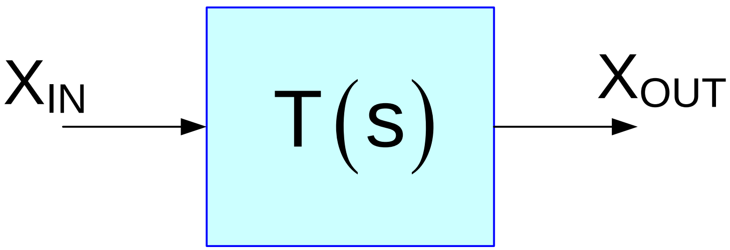
EE 508
Lecture 5

- Dead Networks
- Root Characterizations
- Scaling, Normalization and Transformations
- Degrees of Freedom and Systematic Design

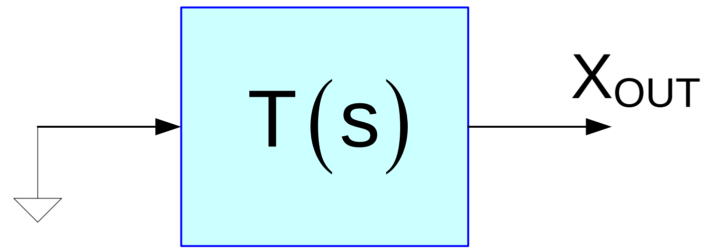
Filter Concepts and Terminology

- 2-nd order polynomial characterization
- Biquadratic Factorization
- Op Amp Modeling
- Stability and Instability
- Roll-off characteristics
- Distortion
- Dead Networks
- Root Characterization
- Scaling, normalization, and transformation

Dead Networks



$$T(s) = \frac{N(s)}{D(s)}$$



$$D(s)$$

The “dead network” of any linear circuit is obtained by setting ALL independent sources to zero.

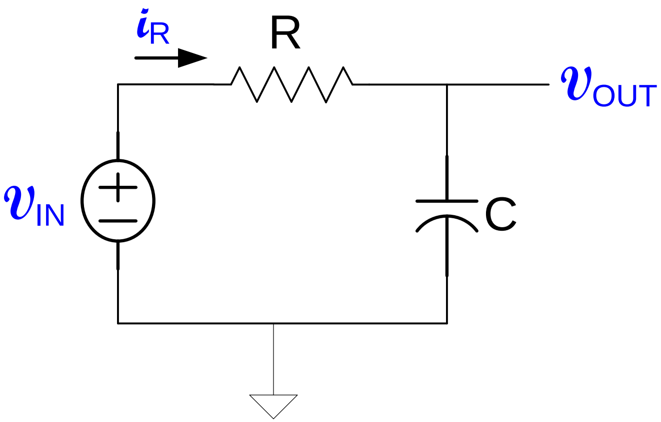
- Replace independent current sources with opens
- Replace independent voltage sources with shorts
- Dependent sources remain intact

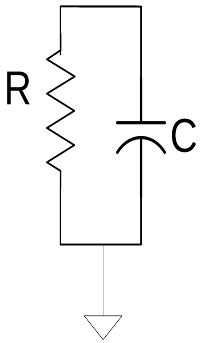
D(s) is characteristic of the dead network and is independent of where the excitation is applied or where the response is measured

D(s) is the same for ALL transfer functions of a given “dead network”

Dead Networks

Example:



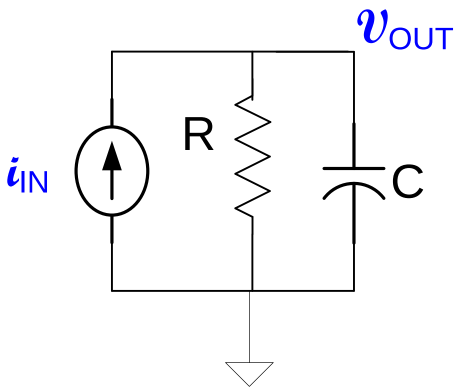


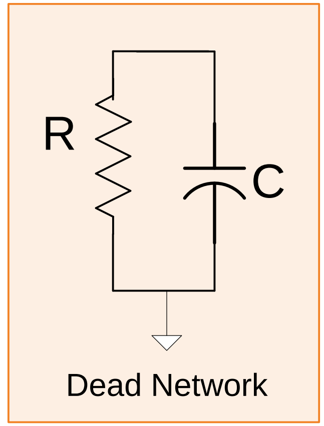
Dead Network

$$T(s) = \frac{1}{1+RCs}$$

$$D(s) = 1+RCs$$

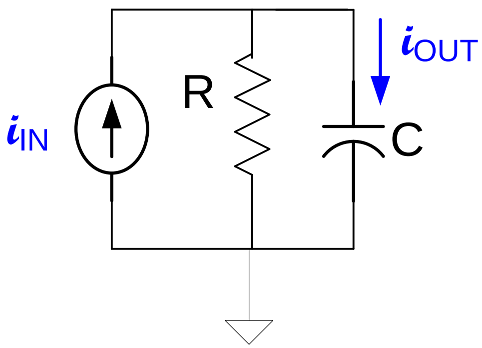
Dead Networks

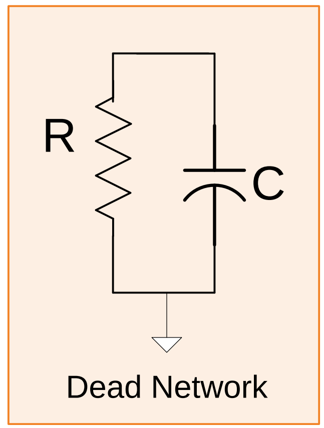




$$\frac{v_{OUT}}{i_{IN}} = T(s) = \frac{R}{1+RCs}$$

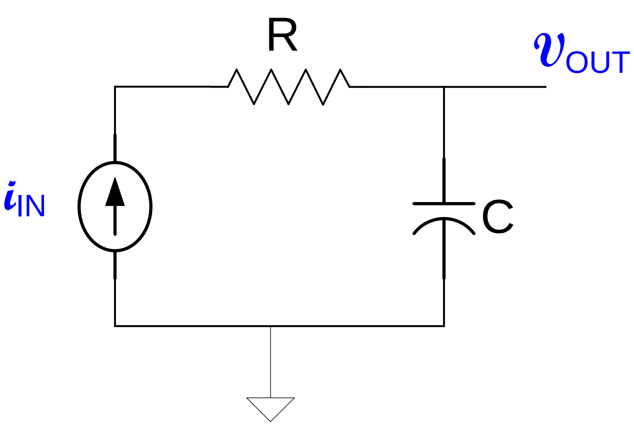
$$D(s) = 1+RCs$$

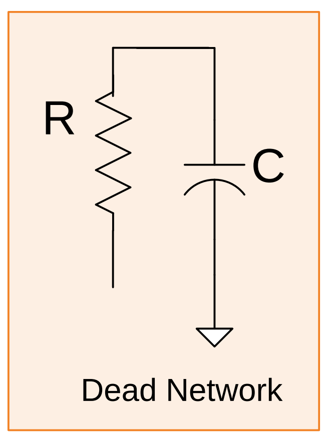




$$\frac{i_{OUT}}{i_{IN}} = T(s) = \frac{RCs}{1+RCs}$$

$$D(s) = 1+RCs$$



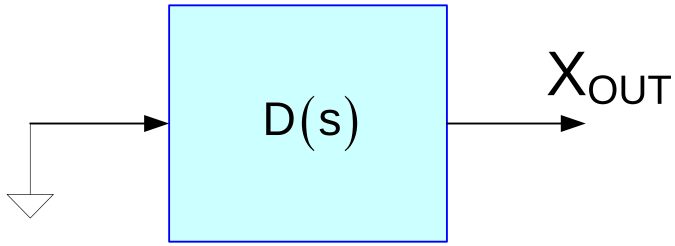


$$\frac{v_{OUT}}{i_{IN}} = T(s) = \frac{1}{Cs}$$

$$D(s) = Cs$$

Note: This has a different dead network!

D(s) is the same for ALL transfer functions of a given "dead network"



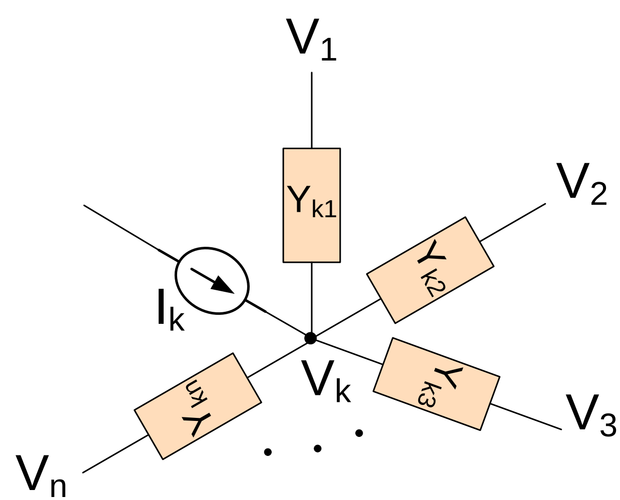
This is an important observation. Why is it true?

Plausibility argument:

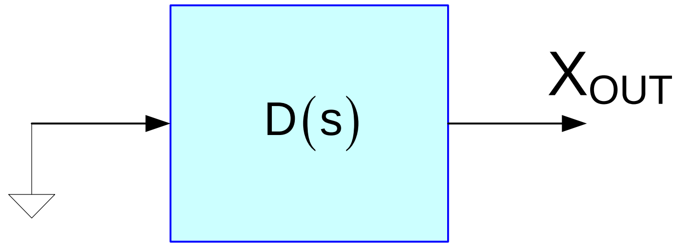
Consider a network with only admittance elements and independent current sources

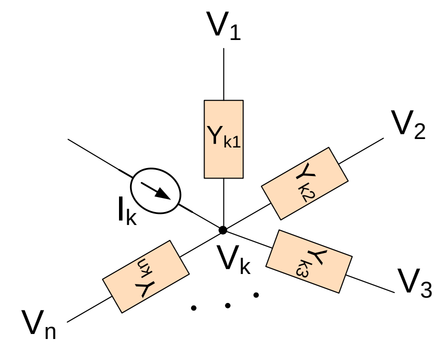
At node k, can write the equation

$$\sum_{\substack{i=1 \\ i\neq k}}^{n} Y_{ki}\left(V_k - V_i\right) = I_k$$



D(s) is the same for ALL transfer functions of a given “dead network”





Plausibility argument:

Doing this at each node results in the set of equations

$$\begin{bmatrix} Y_{11} & Y_{12} & \cdots\cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots\cdots & Y_{2n} \\ . & & & \\ . & & & \\ Y_{n1} & Y_{n2} & \cdots\cdots & Y_{nn} \end{bmatrix} \bullet \begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ . \\ V_n \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ . \\ . \\ . \\ I_n \end{bmatrix}$$

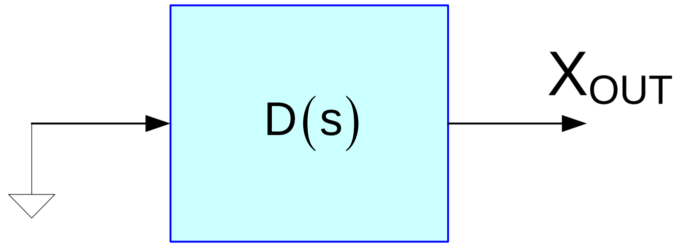
In matrix form

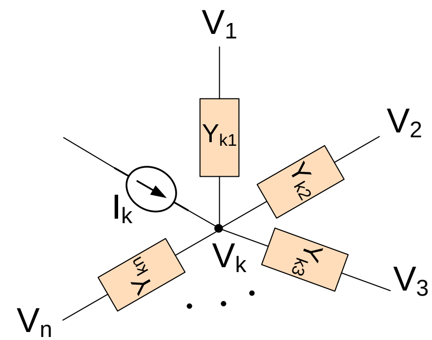
$$\mathbf{Y} \bullet \mathbf{V} = \mathbf{I}$$

The nodal voltages are given by

$$\mathbf{V} = \mathbf{Y}^{-1} \bullet \mathbf{I}$$

D(s) is the same for ALL transfer functions of a given “dead network”





Plausibility argument:

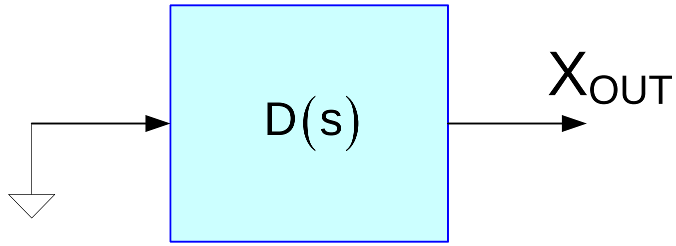
$$\mathbf{V} = \mathbf{Y}^{-1} \bullet \mathbf{I}$$

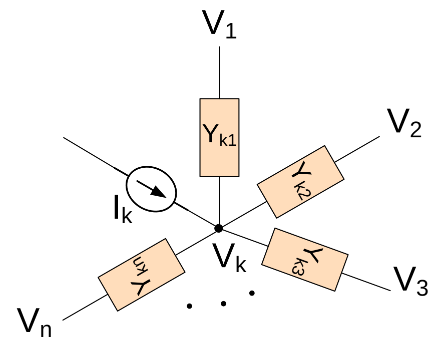
The nodal voltage V_k in this solution is given by the ratio of two determinates where the one in the numerator is obtained by replacing the kth column with the excitation vector and the one in the denominator is the determinate of the indefinite admittance matrix **Y**

$$V_k = \frac{\begin{vmatrix} Y_{11} & Y_{12} & ..I_1. & Y_{1n} \\ Y_{21} & Y_{22} & ..I_2. & Y_{2n} \\ . & & & \\ . & & & \\ Y_{n1} & Y_{n2} & ..I_n. & Y_{nn} \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} & ..... & Y_{1n} \\ Y_{21} & Y_{22} & ..... & Y_{2n} \\ . & & & \\ . & & & \\ Y_{n1} & Y_{n2} & ..... & Y_{nn} \end{vmatrix}}$$

Note the denominator is the same for all nodal voltages and is independent of the excitations – that is, it is dependent only upon the dead network

D(s) is the same for ALL transfer functions of a given “dead network”





Plausibility argument:

Note the denominator is the same for all nodal voltages and is independent of the excitations – that is, it is dependent only upon the dead network

Thus all branch voltages and all branch currents have the same denominator and this (after multiplying through by the correct power of s to make V_k a rational fraction) is the characteristic polynomial D(s)

This concept can be extended to include independent voltage sources as well as dependent sources

$$V_k = \frac{\begin{vmatrix} Y_{11} & Y_{12} & ..I_1. & Y_{1n} \\ Y_{21} & Y_{22} & ..I_2. & Y_{2n} \\ . & & & \\ . & & & \\ Y_{n1} & Y_{n2} & ..I_n. & Y_{nn} \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} & ..... & Y_{1n} \\ Y_{21} & Y_{22} & ..... & Y_{2n} \\ . & & & \\ . & & & \\ Y_{n1} & Y_{n2} & ..... & Y_{nn} \end{vmatrix}}$$

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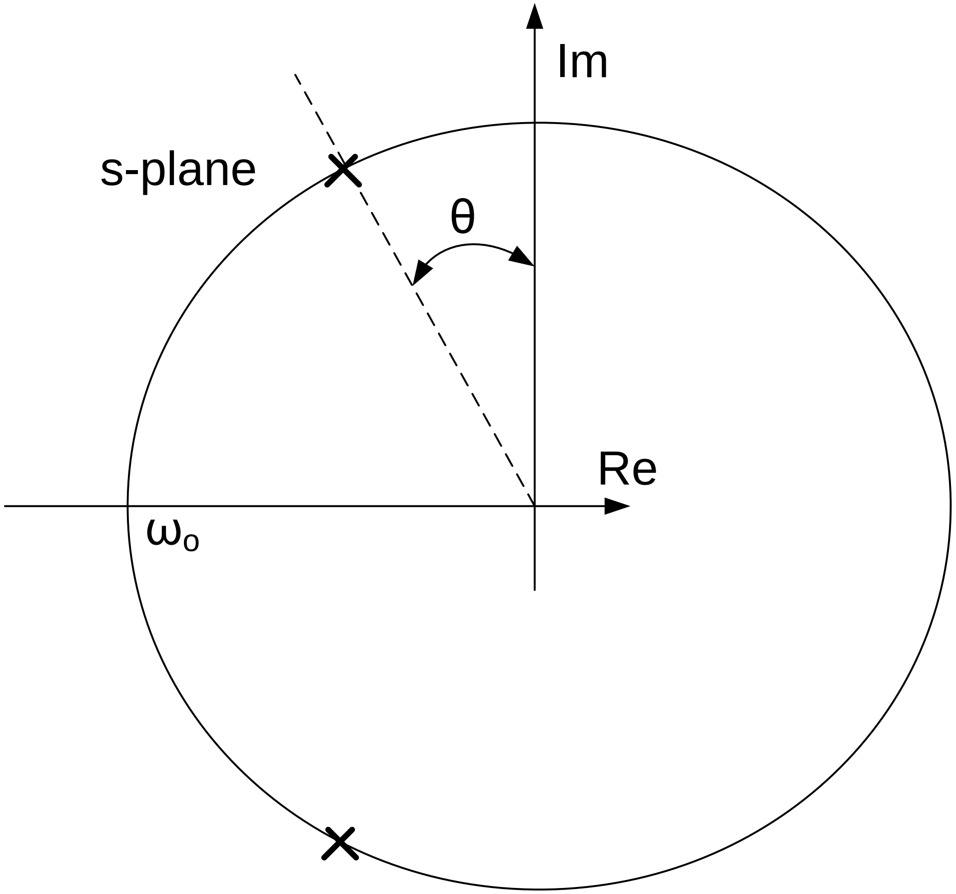
Root characterization in s-plane
(for complex-conjugate roots)

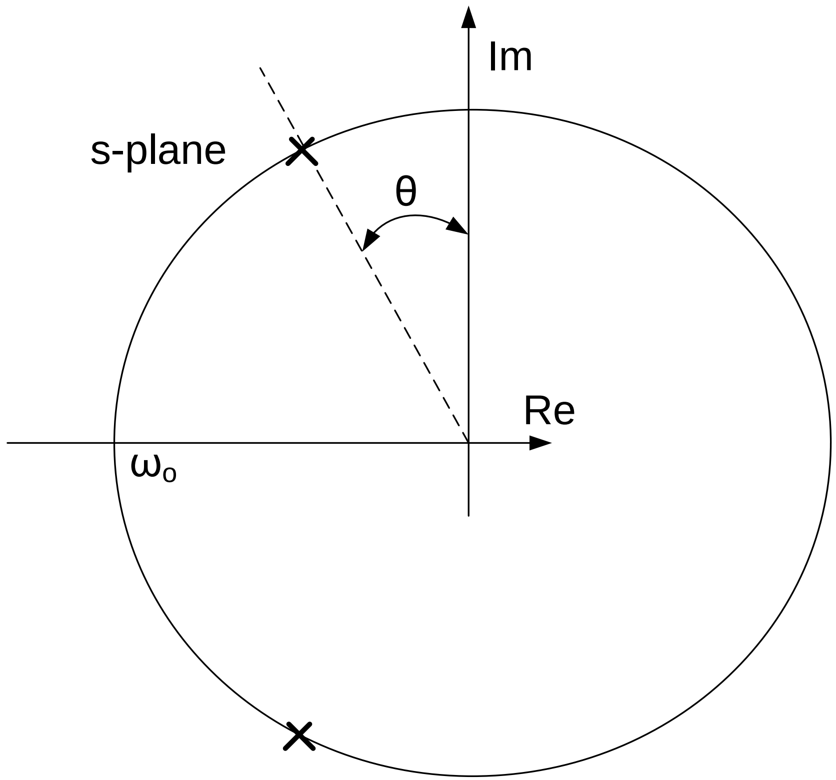


$$s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2$$

1-1 relationship between angle θ and Q of root

For low Q, θ is large
For high Q, θ is small

Root characterization in s-plane
(for complex-conjugate roots)



$$s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2$$

for θ=45°, Q=1/√2

for θ=90°, Q=1/2

roots located at
$$s = -\frac{\omega_0}{2Q} \pm \frac{1}{2}\sqrt{\left(\frac{\omega_0}{Q}\right)^2 - 4\omega_0^2} = \omega_0\left(-\frac{1}{2Q} \pm \frac{1}{2}\sqrt{\left(\frac{1}{Q}\right)^2 - 4}\right)$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{4Q^2 - 1}}\right)$$

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Scaling, Normalization and Transformations

- Frequency scaling
- Frequency Normalization
- Impedance scaling
- Transformations
  - LP to BP
  - LP to HP
  - LP to BR

Scaling, Normalization and Transformations

Frequency normalization: $$s_n = \frac{s}{\omega_0}$$

Frequency scaling: $$s = \omega_0 s_n$$

Purpose:

ω_0 independent approximations

ω_0 independent synthesis

Simplifies analytical expressions for T(s)

Simplifies component values in synthesis

Use single table of normalized filter circuits for given normalized approximating function

Note: The normalization subscript "n" is often dropped

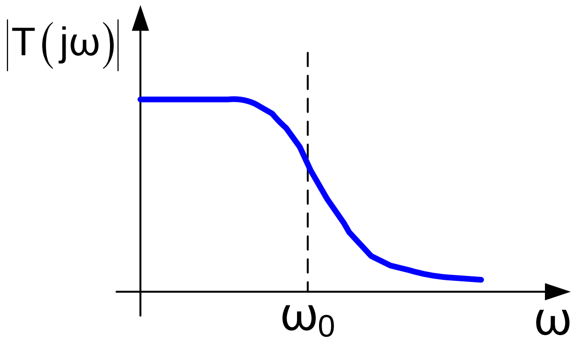
Frequency normalization/scaling example

$$T(s) = \frac{6000}{s + 6000}$$

Define ω_0=6000

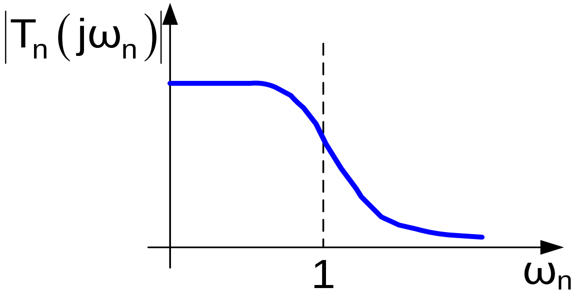
$$s_n = \frac{s}{\omega_0}$$

$$T(s) = \frac{\omega_0}{s + \omega_0}$$



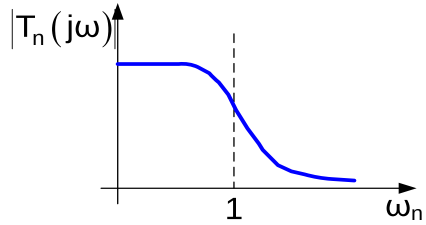
Normalized transfer function:

$$T_n(s_n) = \frac{1}{s_n + 1}$$

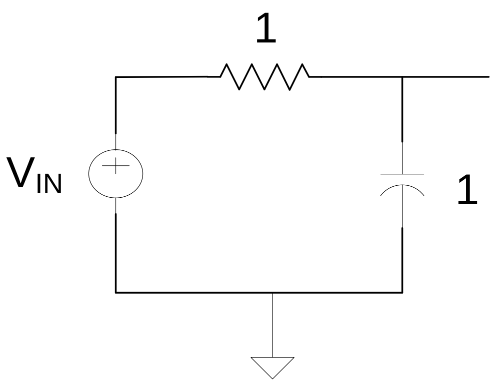


Frequency normalization/scaling example

$$T_n(s_n) = \frac{1}{s_n + 1}$$



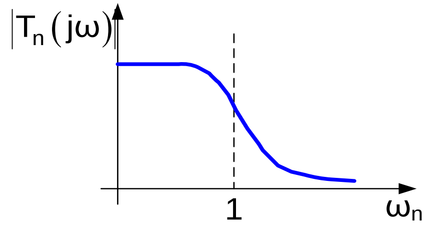
Synthesis of normalized function



$$T(s) = \frac{1}{s+1}$$

Frequency normalization/scaling example

$$T_n(s_n) = \frac{1}{s_n + 1}$$



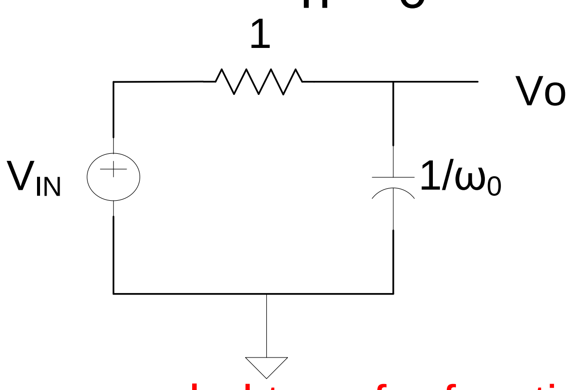
Frequency scaling transfer function by ω_0

$$s = \omega_0 s_n$$

$$T(s) = \frac{1}{\left(\frac{s}{\omega_0}\right) + 1} \quad \Rightarrow \quad T(s) = \frac{\omega_0}{s + \omega_0}$$

Frequency scaling circuit by ω_0 (actually magnitude of ω_0) (scale all energy storage elements in circuit)

$$C = C_n/\omega_0$$

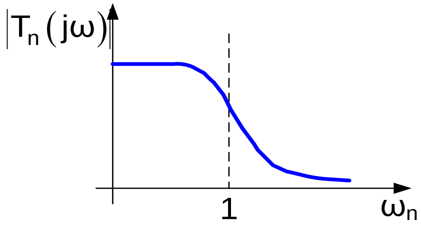


$$\Rightarrow \quad T(s) = \frac{\omega_0}{s + \omega_0}$$

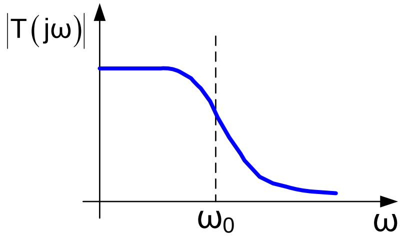
Frequency scaled transfer function is that of the frequency scaled circuit !

Frequency normalization/scaling example

$$T_n(s_n) = \frac{1}{s_n + 1}$$



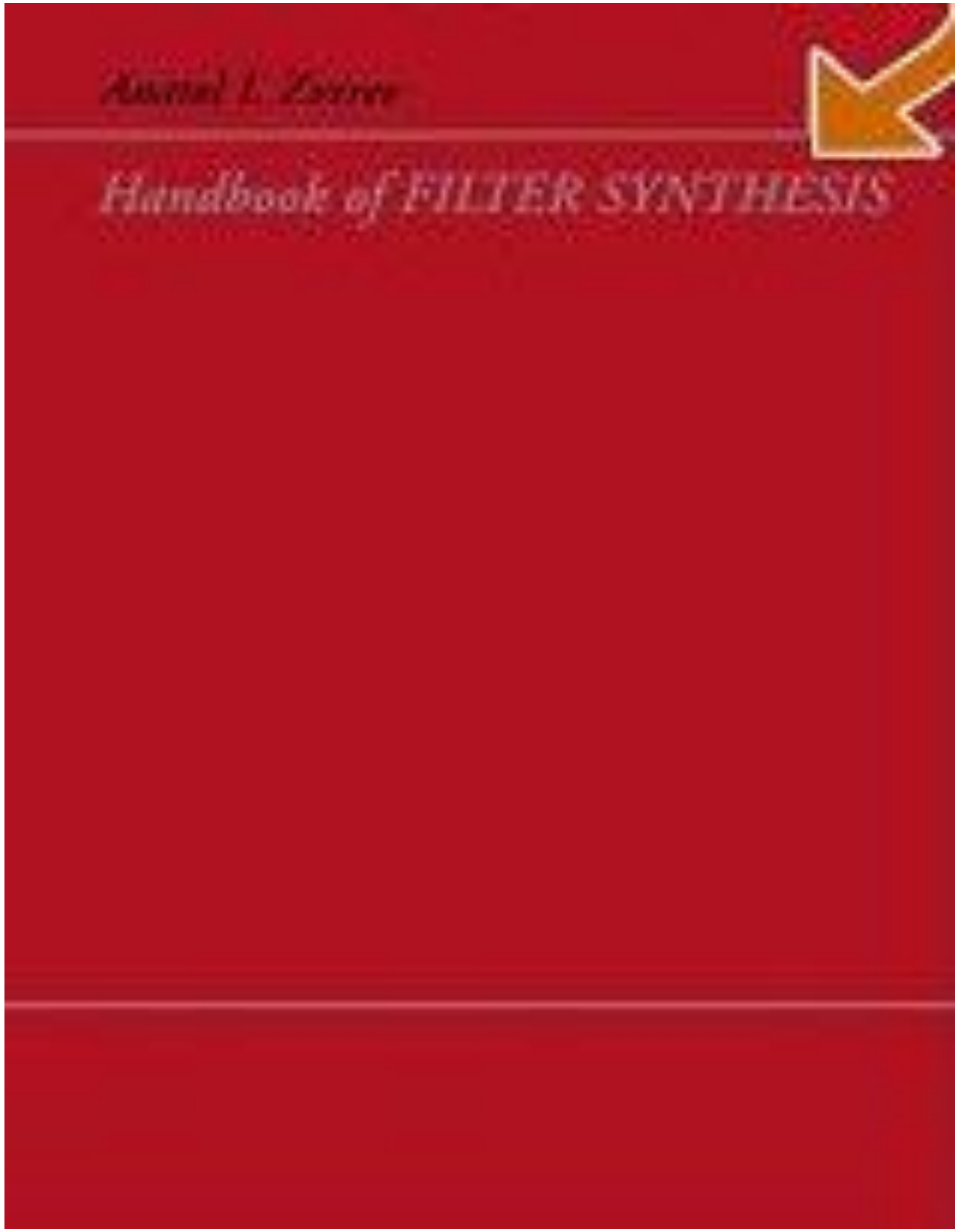
$$T(s) = \frac{\omega_0}{s + \omega_0}$$



Frequency scaling / normalization does not change the shape of the transfer function, it only scales the frequency axis linearly

The frequency scaled circuit can be obtained from the normalized circuit simply by scaling the frequency dependent impedances (up or down) by the scaling factor

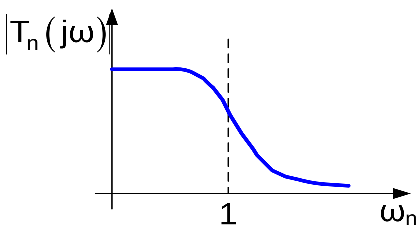
This makes the use of filter design tables for the design of lowpass filters practical whereby the circuits in the table all have a normalized band edge of 1 rad/sec.


Handbook of FILTER SYNTHESIS

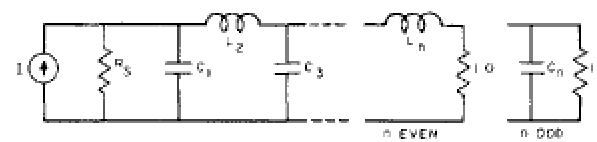
Frequency normalization/scaling

Example: Table for passive LC ladder Butterworth filter with 3dB band edge of 1 rad/sec and resistive source/load terminations

$$T_n(s_n) = \frac{1}{s_n + 1}$$



LOW PASS ELEMENT VALUES

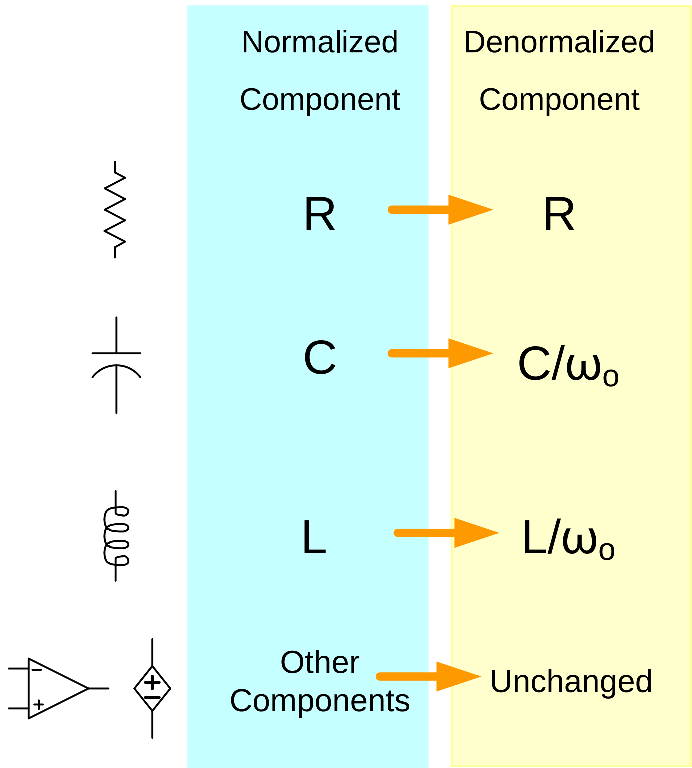


n	R_S	C_1	L_2	C_3	L_4
2	1.0000	1.4142	1.4142		
	1.1111	1.0353	1.9352		
	1.7500	0.8485	2.1213		
	1.4286	0.6971	2.4387		
	1.6667	0.5657	2.8284		
	2.0000	0.4483	3.3461		
	2.5000	0.3419	4.0951		
	3.3333	0.2447	5.3126		
	5.0000	0.1557	7.7067		
	10.0000	0.0743	14.8130		
	INF.	1.4142	0.7071		
3	1.0000	1.0000	2.0000	1.0000	
	0.9000	0.8082	1.6332	1.5994	
	0.8000	0.8442	1.3840	1.9259	
	0.7000	0.9152	1.1652	2.2774	
	0.6000	1.0225	0.9650	2.7024	
	0.5000	1.1811	0.7789	3.2612	
	0.4000	1.4254	0.6042	4.0642	
	0.3000	1.8380	0.4396	5.3634	
	0.2000	2.6687	0.2842	7.9102	
	0.1000	5.1672	0.1377	15.4554	
	INF.	1.5000	1.3333	0.5000	
4	1.0000	0.7654	1.8478	1.8478	0.7654
	1.1111	0.4657	1.5924	1.7439	1.4690
	1.2500	0.3983	1.6946	1.5110	1.8109
	1.4286	0.3251	1.8615	1.2913	2.1752
	1.6667	0.2693	2.1029	1.0824	2.6131
	2.0000	0.2175	2.4524	0.8826	3.1868
	2.5000	0.1692	2.9858	0.6911	4.0094
	3.3333	0.1237	3.8826	0.5072	5.1391
	5.0000	0.0804	5.6835	0.3307	7.9397
	10.0000	0.0392	11.0942	0.1616	15.6421
	INF.	1.5307	1.5772	1.0824	0.3827
n	$1/R_S$	L_1	C_2	L_3	C_4

Frequency normalization/scaling

The frequency scaled circuit can be obtained from the normalized circuit simply by scaling the frequency dependent impedances (up or down) by the scaling factor

Component denormalization by factor of ω_0



Component values of energy storage elements are scaled down by a factor of ω_0

Desgin Strategy

Theorem: A circuit with transfer function T(s) can be obtained from a circuit with normalized transfer function $T_n(s_n)$ by denormalizing all frequency dependent components.

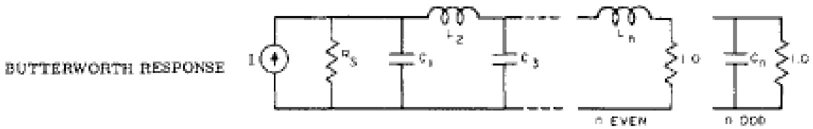
$C \rightarrow C/\omega_o$

$L \rightarrow L/\omega_o$

Example: Design a V-V passive 3rd-order Lowpass Butterworth filter with a 3-db band-edge of 1K rad/sec and equal source and load terminations.

(from the BW approximation which will be discussed later:) $T(s)=\dfrac{1}{s^3+2s^2+2s+1}$

LOW PASS ELEMENT VALUES



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	INF.	1.4142	0.7071		
3	1.0000	1.0000	2.0000	1.0000	
	[illegible]	[illegible]	[illegible]	[illegible]	
	0.8000	0.8442	1.3840	1.9259	
	0.7000	0.9152	1.1652	2.2774	
	0.6000	1.0225	0.9650	2.7024	
	0.5000	1.1811	0.7789	3.2612	
	0.4000	1.4254	0.6042	4.0642	
	0.3000	1.8380	0.4396	5.3634	
	0.2000	2.6687	0.2842	7.9102	
	0.1000	5.1672	0.1377	15.4554	
	INF.	1.5000	1.3333	0.5000	
4	1.0000	0.7654	1.8478	1.8478	0.7654
	1.1111	0.4657	1.5924	1.7439	1.4690
	1.2500	0.3982	1.6946	1.5110	1.8109
	1.4286	0.3251	1.8618	1.2913	2.1752
	1.6667	0.2693	2.1029	1.0824	2.6131
	2.0000	0.2175	2.4524	0.8826	3.1868
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	3.3333	0.1237	3.8826	0.5072	5.3381
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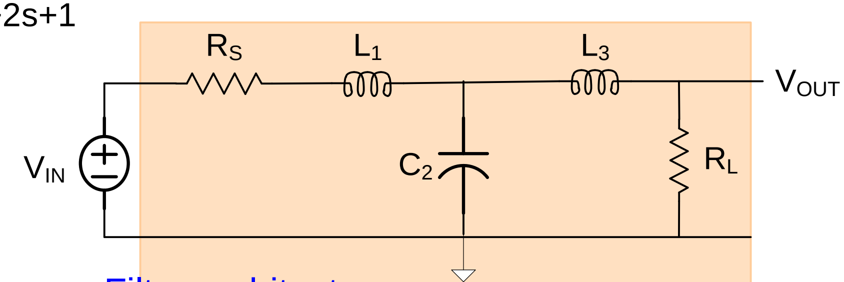
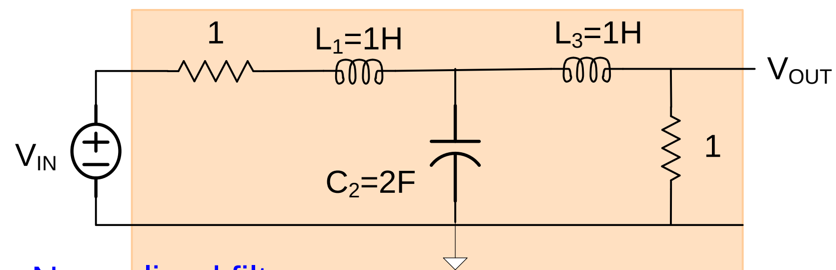
n	$1/R_s$	L_1	C_2	L_3	C_4



Filter architecture

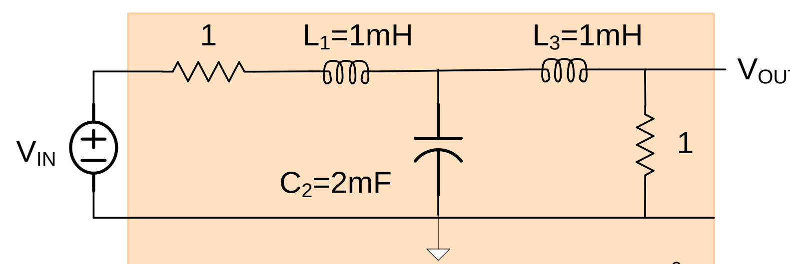


Normalized filter

$T(s)=K\dfrac{1}{s^3+2s^2+2s+1}$

C → C/θ

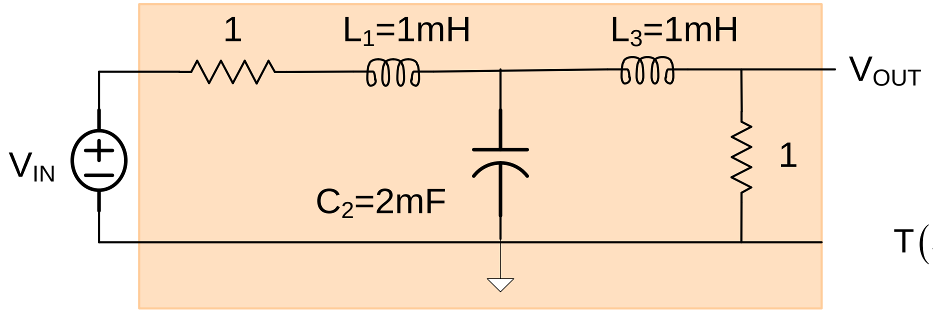
L → L/θ



Denormalized filter

$T(s)=K\dfrac{10^9}{s^3+2\bullet 10^3 s^2+2\bullet 10^6 s+10^9}$

Example: Design a V-V passive 3rd-order Lowpass Butterworth filter with a band-edge of 1K Rad/Sec and equal source and load terminations.



$$T(s)=K\frac{10^9}{s^3+2\bullet10^3s^2+2\bullet10^6s+10^9}$$

Is this solution practical?

Some component values are too big and some are too small !

Filter Concepts and Terminology

- Frequency scaling
- Frequency Normalization
- Impedance scaling
- Transformations
  - LP to BP
  - LP to HP
  - LP to BR

Impedance Scaling

Impedance scaling of a circuit is achieved by multiplying ALL impedances in the circuit by a constant

$R \rightarrow \theta R$

$C \rightarrow C/\theta$

$L \rightarrow L\theta$

$A \rightarrow$

- θA for transresistance gain
- A for dimensionless gain
- A/θ for transconductance gain

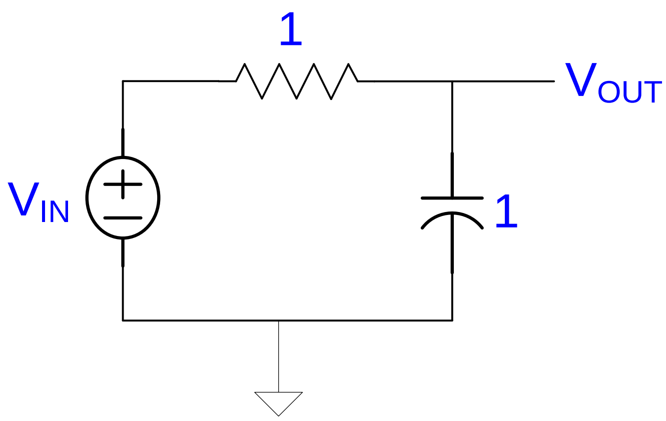
Impedance Scaling

Theorem: If all impedances in a circuit are scaled by a constant θ, then

a) All dimensionless transfer functions are unchanged
b) All transresistance transfer functions are scaled by θ
c) All transconductance transfer functions are scaled by θ^{-1}

Impedance Scaling

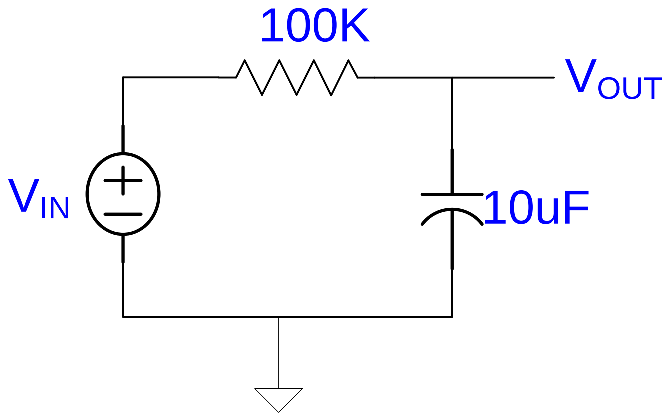
Example:



$$T(s) = \frac{1}{s+1}$$

T(s) is dimensionless

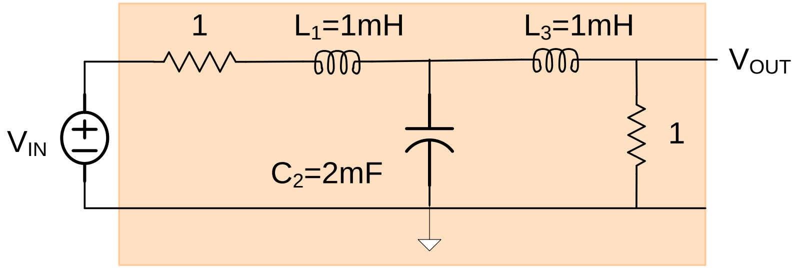
Impedances scaled by $\theta=10^5$



$$T(s) = \frac{1}{s+1}$$

Note second circuit much more practical than the first

Example: Design a V-V passive 3rd-order Lowpass Butterworth filter with a band-edge of 1K Rad/Sec and equal source and load terminations.



$$T(s)=K\frac{10^9}{s^3+2\bullet10^3s^2+2\bullet10^6s+10^9}$$

Is this solution practical?

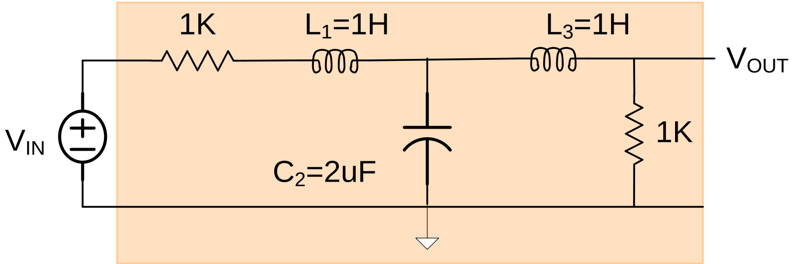
Some component values are too big and some are too small !

Impedance scale by θ=1000

$R \rightarrow \theta R$

$C \rightarrow C/\theta$

$L \rightarrow \theta L$



$$T(s)=K\frac{10^9}{s^3+2\bullet10^3s^2+2\bullet10^6s+10^9}$$

Component values more practical

Transformations

– LP to BP

– LP to HP

– LP to BR

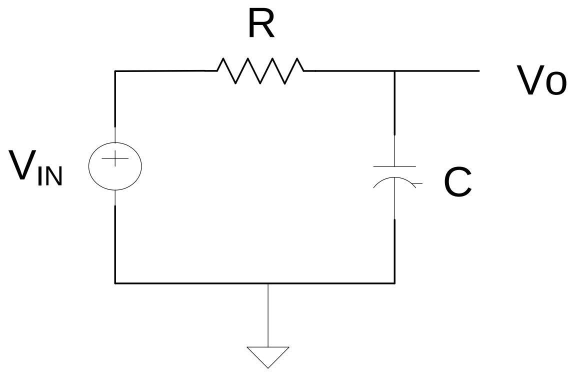
It can be shown the standard HP, BP, and BR approximations can be obtained by a frequency transformation of a standard LP approximating function

Will address the LP approximation first, and then provide details about the frequency transformations

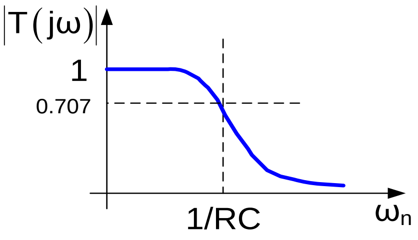
Typical approach to filter design

1. Transform requirements to a low-pass filter problem
2. Obtain normalized low-pass approximating function
3. Synthesize circuit to realize normalized approximating function
4. Denormalize circuit obtained in step 3
5. Impedance scale to obtain acceptable component values
6. Transform to desired filter type (LP, BP, BR)

Degrees of Freedom



$$T(s) = \frac{V_O}{V_{IN}} = \frac{1}{RCs+1}$$



Circuit has two design variables: {R,C}

Circuit has one key controllable performance characteristic: $\omega_0 = \frac{1}{RC}$

If ω_0 is specified for a design, circuit has

2 design variables
1 constraint

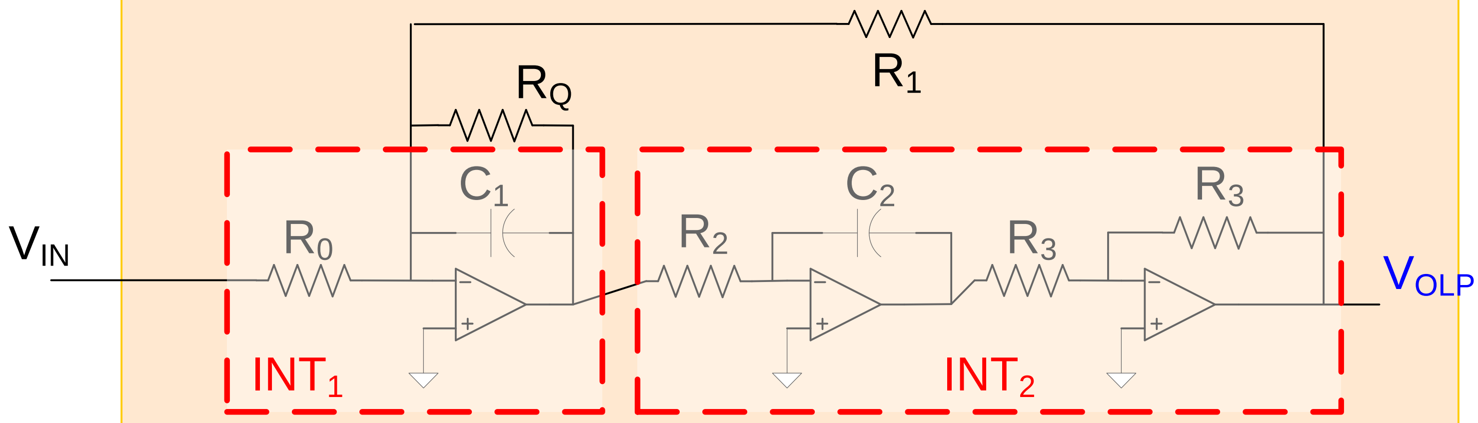
1 Degree of Freedom

Performance/Cost strongly affected by how degrees of freedom in a design are used !

Example: Design a 2nd order lowpass Butterworth filter with 3dB passband attenuation, a dc gain of 5, and a 3dB bandedge of 4KHz

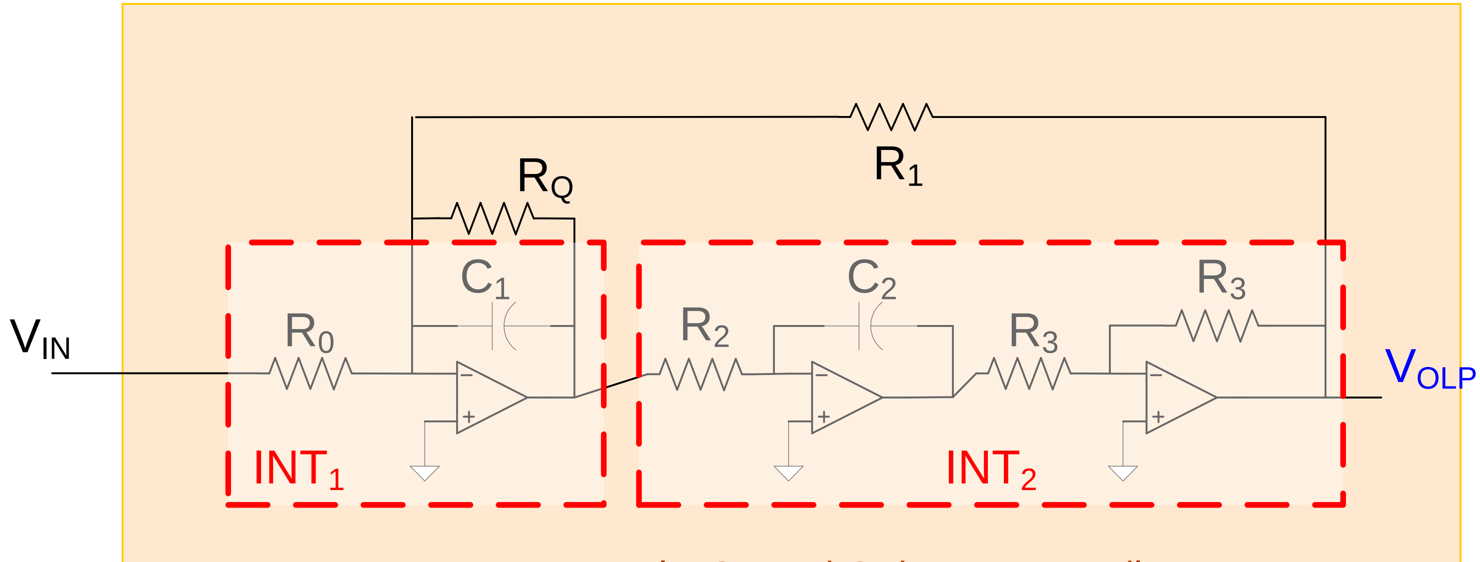
Note: We have not discussed the Butterworth approximation yet so some details here will be based upon concepts that will be developed later

$$T_{BWn} = \left(\frac{1}{s^2+\sqrt{2}s+1}\right)\bullet 5 \quad \Rightarrow \quad \omega_0=1, \quad Q=\frac{1}{\sqrt{2}}=0.707$$



A Popular Second-Order Lowpass Filter

Example: Design a 2nd order lowpass Butterworth filter with 3dB passband attenuation, a dc gain of 5, and a 3dB band edge of 4KHz



A Popular Second-Order Lowpass Filter

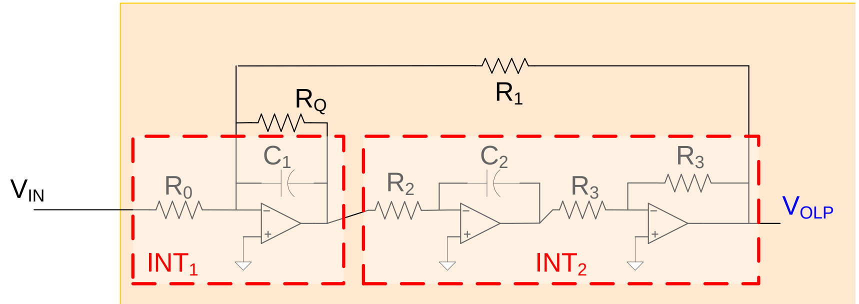
$$T(s) = \frac{\dfrac{1}{R_2 R_0 C_1 C_2}}{s^2 + s\left(\dfrac{1}{R_Q C_1}\right) + \dfrac{1}{R_2 R_1 C_1 C_2}}$$

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \qquad Q = \frac{R_Q}{\sqrt{R_1 R_2}}\sqrt{\frac{C_1}{C_2}}$$

7 design variables and only two constraints (ignoring the gain right now)

Circuit has 5 Degrees of Freedom!

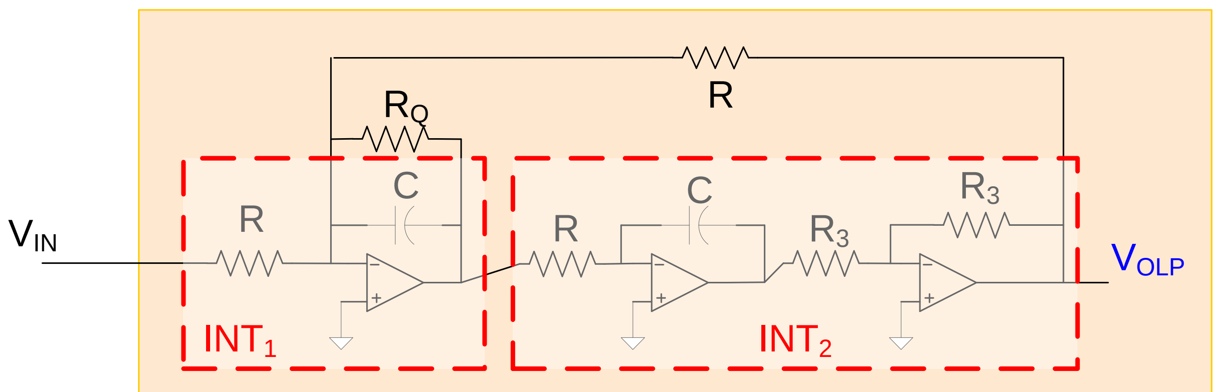
Example: Design a 2nd order lowpass Butterworth filter with 3dB passband attenuation, a dc gain of 5, and a 3dB band edge of 4KHz



A Popular Second-Order Lowpass Filter

If $C_1=C_2=C$ and $R_1=R_2=R_0=R$, this reduces to

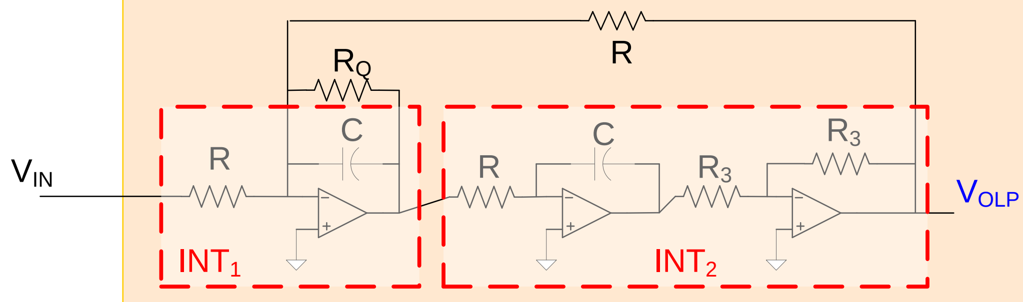
$$T(s)=\frac{\frac{1}{(RC)^2}}{s^2+s\left(\frac{R}{R_Q}\frac{1}{RC}\right)+\frac{1}{(RC)^2}}$$



A Popular Second-Order Lowpass Filter

How many degrees of freedom remain? 2

Example: Design a 2nd order lowpass Butterworth filter with 3dB passband attenuation, a dc gain of 5, and a 3dB bandedge of 4KHz



A Popular Second-Order Lowpass Filter

$$T(s) = \frac{\frac{1}{(RC)^2}}{s^2 + s\left(\frac{R}{R_Q}\frac{1}{RC}\right) + \frac{1}{(RC)^2}}$$

$$\omega_0 = \frac{1}{RC}$$

$$Q = \frac{R_Q}{R}$$

Normalizing by the factor ω_0, we obtain

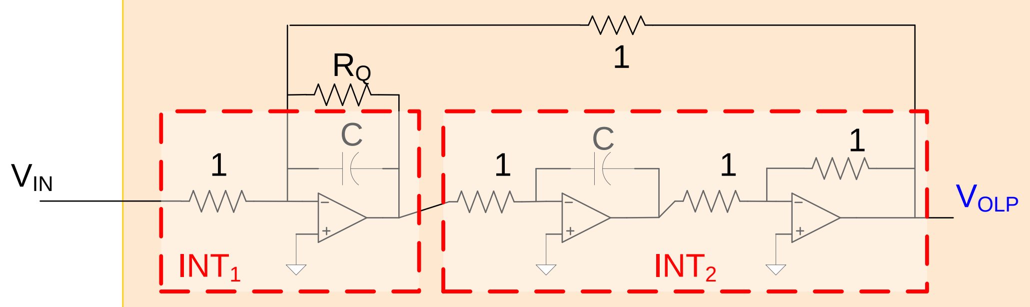
$$T(s_n) = \frac{1}{s^2 + s\left(\frac{1}{Q}\right) + 1}$$

Lets now use up the two degrees of freedom in the circuit:

Setting $R=R_3=1$ obtain the following normalized circuit

Example: Design a 2nd order lowpass Butterworth filter with 3dB passband attenuation, a dc gain of 5, and a 3dB bandedge of 4KHz

Setting $R=R_3=1$ obtain the following circuit



A Popular Second-Order Lowpass Filter

The two constraints become

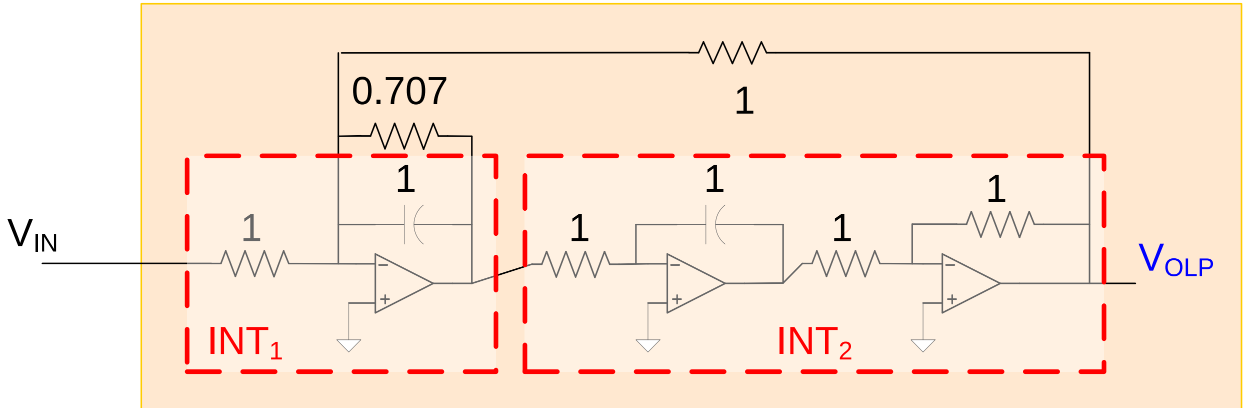
$$\omega_0 = \frac{1}{RC} = \frac{1}{C} \qquad Q = \frac{R_Q}{R} = R_Q$$

This leaves 2 unknowns, R_Q and C and two constraints (i.e. no remaining degrees of freedom)

Example: Design a 2^{nd} order lowpass Butterworth filter with 3dB passband attenuation, a dc gain of 5, and a 3dB bandedge of 4KHz

$$T(s_n) = \frac{1}{s^2 + s\left(\frac{1}{Q}\right) + 1} \qquad \omega_{0n} = 1 \qquad Q_N = \frac{1}{\sqrt{2}}$$

To satisfy the 2 constraints, must now set $R_Q = Q$ $\quad C = 1$



A Popular Second-Order Lowpass Filter

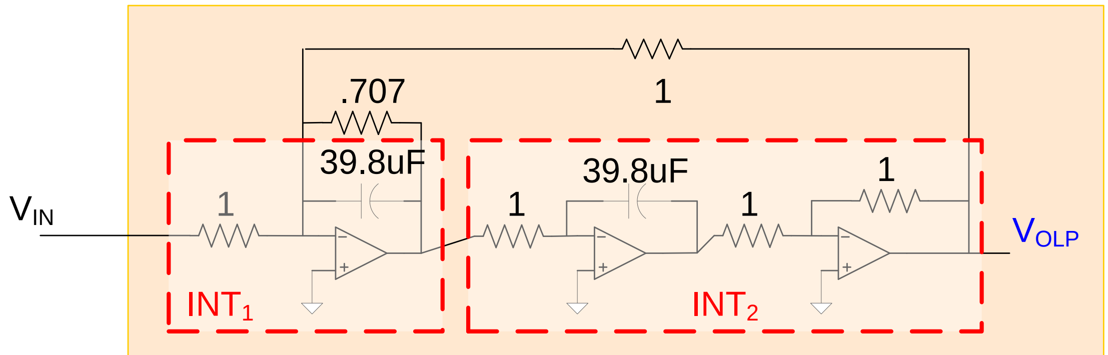
Now we can do frequency scaling $\quad C \rightarrow C/\omega_o$

$L \rightarrow L/\omega_o$

$C=1 \rightarrow 1/(2\pi \bullet 4K) = 39.8uF$

Example: Design a 2^{nd} order lowpass Butterworth filter with 3dB passband attenuation, a dc gain of 5, and a 3dB bandedge of 4KHz

Denormalized circuit with bandedge of 4 KHz



A Popular Second-Order Lowpass Filter

This has the right transfer function (but unity gain)

Can now do impedance scaling to get more practical component values

$R \rightarrow \theta R$

$C \rightarrow C/\theta$

$L \rightarrow \theta L$

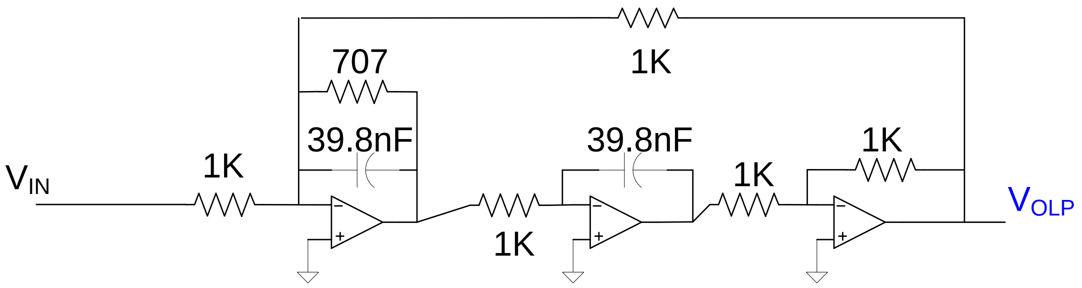
A good impedance scaling factor may be θ=1000

R → 1K

C → 39.8nF

Example: Design a 2nd order lowpass Butterworth filter with 3dB passband attenuation, a dc gain of 5, and a 3dB bandedge of 4KHz

Denormalized circuit with bandedge of 4 KHz



This has the right transfer function (but unity gain)

To finish the design, preceed or follow this circuit with an amplifier with a gain of 5 to meet the dc gain requirements

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It can be shown the standard HP, BP, and BR approximations can be obtained by a frequency transformation of a standard LP approximating function

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