EE 508
Lecture 4

Filter Concepts/Terminology
Basic Properties of Electrical Circuits
Filter Concepts and Terminology

Reflecting poles and zeros to maintain stability or establish minimum phase

Note: magnitude of real part is preserved in reflection, imaginary part remains unchanged
Review from Last Time

Filter Concepts and Terminology

Reflecting poles and zeros to maintain stability or establish minimum phase

Note: complex conjugate reciprocal reflection maintains angle but magnitude of reflected root is the reciprocal of the magnitude of the original root
Filter Concepts and Terminology

- 2-nd order polynomial characterization
- Biquadratic Factorization
- Op Amp Modeling
- Stability and Instability
- Roll-off characteristics
- Dead Networks
- Root Characterization
- Scaling, normalization, and transformation
2-nd order polynomial characterization

\[ s^2 + as + b \]

\[ s^2 + \frac{\omega_0}{Q} s + \omega_0^2 \]

\[ s^2 + 2\zeta \omega_0 s + \omega_0^2 \]

\[ s^2 + (p_1 + p_2) s + p_1 p_2 = (s + p_1)(s + p_2) \]

with complex conjugate roots

\[ s^2 + 2\alpha s + \alpha^2 + \beta^2 = (s + \alpha + j\beta)(s + \alpha - j\beta) \]

\[ s^2 + 2r\cos(\theta)s + r^2 = (s + re^{j\theta})(s + re^{-j\theta}) \]
2-nd order polynomial characterization

\{a,b\} \quad \{\omega_o,Q\} \quad \{\zeta, \omega_o\} \quad \{p_1,p_2\}

\{\alpha,\beta\} \quad \{r,\theta\}

Alternate equivalent parameter sets

Widely used interchangeably

Easy mapping from one to another

Defined irrespective of whether polynomial appears in numerator or denominator of transfer function

If order is greater than 2, often multiple root pairing options so these parameter sets will not be unique for a given polynomial or transfer function

If cc roots exist, these will almost always be paired together (unique)
Biquadratic Factorization

\[ T(s) = \frac{\sum_{i=1}^{m} a_i s^i}{\sum_{i=1}^{n} b_i s^i} = \frac{N(s)}{D(s)} \]

If \( m \) or \( n \) is even, integer-monic polynomials \( N(s) \) or \( D(s) \) can be expressed as

\[ P(s) = \sum_{i=0}^{k} c_i s^i = \prod_{i=1}^{k/2} (s^2 + d_{1i} s + d_{2i}) \]

If \( m \) or \( n \) is odd, integer-monic polynomials \( N(s) \) or \( D(s) \) can be expressed as

\[ P(s) = \sum_{i=0}^{k} c_i s^i = (s + d_0) \prod_{i=1}^{(k-1)/2} (s^2 + d_{1i} s + d_{2i}) \]

- These are termed quadratic factorizations

- If both \( N(s) \) and \( D(s) \) are expressed as quadratic factorizations, quadratic pairs can be grouped to obtain a Biquadratic factorization of \( T(s) \)
Biquadratic Factorization

Pole and zero pairings can always be made so that all coefficients in the biquadratic factorizations are real.

In general, the biquadratic factorizations are not unique.

- If roots are real, multiple choices for first-order factor and remaining roots can be partitioned into groups of 2 in different ways.

- Complex conjugate root pairs are generally grouped together so that all coefficients are real.
Biquadratic Factorization

If $n$ is even and $n \geq m$, 

$$T(s) = \sum_{i=1}^{m} a_s^i s^i \sum_{i=1}^{n} b_s^i s^i = K \cdot \prod_{i=1}^{n/2} T_{BQi}(s)$$

If $n$ is odd and $n \geq m$, 

$$T(s) = \sum_{i=1}^{m} a_s^i s^i \sum_{i=1}^{n} b_s^i s^i = K \cdot \left( \frac{a_{10} s + a_{00}}{s + b_{00}} \right) \cdot \prod_{i=1}^{(n-1)/2} T_{BQi}(s)$$

where 

$$T_{BQi}(s) = \frac{a_{2i} s^2 + a_{1i} s + a_{0i}}{s^2 + b_{1i} s + b_{0i}}$$

and where $K$ is a real constant and all coefficients are real (some may be 0)

- Factorization is not unique
- $H(z)$ factorizations not restricted to have $m \leq n$
- Each biquatracic factor can be represented by any of the 6 alternative parameter sets in the numerator or denominator.
Common Filter Architectures

Cascaded Biquads

\[ V_{IN} \rightarrow T_1(s) \rightarrow T_2(s) \rightarrow \cdots \rightarrow T_k(s) \rightarrow \cdots \rightarrow T_m(s) \rightarrow V_{OUT} \]

Leapfrog

\[ V_{IN} \rightarrow I_1(s) \rightarrow I_2(s) \rightarrow I_3(s) \rightarrow I_4(s) \rightarrow \cdots \rightarrow I_{k-1}(s) \rightarrow I_k(s) \rightarrow V_{OUT} \]

Multiple-loop Feedback

\[ X_{IN} \rightarrow \alpha_0 \rightarrow \alpha_F \rightarrow \alpha_1 \rightarrow \alpha_2 \rightarrow \alpha_k \rightarrow X_{OUT} \]

- Three classical filter architectures are shown
- The Cascaded Biquad and the Leapfrog approaches are most common
- The Cascaded Biquad structure follows directly from the Biquadratic Factorization
Common Filter Architectures

Cascaded Biquads

\[ V_{IN} \rightarrow T_1(s) \rightarrow T_2(s) \rightarrow \cdots \rightarrow T_k(s) \rightarrow \cdots \rightarrow T_m(s) \rightarrow V_{OUT} \]

\[ T(s) = T_1 T_2 \cdots T_m \]

- Sequence in Cascade often affect performance
- Different biquadratic factorizations will provide different performance
- Although some attention was given to the different alternatives for biquadratic factorization, a solid general formulation of the cascade sequencing problem or the biquadratic factorization problem never evolved
Gain, Bandwidth and GB

Frequency Dependent Model of Op Amps

Most op amps are designed so that they behave as a first-order circuit at frequencies up to the unity gain frequency or beyond.

\[
A_1(s) = \frac{GB}{s+BW_A}
\]

where

\[
BW_A = \frac{GB}{A_0} << 1
\]

Can usually model with a more-simplified gain expression:

\[
A(s) = \frac{GB}{s}
\]

Adequate model for most applications
Gain, Bandwidth and GB

Effects of GB on closed-loop Amplifiers

\[ A_1(s) = \frac{GB}{s + BW_A} \]

GB = \( A_0 \cdot BW_A \)

\[ A(s) = \frac{GB}{s + BW_A} \]

Adequate model for most applications

\[ V_1 = \frac{V_{OUT}}{K_0} \]

\[ V_{OUT} = A(s)(V_1 - V_{IN}) \]

\[ A(s) = \frac{GB}{s + BW_A} \]

\[ A_{FB}(s) = \frac{V_{OUT}}{V_{IN}} = \frac{K_0}{s \frac{K_0}{GB} + \left(1 + \frac{K_0}{GB} \right) BW_A} \]

\[ A_{FB}(s) \approx \frac{K_0}{1 + \frac{K_0}{GB} s} \]

\[ BW = \frac{GB}{K_0} \]

Basic Noninverting Amplifier

\[ K_0 = 1 + \frac{R_2}{R_1} \]
Gain, Bandwidth and GB

Effects of GB on closed-loop Amplifiers

\[ A_1(s) = \frac{GB}{s + BW_A} \]

\[ GB = A_0 \cdot BW_A \]

\[ A(s) = \frac{GB}{s} \]

Adequate model for most applications

Basic Noninverting Amplifier

\[ K_0 = 1 + \frac{R_2}{R_1} \]

\[ A_{FB}(s) = \frac{K_0}{1 + s \frac{K_0}{GB}} \]

\[ BW = \frac{GB}{K_0} \]

\[ A_{FB}(s) \]

20 dB/decade

\[ A_0 \]

\[ A(s) \]

\[ K_0 \]

\[ A_{FB}(s) \]

\[ 0dB \]

\[ BW_A \]

\[ BW \]

s-plane

Re

Im

p_{CL}

\[ GB/K_0 \]

p_{OL}

\[ BW_A \]

\[ \omega \]
Gain, Bandwidth and GB

Summary of Effects of GB on Basic Inverting and Noninverting Amplifiers

Basic Noninverting Amplifier

\[ K_0 = 1 + \frac{R_2}{R_1} \]

\[ BW = \frac{GB}{K_0} \]

\[ A_{FB}(s) = \frac{K_0}{1 + s \frac{K_0}{GB}} \]

Basic Inverting Amplifier

\[ K_0 = \frac{R_2}{R_1} \]

\[ BW = \frac{GB}{1 + K_0} \]

\[ A_{FB}(s) = -\frac{K_0}{1 + s \frac{1 + K_0}{GB}} \]
End of Lecture 4