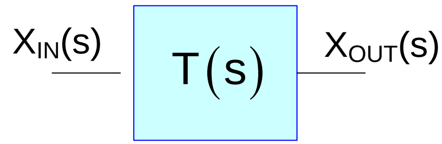
EE 508
Lecture 4

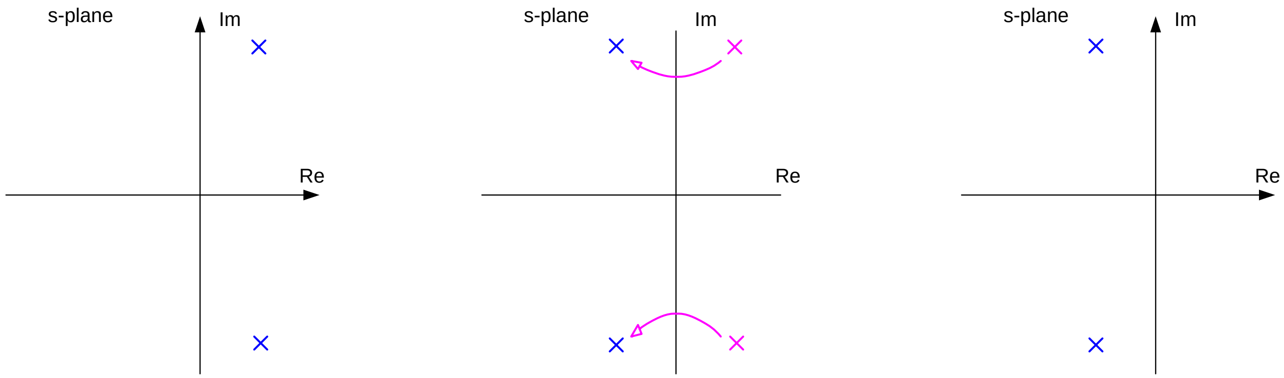
Filter Concepts/Terminology

Basic Properties of Electrical Circuits

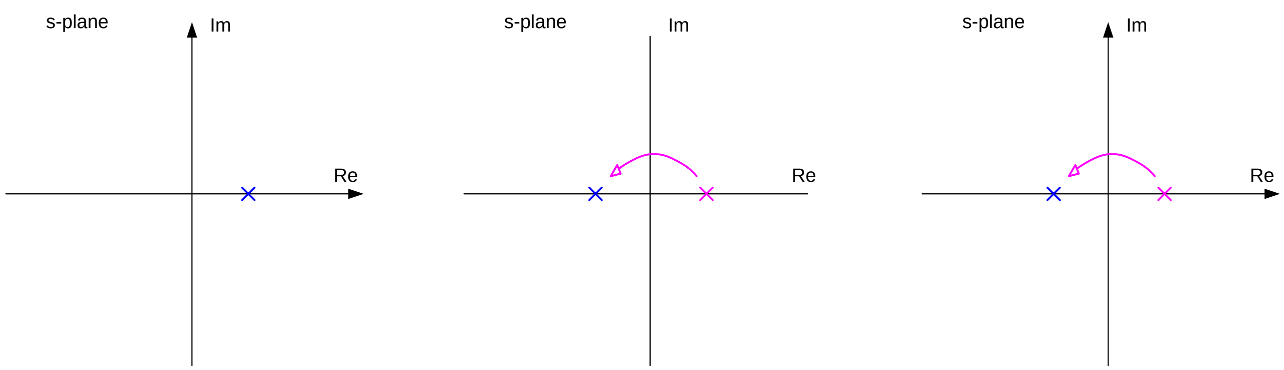
Filter Concepts and Terminology



Reflecting poles and zeros to maintain stability or establish minimum phase



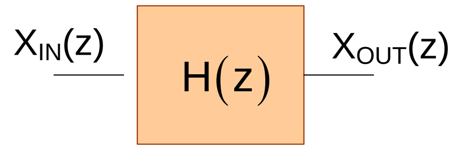
Not minimum Phase | Reflection | Minimum Phase



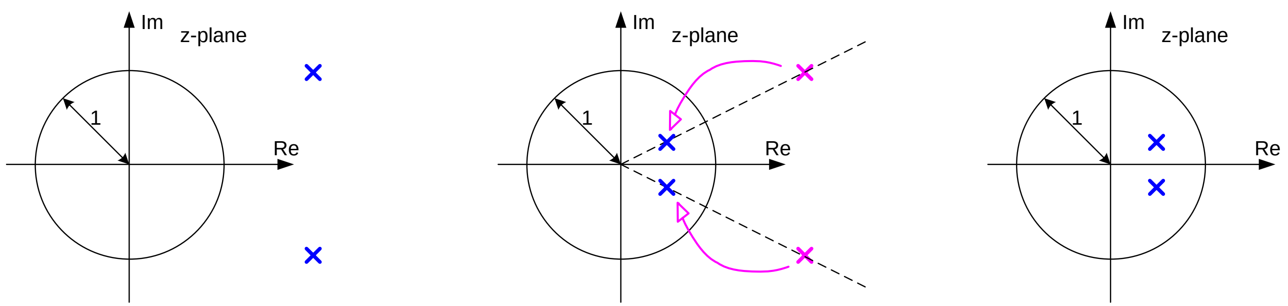
Note: magnitude of real part is preserved in reflection, imaginary part remains unchanged

Review from Last Time

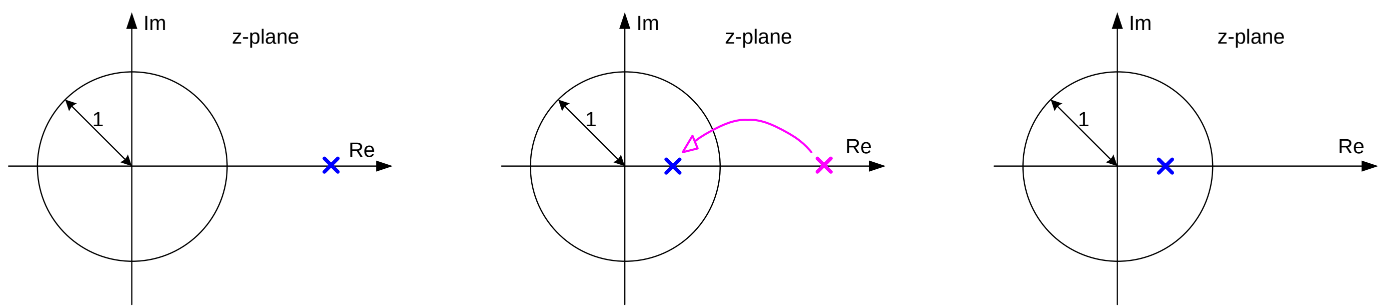
Filter Concepts and Terminology



Reflecting poles and zeros to maintain stability or establish minimum phase



Not minimum Phase | Reflection | Minimum Phase



Note: complex conjugate reciprocal reflection maintains angle but magnitude of reflected root is the reciprocal of the magnitude of the original root

Filter Concepts and Terminology

- 2-nd order polynomial characterization
- Biquadratic Factorization
- Op Amp Modeling
- Stability and Instability
- Roll-off characteristics
- Dead Networks
- Root Characterization
- Scaling, normalization, and transformation

2-nd order polynomial characterization

$$s^2+as+b \qquad \{a,b\}$$

$$s^2+\frac{\omega_o}{Q}s+\omega_0^2 \qquad \{\omega_o,Q\}$$

$$s^2+2\zeta\omega_0 s+\omega_0^2 \qquad \{\zeta, \omega_o\}$$

$$s^2+(p_1+p_2)s+p_1p_2 = (s+p_1)(s+p_2) \qquad \{p_1,p_2\}$$

with complex conjugate roots

$$s^2+2\alpha s+\alpha^2+\beta^2 = (s+\alpha+j\beta)(s+\alpha-j\beta) \qquad \{\alpha,\beta\}$$

$$s^2+2r\cos(\theta)s+r^2 = (s+re^{j\theta})(s+re^{-j\theta}) \qquad \{r,\theta\}$$

2-nd order polynomial characterization

$\{a,b\}$ $\{\omega_o,Q\}$ $\{\zeta, \omega_o\}$ $\{p_1,p_2\}$

$\{\alpha,\beta\}$ $\{r,\theta\}$

Alternate equivalent parameter sets

Widely used interchangeably

Easy mapping from one to another

Defined irrespective of whether polynomial appears in numerator or denominator of transfer function

If order is greater than 2, often multiple root pairing options so these parameter sets will not be unique for a given polynomial or transfer function

If cc roots exist, these will almost always be paired together (unique)

Biquadratic Factorization

$$T(s) = \frac{\sum_{i=1}^{m} a_i s^i}{\sum_{i=1}^{n} b_i s^i} = \frac{N(s)}{D(s)}$$

If m or n is even, integer-monic polynomials N(s) or D(s) can be expressed as

$$P(s) = \sum_{i=0}^{k} c_i s^i = \prod_{i=1}^{k/2} (s^2 + d_{1i} s + d_{2i})$$

If m or n is odd, integer-monic polynomials N(s) or D(s) can be expressed as

$$P(s) = \sum_{i=0}^{k} c_i s^i = (s + d_0) \prod_{i=1}^{(k-1)/2} (s^2 + d_{1i} s + d_{2i})$$

- These are termed quadratic factorizations
- If both N(s) and D(s) are expressed as quadratic factorizations, quadratic pairs can be grouped to obtain a Biquadratic factorization of T(s)

Biquadratic Factorization

Pole and zero pairings can always be made so that all coefficients
In the biquadratic factorizations are real

In general, the biquadratic factorizations are not unique

-If roots are real, multiple choices for first-order factor and remaining roots can be partitioned into groups of 2 in different ways

-Complex conjugate root pairs are generally grouped together so that all Coefficients are real

Biquadratic Factorization

If n is even,

$$T(s)=\frac{\sum_{i=1}^{m} a_i s^i}{\sum_{i=1}^{n} b_i s^i}=K\bullet\prod_{i=1}^{n/2} T_{BQi}(s)$$

If n is odd,

$$T(s)=\frac{\sum_{i=1}^{m} a_i s^i}{\sum_{i=1}^{n} b_i s^i}=K\bullet\left(\frac{a_{10}s+a_{00}}{s+b_{00}}\right)\bullet\prod_{i=1}^{(n-1)/2} T_{BQi}(s)$$

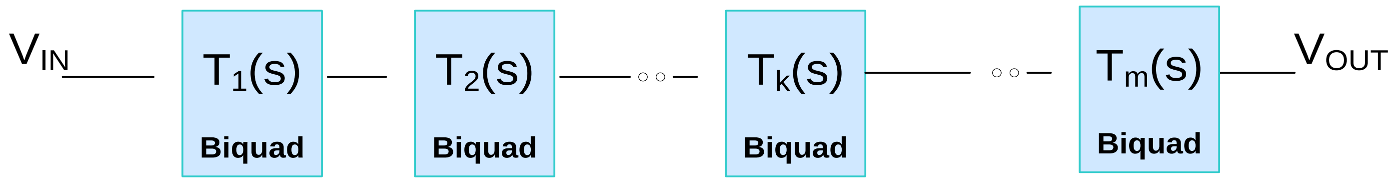
where $$T_{BQi}(s)=\frac{a_{2i}s^2+a_{1i}s+a_{0i}}{s^2+b_{1i}s+b_{0i}}$$

and where K is a real constant and all coefficients are real (some may be 0)

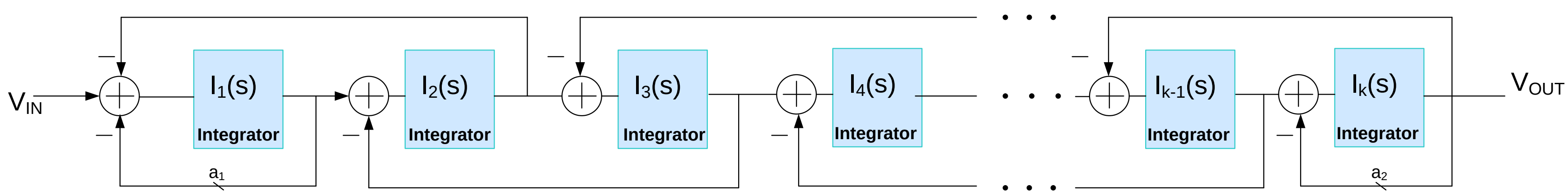
- Factorization is not unique
- H(z) factorizations not restricted to have m<n
- Each biquatratic factor can be represented by any of the 6 alternative parameter sets in the numerator or denominator

Common Filter Architectures

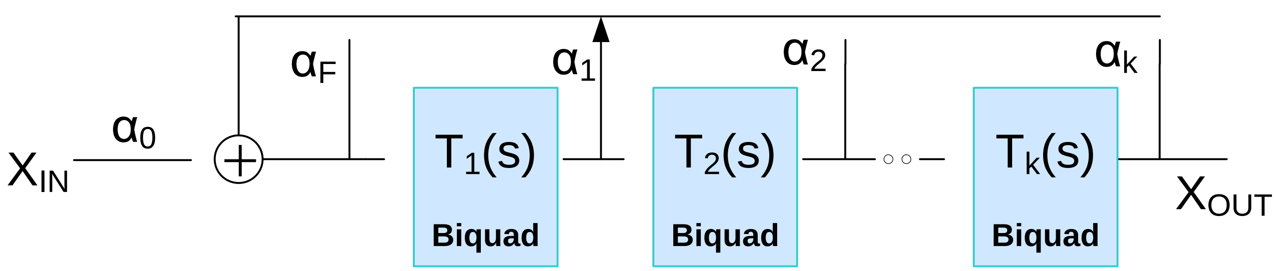
Cascaded Biquads



Leapfrog



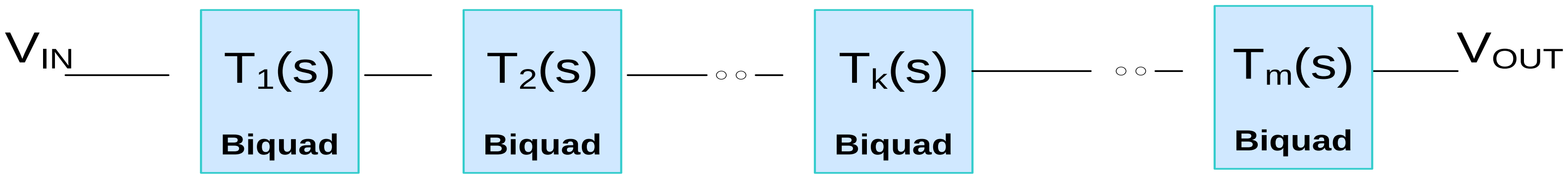
Multiple-loop Feedback



- Three classical filter architectures are shown
- The Cascaded Biquad and the Leapfrog approaches are most common

Common Filter Architectures

Cascaded Biquads



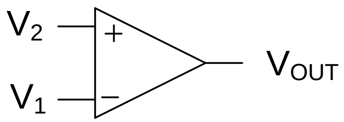
$$T(s) = T_1 T_2 \cdots T_m$$

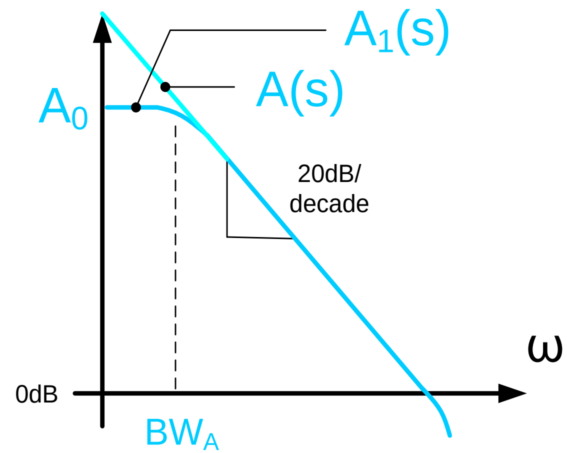
- Sequence in Cascade often affect performance
- Different biquadratic factorizations will provide different performance
- Although some attention was given to the different alternatives for biquadratic factorization, a solid general formulation of the cascade sequencing problem or the biquadratic factorization problem never evolved

Gain, Bandwidth and GB

Frequency Dependent Model of Op Amps

Most op amps are designed so that they behave as a first-order circuit at frequencies up to the unity gain frequency or beyond





$$A_1(s)=\frac{GB}{s+BW_A}$$

$$GB = A_0 \bullet BW_A$$

where $$BW_A=\frac{GB}{A_0} <<1$$

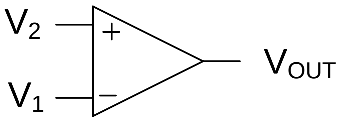
Can usually model with a more-simplified gain expression

$$A(s)=\frac{GB}{s}$$

Adequate model for most applications

Gain, Bandwidth and GB

Effects of GB on closed-loop Amplifiers



$$A_1(s)=\frac{GB}{s+BW_A}$$

$$GB = A_0 \bullet BW_A$$

$$A(s)=\frac{GB}{s}$$

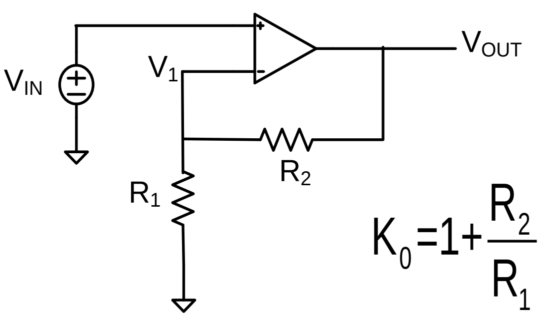
Adequate model for most applications

$$\left.\begin{aligned} V_1 &= \frac{V_{OUT}}{K_0} \\ V_{OUT} &= A(s)(V_1 - V_{IN}) \\ A(s) &= \frac{GB}{s+BW_A} \end{aligned}\right\}$$

$$A_{FB}(s)=\frac{V_{OUT}}{V_{IN}}=\frac{K_0}{s\frac{K_0}{GB}+\left(1+K_0\frac{BW_A}{GB}\right)}$$

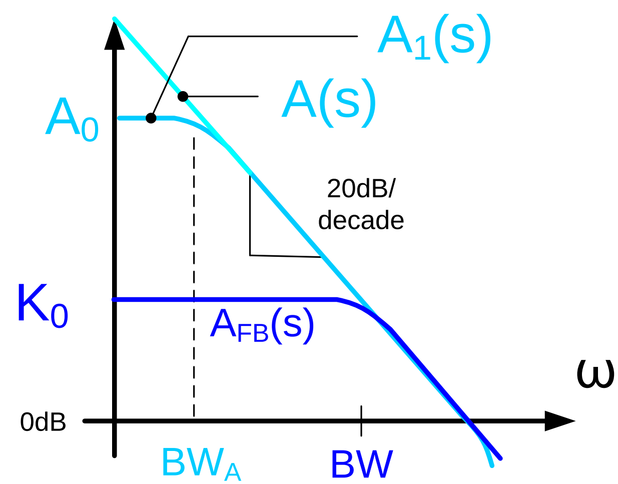
$$A_{FB}(s)\approx\frac{K_0}{1+s\frac{K_0}{GB}}$$

$$BW=\frac{GB}{K_0}$$



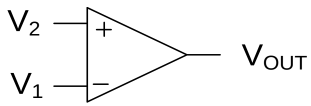
$$K_0=1+\frac{R_2}{R_1}$$

Basic Noninverting Amplifier



Gain, Bandwidth and GB

Effects of GB on closed-loop Amplifiers



$$A_1(s)=\frac{GB}{s+BW_A}$$

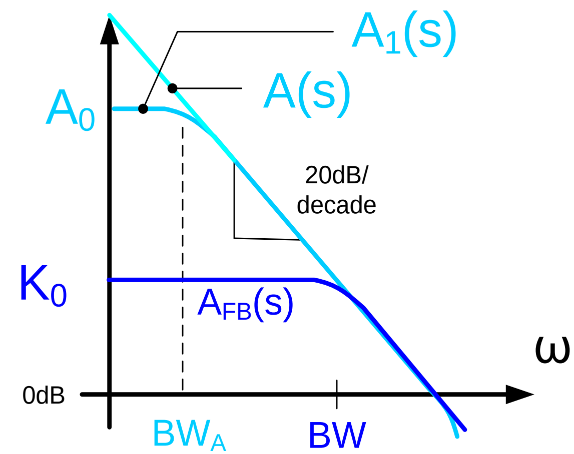
$$GB = A_0 \bullet BW_A$$

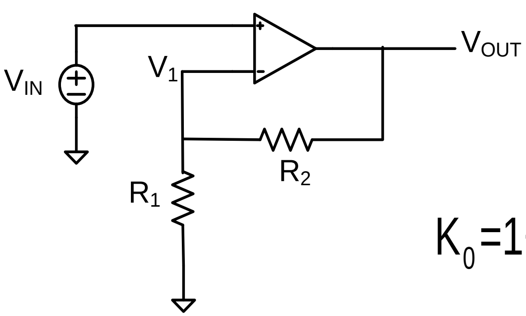
$$A(s)=\frac{GB}{s}$$

Adequate model for most applications

$$A_{FB}(s) \cong \frac{K_0}{1+s\frac{K_0}{GB}}$$

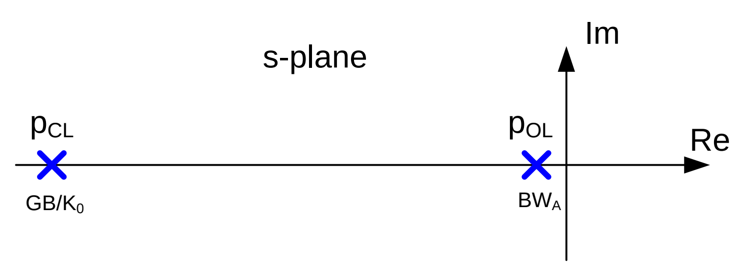
$$BW = \frac{GB}{K_0}$$





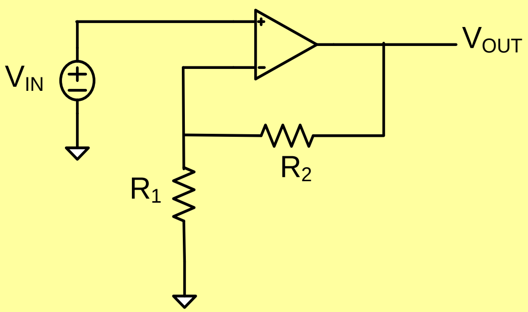
$$K_0=1+\frac{R_2}{R_1}$$

Basic Noninverting Amplifier



Gain, Bandwidth and GB

Summary of Effects of GB on Basic Inverting and Noninverting Amplifiers

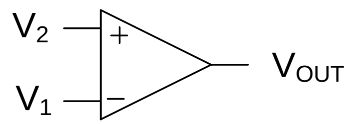


Basic Noninverting Amplifier

$$K_0 = 1 + \frac{R_2}{R_1}$$

$$BW = \frac{GB}{K_0}$$

$$A_{FB}(s) = \frac{K_0}{1 + s\frac{K_0}{GB}}$$

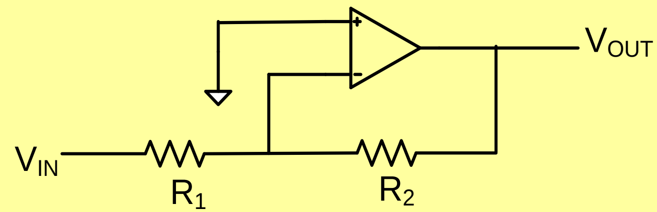


$$A_1(s) = \frac{GB}{s + BW_A}$$

$$GB = A_0 \bullet BW_A$$

$$A(s) = \frac{GB}{s}$$

Adequate model for most applications

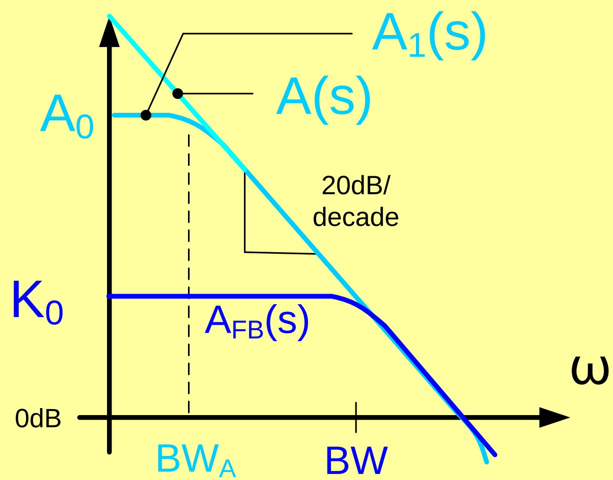


Basic Inverting Amplifier

$$K_0 = \frac{R_2}{R_1}$$

$$BW = \frac{GB}{1 + K_0}$$

$$A_{FB}(s) = -\frac{K_0}{1 + s\frac{(1 + K_0)}{GB}}$$



Stability and Instability

True or False?

An unstable circuit will oscillate

False – unstable circuits will either latch up or oscillate. Latch-up is often the consequence of saturating nonlinearities of circuits that have positive real axis poles

Achieving stability is a major goal of the filter designer

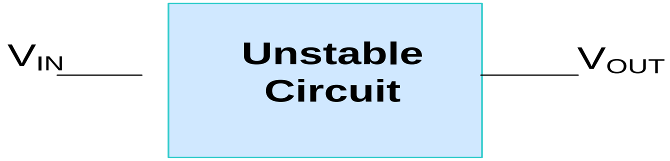
False – a filter is usually of little practical use if there are concerns about stability

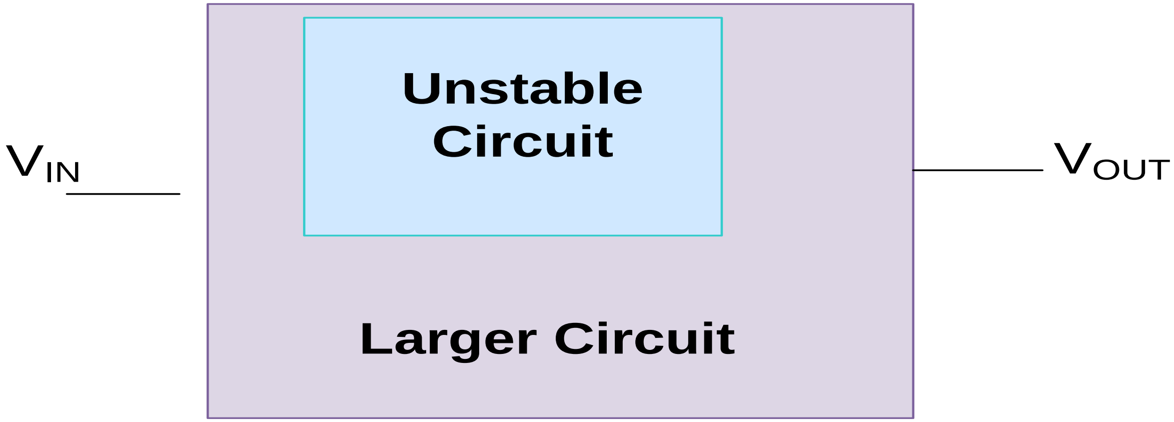
Unstable circuits are of little use in designing filters

False – will discuss details later

Theorem:

If a circuit is unstable, then if this circuit is included as a subcircuit in a larger circuit structure, the larger circuit will also be unstable.



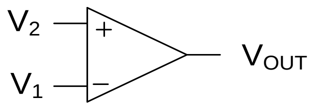


Proof ?:

Consider First Some Related Concepts

Gain, Bandwidth and GB

Consider “positive feedback” closed-loop amplifier

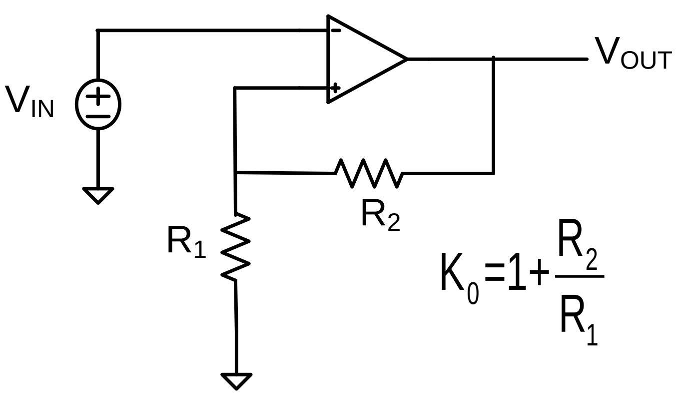


$$A_1(s)=\frac{GB}{s+BW_A}$$

$$GB = A_0 \bullet BW_A$$

$$A(s)=\frac{GB}{s}$$

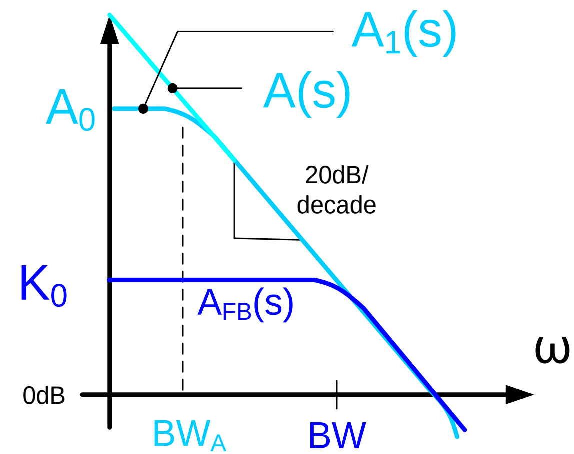
Adequate model for most applications

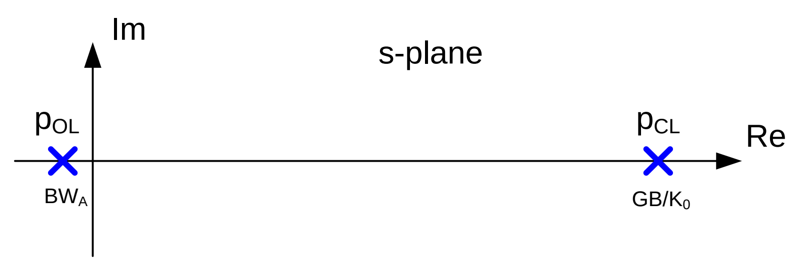


$$K_0=1+\frac{R_2}{R_1}$$

$$A_{FB}(s) \approx \frac{K_0}{1-s\frac{K_0}{GB}}$$

$$BW=\frac{GB}{K_0}$$

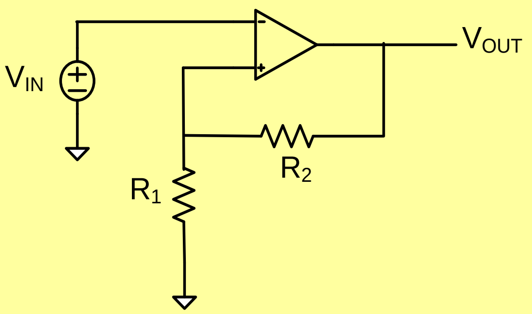




Feedback Amplifier is Unstable !

Gain, Bandwidth and GB

Summary of Effects of GB on Basic Inverting and Noninverting Amplifiers with “Positive Feedback”

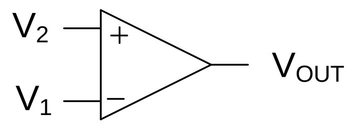


Basic Noninverting Amplifier

$$K_0 = 1 + \frac{R_2}{R_1}$$

$$BW = \frac{GB}{K_0}$$

$$A_{FB}(s) = \frac{K_0}{1 - s\frac{K_0}{GB}}$$

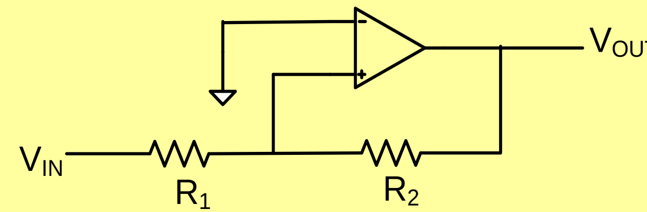


$$A_1(s) = \frac{GB}{s + BW_A}$$

$$GB = A_0 \bullet BW_A$$

$$A(s) = \frac{GB}{s}$$

Adequate model for most applications

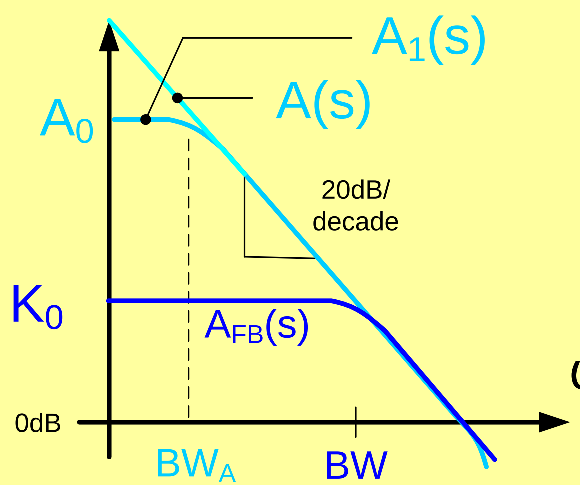


Basic Inverting Amplifier

$$K_0 = \frac{R_2}{R_1}$$

$$BW = \frac{GB}{1 + K_0}$$

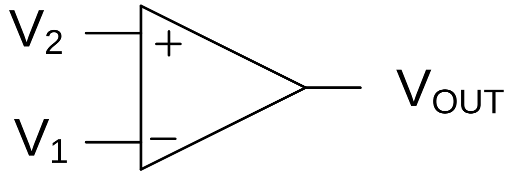
$$A_{FB}(s) = -\frac{K_0}{1 - s\frac{(1+K_0)}{GB}}$$



Both FB Amplifiers are Unstable

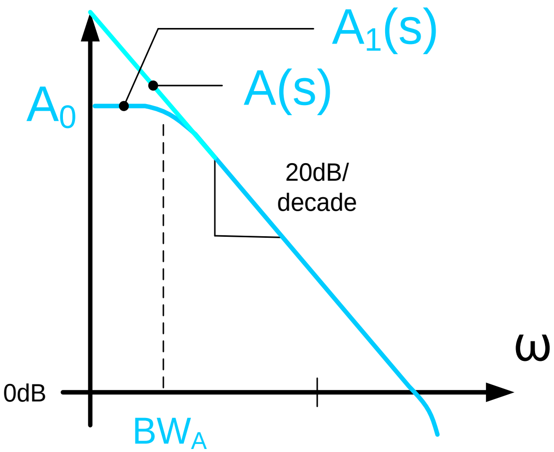
Gain, Bandwidth and GB

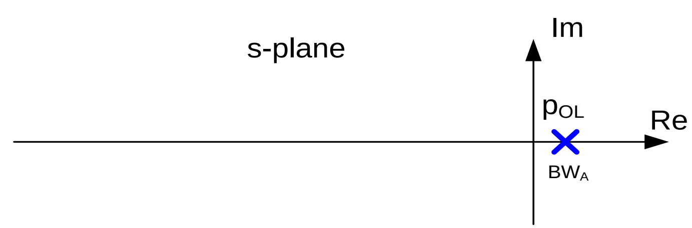
Consider Op Amp with RHP Pole (Unstable Op Amp)



$$A_1(s)=\frac{GB}{s - BW_A}$$

$$|GB| = A_0 \bullet BW_A$$

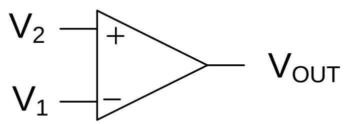




Op Amp is Unstable, dc gain is negative

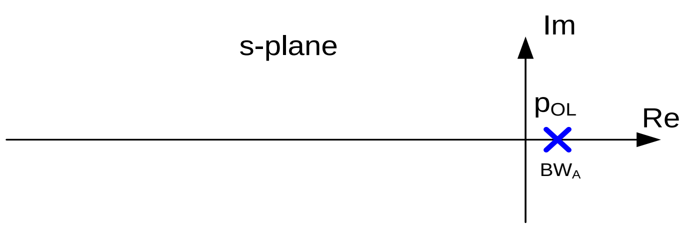
Gain, Bandwidth and GB

Consider Op Amp with RHP Pole (Unstable Op Amp)



$$A_1(s)=\frac{GB}{s - BW_A}$$

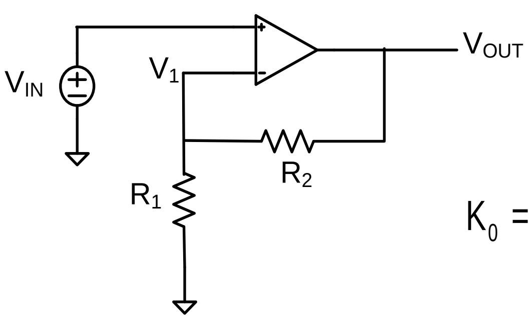
$$|GB| = A_0 \bullet BW_A$$



$$\left.\begin{aligned} V_1 &= \frac{V_{OUT}}{K_0} \\ V_{OUT} &= A(s)(V_1 - V_{IN}) \\ A(s) &= \frac{GB}{s - BW_A} \end{aligned}\right\}$$

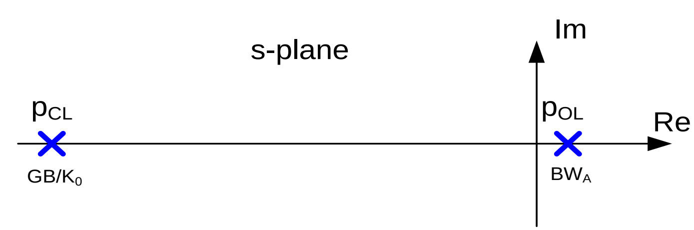
$$A_{FB}(s)=\frac{V_{OUT}}{V_{IN}}=\frac{K_0}{s\frac{K_0}{GB}+\left(1-K_0\frac{BW_A}{GB}\right)}$$

$$A_{FB}(s) \approx \frac{K_0}{1+s\frac{K_0}{GB}} \qquad BW=\frac{GB}{K_0}$$



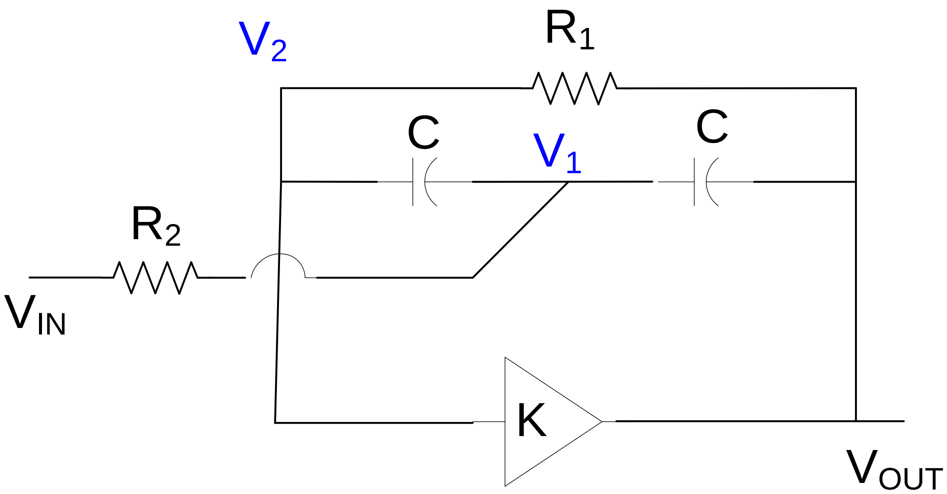
Basic Noninverting Amplifier

$$K_0 = 1+\frac{R_2}{R_1}$$



- Feedback Amplifier is stable and performs very well!
- Serves as counter-example for “Theorem”!

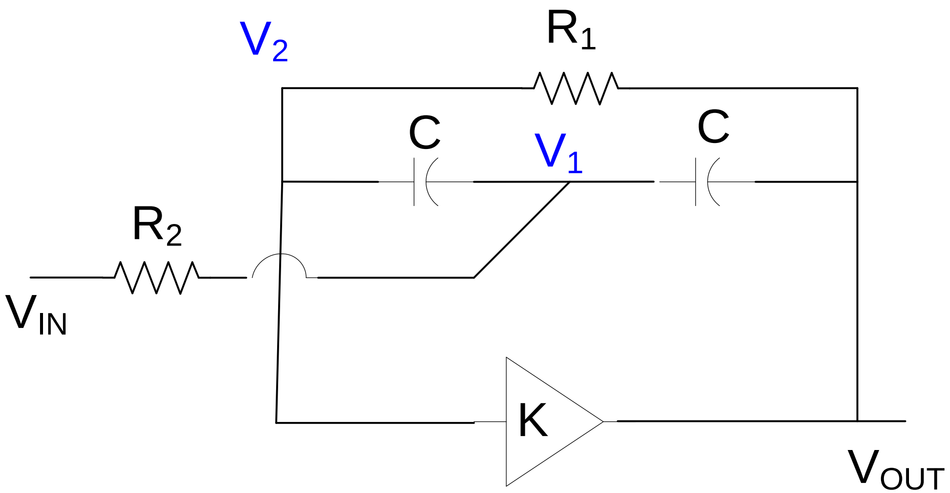
Consider another Filter Example:



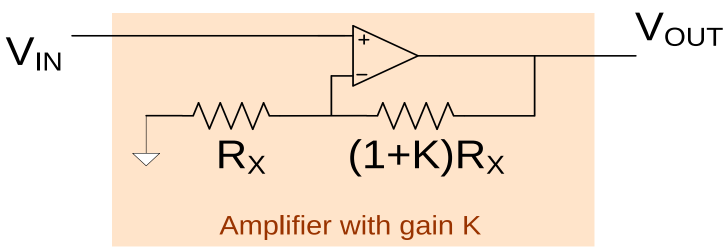
$$V_1\left(sC+sC+G_2\right) = V_{IN}G_2+V_2sC+V_{OUT}sC$$
$$V_2\left(sC+G_1\right) = V_1sC+V_{OUT}G_1$$
$$V_{OUT} = KV_2$$

$$T(s) = \frac{s\left(\frac{K}{CR_2[1-K]}\right)}{s^2+s\left(\frac{2}{CR_1}-\frac{1}{CR_2[1-K]}\right)+\frac{1}{C^2R_1R_2}}$$

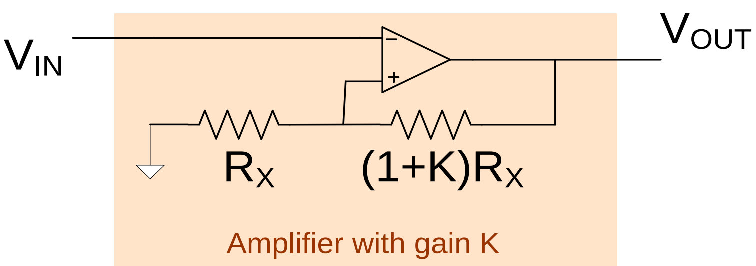
Consider Filter Example:



$$T(s)=\frac{s\left(\frac{K}{CR_2[1-K]}\right)}{s^2+s\left(\frac{2}{CR_1}-\frac{1}{CR_2[1-K]}\right)+\frac{1}{C^2R_1R_2}}$$



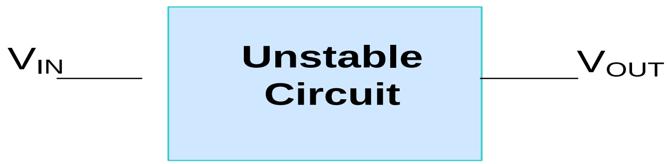
- Stable Amplifier
- But if used in above, filter will be unstable

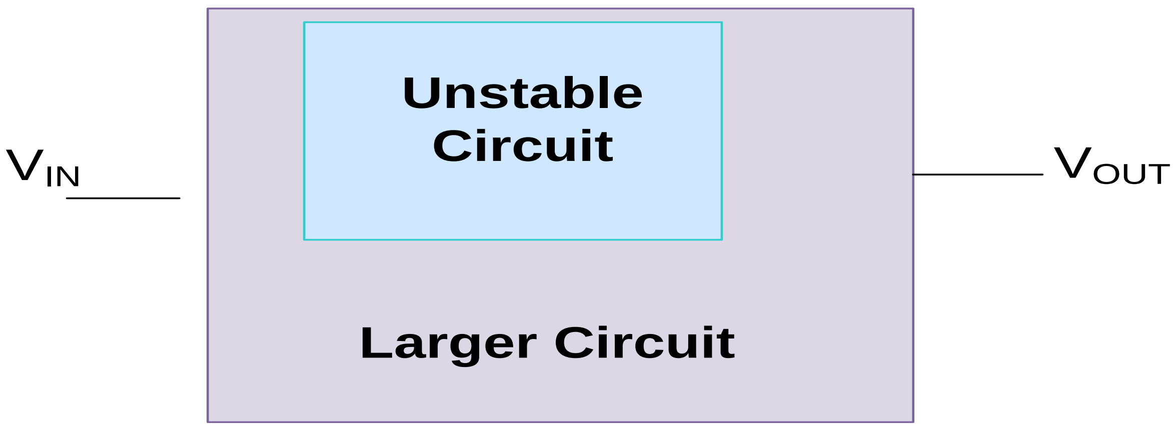


- Unstable Amplifier
- But if used in above, filter will be stable
- Serves as another counter example for "theorem"

Theorem:

If a circuit is unstable, then if this circuit is included as a subcircuit in a larger circuit structure, the larger circuit will also be unstable.

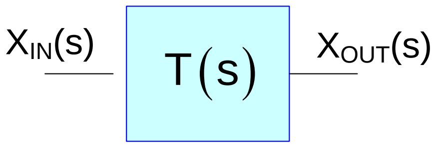


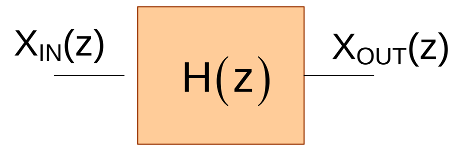


Proof:

This theorem is not valid though many circuit and filter designers believe it to be true !

Filter Concepts and Terminology





Stability Issues:

Is stability or instability good or bad?

Often there is an impression that instability is bad - but why?

Some observations:

- An unstable filter does not behave as a filter
- Unstable filter circuits are often used as waveform generators
- If an unstable circuit is embedded in a larger system, the larger system may be stable or it may be unstable
- If a stable circuit is embedded in a larger system, the larger system may be stable or it may be unstable
- Digital latches, RAMs, etc. are unstable amplifiers
- Some of the best filter circuits include an embedded unstable filter

Stability or Instability is neither good or bad, but it is important for the designer to be aware of the opportunities and limitations associated with this issue

End of Lecture 4