EE 508
Lecture 5

- Dead Networks
- Scaling, Normalization and Transformations
Filter Concepts and Terminology

- 2-nd order polynomial characterization
- Biquadratic Factorization
- Op Amp Modeling
- Stability and Instability
- Roll-off characteristics
- Distortion
- Dead Networks
- Root Characterization
- Scaling, normalization, and transformation
The “dead network” of any linear circuit is obtained by setting ALL independent sources to zero.

- Replace independent current sources with opens
- Replace independent voltage sources with shorts
- Dependent sources remain intact

\[ T(s) = \frac{N(s)}{D(s)} \]

\( D(s) \) is characteristic of the dead network and is independent of where the excitation is applied or where the response is measured.

\( D(s) \) is the same for ALL transfer functions of a given “dead network”
Dead Networks

Example:

\[ T(s) = \frac{1}{1+RCs} \]

\[ D(s) = 1+RCs \]
Dead Networks

\[ \frac{v_{\text{OUT}}}{i_{\text{IN}}} = T(s) = \frac{R}{1+RCs} \]

\[ D(s) = 1+RCs \]

\[ \frac{i_{\text{OUT}}}{i_{\text{IN}}} = T(s) = \frac{RCs}{1+RCs} \]

\[ D(s) = 1+RCs \]

\[ \frac{v_{\text{OUT}}}{i_{\text{IN}}} = T(s) = \frac{1}{Cs} \]

\[ D(s) = Cs \]

Note: This has a different dead network!
D(s) is the same for ALL transfer functions of a given “dead network”

This is an important observation. Why is it true?

Plausibility argument:

Consider a network with only admittance elements and independent current sources

At node k, can write the equation

\[ \sum_{i=1}^{n} Y_{ki} (V_k - V_i) = I_k \]
D(s) is the same for ALL transfer functions of a given “dead network”

Plausibility argument:

Doing this at each node results in the set of equations

\[
\begin{bmatrix}
Y_{11} & Y_{12} & \cdots & Y_{1n} \\
Y_{21} & Y_{22} & \cdots & Y_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
Y_{n1} & Y_{n2} & \cdots & Y_{nn}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_n
\end{bmatrix}
= 
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_n
\end{bmatrix}
\]

In matrix form

\[Y \cdot V = I\]

The nodal voltages are given by

\[V = Y^{-1} \cdot I\]
D(s) is the same for ALL transfer functions of a given “dead network”

Plausibility argument:

\[ V = Y^{-1} \cdot I \]

The nodal voltage \( V_k \) in this solution is given by the ratio of two determinates where the one in the numerator is obtained by replacing the kth column with the excitation vector and the one in the denominator is the determinate of the indefinite admittance matrix \( Y \).

Note the denominator is the same for all nodal voltages and is independent of the excitations – that is, it is dependent only upon the dead network.
D(s) is the same for ALL transfer functions of a given “dead network”

Plausibility argument:

Note the denominator is the same for all nodal voltages and is independent of the excitations – that is, it is dependent only upon the dead network.

Thus all branch voltages and all branch currents have the same denominator and this (after multiplying through by the correct power of s to make $V_k$ a rational fraction) is the characteristic polynomial $D(s)$.

This concept can be extended to include independent voltage sources as well as dependent sources.
Filter Concepts and Terminology

• 2-nd order polynomial characterization
• Biquadratic Factorization
• Op Amp Modeling
• Stability and Instability
• Roll-off characteristics
• Distortion
• Dead Networks

Root Characterization
• Scaling, normalization, and transformation
Root characterization in s-plane
(for complex-conjugate roots)

For low Q, $\theta$ is large
For high Q, $\theta$ is small

1-1 relationship between angle $\theta$ and Q of root

$s^2 + s \left( \frac{\omega_0}{Q} \right) + \omega_0^2$
Root characterization in s-plane
(for complex-conjugate roots)

$s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2$

for $\theta=45^\circ$, $Q=1/\sqrt{2}$

for $\theta=90^\circ$, $Q=1/2$

roots located at

$s = -\frac{\omega_0}{2Q} \pm \frac{1}{2} \sqrt{\left(\frac{\omega_0}{Q}\right)^2 - 4\omega_0^2} = \omega_0 \left(-\frac{1}{2Q} \pm \frac{1}{2} \sqrt{\left(\frac{1}{Q}\right)^2 - 4}\right)$

$\theta = \tan^{-1}\left(\frac{1}{\sqrt{4Q^2 - 1}}\right)$
Filter Concepts and Terminology

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Scaling, Normalization and Transformations

- Frequency scaling
- Frequency Normalization
  - Impedance scaling
  - Transformations
    - LP to BP
    - LP to HP
    - LP to BR
Scaling, Normalization and Transformations

Frequency normalization:

\[ s_n = \frac{s}{\omega_0} \]

Frequency scaling:

\[ s = \omega_0 s_n \]

Purpose:

- \( \omega_0 \) independent approximations
- \( \omega_0 \) independent synthesis
- Simplifies analytical expressions for \( T(s) \)
- Simplifies component values in synthesis
- Use single table of normalized filter circuits for given normalized approximating function

Note: The normalization subscript “n” is often dropped
Frequency normalization/scaling example

\[ T(s) = \frac{6000}{s + 6000} \]

Define \( \omega_0 = 6000 \)

\[ s_n = \frac{s}{\omega_0} \]

\[ T(s) = \frac{\omega_0}{s + \omega_0} \]

Normalized transfer function:

\[ T_n(s_n) = \frac{1}{s_n + 1} \]
Frequency normalization/scaling example

\[ T_n(s_n) = \frac{1}{s_n + 1} \]

Synthesis of normalized function

\[ T(s) = \frac{1}{s + 1} \]
Frequency normalization/scaling example

\[ T_n(s_n) = \frac{1}{s_n + 1} \]

Frequency scaling transfer function by \( \omega_0 \)

\[ S = \omega_0 s_n \]

\[ T(s) = \frac{1}{\left( \frac{s}{\omega_0} \right) + 1} \]

\[ \Rightarrow \quad T(s) = \frac{\omega_0}{s + \omega_0} \]

Frequency scaling circuit by \( \omega_0 \) (actually magnitude of \( \omega_0 \)) (scale all energy storage elements in circuit)

\[ C = \frac{C_n}{\omega_0} \]

Frequency scaled transfer function is that of the frequency scaled circuit!
Frequency scaling / normalization does not change the shape of the transfer function, it only scales the frequency axis linearly.

The frequency scaled circuit can be obtained from the normalized circuit simply by scaling the frequency dependent impedances (up or down) by the scaling factor.

This makes the use of filter design tables for the design of lowpass filters practical whereby the circuits in the table all have a normalized band edge of 1 rad/sec.
Frequency normalization/scaling

Example: Table for passive LC ladder Butterworth filter with 3dB band edge of 1 rad/sec and resistive source/load terminations

\[ T_n(s_n) = \frac{1}{s_n + 1} \]

\[ |T_n(j\omega)| \]

<table>
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<th>e</th>
<th>R_s</th>
<th>C_1</th>
<th>L_2</th>
<th>C_3</th>
<th>L_4</th>
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<td>1.000</td>
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</tr>
</tbody>
</table>
Frequency normalization/scaling

The frequency scaled circuit can be obtained from the normalized circuit simply by scaling the frequency dependent impedances (up or down) by the scaling factor $\omega_0$. Component denormalization by factor of $\omega_0$.

Component values of energy storage elements are scaled down by a factor of $\omega_0$. Other components remain unchanged.
Desgin Strategy

Theorem: A circuit with transfer function $T(s)$ can be obtained from a circuit with normalized transfer function $T_n(s_n)$ by denormalizing all frequency dependent components.

\[
\begin{align*}
C & \rightarrow \frac{C}{\omega_0} \\
L & \rightarrow \frac{L}{\omega_0}
\end{align*}
\]
Example: Design a V-V passive 3\textsuperscript{rd}-order Lowpass Butterworth filter with a 3-db band-edge of 1K rad/sec and equal source and load terminations.

(from the BW approximation which will be discussed later:)

\[ T(s) = \frac{1}{s^3 + 2s^2 + 2s + 1} \]
Example: Design a V-V passive 3\textsuperscript{rd}-order Lowpass Butterworth filter with a band-edge of 1K Rad/Sec and equal source and load terminations.

![Filter Circuit Diagram]

Is this solution practical?

Some component values are too big and some are too small!
Filter Concepts and Terminology

- Frequency scaling
- Frequency Normalization
- Impedance scaling
- Transformations
  - LP to BP
  - LP to HP
  - LP to BR
Impedance Scaling

Impedance scaling of a circuit is achieved by multiplying ALL impedances in the circuit by a constant

\[ \frac{C}{\theta} \]

\[ \frac{L}{\theta} \]

\[ R \rightarrow \theta R \]

\[ C \rightarrow \frac{C}{\theta} \]

\[ L \rightarrow \frac{L}{\theta} \]

\[ A \rightarrow \frac{\theta A}{\theta} \text{ for transresistance gain} \]

\[ A \rightarrow \frac{A}{\theta} \text{ for dimensionless gain} \]

\[ A \rightarrow A/\theta \text{ for transconductance gain} \]
Impedance Scaling

Theorem: If all impedances in a circuit are scaled by a constant $\theta$, then

a) All dimensionless transfer functions are unchanged
b) All transresistance transfer functions are scaled by $\theta$
c) All transconductance transfer functions are scaled by $\theta^{-1}$
Impedance Scaling

Example:

\[ T(s) = \frac{1}{s+1} \]

Impedances scaled by \( \theta = 10^5 \)

\[ T(s) = \frac{1}{s+1} \]

Note second circuit much more practical than the first
Example: Design a V-V passive 3\textsuperscript{rd}-order Lowpass Butterworth filter with a band-edge of 1K Rad/Sec and equal source and load terminations.

\[
T(s) = K \frac{10^9}{s^3 + 2 \cdot 10^3 s^2 + 2 \cdot 10^6 s + 10^9}
\]

Is this solution practical?

\textbf{Some component values are too big and some are too small!}

Impedance scale by $\theta=1000$

\begin{align*}
R & \rightarrow \theta R \\
C & \rightarrow C/\theta \\
L & \rightarrow \theta L
\end{align*}

Component values more practical
End of Lecture 5