VALIDATION OF THREE DIMENSIONAL EDDY-CURRENT PROBE-FLAW INTERACTION MODEL USING ANALYTICAL RESULTS

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Abstract

A volume element calculation has been used to predict the field at the slot in a conductor excited by an eddy current probe. The slot simulates the presence of a surface crack in a metal component. Interaction between the simulated flaw and the electromagnetic field is computed in order to model the fundamental physical process of eddy-current nondestructive evaluation. The results are used to suggest methods for improving testing techniques and to develop procedures for processing inspection data. As part of an on-going program to validate the computer model, we compare predictions of the volume element calculation with analytical results for a test case. In the test problem a constant, uni-directional eddy-current distribution is perturbed by a semicircular crack of negligible opening at the surface of a half-space conductor. By taking the low frequency limit, the field at the flaw and the change in probe impedance due to the presence of the flaw, are given explicitly by simple analytical expressions. Numerical results found using the volume element calculation are compared with values found from the analysis.

Introduction

The task of predicting the behaviour of probe–flaw interaction in eddy–current nondestructive evaluation(NDE) is motivated by a need to improve inspection techniques and develop methods for analyzing data. In order to get reliable predictions, the theoretical assumptions, the numerical algorithm and the computer code must undergo extensive and rigorous testing. As part of a program to validate a volume element scheme for eddy–current NDE, numerical results have been compared with an analytic solution for a semicircular surface breaking crack. Although the three-dimensional volume integral code is design to give results at arbitrary frequencies, the analytical solution is found by considering the low frequency limit where the induced current can be represented by a potential satisfying the Laplace equation.

In modeling single frequency inspections, one needs to consider time–harmonic solutions of Maxwell's equations. These solutions are characterised by a complex parameter k related to the material properties and to the excitation frequency. In conventional notation $k = \sqrt{i\omega\mu\sigma} = (1+i)/\delta$ where δ is the skin depth and k has the dimensions of reciprocal length. To get low frequency eddy-current solutions in the vicinity of a flaw whose characteristic dimension is a, we can assume the dimensionless parameter $|ka|$ is small. As in wave scattering theory where k is the wavenumber, it is possible to seek a solution in the form of a power series expansion in ka using perturbation theory. For present purposes we shall not develop a full perturbation expansion but consider only the lowest order non–vanishing term in the series for a circular crack comparing the solution with numerical predictions found using the volume element technique.

Volume Element Scheme

The overall structure of the volume element scheme used here is fairly conventional, based on an integral equation for the electric field[1] valid at any frequency in the range where displacement current is negligible. The integral kernel is chosen to ensure that the electric field satisfies the correct continuity conditions at the conductor–air interface. Assuming a half–space conductor with an interface at $z = 0$, the formalism uses a half–space dyadic Green's function[2] which guarantees that the x- and y–components of the electric and magnetic field are continuous at the xy–plane regardless of the size, shape and material properties of the flaw. A discrete approximation of the integral equation is found by postulating that the field may be represented as piece-wise constant. In this way a discrete field representation is defined on a three dimensional lattice of volume elements filling a region in the form of a rectangular parallelepiped enclosing the flaw. Formally, the volume elements are introduced by expanding the electric field in the flaw region using a set of three–dimensional pulse functions. Taking moments of the integral equation completes the discretisation. A numerical solution of the resulting matrix equation is then sought using a conjugate–gradient algorithm.

It is only at the region of the slot that a discrete representation of the field is needed and this is defined on a simple regular grid of rectangular parallelepipeds. Thus one can have any desired flaw shape that can be constructed from a array of blocks each with an assigned conductivity. The conductivity is assumed constant within a given block with a value that can range from zero, if it wholly inside a crack or cavity, to many times the host conductivity. The upper limit of the flaw conductivity really depends on how many volume elements can be tolerated. A high conductivity region will be associated with a small skin depth and taking account of field variations over short distances without introducing significant discretisation errors requires a large number of elements. Fortunately high conductivity flaws are of limited practical importance.

Integral Formulation

There is some flexibility as to the choice of the unknowns in setting up a linear system based on a volume element discretisation scheme. One could for example, derive a matrix equation whose solution gives the electric field in the flaw region, or rather a piece-wise constant approximation of the electric field. However we prefer to consider a flaw in an electromagnetic field in terms of an equivalent source distribution and find an equation for the source density. Treating the flaw as an equivalent source is not a new conception in eddy–current theory, quite the contrary. For example Burrows[2] pointed out, many years ago that a small defect such as a tiny spheroidal cavity gives rise to a perturbed field that is the same as that due to a current dipole. In generalising this idea one can consider a far–field distribution in terms of a multipole expansion rather as is done in radiation scattering theory but this would not be particularly useful in eddy–current simulation since the flaw signals would be very small in situations permitting far field approximations. An alternative extension of the equivalent source concept treats the source of the perturbed field as an induced volume or surface current dipole distribution, depending on whether the flaw acts as a surface barrier or a volumetric flaw. The equivalent source concept does not introduce approximations since it is consistent with an exact solution of Maxwell's equations. It is not essential to the theory but provides a physically appealing and intuitive way of understanding the equations. Ultimately the volume element model depends for its validity on the making correct assumptions and reasonable approximations, rather than on a particular way of interpreting the relationships.

Suppose we have a half-space conductor of uniform conductivity σ_0 containing a finite flaw whose conductivity $\sigma(\mathbf{r})$ may be a function of position. In effect the flaw behaves as an induced secondary source, $\mathbf{P}(\mathbf{r}) = [\sigma(\mathbf{r}) - \sigma_0]\mathbf{E}(\mathbf{r})$. $\mathbf{P}(\mathbf{r})$, the electric current dipole density, is zero except at a flaw where the conductivity differs from the host conductivity σ_0. With the origin of the scattered field defined in this way, a formal solution of Maxwell's equations for the electric field can then be written as

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^{(i)}(\mathbf{r}) + \int_{flaw} \mathbf{G}^{(ee)}(\mathbf{r}|\mathbf{r}') \cdot \mathbf{P}(\mathbf{r}')d\mathbf{r}', \qquad (1)$$

where $\mathbf{E}^{(i)}(\mathbf{r})$ is the incident field and the integral represents the scattered field. $\mathbf{G}^{(ee)}(\mathbf{r}|\mathbf{r}')$ is a half-space dyadic Green's function[3]. An equation for $\mathbf{P}(\mathbf{r})$ is found by multiplying by $\sigma(\mathbf{r}) - \sigma_0$ to give,

$$\mathbf{P}(\mathbf{r}) = \mathbf{P}^{(i)}(\mathbf{r}) + \sigma_0 v(\mathbf{r}) \int_{flaw} \mathbf{G}^{(ee)}(\mathbf{r}|\mathbf{r}') \cdot \mathbf{P}(\mathbf{r}')d\mathbf{r}', \qquad (2)$$

where $v(\mathbf{r}) = [\sigma(\mathbf{r}) - \sigma_0]/\sigma_0$ is the flaw function and $\mathbf{P}^{(i)}(\mathbf{r}) = [\sigma(\mathbf{r}) - \sigma_0]\mathbf{E}^{(i)}(\mathbf{r})$.

Matrix Approximation

(2) is approximated by a matrix equation using the method of moments[4] and a solution found by applying a conjugate-gradient algorithm[5]. A similar procedure has been used previously to determine the induced magnetisation of the ferrite-cores of eddy-current probes[6]. The discrete approximation of the integral equation (2) is derived by subdividing the region occupied by the flaw into a regular lattice of $N_x \times N_y \times N_z$ cells each with a volume $\delta_x \times \delta_y \times \delta_z$. Expanding the solution in pulse functions defined on this lattice we have

$$\mathbf{P}(\mathbf{r}) \simeq \sum_{K=0}^{N_x-1} \sum_{L=0}^{N_y-1} \sum_{M=0}^{N_z-1} \mathbf{P}_{KLM} \mathcal{P}_{KLM}\left(\frac{x}{\delta_x}, \frac{y}{\delta_y}, \frac{z-z_0}{\delta_z}\right). \qquad (3)$$

Here $\mathcal{P}_{KLM}(u, v, w) = 1.0$, $(K, L, M = 0, 1, 2, 3...)$ wherever $K \le u < K+1$ and $L \le v < L+1$ and $M \le w < M+1$, otherwise it is zero. z_0 is a reference coordinate for the source lattice, usually the coordinate of the lowest point on the flaw. For example, in the case of a surface breaking flaw, a suitable choice would be $z_0 = -N_z\delta_z$. Then the topmost layer of cells would extend to the surface of the conductor in the plane $z = 0$. Similarly expanding the flaw function $v(\mathbf{r})$, we have

$$v(\mathbf{r}) \simeq \sum_{K=0}^{N_x-1} \sum_{L=0}^{N_y-1} \sum_{M=0}^{N_z-1} v_{KLM} \mathcal{P}_{KLM}\left(\frac{x}{\delta_x}, \frac{y}{\delta_y}, \frac{z-z_0}{\delta_z}\right), \qquad (4)$$

the values of v_{KLM} being assigned in such a way as to give the best approximation to the flaw.

To complete the conversion to a discrete form, the integral equation is multiplied by testing functions and integrated over the field coordinates. Here we choose the same pulse functions for testing as are used for expanding the solution thus following the Galerkin variant of the method of moments. Substituting (3) and (4) into (2), multiplying by $\mathcal{P}_{klm}\left(\frac{x}{\delta_x}, \frac{y}{\delta_y}, \frac{z-z_0}{\delta_z}\right) \Big/ \delta_x\delta_y\delta_z$ and integrating with respect to x, y and z gives

$$\mathbf{P}_{klm}^{(i)} = \mathbf{P}_{klm} - v_{klm} \sum_{K=0}^{N_x-1} \sum_{L=0}^{N_y-1} \sum_{M=0}^{N_z-1} \mathbf{G}_{klm,KLM} \cdot \mathbf{P}_{KLM}, \qquad (5)$$

where $\mathbf{P}_{klm}^{(i)}$ is predefined in terms of the unperturbed electric field in the flaw region due to the probe. In general

$$\begin{aligned} \mathbf{P}_{klm}^{(i)} &= \frac{1}{\delta_x\delta_y\delta_z} \int_{x_k}^{x_{k+1}} \int_{x_l}^{x_{l+1}} \int_{z_m}^{z_{m+1}} \mathbf{P}^{(i)}\mathbf{r})dx\,dy\,dz, \\ &= \frac{\sigma_0 v_{klm}}{\delta_x\delta_y\delta_z} \int_{x_k}^{x_{k+1}} \int_{x_l}^{x_{l+1}} \int_{z_m}^{z_{m+1}} \mathbf{E}^{(i)}(\mathbf{r})dx\,dy\,dz. \end{aligned} \qquad (6)$$

Here $x_k = k\delta_x$, $y_l = l\delta_y$ and $z_m = m\delta_z + z_0$ etc. $(k, l, m = 0, 1, 2, 3....)$. The matrix in (5) is given by

$$\begin{aligned} \mathbf{G}_{klm,KLM} &= \frac{1}{\delta_x\delta_y\delta_z} \int_{x_{K-1}}^{x_{K+1}} \int_{y_{L-1}}^{y_{L+1}} \int_{z_{M-1}}^{z_{M+1}} \mathbf{G}(x_k, y_l, z_m|x', y', z') \\ &\quad \beta_K\left(\frac{x}{\delta_x}\right)\beta_L\left(\frac{y}{\delta_y}\right)\beta_M\left(\frac{z-z_0}{\delta_z}\right) dx'dy'dz' \end{aligned} \qquad (7)$$

where $\beta_j(u)$, $(j = 0, 1, 2, 3...)$ is a convolution of pulse functions given by

$$\beta_j(u) = \begin{cases} u - j + 1 & \text{if } j - 1 \le u < j \\ j - u + 1 & \text{if } j \le u < j + 1 \\ 0 & \text{otherwise} \end{cases}, \qquad (8)$$

The matrix elements are found from the integral (8) using a numerical quadrature scheme with evaluation of the singular element carried out following a method devised by Lee *et al* [7]. Although the discrete representation of (2) requires a dense matrix of say $N \times N$ elements, the symmetry properties of the Green's function plus the choice of a regular cell array leads to redundancy in the matrix which means that we do not have to store N^2 elements on the computer. For example the free-space Green's function is dependent on $x - x'$, $y - y'$ and $z - z'$, consequently its matrix representation has a Toeplitz structure; elements on any leading diagonal are equal. In (2), the Green's function can be written as a sum of two terms, one of which is the free space Green's function with its $x - x'$, $y - y'$, $z - z'$-dependence and the other term, due to reflection from the surface of the conductor, is $x - x'$, $y - y'$, $z + z'$-dependent. (2) therefore contains a combination of a convolutional and a correlational integral. As a result the corresponding matrix has a structure permitting matrix-vector products to be carried out using fast Fourier transforms[6]. The efficient execution of these products enhances the performance of the conjugate-gradient calculation.

Validation Test

The aim of this study is to compare predictions for the effective source distribution at a flaw determined from a volume element model, with the corresponding analytical result for an ideal crack in the low frequency limit. Here we define an ideal crack as having a negligible opening, acting as a surface barrier inpenetrable to eddy-current. For the ideal crack the equivalent source is a *surface* distribution of current dipoles[8]. In order to compare volume and surface source distributions, we shall examine the behavior of the volume dipole density $\mathbf{P}(\mathbf{r})$ as a three dimensional flaw collapses to a surface.

Consider a penny-shaped crack of radius a with its axis in the x-direction having a finite opening Δ_c that is small compared with a. Assuming that the unperturbed field at the crack is fairly uniform, the electric field on the inside at a section through the diameter has the form indicated schematically in Figure 1. Note that $\mathbf{E}(\mathbf{r})$ is largely directed normal to the crack faces and that the tangential components are relatively small. Current at the outside surface of the crack is dependent on the tangential components of the electric field, the normal component of the electric field at the external surface being zero. Because the tangential components are continuous across the crack-conductor interface the y- and z-components of the electric field at the inner surface of the flaw are proportional to the external current density. Thus the behavior of these electric field components on the inside surface can be understood in terms of the external current distribution.

The x-directed electric field in the flaw depends mainly on the potential drop across the flaw faces. (An alternating electric field cannot generally be expressed solely as the gradiant of a scalar function but the dominate contribution in the flaw can be represented in this way.) Because the eddy- current distribution does not change much if an initially small crack opening Δ_c is further reduced, the potential difference across the crack is insensitive to variations in Δ_c. In the limit of small crack opening the line integral of E_x and therefore P_x

across the crack in the x- direction from face to face tends to a value $p(y,z)$ that is independent of the path length Δ_c. In this limit the penny-shaped flaw collapses to a disc and its effective source becomes a surface current dipole distribution whose dipole density is $p(y,z)$. Formally we have

$$\lim_{\Delta_c \to 0} \int_{-\Delta_c/2}^{\Delta_c/2} P_x(x,y,z)dx = p(y,z). \quad (9)$$

(9), or an approximation of it, allows us to compare $p(y,z)$ found from analysis with the source distribution given by the volume element model.

The appropriate solution of Laplace equation for a disc in an unbounded domain can be found in standard texts on boundary value problems[9] or hydrodynamics[10]. Assuming that the unperturbed field normal to the disc is a constant E_0 we get

$$p(y,z) = -\frac{4\sigma_0 E_0}{\pi}\sqrt{a^2-(y^2+z^2)} \quad (10)$$

a result that is found assuming that the normal component of the current density at the disc surface is zero. This dipole distribution also applies to a semicircular disc at the surface of a half space conductor and normal to the interface. Supposing that the interface is in the plane $z=0$, the dipole distribution gives rise to an electric field that satisfies the condition $E_z = 0$ at $z=0$. Hence it is an appropriate solution of the half-space Laplace problem for a half-disc.

Comparison of Results

To make a meaningful comparison of the volume integral prediction with a "low frequency" boundary integral solution, it is necessary to ensure that the two inequalities hold. Firstly we need to choose flaw parameters such that $\Delta_c \ll a$ and secondly the low frequency assumption means that $a \ll \delta$. These conditions have been approximated using a simulated crack of radius $a = 10mm.$, with an opening $\Delta_c = 1mm.$ in a uniform field whose skin depth is $\delta = 1000mm$. For the computation, a flaw grid consisting of $16 \times 4 \times 8$ volume elements was used, although the flaw itself was only 14 cells long 3 cells wide and 7 cells deep. Figure 1 shows the value of $p(y,z)$ at the flaw compared with the theoretical result normalised to a maximum value of 1.0 by choosing the unperturbed field as $E_0 = \pi/(4a\sigma_0)$. The volume integral result is found by approximating the line integral of $P_x(x,y,z)$ across the crack opening, equation (9), by the sum of contributions from the three cells. Clearly the agreement between the two results is very good.

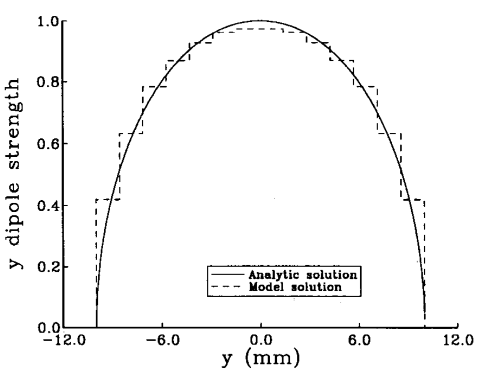

Figure 1: Comparison of volume element and analytical result, for the effective current dipole distribution along the diameter of a semicircular surface crack in a uniform field.

Figure 2 shows results of the volume element calculation of $\Delta_c \mathbf{P}(x,y,z)$ at one of the crack faces as a surface plot. A characteristic of the distribution is that P_x tends to a small value at the edge of the crack with a small opening. This is consistent with the fact that the surface dipole distribution vanishes at the crack edge[8]. As discussed earlier the tangential components of $\mathbf{P}(x,y,z)$ are dependent on the current density at the crack faces. For a narrow crack P_y and P_z are smaller than the axial component P_x, showing an indication of a weak singularity at the corners. These tangential components are indicative of the external current density, showing that the flow pattern is directed radially towards or away from the edge of the crack and that a maximum in the current density arises at the edge.

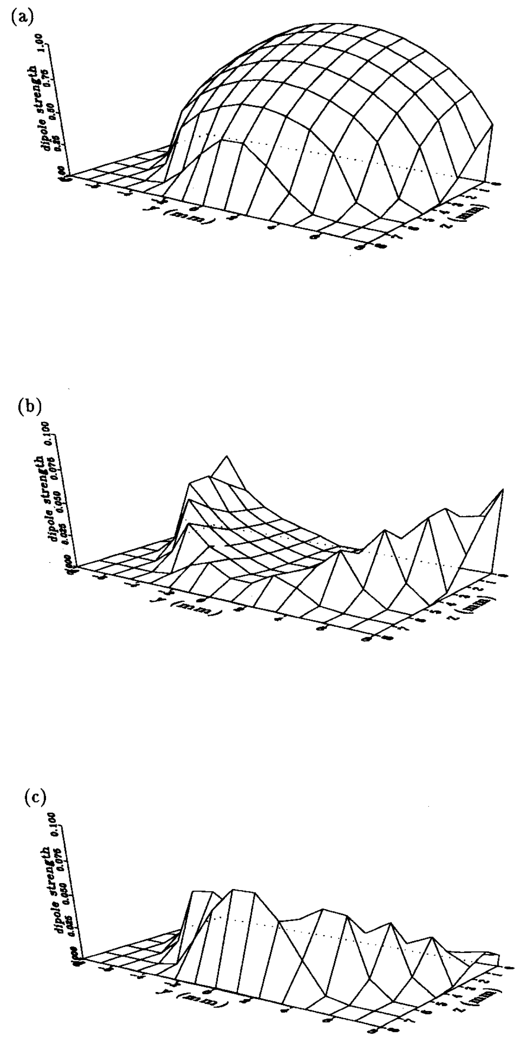

Figure 3: Components of the current dipole density at the face of a semicircular crack computed using volume elements. (a) x-component (b) y-component (c) z-component

Conclusion

A volume element model, designed for applications in eddy–current nondestructive evaluation, has been tested by comparing predictions with an analytical solution for a semicircular crack. The analytical result is adapted from a well known hydrodynamic problem where incompressible fluid flows around a circular disc. Just as the effects of the disc on the fluid may be viewed in terms of a layer of fluid dipoles, a circular crack has the effect of a layer of electric current dipoles. The prediction of the volume element code for the dipole density in the low frequency limit shows good agreement with the analytical result.

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