A NEW INTEGRATOR WITH REDUCED AMPLIFIER DEPENDENCE FOR USE IN ACTIVE RC-FILTER SYNTHESIS

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ABSTRACT

The integrator Q factor has been recognized as a figure of merit for predicting performance of filter structures which employ integrator blocks. This factor optimizes the phase response of the integrator. It is shown that the filter transfer function is dependent on both the integrator's magnitude and phase responses. The paper presents a new circuit for use as an integrator block in filters. It is designed to eliminate first-order operational amplifier time constant effects on the filter transfer function without requiring matched operational amplifiers. This circuit optimized both the magnitude and phase response of the integrator, while integrators designed for a high Q factor optimize only the phase response. The paper shows the improvement in magnitude response of the proposed integrator over the state of the art circuits. It also shows that the new circuit's phase response closely matches that of the state of the art integrators with the highest Q. The new circuit achieves this improvement in performance with a lower component count than the high Q designs.

INTRODUCTION

Integrator blocks find use in many popular filter structures [1] - [7] that typically have low passive sensitivities, low component count, or a choice of low-, high-, and band-pass outputs. The non-zero time constant of the operational amplifiers employed in these integrators can seriously degrade filter performance at high frequencies.

The integrator transfer function, $I(j\omega)$, may be written as follows:

$$I(j\omega) = \frac{1}{R(\omega) + jX(\omega)} \tag{1}$$

The quality factor Q, is defined to be [7], [8]

$$Q = \frac{X(\omega)}{R(\omega)} . \tag{2}$$

Note that the phase angle, θ, of $I(j\omega)$ is defined as

$$\theta = - \arctan \left[\frac{X(\omega)}{R(\omega)}\right] = - \arctan (Q). \tag{3}$$

An integrator's phase is ideally 90 degrees; therefore, the Q is ideally infinite. The state of the art circuits [8] - [10] seek maximize the Q thus optimizing the phase response of the integrator.

SENSITIVITY ANALYSIS

The performance of active filters employing these integrators rather than the characteristics of the integrators themselves is of prime importance. The filter transfer function is dependent upon both the integrator's magnitude and phase responses. If the integrator is characterized by a transfer function of the form

$$I(j\omega) = I\, e^{j\theta} \tag{4}$$

then a filter structure with two integrator blocks has a transfer function which may be written functionally as

$$T_F(j\omega) = F(I_1, \theta_1, I_2, \theta_2). \tag{5}$$

It follows from a MacLaurin series expansion of F in the four variables I_1, I_2, θ_1, and θ_2 that the relative change in F due to deviations of the integrators from ideal can be approximated by

$$\frac{\Delta F}{F} = \frac{\partial F}{\partial I}\frac{\Delta I_1}{F} + \frac{\partial F}{\partial \theta_1}\frac{\Delta \theta_1}{F} + \frac{\partial F}{\partial I_2}\frac{\Delta I_2}{F} + \frac{\partial F}{\partial \theta_2}\frac{\Delta \theta_2}{F} \tag{6}$$

where F and all partial derivatives are evaluated at the nominal or ideal values for the integrators. This may be expressed in terms of the sensitivities:

$$\frac{\Delta F}{F} = \frac{\Delta I_1}{I_1} S^F_{I_1} + \frac{\Delta \theta_1}{\theta_1} S^F_{\theta_1} + \frac{\Delta I_2}{I_2} S^F_{I_2} + \frac{\Delta \theta_2}{I_2} S^F_{\theta_2} \tag{7}$$

This analysis can be extended for filters employing any number of integrator blocks.

For the second-order, state-variables band-pass filter shown in Figure 1, the gain is given by the expression

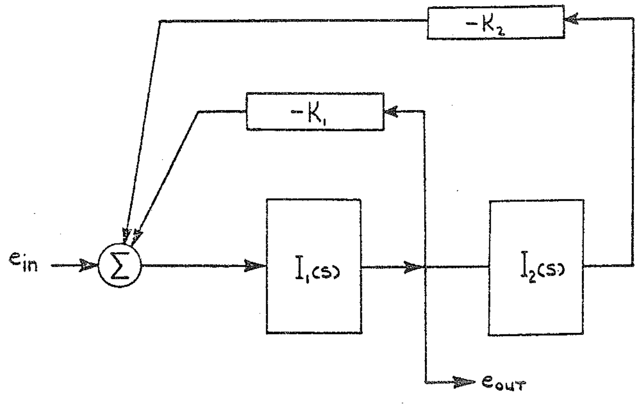

Figure 1: Second-order, State-variable Filter Structure with Integrators $I_1(s)$ and $I_2(s)$.

$$T_F(j\omega) = F(I_1, \theta_1, I_2, \theta_2) = \frac{I_1 e^{j\theta_1}}{1 + K_1 I_1 e^{j\theta_1} + K_2 I_1 I_2 e^{j(\theta_1+\theta_2)}} \quad (8)$$

For typical choices of I_1, θ_1, I_2, and θ_2 in (8) it can be shown that the terms multiplying $\Delta I_1/I_1$ and $\Delta I_2/I_2$ are comparable in magnitude to those multiplying $\Delta\theta_1/\theta_1$ and $\Delta\theta_2/\theta_2$. Since the sensitivities in (7) are dependent upon the topology of the filter structure rather than the topology of the specific integrator employed, it is desirable to design the integrator blocks such that both $\Delta I/I$ and $\Delta\theta/\theta$ approach zero for optimum filter performance.

PROPOSED INTEGRATOR

An inverting integrator circuit with first-order Op Amp time constant effects eliminated is shown in Figure 2. Given the normalization s_n = sRC and $\tau_n = \tau/RC$, the transfer function is

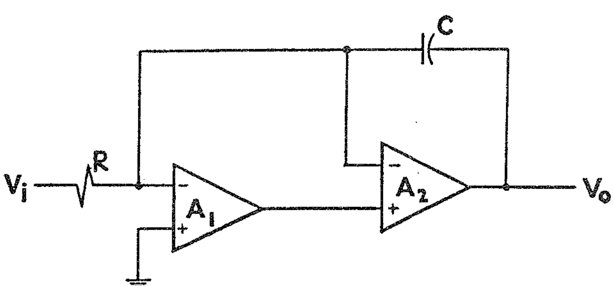

Figure 2: Integrator with First-order Op Amp Time Constant Effects Eliminated.

$$I(s_n) = \frac{-(1 + \tau_{1n}s_n)}{s_n[1 + s_n\tau_{1n}] + s_n^2\tau_{1n}\tau_{2n} + s_n^3\tau_{1n}\tau_{2n}} \quad (9)$$

Ignoring second-order terms and without requiring matched Op Amps, this reduces to the ideal

$$I(s_n) = \frac{-1}{s_n} \quad (10)$$

This demonstrates that all first-order Op Amp time constant effects are eliminated for both the integrator and for any filter employing this circuit.

COMPUTER ANALYSIS

Figure 3, and 4 are computer derived plots of the percent difference from the ideal versus frequency for the magnitude and phase angle of the circuit of Figure 2 and a selection of the state of the art integrators [8], [9]. Two different values of normalized τ are used. It can be seen that the magnitude response of the circuit of Figure 2 is much improved over any other integrator graphed even for the τ τ_n = 0.050 case. For the state-variable filter of Figure 1, this value of RC would correspond to a center frequency of 50 KHz for Op Amps with gain bandwidths of 1 MHz. It can be seen from these plots that the phase response of the circuit of Figure 2 closely matches that of the state of the art circuits with the highest Q factor.

Another advantage of the integrator of Figure 2 is that it has a component count of four. This is just one more Op Amp than the popular single Op Amp integrator with a much improved performance. This component count is the same as Brackett and Sedra's High Q inverting integrator ([8] circuit 1e.) while the new integrator has an improved magnitude response. The circuit of Figure 2 has less components than Soliman and Ismail's actively compensated Balanced Time Constant Integrator [9] which has the further disadvantage of requiring matched RC products.

CONCLUSION

In conclusion, the circuit of Figure 2 demonstrated a much improved magnitude response and a phase response closely matching the best of the state of the art integrators. It achieves this with a low component count.

REFERENCES

[1] L.T. Bruton, "Multiple-amplifier RC-active filter design with emphasis on GIC realizations," IEEE Trans. Circuit and Syst., Vol. CAS-25, pp. 830-845, Oct. 1978.

[2] ________, "Topological equivalence of inductorless ladder structures using integrators," IEEE Trans. Circuit Theory, vol. CT-20, pp. 434-437, July 1973.

[3] F.E.J. Girling and E.F. Good, "Active Filters-7. The two integrator loop," Wireless World, pp. 76-80, Feb. 1970; "Active Filters-8. The two integrator loop, continued," pp. 134-139, Mar. 1970.

[4] ________, "Active filters - 12. The leap-frog or active ladder synthesis," Wireless World, vol. 76, pp. 117-119, Mar. 1970; "Active filters - 13. Applications of the active ladder synthesis," pp. 134-139, Apr. 1970.

[5] D. Akerberg and K. Mossberg, "Low sensitive easily trimmed standard building block for active RC filters," Electron. Letter, vol. 5, pp. 528-529, 1969.

[6] L.C. Thomas, "The biquadi Part I - Some practical design considerations, Part II - A multipurpose active filtering system," IEEE Trans. Circuit Theory, vol. CT-18, pp. 350-361, May 1971.

[7] F.E.J. Girling and E.F. Good, "Active Filters - 5. An integrator and a lag in a loop," Wireless World, pp. 568-572, Dec. 1969.

[8] P.O. Brackett and A.S. Sedra, "Active compensation for high frequency effects in op-amp circuits with applications to active RC-filters," IEEE Trans. Circuits and System, vol. CAS-23, pp. 68-72, Feb. 1976.

[9] A.M. Soliman and M. Ismail, "On the active compensation of noninverting integrators," Proc. IEEE, vol. 67, pp. 961-963, June 1979.

[10] W. Kellner, "Noninverting integrators with improved high-frequency response," Electron. Letter, vol. 15, No. 9, pp 257-258, April 26, 1979.

Key for Figures 3 & 4

Circuit of Figure 2

Soliman, active compensated BTC [9]

Brackett and Sedra, Circuit 1e [8]

Brackett and Sedra, Circuit 1f [8]

Brackett and Sedra, Circuit 1d [8]

Popular Single Op Amp Integrator

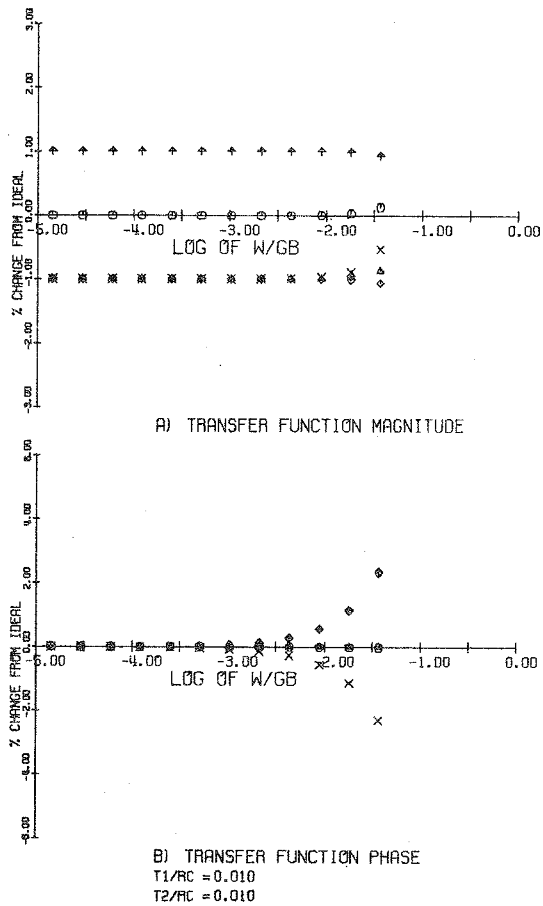

Figure 3

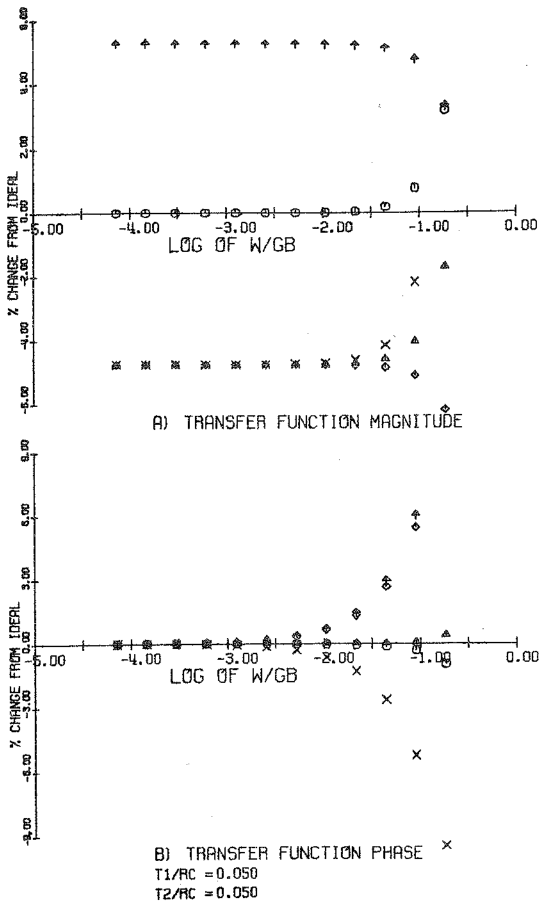
% CHANGE FROM IDEAL
LOG OF W/GB
A) TRANSFER FUNCTION MAGNITUDE
% CHANGE FROM IDEAL
LOG OF W/GB
B) TRANSFER FUNCTION PHASE
T1/RC = 0.050
T2/RC = 0.050

Figure 4