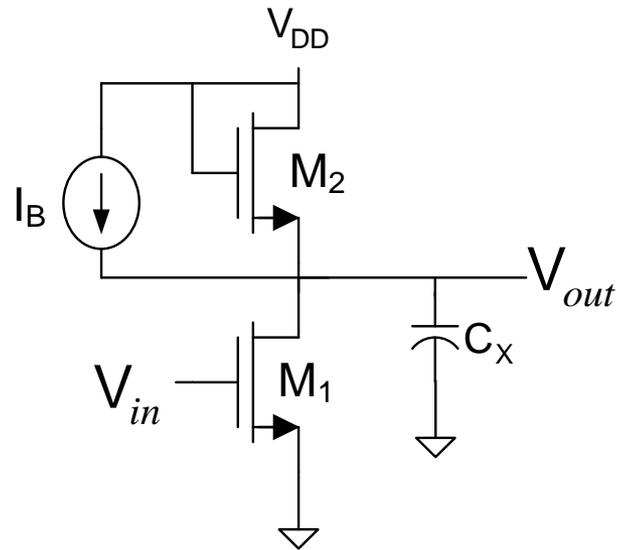


# EE 508

## Lecture 37

### High Frequency Filters

## A lossy integrator stage



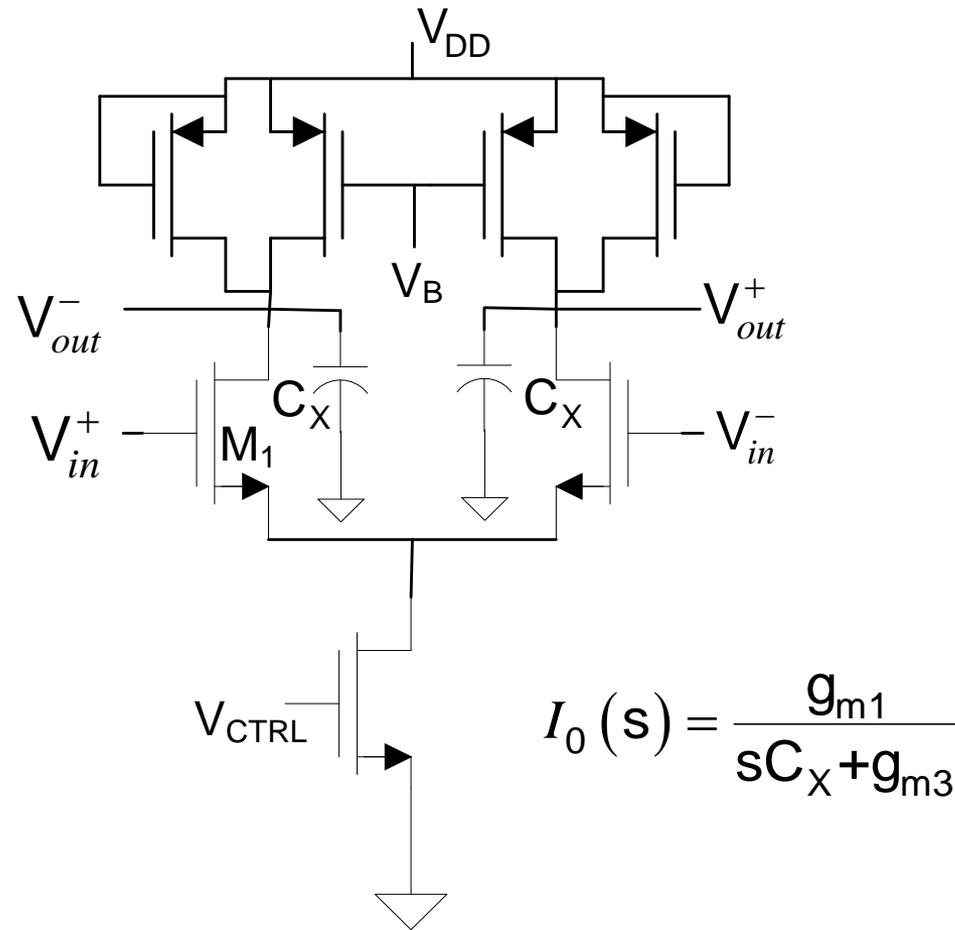
$$I(s) = \frac{-g_{m1}/C_X}{s + g_{m2}/C_X}$$

$$I_0 = g_{m1}/C_X$$

$$\alpha_L = g_{m2}/C_X$$



## A fully-differential voltage-controlled integrator stage with loss



Will need CMFB circuit

# Example:

Using the single-stage lossy integrator, design the integrator to meet a given  $\omega_0$  and  $Q$  requirement

Recall:

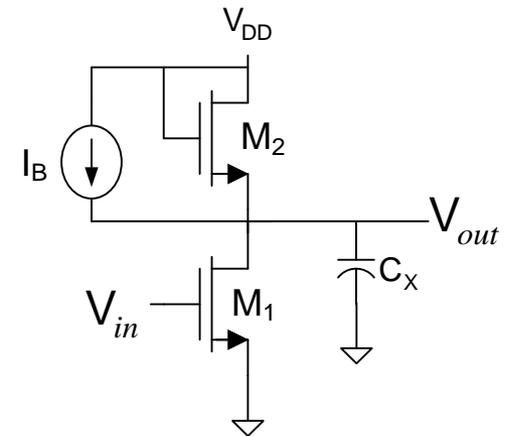
$$I_0 = \frac{\omega_0}{(\sin\theta)2Q} \sqrt{4Q^2 - 1} \quad (1)$$

$$\alpha_L = \frac{\omega_0}{2Q} + \frac{\omega_0}{2Q(\tan\theta)} \sqrt{4Q^2 - 1} \quad (2)$$

Substituting for  $I_0$  and  $\alpha_L$  we obtain:

$$\frac{g_{m1}}{C_X} = \frac{\omega_0}{(\sin\theta)2Q} \sqrt{4Q^2 - 1} \quad (3)$$

$$\frac{g_{m2}}{C_X} = \frac{\omega_0}{2Q} + \frac{\omega_0}{2Q(\tan\theta)} \sqrt{4Q^2 - 1} \quad (4)$$



$$I_0 = g_{m1}/C_X$$

$$\alpha_L = g_{m2}/C_X$$

# Example:

Using the single-stage lossy integrator, design the integrator to meet a given  $\omega_0$  and  $Q$  requirement

Expressing  $g_{m1}$  and  $g_{m2}$  in terms of design parameters:

$$\frac{\mu C_{OX} W_1 V_{EB1}}{L_1 C_X} = \frac{\omega_0}{(\sin\theta) 2Q} \sqrt{4Q^2 - 1} \quad (5)$$

$$\frac{\mu C_{OX} W_2 V_{EB2}}{L_2 C_X} = \frac{\omega_0}{2Q} + \frac{\omega_0}{2Q(\tan\theta)} \sqrt{4Q^2 - 1} \quad (6)$$

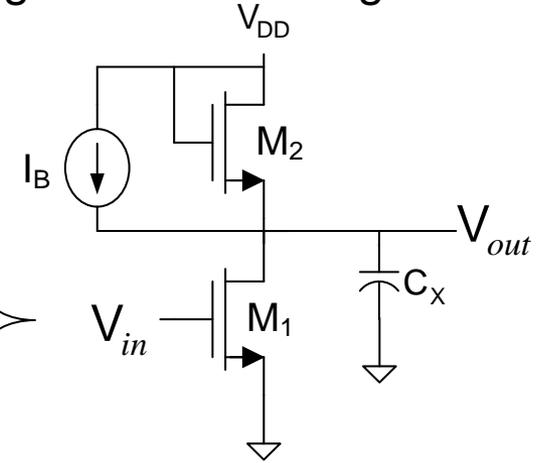
If we assume  $I_B = 0$ , equating drain currents obtain:

$$V_{EB2} = V_{EB1} \sqrt{\frac{W_1 L_2}{W_2 L_1}} \quad (7)$$

Thus the previous two expressions can be rewritten as :

$$\frac{\mu C_{OX} V_{EB1}}{C_X} \left[ \frac{W_1}{L_1} \right] = \frac{\omega_0}{(\sin\theta) 2Q} \sqrt{4Q^2 - 1} \quad (8)$$

$$\frac{\mu C_{OX} V_{EB1}}{C_X} \left[ \frac{W_1 W_2}{L_1 L_2} \right] = \frac{\omega_0}{2Q} + \frac{\omega_0}{2Q(\tan\theta)} \sqrt{4Q^2 - 1} \quad (9)$$



$$I_0 = g_{m1}/C_X$$

$$\alpha_L = g_{m2}/C_X$$

# Example:

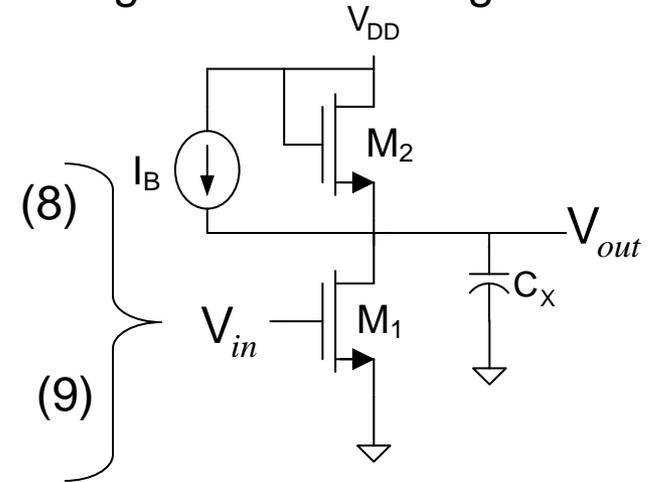
Using the single-stage lossy integrator, design the integrator to meet a given  $\omega_0$  and  $Q$  requirement

$$\frac{\mu C_{OX} V_{EB1}}{C_X} \left[ \frac{W_1}{L_1} \right] = \frac{\omega_0}{(\sin\theta) 2Q} \sqrt{4Q^2 - 1}$$

$$\frac{\mu C_{OX} V_{EB1}}{C_X} \left[ \frac{W_1 W_2}{L_1 L_2} \right] = \frac{\omega_0}{2Q} + \frac{\omega_0}{2Q(\tan\theta)} \sqrt{4Q^2 - 1}$$

Taking the ratio of these two equations we obtain:

$$\frac{W_2}{L_2} = \frac{\sin\theta + \cos\theta \sqrt{4Q^2 - 1}}{\sqrt{4Q^2 - 1}} \quad (10)$$



$$I_0 = g_{m1}/C_X$$

$$\alpha_L = g_{m2}/C_X$$

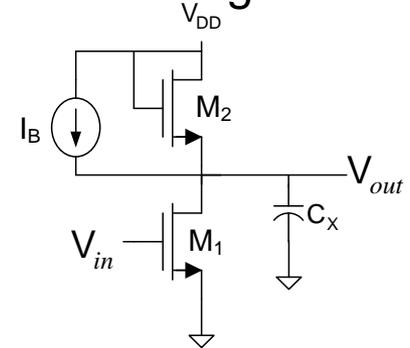
Observe that the pole  $Q$  is determined by the dimensions of the lossy device !

# Example:

Using the single-stage lossy integrator, design the integrator to meet a given  $\omega_0$  and  $Q$  requirement

$$\frac{\mu C_{OX} V_{EB1}}{C_X} \left[ \frac{W_1}{L_1} \right] = \frac{\omega_0}{(\sin\theta) 2Q} \sqrt{4Q^2 - 1} \quad (8)$$

$$\frac{W_2}{L_2} = \frac{\sin\theta + \cos\theta \sqrt{4Q^2 - 1}}{\sqrt{4Q^2 - 1}} \quad (10)$$



Still must obtain  $W_1/L_1$ ,  $V_{EB1}$ , and  $C_X$  from either of these equations

Although it appears that there might be 3 degrees of freedom left and only one constraint (one of these equations), if these integrators are connected in a loop, the operating point (Q-point) will be the same for all stages and will be that value where  $V_{out} = V_{in}$ . So, this adds a second constraint.

Setting  $V_{out} = V_{in}$ , and assuming  $V_{T1} = V_{T2}$ , we obtain from KVL

$$V_{DD} = V_{EB1} + V_{EB2} + 2V_T \quad (11)$$

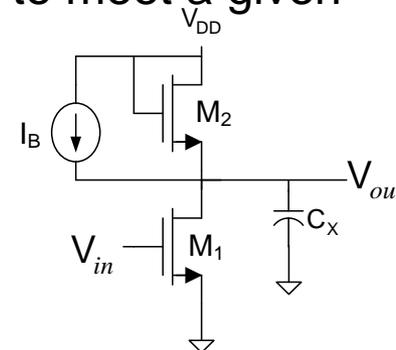
But  $V_{EB1}$  and  $V_{EB2}$  are also related in (7)

# Example:

Using the single-stage lossy integrator, design the integrator to meet a given  $\omega_0$  and  $Q$  requirement

$$\frac{\mu C_{OX} V_{EB1}}{C_X} \left[ \frac{W_1}{L_1} \right] = \frac{\omega_0}{(\sin\theta) 2Q} \sqrt{4Q^2 - 1} \quad (8)$$

$$\frac{W_2}{L_2} = \frac{\sin\theta + \cos\theta \sqrt{4Q^2 - 1}}{\sqrt{4Q^2 - 1}} \quad (10)$$



Still must obtain  $W_1/L_1$ ,  $V_{EB1}$ , and  $C_X$  from either of these equations

$$V_{DD} = V_{EB1} + V_{EB2} + 2V_T \quad (11)$$

$$V_{EB2} = V_{EB1} \sqrt{\frac{W_1 L_2}{W_2 L_1}} \quad (7)$$

$$V_{EB1} = \frac{V_{DD} - 2V_T}{1 + \sqrt{\frac{W_2 L_1}{W_1 L_2}}} \quad (12)$$

Substituting (10) into (12) and then into (8) we obtain

$$\frac{\mu C_{OX}}{C_X} \left[ \frac{W_1}{L_1} \right] \left( \frac{V_{DD} - 2V_T}{1 + \sqrt{\left(\frac{W_1}{L_1}\right)^{-1} \left( \frac{\sin\theta + \cos\theta \sqrt{4Q^2 - 1}}{\sqrt{4Q^2 - 1}} \right)}} \right) = \frac{\omega_0}{(\sin\theta) 2Q} \sqrt{4Q^2 - 1} \quad (13)$$

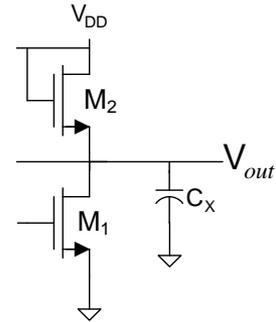
# Example:

Using the single-stage loss  $\omega_0$  and Q requirement

Lecture 38

## High Frequency Filter Design

iven



$$\frac{W_2}{L_2} = \frac{\sin\theta + \cos\theta\sqrt{4Q^2-1}}{\sqrt{4Q^2-1}}$$

$$\frac{\mu C_{OX}}{C_X} \left[ \frac{W_1}{L_1} \right] \left( \frac{V_{DD} - 2V_T}{1 + \sqrt{\left(\frac{W_1}{L_1}\right)^{-1} \left( \frac{\sin\theta + \cos\theta\sqrt{4Q^2-1}}{\sqrt{4Q^2-1}} \right)}} \right)$$

There is still one degree of freedom remaining. Can either pick  $W_1/L_1$  and solve for  $C_X$  or pick  $C_X$  and solve for  $W_1/L_1$ .

Explicit expression for  $W_1/L_1$  not available

Tradeoffs between  $C_X$  and  $W_1/L_1$  will often be made

Since  $V_{OUTQ} = V_T + V_{EB1}$ , it may be preferred to pick  $V_{EB1}$ , then solve (12) for  $W_1/L_1$  and then solve (13) for  $C_X$

Adding  $I_B$  will provide one additional degree of freedom and will relax the relationship between  $V_{OUTQ}$  and  $W_1/L_1$  since (7) will be modified

# High Frequency Filter Design

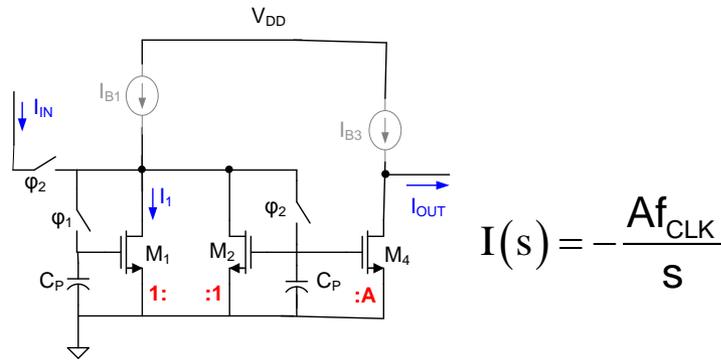
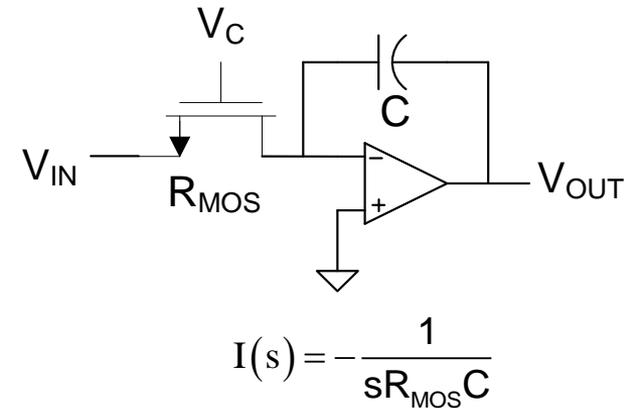
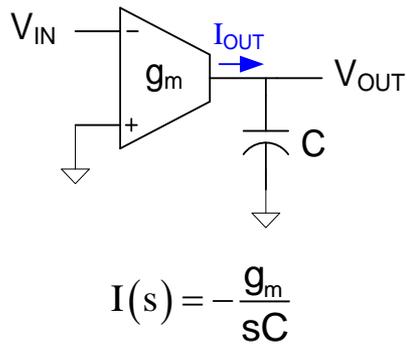
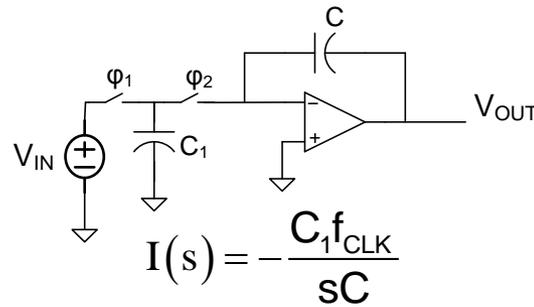
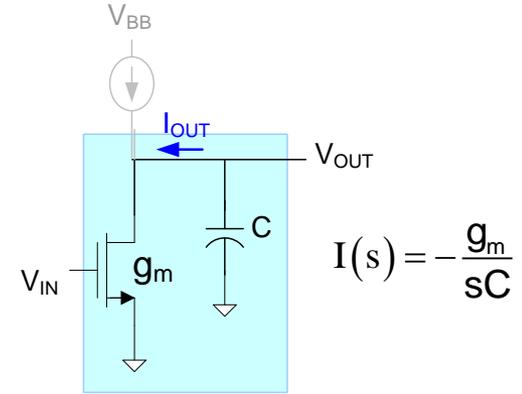
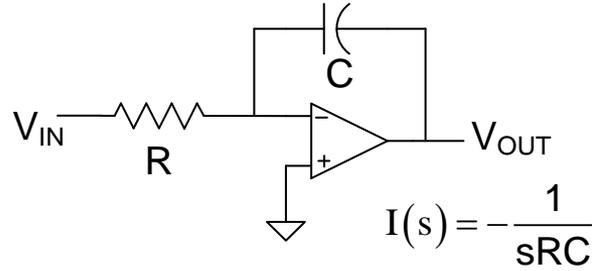
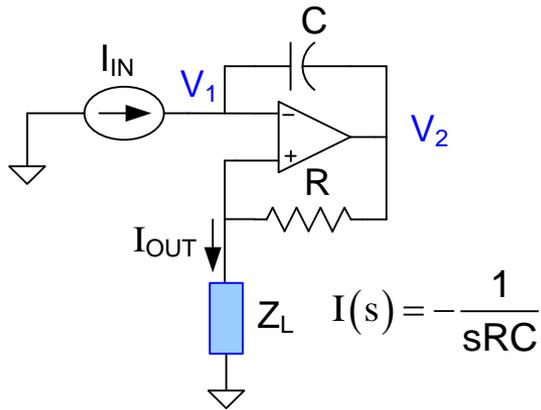
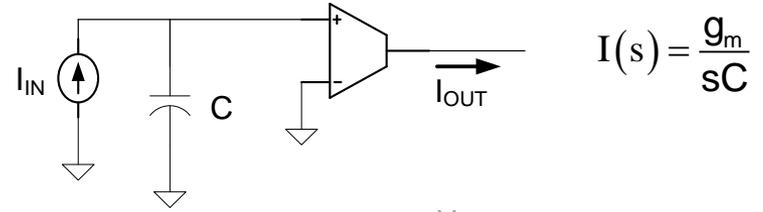
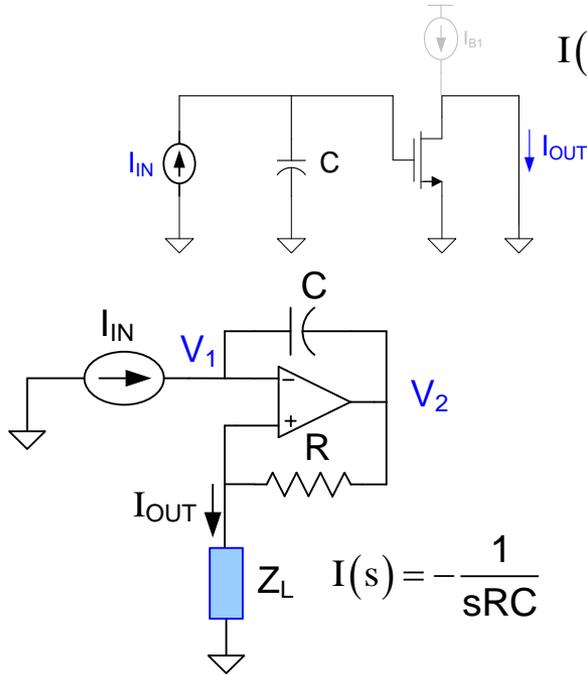
- Architecture selection is critical
- At high frequencies, simplicity of the structures is important
- Parasitic capacitances and their relationship to the time constants that can be achieved provide the ultimate limit on speed
- Will limit discussions to “inductorless” structures

# High Frequency Filter Design

Following two methods will provide highest frequency of operation

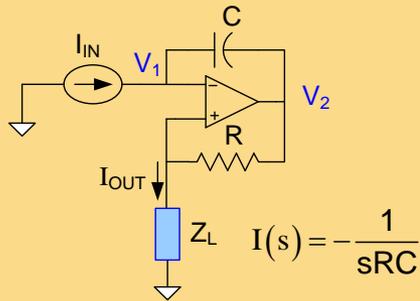
- Degenerate VCOs
- Simple high-frequency integrator-based filters

# Integrator Architecture Selection

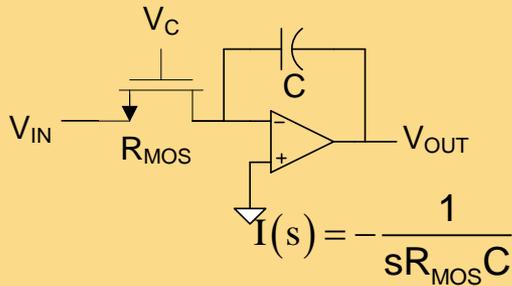


# Integrators for High-Speed Operation

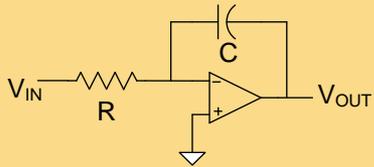
Slow



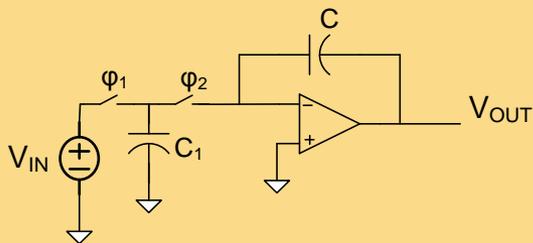
$$I(s) = -\frac{1}{sRC}$$



$$I(s) = -\frac{1}{sR_{MOS}C}$$

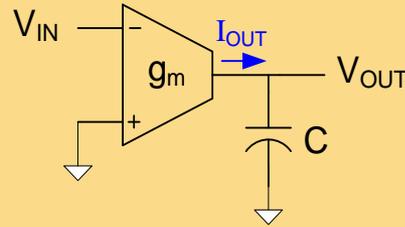


$$I(s) = -\frac{1}{sRC}$$

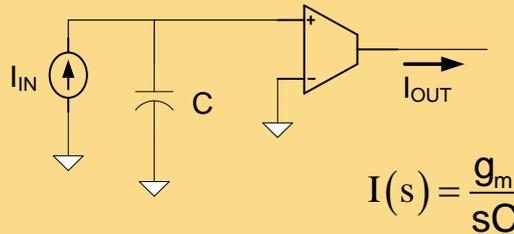


$$I(s) = -\frac{C_1 f_{CLK}}{sC}$$

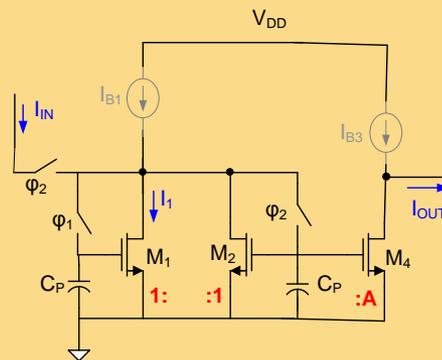
Reasonably Fast



$$I(s) = -\frac{g_m}{sC}$$

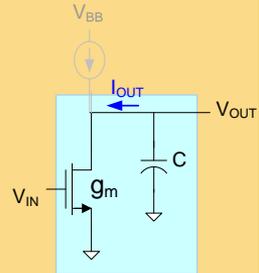


$$I(s) = \frac{g_m}{sC}$$

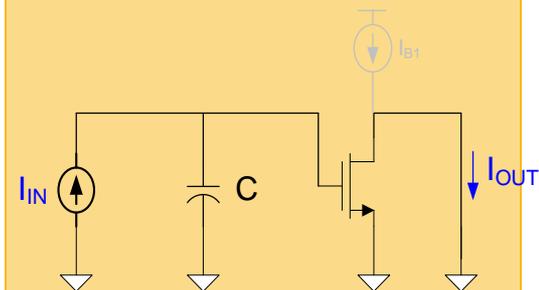


$$I(s) = -\frac{Af_{CLK}}{s}$$

Very Fast



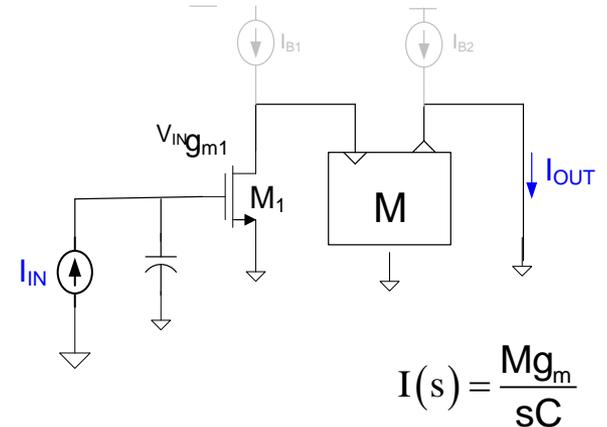
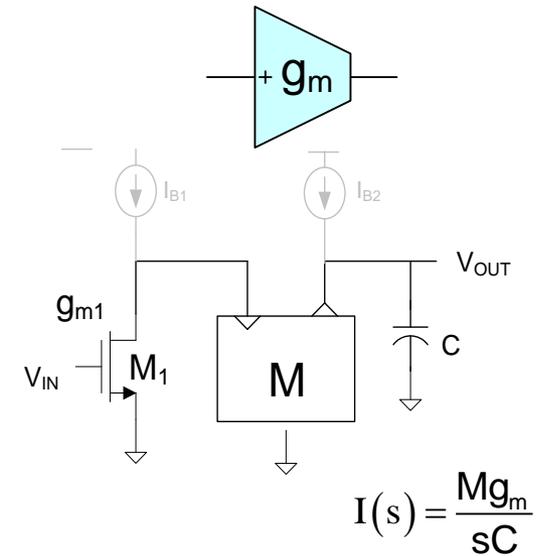
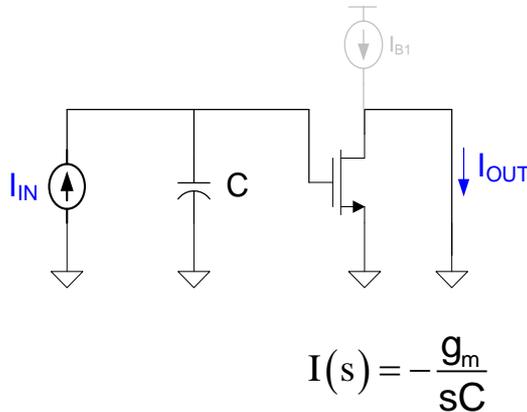
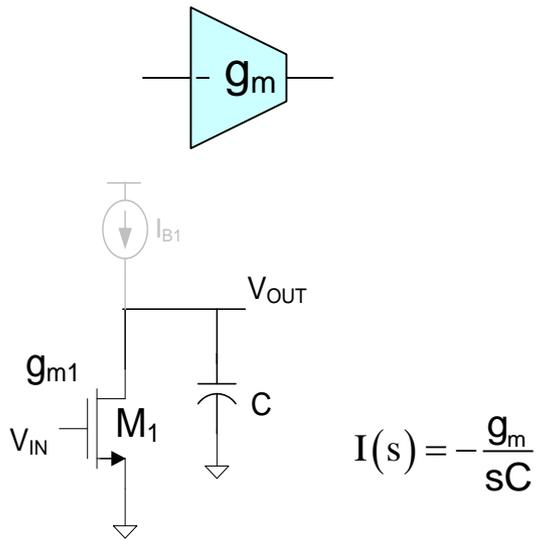
$$I(s) = -\frac{g_m}{sC}$$



$$I(s) = -\frac{g_m}{sC}$$

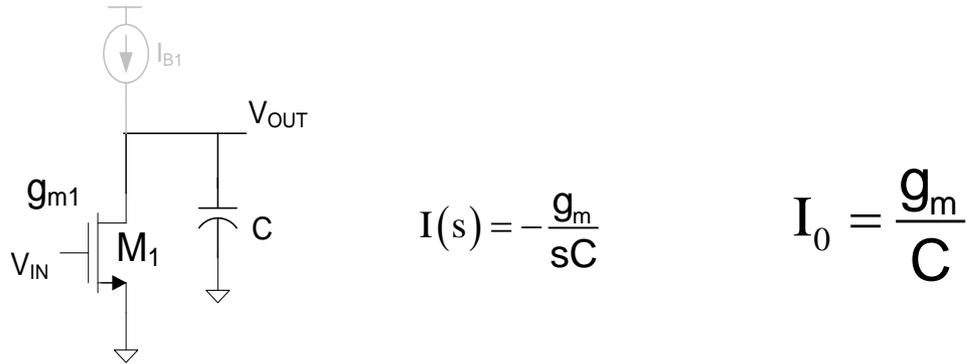
# Single-ended High-Frequency TA Integrators

Structures of choice for highest-frequency of operation



Some authors focus on voltage mode and others on current mode  
But overall structures and performance appears to be identical

# Single-ended High-Frequency TA Integrators



Recall:  $\omega_0$  for integrator-based filters generally proportional to  $I_0$

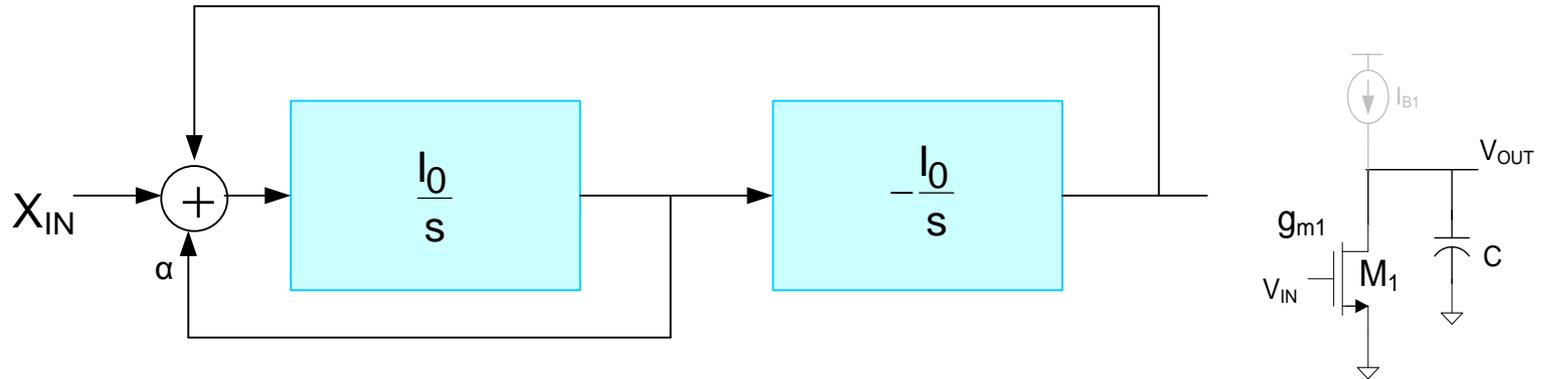
How high can  $I_0$  be?

$$I_0 = \frac{\mu C_{OX} W / L V_{EB}}{C}$$

Looks like we can make  $I_0$  as large as we want by making  $V_{EB}$  large,  $C$  small,  $L$  small, and  $W$  large

# Single-ended High-Frequency TA Integrators

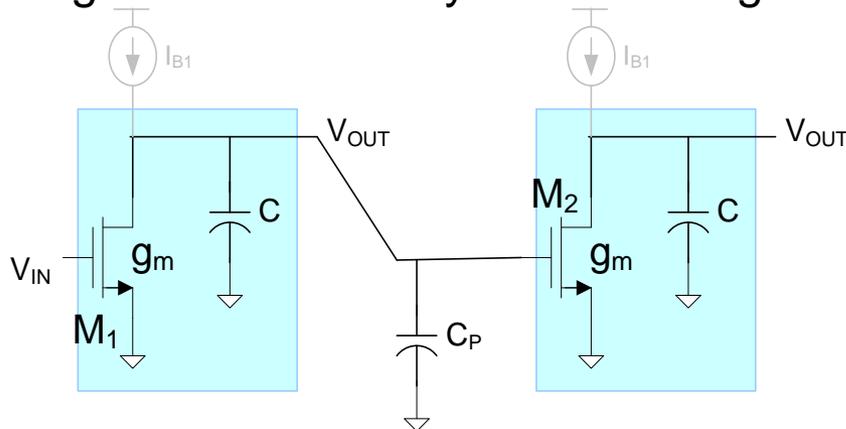
How high can  $I_0$  be?



Consider a typical filter – the two integrator loop

$$I_0 = \frac{\mu C_{OX} W / L V_{EB}}{C}$$

Integrator is loaded by another integrator!



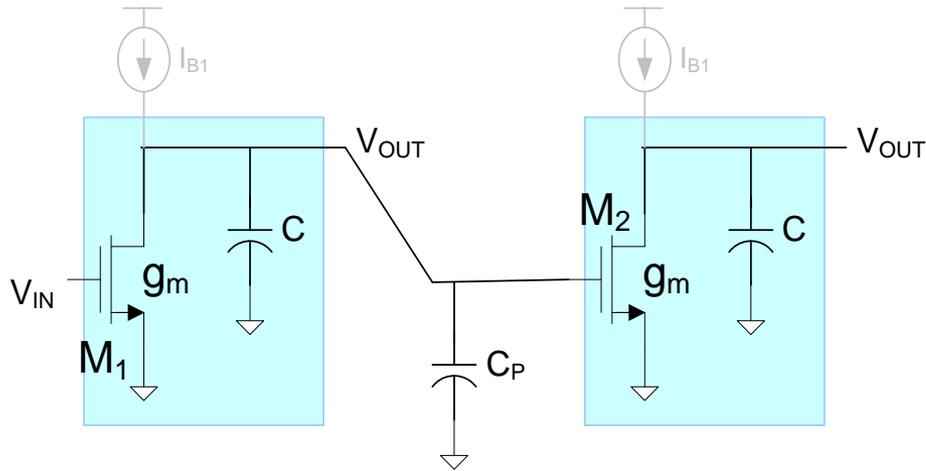
$$I_0 = \frac{\mu C_{OX} W_1 / L_1 V_{EB1}}{C + C_P + C_{OX} W_2 L_2}$$

Even if  $C$  goes to 0,  $I_0$  is limited!

$C_P$  is the parasitic capacitances on the output node

# Single-ended High-Frequency TA Integrators

How high can  $I_0$  be?



$$I_0 = \frac{\mu C_{OX} W_1 / L_1 V_{EB1}}{C + C_P + C_{OX} W_2 L_2}$$

Setting  $C$  to 0 and assuming  $C_p$  is small,

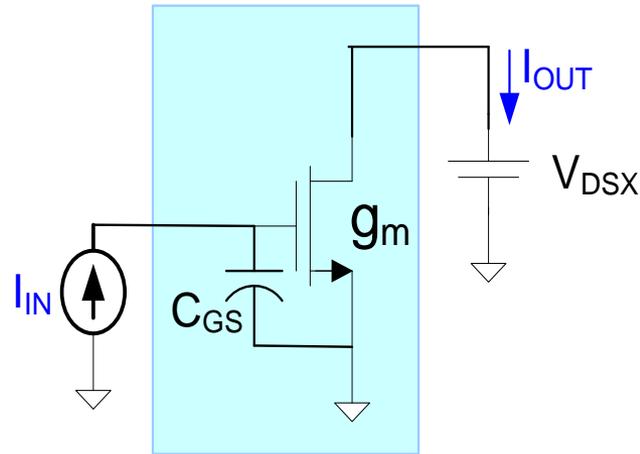
$$I_0 = \frac{\mu C_{OX} W_1 / L_1 V_{EB1}}{C_{OX} W_2 L_2}$$

$$I_0 = \frac{\mu W_1 V_{EB1}}{W_2 L_1 L_2}$$

Assuming the integrator stages are identical, it follows that

$$I_0 = \frac{\mu V_{EB1}}{L_{\min}^2}$$

# Transition (transit) frequency ( $f_T$ ) of a process



The transit frequency of a process is the frequency where the short-circuit current gain of the common-source configuration drops to 1.

$$i_{OUT} = g_m v_{gs}$$

$$i_{IN} \cdot \frac{1}{sC_{GS}} = v_{gs}$$

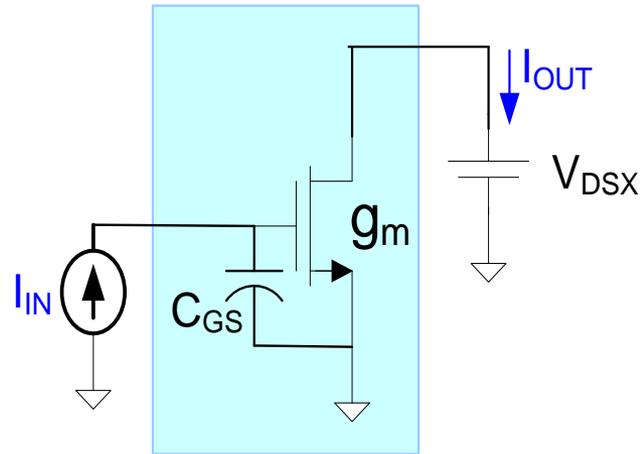
$$\frac{i_{OUT}}{i_{IN}} = \frac{g_m}{sC_{GS}}$$

$$1 = \frac{g_m}{C_{GS} \omega_T}$$

$$\omega_T = \frac{g_m}{C_{GS}} = \frac{\left( \mu C_{OX} \frac{W}{L} V_{EB} \right)}{C_{OX} WL} = \frac{\mu V_{EB}}{L^2}$$

$$\omega_T = \frac{\mu V_{EB}}{L_{min}^2}$$

# Transition (transit) frequency ( $f_T$ ) of a process



The transit frequency of a process is the frequency where the short-circuit current gain of the common-source configuration drops to 1.

$$\omega_T = \frac{\mu V_{EB}}{L_{min}^2}$$

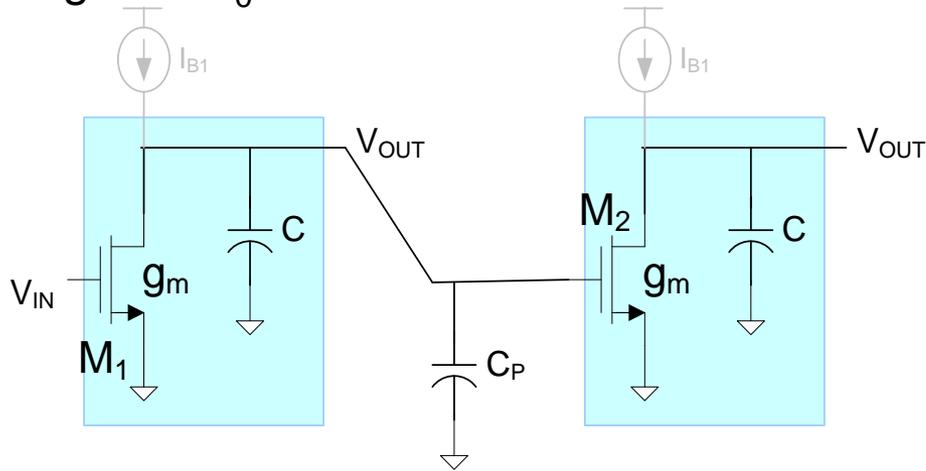
This is dependent upon  $V_{EB}$

Does not include effects of diffusion capacitances or overlap capacitances

$f_{MAX}$  is another figure that characterizes the speed of a process

# Single-ended High-Frequency TA Integrators

How high can  $I_0$  be?



$$I_{0M} = \frac{\mu V_{EB1}}{L_{min}^2}$$

$$I_{0M} = \omega_T$$

Speed of operation increases with  $V_{EB1}$

$V_{EB1}$  is limited by signal swing requirements and  $V_{DD}$

Signal Swing:

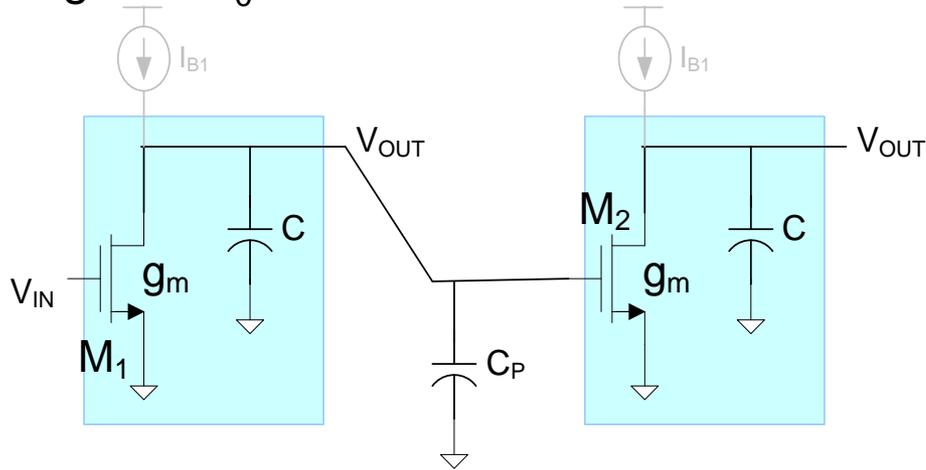
$$V_{SW-OP} \approx \min\{V_{DD} - V_{OQ}, V_{OQ} - (V_T + 100mV)\}$$

$$V_{OQ} = V_T + V_{EB}$$

$$V_{SW-OP} \approx \min\{V_{DD} - V_T - V_{EB}, V_T + V_{EB} - (V_T + 100mV)\}$$

# Single-ended High-Frequency TA Integrators

How high can  $I_0$  be?



$$I_{0M} = \frac{\mu V_{EB1}}{L_{min}^2}$$

$$I_{0M} = \omega_T$$

Speed of operation increases with  $V_{EB}$

$V_{EB}$  is limited by signal swing requirements and  $V_{DD}$

Signal Swing:

$$V_{DD} - V_T - V_{EB} = V_T + V_{EB} - (V_T + 100\text{mV})$$

$$V_{EB} = \frac{V_{DD} + 100\text{mV} - V_T}{2}$$

$$I_{0MAX} \approx \frac{\mu(V_{DD} + 100\text{mV} - V_T)}{2L_{min}^2}$$

