OPERATIONAL AMPLIFIER GAIN-BANDWIDTH PRODUCT EFFECTS ON THE PERFORMANCE OF SWITCHED-CAPACITOR NETWORKS

Randall L. Geiger
Department of Electrical Engineering
Texas A&M University

Edgar Sanchez-Sinencio
Department of Electrical Engineering
INAOE, Puebla, Puebla MEXICO

ABSTRACT

A method of analyzing switched-capacitor filters which incorporates a single-pole model of the operational amplifiers is presented. Closed-form algebraic expressions for filter transfer functions in the z-domain are obtained. This approach to filter analysis is computationally very efficient when compared to time domain methods. The necessity of including a frequency dependent model of the OP AMP rather than the common finite gain model when considering stability and doing a performance analysis is emphasized.

To illustrate the method of analysis an analog second-order bandpass filter is transformed by two different methods into a switched-capacitor counterpart. The s-domain performance of the analog circuits is compared with the z-domain performance of the sampled-data configurations to show how the finite gain-bandwidth product of the operational amplifiers affects the respective topologies. These comparisons show that the effects of switching rates and switching arrangements on filter performance are strongly dependent upon the characteristics of the operational amplifiers. These comparisons also show that it is not sufficient to investigate the effect of the operational amplifiers on the performance of an analog filter to predict how they will affect the performance of a switched-capacitor filter derived from the analog configuration.

I. INTRODUCTION

The effects of the finite gain-bandwidth product (GB) of the Operational Amplifier (OP AMP) on the performance of active filters has received considerable attention in the literature. The effects of GB on the performance of analog sampled data filters employing OP AMPs and switched capacitors (SC) are investigated here.

Several authors have recently presented systematic analysis procedures for obtaining closed-form expressions for the z-domain transfer functions of SC filters [1] - [4]. The OP AMPs have either been assumed to be ideal or to have a finite frequency independent gain in these analyses. Generally one of these two assumptions was standard when analyzing active RC filters up until the early 70's at which time it was generally agreed that a frequency independent model of the OP AMP was inadequate. Since that time, the single-pole model of the OP AMP has received widespread acceptance. One would suspect that it is equally important to include at least a single-pole model of the OP AMP when analyzing analog sampled-data filters. Such a model has been employed by Martin & Sedra [5], Temes [6] and Geiger & Sanchez [7] to analyze SC integrators but extension of these works to second-and higher-order networks is only possible for a special restrictive class of filter structures.

Here a continuous time second-order state variable filter is transformed into a sampled-data filter by replacing the integrators by two popular different sampled data integrators. An examination of these two SC filters follows. It is concluded from this investigation that stability of the analog filter does not guarantee stability of the derived sampled-data filter when the OP AMP effects are included even when stable mappings are used (i.e. Bilinear Mapping).

II. DETERMINATION OF OPERATIONAL AMPLIFIER GB EFFECTS IN ACTIVE NETWORKS

All operational amplifiers will be assumed to have infinite input impedance, infinite CMRR, zero output impedance, and gain given by

$$A(s) = GB/s \tag{1}$$

where GB is the gain-bandwidth product of the OP AMP. Since the OP AMP has been modeled as an ideal integrator, it can be concluded that $v_0(t)$ is continuous for all time (i.e. no jumps occur at switch transitions) with bounded inputs.

The temptation exists to analyze filters employing the SC integrators in terms of flow-diagrams and the integrator gains of the individual integrators which have already been derived [5 - 7] as is done for continuous time filters and ideal sampled-data filters. Unfortunately this method of analysis can not be used in general since the integrator outputs (which often are actually inputs to the next integrator) do not remain constant throughout the interval ([n-1]T, nT) which was required in the analysis of the integrators.

Bandpass Sampled-Data Filters

A second-order integrator-based bandpass filter is shown in Fig. 1a. This analog filter and the subsequently derived sampled-data filters were selected because they are topologically relatively simple but serve as good examples for demonstrating the importance of considering gain-bandwidth product effects when designing analog sampled-data filters.

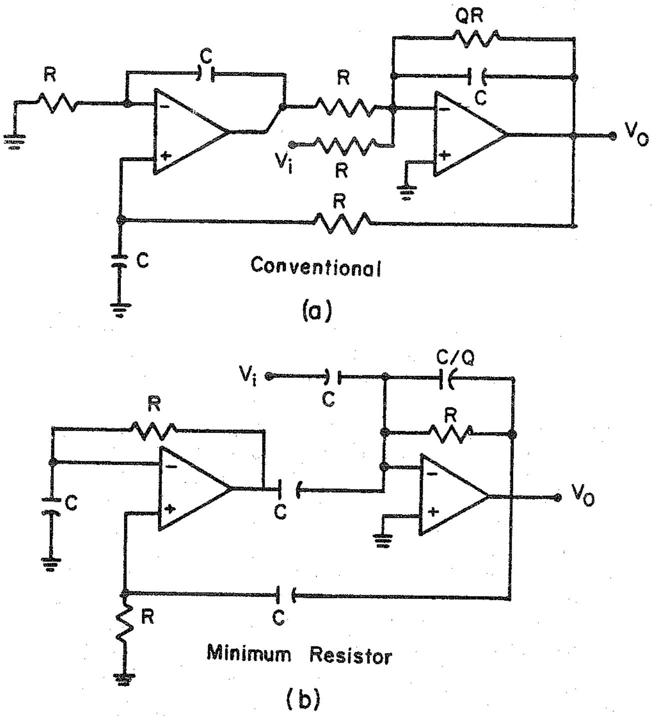

Fig. 1 Second-Order Analog Bandpass Filters

The transfer function of this analog filter is ideally

$$T(s) = \frac{-s\,\omega_o}{s^2 + s\,\frac{\omega_o}{Q} + \omega_o^2} \tag{2}$$

where $\omega_o = 1/RC$.

A sampled-data filter derived from this analog filter by using the forward integrator is shown in Fig. 2(a).

A bilinear switching transformation of this analog filter will also be considered due to the attractive properties of the bilinear s-plane to z-plane transformation. Because of the large number of resistors in the analog filter of Fig. 1a it is transformed to the minimum resistor configuration of Fig. 1(b) to reduce the number of capacitors in the ensuing bilinear transformed switched capacitor filter of Fig. 2b. The circuit of Fig. 2(b) will now be analyzed.

For $(n-1)T \leq t < nT$, the circuit of Fig. 2(b) is linear and can be redrawn as in Fig. 3 where the voltages $e_i(t)$, $i = 1, \ldots 8$ are the capacitor voltages at time t. From the model of the OP AMPs it follows that

$$v_2(t) = -GB_2 \int_{nT-T}^{t} (v_a(\tau) - v_b(\tau))\, d\tau + v_2(nT-T)$$
$$v_o(t) = -GB_1 \int_{nT-T}^{t} v_c(\tau)\, d\tau + v_o(nT-T) \tag{3}$$

By applying conservation of charge during linear operation and at switching these integral equations can be solved for $v_o(t)$. This solution can be evaluated at $t = nT$ because of the continuity of $v_o(t)$ to obtain a difference equation relating $v_o(nt)$ to $v_i(nt)$. The transfer function in the z-domain follows by taking the z-transform of the difference equation. The details of this derivation are tedious and appear in [8]. Due to space limitations, the results of the analysis for both circuits of Fig. 2 will only be presented in graphical form here.

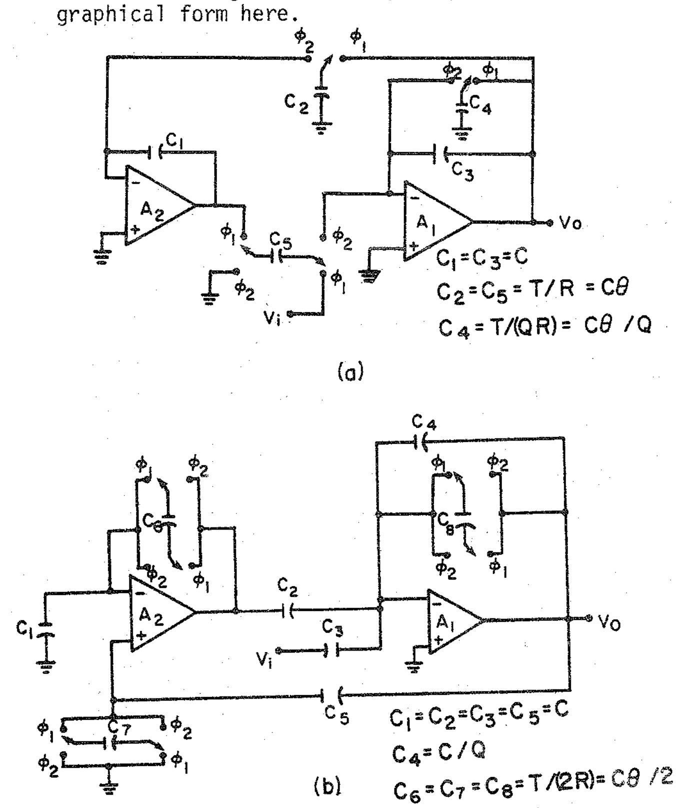

Fig. 2 Sampled-Data Bandpass Filters (a) Forward, (b) Bilinear

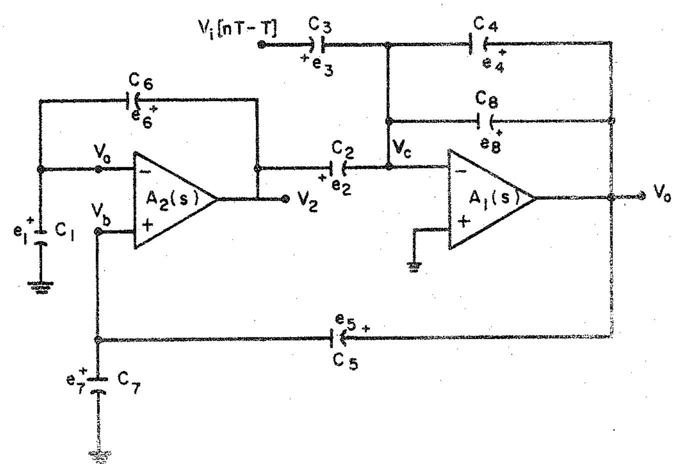

Fig. 3 Circuit of Fig. 2(b) for $nT-T < t < nT$

III. OPERATIONAL AMPLIFIER EFFECTS ON PERFORMANCE OF ANALOG SAMPLED DATA BANDPASS FILTER

In this section a comparison of the performance of the analog bandpass filter of Fig. 1 is made with that of the two derived switched-capacitor filters of Fig. 2. A design pole Q of 10 is assumed throughout. The symbol ω_0 (rad/sec.) denotes the design center frequency of the bandpass filters and T the time between sampled-data points. The normalized gain-bandwidth product is defined by $GB_n = GB/\omega_0$. Fig 4. shows a plot of the upper half-plane desired poles for the analog filter of Fig. 1. Fig. 5 and Fig. 6 are plots of the upper half plane desired poles for values of $\theta = (0.01)\ 2\pi$ rad/sec. and $(.05)\ 2\pi$ rad/sec. where $\theta = T\omega_0$. Note that the clock frequency for the switch control is related to θ by $f_c = \frac{\omega_0}{\theta}$ for the circuit of Fig. 2(a) and $f_c = \frac{\omega_0}{2\theta}$ for the circuit of Fig. 2(b). It should be noted that the $GB_n = \infty$ pole locations differ for the two filters of Fig. 2. The absence of pole-locus plots for the circuit of Fig. 2(a) when $\theta = (0.05) \times 2\pi$ is due to the fact that this circuit is unstable with this slow switching rate even if the OP AMPs are ideal.

Several observations about the performance and stability of these filters can be made from the pole-locus plots.

1) The value of GB_n required for stability of the analog filter is different than that for either of the sampled-data filters.
2) The pole-locus and stability criterion for the SC filter appears to be strongly dependent upon the particular switching arrangement employed.
3) Larger values of θ (slower switch clock rates) significantly reduce OP AMP requirements as can be seen from a comparison of Fig. 5 and 6. This, however, is done the expense of increased warping in the s-plane to z-plane transformation. It is interesting to note that for $\theta = (.01)\ 2\pi$ and $\theta = (.05)\ 2\pi$ the sampled-data filter of Fig. 2(b) is unstable for certain values of GB_n whereas the analog filter from which it was derived remains stable.
4) The spiraling nature of the pole-locus for the circuit of Fig. 2(b) is most interesting. It demonstrates that improving GB locally may well result in a deterioration in filter performance or actually cause instability.
5) The spiraling nature of the desired poles can not be obtained by employing the finite gain model of the OP AMP.

The combined effects of the parasitic poles and desired poles as well as the zero locations can be seen in the transfer function magnitude plots of Fig. 7. The plots show the gains $|T(j\omega)|$ and $|H(e^{j\omega})|$ for the analog and sampled-data circuits respectively on the same set of axes. One is cautioned that the pole plots should be considered in conjunction with the magnitude

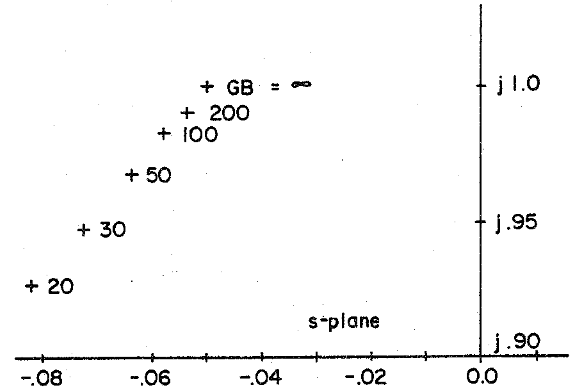

Fig. 4 Desired Poles of Analog Filter of Fig. 1(b) GB_n values indicated on plot.

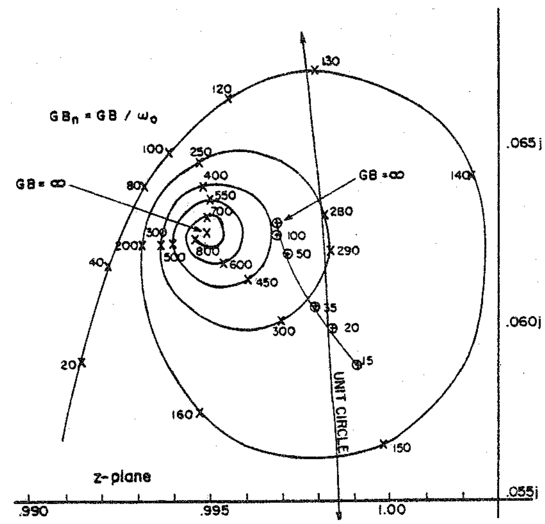

Fig. 5 Poles of Sampled-Data Filters $\theta = 0.01 \times 2\pi$, $GB_n = \frac{GB}{\omega_0}$ values indicated on plot, X - circuit of Fig. 2(b), + - Circuit of Fig. 2(a)

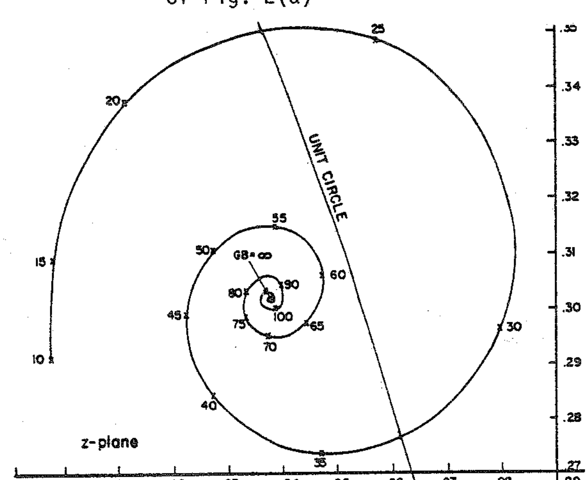

Fig. 6 Poles of Sampled-Data Filters $\theta = 0.05 \times 2\pi$, $GB_n = \frac{GB}{\omega_0}$ values indicated on plot, X - Circuit of Fig. 2(b).

plots since the apparently good performance for some values of GB_n in Fig. 2(e.g. $\theta = .01 \times 2\pi$, $GB_n = 100$) are only obtained because the pole locations on the spiral are locally close to the desired pole locations but this performance deteriorates rapidly for small changes of GB in either direction.

A general SC filter analysis employing any number of OP AMPs is outlined in [8].

The economic advantages of conducting a closed form analysis of sampled-data filters are significant. The total CPU time required to generate the data for all root locus and transfer function magnitude plots presented in this paper is well under 1 minute on a Amdahl 460V and no attempt was made to optimize the algorithms employed. Aside from the economic advantages when compared to existing time domain approaches, a time domain analysis, which may even be based upon a single value of GB, may miss the spiraling motion such as was obtained for the circuit of Fig. 2(b) giving false encouragement for acceptable filter performance.

IV. EXPERIMENTAL RESULTS

The switched-capacitor filter of Fig. 2(b) was designed for a center frequency of 200Hz, $Q = 10$, and $\theta = (.01)(2\pi)$. The low center frequency was picked so that the OP AMPs could be replaced by a GB controllable [8] macromodel to provide a convenient means of experimental GB adjustment.

Close agreement with the theoretical performance was observed. The circuit went into low frequency oscillation (approx. 200 Hz) for $145 < GB_n < 166$ as expected. Additional details of the experimental evaluation appear in [8].

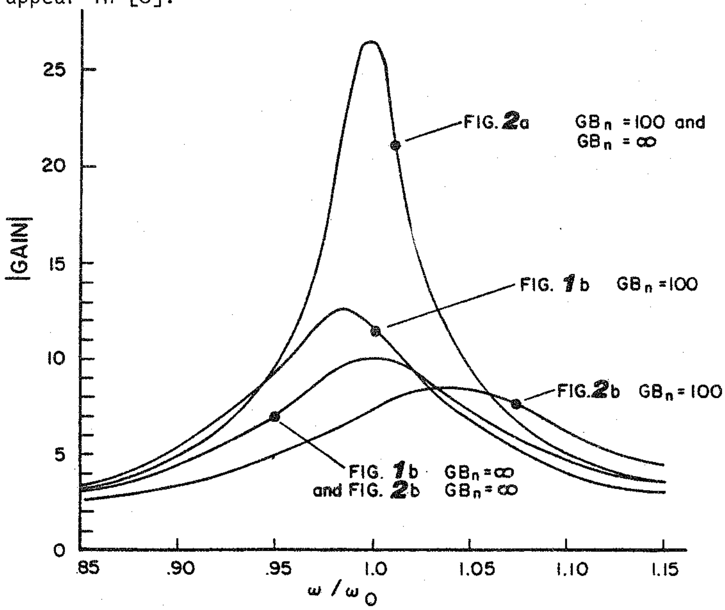

Fig. 7 Magnitude Responses for Sampled-Data Filters $\theta = .01 \times 2\pi$, $GB_n = 100$ and ∞

V. REFERENCES

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