EE 508

Lecture 43

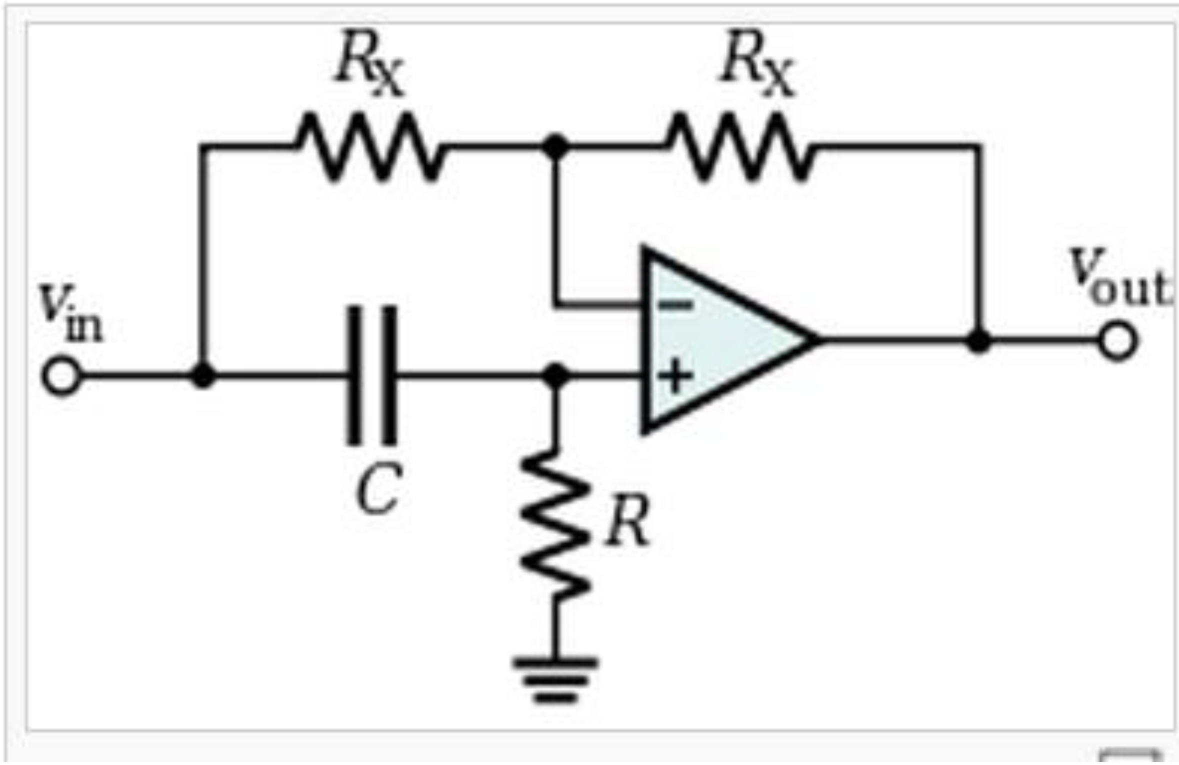
Basic Filter Components

- All Pass Networks
- Arbitrary Transfer Function Synthesis
- Impedance Transformation Circuits
- Equalizers

All-Pass Circuits

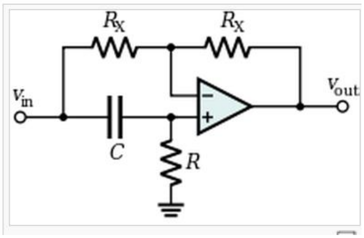
- Magnitude of Gain is Constant
- Phase Changes with Frequency
- Used to correct undesired phase characteristics of a filter

First-Order All Pass

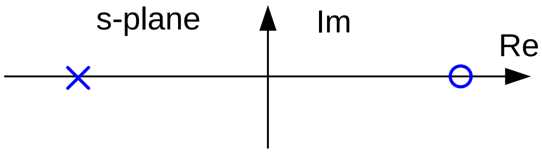


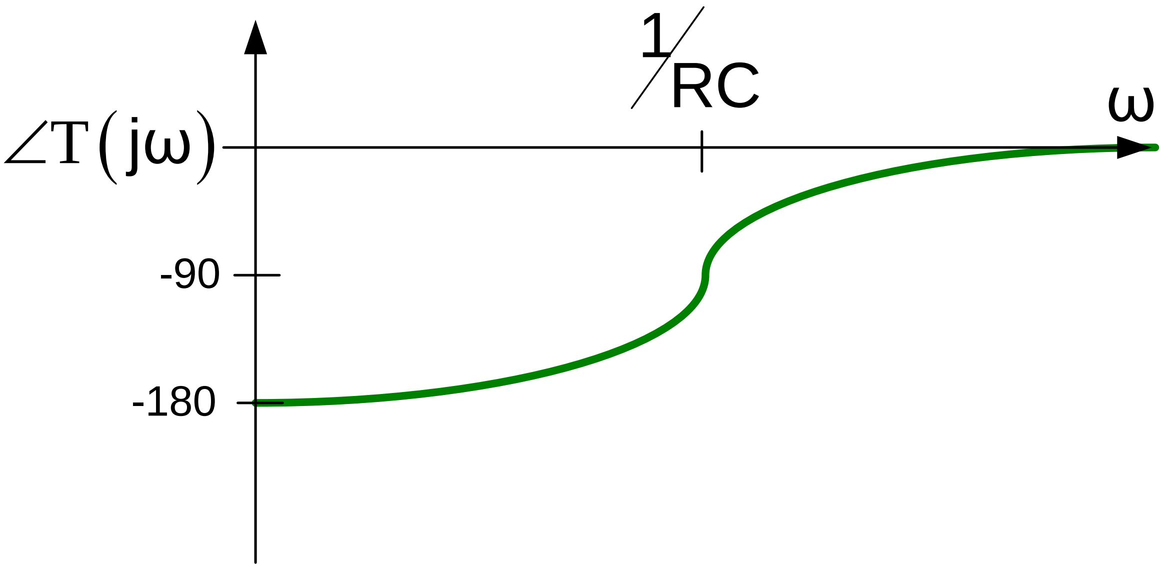
$$T(s) = \frac{s - \frac{1}{RC}}{s + \frac{1}{RC}}$$

First-Order All Pass

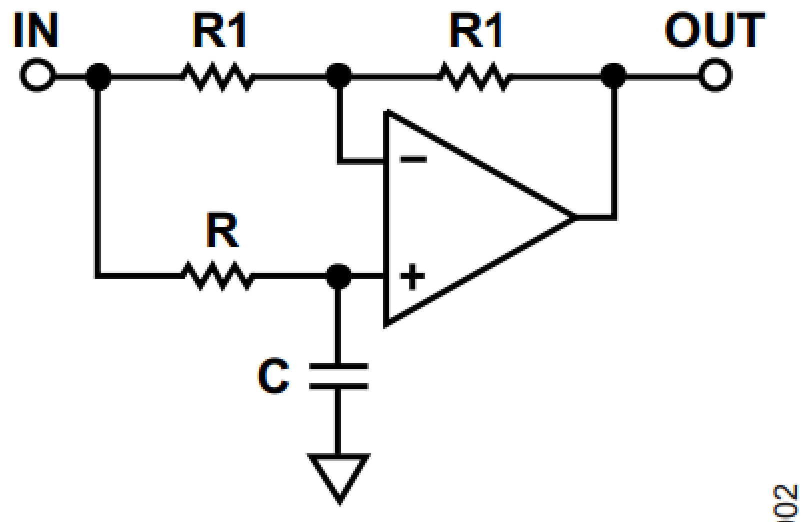


$$T(s) = \frac{s - \frac{1}{RC}}{s + \frac{1}{RC}}$$



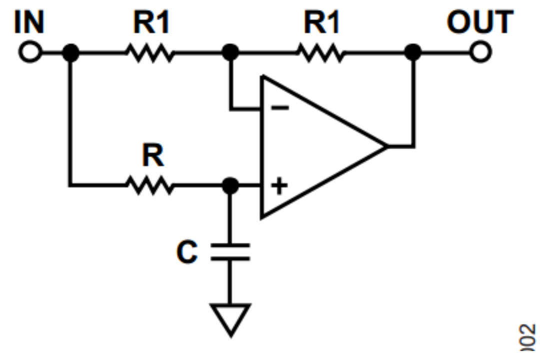


First-Order All Pass

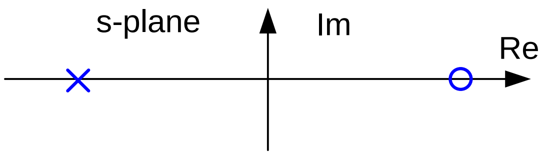


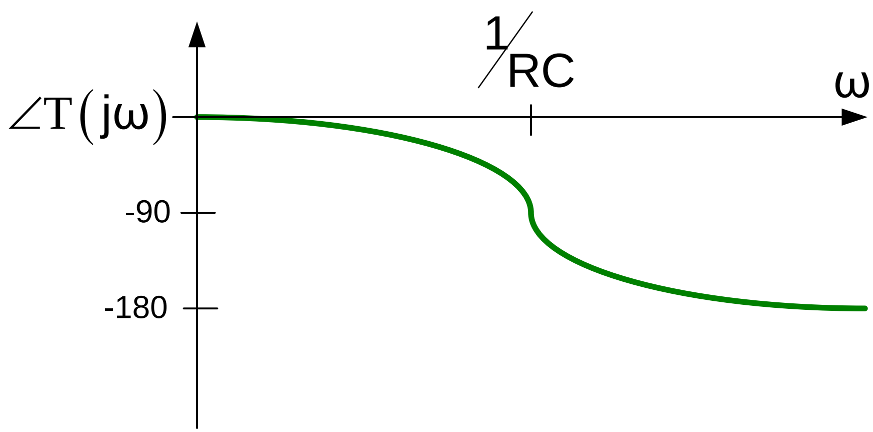
$$T(s) = -\frac{s - \frac{1}{RC}}{s + \frac{1}{RC}}$$

First-Order All Pass

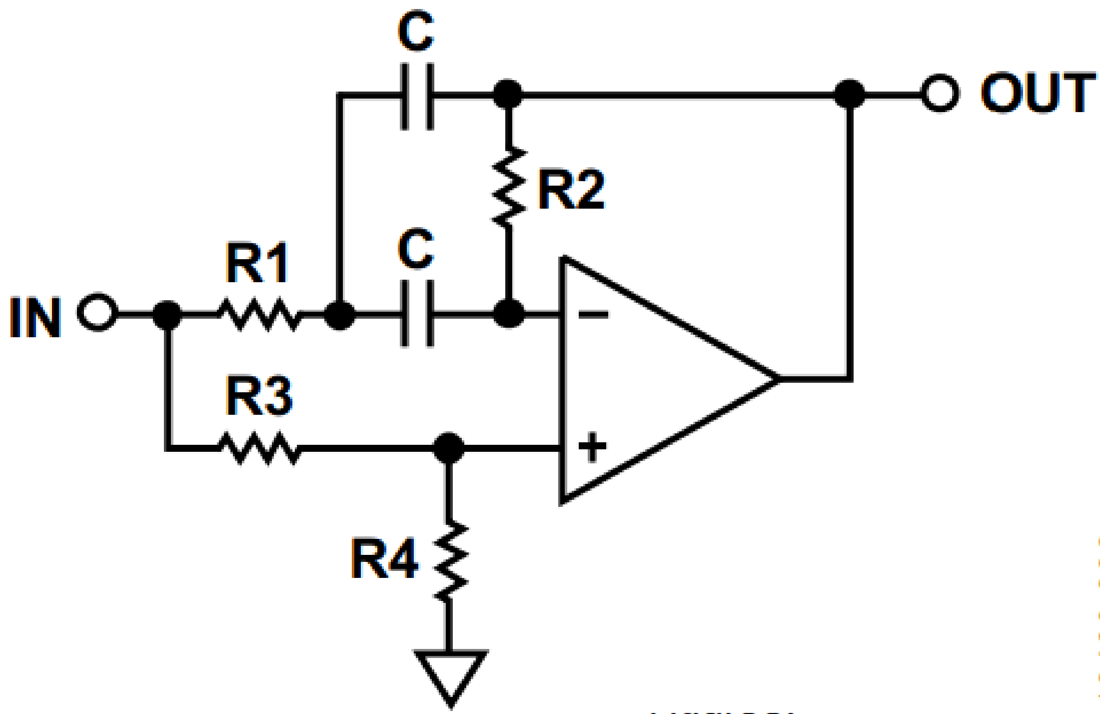


$$T(s) = -\frac{s - \frac{1}{RC}}{s + \frac{1}{RC}}$$





Second-Order All Pass

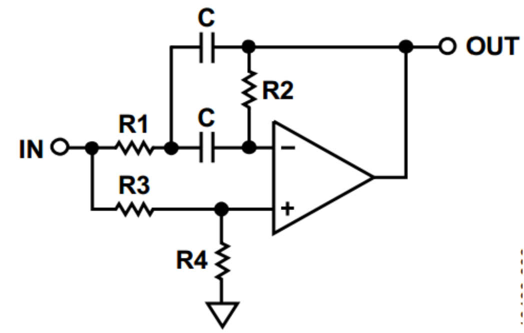


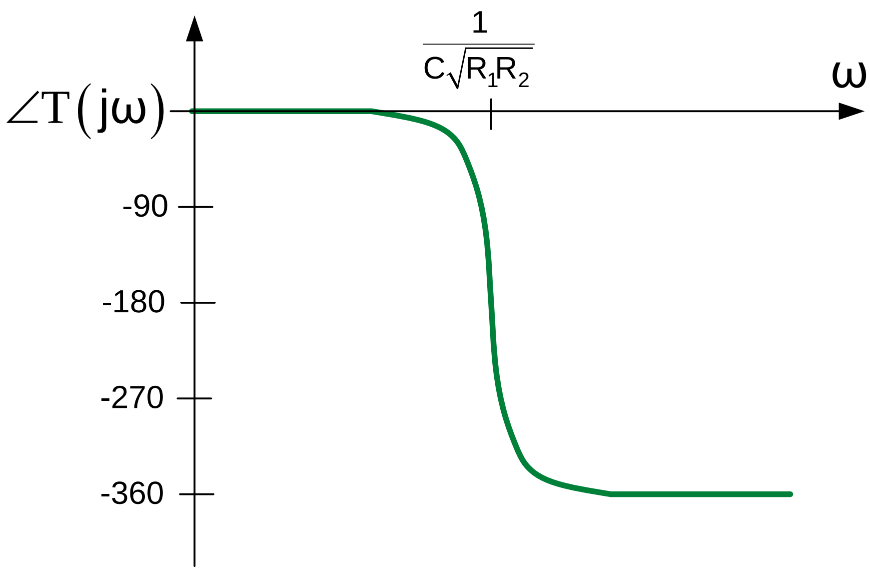
$$\frac{V_O}{V_{IN}} = \frac{s^2 - s\left(\frac{2}{R2\,C}\right) + \frac{1}{R1\;R2\;C^2}}{s^2 + s\left(\frac{2}{R2\,C}\right) + \frac{1}{R1\;R2\;C^2}}$$

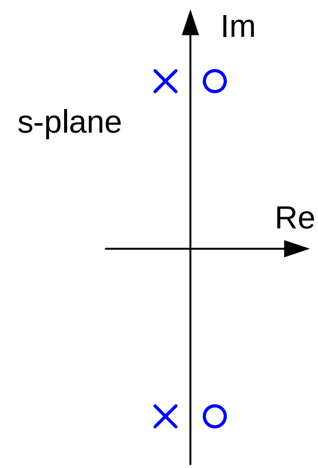
Based upon Bridged-T Feedback Structure

Second-Order All Pass

$$\frac{V_O}{V_{IN}} = \frac{s^2 - s\left(\frac{2}{R2\,C}\right) + \frac{1}{R1\;R2\;C^2}}{s^2 + s\left(\frac{2}{R2\,C}\right) + \frac{1}{R1\;R2\;C^2}}$$



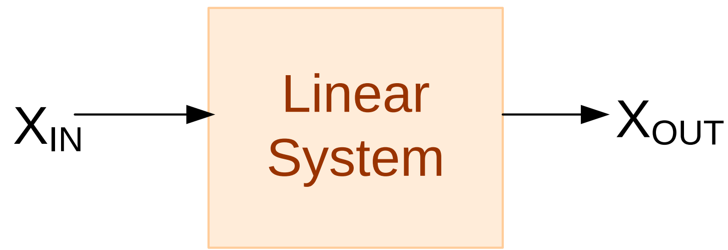




Arbitrary Transfer Function Synthesis

- Based upon coefficient derivation
- Can be used to implement/solve an arbitrary differential equation
- Versatile
- Basic concept of Analog Computer

Applications of integrators to solving differential equations



Standard Integral form of a differential equation

$$X_{OUT} = b_1 \int X_{OUT} + b_2 \iint X_{OUT} + b_3 \iiint X_{OUT} + \ldots + a_0 X_{IN} + \int X_{IN} + \iint X_{IN} + \ldots$$

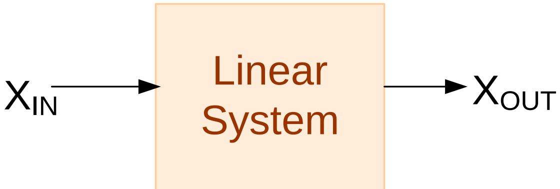
Standard differential form of a differential equation

$$X_{OUT} = \alpha_1 X_{OUT}^{'} + \alpha_2 X_{OUT}^{''} + \alpha_3 X_{OUT}^{'''} + \ldots + \beta_1 X_{IN} + \beta_2 X_{IN}^{'} + \beta_3 X_{IN}^{''} + \ldots$$

Initial conditions not shown

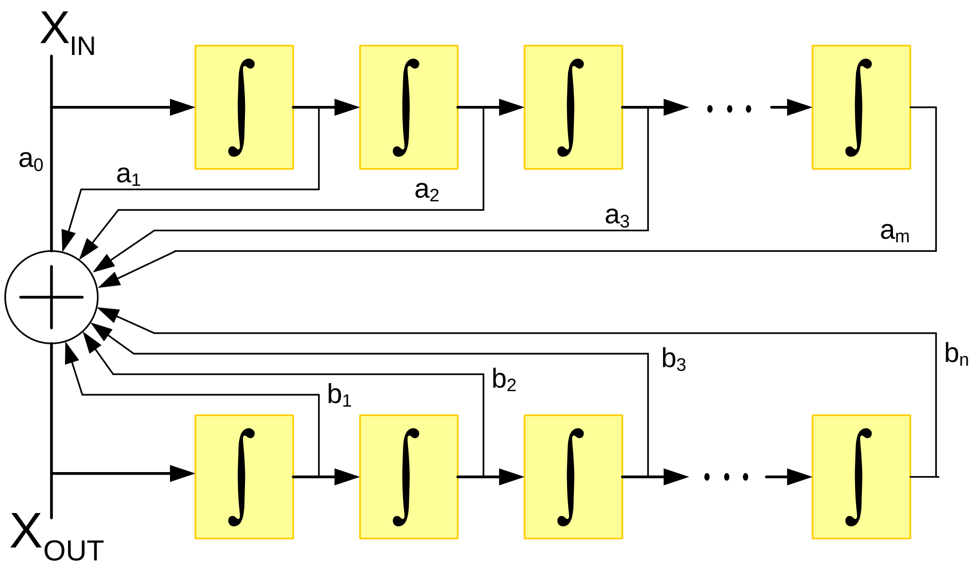
Can express any system in either differential or integral form

Applications of integrators to solving differential equations



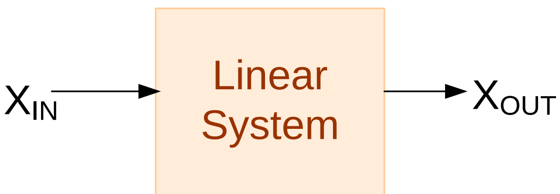
Consider the standard integral form

$$X_{OUT} = b_1 \int X_{OUT} + b_2 \iint X_{OUT} + b_3 \iiint X_{OUT} + \ldots + a_0 X_{IN} + \int X_{IN} + \iint X_{IN} + \ldots$$



This circuit is comprised of summers and integrators
Can solve an arbitrary linear differential equation
This concept was used in Analog Computers in the past

Applications of integrators to solving differential equations



Consider the standard integral form

$$X_{OUT} = b_1 \int X_{OUT} + b_2 \iint X_{OUT} + b_3 \iiint X_{OUT} + \ldots + a_0 X_{IN} + \int X_{IN} + \iint X_{IN} + \ldots$$

Take the Laplace transform of this equation

$$\mathcal{X}_{OUT} = b_1 \frac{1}{s} \mathcal{X}_{OUT} + b_2 \frac{1}{s^2} \mathcal{X}_{OUT} + b_3 \frac{1}{s^3} \mathcal{X}_{OUT} + \ldots + b_n \frac{1}{s^n} + a_0 \mathcal{X}_{IN} + a_1 \frac{1}{s} \mathcal{X}_{IN} + a_2 \frac{1}{s^2} \mathcal{X}_{IN} + a_3 \frac{1}{s^3} \mathcal{X}_{IN} + \ldots + a_m \frac{1}{s^m}$$

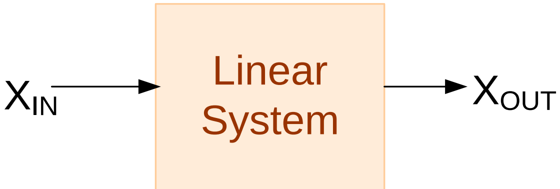
Multiply by s^n and assume m=n   (some of the coefficients can be 0)

$$s^n \mathcal{X}_{OUT} = b_1 s^{n-1} \mathcal{X}_{OUT} + b_2 s^{n-2} \mathcal{X}_{OUT} + b_3 s^{n-3} \mathcal{X}_{OUT} + \ldots + b_n + a_0 s^n \mathcal{X}_{IN} + a_1 s^{n-1} \mathcal{X}_{IN} + a_2 s^{n-2} \mathcal{X}_{IN} + a_3 s^{n-3} \mathcal{X}_{IN} + \ldots + a_n$$

$$\mathcal{X}_{OUT}\left(s^n - b_1 s^{n-1} - b_2 s^{n-2} - b_3 s^{n-3} - \ldots - b_n\right) = \mathcal{X}_{IN}\left(a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3} + \ldots + a_n\right)$$

$$T(s) = \frac{\mathcal{X}_{OUT}}{\mathcal{X}_{IN}} = \frac{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3} + \ldots + a_n}{s^n - b_1 s^{n-1} - b_2 s^{n-2} - b_3 s^{n-3} - \ldots - b_n}$$

Applications of integrators to solving differential equations



Consider the standard integral form

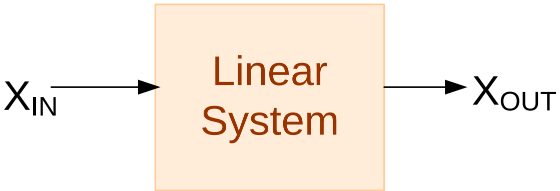
$$X_{OUT} = b_1 \int X_{OUT} + b_2 \iint X_{OUT} + b_3 \iiint X_{OUT} + \ldots + a_0 X_{IN} + \int X_{IN} + \iint X_{IN} + \ldots$$

$$T(s) = \frac{\mathcal{X}_{OUT}}{\mathcal{X}_{IN}} = \frac{a_0 s^{n} + a_1 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3} + \ldots + a_n}{s^{n} - b_1 s^{n-1} - b_2 s^{n-2} - b_3 s^{n-3} - \ldots - b_n}$$

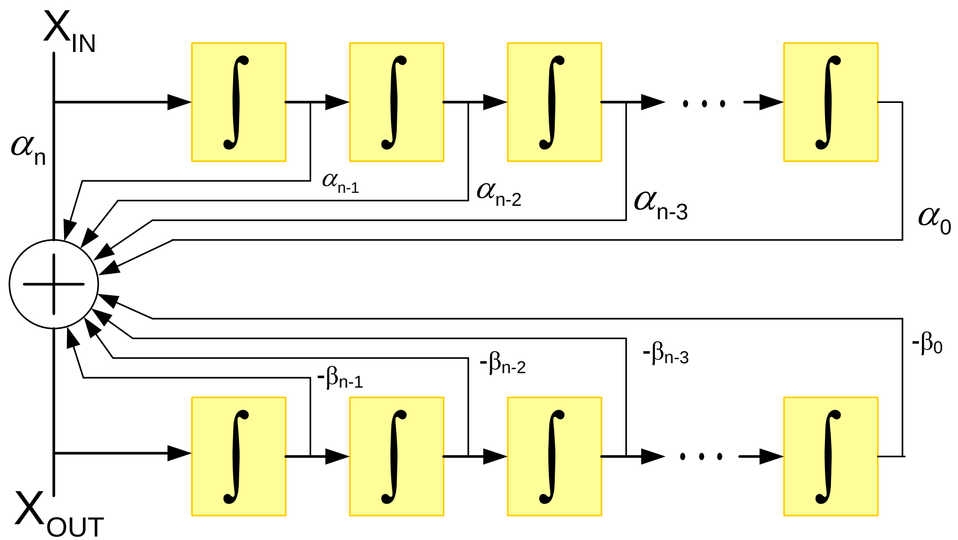
This can be written in more standard form

$$T(s) = \frac{\alpha_n s^{n} + \alpha_{n-1} s^{n-1} + \ldots \alpha_1 s \ + \alpha_0}{s^{n} + \beta_{n-1} s^{n-1} + \ldots + \beta_1 s + \beta_0}$$

Applications of integrators to filter design



$$T(s) = \frac{\alpha_n s^n + \alpha_{m-1} s^{n-1} + \ldots \alpha_1 s \; + \alpha_0}{s^n + \beta_{n-1} s^{n-1} + \ldots + \beta_1 s + \beta_0}$$



Can design (synthesize) any T(s) with just integrators and summers !

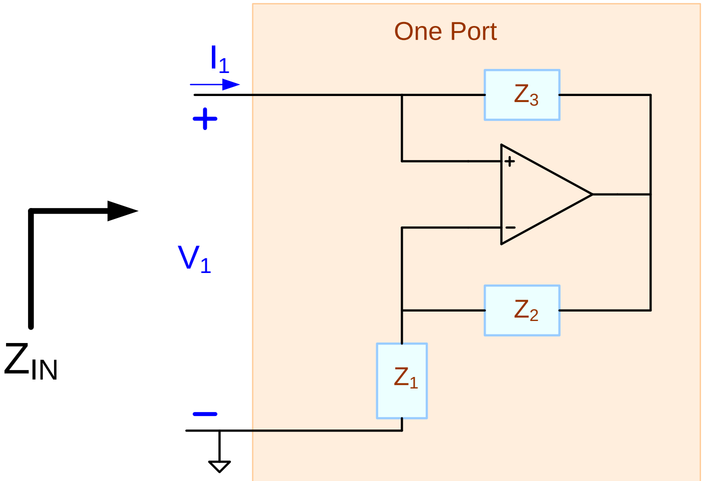
Integrators are not used “open loop” so loss is not added

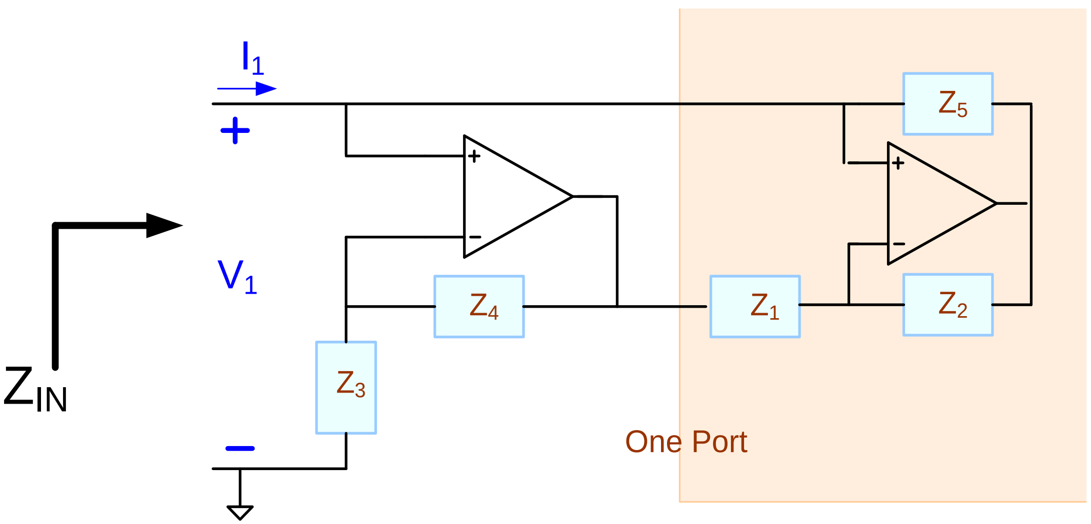
Although this approach to filter design works, often more practical methods are used

Impedance Synthesis

- Focus on synthesizing impedance rather than transfer function
- Gyrators will provide inductance simulation
- Capacitance Multiplication
- Synthesis of super components

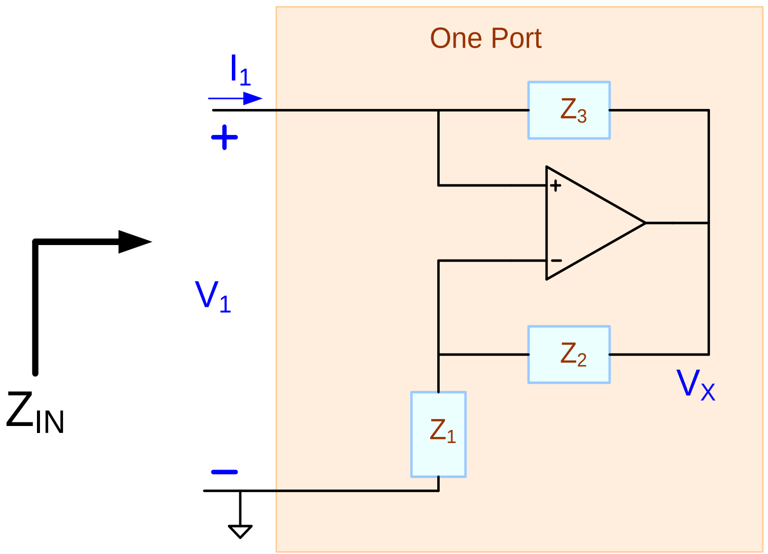
Impedance Converters





Note these circuits are strictly one-ports and have no output node

Impedance Converters



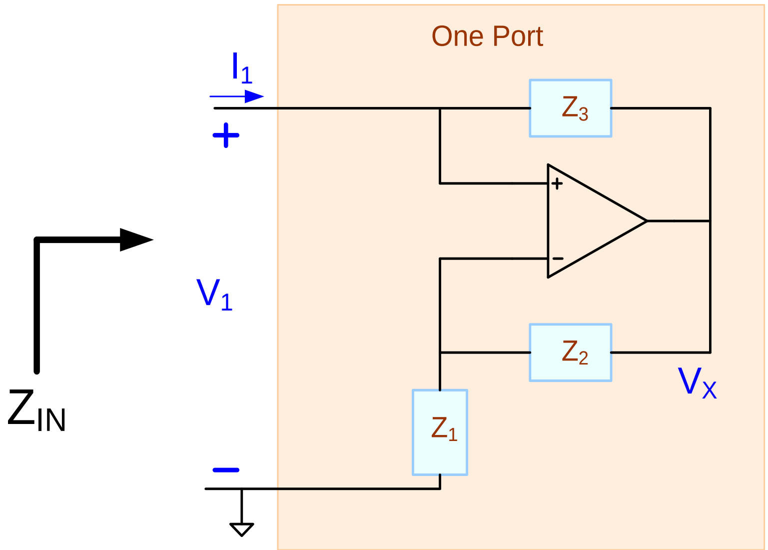
$$V_1(G_1+G_2) = V_X G_2$$

$$I_1=(V_1-V_X)G_3$$

$$Z_{IN} = -\frac{Z_1 Z_3}{Z_2}$$

Observe this input impedance is negative!

Impedance Converters



$$Z_{IN} = -\frac{Z_1 Z_3}{Z_2}$$

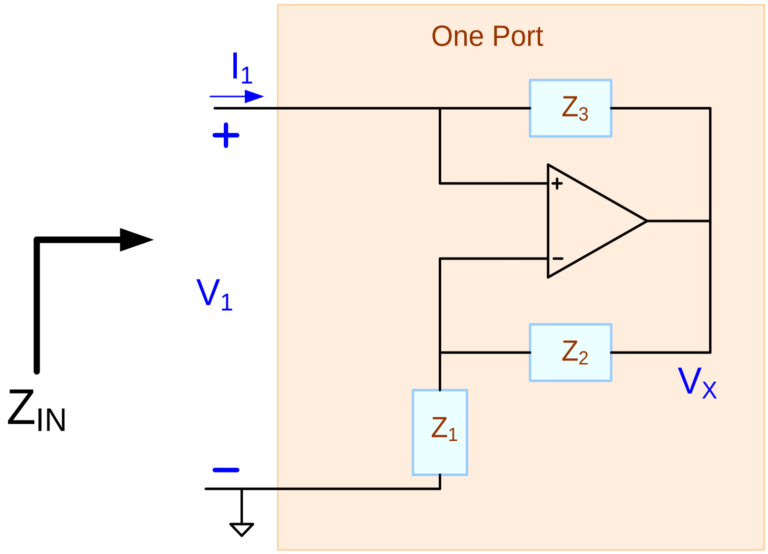
If $Z_1=R_1$, $Z_2=R_2$ and $Z_3=R_3$, $\quad Z_{IN} = -\frac{R_1 R_3}{R_2}$ This is a negative resistor !

If $Z_2=1/sC$, $Z_1=R_1$ and $Z_3=R_3$, $\quad Z_{IN} = -sCR_1R_3$ This is a negative inductor !

If $Z_2=R_2$, $Z_1=1/sC$ and $Z_3=R_3$, $\quad Z_{IN} = -\frac{R_3}{sCR_2}$ This is a negative capacitor !

This is termed a Negative Impedance Converter

Impedance Converters



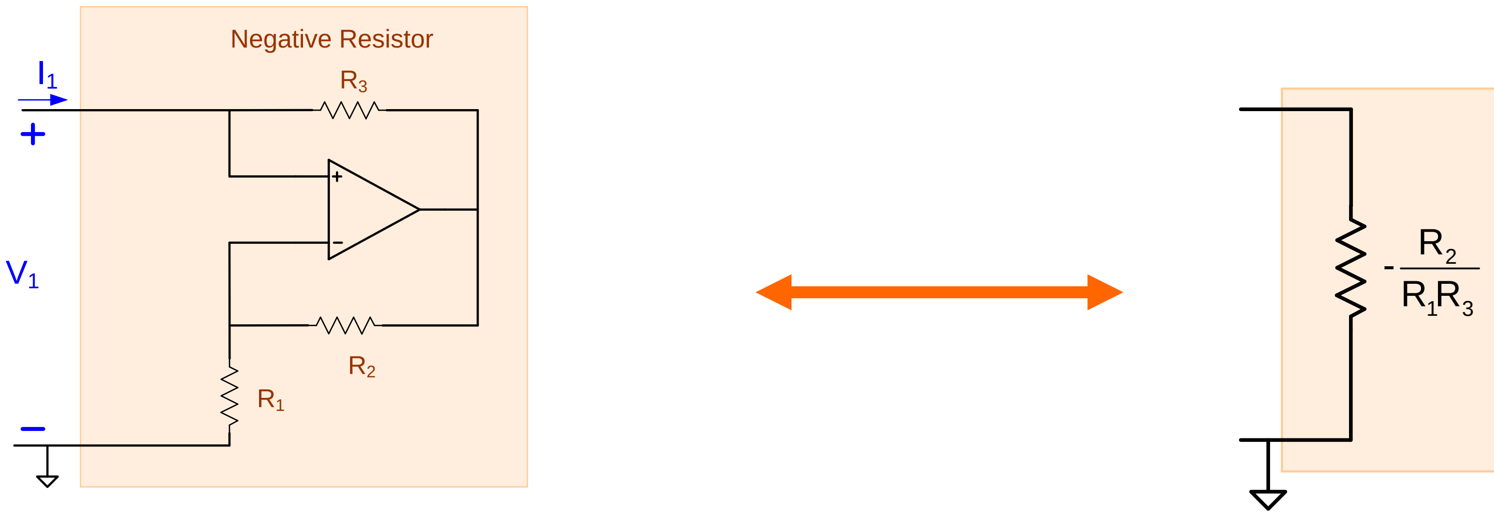
$$Z_{IN} = -\frac{Z_1 Z_3}{Z_2}$$

If Z_2=1/sC, Z_1=R_1 and Z_3=R_3, Z_{IN}= -sCR_1R_3

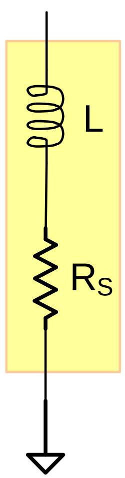
Modification of NIC to provide a positive inductance:

Replace Z_1 itself with a second NIC that has a negative input impedance

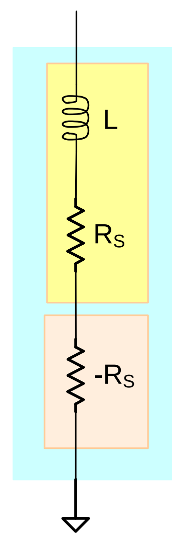
Negative Impedance Converter



One application of NIC



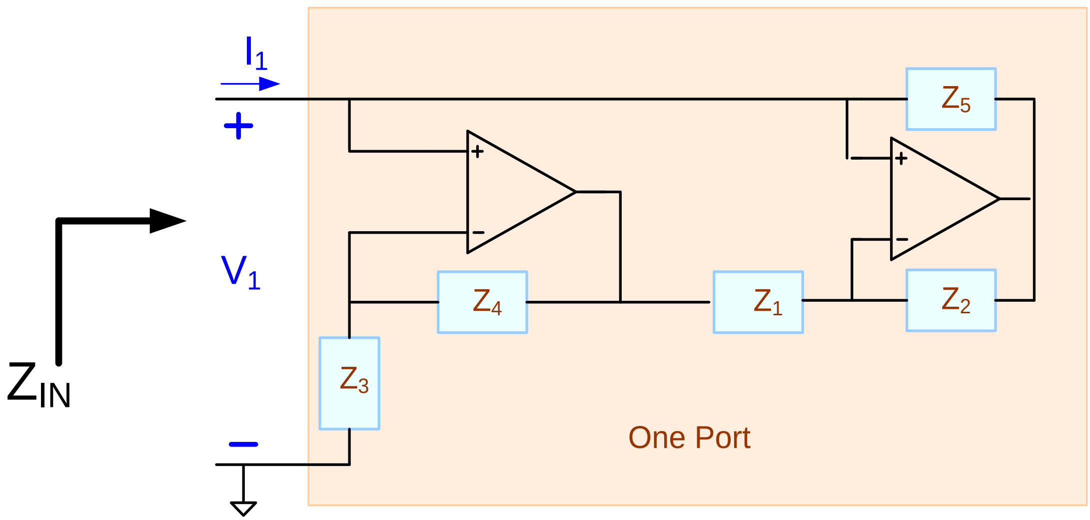
Lossy Inductor



If select components so that $R_S = \frac{R_2}{R_1R_3}$

Lossless Inductor

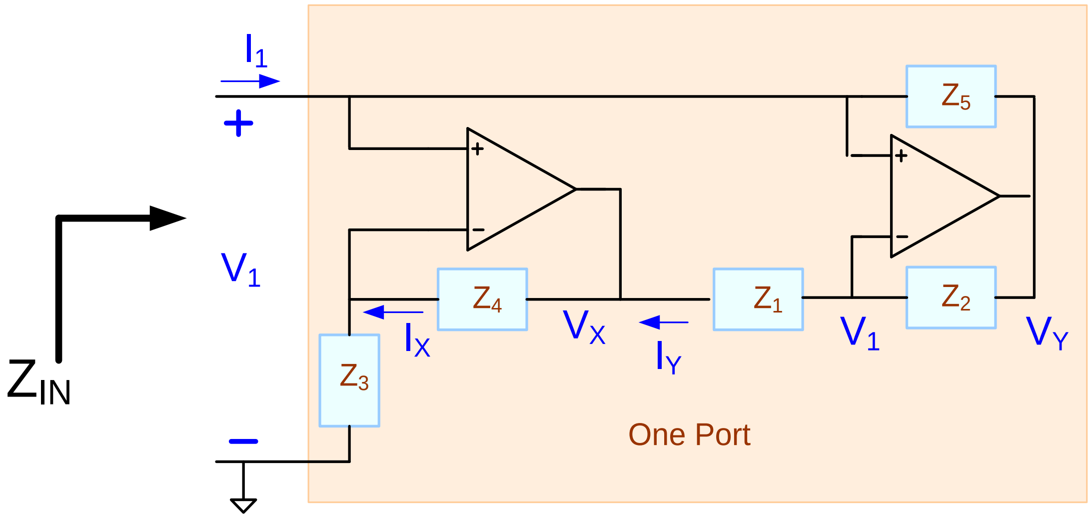
Impedance Converters



$$Z_{IN} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$

This circuit is often called a Gyrator

Gyrator Analysis



$$I_X = V_1 G_3$$

$$V_X = V_1 + V_1 G_3 / G_4 = V_1\left(1 + \frac{G_3}{G_4}\right)$$

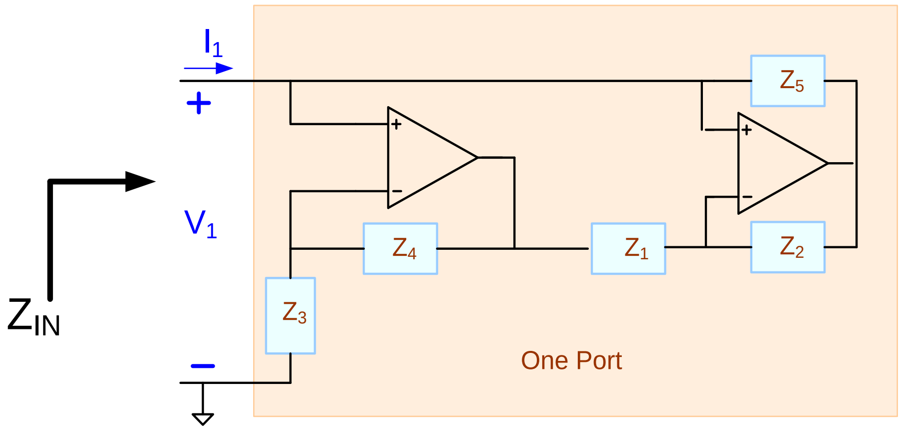
$$I_Y = (V_1 - V_X) G_1 = V_1\left(-\frac{G_3}{G_4}\right) G_1$$

$$V_Y = V_1 + I_Y / G_2 = V_1\left(1 - \frac{G_3}{G_4}\left(\frac{G_1}{G_2}\right)\right)$$

$$I_1 = (V_1 - V_Y) G_5 = V_1\left(\frac{G_3}{G_4}\left(\frac{G_1}{G_2}\right)\right) G_5$$

$$Z_{IN} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$

Gyrator Applications



$$Z_{IN} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$

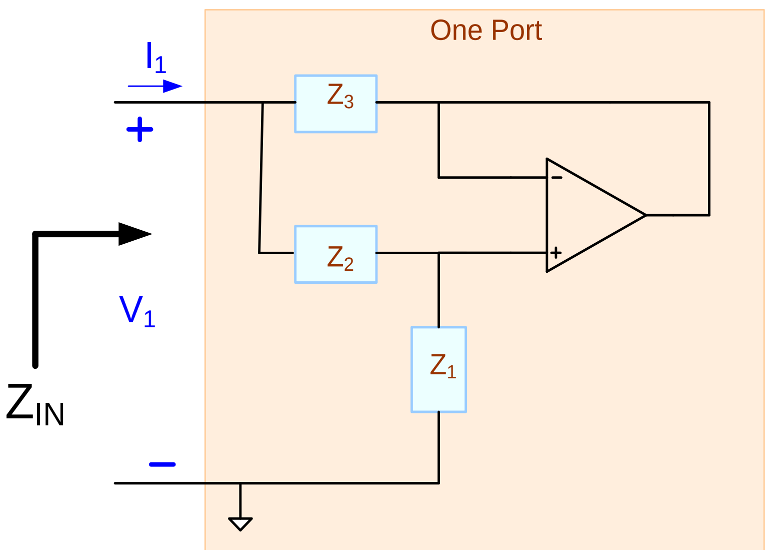
If $Z_1 = Z_3 = Z_4 = Z_5 = R$ and $Z_2 = 1/sC$ $\quad Z_{IN} = \left(R^2 C\right) s$ $\quad$ This is an inductor of value $L = R^2 C$

If $Z_2 = R_2$, $Z_3 = R_3$, $Z_4 = R_4$, $Z_5 = R_5$ and $Z_1 = 1/sC$ $\quad Z_{IN} = \dfrac{R_3 R_5}{s C R_2 R_4}$

This is a capacitor of value $\quad C_{EQ} = C \dfrac{R_2 R_4}{R_3 R_5}$ $\quad$ (can scale capacitance up or down)

If $Z_2 = Z_4 = Z_5 = R$ and $Z_1 = Z_3 = 1/sC$ $\quad Z_{IN} = \left(R^3 C^2\right) s^2$ $\quad$ This is a “super” capacitor of value $R^3 C^2$

Impedance Converters

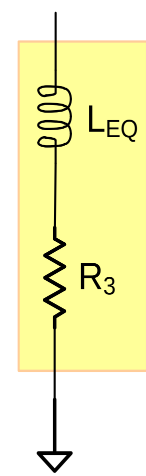


$$I_1 = \left(V_1 - \left(\frac{Z_1}{Z_1 + Z_2} \right) V_1 \right) G_3$$

$$Z_{IN} = Z_3 \left(1 + \frac{Z_2}{Z_1} \right)$$

If $Z_3 = R_3$, $Z_2 = R_2$ and $Z_1 = 1/sC$

$$Z_{IN} = R_3 + s\left(CR_2R_3\right)$$

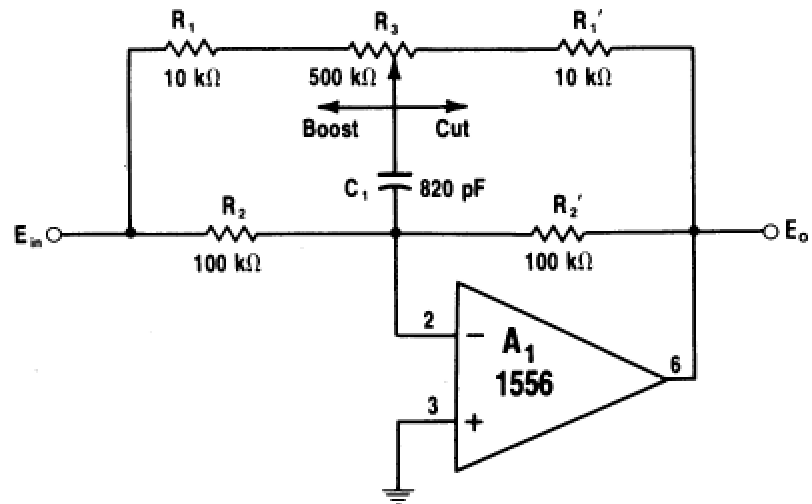


$$L_{EQ} = CR_2R_3$$

Shelving Equalizers

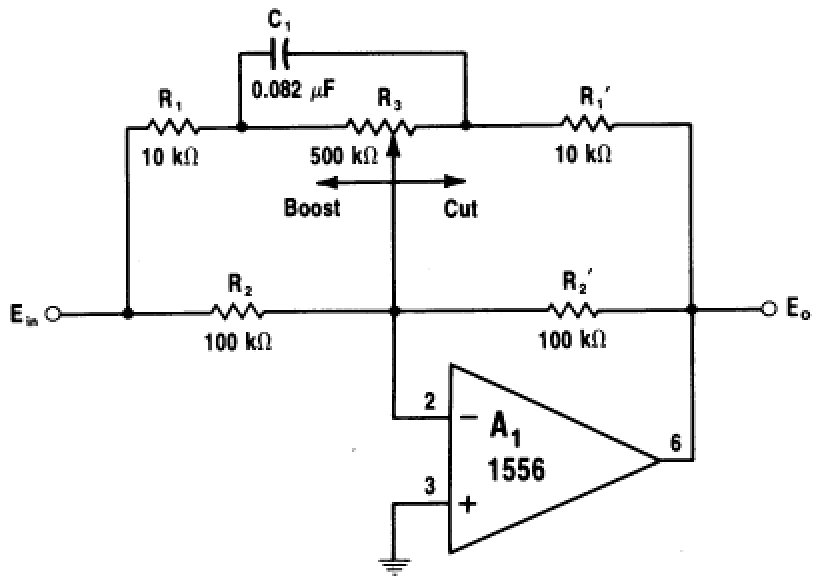
- Widely used in audio applications
- User-programmable filter response

Shelving Equalizers



(A) High frequency.

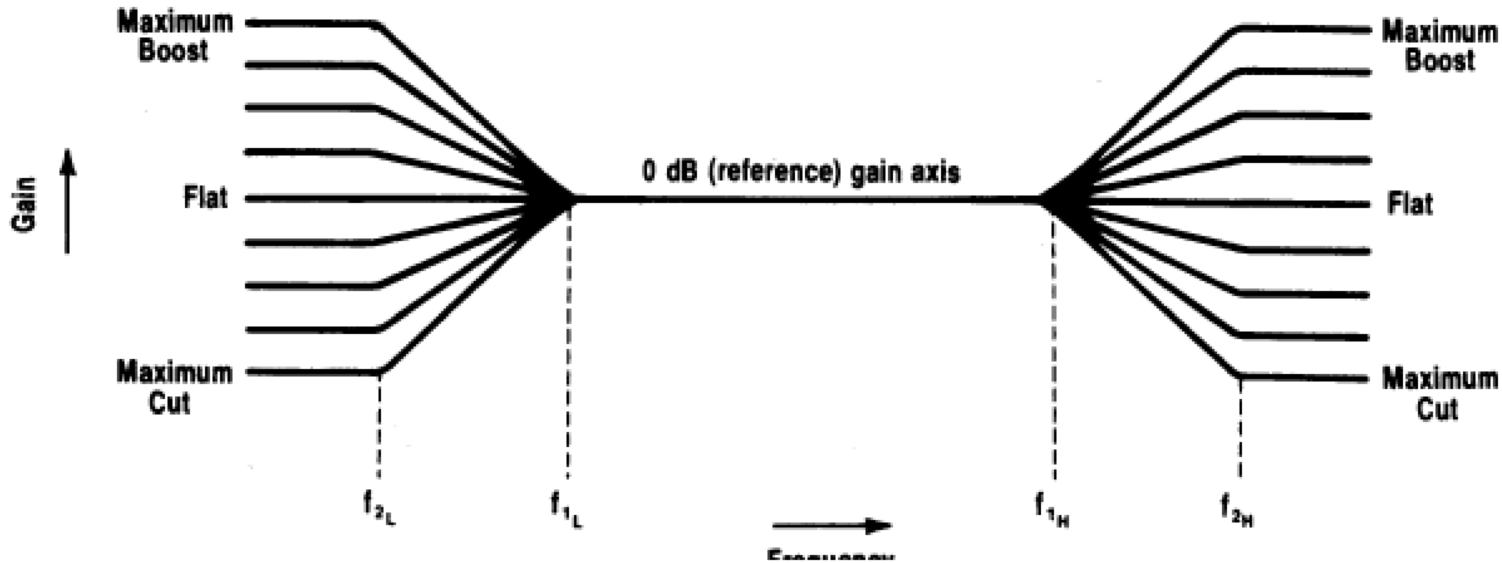
Shelving Equalizers



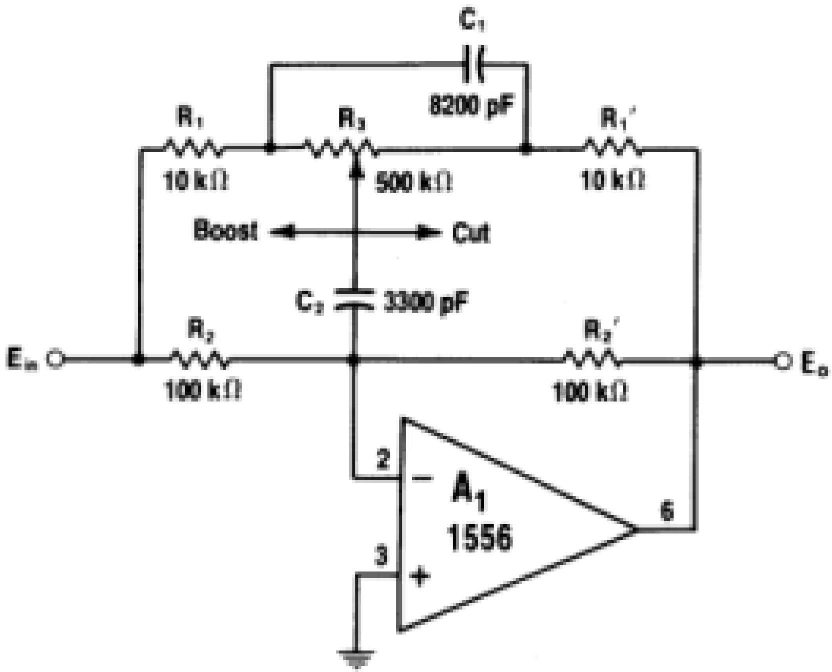
(B) Low frequency.

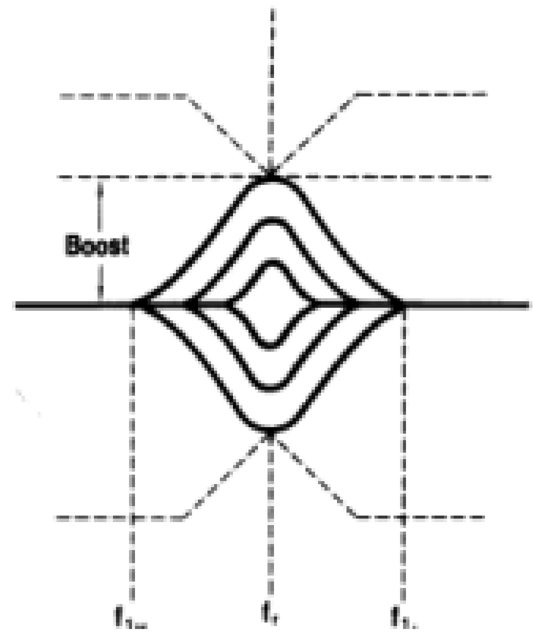
Fig. 6-37. Shelving equalizers.

Shelving Equalizers



- The expressions for f_L and f_H for the previous two circuits show a small movement with the potentiometer position in contrast to the fixed point location depicted in this figure
- The OTA-C filters discussed earlier in the course can be designed to have fixed values for f_L and f_H when cut or boost is used.


C1
8200 pF
R1
10 kΩ
R3
500 kΩ
R1'
10 kΩ
Boost
Cut
C2
3300 pF
Ein
R2
100 kΩ
R2'
100 kΩ
Eo
2
3
6
A1
1556


Boost

End of Lecture 43