EE 508
Lecture 6
Scaling, Normalization and Transformation
The “dead network” of any linear circuit is obtained by setting ALL independent sources to zero.

- Replace independent current sources with opens
- Replace independent voltage sources with shorts
- Dependent sources remain intact

\[ T(s) = \frac{N(s)}{D(s)} \]

\[ D(s) \] is characteristic of the dead network and is independent of where the excitation is applied or where the response is measured.

\[ D(s) \] is the same for ALL transfer functions of a given “dead network”

Review from Last Time
Root characterization in s-plane
(for complex-conjugate roots)

$s^2 + s \left( \frac{\omega_0}{Q} \right) + \omega_0^2$

for $\theta = 90^\circ, \; Q = 1/\sqrt{2}$

roots located at

$s = \frac{-\omega_0}{2Q} \pm \frac{1}{2} \sqrt{\left( \frac{\omega_0}{Q} \right)^2 - 4\omega_0^2} = \omega_0 \left( \frac{-1}{2Q} \pm \frac{1}{2} \sqrt{\left( \frac{1}{Q} \right)^2 - 4} \right)$

$\theta = \tan^{-1} \left( 4Q^2 - 1 \right)$
Scaling, Normalization and Transformations

- Frequency scaling
- Frequency Normalization
  - Impedance scaling
  - Transformations
    - LP to BP
    - LP to HP
    - LP to BR
Scaling, Normalization and Transformations

Frequency normalization:

\[ s_n = \frac{s}{\omega_0} \]

Frequency scaling:

\[ s = \omega_0 s_n \]

Purpose:

- \( \omega_0 \) independent approximations
- \( \omega_0 \) independent synthesis

Simplifies analytical expressions for \( T(s) \)

Simplifies component values in synthesis

Use single table of normalized filter circuits for given normalized approximating function

Note: The normalization subscript “n” is often dropped
Frequency normalization/scaling example

\[ T(s) = \frac{6000}{s + 6000} \]

Define \( \omega_0 = 6000 \)

\[ s_n = \frac{s}{\omega_0} \]

\[ T(s) = \frac{\omega_0}{s + \omega_0} \]

Normalized transfer function:

\[ T_n(s_n) = \frac{1}{s_n + 1} \]
Frequency normalization/scaling example

\[ T_n(s_n) = \frac{1}{s_n + 1} \]

Synthesis of normalized function

\[ T(s) = \frac{1}{s + 1} \]
Frequency normalization/scaling example

\[ T_n(s_n) = \frac{1}{s_n + 1} \]

Frequency scaling by \( \omega_0 \) (of transfer function)

\[ s = \omega_0 s_n \]

\[ T(s) = \frac{1}{\left(\frac{s}{\omega_0}\right) + 1} \]

\[ T(s) = \frac{\omega_0}{s + \omega_0} \]

Frequency scaling by \( \omega_0 \) (actually magnitude of \( \omega_0 \)) (scale all energy storage elements in circuit)

\[ C = \frac{C_n}{\omega_0} \]

\[ T(s) = \frac{\omega_0}{s + \omega_0} \]
Frequency normalization/scaling example

\[ T_n(s_n) = \frac{1}{s_n + 1} \]

\[ T(s) = \frac{\omega_0}{s + \omega_0} \]

Frequency scaling / normalization does not change the shape of the transfer function, it only scales the frequency axis linearly.

The frequency scaled circuit can be obtained from the normalized circuit simply by scaling the frequency dependent impedances (up or down) by the scaling factor.

This makes the use of filter design tables for the design of lowpass filters practical whereby the circuits in the table all have a normalized band edge of 1 rad/sec.
Frequency normalization/scaling

Example: Table for passive LC ladder Butterworth filter with 3dB band edge of 1 rad/sec and resistive source/load terminations

\[ T_n(s_n) = \frac{1}{s_n + 1} \]

\[ |T_n(j\omega)| \]

\[ \omega_n \]

\[ \frac{1}{\frac{1}{\omega_s}} \]

\[ \omega_s \]

\[ L_1 \]

\[ C_1 \]

\[ L_2 \]

\[ C_2 \]

\[ L_3 \]

\[ C_3 \]

\[ L_4 \]

\[ C_4 \]
Frequency normalization/scaling

The frequency scaled circuit can be obtained from the normalized circuit simply by scaling the frequency dependent impedances (up or down) by the scaling factor $\omega_0$.

Component denormalization by factor of $\omega_0$:

<table>
<thead>
<tr>
<th>Normalized Component</th>
<th>Denormalized Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>$R$</td>
</tr>
<tr>
<td>$C$</td>
<td>$C/\omega_0$</td>
</tr>
<tr>
<td>$L$</td>
<td>$L/\omega_0$</td>
</tr>
<tr>
<td>Other Components</td>
<td>Unchanged</td>
</tr>
</tbody>
</table>

Component values of energy storage elements are scaled down by a factor of $\omega_0$. 
Theorem: A circuit with transfer function $T(s)$ can be obtained from a circuit with normalized transfer function $T_n(s_n)$ by denormalizing all frequency dependent components.

$C \rightarrow C/\omega_0$

$L \rightarrow L/\omega_0$
Example: Design a V-V passive 3rd-order Lowpass Butterworth filter with a 3-db band-edge of 1K rad/sec and equal source and load terminations.

(from the BW approximation which will be discussed later:)

\[ T(s) = \frac{1}{s^3 + 2s^2 + 2s + 1} \]
Example: Design a V-V passive 3\textsuperscript{rd}-order Lowpass Butterworth filter with a band-edge of 1K Rad/Sec and equal source and load terminations.

Is this solution practical?

Some component values are too big and some are too small!
Filter Concepts and Terminology

• Frequency scaling
• Frequency Normalization
• Impedance scaling
• Transformations
  – LP to BP
  – LP to HP
  – LP to BR
Impedance Scaling

Impedance scaling of a circuit is achieved by multiplying ALL impedances in the circuit by a constant $\frac{C}{\theta}$

- $R \rightarrow \theta R$ for transresistance gain
- $C \rightarrow C/\theta$ for dimensionless gain
- $L \rightarrow L\theta$ for transconductance gain
- $A \rightarrow \theta A$ for transresistance gain
- $A \rightarrow A/\theta$ for dimensionless gain
Impedance Scaling

Theorem: If all impedances in a circuit are scaled by a constant $\theta$, then

a) All dimensionless transfer functions are unchanged
b) All transresistance transfer functions are scaled by $\theta$
c) All transconductance transfer functions are scaled by $\theta^{-1}$
Impedance Scaling

Example:

\[ T(s) = \frac{1}{s+1} \]

T(s) is dimensionless

Impedances scaled by \( \theta = 10^5 \)

Note second circuit much more practical than the first
Example: Design a V-V passive 3rd-order Lowpass Butterworth filter with a band-edge of 1K Rad/Sec and equal source and load terminations.

Is this solution practical?

Some component values are too big and some are too small!

Impedance scale by $\theta = 1000$

- $R \rightarrow \theta R$
- $C \rightarrow C/\theta$
- $L \rightarrow \theta L$

Component values more practical
Typical approach to lowpass filter design

1. Obtain normalized approximating function

2. Synthesize circuit to realize normalized approximating function

3. Denormalize circuit obtained in step 2

4. Impedance scale to obtain acceptable component values
End of Lecture 6