Integrator Design

Current-Mode Integrators

s-domain to z-domain mappings
How can integrator performance be improved?

- Better op amps
- Better Integrator Architectures

How can the performance of integrator structures be compared?

Need metric for comparing integrator performance
Are there other integrators in the basic classes that have been considered?

**Miller Inverting**

\[ A_V(s) = - \frac{1}{RC_s} \]

**High-Q Inverting**

\[ A_V(s) = - \frac{1}{RC_s} \]

**Cascaded Inverting**

\[ A_V(s) = - \frac{1}{RC_s} \]

**Zero Second Derivative Inverting**

\[ A_V(s) = - \frac{1}{RC_s} \]

**Zero Sensitivity Inverting**

\[ A_V(s) = - \frac{1}{RC_s} \]

Review from last time
Are there other integrators in the basic classes that have been considered?

- **Miller Noninverting**
  
  \[ A_V(s) = \frac{1}{RC_s} \]

- **Modified High-Q Noninverting**
  
  \[ A_V(s) = \frac{1}{RC_s} \]

- **Phase Lead Integrator**
  
  \[ A_V(s) = \frac{1}{RC_s} \]

- **High-Q NonInverting**
  
  \[ A_V(s) = \frac{1}{RC_s} \]

- **Zero Sensitivity Noninverting**
  
  \[ A_V(s) = \frac{1}{RC_s} \]

- **Modified Miller Noninverting**
  
  \[ A_V(s) = \frac{1}{RC_s} \]
Are there other integrators in the basic classes that have been considered?

Miller Noninverting

\[ AV(s) = \frac{1}{RCs} \]

Zero Sensitivity Noninverting

\[ AV(s) = \frac{2}{RCs} \]

If \( R_1 = R_2 \) and \( R_3 = R_4 \)

(note this has a grounded integrating capacitor!)

Review from last time
Review from last time

De Boo Integrator

Howland Current Source

\[ A_V(s) = \frac{1}{s} \left[ \frac{1}{R_1 C} \left( 1 + \frac{R_{22}}{R_{11}} \right) \right] \]

\[ A_V(s) = -\frac{1}{s} \left[ \frac{R_B}{R_A R_1 C} \left( 1 + \frac{R_{22}}{R_{11}} \right) \right] \]
How can the performance of an integrator be characterized and how can integrators be compared?

Consider Ideal Integrator Gain Function

\[ A_V(s) = \frac{l_0}{s} \]
\[ A_V(j\omega) = \frac{l_0}{j\omega} \]

Ideal Integrator

Consider a nonideal integrator Gain Function

\[ A_V(s) = \frac{\alpha l_0}{s+\alpha} A_{OO}(s) \]

Nonideal Integrator

Key characteristics of an ideal integrator:

- Magnitude of the gain at \( l_0 = 1 \)
- Phase of integrator always 90°
- Gain decreases with \( 1/\omega \)

Are any of these properties more critical than others?

Key property of ideal integrator is a phase shift of 90° at frequencies around \( l_0 \)!
How can the performance of an integrator be characterized and how can integrators be compared?

Express $A_V(j\omega)$ as

$$A_V(j\omega) = \frac{\pm 1}{R(\omega) + jX(\omega)}$$

where $R(\omega)$ and $X(\omega)$ are real and represent the real and imaginary parts of the denominator respectively.

$$\text{Phase} = -\tan^{-1}\left(\frac{X(\omega)}{R(\omega)}\right)$$

Ideally $R(\omega) = 0$

Definition: The Integrator Q factor is the ratio of the imaginary part of the denominator to the real part of the denominator

$$Q_{\text{INT}} = \left(\frac{X(\omega)}{R(\omega)}\right)$$

Typically most interested in $Q_{\text{INT}}$ at the nominal unity gain frequency of the integrator.
How can the performance of an integrator be characterized and how can integrators be compared?

Express $A_V(j\omega)$ as

$$V(j\omega) = \frac{\pm 1}{R(\omega) + jX(\omega)}$$

Lead/Lag Characteristics for Inverting Integrators (inverting gain at $\omega_0$ should be 1 at angle of 90°)

For Phase Lag Integrators, $R(\omega)$ is negative
For Phase Lead integrators, $R(\omega)$ is positive
How can the performance of an integrator be characterized and how can integrators be compared?

Lead/Lag Characteristics for Inverting Integrators

\[ I_V(j\omega) = \frac{-1}{R(\omega) + jX(\omega)} \]

For Phase Lag Integrators, \( R(\omega) \) and \( X(\omega) \) have opposite signs. For Phase Lead integrators, \( R(\omega) \) and \( X(\omega) \) have the same sign. Phase shift ideally 90°.

Lead/Lag Characteristics for Noninverting Integrators

(Noninverting gain at \( \omega_0 \) should be 1 at angle of -90°)

\[ I_V(j\omega) = \frac{1}{R(\omega) + jX(\omega)} \]

For Phase Lag Integrators, \( R(\omega) \) and \( X(\omega) \) have opposite signs. For Phase Lead integrators, \( R(\omega) \) and \( X(\omega) \) have the same sign. Phase shift ideally 270°.
Consider Miller Inverting Integrator

\[ V_{IN} \quad \text{R} \quad \text{C} \quad A(s) = \frac{1}{\tau s} \]

\[ V_{OUT} \]

\[
A_V(s) = \frac{-1}{RCs + \tau s(1 + RCs)}
\]

\[
A_V(s) = \frac{-1}{RCj\omega + \tau j\omega(1 + RCj\omega)}
\]

\[
A_V(s) = \frac{-1}{-\tau \omega^2 RC + j(\omega[RC + \tau])}
\]

Normalizing by \( \omega_n = \omega RC \) and \( \tau_n = \tau / RC = I_{on}/GB \)

\[
A_V(s) = \frac{-1}{-\tau_n \omega_n^2 + j(\omega_n[1 + \tau_n])}
\]

Observe this integrator has excess phase shift (more than 90° in the denominator) at all frequencies.
Integrator Q Factor

\[ A_V(j\omega) = \frac{\pm 1}{R(\omega) + jX(\omega)} \]

\[ Q_{\text{INT}} = \left( \frac{X(\omega)}{R(\omega)} \right) \]

Consider Miller Inverting Integrator

A (s) = \frac{1}{\tau s}

Since the phase is less than 90°, the Miller Inverting Integrator is a Phase Lag Integrator and \( Q_{\text{INT}} \) is negative

\[ A_V(s) = \frac{-1}{-\tau_n \omega_n^2 + j(\omega_n[1+\tau_n])} \]

\[ Q_{\text{INT}} = -\frac{\omega_n[1+\tau_n]}{\tau_n \omega_n^2} \approx -\frac{1}{\tau_n \omega_n} \]

\[ Q_{\text{INT}}\big|_{\omega_N=1} \approx -\frac{1}{\tau_n \omega_n}\big|_{\omega_N=1} \approx -\frac{1}{\tau_n} = -\frac{\text{GB}}{\omega_0} = -A(I_{0N}) = -A \]
Consider Miller Inverting Integrator

\[ AV(s) = \frac{-1}{RCs + \tau s (1 + RCs)} \]

\[ I_0 = \frac{1}{RC} \]

Poles at \( s = 0 \) and \( s = -I_0 (1 + GB/I_0) \approx -GB \)
Is the integrator Q factors simply a metric or does it have some other significance?

It can be shown that the pole Q for the TT Biquad can be approximated by

\[ Q_P \approx \frac{1}{\frac{1}{Q_{PN}} + \frac{1}{Q_{INT1}} + \frac{1}{Q_{INT2}}} \]

where \( Q_{INT1} \) and \( Q_{INT2} \) are evaluated at \( \omega = \omega_0 \)
Is the integrator $Q$ factors simply a metric or does it have some other significance?

It can be shown that the pole $Q$ for the TT Biquad can be approximated by

$$Q_P \approx \frac{1}{1 + \frac{1}{Q_{PN}} + \frac{1}{Q_{INT1}(\omega_0)} + \frac{1}{Q_{INT2}(\omega_0)}}$$

Similar expressions for other second-order biquads

Observe that the integrator $Q$ factors adversely affect the pole $Q$ of the filter

Observe that if $Q_{INT1}$ and $Q_{INT2}$ are of opposite signs and equal magnitudes, nonideal effects of integrator can cancel
What can be done to correct the phase problems of an integrator?

One thing that can help the Miller Integrator is phase-lead compensation. $R_x$ and $C_x$ will add phase-lead by introduction of a zero. $R_x$ and $C_x$ will be small components.
Consider Miller Noninverting Integrator

\[ A_V(s) = \frac{-1}{RCs + \tau s(1 + RCs)} \cdot \frac{-1}{1 + 2\tau s} \]

\[ A_V(j\omega) = \frac{1}{-3\tau RC\omega^2 + j[\omega RC(1 + 2\tau \omega)]} \]

\[ A_V(j\omega) \approx \frac{1}{-3\tau RC\omega^2 + j\omega RC} \]

Observe this integrator has excess phase shift (more than 90° in the denominator) at all frequencies

\[ Q_{\text{INT}} \approx \frac{\omega RC}{-3\tau RC\omega^2} = \frac{-1}{3\tau \omega} \]

\[ Q_{\text{INT}} \approx \frac{-1}{3\left(\frac{\omega}{GB}\right)} = \frac{-1}{3} |A(j\omega)| \]

Note: The Miller Noninverting Integrator has a modestly poorer \( Q_{\text{INT}} \) than the Miller Inverting Integrator
Example:

If $f_0=10\text{KHz}$, $GB=1\text{MHz}$, $Q_{\text{NOM}}=10$, estimate the pole $Q$ for the Tow-Thomas Biquad if the Miller Integrator and the Miller Noninverting Integrators are used. Also determine the relative degradation in performance due to each of the integrators.

$$Q_P \approx \frac{1}{\frac{1}{Q_{\text{PN}}} + \frac{1}{Q_{\text{INT1}}} + \frac{1}{Q_{\text{INT2}}}}$$
Example:

If $f_0 = 10\text{KHz}$, $GB = 1\text{MHz}$, $Q_{\text{NOM}} = 10$, estimate the pole $Q$ for the Tow-Thomas Biquad if the Miller Integrator and the Miller Noninverting Integrators are used. Also determine the relative degradation in performance due to each of the integrators.

$$Q_P \cong \frac{1}{1 + \frac{1}{Q_{\text{PN}}} + \frac{1}{Q_{\text{INT1}}} + \frac{1}{Q_{\text{INT2}}}}$$

$$Q_{\text{INT1}} = -A = -\frac{GB}{\omega} = -\frac{1\text{MHz}}{10\text{KHz}} = -100$$

$$Q_{\text{INT2}} \cong -\frac{1}{3} |A(j\omega)| = -\frac{1}{3} \left( \frac{GB}{\omega} \right) = -\frac{1\text{MHz}}{3 \cdot 10\text{KHz}} = -33$$

$$Q_P \cong \frac{1}{1 - 0.01 - 0.033} = 17.5$$

Note the nonideal integrators cause about a 75% shift in $Q_P$.

Note that 3 times as much of the shift is due to the noninverting integrator as is due to the inverting integrator!

Similar effects of the integrators will be seen on other filter structures.
Example:

If \( f_0 = 10\text{KHz} \), \( GB = 1\text{MHz} \), \( Q_{\text{NOM}} = 10 \), estimate the pole \( Q \) for the Tow-Thomas Biquad if the Miller Integrator and the Miller Noninverting Integrators are used. Also determine the relative degradation in performance due to each of the integrators.

\[
Q_P \approx \frac{1}{1 + \frac{1}{Q_{\text{PN}}} + \frac{1}{Q_{\text{INT1}}} + \frac{1}{Q_{\text{INT2}}}}
\]

\[
Q_{\text{INT1}} = -A = -\frac{GB}{\omega} = -\frac{1\text{MHz}}{10\text{KHz}} = -100
\]

\[
Q_{\text{INT2}} = -\frac{1}{3} |A(j\omega)| = -\frac{1}{3} \left( \frac{GB}{\omega} \right) = -\frac{1\text{MHz}}{3 \times 10\text{KHz}} = -33
\]

\[
Q_P \approx \frac{1}{1 - 0.01 - 0.033} = 17.5
\]

How can the problem be solved?

1. Compensate Integrator
2. Use better integrators
3. Use phase-lead and phase/lag pairs
Example:

If \( f_0 = 10\text{KHz} \), \( GB = 1\text{MHz} \), \( Q_{\text{NOM}} = 10 \), estimate the pole \( Q \) for the Tow-Thomas Biquad if the Miller Integrator and the Miller Noninverting Integrators are used. Also determine the relative degradation in performance due to each of the integrators.

\[
Q_P \approx \frac{1}{\frac{1}{Q_{\text{PN}}} + \frac{1}{Q_{\text{INT1}}} + \frac{1}{Q_{\text{INT2}}}}
\]

\( Q_{\text{INT1}} = -100 \quad Q_{\text{INT2}} = -33 \)

How can the problem be solved?

Phase Compensation of \( \text{INT1} \)

\[
A_V(s) = \frac{-(1+R_XCs)}{RCs+\tau s(1+[R+R_X]Cs)}
\]

\( Q_{\text{INT1}} = \frac{-GB}{\omega} \frac{1}{1-GB\cdot CR_X} \)

Pick \( R_X \) so that \( Q_{\text{INT1}} = 33 \) at \( \omega = 1/(RC) \)

Solving, obtain \( CR_X = 4/GB \)

Useful for hand calibration but not practical for volume production because of variability in components.
What are the integrator Q factors for other integrators that have been considered?

**Miller Inverting**

\[ V_{IN} \rightarrow R \rightarrow C \rightarrow V_{OUT} \]

\[ A_V(s) = -\frac{1}{RCs} \]

\[ Q_{INT} = -A \]

**High-Q Inverting**

\[ V_{IN} \rightarrow R \rightarrow C \rightarrow V_{OUT} \]

\[ A_V(s) = -\frac{1}{RCs} \]

\[ Q_{INT} = -A^2 \]

**Zero Second Derivative Inverting**

\[ V_{IN} \rightarrow R \rightarrow C \rightarrow V_{OUT} \]

\[ A_V(s) = -\frac{1}{RCs} \]

\[ Q_{INT} = -A^2 \]

**Cascaded Inverting**

\[ V_{IN} \rightarrow R \rightarrow C \rightarrow V_{OUT} \]

\[ A_V(s) = -\frac{1}{RCs} \]

\[ (\text{stability problems}) \]

**Zero Sensitivity Inverting**

\[ V_{IN} \rightarrow R \rightarrow C \rightarrow V_{OUT} \]

\[ A_V(s) = -\frac{1}{RCs} \]

\[ Q_{INT} = -A^2 \]
What are the integrator Q factors for other integrators that have been considered?

**Miller Inverting**

\[ A_V(s) = - \frac{1}{RC_s} \]

\[ Q_{INT} = -A \]

**Howland Current Source**

\[ A_V(s) = \frac{1 + \frac{R_2}{R_1}}{R_1C_s + \tau_1s\left(1 + \frac{R_2}{R_1}\right)\left(1 + \frac{R_1}{R_2} + sC_R_1\right)} \]

\[ Q_{INT} = -\frac{A}{1 + \frac{R_2}{R_1}} \]

If \( R_1 = R_2 = R \)

\[ A_V(s) = \frac{2}{RC_s} \]

\[ Q_{INT} = -\frac{A}{2} \]
What are the integrator Q factors for other integrators that have been considered?

**Miller Noninverting**

\[ Q_{\text{INT}} = -\frac{1}{3}|A(j\omega)| \]

\[ A_V(s) = \frac{1}{RC_s} \]

**Phase Lead Integrator**

\[ Q_{\text{INT}} = |A(j\omega)| \]

\[ A_V(s) = \frac{1}{RC_s} \]

**Modified High-Q Noninverting**

\[ Q_{\text{INT}} = -|A(j\omega)|^3 \]

\[ A_V(s) = \frac{1}{RC_s} \]

**High-Q NonInverting**

\[ Q_{\text{INT}} = -|A(j\omega)| \]

\[ A_V(s) = \frac{1}{RC_s} \]

**Zero Sensitivity Noninverting**

\[ Q_{\text{INT}} = -|A(j\omega)|^3 \]

\[ A_V(s) = \frac{1}{RC_s} \]

**Modified Miller Noninverting**

\[ Q_{\text{INT}} = -|A(j\omega)| \]

\[ A_V(s) = \frac{1}{RC_s} \]
Improving Integrator Performance:

1. Compensate Integrator
2. Use better integrators
3. Use phase-lead and phase/lag pairs

- These methods all provide some improvements in integrator performance
- But both magnitude and phase of an integrator are important so focusing only on integrator Q factor only may only improve performance to a certain level
- In higher-order integrator-based filters, the linearity in $1/\omega$ of the integrator gain is also important. The integrator magnitude and Q factor at $\omega_0$ ignore the frequency nonlinearity that may occur in the $1/\omega$ dependence
- There is little in the literature on improving the performance of integrated integrators within a basic class. At high frequencies where the active device performance degrades, particularly in finer-feature processes, there may be some benefits that can be derived from architectural modifications along the line of those discussed in this lecture
Selected Current Mode, Transresistance Mode, and Transconductance Mode Integrators
Current-Mode Filters

Basic Concepts of Benefits of Current-Mode Filters:

• Large voltage swings difficult to maintain in integrated processes because of linearity concerns
• Large voltage swings slow a circuit down because of time required to charge capacitors
• Voltage swings can be very small when currents change
• Current swings are not inherently limited in integrated circuits (only voltage swings)
• With low voltage swings, current-mode circuits should dissipate little power

\[ T(s) = \frac{I_{OUT}}{I_{IN}} = \frac{-I_0^2}{s^2 + \alpha I_0 s + I_0^2} \]
Concept of Current-Mode Filters is Somewhat Recent:

Key paper that generated interest in current-mode filters (ISCAS 1989):

Switched currents - a new technique for analog sampled-data signal processing

This paper is one of the most significant contributions that has ever come from ISCAS
End of Lecture 30