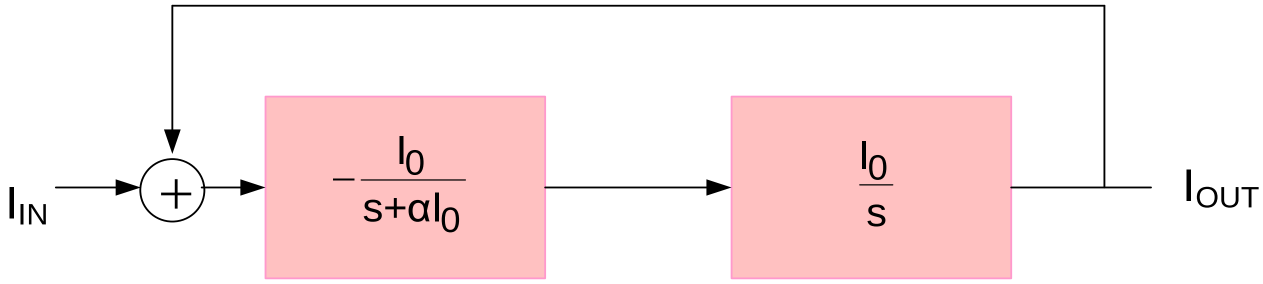
EE 508
Lecture 31

Switched Current Filters

Review from last time

Current-Mode Filters



$$T(s)=\frac{I_{OUT}}{I_{IN}}=\frac{-I_0^2}{s^2+\alpha I_0 s+I_0^2}$$

Basic Concepts of Benefits of Current-Mode Filters:

- Large voltage swings difficult to maintain in integrated processes because of linearity concerns
- Large voltage swings slow a circuit down because of time required to charge capacitors
- Voltage swings can be very small when currents change
- Current swings are not inherently limited in integrated circuits (only voltage swings)
- With low voltage swings, current-mode circuits should dissipate little power

Review from last time

Current-Mode Filters

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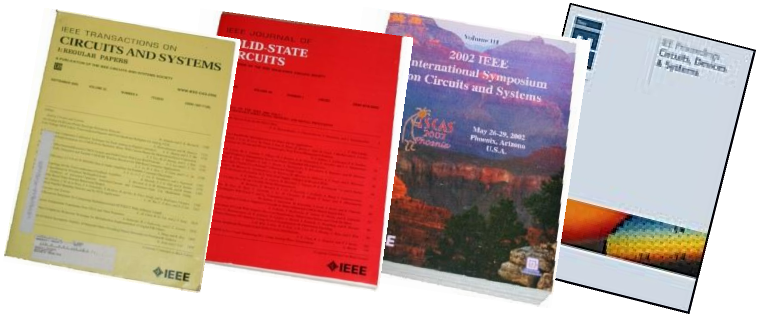

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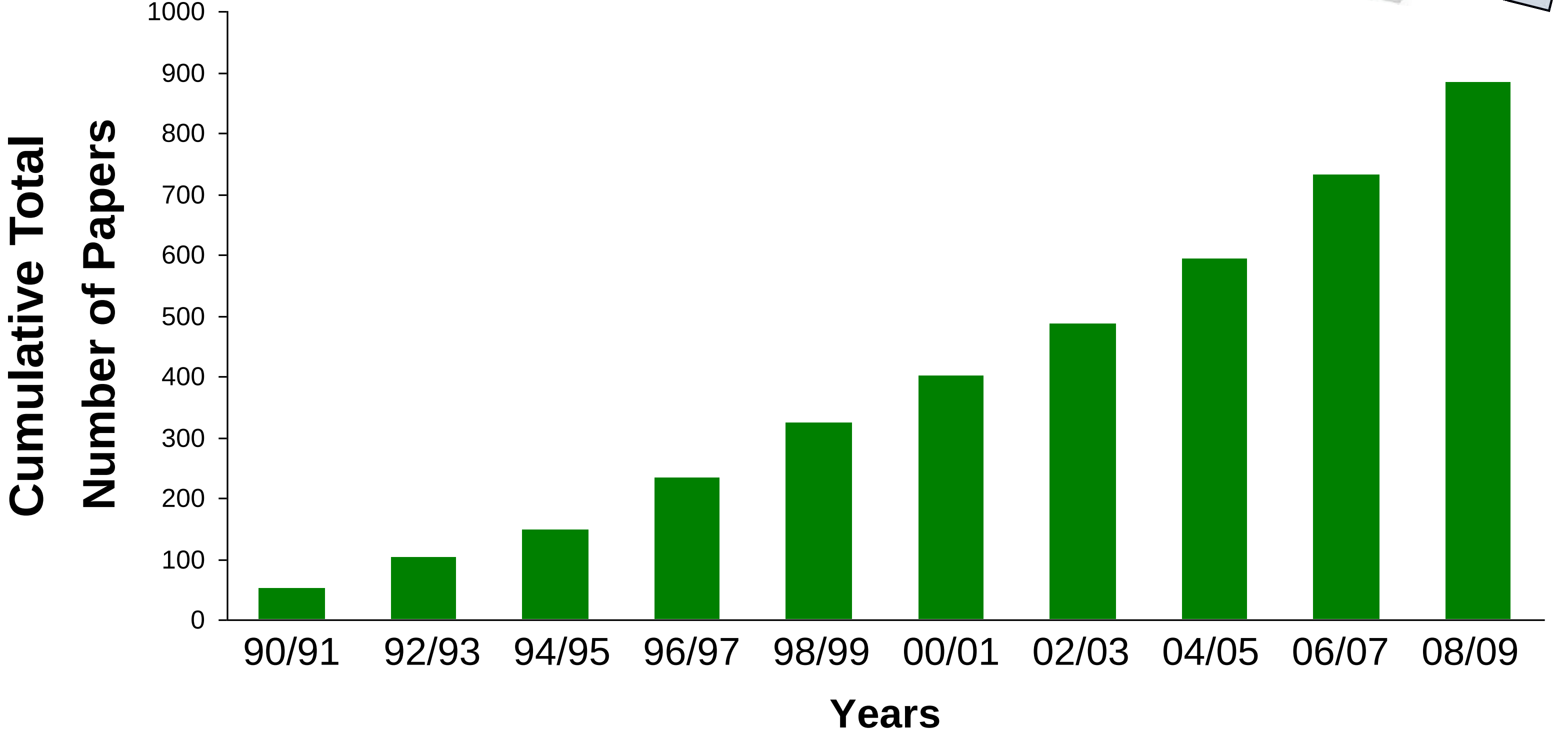


Steady growth in research in the area since 1990 and publication rate is growing with time !!

Review from last time

Current-Mode Filters

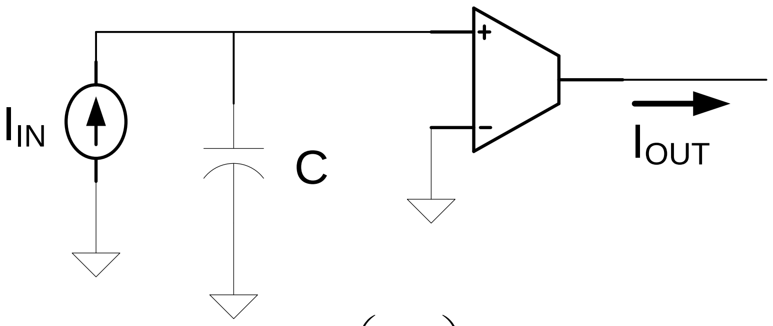
The Conventional Wisdom:

- Current-Mode circuits operate at higher-frequencies than voltage-mode counterparts
- Current-Mode circuits operate at lower supply voltages and lower power levels than voltage-mode counterparts
- Current-Mode circuits are simpler than voltage-mode counterparts
- Current-Mode circuits offer better linearity than voltage-mode counterparts

This represents four really significant benefits of current-mode circuits!

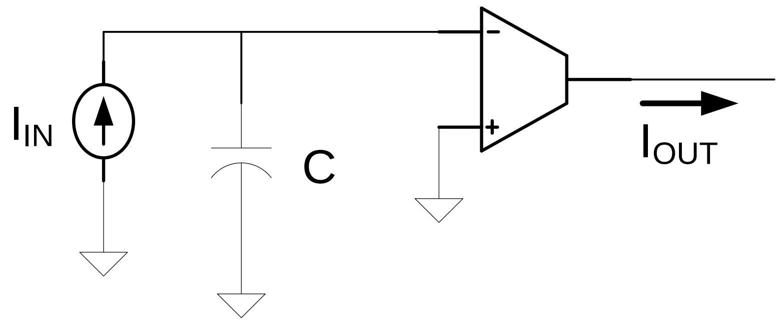
Some Current-Mode Integrators

OTA-C



$$I_{OUT} = \left(\frac{g_m}{Cs}\right) I_{IN}$$

Noninverting



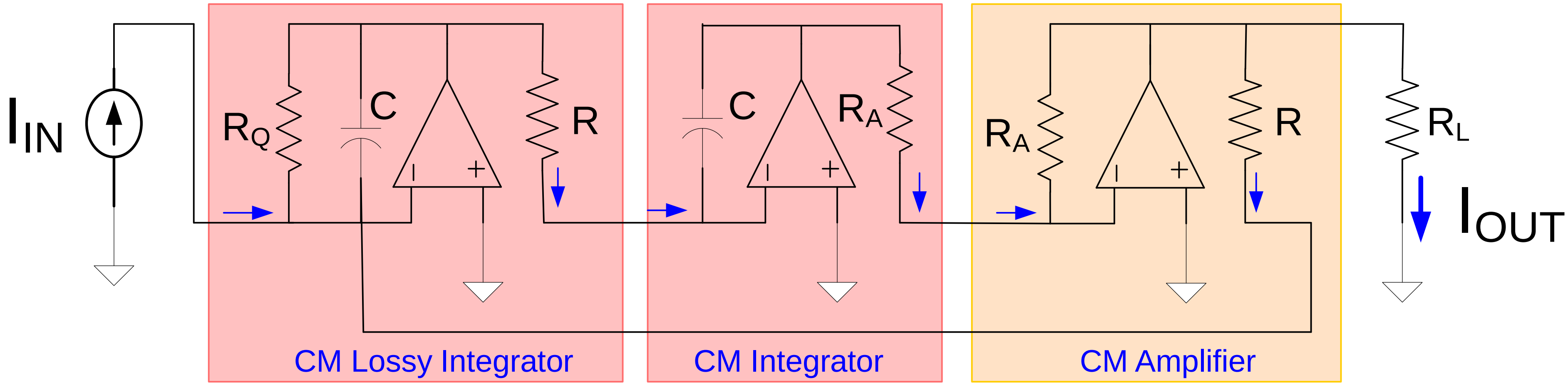
$$I_{OUT} = \left(\frac{-g_m}{Cs}\right) I_{IN}$$

Inverting

- Summing inputs really easy to obtain
- Loss is easy to add
- Same component count as voltage-mode integrators
- Many argue that since only interested in currents, can operate at lower voltages and higher frequencies

Review from last time

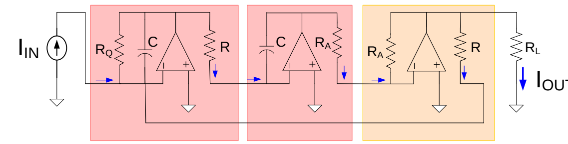
Current-Mode Two Integrator Loop

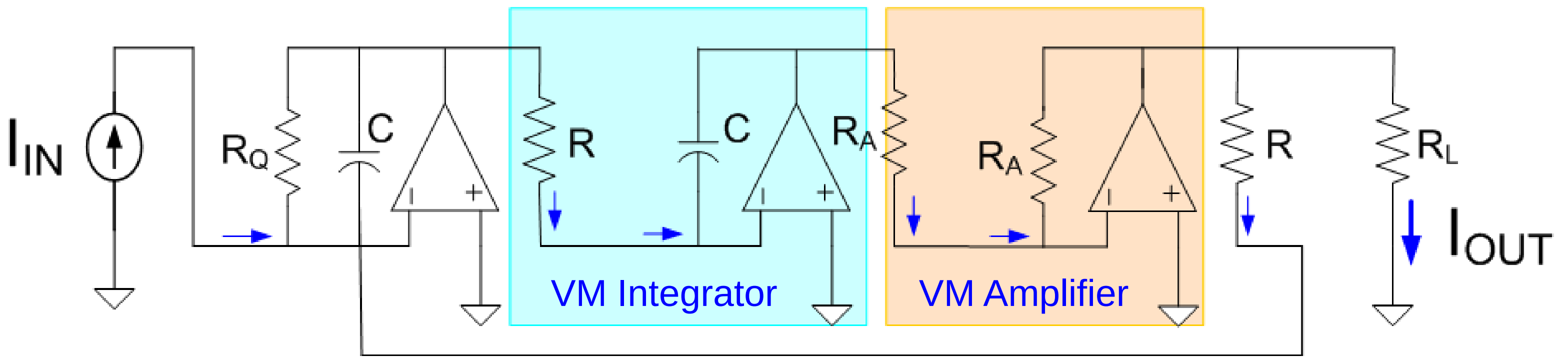


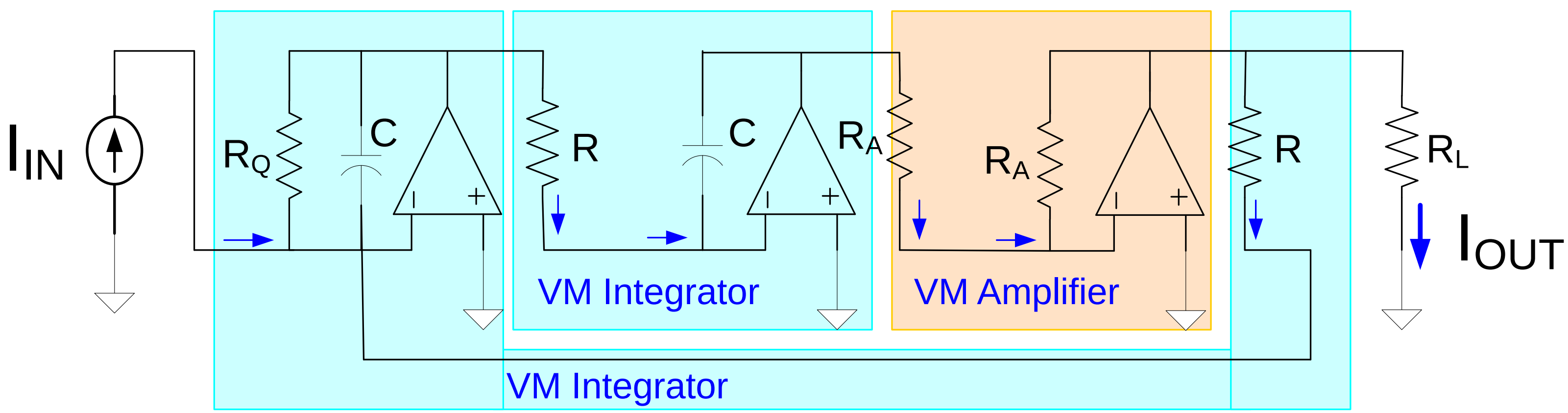
- Straightforward implementation of the two-integrator loop
- Simple structure

Current-Mode Two Integrator Loop

An Observation:







This circuit is identical to another one with two voltage-mode integrators and a voltage-mode amplifier !

Observation

- Many papers have appeared that tout the performance advantages of current-mode circuits
- In all of the current-mode papers that this instructor has seen, no attempt is made to provide a quantitative comparison of the key performance features of current-mode circuits with voltage-mode counterparts
- All justifications of the advantages of the current-mode circuits this instructor has seen are based upon qualitative statements

Observations (cont.)

- It appears easy to get papers published that have the term “current-mode” in the title
- Over 900 papers have been published in IEEE forums alone !
- Some of the “current-mode” filters published perform better than other “voltage-mode” filters that have been published
- We are still waiting for even one author to quantitatively show that current-mode filters offer even one of the claimed four advantages over their voltage-mode counterparts

Will return to a discussion of Current-Mode filters later

Switched-Current Filters

Basic idea introduced by Hughes and Bird at ISCAS 1989

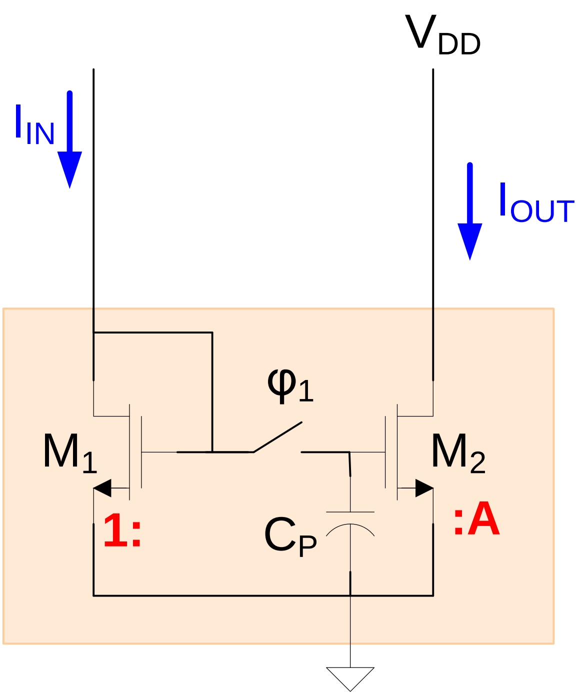
Switched currents-a new technique for analog sampled-data signal processing
JB Hughes, NC Bird... - Circuits and Systems, 1989 ..., 2002 - ieeexplore.ieee.org
NTRODUCTION The enormous complexity available in state-of-the-art CMOS processing has made possible the integration of complete systems, including both digital and analog signal processing functions, within the same chip Through the last decade, the **switched** capacitor technique **...**
Cited by 151 - Related articles

Technique introduced directly in the z-domain

Review from last time

Switched-Current Filters

Basic idea introduced by Hughes and Bird at ISCAS 1989



$$I_{OUT}=\begin{cases} AI_{IN}(t) & \text{for } \varphi_1 \text{ closed} \\ AI_{IN}(T_{SW}) & \text{for } \varphi_1 \text{ open} \end{cases}$$

If Φ_1 is a periodic signal and if I_{IN} is also appropriately clocked, the input/output currents of this circuit can be represented with the difference equation

$$I_{OUT}(nT) = AI_{IN}(nT-T)$$

This switched mirror becomes a delay element

"Gain" A is that of a current mirror

A can be accurately controlled

Circuit is small and very fast

Concept can be extended to implement arbitrary difference equation

Difference equation characterizes filter H(z)

Need only current mirrors and switches

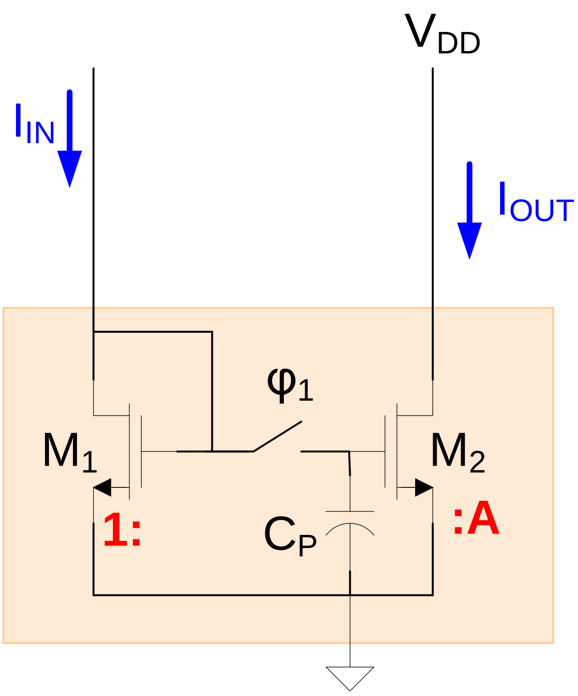
Truly a "current-mode" circuit

Review from last time

Switched-Current Filters

Basic idea introduced by Hughes and Bird at ISCAS 1989

$$I_{OUT}(nT) = AI_{IN}(nT-T)$$



Cp is parasitic gate capacitance on M_2

Very low power dissipation

Potential to operate at very low voltages

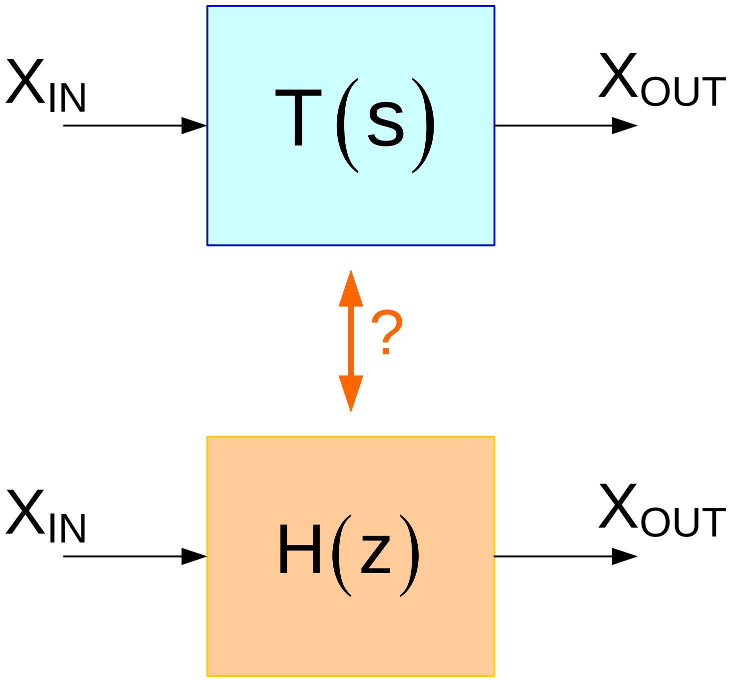
Potential for accuracy of a SC circuit at both low and high frequencies but without the Op Amp and large C ratios

Neither capacitor or resistor values needed to do filtering!

A completely new approach to designing filters that offers potential for overcoming most of the problems plaguing filter designers for decades !

Before developing Switch-Current concept, need to review background information in s to z domain transformations

s-domain to z-domain transformations

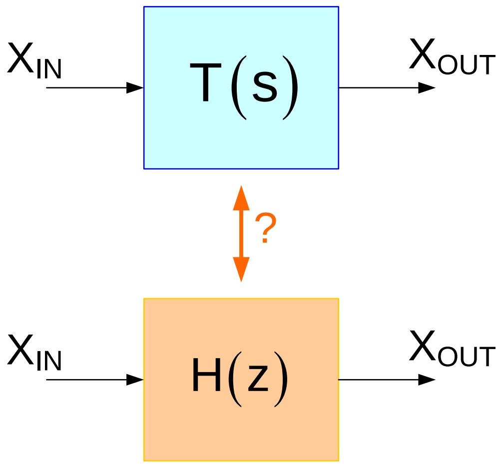


For a given T(s) would like to obtain a function H(z) or for a given H(z) would like to obtain a T(s) such that preserves the magnitude and phase response

Mathematically, would like to obtain the relationship:

$$T(s)\Big|_{s=j\omega} = H(z)\Big|_{z=e^{j\omega T}}$$

s-domain to z-domain transformations



want: $$T(s)\big|_{s=j\omega} = H(z)\big|_{z=e^{j\omega T}}$$

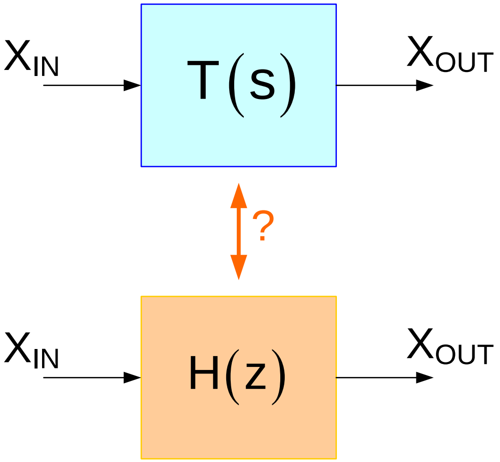
equivalently, want:

$$T(s) = H(z)\big|_{z=e^{sT}}$$

But if this were to happen, T(s) would not be a rational fraction in s with real coeff.

Thus, it is impossible to obtain this mapping between T(s) and H(z)

s-domain to z-domain transformations



goal: $$T(s) = H(z)\Big|_{z=e^{sT}}$$

If can't achieve this goal, would like to map imaginary axis to unit circle and map stable filters to stable filters

consider: $$z = e^{sT}$$

Case 1:

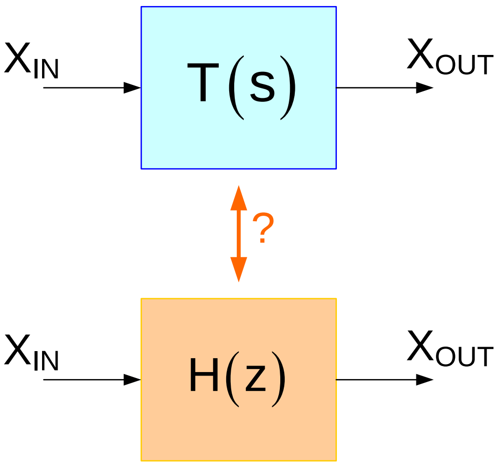
$$z = e^{sT} \simeq \sum_{i=0}^{\infty} \frac{1}{i!}(sT)^i$$

$$z = \sum_{i=0}^{\infty} \frac{1}{i!}(sT)^i \simeq 1 + sT$$

$$s = \frac{z - 1}{T}$$

Termed the Forward Euler transformation

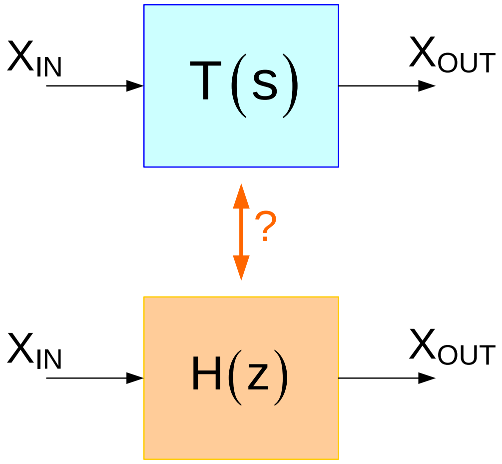
s-domain to z-domain transformations



$$s = \frac{z - 1}{T}$$ Forward Euler transformation

- Doesn’t map imaginary axis in s-plane to unit circle in z-plane
- Doesn’t <u>guarantee</u> stable filter will map to stable filter
- But mapping may give stable filter with good frequency response

s-domain to z-domain transformations



consider: $z = e^{sT}$

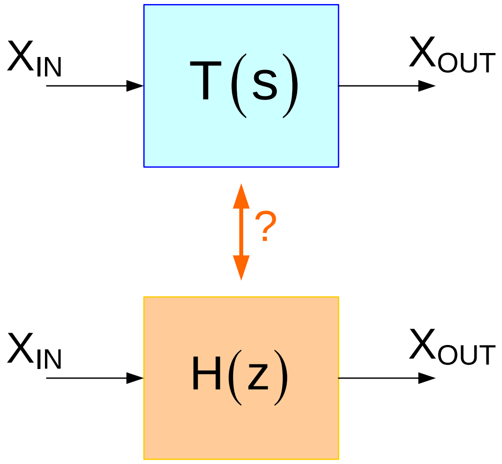
Case 2: $z = e^{sT} = \frac{1}{e^{-sT}} = \frac{1}{\sum_{i=0}^{\infty} \frac{1}{i!}(-sT)^i} \simeq \frac{1}{1-sT}$

$$z \simeq \frac{1}{1-sT}$$

$$s = \left(\frac{1}{T}\right)\frac{z-1}{z}$$

Termed the Backward Euler transformation

s-domain to z-domain transformations

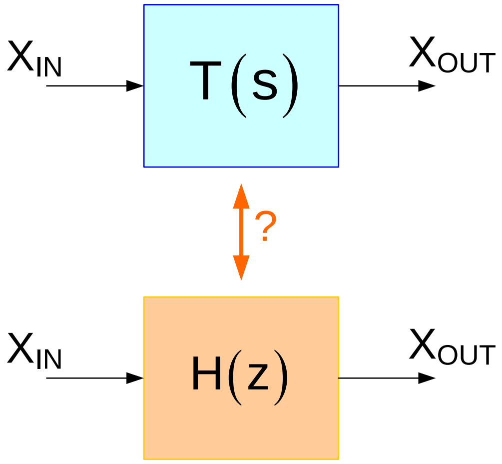


$$s = \left(\frac{1}{T}\right)\frac{z-1}{z}$$

Backward Euler transformation

- Doesn’t map imaginary axis in s-plane to unit circle in z-plane
- Does <u>guarantee</u> stable filter will map to stable filter

s-domain to z-domain transformations



consider: $z = e^{sT}$

Case 3:

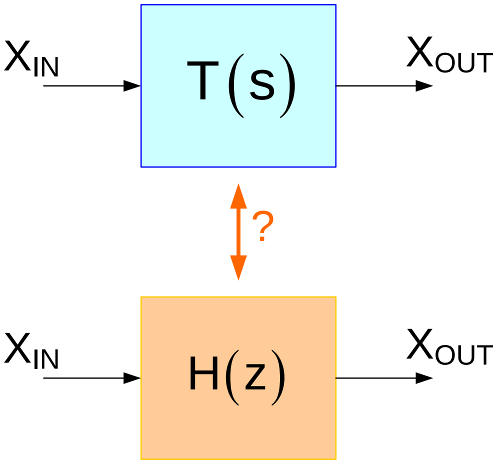
$$z = e^{sT} = \frac{e^{s\frac{T}{2}} 1}{e^{-s\frac{T}{2}}} = \frac{\sum_{i=0}^{\infty} \frac{1}{i!}\left(s\frac{T}{2}\right)^{i}}{\sum_{i=0}^{\infty} \frac{1}{i!}\left(-s\frac{T}{2}\right)^{i}} \simeq \frac{1+s\frac{T}{2}}{1-s\frac{T}{2}}$$

solving for s, obtain

$$s = \frac{2}{T} \bullet \frac{z-1}{z+1}$$

Termed the Bilinear z transformation

s-domain to z-domain transformations

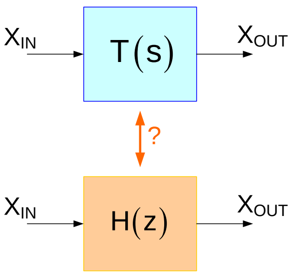


$$s = \frac{2}{T} \bullet \frac{z-1}{z+1}$$

Bilinear z transformation

- Maps imaginary axis in s-plane to unit circle in z-plane (preserves shape, distorts frequency axis)
- Does <u>guarantee</u> stable filter will map to stable filter
- Bilinear z transformation is widely used

s-domain to z-domain transformations



consider: $z = e^{sT}$

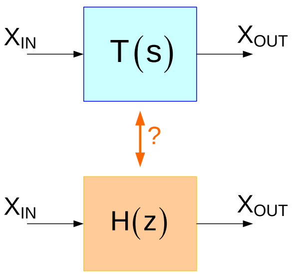
Three Popular Transformations

$$s = \frac{z - 1}{T}$$ Forward Euler $$s = \frac{1 - z^{-1}}{Tz^{-1}}$$

$$s = \frac{z-1}{Tz}$$ Backward Euler $$s = \frac{1-z^{-1}}{T}$$

$$s = \frac{2}{T} \bullet \frac{z-1}{z+1}$$ Bilinear z transform $$s = \frac{2}{T} \bullet \frac{1-z^{-1}}{1+z^{-1}}$$

s-domain to z-domain transformations

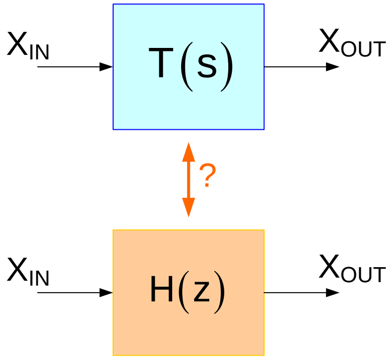


Three Popular Transformations

$s=\frac{z-1}{T}$	Forward Euler	$s=\frac{1-z^{-1}}{Tz^{-1}}$
$s=\frac{z-1}{Tz}$	Backward Euler	$s=\frac{1-z^{-1}}{T}$
$s=\frac{2}{T}\bullet\frac{z-1}{z+1}$	Bilinear z transform	$s=\frac{2}{T}\bullet\frac{1-z^{-1}}{1+z^{-1}}$

- Transformations of standard approximations in s-domain are the corresponding transformations in the z-domain
- Transformations are not unique
- Transformations cause warping of the imaginary axis and may cause change in basic shape
- Transformations do not necessarily guarantee stability
- These transformations preserve order

z-domain integrators



$$T(s) = \frac{I_0}{s}$$

Some z-domain integrators

$$H(z) = \begin{cases} \dfrac{TI_0}{z-1} & \text{Forward Euler} \\ \dfrac{I_0 Tz}{z-1} & \text{Backward Euler} \\ \dfrac{TI_0}{2}\left(\dfrac{z+1}{z-1}\right) & \text{Bilinear z} \end{cases}$$

Three Popular Transformations

$s = \dfrac{z-1}{T}$	Forward Euler	$s = \dfrac{1-z^{-1}}{Tz^{-1}}$
$s = \dfrac{z-1}{Tz}$	Backward Euler	$s = \dfrac{1-z^{-1}}{T}$
$s = \dfrac{2}{T} \bullet \dfrac{z-1}{z+1}$	Bilinear z transform	$s = \dfrac{2}{T} \bullet \dfrac{1-z^{-1}}{1+z^{-1}}$

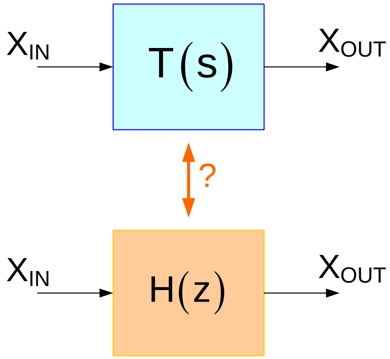
Corresponding difference equations:

$$V_{OUT}(nT+T) = TI_0 V_{IN}(nT) + V_{OUT}(nT) \quad \text{Forward Euler}$$

$$V_{OUT}(nT+T) = I_0 T V_{IN}(nT+T) + V_{OUT}(nT) \quad \text{Backward Euler}$$

$$V_{OUT}(nT+T) = \frac{TI_0}{2}\left(V_{IN}(nT+T) + V_{IN}(nT)\right) + V_{OUT}(nT) \quad \text{Bilinear z}$$

z-domain lossy integrators



$$T(s) = \frac{I_0}{s+\alpha}$$

Three Popular Transformations

$s = \frac{z-1}{T}$	Forward Euler	$s = \frac{1-z^{-1}}{Tz^{-1}}$
$s = \frac{z-1}{Tz}$	Backward Euler	$s = \frac{1-z^{-1}}{T}$
$s = \frac{2}{T} \bullet \frac{z-1}{z+1}$	Bilinear z transform	$s = \frac{2}{T} \bullet \frac{1-z^{-1}}{1+z^{-1}}$

Some z-domain lossy integrators

$$H(z) = \begin{cases} \dfrac{TI_0}{z-1+\alpha T} & \text{Forward Euler} \\ \dfrac{I_0 Tz}{z(1+\alpha T)-1} & \text{Backward Euler} \\ \dfrac{TI_0}{2}\left(\dfrac{z+1}{z\left(1+\dfrac{\alpha T}{2}\right)+\left(\dfrac{\alpha T}{2}-1\right)}\right) & \text{Bilinear z} \end{cases}$$

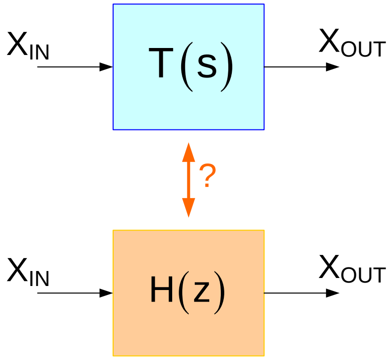
Corresponding difference equations:

$$V_{OUT}(nT+T) = TI_0 V_{IN}(nT) + [1-\alpha T] V_{OUT}(nT)$$ Forward Euler

$$(1+\alpha T) V_{OUT}(nT+T) = I_0 T V_{IN}(nT+T) + V_{OUT}(nT)$$ Backward Euler

$$\left(1+\frac{\alpha T}{2}\right) V_{OUT}(nT+T) = \frac{TI_0}{2}\left(V_{IN}(nT+T) + V_{IN}(nT)\right) + \left[1-\frac{\alpha T}{2}\right] V_{OUT}(nT)$$ Bilinear z

z-domain lossy integrators



Some z-domain lossy integrators

$$T(s) = \frac{I_0}{s+\alpha}$$

Functional Form

$$H(z) = \begin{cases} \dfrac{TI_0}{z-1+\alpha T} \\[2ex] \dfrac{I_0 Tz}{z(1+\alpha T)-1} \\[2ex] \dfrac{TI_0}{2}\left(\dfrac{z+1}{z\left(1+\dfrac{\alpha T}{2}\right)+\left(\dfrac{\alpha T}{2}-1\right)}\right) \end{cases}$$

	Functional Form	
$\dfrac{TI_0}{z-1+\alpha T}$	$\dfrac{G}{z-H}$	Forward Euler
$\dfrac{I_0 Tz}{z(1+\alpha T)-1}$	$\dfrac{Gz}{zH-1}$	Backward Euler
$\dfrac{TI_0}{2}\left(\dfrac{z+1}{z\left(1+\dfrac{\alpha T}{2}\right)+\left(\dfrac{\alpha T}{2}-1\right)}\right)$	$G\left(\dfrac{z+1}{z-H}\right)$	Bilinear z

Corresponding difference equations:

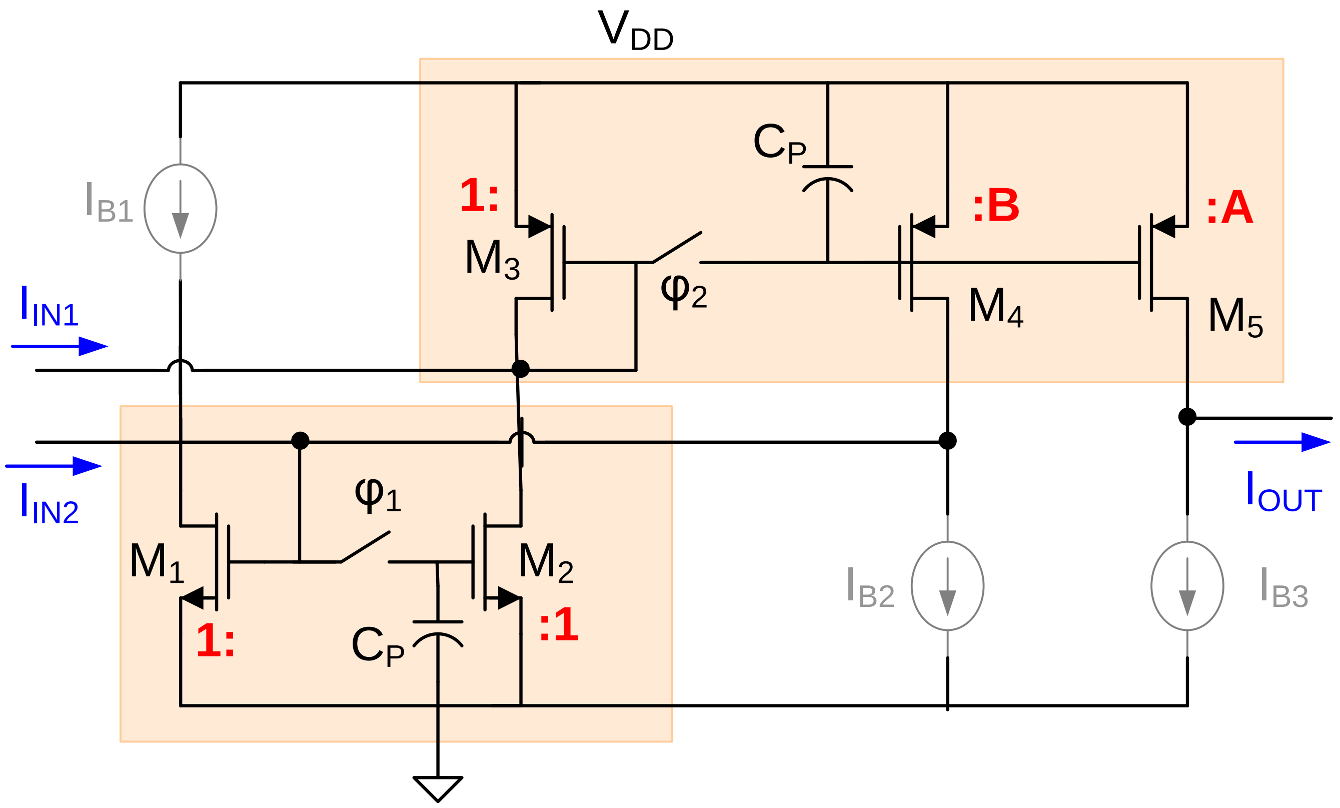
$$V_{OUT}(nT+T) = GV_{IN}(nT) + HV_{OUT}(nT)$$ Forward Euler

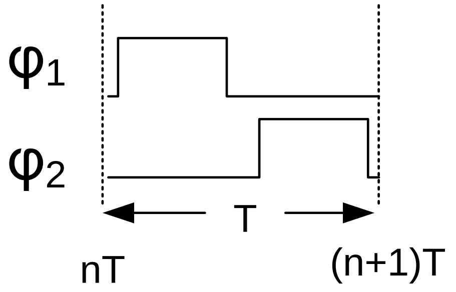
$$HV_{OUT}(nT+T) = GV_{IN}(nT+T) + V_{OUT}(nT)$$ Backward Euler

$$V_{OUT}(nT+T) = G\left(V_{IN}(nT+T) + V_{IN}(nT)\right) + HV_{OUT}(nT)$$ Bilinear z

Switched-Current Integrator

Consider this circuit



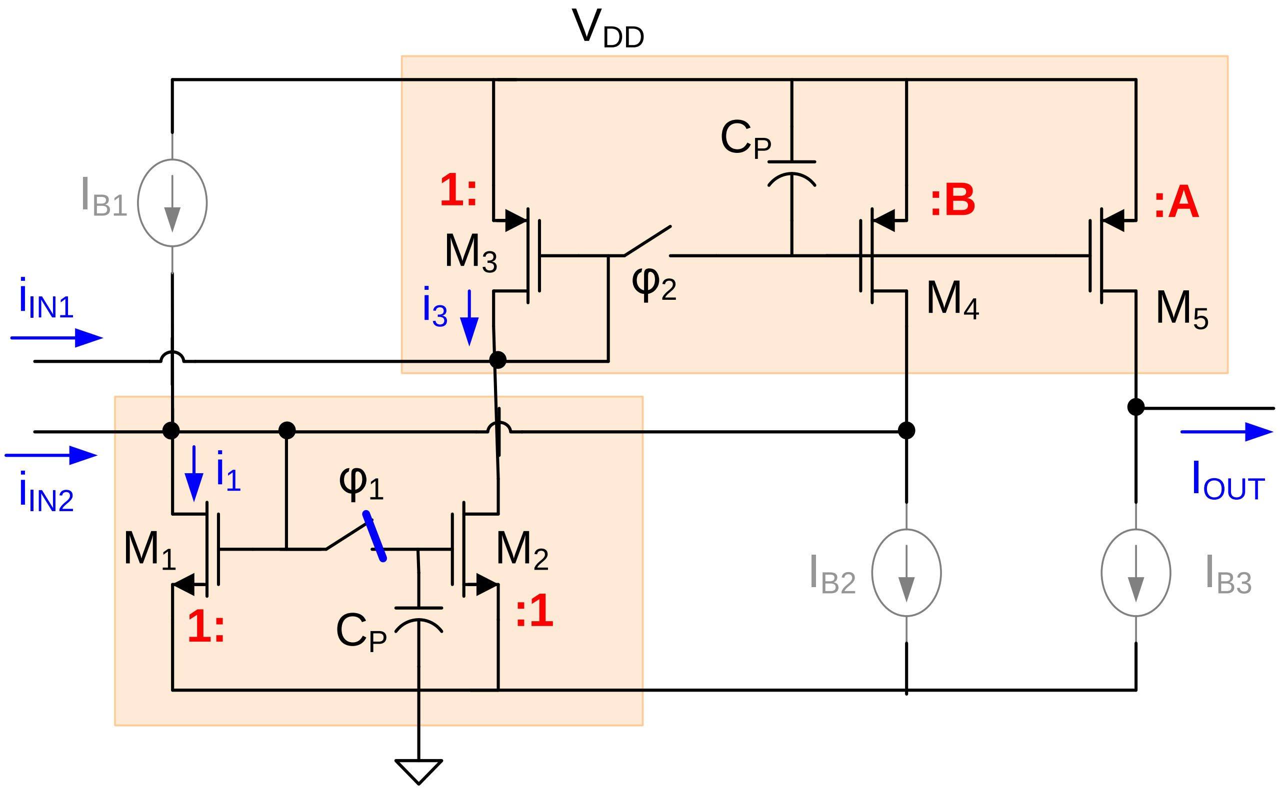


- Clocks complimentary, nonoverlapping
- Phase not critical

Assume inputs change only during phase Φ_2

(<u>may</u> be outputs from other like stages)

Switched-Current Integrator



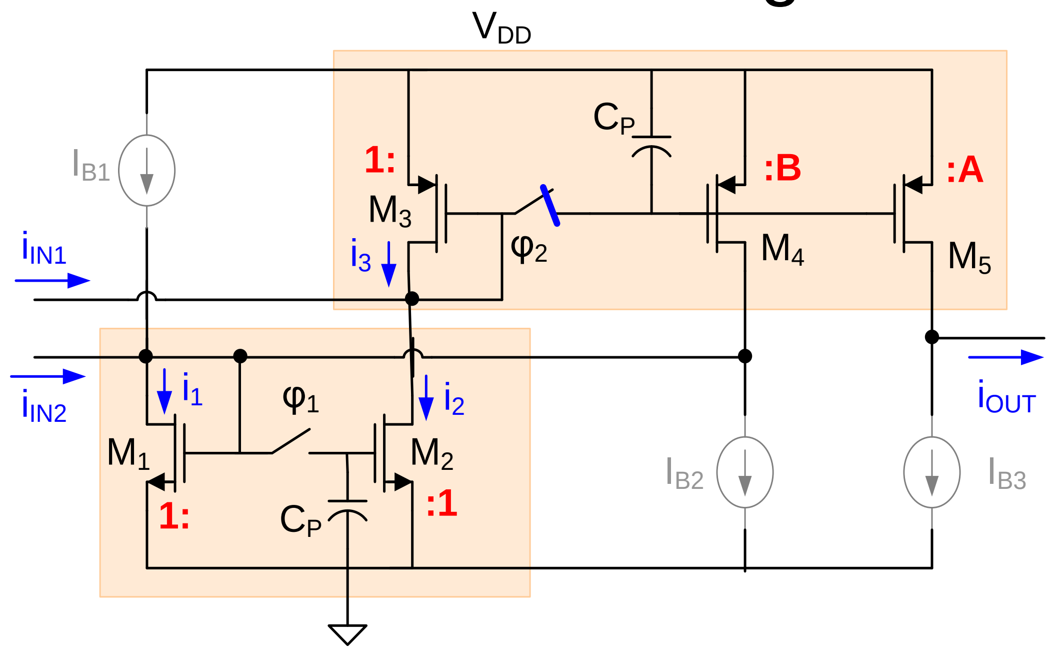
Consider Φ_1 closed, Φ_2 open (nT-T < t < nT-T/2)

$$i_1(t) = Bi_3(nT-T) + i_{iN2}(t)$$

Since current does not change during this interval

$$i_1(nT-T) = Bi_3(nT-T) + i_{iN2}(nT-T)$$

Switched-Current Integrator



Consider Φ_2 closed, Φ_1 open (nT-T/2 < t < nT)

$$i_2(t) = i_1(nT-T)$$

$$i_2(t) = i_3(t) + i_{IN1}(t)$$

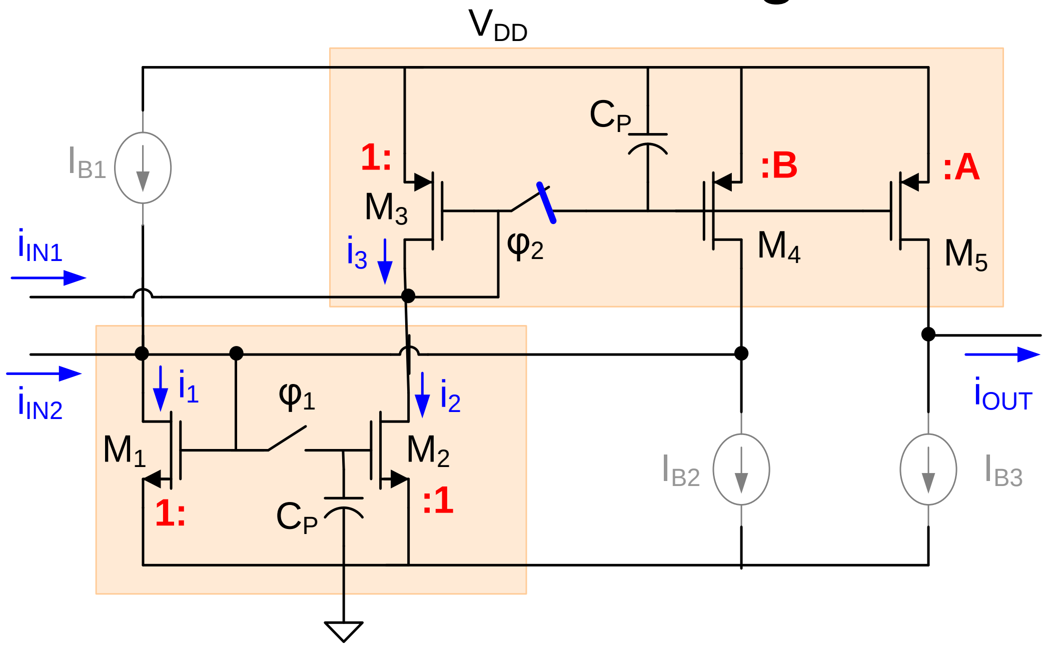
$$i_{OUT}(t) = Ai_3(t)$$

$$i_1(nT-T) = Bi_3(nT-T) + i_{iN2}(nT-T)$$

(from first phase)

$$\left(\frac{1}{A}\right)i_{OUT}(t) + i_{IN1}(t) = \frac{B}{A}i_{OUT}(nT-T) + i_{IN2}(nT-T)$$

Switched-Current Integrator



Consider Φ_2 closed, Φ_1 open (nT-T/2 < t < nT)

$$\left(\frac{1}{A}\right) i_{OUT}(t) + i_{IN1}(t) = \frac{B}{A} i_{OUT}(nT-T) + i_{IN2}(nT-T)$$

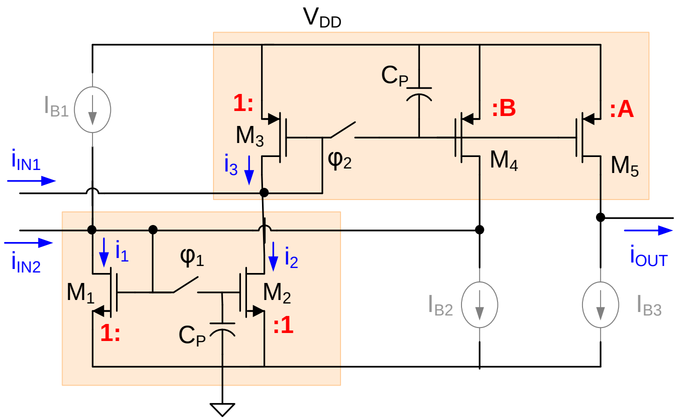
Evaluating at t=nT, we have

$$\left(\frac{1}{A}\right) i_{OUT}(nT) + i_{IN1}(nT) = \frac{B}{A} i_{OUT}(nT-T) + i_{IN2}(nT-T)$$

Taking z-transform, obtain

$$I_{OUT}(z) = \left(\frac{Az^{-1}}{1-Bz^{-1}}\right) I_{IN2}(z) - \left(\frac{A}{1-Bz^{-1}}\right) I_{IN1}(z)$$

Switched-Current Integrator



Recall lossy integrators:

$$H(z) = \begin{cases} \dfrac{Gz^{-1}}{1 - Hz^{-1}} & \text{Forward Euler} \\ \dfrac{G}{1 - Hz^{-1}} & \text{Backward Euler} \\ G\left(\dfrac{1 + z^{-1}}{1 - Hz^{-1}}\right) & \text{Bilinear z} \end{cases}$$

For H=1 becomes lossless

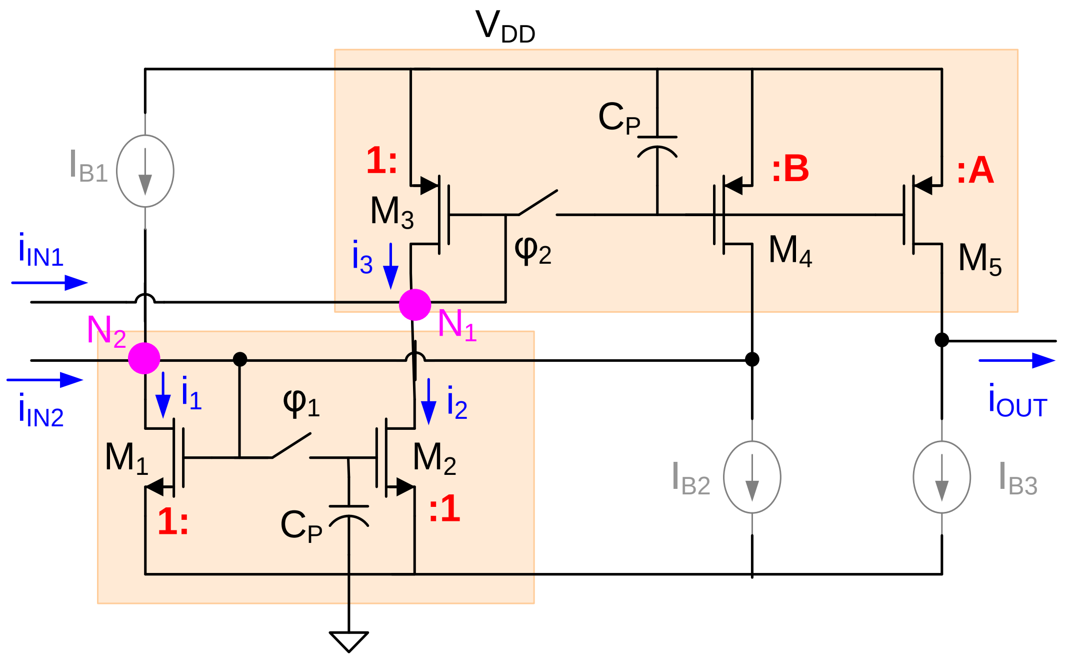
$$I_{OUT}(z) = \left(\frac{Az^{-1}}{1-Bz^{-1}}\right) I_{IN2}(z) - \left(\frac{A}{1-Bz^{-1}}\right) I_{IN1}(z)$$

If $I_{IN1}=0$, becomes Forward Euler integrator

If $I_{N2}=0$, becomes Backward Euler integrator

If $I_{N1}= - I_{IN2}$, becomes Bilinear Integrator

Switched-Current Integrator

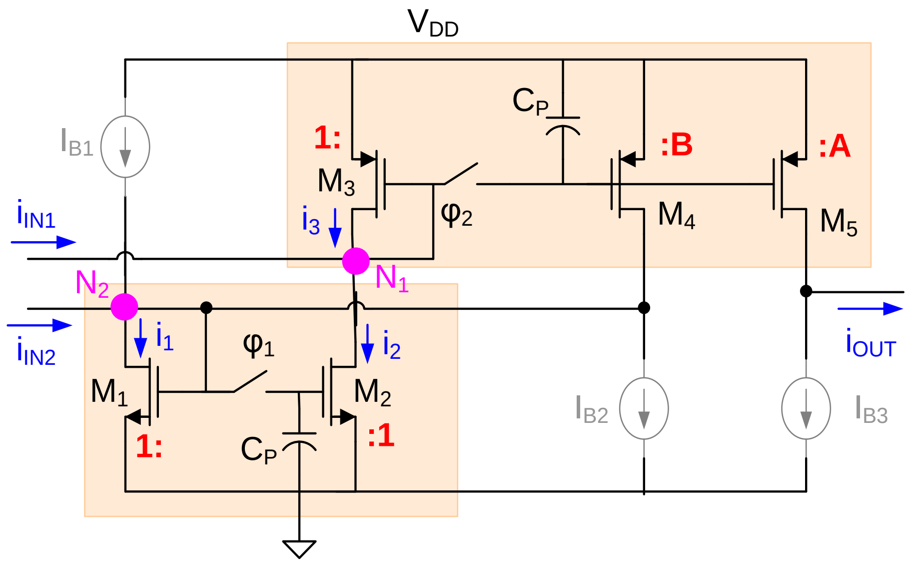


$$I_{OUT}(z) = \left(\frac{Az^{-1}}{1-Bz^{-1}}\right) I_{IN2}(z) - \left(\frac{A}{1-Bz^{-1}}\right) I_{IN1}(z)$$

- Summing inputs can be provided by summing currents on N_1 or N_2 or both
- Multiple outputs can be provided by adding outputs to upper mirror
- Amount of loss determined by mirror gain B

Switched-Current Integrator

Sensitivity Analysis



Consider Forward Euler

$$I_{OUT}(z) = \left(\frac{Az^{-1}}{1-Bz^{-1}}\right) I_{IN2}(z)$$

$$H(z) = \frac{TI_0}{z-1+\alpha T}$$

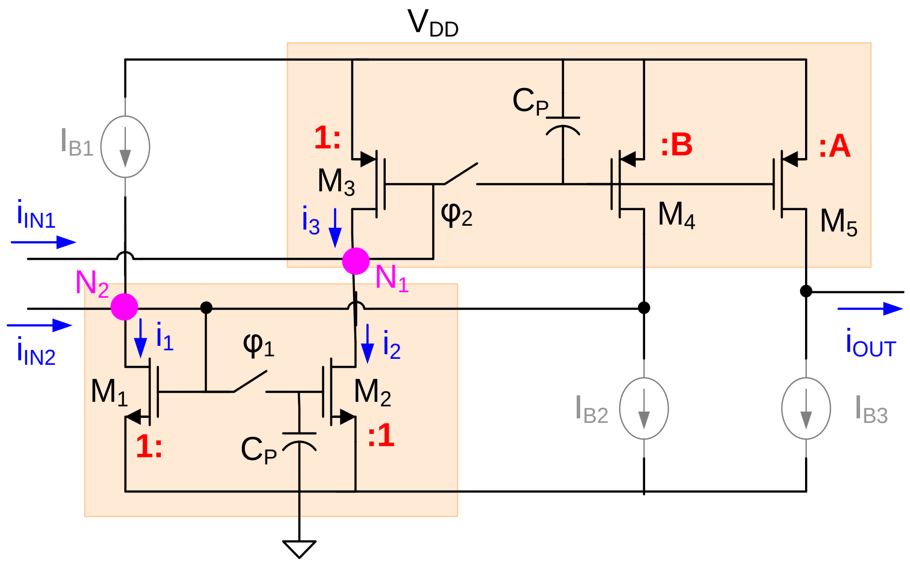
$$I_0 = \frac{A}{T} \qquad \frac{1-B}{T} = \alpha$$

$$S_A^{I_0} = 1 \qquad S_B^{\alpha} = \frac{-B}{1-B}$$

For low loss integrator (e.g. ideal integrator), the sensitivity of α is very large!

Switched-Current Integrator

Sensitivity Analysis



Consider Bilinear z

$$I_{OUT}(z) = A\left(\frac{z^{-1}+1}{1-Bz^{-1}}\right) I_{IN}(z) \qquad H(z) = \frac{TI_0}{2}\left(\frac{z+1}{z\left(1+\frac{\alpha T}{2}\right)+\left(\frac{\alpha T}{2}-1\right)}\right)$$

$$I_0 = A\frac{2}{T(1+B)} \qquad \alpha = \frac{2}{T}\frac{1-B}{1+B}$$

$$S_A^{I_0} = 1 \qquad S_B^{\alpha} = \frac{-B}{(1-B)(1+B)}$$

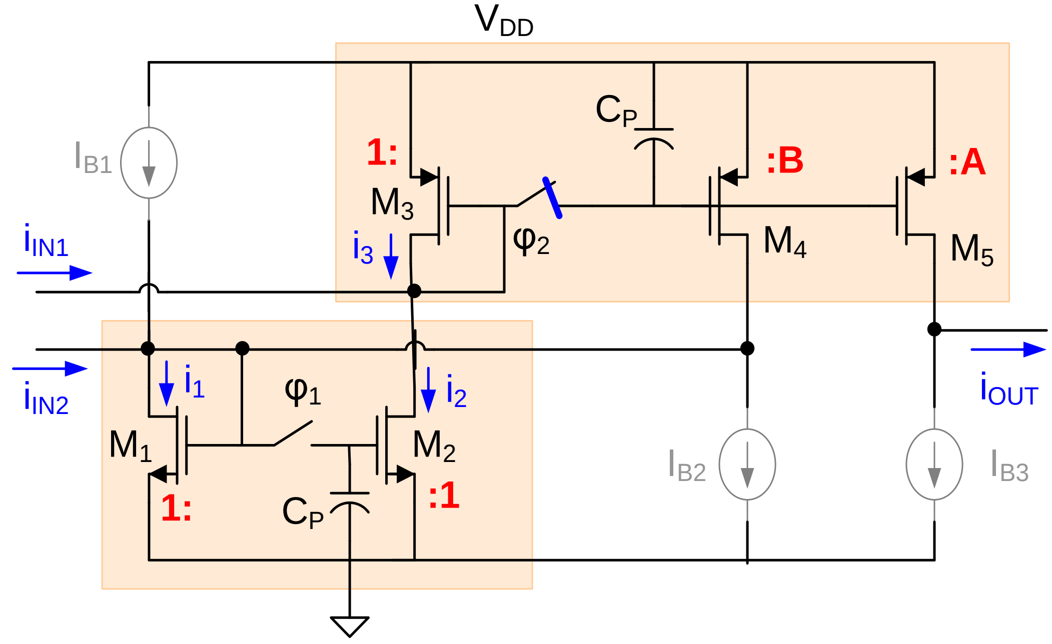
For low loss integrator (e.g. ideal integrator), the sensitivity of α is very large!

What about the sensitivity to the gain of the lower current mirror?

Switched-Current Integrator

Define A_1 to be the gain of the lower mirror

Sensitivity to A_1?



Consider Φ_2 closed, Φ_1 open (nT-T/2 < t < nT)

$$i_2(t) = A_1\, i_1(nT-T)$$

$$i_2(t) = i_3(t) + i_{IN1}(t)$$

$$i_{OUT}(t) = A i_3(t)$$

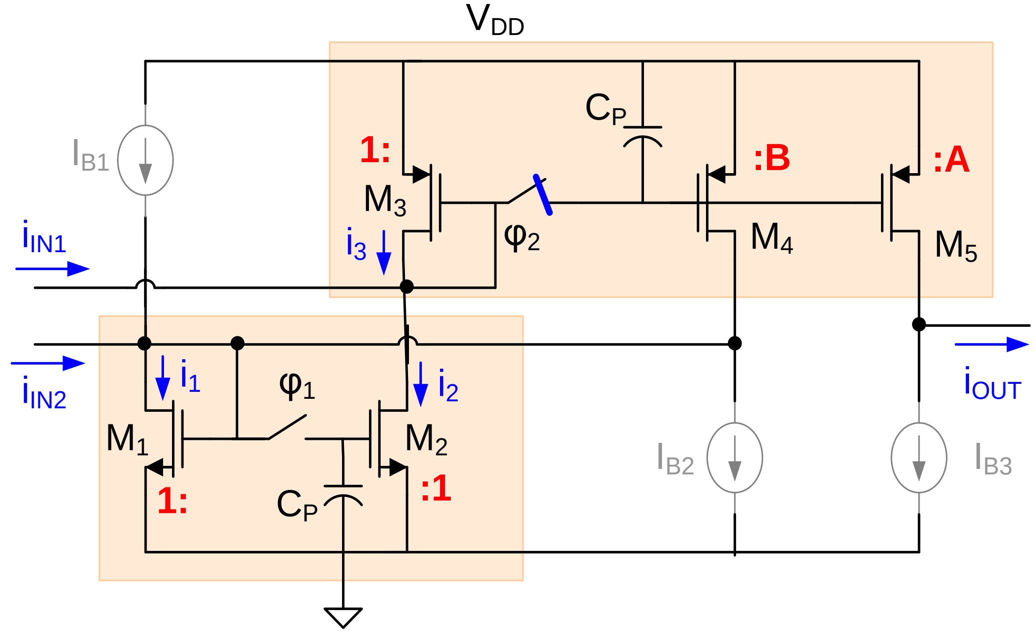
$$i_1(nT-T) = B i_3(nT-T) + i_{IN2}(nT-T) \quad \text{(from first phase)}$$

$$\left(\frac{1}{A}\right) i_{OUT}(t) + i_{IN1}(t) = \frac{A_1 B}{A} i_{OUT}(nT-T) + A_1 i_{IN2}(nT-T)$$

Switched-Current Integrator

Define A_1 to be the gain of the lower mirror

Sensitivity to A_1?



$$\left(\frac{1}{A}\right)i_{OUT}(nT)+i_{IN1}(nT)=\frac{A_1B}{A}i_{OUT}(nT-T)+A_1i_{IN2}(nT-T)$$

Taking z-transform, obtain

$$I_{OUT}(z)=\left(\frac{A_1Az^{-1}}{1-BA_1z^{-1}}\right)I_{IN2}(z)-\left(\frac{A}{1-BA_1z^{-1}}\right)I_{IN1}(z)$$

Consider Forward Euler

$$\frac{1-BA_1}{T}=\alpha \qquad S_B^{\alpha}=\frac{-BA_1}{1-BA_1} \qquad S_{A_1}^{\alpha}=\frac{-BA_1}{1-BA_1}$$

Sensitivity to A_1 is also large for low-loss or lossless integrator