Basic Filter Components

• All Pass Networks
• Arbitrary Transfer Function Synthesis
• Impedance Transformation Circuits
• Equalizers
All-Pass Circuits

- Magnitude of Gain is Constant
- Phase Changes with Frequency
- Used to correct undesired phase characteristics of a filter
First-Order All Pass

\[ T(s) = \frac{s - \frac{1}{RC}}{s + \frac{1}{RC}} \]
First-Order All Pass

\[ T(s) = \frac{s - \frac{1}{RC}}{s + \frac{1}{RC}} \]

Diagram of the frequency response with the angle \( \angle T(j\omega) \) plotted against \( \omega \) and the real and imaginary axes labeled. The angle changes from -180 to -90 degrees as \( \omega \) increases.
First-Order All Pass

\[ T(s) = - \frac{s - \frac{1}{RC}}{s + \frac{1}{RC}} \]
First-Order All Pass

\[ T(s) = -\frac{s \frac{1}{RC}}{s + \frac{1}{RC}} \]

\[ T(j\omega) \]

\[ -180 \]

\[ -90 \]

\[ \omega \]

\[ 1/RC \]
Second-Order All Pass

Based upon Bridged-T Feedback Structure
Second-Order All Pass

\[
\frac{V_O}{V_{IN}} = \frac{s^2 - s\left(\frac{2}{R_2 C}\right) + \frac{1}{R_1 R_2 C^2}}{s^2 + s\left(\frac{2}{R_2 C}\right) + \frac{1}{R_1 R_2 C^2}}
\]

\[\angle T(j\omega) = \frac{1}{C\sqrt{R_1 R_2}}\]
Arbitrary Transfer Function Synthesis

- Based upon coefficient derivation
- Can be used to implement/solve an arbitrary differential equation
- Versatile
- Basic concept of Analog Computer
Applications of integrators to solving differential equations

Standard Integral form of a differential equation

\[
X_{OUT} = b_1 \int X_{OUT} + b_2 \iint X_{OUT} + b_3 \iiint X_{OUT} + \ldots + a_0 X_{IN} + \int X_{IN} + \iint X_{IN} + \ldots
\]

Standard differential form of a differential equation

\[
X_{OUT} = \alpha_1 X'_{OUT} + \alpha_2 X''_{OUT} + \alpha_3 X'''_{OUT} + \ldots + \beta_1 X_{IN} + \beta_2 X'_{IN} + \beta_3 X''_{IN} + \ldots
\]

Initial conditions not shown

Can express any system in either differential or integral form
Applications of integrators to solving differential equations

Consider the standard integral form

\[ X_{OUT} = b_1 \int X_{OUT} + b_2 \int \int X_{OUT} + b_3 \int \int \int X_{OUT} + \ldots + a_0 X_{IN} + \int X_{IN} + \int \int X_{IN} + \ldots \]

This circuit is comprised of summers and integrators
Can solve an arbitrary linear differential equation
This concept was used in Analog Computers in the past
Applications of integrators to solving differential equations

Consider the standard integral form

\[ X_{OUT} = b_1 \int X_{OUT} + b_2 \int \int X_{OUT} + b_3 \int \int \int X_{OUT} + \ldots + a_0 X_{IN} + \int X_{IN} + \int \int X_{IN} + \ldots \]

Take the Laplace transform of this equation

\[ \mathcal{X}_{OUT} = \frac{b_1}{s} \mathcal{X}_{OUT} + \frac{b_2}{s^2} \mathcal{X}_{OUT} + \frac{b_3}{s^3} \mathcal{X}_{OUT} + \ldots + b_n \frac{1}{s^n} + a_0 \mathcal{X}_{IN} + a_1 \frac{1}{s} \mathcal{X}_{IN} + a_2 \frac{1}{s^2} \mathcal{X}_{IN} + a_3 \frac{1}{s^3} \mathcal{X}_{IN} + \ldots + a_m \frac{1}{s^m} \]

Multiply by \( s^n \) and assume \( m=n \) (some of the coefficients can be 0)

\[ s^n \mathcal{X}_{OUT} = b_1 s^{n-1} \mathcal{X}_{OUT} + b_2 s^{n-2} \mathcal{X}_{OUT} + b_3 s^{n-3} \mathcal{X}_{OUT} + \ldots + b_n s^n \mathcal{X}_{IN} + a_0 s^n \mathcal{X}_{IN} + a_1 s^{n-1} \mathcal{X}_{IN} + a_2 s^{n-2} \mathcal{X}_{IN} + a_3 s^{n-3} \mathcal{X}_{IN} + \ldots + a_n \]

\[ \mathcal{X}_{OUT} \left( s^n - b_1 s^{n-1} - b_2 s^{n-2} - b_3 s^{n-3} - \ldots - b_n \right) = \mathcal{X}_{IN} \left( a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3} + \ldots + a_n \right) \]

\[ T(s) = \frac{\mathcal{X}_{OUT}}{\mathcal{X}_{IN}} = \frac{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3} + \ldots + a_n}{s^n - b_1 s^{n-1} - b_2 s^{n-2} - b_3 s^{n-3} - \ldots - b_n} \]
Applications of integrators to solving differential equations

Consider the standard integral form

\[ X_{OUT} = b_1 \int X_{OUT} + b_2 \int \int X_{OUT} + b_3 \int \int \int X_{OUT} + \ldots + a_0 X_{IN} + \int X_{IN} + \int \int X_{IN} + \ldots \]

\[ T(s) = \frac{X_{OUT}}{X_{IN}} = \frac{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3} + \ldots + a_n}{s^n - b_1 s^{n-1} - b_2 s^{n-2} - b_3 s^{n-3} - \ldots - b_n} \]

This can be written in more standard form

\[ T(s) = \frac{\alpha_n s^n + \alpha_{n-1} s^{n-1} + \ldots + \alpha_1 s + \alpha_0}{s^n + \beta_{n-1} s^{n-1} + \ldots + \beta_1 s + \beta_0} \]
Applications of integrators to filter design

Can design (synthesize) any \( T(s) \) with just integrators and summers!

Integrators are not used “open loop” so loss is not added

Although this approach to filter design works, often more practical methods are used
Impedance Synthesis

- Focus on synthesizing impedance rather than transfer function
- Gyrators will provide inductance simulation
- Capacitance Multiplication
- Synthesis of super components
Note these circuits are strictly one-ports and have no output node.
Impedance Converters

\[ V_1(G_1 + G_2) = V_x G_2 \]

\[ I_1 = (V_1 - V_x) G_3 \]

\[ Z_{IN} = -\frac{Z_1 Z_3}{Z_2} \]

Observe this input impedance is negative!
Impedance Converters

\[ Z_{IN} = -\frac{Z_1Z_3}{Z_2} \]

If \( Z_1 = R_1 \), \( Z_2 = R_2 \) and \( Z_3 = R_3 \),
\[ Z_{IN} = -\frac{R_1R_3}{R_2} \]
This is a negative resistor!

If \( Z_2 = 1/sC \), \( Z_1 = R_1 \) and \( Z_3 = R_3 \),
\[ Z_{IN} = -sCR_1R_3 \]
This is a negative inductor!

If \( Z_2 = R_2 \), \( Z_1 = 1/sC \) and \( Z_3 = R_3 \),
\[ Z_{IN} = -\frac{R_3}{sCR_2} \]
This is a negative capacitor!

This is termed a Negative Impedance Converter
Impedance Converters

If \( Z_2 = 1/sC \), \( Z_1 = R_1 \) and \( Z_3 = R_3 \),

\[
Z_{IN} = -sCR_1R_3
\]

Modification of NIC to provide a positive inductance:

Replace \( Z_1 \) itself with a second NIC that has a negative input impedance.
Negative Impedance Converter

If select components so that \( R_s = \frac{R_2}{R_1R_3} \)

One application of NIC

Lossy Inductor

Lossless Inductor
Impedance Converters

This circuit is often called a Gyrator

\[ Z_{IN} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4} \]
Gyrator Analysis

\[ I_X = V_1 G_3 \]
\[ V_X = V_1 + \frac{V_1 G_3}{G_4} = V_1 \left(1 + \frac{G_3}{G_4}\right) \]
\[ I_Y = \left(V_1 - V_X\right) G_1 = V_1 \left(\frac{-G_3}{G_4}\right) G_1 \]
\[ V_Y = V_1 + \frac{I_Y}{G_2} = V_1 \left(1 - \frac{G_3}{G_4} \frac{G_1}{G_2}\right) \]

\[ I_1 = (V_1 - V_Y) G_5 = V_1 \left(\frac{G_3}{G_4} \left(\frac{G_1}{G_2}\right)\right) G_5 \]

\[ Z_{IN} = \frac{Z_1}{Z_3} \frac{Z_2}{Z_4} \]
Gyrator Applications

If $Z_1 = Z_3 = Z_4 = Z_5 = R$ and $Z_2 = 1/sC$,

$$Z_{IN} = \left(R^2C\right)s$$

This is an inductor of value $L = R^2C$.

If $Z_2 = R_2$, $Z_3 = R_3$, $Z_4 = R_4$, $Z_5 = R_5$ and $Z_1 = 1/sC$,

$$Z_{IN} = \frac{R_3R_5}{sC R_2 R_4}$$

This is a capacitor of value $C_{EQ} = C \frac{R_2 R_4}{R_3 R_5}$ (can scale capacitance up or down).

If $Z_2 = Z_4 = Z_5 = R$ and $Z_1 = Z_3 = 1/sC$,

$$Z_{IN} = \left(R^3C^2\right)s^2$$

This is a “super” capacitor of value $R^3C^2$. 

$$Z_{IN} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$
Impedance Converters

$\begin{align*}
I_1 & = \left( V_1 - \frac{Z_1}{Z_1 + Z_2} \right) V_1 \left[ G_3 \right] \\
Z_{IN} & = Z_3 \left( 1 + \frac{Z_2}{Z_1} \right)
\end{align*}$

If $Z_3 = R_3$, $Z_2 = R_2$ and $Z_1 = 1/sC$

$Z_{IN} = R_3 + s \left( CR_2 R_3 \right)$

$L_{EQ} = CR_2 R_3$
Shelving Equalizers

• Widely used in audio applications
• User-programmable filter response
Shelving Equalizers

(A) High frequency.
Shelving Equalizers

(B) Low frequency.

Fig. 6-37. Shelving equalizers.
Shelving Equalizers

- The expressions for $f_L$ and $f_H$ for the previous two circuits show a small movement with the potentiometer position in contrast to the fixed point location depicted in this figure.

- The OTA-C filters discussed earlier in the course can be designed to have fixed values for $f_L$ and $f_H$ when cut or boost is used.
End of Lecture 43