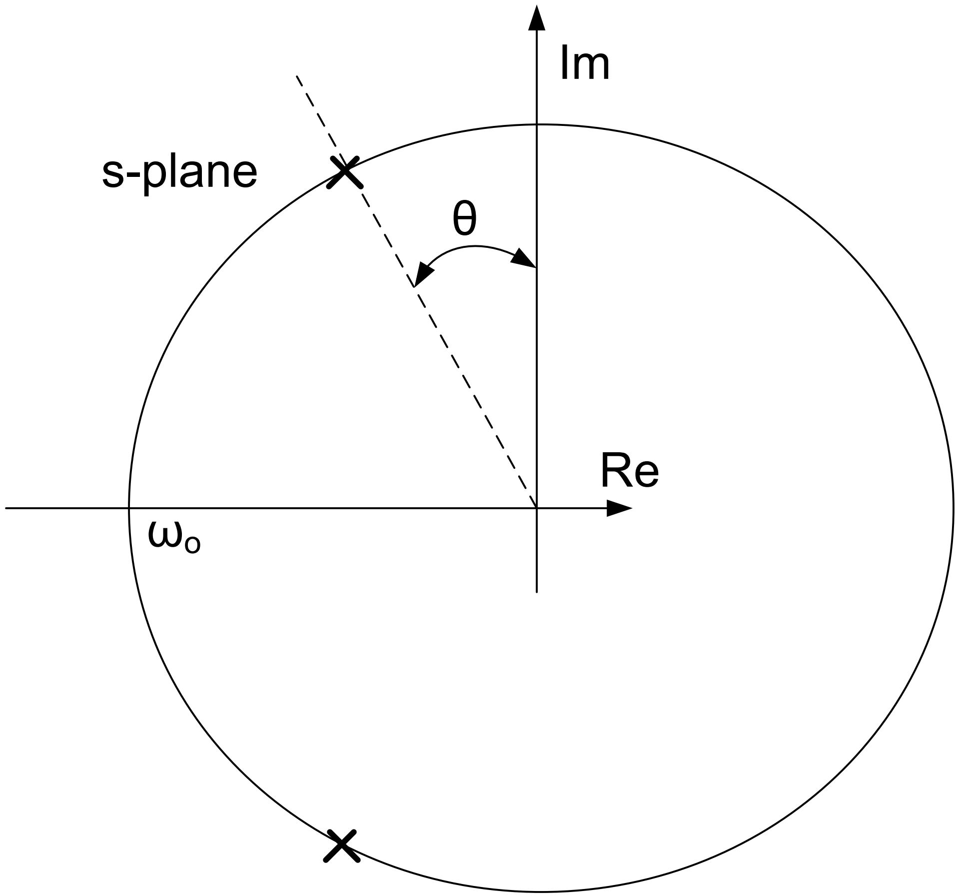
EE 508
Lecture 6

Filter Concepts/Terminology

Approximation Problem

Root characterization in s-plane
(for complex-conjugate roots)

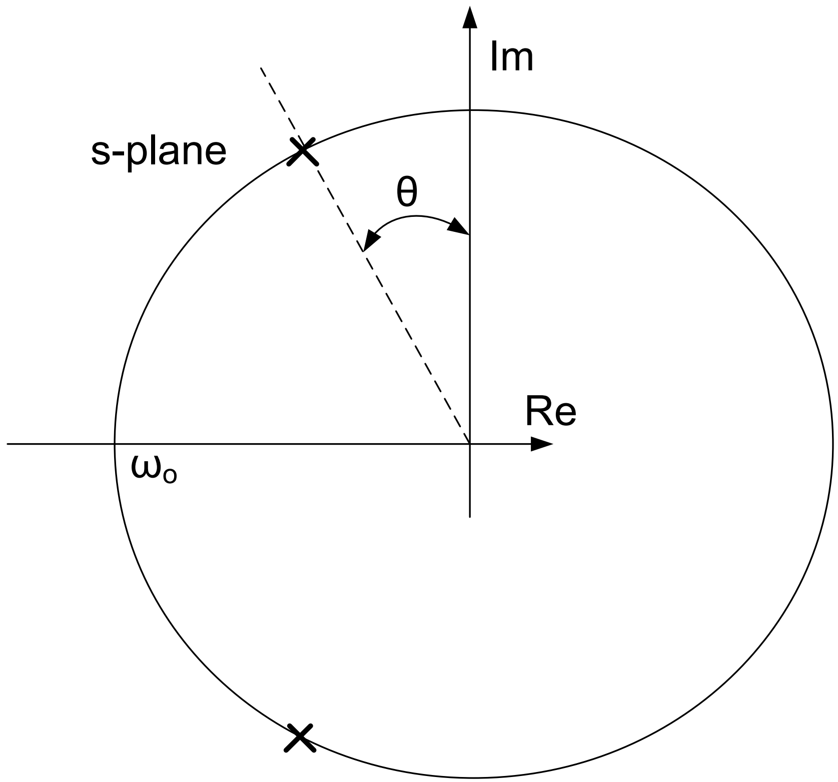


$$s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2$$

1-1 relationship between angle θ and Q of root

For low Q, θ is large
For high Q, θ is small

Root characterization in s-plane
(for complex-conjugate roots)



$$s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2$$

for θ=90°, Q=1/√2

roots located at $$s = -\frac{\omega_0}{2Q} \pm \frac{1}{2}\sqrt{\left(\frac{\omega_0}{Q}\right)^2 - 4\omega_0^2} = \omega_0\left(-\frac{1}{2Q} \pm \frac{1}{2}\sqrt{\left(\frac{1}{Q}\right)^2 - 4}\right)$$

$$\theta = \tan^{-1}\left(4Q^2 - 1\right)$$

Filter Concepts and Terminology

- Frequency scaling
- Frequency Normalization
- Impedance scaling
- Transformations
  - LP to BP
  - LP to HP
  - LP to BR

Frequency Scaling and normalization

Frequency normalization: $s_n = \dfrac{s}{\omega_0}$

Frequency scaling: $s = \omega_0 s_n$

Purpose:

ω_0 independent approximations

ω_0 independent synthesis

Simplifies analytical expressions for T(s)

Simplifies component values in synthesis

Use single table of normalized filter circuits for given normalized approximating function

Note: The normalization subscript "n" is often dropped

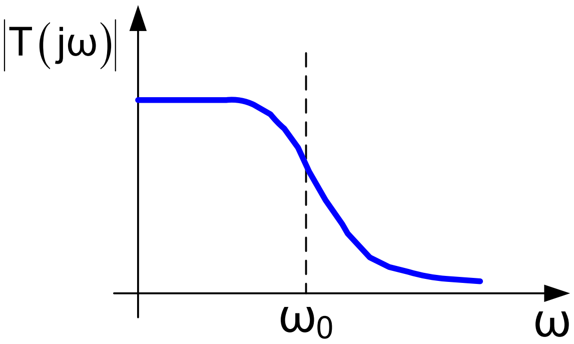
Frequency normalization/scaling example

$$T(s) = \frac{6000}{s + 6000}$$

Define ω_0=6000

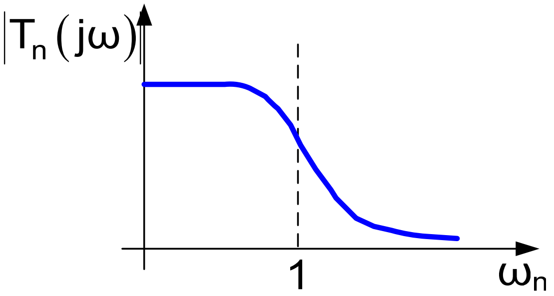
$$s_n = \frac{s}{\omega_0}$$

$$T(s) = \frac{\omega_0}{s + \omega_0}$$



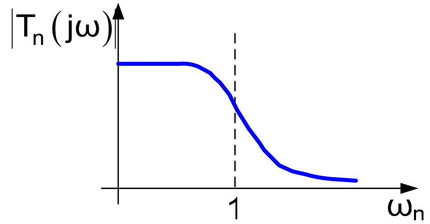
Normalized transfer function:

$$T_n(s_n) = \frac{1}{s_n + 1}$$

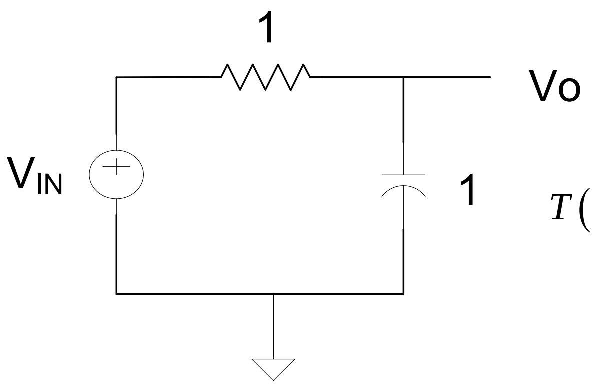


Frequency normalization/scaling example

$$T_n(s_n) = \frac{1}{s_n + 1}$$



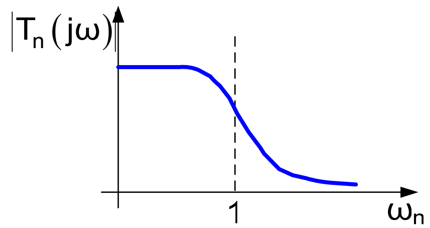
Synthesis of normalized function



$$T(s) = \frac{1}{s+1}$$

Frequency normalization/scaling example

$$T_n(s_n) = \frac{1}{s_n + 1}$$



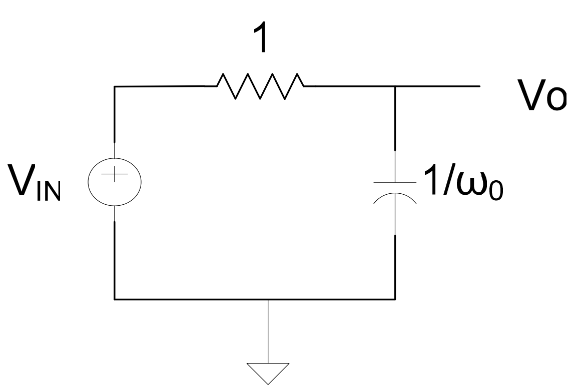
Frequency scaling by ω_0 (of transfer function)

$$s = \omega_0 s_n$$

$$T(s) = \frac{1}{\left(\frac{s}{\omega_0}\right) + 1} \quad \Rightarrow \quad T(s) = \frac{\omega_0}{s + \omega_0}$$

Frequency scaling by ω_0 (of energy storage elements in circuit)

$$C = C_n/\omega_0$$

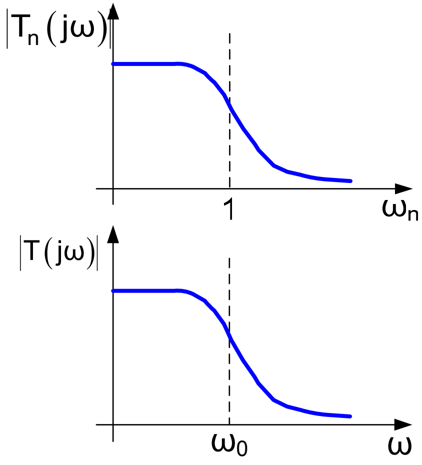


$$\Rightarrow \quad T(s) = \frac{\omega_0}{s + \omega_0}$$

Frequency normalization/scaling example

$$T_n(s_n) = \frac{1}{s_n + 1}$$

$$T(s) = \frac{\omega_0}{s + \omega_0}$$



Frequency scaling / normalization does not change the shape of the transfer function, it only scales the frequency axis linearly

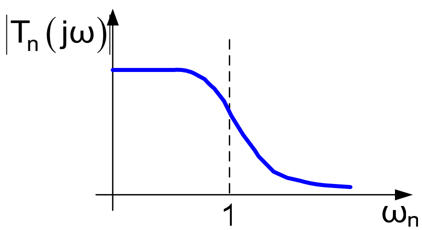
The frequency scaled circuit can be obtained from the normalized circuit simply by scaling the frequency dependent impedances (up or down) by the scaling factor

This makes the use of filter design tables for the design of lowpass filters practical whereby the circuits in the table all have a normalized band edge of 1 rad/sec.

Frequency normalization/scaling

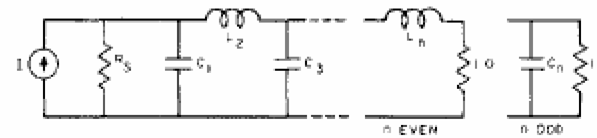
Example: Table for passive ladder Butterworth filter with 3dB band edge of 1 rad/sec

$$T_n(s_n) = \frac{1}{s_n + 1}$$



LOW PASS ELEMENT VALUES

BUTTERWORTH RESPONSE

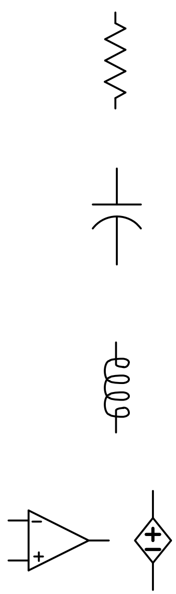


n	R_s	C_1	L_2	C_3	L_4
2	1.0000	1.4142	1.4142		
	1.1111	1.0353	1.9352		
	1.2500	0.8485	2.1213		
	1.4286	0.6971	2.4387		
	1.6667	0.5657	2.8284		
	2.0000	0.4483	3.3461		
	2.5000	0.3419	4.0951		
	3.3333	0.2447	5.3126		
	5.0000	0.1557	7.7067		
	10.0000	0.0743	14.8136		
	INF.	1.4142	0.7071		
3	1.0000	1.0000	2.0000	1.0000	
	0.9000	0.8082	1.6332	1.5994	
	0.8000	0.8442	1.3840	1.9259	
	0.7000	0.9152	1.1652	2.2774	
	0.6000	1.0225	0.9650	2.7024	
	0.5000	1.1811	0.7789	3.2612	
	0.4000	1.4254	0.6042	4.0642	
	0.3000	1.8380	0.4396	5.3634	
	0.2000	2.6687	0.2842	7.9102	
	0.1000	5.1672	0.1377	15.4554	
	INF.	1.5000	1.3333	0.5000	
4	1.0000	0.7654	1.8478	1.8478	0.7654
	1.1111	0.4657	1.5924	1.7439	1.4690
	1.2500	0.3983	1.6946	1.5110	1.8109
	1.4286	0.3251	1.8618	1.2913	2.1752
	1.6667	0.2693	2.1029	1.0824	2.6131
	2.0000	0.2175	2.4524	0.8826	3.1868
	2.5000	0.1692	2.9858	0.6911	4.0094
	3.3333	0.1237	3.8826	0.5072	5.3391
	5.0000	0.0804	5.6835	0.3307	7.9397
	10.0000	0.0392	11.0942	0.1616	15.6421
	INF.	1.5307	1.5772	1.0824	0.3827
n	$1/R_s$	L_1	C_2	L_3	C_4

Frequency normalization/scaling

The frequency scaled circuit can be obtained from the normalized circuit simply by scaling the frequency dependent impedances (up or down) by the scaling factor

Component denormalization by factor of ω_0



Normalized Component		Denormalized Component
R	→	R
C	→	C/ω_o
L	→	L/ω_o
Other Components	→	Unchanged

Component values of energy storage elements are scaled down by a factor of ω_0

Desgin Strategy

Theorem: A circuit with transfer function T(s) can be obtained from a circuit with normalized transfer function $T_n(s_n)$ by denormalizing all frequency dependent components.

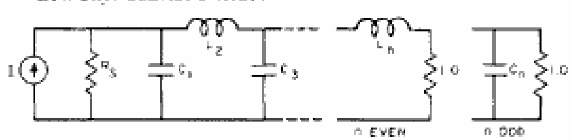
$$C \rightarrow C/\omega_o$$

$$L \rightarrow L\omega_o$$

Example: Design a V-V passive 3rd-order Lowpass Butterworth filter with a band-edge of 1K Rad/Sec and equal source and load terminations.

LOW PASS ELEMENT VALUES

BUTTERWORTH RESPONSE



n	R_s	C_1	L_2	C_3	L_4
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	INF.	1.4142	0.7071		
3	1.0000	1.0000	2.0000	1.0000	
	[illegible]	[illegible]	[illegible]	[illegible]	
	0.8000	0.8442	1.3840	1.9259	
	0.7000	0.9152	1.1652	2.2774	
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	2.5000	0.1692	2.9858	0.6911	4.0094
	3.3333	0.1237	3.8826	0.5072	5.3381
	5.0000	0.0804	5.6835	0.3307	7.9397
	10.0000	0.0392	11.0942	0.1616	15.6421
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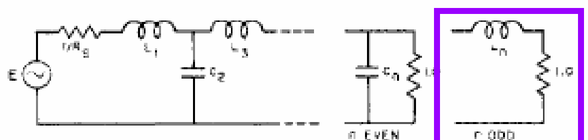
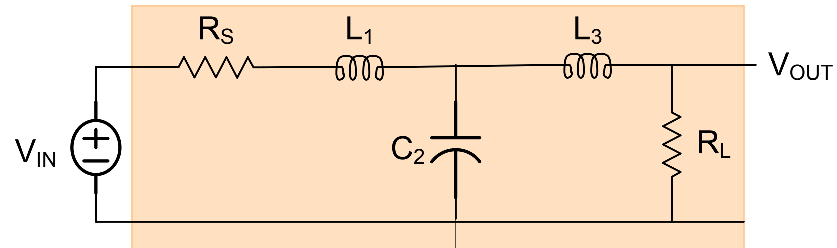
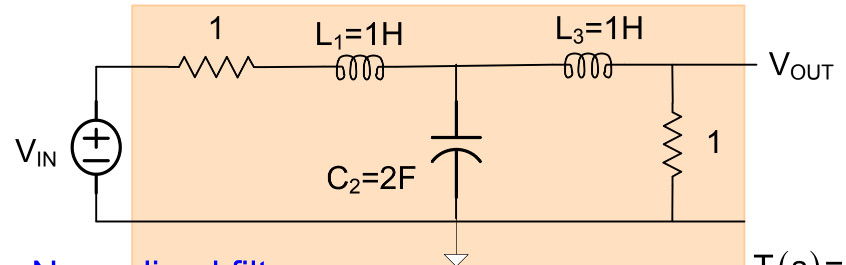
n	$1/R_s$	L_1	C_2	L_3	C_4





Filter architecture

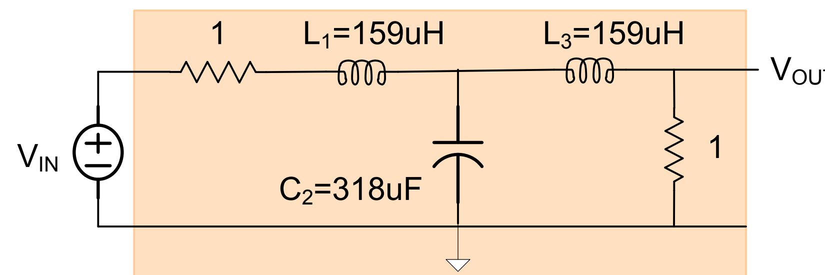


Normalized filter

$$T(s)=\frac{1}{s+1}$$

C → C/θ

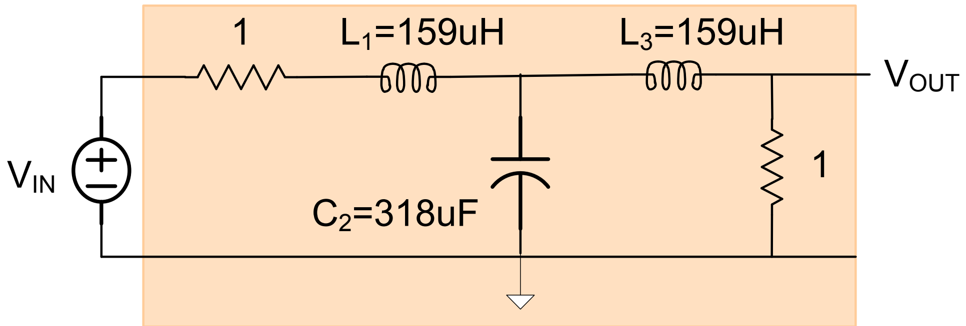
L → L/θ



Denormalized filter

$$T(s)=\frac{1000}{s+1000}$$

Example: Design a V-V passive 3rd-order Lowpass Butterworth filter with a band-edge of 1K Rad/Sec and equal source and load terminations.



$$T(s)=\frac{1000}{s+1000}$$

Is this solution practical?

Some component values are too big and some are too small !

Filter Concepts and Terminology

- Frequency scaling
- Frequency Normalization
- Impedance scaling
- Transformations
  - LP to BP
  - LP to HP
  - LP to BR

Impedance Scaling

Impedance scaling of a circuit is achieved by multiplying ALL impedances in the circuit by a constant

$R \rightarrow \theta R$

$C \rightarrow C/\theta$

$L \rightarrow L\theta$

$A \rightarrow$

- θA for transresistance gain
- A for dimensionless gain
- A/θ for transconductance gain

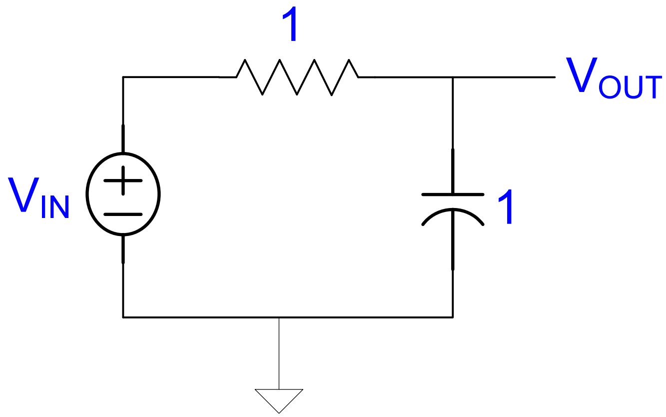
Impedance Scaling

Theorem: If all impedances in a circuit are scaled by a constant θ, then

a) All dimensionless transfer functions are unchanged
b) All transresistance transfer functions are scaled by θ
c) All transconductance transfer functions are scaled by θ^{-1}

Impedance Scaling

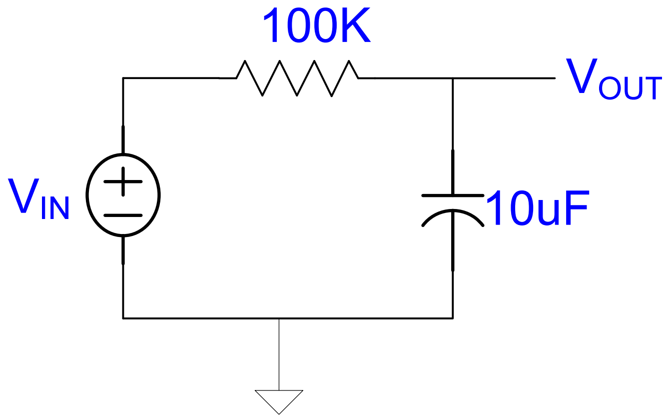
Example:



$$T(s) = \frac{1}{s+1}$$

T(s) is dimensionless

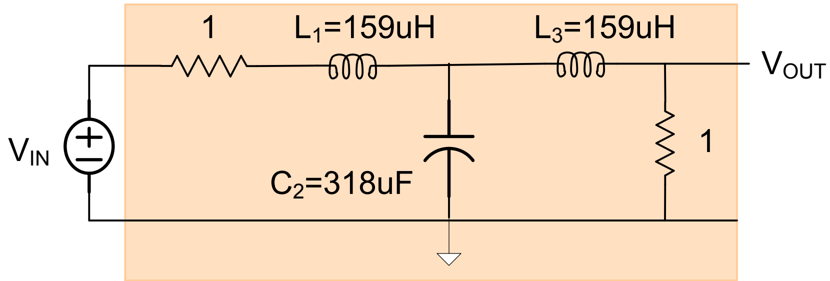
Impedances scaled by $\theta=10^5$



$$T(s) = \frac{1}{s+1}$$

Note second circuit much more practical than the first

Example: Design a V-V passive 3rd-order Lowpass Butterworth filter with a band-edge of 1K Rad/Sec and equal source and load terminations.



$$T(s)=\frac{1000}{s+1000}$$

Is this solution practical?

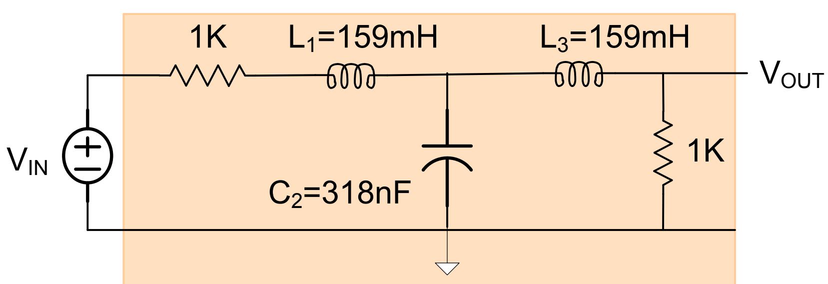
Some component values are too big and some are too small !

Impedance scale by θ=1000

$R \rightarrow \theta R$

$C \rightarrow C/\theta$

$L \rightarrow \theta L$



$$T(s)=\frac{1000}{s+1000}$$

Component values more practical

Typical approach to lowpass filter design

1. Obtain normalized approximating function

2. Synthesize circuit to realize normalized approximating function

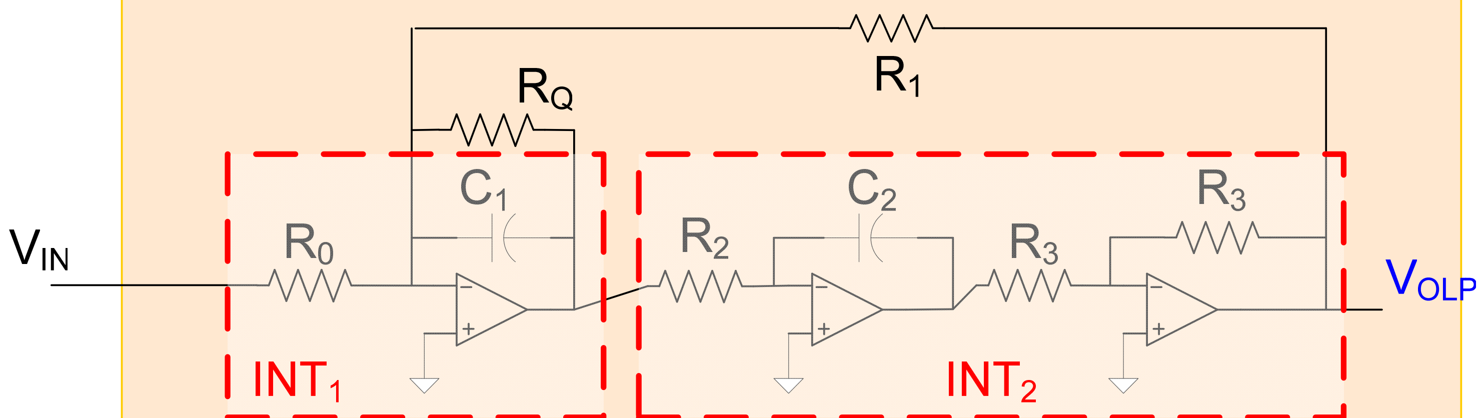
3. Denormalize circuit obtained in step 2

4. Impedance scale to obtain acceptable component values

Example: Design a 2nd order lowpass Butterworth filter with 3dB passband attenuation, a dc gain of 5, and a 3dB bandedge of 4KHz

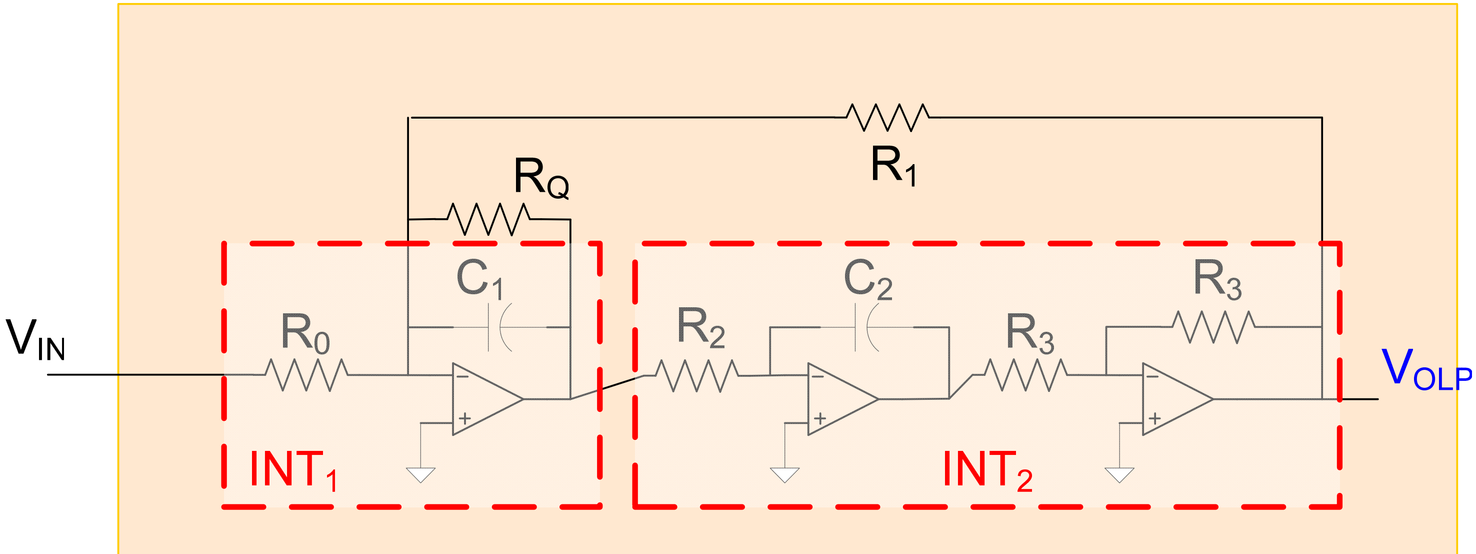
Note: We have not discussed the Butterworth approximation yet so some details here will be based upon concepts that will be developed later

$$T_{BWn} = \left(\frac{1}{s^2 + \sqrt{2}s + 1} \right) \bullet 5$$



A Popular Second-Order Lowpass Filter

Example: Design a 2^{nd} order lowpass Butterworth filter with 3dB passband attenuation, a dc gain of 5, and a 3dB bandedge of 4KHz



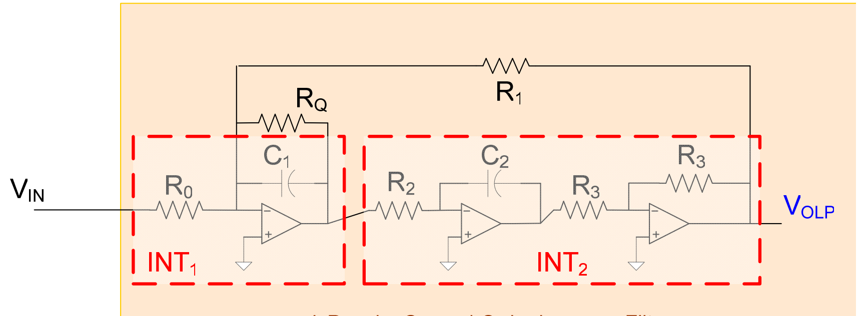
A Popular Second-Order Lowpass Filter

$$T(s) = \frac{\frac{1}{R_2R_0C_1C_2}}{s^2+s\left(\frac{1}{R_QC_1}\right)+\frac{1}{R_2R_1C_1C_2}}$$

$$\omega_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}} \qquad Q=\frac{R_Q}{\sqrt{R_1R_2}}\sqrt{\frac{C_1}{C_2}}$$

8 design variables and only two constraints (ignoring the gain right now)

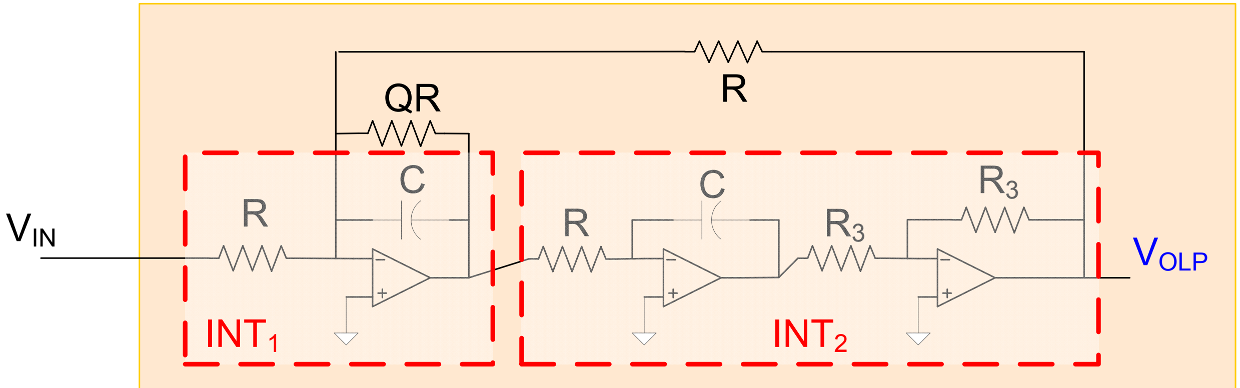
Example: Design a 2^{nd} order lowpass Butterworth filter with 3dB passband attenuation, a dc gain of 5, and a 3dB bandedge of 4KHz



A Popular Second-Order Lowpass Filter

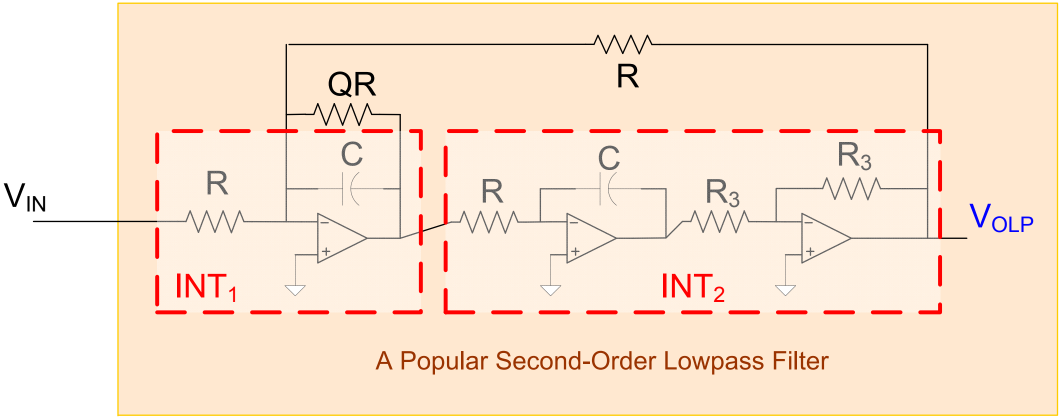
If $C_1=C_2=C$ and $R_1=R_2=R_0=R$, this reduces to

$$T(s) = \frac{\frac{1}{(RC)^2}}{s^2+s\left(\frac{R}{R_Q}\frac{1}{RC}\right)+\frac{1}{(RC)^2}}$$



A Popular Second-Order Lowpass Filter

Example: Design a 2nd order lowpass Butterworth filter with 3dB passband attenuation, a dc gain of 5, and a 3dB bandedge of 4KHz



A Popular Second-Order Lowpass Filter

$$T(s) = \frac{\frac{1}{(RC)^2}}{s^2 + s\left(\frac{R}{R_Q}\frac{1}{RC}\right) + \frac{1}{(RC)^2}}$$

$$\omega_0 = \frac{1}{RC}$$

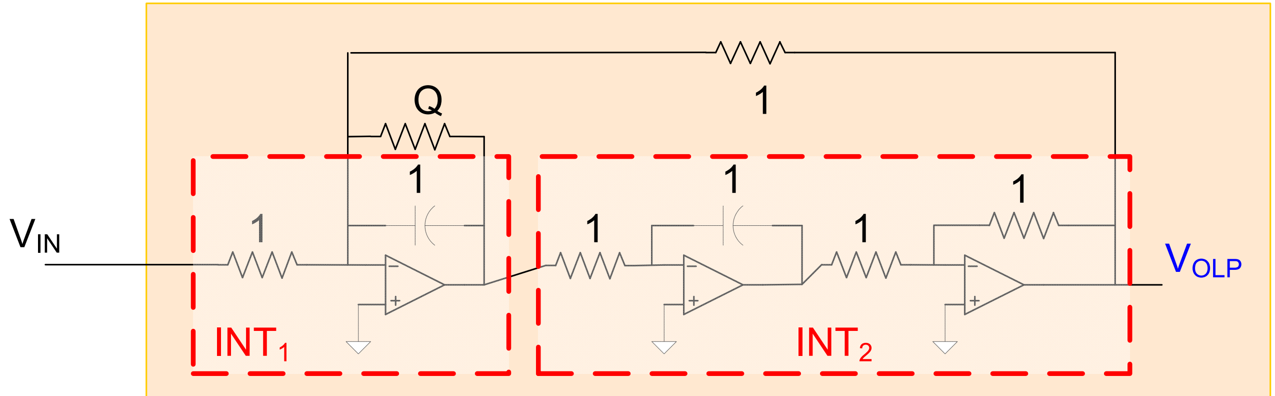
$$Q = \frac{R_Q}{R}$$

Normalizing by the factor ω_0, we obtain

$$T(s_n) = \frac{1}{s^2 + s\left(\frac{1}{Q}\right) + 1}$$

Setting R=C=1 obtain the following normalized circuit

Example: Design a 2nd order lowpass Butterworth filter with 3dB passband attenuation, a dc gain of 5, and a 3dB bandedge of 4KHz



A Popular Second-Order Lowpass Filter

$$T(s_n) = \frac{1}{s^2 + s\left(\frac{1}{Q}\right) + 1}$$

$\omega_{0n} = 1$

Must now set $Q = \frac{1}{\sqrt{2}}$

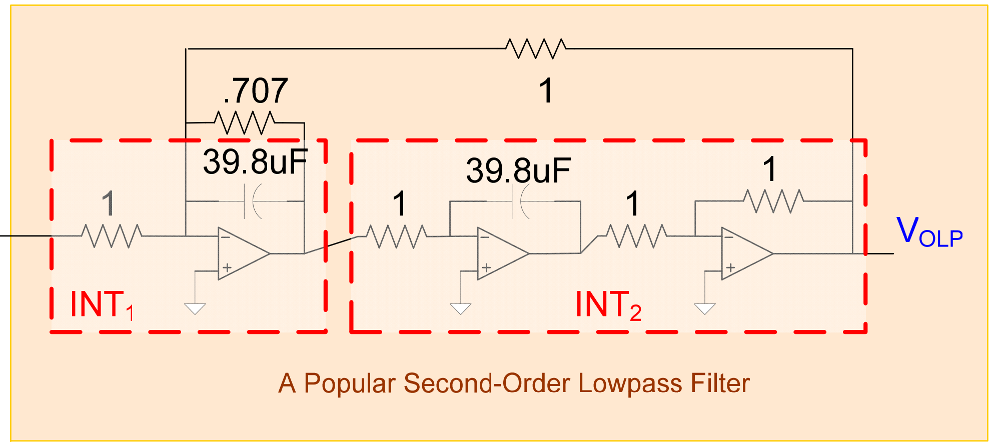
Now we can do frequency scaling $C \rightarrow C/\omega_o$

$L \rightarrow L\omega_o$

$C=1 \rightarrow 1/(2\pi \bullet 4K) = 39.8uF$

Example: Design a 2nd order lowpass Butterworth filter with 3dB passband attenuation, a dc gain of 5, and a 3dB bandedge of 4KHz

Denormalized circuit with bandedge of 4 KHz



This has the right transfer function (but unity gain)

Can now do impedance scaling to get more practical component values

$R \rightarrow \theta R$

$C \rightarrow C/\theta$

$L \rightarrow \theta L$

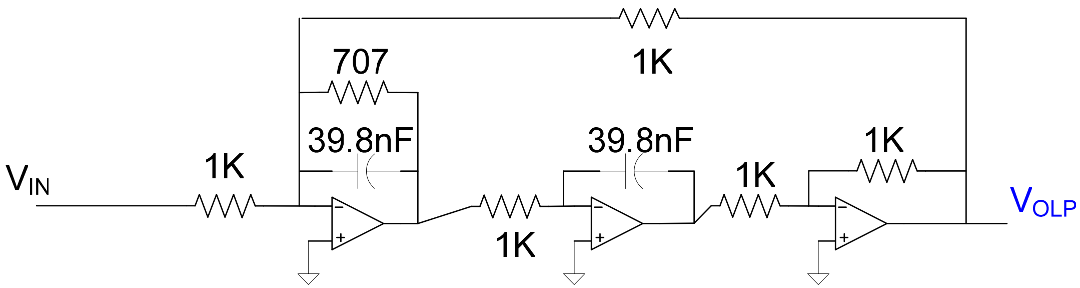
A good impedance scaling factor may be θ=1000

R → 1K

C → 39.8nF

Example: Design a 2^{nd} order lowpass Butterworth filter with 3dB passband attenuation, a dc gain of 5, and a 3dB bandedge of 4KHz

Denormalized circuit with bandedge of 4 KHz



This has the right transfer function (but unity gain)

To finish the design, preceed or follow this circuit with an amplifier with a gain of 5 to meet the dc gain requirements