POLE-ZERO PAIRING STRATEGY FOR AREA AND SENSITIVITY REDUCTION IN CASCADE SC FILTERS

Cai Xuexiang, Edgar Sánchez-Sinencio and Randall L. Geiger

Texas A & M University
Department of Electrical Engineering
College Station, TX 77843

Abstract.- The effects of pole-zero pairing on the total capacitance of cascaded biquad SC filters is investigated. It is shown that significant reductions in total capacitance are possible through optimal pole-zero pairing without causing degradations in filter performance. Gain distribution between the biquadratic sections is also shown to significantly affect total capacitance.

I. Introduction

It is generally the goal of the SC circuit designer to optimize circuit performance while simultaneously minimizing silicon area. Unfortunately this problem is sufficiently challenging that most designers commit suboptimal designs to fabrication. A method of reducing and/or minimizing silicon area while maintaining a given set of performance specifications for practical cascaded SC biquad structures by using an optimal pole-zero pairing strategy is discussed in this paper.

II. Development

It will be assumed that a function $H(z)$ has been obtained which meets given desired performance specifications and that a circuit is to be synthesized to realize $H(z)$. It will be further assumed that $H(z)$ is to be realized as a cascade of biquadratic sections. The conventional approach in realizing $H(z)$ as a cascade of biquads is to express $H(z)$ as a product of biquadratic fractions as

$$H(z) = \prod_{k=1}^{n} \frac{a_{jk}z^2 + a_{jk}z + a_{0k}}{\beta_{jk}z^2 + \beta_{jk}z + \beta_{0k}} = \prod_{k=1}^{n_1} H_k(z)$$

where $a_{ij}$ and $\beta_{ij}$ are real for all $i, j$. The degree of $H(z)$ is $2n_1$ for $n_1$ even and $2n_1 - 1$ for $n_1$ odd. $\beta_{2k} = 1$ for all $k$ when $n_1$ is even and

$$\beta_{2k} = \begin{cases} k < n_1 \\ 0 & \text{else} \end{cases}$$

where $n_1$ is odd. $H_k(z)$ represents the corresponding biquadratic fraction in $z$. The SC structure is synthesized by cascading $n$ biquadratic blocks with transfer functions $H_k(z)$, $z = 1, \cdots, n_1$.

It should be noted from (1) that the pole pair and zero pair pairings are not unique and that if there are real poles and/or zeros, even the pole pairings or zero pairings into biquadratic polynomials are not unique. The number of possible pole-zero pairings into biquadratic fractions can be quite large.

To determine the number of second-order pole-zero pairings, assume that all poles and zeros are unique and that there are $n$ and $m$ poles and zeros and $nr$ and $mr$ real axis poles and zeros respectively. Define $\phi(k)$ and $\theta(k)$ by the expressions

$$\phi(k) = \begin{cases} k/2 & \text{for } k \text{ even} \\ k/2 + 1 & \text{for } k \text{ odd} \end{cases}$$

and

$$\theta(k) = \begin{cases} \frac{k}{2} & \text{for } k \text{ even} \\ \frac{k+1}{2} & \text{for } k \text{ odd} \end{cases}$$

It follows that there are $\phi(k)$ unique decompositions of a group of $k$ distinct elements into a group of element pairs and that each group contains $\theta(k)$ pairs.

Assuming the number of poles is greater than or equal to the number of zeros and that all zeros are paired (with the exception of the zero for $m$ odd), it can be shown that the number of unique second-order biquadratic decompositions of $H(z)$ is given by

$$N = \phi(nr)\phi(nr)\theta(n_1)\theta(m)!$$

In the case that some of the real zeros are allowed to be singly decomposed rather than paired, as is often done when $m < n$, the number of unique second-order biquadratic decompositions of $H(z)$ will increase significantly. Correspondingly, in the case that some poles or zeros are repeated, the number of unique decompositions will decrease.

To obtain an appreciation for the number of possible pole-zero pairings, it follows from (4) that if $H(z)$ has 4 unique real zeros and 4 unique real poles, then $N = 18$ whereas if there are 6 unique real zeros and 6 unique real poles, $N = 1350$.

It should be apparent that for large $m$ and $n$, the number of possible pairings is quite large. The questions naturally arises, "How should the pole-zero pairings be made?" and "How significant are the effects of a non-optimal pole-zero pairing strategy?"

As a simple example, consider the 4th order Chebyshev bandpass approximation which has two distinct complex conjugate pole pairs located at $P_1$, $P_2$, $P_3$ and $P_4$ and four real axis zeros. Two zeros are located at $z = \pm 1$ and the other two are located at $z = \pm i$. There are three possible pole-zero pairing strategies for the biquadratic decomposition indicated pictorially in Fig. 1. It is reasonable to anticipate that there will be appreciably different characteristics of the circuits that are synthesized based upon the different decomposition schemes.

In an attempt to isolate the pole-zero pairing effects, it will be assumed that the topology for implementing all biquadratic sections is fixed and that the sections are all designed to have identical $\omega_0$ and $Q$ sensitivities. It will be further assumed that the gain of the individual biquads is distributed by one of the two strategies.

609

CH2255-8/86/0000-0609 $1.00 © 1986 IEEE
the biquadratic block, the specific sensitivity requirements, the
gain distribution strategy, the sequence of the biquadratic blocks
in the cascade, and the system clock frequency. An analytical
treatment of the problem appears to be unwieldy. The signifi-
cance of the pole-zero pairing strategy is demonstrated in the
following section by considering practical examples.

III. Significance of Pole-Zero Pairing Strategy

It will be assumed that all even-order biquadratic sections are
realized with the low GB sensitivity structures of [1] shown
in Fig. 2 and that partial positive feedback is used in these
structures to reduce total capacitance [2] - [4]. It will be fur-
ther assumed that the partial positive feedback is restricted in
each block to the extent that $S^2 \leq 1$ for all capacitor ratios,
$\alpha$, in the structure. The pole-zero pairing problem is addressed
by using AROMA to synthesize the cascaded biquadratic struc-
ture using all possible pole-zero pairing schemes. The initial SC
design program, AROMA [5], has been extended to include the
gain distribution strategies of the previous section. The total
capacitance for each pole-zero pairing scheme is used as a figure
of merit for evaluating the pairings. The ratio $T^2$, which is the
ratio of the largest total capacitance to the smallest total capa-

citance is used to demonstrate the significance of the pole-zero
pairing strategy.

Consider initially the bandpass filter requirement depicted in
Fig. 3 with $f_1 = 900$ Hz, $f_3 = 1.0$ KHz, $A_{max} = 0$ dB,
$A_{min} = -30$ dB, and with a Passband Ripple = 0.5 dB. The
effects of the pole-zero pairings using a Chebyshev approxima-
tion and an Elliptic approximation obtained by using the bilin-
ear $s$-transform of the corresponding s-domain approximations to
obtain $H(s)$ and the two gain distribution strategies discussed
previously are shown in Table 1 for several values of $n$ (the
order of $H(s)$) for a clock frequency of $f_3 = 8f_2 = 8\sqrt{f_1f_3}$. The
stop-band corners, $f_2$ and $f_4$, vary with $n$. It can be seen from
this table that significant reductions in total capacitance can be
made without sacrificing performance through judicious pole-
zero pairings. It should be noted from this table that the gain
distribution strategy also impacts the total capacitance.

All possible pole-zero pairing strategies were exhaustively
considered to obtain the data presented in Table 1. As was
demonstrated previously, the number of combinations becomes
unwieldy large as the order of the approximation increases mak-
ing an exhaustive investigation impractical in these cases.

### Table 1 Comparison of Different Pole-Zero Pairing Strategies

<table>
<thead>
<tr>
<th>Approximation Type</th>
<th>Chebyshev</th>
<th>Elliptic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order of Approximation</td>
<td>Equal Peak Gain</td>
<td></td>
</tr>
<tr>
<td>Number of Biquadratic Blocks</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Number of Unique Pole-zero Pairings</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>c</td>
<td>0.75</td>
<td>0.68</td>
</tr>
<tr>
<td>C</td>
<td>0.75</td>
<td>0.68</td>
</tr>
<tr>
<td>$S^2$</td>
<td>1.01</td>
<td>1.02</td>
</tr>
<tr>
<td>$T^2$</td>
<td>1.06</td>
<td>1.07</td>
</tr>
<tr>
<td>$W_B$</td>
<td>1.27</td>
<td>1.32</td>
</tr>
</tbody>
</table>
$TCK_{min} \approx TC_1 + \frac{(TC_M - TC_1)}{K+1}$

Iterative computer search may be justified and practical for low order, it becomes impractical for higher-order systems. A statistical pairing strategy which will, in the average sense, give a relatively low total capacitance and which will require minimal computational overhead has been proposed for higher-order systems.

Finally, the effects of gain distribution among the biquadratic blocks was considered. Two gain-distribution strategies were discussed. The gain distribution strategy was also shown to have an appreciable effect on total capacitance. This bears out that both total capacitance and dynamic range should be considered when defining a gain distribution scheme.

The pole-zero pairing strategy is thus to randomly pick $K$ pole-zero pairings and select the one from this set which results in minimum total capacitance. Although the minimum total capacitance will not, in general, be obtained, significant reductions in total capacitance relative to an arbitrary pole-zero pairing strategy can be anticipated in the average sense.

IV. CONCLUSIONS

The effects of pole-zero pairing in cascaded biquad SC filters has been investigated. It was shown that significant reductions in total capacitance can be anticipated through an optimal pole-zero pairing scheme relative to a random pairing strategy without deterioration in filter performance. The reduction in total capacitance should result directly in a silicon area reduction at fabrication. The importance of considering the pole-zero pairing problem becomes more significant as the degree of the approximating function $H(z)$ increases.

For higher order $H(z)$ the number of possible pole-zero pairing combinations becomes unwieldy large. Although an exhaust-

REFERENCES