EE 505
Lecture 8

Mismatch Effects
Matching Requirements in Data Converters
Recall

\[ \frac{1}{\mathcal{J}_R} = \frac{L}{w^3} A_p^2 \]

\[ \frac{1}{\mathcal{J}^{br}_R} = \frac{A_p^2}{w \cdot L \cdot P_{nom}^2} \]

How is \( A_p \) obtained?

- build test structure to obtain \( A_p \)
- Get \( A_p \) from the process development group
Case 1. (How about this?)

1) Take a large number, n, of test resistors with length and width equal to 1μm

2) Measure $R_1, R_2, \ldots, R_n$

3) Calculate the sample standard deviation

$$\hat{\rho} = \hat{\sigma}_{\text{sample}}$$

$$\hat{\rho}_{\text{nom}} = \mu_{\text{sample}}$$

There are some serious problems with this approach!

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- Fringe effects will significantly skew $\hat{\rho}$
- Increasing size can reduce/minimize this concern
\[ J^2_r = \frac{L A_p^2}{\omega^3} \]

This strategy significantly reduces the boundary problem associated with the \( l \times 1 \mu \) structure.

- but, this approach still has significant problems.

Gradient effects will be particularly significant for large cells!
Consider Gradient Direction

\[ R_1 \quad R_2 \]

\[ R_3 \quad R_4 \quad \ldots \]

Gradient effects will dramatically skew Ap extraction!

- Need large test structures that are insensitive to gradient effects!

- Consider a two-resistor test cell
How does the ratio matching of two resistors relate to the standard deviation of a single resistor?

\[ \Theta = \frac{R_1 - R_2}{R_N} = \frac{R_N + R_{1R} - R_N - R_{2R}}{R_N} \]

\[ \Theta = \frac{R_{1R} - R_{2R}}{R_N} \]

\[ \therefore \sigma_\Theta^2 = \frac{1}{R_N^2} \left( \sigma_{R_{1R}}^2 + \sigma_{R_{2R}}^2 \right) \]

\[ \sigma_\Theta^2 = \frac{2 \sigma_{R_R}^2}{R_N^2} \]
\[ \mathcal{V}_\Theta = \frac{2 \mathcal{V}_{R_R}}{R_N^2} \]

\[ \Rightarrow \mathcal{V}_\Theta = \sqrt{2} \frac{\mathcal{V}_{R_R}}{R_N} \]

**Strategy for test structures**

- large cells but not too big to create nonlinear gradients
- spread a large number of these test structures on a die
- generate \( \Theta_1, \Theta_2, \ldots, \Theta_n \)
\[ A_p = \frac{\sigma_{OR}}{R} \sqrt{W \cdot L} \quad P_{\text{Nom}} \]

\[ \hat{A}_p = \frac{\sigma_{OR}}{R} \left| \frac{\sqrt{W \cdot L}}{\text{Sample}} \right| P_{\text{Nom}} \]

\[ \hat{A}_p = \frac{\sigma_{\theta}}{\sqrt{2}} \sqrt{W \cdot L} \quad P_{\text{Nom}} \]

\( \hat{A}_p \) is a good estimator of \( A_p \) if \( n \) is reasonably large!
Example: If a ratio of 10:1 is desired, determine the ratio matching accuracy relative to the standard deviation of a single resistor.

Consider:

\[ V_i \frac{R}{V_o} \]

\[ V_i \frac{10R}{V_o} \]

Assume realized as series connection of 10 resistors.

Question: What is the "yield" of these two amplifiers and how do they compare if a given gain accuracy requirement is specified?
Ratio Matching Effects
in Data Converters
Example

\[ R_2 \]
\[ R_1 \]
\[ V_1 \]
\[ - \]
\[ + \]
\[ V_0 \]

How does the ratio matching accuracy for ratios different than 1 relate to the accuracy for ratios of 1?

\[ R_2 : \quad \frac{R_{21}}{R_{22}} \ldots \frac{R_{2k}}{R_2} \]

\[ R_1 : \quad \frac{R_{1}}{R_{11}} \]

Assume: \( R_{2k} = R_{11} = R_0 \) for all \( k \)

\[ \left| \frac{V_0}{V_1} \right|_n = \frac{R_2}{R_1} = \frac{kR_0}{R_0} = k \]

define \( \Theta \) to be the gain error
\[ \Theta = \frac{R_2}{R_1} \left| \begin{array}{c} \text{Nom} \\ \text{Act} \end{array} \right| - \frac{R_2}{R_1} \left| \begin{array}{c} \text{Nom} \\ \text{Act} \end{array} \right| \\
\]

\[ \Theta = K - \sum_{i=1}^{K} \frac{R_{2i}}{R_{1i}} \]

but \[ R_{2i} = R_0 + R_{R2i} \]

\[ R_{1i} = R_0 + R_{R1i} \]

\[ \Theta = K - \left( \frac{\sum_{i=1}^{K} R_0 + \sum_{i=1}^{K} R_{R2i}}{R_0 + R_{R1i}} \right) \]

\[ \Theta = K - \left( K \frac{R_0}{R_0} + \frac{\sum_{i=1}^{K} R_{R2i}}{R_0 \left( 1 + \frac{R_{R1i}}{R_0} \right)} \right) \]

\[ \Theta = K - \left( K + \frac{\sum_{i=1}^{K} R_{R2i}}{R_0} \right) \left( 1 - \frac{R_{R1i}}{R_0} \right) \]
$$\Theta = K - \left( K + \sum_{i=1}^{N} \frac{R_{R_{2i}}}{R_0} - K \frac{RR_{i1}}{R_0} + \theta \right)$$

$$\Theta = \sum_{i=1}^{N} \frac{R_{R_{2i}}}{R_0} - K \frac{RR_{i1}}{R_0}$$

$$\sigma^2 = K \sigma^2_{\frac{R_R}{R_N}} + K^2 \sigma^2_{\frac{R_R}{R_N}}$$

$$\sigma_\Theta = \sigma_{\frac{R_R}{R_N}} \sqrt{K + K^2}$$

Note: $K$ is simply the nominal magnitude of the $dc$ gain

If $K = 1$  \hspace{1cm} $\sigma_\Theta = \sigma_{\frac{R_R}{R_N}} \sqrt{2}$

\hspace{1cm} $K = 10$  \hspace{1cm} $\sigma_\Theta = \sigma_{\frac{R_R}{R_N}} \sqrt{110}$

\hspace{1cm} $\sigma_\Theta \approx 10.5 \sigma_{\frac{R_R}{R_N}}$
Question: Which will have the lowest $\theta$?

Note: $R_{\text{total}} = \begin{cases} 17R & \text{for case 1} \\ 10R & \text{for case 2} \end{cases}$
Example:

\[ u_i \xrightarrow{R} u_o \]

\[ u'_i \xrightarrow{R} u'_o \]

\[ v'_i \xrightarrow{R} v'_o \]
Consider \( \text{INL}_k = \text{Actual Output} - \text{Fit line Output} \)

\[
V_{ok} = \frac{M^{-1} R_i}{M^2 R_i} \cdot \frac{V_{\text{REF}}}{R_i} \\
V_{\text{FIT}}(k) = \frac{k-1}{N-1} \sum_{i=1}^{N-1} \frac{R_i}{N} \cdot \frac{V_{\text{REF}}}{R_i}
\]
\[ \text{INL}_k = \left( \frac{\sum_{i=1}^{k-1} R_i}{\sum_{i=1}^{N} R_i} - \frac{k-1}{N-1} \frac{\sum_{i=1}^{N} R_i}{2^n} \right) V_{\text{REF}} \]

\[ \frac{V_{\text{REF}}}{2^n} \]

\[ = \frac{\sum_{i=1}^{k-1} R_i}{\sum_{i=1}^{N} R_i} - \frac{k-1}{N-1} \frac{\sum_{i=1}^{N} R_i}{2^n} \]

\[ \frac{\sum_{i=1}^{N} R_i}{2^n} \]

\[ \text{INL}_k = \sum_{i=1}^{k-1} R_i - \frac{k-1}{N-1} \sum_{i=1}^{N} R_i - \frac{k-1}{N-1} \sum_{i=k}^{N-1} R_i \]

\[ \frac{2^n \sum_{i=1}^{N} R_i}{2^n} \]

\[ \text{INL}_k = \sum_{i=1}^{k-1} R_i (1 - \frac{k-1}{N-1}) - \frac{k-1}{N-1} \sum_{i=k}^{N-1} R_i \]

\[ \frac{2^n \sum_{i=1}^{N} R_i}{2^n} \]

Let \( R_i = R_N + R_R_i \)

\[ \text{INL}_k = \sum_{i=1}^{k-1} R_i (1 - \frac{k-1}{N-1}) - \frac{k-1}{N-1} \sum_{i=k}^{N-1} R_i \]

\[ \frac{2^n \sum_{i=1}^{N} R_i}{2^n} \]

Let \( R_i = R_N + R_R_i \)
\[\text{INL}_k = \sum_{i=1}^{k-1} \frac{R_{R_i}}{R_N} \left( \frac{N-k}{N-1} \right) - \sum_{i=k}^{N-1} \frac{R_{R_i}}{R_N} \left( \frac{k-1}{N-1} \right)\]

\[\phi_{\text{INL}_k} = \phi_{\frac{R_{R_k}}{R_N}} \sqrt{\frac{(N-k)(k-1)}{N-1}} \quad 2 \leq k \leq N-2\]

\[\phi^2_{\text{INL}_k} = \phi^2_{\frac{R_{R_k}}{R_N}} \left( \frac{[N-k]^2}{[N-1]^2} (k-1) + \frac{(k-1)^2}{[N-1]^2} (N-k) \right)\]

\[\text{INL} = \max_k \left| \text{INL}_k \right|\]

\text{INL is an order statistic}

Distribution functions for order statistics are very complicated

\[\sigma_{\text{INL}_k}\]
\[ \sigma_{\text{INL}} \]

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\text{max of } \sigma_{\text{INL}} \text{ occurs at } K = \frac{N}{2}
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\[ \sigma_{\text{INL max}} = \sigma_{\text{BR}} \frac{\sqrt{N}}{2N} \]