Eddy-current interaction with an ideal crack. II. The inverse problem

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Eddy-current inversion is the process whereby the geometry of a flaw in a metal is derived from electromagnetic probe measurements. An inversion scheme is described for finding the shape and size of cracks from eddy-current probe impedance measurements. The approach is based on an optimization scheme that seeks to minimize a global error function quantifying the difference between predicted and observed probe impedances. The error minimum is sought using a standard descent algorithm that requires a knowledge of the gradient of the error with respect to a variation of the flaw geometry. Computation of the gradient is based on a provisional flaw estimate, then the flaw estimate is updated in a “direction” that reduces the error. The process continues iteratively until a convergence criterion has been satisfied. Then the final flaw estimate should match the shape of the real defect. An equation for the gradient has been derived using an integral formulation of the ideal crack problem. Numerical estimates of the error gradient and the probe impedances have been calculated using approximations based on the moment method. Tests of the inversion scheme using single frequency probe impedance measurements have been carried out by calculating the shapes of narrow slots in aluminum alloy plates. Good agreement is found between the optimum profiles and the measured slot shapes.

I. INVERSION

In eddy-current nondestructive evaluation, a defect such as a crack in a metal is detected through changes of the probe signal caused by perturbations of induced current (Fig. 1). The task of predicting the signals from a knowledge of the probe and flaw is a direct or forward problem. In the corresponding inverse problem, the aim is to determine the shape and size of the flaw from probe signals measured at different positions and possibly at different frequencies. Time-domain measurements could also be used but the vast majority of eddy-current inspections are performed at a single frequency. Therefore, the primary interest is in the inversion of oscillatory signals.

An optimization approach to inversion seeks the flaw which minimizes the difference between tentative predictions of the probe signals and the measurements. If the agreement is unsatisfactory, then the flaw is updated and a new prediction made. The process continues through a number of iterations until predictions and observations match to within a reasonable tolerance. When a tolerable agreement has been found, the final flaw should be close to an optimum estimate of the real defect.

An improved estimate of the flaw is obtained if a global error quantifying the difference between predictions and the measurements is reduced. Generally it is not possible to reduce the error to zero owing to the effects of inaccuracies in the measurements or deficiencies in the predictions. However, a minimization search can be carried out using a descent algorithm that terminates once the error is below a predefined threshold. Because the inversion is inherently nonlinear, it is a possible that a false minimum is found in which case the final flaw estimate may be inaccurate. There are standard methods of dealing with local minima; for example, simulated annealing can be used to distinguish the global minimum, but they can be computationally expensive and are best avoided. Although the possibility of false minima cannot be discounted, they have not been encountered in the present investigation of ideal crack inversion.

In general, a flaw is represented by a flaw function defined in terms of a variation of the electrical properties of the material. There are a number of possible representations, two of which were explored in an earlier article. Any of these possibilities, the flaw function is defined as the electrical conductivity expressed as a function of position. A discrete version of this representation would be a piecewise constant approximation of the conductivity on a three-dimensional rectangular grid, which means that the inversion would seek the conductivity of each volume element in three dimensions. An unconstrained search for the conductivity of each cell is likely to be computationally expensive, therefore it would probably be necessary to limit the process in some way. The second possibility, leading to a more restricted optimization problem, arises for cases where it is known a priori that the defect is homogeneous; for example, if it is a cavity or a uniform inclusion. The inversion may then be specified as a search for the bounding surface. Ideal crack inversion presents us with a third possible flaw function: the equation of the line following the crack edge. Here the aim is to devise a means of finding the shape of a crack in a known plane by calculating the position of the edge from a set of probe measurements.

With the above conception of the task, the search for the optimum crack shape takes place in a function space spanned...
FIG. 1. A normal eddy-current coil over a surface breaking crack in a half-space conductor.

by all possible flaw functions in a particular class. In order to
find the minimum error via a descent algorithm, it is neces-
sary to determine the error gradient in this space with respect
to a variation of the flaw. A knowledge of the gradient en-
ables one to update the flaw in a "direction" that will guar-
antee an error reduction at each iteration. In crack inversion
based on probe impedance measurements, the error gradient
is dependent on the gradient of the impedance with respect to
a variation in the location of the crack edge. Clearly the
central requirement of the scheme is the determination of the
impedance gradient.

In part I, the interaction of eddy currents with an ideal
crack was calculated using a boundary integral method.2 In
the problem formulation, an integral equation for the field at
the crack is derived, the equation is approximated using the
moment method and a solution of the resulting linear system
found by LU decomposition. This concluding part concerns
the corresponding inverse problem. Previous work on crack
inversion has been based on the thin skin limit where the
skin depth is small compared with the depth of the crack.3T4
In the present scheme, the skin depth is arbitrary. By devel-
oping the boundary integral theory further, an equation for
the impedance gradient has been derived. Using this gradient
and an extension of the numerical scheme for approximating
the forward problem, inversions have been performed to find
the shapes of simulated cracks from probe impedance mea-
surements. Our account of ideal crack inversion begins with
a brief review of the forward problem. Further details are
given in part I.

II. FORWARD PROBLEM

An ideal crack is a perfect surface barrier to electrical
current but has zero thickness. The current density on oppo-
site sides of the surface will usually be different, therefore an
ideal crack supports a discontinuity in the tangential electric
field. A solution of the appropriate boundary-value problem
may be found by representing the discontinuity by an equiva-
 lent source distribution that gives rise to the same field. A
suitable source equivalent of an ideal crack in an electromag-
netic field is a current dipole layer where the dipoles are
orientated normal to the surface.2

The electric field perturbed by a crack at an open surface
$S_0$ may be expressed as a surface integral using Green’s sec-
ond theorem. By applying a vector version of the theorem, it
is found that the electric field in conductor containing an
ideal crack is given by

$$E(x) = E^{(0)}(x) + t \omega \mu_0 \int_{S_0} \mathcal{S}(r|r') \cdot p(r') dS',$$  

where $E^{(0)}(x)$ is the unperturbed electric field, $p(r') = \hat{n} \rho(r')$
is the dipole distribution on the crack surface $S_0$, and $\hat{n}$ is a
unit vector normal to the surface. The half-space dyadic
Green’s function, $\mathcal{S}(r|r')$, ensures that the solution satisfies
the correct continuity and boundary conditions for a half-
space conductor. Setting the normal component of the elec-
tric field to zero in the limit as the field point approaches
the surface of the crack gives

$$E_n^{(0)}(r_\pm) = -i \alpha \omega \mu_0 \lim_{r - r_\pm} \mathcal{G}(r_\pm|r') p(r') dS',$$  

where $r_\pm$ denotes the limit as the crack surface $S_0$ is ap-
proached from one side or the other. This equation for the
dipole density will be written as

$$E_n^{(0)}(r_\pm) = -i \omega \mu_0 \int_{S_0} G^{\alpha}(r_\pm|r') p(r') dS',$$  

where $G^{\alpha}(r|r') = \hat{n} \cdot \mathcal{S}(r|r') \cdot \hat{n}$. Because of the nature of the singular-
ity of the Green’s function, the integral is to be in-
terpreted using the Hadamard theory of the finite part.5

Equation (3) has been approximated using the moment
method and the resulting linear system solved to give a
piecewise constant estimate of the dipole density.2 The probe
impedance change due to an ideal crack is given by

$$Z = -\int_{S_0} E_n^{(0)}(r) p(r) dS$$  

for unit probe current. A discrete approximation of Eq. (4) is
used to predict the measurements.

III. NONLINEAR OPTIMIZATION

In ideal crack inversion the task is to find the boundary
of the defect from a knowledge of the scattered field as ac-
quired, for example, by measuring probe impedance as a
function of position and frequency. It is assumed that the
crack lies in a known plane but the shape and size are un-
known and must be inferred from measurements. The crack
geometry is represented by the equation of the line of the
crack edge written as $\xi(s,t) = 0$, where $s$ and $t$ are coor-
dinates of a point in the crack plane. The probe impedance due
to the flaw is a continuous function of position and the exci-
tation frequency $\omega$. These coordinates are combined in the
vector $m = (x,y,z,\omega)$. The impedance is written as $Z[\xi(m)]$, where
the square bracket containing $\xi$ denotes a functional
dependence on the flaw function.

A global "error" $\mathcal{E} [\xi]$ is defined by

$$\mathcal{E} [\xi] = \sum_i W(m_i) |Z[\xi(m_i)] - Z_{obs}(m_i)|^2,$$  

where $W(m_i)$ is a weight function.
where the summation is over all the observations and $Z_{\text{obs}}(m_i)$ is the observed impedance at $m_i$. The weighting function $W(m_i)$ is used to give more or less weight to the data as necessary. The aim is to find the flaw function $\xi$ that minimizes the global error.

Whether it is a steepest descent, a conjugate-gradient, or Levenberg–Marquardt scheme that is chosen, it is necessary to calculate the gradient of the error with respect to a variation of the flaw function. An incremental variation of the flaw function means that the location of the crack edge is changed by adding or subtracting a small strip of territory to the perimeter of the crack domain. Suppose a variation in the flaw function $d\delta(t)$ results in the addition to the crack domain of a strip $d\delta(t)$ wide, where $t$ is a coordinate measured along the crack edge and $s$ is measured in the orthogonal direction in the crack plane. Then

$$d\delta(t) = -\frac{d\xi}{ds} \delta s(t).$$

Without loss of generality, the flaw function will be scaled in order that $\delta \xi/\delta s = -1$ enabling us to equate $\delta\xi(t)$ with the width of the strip.

Adding the strip to the perimeter of the crack domain gives rise to a change in the global error. Each part of the strip contributes to the change and by integrating the contributions, the total error increment is expressed as

$$d\zeta[\xi,\delta\xi] = \int_{\text{edge}} \nabla_{\xi} \zeta(t) \delta\xi(t) dt.$$ (6)

This relationship introduces $\nabla_{\xi} \zeta(t)$, the functional gradient of the error with respect to a variation of the flaw function. It represents the change in the global error due to a variation of the crack boundary at $t$. In a similar way the impedance gradient may be defined such that a change of impedance due to an incremental but arbitrary flaw variation is given by

$$dZ[\xi,\delta\xi,m] = \int_{\text{edge}} \nabla_{\xi} \zeta(m,t) \delta\xi(t) dt.$$ (7)

The impedance gradient $\nabla_{\xi} \zeta(m,t)$ may be viewed as sensitivity function since it represents the effect on the impedance at $m$ due to a change in the location of the edge of the crack at a point whose coordinate is $t$.

Substituting Eq. (5) into the definition of the functional derivative,

$$d\zeta[\xi,\delta\xi] = \lim_{\beta \to 0} \frac{\zeta[\xi + \beta \delta\xi] - \zeta[\xi]}{\beta} = \frac{d}{d\beta} \zeta[\xi + \beta \delta\xi] |_{\beta = 0},$$ (8)

and using Eq. (7) gives

$$d\zeta[\xi,\delta\xi] = \int_{\text{edge}} \left( 2 \Re \sum_i [Z[\xi,m_i] - Z_{\text{obs}}(m_i)]^* \nabla_{\xi} \zeta(m_i,t) \right) \delta\xi(t) dt.$$ (9)

Hence, comparing Eq. (6), one identifies

$$\nabla_{\xi} \zeta(t) = 2 \Re \sum_i [Z[\xi,m_i] - Z_{\text{obs}}(m_i)]^* \nabla_{\xi} \zeta(m_i,t).$$ (10)

Equation (10) enables one to calculate the error gradient from a knowledge of the impedance gradient at all the observation points.

In order to find the optimum flaw function, the error gradient is evaluated and the boundary of the flaw updated using a formula that depends on the descent algorithm. Let the modified flaw function be $\xi + \Delta\xi$, then the steepest descent update is given by

$$\Delta \xi(t) = -\alpha \nabla_{\xi} \zeta(t),$$ (11)

where $\alpha$ governs the step size and is chosen to minimize the error functional $\zeta[\xi]$ in the direction of the gradient. Iteration continues until the condition $\zeta[\xi] < \epsilon$ has been satisfied, where $\epsilon$ is a real positive constant representing a tolerable residual error. Alternatively the process is terminated when the error no longer decreases significantly.

Evaluation of the error gradient from Eq. (10) requires first the impedance predictions $Z[\xi,m_i]$, for all $i$ and second the impedance gradient $\nabla_{\xi} \zeta(m_i,t)$. The predictions are found from the solution of multiple forward problems. Assuming a suitable solver is available, the optimization problem reduces to one of finding the impedance gradient. A possible way forward is to approximate the impedance gradient as a discrete form at this point, replacing it with a sensitivity matrix. The matrix elements could be evaluated using a finite difference scheme but the use of finite differences for this purpose has potentially a very high computational cost that is best avoided. A more elegant approach is to postpone the discretization and extend the field theory with the aim of expressing the functional gradients in terms of the electromagnetic field at the flaw.

In an earlier article it was shown that the impedance gradient can be found in general from the same solutions that are used in predicting the impedances, plus an equal number of solutions from the corresponding adjoint problems. These adjoint problems are defined in terms of an operator acting on the dipole density, as in Eq. (2), where an integral form for the operator is employed. If the operator is self-adjoint, the expressions for the impedance gradient simplify a little, but more significantly the computational effort needed to calculate it is halved because the adjoint problems are identical to the regular problems. Below we treat a special case of the ideal crack in which the surface of the crack is perpendicular to the surface of a half-space conductor. For this and similar calculations, each standard forward problem and its adjoint are the same. This means that an inversion from $N$ measurements requires the solution of only $N$ rather than $2N$ forward problems to give both the predictions and the impedance gradient at each iteration. If the crack were inclined to the air-conductor interface, the operator would not be self-adjoint and it would be necessary to solve two forward problems for each measurement.
IV. IMPEDANCE VARIATION

Consider an ideal crack in the plane \( x=0 \) perpendicular to the surface of a half-space conductor occupying the region \( z<0 \). A variation of the flaw will be defined in terms of a flaw characteristic function \( \gamma(r) \) where

\[
\gamma(r) = \begin{cases} 
1, & r \in S_0, \\
0 & \text{otherwise}.
\end{cases}
\]

The impedance variation resulting from an arbitrary incremental change in the flaw characteristic is

\[
dZ = - \int_{S_E} E_x^{(0)}(r) dp(r) dS. \tag{13}
\]

Here and throughout this section the functional dependence on \( \xi \) is implied but not stated and \( r \in S_E \). We have chosen to integrate over an extended domain in the plane of the crack, denoted by \( S_E \), rather than the crack region \( S_0 \). This is simply a device for making the region of integration flaw independent. The extended domain is larger than that of the flaw, either before or after variation, but it is otherwise arbitrary. By defining \( p(r) \) and \( dp(r) \) as nonzero over the flaw region and zero otherwise, the extension of the region of integration makes no difference to the integral. In order to identify the impedance gradient introduced in Eq. (7), Eq. (13) is transformed into a line integral whose path is the edge of the crack.

An equation for \( dp(r) \) is found by considering the equivalent of Eq. (2) for the varied flaw, namely

\[
E_x^{(0)}[\gamma(r) + \delta\gamma(r)]
= -i\omega\mu_0 [\gamma(r) + \delta\gamma(r)]
\times \int_{S_E} G^{zz}(r|r')[p(r') + dp(r')] dS',
\]

where \( \delta\gamma(r) \) is unity in the region of the flaw variation, the strip region, and zero elsewhere. The presence of the flaw characteristic in this equation means that it is valid for any field point in the plane of the flaw. Take the normal component of Eq. (1), multiply it by \( \gamma(r) + \delta\gamma(r) \), and integrate over the width of the strip. In order to do this integration explicitly, the variation of the dipole density at the edge of the crack and the field at the crack tip is needed.

\[
E_z(r) \delta\gamma(r) = -i\omega\mu_0 \int_{S_E} G^{zz}(r|r') dp(r') dS',
\]

ensures the normal electric field at the strip surface of the varied crack is zero. The second relationship follows by restricting Eq. (15) to the original crack domain, giving

\[
-i\omega\mu_0 \gamma(r) \int_{S_E} G^{zz}(r|r') dp(r') dS' = 0,
\]

which means that the normal electric field at the surface of the unvaried crack is unchanged, in fact remains zero, after the crack is varied. In other words the normal field due to \( dp(r) \) is zero over the surface of the original crack.

Equation (13) is transformed by substituting for \( E_x^{(0)}(r) \) from Eq. (14). This gives

\[
dZ = - \int_{S_E} \left[ E_x^{(0)}(r) dp(r) dS = i\omega\mu_0 \left[ \int_{S_E} \gamma(r) \delta\gamma(r) \times \right. \int_{S_E} G^{zz}(r|r') [p(r') + dp(r')] dS' dp(r') \right. \}
\]

Reversing the order of integration and using Eq. (16) gives

\[
dZ = - \int_{S_E} \left[ [p(r') + dp(r')] E_x(r') \delta\gamma(r') dS' \right.
- \int_{S_E} \left. dp(r') E_x(r') \delta\gamma(r') dS', \right.
\]

where we have used the fact that \( G^{zz}(r|r') = G^{zz}(r'|r) \).

The symmetry property of the \( xx \) component of the dyadic Green's function means that the integral operator is self-adjoint in the sense considered here.1 It is this property that makes the regular and adjoint forward problems identical. Indeed, the same is true for any crack whose surface is normal to the conductor-air interface. On the other hand, the excitation of \( z \) components of the dipole distribution on an inclined crack will bring into play components of the half-space dyad that do not exhibit the self-adjoint property.2 Therefore, a calculation of the impedance gradient in such cases requires the solution of distinct adjoint problems.

Equation (19) can be cast in the form of Eq. (7) by integrating over the width of the strip. In order to do this integration explicitly, the variation of the dipole density at the edge of the crack and the field at the crack tip is needed.

V. LOCAL EDGE SOLUTION

In a region whose dimensions are small compared with a skin depth, the electric field can be approximated as the gradient of a scalar potential satisfying the Laplace equation. In such a region, close to a crack with a smooth edge, the curvature of the edge can be neglected and the field described adequately by a scalar potential of the form

\[
V(\rho, \phi) = -\mathcal{F} \rho^{1/2} \cos \left( \frac{\phi}{2} \right),
\]

where \( \phi \) is measured from the positive crack face and \( \rho \) is a radial coordinate defined with respect to an axis along the
Substituting Eqs. (25) and (26) into Eq. (19) and using the definition (8) gives

\[ dZ[\xi, \delta \xi, \mathbf{m}] = \sigma \lim_{\beta \to 0} \frac{1}{\beta} \int_{0}^{\xi} \beta \delta \xi(t) \mathcal{F}(\mathbf{m}, t) \delta \xi(t) dt, \]

which is the desired form for \( dZ[\xi, \delta \xi, \mathbf{m}] \). Comparing with Eq. (7), one identifies

\[ \nabla \varphi(\mathbf{m}, t) = \frac{\pi \sigma}{2} \mathcal{F}(\mathbf{m}, t). \]

This central result gives the impedance gradient in terms of a function \( \mathcal{F}(\mathbf{m}, t) \) which weights the dipole distribution at the edge of the crack.

The inversion algorithm begins by solving the forward problem for some initial estimate of the flaw shape. The solution is computed for each probe position giving a set of dipole distributions and impedance predictions. The dipole distribution at the edge of the crack is used to determine \( \mathcal{F}(\mathbf{m}, t) \) and the corresponding impedance gradient is calculated from Eq. (28). Next the error gradient is evaluated using Eq. (10) and the flaw is updated using the steepest descent formulas, Eq. (11). The process continues iteratively until a convergence condition is satisfied.

**VI. IMPEDANCE GRADIENT**

In evaluating Eq. (19) for an arbitrary ideal crack, the dipole density at the crack edge will be written, in accordance with the preceding discussions, as

\[ \delta \mathcal{F}(s, t) = 2\sigma \mathcal{F}(\mathbf{m}, t) \sqrt{\delta \xi(t)} - s, \quad 0 \leq s \leq \delta \xi, \]

where \( \delta \mathcal{F}(s, t) = \mathcal{F}(\mathbf{m}, t) \sqrt{\delta \xi(t)} - s \) is the jump in the electric field at the crack edge.

The singular electric field normal to the crack plane at the tip of the unvaried crack has the form given by the \( \phi \) component of Eq. (23) with \( \phi = \pi \). Thus,

\[ E_\phi(s, t) = \frac{1}{\sqrt{s}}. \]

**TABLE I. Probe and flaw parameters.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Coil parameters</strong></td>
<td></td>
</tr>
<tr>
<td>inner radius ( a_1 )</td>
<td>7.38 ± 0.01 mm</td>
</tr>
<tr>
<td>outer radius ( a_2 )</td>
<td>4.99 ± 0.01 mm</td>
</tr>
<tr>
<td>length ( 2b )</td>
<td>4.99 ± 0.01 mm</td>
</tr>
<tr>
<td>lift-off ( l )</td>
<td>0.313 ± 0.01 mm</td>
</tr>
<tr>
<td>number of turns</td>
<td>4000</td>
</tr>
<tr>
<td><strong>Frequency</strong></td>
<td>350 Hz</td>
</tr>
<tr>
<td><strong>Conductor</strong></td>
<td></td>
</tr>
<tr>
<td>conductivity</td>
<td>22.62 ± 0.06 MS/m</td>
</tr>
<tr>
<td>thickness</td>
<td>24 mm</td>
</tr>
<tr>
<td><strong>Flaw</strong></td>
<td></td>
</tr>
<tr>
<td>shape</td>
<td>irregular</td>
</tr>
<tr>
<td>length ( d )</td>
<td>22.1 ± 0.02 mm</td>
</tr>
<tr>
<td>depth ( h )</td>
<td>8.61 ± 0.05 mm</td>
</tr>
<tr>
<td>opening ( c )</td>
<td>0.33 ± 0.01 mm</td>
</tr>
<tr>
<td><strong>Derived quantities</strong></td>
<td></td>
</tr>
<tr>
<td>skin depth ( \delta )</td>
<td>5.65 ± 0.02 mm</td>
</tr>
<tr>
<td>opening/skin depth ( c/\delta )</td>
<td>0.058 ± 0.002</td>
</tr>
</tbody>
</table>

A number of tests of the inversion scheme have been carried out using experimental data obtained by measuring coil impedance due to interactions with simulated defects in metal plates. The skin depth was much smaller than the plate thickness for all observations, therefore no significant errors are introduced by treating the conductor as a half-space.
Single frequency impedance measurements were made on simulated cracks in the form of narrow slots using a normal coil. These observations were made with the coil at a number of equally spaced locations along the length of the defect. The axis of the coil was aligned with the vertical plane of the slot. Both the probe position and data collection were controlled automatically by computer. The dimensions of the coil and slots, together with other experimental parameters, are given in Table 1.

A discrete approximate solution of the forward problem, found using the boundary element scheme described in part I, gives the predicted probe impedance and a piecewise constant dipole density on a regular grid of rectangular elements. In order to evaluate an estimate of the impedance gradient from these results, the dipole density is sampled by interpolation at points that are at a small fixed distance from the edge of the crack. The distance being of the order of the dimensions of a boundary element. From these samples, the function $\mathcal{F}(m,t)$ is estimated using the half-power edge variation of the dipole density, Eq. (22). The estimates give the impedance gradient at points where vertical lines through the centers of boundary elements intersect the crack edge. The steepest descent formula is used to relocate these points and the crack profile redefined as a continuous line using a cubic spline interpolation.

Clearly there are discretization errors in this procedure, particularly as the piecewise constant approximation of the dipole density is likely to distort the dipole distribution quite...
It is evident from Fig. 5 that some distortion of the crack shape is produced in the regions where the edge is inclined at around 45° to the surface of the conductor. This may be due to discretization errors arising from a jagged edge approximation of the crack shape by the rectangular grid.

Observations made on an irregular simulated defect are shown in Figs. 6 and 7 where they are compared with the boundary element predictions. The inversion results using 71 observations are shown in Fig. 8. Again the initial or seed crack was semicircular, with a 5 mm radius. The boundary element grid for the inversion consisted of 32×16 cells. After 100 iterations, the calculation was terminated with crack profile that was in good agreement with the measure shape. The final calculated flaw length was 48.9 mm, compared with a measured value of 49.78 mm. The maximum depth found by inversion was 8.95 mm compared with a measured value of 8.94 mm.

VIII. CONCLUSION

An inversion scheme has been developed to reconstruct the geometry of cracks using probe impedance measurements. Tests of the inversion using observations on simulated cracks of known dimension show that the shapes can be reconstructed in reasonable time even with quite modest computer resources. All the results presented in this article could be calculated in less than 2 h on a personal computer.

Flaw inversion by optimization contains two central requirements: an effective means of predicting the observations and an efficient method of finding the gradient of the predictions with respect to a variation of the flaw. Although a boundary element scheme has been used in the present study to make the predictions, any suitable forward problem solver could be used instead. Similarly, the impedance gradient is determined by the behavior of the dipole density at the edge of the crack, therefore any numerical scheme that can be used to calculate the jump in the field or the dipole density at an ideal crack surface can be used to calculate the impedance gradient. It is not necessary to use a calculation based on an integral formulation.

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