Switched Current Filters
Current-Mode Filters

Basic Concepts of Benefits of Current-Mode Filters:

- Large voltage swings difficult to maintain in integrated processes because of linearity concerns
- Large voltage swings slow a circuit down because of time required to charge capacitors
- Voltage swings can be very small when currents change
- Current swings are not inherently limited in integrated circuits (only voltage swings)
- With low voltage swings, current-mode circuits should dissipate little power
Review from last time

Current-Mode Filters

Steady growth in research in the area since 1990 and publication rate is growing with time !!
Current-Mode Filters

The Conventional Wisdom:

– Current-Mode circuits operate at higher-frequencies than voltage-mode counterparts
– Current-Mode circuits operate at lower supply voltages and lower power levels than voltage-mode counterparts
– Current-Mode circuits are simpler than voltage-mode counterparts
– Current-Mode circuits offer better linearity than voltage-mode counterparts

This represents four really significant benefits of current-mode circuits!
Some Current-Mode Integrators

OTA-C

\[ I_{OUT} = \left( \frac{g_m}{C_s} \right) I_{IN} \]

Noninverting

\[ I_{OUT} = \left( \frac{-g_m}{C_s} \right) I_{IN} \]

Inverting

- Summing inputs really easy to obtain
- Loss is easy to add
- Same component count as voltage-mode integrators
- Many argue that since only interested in currents, can operate at lower voltages and higher frequencies
• Straightforward implementation of the two-integrator loop

• Simple structure
Current-Mode Two Integrator Loop

An Observation:

This circuit is identical to another one with two voltage-mode integrators and a voltage-mode amplifier!
Observation

• Many papers have appeared that tout the performance advantages of current-mode circuits
• In all of the current-mode papers that this instructor has seen, no attempt is made to provide a quantitative comparison of the key performance features of current-mode circuits with voltage-mode counterparts
• All justifications of the advantages of the current-mode circuits this instructor has seen are based upon qualitative statements
Observations (cont.)

- It appears easy to get papers published that have the term “current-mode” in the title.
- Over 900 papers have been published in IEEE forums alone!
- Some of the “current-mode” filters published perform better than other “voltage-mode” filters that have been published.
- We are still waiting for even one author to quantitatively show that current-mode filters offer even one of the claimed four advantages over their voltage-mode counterparts.

Will return to a discussion of Current-Mode filters later.
Switched-Current Filters

Basic idea introduced by Hughes and Bird at ISCAS 1989

**Switched currents** - a new technique for analog sampled-data signal processing

INTRODUCTION The enormous complexity available in state-of-the-art CMOS processing has made possible the integration of complete systems, including both digital and analog signal processing functions, within the same chip. Through the last decade, the **switched** capacitor technique...

Technique introduced directly in the z-domain
Switched-Current Filters

Basic idea introduced by Hughes and Bird at ISCAS 1989

If $\Phi_1$ is a periodic signal and if $I_{IN}$ is also appropriately clocked, the input/output currents of this circuit can be represented with the difference equation

$$I_{OUT}(nT) = AI_{IN}(nT-T)$$

This switched mirror becomes a delay element

“Gain” $A$ is that of a current mirror

$A$ can be accurately controlled

Circuit is small and very fast

Concept can be extended to implement arbitrary difference equation

Difference equation characterizes filter $H(z)$

Need only current mirrors and switches

Truly a “current-mode” circuit
Before developing Switch-Current concept, need to review background information in s to z domain transformations

Switched-Current Filters

Basic idea introduced by Hughes and Bird at ISCAS 1989

\[ I_{OUT}(nT) = A I_{IN}(nT-T) \]

- \( I_{OUT} \) is parasitic gate capacitance on \( M_2 \)
- Very low power dissipation
- Potential to operate at very low voltages
- Potential for accuracy of a SC circuit at both low and high frequencies but without the Op Amp and large C ratios
- Neither capacitor or resistor values needed to do filtering!

A completely new approach to designing filters that offers potential for overcoming most of the problems plaguing filter designers for decades!
s-domain to z-domain transformations

For a given $T(s)$ would like to obtain a function $H(z)$ or for a given $H(z)$ would like to obtain a $T(s)$ such that preserves the magnitude and phase response

Mathematically, would like to obtain the relationship:

$$T(s)igg|_{s=j\omega} = H(z)igg|_{z=e^{j\omega\tau}}$$
s-domain to z-domain transformations

\[ s=j\omega \quad \text{s-domain} \]
\[ z=e^{j\omega T} \quad \text{z-domain} \]

want:
\[ T(s)\big|_{s=j\omega} = H(z)\big|_{z=e^{j\omega T}} \]

equivalently, want:
\[ T(s) = H(z)\big|_{z=e^{sT}} \]

But if this were to happen, \( T(s) \) would not be a rational fraction in \( s \) with real coeff.

Thus, it is impossible to obtain this mapping between \( T(s) \) and \( H(z) \).
s-domain to z-domain transformations

\[ X_{IN} \rightarrow T(s) \rightarrow X_{OUT} \]

\[ X_{IN} \rightarrow H(z) \rightarrow X_{OUT} \]

goal: \( T(s) = H(z) \big|_{z=e^{sT}} \)

If can’t achieve this goal, would like to map imaginary axis to unit circle and map stable filters to stable filters

consider: \( z = e^{sT} \)

Case 1:
\[
\begin{align*}
z &= e^{sT} \\
&\approx \sum_{i=0}^{\infty} \frac{1}{i!} (sT)^i \\
z &= \sum_{i=0}^{\infty} \frac{1}{i!} (sT)^i \approx 1 + sT
\end{align*}
\]

\[ s = \frac{z - 1}{T} \]

Termed the Forward Euler transformation
s-domain to z-domain transformations

\[ s = \frac{z - 1}{T} \]

Forward Euler transformation

- Doesn’t map imaginary axis in s-plane to unit circle in z-plane
- Doesn’t guarantee stable filter will map to stable filter
- But mapping may give stable filter with good frequency response
s-domain to z-domain transformations

Consider: \( z = e^{sT} \)

Case 2: \( z = e^{sT} = \frac{1}{e^{-sT}} = \frac{1}{\sum_{i=0}^{\infty} \frac{1}{i!}(-sT)^i} \approx \frac{1}{1-sT} \)

\( z \approx \frac{1}{1-sT} \)

\( s = \left( \frac{1}{T} \right) \frac{z^{-1}}{z} \)

Termed the Backward Euler transformation
s-domain to z-domain transformations

\[ s = \left( \frac{1}{T} \right) \frac{z-1}{z} \]  
Backward Euler transformation

- Doesn’t map imaginary axis in s-plane to unit circle in z-plane
- Does guarantee stable filter will map to stable filter
s-domain to z-domain transformations

Consider: $z = e^{sT}$

Case 3: $z = e^{sT} = e^{s^2T/2} = \frac{1}{e^{-s^2T/2}} \sum_{i=0}^{\infty} \frac{1}{i!} \left(\frac{sT}{2}\right)^i \approx \frac{1+sT/2}{1-sT/2}$

Solving for $s$, obtain

$$s = \frac{2}{T} \cdot \frac{z-1}{z+1}$$

Termed the Bilinear $z$ transformation
s-domain to z-domain transformations

\[ s = \frac{2}{T} \cdot \frac{z-1}{z+1} \]

Bilinear z transformation

- Maps imaginary axis in s-plane to unit circle in z-plane (preserves shape, distorts frequency axis)
- Does guarantee stable filter will map to stable filter
- Bilinear z transformation is widely used
s-domain to z-domain transformations

consider: \( z = e^{sT} \)

Three Popular Transformations

\[
\begin{align*}
S &= \frac{z^{-1}}{T} \\
\text{Forward Euler} \\
S &= \frac{z^{-1}}{Tz} \\
\text{Backward Euler} \\
S &= \frac{2}{T} \cdot \frac{z^{-1}}{z + 1} \\
\text{Bilinear z transform} \\
S &= \frac{1 - z^{-1}}{TZ^{-1}} \\
S &= \frac{1 - z^{-1}}{T} \\
S &= \frac{2}{T} \cdot \frac{1 - z^{-1}}{1 + z^{-1}}
\end{align*}
\]
s-domain to z-domain transformations

Three Popular Transformations

- **Forward Euler**: 
  \[ s = \frac{z - 1}{T} \]
- **Backward Euler**: 
  \[ s = \frac{1 - z^{-1}}{T} \]
- **Bilinear z-transform**: 
  \[ s = \frac{2}{T} \cdot \frac{z - 1}{z + 1} \]

- Transformations of standard approximations in s-domain are the corresponding transformations in the z-domain.
- Transformations are not unique.
- Transformations cause warping of the imaginary axis and may cause change in basic shape.
- Transformations do not necessarily guarantee stability.
- These transformations preserve order.
z-domain integrators

Three Popular Transformations

Forward Euler
\[ s = \frac{z - 1}{T} \]

Backward Euler
\[ s = \frac{1 - z^{-1}}{T} \]

Bilinear z transform
\[ s = \frac{2}{T} \cdot \frac{z^{-1}}{1 + z^{-1}} \]

Corresponding difference equations:

Forward Euler
\[ V_{\text{OUT}}(nT+T) = T l_0 V_{\text{IN}}(nT) + V_{\text{OUT}}(nT) \]

Backward Euler
\[ V_{\text{OUT}}(nT+T) = l_0 T V_{\text{IN}}(nT+T) + V_{\text{OUT}}(nT) \]

Bilinear z
\[ V_{\text{OUT}}(nT+T) = \frac{T l_0}{2} \left( V_{\text{IN}}(nT+T) + V_{\text{IN}}(nT) \right) + V_{\text{OUT}}(nT) \]
z-domain lossy integrators

Three Popular Transformations

- Forward Euler: \( s = \frac{z^{-1}}{T} \)
- Backward Euler: \( s = 1 - \frac{z^{-1}}{T} \)
- Bilinear z transform: \( s = \frac{2}{T} \cdot \frac{z^{-1}}{1+z^{-1}} \)

Corresponding difference equations:

**Forward Euler**

\[
V_{\text{out}}(nT+T) = TI_0 V_{\text{in}}(nT) + [1 - \alpha T] V_{\text{out}}(nT)
\]

**Backward Euler**

\[
(1 + \alpha T) V_{\text{out}}(nT+T) = I_0 TV_{\text{in}}(nT+T) + V_{\text{out}}(nT)
\]

**Bilinear z transform**

\[
\left(1 + \frac{\alpha T}{2}\right) V_{\text{out}}(nT+T) = \frac{T I_0}{2} \left(V_{\text{in}}(nT+T) + V_{\text{in}}(nT)\right) + \left[1 - \frac{\alpha T}{2}\right] V_{\text{out}}(nT)
\]
Some z-domain lossy integrators

\[
T(s) = \frac{I_0}{s + \alpha}
\]

\[
H(z) = \begin{cases}
  \frac{TI_0}{z - 1 + \alpha T} \\
  \frac{I_0 Tz}{z (1 + \alpha T) - 1} \\
  \frac{TI_0}{2} \left( \frac{z + 1}{z} \left( 1 + \frac{\alpha T}{2} \right) + \left( \frac{\alpha T}{2} - 1 \right) \right)
\end{cases}
\]

Corresponding difference equations:

- \[ V_{OUT}(nT+T) = GV_{IN}(nT) + HV_{OUT}(nT) \]  \quad \text{Forward Euler}
- \[ HV_{OUT}(nT+T) = GV_{IN}(nT+T) + V_{OUT}(nT) \]  \quad \text{Backward Euler}
- \[ V_{OUT}(nT+T) = G \left( V_{IN}(nT+T) + V_{IN}(nT) \right) + HV_{OUT}(nT) \]  \quad \text{Bilinear z}
Switched-Current Integrator

Consider this circuit

- Clocks complimentary, nonoverlapping
- Phase not critical

**Assume inputs change only during phase \( \Phi_2 \)**

*(may be outputs from other like stages)*
Consider $\Phi_1$ closed, $\Phi_2$ open ($nT-T < t < nT-T/2$)

$$i_1(t) = Bi_3(nT-T) + i_{IN2}(t)$$

Since current does not change during this interval

$$i_1(nT-T) = Bi_3(nT-T) + i_{IN2}(nT-T)$$
Consider $\Phi_2$ closed, $\Phi_1$ open ($nT-T/2 < t < nT$)

\[
i_2(t) = i_1(nT-T)
\]

\[
i_2(t) = i_3(t) + i_{IN1}(t)
\]

\[
i_{OUT}(t) = Ai_3(t)
\]

\[
i_1(nT-T) = Bi_3(nT-T) + i_{IN2}(nT-T) \quad \text{(from first phase)}
\]

\[
\left(\frac{1}{A}\right)i_{OUT}(t) + i_{IN1}(t) = \frac{B}{A}i_{OUT}(nT-T) + i_{IN2}(nT-T)
\]
Consider $\Phi_2$ closed, $\Phi_1$ open ($nT-T/2 < t < nT$)

\[
\left(\frac{1}{A}\right)i_{OUT}(t) + i_{IN1}(t) = \frac{B}{A}i_{OUT}(nT-T) + i_{IN2}(nT-T)
\]

Evaluating at $t=nT$, we have

\[
\left(\frac{1}{A}\right)i_{OUT}(nT) + i_{IN1}(nT) = \frac{B}{A}i_{OUT}(nT-T) + i_{IN2}(nT-T)
\]

Taking z-transform, obtain

\[
i_{OUT}(z) = \left(\frac{Az^{-1}}{1-Bz^{-1}}\right)i_{IN2}(z) - \left(\frac{A}{1-Bz^{-1}}\right)i_{IN1}(z)
\]
Switched-Current Integrator

Recall lossy integrators:

$$H(z) = \begin{cases} 
\frac{Gz^{-1}}{1 - Hz^{-1}} & \text{Forward Euler} \\
\frac{G}{1 - Hz^{-1}} & \text{Backward Euler} \\
G \left( \frac{1 + z^{-1}}{1 - Hz^{-1}} \right) & \text{Bilinear } z
\end{cases}$$

For $H=1$ becomes lossless

$$I_{OUT}(z) = \left( \frac{Az^{-1}}{1 - Bz^{-1}} \right) I_{IN2}(z) - \left( \frac{A}{1 - Bz^{-1}} \right) I_{IN1}(z)$$

If $I_{IN1}=0$, becomes Forward Euler integrator
If $I_{N2}=0$, becomes Backward Euler integrator
If $I_{N1} = -I_{IN2}$, becomes Bilinear Integrator
Switched-Current Integrator

\[ I_{OUT}(z) = \left( \frac{Az^{-1}}{1-Bz^{-1}} \right) I_{IN2}(z) - \left( \frac{A}{1-Bz^{-1}} \right) I_{IN1}(z) \]

- Summing inputs can be provided by summing currents on N_1 or N_2 or both
- Multiple outputs can be provided by adding outputs to upper mirror
- Amount of loss determined by mirror gain B
Switched-Current Integrator

Sensitivity Analysis

Consider Forward Euler

\[ I_{OUT}(z) = \left( \frac{Az^{-1}}{1-Bz^{-1}} \right) I_{IN2}(z) \]

\[ H(z) = \frac{Tl_0}{z - 1 + \alpha T} \]

\[ l_0 = \frac{A}{T}, \quad \frac{1-B}{T} = \alpha \]

\[ S_{l_0}^l = 1 \quad S_{B}^\alpha = \frac{-B}{1-B} \]

For low loss integrator (e.g. ideal integrator), the sensitivity of \( \alpha \) is very large!
Switched-Current Integrator

Sensitivity Analysis

Consider Bilinear $z$

$$I_{OUT}(z) = A \left( \frac{z^{-1} + 1}{1 - Bz^{-1}} \right) I_{IN}(z)$$

$$I_0 = A \frac{2}{T(1 + B)}$$

$$\alpha = \frac{2}{T} \frac{1 - B}{1 + B}$$

$$S_A^{I_0} = 1$$

$$S_B^\alpha = \frac{-B}{(1 - B)(1 + B)}$$

For low loss integrator (e.g. ideal integrator), the sensitivity of $\alpha$ is very large!

What about the sensitivity to the gain of the lower current mirror?
Switched-Current Integrator

Define $A_1$ to be the gain of the lower mirror

**Sensitivity to $A_1$?**

Consider $\Phi_2$ closed, $\Phi_1$ open ($nT-T/2 < t < nT$)

\[
\begin{align*}
    i_2(t) &= A_1 i_1(nT-T) \\
    i_2(t) &= i_3(t) + i_{IN1}(t) \\
    i_{OUT}(t) &= A i_3(t) \\
    i_1(nT-T) &= B i_3(nT-T) + i_{IN2}(nT-T) \quad \text{(from first phase)} \\
    \left(\frac{1}{A}\right) i_{OUT}(t) + i_{IN1}(t) &= \frac{A_1 B}{A} i_{OUT}(nT-T) + A_1 i_{IN2}(nT-T)
\end{align*}
\]
Define $A_1$ to be the gain of the lower mirror

**Sensitivity to $A_1$?**

\[
\frac{1}{A} i_{\text{OUT}}(nT) + i_{\text{IN}}(nT) = \frac{A_1 B}{A} i_{\text{OUT}}(nT-T) + A_1 i_{\text{IN2}}(nT-T)
\]

Taking z-transform, obtain

\[
i_{\text{OUT}}(z) = \left(\frac{A_1 Az^{-1}}{1 - BA_1 z^{-1}}\right) i_{\text{IN2}}(z) - \left(\frac{A}{1 - BA_1 z^{-1}}\right) i_{\text{IN1}}(z)
\]

Consider Forward Euler

\[
\frac{1 - BA_1}{T} = \alpha
\]

\[
S_B^\alpha = \frac{-BA_1}{1 - BA_1} \quad S_{A_1}^\alpha = \frac{-BA_1}{1 - BA_1}
\]

**Sensitivity to $A_1$ is also large for low-loss or lossless integrator**