Transconductor Design and Applications
Simple single-ended OTA

\[ V_{in} \rightarrow M_2 \rightarrow M_3 \rightarrow M_4 \rightarrow -V_{G1} \]

\[ V_G \rightarrow M_1 \rightarrow V_{DD} \]

\[ V_{SS} \rightarrow \]

\[ I_{out} \rightarrow \]
Simple single-ended OTA

\[
\begin{align*}
I_0 &= I_1 - I_2 \\
I_1 &= \beta_1 \left( V_G - V_X - V_{Tn} \right)^2 \\
I_2 &= \beta_2 \left( V_X - V_{in} + V_{Tp} \right)^2 \\
I_2 &= \beta_3 \left( V_{in} - V_Y - V_{Tn} \right)^2 \\
I_2 &= \beta_4 \left( V_Y + V_{G1} + V_{Tp} \right)^2
\end{align*}
\]

Taking the square root of the two \( I_1 \) equations

\[
\begin{align*}
\sqrt{\frac{1}{\beta_1}} \sqrt{I_1} &= (V_G - V_X - V_{Tn}) \\
\sqrt{\frac{1}{\beta_2}} \sqrt{I_1} &= (V_X - V_{in} + V_{Tp})
\end{align*}
\]

Adding these two equations, we obtain

\[
\left( \sqrt{\frac{1}{\beta_2}} + \sqrt{\frac{1}{\beta_1}} \right) \sqrt{I_1} = \left( V_G - V_{in} + V_{Tp} - V_{Tn} \right)
\]

Similarly, for the last two equations, obtain

\[
\left( \sqrt{\frac{1}{\beta_3}} + \sqrt{\frac{1}{\beta_4}} \right) \sqrt{I_2} = \left( V_{G1} + V_{in} + V_{Tp} - V_{Tn} \right)
\]
Simple single-ended OTA

\[ I_0 = I_1 - I_2 \]

\[ \left( \sqrt{\frac{1}{\beta_2}} + \sqrt{\frac{1}{\beta_1}} \right) \sqrt{I_1} = (V_G - V_{in} + V_{Tp} - V_{Tn}) \]

\[ \left( \sqrt{\frac{1}{\beta_3}} + \sqrt{\frac{1}{\beta_4}} \right) \sqrt{I_2} = (V_{G1} + V_{in} + V_{Tp} - V_{Tn}) \]

Squaring the last two equations we obtain

\[ I_1 = \beta_5 \left( V_G - V_{in} + V_{Tp} - V_{Tn} \right)^2 \]

\[ I_2 = \beta_6 \left( V_{G1} + V_{in} + V_{Tp} - V_{Tn} \right)^2 \]

Define

\[ \left( \sqrt{\frac{1}{\beta_2}} + \sqrt{\frac{1}{\beta_1}} \right) = \sqrt{\beta_5} \]

\[ \left( \sqrt{\frac{1}{\beta_3}} + \sqrt{\frac{1}{\beta_4}} \right) = \sqrt{\beta_6} \]

Equating the difference to \( I_0 \), we obtain

\[ I_0 = (\beta_5 - \beta_6) V_{in}^2 \]

\[ + V_{in} \left( 2 \beta_5 \left[ V_{Tn} - V_{Tp} - V_G \right] + 2 \beta_6 \left[ V_{Tn} - V_{Tp} + V_{G1} \right] \right) \]

\[ + \beta_5 \left[ V_{Tp} - V_{Tn} + V_G \right]^2 - \beta_6 \left[ V_{Tp} - V_{Tn} + V_{G1} \right]^2 \]
Simple single-ended OTA

\[ I_0 = (\beta_5 - \beta_6) V_{in}^2 \]
\[ + V_{in} \left( 2\beta_5 \left[ V_{Tn} - V_{Tp} - V_G \right] + 2\beta_6 \left[ V_{Tn} - V_{Tp} + V_{G1} \right] \right) \]
\[ + \beta_5 \left[ V_{Tp} - V_{Tn} + V_G \right]^2 - \beta_6 \left[ V_{Tp} - V_{Tn} + V_{G1} \right]^2 \]

If size devices so that \( \beta_5 = \beta_6 \) and \( V_G = V_{G1} \), this simplifies to

\[ I_0 = V_{in} \left( 4\beta_5 \left[ V_{Tn} - V_{Tp} - V_G \right] \right) \]

Note this behaves as a linear transconductor!

\[ g_m = 4\beta_5 \left[ V_{Tn} - V_{Tp} - V_G \right] \]

• Since both \( M_2 \) and \( M_3 \) are driven, this is a power-efficient method for generating a given \( g_m \)

• Behavior will degrade with bulk-dependent threshold voltages of n-channel devices

• Would like to generate \( V_G \) and \( V_{G1} \) independent of \( V_{DD} \)
**V\textsubscript{DD} Independent Bias Generators**

Two widely-used \( V\textsubscript{DD} \) independent bias generators (start-up ckts not shown)

\[
\begin{align*}
I\text{D}_2 &= I\text{D}_1 = M I\text{D}_3 \\
I\text{D}_4 &= I\text{D}_3 = \frac{\mu C\text{O}X W_4}{2L_4} (V_{01} - V_{Tn})^2 \\
I\text{D}_1 &= \frac{\mu C\text{O}X W_1}{2L_1} (V_{01} - V_X - V_{Tn})^2 \\
V_X &= I\text{D}_1 R_1
\end{align*}
\]

4 equations and 4 unknowns \( \{I\text{D}_1, I\text{D}_3, V_{01}, V_X\} \)

Define:

\( M \) is the \( M_3:M_2 \) mirror gain

\[
\beta_k = \frac{\mu C\text{O}X W_k}{2L_k}
\]

\[
\begin{align*}
V_X &= \frac{1}{R_1} \left( \sqrt{\frac{1}{M\beta_4}} - \sqrt{\frac{1}{\beta_1}} \right)^2 \\
V_{01} &= V_{Tn} + \left( \frac{1}{MR} \right) \left( \frac{1}{\beta_4} - \frac{1}{\sqrt{M\beta_1\beta_4}} \right)
\end{align*}
\]
V\textsubscript{DD} Independent Bias Generators

Two widely-used V\textsubscript{DD} independent bias generators (start-up ckts not shown)

Define:

- \( M \) is the \( M_3 : M_2 \) mirror gain

\[
\beta_k = \frac{\mu C_{OX} W_k}{2L_k}
\]

\[
V_X = \frac{1}{R_1} \left( \sqrt{\frac{1}{M \beta_4}} - \sqrt{\frac{1}{\beta_1}} \right)^2
\]

\[
V_{01} = V_{Tn} + \left( \frac{1}{MR} \right) \left( \frac{1}{\beta_4} - \frac{1}{\sqrt{M \beta_1 \beta_4}} \right)
\]

Observe \( V_X \) is independent of both \( V_T \) and \( V_{DD} \)

Offers some attractive properties when used as part of a temperature sensor as well

Requires Start-up Circuit

May need compensation for stability

Pseudo-static operation so frequency response of little concern
**V_{DD} Independent Bias Generators**

**Stability Concerns**

Since there is a local feedback loop, the issue of stability must be addressed. To do this, consider the small-signal equivalent circuit.
V_{DD} Independent Bias Generators

Stability Concerns

Summing current at the three nodes, we obtain

\[
\begin{align*}
\nu_A (g_{01} + g_{02} + sC_A) + g_{m2} \nu_B + g_{m1} (\nu_A - \nu_S) &= g_{01} \nu_S \\
\nu_B (g_{m3} + g_{01} + g_{03} + sC_B) + g_{m1} \nu_A &= 0 \\
\nu_S (g_{01} + g_1) &= g_{01} \nu_A + g_{m1} (\nu_A - \nu_S)
\end{align*}
\]

Solving and neglecting \(g_0\) terms compared to \(g_m\) terms, we obtain the characteristic polynomial

\[
D(s) = s^2 C_A C_B + s \left( C_A g_{m3} - C_B \frac{g_{m1}^2}{g_1 + g_{m1}} \right) + g_{m1} \left( g_{m3} - g_{m2} \right)
\]
\( V_{DD} \) Independent Bias Generators

Stability Concerns

\[
D(s) = s^2 C_A C_B + s \left( C_A g_{m3} - C_B \frac{g_{m1}^2}{g_1 + g_{m1}} \right) + g_{m1} (g_{m3} - g_{m2})
\]

Thus, for stability, must have

\[ g_{m3} > g_{m2} \]

\[ C_A g_{m3} > C_B \frac{g_{m1}^2}{g_1 + g_{m1}} \]
**$V_{DD}$ Independent Bias Generators**

Two widely-used $V_{DD}$ independent bias generators (start-up ckts not shown)

Define:
M to be the $M_3:M_2$ mirror gain

$I_{D2} = I_{D1} = I_{D5} = MI_{D3}$

$I_{D4} = I_{D3} = \frac{\mu C_{OX} W_4}{2L_4} (V_{01} - V_{Tn})^2$

$I_{D1} = \frac{\mu C_{OX} W_1}{2L_1} (V_{01} - V_Y - V_{Tn})^2$

$I_{D5} = \frac{\mu C_{OX} W_5}{2L_5} (V_Y - V_{Tn})^2$

4 equations and 4 unknowns
\{I_{D1}, I_{D3}, V_{01}, V_Y\}

$V_{01} = V_{Tn} \left( \frac{\sqrt{M} \left(1 + \sqrt{\frac{W_1 L_5}{W_5 L_1}}\right) - 2}{\sqrt{M} \left(1 + \sqrt{\frac{W_1 L_5}{W_5 L_1}}\right) - 1} \right)$

$V_Y = V_{Tn} \left[ \frac{\sqrt{M} \left(1 + \sqrt{\frac{W_1 L_5}{W_5 L_1}}\right) - 2}{\sqrt{M} \left(1 + \sqrt{\frac{W_1 L_5}{W_5 L_1}}\right) - 1} \right] + \frac{1}{1 + \sqrt{\frac{W_3 L_1}{W_3 L_5}}} \left( \frac{\sqrt{M} \left(1 + \sqrt{\frac{W_1 L_5}{W_5 L_1}}\right) - 2}{\sqrt{M} \left(1 + \sqrt{\frac{W_1 L_5}{W_5 L_1}}\right) - 1} \right)$
**V<sub>DD</sub> Independent Bias Generators**

Two widely-used V<sub>DD</sub> independent bias generators *(start-up ckts not shown)*

Define:
M to be the M<sub>3</sub>:M<sub>2</sub> mirror gain

Note V<sub>01</sub> and V<sub>Y</sub> are dependent only upon V<sub>T</sub>

Applications well beyond this biasing requirement

Requires Start-up Circuit

May need compensation for stability

Pseudo-static operation so frequency response of little concern
Two widely-used $V_{DD}$ independent bias generators (start-up ckts not shown)

$$V_X = \frac{1}{R_1} \left( \sqrt{\frac{1}{M\beta_4}} - \sqrt{\frac{1}{\beta_1}} \right)^2$$

$$V_{01} = V_{Tn} + \left( \frac{1}{MR} \right) \left( \frac{1}{\beta_4} - \frac{1}{\sqrt{M\beta_1\beta_4}} \right)$$

where $\beta_k = \frac{\mu C_{OX} W_k}{2L_k}$ and M is the $M_3:M_2$ mirror gain
Transconductance Linearization Strategies

Recall with $R_S=0$

Widely used source degeneration
Transconductance Linearization Strategies

\[
\begin{align*}
I_{D1} &= \beta (V_1 - V_{S1} - V_T)^2 \\
I_{D2} &= \beta (V_2 - V_{S2} - V_T)^2 \\
V_{S1} - I_{D1}R_{S1} &= V_{S2} - I_{D2}R_{S2} \\
I_{D1} + I_{D2} &= I_T
\end{align*}
\]

With a straightforward analysis, we obtain the expression

\[
\sqrt{\frac{1}{\beta}} \left( \sqrt{I_T - I_{D1}} - \sqrt{I_{D1}} \right) + R_s (I_T - 2I_{D1}) = V_d
\]

The first term on the right is the nonlinear term of the original source coupled pair and the second is linear in \( I_{D1} \)

The larger the second term becomes, the more linear the transfer characteristics are
Transconductance Linearization Strategies

The transconductance of this structure can be readily derived to obtain

\[
\sqrt{\frac{1}{\beta}} \left( \sqrt{I_T - I_{D1}} - \sqrt{I_{D1}} \right) + R_S (I_T - 2I_{D1}) = V_d
\]

This can be expressed as

\[
g_m = \left( \frac{\partial V_d}{\partial I_{D1}} \right)^{-1}_{Q-pt} = \left[ \sqrt{\frac{1}{\beta}} \cdot \frac{1}{2} \left( -\left( I_T - I_{D1} \right)^{-1/2} - I_{D1}^{-1/2} \right) - 2R_S \right]^{-1}_{Q-pt}
\]
Transconductance Linearization Strategies

\[ \sqrt{\frac{1}{\beta}} \left( \sqrt{I_T - I_{D1}} - \sqrt{I_{D1}} \right) + R_S (I_T - 2I_{D1}) = V_d \]
Transconductance Linearization Strategies

There are a host of transconductance linearization strategies that have been discussed in the literature. Some are shown below.

Many are strongly dependent upon a precise square-law model of the MOS devices and do not provide practical solutions when the devices are not square-law devices.

Analysis or simulation with a more realistic model is necessary to validate linearity and practical applications of these structures.
Transconductance Linearization Strategies

How good is the square-law model that we have been using for predicting filter performance?

It is reasonably good when analyzing structures whose linearity characteristics are not strongly dependent upon the device model

The circuits considered to date are not particularly linear so the square-law model probably does a pretty good job of predicting their performance

More accurate models are usually unwieldy for hand analysis
Fig. 1 Linearised CMOS transconductance circuit
Transconductance Linearization Strategies

From CAS 2006 P 811  Jose Silva
Transconductance Linearization Strategies

Linearity Enhancement with Source Degeneration
Transconductance Linearization Strategies

Linearization with active source degeneration
CMOS transconductance amplifiers, architectures and active filters: a tutorial

E. Sánchez-Sinencio and J. Silva-Martínez

Abstract: An updated version of a 1985 tutorial paper on active filters using operational transconductance amplifiers (OTAs) is presented. The integrated circuit issues involved in active filters (using CMOS transconductance amplifiers) and the progress in this field in the last 15 years is addressed. CMOS transconductance amplifiers, nonlinearised and linearised, as well as frequency limitations and dynamic range considerations are reviewed. OTA-C filter architectures, current-mode filters, and other potential applications of transconductance amplifiers are discussed.
Linearity compensation with cross-coupled feedback
Single-ended input TAs
Differential input OTAs

Differential input and output OTAs
Parasitic Capacitances and Floating Nodes

There is invariably a parasitic capacitance associated with every terminal of every element in a filter.

These parasitic capacitances can be significant in integrated filters.

These can be combined into a single parasitic capacitance on each node.
Parasitic Capacitances and Floating Nodes

A floating node is a node that is not connected to either a zero-impedance element or across a null-port.

Floating nodes are generally avoided in integrated filters because the parasitic capacitances on the floating nodes usually degrade filter performance and often increase the order of the filter.

Some filter architectures inherently have no floating nodes, specifically, most of the basic integrator-based filters have no floating nodes.
Parasitic Capacitances and Floating Nodes
Parasitic Capacitances and Floating Nodes

No floating nodes!
Signal Swing in OTA Circuits

The signal swing for the basic bipolar OTA is limited to a few mV for reasonably linear operation.

This limited signal swing limits the use of the OTS.

The following circuit (with maybe a 100:1 or more attenuation) can be used to increase the input signal swing to the volt range and although it involves quite a few more components, the functionality can be most significant.

Program range is not affected by adding the attenuators.

\[ g_{meq} = \theta g_m \]

**Active Filter Design Using Operational Transconductance Amplifiers: A Tutorial**

Randall L. Geiger and Edgar Sánchez-Sinencio
Programmable Filter Structures

It will be assumed that the transconductance gain can be programmed with either a dc current or a dc voltage

\[ V_{in} \quad g_m \quad V_{out} \]

\[ T(s) = \frac{g_m}{g_m + sC} \]

Programmable First-Order Low-Pass Filter
Programmable Filter Structures

\[
T(s) = \frac{sC}{g_m + sC}
\]

Programmable First-Order High-Pass Filter
End of Lecture 35