EE 508
Lecture 4
Filter Concepts/Terminology
Basic Properties of Electrical Circuits
Review from Last Time

Filter Concepts and Terminology

• 2-nd order polynomial characterization
• Biquadratic Factorization
• Op Amp Modeling
• Stability and Instability
• Roll-off characteristics
• Dead Networks
• Root Characterization
• Scaling, normalization, and transformation
2-nd order polynomial characterization

\[ s^2 + as + b \]
\[ \{a, b\} \]

\[ s^2 + \frac{\omega}{Q} s + \omega^2 \]
\[ \{\omega, Q\} \]

\[ s^2 + 2\zeta \omega_0 s + \omega^2 \]
\[ \{\zeta, \omega_0\} \]

\[ s^2 + (p_1 + p_2)s + p_1 p_2 = (s + p_1)(s + p_2) \]
\[ \{p_1, p_2\} \]

with complex conjugate roots

\[ s^2 + 2\alpha s + \alpha^2 + \beta^2 = (s + \alpha + j\beta)(s + \alpha - j\beta) \]
\[ \{\alpha, \beta\} \]

\[ s^2 + 2r\cos(\theta)s + r^2 = (s + re^{i\theta})(s + re^{-i\theta}) \]
\[ \{r, \theta\} \]
Biquadratic Factorization

If \( n \) is even and \( n \geq m \),

\[
T(s) = \sum_{i=1}^{m} a_i s^i + \sum_{i=1}^{n} b_i s^i = K \cdot \prod_{i=1}^{n/2} T_{BQ_i}(s)
\]

If \( n \) is odd and \( n \geq m \),

\[
T(s) = \sum_{i=1}^{m} a_i s^i + \sum_{i=1}^{n} b_i s^i = K \cdot \left( \frac{a_{10} s + a_{00}}{s + b_{00}} \right) \cdot \prod_{i=1}^{(n-1)/2} T_{BQ_i}(s)
\]

where \( T_{BQ_i}(s) = \frac{a_{2i} s^2 + a_{1i} s + a_{0i}}{s^2 + b_{1i} s + b_{0i}} \)

and where \( K \) is a real constant and all coefficients are real (some may be 0)

- Factorization is not unique
- \( H(z) \) factorizations not restricted to have \( m \leq n \)
- Each biquatradic factor can be represented by any of the 6 alternative parameter sets in the numerator or denominator
Three classical filter architectures are shown:

- The Cascaded Biquad and the Leapfrog approaches are most common.
- The Cascaded Biquad structure follows directly from the Biquadratic Factorization.
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Gain, Bandwidth and GB

Frequency Dependent Model of Op Amps

Most op amps are designed so that they behave as a first-order circuit at frequencies up to the unity gain frequency or beyond.

\[ A_1(s) = \frac{GB}{s + BW_A} \]

\[ GB = A_0 \cdot BW_A \]

where

\[ BW_A = \frac{GB}{A_0} \ll 1 \]

Can usually model with a more-simplified gain expression

\[ A(s) = \frac{GB}{s} \]

Adequate model for most applications.
Gain, Bandwidth and GB

Effects of GB on closed-loop Amplifiers

\[ A_1(s) = \frac{GB}{s + BW_A} \]

GB = \( A_0 \cdot BW_A \)

\[ A(s) = \frac{GB}{s} \]

Adequate model for most applications

Basic Noninverting Amplifier

\[ V_1 \]
\[ R_2 \]
\[ R_1 \]
\[ V_{IN} \]
\[ V_{OUT} \]

\[ K_0 = 1 + \frac{R_2}{R_1} \]

\[ V_1 = \frac{V_{OUT}}{K_0} \]
\[ V_{OUT} = A(s)(V_1 - V_{IN}) \]
\[ A(s) = \frac{GB}{s + BW_A} \]

\[ A_{FB}(s) = \frac{V_{OUT}}{V_{IN}} = \frac{K_0}{s \frac{K_0}{GB} + \left(1 + K_0 \frac{BW_A}{GB}\right)} \]

\[ A_{FB}(s) \approx \frac{K_0}{1 + s \frac{K_0}{GB}} \]

\[ BW = \frac{GB}{K_0} \]

Basic Noninverting Amplifier

\[ \omega \]

0dB

20dB/decade

BW_A

BW

\[ 0 \]

A_0

A(s)

A_{FB}(s)

A_1(s)
Gain, Bandwidth and GB

Effects of GB on closed-loop Amplifiers

**Basic Noninverting Amplifier**

\[ K_0 = \frac{R_2}{R_1} \]

GB = \( A_0 \cdot BW_A \)

\[ A(s) = \frac{GB}{s} \]

**Adequate model for most applications**

\[ A_{FB}(s) \approx \frac{K_0}{1 + s \frac{GB}{K_0}} \]

\[ BW = \frac{GB}{K_0} \]

\[ A(s) = A_0 \cdot A(s) \]

\[ K_0 \]

\[ BW \]

\[ BW_A \]

\[ s\text{-plane} \]

\[ \text{Re} \]

\[ \text{Im} \]
Gain, Bandwidth and GB

Summary of Effects of GB on Basic Inverting and Noninverting Amplifiers

\[ A_0(s) = \frac{GB}{s + BW_A} \]

Adequate model for most applications

\[ K_0 = 1 + \frac{R_2}{R_1} \]

\[ BW = \frac{GB}{K_0} \]

\[ A_{FB}(s) = \frac{K_0}{1 + s \frac{K_0}{GB}} \]

\[ BW = \frac{GB}{1 + K_0} \]

\[ A_{FB}(s) = -\frac{K_0}{1 + s \frac{1 + K_0}{GB}} \]
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Stability and Instability

True or False?

An unstable circuit will oscillate

False – unstable circuits will either latch up or oscillate. Latch-up is often the consequence of saturating nonlinearities of circuits that have positive real axis poles.

Achieving stability is a major goal of the filter designer

False – a filter is usually of little practical use if there are concerns about stability.

Unstable circuits are of little use in designing filters

False – will discuss details later.
Theorem ?: If a circuit is unstable, then if this circuit is included as a subcircuit in a larger circuit structure, the larger circuit will also be unstable.

Proof ?: Consider First Some Related Concepts
Gain, Bandwidth and GB

Consider “positive feedback” closed-loop amplifier

\[ A_1(s) = \frac{GB}{s + BW_A} \]

\[ GB = A_0 \cdot BW_A \]

\[ A(s) = \frac{GB}{s} \]

Adequate model for most applications

Feedback Amplifier is Unstable!
Gain, Bandwidth and GB

Summary of Effects of GB on Basic Inverting and Noninverting Amplifiers with “Positive Feedback"

\[ K_0 = 1 + \frac{R_2}{R_1} \]

\[ BW = \frac{GB}{K_0} \]

\[ A_{FB}(s) = \frac{K_0}{1 - s \frac{GB}{K_0}} \]

\[ A(s) = \frac{GB}{s} \]

\[ A_0 \]

\[ \omega \]

Both FB Amplifiers are Unstable
Gain, Bandwidth and GB

Summary of Effects of GB on Basic Inverting and Noninverting Amplifiers with “Positive Feedback”

Both FB Amplifiers are Unstable

Is “Positive Feedback” bad?

- Engineers often make the assumption that positive feedback is bad and must be avoided
- Positive feedback in these stand-alone amplifiers resulted in unstable circuits
- Positive feedback is often very beneficial and should not be unilaterally avoided
Gain, Bandwidth and GB

Consider Op Amp with RHP Pole (Unstable Op Amp)

\[ A_1(s) = \frac{GB}{s - BW_A} \]

\[ |GB| = A_0 \cdot BW_A \]

Op Amp is Unstable, dc gain is negative
Gain, Bandwidth and GB

Consider Op Amp with RHP Pole (Unstable Op Amp)

\[ V_1 = \frac{V_{OUT}}{K_0} \]
\[ V_{OUT} = A(s)(V_1 - V_{IN}) \]
\[ A(s) = \frac{GB}{s - BW_A} \]

\[ |GB| = A_0 \cdot BW_A \]

\[ A_{FB}(s) = \frac{V_{OUT}}{V_{IN}} = \frac{K_0}{s \cdot GB + \left(1 - K_0 \frac{BW_A}{GB}\right)} \]

\[ A_{FB}(s) \approx \frac{K_0}{1 + s \cdot \frac{GB}{K_0}} \]

\[ BW = \frac{GB}{K_0} \]

- Feedback Amplifier is stable and performs very well!
- Serves as counter-example for “Theorem”!
Consider another Filter Example:

\[ V_1(sC+sC+G_2) = V_{IN}G_2 + V_2sC + V_{OUT}sC \]
\[ V_2(sC+G_1) = V_1sC + V_{OUT}G_1 \]
\[ V_{OUT} = KV_2 \]

\[ T(s) = \frac{s \left( \frac{K}{CR_2[1-K]} \right)}{s^2 + s \left( \frac{2}{CR_1} - \frac{1}{CR_2[1-K]} \right) + \frac{1}{C^2R_1R_2}} \]
Consider Filter Example:

$$T(s) = \frac{s \left( \frac{K}{CR_2[1-K]} \right)}{s^2 + s \left( \frac{2}{CR_1} - \frac{1}{CR_2[1-K]} \right) + \frac{1}{C^2R_1R_2}}$$

- Stable Amplifier
- But if used in above, filter will be unstable

- Unstable Amplifier
- But if used in above, filter will be stable
- Serves as another counter example for "theorem"
Theorem:
If a circuit is unstable, then if this circuit is included as a subcircuit in a larger circuit structure, the larger circuit will also be unstable.

Proof:
This theorem is not valid though many circuit and filter designers believe it to be true!
Filter Concepts and Terminology

Stability Issues:

Is stability or instability good or bad?

Often there is an impression that instability is bad - but why?

Some observations:
- An unstable filter does not behave as a filter
- Unstable filter circuits are often used as waveform generators
- If an unstable circuit is embedded in a larger system, the larger system may be stable or it may be unstable
- If a stable circuit is embedded in a larger system, the larger system may be stable or it may be unstable
- Digital latches, RAMs, etc. are unstable amplifiers
- Some of the best filter circuits include an embedded unstable filter

Stability or Instability is neither good or bad, but it is important for the designer to be aware of the opportunities and limitations associated with this issue
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Roll-off characteristics

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Single-pole roll-off characterization

Consider: \( T(s) = \frac{\omega_p}{s + \omega_p} \)

\[
T(j\omega) = \frac{\omega_p}{j\omega + \omega_p}
\]

\[
|T(j\omega)| = \frac{\omega_p}{\sqrt{\omega^2 + \omega_p^2}}
\]

\[
\angle T(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_p}\right)
\]

\( m = -20\text{dB/decade} \)

\( m = -6\text{dB/octave} \)
Single-pole roll-off characterization

Consider:

\[ T(s) = \frac{\omega_p}{s + \omega_p} \]

\[ T(j\omega) = \frac{\omega_p}{j\omega + \omega_p} \]

\[ |T(j\omega)| = \frac{\omega_p}{\sqrt{\omega^2 + \omega_p^2}} \]

\[ \angle T(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_p}\right) \]
Roll-off characterization

At frequencies well-past a pole or zero, each LHP pole (real or complex) causes a roll-off in magnitude on a log-log axis of -20dB/decade and each LHP zero causes a roll-off of +20dB/decade

At frequencies of magnitude comparable to that of a pole or zero, it is not easy to predict the roll-off in the magnitude characteristics by some simple expression
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Distortion in Filters

- **Magnitude Distortion**
  - frequency dependent change in gain of a circuit
  (usually bad if building an amplifier but critical if building a filter)

- **Phase Distortion**
  - a circuit has phase distortion if the phase of the transfer function is not linear with frequency

- **Nonlinear Distortion**
  - Presence of frequency components in the output that are not present in the input (generally considered bad in filters but necessary in many other circuits)
End of Lecture 4