EE 508 Lecture 33

Leapfrog Networks

Leapfrog Filters



Introduced by Girling and Good, Wireless World, 1970

This structure has some very attractive properties and is widely used though the real benefits and limitations of the structure are often not articulated

Background Information for Leapfrog Filters



Assume the impedance R_S is fixed



This theorem will follow after we prove the following theorem:

Implications of Theorem 1

Many passive LC filter such as that shown below exist that have near maximum power transfer in the passband



If a component in the LC network changes a little, there is little change in the passband gain characteristics (depicted as bandpass)



Implications of Theorem 1



Cascaded Biquad has a response that is the product of the individual second-order transfer functions



If a component in a biquad changes a little, there is often a large change in the passband gain characteristics (depicted as bandpass)

Implications of Theorem 1



If a component in a biquad changes a little, there is often a large change in the passband gain characteristics (depicted as bandpass)



Consider now only the set of equations and disassociate them from the circuit from where they came





Each equation corresponds to either an integrator or summer with the output voltage output variables and the gain indicated (don't worry about the units)

$$V_{0} - \underbrace{ \begin{bmatrix} + & 1 \\ R_{1} \end{bmatrix}}_{R_{1}} - \underbrace{ V_{1}^{'}}_{I} \underbrace{ \frac{+ 1}{sC_{2}}}_{R_{2}} - \underbrace{ V_{2}}_{2} \underbrace{ \frac{+ 1}{sL_{3}}}_{SL_{3}} - \underbrace{ V_{3}^{'}}_{S} \underbrace{ \frac{+ 1}{sC_{4}}}_{SC_{4}} - \underbrace{ V_{4}}_{4} \underbrace{ \frac{+ 1}{sL_{5}}}_{SL_{5}} - \underbrace{ V_{5}^{'}}_{S} \underbrace{ \frac{+ 1}{sC_{6}}}_{SL_{7}} - \underbrace{ V_{7}^{'}}_{R_{8}} \underbrace{ \frac{+ R_{8}}{R_{8}}}_{N_{8}} - \underbrace{ V_{8}}_{S} \underbrace{ V_{8}}_{S} = V_{0ut}$$

The Leapfrog Configuration





In the general case, this can be redrawn as shown below



Note the first and last integrators become lossy because of the local feedback

Doubly-terminated LC filters with near maximum power transfer in the passband were developed from the 30's to the 60's

Seldom discussed in current texts but older texts and occasionally software tools provide the passive structures needed to synthesize leapfrog networks

One good book is that by Zverev





The Butterworth Low-Pass Filters



Leading element is a shunt capacitor

(appear from top to bottom in table)

n ODD

1.0

n EVEN



Leading element is a series capacitor

(appear from bottom to top in table)

Can do Thevenin-Norton Transformations

n	Rs	C ₁	L ₂	C ₃	L ₄
2	1.0000 1.1111 1.2500 1.4286 1.6667 2.0000 2.5000 3.3333 5.0000 10.0000 INF.	1.4142 1.0353 0.8485 0.6971 0.5657 0.4483 0.3419 0.2447 0.1557 0.0743 1.4142	1.4142 1.8352 2.1213 2.4387 2.8284 3.3461 4.0951 5.3126 7.7067 14.8138 0.7071		
3	1.0000 0.9000 0.8000 0.7000 0.6000 0.5000 0.4000 0.3000 0.2000 0.1000 INF.	1.0000 0.8082 0.8442 0.9152 1.0225 1.1811 1.4254 1.8380 2.6687 5.1672 1.5000	2.0000 1.6332 1.3840 1.1652 0.9650 0.7789 0.6042 0.4396 0.2842 0.1377 1.3333	1.0000 1.5994 1.9259 2.2774 2.7024 3.2612 4.0642 5.3634 7.9102 15.4554 0.5000	
4	1.0000 1.1111 1.2500 1.4286 1.6667 2.0000 2.5000 3.3333 5.0000 10.0000 INF.	0.7654 0.4657 0.3882 0.3251 0.2690 0.2175 0.1692 0.1237 0.0804 0.0392 1.5307	1.8478 1.5924 1.6946 1.8618 2.1029 2.4524 2.9858 3.8826 5.6835 11.0942 1.5772	1.8478 1.7439 1.5110 1.2913 1.0824 0.8826 0.6911 0.5072 0.3307 0.1616 1.0824	0.7654 1.4690 1.8109 2.1752 2.6131 3.1868 4.0094 5.3381 7.9397 15.6421 0.3827
n	1/R	L ₁	C ₂	L ₃	C ₄

Normalized so $R_L=1$

n	Rs	C ₁	L ₂	C ₃	L ₄	C ₅	L ₆	C ₇
		0.0100	1 0100	0.0000	1 0100	0 0100		
	1.0000	0.6180	1.6180	2.0000	1.6180	0.0180		
	0.9000	0.4416	1.0265	1.9095	1.7562	1.3887		1
	0.8000	0.4698	0.8660	2.0605	1.5443	1.7380		T
	0.7000	Q.5173	0.7313	2.2849	1.3326	2.1083		
5	0.6000	0.5860	0.6094	2.5998	1.1255	2.5524		
	0.5000	0.6857	0.4955	3.0510	0.9237	3.1331		
	0.4000	0.8378	0.3877	3.7357	0.7274	3.9648		
	0.3000	1.0937	0.2848	4.8835	0.5367	5.3073		
	0.2000	1.5077	0.1861	7.1849	0.3518	7.9345		
	0.1000	3.1522	0.0912	14.0945	0.1727	15.7103		
	INF.	1.5451	1.6944	1.3820	0.8944	0.3090		
	1 0000	0.5150	- h-h-			<u> </u>		
	1,0000	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176	
	1.1111	0.2890	1.0403	1.3217	2.0539	1.7443	1.3347	
	1.2500	0.2445	1.1163	1.1257	2.2389	1.5498	1.6881	1.
	1.4286	0.2072	1.2363	0.9567	2.4991	1.3464	2.0618	
	1.6667	0.1732	1.4071	0.8011	2.8580	1.1431	2.5092	
6	2.0000	0.1412	1.6531	0.6542	3.3687	0.9423	3.0938	
183.4	2.5000	0.1108	2.0275	0.5139	4.1408	0.7450	3.9305	
515.1	3.3333	0.0816	2.6559	0.3788	5.4325	0.5517	5.2804	1
10-1	5.0000	0.0535	3.9170	0.2484	8.0201	0.3628	7.9216	
80	10.0000	0.0263	7.7053	0.1222	15.7855	0.1788	15.7375	
0	INF.	1.5529	1.7593	1.5529	1.2016	0.7579	0.2588	
	1 0000							
	1.0000	0.4450	1.2470	T-8018	2.0000	T 8019	1.2470	0.4450
	0.9000	0.2985	0.7111	1.4043	1.4891	2.1249	1.7268	1.2961
	0.8000	0.3215	0.6057	1.5174	1.2777	2.3338	1.5461	1.6520
8	0.7000	0.3571	0.5154	1.6883	1.0910	2.6177	1.3498	2.0277
7	0.6000	0.4075	0.4322	1.9284	0.9170	3.0050	1.1503	2.4771
1	0.5000	0.4799	0.3536	2.2726	0.7512	3.5532	0.9513	3.0640
	0.4000	0.5899	0.2782	2.7950	0.5917	4.3799	0.7542	3.9037
	0.3000	0.7745	0.2055	3.6706	0.4373	5.7612	0.5600	5.2583
	0.2000	1.1448	0.1350	5.4267	0.2874	8.5263	0.3692	7.9079
	0.1000	2.2571	0.0665	10.7004	0.1417	16.8222	0.1823	15.7480
	INF.	1.5576	1.7988	1.6588	1.3972	1.0550	0.6560	0.2225
n	1/R	L ₁	C,	L	C,	L_	C.	T.

Example:

Design a 6th-order BW lowpass Leapfrog filter with a leading capacitor, with equal source and load terminations, and with a 3dB band edge of 4KHz.

Start with the normalized BW lowpass filter



Do Norton to Thevenin transformation at input

n	R _s	C ₁	L ₂	C ₃	L ₄	C ₅	L ₆	C ₇
5	1.0000 0.9000 0.8000 0.7000 0.6000 0.5000 0.4000 0.3000 0.2000 0.1000 INF.	0.6180 0.4416 0.4698 0.5173 0.5860 0.6857 0.8378 1.0937 1.6077 3.1522 1.5451	1.6180 1.0265 0.8660 0.7313 0.6094 0.4955 0.3877 0.2848 0.1861 0.0912 1.6944	2.0000 1.9095 2.0605 2.2849 2.5998 3.0510 3.7357 4.8835 7.1849 14.0945 1.3820	1.6180 1.7562 1.5443 1.3326 1.1255 0.9237 0.7274 0.5367 0.3518 0.1727 0.8944	0.6180 1.3887 1.7380 2.1083 2.5524 3.1331 3.9648 5.3073 7.9345 15.7103 0.3090		
	1.0000	0.5176	1.4142	1,9319	1.9319	1.4142	0.5176	
6	1.111 1.2500 1.4286 1.6667 2.0000 2.5000 3.3333 5.0000 10.0000 INF.	0.2000 0.2445 0.2072 0.1732 0.1412 0.1108 0.0816 0.0535 0.0263 1.5529	1.0403 1.1163 1.2363 1.4071 1.6531 2.0275 2.6559 3.9170 7.7053 1.7593	1.3217 1.1257 0.9567 0.8011 0.6542 0.5139 0.3788 0.2484 0.1222 1.5529	2.0539 2.2389 2.4991 2.8580 3.3687 4.1408 5.4325 8.0201 15.7855 1.2016	1.7443 1.5498 1.3464 1.1431 0.9423 0.7450 0.5517 0.3628 0.1788 0.7579	1.3347 1.6881 2.0618 2.5092 3.0938 3.9305 5.2804 7.9216 15.7375 0.2588	
7	1.0000 0.9000 0.8000 0.7000 0.6000 0.5000 0.4000 0.3000 0.2000 0.1000 INF.	0.4450 0.2985 0.3215 0.3571 0.4075 0.4799 0.5899 0.7745 1.1448 2.2571 1.5576	1.2470 0.7111 0.6057 0.5154 0.4322 0.3536 0.2782 0.2055 0.1350 0.0665 1.7988	1.8019 1.4043 1.5174 1.6883 1.9284 2.2726 2.7950 3.6706 5.4267 10.7004 1.6588	2.0000 1.4891 1.2777 1.0910 0.9170 0.7512 0.5917 0.4373 0.2874 0.1417 1.3972	1.8019 2.1249 2.3338 2.6177 3.0050 3.5532 4.3799 5.7612 8.5263 16.8222 1.0550	1.2470 1.7268 1.5461 1.3498 1.1503 0.9513 0.7542 0.5600 0.3692 0.1823 0.6560	0.4450 1.2961 1.6520 2.0277 2.4771 3.0640 3.9037 5.2583 7.9079 15.7480 0.2225
n	1/R _s	L ₁	C ₂	L ₃	C4	L ₅	с ₆	L ₇

 $R_s=1$, $C_1=.5176$, $L_2=1.414$, $C_3=1.939$, $L_4=1.9319$, $C_5=1.4142$, $L_6=0.5176$ Note index differs by 1 from that used for Leapfrog formulation



Labeled voltages are single-ended voltages at "+" inputs to the integrators

Changing the index notation:

 $R_1=1, C_2=.5176, L_3=1.414, C_4=1.939, L_5=1.9319, C_6=1.4142, L_7=0.5176$

Implement in the technology of choice

Combine loss on input and output integrators to eliminate two stages

Do frequency denormalization to obtain band-edge at 4KHz

Do impedance scaling to obtain acceptable component values

Consider lowpass to bandpass transformations



Bandpass Leapfrog Structure obtained by replacing integrators by the corresponding transformed block

Zero sensitivity to parameters in the transformed blocks will be retained at the image frequencies of the bandpass filter

$$s_n \rightarrow \frac{s^2 + \omega_0^2}{sBW}$$

$$\frac{1}{s_n} \rightarrow \frac{sBW}{s^2 + \omega_0^2} \qquad \qquad \frac{1}{s_n + \alpha} \rightarrow \frac{sBW}{s^2 + s\alpha BW + \omega_0^2}$$

Integrators map to bandpass biquads with infinite Q

Lossy integrators map to bandpass biquads with finite Q









"Loss" at input and/or output can usually be incorporated into finite-Q terminating biquads instead of requiring additional voltage amplifiers



- The bandpass biquads can be implemented with various architectures and the architecture does not ideally affect the passband sensitivity of the filter
- Integrator-based biquads are often used in integrated applications
- Note the lossless biquads are infinite Q structures !

Is it easy or practical to implement infinite Q biquads?

Are there stability concerns about the infinite Q biquads?



Integrator-based biquads







$$T(s) = \frac{s(BW/C)}{s^2 + s\alpha BW + \omega_0^2}$$

Integrator-based biquads

OTA-C Implementations (Concept)



$$T(s) = \frac{s(BW/C)}{s^2 + \omega_0^2}$$



Finite Q bandpass biquad



$$T(s) = \frac{s(BW/C)}{s^2 + s\alpha BW + \omega_0^2}$$



(Not Differential)

Integrator-based biquads

OTA-C Implementations

Infinite Q bandpass biquad



Multiple inputs can be added to lossy integrator too!



Note the lossless biquads are infinite Q structures !

Is it easy or practical to implement infinite Q biquads?

Yes – have shown by example in g_m -C family and also easy in other families

Are there stability concerns about the infinite Q biquads?

Stability of overall leapfrog structure of concern, not stability of individual biquads Overall leapfrog structure is robust with low passband sensitivities !

Leapfrog Implementations

Fifth-order Lowpass Leapfrog with OTAs



Practically can either fix g_ms and vary capacitors or fix capacitors and vary g_m's

Some leapfrog properties



What can be said about sensitivities of parameters such as band edges of leapfrog filters? If these sensitivities are not at or near 0, are they at least very small?

No! Nothing can be said about these sensitivities and they are not necessarily any smaller than what one may have for other structures such as cascaded biquads

Instead of having components (such as L's or C's) in the image of the lossless ladder network there are circuits such as integrators or biquads with more than one characterization parameters. Are the sensitivities of $|T(j\omega)|$ to these components also 0 at frequencies where the "parent" passive filter are 0?

Yes! The following theorem addresses this issue in the case of integrators

Theorem: If f(u) is a function of a variable u where u=x_1x_2, then $S_u^f = S_{x_1}^f = S_{x_2}^f$

Note: Although the results are the same as for the sensitivity of kf, in this case both x_1 and x_2 are variables whereas in the former case k is a constant.

As a consequence, if the unity gain frequency of an integrator which may be expressed (for example) as 1/RC, the transfer function magnitude sensitivity to both R and C vanish at frequencies where the sensitivity to I₀ vanishes