EE 505
Lecture 11

Statistical Characterization of DACs
Review from last lecture:

Spectral characterization of data converters is widely used to characterize both linearity and high frequency performance.

Spectral characteristics obtained from Fourier Series representation of output with a periodic input.

Fourier Series coefficients can be obtained either by integral equation or in a more computationally-efficient way from the DFT provided some very stringent conditions are satisfied when calculating the DFT:
- sample over an integral number of periods
- input signal is appropriately band limited
- jitter occurs when samples are taken

Dramatic errors are introduced if DFT (or FFT) methods are used and these stringent conditions are not satisfied.

In circuit simulators such as SPICE or SPECTRE, it is critical that the actual DFT sampling points be included as simulation points in a transient analysis (can use STROBE PERIOD to achieve this in SPECTRE).
Review from last lecture:

Windowing is often used to compensate for imperfect sampling but windowing itself can skew the actual spectral performance of the data converter.

Time and amplitude quantization contribute primarily to an increased noise floor but little to harmonic distortion in $n>5$.

Linear settling in a DAC may introduce an error in DAC output but does not significantly contribute to spectral distortion.

Return-to-Zero operation of a DAC can reduce the harmonic distortion at the Expense of a reduction in the effective output power level of the DAC.
Theorem: If $X_1, \ldots, X_n$ are uncorrelated random variables and $a_1, \ldots, a_n$ are real numbers, then the random variable $Y$ defined by

$$Y = \sum_{i=1}^{n} a_i X_i$$

has mean and variance given by

$$\mu_Y = \sum_{i=1}^{n} a_i \mu_i$$

$$\sigma_Y = \sqrt{\sum_{i=1}^{n} (a_i \sigma_i)^2}$$

where $\mu_i$ and $\sigma_i$ are the mean and variance of $X_i$ for $i=1, \ldots, n$. 
Homework Problem 1  (for next assignment)

Identify/Generate a set of test structures, measurement procedures and data extraction algorithms that can be used to obtain Ap for a given process
Recall
for R-string DACs

\[ \sigma_{\text{INL}_k} = \frac{\sigma_{R_a}}{R_N} \sqrt{\frac{(N-k)(k-1)}{N-1}} \]

\[ \sigma_{\text{INL}_{k\text{max}}} = \frac{\sigma_{R_a}}{R_N} \sqrt{\frac{N}{2}} \]

\[ \sigma_{\text{INL}} = 4 \sigma_{\text{INL}_{k\text{max}}} \]

for I-steering DAC

\[ \sigma_{\text{INL}_k} = \frac{\sigma_{I_R}}{I_N} \sqrt{\frac{(N-k)(k-1)}{N-1}} \]

\[ \sigma_{\text{INL}_{k\text{max}}} = \frac{\sigma_{I_R}}{I_N} \sqrt{\frac{N}{2}} \]

\[ \sigma_{\text{INL}} = 4 \sigma_{\text{INL}_{k\text{max}}} \]
For Resistors

\[ \frac{U_{R_{\text{b}}}}{R_{\text{b}}} = \frac{A_{p}}{P_{\text{nom}} \sqrt{W \cdot L}} \]

where \( A_{p} \) is a constant determined by the process

\[ P_{\text{nom}} = \text{Nominal sheet resistance} \]

\( W, L \) geometric parameters

Note: Have neglected

a) contact resistance
b) edge roughness
For transistors, need \( \frac{\Delta I_R}{\Delta I_N} \)

Assume MOSFET in saturation

\[
I_D = \frac{m_{Cox}W}{2L} (V_{GS} - V_T)^2
\]

Assume layout that cancels gradient effects

(The following analysis would not be accurate if gradient effects are not cancelled – gradient effects would determine matching performance)
\( \mu(x, y), \ \text{Cov}(x, y), \ \nu_T(x, y) \)

It would be best to consider physical parameters rather than model parameters because physical parameters are statistically uncorrelated whereas the model parameters are somewhat correlated.
Assume $I_D$ can be modeled by the equation

$$I_D = \frac{\mu C_{ox} W}{2L} (V_{GS} - V_T)^2$$

where $\mu$, $C_{ox}$, and $V_T$ are given by the equations

$$\mu = \frac{\int_A \mu(x,y) \, dx \, dy}{A}$$

$$C_{ox} = \frac{\int_A C_{ox}(x,y) \, dx \, dy}{A}$$

$$V_T = \frac{\int_A V_T(x,y) \, dx \, dy}{A}$$
The previous assumptions are not precise as following example shows.

\[ I_D = \frac{\mu C_{ox} W}{2L} (V_{GS} - V_{T1}) \]

\[ V_{TEQ} = \frac{\int_A V_T(x,y) \, dx \, dy}{A} = \frac{V_{T1} + V_{T2}}{2} \]

If \( V_{T2} = 2V \), \( V_{T1} = 1V \)

\[ V_{TEQ} = 1.5V \]
Practical Comment:
If the nominal current density is constant throughout the channel region, the integral models for parameters are good approximations.

\[ I = \frac{\mu C_{ox} W (V_{gs} - V_T)^2}{2L} \]

\[ I = \frac{(\mu_n + \mu_R) (C_{oxN} + C_{oxR}) W (V_{gs} - V_{TN} - V_{TR})^2}{2L} \]

\[ I = \left[ \mu_N C_{oxN} (V_{gs} - V_{TN})^2 + \mu_R \left[ C_{oxN} (V_{gs} - V_{TN})^2 \right] \right] \\
+ C_{oxR} U_N (V_{gs} - V_{TN})^2 - 2V_{TR} (V_{gs} - V_{TN}) \mu_N C_{oxN} \\
+ E (\mu_R, C_{oxR} V_{TR}) \frac{W}{2L} \]
$$I_R = I - I_N = \left[ M_R \left[ \text{CoxN} (V_{G, S} - V_{T, N})^2 \right] + C_{OX} \left[ V_{G, S} - V_{T, N} \right] \right] \frac{W}{2L}$$

$$\frac{I_R}{I_N} = \left( \frac{M_R \left[ \text{CoxN} (V_{G, S} - V_{T, N})^2 \right] + C_{OX} \left[ V_{G, S} - V_{T, N} \right]}{-V_T \left[ 2(V_{G, S} - V_{T, N}) M_N C_{OXN} \right]} \right) \frac{W}{2L}$$

$$M_N \text{CoxN} (V_{G, S} - V_{T, N})^2 \frac{W}{2L}$$

$$\frac{I_R}{I_N} = \frac{M_R}{M_N} + \frac{C_{OX}}{\text{CoxN}} - 2 \frac{V_T}{(V_{G, S} - V_{T, N})}$$

If we assume variables are uncorrelated

$$\sigma \frac{I_R}{I_N} = \sqrt{\sigma \frac{M_R}{M_N} + \sigma \frac{C_{OX}}{\text{CoxN}} + \frac{4}{(V_{G, S} - V_{T, N})^2} \sigma \frac{V_T}{V_{T, N}}}$$