EE 505
Lecture 9

• Statistical Circuit Modeling
Statistical Analysis Strategy

Will first focus on statistical characterization of resistors, then extend to capacitors and transistors

Every resistor $R$ can be expressed as

$$R = R_N + R_{RP} + R_{RW} + R_{RD} + R_{RGRAD} + R_{RL}$$

where $R_N$ is the nominal value of the resistor and the remaining terms are all random variables

- $R_{RP}$: Random process variations
- $R_{RW}$: Random wafer variations
- $R_{RD}$: Random die variations
- $R_{RGRAD}$: Random gradient variations
- $R_{RL}$: Local Random Variations
Theorem: If $X_1, \ldots, X_n$ are uncorrelated random variables and $a_1, \ldots, a_n$ are real numbers, then the random variable $Y$ defined by

$$Y = \sum_{i=1}^{n} a_i X_i$$

has mean and variance given by

$$\mu_Y = \sum_{i=1}^{n} a_i \mu_i$$

$$\sigma_Y = \sqrt{\sum_{i=1}^{n} (a_i \sigma_i)^2}$$

where $\mu_i$ and $\sigma_i$ are the mean and variance of $X_i$ for $i=1, \ldots, n$. 
## Summary of Results

<table>
<thead>
<tr>
<th>Structure</th>
<th>Nominal Resistance</th>
<th>Standard Deviation</th>
<th>Normalized Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>$R_N$</td>
<td>$\sigma_R = \sigma_{R_R}$</td>
<td>$\frac{\sigma_{R_R}}{R_N}$</td>
</tr>
<tr>
<td>Ser nR</td>
<td>$nR_N$</td>
<td>$\sqrt{n}\sigma_{R_R}$</td>
<td>$\frac{1}{\sqrt{n}} \frac{\sigma_{R_R}}{R_N}$</td>
</tr>
<tr>
<td>Par nR</td>
<td>$\frac{R_N}{n}$</td>
<td>$\frac{1}{n^{3/2}} \sigma_{R_R}$</td>
<td>$\frac{1}{\sqrt{n}} \frac{\sigma_{R_R}}{R_N}$</td>
</tr>
</tbody>
</table>

Note increasing or decreasing the resistance by a factor of $n$ decreased the normalized standard deviation by $\sqrt{n}$.

Note increasing the area by a factor of $n$ decreased the normalized standard deviation by $\sqrt{n}$.

What is the relationship between resistance, area, and standard deviation?
Have considered in previous examples the following scenarios:

- Current density is uniform in each structure
- Aspect ratio plays no role in normalized performance
- Resistance value plays no role in normalized performance
- Only factor in normalized performance is area
- For a given resistance, each factor of 2 reduction in $\sigma$ requires a factor of 4 increase in area
Key Implications:

If yield of a data converter is dominated by matching performance, then every bit improvement in performance will require at least a factor of 2 reduction in \( \sigma \) and thus a factor of 4 increase in the area for the matching critical components if the same yield is to be obtained.
Consider the normalized resistance \( \frac{R}{R_N} \)

\[
\sigma_R^2 \left( \frac{1}{R_N^2} \right) \sigma_R^2 = \left( \frac{W^2}{L^2 R_{\text{\scriptsize{N}}}^2} \right) \sigma_{\text{\scriptsize{REF}}}^2 \cdot \frac{L}{W^3} = \frac{1}{WL} \left[ \frac{\sigma_{\text{\scriptsize{REF}}}^2}{R_{\text{\scriptsize{N}}}^2} \right]
\]

Define the parameter \( A_R \) by the expression \( A_R = \frac{\sigma_{\text{\scriptsize{REF}}}}{R_{\text{\scriptsize{N}}}^2} \)

\( A_R \) is also a process parameter but more convenient to use than both \( R_{\text{\scriptsize{N}}} \) and \( \sigma_{\text{\scriptsize{REF}}} \)

\[
\frac{\sigma_R}{R_N} = \frac{A_R}{\sqrt{WL}} = \frac{A_R}{\sqrt{A}}
\]

Note the normalized variance is independent of the resistor value!
How can $A_R$ be obtained?

Recall: 

\[ \sigma_{R \frac{R}{R_N}} = \frac{A_R}{\sqrt{A}} \quad \text{where} \quad A_R = \frac{\sigma_{REF}}{R_{\parallel N}} \]

1. Obtain $A_R$ from a PDK

2. Build a test structure to obtain $A_R$
Case 1. (How about this?)

1) Take a large number, $n$, of test resistors with length and width equal to 1 $\mu$

2) Measure $R_1, R_2, \ldots, R_n$

3) Calculate the sample standard deviation

\[ \hat{\sigma}_{\text{REF}} \approx \sigma_{\text{SAMPLE}} \]

\[ R_{\text{MN}} \approx \mu_{\text{SAMPLE}} \]

There are some serious problems with this approach!

- Fringe effects will significantly skew $\hat{\sigma}_{\text{REF}}$
- Increasing size can reduce/minimize this concern
\[ \sigma_R^2 = \sigma_{REF}^2 \cdot \frac{L}{W^3} \]

This strategy significantly reduces the boundary problem associated with the \textit{LxLxL} structure.

- But, this approach still has significant problems.

Gradient effects will be particularly significant for large cells.
Consider Gradient Direction

\[ R_1 \quad R_2 \quad R_3 \quad R_4 \quad \ldots \]

Gradient effects will dramatically skew \( \sigma_{REF} \) extraction!

- Need large test structures that are insensitive to gradient effects!
- Consider a two-resistor test cell
How does the ratio matching of two resistors relate to the standard deviation of a single resistor?

\[ R \rightarrow \sigma_R \text{ or } \frac{\sigma_R}{R_N} \]

\[
\begin{align*}
\begin{cases}
R_1 = R_2 = R_N \\
R_{1N} = R_{2N} = R_N
\end{cases}
\end{align*}
\]

\[ \Theta = \frac{R_1 - R_2}{R_N} = \frac{R_{1N} + R_{1R} - R_N - R_{2R}}{R_N} \]

\[ \Theta = \frac{R_{1R} - R_{2R}}{R_N} \]

\[ \therefore \sigma^2_\Theta = \frac{1}{R_N^2} \left( \sigma^2_{R_{1R}} + \sigma^2_{R_{2R}} \right) \]

\[ \sigma^2_\Theta = \frac{2 \sigma^2_{R_{1R}}}{R_N^2} \]

\[ \sigma^2_{\Delta R} = 2 \sigma^2_{\frac{R}{R_N}} \]
Measurement of $A_R$

$$\sigma_{\Delta R} = \sqrt{2} \frac{\sigma_R}{R_N}$$

Strategy for test structures

- large cells but not too big to create nonlinear gradients
- spread a large number of these test structures on a chip
- generate $\frac{\Delta R_1}{R_N}$, $\frac{\Delta R_2}{R_N}$, ..., $\frac{\Delta R_k}{R_N}$
- calculate variance of these samples $\hat{\sigma}_{\Delta R}$

$R_{1A}$ $R_{2A}$

$R_{2B}$ $R_{1B}$

common centroid
Measurement of $A_R$

$$\frac{\sigma_R}{R_N} = \frac{1}{\sqrt{2}} \frac{\sigma_{\Delta R}}{R_N} \approx \frac{1}{\sqrt{2}} \hat{\sigma}_{\Delta R}$$

$$\sigma_R = \frac{A_R}{\sqrt{A}}$$

$$A_R = \sqrt{\frac{A}{2}} \hat{\sigma}_{\Delta R}$$
Measurement of $A_R$

What about just taking a large number of resistors at multiple sites on a die, at multiple die locations on a wafer, and on many wafers and wafer lots:

\[
\sigma_{\frac{R}{R_N}} \approx \hat{\sigma}_{\frac{R}{R_N}}
\]

\[
\sigma_{\frac{R}{R_N}} = \frac{A_R}{\sqrt{A}}
\]

\[
A_R = \sqrt{A} \hat{\sigma}_{\frac{\Delta R}{R_N}}
\]
Example: If a ratio of 10:1 is desired, determine the ratio matching accuracy relative to the standard deviation of a single resistor.

Consider:

```
\[ V_{in} \rightarrow R \rightarrow \text{Op Amp} \rightarrow V_o \]
```

and

```
\[ V_{in} \rightarrow R \rightarrow \text{Op Amp} \rightarrow V_o \]
```

assume realized as series connection of 10 resistors.

Question: What is the "yield" of these two amplifiers and how do they compare if a given gain accuracy requirement is specified?
Ratio Matching Effects in Data Converters

Ratio matching is often critical in ADCs and DACs

Accuracy and matching of gains is also critical in some data converters
Amplifier Gain Accuracy

Example

\[
\frac{V_o}{V_i} = \frac{R_2}{R_1} = k \frac{R_w}{R_o} = k
\]

Assume: \( R_{ak_n} = R_{in} = R_o \) for all \( k \)

How does the ratio matching accuracy for ratios different than 1 relate to the accuracy for ratios of 1?
Amplifier Gain Accuracy

\[ \Theta = \frac{R_2}{R_1} \left| \frac{R_2}{R_{1,\text{Nom}}} - \frac{R_2}{R_{1,\text{Act}}} \right| \]

\[ \Theta = K - \sum_{i=1}^{K} \frac{R_{a,i}}{R_{1,i}} \]

But \( R_{a,i} = R_o + R_{R_{a,i}} \quad 1 \leq i \leq K \)

\[ R_{1,i} = R_o + R_{R_{1,i}} \]

\[ \Theta = K - \left( \sum_{i=1}^{K} \frac{R_o + \sum_{i=1}^{K} R_{R_{a,i}}}{R_o + R_{R_{1,i}}} \right) \]

\[ \Theta = K - \left( K R_o + \sum_{i=1}^{K} R_{R_{a,i}} \right) \]

\[ \Theta = K - \left( K R_o + \sum_{i=1}^{K} \frac{R_{R_{a,i}}}{R_o} \right) \]

\[ \Theta = K - \left( K + \sum_{i=1}^{K} \frac{R_{R_{a,i}}}{R_o} \right) \left( 1 - \frac{R_{R_{1,i}}}{R_o} \right) \]
Amplifier Gain Accuracy

\[ \Theta = K - \left( K + \frac{M}{R_o} \right) - K \frac{R_{\text{in}}}{R_o} + \Theta_c \]

\[ \Theta = \frac{M}{R_o} - K \frac{R_{\text{in}}}{R_o} \]

\[ \Theta^2 = K \frac{\sigma_{R_{\text{in}}}}{R_N} + K^2 \frac{\sigma_{R_{\text{in}}}}{R_N} \]

\[ \sigma_{\Theta} = \frac{\sigma_{R_{\text{in}}}}{R_N} \sqrt{K + K^2} \]

Note: \( K \) is simply the nominal magnitude of the \( \Theta \)c gain.

If \( K = 1 \), \( \sigma_{\Theta} = \sigma_{R_{\text{in}}} \sqrt{2} \)

If \( K = 10 \), \( \sigma_{\Theta} = \sigma_{R_{\text{in}}} \sqrt{110} \)

\[ \sigma_{\Theta} \approx 10.5 \sigma_{R_{\text{in}}} \]
Amplifier Gain Accuracy

Question: Which will have the lowest $\delta$?

Note: $R_{\text{Total}} = \begin{cases} 17R & \text{for case 1} \\ 10R & \text{for case 2} \end{cases}$
Amplifier Gain Accuracy

Question: Which will have the lowest δ?

Note: \[ R_{\text{TOTAL}} = \begin{cases} 17R & \text{for case 1} \\ 10R & \text{for case 2} \end{cases} \]
Amplifier Gain Accuracy

Many different ways to achieve a given gain with a given resistor area
String DAC Statistical Performance

Resistors are uncorrelated but identically distributed, typically zero mean Gaussian

Consider $V_{NL,k} = \text{Actual Output} - \text{Fit Line Output}$

$$V_{OUT}(k) = \begin{cases} 0 & k = 0 \\ \sum_{j=1}^{k} R_j V_{REF} & 1 \leq k \leq N-1 \\ \sum_{j=1}^{N} R_j V_{REF} & k = N \end{cases}$$

$$V_{FIT}(k) = \frac{k}{N-1} \sum_{j=1}^{N} R_j V_{REF} \quad 0 \leq k \leq N-1$$

$V_{out}$ is of considerable interest

$INL = \max \left| V_{NL,k} \right|$
String DAC Statistical Performance

\[
\begin{align*}
INL_k &= \frac{\left(\sum_{j=1}^{k} R_j - k \sum_{j=1}^{N-1} R_j \right)}{V_{REF}} V_{REF} \quad 1 \leq k \leq N - 1 \\
&= \frac{\sum_{j=1}^{k} R_j - k \sum_{j=1}^{N-1} R_j}{\sum_{j=1}^{N} R_j} 2^n \quad 1 \leq k \leq N - 1 \\
&= \frac{\sum_{j=1}^{k} R_j - k \sum_{j=1}^{N-1} R_j - k \sum_{j=k+1}^{N-1} R_j}{\sum_{j=1}^{N} R_j} 2^n \quad 1 \leq k \leq N - 1 \\
&= \frac{\sum_{j=1}^{k} R_j \left(1 - k \frac{1}{N-1}\right) - k \sum_{j=k+1}^{N-1} R_j}{\sum_{j=1}^{N} R_j} 2^n \quad 1 \leq k \leq N - 1
\end{align*}
\]

Let \( R_j = R_{NOM} + R_{Rj} \)
String DAC Statistical Performance

If we do a Taylor’s series expansion of the reciprocal of the denominator and eliminate second-order and higher terms it follows that

Note that INK_k is a zero-mean multivariate Gaussian distribution

\[
INL_k = \left( \frac{k}{R_{NOM}} \left(1 - \frac{k}{N-1}\right) - \frac{k}{N-1} \sum_{j=k+1}^{N} R_j \right) + \left( \frac{k}{R_{NOM}} \left(1 - \frac{k}{N-1}\right) - \frac{k}{N-1} \sum_{j=k+1}^{N} R_j \right) 2^n \quad 1 \leq k \leq N - 1
\]

\[
INL_k = \frac{\sum_{j=1}^{k} R_j \left(1 - \frac{k}{N-1}\right) - \frac{k}{N-1} \sum_{j=k+1}^{N} R_j}{1 + \frac{1}{NR_{NOM}} \sum_{j=1}^{N} R_j} \quad 1 \leq k \leq N - 1
\]
String DAC Statistical Performance

\[ \text{INL}_k = \frac{1}{R_{\text{NOM}}} \left[ \sum_{j=1}^{k} R_{Rj} \left(1 - \frac{k}{N-1}\right) - \frac{k}{N-1} \sum_{j=k+1}^{N-1} R_{Rj} \right] \quad 1 \leq k \leq N-1 \]

Since the resistors are identically distributed and the coefficients are not a function of the index \(i\), it follows that

\[ \sigma^2_{\text{INL}_k} = \sigma^2_{\text{R}_R} \frac{1}{R_{\text{NOM}}} \left[ \sum_{j=1}^{k} \left(1 - \frac{k}{N-1}\right)^2 - \sum_{j=k+1}^{N-1} \left(\frac{k}{N-1}\right)^2 \right] \quad 1 \leq k \leq N-1 \]

Since the index in the sum does not appear in the arguments, this simplifies to

\[ \sigma_{\text{INL}_k} = \sigma_{\text{R}_R} \frac{1}{R_{\text{NOM}}} \sqrt{\frac{(N-1-k)k}{N-1}} \quad 1 \leq k \leq N-1 \]

Note there is a nice closed-form expression for the INL\(_k\) for a string DAC !!
INL$_k$ assumes a maximum variance at mid-code

\[ \sigma_{\text{INL}_k} \]

\[ \max \text{ of } \sigma_{\text{INL}_k} \text{ occurs at } k = \frac{N}{2} \]

\[ \sigma_{\text{INL}_k}^{\text{max}} = \sigma \frac{R_R}{R_{\text{NOM}}} \frac{\sqrt{N}}{2} \]
String DAC Statistical Performance

How about statistics for the INL?

\[
INL = \max_k |INL_k|
\]

\[
INL_k = \sum_{i=1}^{k-1} \frac{R_{R_i}}{R_N} \left( \frac{N-k}{N-1} \right) - \sum_{i=k}^{N-1} \frac{R_{R_i}}{R_N} \left( \frac{k-1}{N-1} \right)
\]

INL is an order statistic

Distribution functions for order statistics are very complicated and closed form solutions do not exist

INL is not zero-mean and not Gaussian
Current Steering DAC Statistical Characterization

Unary weighted

Assume unary current source array and define $I_0 = 0$

$$V_{OUT}(k) = -R \sum_{j=0}^{k-1} I_j$$

$1 \leq k \leq N$

For notational convenience will normalize by $-R$ to obtain

$$I_{OUTX}(k) = \sum_{i=0}^{k-1} I_i$$

$1 \leq k \leq N$

Assume current sources are random variables with identical distributions

$$I_j = I_{NOM} + I_{Rj} \quad I_{Rj} \sim N(0, \sigma_1)$$
Current Steering DAC Statistical Characterization

Unary weighted

\[ \text{INL}_k(k) = \frac{\sum_{j=0}^{k-1} I_j - I_{\text{FIT}}(k)}{I_{\text{NOM}}} \quad 1 \leq k \leq N \]

\[ I_{\text{FIT}}(k) = \frac{k - 1}{N - 1} \left( \sum_{j=1}^{N-1} I_j \right) \quad 1 \leq k \leq N \]

\[ \text{INL}_k(k) = \frac{\sum_{j=1}^{k-1} I_j - \frac{k - 1}{N - 1} \left( \sum_{j=1}^{N-1} I_j \right)}{I_{\text{NOM}}} \]

\[ \text{INL}_k = \frac{\sum_{i=1}^{k-1} \left( 1 - \frac{k - 1}{N - 1} \right) I_i - \frac{k - 1}{N - 1} \sum_{i=k}^{N-1} I_i}{I_{\text{NOM}}} \]
Current Steering DAC Statistical Characterization

Unary weighted

\[ \text{INL}_k = \frac{\sum_{i=1}^{k-1} \left( 1 - \frac{k-1}{N-1} \right) I_i - \frac{k-1}{N-1} \sum_{i=k}^{N-1} I_i}{I_{\text{NOM}}} \]

Model the current sources as

\[ I_j = I_{\text{NOM}} + I_{R_j} \]

\[ \text{INL}_k = \frac{\sum_{i=1}^{k-1} \left( 1 - \frac{k-1}{N-1} \right) \left( I_{\text{NOM}} + I_{R_k} \right) - \frac{k-1}{N-1} \sum_{i=k}^{N-1} \left( I_{\text{NOM}} + I_{R_k} \right)}{I_{\text{NOM}}} \]

It can be shown that the nominal part cancels, thus

\[ \text{INL}_k = \sum_{i=1}^{k-1} \left( \frac{N-k}{N-1} \right) \left( \frac{I_{R_k}}{I_{\text{NOM}}} \right) - \frac{k-1}{N-1} \sum_{i=k}^{N-1} \left( \frac{I_{R_k}}{I_{\text{NOM}}} \right) \]

This is a sum of uncorrelated random variables
The variance of $I_{NKk}$ can be readily calculated

$$\sigma^2_{\text{INL}_k} = \sum_{i=1}^{k-1} \left( \frac{N - k}{N - 1} \right)^2 \sigma^2_{I_{Rk}/I_{\text{NOM}}} - \left( \frac{k - 1}{N - 1} \right)^2 \sum_{i=k}^{N-1} \sigma^2_{I_{Rk}/I_{\text{NOM}}}$$

$$\sigma^2_{\text{INL}_k} = \left[ (k - 1) \left( \frac{N - k}{N - 1} \right)^2 + (N - k) \left( \frac{k - 1}{N - 1} \right)^2 \right] \sigma^2_{I_{Rk}/I_{\text{NOM}}}$$

This simplifies to

$$\sigma^2_{\text{INL}_k} = \frac{(k - 1)(N - k)}{(N - 1)} \sigma^2_{I_{Rk}/I_{\text{NOM}}}$$
As for the string DAC, the maximum $\text{INL}_k$ occurs near mid-code at about $k=N/2$ thus

\[
\sigma_{\text{INL}_{k-\text{MAX}}} = \sigma_{\frac{I_R}{I_{\text{NOM}}}} \left[ \frac{\sqrt{N}}{2} \right]
\]

And, as for the string DAC, the INL is an order statistic and thus a closed-form solution does not exist.
The structure looks about the same as for the unary structure but now the current sources are binary weighted.

Ideally

\[ I_j = 2^{j-1} I_{NOM} \]

\[ 1 \leq j \leq n-1 \]

Define the decimal equivalent of \( b, k_b \), by

\[ k_b = \sum_{j=1}^{n} b_j 2^{j-1} \]

For notational convenience will normalize by \(-R\) to obtain

\[ I_{OUTX}(b) = \sum_{i=1}^{n} b_i I_i \]

for \(<0,0,...0> \leq b \leq <1,1,...1>\)
Current Steering DAC Statistical Characterization
Binary Weighted

\[ I_{FIT}(b) = \frac{k_b}{N-1} \sum_{i=1}^{n} I_i \]  
\[ 0 \leq k_b \leq N-1 \]

Thus

\[ I_{INL_k}(b) = \frac{I_{OUTX}(b) - I_{FIT}(b)}{I_{LSBX}} \]

for \( <0,0,\ldots0> \leq b \leq <1,1,\ldots1> \) or equivalently for \( 0 \leq k_b \leq N-1 \)

\[ I_{INL_k}(b) = \frac{\sum_{i=1}^{n} b_i I_i - k_b \sum_{i=1}^{n} I_i}{N-1 \sum_{i=1}^{n} I_i} \]
Bundled current sources assume comprised of unary current sources

\[ I_m = \sum_{k=2^{m-1}}^{2^m-1} I_{GK} \quad I_{Gk} = I_{NOM} + I_{RGk} \]

Thus

\[ INL_b = \frac{\sum_{i=1}^{n} \left( b_i \left( \sum_{k=2^{i-1}}^{2^i-1} I_{GK} \right) \right) - \frac{k_b}{N-1} \sum_{i=1}^{2^n-1} I_{Gi}}{I_{LSBX}} \]

Substituting the values for \( I_{GK} \), it can be shown that the nominal parts cancel thus

\[ INL_b = \frac{\sum_{i=1}^{n} \left( b_i \left( \sum_{k=2^{i-1}}^{2^i-1} I_{RGk} \right) \right) - \frac{k_b}{N-1} \sum_{i=1}^{2^n-1} I_{RGi}}{I_{LSBX}} \]
Current Steering DAC Statistical Characterization
Binary Weighted

This can be expressed as

\[
\text{INL}_b = \sum_{i=1}^{n} \sum_{k=2^{i-1}}^{2^i-1} b_i - \frac{k_b}{N-1} \frac{I_{RGk}}{I_{LSBX}}
\]

This is now a sum of uncorrelated random variables, thus

\[
\sigma_{\text{INL}_b} = \sqrt{\sum_{i=1}^{n} \sum_{k=2^{i-1}}^{2^i-1} \left( b_i - \frac{k_b}{N-1} \right)^2} \cdot \sigma_{\frac{I_{RGk}}{I_{LSBX}}}
\]

This reduces to

\[
\sigma_{\text{INL}_b} = \sqrt{\sum_{i=1}^{n} 2^{i-1} \left( b_i - \frac{k_b}{N-1} \right)^2} \cdot \sigma_{\frac{I_{RGk}}{I_{LSBX}}}
\]
Current Steering DAC Statistical Characterization

Binary Weighted

It can be shown that this assumes a maximum INL\(_b\) occurs at \(b=<011\ldots11111\) or \(b=<100\ldots0000>\)

Substituting \(b=<1000\ldots000>\)

\[
\sigma_{INL_{b=<1000..0>}} = \sqrt{2^{n-1} \left[ 1 - \frac{N/2}{N-1} \right]^2 + \sum_{i=1}^{n-1} 2^{i-1} \left[ \frac{N/2}{N-1} \right]^2} \cdot \sigma_{I_{RGk}/I_{LSBX}}
\]

This simplifies to

\[
\sigma_{INL_{b=<1000..0>}} = \sqrt{2^{n-1} \left[ 1 - \frac{N/2}{N-1} \right]^2 + \sum_{i=1}^{n-1} 2^{i-1} \left[ \frac{N/2}{N-1} \right]^2} \cdot \sigma_{I_{RGk}/I_{LSBX}}
\]

This can be expressed as

\[
\sigma_{INL_{b=<1000..0>}} = \sqrt{\frac{N}{2} \left[ 1 - \frac{N/2}{N-1} \right]^2 + \left( \frac{N}{2} - 1 \right) \left[ \frac{N/2}{N-1} \right]^2} \cdot \sigma_{I_{RGk}/I_{LSBX}}
\]
Current Steering DAC Statistical Characterization

Binary Weighted

\[
s_{INL_{b=<1000..0>}} = \sqrt{\frac{N}{2}} \left[ 1 - \frac{N/2}{N-1} \right]^2 + \left( \frac{N}{2} - 1 \right) \left[ \frac{N/2}{N-1} \right]^2 \cdot \sigma_{\frac{I_{RGK}}{I_{LSBX}}} \]

\[
s_{INL_{MAX}} \cong s_{INL_{b=<1,0,\ldots,0>}} \cong \frac{\sqrt{N}}{2} \sigma_{\frac{I_{RG}}{I_{LSBX}}}^2
\]

Note this is the same result as obtained for the unary DAC

But closed form expressions do not exist for the INL of this DAC since the INL is an order statistic
End of Lecture 9