\omega)}$$

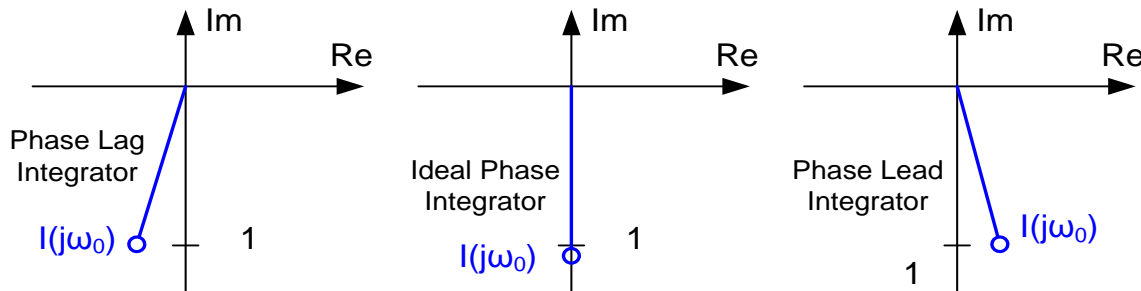


For Phase Lag Integrators, $R(\omega)$ and $X(\omega)$ have opposite signs. For Phase Lead integrators,

$R(\omega)$ and $X(\omega)$ have the same sign. Phase shift ideally 90°

Lead/Lag Characteristics for Noninverting Integrators

$$I_V(j\omega) = \frac{1}{R(\omega) + jX(\omega)}$$

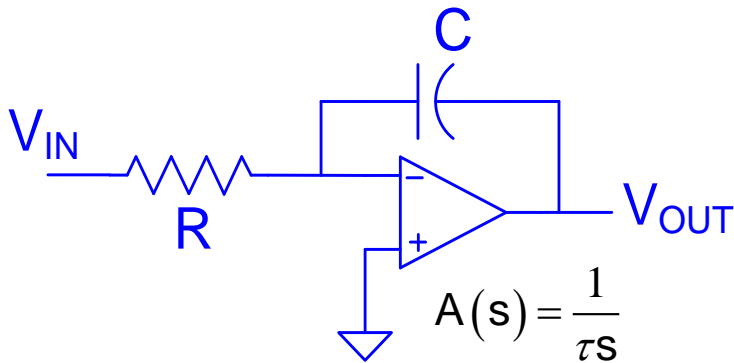


For Phase Lag Integrators, $R(\omega)$ and $X(\omega)$ have opposite signs. For Phase Lead integrators,

$R(\omega)$ and $X(\omega)$ have the same sign. Phase shift ideally 270°

Integrator Q Factor

Consider Miller Inverting Integrator



$$A_V(s) = \frac{-1}{RCs + \tau s(1 + RCs)}$$

$$A_V(s) = \frac{-1}{RCj\omega + \tau j\omega(1 + RCj\omega)}$$

$$A_V(s) = \frac{-1}{-\tau\omega^2 RC + j(\omega[RC + \tau])}$$

Normalizing by $\omega_n = \omega RC$ and $\tau_n = \tau / RC = I_{on} / GB$

$$A_V(s) = \frac{-1}{-\tau_n \omega_n^2 + j(\omega_n [1 + \tau_n])}$$

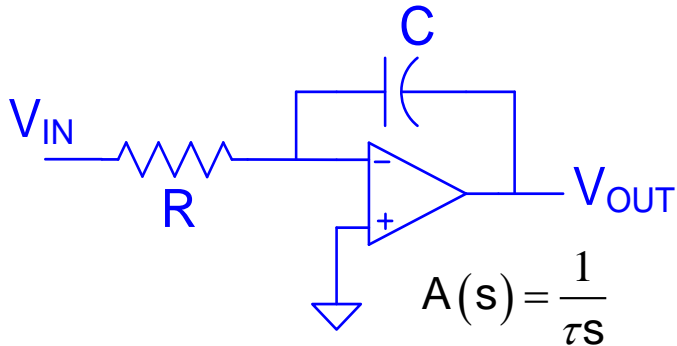
Observe this integrator has excess phase shift (more than 90° in the denominator) at all frequencies

Integrator Q Factor

$$A_V(j\omega) = \frac{\pm 1}{R(\omega) + jX(\omega)}$$

$$Q_{INT} = \left(\frac{X(\omega)}{R(\omega)} \right)$$

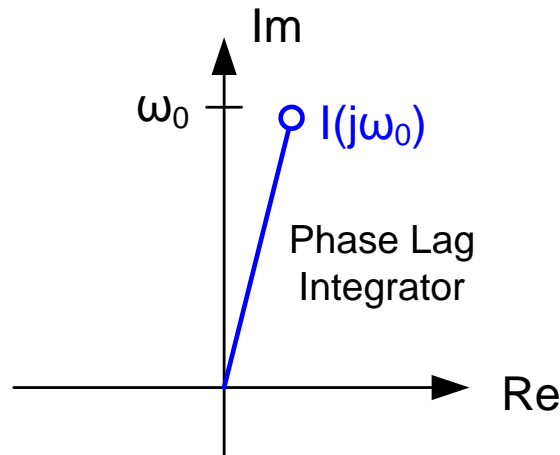
Consider Miller Inverting Integrator



$$A_V(s) = \frac{-1}{-\tau_n \omega_n^2 + j(\omega_n [1 + \tau_n])}$$

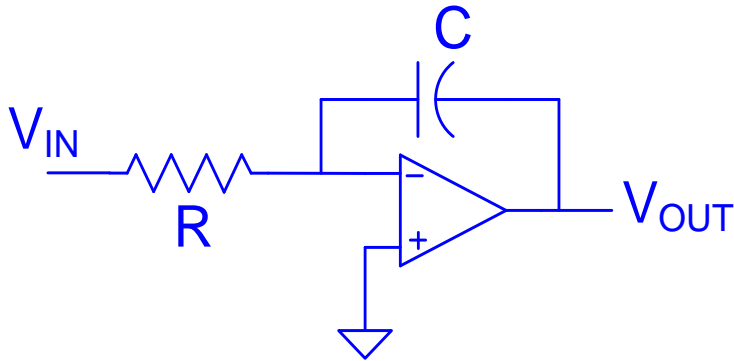
$$Q_{INT} = -\frac{\omega_n [1 + \tau_n]}{\tau_n \omega_n^2} \approx -\frac{1}{\tau_n \omega_n} = -\frac{GB}{\omega} = -A$$

Since the phase is less than 90° , the Miller Inverting Integrator is a Phase Lag Integrator and Q_{INT} is negative



Integrator Pole Locations

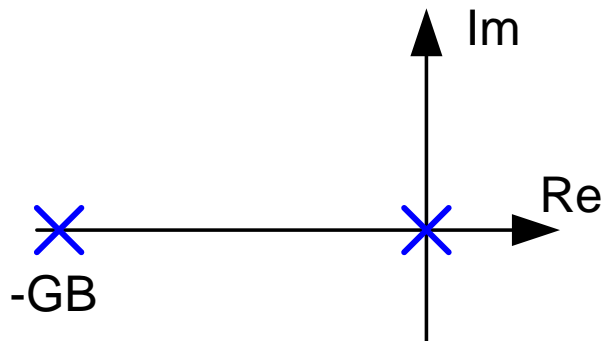
Consider Miller Inverting Integrator



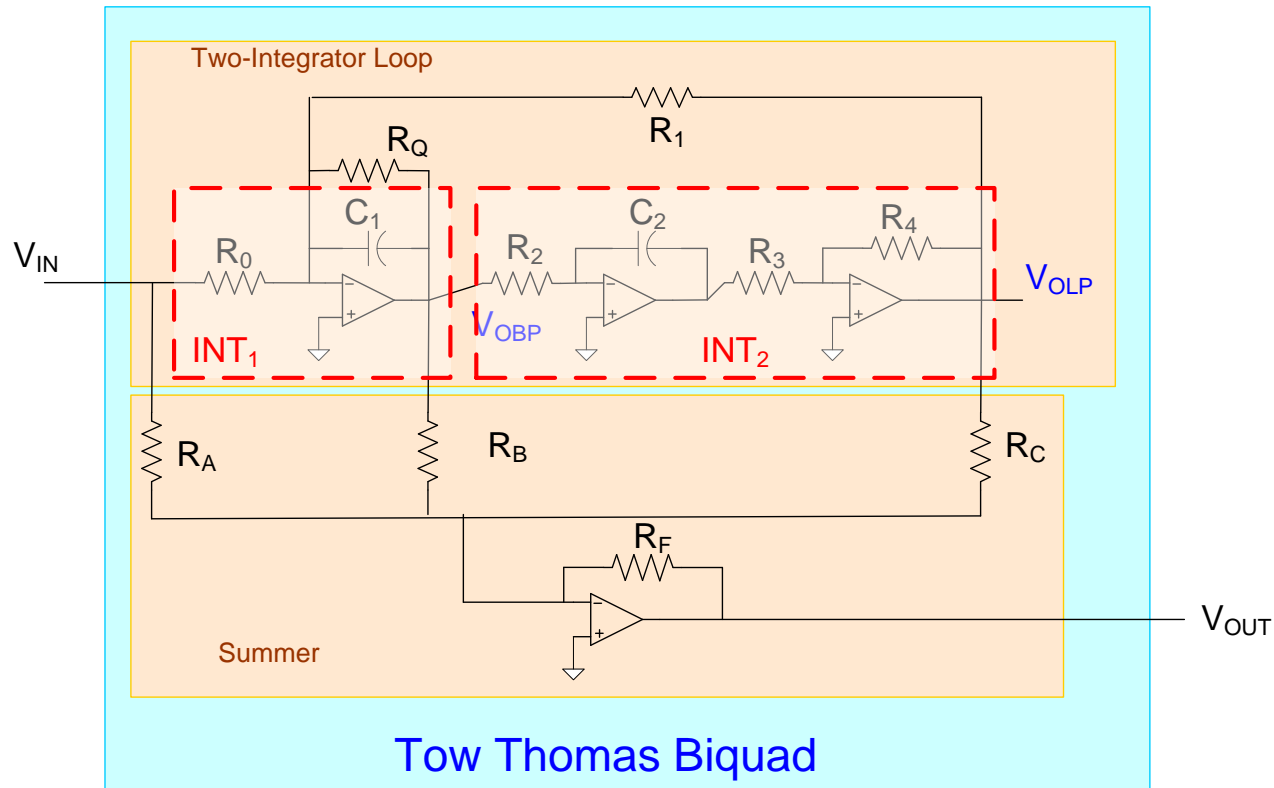
$$A_V(s) = \frac{-1}{RCs + \tau s(1 + RCs)}$$

$$I_0 = 1/(RC)$$

Poles at $s=0$ and $s = -I_0(1 + GB/I_0) \simeq -GB$



Is the integrator Q factors simply a metric or does it have some other significance?

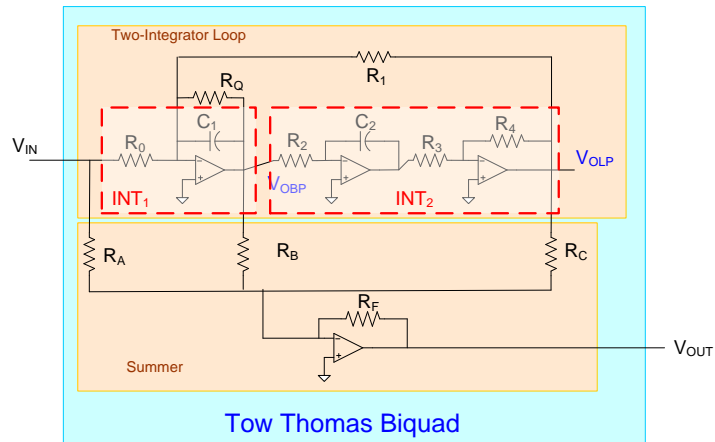


It can be shown that the pole Q for the TT Biquad can be approximated by

$$Q_P \cong \frac{1}{\frac{1}{Q_{PN}} + \frac{1}{Q_{INT1}} + \frac{1}{Q_{INT2}}}$$

where Q_{INT1} and Q_{INT2} are evaluated at $\omega = \omega_0$

Is the integrator Q factors simply a metric or does it have some other significance?



It can be shown that the pole Q for the TT Biquad can be approximated by

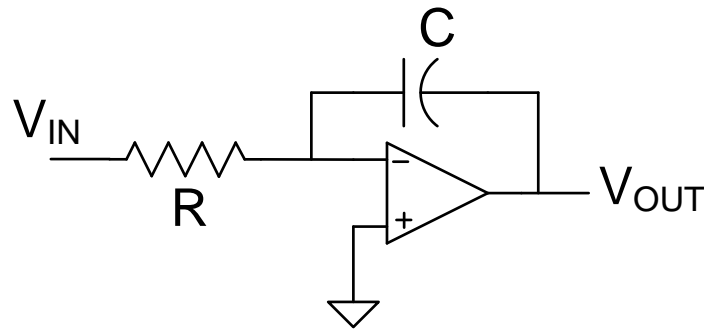
$$Q_P \cong \frac{1}{\frac{1}{Q_{PN}} + \frac{1}{Q_{INT1}(\omega_0)} + \frac{1}{Q_{INT2}(\omega_0)}}$$

Similar expressions for other second-order biquads

Observe that the integrator Q factors adversely affect the pole Q of the filter

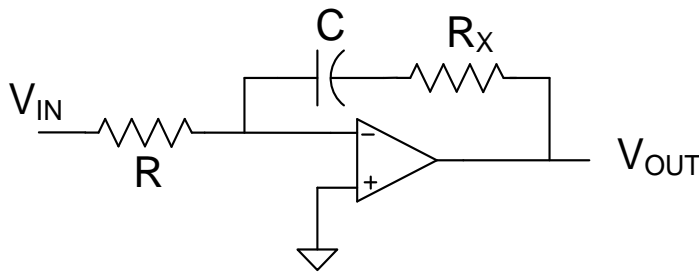
Observe that if Q_{INT1} and Q_{INT2} are of opposite signs and equal magnitudes, nonideal effects of integrator can cancel

What can be done to correct the phase problems of an integrator?

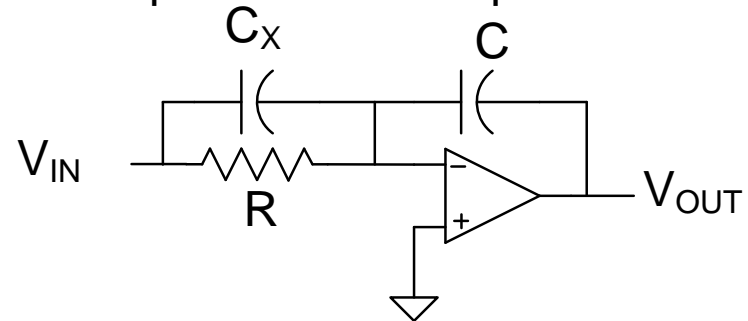


$$A_V(s) = \frac{-1}{RCs + \tau s(1 + RCs)}$$

One thing that can help the Miller Integrator is phase-lead compensation



$$A_V(s) = \frac{-(1 + R_x Cs)}{RCs + \tau s(1 + [R + R_x]Cs)}$$



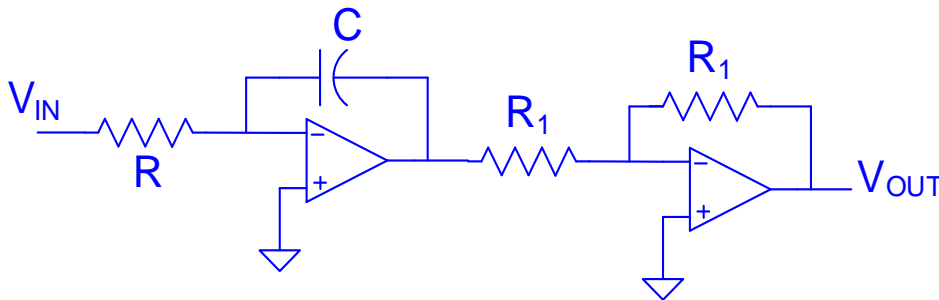
$$A_V(s) = \frac{-(1 + RC_x s)}{RCs + \tau s(1 + R[C + C_x]s)}$$

R_x and C_x will add phase-lead by introduction of a zero

R_x and C_x will be small components

Integrator Q Factor

Consider Miller Noninverting Integrator



$$A_V(s) = \frac{-1}{RCs + \tau s(1 + RCs)} \cdot \frac{-1}{1 + 2\tau s}$$

$$A_V(j\omega) = \frac{1}{-3\tau RC\omega^2 + j[\omega RC(1 + 2\tau\omega)]}$$

$$A_V(j\omega) \approx \frac{1}{-3\tau RC\omega^2 + j\omega RC}$$

Observe this integrator has excess phase shift (more than 90° in the denominator) at all frequencies

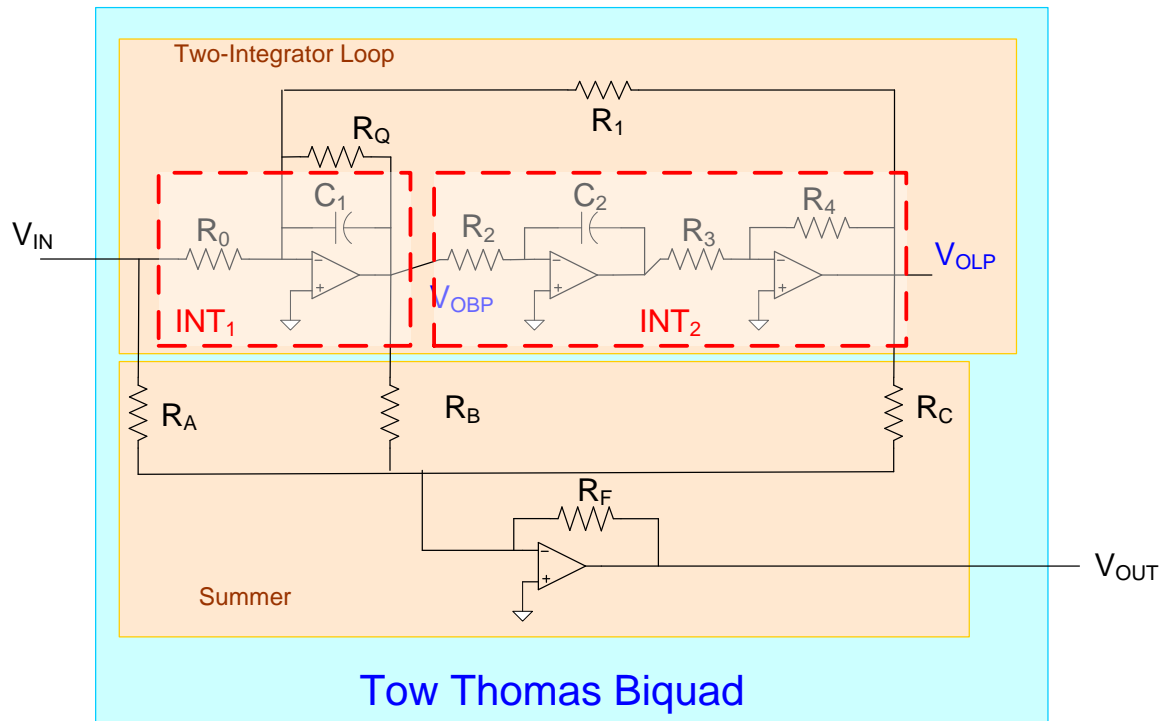
$$Q_{INT} \approx \frac{\omega RC}{-3\tau RC\omega^2} = \frac{-1}{3\tau\omega}$$

$$Q_{INT} \approx \frac{-1}{3} \left(\frac{GB}{\omega} \right) = -\frac{1}{3} |A(j\omega)|$$

Note: The Miller Noninverting Integrator has a modestly poorer Q_{INT} than the Miller Inverting Integrator

Example:

If $f_0=10\text{KHz}$, $GB=1\text{MHz}$, $Q_{\text{NOM}}=10$, estimate the pole Q for the Tow-Thomas Biquad if the Miller Integrator and the Miller Noninverting Integrators are used. Also determine the relative degradation in performance due to each of the integrators.



$$Q_P \cong \frac{1}{\frac{1}{Q_{PN}} + \frac{1}{Q_{INT1}} + \frac{1}{Q_{INT2}}}$$

Example:

If $f_0=10\text{KHz}$, $GB=1\text{MHz}$, $Q_{\text{NOM}}=10$, estimate the pole Q for the Tow-Thomas Biquad if the Miller Integrator and the Miller Noninverting Integrators are used. Also determine the relative degradation in performance due to each of the integrators.

$$Q_P \cong \frac{1}{\frac{1}{Q_{PN}} + \frac{1}{Q_{\text{INT1}}} + \frac{1}{Q_{\text{INT2}}}}$$

$$Q_{\text{INT1}} = -A = -\frac{GB}{\omega} = -\frac{1\text{MHz}}{10\text{KHz}} = -100$$

$$Q_{\text{INT2}} \cong -\frac{1}{3}|A(j\omega)| = -\frac{1}{3}\left(\frac{GB}{\omega}\right) = -\frac{1\text{MHz}}{3 \cdot 10\text{KHz}} = -33$$

$$Q_P \cong \frac{1}{\frac{1}{10} - .01 - .033} = 17.5$$

Note the nonideal integrators cause about a 75% shift in Q_P

Note that 3 times as much of the shift is due to the noninverting integrator as is due to the inverting integrator!

Similar effects of the integrators will be seen on other filter structures

Example:

If $f_0=10\text{KHz}$, $GB=1\text{MHz}$, $Q_{\text{NOM}}=10$, estimate the pole Q for the Tow-Thomas Biquad if the Miller Integrator and the Miller Noninverting Integrators are used. Also determine the relative degradation in performance due to each of the integrators.

$$Q_P \cong \frac{1}{\frac{1}{Q_{PN}} + \frac{1}{Q_{\text{INT1}}} + \frac{1}{Q_{\text{INT2}}}}$$

$$Q_{\text{INT1}} = -A = -\frac{GB}{\omega} = -\frac{1\text{MHz}}{10\text{KHz}} = -100$$

$$Q_{\text{INT2}} \cong -\frac{1}{3}|A(j\omega)| = -\frac{1}{3}\left(\frac{GB}{\omega}\right) = -\frac{1\text{MHz}}{3 \cdot 10\text{KHz}} = -33$$

$$Q_P \cong \frac{1}{\frac{1}{10} - .01 - .033} = 17.5$$

How can the problem be solved?

1. Compensate Integrator
2. Use better integrators
3. Use phase-lead and phase/lag pairs

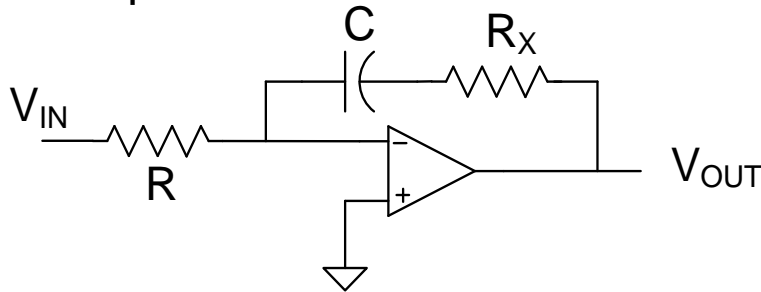
Example:

If $f_0=10\text{KHz}$, $GB=1\text{MHz}$, $Q_{\text{NOM}}=10$, estimate the pole Q for the Tow-Thomas Biquad if the Miller Integrator and the Miller Noninverting Integrators are used. Also determine the relative degradation in performance due to each of the integrators.

$$Q_P \cong \frac{1}{\frac{1}{Q_{PN}} + \frac{1}{Q_{\text{INT1}}} + \frac{1}{Q_{\text{INT2}}}} \quad Q_{\text{INT1}} = -100 \quad Q_{\text{INT2}} \approx -33$$

How can the problem be solved?

Phase Compensation of INT1



$$A_V(s) = \frac{-(1+R_XCs)}{RCs + \tau s(1+[R+R_X]Cs)}$$

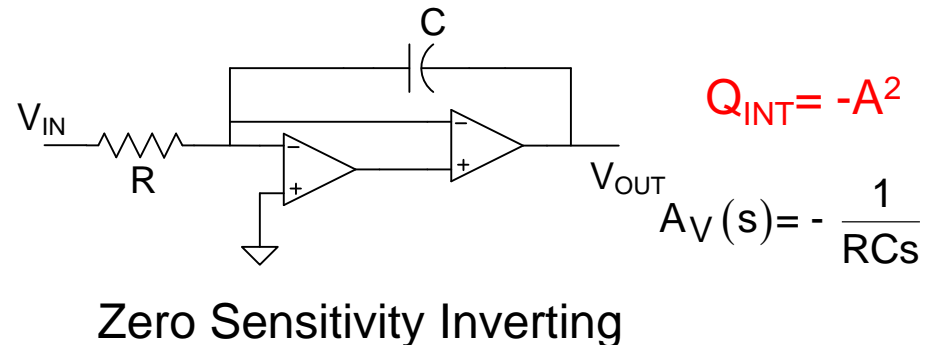
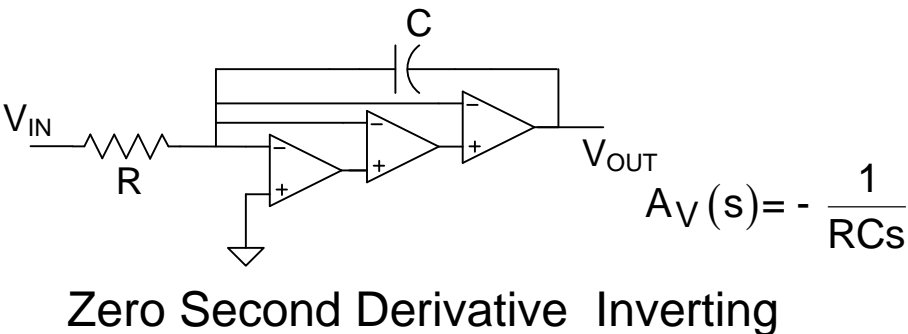
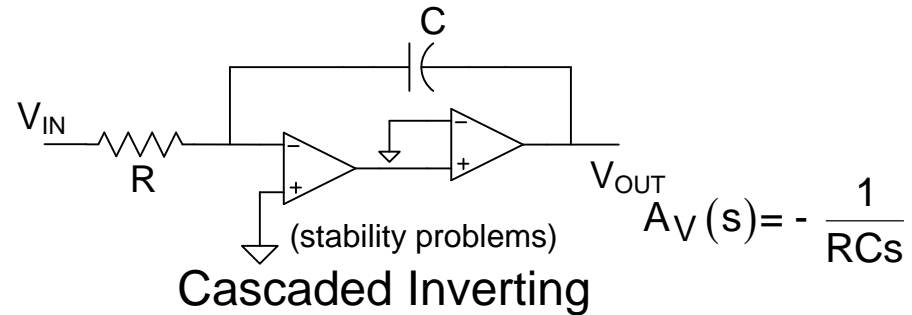
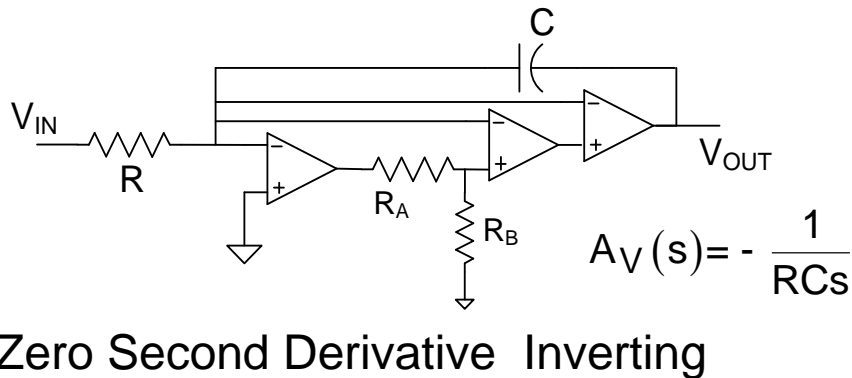
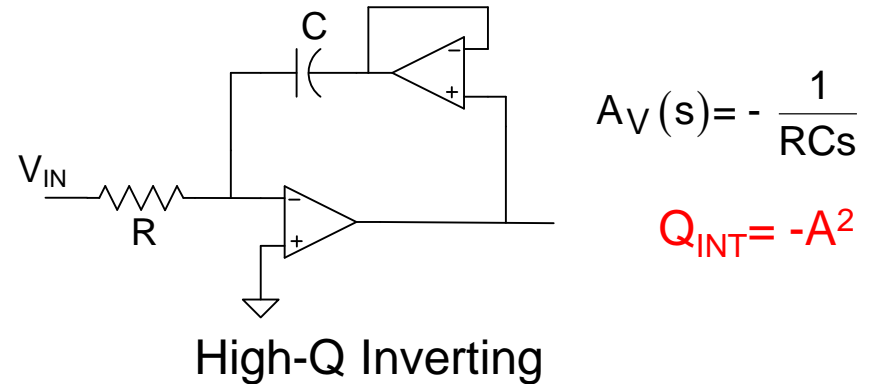
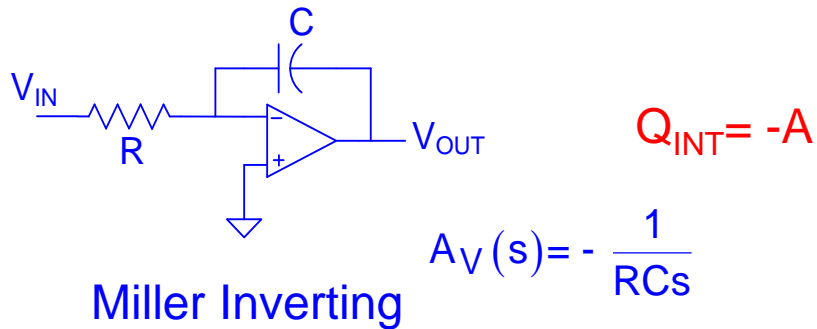
$$Q_{\text{INT1}} = \frac{-GB/\omega}{1-GB \cdot CR_X}$$

Pick R_X so that $Q_{\text{INT1}}=33$ at $\omega=1/(RC)$

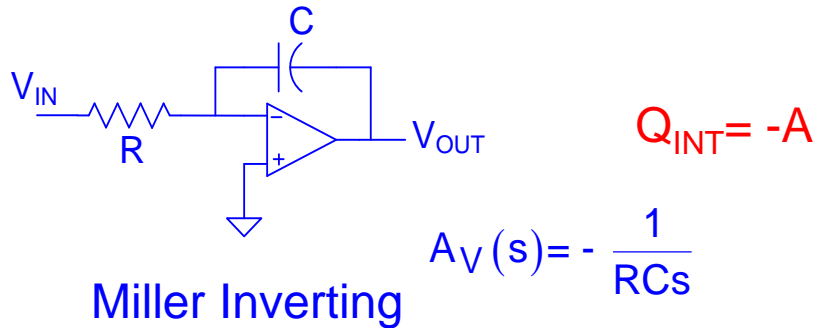
Solving, obtain $CR_X=4/GB$

Useful for hand calibration but not practical for volume production because of variability in components

What are the integrator Q factors for other integrators that have been considered?

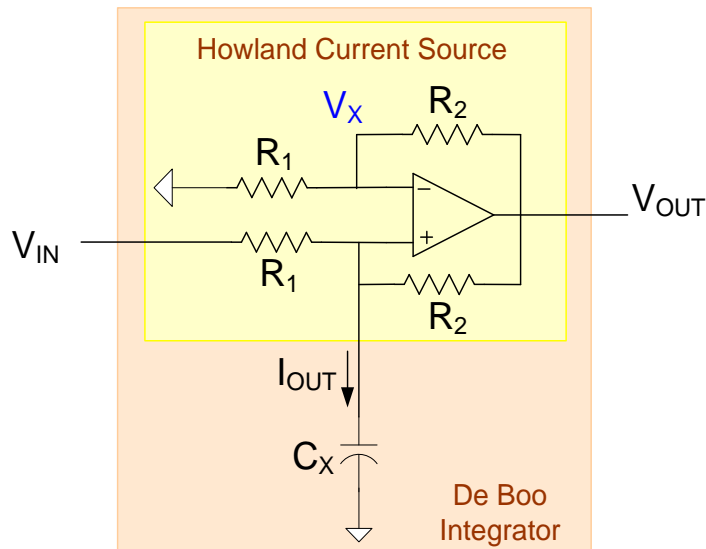


What are the integrator Q factors for other integrators that have been considered?



$$A_V(s) = -\frac{1}{RCs}$$

$$Q_{INT} = -A$$



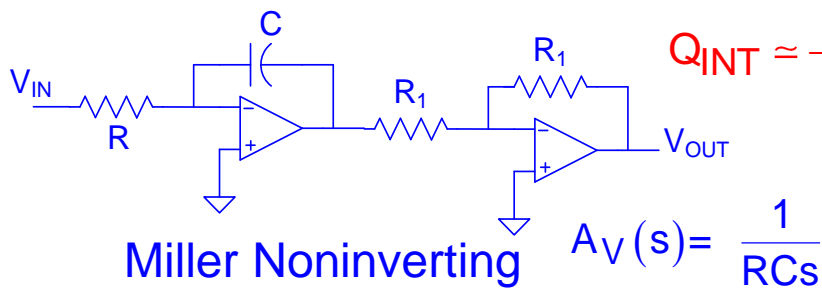
$$A_V(s) = \frac{1 + \frac{R_2}{R_1}}{R_1Cs + \tau_1s \left(1 + \frac{R_2}{R_1}\right) \left(1 + \frac{R_1}{R_2} + sCR_1\right)}$$

$$Q_{INT} = -\frac{A}{1 + \frac{R_2}{R_1}}$$

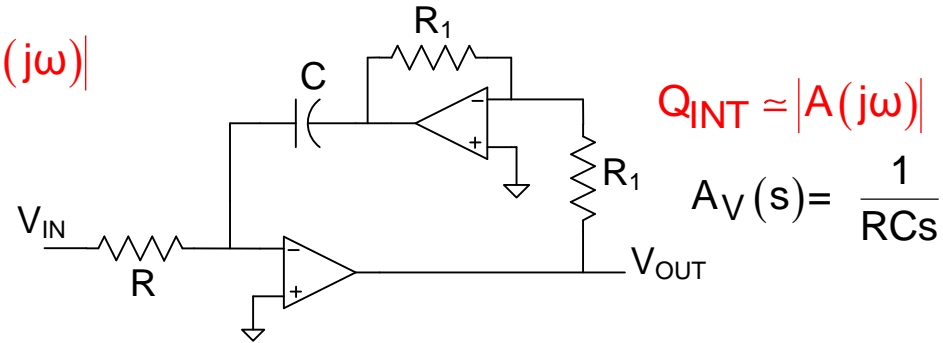
If $R_1 = R_2 = R$ $A_V(s) = \frac{2}{RCs}$

$$Q_{INT} = -\frac{A}{2}$$

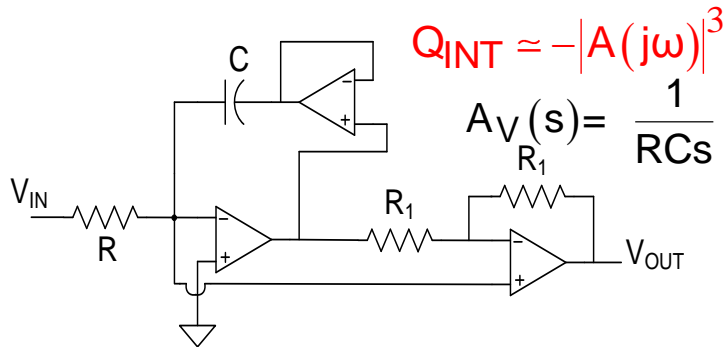
What are the integrator Q factors for other integrators that have been considered?



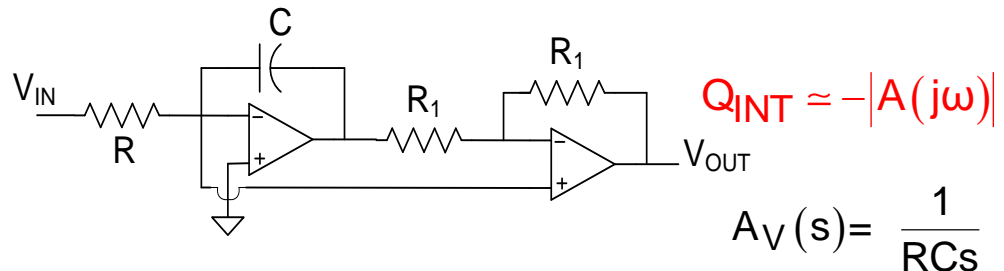
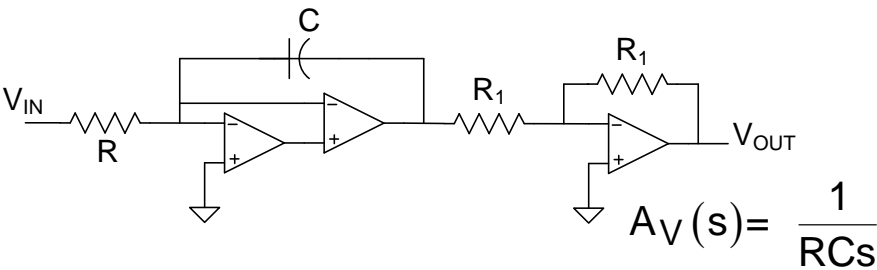
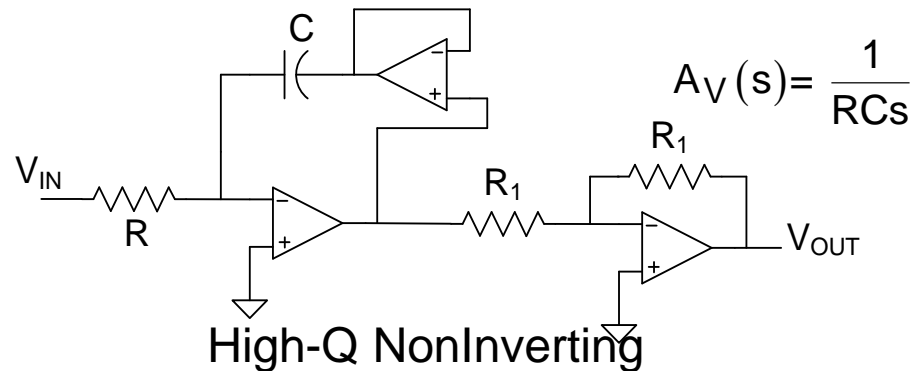
$$Q_{INT} \approx -\frac{1}{3}|A(j\omega)|$$



$$Q_{INT} \approx |A(j\omega)|$$



$$Q_{INT} \approx -|A(j\omega)|^3$$



$$Q_{INT} \approx -|A(j\omega)|$$

Improving Integrator Performance:

1. Compensate Integrator
2. Use better integrators
3. Use phase-lead and phase/lag pairs

- These methods all provide some improvements in integrator performance
- But both magnitude and phase of an integrator are important so focusing only on integrator Q factor only may only improve performance to a certain level
- In higher-order integrator-based filters, the linearity in $1/\omega$ of the integrator gain is also important. The integrator magnitude and Q factor at ω_0 ignore the frequency nonlinearity that may occur in the $1/\omega$ dependence
- There is little in the literature on improving the performance of integrated integrators within a basic class. At high frequencies where the active device performance degrades, particularly in finer-feature processes, there may be some benefits that can be derived from architectural modifications along the line of those discussed in this lecture

