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Tip trajectory tracking of a flexible-joint robot using stable inversion*

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Abstract

Design of output tracking controllers for nonlinear nonminimum phase systems is challenging. Among existing methods the regulation approach usually leads to large transient errors while the classical inversion approach results in unbounded internal dynamics for nonminimum phase systems. In this paper, stable inversion is applied to the design of tip trajectory tracking for a single-link flexible-joint robot mounted on a wobbly platform. This new type of tracking controller achieves remarkably accurate output tracking without any transient or steady-state errors together with guaranteed stability of both external and internal signals. After development of system dynamics for the robot system, this paper defines the stable inversion problem for such a system which is followed by construction of the unique inverse solution to this problem. Then, this stable inversion approach is applied to the design of a tip trajectory tracking controller with only partial state measurements. Simulation study demonstrates the effectiveness of this approach in output tracking.

1. Introduction

The most elementary task that appears in modern robot control is to drive the end-effector of a robot arm to follow a given desired trajectory without any overshoot or residual vibration. This paper presents a closed-loop tracking control law using a nominal control input generated by stable inversion as a feed-forward signal and a stabilizing signal from a feedback stabilizer. The design should avoid the transient error phenomenon and the unstable internal dynamics which are the fundamental limitations of the nonlinear regulation approach and the classical inversion approach. Therefore, a stable and remarkably accurate output tracking will be achieved.

There has been considerable work in the area of output tracking controller design. For nonlinear systems there are basically two approaches. The first is the classical inversion approach that controls the transient behavior precisely by using stabilizing feedback together with feed-forward signals generated by an inverse system. The classical inversion was first studied by Brockett and Mesarovic [1]. Later, Silverman developed an easy-to-follow step-by-step procedure for the inversion of a class of linear multivariable systems [2]. These linear results were extended to nonlinear real-analytic systems by Hirschorn [3] and Singh [4]. For a given desired output and a fixed initial condition, all these inversion algorithms produce causal inverses that are unbounded for nonminimum phase systems.

Another is the nonlinear regulation approach recently developed by Isidori and Byrnes [5]. This approach also uses the structure of feed-forward plus feedback and it provides asymptotic output tracking for a class of reference trajectories generated by a given autonomous exosystem. The feed-forward signals are calculated by solving a set of nonlinear partial differential equations of the same order as the forward system dynamics. Besides the numerical tractability of nonlinear PDEs, a major concern is the possibly large transient error that is not controlled in this approach.

The approach to output tracking by stable inversion avoids difficulties in both regulation and classical inversion while preserves advantages of both, and is applied to achieve tip trajectory tracking for a flexible-joint robot in this paper. The remainder of this paper is organized as follows. The system dynamics of a single-link flexible-joint robot mounted on a wobbly platform is developed in section 2 using the Lagrange's method. In section 3, following the general framework of stable inversion in [6], a stable inversion problem for this system is defined which is followed by construction of the inverse solution to this problem. In section 4 we apply this stable inversion approach to design a tip trajectory tracking controller. Simulation study demonstrates the effectiveness of this approach in achieving excellent output tracking. A conclusion is finally given in section 5.

2. System Dynamics

Consider a flexible-joint robot mounted on a wobbly platform (see Figure 1). It is assumed that there is no motion in the vertical direction. Thus, only the motion in the horizontal plane will be considered and modeled. There is a total of five degrees of freedom in the model: linear

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displacement (x_p, y_p) and angular displacement θ_p of the platform, angular displacement θ_r of the rotor of the motor, angular displacement θ_l of the link. The three angles θ_p, θ_r, and θ_l are measured with respect to the X-axis

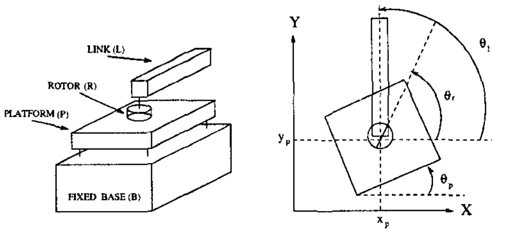

Figure 1: Flexible-joint robot on wobbly platform

as shown in Figure 1. We assume that the point (x_p, y_p) is the center of mass of both the platform and the rotor. We also assume that the platform is subject to linear and angular restoring forces proportional to its deviation from an initial position, and the link is subject to an angular restoring force proportional to its deviation $(\theta_l - \theta_r)$ from alignment with the rotor. All motions are also assumed to subject to viscous friction forces proportional to their velocities respectively.

The angle of the link relative to the platform, $\theta_l - \theta_p$, is defined as the system's output while u, the torque generated by the motor, is the control input. The system with parameters listed in Table 1 is utilized as the physical model in this paper.

$M(5.0)$	total mass of L, R, and P
$m(0.5)$	mass of L
$r(0.3)$	distance from L-center to (x_p, y_p)
$I_l(0.6)$	moment of inertia of L w/ (x_p, y_p)
$I_r(0.05)$	moment of inertia of R w/ (x_p, y_p)
$I_p(5.0)$	moment of inertia of P w/ (x_p, y_p)
$k_1(2600)$	linear spring const btwn P and B
$k_2(2960)$	angular spring const btwn P and B
$k_3(8.0)$	spring const btwn L and R
$b_1(14.0)$	linear friction coef btwn P and B
$b_2(15.0)$	angular friction coef btwn P and B
$b_3(0.04)$	friction coef btwn L and R
$b_4(0.007)$	friction coef btwn P and R

Table 1: Details of the flexible-joint robot Model

To apply the Lagrange's method, the kinetic energy of the whole system containing three bodies (platform, rotor and link) is firstly found as follows:

$$T = \frac{1}{2}M[\dot{x}_p^2 + \dot{y}_p^2] + mr\dot{\theta}_l[-\dot{x}_p \sin\theta_l + \dot{y}_p \cos\theta_l] + \frac{1}{2}I_l\dot{\theta}_l^2 + \frac{1}{2}I_r\dot{\theta}_r^2 + \frac{1}{2}I_p\dot{\theta}_p^2. \quad (1)$$

Secondly, the total potential energy stored in all the springs is given by

$$P = \frac{1}{2}k_1[x_p^2 + y_p^2] + \frac{1}{2}k_2\theta_p^2 + \frac{1}{2}k_3[\theta_l - \theta_r]^2. \quad (2)$$

Let $\psi = (x_p, y_p, \theta_l, \theta_r, \theta_p)$ be the system's generalized coordinates. Invoking the extended Hamilton's principle in the form of Lagrange's equation

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\psi}} - \frac{\partial L}{\partial \psi} = F, \quad (3)$$

where $L = T - P$ is the Lagrangian, and F is the generalized force including motor driving force and viscous friction forces, we obtain the system dynamics as follows:

$$\begin{cases} M\ddot{x}_p - mr[\ddot{\theta}_l \sin\theta_l + \dot{\theta}_l^2 \cos\theta_l] + b_1\dot{x}_p + k_1 x_p = 0 \\ M\ddot{y}_p + mr[\ddot{\theta}_l \cos\theta_l - \dot{\theta}_l^2 \sin\theta_l] + b_1\dot{y}_p + k_1 y_p = 0 \\ I_l\ddot{\theta}_l - mr[\ddot{x}_p \sin\theta_l - \ddot{y}_p \cos\theta_l] + b_3[\dot{\theta}_l - \dot{\theta}_r] \\ \quad +k_3[\theta_l - \theta_r] - mr\dot{\theta}_l[\dot{x}_p \cos\theta_l + \dot{y}_p \sin\theta_l] = 0 \\ I_r\ddot{\theta}_r - b_3[\dot{\theta}_l - \dot{\theta}_r] - k_3[\theta_l - \theta_r] + b_4[\dot{\theta}_r - \dot{\theta}_p] = u \\ I_p\ddot{\theta}_p + b_2\dot{\theta}_p + k_2\theta_p - b_4[\dot{\theta}_r - \dot{\theta}_p] = -u. \end{cases} \quad (4)$$

Combining the above dynamics equations with the definition of system's output, the forward dynamics of our robot system can thus be written in the following compact form:

$$M_1(\psi)\ddot{\psi} + H(\dot{\psi}, \psi) + M_2\dot{\psi} + M_3\psi = Du, \quad (5)$$

$$y = h(\psi), \quad (6)$$

where $h(\psi) := \theta_l - \theta_p$, and the inertia matrix M_1, centrifugal/Coriolis term H, damping matrix M_2, stiffness matrix M_3, and torque distribution matrix D can all be directly derived accordingly.

3. Stable Inversion Problem

The forward dynamics (5)-(6) of our robot system can be written in a state-space form:

$$\begin{cases} \dot{\psi} = \dot{\psi} \\ \ddot{\psi} = -M_1^{-1}[H + M_2\dot{\psi} + M_3\psi] + M_1^{-1}Du, \end{cases} \quad (7)$$

$$y = h(\psi). \quad (8)$$

It is noticed from this form that this SISO nonlinear system is affine in its control input. Furthermore, the left hand sides of both dynamics and output equations are smooth on $(\psi, \dot{\psi})$. Thus, it fits into the general framework of the stable inversion problem developed in [6]. Following the procedure in that framework, we define the stable inversion problem for this robot system as follows: *given any smooth reference output trajectory y_d with a compact support on $[t_0, t_f]$, find a bounded input $u_d(t)$ and a bounded state trajectory $(\psi_d, \dot{\psi}_d)$ such that they approach zero as time tends to plus or minus infinity and map to the exact* *desired output trajectory y_d through the forward dynamics (7)-(8).*

Here the pair, u_d and $(\psi_d, \dot{\psi}_d)$, is referred to as the stable inverse solution for a given reference output y_d. It is called stable inverse because of the boundedness and convergence of the inverse solution. Besides, $(\psi_d, \dot{\psi}_d)$ is called the desired state trajectory and u_d the nominal control input. Later on in this paper, they will be incorporated into a tracking controller which achieves stable and accurate output trajectory tracking.

In order to solve the problem to find the stable inverse pair, we again follow the procedure in the stable inversion framework in [6]. Firstly, we compute the time-derivatives of the output until the input appears explicitly:

$$\dot{y} = \dot{\psi}_3 - \dot{\psi}_5, \tag{9}$$

$$\ddot{y} = \alpha(\psi, \dot{\psi}) + \frac{1}{I_p} u, \tag{10}$$

where the expression of $\alpha(\psi, \dot{\psi})$ can be obtained after some algebra from the forward dynamics. It is clear from the above equation (10) that this system has a well-defined relative degree two. Secondly, a coordinate transformation is made to the forward dynamics. In addition to the output and its first derivative, we also choose all the flexible modes of the system $\eta = (x_p, y_p, \theta_l - \theta_r, \theta_p)$ together with their first derivatives as the new set of coordinates whose linear independence can be verified easily. It turns out that the transformation is linear and can be written as follows:

$$\begin{bmatrix} y \\ \dot{y} \\ \eta \\ \dot{\eta} \end{bmatrix} = M_\phi \begin{bmatrix} \psi \\ \dot{\psi} \end{bmatrix}, \qquad \eta \in \mathbf{R}^4, \tag{11}$$

where the transformation matrix is given by

$$M_\phi = \begin{bmatrix} M_{\phi 1} & O_{1\times 5} \\ O_{1\times 5} & M_{\phi 1} \\ M_{\phi 2} & O_{4\times 5} \\ O_{4\times 5} & M_{\phi 2} \end{bmatrix}, \tag{12}$$

and

$$M_{\phi 1} = \begin{bmatrix} 0 & 0 & 1 & 0 & -1 \end{bmatrix}, \tag{13}$$

$$M_{\phi 2} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{14}$$

Set $y \equiv y_d$. The output of the inverse equation yields

$$u = I_p \left[\ddot{y}_d - \alpha(\psi, \dot{\psi}) \right], \tag{15}$$

and the system dynamics under the new coordinates is given by

$$\begin{cases} \dot{y} = \dot{y}_d \\ \ddot{y} = \ddot{y}_d \\ \dot{\eta} = \dot{\eta} \\ \ddot{\eta} = p(y_d, \dot{y}_d, \ddot{y}_d, \eta, \dot{\eta}). \end{cases} \tag{16}$$

A convenient way to find the expression for $p(\cdot)$ is to derive it directly from dynamics equation (4). Adding the last two equations together and then substituting θ_l by $\theta_p + y_d$ to all the equations in (4) yields

$$\begin{cases} M\ddot{x}_p - mr[\ddot{\theta}_p + \ddot{y}_d]\sin(\theta_p + y_d) \\ \quad +mr[\dot{\theta}_p + \dot{y}_d]^2\cos(\theta_p + y_d) + b_1\dot{x}_p + k_1 x_p = 0 \\ M\ddot{y}_p + mr[\ddot{\theta}_p + \ddot{y}_d]\cos(\theta_p + y_d) \\ \quad -mr[\dot{\theta}_p + \dot{y}_d]^2\sin(\theta_p + y_d) + b_1\dot{y}_p + k_1 y_p = 0 \\ I_l[\ddot{\theta}_p + \ddot{y}_d] + mr[-\ddot{x}_p\sin(\theta_p + y_d) + \ddot{y}_p\cos(\theta_p + y_d)] \\ \quad -mr[\dot{\theta}_p + \dot{y}_d][\dot{x}_p\cos(\theta_p + y_d) + \dot{y}_p\sin(\theta_p + y_d)] \\ \quad +b_3\dot{\theta}_{lr} + k_3\theta_{lr} = 0 \\ [I_r + I_p]\ddot{\theta}_p - I_r\ddot{\theta}_{lr} + I_r\ddot{y}_d - b_3\dot{\theta}_{lr} + b_2\dot{\theta}_p \\ \quad -k_3\theta_{lr} + k_2\theta_p = 0, \end{cases} \tag{17}$$

where $\theta_{lr} := \theta_l - \theta_r$. The last two equations in (16) is its state-space form that is called *reference dynamics.*

By setting $y_d \equiv 0$ in the reference dynamics, we obtain the *zero dynamics* [7]:

$$\begin{cases} \dot{\eta} = \dot{\eta} \\ \ddot{\eta} = p(0, 0, 0, \eta, \dot{\eta}). \end{cases} \tag{18}$$

Using parameters in Table 1, eigenvalues of the first approximation at the origin of the zero dynamics are calculated as shown in Table 2. Hyperbolicity of the equilibrium point at the origin can be easily seen since there is no eigenvalues with zero real part. It is also noticed that this system is nonminimum phase due to the existence of two unstable eigenvalues. From theory of differential equations [8], locally there exist a stable submanifold of dimension six and an unstable submanifold of dimension two, both of which may be expressed by $w^u(\eta, \dot{\eta}) = 0$ and $w^s(\eta, \dot{\eta}) = 0$ respectively.

Eigenvalues of Zero Dynamics	
$-1.40 \pm j22.76$	$-1.41 \pm j22.82$
$6.12 \pm j29.84$	$-9.91 \pm j27.47$

Table 2: Eigenvalues of zero dynamics

Let us consider the following two-point boundary value problem (TPBVP):

$$\ddot{\eta} = p(y_d, \dot{y}_d, \ddot{y}_d, \eta, \dot{\eta}), \tag{19}$$

subject to

$$\begin{cases} w^s(\eta(t_0), \dot{\eta}(t_0)) = 0 \\ w^u(\eta(t_f), \dot{\eta}(t_f)) = 0. \end{cases} \tag{20}$$

The boundary condition (20) basically says that at $t = t_0$ the desired state trajectory should stay inside the unstable manifold while at $t = t_f$ the stable manifold. It has been shown [9] that this TPBVP locally has a unique solution $(\eta_d, \dot{\eta}_d)$ under a mild sufficient condition on y_d: the norm $\| (y_d(t), \dot{y}_d(t)) \|_2$ is not too large for all $t \in [t_0, t_f]$. It has also been shown [6] that the stable inverse pair can be constructed from $(\eta_d, \dot{\eta}_d)$ through equations (11) and (15):

$$\begin{bmatrix} \psi_d \\ \dot{\psi}_d \end{bmatrix} = M_\phi^{-1} \begin{bmatrix} y_d \\ \dot{y}_d \\ \eta_d \\ \dot{\eta}_d \end{bmatrix}, \tag{21}$$

and

$$u_d = I_p \left[\ddot{y}_d - \alpha(\psi_d, \dot{\psi}_d) \right]. \tag{22}$$

4. Application to Output Tracking

In this section we are first constructing the stable inverse pair through approximately solving the corresponding TPBVP (19)-(20) defined in the previous section. Then, a controller is designed by using the stable inverse solution to drive the tip of the link to track a prescribed reference trajectory.

Let the desired trajectory be defined as follows with $t_0 = 1$ and $t_f = 2$:

$$y_d = \begin{cases} 0, & t \le t_0, \\ 2[t - t_0] - \frac{1}{\pi}\sin(2\pi[t - t_0]), & t_0 < t \le t_f, \\ 2, & t > t_f. \end{cases}$$

To find the stable inverse pair, i.e. the u_d and the $(\psi_d, \dot{\psi}_d)$, certain algorithms [10] have been developed to solve the corresponding TPBVP which include decoupling stable/unstable manifolds method and minimum-energy optimization approach. In this paper, to avoid detailed description of any algorithm, we choose to solve the TPBVP simply by approximately decoupling the stable/unstable manifolds via a linear coordinate transformation. The approximated solution will be used to construct the stable inverse provided that the tracking accuracy is satisfactory. Details are as follows.

Rewrite the differential equation (19) in the TPBVP in the following state-space form:

$$\begin{bmatrix} \dot{\eta} \\ \ddot{\eta} \end{bmatrix} = A_\eta \begin{bmatrix} \eta \\ \dot{\eta} \end{bmatrix} + R(y_d, \dot{y}_d, \ddot{y}_d, \eta, \dot{\eta}), \tag{23}$$

where $R(0,0,0,\eta,\dot{\eta}) = \mathcal{O}(|(\eta, \dot{\eta})|^2)$ and A_η is the linear part of the zero dynamics at the origin. From elementary linear algebra, there exists a linear transformation $(\eta, \dot{\eta}) = T(z_1, z_2)$ which transforms equation (23) into

$$\begin{cases} \dot{z}_1 = A_{z1} z_1 + R_{z1}(y_d, \dot{y}_d, \ddot{y}_d, z_1, z_2) \\ \dot{z}_2 = A_{z2} z_2 + R_{z2}(y_d, \dot{y}_d, \ddot{y}_d, z_1, z_2), \end{cases} \tag{24}$$

where both A_{z1} and $-A_{z2}$ are Hurwitz. Recall that the boundary condition requires that at t_0 the $(\eta_d, \dot{\eta}_d)$ stays in the unstable manifold while at t_f the stable manifold. We approximate the boundary condition simply by $z_1(t_0) = 0$ and $z_2(t_f) = 0$ since, roughly speaking, z_1 and z_2 pick up the stable and unstable parts of the zero dynamics respectively. The stable inverse pair is then obtained approximately through the following iterative steps:

- 1: Set $z_1^0(t) = 0$ for all t.
- 2: Integrate the unstable part of equation (24) from $t = t_f$ to $t = 0$ backward in time to obtain $z_2(t)$.
- 3: Integrate the stable part of equation (24) from $t = t_0$ to $t = 3$ forward in time to obtain $z_1(t)$.
- 4: If $\|z_1 - z_1^0\|$ is greater than a given threshold, set $z_1^0 = z_1$ and go to step 2, otherwise step 5.
- 5: Use linear transformation $(\eta_d, \dot{\eta}_d) = T(z_1, z_2)$ to find the unique solution to TPBVP.
- 6: Construct $(\psi_d, \dot{\psi}_d)$ via equation (21) and u_d (22).

Using only $\theta_r - \theta_p$ and $\dot{\theta}_r - \dot{\theta}_p$, the measurements of rotor position and velocity relative to the platform, controller by stable inversion is simply designed as follows: use u_d as a feed-forward signal that is superimposed by a PD stabilizing feedback. The control law is therefore given by

$$\begin{aligned} u = u_d &+ a_p[(\theta_r - \theta_p)_d - (\theta_r - \theta_p)] \\ &+ a_d[(\dot{\theta}_r - \dot{\theta}_p)_d - (\dot{\theta}_r - \dot{\theta}_p)], \end{aligned} \tag{25}$$

where a_p and a_d are two design parameters. Noticed that $(\theta_r - \theta_p)_d$ and $(\dot{\theta}_r - \dot{\theta}_p)_d$ are part of the desired state trajectory $(\psi_d, \dot{\psi}_d)$. The stability analysis of this type of feedback control is discussed in [11]. The forward simulation starts from $t = 0.5$ second from a rest initial condition and its results using $a_p = 30300$ and $a_d = 1616$ are shown in Figure 2. It is seen the excellent tracking per-

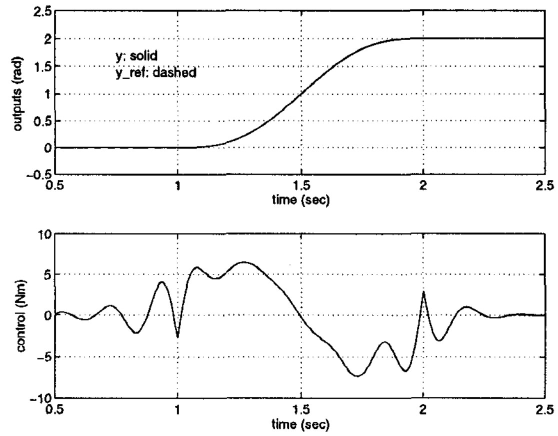

Figure 2: Simulation by Stable Inversion

formance by this controller: no transient and steady-state error with stability of all signals. It is also noticed the simple structure of the feedback control with only partial state measurements.

As mentioned in the introduction, classical inversion is another approach to obtain accurate output tracking. However, the method is not applicable to systems with unstable zero dynamics such as this robot system due to the unboundedness of internal signals it would generate. In addition, it is obvious that the nonlinear regulator approach is not applicable here either since solving its associated partial differential equations is, if not impossible, extremely difficult in this highly nonlinear robot system. Another restriction is that the regulator method can not be used to track an arbitrary smooth reference trajectory.

It is interesting to notice that in this robot system the angular motion, $(\theta_p, \dot{\theta}_p)$, of the platform is rather small. By neglecting this motion the system may be approximated by a minimum phase model with its order reduced by two. (See [12] for a detailed description of this model reduction and the following control law design.) A feedback control law can be designed directly through input/output linearization based on this approximated model. The control achieves output tracking with bounded internal signals. Simulation results by this approximate approach are shown in Figure 3. It is noticed

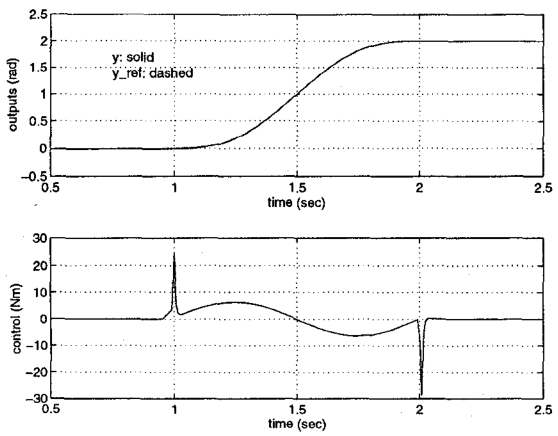

Figure 3: Simulation by input/output linearization

that this input/output linearization method based on the reduced minimum phase model also achieves output trajectory tracking with a satisfactorily small tracking error. It also be noticed that this method, unlike the stable inversion approach, requires a full-state measurement.

5. Conclusion

Stable inversion, an approach to the design of output tracking controller for nonlinear nonminimum phase systems, is successfully applied to the tip trajectory tracking of a single-link flexible-joint robot mounted on a wobbly platform. The key assumptions, a well-defined relative degree and hyperbolicity of the fixed point of the zero dynamics, in using stable inversion are both satisfied by this system. Simulation results demonstrate that the stable inversion approach is very effective for obtaining accurate output tracking with only partial state measurements for this nonminimum phase system. The approach is expected to perform equivalently well for other many realistic nonlinear nonminimum phase systems.

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