EE 505

Lecture 6

Performance Limitations Associated with Nonideal Components

• Active Component Modeling
• Passive Component Modeling
• Offset Voltages
• Statistical Characterization and Impact on Yield
Note: I will be on travel next week but will have email contact most of the time. Normally two lectures on spectral characterization would appear here. Will delay these two lectures until after I return.
Linearity Measurements (testing)

Code density testing:

\[
DNL_i = \frac{\hat{C}_i - \bar{C}}{\bar{C}} \\
INL_i = \begin{cases} 
0 & i = 0, N-2 \\
\sum_{k=1}^{i} \hat{C}_k - i\bar{C} & 1 \leq i \leq N-3
\end{cases}
\]

\[
DNL = \max_{1 \leq i \leq N-2} \{ |DNL_i| \} \\
INL = \max_{1 \leq i \leq N-3} \{ |INL_i| \}
\]

- Code Density Measurements are Indirect Measurements of the INL and DNL
- Can give very wrong information under some nonmonotone missing code scenarios
- Often use an average of 16 or 32 samples per code
- Measurement noise often 1 lsb or larger but averages out
- Sometimes use good sinusoidal waveform but must correct code density for this distinction
- Full code-density testing is costly for high-resolution low-speed data converters because of data acquisition costs
- Reduced code testing using servo methods is often a less costly alternative but may miss some errors
We will define “Noise” to be the difference between the actual output and the desired output of a system

Types of noise:

- Random noise due to movement of electrons in electronic circuits
- Interfering signals generated by other systems
- Interfering signals generated by a circuit or system itself
- Error signals associated with imperfect signal processing algorithms or circuits
Quantization Noise in ADC
(same concepts apply to DACs)

Consider an Ideal ADC with first transition point at $0.5X_{\text{LSB}}$

If the input is a low frequency sawtooth waveform of period $T$ that goes from 0 to $X_{\text{REF}}$, the error signal in the time domain will be:

$$\epsilon_Q$$

where $T_1 = T/2^n$

This time-domain waveform is termed the Quantization Noise for the ADC with a sawtooth (or triangular) input.
Quantization Noise in ADC

\[ E_{\text{RMS}} = \sqrt{\frac{1}{T_1} \int_{-T_1/2}^{T_1/2} \varepsilon_Q^2(t) dt} \]

\[ E_{\text{RMS}} = \sqrt{\frac{1}{T_1} \int_{-T_1/2}^{T_1/2} \left( \frac{-X_{\text{LSB}}}{T_1} \right)^2 t^2 dt} \]

\[ E_{\text{RMS}} = X_{\text{LSB}} \sqrt{\frac{1}{T_1^3} \int_{-T_1/2}^{T_1/2} t^3 dt} \]

\[ E_{\text{RMS}} = \frac{X_{\text{LSB}}}{\sqrt{12}} \]
Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $X_{\text{REF}}$ centered at $X_{\text{REF}}/2$?

![Graph showing time and amplitude quantization points](image-url)
Quantization Noise in ADC

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Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $X_{\text{REF}}$ centered at $X_{\text{REF}}/2$?

For low $f_{\text{SIG}}/f_{\text{CL}}$ ratios, bounded by $\pm X_{\text{LB}}$ and at any point in time, behaves almost as if a uniformly distributed random variable:

$$\epsilon_Q \sim U[-0.5X_{\text{LSB}}, 0.5X_{\text{LSB}}]$$
Recall:

If the random variable $f$ is uniformly distributed in the interval $[A,B]$ then the mean and standard deviation of $f$ are given by

$$\mu_f = \frac{A+B}{2}, \quad \sigma_f = \frac{B-A}{\sqrt{12}}$$

Theorem: If $n(t)$ is a random process, then for large $T$,

$$V_{RMS} = \sqrt{\frac{1}{T} \int_{t_1}^{t_1+T} n^2(t) dt} = \sqrt{\sigma_n^2 + \mu_n^2}$$
Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $X_{\text{REF}}$ centered at $X_{\text{REF}}/2$?

$\varepsilon_Q \sim U[-0.5X_{\text{LSB}}, 0.5X_{\text{LSB}}]$

$\mu_{\varepsilon_Q} = \frac{A+B}{2} = 0 \quad \sigma_f = \frac{B-A}{\sqrt{12}} = \frac{X_{\text{LSB}}}{\sqrt{12}}$

$V_{\text{RMS}} = \sqrt{\frac{1}{T} \int_{t_1}^{t_1+T} n^2(t)dt} = \sqrt{\sigma_n^2 + \mu_n^2}$

$V_{\text{RMS}} = \sigma_{\varepsilon_Q} = \frac{X_{\text{LSB}}}{\sqrt{12}}$

Note this is the same RMS noise that was present with a triangular input.
ENOB based upon Quantization Noise

\[ \text{SNR} = 6.02 \, n + 1.76 \]

Solving for \( n \), obtain

\[ \text{ENOB} = \frac{\text{SNR}_{\text{dB}} - 1.76}{6.02} \]

Note: could have used the \( \text{SNR}_{\text{dB}} \) for a triangle input and would have obtained the expression

\[ \text{ENOB} = \frac{\text{SNR}_{\text{dB}}}{6.02} \]

But the earlier expression is more widely used when specifying the ENOB based upon the noise level present in a data converter.
Quantization Noise

Effects of quantization noise can be very significant, even at high resolution, when signals are not of maximum magnitude.

Quantization noise remains constant but signal level is reduced.

The desire to use a data converter at a small fraction of full range is one of the major reasons high resolution is required.
ENOB Summary

Resolution:

\[ \text{ENOB} = \frac{\log_{10} N_{\text{ACT}}}{\log_{10} 2} = \log_2 N_{\text{ACT}} \]

INL:

\[ \text{ENOB} = n_R - \log_2 (\nu) - 1 \quad \text{where} \quad n_R \text{ specified res, } \nu \text{ INL in LSB} \]

\[ \text{ENOB} = - \log_2 (\text{INL}_{\text{REF}}) - 1 \quad \text{where} \quad \text{INL}_{\text{REF}} \text{ INL rel to } X_{\text{REF}} \]

DNL:

HW problem

Quantization noise:

\[ \text{ENOB} = \frac{\text{SNR}_{\text{dB}}}{6.02} \quad \text{rel to triangle/sawtooth} \]

\[ \text{ENOB} = \frac{\text{SNR}_{\text{dB}} - 1.76}{6.02} \quad \text{rel to sinusoid} \]
Consider a flash ADC

- Resistor values and offset voltages of Comparators are all random variables at design level.
- Variations of these RVs affect the break point and thus the yield.
Consider Current-Steering DAC

Ideally

\[ V_{\text{OUT}} = -I_1 \cdot R_F \sum_{i=0}^{n-1} \frac{b_i}{2^{n-i}} \]
Consider Current-Steering DAC

Basic Implementation of Current Sources

Ideally

\[ I_k \approx \frac{\mu_{COX}W_k}{2L_k} \left( V_R - V_{Tpk} \right)^2 \]

L_k = L_0

Actually

\[ I_k \approx \frac{\mu_kC_{OXk}W_k}{2L_k} \left( V_R - V_{Tpk} \right)^2 \]

\( I_k \) is a random variable and is a function of the model parameters \( \mu_k, C_{OXk}, W_k, L_k, \) and \( V_{Tpk} \)

\( \mu_k, C_{OXk}, W_k, L_k, \) and \( V_{Tpk} \) are all random variables
Recall from previous lecture

How important is statistical analysis?

Example: 7-bit FLASH ADC with R-string DAC

Assume R-string is ideal, $V_{\text{REF}} = 1\text{V}$ and $V_{\text{OS}}$ for each comparator must be at most $\pm \frac{1}{2}$ LSB

Case 1

Standard deviation is 5mV

$P_{\text{COMP}} = 0.565$

$Y_{\text{ADC}} = 3.2 \times 10^{-32}$

Case 2

Standard deviation is 1mV

$P_{\text{COMP}} = 0.999904$

$Y_{\text{ADC}} = 0.988$

Statistics play a key role in the performance and consequently yield of a data converter
Resistor Characterization

Resistors are generally made of thin films of conductive or semiconductor materials

Film Characterized by Resistivity: \( \rho(x, y, z) \)

Generally \( h \) is very small compared to \( L \) and \( W \)

Films are often characterized by Sheet Resistance \( R_{\square}(x, y) = \frac{\rho(x, y, z)}{h(x, y)} \)

Ideally \( \rho(x, y, z) \) is independent of position as is \( R_{\square}(x, y) \)

In the ideal case \( R = \rho \left( \frac{1}{h} \cdot \frac{L}{W} \right) = R_{\square} \left( \frac{L}{W} \right) \)
Resistor Characterization

Resistors are generally made of thin films of conductive or semiconductor materials.

Ideally, the resistance of a resistor can be expressed as:

$$R = R_{\square} \left( \frac{L}{W} \right)$$

$$R_{\square} = \frac{\rho}{h}$$
Resistor Characterization

Ideally

Actually

Boundary of resistor varies \( \rho(x,y,z) \) varies with position