POLE-ZERO PAIRING STRATEGY FOR AREA AND SENSITIVITY REDUCTION IN CASCADE SC FILTERS

Cai Xuexiang, Edgar Sánchez-Sinencio and Randall L. Geiger

Texas A & M University
Department of Electrical Engineering
College Station, TX 77843

Abstract.- The effects of pole-zero pairing on the total capacitance of cascaded biquad SC filters is investigated. It is shown that significant reductions in total capacitance are possible through optimal pole-zero pairing without causing degradations in filter performance. Gain distribution between the biquadratic sections is also shown to significantly affect total capacitance.

I. Introduction

It is generally the goal of the SC circuit designer to optimize circuit performance while simultaneously minimizing silicon area. Unfortunately this problem is sufficiently challenging that most designers commit suboptimal designs to fabrication. A method of reducing and/or minimizing silicon area while maintaing a given set of performance specifications for practical cascaded SC biquad structures by using an optimal pole-zero pairing strategy is discussed in this paper.

II. Development

It will be assumed that a function H(z) has been obtained which meets given desired performance specifications and that a circuit is to be synthesized to realize H(z). It will be further assumed that H(z) is to be realized as a cascade of biquadratic sections. The conventional approach in realizing H(z) as a cascade of biquads is to express H(z) as a product of biquadratic fractions as

$$H(z) = \prod_{k=1}^{n_1} \left(\frac{\alpha_{2k} z^2 + \alpha_{1k} z + \alpha_{0k}}{\beta_{2k} z^2 + \beta_{1k} z + \beta_{0k}} \right) = \prod_{k=1}^{n_1} H_k(z) \qquad (1)$$

where α_{ij} and β_{ij} are real for all i, j. The degree of H(z) is $2n_1$ for n_1 even and $2n_1 - 1$ for n_1 odd. $\beta_{2k} = 1$ for all k when n_1 is even and

$$\beta_{2k} = \left\{ \begin{matrix} 1 & k < n_1 \\ 0 & k = n_1 \end{matrix} \right\}$$

when n_1 is odd. $H_k(z)$ represents the corresponding biquadratic fraction in z. The SC structure is synthesized by cascading n biquadratic blocks with transfer functions $H_k(z)$, $z = 1, \cdots, n_1$.

It should be noted from (1) that the pole pair and zero pair pairings are not unique and that if there are real poles and/or zeros, even the pole pairings or zero pairings into biquadratic polynomials are not unique. The number of possible pole-zero pairings into biquadratic fractions can be quite large.

To determine the number of second-order pole-zero pairings, assume that all poles and zeros are unique and that there are n and m poles and zeros and nr and mr real axis poles and zeros respectively. Define $\phi(k)$ and $\theta(k)$ by the expressions

$$\phi(k) = \begin{cases} \prod_{i=0}^{\frac{k-2}{2}} (k-1-2i) & \text{for k even} \\ \prod_{i=0}^{\frac{k-1}{2}} (k-2i) & \text{for k odd} \end{cases} \qquad (2)$$

and

$$\theta(k) = \begin{cases} \frac{k}{2} & \text{for k even} \\ \frac{k+1}{2} & \text{for k odd} \end{cases} \qquad (3)$$

It follows that there are $\phi(k)$ unique decompositions of a group of k distinct elements into a group of element pairs and that each group contains $\theta(k)$ pairs.

Assuming the number of poles is greater than or equal to the number of zeros and that all zeros are paired (with the exception of the zero for m odd), it can be shown that the number of unique second-order biquadratic decompositions of $H(z)$ is given by

$$N = \phi(mr)\phi(nr)\binom{\theta(n)}{\theta(m)}\theta(m)! \qquad (4)$$

In the case that some of the real zeros are allowed to be singly decomposed rather than paired, as is often done when $m < n$, the number of unique second-order biquadratic decompositions of H(z) will increase significantly. Correspondingly, in the case that some poles or zeros are repeated, the number of unique decompositions will decrease.

To obtain an appreciation for the number of possible pole-zero pairings, it follows from (4) that if H(z) has 4 unique real zeros and 4 unique real poles, then N=18 whereas if there are 6 unique real zeros and 6 unique real poles, N=1350.

It should be apparent that for large m and n, the number of possible pairings is quite large. The questions naturally arises, "How should the pole-zero pairings be made?" and "How significant are the effects of a non-optimal pole zero pairing strategy?"

As a *simple* example, consider the 4th order Chebyschev bandpass approximation which has two distinct complex conjugate pole pairs located at P_1, P_1^*, P_2 and P_2^* and four real axis zeros. Two zeros are located at z=+1 and the other two are located at z=-1. There are three possible pole-zero pairing strategies for the biquadratic decomposition indicated pictorially in Fig. 1. It is reasonable to anticipate that there will be appreciably different characteristics of the circuits that are synthesized based upon the different decomposition schemes.

In an attempt to isolate the pole-zero pairing effects, it will be assumed that the topology for implementing all biquadratic sections is fixed and that the sections are all designed to have identical ω_0 and Q sensitivities. It will be further assumed that the gain of the individual biquads is distributed by one of the two strategies:

CH2255-8/86/0000-0609 $1.00 © 1986 IEEE

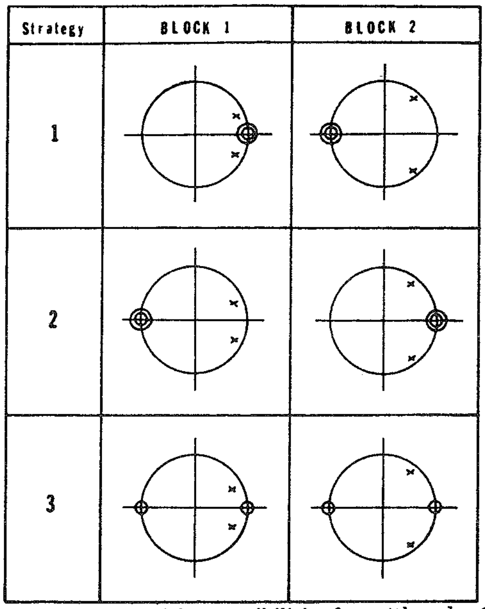

Fig. 1 Pole-zero pairing possibilities for a 4th-order Chebyshev Filter

Gain Distribution Strategy 1 (Single frequency)

$$|H_k(z)|_{z=e^{j\omega_1}} = | \; |H(z)|_{z=e^{j\omega_1}}|^{1/n}$$

for some ω_1 and for all $1 \leq k \leq n$.

Gain Distribution Strategy 2 (Equal peak gain)

$$\text{average}\left(\max_{\omega} |H_k(z)|_{z=e^{j\omega}}|\right) = H_m$$

for some constant H_m and for all $1 \leq k \leq n$.

Note that both gain distribution strategies are independent of how the individual biquads are ordered in the cascade. These types of gain distribution strategies were intentionally adopted, possibly at the expense of some dynamic range reduction, to circumvent the need for considering position in the cascade as another variable.

It is our conjecture that an optimal pole-zero pairing strategy, in the general case, is a function of H(z), the topology of the biquadratic block, the specific sensitivity requirements, the gain distribution strategy, the sequence of the biquadratic blocks in the cascade, and the system clock frequency. An analytical treatment of the problem appears to be unwieldly. The significance of the pole-zero pairing strategy is demonstrated in the following section by considering practical examples.

III. Significance of Pole-Zero Pairing Strategy

It will be assumed that all even-order biquadratic sections are realized with the low GB sensitivity structures of [1] shown in Fig. 2 and that partial positive feedback is used in these structures to reduce total capacitance [2] - [4]. It will be further assumed that the partial positive feedback is restricted in each block to the extent that $S_{\alpha}^{Q} \leq 1$ for all capacitor ratios, α, in the structure. The pole-zero pairing problem is addressed by using AROMA to synthesize the cascaded biquadratic structure using all possible pole-zero pairing schemes. The initial SC design program, AROMA [5], has been extended to include the gain distribution strategies of the previous section. The total capacitance for each pole-zero pairing scheme is used as a figure of merit for evaluating the pairings. The ratio TC_R, which is the ratio of the largest total capacitance to the smallest total capacitance is used to demonstrate the significance of the pole-zero pairing strategy.

Consider initially the bandpass filter requirement depicted in Fig. 3 with $f_1 = 900Hz$, $f_2 = 1.0\ KHz$, $A_{max} = 0dB$, $A_{min} = -30dB$ and with a Passband Ripple $= 0.5dB$. The effects of the pole-zero pairings using a Chebyschev approximation and an Elliptic approximation obtained by using the bilinear z-transform of the corresponding s-domain approximations to obtain $H(z)$ and the two gain distribution strategies discussed previously are shown in Table 1 for several values of n (the order of $H(z)$) for a clock frequency of $f_s = 8f_0 = 8\sqrt{f_1 f_2}$. The stop-band corners, f_3 and f_4, vary with n. It can be seen from this table that significant reductions in total capacitance can be made without sacrificing performance through judicious pole-zero pairings. It should be noted from this table that the gain distribution strategy also impacts the total capacitance.

All possible pole-zero pairing strategies were exhaustively considered to obtain the data presented in Table 1. As was demonstrated previously, the number of combinations becomes unwieldly large as the order of the approximation increases making an exhaustive investigation impractical in these cases.

Table 1 Comparison of Different Pole-zero Pairing Strategies

Approximation Type	Chebyschev										Elliptic							
Gain Distribution Strategy	Equal Peak Gain					Single Frequency					Equal Peak Gain				Single Frequency			
Order of Approximation	4	6	8	10	12	4	6	8	10	12	4	6	8	10	4	6	8	10
Number of Biquadratic Blocks	2	3	4	5	6	2	3	4	5	6	2	3	4	5	2	3	4	5
Number of Unique Pole-zero Pairings	3	7	19	91	281	3	7	19	91	281	2	6	24	120	2	6	24	120
f_3	600	800	850	870	880	600	800	850	870	880	800	850	880	888	800	850	880	888
f_4	1300	1100	1050	1030	1020	1300	1100	1050	1030	1020	1100	1050	1020	1012	1100	1050	1020	1012
TC_{min}	157	290	507	828	1117	157	320	420	779	1087	109	310	695	1351	109	350	694	1466
TC_{max}	199	441	809	1286	1877	199	441	817	1286	1877	110	408	852	2162	110	441	831	1874
TC_R	1.27	1.52	1.59	1.56	1.67	1.27	1.45	1.96	1.64	1.72	1.01	1.32	1.22	1.59	1.01	1.25	1.19	1.28

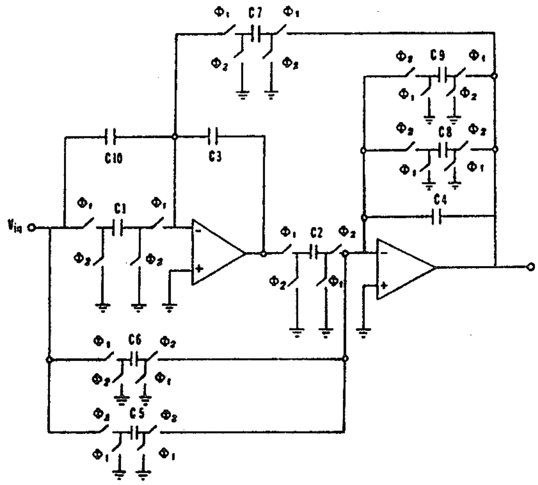

Fig. 2 Low sensitivity biquadratric building block

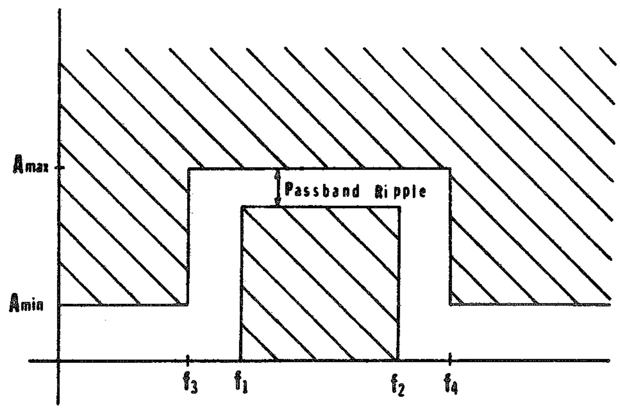

Fig. 3 Design specifications for a bandpass filter

Others have investigated the pole-zero pairing problem in related applications [6] - [8]. The resultant pole-zero pairing strategies were typically based upon a readily computable metric relating to the relative spacing between the poles and zeros.

We have been looking for a similar spacing-based pole-zero pairing strategy for total capacitance minimization in SC circuits. To date, we have not been successful at finding such a strategy. Numerous examples involving low-pass, bandpass and highpass functions with varied approximating functions, orders, clock frequencies, etc. have been investigated. Space limitations preclude discussing these investigations in this manuscript. Suffice it to say that in some examples pairing closely spaced poles and zeros resulted in a relatively low total capacitance whereas in other examples such a strategy resulted in a relatively large total capacitance.

As an alternative, we are suggesting an exhaustive investigation when the number of pairings is small and a statistical approach when the number of combinations is large.

It can be shown that if K pole-zero pairing strategies are randomly selected from the group of M possible structures, then the expected value of the total capacitance for the pairing strategy in this group of K elements with minimum total capacitance is, for large K, approximately given by

$$TC_{K,\ min} \simeq TC_1 + \frac{(TC_M - TC_1)}{K+1}$$

tive computer search may be justified and practical for low order, it becomes impractical for higher-order systems. A statistical pairing strategy which will, in the average sense, give a relatively low total capacitance and which will require minimal computational overhead has been proposed for higher-order systems.

Finally, the effects of gain distribution among the biquadratic blocks was considered. Two gain-distribution strategies were discussed. The gain distribution strategy was also shown to have an appreciable effect on total capacitance. This bears out that both total capacitance and dynamic range should be considered when defining a gain distribution scheme.

The pole-zero pairing strategy is thus to randomly pick K pole-zero pairings and select the one from this set which results in minimum total capacitance. Although the minimum total capacitance will not, in general, be obtained, significant reductions in total capacitance relative to an arbitrary pole-zero pairing strategy can be anticipated in the average sense.

IV. CONCLUSIONS

The effects of pole-zero pairing in cascaded biquad SC filters has been investigated. It was shown that significant reductions in total capacitance can be anticipated through an optimal pole-zero pairing scheme relative to a random pairing strategy without deterioration in filter performance. The reduction in total capacitance should result directly in a silicon area reduction at fabrication. The importance of considering the pole-zero pairing problem becomes more significant as the degree of the approximating function H(z) increases.

For higher order H(z) the number of possible pole-zero pairing combinations becomes unwieldly large. Although an exhaus-

REFERENCES

[1] E. Sánchez-Sinencio, J. Silva-Martínez, and R.L. Geiger, "Biquadratic SC Filters with Small GB Effects" *IEEE Trans. Circuits Syst.* vol. CAS-31, pp 876-884, October 1984.

[2] G. Fisher and G.S. Moschytz, "High-Q SC Biquad with a minimum Capacitor Spread", *Electron. Lett.*, vol. 18 pp 1087-1089, December 1982.

[3] E. Sánchez-Sinencio, P.E. Allen, A.W.T. Ismail and E. Klinkovsky, "Switched-Capacitor Filters with Partial Positive Feedback". *AEU*, band 38, pp 331-339, September/October 1984.

[4] E. Sánchez-Sinencio, R.L. Geiger and J. Silva-Martínez "Tradeoffs Between Passive Sensitivity, Output Voltage Swing and Total Capacitance in Biquadratic SC Filters" *IEEE Trans. Circuits Syst.*, vol. CAS-31, pp 984-987, November 1984.

[5] E. Sánchez-Sinencio, and Jaime Ramírez-Angulo. "AROMA: An Area Optimized CAD Program for Cascade SC Filter Design", *IEEE Trans. on Computer-Aided Design,* vol. CAD-4, pp 296-303, July 1985.

[6] E. Lueder, "A Decomposition of a Transfer Function Minimizeing Distortion and Inband Losses" *Bell Syst. Tech. J.*, vol. 49, pp 455-469, March 1970.

[7] E. Lueder. "Optimization of the Dynamic Range and the Noise Distance of RC-Acitve Filters by Dynamic Programming" *Circuit Theory and Applications*, vol. 3, pp 365-370, June 1975.

[8] G.S. Moschytz, *Linear Integrated Circuits: Design*, New York: Van Nostrand Reinhold, 1975.