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Amplifiers with Maximum Bandwidth

RANDALL L. GEIGER

***Abstract*—A novel two-operational amplifier voltage amplifier is presented that has a bandwidth larger than any other possible single- or two-operational amplifier configuration. The maximum bandwidth single-operational amplifier amplifiers are also discussed.**

I. Introduction

THE HIGH frequency roll-off of finite gain voltage amplifiers limits the use of these devices. It is desirable to obtain the circuit configuration that will yield the maximum bandwidth for a given number of active devices. The use of maximum bandwidth voltage amplifiers in active filters requiring finite gain amplifiers increases the useful frequency range of these filters [1], [2].

The active elements used in the amplifiers discussed here will be internally compensated operational amplifiers with infinite input impedance and zero output impedance that can be modeled [3] with the gain function

$$A(s)=\frac{\mathrm{GB}}{s}=\frac{1}{s_n} \tag{1}$$

where GB is the gain-bandwidth product of the operational amplifier and s_n the normalized complex frequency. If more than one operational amplifier is used, they are assumed to be identical.

Five voltage amplifiers along with the bandwidth of each are shown in Fig. 1. The first four are popular configurations and the fifth is believed to be novel. Some comments will be made about these amplifiers.

II. Discussion

1) Of all amplifiers with dc gain $K_0 \geq 1$ using only resistors, a single operational amplifier, and with output taken from the output of the operational amplifier, Circuit 1 has the largest bandwidth for a given K_0.

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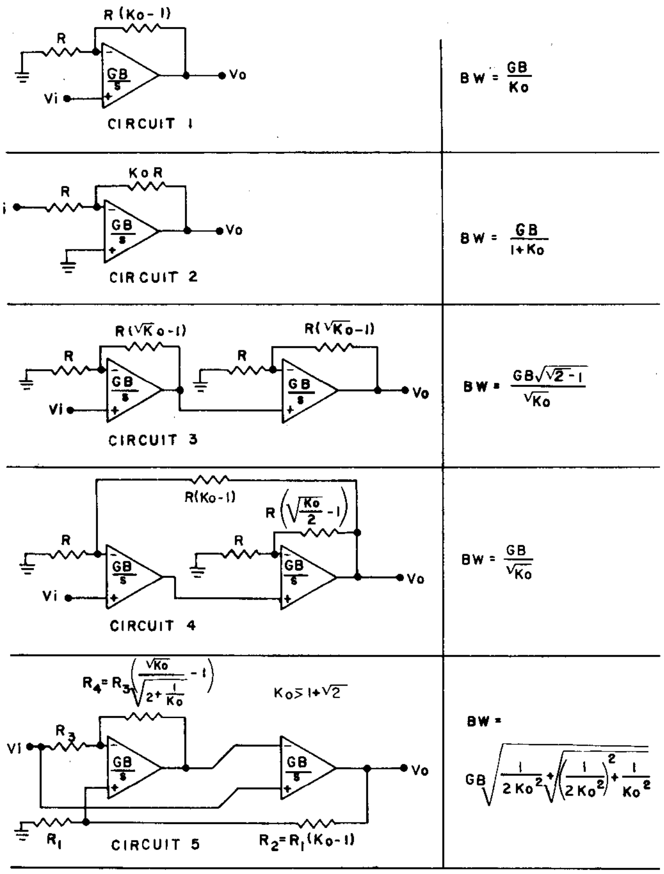

Fig. 1. Voltage amplifier configurations.

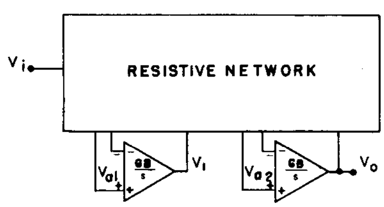

Fig. 2. Two-operational amplifier amplifiers.

2) Of all amplifiers with a dc gain $K_0 \leqslant -1$ using only resistors, a single operational amplifier, and with output taken from the output of the operational amplifier, Circuit 2 has the largest bandwidth for a given K_0.

3) In the class of all amplifiers with dc gain $K_0 \geqslant 1+\sqrt{2}$, monotone frequency response, using only resistors and at most two operational amplifiers, and with output taken from the output of an operational amplifier, Circuit 5 has the largest bandwidth. It can be observed in Fig. 3 that Circuit 5 exhibits a significant improvement in bandwidth over the other popular multiple amplifier configurations, Circuit 3 and Circuit 4 [4], which are included for comparison with the new circuit.

Comments 1) and 2) imply that for a given dc gain K_0, all other single-operational amplifier amplifiers have bandwidth smaller than those of Circuit 1 or 2. Comment 3 will now be established.

Fig. 2 shows the general amplifier using only two operational amplifiers and resistors and with output taken from one of the operational amplifiers.

The above model of the operational amplifiers requires that

$$V_{a1} = V_1 s_n \quad \text{and} \quad V_{a2} = V_0 s_n. \tag{2}$$

Since the transfer function of any resistive network has magnitude $\leqslant 1$, it follows by superposition that

$$V_{a1} = r_1 V_0 + r_2 V_i + r_3 V_1 \tag{3}$$

$$V_{a2} = r_4 V_0 + r_5 V_i + r_6 V_1 \tag{4}$$

where $r_i \in [-1,1]$ for $i = 1,\cdots,6$. The resulting gain of the amplifier is thus

$$K(s_n) = \frac{V_0}{V_i} = \frac{r_5 s_n + r_2 r_6 - r_5 r_3}{s_n^2 + s_n(-r_3 - r_4) + r_3 r_4 - r_1 r_6} = \frac{d(s_n + c)}{s_n^2 + \frac{\omega_0}{Q} s_n + \omega_0^2} \tag{5}$$

where $d = r_5$, $c = (r_2 r_6 / r_5) - r_3$, $\omega_0^2 = r_3 r_4 - r_1 r_6$, and $\omega_0 / Q = -r_3 - r_4$.

However, the 3-dB bandwidth of the transfer function written in (5) satisfies

$$\mathrm{BW}_n^2 = \frac{(2 - 1/Q^2)\omega_0^2}{2} + \frac{d^2}{K_0^2} + \sqrt{\left(\frac{(2-1/Q^2)\omega_0^2}{2} + \frac{d^2}{K_0^2}\right)^2 + \frac{d^2 c^2}{K_0^2}} \tag{6}$$

where $K_0 = dc/\omega_0^2$ is the dc gain of the amplifier system of Fig. 2 and $\mathrm{BW}_n = \mathrm{BW}/\mathrm{GB}$. For monotone $|K(j\omega)|$, it is easy to show that

$$2 - \frac{1}{Q^2} \leqslant -\frac{\omega_0^2}{c^2}. \tag{7}$$

Consequently, for a monotone $K(s)$ it follows that the bandwidth is bounded by

$$\mathrm{BW}_n \leqslant \sqrt{\frac{d^2}{2K_0^2} + \sqrt{\left(\frac{d^2}{2K_0^2}\right)^2 + \left(\frac{dc}{K_0}\right)^2}}\,. \tag{8}$$

Since d and dc have magnitudes less than or equal to unity, it follows that

$$\mathrm{BW}_n \leqslant \sqrt{\frac{1}{2K_0^2} + \sqrt{\left(\frac{1}{2K_0^2}\right)^2 + \frac{1}{K_0^2}}}\,. \tag{9}$$

It now follows from (5) and (7) that for a circuit to attain the upper bound given in (9) it is necessary that

$$|r_5| = 1$$

$$|r_2 r_6 - r_3 r_5| = 1$$

$$r_3 r_4 - r_1 r_6 = 1/K_0$$

$$-r_3 - r_4 = \frac{\sqrt{2 + 1/K_0}}{\sqrt{K_0}}. \tag{10}$$

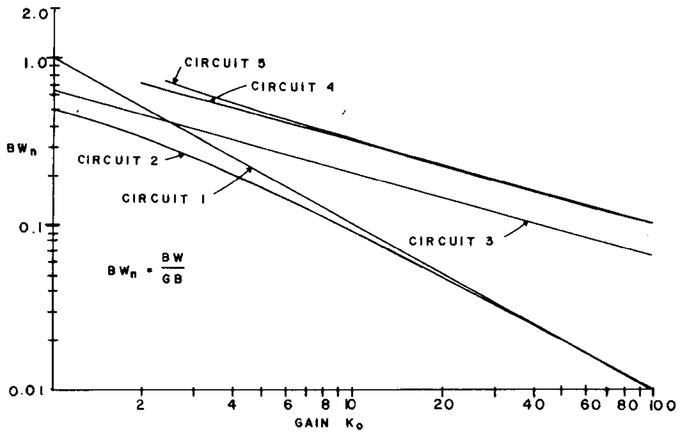

Fig. 3. Normalized bandwidth versus gain for circuits of Fig. 1.

After some lengthy algebraic manipulations, it can be shown that the only realizable solution to (10) is

$$r_1=\frac{1}{K_0}, \qquad r_4=0$$

$$r_2=\frac{\sqrt{2+1/K_0}}{\sqrt{K_0}}-1, \qquad r_5=1$$

$$r_3=-\frac{\sqrt{2+1/K_0}}{\sqrt{K_0}}, \qquad r_6=-1. \tag{11}$$

It now follows from (3), (4), and Fig. 2 that Circuit 5 of Fig. 1 is a circuit that attains the upper bound set in (9).

Circuit 5 can be easily tuned as follows. Pick R_2 and R_4. With a dc input adjust R_1 so that the gain is K_0, then adjust R_3 for maximum bandwidth with no peaking of the frequency response.

In Fig. 3, the normalized bandwidths of all the circuits presented in Fig. 1 are plotted against K_0, which represents the magnitude of the overall dc gain.

The bandwidth sensitivity in Circuits 1 and 2 with respect GB is unity; whereas, in Circuits 3–5, the bandwidth sensitivity with respect to the GB of either operational amplifier is $\frac{1}{2}$. In particular, the active bandwidth sensitivities of the maximum bandwidth amplifier of Circuit 5 are at least as low as those of the other amplifiers shown in Fig. 1.

III. Experimental Results

The maximum bandwidth amplifier has been constructed using two 741-type operational amplifiers with measured gain-bandwidth products of 860 kHz ± 1 percent. R_4 and R_2 were chosen to be 10.1 kΩ. R_1 was selected for $K_0=3$, 4, 6, 10, 40, and 100, and R_3 then adjusted to make the frequency response magnitude monotone. In all cases, the measured bandwidth of the amplifier exceeded the theoretical value given in Fig. 1.

IV. Conclusion

An argument similar to the above can establish that in the class of single-operational amplifier configurations, Circuits 1 and 2 have the optimal bandwidth for the noninverting and inverting cases respectively. Circuit 5, which uses only four resistors and has gain determined by the ratio of two resistors, has been shown to have bandwidth greater than any other single- or two-operational amplifier noninverting amplifier with dc gain K_0, for $K_0 \geq 1+\sqrt{2}$. The bandwidths predicted in Fig. 3 have been experimentally verified.

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