Pole-zero pairing strategies for cascaded switched-capacitor filters

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Abstract: The effect of pole-zero pairing on the total capacitance of cascaded biquad SC filters is investigated. It is shown that significant reductions in total capacitance and hence corresponding reductions in silicon area are possible through optimal pole-zero pairing without causing significant degradations in filter performance.

1 Introduction

Switched-capacitor (SC) filter design methods have reached a certain level of maturity [1–2]. Specific SC biquadratic structures suitable for cascaded biquad designs have evolved [1–8] which are now widely used because of the recognised low sensitivity characteristics of these structures relative to those attainable with existing alternative schemes. Methods which allow designers to compute the capacitor ratios for these structures rapidly and exactly, to realise a predetermined set of design specifications are also available. Several groups have actually created a 'SC silicon compiler' with programs which automatically select a good filter structure based upon the given design specifications, size all components, and generate a circuit layout which can be submitted for fabrication. The problems of pole-zero pairing and the minimisation of total capacitance still prevail.

Pole-zero pairing has been discussed in active *RC* filter structures for optimal dynamic range and inband losses [9–11]. For the case of SC filter implementations, the pole-zero pairing scheme has not been discussed in detail in the literature. The pole-zero pairing problem can be seen as the availability of an additional degree of freedom for the designer to use to optimise performance. This additional degree of freedom, conceptually, affects several characteristics of concern to the designer such as sensitivity, dynamic range, total capacitance etc. Trade-offs between these characteristics must be made in any design.

It is generally a goal of the SC circuit designer to optimise circuit performance while simultaneously minimising silicon area [3–6]. Unfortunately this is such a challenging problem that most designers commit sub-optimal designs to fabrication. A method of reducing and/or minimising silicon area while maintaining a given set of performance specifications for practical cascaded SC biquad structures by using an optimal pole-zero pairing strategy is discussed in this paper.

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2 Development

It will be assumed that a function $H(z)$ has been obtained which meets given desired performance specifications and that a circuit is to be synthesised to realise $H(z)$. It will be further assumed that the degree of the numerator of $H(z)$ is less than or equal to that of the denominator and that $H(z)$ is to be realised as a cascade of biquadratic sections. The conventional approach in realising $H(z)$ as a cascade of biquads is to express $H(z)$ as a product of biquadratic fractions:

$$H(z) = \prod_{k=1}^{n_1} \left(\frac{\alpha_{2k} z^2 + \alpha_{1k} z + \alpha_{0k}}{\beta_{2k} z^2 + \beta_{1k} z + \beta_{0k}} \right) = \prod_{k=1}^{n_1} H_k(z) \tag{1}$$

where α_{ij} and β_{ij} are real for all i, j. The degree of $H(z)$ is $2n_1$ provided that $\beta_{2k} = 1$ for all k and $2n_1 - 1$ when the β_{2k} variables satisfy the equation

$$\beta_{2k} = \begin{Bmatrix} 1 & k < n_1 \\ 0 & k = n_1 \end{Bmatrix}$$

$H_k(z)$ represents the corresponding biquadratic fraction in z. The SC structure is synthesised by cascading n_1 biquadratic blocks with transfer functions $H_k(z)$, $k = 1, \ldots, n_1$.

It should be noted from eqn. 1 that the pole pair and zero pair pairings are not unique and that if there are real poles and/or zeros, even the pole pairings or zero pairings into biquadratic polynomials are not unique. The number of possible pole-zero pairings into biquadratic fractions can be quite large.

To determine the number of 2nd-order pole-zero pairings, assume that all poles and zeros are unique and that there are n poles, m zeros and n_r and m_r real axis poles and zeros, respectively. Define $\phi(k)$ and $\theta(k)$ by the expressions

$$\phi(k) \triangleq \begin{cases} \displaystyle\prod_{i=0}^{(k-2)/2} (k - 1 - 2i) & \text{for } k \text{ even} \\ \displaystyle\prod_{i=0}^{(k-1)/2} (k - 2i) & \text{for } k \text{ odd} \end{cases} \tag{2}$$

and

$$\theta(k) \triangleq \begin{cases} \dfrac{k}{2} & \text{for } k \text{ even} \\ \dfrac{k+1}{2} & \text{for } k \text{ odd} \end{cases} \quad (3)$$

There are $\phi(k)$ unique decompositions of a set of k elements into disjoint subsets of element pairs, and the number of disjoint subsets in each decomposition is $\theta(k)$. Assuming the number of poles is greater than or equal to the number of zeros and that all zeros are paired (with the exception of the odd zero for m odd), it can be shown that the number of unique 2nd-order biquadratic decompositions $H(z)$ is given by

$$N = \phi(m_r)\phi(n_r)\binom{\theta(n)}{\theta(m)}\theta(m)! \quad (4)$$

In the case that some of the real zeros are allowed to be singly decomposed rather than paired, as is often done when the number of unique 2nd-order biquadratic decompositions of $H(z)m_n$ increases significantly. Correspondingly, in the case that some poles or zeros are repeated, the number of unique decompositions will decrease.

To obtain an appreciation of the number of possible pole-zero pairings, it follows from eqn. 4 that if $H(z)$ has four unique real zeros and four unique real poles, $N = 18$ whereas if there are six unique real zeros and six unique real poles, $N = 1350$.

It should be apparent that for large m and n, the number of possible pairings is quite large. Many practical SC applications exist in which the number of possible pole-zero pairings is very large. The questions naturally arises, 'How should the pole-zero pairings be made?' and 'How significant are the effects of a nonoptimal pole zero pairing strategy?'

As a simple example, consider the 4th-order Chebyšchev bandpass approximation which has two distinct complex conjugate pole pairs located at P_1, P_1^*, P_2 and P_2^* and four real axis zeros. Two zeros are located at $z = +1$ and the other two are located at $z = -1$. There are three possible pole-zero pairing strategies for the biquadratic decomposition indicated pictorially in Fig. 1. It is reasonable to anticipate that there will be appreciably different characteristics of the circuits that are synthesised based on the different decomposition schemes.

It is known that in SC filter designs there are tradeoffs between maximum voltage swing, capacitance, clock frequency and passive sensitivity [5–6]. In an attempt to isolate the pole-zero pairing effects, it will be assumed that the topology for implementing all biquadratic sections [4, 8] is fixed and that the sections are all designed to have identical ω_0 and Q sensitivities. It will also be assumed that the gain of the individual biquads is distributed by one of the following two strategies:

(a) Gain distribution strategy 1 (single frequency):

$$|H_k(z)|\Big|_{z=e^{j\omega_1 T}} = \left[\,|H(z)|\Big|_{z=e^{j\omega_1 T}}\right]^{1/n} \quad (5)$$

for some ω_1 and for all $1 \leqslant k \leqslant n$. In this expression T is the clock period. ω_1 could reasonably be chosen at the middle of the passband.

(b) Gain distribution strategy 2 (equal peak gain):

$$\max_{\omega T}\left| H_k(z)\Big|_{z=e^{j\omega T}}\right| = H_{max} \quad (6)$$

for all $1 \leqslant k \leqslant n$ where H_{max} is defined by the expression

$$H_{max} = \left(\prod_{j=1}^{n} H_{max_j}\right)^{1/n} \quad (7)$$

where H_{max_j} is the maximum gain of the jth stage.

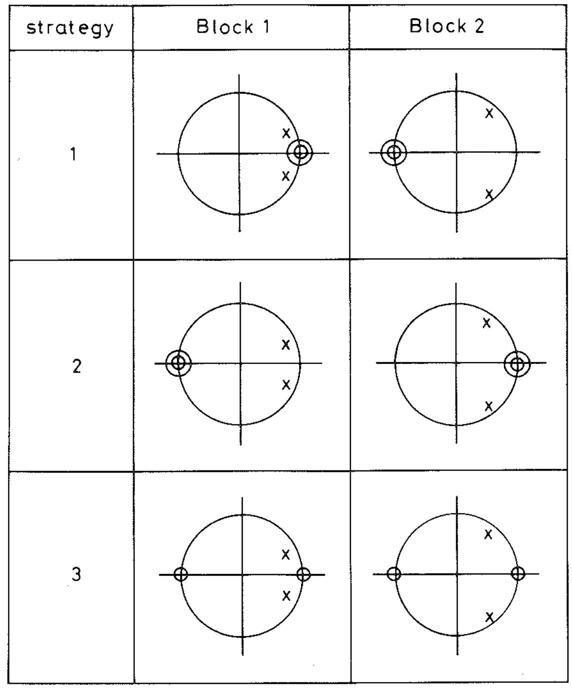

Fig. 1 *Pole-zero pairing possibilities for a 4th-order Chebyšchev filter*

Note that both gain distribution strategies are independent of how the individual biquads are ordered in the cascade. These types of gain distribution strategies were intentionally adopted, possibly at the expense of some dynamic range reduction, to circumvent the need for considering position in the cascade as another variable. In the actual implementation, the sequence of the biquads in the cascade should be chosen to maximise dynamic range [8–10].

It is our conjecture that an optimal pole-zero pairing strategy, in the general case, is a function of:
(i) $H(z)$
(ii) the topology of the biquadratic block
(iii) the specific sensitivity requirements and the gain distribution strategy
(iv) the sequence of the biquadratic blocks in the cascade
(v) the system clock frequency.

An analytical treatment of the problem, in general, appears to be unwieldy. The significance of the pole-zero pairing strategy is demonstrated in the following Section by considering practical examples.

3 Significance of pole-zero pairing strategy

It will be assumed that all even-order biquadratic sections are realised with the low GB sensitivity and low power consumption structure [4, 8] shown in Fig. 2 and that partial positive feedback is used in these structures to reduce total capacitance [4–7]. Relevant design equations appear in the Appendix. Further it will be assumed that the partial positive feedback is restricted in each block to the extent that $|S_\alpha^Q| \leqslant 1$ and $|S_\alpha^{\omega_0}| < 1$ for all capacitor ratios α in the structure. The pole-zero pairing problem is initially addressed by using AROMA [7] to synthesise $H(z)$ with the cascaded biquadratic structure using all possible pole-zero pairing schemes.

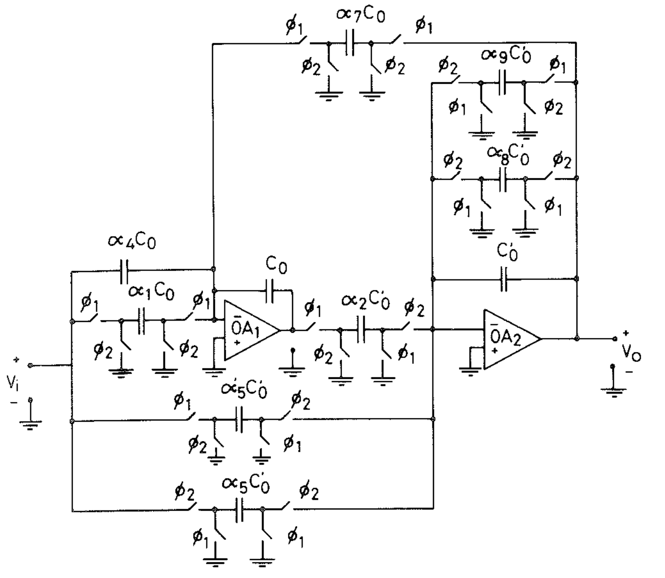

Fig. 2 *General SC Biquad Structure [4, 8]*

The initial SC design program, AROMA [7], has been extended to include the gain distribution strategies of the preceding Section. The total capacitance for each pole-zero pairing scheme is used as a figure of merit for evaluating the pairings. The ratio TC_r, which is the ratio of the largest total capacitance to the smallest total capacitance in the exhaustive comparison, is used to demonstrate the significance of adopting a good pole-zero pairing strategy.

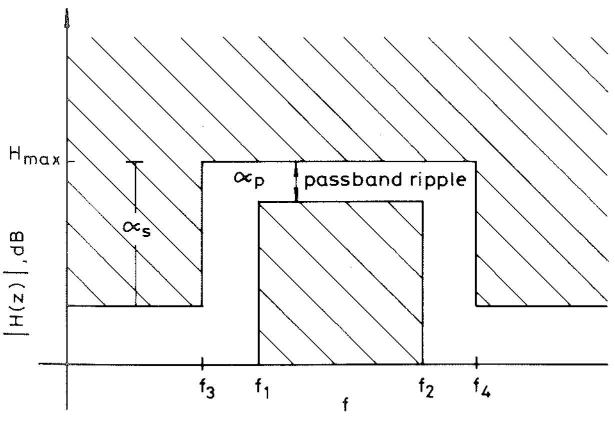

Fig. 3 *Design specifications for a bandpass filter*

Consider initially the bandpass filter requirement depicted in Fig. 3 where H_{max} is the peak gain in the passband, α_p is the maximum ripple in the passband and α_s is the minimum loss in the stopband. Assume for comparison purposes that $f_1 = 900$ Hz, $f_2 = 1.0$ kHz, $H_{max} = 0$ dB, $\alpha_s = 30$ dB and $\alpha_p = 0.5$ dB. The effects of the pole-zero pairings using a Chebÿchev approximation and an elliptic approximation (with 0 dB peak gain, 0.5 dB passband ripple and 0.5 dB cutoff frequencies at f_1 and f_2) obtained from standard tables of the poles of the approximating functions [12] by using the bilinear z-transform of the corresponding s-domain approximations to obtain $H(z)$ and the two gain distribution strategies discussed previously are shown in Table 1 for several values of n (the order of $H(z)$) for a clock frequency of $f_c = 8f_0 = 8\sqrt{f_1 f_2}$. The stopband corners f_3 and f_4 vary with n. It can be seen from this Table that significant reductions in total capacitance can be made without sacrificing performance through judicious pole-zero pairings. For example, almost a 50% reduction in total capacitance was observed for the 8th-order Chebÿchev single-frequency approximation by using the minimum total capacitance structure rather than the maximum total capacitance structure. Furthermore, for the Chebÿchev approximation the single frequency gain distribution provides a greater reduction in total capacitance. For the elliptic approximation the opposite occurs, the greater reduction is obtained when the equal peak gain distribution is used.

From this table it can be observed that, in general, the capacitance reduction potential increases with the order of the transfer functions. It can also be seen that the total capacitance varies with the gain distribution strategy and that for a given order of approximation, the total capacitance required for the Chebÿchev approximation can differ significantly from that required for the elliptic approximation. It can also be observed that for low orders the elliptic approximation requires less total capacitance, whereas for higher orders, the total capacitance required for the elliptic approximation becomes larger. Finally, although not directly related to the pole-zero pairing problem, this Table bears out the fact that the total capacitance for a given approximating function increases significantly with increasing order.

All possible pole-zero pairing strategies were exhaustively considered to obtain the data presented in Table 1 (e.g., for $n = 12$, 281 pole-zero pairings were investigated for the Chebÿchev approximation). As was demonstrated previously, the number of combinations becomes unwieldy as the order of the approximation increases, making an exhaustive investigation impractical in these cases.

Table 1 : Comparison for different pole-zero pairing strategies, approximations and gain distributions

Approximation type	Chebÿchev										Elliptic							
Gain distribution strategy	Equal peak gain					Single frequency					Equal peak gain				Single frequency			
Order of approximation	4	6	8	10	12	4	6	8	10	12	4	6	8	10	4	6	8	10
Number of biquadratic blocks	2	3	4	5	6	1	3	4	5	6	2	3	4	5	2	3	4	5
Number of unique pole-zero pairings	3	7	19	91	281	3	7	19	91	281	2	6	24	120	2	6	24	120
f_3	550	750	800	860	870	550	750	800	860	870	800	850	880	888	800	850	880	888
f_4	1350	1150	1100	1040	1030	1350	1150	1100	1040	1030	1100	1050	1020	1012	1100	1050	1020	1012
TC_{min}	157	355	507	813	1137	157	320	420	779	1087	111	324	707	1392	111	355	709	1499
TC_{max}	199	441	809	1286	1877	199	462	817	1286	1877	111	486	869	2183	111	447	851	1917
TC_r	1.27	1.24	1.59	1.56	1.65	1.27	1.44	1.96	1.64	1.72	1.00	1.50	1.23	1.57	1.00	1.26	1.20	1.28

Others have investigated the pole-zero pairing problem in related applications [9–11]. Some investigators have been concerned with sensitivity reduction whereas others were concerned about maximising dynamic range. In these discrete applications, the component spread, which is related to total capacitance in SC applications, did not serve as the focal point of the investigations. The component spread was not of major concern from an economic viewpoint because the actual cost of discrete components is not strongly dependent on the nominal component values in audio frequency filtering applications. These researchers were able to develop 'rules of thumb' for pole-zero pairing which resulted in reasonable filter performance. The resultant pole-zero pairing strategies were typically based on a readily computable metric relating to the relative spacing between the poles and zeros. Such a strategy is useful because the pole-zero pairings can be achieved easily with relatively minimal computational overheads.

We have been looking for a similar spacing-based pole-zero pairing strategy for total capacitance minimisation in SC circuits. To date, we have not been successful at finding such a strategy. Numerous examples involving lowpass, bandpass and highpass functions with varied approximating functions, orders, clock frequencies etc. have been investigated. Space limitations preclude discussing these investigations in this paper. Suffice it to say that in some examples pairing closely spaced poles and zeros resulted in a relatively low total capacitance whereas in other examples such a strategy resulted in a relatively large total capacitance.

3.1 Statistical approach

As an alternative, we are suggesting an exhaustive investigation when the number of pairings is small and a statistical approach when the number of combinations is large. Remarks on the statistical approach follow.

As the number of unique pole-zero pairing strategies is finite, they can be rank ordered relative to the total capacitance TC by the sequence $S_1, \ldots, S_M$ where S_i, $i = 1, \ldots, M$ denotes a specific pole-zero pairing strategy and $TC_i \geqslant TC_j$ for $i \geqslant j$. Relative to this rank ordering, the total capacitance ratio, defined previously, is given by $TC_r = TC_M/TC_1$. Ideally, we would like to obtain the pairing strategy S_1.

Assume initially that $TC_i \simeq (TC_M - TC_i)(i-1)/(M-i) + TC_1$. This assumption has been made because in the examples considered, a reasonably uniform distribution between TC_1 and TC_M was observed. If it is now assumed that K pole-zero pairing strategies are randomly selected from the group of M possible structures, the expected value of the pairing strategy in this group of K elements with minimum total capacitance is, for large K, approximately given by

$$TC_{K,\,min} \simeq TC_1 + \frac{(TC_M - TC_1)}{K+1} \qquad (8)$$

The pole-zero pairing strategy is thus to pick K pole-zero pairings randomly and select the one from this set which results in minimum total capacitance. Although the minimum total capacitance will not, in general, be obtained with this scheme, significant reductions in total capacitance relative to an arbitrary pole-zero pairing strategy can be anticipated in the average sense.

3.2 Effects of clock frequency and filter specifications on TC_r

It was seen in the example presented in the preceding Section that an increase of nearly 100% in total capacitance can result in practical SC applications with a non-optimal pole-zero pairing strategy. In this Section, the influence of changing clock frequencies and filter specifications will be investigated.

To investigate the effect of clock frequencies, consider for comparison purposes a 6th-order Chebÿchev filter which has the same specifications as considered previously, namely $H_{max} = 0$ dB, $\alpha_p = 0.5$ dB, $\alpha_s = 30$ dB, $f_1 = 900$ Hz and $f_2 = 1000$ Hz. In Table 2, the values of TC_r for this circuit with three different clock frequencies using the equal peak gain distribution strategy are presented. Note that although major differences in TC_r are not observed, the pole-zero pairing does become more critical for high f_c/f_0 because total capacitance increases significantly with f_c.

Table 2: Effects of clock frequency on total capacitance for different pole-zero pairings

	$f_c = 8$ kHz	$f_c = 16$ kHz	$f_c = 32$ kHz
TC_{min}	355	780	2209
TC_{max}	441	1036	3182
TC_r	1.24	1.33	1.44

TC_{max} and TC_{min} are in terms of unit capacitances

To consider the effects of changing filter specifications, the bandwidth of the Chebÿchev and elliptic bandpass responses were varied. The ripple, gain and clock frequency were held constant ($\alpha_p = 0.5$ dB, $\alpha_s = 30$ dB and $f_c = 8.4\,f_0$). The TC_r values are compared in Table 3. Note that although rather significant changes in the specifications were made, the TC_r changes are comparable in magnitude to those observed for changing clock frequency, order, gain distribution strategy or type of approximation.

4 Conclusions

The effects of pole-zero pairing in cascaded biquad SC filters have been investigated. To compare different pole-zero pairings with the same circuit properties several design constraints were imposed. It was shown that significant reductions in total capacitance can be anticipated through an optimal pole-zero pairing scheme relative to a

Table 3: Effects of changing filter specifications on TC_r

	Chebÿchev									Elliptic									
BW, Hz,	100				300			500		100				300			500		
f_1, Hz	900				800			700		900				800			700		
f_2, Hz	1000				1100			1200		1000				1100			1200		
Order	4	6	8	10	6	8	10	8	10	4	6	8	10	6	8	10	6	8	10
TC_{max}	199	441	809	1286	160	288	455	188	294	111	486	869	2183	169	309	760	120	238	577
TC_{min}	157	355	507	813	113	194	307	134	206	111	324	707	1392	131	237	493	91	165	370
TC_r	1.27	1.24	1.59	1.56	1.42	1.49	1.48	1.41	1.43	1.0	1.5	1.23	1.57	1.3	1.3	1.54	1.31	1.44	1.56

TC_{max} and TC_{min} are in terms of unit capacitance random pairing strategy without deterioration in filter performance. The reduction in total capacitance should result directly in a silicon area reduction at fabrication. The importance of considering the pole-zero pairing problem becomes more significant both at high clock frequencies and as the degree of the approximating function $H(z)$ increases. From the examples presented, it appears that the magnitude of the pole-zero pairing problem is not strongly dependent on the exact filter specifications or gain distribution strategy.

For higher-order $H(z)$ the number of possible pole-zero pairing combinations becomes unwieldy. Although an exhaustive computer search may be justified in some cases, a simple statistical approach which offers, on average, appreciable reductions in total capacitance was introduced.

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6 Appendix: Design equations for the SC biquad [4, 8]

The transfer function when input and output are sampled at the same time for the SC biquad of Fig. 2 is given by

$$H(z) = \frac{\alpha'_5}{1+\alpha_8} \frac{z^2 - z[\alpha_5 + \alpha'_5 - \alpha_2(\alpha_1 + \alpha_4)]/\alpha'_5 + (\alpha_5 - \alpha_2\alpha_4)/\alpha'_5}{z^2 - z[2 + \alpha_8 + \alpha_9 - \alpha_2\alpha_7]/(1+\alpha_8) + \dfrac{1+\alpha_9}{1+\alpha_8}} \tag{9}$$

Eqn. 9 can be also written, for convenience, as

$$H(z) = K \frac{z^2 + \varepsilon z + \delta}{z^2 - 2r\cos\theta z + r^2} \tag{10}$$

A simple solution of the design equations for the poles follows. Let $\alpha_9 = k\alpha_8$ and $0 \leqslant k < 1$:

$$\alpha_9 = \frac{1 - r^2}{r^2/k - 1} \tag{11}$$

$$\alpha_8 = \frac{1 - r^2}{r^2 - k} \tag{12}$$

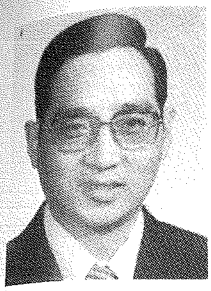
select α_2 and obtain

$$\alpha_7 = (1+\alpha_8)(1 + r^2 - 2r\cos\theta)/\alpha_2 \tag{13}$$

k is related to $S^Q_{\alpha_9}$ i.e., $k = S^Q_{\alpha_9}/(1 + S^Q_{\alpha_9})$. Table 4 shows the zero placement formulas for different type of filters used in this paper. The notation used is described on p. 163 of Reference 2. $K = \alpha'_5/(1+\alpha_8)$ unless otherwise specified.

Table 4: Equations for filters used in the paper

Filter type	Design equations	Simple solution
LP 20	$\frac{\alpha_5 - \alpha_2\alpha_4}{\alpha'_5} = 1$	$\alpha_4 = 0$, $\alpha_5 = \alpha'_5$
	$\frac{\alpha_5 + \alpha'_5 - \alpha_2(\alpha_1+\alpha_4)}{\alpha'_5} = -2$	$\alpha'_5 = K(1+\alpha_8)$ $\alpha_1\alpha_2 = 4\alpha_5$
BP 20	$\alpha_5 + \alpha'_5 - \alpha_2(\alpha_1+\alpha_4) = 0$	$\alpha_5 = 0$, $\alpha_1 = 0$ $\alpha'_5 = K(1+\alpha_8)$
	$\frac{\alpha_5 - \alpha_2\alpha_4}{\alpha'_5} = -1$	$\alpha_2\alpha_4 = \alpha'_5$
HP	$\frac{\alpha_5 + \alpha'_5 - \alpha_2(\alpha_1+\alpha_4)}{\alpha'_5} = 2$	$\alpha_1 = \alpha_4 = 0$
	$\frac{\alpha_5 - \alpha_2\alpha_4}{\alpha'_5} = 1$	$\alpha'_5 = K(1+\alpha_8)$ $\alpha_5 = \alpha'_5$
General	$K = \frac{\alpha'_5}{1+\alpha_8}$	$\alpha'_5 = K(1+\alpha_8)$
(except BP 20)	$\varepsilon = \frac{\alpha_2(\alpha_1+\alpha_4) - \alpha_5 - \alpha'_5}{K(1+\alpha_8)}$	$\alpha_4 = 0$ $\alpha_1 = \frac{K(1+\alpha_8)(\varepsilon + K + K\delta)}{\alpha_2}$
	$\delta = \frac{\alpha_5 - \alpha_2\alpha_4}{\alpha'_5}$	$\alpha_5 = \delta\alpha'_5$



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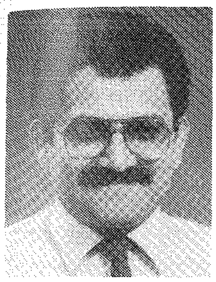

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