

EE 508

Lecture 5

Filter Concepts/Terminology
Basic Properties of Electrical Circuits

2-nd order polynomial characterization

$$s^2 + as + b$$

$$\{a, b\}$$

$$s^2 + \frac{\omega_0}{Q} s + \omega_0^2$$

$$\{\omega_0, Q\}$$

$$s^2 + 2\zeta\omega_0 s + \omega_0^2$$

$$\{\zeta, \omega_0\}$$

$$s^2 + (p_1 + p_2)s + p_1 p_2 = (s + p_1)(s + p_2)$$

$$\{p_1, p_2\}$$

with complex conjugate roots

$$s^2 + 2\alpha s + \alpha^2 + \beta^2 = (s + \alpha + j\beta)(s + \alpha - j\beta)$$

$$\{\alpha, \beta\}$$

$$s^2 + 2r\cos(\theta)s + r^2 = (s + re^{j\theta})(s + re^{-j\theta})$$

$$\{r, \theta\}$$

Review from Last Time

Biquadratic Factorization

If n is even,

$$T(s) = \frac{\sum_{i=1}^m a_i s^i}{\sum_{i=1}^n b_i s^i} = K \cdot \prod_{i=1}^{n/2} T_{BQi}(s)$$

If n is odd,

$$T(s) = \frac{\sum_{i=1}^m a_i s^i}{\sum_{i=1}^n b_i s^i} = K \cdot \left(\frac{a_{10} s + a_{00}}{s + b_{00}} \right) \cdot \prod_{i=1}^{(n-1)/2} T_{BQi}(s)$$

where

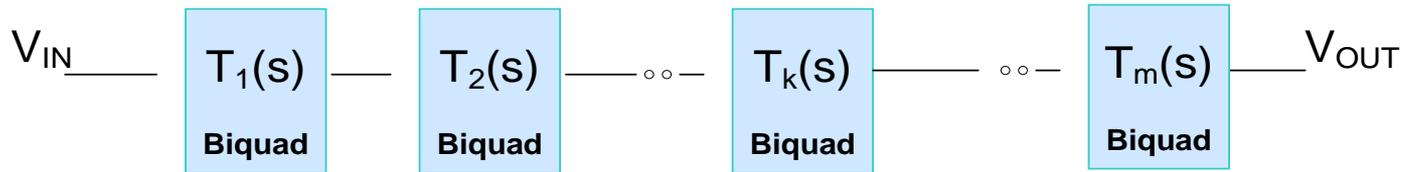
$$T_{BQi}(s) = \frac{a_{2i} s^2 + a_{1i} s + a_{0i}}{s^2 + b_{1i} s + b_{0i}}$$

and where K is a real constant and all coefficients are real (some may be 0)

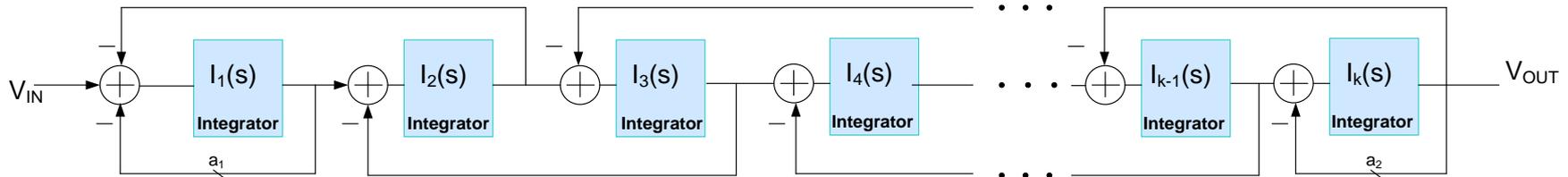
- Factorization is not unique
- $H(z)$ factorizations not restricted to have $m < n$
- Each biquadratic factor can be represented by any of the 6 alternative parameter sets in the numerator or denominator

Common Filter Architectures

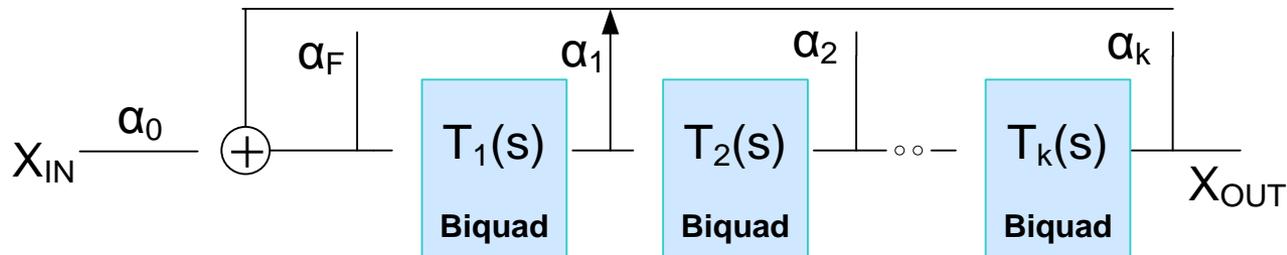
Cascaded Biquads



Leapfrog



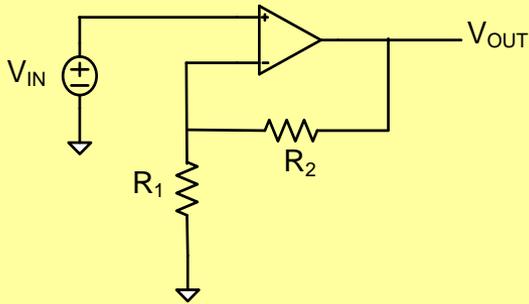
Multiple-loop Feedback



- Three classical filter architectures are shown
- The Cascaded Biquad and the Leapfrog approaches are most common

Gain, Bandwidth and GB

Summary of Effects of GB on Basic Inverting and Noninverting Amplifiers

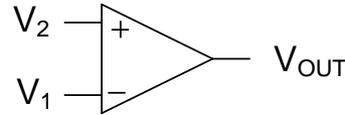


Basic Noninverting Amplifier

$$K_0 = 1 + \frac{R_2}{R_1}$$

$$BW = \frac{GB}{K_0}$$

$$A_{FB}(s) = \frac{K_0}{1 + s \frac{K_0}{GB}}$$

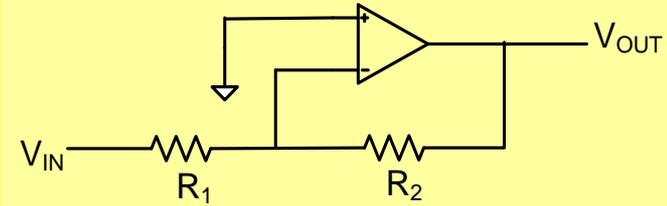


$$A_1(s) = \frac{GB}{s + BW_A}$$

$$GB = A_0 \cdot BW_A$$

$$A(s) = \frac{GB}{s}$$

Adequate model for most applications

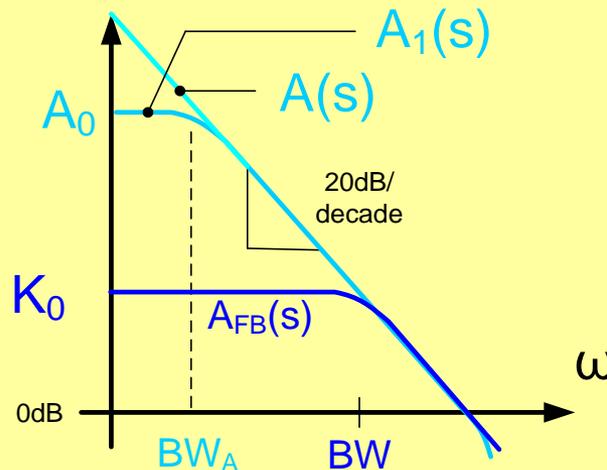


Basic Inverting Amplifier

$$K_0 = \frac{R_2}{R_1}$$

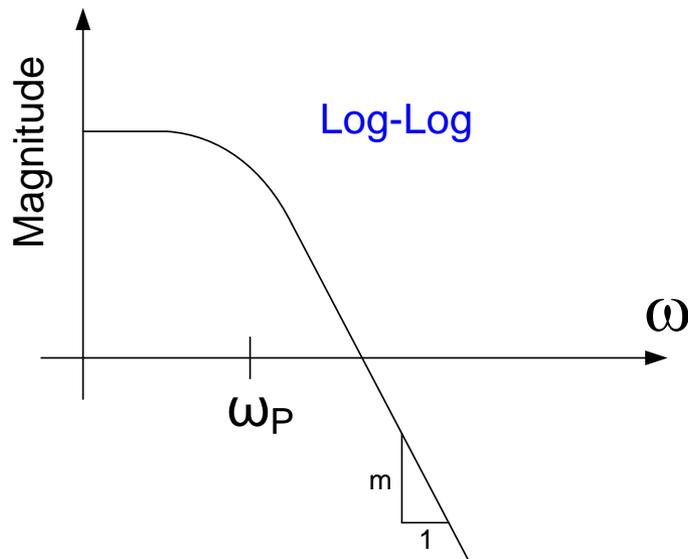
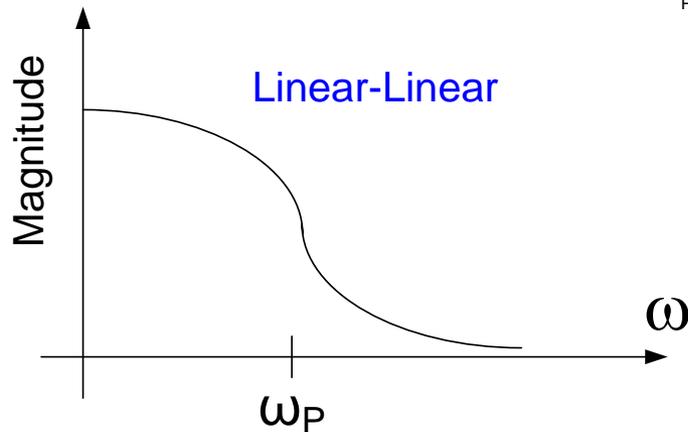
$$BW = \frac{GB}{1 + K_0}$$

$$A_{FB}(s) = -\frac{K_0}{1 + s \frac{(1 + K_0)}{GB}}$$



Single-pole roll-off characterization

Consider: $T(s) = \frac{\omega_p}{s + \omega_p}$



$$T(j\omega) = \frac{\omega_p}{j\omega + \omega_p}$$

$$|T(j\omega)| = \frac{\omega_p}{\sqrt{\omega^2 + \omega_p^2}}$$

$$m = -20\text{dB/decade}$$

$$m = -6\text{dB/octave}$$

$$\angle T(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_p}\right)$$

Single-pole roll-off characterization

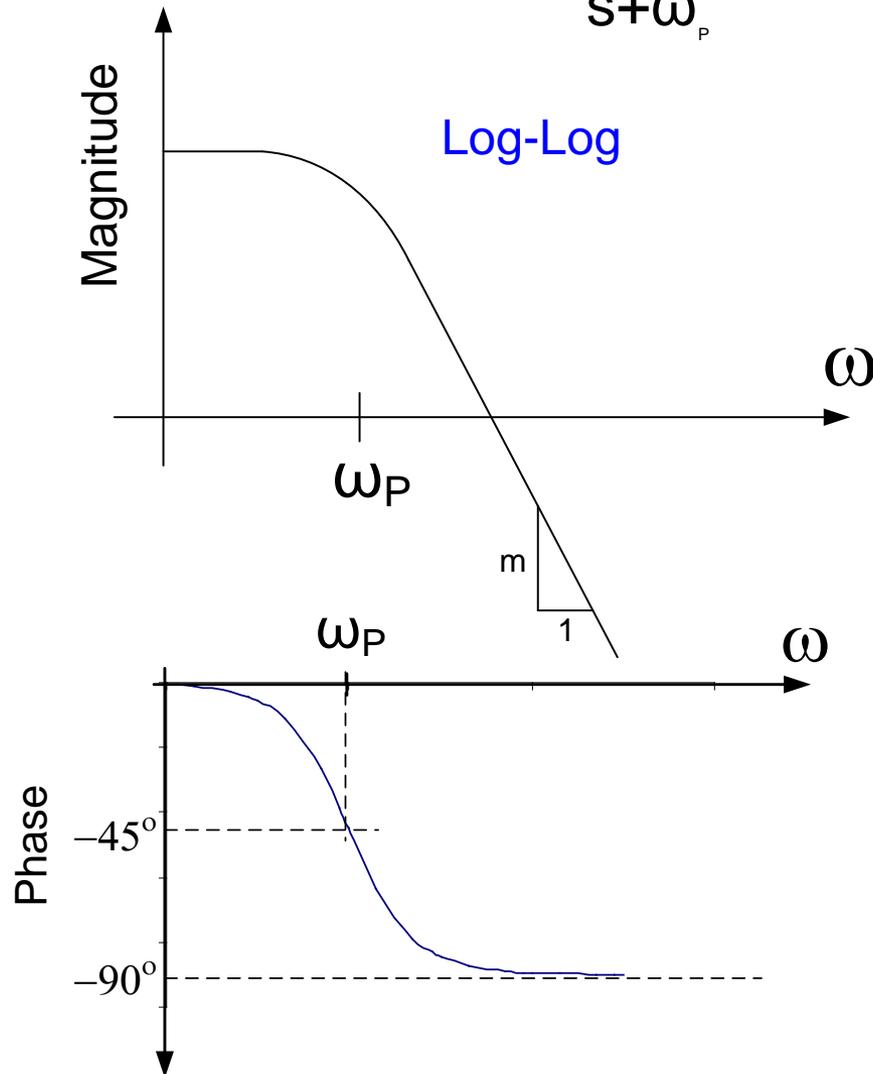
Consider:

$$T(s) = \frac{\omega_p}{s + \omega_p}$$

$$T(j\omega) = \frac{\omega_p}{j\omega + \omega_p}$$

$$|T(j\omega)| = \frac{\omega_p}{\sqrt{\omega^2 + \omega_p^2}}$$

$$\angle T(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_p}\right)$$



Roll-off characterization

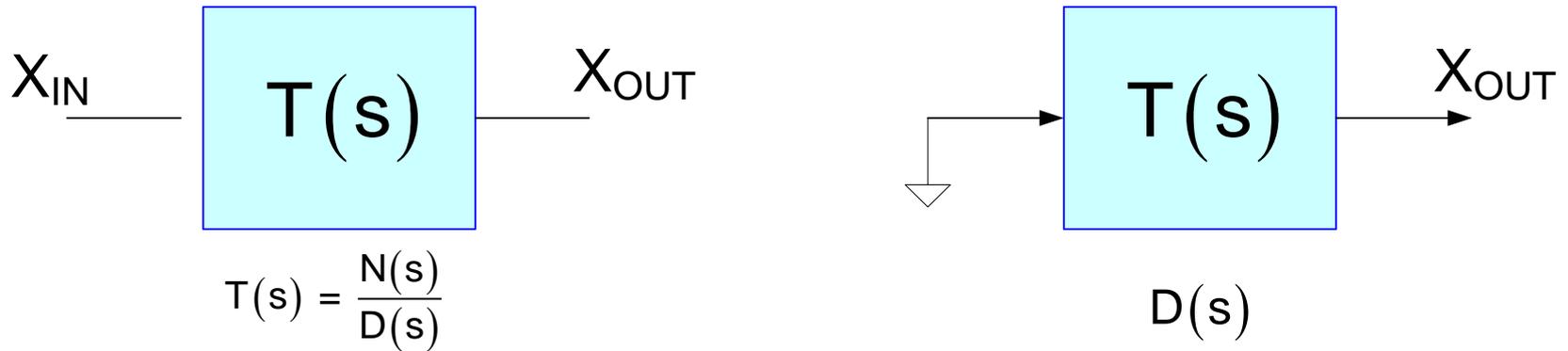
At frequencies well-past a pole or zero, each LHP pole (real or complex) causes a roll-off in magnitude on a log-log axis of -20dB/decade and each LHP zero causes a roll-off of $+20\text{dB/decade}$

At frequencies of magnitude comparable to that of a pole or zero, it is not easy to predict the roll-off in the magnitude characteristics by some simple expression

Distortion in Filters

- Magnitude Distortion
 - frequency dependent change in gain of a circuit (usually bad if building amplifier but critical if building a filter)
- Phase Distortion
 - a circuit has phase distortion if the phase of the transfer function is not linear with frequency
- Nonlinear Distortion
 - Presence of frequency components in the output that are not present in the input (generally considered bad in filters but necessary in many other circuits)

Dead Networks



The “dead network” of any linear circuit is obtained by setting ALL independent sources to zero.

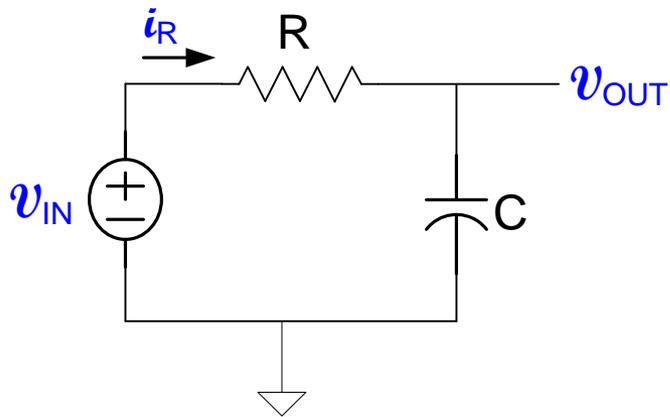
- Replace independent current sources with opens
- Replace independent voltage sources with shorts
- Dependent sources remain intact

$D(s)$ is characteristic of the dead network and is independent of where the excitation is applied or where the response is measured

$D(s)$ is the same for ALL transfer functions of a given “dead network”

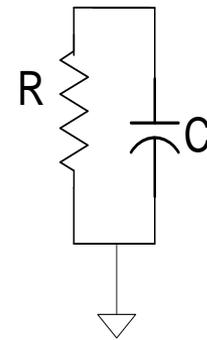
Dead Networks

Example:



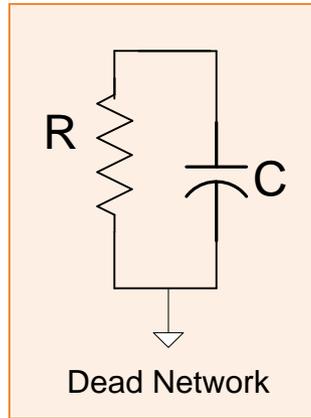
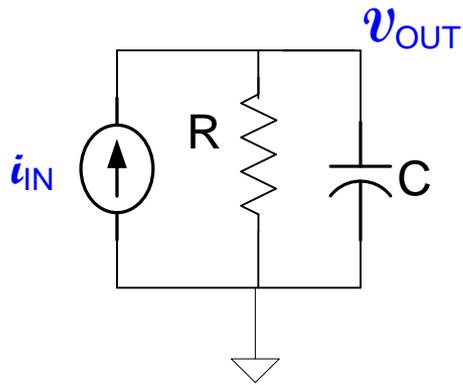
$$T(s) = \frac{1}{1+RCs}$$

$$D(s) = 1+RCs$$



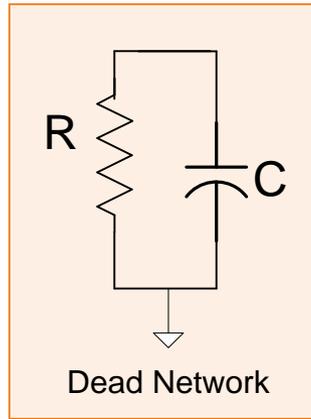
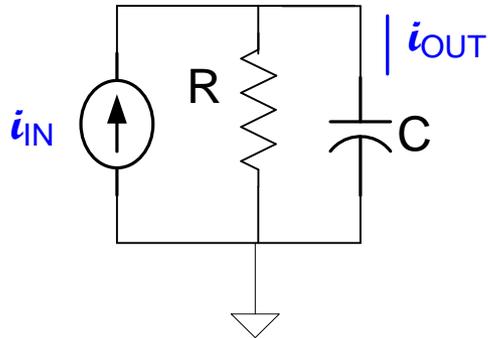
Dead Network

Dead Networks



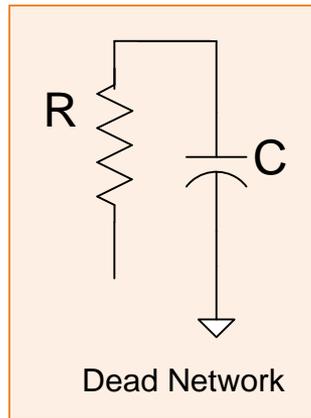
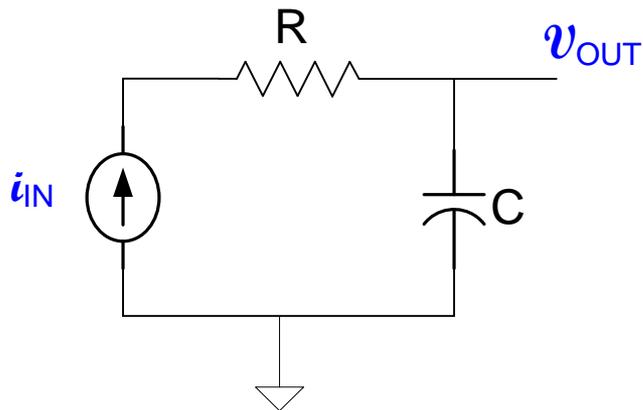
$$\frac{v_{OUT}}{i_{IN}} = T(s) = \frac{R}{1+RCs}$$

$$D(s) = 1+RCs$$



$$\frac{i_{OUT}}{i_{IN}} = T(s) = \frac{RCs}{1+RCs}$$

$$D(s) = 1+RCs$$

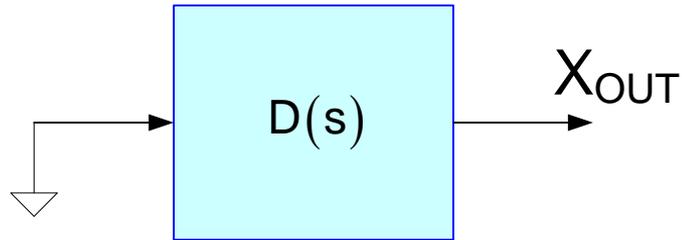


$$\frac{v_{OUT}}{i_{IN}} = T(s) = \frac{1}{Cs}$$

$$D(s) = Cs$$

Note: This has a different dead network!

$D(s)$ is the same for ALL transfer functions of a given “dead network”



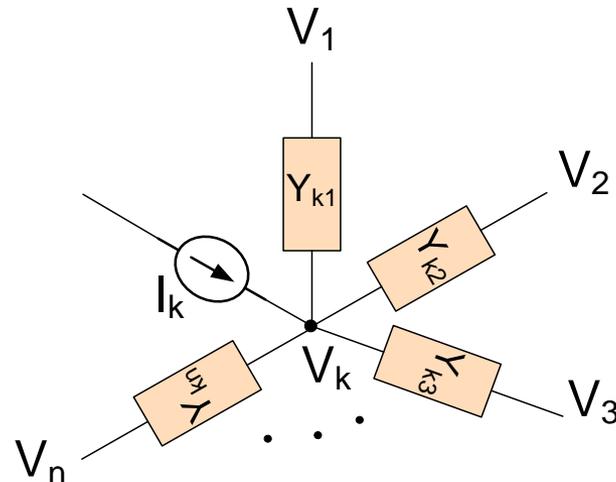
This is an important observation. Why is it true?

Plausibility argument:

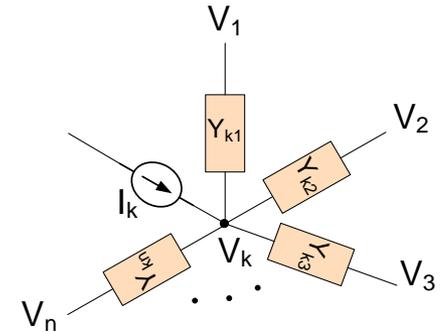
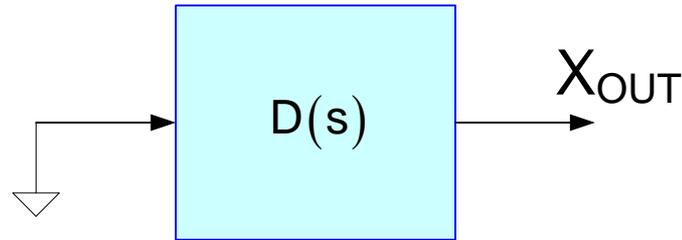
Consider a network with only admittance elements and independent current sources

At node k , can write the equation

$$\sum_{\substack{i=1 \\ i \neq k}}^n Y_{ki} (V_k - V_i) = I_k$$



$D(s)$ is the same for ALL transfer functions of a given “dead network”



Plausibility argument:

Doing this at each node results in the set of equations

$$\begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1n} \\ Y_{21} & Y_{22} & \dots & Y_{2n} \\ \cdot & & & \\ \cdot & & & \\ Y_{n1} & Y_{n2} & \dots & Y_{nn} \end{bmatrix} \bullet \begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ V_n \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ \cdot \\ \cdot \\ I_n \end{bmatrix}$$

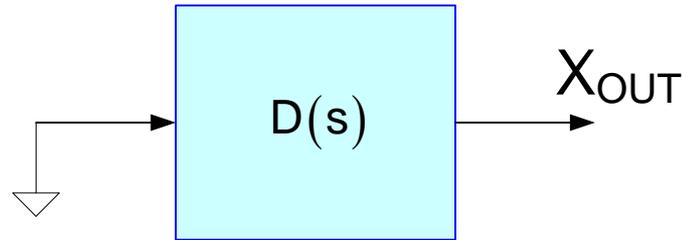
In matrix form

$$\mathbf{Y} \bullet \mathbf{V} = \mathbf{I}$$

The nodal voltages are given by

$$\mathbf{V} = \mathbf{Y}^{-1} \bullet \mathbf{I}$$

$D(s)$ is the same for ALL transfer functions of a given “dead network”

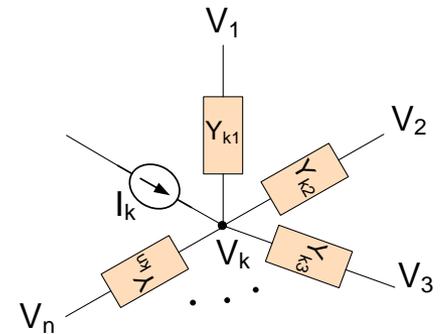


Plausibility argument:

$$\mathbf{V} = \mathbf{Y}^{-1} \cdot \mathbf{I}$$

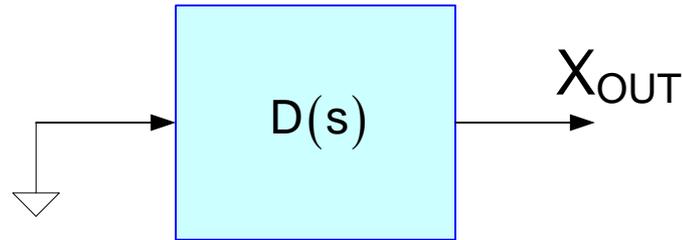
The nodal voltage V_k in this solution is given by the ratio of two determinates where the one in the numerator is obtained by replacing the k th column with the excitation vector and the one in the denominator is the determinate of the indefinite admittance matrix \mathbf{Y}

Note the denominator is the same for all nodal voltages and is independent of the excitations – that is, it is dependent only upon the dead network



$$V_k = \frac{\begin{vmatrix} Y_{11} & Y_{12} & \dots & I_1 & Y_{1n} \\ Y_{21} & Y_{22} & \dots & I_2 & Y_{2n} \\ \cdot & & & & \\ \cdot & & & & \\ Y_{n1} & Y_{n2} & \dots & I_n & Y_{nn} \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} & \dots & Y_{1n} \\ Y_{21} & Y_{22} & \dots & Y_{2n} \\ \cdot & & & \\ \cdot & & & \\ Y_{n1} & Y_{n2} & \dots & Y_{nn} \end{vmatrix}}$$

$D(s)$ is the same for ALL transfer functions of a given “dead network”

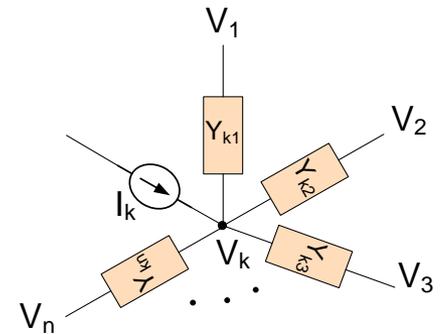


Plausibility argument:

Note the denominator is the same for all nodal voltages and is independent of the excitations – that is, it is dependent only upon the dead network

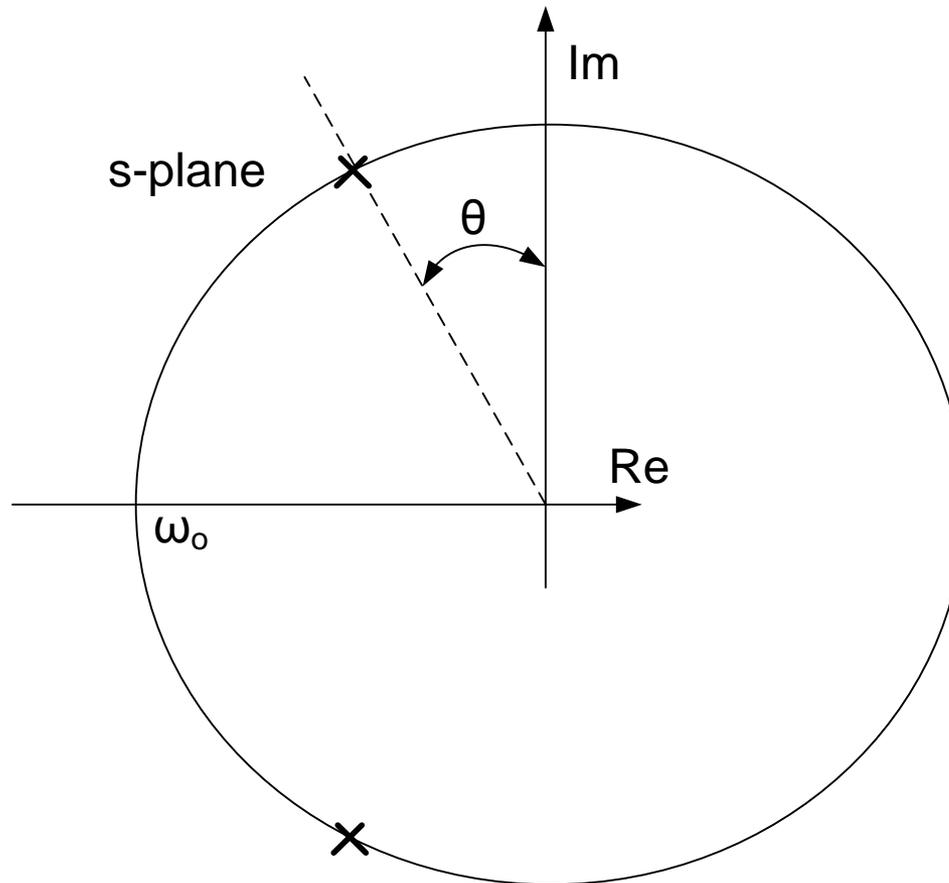
Thus all branch voltages and all branch currents have the same denominator and this (after multiplying through by the correct power of s to make V_k a rational fraction) is the characteristic polynomial $D(s)$

This concept can be extended to include independent voltage sources as well as dependent sources



$$V_k = \frac{\begin{vmatrix} Y_{11} & Y_{12} & \dots & I_1 & Y_{1n} \\ Y_{21} & Y_{22} & \dots & I_2 & Y_{2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ Y_{n1} & Y_{n2} & \dots & I_n & Y_{nn} \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} & \dots & Y_{1n} \\ Y_{21} & Y_{22} & \dots & Y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{n1} & Y_{n2} & \dots & Y_{nn} \end{vmatrix}}$$

Root characterization in s-plane (for complex-conjugate roots)



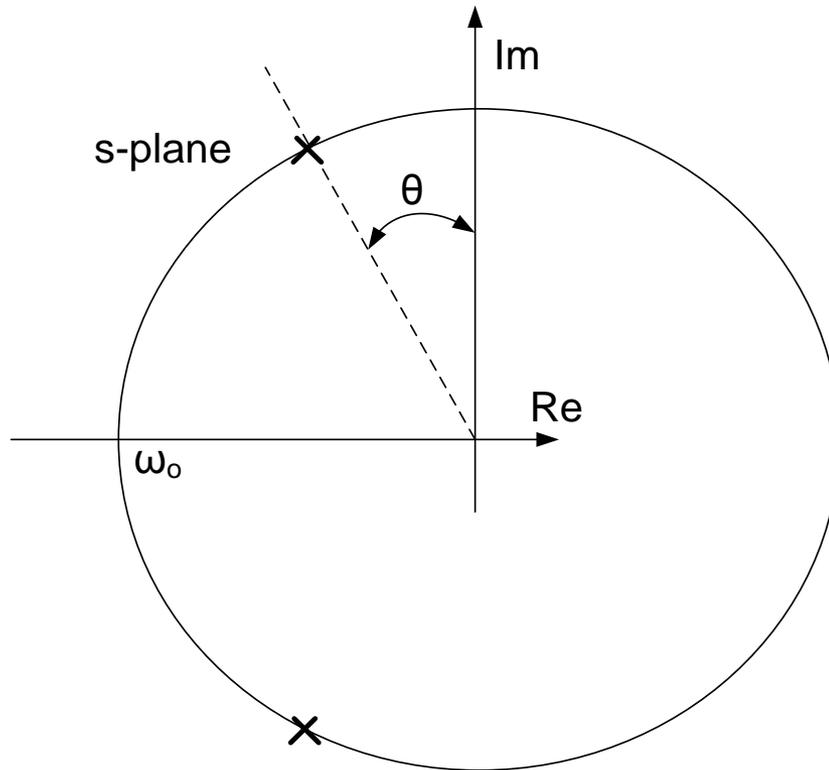
$$s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2$$

1-1 relationship between angle θ and Q of root

For low Q , θ is large

For high Q , θ is small

Root characterization in s-plane (for complex-conjugate roots)



$$s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2$$

for $\theta=90^\circ$, $Q=1/\sqrt{2}$

roots located at

$$s = -\frac{\omega_0}{2Q} \pm \frac{1}{2} \sqrt{\left(\frac{\omega_0}{Q}\right)^2 - 4\omega_0^2} = \omega_0 \left(-\frac{1}{2Q} \pm \frac{1}{2} \sqrt{\left(\frac{1}{Q}\right)^2 - 4} \right)$$

$$\theta = \tan^{-1}(4Q^2 - 1)$$

End of Lecture 5