ZERO SENSITIVITY ACTIVE FILTERS EMPLOYING FINITE GAIN VOLTAGE AMPLIFIERS

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ABSTRACT

A completely general characterization of active filters employing finite-gain amplifiers constructed from operational amplifiers is presented. Conditions for zero pole-, ω_o-, and Q-sensitivities with respect to the time constraints of the operational amplifiers are developed. These conditions impose constraints only upon the gain functions of the finite-gain amplifiers. Several finite-gain amplifiers are presented that satisfy the zero-sensitivity constraints and thus their use in any existing active filter employing finite-gain amplifiers results in a zero-sensitivity design. A performance comparison of an existing active filter using a conventional finite-gain amplifier with the same filter using the zero-sensitivity finite-gain amplifiers is made.

INTRODUCTION

At high frequencies the performance of active filters using operational amplifiers is affected by the properties of the operational amplifiers themselves. Geiger and Budak (1) have recently shown that it is possible to design active filters that possess zero pole-, ω_o-, and Q-sensitivity with respect to the operational amplifier (OP AMP) time constants. These zero-sensitivity filters were shown to be much less dependent on the properties of the OP AMPs than are conventional filters. It is shown in this paper that an interesting subset of these zero-sensitivity active filters are those that result from replacing the finite-gain amplifiers of <u>any</u> existing design employing finite-gain amplifiers with special finite-gain amplifiers henceforth referred to as zero-sensitivity amplifiers. As in the general zero-sensitivity design, no matching of the OP AMPs is required to obtain the zero-sensitivity properties of the filters employing the zero-sensitivity amplifiers.

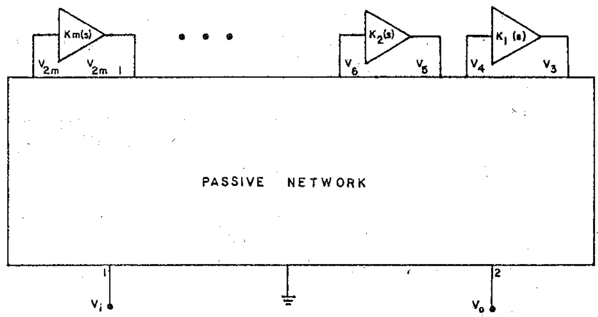
In Part I the general characterization of active filters employing finite-gain amplifiers is discussed. The transfer function of the active filter is written in a form that explicitly shows the effects of the frequency dependent nature of the finite-gain amplifiers.

A discussion of existing finite-gain amplifiers appears in Part II. The constraints on the gain function necessary to obtain a zero-sensitivity amplifier are developed and several zero-sensitivity amplifiers are subsequently introduced.

A comparison of the performance of an active filter using a conventional finite-gain amplifier is made with the same active filter using a zero-sensitivity amplifier in Part III. This comparison may be considered as representative for comparing active filters using conventional versus those using zero-sensitivity amplifiers.

PART I

The general active filter employing finite-gain amplifiers is shown in Fig. 1. The frequency dependent nature of the finite gain amplifiers is emphasized by the gain functions $K_i(s)$, $i = 1,\ldots m$. Ideally $K_i(s)$ should be independent of s.



General active filter employing m finite gain amplifiers

Fig. 1

Since most existing active filter sections use at most two finite-gain amplifiers and many use only one, the analysis here will be done for a single finite-gain amplifier. Results only will be stated for multiple finite-gain-amplifier active filters; however, these results are easily attainable by a straightforward extension of the analysis of the single-amplifier case.

With m = 1 in Fig., 1, the transfer function of the active filter may be written in the form

$$\frac{V_o}{V_i} = \frac{T_{21} + K_{10}(T_{41}T_{23} - T_{21}T_{43}) + (K_1(s) - K_{10})(T_{41}T_{23} - T_{21}T_{43})}{1 - K_{10}T_{43} + [K_{10} - K_1(s)]T_{43}} \qquad [1]$$

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where Ts are transfer functions associated with the passive network, K_{10} is the desired gain (ideal) of the finite-gain amplifier, and $K_1(s)$ is the actual gain.

After multiplying through by the least common multiple of all denominator polynomials of the passive transfer functions, [1] may be expressed as:

$$\frac{V_0}{V_i} = \frac{N_0 + K_{10}N_1 + [K_1(s) - K_{10}]N_2}{D_0 + K_{10}D_1 + [K_1(s) - K_{10}]D_1} \qquad [2]$$

where Ns and Ds are polynomials in s dependent only upon the passive network of Fig. 1. The effect on the transfer function of any departure of $K_1(s)$ from K_{10} is explicitly brought out in [2].

PART II

Various means have been used in the past to improve the performance of finite-gain amplifiers. First- and second-order compensation of uncompensated operational amplifiers requiring one and two external capacitors respectively whose values depend upon the properties of the OP AMP is discussed in text books and manufacturer's applications notes. Improved finite-gain amplifiers using only resistors and internally compensated OP AMPs have been discussed recently (2), (3), and (4). Reddy (3) has contributed designs requiring two OP AMPs that have a variable phase shift that can be adjusted by changing an external resistor, and Geiger (4) has introduced the maximum-bandwidth amplifiers.

In this paper consideration will be given only to those finite-gain amplifiers that are realized with resistors and internally compensated OP AMPs. The design of several two-OP AMP finite-gain amplifiers that result in zero-sensitivity active filters when used in any existing active filter employing finite gain voltage amplifiers are presented.

It is assumed that the OP AMPs discussed here are all internally compensated with zero output and infinite input impedances, infinite common-mode rejection ratio, and with a voltage gain function that can be approximated by (5)

$$A(s) = \frac{1}{\tau s} \qquad [3]$$

where the OP AMP time constant τ (in seconds) is the reciprocal of the gain-bandwidth product (in radians per second). The OP AMP is thus ideal when $\tau = 0$.

If p is a pole (or zero) of [2] which is the transfer function of a network containing n OP AMPs, then it can be expressed in a Maclaurin expansion as

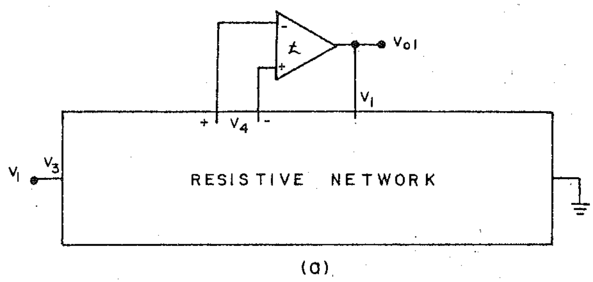
$$p = p\Big|_{\tau_1 = \cdots = \tau_n = 0} + \sum_{i=1}^{n} \frac{\partial p}{\partial \tau_i}\Big|_{\tau_1 = \cdots = \tau_n = 0} \tau_i$$

$$+ \text{ higher order terms.} \qquad [4]$$

The pole sensitivity is defined to be the pole derivative (1), (6). Thus if the pole sensitivity of each desired pole with respect to each OP AMP time constant is zero, only second- and higher-order time constant terms appear in the Maclaurin expansion for the pole positions rendering the transfer function quite insensitive to the OP AMP time constants.

The ω_0 and Q sensitivities are likewise defined to be equal to the derivatives of the respective functions where ω_0 and Q are respectively the pole resonant frequency and pole Q. It follows that the ω_0 and Q sensitivities are zero when the pole sensitivity is zero.

It can be shown that to obtain zero pole-sensitivity and zero zero-sensitivity for the transfer function given in (2) it is necessary and sufficient that the numerator polynomial of $[K_{10} - K_1(s)]$ contains no first-order time-constant terms (1).



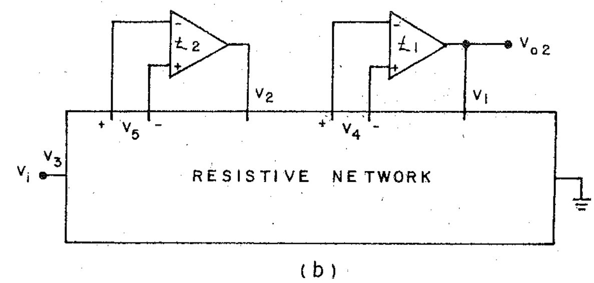


Fig. 2 General finite gain amplifiers

The general single-OP AMP and two-OP AMP finite-gain amplifiers employing only resistors and OP AMPs are shown in Fig. 2. The gain functions for these general amplifiers are respectively:

$$\frac{V_{01}}{V_i} = \frac{-c_{43}}{c_{41} + \tau_1 s} \qquad [5]$$

$$\frac{V_{02}}{V_i} = \frac{c_{42}c_{53} - c_{43}c_{52} - \tau_2 s c_{43}}{c_{41}c_{52} - c_{42}c_{51} + \tau_1 s c_{52} + \tau_2 s c_{41} + \tau_1\tau_2 s^2} \qquad [6]$$

where $c_{ij} = \frac{V_i}{V_j}\Big|_{V_k = 0}$ for $k \epsilon \{1,2,3\} - \{j\}$ is the transfer function of the resistive network from terminal j to terminal i. The lower case "c" has been used in the transfer functions to emphasize the frequency independent nature of these transfer functions.

The numerators of $[K_{10} - K_1(s)]$ are for the single- and two-OP AMP cases respectively:

$$N_1(s) = K_{10}c_{41} + c_{43} + \tau s K_{10} \qquad [7]$$

$$N_2(s) = K_{10}[c_{41}c_{52} - c_{42}c_{51}] + c_{40}c_{52} - c_{42}c_{53} + \tau_1 s K_{10} c_{52} + \tau_2 s(c_{41}K_{10} - c_{43}) + K_{10}\tau_1\tau_2 s^2. \qquad [8]$$

Since K_{10} is the desired dc gain, it follows from [7] that it is impossible in the single-OP AMP case to satisfy the zero amplifier-sensitivity condition that $N_1(s)$ contains no first-order time-constant terms.

The conditions for zero sensitivity with respect to both OP AMP time constants in the two OP AMP case

$$\left.\begin{array}{l} c_{52} = 0 \\ \text{and} \\ c_{41}K_{10} - c_{43} = 0 \end{array}\right\} \qquad [9]$$

are, however, realizable. It follows from [8] that the zero sensitivity properties are obtained without requiring matched OP AMPs.

The finite gain amplifiers shown in Fig. 3 all satisfy the zero amplifier-sensitivity conditions of [9] and thus their use in any active filter requiring finite-gain amplifiers results in a zero-sensitivity filter.

Several observations are to be made about the zero-sensitivity amplifiers.

1. The finite-gain amplifier of Fig. 3a and Fig. 3c are special cases of the phase shift adjustable amplifier introduced by Reddy (3).

2. No matching of the OP AMPs is required to obtain the zero-sensitivity amplifiers.

3. The zero-sensitivity amplifier constraints, [9], say nothing about the stability of active filters employing the zero-sensitivity amplifiers of Fig. 3.

4. The amplifier of Fig. 3a has infinite input impedance whereas the remaining two amplifiers do have a finite input impedance that may need to be included in filter designs using these amplifiers.

Finally, some justification for using only internally compensated OP AMPs and resistors to build zero-sensitivity amplifiers instead of externally compensating uncompensated OP AMPs is warranted. The obvious advantage of using a single OP AMP in an entire system to keep component inventories and component count down leads to the use of internally compensated OP AMPs. This fact aside, external compensation of an uncompensated OP AMP with either one or two capacitors requires the undesirable process of choosing capacitor (and sometimes resistor) values that are dependent on the parameters of the particular OP AMP used whereas the zero-sensitivity amplifiers of Fig. 3 require no matching of passive components to parameters of the active devices.

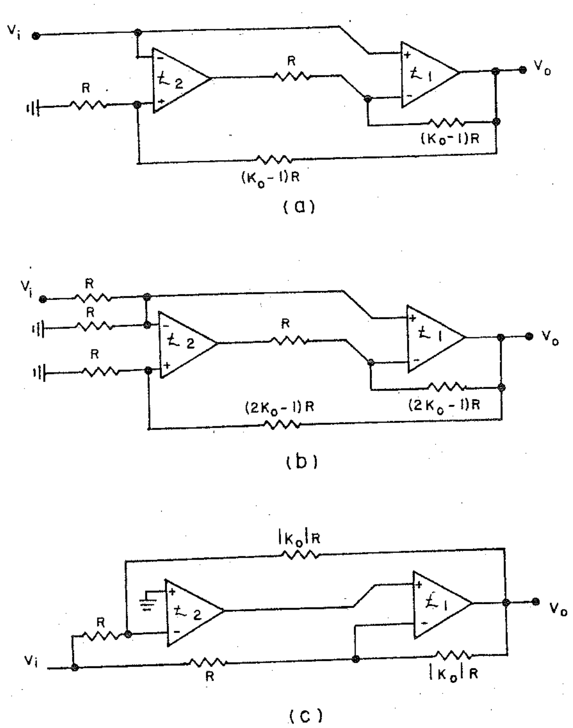


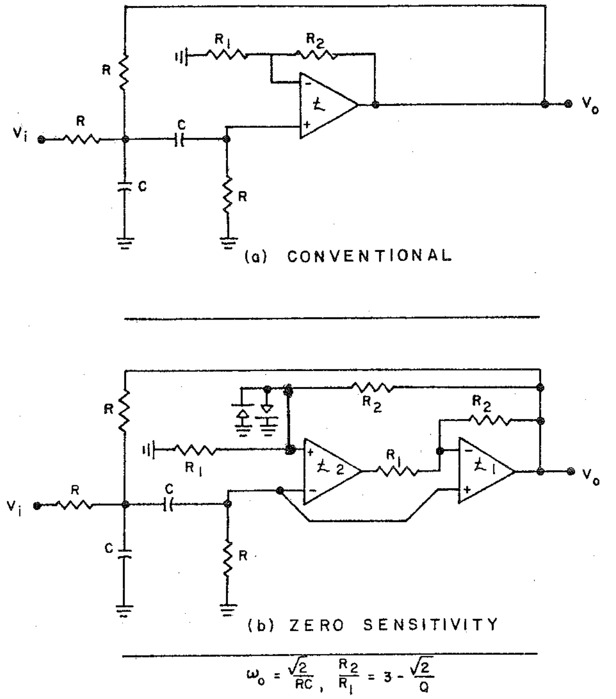
Fig. 3 Zero sensitivity amplifiers

It is easy to show that the conditions for zero sensitivity for active filters employing m finite-gain amplifiers are that the numerator polynomials of $[K_{i0} - K_i(s)]$, $1 \leq i \leq m$, all contain no first-order time-constant terms. It thus follows that if all m finite-gain amplifiers in Fig. 1 are replaced with the zero-sensitivity amplifiers of Fig. 3, then the resulting active filter will possess zero sensitivity with respect to each of the 2m OP AMP time constants and that this zero sensitivity is obtained without requiring matched OP AMPs.

PART III

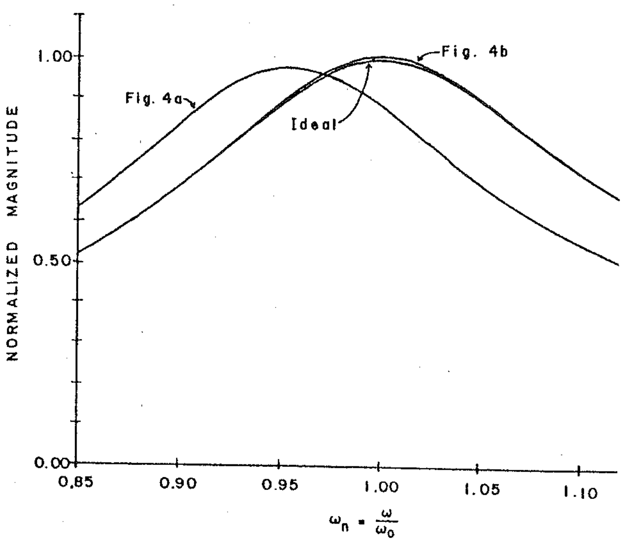
The performance of an active filter employing a unity gain amplifier (2), (8), (9), is discussed in this section. The zero-sensitivity amplifier of Fig. 3a and the conventional unity-gain amplifier are used in the bandpass filters shown in Fig. 4. The transfer function magnitudes for $\tau\omega_o = .01$ and $Q = 5$ of these two filters are compared to the response obtained with ideal OP AMPs in Fig. 5. The performance improvement obtained from using the zero-sensitivity amplifier is quite obvious.

The performance of this zero-sensitivity filter is typical of that reported for other general zero-sensitivity filters in (1) and (7). The results are also similar to those presented by Reddy (3) who chose the phase shift in his Wien Bridge oscillator example so that the finite-gain



Example of a bandpass filter employing conventional and zero-sensitivity unity-gain amplifiers

Fig. 4



Comparison of transfer magnitude for circuits of Fig. 4, $Q = 5$, $\tau\omega_o = .01$

Fig. 5

amplifier was actually a zero-sensitivity amplifier.

It is to be concluded that although performance comparisons have been made for only a single filter, the improvement may be considered representative for that expected using zero-sensitivity amplifiers.

EXPERIMENTAL RESULTS

The zero-sensitivity bandpass filters of Fig. 4 were built using 741 type OP AMPs with measured GBs of $1.17 \times 2\pi \pm 3\%$ rad/sec. The following measured component values were used:

$$R = 10.095 \text{ K}\Omega \pm 5\Omega$$

$$C = 1.232 \text{ nf} \pm 1\%$$

$$R_2 = 10.095 \text{ K}\Omega \pm 5\Omega$$

$$R_1 = 3.795 \text{ K}\Omega \pm 5\Omega$$

The experimental and ideal values of f_0 and Q are compared in Table 1. The clamping diodes (3) do not affect the filter performance under normal operating conditions but are necessary when using the 741s to prevent oscillation when the power supplies are switched on or when large noise spikes appear on the input.

Table 1. Experimental Performance of Bandpass Filters of Fig. 4

	$f_o = \omega_o/2\pi$	Q	% diff in f_o
Fig. 4a	16.737 KHz.	4.07	7.5%
Fig. 4b	17.983 KHz.	4.04	0.6%
Ideal OP AMPs	18.097 KHz.	4.16	

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