TABLE I
FILTER COEFFICIENTS W_i

N	W_0	W_1	W_3	W_5	W_7	W_9	W_{11}	W_{13}	W_{15}	W_{17}	W_{19}
0	½	¼									
1	½	$\frac{9}{32}$	$\frac{-1}{32}$								
2	½	$\frac{150}{512}$	$\frac{-25}{512}$	$\frac{3}{512}$							
3	½	$\frac{1225}{4096}$	$\frac{-245}{4096}$	$\frac{49}{4096}$	$\frac{-5}{4096}$						
4	½	$\frac{19845}{2^{16}}$	$\frac{-2205}{2^{15}}$	$\frac{567}{2^{15}}$	$\frac{-405}{2^{17}}$	$\frac{35}{2^{17}}$					
5	½	$\frac{160083}{2^{19}}$	$\frac{-38115}{2^{19}}$	$\frac{22869}{2^{20}}$	$\frac{-5445}{2^{20}}$	$\frac{847}{2^{20}}$	$\frac{-63}{2^{20}}$				
6	½	$\frac{7(429)^2}{2^{22}}$	$\frac{-7(429)^2}{2^{24}}$	$\frac{21(143)^2}{2^{24}}$	$\frac{-3(143)^2}{2^{23}}$	$\frac{13013}{2^{23}}$	$\frac{-3549}{2^{24}}$	$\frac{231}{2^{24}}$			
7	½	$\frac{(6435)^2}{2^{27}}$	$\frac{-21(715)^2}{2^{27}}$	$\frac{21(429)^2}{2^{27}}$	$\frac{-33(195)^2}{2^{27}}$	$\frac{77(65)^2}{2^{27}}$	$\frac{-61425}{2^{27}}$	$\frac{7425}{2^{27}}$	$\frac{-429}{2^{27}}$		
8	½	$\frac{(36465)^2}{2^{32}}$	$\frac{-15(2431)^2}{2^{30}}$	$\frac{77(663)^2}{2^{30}}$	$\frac{-55(663)^2}{2^{31}}$	$\frac{1001(85)^2}{2^{31}}$	$\frac{-13(255)^2}{2^{30}}$	$\frac{55(51)^2}{2^{30}}$	$\frac{-123981}{2^{33}}$	$\frac{6435}{2^{33}}$	
9	½	$\frac{5(46189)^2}{2^{35}}$	$\frac{-165(4199)^2}{2^{35}}$	$\frac{33(4199)^2}{2^{34}}$	$\frac{-2145(323)^2}{2^{34}}$	$\frac{715(323)^2}{2^{34}}$	$\frac{-195(323)^2}{2^{34}}$	$\frac{165(323)^2}{2^{36}}$	$\frac{-7293(19)^2}{2^{36}}$	$\frac{715(19)^2}{2^{36}}$	$\frac{-12155}{2^{36}}$

APPENDIX
SOLUTION TO (3)

Formally the solution to (3) is

$$A_{k,N}=\frac{\Delta k}{\Delta}.$$

Muir [4] has shown that the determinant Δ can be expanded as the difference products of the x_k^2's. When the ratio Δ_k/Δ is thus formed and the common factors are cancelled we obtain

$$A_{k,N}=-\frac{1}{x_k^2}\prod_{i=1}^{k-1}\frac{1-x_{k-1}^2}{x_k^2-x_{k-i}^2}\prod_{j=k+1}^{N}\frac{x_j^2-1}{x_j^2-x_k^2}. \tag{A1}$$

After substituting $x_k=2k+1$ into (A1) and performing the products indicated, it can be readily seen that

$$-\frac{1}{x_k^2}\prod_{i=1}^{k-1}\frac{1-x_{k-i}^2}{x_k^2-x_{k-i}^2}=\frac{(-1)^k}{2k+1}\div\binom{2k+1}{k}. \tag{A2}$$

Similarly the second product can be shown to be given by

$$\prod_{j=k+1}^{N}\frac{x_j^2-1}{x_j^2-x_k^2}=\binom{2k+1}{k}\frac{N!(N+1)!}{(N-k)!(N+k+1)!}. \tag{A3}$$

Combining (A2) and (A3) we finally obtain the solution

$$A_{k,N}=(-1)^k\frac{N!(N+1)!}{(2k+1)(N-k)!(N+k+1)!}. \tag{A4}$$

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A Grounded Constant Current Source with Improved Bandwidth

A. BUDAK AND R. GEIGER

***Abstract*—A grounded constant current source employing a single operational amplifier is discussed. It is shown that improved bandwidth and reduced slew rate limitations are obtained with this circuit.**

I. INTRODUCTION

Circuit 1 of Fig. 1 is a commonly used single OP AMP voltage-controlled current source that has a grounded terminal. It is listed in many texts and application notes, some of the more recent of which are [1]–[5]. At low frequencies, this circuit acts as an ideal current source. However, at high frequencies, the load current tends toward zero if the OP AMP gain is characterized

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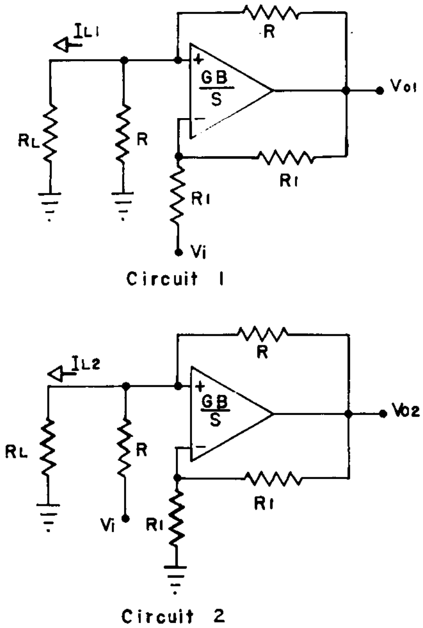

Fig. 1. Single OP AMP current sources.

by a single pole. Circuit 2 of Fig. 1 [5] has the same circuit topology as that of circuit 1 except that the input is applied at a differen node. The load current for circuit 2 decreases monotonically to a constant value rather than zero as the frequency increases. This constant can often be chosed quite close to the value based on an ideal amplifier. The improved frequency response of circuit 2 is obtained at the expense of increased current drawn from V_i [5].

In what follows, the term bandwidth will refer to the half-power frequency (in radians per second) of the current source. Further, the OP AMP is assumed to have infinite input impedance and a gain function given by

$$A(s)=\frac{GB}{s} \tag{1}$$

where GB is the gain–bandwidth product of the OP AMP in radians per second.

II. Discussion

Circuit 1 has a load current given by

$$I_{L1}=-\frac{V_i}{R}\left[\frac{\frac{1}{2}}{\frac{s}{GB}\left[1+\frac{2R_L}{R}\right]+\frac{1}{2}}\right] \tag{2}$$

and bandwidth given by

$$BW_1=\frac{GB}{2\left(1+\frac{2R_L}{R}\right)}. \tag{3}$$

Circuit 2 has a load current given by

$$I_{L2}=\frac{V_i}{R}\left[\frac{\frac{s}{GB}+\frac{1}{2}}{\frac{s}{GB}\left[1+\frac{2R_L}{R}\right]+\frac{1}{2}}\right] \tag{4}$$

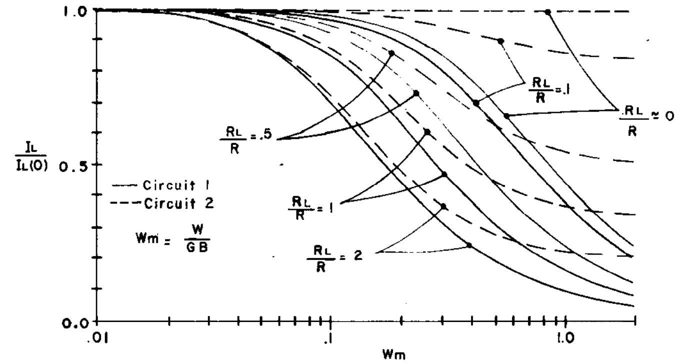

Fig. 2. Normalized current versus normalized frequency of circuits 1 and 2 for different values of R_L/R.

and bandwidth given by

$$BW_2=\begin{cases}\frac{1}{2}\cdot\frac{GB}{\sqrt{\left(1+\frac{2R_L}{R}\right)^2-2}}, & R_L>\frac{R}{2}(\sqrt{2}-1)\\ \infty, & R_L\leqslant\frac{R}{2}(\sqrt{2}-1)\end{cases}. \tag{5}$$

Since the pole structure of the circuit is independent of where the input is applied, circuit 1 and circuit 2 have identical poles. Because circuit 1 has an all-pole transfer function, I_{L1} tends to zero as the frequency goes to infinity. However, from (4), the zero in the transfer function of circuit 2 assures that for all frequencies

$$I_{L2}(0)\geqslant I_{L2}\geqslant I_{L2}(0)\left[1+\frac{2R_L}{R}\right]^{-1} \tag{6}$$

where $I_{L2}(0)=V_i/R$ is the dc load current of circuit 2. From (6) it follows that circuit 2 is an ideal current source at low frequencies and at least as good as a passive current source with a source impedance of value $R/2$ at high frequencies. The normalized frequency response of circuits 1 and 2 are compared in Fig. 2 for several values of R_L/R. It is clear that the circuit should be designed with as high an R as possible.

From (3) and (5) it can be seen that the bandwidth improvement is given by

$$\frac{BW_2}{BW_1}=\begin{cases}\frac{1+\frac{2R_L}{R}}{\sqrt{\left(1+\frac{2R_L}{R}\right)^2-2}}, & R_L>\frac{R}{2}(\sqrt{2}-1)\\ \infty, & R_L\leqslant\frac{R}{2}(\sqrt{2}-1)\end{cases}. \tag{7}$$

In many applications of a VCCS, the relationship between the phase of the input voltage and output current is also of interest. From (2) and (4) it can be seen that the phase of circuit 2 is much closer to the value obtained by assuming the OP AMP to be ideal than is the phase of circuit 1.

It is interesting to note that for either circuit of Fig. 1, at low frequencies, the impedance facing the load R_L is approximately $R//-R=\infty$. Consequently, the Norton equivalent of the circuit facing R_L is an ideal current source.

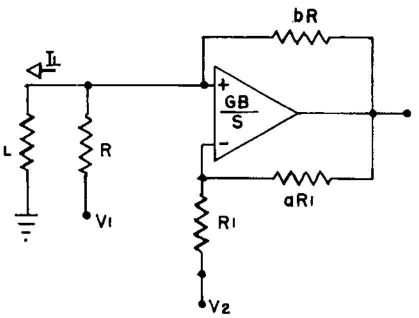

Fig. 3. General current source.

The current source given in Fig. 1 is a special case of the more general circuit shown in Fig. 3. It can be shown that $b=a$ in order for the circuit to act as a current source. If the four resistors associated with the current source (R, bR, R_1, aR_1) were not appropriately matched, $b \neq a$, the internal resistance of the current source would not be infinite even for an ideal amplifier. Furthermore, with the one-pole rolloff model for the operational amplifier given in (1) the circuit would possess a right half-plane pole if

$$\frac{b}{a} > \frac{R_L}{R+R_L}. \tag{8}$$

However, unless $R_L \cong \infty$, this condition does not arise without excessive resistor mismatch and the circuit remains stable.

It is easy to show that if both circuits are to deliver the same load current, then the ratio of the output voltage of the OP AMP in circuit 1 to that in circuit 2 is given by

$$\frac{V_{o1}}{V_{o2}} = \left(1 + \frac{R}{2R_L}\right)\left(1 + \frac{2s}{GB}\right). \tag{9}$$

It now follows from (9) that for low R_L/R or high frequencies the output voltage ratio is indeed large. Since slew rate limitations are dependent upon the magnitude of the output voltage, it follows that circuit 2 is much less affected by slew rate than is circuit 1 or, alternately, that circuit 2 is useful at much higher load currents than is circuit 1.

III. Experimental Results

Both circuits of Fig. 1 were tested for several different load currents using a 741 OP AMP with a measured GB of 1.14 MHz and with $R = R_1 = 3.9$ kΩ. The measured bandwidths agreed quite closely with those predicted by (3) and (5). The significantly improved current range of circuit 2 due to slew rate limitations of the OP AMP was also observed.

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Realization of a Nonoriented 2*n*-Terminal Communication Network

H. O. GUPTA

A method for realizing a $2n$-terminal nonoriented network which satisfies maximum two flows at a time and having the maximum local communication was developed by Matsui [2]. This method is based on the determination of cutsets taking one and two nodes at a time such that a condition is imposed by taking one more cutset between source and sink networks. Theorem 4 presented in [2] is not true and the given condition (15) is insufficient because it does not take into account the other cutsets; as such it must be replaced with the following condition,

$$\sum_{i \in U_p(k)} \sum_{\substack{j \in U_p(k) \\ j>i}} t_{ij} - (p-2) \sum_{l \in U_p(k)} t_{ll} \geqslant \max_{i,j \in U_p(k)} t_{ij}, \qquad p = 3,4,\cdots,n \tag{1}$$

where t_{ij} is the simultaneous flow requirement from node i and j and $U_p(k)$ is the kth subset of source nodes having size p.

Consider a requirement matrix T for maximum two flows at a time, which satisfied all the conditions imposed in [2] but does not satisfy (1).

$$T = [t_{ij}] = \begin{bmatrix} 3 & 17/4 & 17/4 & 23/4 \\ 17/4 & 3 & 17/4 & 23/4 \\ 17/4 & 17/4 & 3 & 23/4 \\ 23/4 & 23/4 & 23/4 & 3 \end{bmatrix}. \tag{2}$$

The realization obtained by Matsui's method is shown in Fig. 1. The capacity of the cutset S^3 shown in Fig. 1 is 15/4, which is

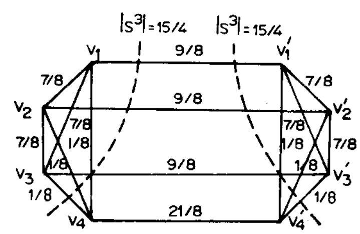

Fig. 1. Communication network obtained by Matsui's method.

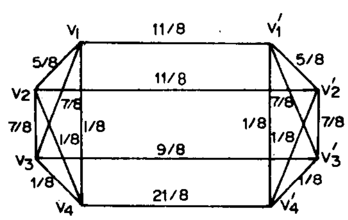

Fig. 2. Changed communication network.

less than the required two flows from the group of nodes (1,2,3), i.e., 17/4. Therefore, Theorem 4 presented in [2] is not true; hence, the condition (15) is insufficient and must be replaced by condition (1) of this letter.

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