

COMPONENT QUANTIZATION EFFECTS ON CONTINUOUS TIME FILTERS

Jaime Ramírez-Angulo, Randall L. Geiger and Edgar Sánchez-Sinencio

Texas A&M University
Dept. of Electrical Engineering, College Station, TX 77843

Abstract.- Recent trends in precision continuous-time active filter design use post fabrication tunable structures. Digitally adjustable quantization components are often used as tunable elements. It is shown that the performance of such filters is strongly dependent upon the circuit topology and component quantization schemes. Tradeoffs between number of bits, quantization scheme and resolution for some representative bandpass active filter structures are addressed. Applications to monolithic continuous-time circuit design are considered.

I. Introduction

Programmable filters find use in a wide variety of applications spanning the areas of audio, instrumentation, communications, medical electronics, etc. Digital control of the filter characteristics is a desirable feature that can be used for very accurate parameter setting. It also allows for digital system compatibility. The quantization of the component values of an RC-active filter results in filters which are characterized by a discrete set of attainable values for the pole frequency Ω_o and Q .

One popular way to digitally control the active filter's characteristics is through the use of multiplying digital to analog converters (MDACs) followed by op-amps [1]. This combination can be used to implement digitally adjustable gain blocks, integrators, leaky integrators and summers. In these blocks the MDAC works essentially as a binarily weighted resistance. In [2] and [3] second-order structures using MDACs for independent control of Ω_o and Q are compared. The structures are based on the State-Variable realization in [2], and on realizations with two gain blocks in [3]. Emphasis is placed on limitations imposed by the non-ideal performance characteristics of the MDAC-op amp combination. The adopted tuning scheme requires a minimum of two op amps for all structures in these comparison.

One simple scheme for achieving digitally tunable RC-active filters involves replacing some or all of the elements in any existing active filter structure with binarily weighted capacitor or resistor arrays. From a practical viewpoint, those active filters which offer independent Ω_o and Q adjustment may prove easier to tune. The implementation with MDACs can be considered to be a particular case with the resistors taking uniformly distributed values according to the binary law:

$$R = R_o(2^{-1}d_1 + 2^{-2}d_2 + \dots + 2^{-N}d_N).$$

where R_o determines the adjustment range and d_1, d_2, \dots, d_N represents the binary input. In any case, the set of all

possible component quantization values maps the poles in the filter structure to a discrete set of pole locations in the s-plane. The ensuing pole patterns are topology dependent. The most popular approach for implementing tunable filters is based on the State-Variable realization [4]-[5]. This is due primarily to attractive tunability characteristics, favorable sensitivities, and the flexibility for obtaining simultaneously BP, LP and HP transfer characteristics.

The tuning range and parameter resolution of digitally tuned filter structures depends primarily and interactively upon:

- 1) The filter structure
- 2) The component quantization scheme
- 3) The number of bits of the digital control

Component range, functional component value-filter characteristic relationship, number of components, interdependence of parameters, and component sensitivity must be considered when selecting the filter structure. The choice of the structure will be strongly dependent upon the required filter specifications. Filters with large component variations or many components should be avoided in monolithic applications due to large area requirements associated with practical realizations. Parameter independence will allow for convenient adjustment of the filter characteristics by individual components, possibly at the expense of increased cost or decreased performance capabilities. Functional component value-filter characteristic relationships will affect the resolution of the filter structure. Parameter independence does not guarantee attractive component value-filter characteristic relationships.

Circuits with low component sensitivities may prove useful for practically making small local adjustments but are inherently unsuitable for global adjustments of filter characteristics. Circuits with high component sensitivity allow for global parameter adjustment with modest changes in component values but have poor resolution.

In this paper we show a comparative study of the root loci for several representative second-order active filter structures obtained using a combination of two basic component quantization schemes. The aim of this study is to show that in addition to other well known properties, root loci as determined by component quantization must be considered as a important factor when selecting a filter structure for a specific digitally tunable application. Differences in the root loci associated with narrow and wide tuning range applications, effects of quantization schemes and the need to include distribution functions for Ω_o and

Q must be considered when optimizing digitally controlled filter designs.

II. Basic Quantization Schemes

Two basic quantization schemes for obtaining component values X within a prespecified range $[X_{MIN}, X_{MAX}]$ are considered here:

a) A direct quantization scheme where the 2^N discrete values of X are distributed uniformly in the closed interval $[X_{MIN}, X_{MAX}]$ according to the expression:

$$X = X_{MIN} + \Delta X [2^0 d_1 + 2^1 d_2 + \dots + 2^{N-1} d_N] \quad (1)$$

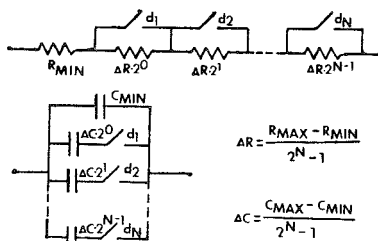
where X_{MIN} and X_{MAX} are the minimum and maximum values needed for the variation of the component, N is the number of bits in the digital word $\mathbf{d} = d_1, d_2, \dots, d_N$, and the incremental value ΔX is given by:

$$\Delta X = \frac{X_{MAX} - X_{MIN}}{2^N - 1} \quad (2)$$

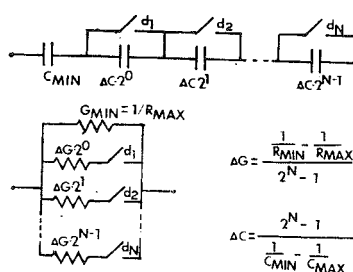
This switching scheme can be implemented for resistors with a series combination and for capacitors with a parallel combination of components as shown in Fig. 1A.

b) An inverse quantization scheme where the 2^N discrete values of $1/X$ are distributed uniformly over the closed interval $[1/X_{MAX}, 1/X_{MIN}]$ according to the expression:

$$\frac{1}{X} = \frac{1}{X_{MAX}} + \frac{1}{\Delta X} [2^0 d_1 + 2^1 d_2 + 2^{N-1} d_N]$$



1A. DIRECT QUANTIZATION SCHEME



1B. INVERSE QUANTIZATION SCHEME

where the inverse incremental value is given in this case by:

$$\frac{1}{\Delta X} = \left(\frac{1}{X_{MIN}} - \frac{1}{X_{MAX}} \right) \frac{1}{2^N - 1} \quad (4)$$

This scheme corresponds to switching arrangements of binarily weighted parallel resistors or series capacitors as is shown in Fig. 1B.

III. Comparative Study of RC-Filter Structures

A comparative study of four representative biquadratic filter structures is now made by using combinations of the two basic switching arrangements of Fig. 1 from both a local (narrow tuning range) and a global (wide tuning range) viewpoint. The biquad structures considered for this study are the bandpass version of the Sallen and Key, the Friend-Deliyannis, the GIC implementation and the KHN-State-Variable Filters. The specific structures along with design equations for independent control of Ω_o and Q are shown in Fig. 2. These were chosen so that single, double and multiple amplifier structures were represented in the comparative study.

In all structures an amplifier gain, K, a set of resistors or a set of capacitors which allow for independent Ω_o and Q adjustment were selected for digital control. For the circuits of Fig. 2B, C and D a single resistor or capacitor value serves as the only free design parameter. In addition to a passive component value, the designer has control of the

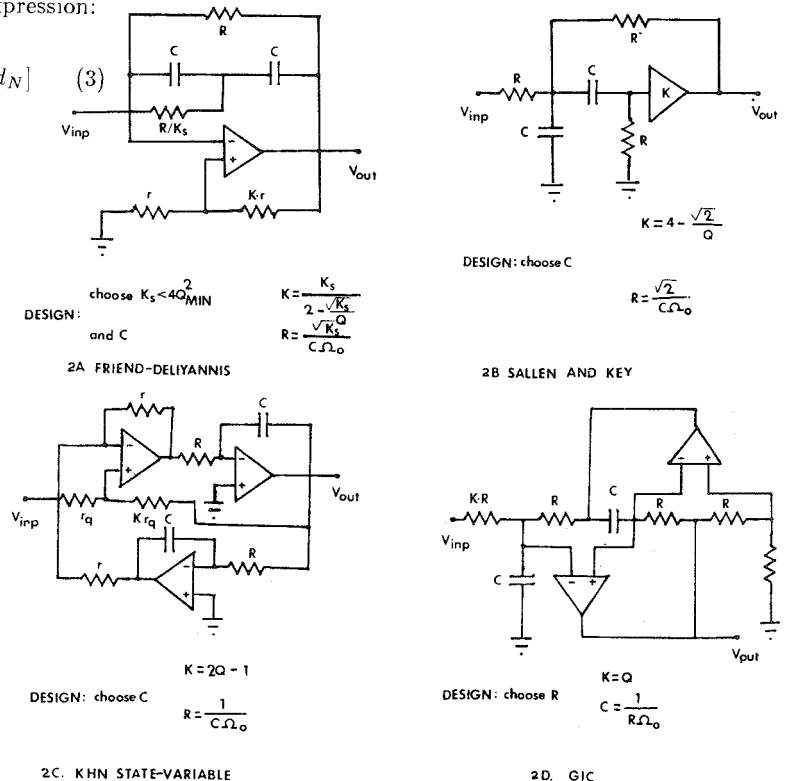


Fig. 1 Component quantization schemes.

Fig. 2 Biquad bandpass RC-active structures.

positive feedback parameter, K_s , in the circuit of Fig. 2A which allows the designer to make tradeoffs between sensitivity and component spread.

The study was done by assigning parameter values K and R or C leading to root loci spanning the same region in the s -plane for all circuits in the comparison. In the global comparison, relatively wide Q and Ω_o tuning ranges were chosen with values for Q adjustable from 0.505 to 100 and Ω_o adjustable over 1 decade from 0.1 to 1.0 rad/sec. Parameter values were quantized to three bits for all structures giving a total of 64 discrete pole positions. In the local comparison, Ω_o and Q were adjustable over a relative narrow range from 0.5 to 1.0 rad/sec (1 octave) and from 1.5 to 6 respectively. In the Friend-Deliyannis structure, the parameter K_s was assigned a value of 1 for the global comparison and 8 for the local comparison. It can be shown that these values are, in both cases, close to the maximum allowed value of $4Q_{MIN}^2$, where Q_{MIN} represents the minimum value of Q attainable with the digital control scheme.

A local comparison of the pole loci of all four circuits using the direct quantization scheme of Fig. 1A for both the Q and Ω_o tuning elements is shown in Fig. 3. Fig. 4 shows a local comparison of the pole loci obtained by using the inverse quantization scheme of Fig. 1B. Note that in the first case the performance of all structures is quite different. In the second case all structures show similar pole loci. Fig. 5 shows a global comparison of the pole loci obtained by using the inverse quantization scheme of Fig. 1B for both tuning parameters K and R or C . Note here that in spite of the fact that all the structures show a similar resolution from the local point of view, in the global

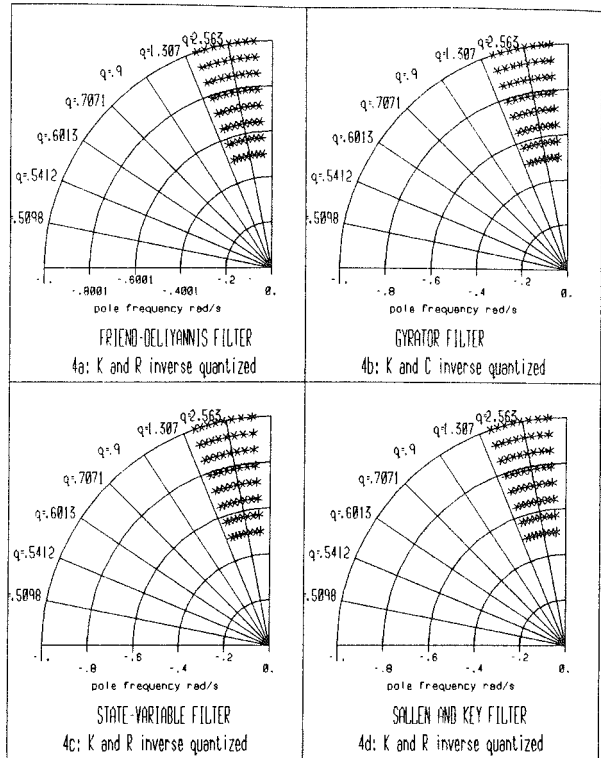


Fig. 4 Local comparison of filter structures with inverse quantized component values.

comparison they show quite different performance.

Next a comparison of the local and global quantization effects is made. For the sake of space this comparison is restricted to the State-Variable and to the Friend-Deliyannis structures. The pole loci are shown in Fig. 6. The inverse quantization scheme of Fig. 1B was used to adjust all parameter values. Note that although the State-Variable structure gives reasonable resolution with this quantization scheme for local adjustment, the resolution is extremely poor for high Q values in the global case.

Although we have restricted this comparison to 3 bit quantization schemes it should be apparent that for a given resolution the number of bits required is strongly a function of both the circuit structure and the quantization scheme. The selection of the optimal structure, switching arrangements and number of quantization bits must be made based primarily on the tuning range and on the specific distribution of the values of Q and Ω_o required for a particular application. The Q -distribution offered by general use commercially available tunable filter units based on the State-Variable realization may be far from optimal for some specific applications. If, for example, a linear Q -distribution over a wide tuning range is required, the GIC structure with K directly quantized is the best choice since $Q=K$. The State-Variable filter shows an approximately linear Q -distribution with the parameter K only for large Q -values. If, on the other hand, a uniform pole distribution (in the angular sense in the s -plane) over a wide tuning range is required, it is evident from Fig. 5 that the State-Variable is the least suitable.

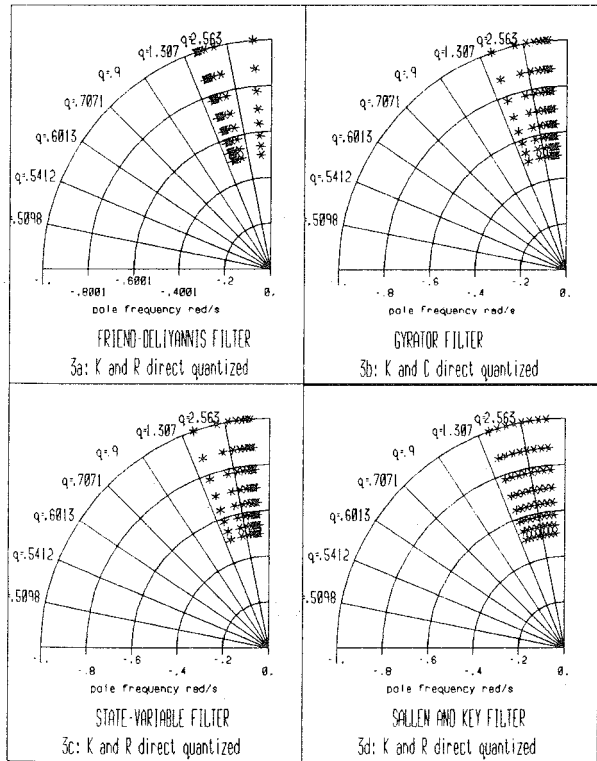


Fig. 3 Local comparison of filter structures with direct quantized component values.

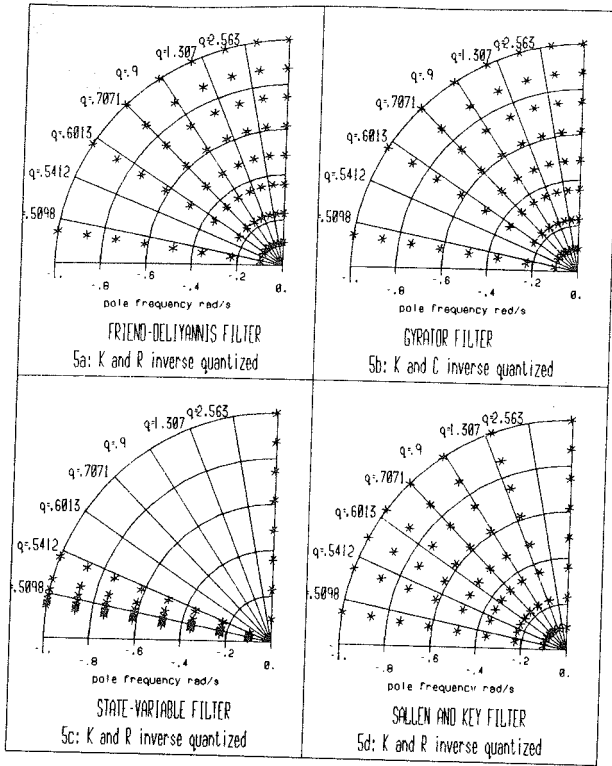


Fig. 5 Global comparison of filter structures with inverse quantized component values.

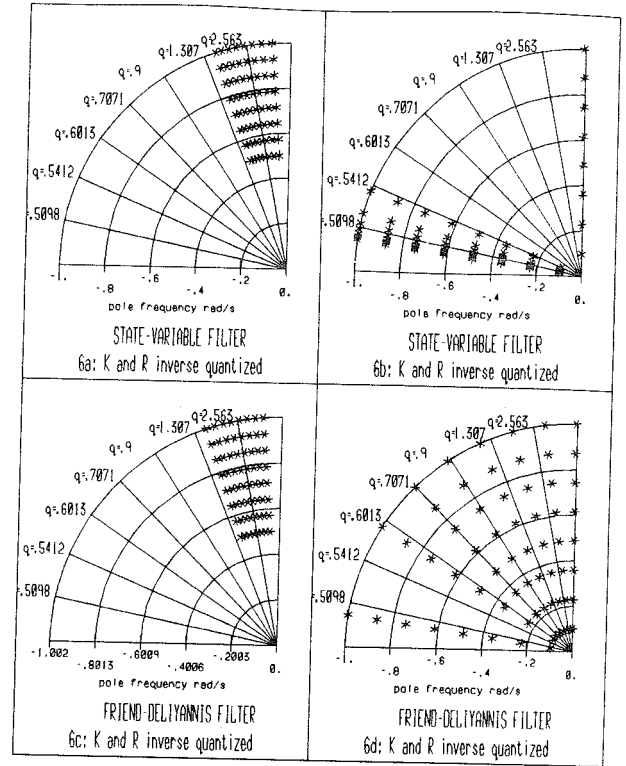


Fig. 6 Comparison of local and global performance for State-Variable and Friend-Deliyannis structures with inverse quantized component values.

Considering the high-Q approximation $Q = \frac{1}{2\alpha \sin \alpha} \approx \frac{1}{2\alpha}$ with α being the angle measured with respect to the imaginary Ω -axis, it can be recognized that an inverse distribution of the values of Q corresponds to a uniform distribution of α even for moderate values of Q . This and the analytical expressions for Q given in Fig.2 can be used to explain the fact that all structures show an approximately uniform pole locus when K is adjusted with the inverse quantization scheme.

IV. Conclusions

It has been shown that quantization schemes strongly affect resolution of filter characteristics. The circuit topology was shown to play a major role in the resolution even with identical quantization schemes. Effects of component quantization were shown to be much different for local and global applications of a given circuit. The importance of judiciously selecting both the circuit topology and component quantization schemes for for digitally controlled filters has been demonstrated.

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