EE 508
Lecture 32

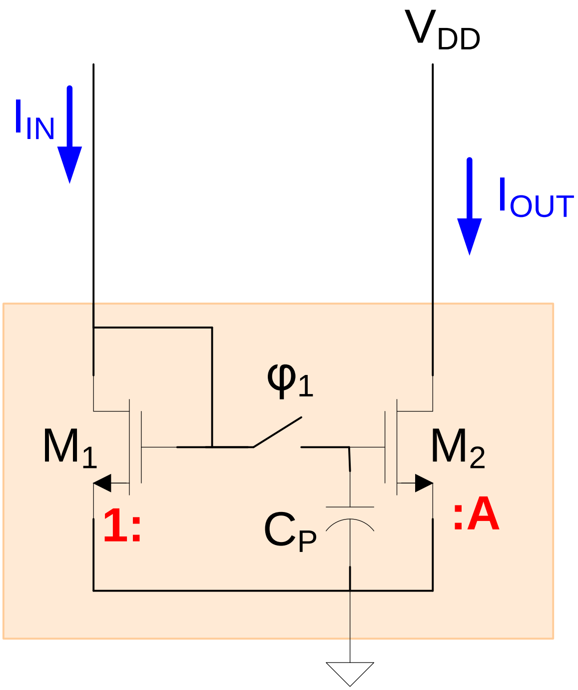
Switched Current Filters
Leapfrog Networks

Review from last time

Switched-Current Filters

Basic idea introduced by Hughes and Bird at ISCAS 1989

$$I_{OUT}(nT) = AI_{IN}(nT-T)$$



Cp is parasitic gate capacitance on M_2

Very low power dissipation

Potential to operate at very low voltages

Potential for accuracy of a SC circuit at both low and high frequencies but without the Op Amp and large C ratios

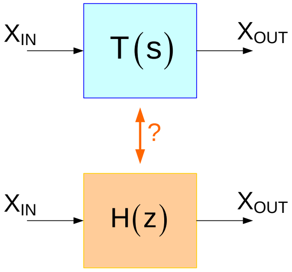
Neither capacitor or resistor values needed to do filtering!

A completely new approach to designing filters that offers potential for overcoming most of the problems plaguing filter designers for decades !

Before developing Switch-Current concept, need to review background information in s to z domain transformations

Review from last time

s-domain to z-domain transformations



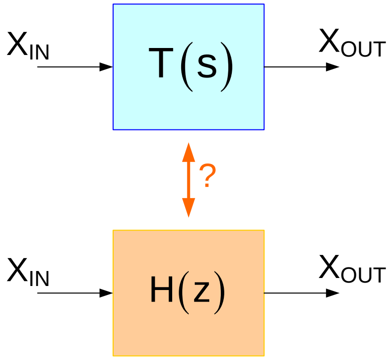
Three Popular Transformations

$s=\frac{z-1}{T}$	Forward Euler	$s=\frac{1-z^{-1}}{Tz^{-1}}$
$s=\frac{z-1}{Tz}$	Backward Euler	$s=\frac{1-z^{-1}}{T}$
$s=\frac{2}{T}\bullet\frac{z-1}{z+1}$	Bilinear z transform	$s=\frac{2}{T}\bullet\frac{1-z^{-1}}{1+z^{-1}}$

- Transformations of standard approximations in s-domain are the corresponding transformations in the z-domain
- Transformations are not unique
- Transformations cause warping of the imaginary axis and may cause change in basic shape
- Transformations do not necessarily guarantee stability
- These transformations preserve order

Review from last time

z-domain lossy integrators



$$T(s) = \frac{I_0}{s+\alpha}$$

Three Popular Transformations

$s = \dfrac{z-1}{T}$	Forward Euler	$s = \dfrac{1-z^{-1}}{Tz^{-1}}$
$s = \dfrac{z-1}{Tz}$	Backward Euler	$s = \dfrac{1-z^{-1}}{T}$
$s = \dfrac{2}{T}\bullet\dfrac{z-1}{z+1}$	Bilinear z transform	$s = \dfrac{2}{T}\bullet\dfrac{1-z^{-1}}{1+z^{-1}}$

Some z-domain lossy integrators

$$H(z) = \begin{cases} \dfrac{TI_0}{z-1+\alpha T} & \text{Forward Euler} \\ \dfrac{I_0 Tz}{z(1+\alpha T)-1} & \text{Backward Euler} \\ \dfrac{TI_0}{2}\left(\dfrac{z+1}{z\left(1+\dfrac{\alpha T}{2}\right)+\left(\dfrac{\alpha T}{2}-1\right)}\right) & \text{Bilinear z} \end{cases}$$

Corresponding difference equations:

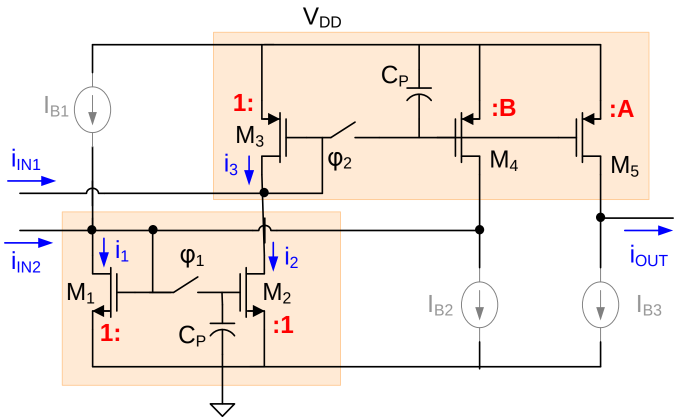
$$V_{OUT}(nT+T) = TI_0 V_{IN}(nT) + [1-\alpha T]V_{OUT}(nT) \qquad \text{Forward Euler}$$

$$(1+\alpha T)V_{OUT}(nT+T) = I_0 T V_{IN}(nT+T) + V_{OUT}(nT) \qquad \text{Backward Euler}$$

$$\left(1+\frac{\alpha T}{2}\right)V_{OUT}(nT+T) = \frac{TI_0}{2}\left(V_{IN}(nT+T)+V_{IN}(nT)\right) + \left[1-\frac{\alpha T}{2}\right]V_{OUT}(nT) \qquad \text{Bilinear z}$$

Review from last time

Switched-Current Integrator



Recall lossy integrators:

$$H(z) = \begin{cases} \dfrac{Gz^{-1}}{1 - Hz^{-1}} & \text{Forward Euler} \\ \dfrac{G}{1 - Hz^{-1}} & \text{Backward Euler} \\ G\left(\dfrac{1 + z^{-1}}{1 - Hz^{-1}}\right) & \text{Bilinear z} \end{cases}$$

For H=1 becomes lossless

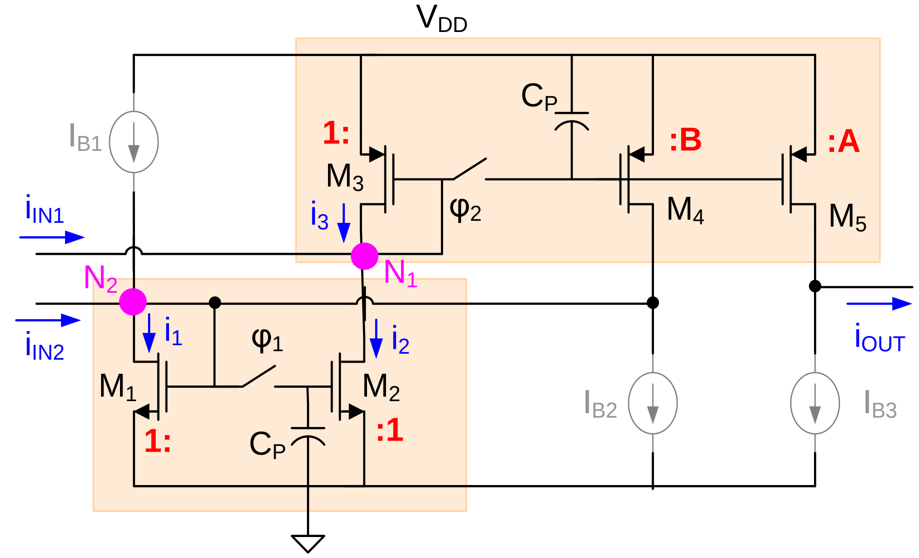
$$I_{OUT}(z) = \left(\frac{Az^{-1}}{1-Bz^{-1}}\right) I_{IN2}(z) - \left(\frac{A}{1-Bz^{-1}}\right) I_{IN1}(z)$$

If I_{IN1}=0, becomes Forward Euler integrator
If I_{N2}=0, becomes Backward Euler integrator
If I_{N1}= - I_{IN2}, becomes Bilinear Integrator

Review from last time

Switched-Current Integrator

Sensitivity Analysis



Consider Forward Euler

$$I_{OUT}(z) = \left(\frac{Az^{-1}}{1-Bz^{-1}}\right) I_{IN2}(z) \qquad H(z) = \frac{TI_0}{z-1+\alpha T}$$

$$I_0 = \frac{A}{T} \qquad \frac{1-B}{T} = \alpha$$

$$S_A^{I_0} = 1 \qquad S_B^{\alpha} = \frac{-TB}{1-B}$$

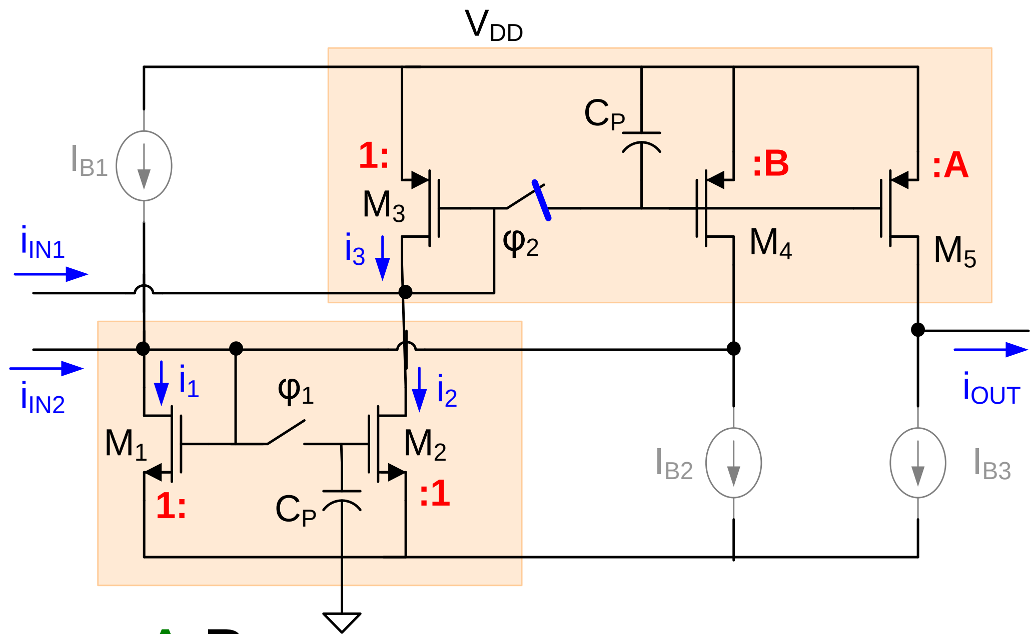
For low loss integrator (e.g. ideal integrator), the sensitivity of α is very large!

Review from last time

Switched-Current Integrator

Define A_1 to be the gain of the lower mirror

Sensitivity to A_1?



$$\left(\frac{1}{A}\right) i_{OUT}(nT) + i_{IN1}(nT) = \frac{A_1 B}{A} i_{OUT}(nT-T) + A_1 i_{IN2}(nT-T)$$

Taking z-transform, obtain

$$I_{OUT}(z) = \left(\frac{A_1 A z^{-1}}{1-BA_1 z^{-1}}\right) I_{IN2}(z) - \left(\frac{A}{1-BA_1 z^{-1}}\right) I_{IN1}(z)$$

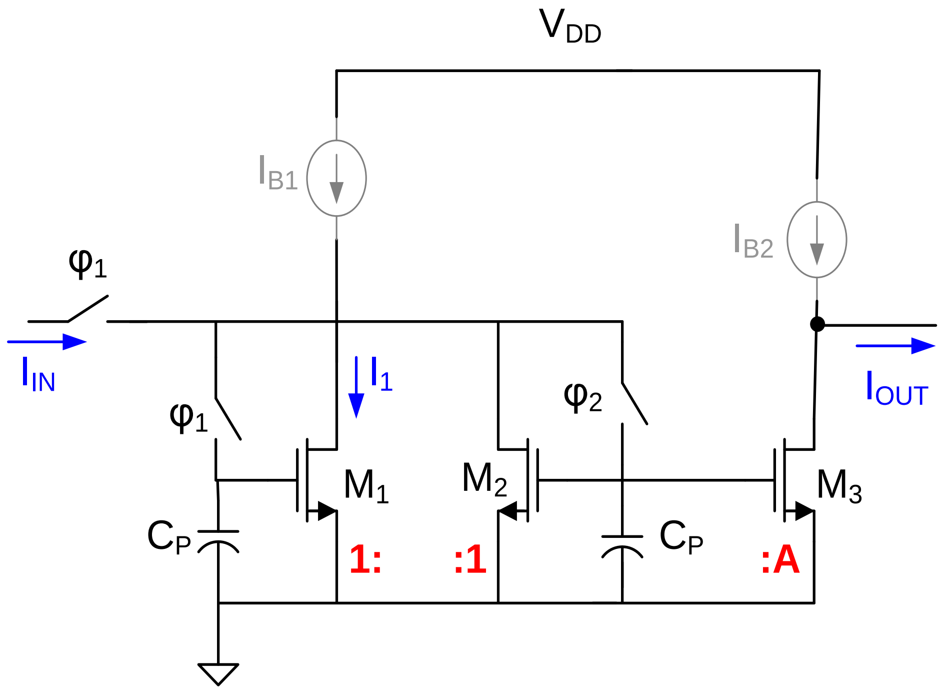
Consider Forward Euler

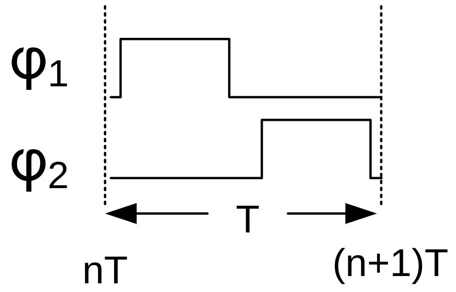
$$\frac{1-BA_1}{T} = \alpha \qquad S_B^{\alpha} = \frac{-BA_1}{1-BA_1} \qquad S_{A_1}^{\alpha} = \frac{-BA_1}{1-BA_1}$$

Sensitivity to A_1 is also large for low-loss or lossless integrator

Switched-Current Integrator

Consider another circuit



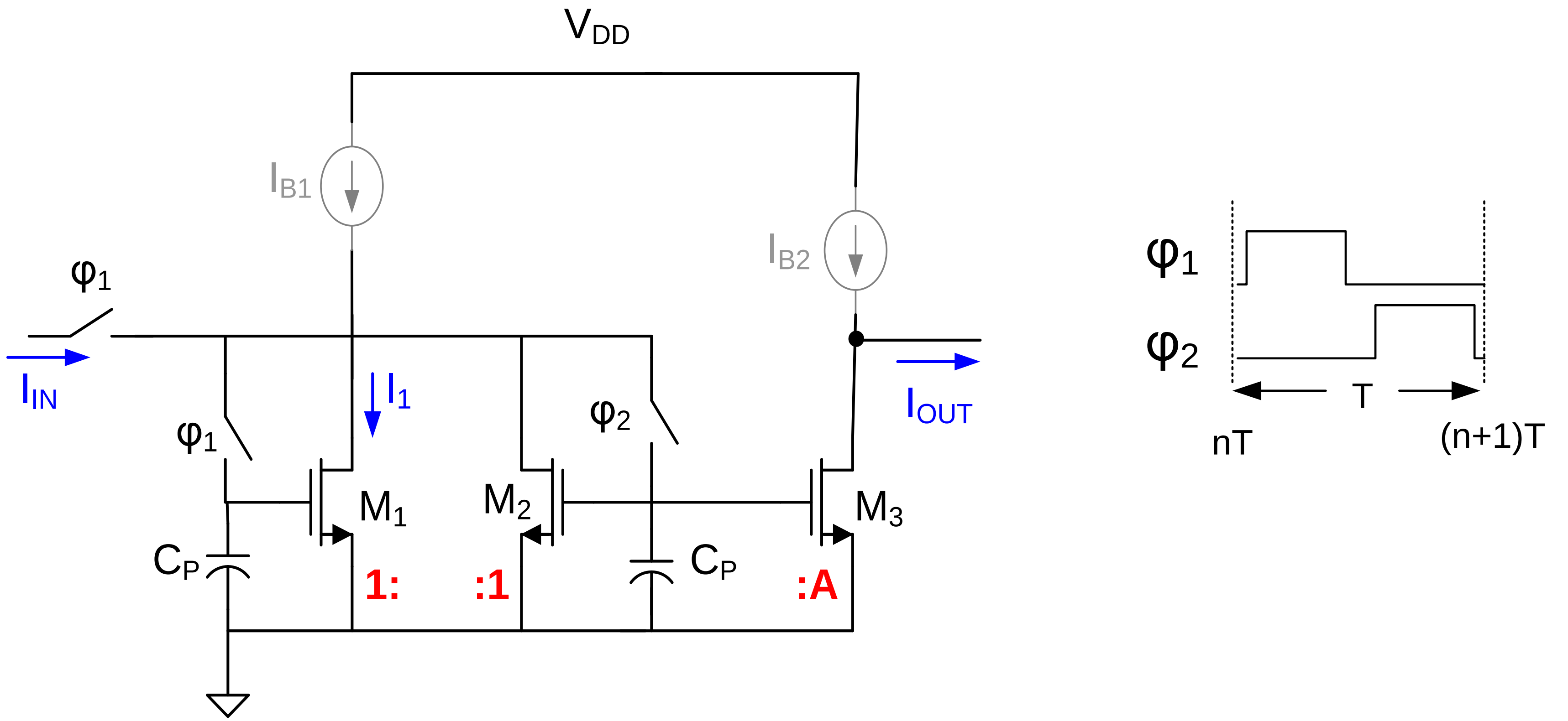


Consider Φ_1 closed, Φ_2 open (nT-T < t < nT-T/2)

$$i_1(t) = \frac{1}{A} i_{OUT}(nT-T) + i_{iN}(t)$$

$$i_1(nT-T) = \frac{1}{A} i_{OUT}(nT-T) + i_{iN}(nT-T) \qquad (1)$$

Switched-Current Integrator



Consider Φ_2 closed, Φ_1 open $(nT\text{-}T/2 < t < nT)$

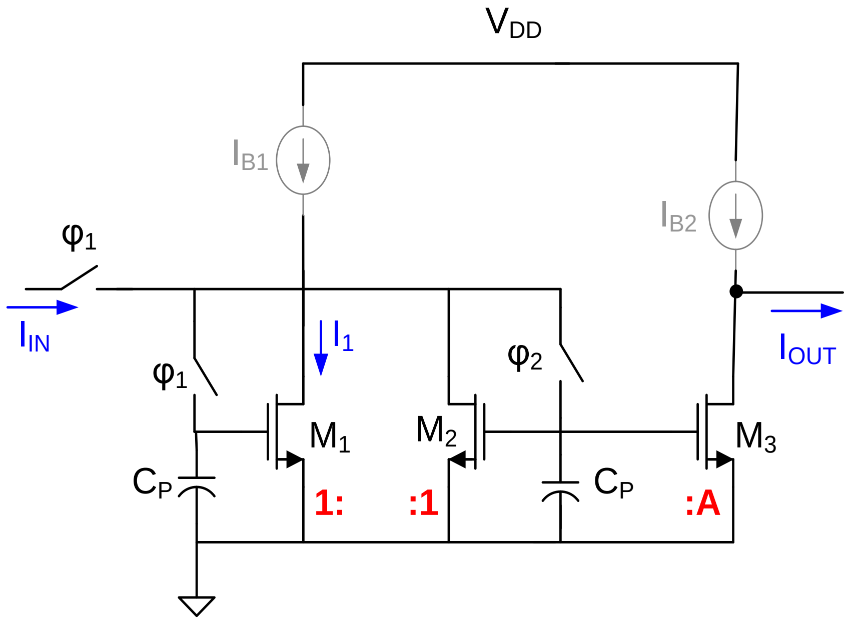
$$i_{OUT}(t) = Ai_1(nT\text{-}T)$$

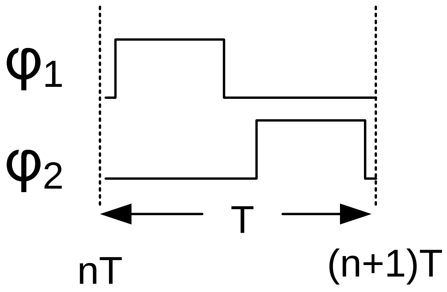
$$i_{OUT}(nT) = Ai_1(nT\text{-}T) \qquad (2)$$

combining (1) and (2), obtain

$$i_{OUT}(nT) = A \bullet \frac{1}{A} i_{OUT}(nT\text{-}T) + Ai_{iN}(nT\text{-}T)$$

Switched-Current Integrator





$$i_{OUT}(nT) = A \bullet \frac{1}{A} i_{OUT}(nT-T) + A i_{iN}(nT-T)$$

$$i_{OUT}(nT) = i_{OUT}(nT-T) + A i_{iN}(nT-T)$$

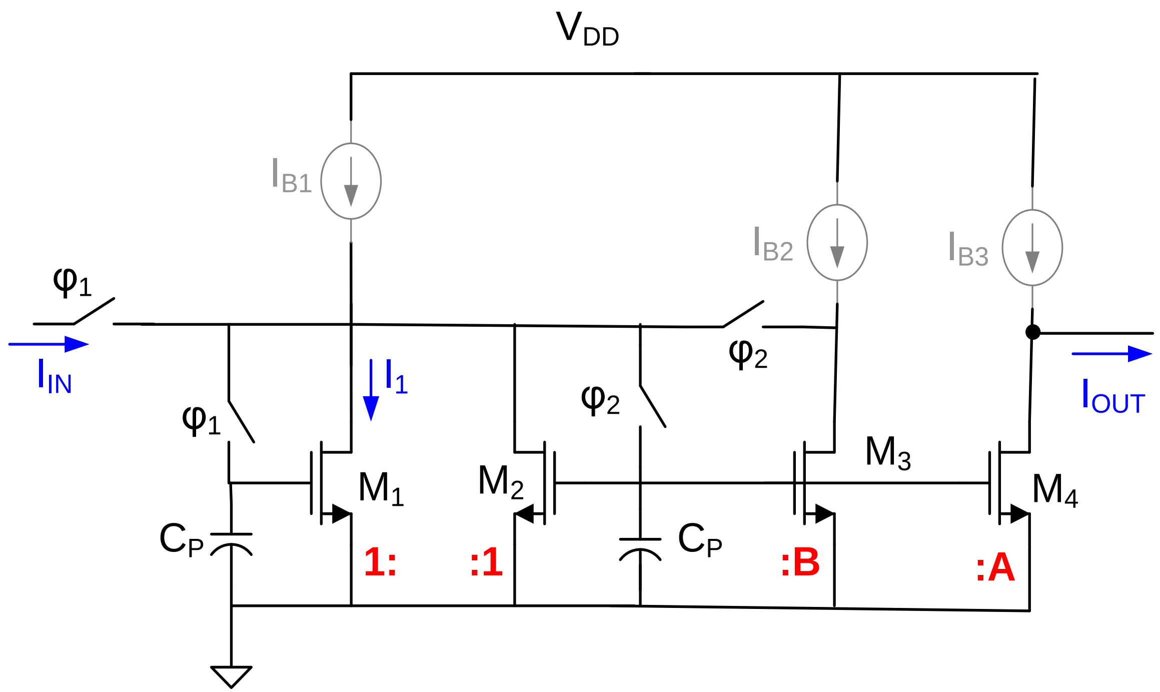
Taking z-transform, obtain

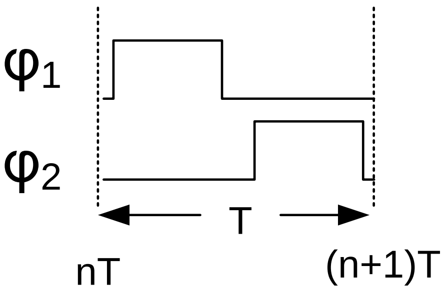
$$I_{OUT}(z) = \left(\frac{Az^{-1}}{1-z^{-1}}\right) I_{IN}(z)$$

Forward Euler Integrator

- Lossless Integrator (no matching required!)
- Matching of M_1 and M_2 not required
- Gain A does not affect coefficient of z^{-1} in the denominator

Switched-Current Integrator



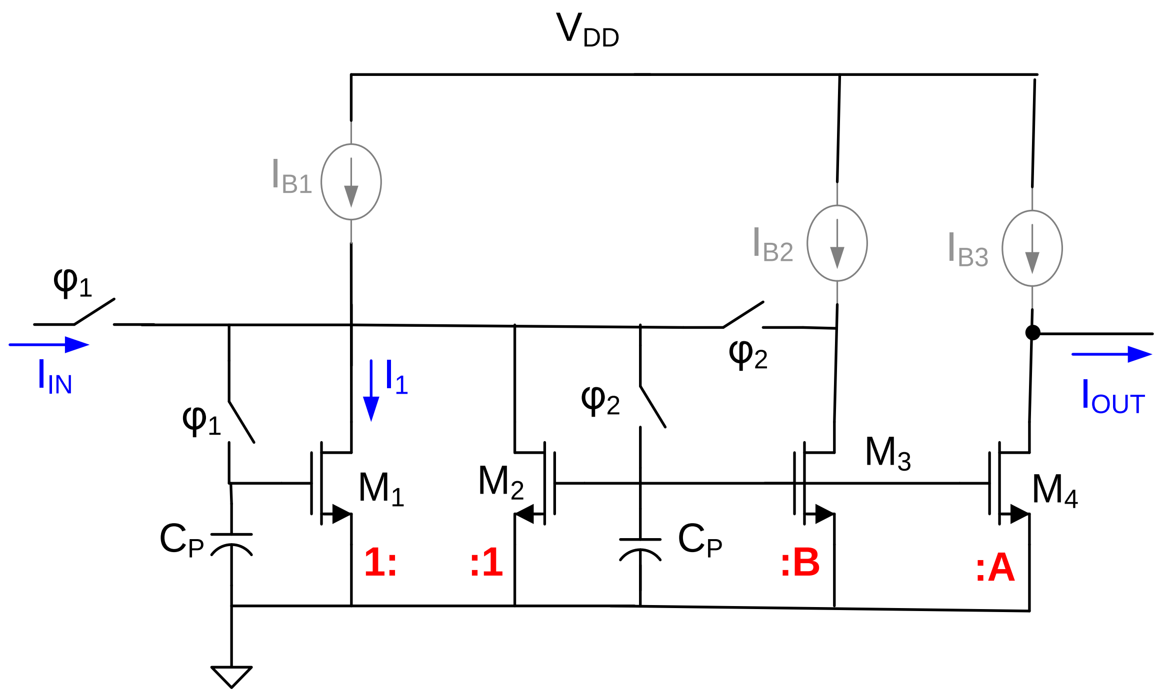


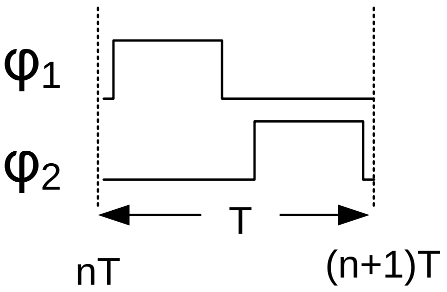
Consider Φ_1 closed, Φ_2 open $(nT\text{-}T < t < nT\text{-}T/2)$

$$i_1(t) = \frac{1}{A} i_{OUT}(nT\text{-}T) + i_{iN}(t)$$

$$i_1(nT\text{-}T) = \frac{1}{A} i_{OUT}(nT\text{-}T) + i_{iN}(nT\text{-}T) \qquad (1)$$

Switched-Current Integrator





Consider Φ_2 closed, Φ_1 open (nT-T/2 < t < nT)

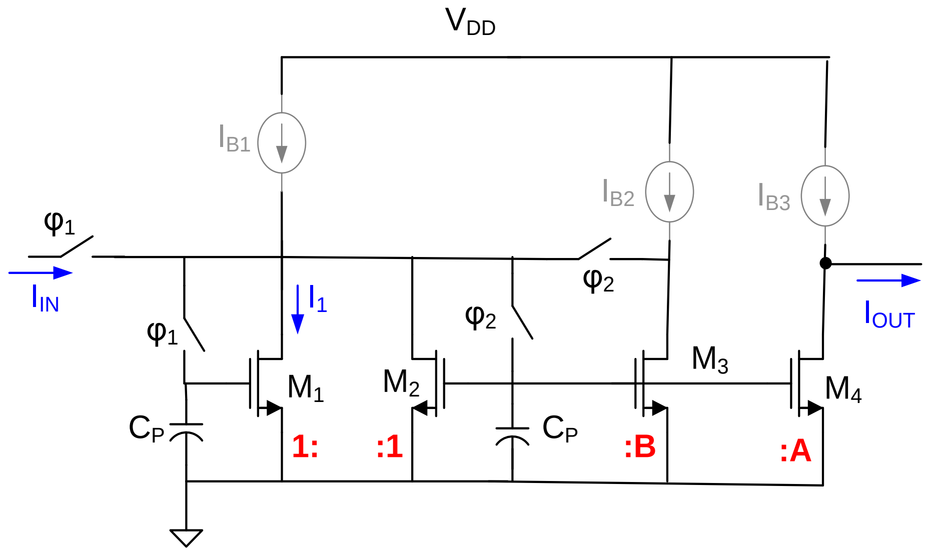
$$i_{OUT}(t) = A\left(i_1(nT-T) - \frac{B}{A} i_{OUT}(t)\right)$$

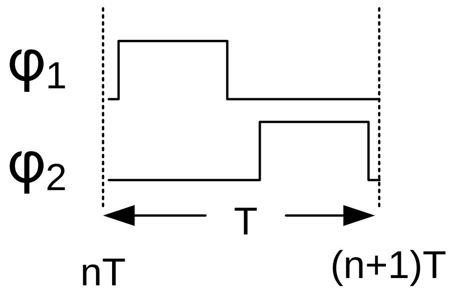
$$i_{OUT}(nT) = A\left(i_1(nT-T) - \frac{B}{A} i_{OUT}(nT)\right) \qquad (2)$$

combining (1) and (2), obtain

$$i_{OUT}(nT) = i_{OUT}(nT-T) - Bi_{OUT}(nT) + Ai_{IN}(nT-T)$$

Switched-Current Integrator





$$i_{OUT}(nT) = i_{OUT}(nT-T) - Bi_{OUT}(nT) + Ai_{IN}(nT-T)$$

Taking z-transform, obtain

$$I_{OUT}(z) = \left(\frac{Gz^{-1}}{1-Hz^{-1}}\right) I_{IN}(z)$$

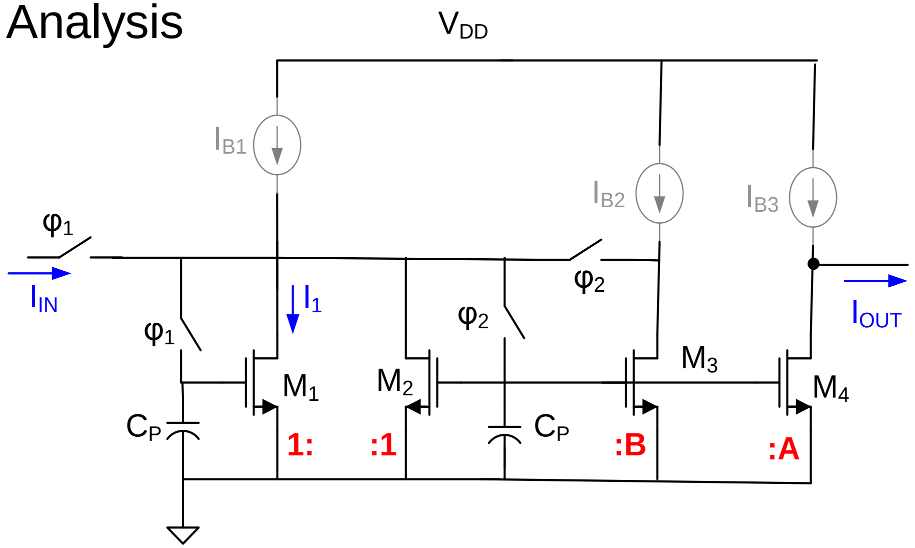
Forward Euler Integrator (Lossy)

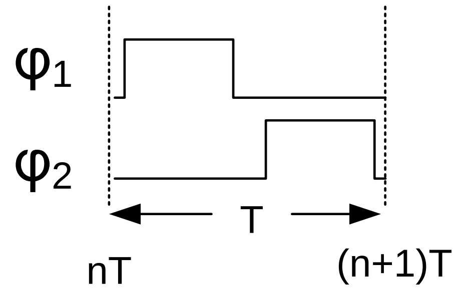
where $G = \dfrac{A}{1+B}$ $H = \dfrac{1}{1+B}$

- Lossy Integrator
- Matching of M_1 and M_2 not required
- Gain A does not affect coefficient of z^{-1} in the denominator

Switched-Current Integrator

Sensitivity Analysis





$$I_{OUT}(z) = \left(\frac{Gz^{-1}}{1-Hz^{-1}}\right) I_{IN}(z) \qquad G = \frac{A}{1+B} \qquad H = \frac{1}{1+B}$$

$$H(z) = \frac{TI_0}{z - 1 + \alpha T}$$

It can be shown that

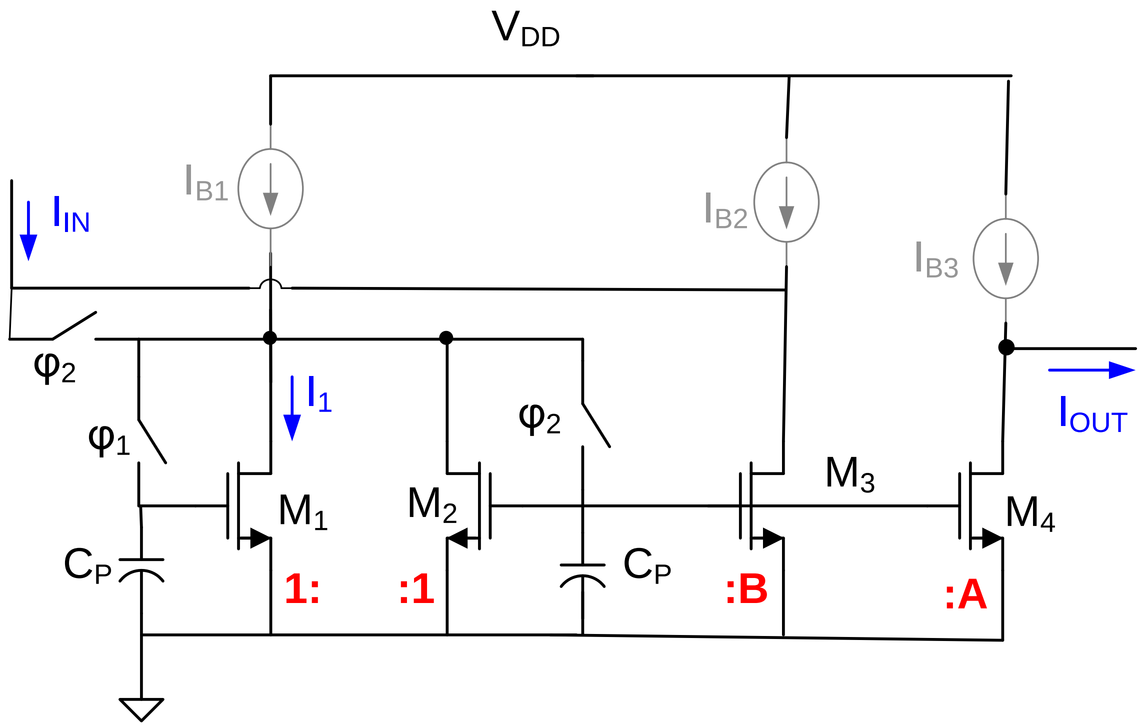
$$\alpha = \frac{1}{T}\left(\frac{B}{B+1}\right)$$

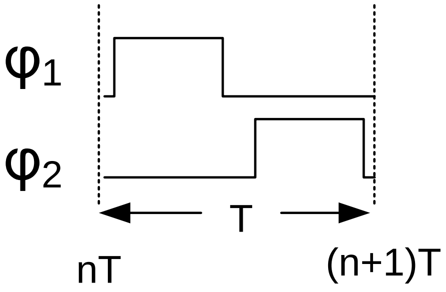
$$S_B^{\alpha} = \frac{T}{1+B}$$

For small loss, B is small and so is the sensitivity

Switched-Current Integrator

Another structure





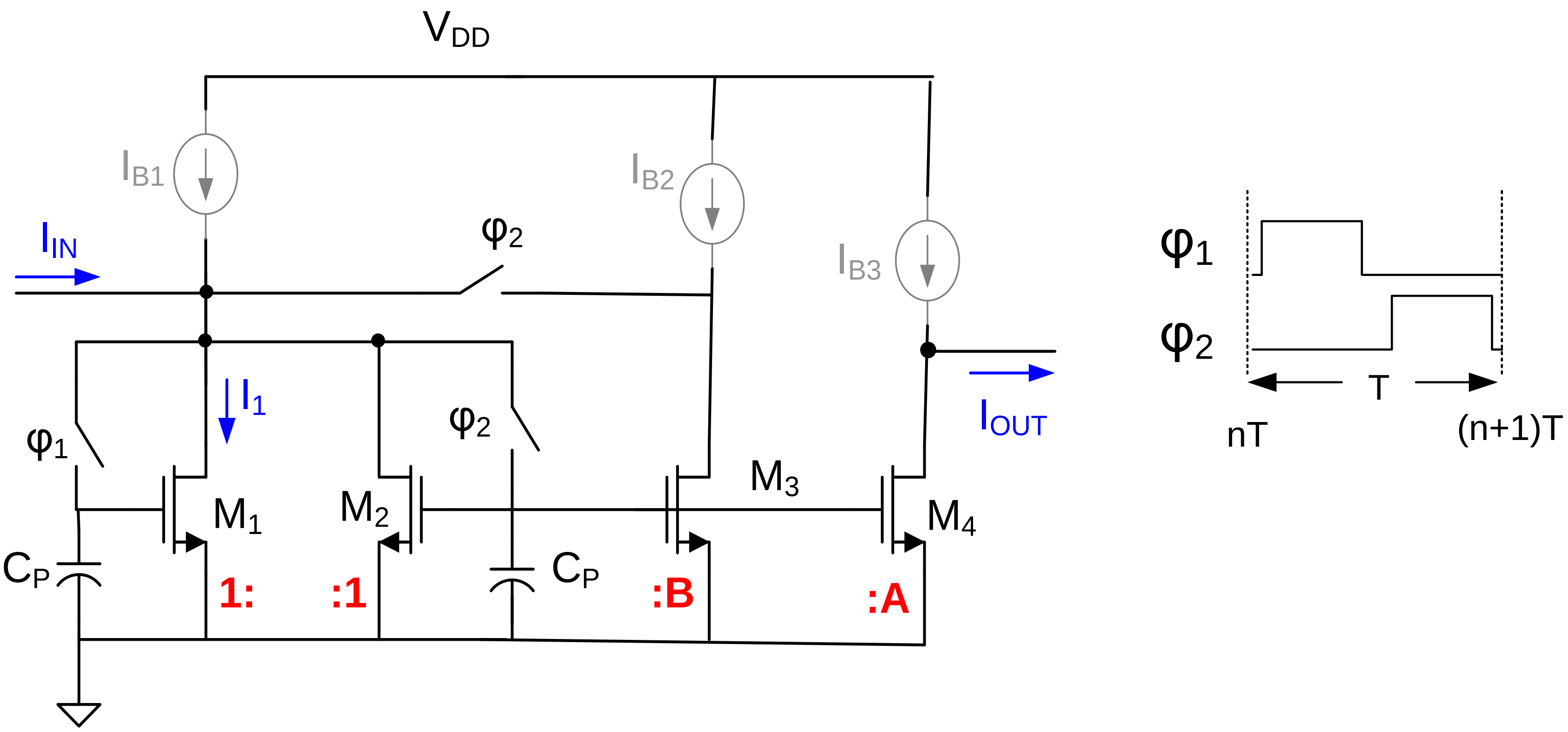
$$I_{OUT}(z) = \left(\frac{-G}{1-Hz^{-1}}\right) I_{IN}(z)$$

Backward Euler Lossy Inverting

$$G = \frac{A}{1+B} \qquad H = \frac{1}{1+B}$$

Switched-Current Integrator

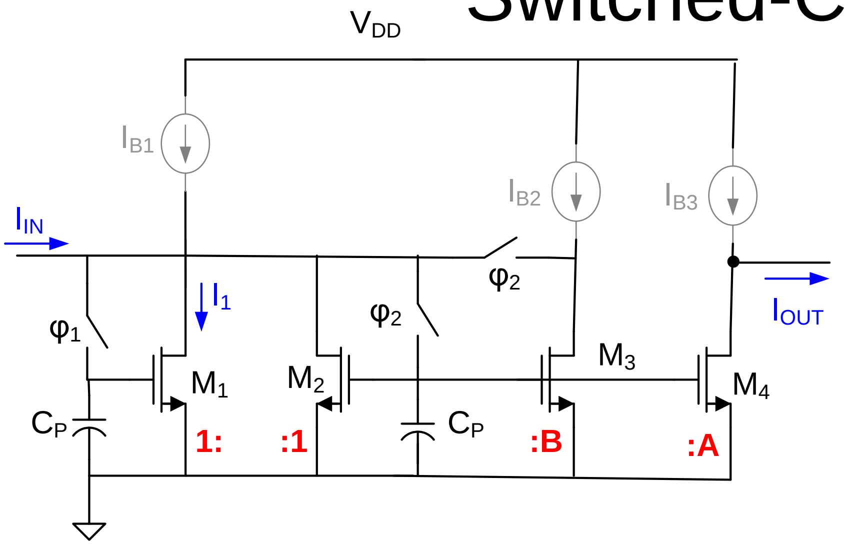
Another structure

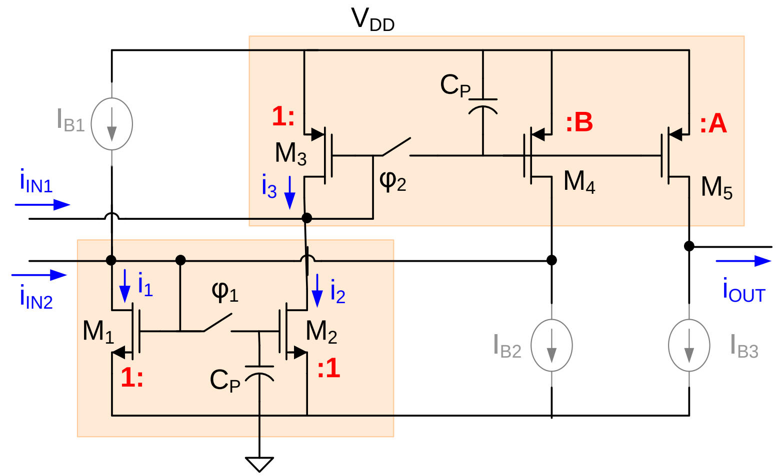


$$I_{OUT}(z) = -G\left(\frac{1-z^{-1}}{1-Hz^{-1}}\right) I_{IN}(z)$$

$$G = \frac{A}{1+B} \qquad H = \frac{1}{1+B}$$

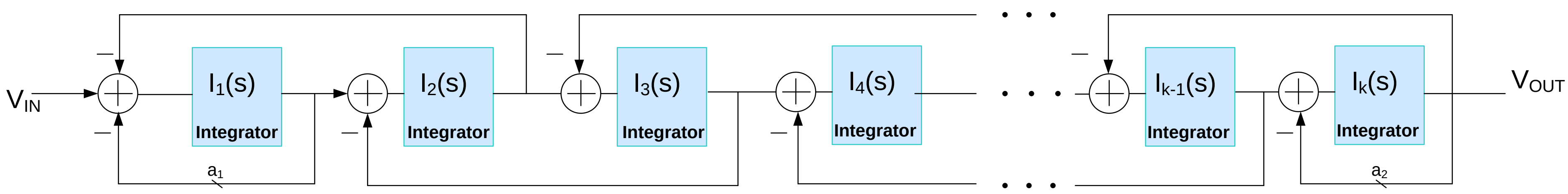
Switched-Current Filters





- Switched-current filters is an entirely different approach to designing filters with potential for overcoming many of the major problems facing the filter designer
- Other switched-current filter and integrator blocks have been proposed
- Integrators can be combined to form filter structures
- Single-ended and fully differential structures are readily formed
- Design of Switched-Current Filters is straightforward
- Beyond Hughes, a few others have looked at switched-current filters
- Hughes demonstrated experimentally modest performance with this technique
- Hughes was a world-class researcher and filter expert
- Hughes spent the better part of a decade trying to perfect the switched-current approach but performance remained modest when he retired
- Limited use of switched-current filters today
- Idea is really unique and there is bound to be some major useful applications of the basic concepts embodies in the switched-current filters!

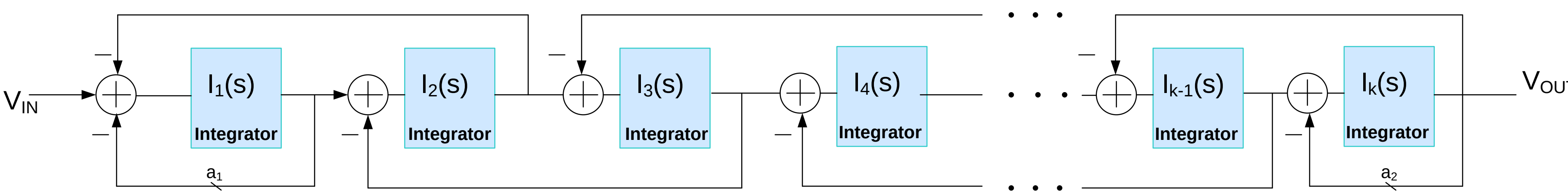
Leapfrog Filters



Introduced by Girling and Good, Wireless World, 1970

This structure has some very attractive properties and is widely used though the real benefits and limitations of the structure are often not articulated

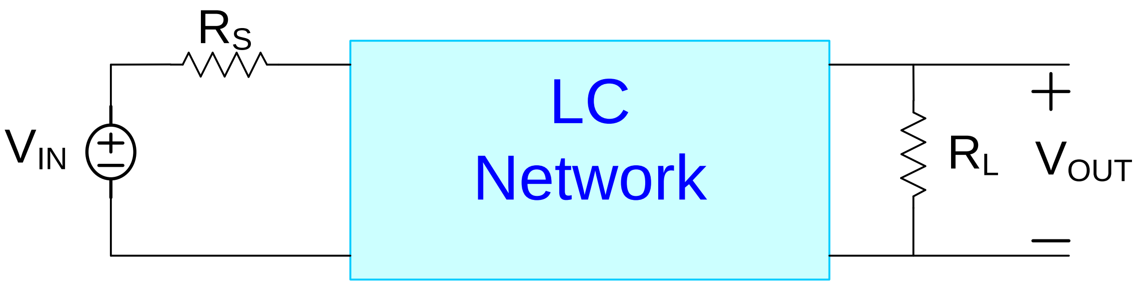
Leapfrog Filters



Observation: This structure appears to be dramatically different than anything else ever reported and it is not intuitive why this structure would serve as a filter, much less, have some unique and very attractive properties

To understand how the structure arose, why it has attractive properties, and to identify limitations, some mathematical background is necessary

Background Information for Leapfrog Filters



Assume the impedance R_S is fixed

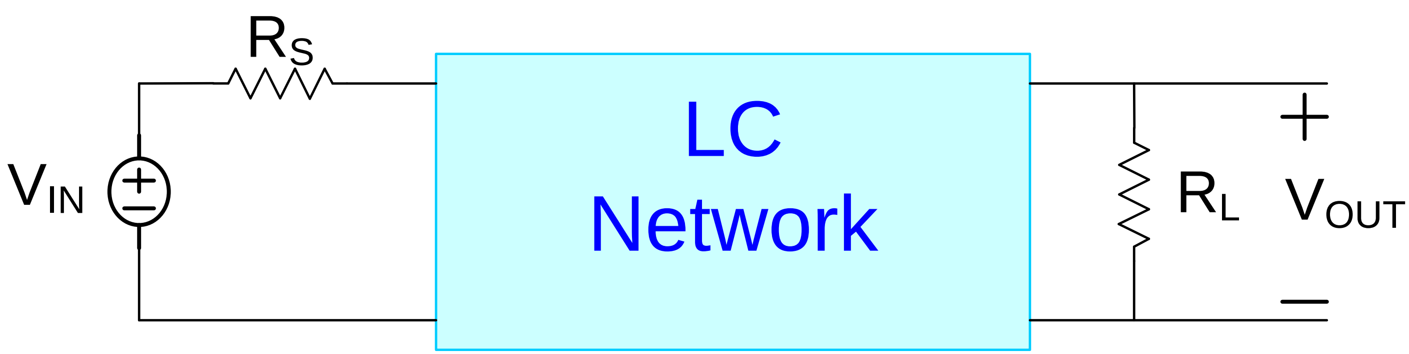
Theorem 1: If the LC network delivers maximum power to the load at a frequency ω, then

$$S_x^{|T(j\omega)|} = 0$$

for any circuit element in the system except for $x = R_L$

This theorem will follow after we prove the following theorem:

Background Information for Leapfrog Filters



Theorem 2: If the LC network delivers maximum power to the load at a frequency ω, then

$$S_x^{P_L(\omega)} = 0$$

where P(ω) is the power delivered to the load at input frequency ω and where x is any circuit element in the system except for x = R_L

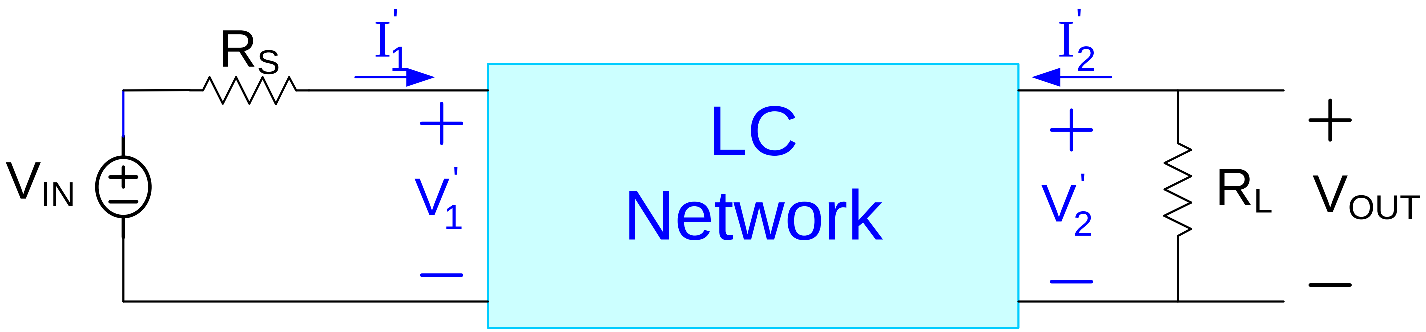
Note: There is no guarantee that there will be any frequencies where maximum power is transferred to the load and whether this does occur depends strongly on the LC circuit structure and the load R_L.

Proof of Theorem 2:

First, we will define the input impedance Z_{11}

Proof of Theorem 2:

Define the port phasors as $\{V_1^{'}, I_1^{'}, V_2^{'}, I_2^{'}\}$



$$Z_{11} = \frac{V_1^{'}}{I_1^{'}}$$

(input impedance to the LC Network)

this can be expressed as

$$Z_{11} = R_1 + jX_1$$

(R_1 and X_1 are real functions of ω and depend on R_L)

Since the LC network is lossless (dissipates no power) we have

$$P_L = \text{Re}\left(V_1^{'} \bullet I_1^{'*}\right)$$

$$P_L = \text{Re}\left(\left[\frac{R_1 + jX_1}{R_S + R_1 + jX_1} V_{in}\right] \bullet \left[\frac{V_{in}}{R_S + R_1 + jX_1}\right]^*\right)$$

$$P_L = |V_{in}|^2 \text{Re}\left(\frac{R_1 + jX_1}{(R_S + R_1)^2 + X_1^2}\right) = |V_{in}|^2 \frac{R_1}{(R_S + R_1)^2 + X_1^2}$$

Proof of Theorem 2:

$$P_L = |V_{in}|^2 \frac{R_1}{(R_S + R_1)^2 + X_1^2}$$

To maximize power delivered to a fixed load at a frequency ω, must have

$$\frac{\partial P_L}{\partial R_1} = 0 \qquad \frac{\partial P_L}{\partial X_1} = 0$$

$$\frac{\partial P_L}{R_1} = |V_{in}|^2 \left[\frac{\left((R_S + R_1)^2 + X_1^2\right) - R_1(2)(R_S + R_1)}{\left((R_S + R_1)^2 + X_1^2\right)^2} \right]$$

$$\frac{\partial P_L}{R_1} = |V_{in}|^2 \left[\frac{\left(R_S^2 + 2R_1R_S + R_S^2 + X_1^2 - 2R_1R_S - 2R_1^2\right)}{\left((R_S + R_1)^2 + X_1^2\right)^2} \right] = |V_{in}|^2 \left[\frac{\left(2\left(R_S^2 - R_1^2\right) + X_1^2\right)}{\left((R_S + R_1)^2 + X_1^2\right)^2} \right]$$

$$\frac{\partial P_L}{\partial R_1} = 0 \longrightarrow 2\left(R_S^2 - R_1^2\right) + X_1^2 = 0$$

$$\frac{\partial P_L}{X_1} = |V_{in}|^2 \left[\frac{-R_1(2X_1)}{\left((R_S + R_1)^2 + X_1^2\right)^2} \right]$$

$$\frac{\partial P_L}{\partial X_1} = 0 \longrightarrow X_1 = 0$$

$$\longrightarrow \quad X_1 = 0 \qquad (1)$$

$$R_1 = R_S \qquad (2)$$

Proof of Theorem 2:

$$X_1 = 0 \qquad (1) \qquad\qquad R_1 = R_S \qquad (2)$$

$$P_L = |V_{in}|^2 \frac{R_1}{(R_S + R_1)^2 + X_1^2}$$

Now let x be any element in the LC network

$$\frac{\partial P_L}{\partial x} = \frac{\partial P_L}{\partial R_1}\frac{\partial R_1}{\partial x} + \frac{\partial P_L}{\partial X_1}\frac{\partial X_1}{\partial x}$$

$$\frac{\partial P_L}{\partial x} = \left[|V_{in}|^2 \left[\frac{\left(2\left(R_S^2 - R_1^2\right) + X_1^2\right)}{\left((R_S + R_1)^2 + X_1^2\right)^2}\right]\right]\frac{\partial R_1}{\partial x} + \left[|V_{in}|^2 \left[\frac{-R_1\left(2X_1\right)}{\left((R_S + R_1)^2 + X_1^2\right)^2}\right]\right]\frac{\partial X_1}{\partial x}$$

It thus follows from (1) and (2) that at maximum power transfer, the two coefficients in this expression vanish, thus

$$\frac{\partial P_L}{\partial x} = \left[|V_{in}|^2 \left[\frac{0}{\left((R_S + R_1)^2 + X_1^2\right)^2}\right]\right]\frac{\partial R_1}{\partial x} + \left[|V_{in}|^2 \left[\frac{0}{\left((R_S + R_1)^2 + X_1^2\right)^2}\right]\right]\frac{\partial X_1}{\partial x} = 0$$

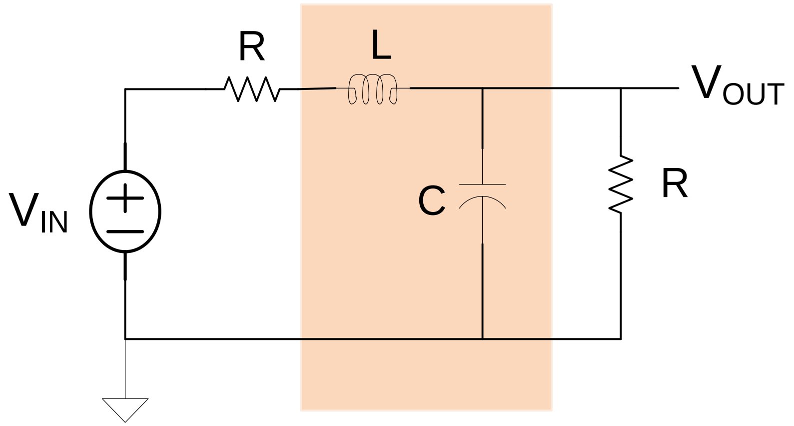
thus

$$S_x^{P_L} = \frac{\partial P_L}{\partial x}\frac{x}{P_L} = 0 \qquad \blacksquare$$

Question: Can we also make the claim that $S_{R_L}^{P_L(\omega)} = 0$ at any frequency where maximum power is transferred to the load?

Yes! Note that the previous analysis is based upon characterizing R_1 and X which are functions of k reactive components, $\{x_1, \quad x_k\}$ and R_L.

The following circuit has maximum power transfer at dc and it can be easily analytically shown that the sensitivity of P to L, C, and R_L is 0 at dc.



Proof of Theorem 1: $S_x^{|T(j\omega)|} = ?$

$$P_L = \mathrm{Re}\left(V_{out} \bullet \left(\frac{V_{out}}{R_L} \right)^* \right)$$

$$P_L = \mathrm{Re}\left(V_{in} T(j\omega) \bullet \left(\frac{V_{in} T(j\omega)}{R_L} \right)^* \right)$$

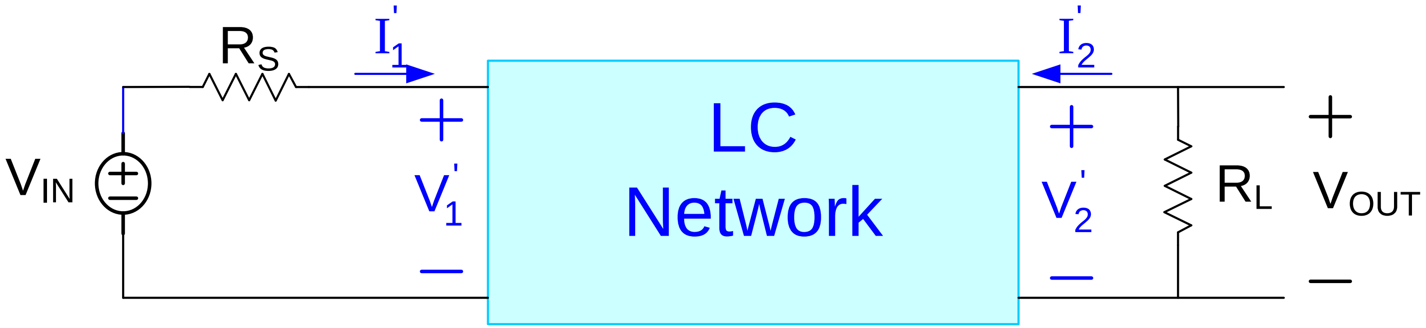
$$P_L = \left(\frac{|V_{in}|^2}{R_L} \right) \bullet |T(j\omega)|^2$$

Recall the following two sensitivity relationships

$$S_x^{kf} = S_x^{f} \qquad\qquad S_x^{f^2} = 2 \bullet S_x^{f}$$

It thus follows that

$$S_x^{P_L} = 2 \bullet S_x^{|T(j\omega)|} \xrightarrow{S_x^{P_L} = 0} S_x^{|T(j\omega)|} = 0 \qquad \blacksquare$$



Lemma: If maximum power is transferred to the load in the doubly-terminated LC network at a frequency ω, then

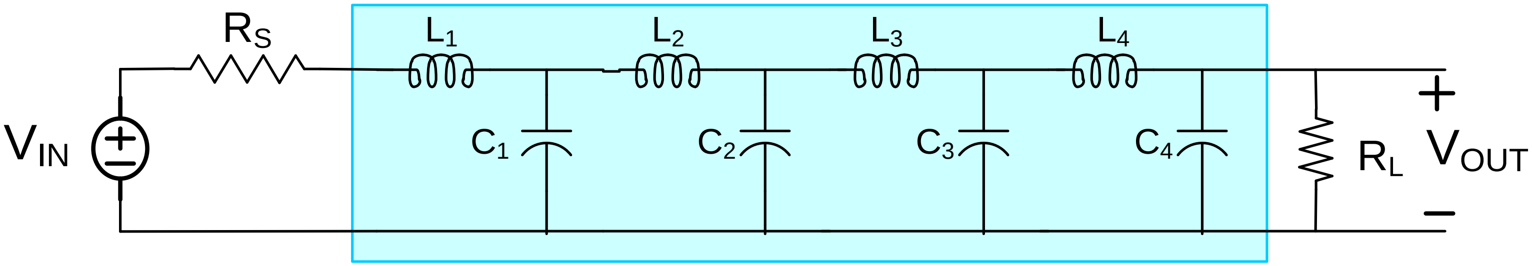
$$\left.\frac{V_2'(j\omega)}{I_2'(j\omega)}\right|_{V_{in}=0} = R_L$$

This lemma indicates that the impedance of the LC network loaded with R_S facing R_L is equal to R_L at frequencies where maximum-power is transferred to R_L

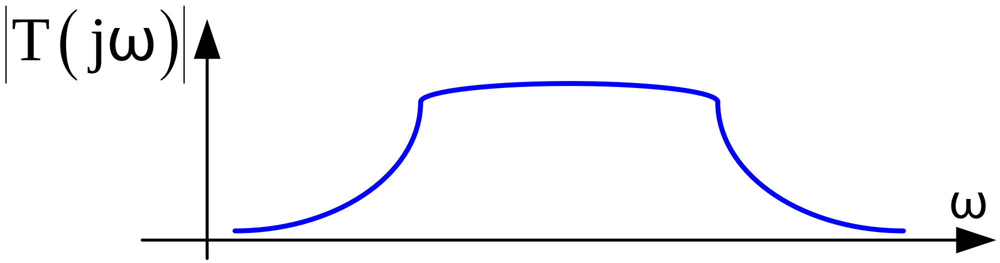
The proof follows directly by considering the Thevenin-equivalent circuit facing R_L

Implications of Theorem 1

Many passive LC filter such as that shown below exist that have near maximum power transfer in the passband

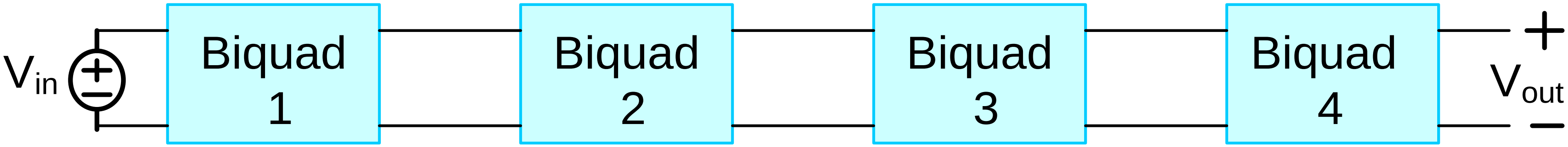


If a component in the LC network changes a little, there is little change in the passband gain characteristics (depicted as bandpass)

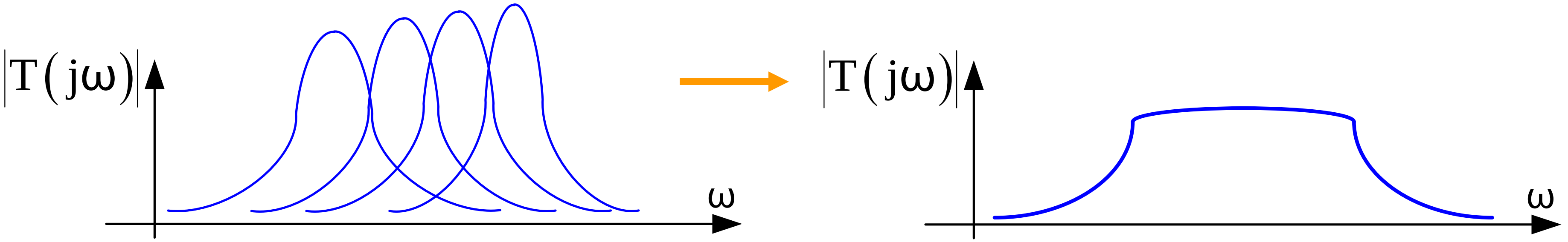


$$S_x^{|T(j\omega)|} \simeq 0$$ in passband

Implications of Theorem 1

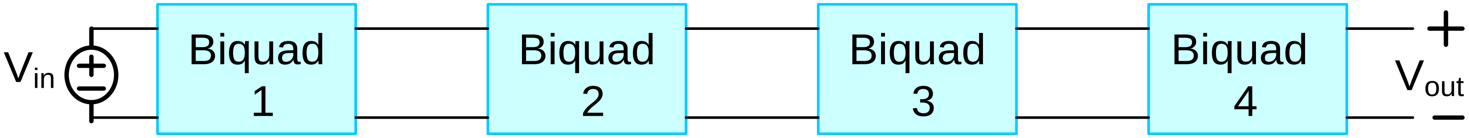


Cascaded Biquad has a response that is the product of the individual second-order transfer functions

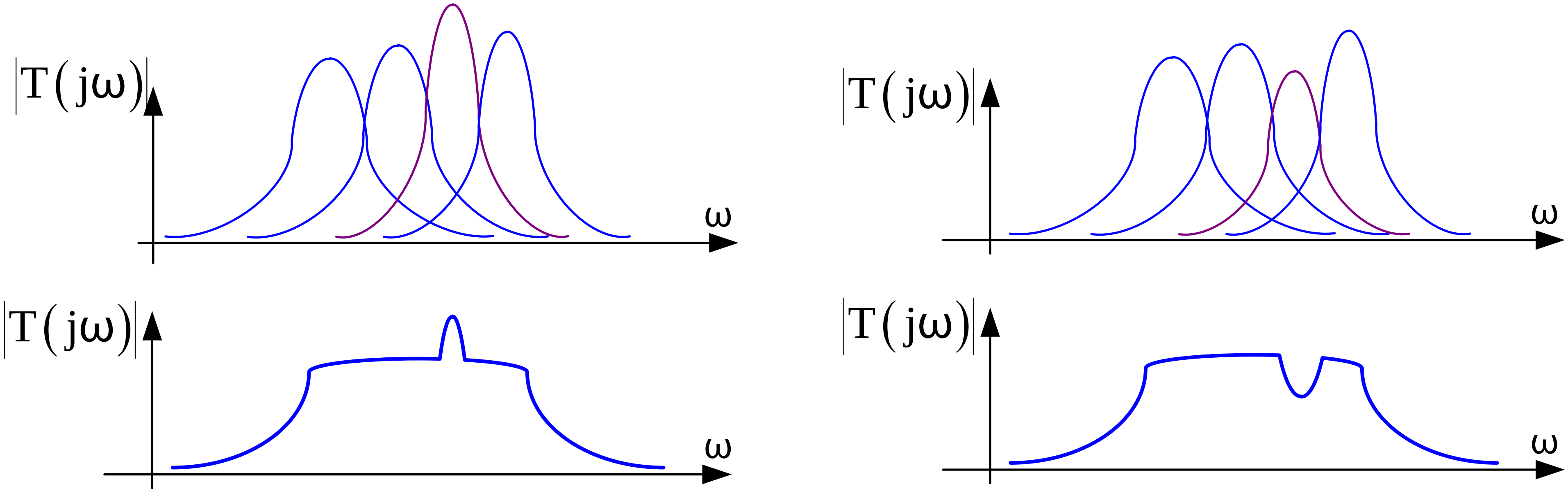


If a component in a biquad changes a little, there is often a large change in the passband gain characteristics (depicted as bandpass)

Implications of Theorem 1

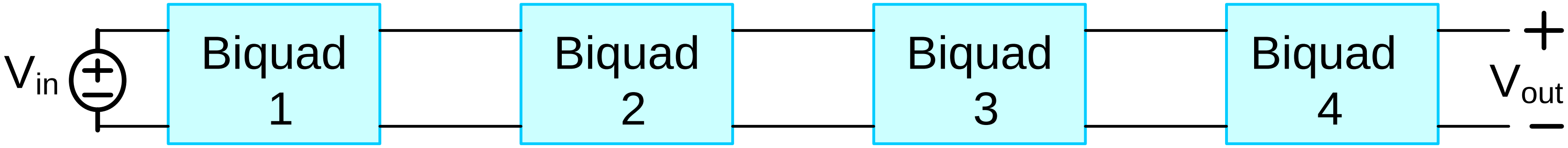


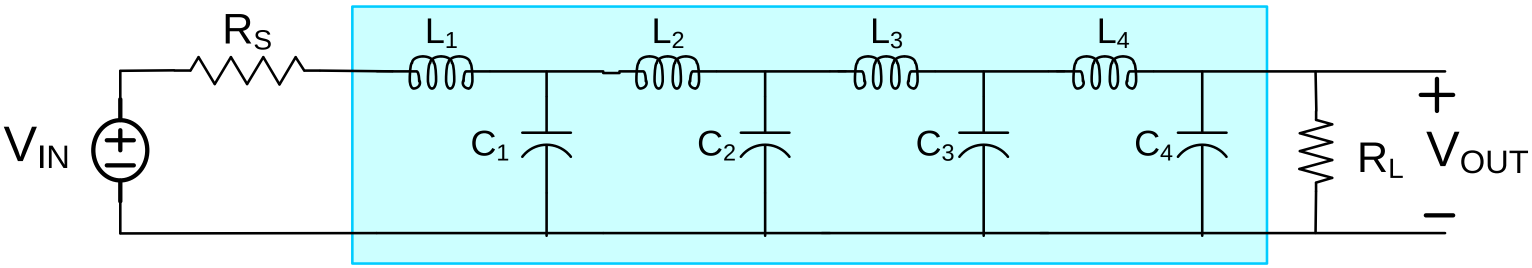
If a component in a biquad changes a little, there is often a large change in the passband gain characteristics (depicted as bandpass)



$$S_x^{|T(j\omega)|} \neq 0$$ in passband

Implications of Theorem 1

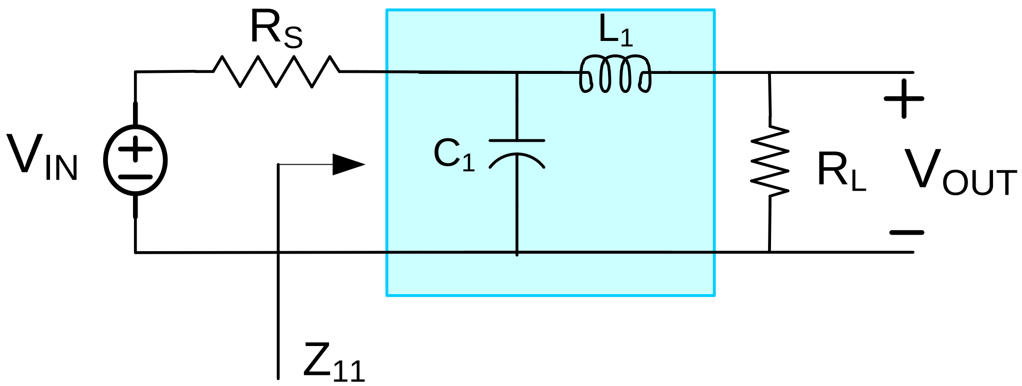




Good doubly-terminated LC networks often much less sensitive to most component values in the passband than are cascaded biquads !

This is a major advantage of the LC networks but can not be applied practically used in most integrated applications or even in pc-board based designs

Example: Determine at what frequencies maximum-power transfer to the load will occur and what value of R_L is needed for this to happen



Recall at maximum-power transfer, Z_{11} is real and equal to R_S

$$Z_{11} = \frac{R_L + sL}{s^2LC + sR_LC + 1}$$

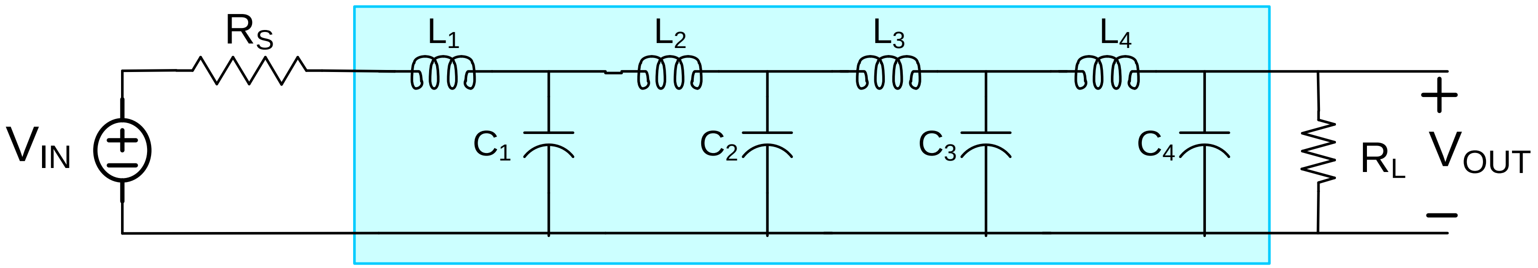
$$Z_{11}(j\omega) = \left(\frac{R_L}{\left(1-\omega^2LC\right)^2 + \omega^2R_L^2C}\right) + j\left(\frac{\omega L - \omega^2R_L^2C - \omega^3L^2C}{\left(1-\omega^2LC\right)^2 + \omega^2R_L^2C}\right)$$

$\mathrm{Im}\left(Z_{11}(j\omega)\right) = 0$ only at $\omega=0$ and one other positive value of ω

To get maximum power transfer at $\omega=0$, must have $R_L=R_S$

Appears not to have maximum power transfer at other frequency where $\mathrm{Im}\left(Z_{11}(j\omega)\right) \neq 0$

Consider again the doubly-terminated circuit that has multiple passband frequencies where maximum power transfer to the load occurs

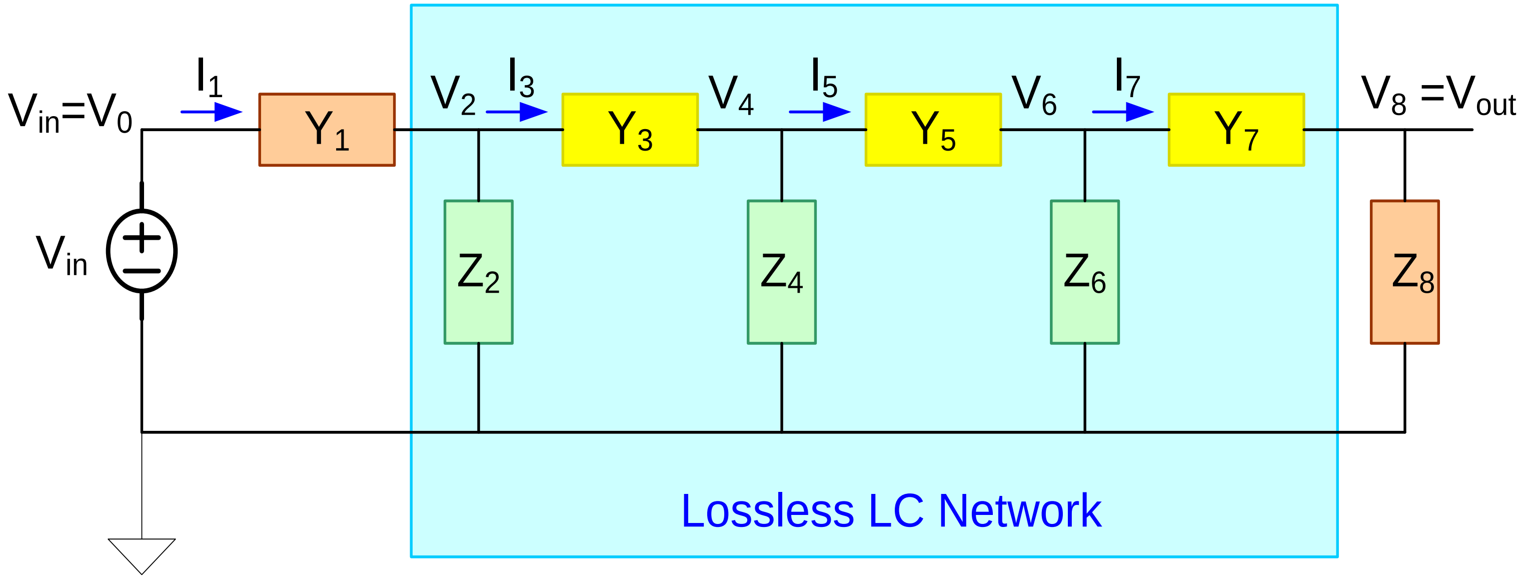


Observe that this structure is completely characterized by a set of equations that characterize the network

All sensitivity properties are inherently determined by this set of equations

Any circuit that has the same set of equations will have the same sensitivity properties

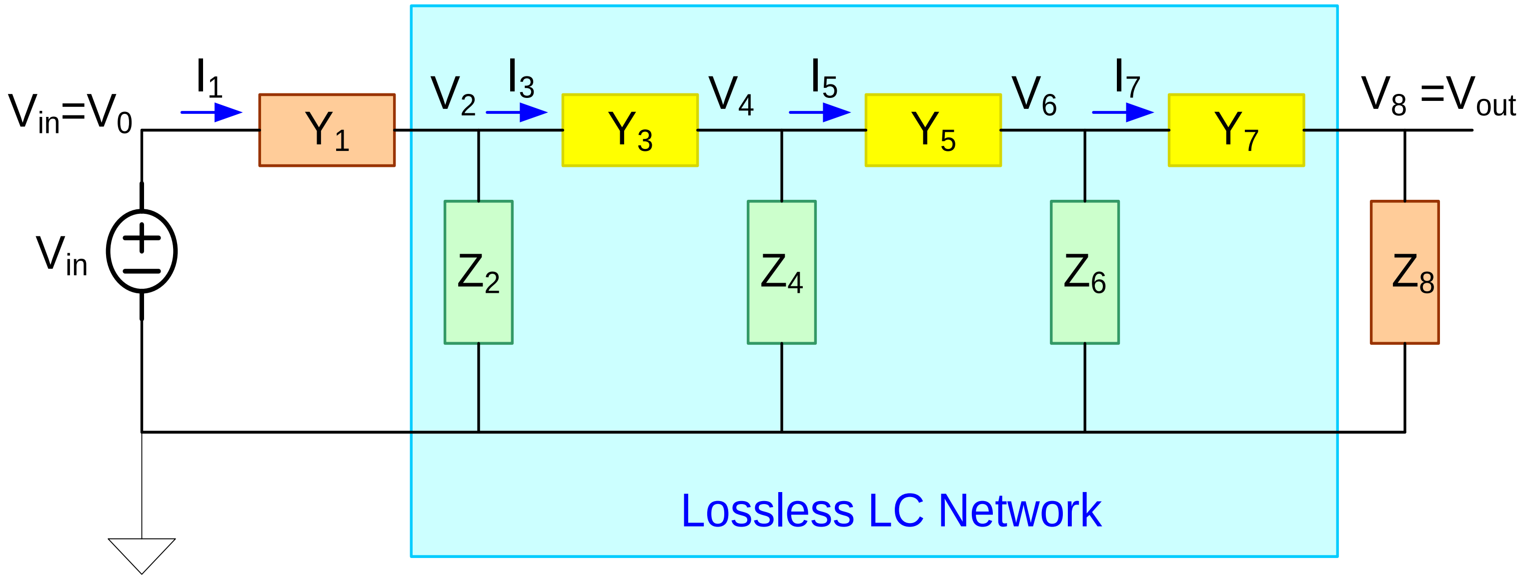
Doubly-terminated Ladder Network with Low Passband Sensitivities



For components in the LC Network observe

$$Y_k = \frac{1}{sL_k} \qquad\qquad Z_k = \frac{1}{sC_k}$$

Doubly-terminated Ladder Network with Low Passband Sensitivities



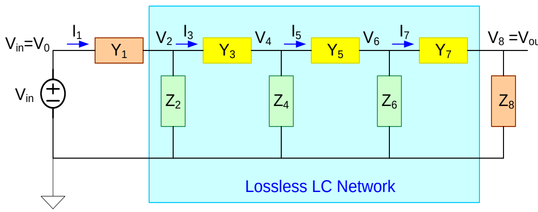
$$\left.\begin{aligned}
I_1 &= \left(V_0 - V_2\right)Y_1 \\
V_2 &= \left(I_1 - I_3\right)Z_2 \\
I_3 &= \left(V_2 - V_4\right)Y_3 \\
V_4 &= \left(I_3 - I_5\right)Z_4 \\
I_5 &= \left(V_4 - V_6\right)Y_5 \\
V_6 &= \left(I_5 - I_7\right)Z_6 \\
I_7 &= \left(V_6 - V_8\right)Y_7 \\
V_8 &= I_7 Z_8
\end{aligned}\right\}$$

Complete set of independent equations that characterize this filter

Solution of this set of equations is tedious

All sensitivity properties of this circuit are inherently embedded in these equations!

Consider now only the set of equations and disassociate them from the circuit from where they came



$$\left.\begin{aligned}
I_1 &= (V_0 - V_2)Y_1 \\
V_2 &= (I_1 - I_3)Z_2 \\
I_3 &= (V_2 - V_4)Y_3 \\
V_4 &= (I_3 - I_5)Z_4 \\
I_5 &= (V_4 - V_6)Y_5 \\
V_6 &= (I_5 - I_7)Z_6 \\
I_7 &= (V_6 - V_8)Y_7 \\
V_8 &= I_7 Z_8
\end{aligned}\right\}$$

Make the associations

$$I_1 = V_1^{'}$$

$$I_3 = V_3^{'}$$

$$I_5 = V_5^{'}$$

$$I_7 = V_7^{'}$$

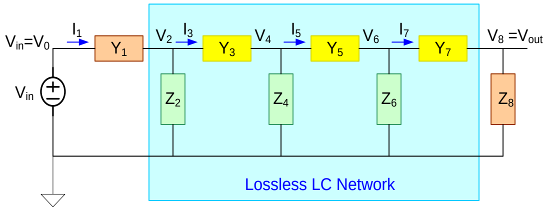
Rewrite the equations as

$$\left.\begin{aligned}
V_1^{'} &= (V_0 - V_2)Y_1 \\
V_2 &= \left(V_1^{'} - V_3^{'}\right)Z_2 \\
V_3^{'} &= (V_2 - V_4)Y_3 \\
V_4 &= \left(V_3^{'} - V_5^{'}\right)Z_4 \\
V_5^{'} &= (V_4 - V_6)Y_5 \\
V_6 &= \left(V_5^{'} - V_7^{'}\right)Z_6 \\
V_7^{'} &= (V_6 - V_8)Y_7 \\
V_8 &= V_7^{'} Z_8
\end{aligned}\right\}$$

This association is nothing more than a renaming of variables so all sensitivities WRT Y's and Z's will remain unchanged!

Consider now only the set of equations and disassociate them from the circuit from where they came

$$\left.\begin{aligned}
V_1' &= (V_0 - V_2)Y_1 \\
V_2 &= (V_1' - V_3')Z_2 \\
V_3' &= (V_2 - V_4)Y_3 \\
V_4 &= (V_3' - V_5')Z_4 \\
V_5' &= (V_4 - V_6)Y_5 \\
V_6 &= (V_5' - V_7')Z_6 \\
V_7' &= (V_6 - V_8)Y_7 \\
V_8 &= V_7' Z_8
\end{aligned}\right\}$$



For the LC filter, recall

$$Y_k = \frac{1}{sL_k} \qquad Z_k = \frac{1}{sC_k}$$

And the source and load termination relationships were

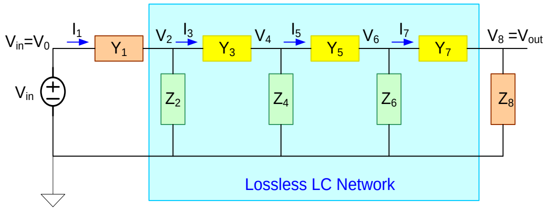
$$Y_1 = \frac{1}{R_1} \qquad Z_8 = R_8$$

These can be written as

$$\left.\begin{aligned}
V_1' &= (V_0 - V_2)\frac{1}{R_1} & V_5' &= (V_4 - V_6)\frac{1}{sL_5} \\
V_2 &= (V_1' - V_3')\frac{1}{sC_2} & V_6 &= (V_5' - V_7')\frac{1}{sC_6} \\
V_3' &= (V_2 - V_4)\frac{1}{sL_3} & V_7' &= (V_6 - V_8)\frac{1}{sL_7} \\
V_4 &= (V_3' - V_5')\frac{1}{sC_4} & V_8 &= V_7' R_8
\end{aligned}\right\}$$

Observe that in the new parameter domain the equations all look like integrator functions if the primed and unprimed variables are all voltages !

Consider now only the set of equations and disassociate them from the circuit from where they came

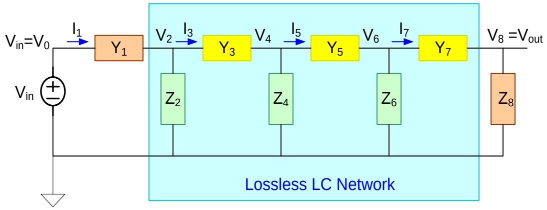


$$\left.\begin{aligned}
V_1' &= (V_0 - V_2)\frac{1}{R_1} & V_5' &= (V_4 - V_6)\frac{1}{sL_5} \\
V_2 &= (V_1' - V_3')\frac{1}{sC_2} & V_6 &= (V_5' - V_7')\frac{1}{sC_6} \\
V_3' &= (V_2 - V_4)\frac{1}{sL_3} & V_7' &= (V_6 - V_8)\frac{1}{sL_7} \\
V_4 &= (V_3' - V_5')\frac{1}{sC_4} & V_8 &= V_7' R_8
\end{aligned}\right\}$$

Observe that in the new parameter domain the equations all look like integrator functions if the primed and unprimed variables are all voltages !

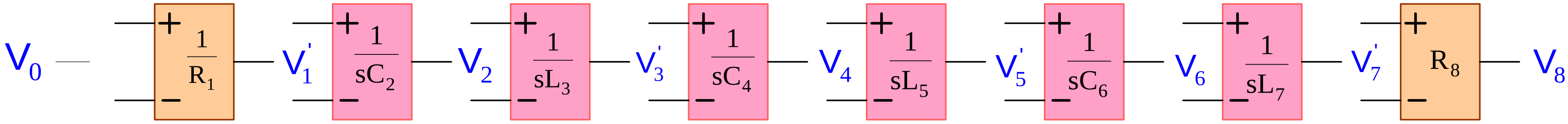
If any circuit is characterized by these equations, the sensitivities to the integrator gains will be identical to the sensitiviies of the original circuit to the Ls and Cs !

Consider now only the set of equations and disassociate them from the circuit from where they came



$$\left.\begin{aligned}
V_1' &= (V_0 - V_2)\frac{1}{R_1} & \qquad V_5' &= (V_4 - V_6)\frac{1}{sL_5}\\
V_2 &= (V_1' - V_3')\frac{1}{sC_2} & \qquad V_6 &= (V_5' - V_7')\frac{1}{sC_6}\\
V_3' &= (V_2 - V_4)\frac{1}{sL_3} & \qquad V_7' &= (V_6 - V_8)\frac{1}{sL_7}\\
V_4 &= (V_3' - V_5')\frac{1}{sC_4} & \qquad V_8 &= V_7' R_8
\end{aligned}\right\}$$

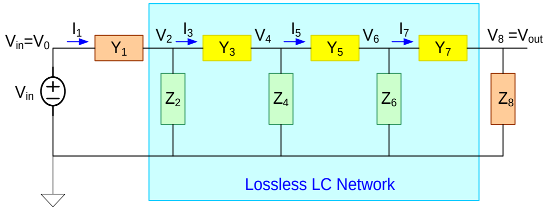
Each equation corresponds to either an integrator or summer with the output voltage output variables and the gain indicated (don't worry about the units)



$$V_0 = V_{in}$$

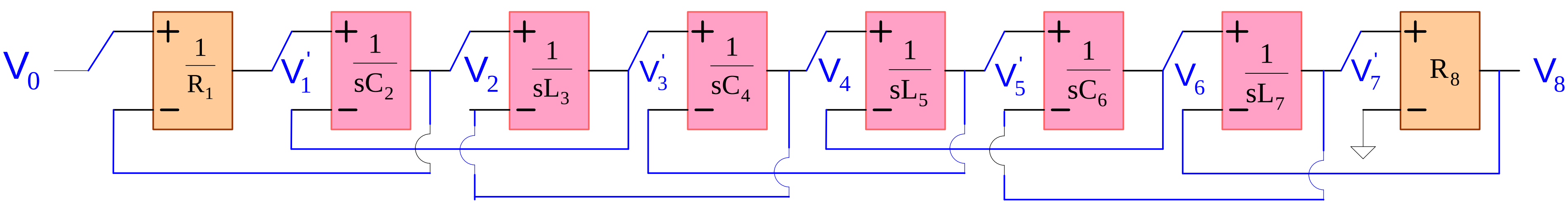
$$V_8 = V_{out}$$

Consider now only the set of equations and disassociate them from the circuit from where they came

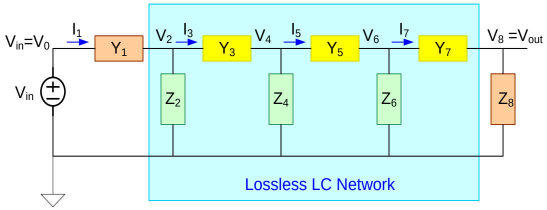


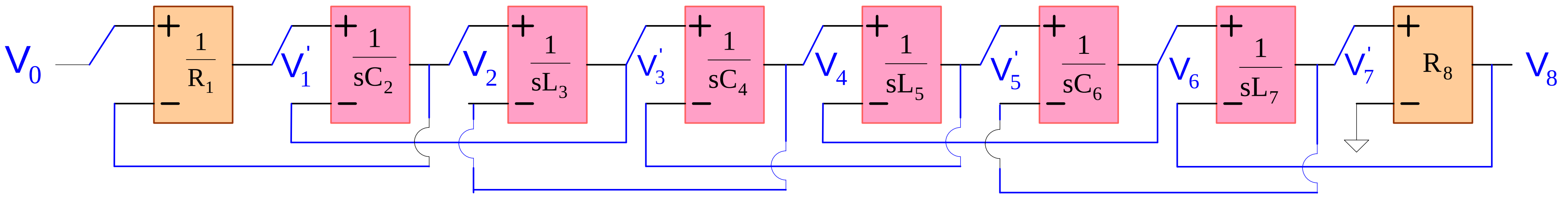
$$
\left.
\begin{array}{ll}
V_1' = (V_0 - V_2)\dfrac{1}{R_1} & V_5' = (V_4 - V_6)\dfrac{1}{sL_5} \\[2ex]
V_2 = (V_1' - V_3')\dfrac{1}{sC_2} & V_6 = (V_5' - V_7')\dfrac{1}{sC_6} \\[2ex]
V_3' = (V_2 - V_4)\dfrac{1}{sL_3} & V_7' = (V_6 - V_8)\dfrac{1}{sL_7} \\[2ex]
V_4 = (V_3' - V_5')\dfrac{1}{sC_4} & V_8 = V_7' R_8
\end{array}
\right\}
$$

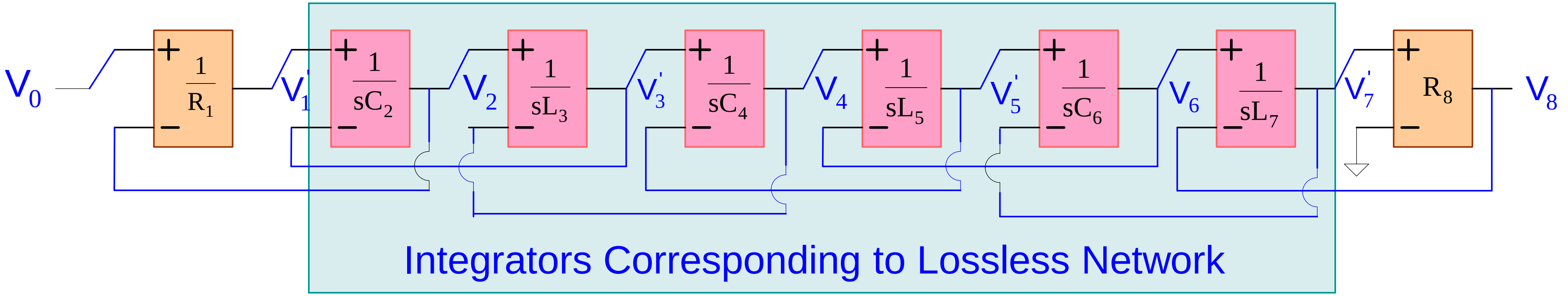
The interconnections that complete each equation can now be added



Consider now only the set of equations and disassociate them from the circuit from where they came



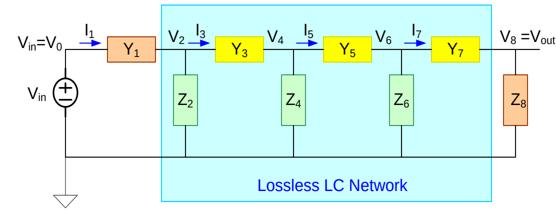


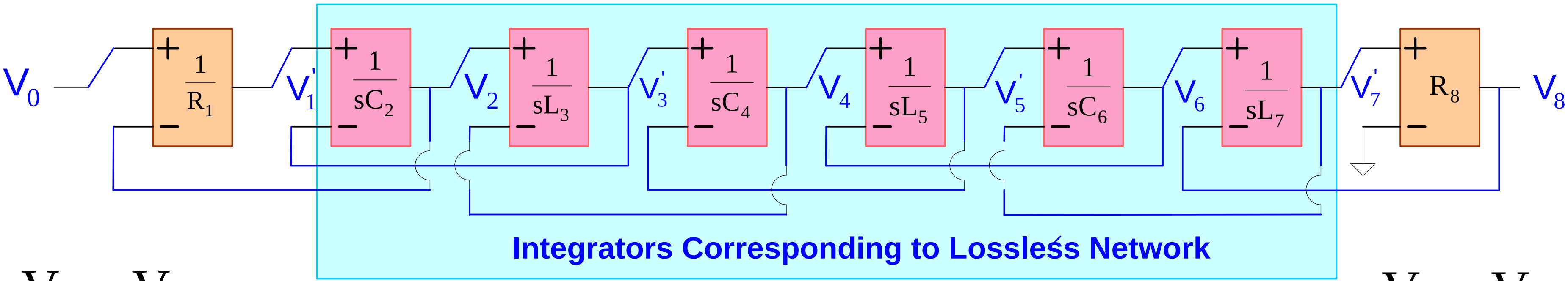


$$V_0 = V_{in}$$

$$V_8 = V_{out}$$

The Leapfrog Configuration



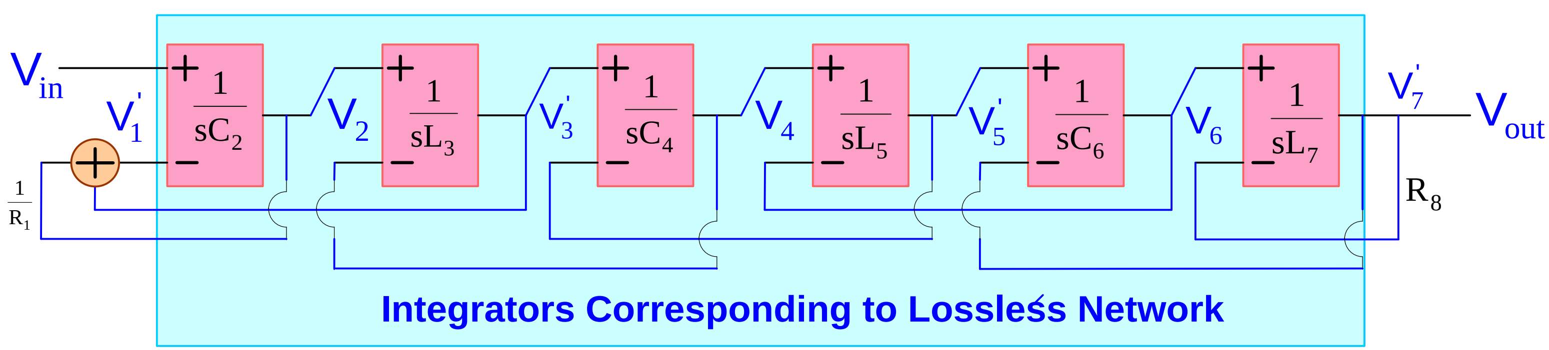


$$V_0 = V_{in}$$

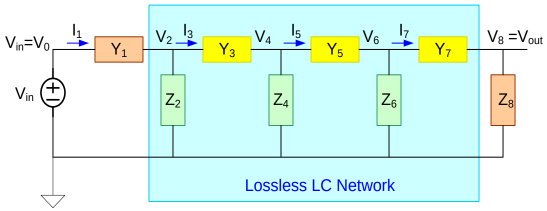
$$V_8 = V_{out}$$

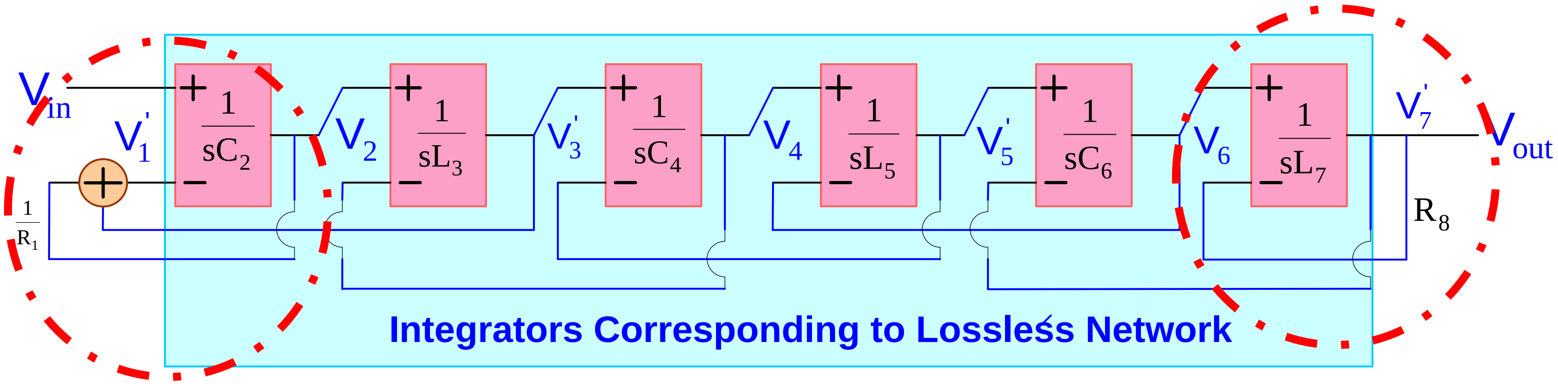
Input summing and weighting can occur at input to the first integrator

The difference between V_8 and V'_7 is only a scale factor that does not affect shape, and the weighting on the Vin input also does not affect shape, thus



The Leapfrog Configuration

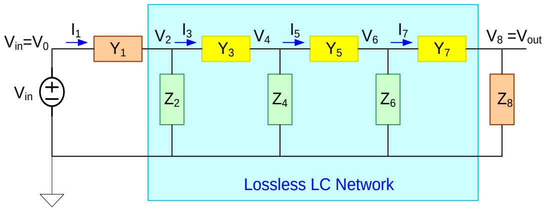


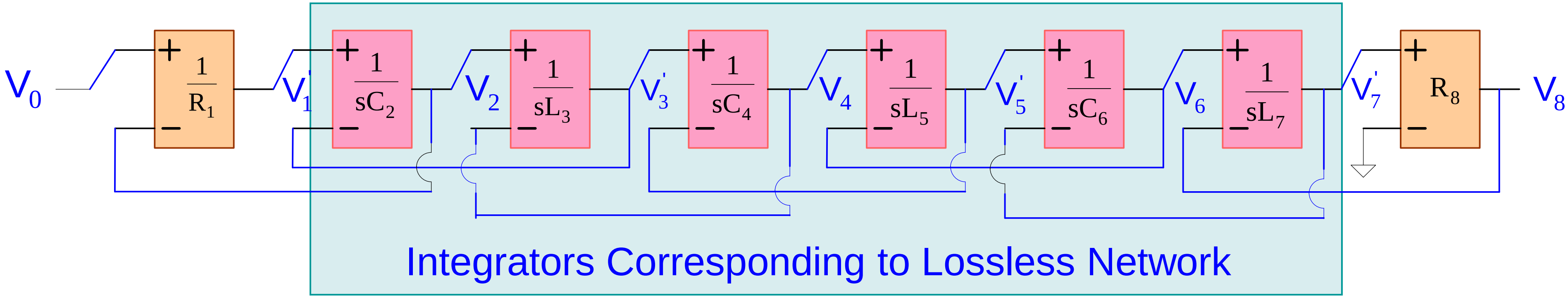


The terminations on both sides have local feedback around an integrator which can be alternately viewed as a lossy integrator

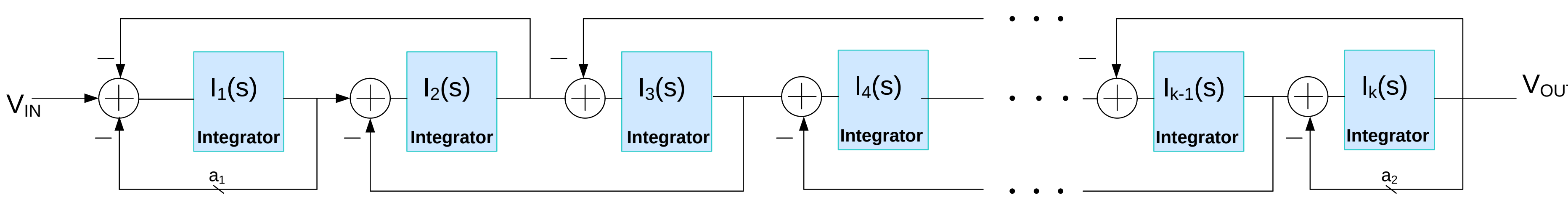
Could redraw the structure as a cascade of internal lossless integrators with terminations that are lossy integrators but since there are so many different ways to implement the integrators and summers, we will not attempt to make that association in the block diagram form but in most practical applications a lossy integrator is often used on the input or the output or both

The Leapfrog Configuration



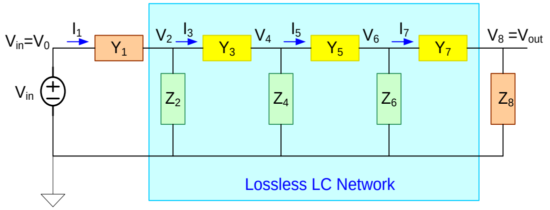


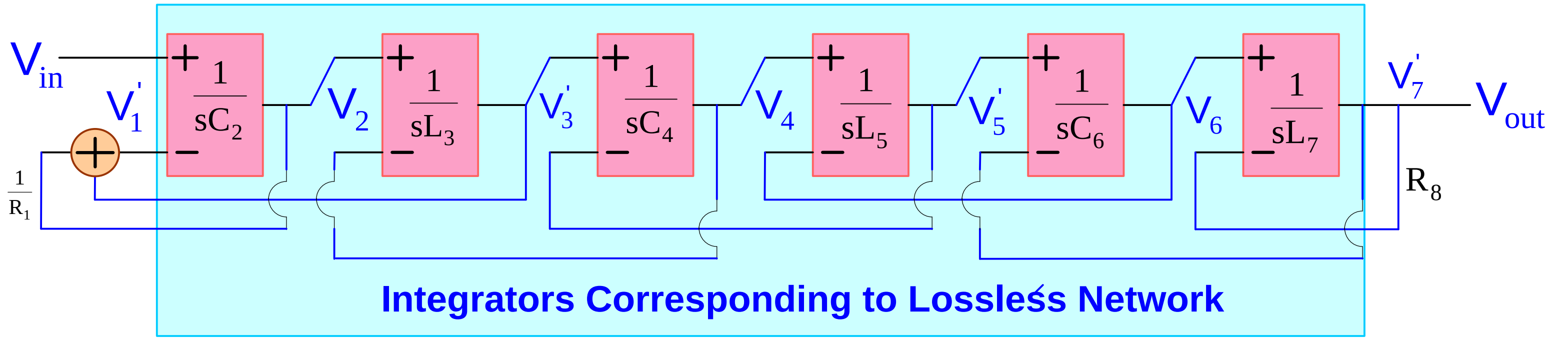
In the general case, this can be redrawn as shown below



Note the first and last integrators become lossy because of the local feedback

The Leapfrog Configuration





The passive prototype filter from which the leapfrog was designed has all shunt capacitors and all series inductors and is thus lowpass.

The resultant leapfrog filter has the same transfer function and is thus lowpass