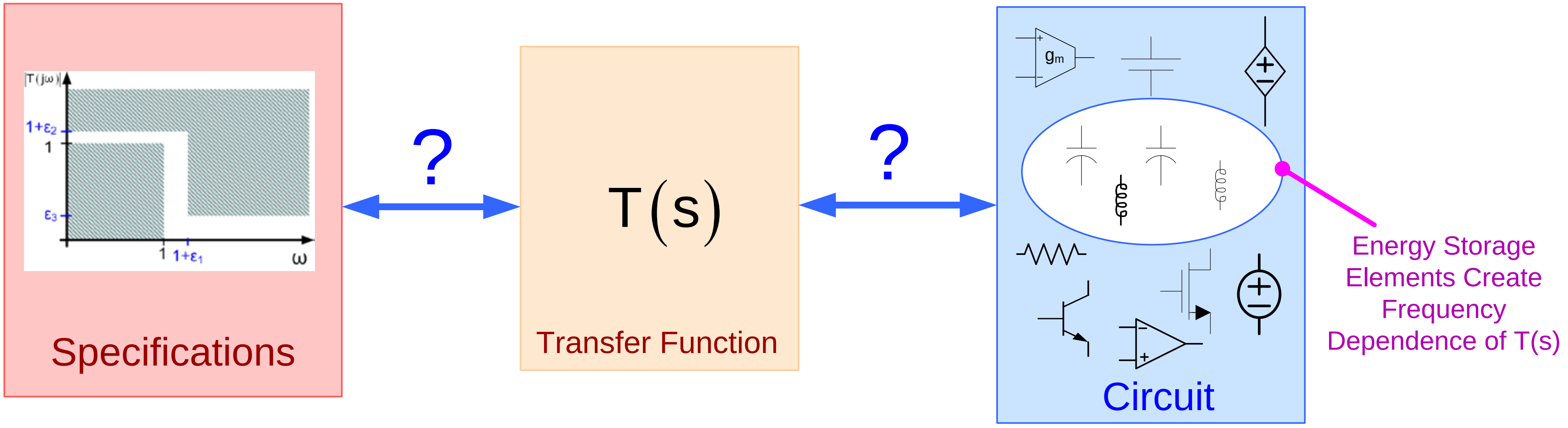
EE 508
Lecture 3

Filter Concepts/Terminology

Basic Properties of Electrical Circuits

Review from Last Time

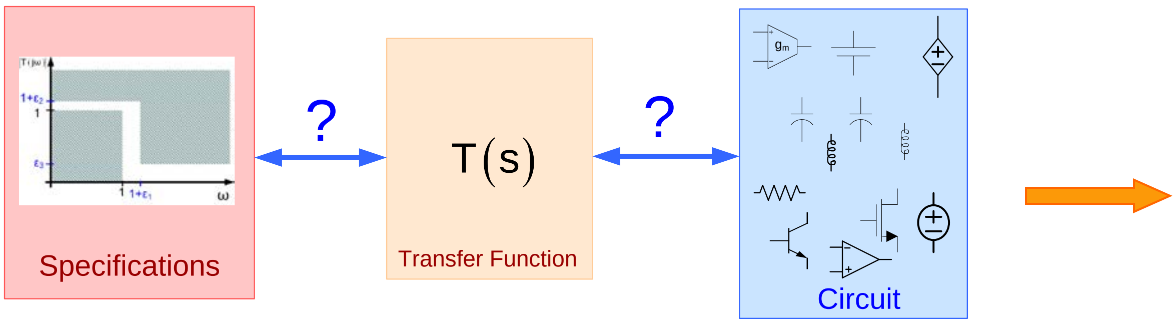
Is there a systematic way to design filters?



Filter Design Strategy: Use the transfer function as an intermediate step between the Specifications and Circuit Implementation

Review from Last Time

Filter Design Process



Filter Design Strategy: Use the transfer function as an intermediate step between the Specifications and Circuit Implementation

Establish Specifications

- possibly $T_D(s)$ or $H_D(z)$
- magnitude and phase characteristics or restrictions
- time domain requirements

Approximation

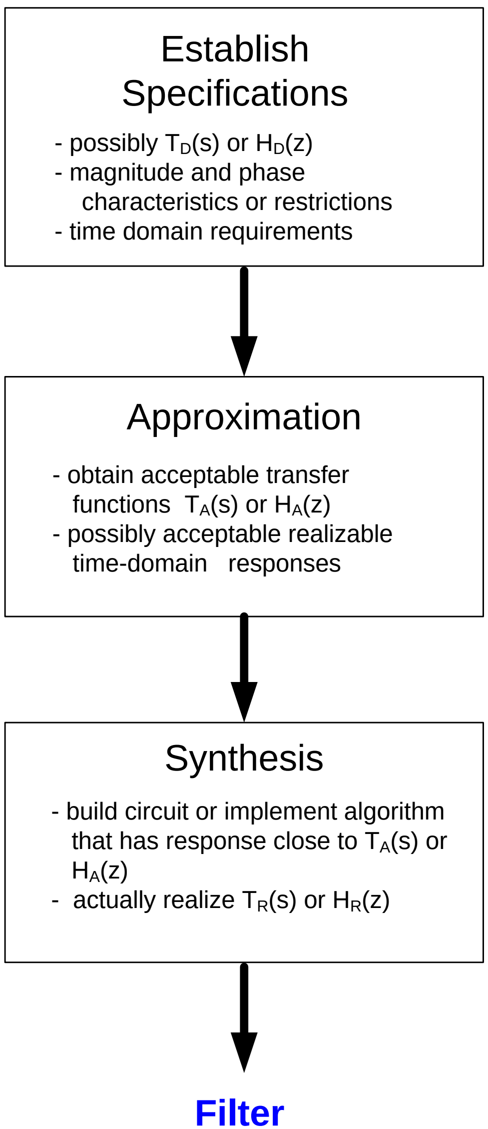
- obtain acceptable transfer functions $T_A(s)$ or $H_A(z)$
- possibly acceptable realizable time-domain responses

Synthesis

- build circuit or implement algorithm that has response close to $T_A(s)$ or $H_A(z)$
- actually realize $T_R(s)$ or $H_R(z)$

Filter

Filter Design Process



Review from Last Time

Must understand the real performance requirements

- **Many acceptable specifications for a given application**
- **Some much better than others**
- **But often difficult to obtain even one that is useful**

Obtain an acceptable approximating function

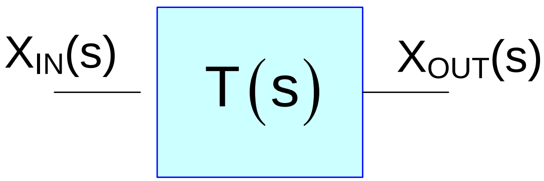
- **Many acceptable approximating functions for a given specification**
- **Some much better than others**
- **But often difficult to obtain even one!**

Design (synthesize) a practical circuit that has a transfer function close to the acceptable transfer function

- **Many acceptable circuits for a given approximating function**
  Some much better than others
- **But often difficult to obtain even one!**

Important to make good decisions at each step in the filter design process because poor decisions will not be absolved in subsequent steps

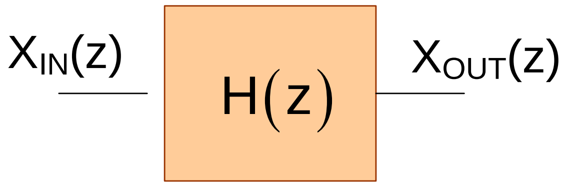
Filter Concepts and Terminology



$$T(s) = \frac{\sum_{i=0}^{m} a_i s^i}{\sum_{i=0}^{n} b_i s^i} = \frac{N(s)}{D(s)}$$

- A polynomial is said to be “integer monic” if the coefficient of the highest-order term is 1
- If D(s) is integer monic, then N(s) and D(s) are unique
- If D(s) is integer monic, then the a_k and b_k terms are unique
- The roots of N(s) are termed the zeros of the transfer function
- The roots of D(s) are termed the poles of the transfer function
- If N(s) and D(s) are of orders m and n respectively, then there are m zeros and n poles in T(s)

Filter Concepts and Terminology



$$H(z) = \frac{\sum_{i=0}^{m} a_i z^i}{\sum_{i=0}^{n} b_i z^i} = \frac{N(z)}{D(z)}$$

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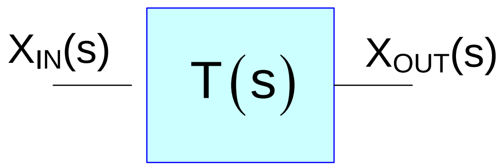
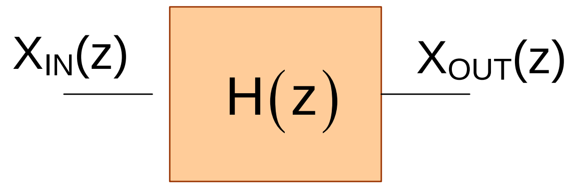
Filter Concepts and Terminology





$$T(s) = \frac{\sum_{i=0}^{m} a_i s^i}{\sum_{i=0}^{n} b_i s^i} = \frac{N(s)}{D(s)}$$

$$H(z) = \frac{\sum_{i=0}^{m} a_i z^i}{\sum_{i=0}^{n} b_i z^i} = \frac{N(z)}{D(z)}$$

- Key Theorem: The continuous-time filter is stable iff all poles lie in the open left half of the s-plane

- Key Theorem: The discrete-time filter is stable iff all poles lie in the open unit circle

- The zeros of T(s) need not lie in the left half plane to maintain stability

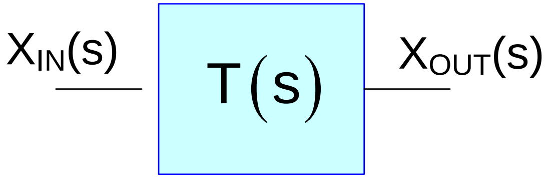
- The zeros of H(z) need not lie in the open unit circle to maintain stability

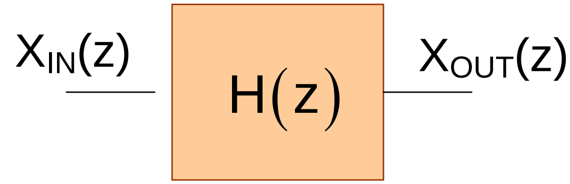
Filter Concepts and Terminology

Minimum Phase Property

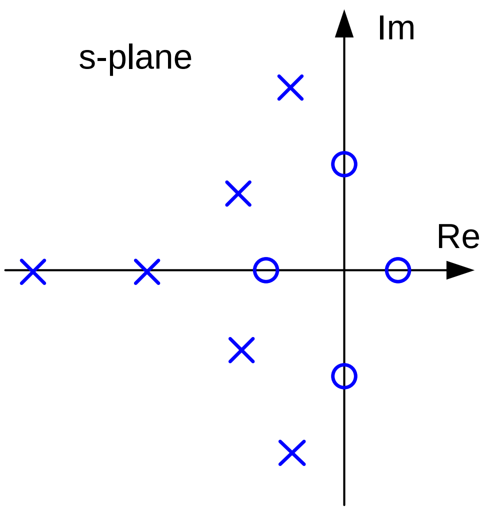
- An s-domain rational fraction is termed minimum-phase if all poles and all zeros have a non-positive real part
- An s-domain rational fraction is minimum-phase if it has no poles or zeros in the RHP or on the imaginary axis
- A z-domain rational fraction is minimum-phase if the magnitude of all poles ad zeros are less that 1
- A z-domain rational fraction is minimum-phase iff no poles or zeros lie on or outside of the unit circcle

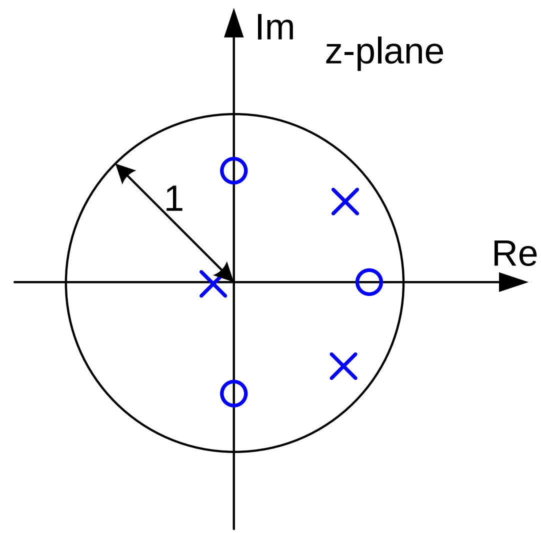
Filter Concepts and Terminology



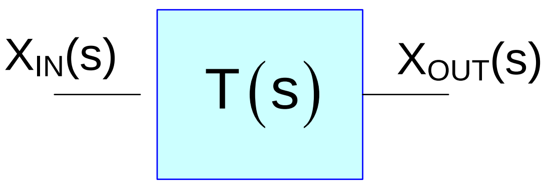


Pole-zero Plots





Filter Concepts and Terminology



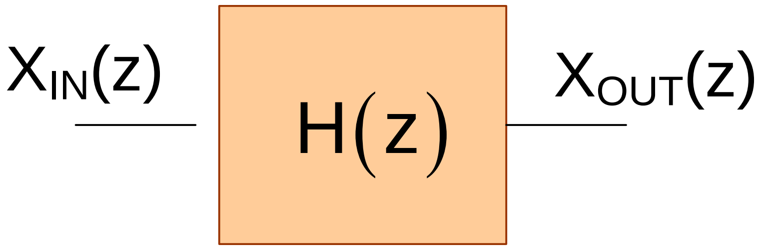
$$T(s) = \frac{\sum_{i=0}^{m} a_i s^i}{\sum_{i=0}^{n} b_i s^i} = \frac{N(s)}{D(s)}$$

- If T(s) is a rational fraction with poles and/or zeros in the RHP, then $\tilde{T}(s)$ obtained by reflecting all RHP roots around the imaginary axis back into the LHP has the following properties

a) minimum phase
b) stable
c) $\left|\tilde{T}(s)\right|_{s=j\omega} = \left|T(s)\right|_{s=j\omega}$ for all ω

Note the phase of T(s) and $\tilde{T}(s)$ will differ

Filter Concepts and Terminology



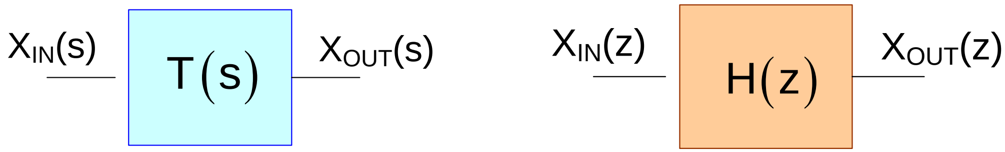
$$H(z) = \frac{\sum_{i=0}^{m} a_i z^i}{\sum_{i=0}^{n} b_i z^i} = \frac{N(z)}{D(z)}$$

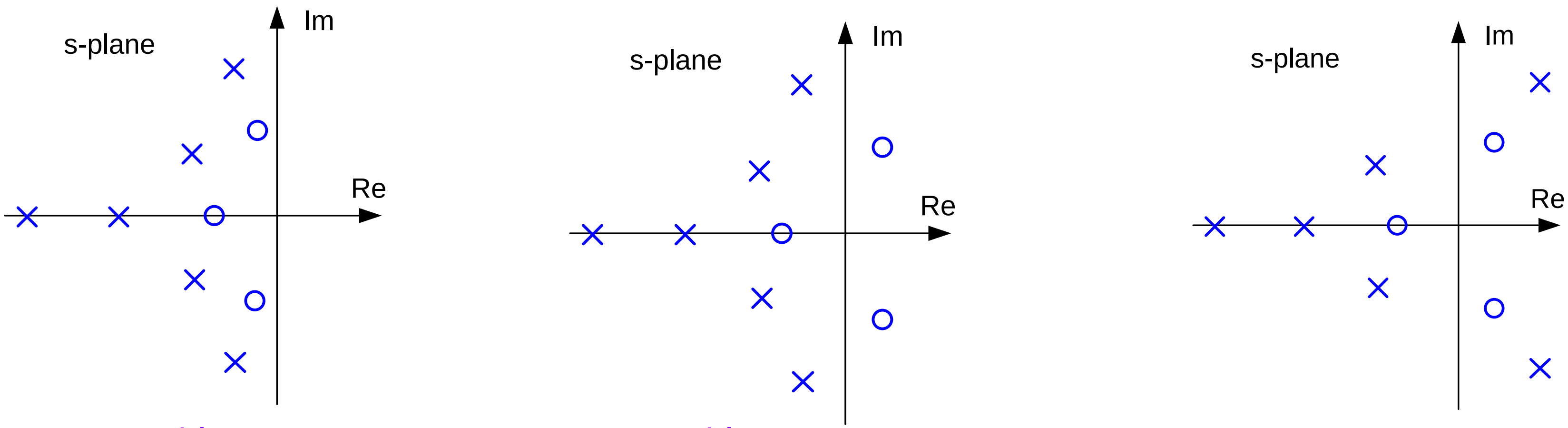
If H(z) is a rational fraction with poles and/or zeros outside the unit circle, then $\tilde{H}(z)$ obtained by reflecting all roots outside the unit circle back into the unit circle by the complex conguate reciprocal reflection and then scaling the transfer function by the magnitude of the reciprocal of the root has the following properties

a) minimum phase
b) stable
c) $\left|\tilde{H}(z)\right|_{z=e^{j\omega T}} = \left|H(z)\right|_{z=e^{j\omega T}}$ for all ω

Note the phase of H(z) and $\tilde{H}(z)$ will differ

Filter Concepts and Terminology

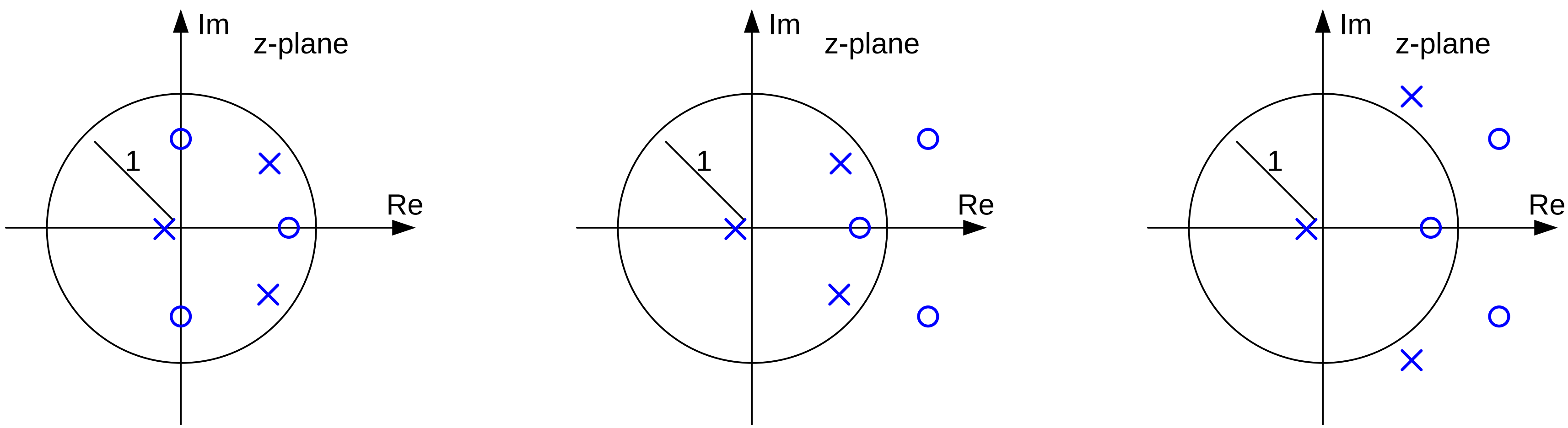




- Stable
- Minimum Phase

- Stable
- Not minimum Phase

- Not stable
- Not minimum Phase



Example: Non-minimum Phase Transfer Function

$$T(s) = \frac{s-1}{s+1}$$

$$\left|T(j\omega)\right| = \sqrt{\frac{\omega^2+(-1)^2}{\omega^2+1^2}}^{2\ 2} = 1$$

$$\angle T(j\omega) = \frac{\tan^{-1}\left(\frac{\omega}{-1}\right)}{\tan^{-1}\left(\frac{\omega}{1}\right)}$$

Beware that arctan function in multi-valued and in CAD tools give “a” principle value that may or may not consider the quadrant of the two arguements

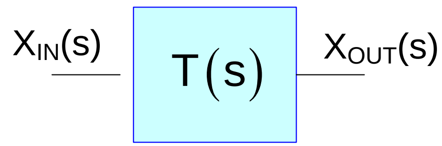
Example: Non-minimum Phase Transfer Function

$$T(s) = \frac{s^2 - as + 1}{s^2 + as + 1}$$

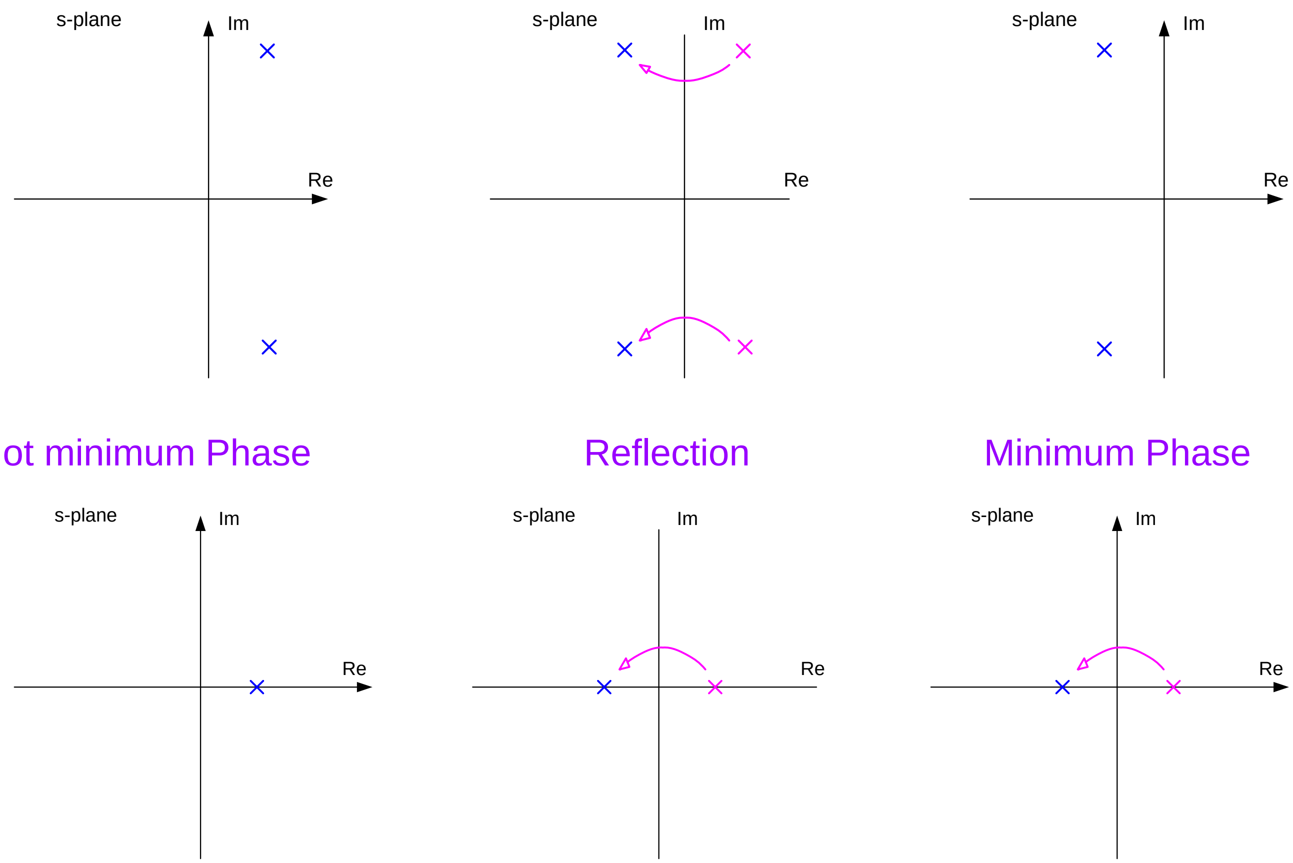
$$\left|T(j\omega)\right| = \sqrt{\frac{\left(1-\omega^2\right)^2 + a^2\omega^2}{\left(1-\omega^2\right)^2 + (-a)^2\,\omega^2}} = 1$$

$$\angle T(j\omega) = \frac{\tan^{-1}\left(\frac{-a\omega}{1-\omega^2}\right)}{\tan^{-1}\left(\frac{a\omega}{1-\omega^2}\right)}$$

Filter Concepts and Terminology

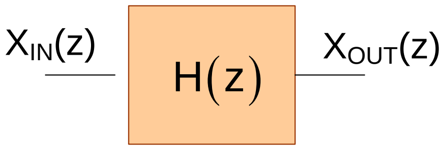


Reflecting poles and zeros to maintain stability or establish minimum phase



Note: magnitude of real part is preserved in reflection, imaginary part remains unchanged

Filter Concepts and Terminology



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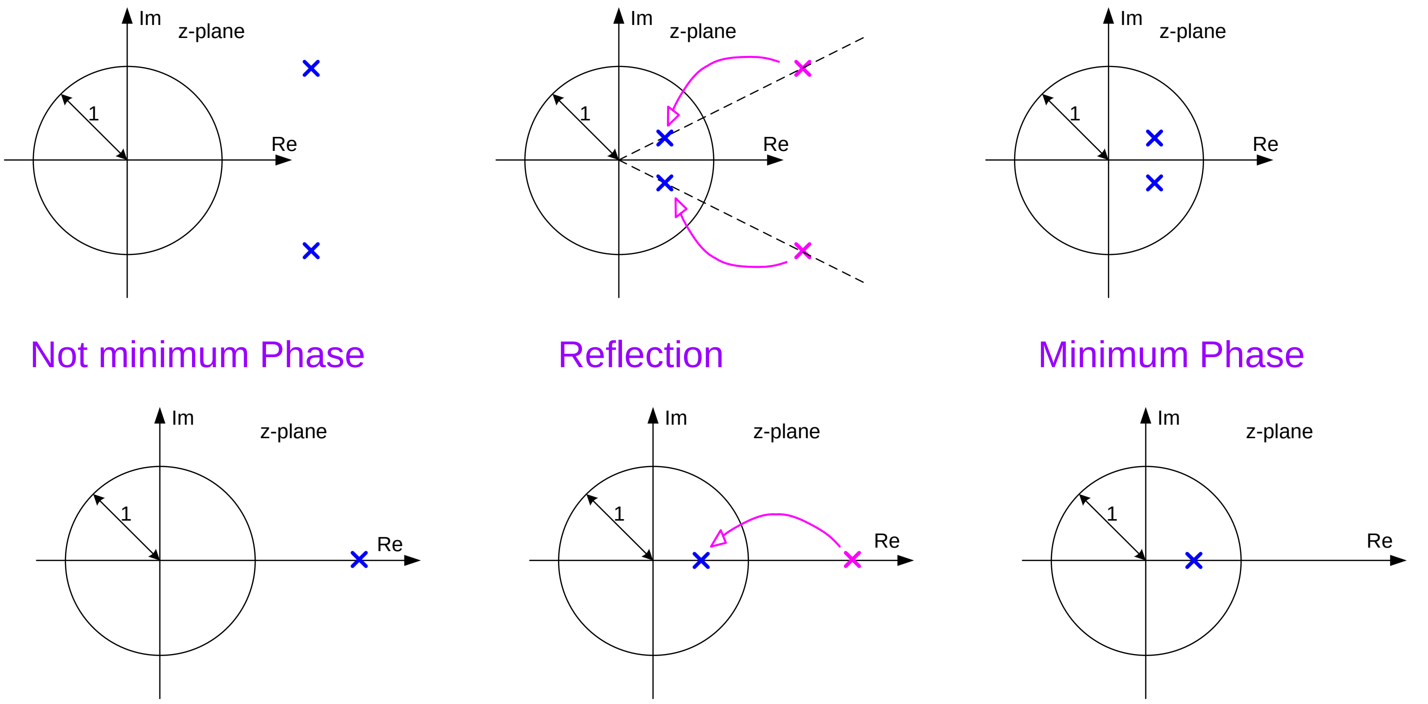

Note: complex conjugate reciprocal reflection maintains angle but magnitude of reflected root is the reciprocal of the magnitude of the original root

End of Lecture 3